Computer Algebra
Independent Integration Tests

Summer 2023 edition

4-Trig-functions/4.3-Tangent/101-4.3.1.2-d-sec-^m-a+b-tan-^n

Nasser M. Abbasi

September 5, 2023 Compiled on September 5, 2023 at 1:27pm

Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [700]. This is test number [101].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (700)	0.00 (0)
Mathematica	100.00 (700)	0.00 (0)
Fricas	81.86 (573)	18.14 (127)
Maple	81.57 (571)	18.43 (129)
Maxima	57.86 (405)	42.14 (295)
Mupad	52.71 (369)	47.29 (331)
Giac	36.86 (258)	63.14 (442)
Sympy	17.71 (124)	82.29 (576)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

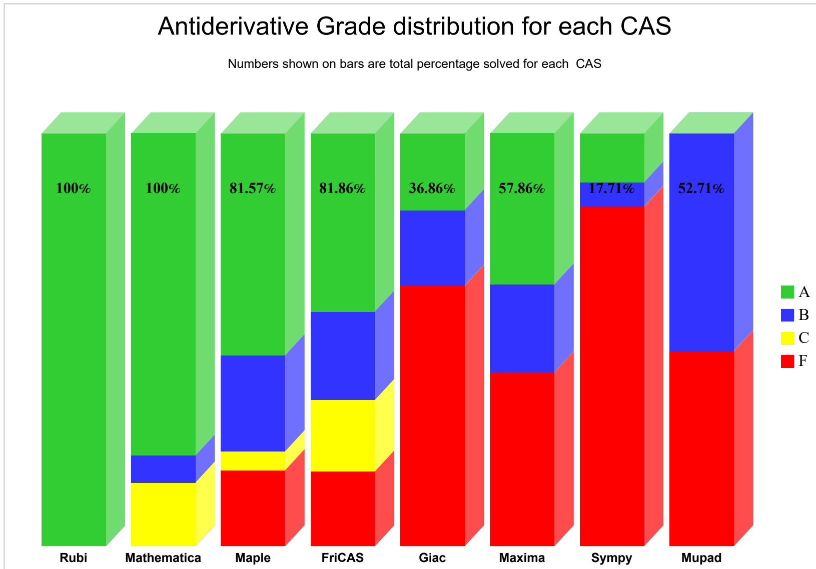
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

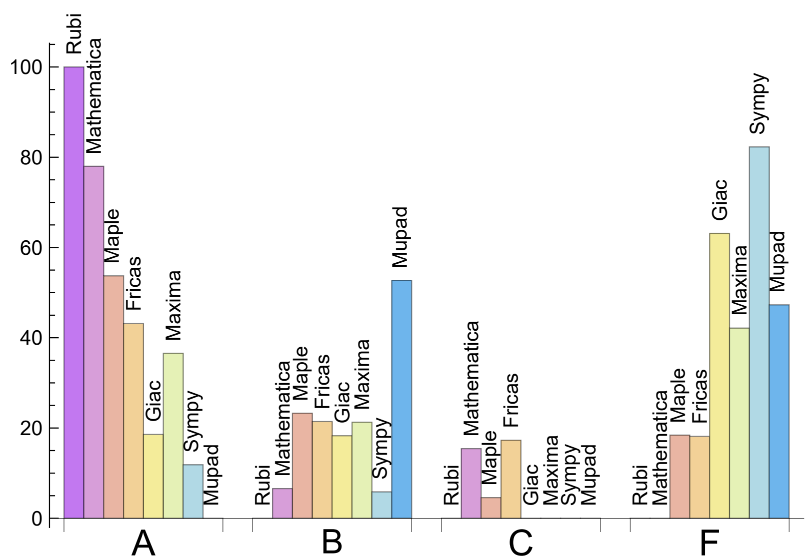
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	78.000	6.571	15.429	0.000
Maple	53.714	23.286	4.571	18.429
Fricas	43.143	21.429	17.286	18.143
Maxima	36.571	21.286	0.000	42.143
Giac	18.571	18.286	0.000	63.143
Sympy	11.857	5.857	0.000	82.286
Mupad	0.000	52.714	0.000	47.286

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	127	78.74	19.69	1.57
Maple	129	91.47	8.53	0.00
Maxima	295	62.03	5.08	32.88
Mupad	331	0.00	100.00	0.00
Giac	442	96.38	1.81	1.81
Sympy	576	73.26	26.39	0.35

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.21
Fricas	0.22
Maxima	1.06
Sympy	1.06
Giac	3.14
Mathematica	3.68
Mupad	5.49
Maple	25.28

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	144.93	1.00	110.00	1.00
Fricas	159.43	1.37	119.00	1.15
Mupad	170.09	1.72	112.00	1.36
Sympy	212.51	2.48	186.00	1.57
Mathematica	353.54	1.50	104.00	0.91
Maxima	371.27	2.40	134.00	1.22
Giac	1467.48	13.97	151.00	1.78
Maple	1500.57	4.17	137.00	1.17

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

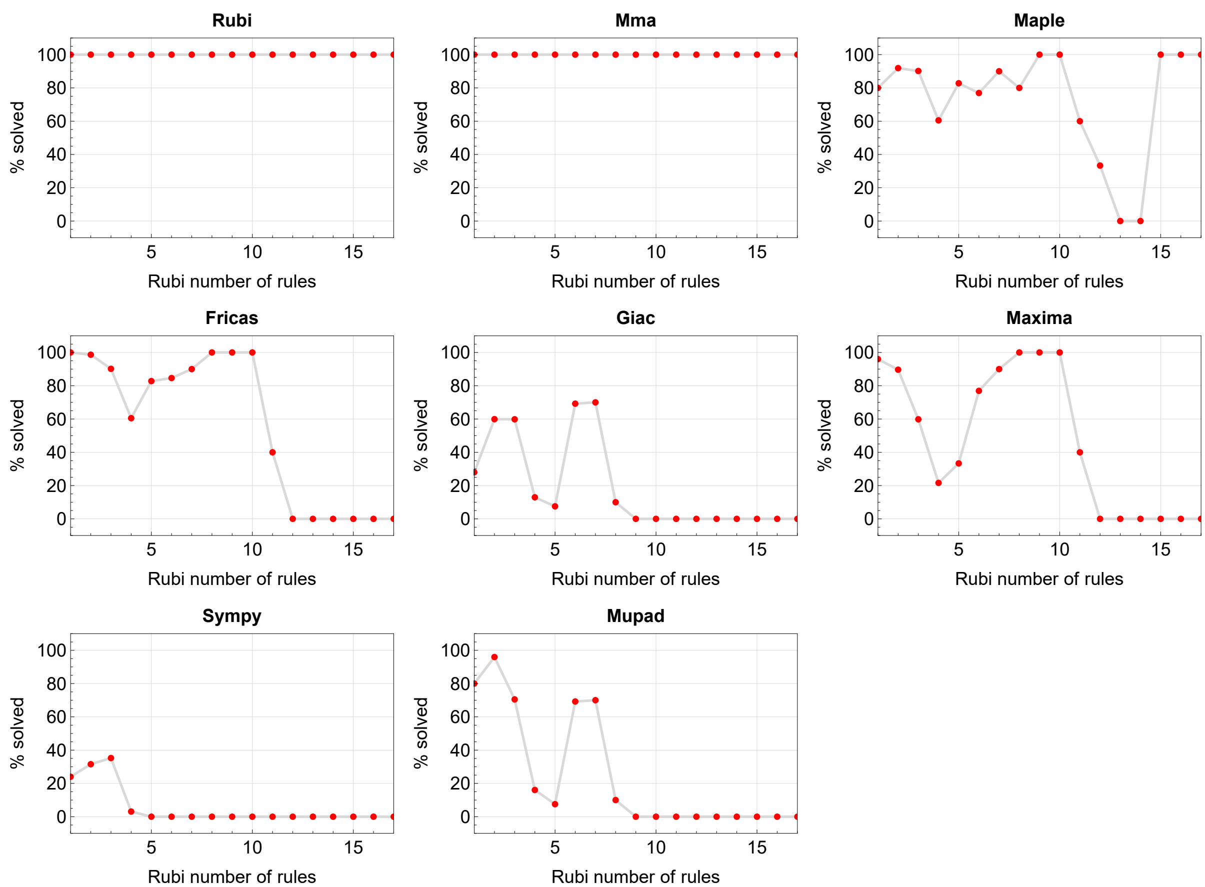


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

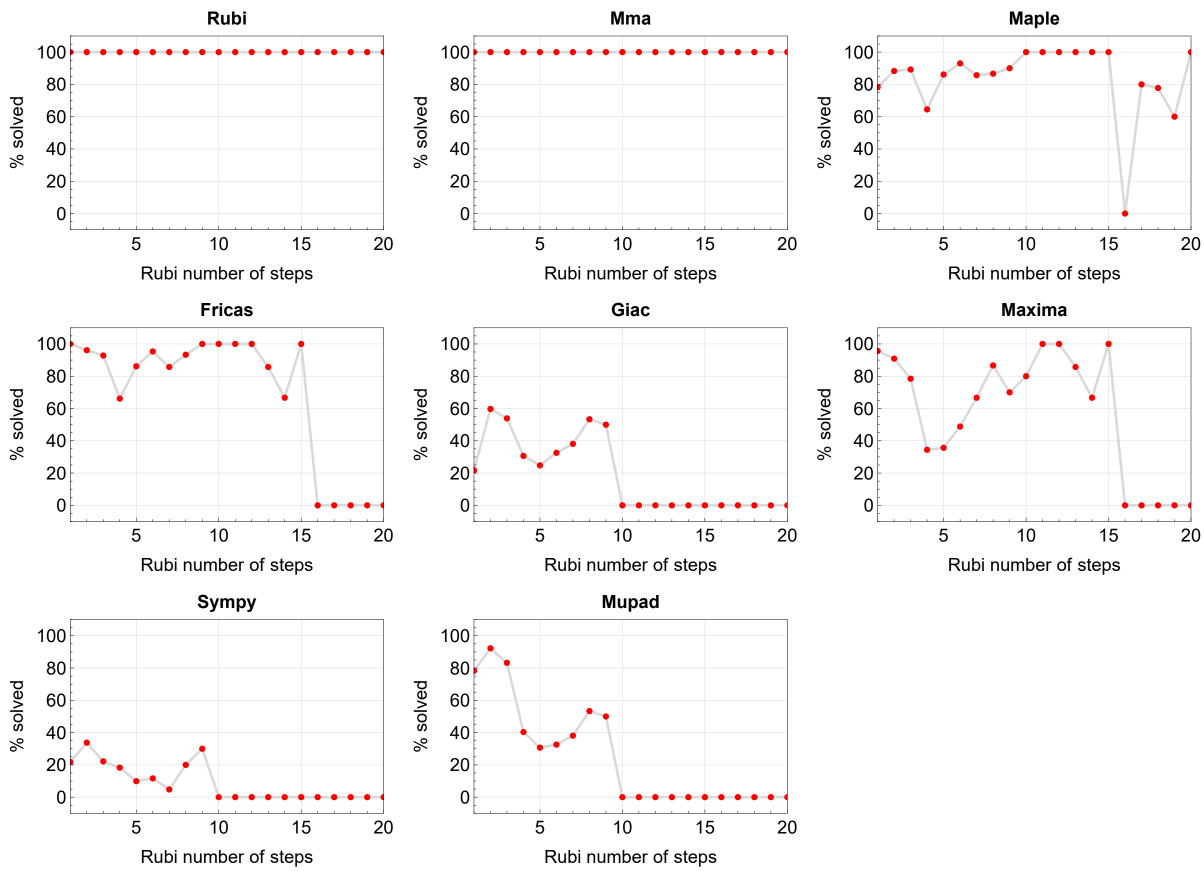


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

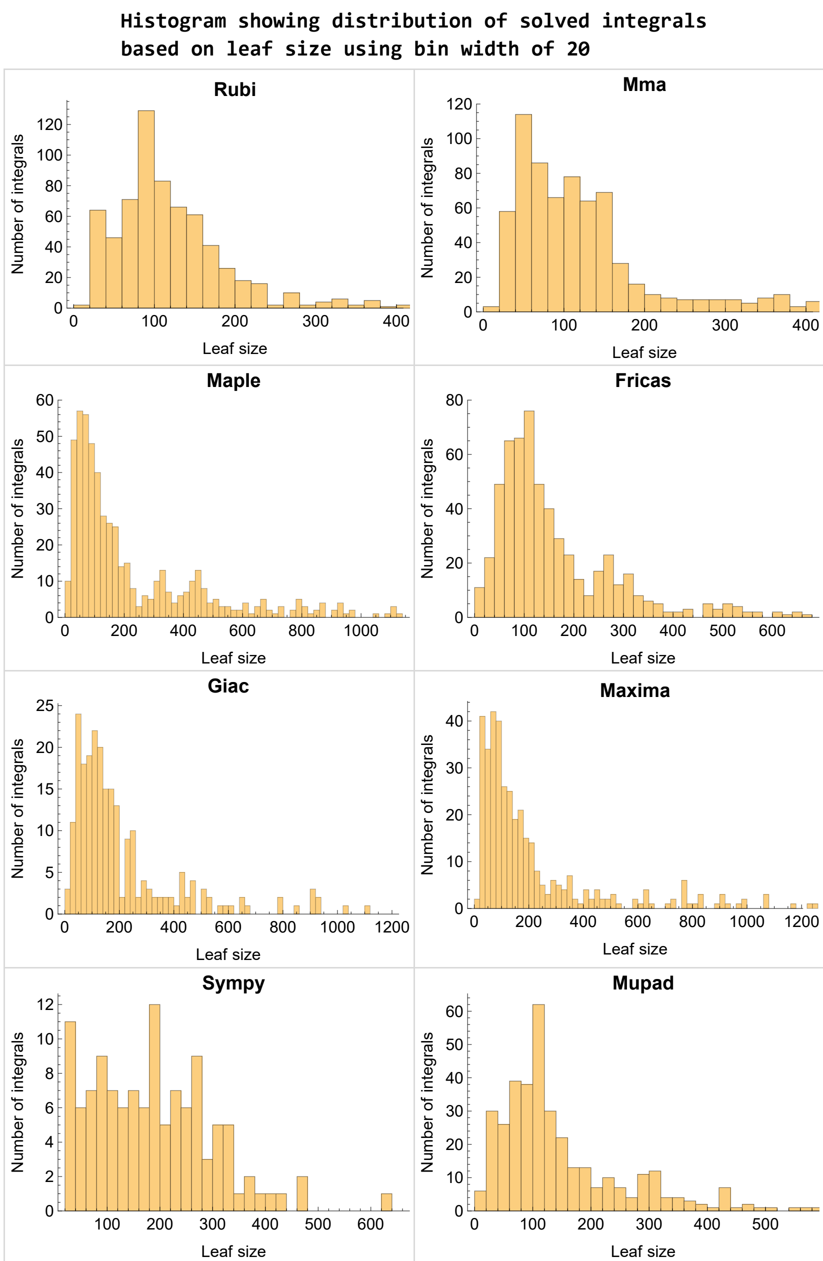


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

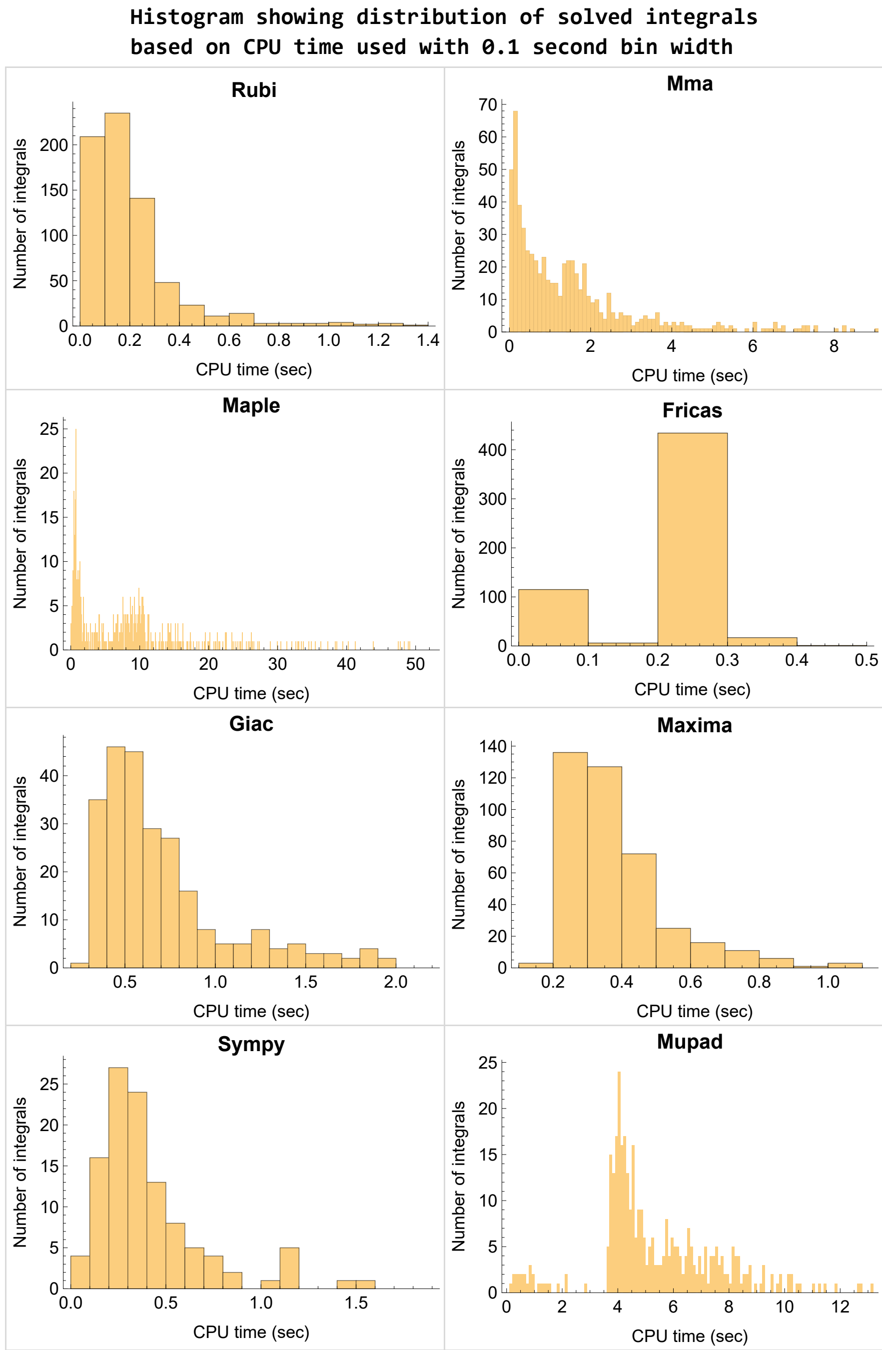


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

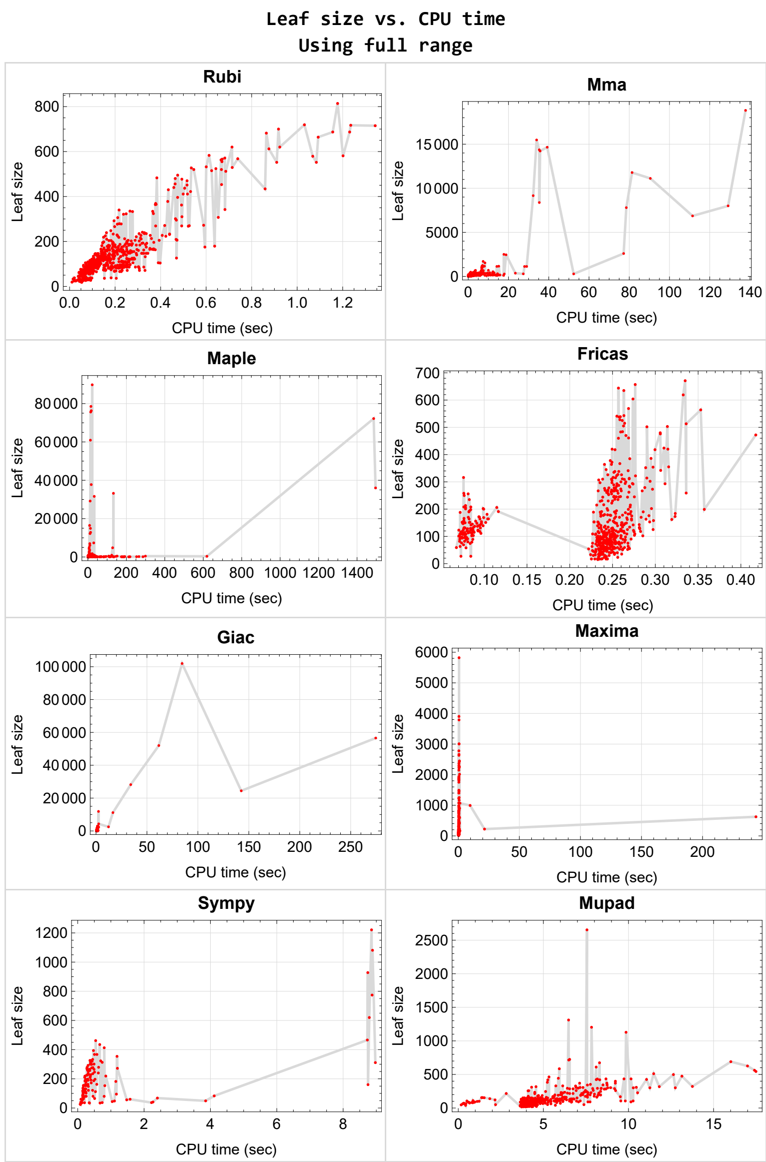


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {92, 93, 176, 177, 431, 476, 603, 605, 607, 609, 610, 612, 614, 616, 617, 619, 621, 623, 632, 633, 634, 635, 636, 637, 638, 639, 643, 644, 645, 651, 652, 653, 654, 658, 662, 664, 670, 691, 698, 699, 700}

Maple {215, 227, 373, 374, 388, 389, 403, 411, 416, 483, 484, 485, 486, 487, 504, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 686}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
```

```
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

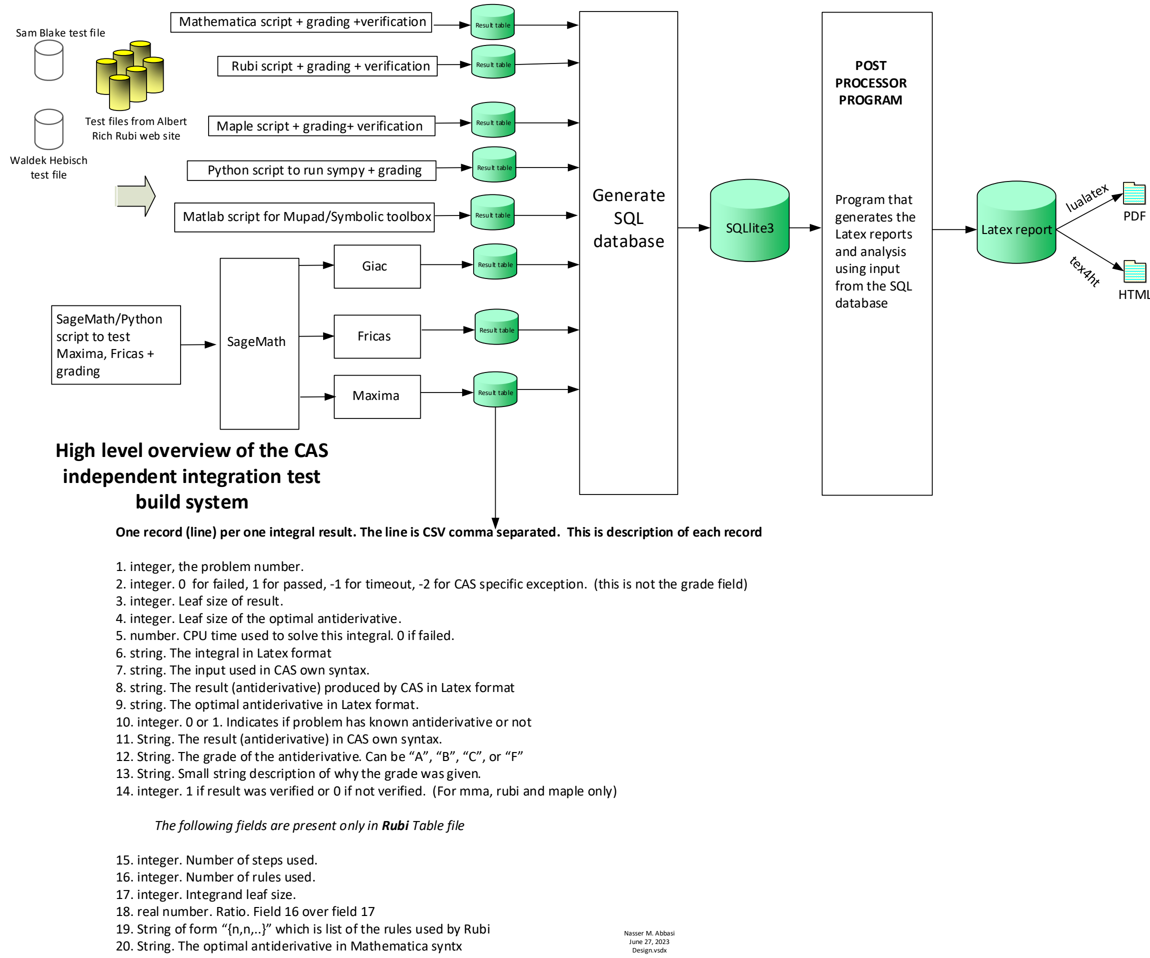
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 124, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 153, 154, 155, 156, 157, 158, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 178, 179, 180, 181, 182, 183, 184, 186, 188, 190, 192, 194, 196, 198, 200, 202, 204, 206, 208, 210, 212, 214, 216, 218, 220, 222, 224, 226, 228, 230, 232, 234, 236, 238, 240, 242, 244, 246, 248, 250, 252, 254, 256, 258, 260, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 279, 280, 281, 282, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 326, 327, 328, 330, 331, 332, 333, 334, 335, 336, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 498, 500, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 521, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 540, 541, 542, 543, 544, 545, 546, 548, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 562, 563, 564, 565, 566, 567, 568, 569, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 602, 606, 624, 625, 626, 627, 628, 629, 630, 631, 640, 641, 642, 646, 647, 648, 649, 650, 655, 657, 661, 663, 665, 667, 669, 671, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 692, 693, 694, 695, 696, 697 }

B grade { 31, 47, 54, 55, 56, 62, 80, 88, 92, 93, 94, 95, 122, 123, 125, 150, 159, 160, 176, 177, 278, 329, 450, 497, 499, 501, 502, 503, 520, 522, 534, 535, 536, 537, 538, 539, 549, 570, 571, 573, 601, 603, 635, 637, 639, 691 }

C grade { 185, 187, 189, 191, 193, 195, 197, 199, 201, 203, 205, 207, 209, 211, 213, 215, 217, 219, 221, 223, 225, 227, 229, 231, 233, 235, 237, 239, 241, 243, 245, 247, 249, 251, 253, 255, 257, 259, 261, 283, 284, 285, 298, 299, 311, 312, 323, 324, 325, 337, 338, 339, 351, 352, 353, 367, 368, 383, 384, 547, 560, 561, 572, 574, 604, 605, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 632, 633, 634, 636, 638, 643, 644, 645, 651, 652, 653, 654, 656, 658, 659, 660, 662, 664, 666, 668, 670, 672, 698, 699, 700 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 60, 61, 63, 64, 65, 66, 70, 71, 72, 73, 74, 81, 82, 83, 91, 92, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 186, 188, 190, 192, 194, 196, 198, 200, 202, 204, 206, 208, 210, 212, 214, 216, 218, 220, 222, 224, 226, 228, 230, 232, 234, 236, 238, 240, 242, 244, 246, 248, 250, 252, 254, 256, 258, 260, 262, 279, 280, 281, 282, 286, 287, 288, 289, 293, 294, 295, 296, 300, 301, 302, 303, 306, 307, 308, 309, 313, 315, 319, 320, 321, 322, 327, 328, 333, 334, 335, 336, 340, 341, 342, 343, 347, 348, 349, 350, 362, 363, 364, 365, 366, 378, 379, 380, 381, 382, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 417, 418, 419, 420, 421, 422, 423, 425, 426, 427, 428, 429, 432, 433, 434, 435, 436, 467, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 573, 575, 576, 577, 648, 655, 657, 658, 665, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 687 }

B grade { 39, 58, 59, 62, 67, 68, 69, 75, 76, 77, 78, 79, 80, 84, 85, 86, 87, 88, 89, 90, 93, 94, 95, 96, 97, 98, 109, 185, 187, 189, 191, 193, 195, 197, 199, 201, 203, 205, 207, 209, 211, 213, 215, 217, 219, 221, 223, 225, 227, 229, 231, 233, 235, 237, 239, 241, 243, 245, 247, 249, 251, 253, 255, 257, 259, 261, 283, 284, 285, 290, 291, 292, 297, 298, 299, 304, 305, 310, 311, 312, 314, 316, 317, 318, 323, 324, 325, 326, 329, 330, 331, 332, 337, 338, 339, 344, 345, 346, 351, 352, 353, 358, 359, 360, 361, 367, 368, 373, 374, 375, 376, 377, 383, 384, 388, 389, 390, 391, 392, 393, 416, 424, 430, 431, 465, 466, 520, 534, 549, 560, 572, 574, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 646, 647, 656, 660, 661, 662, 663, 664, 666, 686 }

C grade { 483, 484, 485, 486, 487, 504, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 659 }

F normal fail { 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 505, 506, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 649, 650, 651, 652, 653, 654, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700 }

F(-1) timedout fail { 354, 355, 356, 357, 369, 370, 371, 372, 385, 386, 387 }

F(-2) exception fail { }

Fricas

A grade { 6, 7, 8, 9, 10, 15, 16, 17, 18, 23, 24, 25, 26, 27, 31, 32, 33, 34, 35, 40, 41, 42, 43, 44, 47, 48, 49, 50, 51, 54, 55, 56, 57, 58, 63, 64, 65, 66, 68, 69, 71, 72, 73, 74, 75, 76, 81, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 110, 111, 112, 113, 117, 118, 119, 120, 121, 125, 126, 127, 128, 129, 135, 136, 137, 138, 139, 142, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 167, 168, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 279, 280, 281, 284, 285, 286, 287, 288, 289, 292, 293, 299, 300, 301, 302, 303, 305, 312, 313, 314, 315, 318, 326, 327, 328, 330, 331, 332, 333, 334, 335, 336, 338, 339, 340, 341, 342, 343, 346, 347, 348, 349, 352, 353, 354, 355, 356, 361, 362, 363, 364, 365, 368, 369, 370, 376, 377, 378, 379, 380, 381, 383, 384, 392, 393, 394, 395, 397, 398, 399, 400, 401, 402, 403, 405, 406, 407, 408, 409, 410, 411, 413, 414, 415, 416, 417, 419, 420, 421, 422, 423, 424, 426, 427, 428, 429, 430, 431, 433, 434, 435, 436, 443, 444, 445, 446, 447, 448, 449, 483, 484, 485, 486, 491, 493, 505, 506, 507, 508, 509, 510, 511, 513, 514, 515, 516, 517, 518, 519, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 547, 548, 549, 553, 558, 559, 564, 565, 647, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 685, 686, 687 }

B grade { 1, 2, 3, 4, 5, 11, 12, 13, 14, 19, 20, 21, 22, 28, 29, 30, 36, 37, 38, 39, 45, 46, 52, 53, 59, 60, 61, 62, 67, 70, 77, 78, 79, 80, 82, 88, 107, 108, 109, 114, 115, 116, 122, 123, 124, 130, 131, 132, 133, 134, 140, 141, 148, 149, 150, 166, 172, 282, 283, 290, 291, 294, 295, 296, 297, 298, 304, 306, 307, 308, 309, 310, 311, 316, 317, 319, 320, 321, 322, 323, 324, 325, 329, 337, 344, 345, 350, 351, 357, 358, 359, 360, 366, 367, 371, 372, 373, 374, 375, 382, 385, 386, 387, 388, 389, 390, 391, 396, 404, 412, 418, 425, 432, 465, 466, 467, 487, 495, 504, 512, 520, 534, 545, 546, 550, 551, 552, 554, 555, 556, 557, 560, 561, 562, 563, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 646, 648, 684 }

C grade { 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672 }

F normal fail { 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 437, 438, 439, 440, 441, 442, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 488, 489, 490, 492, 494, 496, 497, 498, 499, 500, 501, 502, 503, 604, 610, 624, 625, 626, 627, 628, 629, 630, 631, 640, 641, 642, 643, 644, 645, 649, 650, 651, 652, 653, 654, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700 }

F(-1) timedout fail { 603, 606, 607, 608, 609, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 632, 633, 634, 635, 636, 637, 638, 639 }

F(-2) exception fail { 605, 611 }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 62, 63, 64, 65, 68, 69, 70, 71, 72, 75, 76, 80, 81, 82, 83, 84, 89, 90, 99, 100, 101, 102, 103, 110, 114, 115, 116, 117, 118, 126, 130, 131, 132, 134, 135, 136, 143, 144, 148, 149, 151, 152, 154, 160, 161, 162, 166, 167, 168, 172, 178, 179, 180, 181, 182, 279, 280, 281, 282, 283, 284, 285, 293, 294, 295, 296, 297, 298, 299, 306, 307, 308, 309, 310, 311, 312, 319, 320, 321, 322, 323, 324, 325, 335, 336, 337, 338, 339, 347, 348, 349, 350, 351, 352, 353, 362, 363, 364, 365, 366, 367, 368, 378, 379, 380, 381, 382, 383, 384, 397, 398, 399, 405, 406, 407, 413, 414, 415, 417, 419, 420, 421, 422, 426, 427, 428, 429, 433, 434, 435, 436, 467, 483, 484, 485, 486, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 551, 552, 554, 555, 556, 557, 558, 566, 567, 568, 569, 646, 647, 648, 673, 674, 675, 681, 682, 683, 685 }

B grade { 42, 48, 56, 61, 66, 67, 73, 74, 77, 78, 79, 85, 86, 87, 88, 91, 92, 93, 94, 95, 96, 97, 98, 107, 108, 109, 122, 123, 124, 125, 133, 140, 141, 142, 150, 153, 158, 159, 169, 170, 171, 176, 177, 288, 290, 291, 292, 300, 301, 303, 304, 313, 315, 316, 317, 327, 328, 329, 330, 333, 334, 340, 341, 342, 343, 345, 346, 354, 355, 356, 357, 358, 360, 361, 369, 370, 371, 372, 373, 374, 376, 377, 385, 386, 387, 388, 390, 392, 393, 394, 395, 396, 400, 401, 402, 403, 404, 408, 409, 410, 411, 412, 416, 418, 423, 424, 425, 430, 431, 432, 443, 444, 445, 446, 447, 448, 449, 487, 491, 493, 495, 504, 505, 549, 550, 553, 559, 560, 561, 562, 563, 564, 565, 570, 571, 572, 573, 574, 575, 576, 577, 676, 677, 678, 679, 680, 684, 686, 687 }

C grade { }

F normal fail { 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 263, 264, 265, 266, 267, 268, 269, 270, 289, 302, 314, 326, 344, 359, 375, 391, 437, 438, 439, 440, 441, 442, 450, 451, 452, 453, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 488, 489, 490, 492, 494, 496, 497, 498, 499, 500, 503, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 611, 613, 614, 615, 616, 621, 622, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 637, 638, 639, 640, 641, 642, 643, 644, 645, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 688, 689, 690, 693, 694, 695, 696, 697, 698, 699, 700 }

F(-1) timedout fail { 286, 287, 305, 318, 331, 332, 389, 595, 610, 612, 617, 618, 619, 620, 636 }

F(-2) exception fail { 104, 105, 106, 111, 112, 113, 119, 120, 121, 127, 128, 129, 137, 138, 139, 145, 146, 147, 155, 156, 157, 163, 164, 165, 173, 174, 175, 183, 184, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 271, 272, 273, 274, 275, 276, 277, 278, 454, 455, 456, 501, 502, 506, 609, 623, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 691, 692 }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 19, 20, 21, 22, 23, 24, 26, 30, 31, 36, 37, 38, 41, 45, 46, 52, 53, 59, 60, 64, 70, 99, 100, 101, 102, 105, 106, 107, 108, 109, 110, 111, 114, 115, 116, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 167, 168, 169, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 507, 509, 511, 517, 518, 519, 531, 532, 533, 544, 545, 546, 550, 551, 552, 553, 554, 555, 556, 557, 558, 561, 562, 563, 564, 565, 566, 567, 568, 569, 576 }

B grade { 11, 12, 13, 14, 15, 16, 17, 18, 25, 27, 28, 29, 32, 33, 34, 35, 39, 40, 42, 43, 44, 47, 48, 49, 50, 51, 54, 55, 56, 57, 58, 61, 62, 63, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 103, 104, 112, 113, 117, 129, 133, 134, 135, 136, 150, 152, 154, 166, 170, 171, 172, 179, 282, 336, 467, 508, 510, 512, 513, 514, 515, 516, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 547, 548, 549, 559, 560, 570, 571, 572, 573, 574, 575, 577 }

C grade { }

F normal fail { 185, 186, 187, 189, 190, 191, 192, 193, 195, 196, 197, 198, 199, 200, 201, 202, 203, 205, 206, 207, 208, 209, 210, 211, 212, 213, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700 }

F(-1) timedout fail { 297, 298, 299, 311, 312, 323, 324, 325 }

F(-2) exception fail { 188, 194, 204, 214, 646, 647, 648, 659 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 188, 279, 280, 281, 282, 286, 287, 288, 289, 293, 294, 295, 296, 300, 301, 302, 303, 306, 307, 308, 309, 313, 314, 315, 316, 319, 320, 321, 322, 326, 327, 328, 329, 333, 334, 335, 336, 340, 341, 342, 343, 347, 348, 349, 350, 354, 355, 356, 357, 362, 363, 364, 365, 366, 369, 370, 371, 372, 378, 379, 380, 382, 385, 386, 387, 397, 398, 399, 405, 406, 407, 412, 413, 414, 415, 418, 419, 420, 421, 422, 426, 427, 428, 429, 433, 434, 435, 436, 446, 447, 448, 449, 465, 466, 467, 483, 484, 485, 486, 491, 493, 495, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 581, 648, 659, 673, 674, 675, 681, 682, 683 }

C grade { }

F normal fail { }

F(-1) timedout fail { 185, 186, 187, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 283, 284, 285, 290, 291, 292, 297, 298, 299, 304, 305, 310, 311, 312, 317, 318, 323, 324, 325, 330, 331, 332, 337, 338, 339, 344, 345, 346, 351, 352, 353, 358, 359, 360, 361, 367, 368, 373, 374, 375, 376, 377, 381, 383, 384, 388, 389, 390, 391, 392, 393, 394, 395, 396, 400, 401, 402, 403, 404, 408, 409, 410, 411, 416, 417, 423, 424, 425, 430, 431, 432, 437, 438, 439, 440, 441, 442, 443, 444, 445, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 487, 488, 489, 490, 492, 494, 496, 497, 498, 499, 500, 501, 502, 503, 578, 579, 580, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 676, 677, 678, 679, 680, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 14, 15, 23, 24, 25, 26, 27, 31, 32, 40, 41, 43, 44, 47, 48, 49, 50, 54, 55, 56, 57, 63, 64, 65, 66, 68, 69, 71, 72, 73, 74, 75, 76, 81, 82, 83, 84, 85, 86, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 104, 105, 106, 110, 119, 120, 121, 137, 138, 139, 155, 156, 157, 163, 173, 174, 175, 183, 184, 487, 507, 509, 511, 512 }

B grade { 16, 17, 18, 33, 34, 35, 42, 51, 58, 67, 87, 88, 111, 112, 113, 118, 126, 127, 128, 129, 136, 143, 144, 145, 146, 147, 153, 154, 161, 162, 164, 165, 169, 170, 171, 172, 179, 180, 181, 182, 486 }

C grade { }

F normal fail { 11, 12, 13, 19, 20, 21, 22, 28, 29, 30, 36, 37, 38, 39, 45, 46, 52, 53, 59, 60, 61, 62, 70, 77, 78, 79, 80, 99, 100, 101, 102, 103, 107, 108, 109, 114, 115, 116, 117, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 140, 141, 142, 148, 149, 150, 151, 152, 158, 159, 160, 166, 167, 168, 176, 177, 178, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 202, 203, 204, 205, 206, 207, 208, 213, 214, 215, 216, 217, 218, 226, 227, 228, 229, 230, 231, 232, 238, 239, 240, 241, 242, 243, 250, 251, 252, 253, 254, 260, 261, 262, 263, 264, 265, 266, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 294, 295, 296, 297, 300, 301, 302, 303, 309, 314, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 347, 348, 349, 350, 351, 352, 353, 355, 356, 357, 358, 359, 360, 361, 363, 364, 365, 366, 367, 368, 371, 372, 373, 374, 375, 376, 377, 380, 381, 382, 388, 389, 390, 391, 394, 395, 396, 397, 398, 402, 403, 404, 405, 417, 418, 419, 420, 421, 425, 426, 427, 428, 433, 434, 435, 439, 440, 441, 442, 444, 445, 446, 450, 451, 452, 453, 454, 455, 456, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 471, 472, 473, 474, 478, 479, 480, 481, 482, 483, 484, 485, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 508, 510, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 560, 561, 562, 563, 564, 566, 567, 568, 569, 572, 573, 574, 575, 576, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 595, 596, 597, 598, 599, 600, 604, 605, 606, 607, 608, 609, 612, 613, 614, 615, 616, 619, 620, 621, 622, 623, 624, 625, 626, 627, 629, 630, 631, 633, 634, 635, 637, 638, 639, 640, 641, 642, 643, 644, 645, 647, 648, 649, 650, 651, 652, 653, 658, 659, 660, 666, 667, 676, 677, 678, 682, 683, 684, 685, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700 }

F(-1) timedout fail { 185, 199, 200, 201, 209, 210, 211, 212, 219, 220, 221, 222, 223, 224, 225, 233, 234, 235, 236, 237, 244, 245, 246, 247, 248, 249, 255, 256, 257, 258, 259, 267, 291, 292, 293, 298, 299, 304, 305, 306, 307, 308, 310, 311, 312, 313, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 346, 354, 362, 369, 370, 378, 379, 383, 384, 385, 386, 387, 392, 393, 399, 400, 401, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 422, 423, 424, 429, 430, 431, 432, 436, 437, 438, 443, 447, 448, 449, 457, 458, 470, 475, 476, 477, 558, 559, 565, 577, 578, 593, 594, 601, 602, 603, 610, 611, 617, 618, 628, 632, 636, 646, 654, 655, 656, 657, 661, 662, 663, 664, 665, 668, 669, 670, 671, 672, 673, 674, 675, 679, 680, 681, 686, 687 }

F(-2) exception fail { 570, 571 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	79	78	114	189	83	114	106
N.S.	1	1.00	0.84	0.83	1.21	2.01	0.88	1.21	1.13
time (sec)	N/A	0.061	0.312	176.202	0.280	0.236	4.114	0.432	4.023

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	63	67	92	153	68	92	149
N.S.	1	1.00	0.84	0.89	1.23	2.04	0.91	1.23	1.99
time (sec)	N/A	0.055	0.097	63.452	0.237	0.228	2.402	0.427	4.202

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	55	56	70	117	60	70	112
N.S.	1	1.00	0.89	0.90	1.13	1.89	0.97	1.13	1.81
time (sec)	N/A	0.053	0.093	18.230	0.242	0.233	1.579	0.413	3.708

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	43	45	48	81	48	48	48
N.S.	1	1.00	0.93	0.98	1.04	1.76	1.04	1.04	1.04
time (sec)	N/A	0.051	0.036	3.463	0.238	0.232	1.108	0.407	3.995

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	30	30	28	21	45	37	26	23
N.S.	1	1.11	1.11	1.04	0.78	1.67	1.37	0.96	0.85
time (sec)	N/A	0.043	0.012	0.625	0.237	0.228	0.738	0.379	3.873

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	23	17	18	24	18	17
N.S.	1	1.00	1.00	1.21	0.89	0.95	1.26	0.95	0.89
time (sec)	N/A	0.011	0.010	0.062	0.258	0.238	0.084	0.297	3.720

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	48	22	38	23	39	23	22
N.S.	1	1.00	1.07	0.49	0.84	0.51	0.87	0.51	0.49
time (sec)	N/A	0.038	0.036	0.624	0.332	0.240	0.090	0.359	4.170

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	46	53	61	56	136	103	64
N.S.	1	1.00	0.69	0.79	0.91	0.84	2.03	1.54	0.96
time (sec)	N/A	0.054	0.035	3.062	0.337	0.247	0.156	0.383	3.707

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	56	63	82	80	211	127	108
N.S.	1	1.00	0.63	0.71	0.92	0.90	2.37	1.43	1.21
time (sec)	N/A	0.078	0.045	14.070	0.340	0.235	0.210	0.415	4.036

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	68	73	103	104	279	151	152
N.S.	1	1.00	0.61	0.66	0.93	0.94	2.51	1.36	1.37
time (sec)	N/A	0.094	0.149	48.319	0.320	0.240	0.284	0.463	5.553

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	98	74	106	372	0	181	247
N.S.	1	1.00	1.00	0.76	1.08	3.80	0.00	1.85	2.52
time (sec)	N/A	0.073	0.021	36.220	0.254	0.249	0.000	0.410	8.358

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	76	64	86	276	0	139	178
N.S.	1	1.00	1.00	0.84	1.13	3.63	0.00	1.83	2.34
time (sec)	N/A	0.068	0.010	8.212	0.239	0.241	0.000	0.370	7.574

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	51	61	180	0	97	107
N.S.	1	1.00	1.00	0.94	1.13	3.33	0.00	1.80	1.98
time (sec)	N/A	0.050	0.011	1.750	0.236	0.241	0.000	0.358	5.955

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	34	32	82	41	52	39
N.S.	1	1.00	1.00	1.26	1.19	3.04	1.52	1.93	1.44
time (sec)	N/A	0.023	0.007	0.358	0.247	0.243	2.268	0.366	4.042

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	51	17	22	15	26	84	20
N.S.	1	1.00	1.96	0.65	0.85	0.58	1.00	3.23	0.77
time (sec)	N/A	0.030	0.034	0.349	0.256	0.233	0.074	0.361	3.947

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	37	36	42	105	196	54
N.S.	1	1.00	1.00	0.80	0.78	0.91	2.28	4.26	1.17
time (sec)	N/A	0.049	0.006	1.448	0.225	0.251	0.161	0.396	3.706

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	62	47	49	66	184	220	70
N.S.	1	1.00	1.00	0.76	0.79	1.06	2.97	3.55	1.13
time (sec)	N/A	0.051	0.008	6.787	0.253	0.232	0.242	0.534	5.998

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	76	57	58	90	253	244	93
N.S.	1	1.00	1.00	0.75	0.76	1.18	3.33	3.21	1.22
time (sec)	N/A	0.049	0.025	27.385	0.268	0.234	0.321	0.525	6.665

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	79	80	108	189	0	108	151
N.S.	1	1.00	0.72	0.73	0.99	1.73	0.00	0.99	1.39
time (sec)	N/A	0.080	0.544	114.000	0.227	0.228	0.000	0.555	4.316

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	63	69	95	151	0	95	132
N.S.	1	1.00	0.77	0.84	1.16	1.84	0.00	1.16	1.61
time (sec)	N/A	0.084	0.358	34.889	0.260	0.233	0.000	0.545	3.696

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	34	58	56	113	0	56	56
N.S.	1	1.00	0.62	1.05	1.02	2.05	0.00	1.02	1.02
time (sec)	N/A	0.061	0.170	8.362	0.251	0.234	0.000	0.503	3.825

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	50	47	21	75	0	42	35
N.S.	1	1.00	1.85	1.74	0.78	2.78	0.00	1.56	1.30
time (sec)	N/A	0.046	0.092	1.974	0.262	0.227	0.000	0.461	4.062

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	34	40	41	56	53	66	29
N.S.	1	1.00	0.89	1.05	1.08	1.47	1.39	1.74	0.76
time (sec)	N/A	0.021	0.133	0.091	0.368	0.237	0.132	0.382	3.635

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	31	19	32	17	36	17	18
N.S.	1	1.00	1.24	0.76	1.28	0.68	1.44	0.68	0.72
time (sec)	N/A	0.050	0.132	1.348	0.362	0.234	0.102	0.495	3.820

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	52	44	67	41	87	257	50
N.S.	1	1.00	0.83	0.70	1.06	0.65	1.38	4.08	0.79
time (sec)	N/A	0.085	0.207	6.554	0.373	0.237	0.147	0.559	3.818

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	99	79	92	78	185	169	88
N.S.	1	1.00	0.85	0.68	0.79	0.67	1.58	1.44	0.75
time (sec)	N/A	0.110	0.306	26.162	0.355	0.245	0.220	0.605	3.639

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	142	114	115	106	270	342	144
N.S.	1	1.00	0.83	0.67	0.67	0.62	1.58	2.00	0.84
time (sec)	N/A	0.151	0.362	82.579	0.377	0.247	0.275	0.677	4.951

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	108	133	181	364	0	237	290
N.S.	1	1.00	0.92	1.13	1.53	3.08	0.00	2.01	2.46
time (sec)	N/A	0.126	0.243	19.593	0.280	0.249	0.000	0.553	7.897

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	84	111	130	256	0	173	198
N.S.	1	1.00	0.89	1.18	1.38	2.72	0.00	1.84	2.11
time (sec)	N/A	0.098	0.131	3.992	0.285	0.245	0.000	0.513	7.315

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	56	86	83	148	0	107	104
N.S.	1	1.00	0.82	1.26	1.22	2.18	0.00	1.57	1.53
time (sec)	N/A	0.056	0.141	0.788	0.490	0.255	0.000	0.426	4.632

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	180	56	61	52	68	56	41
N.S.	1	1.00	3.91	1.22	1.33	1.13	1.48	1.22	0.89
time (sec)	N/A	0.046	0.522	0.631	0.358	0.240	0.177	0.456	3.748

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	52	38	52	34	75	531	78
N.S.	1	1.00	1.02	0.75	1.02	0.67	1.47	10.41	1.53
time (sec)	N/A	0.051	0.076	3.128	0.654	0.235	0.152	0.520	3.710

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	68	67	79	62	153	613	71
N.S.	1	1.00	0.99	0.97	1.14	0.90	2.22	8.88	1.03
time (sec)	N/A	0.060	0.133	14.235	0.315	0.250	0.232	0.598	5.115

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	85	102	98	90	238	641	256
N.S.	1	1.00	0.98	1.17	1.13	1.03	2.74	7.37	2.94
time (sec)	N/A	0.069	0.146	49.181	0.280	0.232	0.314	0.670	4.098

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	104	131	119	118	314	669	330
N.S.	1	1.00	0.99	1.25	1.13	1.12	2.99	6.37	3.14
time (sec)	N/A	0.070	0.315	135.736	0.271	0.237	0.387	0.721	6.135

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	79	91	108	215	0	108	151
N.S.	1	1.00	0.72	0.83	0.99	1.97	0.00	0.99	1.39
time (sec)	N/A	0.085	0.644	182.417	0.483	0.232	0.000	0.596	3.798

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	63	80	108	177	0	108	151
N.S.	1	1.00	0.77	0.98	1.32	2.16	0.00	1.32	1.84
time (sec)	N/A	0.071	0.367	64.761	0.392	0.239	0.000	0.539	4.170

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	34	69	82	139	0	82	114
N.S.	1	1.00	0.62	1.25	1.49	2.53	0.00	1.49	2.07
time (sec)	N/A	0.056	0.300	19.529	0.510	0.232	0.000	0.543	3.867

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	50	58	21	101	0	56	56
N.S.	1	1.00	1.85	2.15	0.78	3.74	0.00	2.07	2.07
time (sec)	N/A	0.049	0.153	4.088	0.267	0.233	0.000	0.489	3.847

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	46	51	76	97	94	118	41
N.S.	1	1.00	0.73	0.81	1.21	1.54	1.49	1.87	0.65
time (sec)	N/A	0.042	0.148	0.096	0.363	0.239	0.162	0.399	4.199

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	40	50	62	36	61	36	39
N.S.	1	1.00	0.82	1.02	1.27	0.73	1.24	0.73	0.80
time (sec)	N/A	0.061	0.108	2.852	0.380	0.239	0.169	0.614	3.635

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	24	38	57	34	80	135	36
N.S.	1	1.00	0.89	1.41	2.11	1.26	2.96	5.00	1.33
time (sec)	N/A	0.051	0.079	13.760	0.375	0.237	0.174	0.708	3.944

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	65	62	105	55	131	457	77
N.S.	1	1.00	0.72	0.69	1.17	0.61	1.46	5.08	0.86
time (sec)	N/A	0.088	0.245	47.766	0.461	0.234	0.202	0.814	4.152

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	137	97	128	92	226	514	125
N.S.	1	1.00	0.95	0.67	0.89	0.64	1.57	3.57	0.87
time (sec)	N/A	0.121	0.312	133.412	0.428	0.237	0.291	0.919	4.011

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	102	122	155	310	0	189	228
N.S.	1	1.00	0.80	0.96	1.22	2.44	0.00	1.49	1.80
time (sec)	N/A	0.150	1.723	8.870	0.293	0.238	0.000	0.518	7.624

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	93	100	109	202	0	125	136
N.S.	1	1.00	0.94	1.01	1.10	2.04	0.00	1.26	1.37
time (sec)	N/A	0.087	1.476	1.834	0.272	0.244	0.000	0.484	6.255

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	123	93	82	107	107	234	102
N.S.	1	1.00	2.02	1.52	1.34	1.75	1.75	3.84	1.67
time (sec)	N/A	0.061	1.564	1.460	0.291	0.253	0.195	0.604	4.196

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	31	19	75	17	36	901	66
N.S.	1	1.00	0.97	0.59	2.34	0.53	1.12	28.16	2.06
time (sec)	N/A	0.046	0.192	6.812	0.313	0.225	0.132	0.707	3.984

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	145	56	105	48	116	929	130
N.S.	1	1.00	1.65	0.64	1.19	0.55	1.32	10.56	1.48
time (sec)	N/A	0.098	0.746	26.521	0.355	0.227	0.217	0.851	4.788

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	180	85	123	76	190	465	134
N.S.	1	1.00	1.70	0.80	1.16	0.72	1.79	4.39	1.26
time (sec)	N/A	0.107	0.613	81.925	0.406	0.238	0.311	0.941	5.109

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	226	120	145	104	275	1039	330
N.S.	1	1.00	1.82	0.97	1.17	0.84	2.22	8.38	2.66
time (sec)	N/A	0.103	0.910	215.938	0.312	0.251	0.421	0.780	5.731

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	171	133	246	364	0	237	290
N.S.	1	1.00	1.05	0.82	1.51	2.23	0.00	1.45	1.78
time (sec)	N/A	0.234	2.055	20.175	0.278	0.241	0.000	0.598	8.388

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	237	111	180	256	0	173	198
N.S.	1	1.00	1.78	0.83	1.35	1.92	0.00	1.30	1.49
time (sec)	N/A	0.128	1.533	3.806	0.309	0.249	0.000	0.556	7.601

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	906	107	137	162	153	372	159
N.S.	1	1.00	9.34	1.10	1.41	1.67	1.58	3.84	1.64
time (sec)	N/A	0.111	6.766	3.187	0.274	0.267	0.244	0.665	6.342

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	246	79	121	68	109	1299	88
N.S.	1	1.00	3.15	1.01	1.55	0.87	1.40	16.65	1.13
time (sec)	N/A	0.096	0.944	13.925	0.435	0.249	0.244	0.972	4.155

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	145	38	118	34	80	915	130
N.S.	1	1.00	2.20	0.58	1.79	0.52	1.21	13.86	1.97
time (sec)	N/A	0.104	0.552	47.411	0.416	0.239	0.227	1.083	4.878

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	181	74	149	62	156	1327	186
N.S.	1	1.00	1.77	0.73	1.46	0.61	1.53	13.01	1.82
time (sec)	N/A	0.121	0.608	131.887	0.314	0.248	0.286	0.756	4.932

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	216	233	181	90	228	1409	145
N.S.	1	1.00	1.80	1.94	1.51	0.75	1.90	11.74	1.21
time (sec)	N/A	0.129	0.911	0.825	0.295	0.237	0.392	0.809	6.090

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	81	377	160	267	0	160	146
N.S.	1	1.00	0.74	3.46	1.47	2.45	0.00	1.47	1.34
time (sec)	N/A	0.104	0.842	1.150	0.297	0.233	0.000	0.863	4.235

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	63	102	108	229	0	108	151
N.S.	1	1.00	0.77	1.24	1.32	2.79	0.00	1.32	1.84
time (sec)	N/A	0.080	0.529	181.819	0.347	0.236	0.000	0.809	4.047

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	36	91	108	191	0	108	151
N.S.	1	1.00	0.65	1.65	1.96	3.47	0.00	1.96	2.75
time (sec)	N/A	0.053	0.382	66.858	0.335	0.237	0.000	0.777	3.893

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	72	80	21	153	0	82	114
N.S.	1	1.00	2.67	2.96	0.78	5.67	0.00	3.04	4.22
time (sec)	N/A	0.048	0.345	20.703	0.359	0.229	0.000	0.749	3.750

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	71	72	165	177	178	222	73
N.S.	1	1.00	0.61	0.62	1.41	1.51	1.52	1.90	0.62
time (sec)	N/A	0.103	0.179	0.151	0.459	0.248	0.249	0.399	4.399

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	62	85	86	125	131	146	70
N.S.	1	1.00	0.75	1.02	1.04	1.51	1.58	1.76	0.84
time (sec)	N/A	0.079	0.594	14.134	0.451	0.237	0.242	0.939	3.722

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	49	68	88	51	102	450	64
N.S.	1	1.00	0.67	0.93	1.21	0.70	1.40	6.16	0.88
time (sec)	N/A	0.073	0.270	48.941	0.392	0.249	0.252	0.704	4.011

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	36	38	93	34	80	187	53
N.S.	1	1.00	0.65	0.69	1.69	0.62	1.45	3.40	0.96
time (sec)	N/A	0.070	0.116	135.100	0.514	0.234	0.256	0.699	4.173

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	24	301	103	62	162	267	63
N.S.	1	1.00	0.89	11.15	3.81	2.30	6.00	9.89	2.33
time (sec)	N/A	0.048	0.176	1.381	0.594	0.236	0.308	0.785	3.741

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	123	331	164	83	209	857	122
N.S.	1	1.00	0.85	2.30	1.14	0.58	1.45	5.95	0.85
time (sec)	N/A	0.111	0.332	0.696	1.334	0.248	0.373	0.902	4.743

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	159	361	187	120	301	914	171
N.S.	1	1.00	0.80	1.82	0.94	0.61	1.52	4.62	0.86
time (sec)	N/A	0.150	0.639	0.788	0.430	0.260	0.492	0.883	5.556

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	115	122	215	310	0	189	228
N.S.	1	1.00	0.69	0.73	1.29	1.86	0.00	1.13	1.37
time (sec)	N/A	0.169	2.105	8.776	0.330	0.249	0.000	0.722	7.799

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	151	118	173	216	197	510	222
N.S.	1	1.00	1.16	0.91	1.33	1.66	1.52	3.92	1.71
time (sec)	N/A	0.133	1.781	7.326	0.317	0.240	0.292	0.812	8.254

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	130	111	154	122	148	1683	162
N.S.	1	1.00	1.33	1.13	1.57	1.24	1.51	17.17	1.65
time (sec)	N/A	0.113	1.965	26.349	0.301	0.246	0.320	1.176	6.136

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	31	19	152	17	36	1669	104
N.S.	1	1.00	0.97	0.59	4.75	0.53	1.12	52.16	3.25
time (sec)	N/A	0.044	0.420	81.448	0.631	0.233	0.204	0.887	4.313

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	122	56	187	48	121	1697	186
N.S.	1	1.00	1.21	0.55	1.85	0.48	1.20	16.80	1.84
time (sec)	N/A	0.138	0.615	212.039	0.405	0.236	0.321	0.935	4.712

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	146	287	217	76	192	1725	79
N.S.	1	1.00	1.04	2.04	1.54	0.54	1.36	12.23	0.56
time (sec)	N/A	0.154	0.730	0.793	0.309	0.243	0.399	0.947	4.862

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	170	317	246	104	265	1807	139
N.S.	1	1.00	1.07	1.99	1.55	0.65	1.67	11.36	0.87
time (sec)	N/A	0.163	1.043	0.668	0.349	0.250	0.492	1.030	5.811

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	56	611	186	345	0	186	153
N.S.	1	1.00	0.51	5.61	1.71	3.17	0.00	1.71	1.40
time (sec)	N/A	0.113	1.929	0.593	0.282	0.242	0.000	1.200	5.595

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	44	475	173	307	0	173	190
N.S.	1	1.00	0.54	5.79	2.11	3.74	0.00	2.11	2.32
time (sec)	N/A	0.118	1.012	0.582	0.339	0.234	0.000	1.113	5.868

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	34	124	134	269	0	134	107
N.S.	1	1.00	0.62	2.25	2.44	4.89	0.00	2.44	1.95
time (sec)	N/A	0.070	0.721	286.484	0.352	0.241	0.000	1.130	4.728

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	102	113	21	231	0	120	83
N.S.	1	1.00	3.78	4.19	0.78	8.56	0.00	4.44	3.07
time (sec)	N/A	0.070	0.667	111.631	0.302	0.238	0.000	1.064	4.383

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	100	103	121	297	301	378	113
N.S.	1	1.00	0.50	0.52	0.60	1.48	1.50	1.89	0.56
time (sec)	N/A	0.251	0.509	0.338	0.475	0.235	0.379	0.536	4.289

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	96	118	124	245	257	302	102
N.S.	1	1.00	0.72	0.89	0.93	1.84	1.93	2.27	0.77
time (sec)	N/A	0.155	0.890	84.154	0.355	0.258	0.366	0.907	3.795

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	86	114	135	179	216	785	111
N.S.	1	1.00	0.69	0.92	1.09	1.44	1.74	6.33	0.90
time (sec)	N/A	0.140	1.513	219.138	0.366	0.244	0.381	1.044	4.114

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	77	319	146	113	172	799	103
N.S.	1	1.00	0.68	2.80	1.28	0.99	1.51	7.01	0.90
time (sec)	N/A	0.148	0.916	0.839	0.341	0.243	0.406	1.094	4.221

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	31	451	136	17	36	381	66
N.S.	1	1.00	0.72	10.49	3.16	0.40	0.84	8.86	1.53
time (sec)	N/A	0.071	1.610	1.405	0.333	0.244	0.346	1.123	3.842

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	44	588	152	48	121	409	82
N.S.	1	1.00	0.55	7.35	1.90	0.60	1.51	5.11	1.02
time (sec)	N/A	0.077	0.240	1.177	0.354	0.243	0.447	1.229	4.493

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	34	639	162	76	197	437	82
N.S.	1	1.00	0.62	11.62	2.95	1.38	3.58	7.95	1.49
time (sec)	N/A	0.064	0.413	2.809	0.334	0.261	0.522	1.270	3.843

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	116	689	171	104	279	465	105
N.S.	1	1.00	4.30	25.52	6.33	3.85	10.33	17.22	3.89
time (sec)	N/A	0.051	2.932	1.402	0.343	0.276	0.630	1.310	3.898

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	152	739	246	125	323	1457	195
N.S.	1	1.00	0.68	3.28	1.09	0.56	1.44	6.48	0.87
time (sec)	N/A	0.157	0.976	2.592	0.375	0.297	0.693	1.529	5.673

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	186	789	269	162	413	1514	231
N.S.	1	1.00	0.67	2.83	0.96	0.58	1.48	5.43	0.83
time (sec)	N/A	0.219	1.170	2.284	0.393	0.319	0.799	1.629	6.040

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	205	151	396	378	320	924	399
N.S.	1	1.00	0.87	0.64	1.69	1.61	1.36	3.93	1.70
time (sec)	N/A	0.306	2.765	90.461	0.274	0.247	0.434	1.437	9.230

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	1540	147	352	284	277	2835	343
N.S.	1	1.00	7.51	0.72	1.72	1.39	1.35	13.83	1.67
time (sec)	N/A	0.257	8.297	253.242	0.288	0.255	0.458	1.346	8.747

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	A	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	1162	322	326	190	235	2849	281
N.S.	1	1.00	6.72	1.86	1.88	1.10	1.36	16.47	1.62
time (sec)	N/A	0.214	8.064	2.376	0.277	0.252	0.454	1.421	8.578

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	305	385	309	96	187	2863	207
N.S.	1	1.00	2.01	2.53	2.03	0.63	1.23	18.84	1.36
time (sec)	N/A	0.209	3.173	1.562	0.268	0.254	0.484	1.544	8.247

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	146	447	302	34	80	2451	37
N.S.	1	1.00	2.21	6.77	4.58	0.52	1.21	37.14	0.56
time (sec)	N/A	0.088	0.807	1.588	0.265	0.252	0.461	1.615	4.140

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	151	567	355	62	162	2863	65
N.S.	1	1.00	1.11	4.17	2.61	0.46	1.19	21.05	0.48
time (sec)	N/A	0.199	0.959	1.875	0.435	0.260	0.575	1.734	4.812

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	168	617	405	90	240	2891	93
N.S.	1	1.00	0.80	2.92	1.92	0.43	1.14	13.70	0.44
time (sec)	N/A	0.314	1.784	1.807	0.323	0.272	0.621	1.842	4.837

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	173	667	453	118	313	2919	222
N.S.	1	1.00	0.82	3.15	2.14	0.56	1.48	13.77	1.05
time (sec)	N/A	0.242	2.106	1.804	0.500	0.283	0.738	1.889	6.412

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	56	58	87	146	0	87	92
N.S.	1	1.00	0.52	0.54	0.81	1.36	0.00	0.81	0.86
time (sec)	N/A	0.089	0.269	0.451	0.283	0.234	0.000	0.405	4.347

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	46	47	67	109	0	67	114
N.S.	1	1.00	0.58	0.59	0.84	1.36	0.00	0.84	1.42
time (sec)	N/A	0.082	0.168	0.369	0.255	0.227	0.000	0.392	3.944

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	50	36	47	72	0	47	77
N.S.	1	1.00	0.91	0.65	0.85	1.31	0.00	0.85	1.40
time (sec)	N/A	0.067	0.098	0.348	0.299	0.230	0.000	0.379	4.516

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	23	27	33	0	27	25
N.S.	1	1.00	1.00	0.68	0.79	0.97	0.00	0.79	0.74
time (sec)	N/A	0.059	0.054	1.345	0.355	0.224	0.000	0.415	3.901

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	23	20	26	0	57	19
N.S.	1	1.00	1.00	1.00	0.87	1.13	0.00	2.48	0.83
time (sec)	N/A	0.052	0.026	0.536	0.369	0.243	0.000	0.362	4.314

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	35	26	0	32	60	58	29
N.S.	1	1.00	1.06	0.79	0.00	0.97	1.82	1.76	0.88
time (sec)	N/A	0.016	0.081	0.178	0.000	0.230	0.097	0.359	4.242

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	87	61	0	54	151	95	60
N.S.	1	1.00	1.06	0.74	0.00	0.66	1.84	1.16	0.73
time (sec)	N/A	0.095	0.181	0.938	0.000	0.239	0.170	0.369	4.052

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	131	96	0	76	219	116	123
N.S.	1	1.00	0.98	0.72	0.00	0.57	1.63	0.87	0.92
time (sec)	N/A	0.146	0.209	1.589	0.000	0.237	0.218	0.428	4.993

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	60	122	288	266	0	138	193
N.S.	1	1.00	0.71	1.45	3.43	3.17	0.00	1.64	2.30
time (sec)	N/A	0.089	0.819	0.714	0.273	0.251	0.000	0.377	8.363

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	50	100	186	174	0	99	116
N.S.	1	1.00	0.83	1.67	3.10	2.90	0.00	1.65	1.93
time (sec)	N/A	0.071	0.540	0.648	0.357	0.236	0.000	0.427	5.914

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	34	70	83	80	0	58	43
N.S.	1	1.00	1.10	2.26	2.68	2.58	0.00	1.87	1.39
time (sec)	N/A	0.071	0.444	1.189	0.369	0.239	0.000	0.391	4.112

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	25	19	29	17	34	21	25
N.S.	1	1.00	0.89	0.68	1.04	0.61	1.21	0.75	0.89
time (sec)	N/A	0.036	0.191	0.644	0.351	0.232	0.355	0.335	4.048

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	50	49	0	41	126	67	78
N.S.	1	1.00	1.06	1.04	0.00	0.87	2.68	1.43	1.66
time (sec)	N/A	0.057	0.307	0.750	0.000	0.235	0.170	0.412	3.966

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	72	84	0	63	196	119	134
N.S.	1	1.00	1.07	1.25	0.00	0.94	2.93	1.78	2.00
time (sec)	N/A	0.069	0.432	1.267	0.000	0.239	0.238	0.403	5.730

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	94	119	0	85	264	171	188
N.S.	1	1.00	1.11	1.40	0.00	1.00	3.11	2.01	2.21
time (sec)	N/A	0.078	0.606	0.585	0.000	0.246	0.338	0.428	8.209

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	44	47	77	138	0	77	93
N.S.	1	1.00	0.54	0.57	0.94	1.68	0.00	0.94	1.13
time (sec)	N/A	0.086	0.217	0.307	0.373	0.241	0.000	0.484	3.906

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	50	36	47	97	0	47	77
N.S.	1	1.00	0.91	0.65	0.85	1.76	0.00	0.85	1.40
time (sec)	N/A	0.066	0.133	0.346	0.731	0.232	0.000	0.502	3.803

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	50	20	35	54	0	35	33
N.S.	1	1.00	1.85	0.74	1.30	2.00	0.00	1.30	1.22
time (sec)	N/A	0.056	0.054	0.334	0.224	0.222	0.000	0.486	4.260

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	30	32	70	0	100	28
N.S.	1	1.00	1.00	0.79	0.84	1.84	0.00	2.63	0.74
time (sec)	N/A	0.072	0.048	0.663	0.233	0.244	0.000	0.468	3.746

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	19	19	21	17	65	30	22
N.S.	1	1.00	0.73	0.73	0.81	0.65	2.50	1.15	0.85
time (sec)	N/A	0.061	0.117	1.259	0.229	0.229	0.509	0.461	3.888

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	52	44	0	43	117	68	39
N.S.	1	1.00	0.85	0.72	0.00	0.70	1.92	1.11	0.64
time (sec)	N/A	0.041	0.122	0.253	0.000	0.238	0.150	0.337	4.110

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	97	79	0	65	189	103	71
N.S.	1	1.00	0.85	0.69	0.00	0.57	1.66	0.90	0.62
time (sec)	N/A	0.114	0.291	1.746	0.000	0.236	0.231	0.532	3.944

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	142	114	0	87	258	123	149
N.S.	1	1.00	0.86	0.69	0.00	0.53	1.56	0.75	0.90
time (sec)	N/A	0.149	0.306	0.747	0.000	0.240	0.299	0.508	5.252

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	294	133	421	326	0	203	191
N.S.	1	1.00	2.37	1.07	3.40	2.63	0.00	1.64	1.54
time (sec)	N/A	0.123	2.421	0.771	0.257	0.244	0.000	0.492	7.191

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	215	111	295	230	0	151	136
N.S.	1	1.00	2.15	1.11	2.95	2.30	0.00	1.51	1.36
time (sec)	N/A	0.093	1.456	0.668	0.247	0.245	0.000	0.492	6.668

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	146	89	167	134	0	95	104
N.S.	1	1.00	1.97	1.20	2.26	1.81	0.00	1.28	1.41
time (sec)	N/A	0.077	0.765	0.694	0.272	0.240	0.000	0.473	4.915

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	184	54	117	64	0	57	44
N.S.	1	1.00	3.83	1.12	2.44	1.33	0.00	1.19	0.92
time (sec)	N/A	0.060	0.437	0.727	0.563	0.240	0.000	0.419	4.056

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	38	38	45	30	112	47	79
N.S.	1	1.00	0.58	0.58	0.69	0.46	1.72	0.72	1.22
time (sec)	N/A	0.068	0.250	1.067	0.406	0.228	0.535	0.405	3.912

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	68	67	0	52	163	93	90
N.S.	1	1.00	0.96	0.94	0.00	0.73	2.30	1.31	1.27
time (sec)	N/A	0.058	0.495	1.127	0.000	0.235	0.237	0.446	4.847

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	95	102	0	74	231	145	161
N.S.	1	1.00	1.07	1.15	0.00	0.83	2.60	1.63	1.81
time (sec)	N/A	0.077	0.557	0.710	0.000	0.234	0.319	0.518	7.588

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	117	137	0	96	299	197	216
N.S.	1	1.00	1.09	1.28	0.00	0.90	2.79	1.84	2.02
time (sec)	N/A	0.080	0.846	0.705	0.000	0.232	0.400	0.494	6.544

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	56	58	87	194	0	87	119
N.S.	1	1.00	0.51	0.53	0.80	1.78	0.00	0.80	1.09
time (sec)	N/A	0.098	0.281	0.479	0.219	0.255	0.000	0.680	4.539

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	46	47	87	153	0	87	103
N.S.	1	1.00	0.56	0.57	1.06	1.87	0.00	1.06	1.26
time (sec)	N/A	0.079	0.201	0.458	0.223	0.244	0.000	0.766	4.256

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	34	36	67	112	0	67	114
N.S.	1	1.00	0.62	0.65	1.22	2.04	0.00	1.22	2.07
time (sec)	N/A	0.060	0.124	0.439	0.227	0.239	0.000	0.636	3.794

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	50	21	47	69	0	47	77
N.S.	1	1.00	1.85	0.78	1.74	2.56	0.00	1.74	2.85
time (sec)	N/A	0.053	0.097	0.424	0.208	0.235	0.000	0.596	4.449

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	48	68	45	113	0	128	41
N.S.	1	1.00	0.83	1.17	0.78	1.95	0.00	2.21	0.71
time (sec)	N/A	0.060	0.097	0.501	0.219	0.244	0.000	0.587	3.667

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	44	56	66	55	0	100	42
N.S.	1	1.00	0.88	1.12	1.32	1.10	0.00	2.00	0.84
time (sec)	N/A	0.062	0.084	0.438	0.220	0.252	0.000	0.654	3.927

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	24	24	21	30	153	57	20
N.S.	1	1.00	0.89	0.89	0.78	1.11	5.67	2.11	0.74
time (sec)	N/A	0.051	0.130	0.433	0.204	0.235	0.817	0.546	4.372

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	65	62	0	54	155	78	50
N.S.	1	1.00	0.74	0.70	0.00	0.61	1.76	0.89	0.57
time (sec)	N/A	0.062	0.161	0.296	0.000	0.236	0.172	0.386	3.999

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	137	97	0	76	224	115	124
N.S.	1	1.00	0.97	0.69	0.00	0.54	1.59	0.82	0.88
time (sec)	N/A	0.114	0.244	0.731	0.000	0.237	0.266	0.663	5.221

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	159	129	0	98	292	136	173
N.S.	1	1.00	0.82	0.66	0.00	0.50	1.50	0.70	0.89
time (sec)	N/A	0.146	0.365	0.758	0.000	0.234	0.312	0.679	5.634

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	113	122	341	278	0	164	150
N.S.	1	1.00	0.95	1.03	2.87	2.34	0.00	1.38	1.26
time (sec)	N/A	0.139	1.000	0.928	0.234	0.246	0.000	0.728	6.976

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	63	100	215	182	0	112	135
N.S.	1	1.00	0.68	1.08	2.31	1.96	0.00	1.20	1.45
time (sec)	N/A	0.132	0.828	0.853	0.317	0.236	0.000	0.616	6.066

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	108	86	319	112	0	110	105
N.S.	1	1.00	1.66	1.32	4.91	1.72	0.00	1.69	1.62
time (sec)	N/A	0.100	0.715	0.757	0.408	0.251	0.000	0.598	4.421

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	19	29	17	80	36	68
N.S.	1	1.00	1.00	0.59	0.91	0.53	2.50	1.12	2.12
time (sec)	N/A	0.047	0.360	0.794	0.305	0.237	0.797	0.560	3.925

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	54	56	69	41	219	73	133
N.S.	1	1.00	0.55	0.57	0.70	0.42	2.23	0.74	1.36
time (sec)	N/A	0.098	0.345	0.595	0.268	0.246	0.846	0.547	4.737

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	76	85	0	63	197	119	134
N.S.	1	1.00	0.75	0.84	0.00	0.62	1.95	1.18	1.33
time (sec)	N/A	0.109	0.416	0.663	0.000	0.226	0.304	0.637	6.588

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	98	120	0	85	265	171	188
N.S.	1	1.00	0.81	0.99	0.00	0.70	2.19	1.41	1.55
time (sec)	N/A	0.173	0.570	0.709	0.000	0.245	0.363	0.764	7.193

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	120	155	0	107	333	223	136
N.S.	1	1.00	0.86	1.12	0.00	0.77	2.40	1.60	0.98
time (sec)	N/A	0.142	1.070	0.715	0.000	0.230	0.450	0.699	6.768

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	44	47	97	168	0	97	120
N.S.	1	1.00	0.54	0.57	1.18	2.05	0.00	1.18	1.46
time (sec)	N/A	0.103	0.253	0.469	0.223	0.267	0.000	0.869	4.060

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	34	36	67	127	0	67	113
N.S.	1	1.00	0.62	0.65	1.22	2.31	0.00	1.22	2.05
time (sec)	N/A	0.087	0.166	0.456	0.250	0.243	0.000	0.799	4.040

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	58	20	55	84	0	55	93
N.S.	1	1.00	2.15	0.74	2.04	3.11	0.00	2.04	3.44
time (sec)	N/A	0.071	0.130	0.434	0.350	0.234	0.000	0.750	4.135

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	56	84	53	156	0	154	60
N.S.	1	1.00	0.62	0.93	0.59	1.73	0.00	1.71	0.67
time (sec)	N/A	0.105	0.113	0.529	0.284	0.251	0.000	0.727	3.761

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	52	69	95	102	0	146	55
N.S.	1	1.00	0.83	1.10	1.51	1.62	0.00	2.32	0.87
time (sec)	N/A	0.098	0.261	0.454	0.227	0.256	0.000	0.720	4.286

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	36	19	66	17	95	44	25
N.S.	1	1.00	1.24	0.66	2.28	0.59	3.28	1.52	0.86
time (sec)	N/A	0.075	0.054	0.449	0.216	0.235	1.166	0.616	3.915

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	22	24	21	41	272	85	19
N.S.	1	1.00	0.81	0.89	0.78	1.52	10.07	3.15	0.70
time (sec)	N/A	0.070	0.184	0.477	0.189	0.230	1.197	0.627	3.745

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	113	80	0	65	189	88	60
N.S.	1	1.00	0.97	0.69	0.00	0.56	1.63	0.76	0.52
time (sec)	N/A	0.104	0.222	0.338	0.000	0.236	0.243	0.407	4.521

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	142	115	0	87	258	123	90
N.S.	1	1.00	0.84	0.68	0.00	0.51	1.53	0.73	0.53
time (sec)	N/A	0.186	0.289	0.736	0.000	0.232	0.297	0.802	4.598

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	164	143	0	109	326	143	197
N.S.	1	1.00	0.73	0.64	0.00	0.49	1.46	0.64	0.88
time (sec)	N/A	0.212	0.502	0.651	0.000	0.240	0.370	0.808	6.070

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	237	111	295	230	0	151	197
N.S.	1	1.00	1.78	0.83	2.22	1.73	0.00	1.14	1.48
time (sec)	N/A	0.225	1.537	0.899	0.230	0.250	0.000	0.794	7.751

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	988	107	457	160	0	113	162
N.S.	1	1.00	9.23	1.00	4.27	1.50	0.00	1.06	1.51
time (sec)	N/A	0.178	6.644	0.881	0.319	0.247	0.000	0.770	6.256

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	247	71	141	76	0	71	88
N.S.	1	1.00	3.01	0.87	1.72	0.93	0.00	0.87	1.07
time (sec)	N/A	0.123	0.614	0.867	0.298	0.248	0.000	0.647	4.388

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	40	38	53	30	182	73	133
N.S.	1	1.00	0.59	0.56	0.78	0.44	2.68	1.07	1.96
time (sec)	N/A	0.106	0.384	0.816	0.214	0.235	1.148	0.644	4.411

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	73	74	91	52	354	99	64
N.S.	1	1.00	0.55	0.56	0.69	0.39	2.68	0.75	0.48
time (sec)	N/A	0.154	0.396	0.547	0.230	0.231	1.184	0.622	4.221

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	95	103	0	74	231	145	161
N.S.	1	1.00	0.71	0.77	0.00	0.55	1.72	1.08	1.20
time (sec)	N/A	0.154	0.425	0.707	0.000	0.235	0.324	0.807	8.141

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	117	138	0	96	299	197	216
N.S.	1	1.00	0.75	0.88	0.00	0.62	1.92	1.26	1.38
time (sec)	N/A	0.196	0.787	0.677	0.000	0.236	0.421	0.852	6.742

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	139	173	0	118	367	249	262
N.S.	1	1.00	0.80	0.99	0.00	0.68	2.11	1.43	1.51
time (sec)	N/A	0.202	1.235	0.752	0.000	0.240	0.504	0.777	7.931

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	98	124	229	273	0	250	105
N.S.	1	1.00	0.73	0.93	1.71	2.04	0.00	1.87	0.78
time (sec)	N/A	0.105	0.845	0.561	0.209	0.297	0.000	1.925	4.128

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	88	120	212	199	0	224	114
N.S.	1	1.00	0.70	0.95	1.68	1.58	0.00	1.78	0.90
time (sec)	N/A	0.106	0.836	0.515	0.222	0.268	0.000	1.982	3.813

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	79	108	191	124	0	199	104
N.S.	1	1.00	0.68	0.93	1.65	1.07	0.00	1.72	0.90
time (sec)	N/A	0.090	0.796	0.514	0.222	0.252	0.000	1.789	4.528

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	36	19	158	17	160	70	73
N.S.	1	1.00	0.84	0.44	3.67	0.40	3.72	1.63	1.70
time (sec)	N/A	0.055	0.053	0.497	0.229	0.242	8.749	1.574	3.969

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	44	50	141	41	466	137	85
N.S.	1	1.00	0.54	0.62	1.74	0.51	5.75	1.69	1.05
time (sec)	N/A	0.071	0.153	0.529	0.204	0.236	8.730	1.496	4.234

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	34	36	121	63	774	163	85
N.S.	1	1.00	0.62	0.65	2.20	1.15	14.07	2.96	1.55
time (sec)	N/A	0.063	0.125	0.457	0.204	0.236	8.873	1.309	4.345

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	22	24	21	85	1081	189	19
N.S.	1	1.00	0.81	0.89	0.78	3.15	40.04	7.00	0.70
time (sec)	N/A	0.048	0.206	0.415	0.206	0.232	8.885	1.206	4.007

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	152	152	0	109	325	128	198
N.S.	1	1.00	0.66	0.66	0.00	0.48	1.42	0.56	0.86
time (sec)	N/A	0.209	0.482	0.679	0.000	0.230	0.397	0.681	6.005

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	189	169	0	131	394	163	235
N.S.	1	1.00	0.68	0.61	0.00	0.47	1.42	0.59	0.85
time (sec)	N/A	0.227	0.813	0.643	0.000	0.234	0.475	1.280	6.250

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	333	208	197	0	153	462	184	294
N.S.	1	1.00	0.62	0.59	0.00	0.46	1.39	0.55	0.88
time (sec)	N/A	0.277	1.160	0.743	0.000	0.233	0.541	1.248	6.410

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	1704	147	786	267	0	195	344
N.S.	1	1.00	8.31	0.72	3.83	1.30	0.00	0.95	1.68
time (sec)	N/A	0.288	7.392	1.092	0.386	0.285	0.000	1.881	8.501

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	A	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	1244	143	531	182	0	165	284
N.S.	1	1.00	6.80	0.78	2.90	0.99	0.00	0.90	1.55
time (sec)	N/A	0.273	7.084	0.958	0.343	0.268	0.000	1.812	8.674

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	304	115	185	98	0	123	207
N.S.	1	1.00	1.95	0.74	1.19	0.63	0.00	0.79	1.33
time (sec)	N/A	0.253	1.492	0.947	0.323	0.265	0.000	1.644	8.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	40	38	53	30	311	125	37
N.S.	1	1.00	0.59	0.56	0.78	0.44	4.57	1.84	0.54
time (sec)	N/A	0.109	0.513	0.903	0.226	0.241	8.971	1.488	4.216

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	73	74	97	52	620	151	64
N.S.	1	1.00	0.53	0.54	0.70	0.38	4.49	1.09	0.46
time (sec)	N/A	0.235	0.857	0.974	0.221	0.237	8.789	1.422	4.381

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	95	110	141	74	928	177	159
N.S.	1	1.00	0.45	0.52	0.66	0.35	4.36	0.83	0.75
time (sec)	N/A	0.360	0.751	0.779	0.287	0.234	8.743	1.272	5.170

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	117	146	179	96	1221	203	224
N.S.	1	1.00	0.43	0.54	0.67	0.36	4.54	0.75	0.83
time (sec)	N/A	0.378	0.862	0.494	0.276	0.232	8.856	1.167	6.148

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	139	175	0	118	367	249	262
N.S.	1	1.00	0.51	0.65	0.00	0.44	1.35	0.92	0.97
time (sec)	N/A	0.418	1.314	0.732	0.000	0.233	0.579	1.256	7.471

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	161	210	0	140	435	301	308
N.S.	1	1.00	0.53	0.70	0.00	0.47	1.45	1.00	1.02
time (sec)	N/A	0.465	1.631	0.770	0.000	0.236	0.664	1.302	10.461

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	156	434	0	208	0	0	0
N.S.	1	1.00	1.27	3.53	0.00	1.69	0.00	0.00	0.00
time (sec)	N/A	0.130	2.223	27.132	0.000	0.084	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	57	163	0	154	0	0	0
N.S.	1	1.00	0.61	1.73	0.00	1.64	0.00	0.00	0.00
time (sec)	N/A	0.090	0.839	24.867	0.000	0.076	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	102	411	0	116	0	0	0
N.S.	1	1.00	1.13	4.57	0.00	1.29	0.00	0.00	0.00
time (sec)	N/A	0.087	1.013	3.329	0.000	0.073	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	44	96	0	59	0	0	40
N.S.	1	1.00	0.73	1.60	0.00	0.98	0.00	0.00	0.67
time (sec)	N/A	0.075	0.588	7.391	0.000	0.067	0.000	0.000	4.516

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	73	301	0	27	0	0	0
N.S.	1	1.00	1.22	5.02	0.00	0.45	0.00	0.00	0.00
time (sec)	N/A	0.065	0.699	6.644	0.000	0.073	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	62	152	0	76	0	0	0
N.S.	1	1.00	0.65	1.58	0.00	0.79	0.00	0.00	0.00
time (sec)	N/A	0.108	0.790	6.753	0.000	0.074	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	99	320	0	109	0	0	0
N.S.	1	1.00	1.03	3.33	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	0.114	1.129	6.186	0.000	0.075	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	121	179	0	118	0	0	0
N.S.	1	1.00	0.97	1.43	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.158	1.150	5.825	0.000	0.074	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	267	430	0	166	0	0	0
N.S.	1	1.00	1.93	3.12	0.00	1.20	0.00	0.00	0.00
time (sec)	N/A	0.192	3.301	14.594	0.000	0.076	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	67	144	0	108	0	0	0
N.S.	1	1.00	0.63	1.36	0.00	1.02	0.00	0.00	0.00
time (sec)	N/A	0.164	1.605	10.731	0.000	0.073	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	132	807	0	69	0	0	0
N.S.	1	1.00	1.23	7.54	0.00	0.64	0.00	0.00	0.00
time (sec)	N/A	0.135	2.182	10.343	0.000	0.076	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	114	156	0	82	0	0	0
N.S.	1	1.00	1.34	1.84	0.00	0.96	0.00	0.00	0.00
time (sec)	N/A	0.105	1.532	9.085	0.000	0.077	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	114	324	0	94	0	0	0
N.S.	1	1.00	1.34	3.81	0.00	1.11	0.00	0.00	0.00
time (sec)	N/A	0.095	1.986	13.223	0.000	0.074	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	133	172	0	96	0	0	0
N.S.	1	1.00	1.15	1.48	0.00	0.83	0.00	0.00	0.00
time (sec)	N/A	0.121	1.836	11.330	0.000	0.077	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	133	337	0	131	0	0	0
N.S.	1	1.00	1.15	2.91	0.00	1.13	0.00	0.00	0.00
time (sec)	N/A	0.113	2.563	19.859	0.000	0.078	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	155	190	0	142	0	0	0
N.S.	1	1.00	1.05	1.29	0.00	0.97	0.00	0.00	0.00
time (sec)	N/A	0.140	2.258	20.160	0.000	0.085	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	161	488	0	316	0	0	0
N.S.	1	1.00	0.80	2.42	0.00	1.56	0.00	0.00	0.00
time (sec)	N/A	0.271	4.837	11.294	0.000	0.076	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	89	184	0	258	0	0	0
N.S.	1	1.00	0.51	1.05	0.00	1.47	0.00	0.00	0.00
time (sec)	N/A	0.250	2.745	7.500	0.000	0.076	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	129	452	0	208	0	0	0
N.S.	1	1.00	0.74	2.58	0.00	1.19	0.00	0.00	0.00
time (sec)	N/A	0.229	3.367	16.207	0.000	0.076	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	79	157	0	148	0	0	0
N.S.	1	1.00	0.57	1.13	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.188	2.368	14.257	0.000	0.075	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	146	101	1114	0	123	0	0	0
N.S.	1	1.18	0.81	8.98	0.00	0.99	0.00	0.00	0.00
time (sec)	N/A	0.175	2.671	12.234	0.000	0.084	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	123	165	0	82	0	0	0
N.S.	1	1.00	1.11	1.49	0.00	0.74	0.00	0.00	0.00
time (sec)	N/A	0.120	1.971	12.282	0.000	0.071	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	108	324	0	94	0	0	0
N.S.	1	1.00	0.97	2.92	0.00	0.85	0.00	0.00	0.00
time (sec)	N/A	0.135	2.403	14.919	0.000	0.075	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	133	181	0	96	0	0	0
N.S.	1	1.00	1.07	1.46	0.00	0.77	0.00	0.00	0.00
time (sec)	N/A	0.162	1.867	13.451	0.000	0.074	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	118	339	0	108	0	0	0
N.S.	1	1.00	0.95	2.73	0.00	0.87	0.00	0.00	0.00
time (sec)	N/A	0.169	2.949	14.156	0.000	0.079	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	148	201	0	110	0	0	0
N.S.	1	1.00	0.95	1.30	0.00	0.71	0.00	0.00	0.00
time (sec)	N/A	0.208	2.106	20.099	0.000	0.076	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	145	352	0	145	0	0	0
N.S.	1	1.00	0.94	2.27	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.193	4.309	25.767	0.000	0.086	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	170	217	0	156	0	0	0
N.S.	1	1.00	0.91	1.17	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.244	2.588	24.652	0.000	0.090	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	327	480	0	250	0	0	0
N.S.	1	1.00	1.52	2.23	0.00	1.16	0.00	0.00	0.00
time (sec)	N/A	0.312	7.208	24.515	0.000	0.077	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	101	172	0	188	0	0	0
N.S.	1	1.00	0.55	0.94	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.260	2.935	19.377	0.000	0.072	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	178	178	123	1378	0	164	0	0	0
N.S.	1	1.00	0.69	7.74	0.00	0.92	0.00	0.00	0.00
time (sec)	N/A	0.237	5.107	19.089	0.000	0.082	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	130	179	0	129	0	0	0
N.S.	1	1.00	0.89	1.23	0.00	0.88	0.00	0.00	0.00
time (sec)	N/A	0.195	2.973	17.233	0.000	0.076	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	110	1721	0	94	0	0	0
N.S.	1	1.00	0.71	11.03	0.00	0.60	0.00	0.00	0.00
time (sec)	N/A	0.190	4.006	22.493	0.000	0.078	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	133	183	0	96	0	0	0
N.S.	1	1.00	1.06	1.46	0.00	0.77	0.00	0.00	0.00
time (sec)	N/A	0.160	2.711	14.523	0.000	0.079	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	108	339	0	108	0	0	0
N.S.	1	1.00	0.86	2.71	0.00	0.86	0.00	0.00	0.00
time (sec)	N/A	0.170	4.183	25.448	0.000	0.077	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	148	201	0	110	0	0	0
N.S.	1	1.00	0.95	1.29	0.00	0.71	0.00	0.00	0.00
time (sec)	N/A	0.236	2.497	22.542	0.000	0.076	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	121	352	0	122	0	0	0
N.S.	1	1.00	0.78	2.26	0.00	0.78	0.00	0.00	0.00
time (sec)	N/A	0.252	8.208	26.291	0.000	0.087	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	155	217	0	124	0	0	0
N.S.	1	1.00	0.83	1.16	0.00	0.66	0.00	0.00	0.00
time (sec)	N/A	0.265	3.189	33.738	0.000	0.084	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	128	443	0	205	0	0	0
N.S.	1	1.00	0.94	3.26	0.00	1.51	0.00	0.00	0.00
time (sec)	N/A	0.194	2.233	8.742	0.000	0.078	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	62	158	0	152	0	0	0
N.S.	1	1.00	0.59	1.50	0.00	1.45	0.00	0.00	0.00
time (sec)	N/A	0.180	1.579	7.632	0.000	0.074	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	102	410	0	123	0	0	0
N.S.	1	1.00	1.01	4.06	0.00	1.22	0.00	0.00	0.00
time (sec)	N/A	0.123	1.748	6.543	0.000	0.070	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	49	137	0	63	0	0	0
N.S.	1	1.00	0.70	1.96	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	0.102	1.358	6.628	0.000	0.072	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	70	70	74	780	0	96	0	0	0
N.S.	1	1.00	1.06	11.14	0.00	1.37	0.00	0.00	0.00
time (sec)	N/A	0.093	1.369	4.506	0.000	0.072	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	83	160	0	90	0	0	0
N.S.	1	1.00	1.04	2.00	0.00	1.12	0.00	0.00	0.00
time (sec)	N/A	0.094	1.090	6.354	0.000	0.071	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	109	440	0	107	0	0	0
N.S.	1	1.00	1.36	5.50	0.00	1.34	0.00	0.00	0.00
time (sec)	N/A	0.106	1.568	7.539	0.000	0.075	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	125	174	0	115	0	0	0
N.S.	1	1.00	1.10	1.53	0.00	1.01	0.00	0.00	0.00
time (sec)	N/A	0.136	1.611	8.862	0.000	0.083	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	134	477	0	129	0	0	0
N.S.	1	1.00	1.18	4.18	0.00	1.13	0.00	0.00	0.00
time (sec)	N/A	0.125	2.043	10.559	0.000	0.084	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	142	190	0	137	0	0	0
N.S.	1	1.00	0.98	1.31	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.149	1.842	8.916	0.000	0.089	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	302	473	0	256	0	0	0
N.S.	1	1.00	1.65	2.58	0.00	1.40	0.00	0.00	0.00
time (sec)	N/A	0.171	2.881	9.205	0.000	0.081	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	85	174	0	201	0	0	0
N.S.	1	1.00	0.56	1.14	0.00	1.32	0.00	0.00	0.00
time (sec)	N/A	0.146	1.560	8.128	0.000	0.078	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	123	432	0	170	0	0	0
N.S.	1	1.00	0.81	2.84	0.00	1.12	0.00	0.00	0.00
time (sec)	N/A	0.154	1.899	7.878	0.000	0.081	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	67	149	0	115	0	0	0
N.S.	1	1.00	0.56	1.25	0.00	0.97	0.00	0.00	0.00
time (sec)	N/A	0.131	1.384	6.863	0.000	0.073	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	80	431	0	100	0	0	0
N.S.	1	1.00	0.70	3.75	0.00	0.87	0.00	0.00	0.00
time (sec)	N/A	0.114	1.490	8.129	0.000	0.078	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	101	164	0	97	0	0	0
N.S.	1	1.00	1.12	1.82	0.00	1.08	0.00	0.00	0.00
time (sec)	N/A	0.104	1.407	7.510	0.000	0.073	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	102	451	0	108	0	0	0
N.S.	1	1.00	1.13	5.01	0.00	1.20	0.00	0.00	0.00
time (sec)	N/A	0.096	1.821	7.070	0.000	0.072	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	112	177	0	101	0	0	0
N.S.	1	1.00	0.97	1.53	0.00	0.87	0.00	0.00	0.00
time (sec)	N/A	0.120	1.480	5.155	0.000	0.070	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	123	474	0	118	0	0	0
N.S.	1	1.00	1.06	4.09	0.00	1.02	0.00	0.00	0.00
time (sec)	N/A	0.106	2.395	8.478	0.000	0.081	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	134	190	0	126	0	0	0
N.S.	1	1.00	0.89	1.27	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.146	1.543	8.663	0.000	0.080	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	149	511	0	140	0	0	0
N.S.	1	1.00	0.99	3.41	0.00	0.93	0.00	0.00	0.00
time (sec)	N/A	0.149	2.823	11.155	0.000	0.095	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	151	206	0	148	0	0	0
N.S.	1	1.00	0.83	1.14	0.00	0.82	0.00	0.00	0.00
time (sec)	N/A	0.176	1.904	9.955	0.000	0.091	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	128	454	0	213	0	0	0
N.S.	1	1.00	0.72	2.55	0.00	1.20	0.00	0.00	0.00
time (sec)	N/A	0.215	2.328	9.276	0.000	0.078	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	74	160	0	158	0	0	0
N.S.	1	1.00	0.52	1.13	0.00	1.12	0.00	0.00	0.00
time (sec)	N/A	0.189	1.639	8.774	0.000	0.076	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	93	444	0	150	0	0	0
N.S.	1	1.00	0.66	3.15	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.196	1.850	10.353	0.000	0.078	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	125	117	0	97	0	0	0
N.S.	1	1.00	1.08	1.01	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.171	1.448	9.267	0.000	0.074	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	117	473	0	114	0	0	0
N.S.	1	1.00	1.01	4.08	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.169	1.636	8.082	0.000	0.086	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	104	191	0	111	0	0	0
N.S.	1	1.00	0.79	1.45	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.162	1.570	6.607	0.000	0.073	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	140	507	0	120	0	0	0
N.S.	1	1.00	1.06	3.84	0.00	0.91	0.00	0.00	0.00
time (sec)	N/A	0.224	1.742	7.500	0.000	0.080	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	129	142	0	112	0	0	0
N.S.	1	1.00	0.85	0.93	0.00	0.74	0.00	0.00	0.00
time (sec)	N/A	0.259	1.378	6.304	0.000	0.075	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	145	528	0	129	0	0	0
N.S.	1	1.00	0.95	3.47	0.00	0.85	0.00	0.00	0.00
time (sec)	N/A	0.232	2.182	8.695	0.000	0.085	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	151	217	0	137	0	0	0
N.S.	1	1.00	0.81	1.17	0.00	0.74	0.00	0.00	0.00
time (sec)	N/A	0.333	1.670	9.313	0.000	0.092	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	124	466	0	193	0	0	0
N.S.	1	1.00	0.65	2.43	0.00	1.01	0.00	0.00	0.00
time (sec)	N/A	0.253	2.169	11.665	0.000	0.083	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	134	172	0	147	0	0	0
N.S.	1	1.00	0.85	1.10	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.220	1.559	10.102	0.000	0.075	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	106	482	0	114	0	0	0
N.S.	1	1.00	0.65	2.96	0.00	0.70	0.00	0.00	0.00
time (sec)	N/A	0.216	1.594	9.680	0.000	0.077	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	137	191	0	111	0	0	0
N.S.	1	1.00	1.04	1.45	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.191	1.495	7.632	0.000	0.076	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	149	509	0	128	0	0	0
N.S.	1	1.00	1.13	3.86	0.00	0.97	0.00	0.00	0.00
time (sec)	N/A	0.175	1.719	8.038	0.000	0.081	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	144	143	0	125	0	0	0
N.S.	1	1.00	0.88	0.88	0.00	0.77	0.00	0.00	0.00
time (sec)	N/A	0.192	1.614	6.852	0.000	0.086	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	142	541	0	132	0	0	0
N.S.	1	1.00	0.87	3.32	0.00	0.81	0.00	0.00	0.00
time (sec)	N/A	0.209	2.183	8.175	0.000	0.077	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	137	152	0	123	0	0	0
N.S.	1	1.00	0.72	0.80	0.00	0.64	0.00	0.00	0.00
time (sec)	N/A	0.248	1.328	6.550	0.000	0.076	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	71	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.198	0.280	0.000	0.000	0.000	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	61	0	0	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.198	0.177	0.000	0.000	0.000	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	61	0	0	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.197	0.150	0.000	0.000	0.000	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	63	0	0	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.197	0.237	0.000	0.000	0.000	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	111	0	0	0	0	0	0
N.S.	1	1.00	1.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.229	0.472	0.000	0.000	0.000	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	102	0	0	0	0	0	0
N.S.	1	1.00	1.48	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.203	0.375	0.000	0.000	0.000	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	103	0	0	0	0	0	0
N.S.	1	1.00	1.24	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.203	0.355	0.000	0.000	0.000	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	105	0	0	0	0	0	0
N.S.	1	1.00	1.24	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.216	0.514	0.000	0.000	0.000	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	84	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.221	1.498	0.000	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	103	0	0	0	0	0	0
N.S.	1	1.00	1.27	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.216	1.345	0.000	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	112	0	0	0	0	0	0
N.S.	1	1.00	1.58	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.231	1.735	0.000	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	119	0	0	0	0	0	0
N.S.	1	1.00	1.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.253	1.895	0.000	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	87	87	128	0	0	0	0	0	0
N.S.	1	1.00	1.47	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.302	1.620	0.000	0.000	0.000	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	87	87	121	0	0	0	0	0	0
N.S.	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.303	1.410	0.000	0.000	0.000	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	141	0	0	0	0	0	0
N.S.	1	1.00	1.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.307	2.164	0.000	0.000	0.000	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	143	0	0	0	0	0	0
N.S.	1	1.00	2.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.331	1.822	0.000	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	70	82	76	154	0	0	474
N.S.	1	1.00	0.60	0.70	0.65	1.32	0.00	0.00	4.05
time (sec)	N/A	0.109	0.326	2.157	0.293	0.251	0.000	0.000	13.139

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	58	63	58	119	0	0	352
N.S.	1	1.00	0.66	0.72	0.66	1.35	0.00	0.00	4.00
time (sec)	N/A	0.087	0.203	1.201	0.311	0.249	0.000	0.000	7.763

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	48	44	40	84	0	0	230
N.S.	1	1.00	0.81	0.75	0.68	1.42	0.00	0.00	3.90
time (sec)	N/A	0.082	0.136	1.040	0.273	0.253	0.000	0.000	7.365

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	21	46	0	55	82
N.S.	1	1.00	1.00	0.83	0.72	1.59	0.00	1.90	2.83
time (sec)	N/A	0.072	0.070	0.661	0.222	0.240	0.000	0.522	0.641

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	51	384	122	253	0	0	0
N.S.	1	1.00	0.42	3.20	1.02	2.11	0.00	0.00	0.00
time (sec)	N/A	0.122	0.105	38.564	0.307	0.245	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	53	411	176	275	0	0	0
N.S.	1	1.00	0.27	2.13	0.91	1.42	0.00	0.00	0.00
time (sec)	N/A	0.146	0.127	109.911	0.341	0.252	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	53	438	230	297	0	0	0
N.S.	1	1.00	0.20	1.65	0.86	1.12	0.00	0.00	0.00
time (sec)	N/A	0.198	0.235	108.030	0.303	0.249	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	95	116	0	132	0	0	289
N.S.	1	1.00	0.65	0.79	0.00	0.90	0.00	0.00	1.97
time (sec)	N/A	0.344	0.976	8.633	0.000	0.281	0.000	0.000	9.182

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	77	89	0	97	0	0	102
N.S.	1	1.00	0.70	0.81	0.00	0.88	0.00	0.00	0.93
time (sec)	N/A	0.236	0.674	7.643	0.000	0.253	0.000	0.000	6.945

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	63	62	222	62	0	0	88
N.S.	1	1.00	0.86	0.85	3.04	0.85	0.00	0.00	1.21
time (sec)	N/A	0.150	0.526	7.947	21.457	0.236	0.000	0.000	6.646

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	39	37	0	25	0	0	61
N.S.	1	1.00	1.26	1.19	0.00	0.81	0.00	0.00	1.97
time (sec)	N/A	0.039	0.289	5.633	0.000	0.234	0.000	0.000	0.379

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	87	364	774	184	0	0	0
N.S.	1	1.00	1.05	4.39	9.33	2.22	0.00	0.00	0.00
time (sec)	N/A	0.115	0.684	17.334	0.413	0.254	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	126	394	935	245	0	0	0
N.S.	1	1.00	0.82	2.56	6.07	1.59	0.00	0.00	0.00
time (sec)	N/A	0.256	0.718	23.321	0.477	0.275	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	152	421	2220	267	0	0	0
N.S.	1	1.00	0.68	1.89	9.96	1.20	0.00	0.00	0.00
time (sec)	N/A	0.423	0.981	22.484	0.616	0.260	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	73	82	76	170	0	0	544
N.S.	1	1.00	0.62	0.70	0.65	1.45	0.00	0.00	4.65
time (sec)	N/A	0.098	0.429	1.248	0.243	0.283	0.000	0.000	17.489

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	61	63	58	134	0	0	420
N.S.	1	1.00	0.69	0.72	0.66	1.52	0.00	0.00	4.77
time (sec)	N/A	0.088	0.215	1.222	0.249	0.262	0.000	0.000	8.311

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	51	44	40	98	0	0	296
N.S.	1	1.00	0.86	0.75	0.68	1.66	0.00	0.00	5.02
time (sec)	N/A	0.086	0.160	1.230	0.237	0.249	0.000	0.000	7.041

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	21	59	0	0	153
N.S.	1	1.00	1.00	0.83	0.72	2.03	0.00	0.00	5.28
time (sec)	N/A	0.077	0.092	0.998	0.212	0.244	0.000	0.000	1.524

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	92	576	98	244	0	0	0
N.S.	1	1.00	0.99	6.19	1.05	2.62	0.00	0.00	0.00
time (sec)	N/A	0.111	0.170	15.406	0.311	0.239	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	53	659	158	287	0	0	0
N.S.	1	1.00	0.32	3.97	0.95	1.73	0.00	0.00	0.00
time (sec)	N/A	0.138	0.114	14.370	0.314	0.257	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	53	686	212	311	0	0	0
N.S.	1	1.00	0.22	2.87	0.89	1.30	0.00	0.00	0.00
time (sec)	N/A	0.162	0.153	14.053	0.324	0.269	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	109	169	994	125	0	0	293
N.S.	1	1.00	0.74	1.15	6.76	0.85	0.00	0.00	1.99
time (sec)	N/A	0.293	1.260	7.158	9.667	0.267	0.000	0.000	8.402

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	91	123	580	89	0	0	103
N.S.	1	1.00	0.83	1.12	5.27	0.81	0.00	0.00	0.94
time (sec)	N/A	0.217	0.782	7.283	0.610	0.252	0.000	0.000	6.534

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	57	80	0	53	0	0	98
N.S.	1	1.00	0.83	1.16	0.00	0.77	0.00	0.00	1.42
time (sec)	N/A	0.085	0.431	6.757	0.000	0.258	0.000	0.000	5.445

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	47	201	40	0	0	60
N.S.	1	1.00	1.00	1.52	6.48	1.29	0.00	0.00	1.94
time (sec)	N/A	0.063	0.336	15.156	0.396	0.256	0.000	0.000	0.250

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	101	648	884	222	0	0	0
N.S.	1	1.00	0.83	5.31	7.25	1.82	0.00	0.00	0.00
time (sec)	N/A	0.194	0.997	11.931	0.825	0.249	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	160	676	0	274	0	0	0
N.S.	1	1.00	0.83	3.52	0.00	1.43	0.00	0.00	0.00
time (sec)	N/A	0.339	1.608	15.235	0.000	0.260	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	73	82	76	190	0	0	626
N.S.	1	1.00	0.62	0.70	0.65	1.62	0.00	0.00	5.35
time (sec)	N/A	0.117	0.519	1.924	0.216	0.286	0.000	0.000	16.993

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	61	63	58	152	0	0	498
N.S.	1	1.00	0.69	0.72	0.66	1.73	0.00	0.00	5.66
time (sec)	N/A	0.097	0.314	0.160	0.213	0.252	0.000	0.000	12.640

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	51	44	40	114	0	0	370
N.S.	1	1.00	0.86	0.75	0.68	1.93	0.00	0.00	6.27
time (sec)	N/A	0.090	0.260	185.671	0.239	0.251	0.000	0.000	7.129

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	21	73	0	0	242
N.S.	1	1.00	1.00	0.83	0.72	2.52	0.00	0.00	8.34
time (sec)	N/A	0.076	0.133	4.082	0.224	0.254	0.000	0.000	7.426

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	83	603	98	236	0	0	0
N.S.	1	1.00	0.93	6.78	1.10	2.65	0.00	0.00	0.00
time (sec)	N/A	0.109	0.460	18.585	0.321	0.253	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	53	846	140	263	0	0	0
N.S.	1	1.00	0.39	6.18	1.02	1.92	0.00	0.00	0.00
time (sec)	N/A	0.127	0.087	123.884	0.318	0.248	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	53	933	194	309	0	0	0
N.S.	1	1.00	0.25	4.44	0.92	1.47	0.00	0.00	0.00
time (sec)	N/A	0.163	0.122	5.970	0.318	0.264	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	103	224	624	121	0	0	301
N.S.	1	1.00	0.70	1.52	4.24	0.82	0.00	0.00	2.05
time (sec)	N/A	0.317	1.209	41.252	243.694	0.261	0.000	0.000	8.781

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	93	182	0	83	0	0	105
N.S.	1	1.00	0.89	1.75	0.00	0.80	0.00	0.00	1.01
time (sec)	N/A	0.134	0.591	7.418	0.000	0.246	0.000	0.000	7.168

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	46	88	331	45	0	0	64
N.S.	1	1.00	0.71	1.35	5.09	0.69	0.00	0.00	0.98
time (sec)	N/A	0.118	0.534	24.898	0.410	0.231	0.000	0.000	0.445

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	69	66	328	59	0	0	89
N.S.	1	1.00	1.97	1.89	9.37	1.69	0.00	0.00	2.54
time (sec)	N/A	0.073	0.779	35.408	0.460	0.243	0.000	0.000	0.968

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	118	916	1076	244	0	0	0
N.S.	1	1.00	0.74	5.76	6.77	1.53	0.00	0.00	0.00
time (sec)	N/A	0.268	1.414	3.305	0.467	0.268	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	155	945	0	300	0	0	0
N.S.	1	1.00	0.67	4.09	0.00	1.30	0.00	0.00	0.00
time (sec)	N/A	0.439	1.849	3.719	0.000	0.266	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	73	82	76	202	0	0	690
N.S.	1	1.00	0.62	0.70	0.65	1.73	0.00	0.00	5.90
time (sec)	N/A	0.107	0.470	0.609	0.218	0.296	0.000	0.000	16.013

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	63	63	58	164	0	0	562
N.S.	1	1.00	0.72	0.72	0.66	1.86	0.00	0.00	6.39
time (sec)	N/A	0.095	0.431	0.161	0.214	0.272	0.000	0.000	17.405

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	51	44	40	126	0	0	434
N.S.	1	1.00	0.86	0.75	0.68	2.14	0.00	0.00	7.36
time (sec)	N/A	0.087	0.209	187.494	0.207	0.259	0.000	0.000	8.677

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	21	85	0	0	306
N.S.	1	1.00	1.00	0.83	0.72	2.93	0.00	0.00	10.55
time (sec)	N/A	0.085	0.176	4.134	0.364	0.253	0.000	0.000	6.924

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	51	641	117	235	0	0	0
N.S.	1	1.00	0.44	5.53	1.01	2.03	0.00	0.00	0.00
time (sec)	N/A	0.121	0.119	38.355	0.449	0.256	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	53	869	138	261	0	0	0
N.S.	1	1.00	0.39	6.34	1.01	1.91	0.00	0.00	0.00
time (sec)	N/A	0.120	0.114	123.981	0.472	0.243	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	53	1136	176	277	0	0	0
N.S.	1	1.00	0.29	6.28	0.97	1.53	0.00	0.00	0.00
time (sec)	N/A	0.142	0.117	4.652	0.298	0.254	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	109	235	0	109	0	0	286
N.S.	1	1.00	0.78	1.69	0.00	0.78	0.00	0.00	2.06
time (sec)	N/A	0.173	1.069	7.877	0.000	0.267	0.000	0.000	6.743

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	59	122	418	71	0	0	102
N.S.	1	1.00	0.57	1.17	4.02	0.68	0.00	0.00	0.98
time (sec)	N/A	0.190	0.809	29.973	0.401	0.258	0.000	0.000	5.741

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	86	88	504	59	0	0	85
N.S.	1	1.00	1.21	1.24	7.10	0.83	0.00	0.00	1.20
time (sec)	N/A	0.157	0.965	34.537	0.397	0.264	0.000	0.000	0.935

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	73	63	454	73	0	0	112
N.S.	1	1.00	2.09	1.80	12.97	2.09	0.00	0.00	3.20
time (sec)	N/A	0.081	1.309	4.059	0.694	0.258	0.000	0.000	5.756

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	131	1185	1253	258	0	0	0
N.S.	1	1.00	0.67	6.05	6.39	1.32	0.00	0.00	0.00
time (sec)	N/A	0.370	2.410	3.878	0.704	0.256	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	188	1211	0	314	0	0	0
N.S.	1	1.00	0.70	4.52	0.00	1.17	0.00	0.00	0.00
time (sec)	N/A	0.522	3.649	4.768	0.000	0.290	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	342	342	194	1238	0	342	0	0	0
N.S.	1	1.00	0.57	3.62	0.00	1.00	0.00	0.00	0.00
time (sec)	N/A	0.683	6.019	6.019	0.000	0.306	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	73	82	297	150	0	0	434
N.S.	1	1.00	0.62	0.70	2.54	1.28	0.00	0.00	3.71
time (sec)	N/A	0.101	0.260	1.384	0.245	0.261	0.000	0.000	9.736

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	63	63	169	113	0	0	306
N.S.	1	1.00	0.72	0.72	1.92	1.28	0.00	0.00	3.48
time (sec)	N/A	0.092	0.177	1.346	0.238	0.264	0.000	0.000	7.157

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	49	44	79	76	0	0	155
N.S.	1	1.00	0.83	0.75	1.34	1.29	0.00	0.00	2.63
time (sec)	N/A	0.086	0.118	1.118	0.229	0.239	0.000	0.000	1.432

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	21	37	0	55	47
N.S.	1	1.00	1.00	0.89	0.78	1.37	0.00	2.04	1.74
time (sec)	N/A	0.071	0.074	0.770	0.227	0.235	0.000	0.598	0.168

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	51	406	138	271	0	0	0
N.S.	1	1.00	0.35	2.78	0.95	1.86	0.00	0.00	0.00
time (sec)	N/A	0.126	0.135	10.673	0.318	0.253	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	53	540	192	293	0	0	0
N.S.	1	1.00	0.24	2.47	0.88	1.34	0.00	0.00	0.00
time (sec)	N/A	0.149	0.198	11.100	0.321	0.255	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	53	674	246	315	0	0	0
N.S.	1	1.00	0.18	2.31	0.84	1.08	0.00	0.00	0.00
time (sec)	N/A	0.195	0.361	9.443	0.326	0.253	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	95	126	608	153	0	0	301
N.S.	1	1.00	0.65	0.86	4.14	1.04	0.00	0.00	2.05
time (sec)	N/A	0.319	1.200	9.328	0.426	0.293	0.000	0.000	10.282

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	77	99	474	116	0	0	105
N.S.	1	1.00	0.70	0.90	4.31	1.05	0.00	0.00	0.95
time (sec)	N/A	0.230	0.860	6.937	0.383	0.256	0.000	0.000	6.625

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	65	72	340	79	0	0	91
N.S.	1	1.00	0.89	0.99	4.66	1.08	0.00	0.00	1.25
time (sec)	N/A	0.154	0.613	7.516	0.352	0.250	0.000	0.000	10.144

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	40	44	206	40	0	0	98
N.S.	1	1.00	1.14	1.26	5.89	1.14	0.00	0.00	2.80
time (sec)	N/A	0.073	0.439	7.635	0.326	0.248	0.000	0.000	1.111

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	70	122	0	149	0	0	0
N.S.	1	1.00	1.35	2.35	0.00	2.87	0.00	0.00	0.00
time (sec)	N/A	0.049	0.449	7.038	0.000	0.266	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	96	350	837	245	0	0	0
N.S.	1	1.00	0.79	2.87	6.86	2.01	0.00	0.00	0.00
time (sec)	N/A	0.182	0.602	9.742	0.439	0.251	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	117	484	1938	267	0	0	0
N.S.	1	1.00	0.61	2.51	10.04	1.38	0.00	0.00	0.00
time (sec)	N/A	0.322	0.897	11.700	0.517	0.252	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	73	82	76	149	0	0	370
N.S.	1	1.00	0.62	0.70	0.65	1.27	0.00	0.00	3.16
time (sec)	N/A	0.109	0.405	1.369	0.220	0.253	0.000	0.000	8.730

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	59	63	58	108	0	0	242
N.S.	1	1.00	0.67	0.72	0.66	1.23	0.00	0.00	2.75
time (sec)	N/A	0.104	0.180	1.422	0.222	0.249	0.000	0.000	7.568

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	37	44	38	67	0	0	85
N.S.	1	1.00	0.65	0.77	0.67	1.18	0.00	0.00	1.49
time (sec)	N/A	0.102	0.113	1.003	0.226	0.238	0.000	0.000	4.707

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	21	49	0	0	67
N.S.	1	1.00	1.00	0.89	0.78	1.81	0.00	0.00	2.48
time (sec)	N/A	0.093	0.055	0.992	0.231	0.234	0.000	0.000	0.286

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	51	595	153	294	0	0	0
N.S.	1	1.00	0.29	3.40	0.87	1.68	0.00	0.00	0.00
time (sec)	N/A	0.171	0.190	10.010	0.315	0.265	0.000	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	53	729	207	316	0	0	0
N.S.	1	1.00	0.21	2.94	0.83	1.27	0.00	0.00	0.00
time (sec)	N/A	0.181	0.344	8.622	0.311	0.251	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	321	53	811	261	338	0	0	0
N.S.	1	1.00	0.17	2.53	0.81	1.05	0.00	0.00	0.00
time (sec)	N/A	0.237	0.527	9.310	0.320	0.257	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	147	147	108	0	764	184	0	0	301
N.S.	1	1.00	0.73	0.00	5.20	1.25	0.00	0.00	2.05
time (sec)	N/A	0.326	1.899	0.000	0.479	0.323	0.000	0.000	11.256

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	B	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	110	110	92	0	626	143	0	0	105
N.S.	1	1.00	0.84	0.00	5.69	1.30	0.00	0.00	0.95
time (sec)	N/A	0.227	1.501	0.000	0.426	0.281	0.000	0.000	9.548

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	B	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	80	0	488	102	0	0	91
N.S.	1	1.00	1.10	0.00	6.68	1.40	0.00	0.00	1.25
time (sec)	N/A	0.159	1.101	0.000	0.373	0.285	0.000	0.000	7.741

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	B	B	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	35	35	59	0	350	59	0	0	139
N.S.	1	1.00	1.69	0.00	10.00	1.69	0.00	0.00	3.97
time (sec)	N/A	0.076	0.842	0.000	0.344	0.251	0.000	0.000	1.851

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	101	223	813	196	0	0	0
N.S.	1	1.00	1.17	2.59	9.45	2.28	0.00	0.00	0.00
time (sec)	N/A	0.127	1.062	8.938	0.447	0.244	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	95	416	0	246	0	0	0
N.S.	1	1.00	1.09	4.78	0.00	2.83	0.00	0.00	0.00
time (sec)	N/A	0.090	0.784	8.674	0.000	0.259	0.000	0.000	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	120	539	1821	270	0	0	0
N.S.	1	1.00	0.76	3.43	11.60	1.72	0.00	0.00	0.00
time (sec)	N/A	0.252	1.115	9.948	0.549	0.245	0.000	0.000	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	145	673	2632	292	0	0	0
N.S.	1	1.00	0.62	2.89	11.30	1.25	0.00	0.00	0.00
time (sec)	N/A	0.439	1.842	10.019	0.525	0.274	0.000	0.000	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	85	101	94	175	0	0	434
N.S.	1	1.00	0.58	0.69	0.64	1.20	0.00	0.00	2.97
time (sec)	N/A	0.123	0.396	1.286	0.235	0.272	0.000	0.000	10.150

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	73	82	76	134	0	0	306
N.S.	1	1.00	0.62	0.70	0.65	1.15	0.00	0.00	2.62
time (sec)	N/A	0.116	0.321	1.121	0.239	0.270	0.000	0.000	7.327

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	51	63	58	93	0	0	155
N.S.	1	1.00	0.59	0.73	0.67	1.08	0.00	0.00	1.80
time (sec)	N/A	0.113	0.164	1.041	0.239	0.252	0.000	0.000	1.370

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	36	42	44	50	0	0	72
N.S.	1	1.00	0.65	0.76	0.80	0.91	0.00	0.00	1.31
time (sec)	N/A	0.090	0.110	1.033	0.226	0.249	0.000	0.000	0.328

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	21	61	0	0	23
N.S.	1	1.00	1.00	0.83	0.72	2.10	0.00	0.00	0.79
time (sec)	N/A	0.092	0.112	1.216	0.216	0.243	0.000	0.000	4.592

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	51	780	175	305	0	0	0
N.S.	1	1.00	0.25	3.82	0.86	1.50	0.00	0.00	0.00
time (sec)	N/A	0.173	0.344	10.474	0.445	0.249	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	53	914	229	327	0	0	0
N.S.	1	1.00	0.19	3.30	0.83	1.18	0.00	0.00	0.00
time (sec)	N/A	0.210	0.489	9.155	0.440	0.256	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	147	147	112	0	902	199	0	0	301
N.S.	1	1.00	0.76	0.00	6.14	1.35	0.00	0.00	2.05
time (sec)	N/A	0.337	1.995	0.000	1.051	0.357	0.000	0.000	12.751

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	110	110	94	0	764	158	0	0	105
N.S.	1	1.00	0.85	0.00	6.95	1.44	0.00	0.00	0.95
time (sec)	N/A	0.245	1.739	0.000	0.479	0.298	0.000	0.000	9.813

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	B	B	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	80	0	626	117	0	0	91
N.S.	1	1.00	1.10	0.00	8.58	1.60	0.00	0.00	1.25
time (sec)	N/A	0.159	1.615	0.000	0.425	0.275	0.000	0.000	7.064

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	B	B	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	35	35	57	0	488	74	0	0	50
N.S.	1	1.00	1.63	0.00	13.94	2.11	0.00	0.00	1.43
time (sec)	N/A	0.081	1.407	0.000	0.378	0.252	0.000	0.000	2.188

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	123	123	82	268	1074	270	0	0	0
N.S.	1	1.00	0.67	2.18	8.73	2.20	0.00	0.00	0.00
time (sec)	N/A	0.216	1.478	9.840	0.482	0.249	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	86	86	149	396	827	245	0	0	0
N.S.	1	1.00	1.73	4.60	9.62	2.85	0.00	0.00	0.00
time (sec)	N/A	0.189	1.512	9.184	0.704	0.254	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	121	606	0	267	0	0	0
N.S.	1	1.00	0.99	4.97	0.00	2.19	0.00	0.00	0.00
time (sec)	N/A	0.151	1.088	9.230	0.000	0.255	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	143	730	2297	289	0	0	0
N.S.	1	1.00	0.74	3.80	11.96	1.51	0.00	0.00	0.00
time (sec)	N/A	0.318	1.405	10.355	0.512	0.251	0.000	0.000	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	165	864	3789	311	0	0	0
N.S.	1	1.00	0.61	3.20	14.03	1.15	0.00	0.00	0.00
time (sec)	N/A	0.526	1.927	9.111	0.598	0.261	0.000	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	81	101	94	160	0	0	370
N.S.	1	1.00	0.55	0.69	0.64	1.10	0.00	0.00	2.53
time (sec)	N/A	0.113	0.489	1.612	0.246	0.272	0.000	0.000	9.227

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	61	82	76	119	0	0	242
N.S.	1	1.00	0.54	0.73	0.67	1.05	0.00	0.00	2.14
time (sec)	N/A	0.105	0.235	1.317	0.225	0.262	0.000	0.000	7.488

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	49	63	62	77	0	0	110
N.S.	1	1.00	0.58	0.75	0.74	0.92	0.00	0.00	1.31
time (sec)	N/A	0.094	0.136	1.204	0.380	0.244	0.000	0.000	0.863

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	52	44	32	61	0	0	0
N.S.	1	1.00	0.91	0.77	0.56	1.07	0.00	0.00	0.00
time (sec)	N/A	0.087	0.180	1.304	0.408	0.252	0.000	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	21	72	0	0	23
N.S.	1	1.00	1.00	0.83	0.72	2.48	0.00	0.00	0.79
time (sec)	N/A	0.075	0.201	1.317	0.487	0.257	0.000	0.000	4.092

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	51	977	195	316	0	0	0
N.S.	1	1.00	0.22	4.19	0.84	1.36	0.00	0.00	0.00
time (sec)	N/A	0.192	0.445	10.355	0.312	0.251	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	306	53	1111	249	338	0	0	0
N.S.	1	1.00	0.17	3.63	0.81	1.10	0.00	0.00	0.00
time (sec)	N/A	0.211	0.725	9.966	0.315	0.253	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	110	110	92	0	902	173	0	0	105
N.S.	1	1.00	0.84	0.00	8.20	1.57	0.00	0.00	0.95
time (sec)	N/A	0.237	2.003	0.000	0.569	0.323	0.000	0.000	10.273

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	82	0	764	132	0	0	91
N.S.	1	1.00	1.12	0.00	10.47	1.81	0.00	0.00	1.25
time (sec)	N/A	0.149	1.662	0.000	0.469	0.281	0.000	0.000	7.618

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	35	35	59	0	626	89	0	0	50
N.S.	1	1.00	1.69	0.00	17.89	2.54	0.00	0.00	1.43
time (sec)	N/A	0.073	1.473	0.000	0.721	0.262	0.000	0.000	7.382

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	160	160	130	313	1164	326	0	0	0
N.S.	1	1.00	0.81	1.96	7.28	2.04	0.00	0.00	0.00
time (sec)	N/A	0.302	2.049	10.794	0.717	0.261	0.000	0.000	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	121	121	126	425	0	245	0	0	0
N.S.	1	1.00	1.04	3.51	0.00	2.02	0.00	0.00	0.00
time (sec)	N/A	0.276	1.780	10.215	0.000	0.254	0.000	0.000	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	120	795	977	267	0	0	0
N.S.	1	1.00	0.96	6.36	7.82	2.14	0.00	0.00	0.00
time (sec)	N/A	0.251	1.862	10.651	0.457	0.256	0.000	0.000	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	119	798	0	278	0	0	0
N.S.	1	1.00	0.76	5.08	0.00	1.77	0.00	0.00	0.00
time (sec)	N/A	0.202	1.878	9.874	0.000	0.252	0.000	0.000	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	141	923	2779	300	0	0	0
N.S.	1	1.00	0.62	4.07	12.24	1.32	0.00	0.00	0.00
time (sec)	N/A	0.407	2.318	12.797	0.485	0.260	0.000	0.000	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	175	1057	5821	322	0	0	0
N.S.	1	1.00	0.57	3.44	18.96	1.05	0.00	0.00	0.00
time (sec)	N/A	0.653	2.974	11.076	0.621	0.277	0.000	0.000	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	524	524	373	323	1870	418	0	0	0
N.S.	1	1.00	0.71	0.62	3.57	0.80	0.00	0.00	0.00
time (sec)	N/A	0.643	2.123	11.390	0.484	0.253	0.000	0.000	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	277	157	1400	323	0	0	0
N.S.	1	1.00	0.86	0.49	4.33	1.00	0.00	0.00	0.00
time (sec)	N/A	0.249	1.896	11.343	0.482	0.255	0.000	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	32	76	64	0	0	0
N.S.	1	1.00	1.00	0.89	2.11	1.78	0.00	0.00	0.00
time (sec)	N/A	0.077	0.690	8.489	0.317	0.239	0.000	0.000	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	48	52	54	75	0	0	86
N.S.	1	1.00	0.59	0.64	0.67	0.93	0.00	0.00	1.06
time (sec)	N/A	0.171	0.791	10.644	0.377	0.245	0.000	0.000	5.273

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	63	62	130	86	0	0	101
N.S.	1	1.00	0.52	0.51	1.07	0.70	0.00	0.00	0.83
time (sec)	N/A	0.253	0.970	10.524	0.384	0.248	0.000	0.000	5.796

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	80	80	178	97	0	0	109
N.S.	1	1.00	0.49	0.49	1.09	0.59	0.00	0.00	0.66
time (sec)	N/A	0.369	1.262	10.089	0.392	0.240	0.000	0.000	5.785

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	453	453	376	524	3005	644	0	0	0
N.S.	1	1.00	0.83	1.16	6.63	1.42	0.00	0.00	0.00
time (sec)	N/A	0.668	4.189	9.605	0.617	0.257	0.000	0.000	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	571	571	375	422	2367	538	0	0	0
N.S.	1	1.00	0.66	0.74	4.15	0.94	0.00	0.00	0.00
time (sec)	N/A	0.682	3.657	10.541	0.529	0.258	0.000	0.000	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	364	338	359	1871	420	0	0	0
N.S.	1	1.00	0.93	0.99	5.14	1.15	0.00	0.00	0.00
time (sec)	N/A	0.377	3.272	10.476	0.488	0.249	0.000	0.000	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	520	520	401	502	1462	460	0	0	0
N.S.	1	1.00	0.77	0.97	2.81	0.88	0.00	0.00	0.00
time (sec)	N/A	0.546	2.829	10.454	0.502	0.257	0.000	0.000	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	51	76	76	0	0	0
N.S.	1	1.00	1.00	1.34	2.00	2.00	0.00	0.00	0.00
time (sec)	N/A	0.091	1.070	9.750	0.348	0.241	0.000	0.000	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	84	68	59	79	0	0	102
N.S.	1	1.00	1.04	0.84	0.73	0.98	0.00	0.00	1.26
time (sec)	N/A	0.171	1.342	9.542	0.416	0.253	0.000	0.000	5.665

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	98	77	84	91	0	0	110
N.S.	1	1.00	0.78	0.62	0.67	0.73	0.00	0.00	0.88
time (sec)	N/A	0.275	1.452	9.705	0.412	0.246	0.000	0.000	5.852

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	113	95	160	103	0	0	125
N.S.	1	1.00	0.68	0.57	0.96	0.62	0.00	0.00	0.75
time (sec)	N/A	0.370	1.578	9.886	0.416	0.248	0.000	0.000	6.535

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	612	612	386	556	3005	635	0	0	0
N.S.	1	1.00	0.63	0.91	4.91	1.04	0.00	0.00	0.00
time (sec)	N/A	0.876	3.201	9.464	0.650	0.263	0.000	0.000	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	411	411	387	449	2431	525	0	0	0
N.S.	1	1.00	0.94	1.09	5.91	1.28	0.00	0.00	0.00
time (sec)	N/A	0.499	4.090	10.241	0.872	0.261	0.000	0.000	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	563	563	359	661	2013	484	0	0	0
N.S.	1	1.00	0.64	1.17	3.58	0.86	0.00	0.00	0.00
time (sec)	N/A	0.665	3.413	10.548	1.024	0.262	0.000	0.000	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	362	362	343	471	1492	505	0	0	0
N.S.	1	1.00	0.95	1.30	4.12	1.40	0.00	0.00	0.00
time (sec)	N/A	0.492	3.806	10.553	0.504	0.263	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	57	76	80	0	0	104
N.S.	1	1.00	1.00	1.50	2.00	2.11	0.00	0.00	2.74
time (sec)	N/A	0.183	1.161	9.623	0.343	0.240	0.000	0.000	5.026

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	92	76	94	94	0	0	112
N.S.	1	1.00	1.14	0.94	1.16	1.16	0.00	0.00	1.38
time (sec)	N/A	0.181	1.397	9.868	0.434	0.243	0.000	0.000	5.988

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	104	87	96	99	0	0	127
N.S.	1	1.00	0.83	0.70	0.77	0.79	0.00	0.00	1.02
time (sec)	N/A	0.275	1.457	9.877	0.394	0.239	0.000	0.000	6.674

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	121	97	124	99	0	0	133
N.S.	1	1.00	0.72	0.57	0.73	0.59	0.00	0.00	0.79
time (sec)	N/A	0.353	1.605	9.815	0.843	0.241	0.000	0.000	6.803

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	369	369	350	765	2258	461	0	0	0
N.S.	1	1.00	0.95	2.07	6.12	1.25	0.00	0.00	0.00
time (sec)	N/A	0.378	3.779	15.948	0.778	0.269	0.000	0.000	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	483	483	302	157	726	385	0	0	0
N.S.	1	1.00	0.63	0.33	1.50	0.80	0.00	0.00	0.00
time (sec)	N/A	0.383	1.846	14.688	0.468	0.270	0.000	0.000	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	32	76	64	0	0	40
N.S.	1	1.00	1.00	0.89	2.11	1.78	0.00	0.00	1.11
time (sec)	N/A	0.077	0.620	11.600	0.345	0.238	0.000	0.000	5.704

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	48	42	80	78	0	0	78
N.S.	1	1.00	0.60	0.52	1.00	0.98	0.00	0.00	0.98
time (sec)	N/A	0.178	0.664	10.194	0.422	0.234	0.000	0.000	0.845

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	68	61	130	89	0	0	86
N.S.	1	1.00	0.56	0.50	1.07	0.74	0.00	0.00	0.71
time (sec)	N/A	0.268	0.863	10.354	0.413	0.236	0.000	0.000	4.560

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	79	70	178	100	0	0	101
N.S.	1	1.00	0.48	0.42	1.08	0.61	0.00	0.00	0.61
time (sec)	N/A	0.352	1.131	10.228	0.593	0.235	0.000	0.000	5.345

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	87	88	226	111	0	0	109
N.S.	1	1.00	0.42	0.43	1.10	0.54	0.00	0.00	0.53
time (sec)	N/A	0.471	1.399	10.602	0.798	0.240	0.000	0.000	4.938

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	529	529	337	712	1817	483	0	0	0
N.S.	1	1.00	0.64	1.35	3.43	0.91	0.00	0.00	0.00
time (sec)	N/A	0.715	3.655	16.846	0.507	0.259	0.000	0.000	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	365	338	1090	778	539	0	0	0
N.S.	1	1.00	0.93	2.99	2.13	1.48	0.00	0.00	0.00
time (sec)	N/A	0.380	4.028	16.180	0.467	0.259	0.000	0.000	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	55	76	67	0	0	0
N.S.	1	1.00	1.00	1.45	2.00	1.76	0.00	0.00	0.00
time (sec)	N/A	0.092	1.183	11.311	0.376	0.239	0.000	0.000	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	63	59	80	75	0	0	84
N.S.	1	1.00	0.79	0.74	1.00	0.94	0.00	0.00	1.05
time (sec)	N/A	0.154	0.945	14.185	0.429	0.239	0.000	0.000	4.839

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	83	69	130	89	0	0	104
N.S.	1	1.00	0.69	0.57	1.07	0.74	0.00	0.00	0.86
time (sec)	N/A	0.260	1.101	9.737	0.402	0.243	0.000	0.000	4.875

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	100	91	178	100	0	0	112
N.S.	1	1.00	0.61	0.55	1.08	0.61	0.00	0.00	0.68
time (sec)	N/A	0.363	1.414	9.505	0.407	0.242	0.000	0.000	5.095

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	100	97	226	111	0	0	127
N.S.	1	1.00	0.48	0.46	1.08	0.53	0.00	0.00	0.61
time (sec)	N/A	0.468	1.513	9.826	0.764	0.242	0.000	0.000	5.433

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	411	411	370	1612	2449	541	0	0	0
N.S.	1	1.00	0.90	3.92	5.96	1.32	0.00	0.00	0.00
time (sec)	N/A	0.518	5.504	15.103	0.920	0.255	0.000	0.000	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	527	527	357	920	1466	543	0	0	0
N.S.	1	1.00	0.68	1.75	2.78	1.03	0.00	0.00	0.00
time (sec)	N/A	0.535	4.273	15.139	0.665	0.263	0.000	0.000	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	59	76	71	0	0	0
N.S.	1	1.00	1.00	1.55	2.00	1.87	0.00	0.00	0.00
time (sec)	N/A	0.093	1.187	11.109	0.343	0.249	0.000	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	63	73	86	79	0	0	102
N.S.	1	1.00	0.79	0.91	1.08	0.99	0.00	0.00	1.28
time (sec)	N/A	0.177	1.224	14.333	0.408	0.233	0.000	0.000	4.768

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	85	69	130	86	0	0	109
N.S.	1	1.00	0.70	0.57	1.07	0.71	0.00	0.00	0.90
time (sec)	N/A	0.260	1.347	13.263	0.390	0.234	0.000	0.000	5.343

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	102	85	178	89	0	0	118
N.S.	1	1.00	0.63	0.52	1.10	0.55	0.00	0.00	0.73
time (sec)	N/A	0.326	1.448	10.019	0.771	0.247	0.000	0.000	5.121

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	107	102	226	111	0	0	135
N.S.	1	1.00	0.52	0.50	1.10	0.54	0.00	0.00	0.66
time (sec)	N/A	0.472	1.567	8.522	0.643	0.250	0.000	0.000	5.954

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	86	86	118	0	0	0	0	0	0
N.S.	1	1.00	1.37	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.262	1.689	0.000	0.000	0.000	0.000	0.000	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	86	86	116	0	0	0	0	0	0
N.S.	1	1.00	1.35	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.252	1.268	0.000	0.000	0.000	0.000	0.000	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	116	0	0	0	0	0	0
N.S.	1	1.00	1.36	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.240	1.012	0.000	0.000	0.000	0.000	0.000	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	95	0	0	0	0	0	0
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.231	1.113	0.000	0.000	0.000	0.000	0.000	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	95	0	0	0	0	0	0
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.234	1.346	0.000	0.000	0.000	0.000	0.000	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	112	0	0	0	0	0	0
N.S.	1	1.00	1.27	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.254	1.376	0.000	0.000	0.000	0.000	0.000	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	437	437	240	0	3902	528	0	0	0
N.S.	1	1.00	0.55	0.00	8.93	1.21	0.00	0.00	0.00
time (sec)	N/A	0.506	2.443	0.000	0.547	0.259	0.000	0.000	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	378	378	220	0	1906	515	0	0	0
N.S.	1	1.00	0.58	0.00	5.04	1.36	0.00	0.00	0.00
time (sec)	N/A	0.431	1.945	0.000	0.441	0.264	0.000	0.000	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	340	340	161	0	1753	367	0	0	0
N.S.	1	1.00	0.47	0.00	5.16	1.08	0.00	0.00	0.00
time (sec)	N/A	0.217	1.160	0.000	0.467	0.259	0.000	0.000	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	37	37	47	0	107	55	0	0	81
N.S.	1	1.00	1.27	0.00	2.89	1.49	0.00	0.00	2.19
time (sec)	N/A	0.098	0.984	0.000	0.625	0.246	0.000	0.000	5.786

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	70	0	316	58	0	0	90
N.S.	1	1.00	0.86	0.00	3.90	0.72	0.00	0.00	1.11
time (sec)	N/A	0.188	1.093	0.000	0.578	0.244	0.000	0.000	5.075

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	122	100	0	402	107	0	0	122
N.S.	1	1.00	0.82	0.00	3.30	0.88	0.00	0.00	1.00
time (sec)	N/A	0.286	1.499	0.000	0.892	0.244	0.000	0.000	6.360

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	163	116	0	983	133	0	0	303
N.S.	1	1.00	0.71	0.00	6.03	0.82	0.00	0.00	1.86
time (sec)	N/A	0.377	2.039	0.000	0.402	0.236	0.000	0.000	8.950

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	86	86	210	0	0	0	0	0	0
N.S.	1	1.00	2.44	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.180	6.415	0.000	0.000	0.000	0.000	0.000	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	86	86	147	0	0	0	0	0	0
N.S.	1	1.00	1.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.183	1.546	0.000	0.000	0.000	0.000	0.000	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	86	86	114	0	0	0	0	0	0
N.S.	1	1.00	1.33	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.184	0.513	0.000	0.000	0.000	0.000	0.000	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	82	82	67	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.151	0.224	0.000	0.000	0.000	0.000	0.000	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	86	86	152	0	0	0	0	0	0
N.S.	1	1.00	1.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.194	2.046	0.000	0.000	0.000	0.000	0.000	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	86	86	156	0	0	0	0	0	0
N.S.	1	1.00	1.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.200	1.966	0.000	0.000	0.000	0.000	0.000	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	86	86	153	0	0	0	0	0	0
N.S.	1	1.00	1.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.200	2.087	0.000	0.000	0.000	0.000	0.000	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	109	109	186	0	0	0	0	0	0
N.S.	1	1.00	1.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.244	3.291	0.000	0.000	0.000	0.000	0.000	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	109	109	186	0	0	0	0	0	0
N.S.	1	1.00	1.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.242	5.210	0.000	0.000	0.000	0.000	0.000	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	107	186	0	0	0	0	0	0
N.S.	1	1.00	1.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.241	2.739	0.000	0.000	0.000	0.000	0.000	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	107	174	0	0	0	0	0	0
N.S.	1	1.00	1.63	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.234	1.754	0.000	0.000	0.000	0.000	0.000	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	106	106	174	0	0	0	0	0	0
N.S.	1	1.00	1.64	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.247	2.492	0.000	0.000	0.000	0.000	0.000	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	109	109	178	0	0	0	0	0	0
N.S.	1	1.00	1.63	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.256	3.141	0.000	0.000	0.000	0.000	0.000	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	109	109	178	0	0	0	0	0	0
N.S.	1	1.00	1.63	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.261	5.071	0.000	0.000	0.000	0.000	0.000	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	165	0	0	0	0	0	0
N.S.	1	1.00	1.57	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.193	10.023	0.000	0.000	0.000	0.000	0.000	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	80	272	0	247	0	0	168
N.S.	1	1.00	0.82	2.80	0.00	2.55	0.00	0.00	1.73
time (sec)	N/A	0.100	0.881	1.832	0.000	0.252	0.000	0.000	9.520

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	60	150	0	142	0	0	216
N.S.	1	1.00	0.92	2.31	0.00	2.18	0.00	0.00	3.32
time (sec)	N/A	0.074	0.278	1.014	0.000	0.263	0.000	0.000	2.817

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	31	28	60	0	62	104
N.S.	1	1.00	1.00	0.97	0.88	1.88	0.00	1.94	3.25
time (sec)	N/A	0.065	0.187	0.446	0.224	0.241	0.000	0.730	0.506

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	56	54	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.084	0.209	0.000	0.000	0.000	0.000	0.000	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	58	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.075	0.292	0.000	0.000	0.000	0.000	0.000	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	58	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.077	0.542	0.000	0.000	0.000	0.000	0.000	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	94	94	149	0	0	0	0	0	0
N.S.	1	1.00	1.59	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.241	14.720	0.000	0.000	0.000	0.000	0.000	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	149	0	0	0	0	0	0
N.S.	1	1.00	1.62	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.229	13.910	0.000	0.000	0.000	0.000	0.000	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	146	0	0	0	0	0	0
N.S.	1	1.00	1.66	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.187	9.963	0.000	0.000	0.000	0.000	0.000	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	146	0	0	0	0	0	0
N.S.	1	1.00	1.72	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.219	13.431	0.000	0.000	0.000	0.000	0.000	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	94	94	149	0	0	0	0	0	0
N.S.	1	1.00	1.59	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.231	14.216	0.000	0.000	0.000	0.000	0.000	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	94	94	149	0	0	0	0	0	0
N.S.	1	1.00	1.59	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.225	15.681	0.000	0.000	0.000	0.000	0.000	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	96	96	181	0	0	0	0	0	0
N.S.	1	1.00	1.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.226	10.898	0.000	0.000	0.000	0.000	0.000	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	96	96	170	0	0	0	0	0	0
N.S.	1	1.00	1.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.221	11.040	0.000	0.000	0.000	0.000	0.000	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	94	94	170	0	0	0	0	0	0
N.S.	1	1.00	1.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.210	10.434	0.000	0.000	0.000	0.000	0.000	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	91	143	0	0	0	0	0	0
N.S.	1	1.00	1.57	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.227	11.191	0.000	0.000	0.000	0.000	0.000	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	143	0	0	0	0	0	0
N.S.	1	1.00	1.54	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.246	12.154	0.000	0.000	0.000	0.000	0.000	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	98	157	0	0	0	0	0	0
N.S.	1	1.00	1.60	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.252	12.477	0.000	0.000	0.000	0.000	0.000	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	269	269	165	4331	435	335	0	0	511
N.S.	1	1.00	0.61	16.10	1.62	1.25	0.00	0.00	1.90
time (sec)	N/A	0.493	1.889	6.426	0.817	0.252	0.000	0.000	11.480

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	205	205	119	4982	346	265	0	0	425
N.S.	1	1.00	0.58	24.30	1.69	1.29	0.00	0.00	2.07
time (sec)	N/A	0.394	1.891	6.876	0.401	0.244	0.000	0.000	11.061

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	148	148	82	2581	175	178	0	0	227
N.S.	1	1.00	0.55	17.44	1.18	1.20	0.00	0.00	1.53
time (sec)	N/A	0.266	1.378	6.424	0.397	0.247	0.000	0.000	10.534

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	B	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	94	94	58	2484	113	129	246	0	121
N.S.	1	1.00	0.62	26.43	1.20	1.37	2.62	0.00	1.29
time (sec)	N/A	0.151	1.289	7.101	0.377	0.249	0.507	0.000	2.159

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	A	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	37	37	37	842	86	84	49	0	0
N.S.	1	1.00	1.00	22.76	2.32	2.27	1.32	0.00	0.00
time (sec)	N/A	0.060	0.325	5.522	0.332	0.243	3.856	0.000	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	118	118	87	0	0	0	0	0	0
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.262	5.842	0.000	0.000	0.000	0.000	0.000	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	113	113	112	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.229	14.181	0.000	0.000	0.000	0.000	0.000	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	121	116	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.281	17.560	0.000	0.000	0.000	0.000	0.000	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	156	156	122	0	1067	166	0	0	318
N.S.	1	1.00	0.78	0.00	6.84	1.06	0.00	0.00	2.04
time (sec)	N/A	0.273	3.579	0.000	1.366	0.252	0.000	0.000	11.811

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	97	97	166	0	0	0	0	0	0
N.S.	1	1.00	1.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.266	15.718	0.000	0.000	0.000	0.000	0.000	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	98	91	0	595	134	0	0	174
N.S.	1	1.00	0.93	0.00	6.07	1.37	0.00	0.00	1.78
time (sec)	N/A	0.167	2.560	0.000	1.084	0.245	0.000	0.000	7.467

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	97	97	166	0	0	0	0	0	0
N.S.	1	1.00	1.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.266	14.589	0.000	0.000	0.000	0.000	0.000	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	B	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	46	46	59	0	217	109	0	0	106
N.S.	1	1.00	1.28	0.00	4.72	2.37	0.00	0.00	2.30
time (sec)	N/A	0.065	1.966	0.000	0.364	0.251	0.000	0.000	5.628

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	154	0	0	0	0	0	0
N.S.	1	1.00	1.62	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.232	10.340	0.000	0.000	0.000	0.000	0.000	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	65	65	146	0	0	0	0	0	0
N.S.	1	1.00	2.25	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.102	2.241	0.000	0.000	0.000	0.000	0.000	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	157	0	0	0	0	0	0
N.S.	1	1.00	1.65	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.237	14.202	0.000	0.000	0.000	0.000	0.000	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	74	74	151	0	0	0	0	0	0
N.S.	1	1.00	2.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.182	14.688	0.000	0.000	0.000	0.000	0.000	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	97	97	166	0	0	0	0	0	0
N.S.	1	1.00	1.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.255	14.985	0.000	0.000	0.000	0.000	0.000	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	66	165	0	0	0	0	0	0
N.S.	1	1.00	2.50	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.227	13.412	0.000	0.000	0.000	0.000	0.000	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	66	165	0	0	0	0	0	0
N.S.	1	1.00	2.50	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.220	12.187	0.000	0.000	0.000	0.000	0.000	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	150	0	0	0	0	0	0
N.S.	1	1.00	2.38	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.093	2.494	0.000	0.000	0.000	0.000	0.000	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	40	40	40	1261	137	115	0	0	62
N.S.	1	1.00	1.00	31.52	3.42	2.88	0.00	0.00	1.55
time (sec)	N/A	0.070	1.680	8.248	0.337	0.246	0.000	0.000	5.442

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	61	0	304	139	0	0	260
N.S.	1	1.00	0.66	0.00	3.30	1.51	0.00	0.00	2.83
time (sec)	N/A	0.158	2.436	0.000	0.443	0.256	0.000	0.000	9.241

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	148	148	129	0	0	172	0	0	321
N.S.	1	1.00	0.87	0.00	0.00	1.16	0.00	0.00	2.17
time (sec)	N/A	0.259	3.298	0.000	0.000	0.254	0.000	0.000	13.755

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	53	69	70	57	56	70	68
N.S.	1	1.00	0.88	1.15	1.17	0.95	0.93	1.17	1.13
time (sec)	N/A	0.048	0.198	14.753	0.204	0.244	1.477	0.394	4.097

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	63	86	88	0	141	175
N.S.	1	1.00	1.00	0.85	1.16	1.19	0.00	1.91	2.36
time (sec)	N/A	0.064	0.015	8.658	0.350	0.265	0.000	0.406	7.984

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	41	47	48	45	44	48	46
N.S.	1	1.00	0.93	1.07	1.09	1.02	1.00	1.09	1.05
time (sec)	N/A	0.043	0.100	3.540	0.394	0.243	1.040	0.388	4.043

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	50	61	74	0	99	105
N.S.	1	1.00	1.00	0.96	1.17	1.42	0.00	1.90	2.02
time (sec)	N/A	0.048	0.015	2.253	0.281	0.248	0.000	0.385	6.547

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	20	30	34	25	23
N.S.	1	1.00	1.00	0.89	0.71	1.07	1.21	0.89	0.82
time (sec)	N/A	0.037	0.017	0.855	0.200	0.246	0.673	0.353	4.035

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	32	31	54	37	54	38
N.S.	1	1.00	1.00	1.33	1.29	2.25	1.54	2.25	1.58
time (sec)	N/A	0.019	0.013	0.572	0.204	0.262	2.224	0.367	4.531

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	46	23	23	23	0	129	38
N.S.	1	1.00	1.92	0.96	0.96	0.96	0.00	5.38	1.58
time (sec)	N/A	0.027	0.019	0.720	0.209	0.256	0.000	0.358	4.211

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	46	36	38	35	0	146	31
N.S.	1	1.00	1.07	0.84	0.88	0.81	0.00	3.40	0.72
time (sec)	N/A	0.040	0.061	1.136	0.304	0.261	0.000	0.369	4.026

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	36	35	38	0	11886	47
N.S.	1	1.00	1.00	0.82	0.80	0.86	0.00	270.14	1.07
time (sec)	N/A	0.040	0.009	2.894	0.312	0.253	0.000	2.422	4.653

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	62	52	61	51	0	426	41
N.S.	1	1.00	0.95	0.80	0.94	0.78	0.00	6.55	0.63
time (sec)	N/A	0.049	0.101	4.856	0.519	0.262	0.000	0.519	4.060

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	133	138	133	122	0	156	132
N.S.	1	1.00	1.12	1.16	1.12	1.03	0.00	1.31	1.11
time (sec)	N/A	0.127	0.613	113.345	0.659	0.274	0.000	0.532	4.540

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	104	110	104	100	0	118	102
N.S.	1	1.00	1.07	1.13	1.07	1.03	0.00	1.22	1.05
time (sec)	N/A	0.106	0.763	28.988	0.216	0.251	0.000	0.531	4.092

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	54	82	71	79	0	80	71
N.S.	1	1.00	0.72	1.09	0.95	1.05	0.00	1.07	0.95
time (sec)	N/A	0.086	0.222	7.983	0.209	0.264	0.000	0.512	4.212

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	46	48	20	55	0	41	39
N.S.	1	1.00	2.09	2.18	0.91	2.50	0.00	1.86	1.77
time (sec)	N/A	0.044	0.044	2.439	0.249	0.270	0.000	0.455	4.360

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	52	64	55	52	0	245	50
N.S.	1	1.00	1.06	1.31	1.12	1.06	0.00	5.00	1.02
time (sec)	N/A	0.062	0.448	1.998	0.290	0.268	0.000	0.502	3.991

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	197	97	85	75	0	2286	83
N.S.	1	1.00	2.24	1.10	0.97	0.85	0.00	25.98	0.94
time (sec)	N/A	0.103	3.325	8.991	0.623	0.268	0.000	2.597	4.788

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	216	177	220	163	0	437	432
N.S.	1	1.00	1.33	1.09	1.35	1.00	0.00	2.68	2.65
time (sec)	N/A	0.167	0.083	67.511	0.315	0.289	0.000	0.587	8.173

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	168	149	180	142	0	343	328
N.S.	1	1.00	1.28	1.14	1.37	1.08	0.00	2.62	2.50
time (sec)	N/A	0.143	0.069	15.695	0.580	0.274	0.000	0.540	6.891

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	120	118	129	120	0	249	216
N.S.	1	1.00	1.21	1.19	1.30	1.21	0.00	2.52	2.18
time (sec)	N/A	0.127	0.060	4.717	0.212	0.254	0.000	0.509	7.249

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	67	83	82	96	0	122	106
N.S.	1	1.00	1.03	1.28	1.26	1.48	0.00	1.88	1.63
time (sec)	N/A	0.067	0.045	1.068	0.200	0.249	0.000	0.512	4.654

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	84	53	60	62	0	1100	66
N.S.	1	1.00	1.79	1.13	1.28	1.32	0.00	23.40	1.40
time (sec)	N/A	0.042	0.579	1.181	0.209	0.250	0.000	0.702	4.971

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	67	52	52	53	0	11162	77
N.S.	1	1.00	0.74	0.58	0.58	0.59	0.00	124.02	0.86
time (sec)	N/A	0.122	0.061	4.723	0.209	0.242	0.000	16.591	4.240

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	69	88	77	74	0	28204	115
N.S.	1	1.00	0.61	0.77	0.68	0.65	0.00	247.40	1.01
time (sec)	N/A	0.135	0.178	21.460	0.334	0.242	0.000	34.076	4.573

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	92	108	98	94	0	52002	176
N.S.	1	1.00	0.67	0.78	0.71	0.68	0.00	376.83	1.28
time (sec)	N/A	0.138	0.319	68.937	0.358	0.260	0.000	61.655	4.663

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	177	219	176	150	0	220	175
N.S.	1	1.00	0.91	1.13	0.91	0.77	0.00	1.13	0.90
time (sec)	N/A	0.192	2.254	191.796	0.312	0.261	0.000	0.821	4.111

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	115	173	142	128	0	166	139
N.S.	1	1.00	0.83	1.25	1.03	0.93	0.00	1.20	1.01
time (sec)	N/A	0.170	0.670	69.606	0.216	0.282	0.000	0.787	4.856

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	54	127	98	105	0	112	97
N.S.	1	1.00	0.72	1.69	1.31	1.40	0.00	1.49	1.29
time (sec)	N/A	0.094	0.405	16.388	0.206	0.240	0.000	0.754	4.001

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	57	72	20	78	0	57	55
N.S.	1	1.00	2.59	3.27	0.91	3.55	0.00	2.59	2.50
time (sec)	N/A	0.052	0.181	4.500	0.237	0.240	0.000	0.739	4.407

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	401	98	81	79	0	561	141
N.S.	1	1.00	4.66	1.14	0.94	0.92	0.00	6.52	1.64
time (sec)	N/A	0.126	0.837	4.698	0.294	0.257	0.000	0.886	4.418

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	488	114	110	100	0	2496	109
N.S.	1	1.00	5.81	1.36	1.31	1.19	0.00	29.71	1.30
time (sec)	N/A	0.083	1.224	20.988	0.483	0.247	0.000	12.250	4.114

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	177	637	248	208	170	0	465	423
N.S.	1	1.11	4.01	1.56	1.31	1.07	0.00	2.92	2.66
time (sec)	N/A	0.191	3.324	31.313	0.298	0.258	0.000	0.841	8.106

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	144	464	198	157	147	0	333	293
N.S.	1	1.14	3.68	1.57	1.25	1.17	0.00	2.64	2.33
time (sec)	N/A	0.158	2.463	9.239	0.391	0.265	0.000	0.754	8.175

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	109	293	146	111	123	0	171	160
N.S.	1	1.20	3.22	1.60	1.22	1.35	0.00	1.88	1.76
time (sec)	N/A	0.101	2.440	2.562	0.215	0.260	0.000	0.705	6.262

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	102	131	96	84	109	0	4309	116
N.S.	1	1.21	1.56	1.14	1.00	1.30	0.00	51.30	1.38
time (sec)	N/A	0.099	1.818	2.797	0.221	0.276	0.000	2.617	4.693

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	81	75	77	77	0	24430	104
N.S.	1	1.00	1.16	1.07	1.10	1.10	0.00	349.00	1.49
time (sec)	N/A	0.090	0.781	10.368	0.276	0.255	0.000	142.573	4.221

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	121	125	107	102	0	56572	147
N.S.	1	1.00	1.15	1.19	1.02	0.97	0.00	538.78	1.40
time (sec)	N/A	0.127	1.326	40.215	0.214	0.260	0.000	274.601	4.527

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	141	145	126	123	0	101962	214
N.S.	1	1.00	0.99	1.02	0.89	0.87	0.00	718.04	1.51
time (sec)	N/A	0.177	3.084	119.541	0.374	0.260	0.000	84.494	4.491

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	99	106	108	183	0	120	119
N.S.	1	1.00	0.85	0.91	0.93	1.58	0.00	1.03	1.03
time (sec)	N/A	0.136	1.318	32.677	0.348	0.271	0.000	0.384	4.368

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	52	53	53	117	0	54	57
N.S.	1	1.00	0.88	0.90	0.90	1.98	0.00	0.92	0.97
time (sec)	N/A	0.091	0.152	7.497	0.623	0.257	0.000	0.381	4.048

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	59	0	19	18
N.S.	1	1.00	1.00	1.06	1.00	3.28	0.00	1.06	1.00
time (sec)	N/A	0.053	0.018	1.451	0.207	0.256	0.000	0.367	3.938

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	143	120	141	119	0	182	156
N.S.	1	1.00	1.54	1.29	1.52	1.28	0.00	1.96	1.68
time (sec)	N/A	0.185	0.605	2.289	0.310	0.263	0.000	0.363	4.834

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	225	197	271	208	0	322	318
N.S.	1	1.00	1.48	1.30	1.78	1.37	0.00	2.12	2.09
time (sec)	N/A	0.239	1.329	8.190	0.333	0.271	0.000	0.400	4.565

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	152	321	269	361	259	0	278	724
N.S.	1	1.09	2.29	1.92	2.58	1.85	0.00	1.99	5.17
time (sec)	N/A	0.273	2.594	16.287	0.304	0.336	0.000	0.408	6.543

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	109	129	163	191	0	136	310
N.S.	1	1.00	1.38	1.63	2.06	2.42	0.00	1.72	3.92
time (sec)	N/A	0.124	0.526	4.092	0.474	0.272	0.000	0.405	4.308

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	45	43	80	131	0	74	39
N.S.	1	1.00	0.98	0.93	1.74	2.85	0.00	1.61	0.85
time (sec)	N/A	0.042	0.139	1.343	0.514	0.250	0.000	0.378	4.571

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	79	90	142	187	0	118	110
N.S.	1	1.00	0.88	1.00	1.58	2.08	0.00	1.31	1.22
time (sec)	N/A	0.126	0.565	1.595	0.455	0.265	0.000	0.393	4.449

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	137	221	379	262	0	286	342
N.S.	1	1.00	0.83	1.34	2.30	1.59	0.00	1.73	2.07
time (sec)	N/A	0.251	1.715	4.153	0.317	0.269	0.000	0.404	7.586

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	229	201	186	386	0	253	258
N.S.	1	1.00	1.29	1.13	1.04	2.17	0.00	1.42	1.45
time (sec)	N/A	0.191	2.623	265.495	0.212	0.295	0.000	0.496	4.140

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	122	114	115	281	0	149	130
N.S.	1	1.00	1.05	0.98	0.99	2.42	0.00	1.28	1.12
time (sec)	N/A	0.117	5.389	58.769	0.225	0.270	0.000	0.445	4.287

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	51	57	60	178	0	71	67
N.S.	1	1.00	0.84	0.93	0.98	2.92	0.00	1.16	1.10
time (sec)	N/A	0.082	0.170	15.500	0.221	0.262	0.000	0.417	4.563

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	32	21	20	57	0	20	20
N.S.	1	1.00	1.60	1.05	1.00	2.85	0.00	1.00	1.00
time (sec)	N/A	0.050	0.164	3.420	0.340	0.253	0.000	0.429	4.005

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	304	154	282	279	0	250	246
N.S.	1	1.00	2.00	1.01	1.86	1.84	0.00	1.64	1.62
time (sec)	N/A	0.203	4.411	4.168	0.417	0.288	0.000	0.441	5.129

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	416	248	502	424	0	464	463
N.S.	1	1.00	1.77	1.06	2.14	1.80	0.00	1.97	1.97
time (sec)	N/A	0.328	3.599	16.901	0.479	0.310	0.000	0.472	5.330

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	1152	454	827	472	0	530	2654
N.S.	1	1.00	4.90	1.93	3.52	2.01	0.00	2.26	11.29
time (sec)	N/A	0.337	6.762	152.135	0.316	0.417	0.000	0.530	7.560

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	709	259	471	355	0	280	585
N.S.	1	1.00	4.03	1.47	2.68	2.02	0.00	1.59	3.32
time (sec)	N/A	0.201	6.577	32.082	0.331	0.316	0.000	0.497	5.935

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	132	120	135	212	293	0	166	383
N.S.	1	1.45	1.32	1.48	2.33	3.22	0.00	1.82	4.21
time (sec)	N/A	0.126	1.345	7.865	0.296	0.311	0.000	0.478	5.232

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	105	78	118	182	215	0	138	136
N.S.	1	1.28	0.95	1.44	2.22	2.62	0.00	1.68	1.66
time (sec)	N/A	0.091	0.631	1.872	0.358	0.271	0.000	0.435	4.646

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	153	172	348	302	0	286	286
N.S.	1	1.00	0.97	1.10	2.22	1.92	0.00	1.82	1.82
time (sec)	N/A	0.165	0.938	2.315	0.422	0.294	0.000	0.507	6.540

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	249	320	772	418	0	438	674
N.S.	1	1.00	1.03	1.33	3.20	1.73	0.00	1.82	2.80
time (sec)	N/A	0.320	1.803	8.249	0.463	0.300	0.000	0.511	8.298

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	272	195	200	476	0	243	234
N.S.	1	1.00	1.47	1.05	1.08	2.57	0.00	1.31	1.26
time (sec)	N/A	0.200	1.509	1.076	0.236	0.306	0.000	0.621	4.492

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	140	115	128	354	0	140	143
N.S.	1	1.00	1.16	0.95	1.06	2.93	0.00	1.16	1.18
time (sec)	N/A	0.144	4.392	174.192	0.213	0.297	0.000	0.581	4.732

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	57	63	78	284	0	62	80
N.S.	1	1.00	0.83	0.91	1.13	4.12	0.00	0.90	1.16
time (sec)	N/A	0.094	0.225	37.257	0.246	0.270	0.000	0.563	4.288

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	142	0	20	39
N.S.	1	1.00	1.00	0.95	0.91	6.45	0.00	0.91	1.77
time (sec)	N/A	0.055	0.219	7.224	0.232	0.260	0.000	0.516	4.522

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	458	219	458	503	0	439	419
N.S.	1	1.00	2.27	1.08	2.27	2.49	0.00	2.17	2.07
time (sec)	N/A	0.306	6.365	10.265	0.307	0.314	0.000	0.635	5.260

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	596	312	738	671	0	587	715
N.S.	1	1.00	2.02	1.06	2.50	2.27	0.00	1.99	2.42
time (sec)	N/A	0.469	6.295	43.898	0.533	0.335	0.000	0.613	6.490

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	688	444	902	564	0	510	1203
N.S.	1	1.00	2.88	1.86	3.77	2.36	0.00	2.13	5.03
time (sec)	N/A	0.329	3.614	299.230	0.609	0.353	0.000	0.646	7.829

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	189	396	269	518	513	0	314	1311
N.S.	1	1.28	2.68	1.82	3.50	3.47	0.00	2.12	8.86
time (sec)	N/A	0.189	3.682	94.516	0.339	0.336	0.000	0.607	6.480

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	118	132	191	326	294	0	221	260
N.S.	1	1.24	1.39	2.01	3.43	3.09	0.00	2.33	2.74
time (sec)	N/A	0.113	0.487	17.398	0.306	0.273	0.000	0.612	6.927

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	110	280	412	352	0	293	443
N.S.	1	1.00	0.71	1.81	2.66	2.27	0.00	1.89	2.86
time (sec)	N/A	0.139	1.663	3.616	0.304	0.289	0.000	0.540	5.879

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	183	283	658	480	0	399	610
N.S.	1	1.00	0.83	1.28	2.98	2.17	0.00	1.81	2.76
time (sec)	N/A	0.293	2.713	4.994	0.319	0.306	0.000	0.626	8.102

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	371	457	1229	619	0	640	1128
N.S.	1	1.00	1.20	1.47	3.96	2.00	0.00	2.06	3.64
time (sec)	N/A	0.494	3.039	23.375	0.360	0.333	0.000	0.635	9.856

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	69	436	0	145	0	0	0
N.S.	1	1.00	0.57	3.60	0.00	1.20	0.00	0.00	0.00
time (sec)	N/A	0.098	1.382	26.102	0.000	0.096	0.000	0.000	0.000

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	58	162	0	123	0	0	0
N.S.	1	1.00	0.63	1.76	0.00	1.34	0.00	0.00	0.00
time (sec)	N/A	0.081	0.896	24.297	0.000	0.093	0.000	0.000	0.000

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	58	412	0	120	0	0	0
N.S.	1	1.00	0.66	4.68	0.00	1.36	0.00	0.00	0.00
time (sec)	N/A	0.074	0.958	3.589	0.000	0.093	0.000	0.000	0.000

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	42	95	0	74	0	0	39
N.S.	1	1.00	0.72	1.64	0.00	1.28	0.00	0.00	0.67
time (sec)	N/A	0.057	0.813	11.108	0.000	0.084	0.000	0.000	0.431

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	54	306	0	89	0	0	0
N.S.	1	1.00	0.93	5.28	0.00	1.53	0.00	0.00	0.00
time (sec)	N/A	0.065	0.815	10.435	0.000	0.087	0.000	0.000	0.000

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	69	155	0	103	0	0	0
N.S.	1	1.00	0.73	1.65	0.00	1.10	0.00	0.00	0.00
time (sec)	N/A	0.080	0.697	10.849	0.000	0.087	0.000	0.000	0.000

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	74	438	0	111	0	0	0
N.S.	1	1.00	0.79	4.66	0.00	1.18	0.00	0.00	0.00
time (sec)	N/A	0.089	1.419	6.150	0.000	0.096	0.000	0.000	0.000

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	94	178	0	118	0	0	0
N.S.	1	1.00	0.76	1.45	0.00	0.96	0.00	0.00	0.00
time (sec)	N/A	0.106	1.112	6.184	0.000	0.095	0.000	0.000	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	127	317	0	172	0	0	0
N.S.	1	1.00	0.89	2.22	0.00	1.20	0.00	0.00	0.00
time (sec)	N/A	0.211	2.437	146.292	0.000	0.094	0.000	0.000	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	126	833	0	171	0	0	0
N.S.	1	1.00	0.88	5.83	0.00	1.20	0.00	0.00	0.00
time (sec)	N/A	0.191	1.851	21.000	0.000	0.094	0.000	0.000	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	87	239	0	133	0	0	0
N.S.	1	1.00	0.84	2.32	0.00	1.29	0.00	0.00	0.00
time (sec)	N/A	0.162	1.650	15.539	0.000	0.087	0.000	0.000	0.000

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	64	805	0	122	0	0	0
N.S.	1	1.00	0.67	8.47	0.00	1.28	0.00	0.00	0.00
time (sec)	N/A	0.155	2.654	16.287	0.000	0.093	0.000	0.000	0.000

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	101	295	0	131	0	0	0
N.S.	1	1.00	0.73	2.12	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.249	2.433	15.536	0.000	0.095	0.000	0.000	0.000

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	92	863	0	139	0	0	0
N.S.	1	1.00	0.63	5.95	0.00	0.96	0.00	0.00	0.00
time (sec)	N/A	0.241	2.831	23.411	0.000	0.099	0.000	0.000	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	127	336	0	154	0	0	0
N.S.	1	1.00	0.69	1.83	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.293	4.655	21.609	0.000	0.097	0.000	0.000	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	126	931	0	164	0	0	0
N.S.	1	1.00	0.68	5.06	0.00	0.89	0.00	0.00	0.00
time (sec)	N/A	0.309	6.007	33.432	0.000	0.105	0.000	0.000	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	157	349	0	202	0	0	0
N.S.	1	1.00	0.79	1.76	0.00	1.02	0.00	0.00	0.00
time (sec)	N/A	0.189	4.211	619.103	0.000	0.099	0.000	0.000	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-1)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	155	872	0	200	0	0	0
N.S.	1	1.00	0.88	4.95	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	0.173	3.858	22.069	0.000	0.099	0.000	0.000	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	132	298	0	164	0	0	0
N.S.	1	1.00	1.02	2.31	0.00	1.27	0.00	0.00	0.00
time (sec)	N/A	0.130	3.913	21.260	0.000	0.095	0.000	0.000	0.000

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	130	1114	0	169	0	0	0
N.S.	1	1.00	0.73	6.26	0.00	0.95	0.00	0.00	0.00
time (sec)	N/A	0.165	4.903	21.299	0.000	0.099	0.000	0.000	0.000

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	117	323	0	148	0	0	0
N.S.	1	1.00	0.80	2.21	0.00	1.01	0.00	0.00	0.00
time (sec)	N/A	0.144	3.805	23.956	0.000	0.097	0.000	0.000	0.000

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	150	1289	0	164	0	0	0
N.S.	1	1.00	0.74	6.32	0.00	0.80	0.00	0.00	0.00
time (sec)	N/A	0.182	3.943	25.996	0.000	0.104	0.000	0.000	0.000

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	150	366	0	181	0	0	0
N.S.	1	1.00	0.88	2.15	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.171	5.489	23.648	0.000	0.103	0.000	0.000	0.000

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	372	976	0	191	0	0	0
N.S.	1	1.00	2.11	5.55	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	0.172	9.408	32.975	0.000	0.117	0.000	0.000	0.000

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	296	407	0	206	0	0	0
N.S.	1	1.00	1.36	1.87	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.217	10.232	39.402	0.000	0.115	0.000	0.000	0.000

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	456	456	11117	31613	0	0	0	0	0
N.S.	1	1.00	24.38	69.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.464	90.481	33.062	0.000	0.000	0.000	0.000	0.000

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	396	396	290	7347	0	0	0	0	0
N.S.	1	1.00	0.73	18.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.475	27.353	30.430	0.000	0.000	0.000	0.000	0.000

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	334	334	276	3634	0	0	0	0	0
N.S.	1	1.00	0.83	10.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.365	3.428	9.428	0.000	0.000	0.000	0.000	0.000

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	324	324	218	3038	0	0	0	0	0
N.S.	1	1.00	0.67	9.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.368	2.866	8.017	0.000	0.000	0.000	0.000	0.000

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	451	451	285	6968	0	0	0	0	0
N.S.	1	1.00	0.63	15.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.516	4.475	9.987	0.000	0.000	0.000	0.000	0.000

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	422	422	418	6052	0	0	0	0	0
N.S.	1	1.00	0.99	14.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.532	7.171	10.466	0.000	0.000	0.000	0.000	0.000

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	568	568	2596	12841	0	0	0	0	0
N.S.	1	1.00	4.57	22.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.739	77.215	14.007	0.000	0.000	0.000	0.000	0.000

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	480	480	1129	33211	0	0	0	0	0
N.S.	1	1.00	2.35	69.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.464	29.031	132.901	0.000	0.000	0.000	0.000	0.000

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	440	440	378	4812	0	0	0	0	0
N.S.	1	1.00	0.86	10.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.458	12.021	128.711	0.000	0.000	0.000	0.000	0.000

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	477	477	1125	16454	0	0	0	0	0
N.S.	1	1.00	2.36	34.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.493	27.971	8.872	0.000	0.000	0.000	0.000	0.000

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	430	430	422	12206	0	0	0	0	0
N.S.	1	1.00	0.98	28.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.434	10.760	11.210	0.000	0.000	0.000	0.000	0.000

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	555	555	8379	29178	0	0	0	0	0
N.S.	1	1.00	15.10	52.57	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.669	35.310	11.809	0.000	0.000	0.000	0.000	0.000

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	520	520	528	14917	0	0	0	0	0
N.S.	1	1.00	1.02	28.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.667	12.230	13.293	0.000	0.000	0.000	0.000	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	700	700	9161	37753	0	0	0	0	0
N.S.	1	1.00	13.09	53.93	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.918	32.275	17.401	0.000	0.000	0.000	0.000	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	583	583	14225	72185	0	0	0	0	0
N.S.	1	1.00	24.40	123.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.613	35.712	1486.381	0.000	0.000	0.000	0.000	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	532	532	352	36014	0	0	0	0	0
N.S.	1	1.00	0.66	67.70	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.601	10.263	1496.337	0.000	0.000	0.000	0.000	0.000

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	566	566	14364	60919	0	0	0	0	0
N.S.	1	1.00	25.38	107.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.670	35.268	13.111	0.000	0.000	0.000	0.000	0.000

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	515	515	369	75607	0	0	0	0	0
N.S.	1	1.00	0.72	146.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.624	9.699	14.532	0.000	0.000	0.000	0.000	0.000

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	664	664	14652	78532	0	0	0	0	0
N.S.	1	1.00	22.07	118.27	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.092	39.230	15.802	0.000	0.000	0.000	0.000	0.000

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	620	620	409	76277	0	0	0	0	0
N.S.	1	1.00	0.66	123.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.924	12.852	17.743	0.000	0.000	0.000	0.000	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	814	814	15481	89815	0	0	0	0	0
N.S.	1	1.00	19.02	110.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.178	33.968	22.569	0.000	0.000	0.000	0.000	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	69	0	0	0	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.074	0.519	0.000	0.000	0.000	0.000	0.000	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	59	0	0	0	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.064	0.420	0.000	0.000	0.000	0.000	0.000	0.000

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	59	0	0	0	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.068	0.256	0.000	0.000	0.000	0.000	0.000	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	61	0	0	0	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.070	0.320	0.000	0.000	0.000	0.000	0.000	0.000

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	119	109	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.156	0.759	0.000	0.000	0.000	0.000	0.000	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	119	106	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.161	0.901	0.000	0.000	0.000	0.000	0.000	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	119	107	0	0	0	0	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.181	1.015	0.000	0.000	0.000	0.000	0.000	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	119	118	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.178	1.384	0.000	0.000	0.000	0.000	0.000	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	552	552	276	0	0	0	0	0	0
N.S.	1	1.00	0.50	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.084	17.906	0.000	0.000	0.000	0.000	0.000	0.000

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	552	552	280	0	0	0	0	0	0
N.S.	1	1.00	0.51	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.910	14.564	0.000	0.000	0.000	0.000	0.000	0.000

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	579	579	285	0	0	0	0	0	0
N.S.	1	1.00	0.49	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.069	52.388	0.000	0.000	0.000	0.000	0.000	0.000

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	581	581	6862	0	0	0	0	0	0
N.S.	1	1.00	11.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.202	111.464	0.000	0.000	0.000	0.000	0.000	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-1)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	687	687	8003	0	0	0	0	0	0
N.S.	1	1.00	11.65	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.231	129.130	0.000	0.000	0.000	0.000	0.000	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	687	687	7801	0	0	0	0	0	0
N.S.	1	1.00	11.36	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.156	78.547	0.000	0.000	0.000	0.000	0.000	0.000

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	715	715	18832	0	0	0	0	0	0
N.S.	1	1.00	26.34	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.343	137.803	0.000	0.000	0.000	0.000	0.000	0.000

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	717	717	11783	0	0	0	0	0	0
N.S.	1	1.00	16.43	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.235	81.398	0.000	0.000	0.000	0.000	0.000	0.000

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	173	167	159	0	0	0	0	0	0
N.S.	1	0.97	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.226	2.147	0.000	0.000	0.000	0.000	0.000	0.000

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	147	147	119	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.238	0.824	0.000	0.000	0.000	0.000	0.000	0.000

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	65	0	0	0	0	0	0
N.S.	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.076	0.256	0.000	0.000	0.000	0.000	0.000	0.000

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	141	141	1158	0	0	0	0	0	0
N.S.	1	1.00	8.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.248	15.164	0.000	0.000	0.000	0.000	0.000	0.000

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	227	227	2453	0	0	0	0	0	0
N.S.	1	1.00	10.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.314	18.792	0.000	0.000	0.000	0.000	0.000	0.000

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	181	187	699	0	0	0	0	0	0
N.S.	1	1.03	3.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.221	10.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	161	462	286	420	0	0	0
N.S.	1	1.00	1.00	2.87	1.78	2.61	0.00	0.00	0.00
time (sec)	N/A	0.163	3.408	0.115	0.415	0.315	0.000	0.000	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	71	204	116	176	0	0	0
N.S.	1	1.00	0.81	2.32	1.32	2.00	0.00	0.00	0.00
time (sec)	N/A	0.094	0.205	109.936	0.555	0.293	0.000	0.000	0.000

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	27	26	64	0	0	51
N.S.	1	1.00	1.00	1.04	1.00	2.46	0.00	0.00	1.96
time (sec)	N/A	0.055	0.093	8.988	0.190	0.268	0.000	0.000	5.245

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	272	272	225	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.589	1.423	0.000	0.000	0.000	0.000	0.000	0.000

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	434	434	360	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.859	4.712	0.000	0.000	0.000	0.000	0.000	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	159	159	306	0	0	0	0	0	0
N.S.	1	1.00	1.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.233	14.538	0.000	0.000	0.000	0.000	0.000	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	159	159	340	0	0	0	0	0	0
N.S.	1	1.00	2.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.141	5.178	0.000	0.000	0.000	0.000	0.000	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	161	161	341	0	0	0	0	0	0
N.S.	1	1.00	2.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.176	6.574	0.000	0.000	0.000	0.000	0.000	0.000

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	161	161	341	0	0	0	0	0	0
N.S.	1	1.00	2.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.158	8.427	0.000	0.000	0.000	0.000	0.000	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	133	229	0	110	0	0	0
N.S.	1	1.00	1.07	1.85	0.00	0.89	0.00	0.00	0.00
time (sec)	N/A	0.178	1.243	9.940	0.000	0.086	0.000	0.000	0.000

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	250	205	0	80	0	0	0
N.S.	1	1.00	2.78	2.28	0.00	0.89	0.00	0.00	0.00
time (sec)	N/A	0.128	4.265	7.905	0.000	0.081	0.000	0.000	0.000

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	100	168	0	58	0	0	0
N.S.	1	1.00	1.11	1.87	0.00	0.64	0.00	0.00	0.00
time (sec)	N/A	0.122	0.666	5.496	0.000	0.079	0.000	0.000	0.000

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	192	108	0	27	0	0	0
N.S.	1	1.00	3.20	1.80	0.00	0.45	0.00	0.00	0.00
time (sec)	N/A	0.088	3.681	4.256	0.000	0.084	0.000	0.000	0.000

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	143	73	0	89	0	0	74
N.S.	1	1.00	2.38	1.22	0.00	1.48	0.00	0.00	1.23
time (sec)	N/A	0.098	2.237	3.056	0.000	0.079	0.000	0.000	0.661

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	92	190	214	0	151	0	0	0
N.S.	1	1.03	2.13	2.40	0.00	1.70	0.00	0.00	0.00
time (sec)	N/A	0.127	5.023	5.444	0.000	0.083	0.000	0.000	0.000

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	57	260	0	190	0	0	0
N.S.	1	1.00	0.59	2.71	0.00	1.98	0.00	0.00	0.00
time (sec)	N/A	0.128	1.605	6.702	0.000	0.082	0.000	0.000	0.000

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	666	385	0	235	0	0	0
N.S.	1	1.00	5.12	2.96	0.00	1.81	0.00	0.00	0.00
time (sec)	N/A	0.149	7.515	11.274	0.000	0.083	0.000	0.000	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	156	387	0	151	0	0	0
N.S.	1	1.00	0.82	2.04	0.00	0.79	0.00	0.00	0.00
time (sec)	N/A	0.276	2.628	9.441	0.000	0.102	0.000	0.000	0.000

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	464	351	0	140	0	0	0
N.S.	1	1.00	3.01	2.28	0.00	0.91	0.00	0.00	0.00
time (sec)	N/A	0.242	6.001	8.423	0.000	0.094	0.000	0.000	0.000

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	131	315	0	115	0	0	0
N.S.	1	1.00	0.85	2.05	0.00	0.75	0.00	0.00	0.00
time (sec)	N/A	0.230	1.946	7.257	0.000	0.088	0.000	0.000	0.000

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	126	268	277	0	102	0	0	0
N.S.	1	1.05	2.23	2.31	0.00	0.85	0.00	0.00	0.00
time (sec)	N/A	0.194	7.247	6.174	0.000	0.085	0.000	0.000	0.000

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	126	158	239	0	91	0	0	0
N.S.	1	1.05	1.32	1.99	0.00	0.76	0.00	0.00	0.00
time (sec)	N/A	0.195	1.898	5.089	0.000	0.081	0.000	0.000	0.000

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	114	206	0	94	0	0	0
N.S.	1	1.00	1.24	2.24	0.00	1.02	0.00	0.00	0.00
time (sec)	N/A	0.168	3.546	4.182	0.000	0.079	0.000	0.000	0.000

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	116	171	0	79	0	0	0
N.S.	1	1.00	1.26	1.86	0.00	0.86	0.00	0.00	0.00
time (sec)	N/A	0.201	1.557	3.523	0.000	0.086	0.000	0.000	0.000

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	1106	135	0	113	0	0	0
N.S.	1	1.00	9.07	1.11	0.00	0.93	0.00	0.00	0.00
time (sec)	N/A	0.252	7.390	3.715	0.000	0.081	0.000	0.000	0.000

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	67	208	0	152	0	0	0
N.S.	1	1.00	0.53	1.65	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.283	1.575	4.561	0.000	0.083	0.000	0.000	0.000

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	414	321	0	198	0	0	0
N.S.	1	1.00	2.52	1.96	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.296	5.212	7.546	0.000	0.084	0.000	0.000	0.000

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	80	78	202	100	0	0	96
N.S.	1	1.00	0.45	0.44	1.13	0.56	0.00	0.00	0.54
time (sec)	N/A	0.637	2.094	8.878	0.431	0.254	0.000	0.000	6.146

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	63	62	148	86	0	0	84
N.S.	1	1.00	0.48	0.47	1.12	0.65	0.00	0.00	0.64
time (sec)	N/A	0.332	1.709	8.991	0.393	0.265	0.000	0.000	0.836

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	86	56	50	59	68	0	0	88
N.S.	1	1.01	0.66	0.59	0.69	0.80	0.00	0.00	1.04
time (sec)	N/A	0.238	1.202	8.578	0.731	0.249	0.000	0.000	0.583

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	32	76	52	0	0	0
N.S.	1	1.00	1.00	0.89	2.11	1.44	0.00	0.00	0.00
time (sec)	N/A	0.153	1.104	7.312	0.495	0.261	0.000	0.000	0.000

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	335	125	155	1400	313	0	0	0
N.S.	1	1.00	0.37	0.46	4.18	0.93	0.00	0.00	0.00
time (sec)	N/A	0.265	1.794	9.278	0.496	0.269	0.000	0.000	0.000

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	524	620	274	319	1836	470	0	0	0
N.S.	1	1.18	0.52	0.61	3.50	0.90	0.00	0.00	0.00
time (sec)	N/A	0.714	5.149	10.437	0.468	0.266	0.000	0.000	0.000

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	512	512	227	359	2249	569	0	0	0
N.S.	1	1.00	0.44	0.70	4.39	1.11	0.00	0.00	0.00
time (sec)	N/A	0.686	3.708	10.117	0.571	0.269	0.000	0.000	0.000

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	719	719	305	459	2661	657	0	0	0
N.S.	1	1.00	0.42	0.64	3.70	0.91	0.00	0.00	0.00
time (sec)	N/A	1.032	4.506	8.771	0.594	0.277	0.000	0.000	0.000

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	80	70	202	103	0	0	110
N.S.	1	1.00	0.46	0.40	1.15	0.59	0.00	0.00	0.63
time (sec)	N/A	0.595	2.061	7.941	0.799	0.260	0.000	0.000	6.142

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	63	59	136	83	0	0	100
N.S.	1	1.00	0.50	0.47	1.08	0.66	0.00	0.00	0.79
time (sec)	N/A	0.470	1.597	8.204	0.686	0.242	0.000	0.000	1.225

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	48	42	80	68	0	0	82
N.S.	1	1.00	0.60	0.52	1.00	0.85	0.00	0.00	1.02
time (sec)	N/A	0.306	1.032	7.824	0.391	0.247	0.000	0.000	0.797

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	32	76	58	0	0	0
N.S.	1	1.00	1.00	0.89	2.11	1.61	0.00	0.00	0.00
time (sec)	N/A	0.205	1.038	10.041	0.325	0.269	0.000	0.000	0.000

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	495	495	209	161	714	341	0	0	0
N.S.	1	1.00	0.42	0.33	1.44	0.69	0.00	0.00	0.00
time (sec)	N/A	0.475	2.588	12.923	0.489	0.268	0.000	0.000	0.000

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	470	470	250	765	2147	502	0	0	0
N.S.	1	1.00	0.53	1.63	4.57	1.07	0.00	0.00	0.00
time (sec)	N/A	0.516	3.366	12.387	0.489	0.290	0.000	0.000	0.000

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	682	682	245	502	2254	604	0	0	0
N.S.	1	1.00	0.36	0.74	3.30	0.89	0.00	0.00	0.00
time (sec)	N/A	0.865	3.420	12.958	0.873	0.274	0.000	0.000	0.000

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	202	0	0	0	0	0	0
N.S.	1	1.00	1.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.278	14.502	0.000	0.000	0.000	0.000	0.000	0.000

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	86	86	86	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.251	9.094	0.000	0.000	0.000	0.000	0.000	0.000

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	82	82	78	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.207	0.344	0.000	0.000	0.000	0.000	0.000	0.000

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	86	86	433	0	0	0	0	0	0
N.S.	1	1.00	5.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.289	7.522	0.000	0.000	0.000	0.000	0.000	0.000

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	86	86	100	0	0	0	0	0	0
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.326	3.561	0.000	0.000	0.000	0.000	0.000	0.000

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	122	0	0	0	0	0	0
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.390	2.502	0.000	0.000	0.000	0.000	0.000	0.000

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	104	104	143	0	0	0	0	0	0
N.S.	1	1.00	1.38	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.398	2.716	0.000	0.000	0.000	0.000	0.000	0.000

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	183	175	212	0	0	0	0	0	0
N.S.	1	0.96	1.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.337	6.571	0.000	0.000	0.000	0.000	0.000	0.000

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	155	155	140	0	0	0	0	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.283	5.484	0.000	0.000	0.000	0.000	0.000	0.000

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	90	91	75	0	0	0	0	0	0
N.S.	1	1.01	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.128	0.183	0.000	0.000	0.000	0.000	0.000	0.000

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	140	140	1132	0	0	0	0	0	0
N.S.	1	1.00	8.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.295	14.207	0.000	0.000	0.000	0.000	0.000	0.000

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	227	227	2502	0	0	0	0	0	0
N.S.	1	1.00	11.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.354	17.714	0.000	0.000	0.000	0.000	0.000	0.000

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	187	187	365	0	0	0	0	0	0
N.S.	1	1.00	1.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.264	23.409	0.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [616] had the largest ratio of [.6800000000000000049]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.00	22	0.091
2	A	3	2	1.00	22	0.091
3	A	3	2	1.00	22	0.091
4	A	3	2	1.00	22	0.091
5	A	3	3	1.11	22	0.136
6	A	2	1	1.00	13	0.077
7	A	3	3	1.00	22	0.136
8	A	4	3	1.00	22	0.136
9	A	5	3	1.00	22	0.136
10	A	6	3	1.00	22	0.136
11	A	5	3	1.00	22	0.136
12	A	4	3	1.00	22	0.136
13	A	3	3	1.00	22	0.136
14	A	2	2	1.00	20	0.100
15	A	2	2	1.00	20	0.100
16	A	3	2	1.00	22	0.091
17	A	3	2	1.00	22	0.091
18	A	3	2	1.00	22	0.091
19	A	3	2	1.00	24	0.083
20	A	3	2	1.00	24	0.083
21	A	3	2	1.00	24	0.083
22	A	2	2	1.00	24	0.083
23	A	2	2	1.00	15	0.133
24	A	2	2	1.00	24	0.083

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	4	3	1.00	24	0.125
26	A	4	3	1.00	24	0.125
27	A	4	3	1.00	24	0.125
28	A	5	4	1.00	24	0.167
29	A	4	4	1.00	24	0.167
30	A	3	3	1.00	22	0.136
31	A	2	2	1.00	22	0.091
32	A	2	2	1.00	24	0.083
33	A	3	2	1.00	24	0.083
34	A	3	2	1.00	24	0.083
35	A	3	2	1.00	24	0.083
36	A	3	2	1.00	24	0.083
37	A	3	2	1.00	24	0.083
38	A	3	2	1.00	24	0.083
39	A	2	2	1.00	24	0.083
40	A	3	3	1.00	15	0.200
41	A	3	2	1.00	24	0.083
42	A	2	2	1.00	24	0.083
43	A	4	3	1.00	24	0.125
44	A	4	3	1.00	24	0.125
45	A	5	4	1.00	24	0.167
46	A	4	3	1.00	22	0.136
47	A	3	3	1.00	22	0.136
48	A	1	1	1.00	24	0.042
49	A	4	3	1.00	24	0.125
50	A	4	3	1.00	24	0.125
51	A	4	3	1.00	24	0.125
52	A	6	4	1.00	24	0.167
53	A	5	3	1.00	22	0.136
54	A	4	4	1.00	22	0.182
55	A	3	2	1.00	24	0.083
56	A	2	2	1.00	24	0.083
57	A	4	2	1.00	24	0.083
58	A	4	2	1.00	24	0.083
59	A	3	2	1.00	24	0.083

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	3	2	1.00	24	0.083
61	A	3	2	1.00	24	0.083
62	A	2	2	1.00	24	0.083
63	A	5	3	1.00	15	0.200
64	A	3	2	1.00	24	0.083
65	A	3	2	1.00	24	0.083
66	A	3	2	1.00	24	0.083
67	A	2	2	1.00	24	0.083
68	A	4	3	1.00	24	0.125
69	A	4	3	1.00	24	0.125
70	A	6	3	1.00	22	0.136
71	A	5	4	1.00	22	0.182
72	A	4	3	1.00	24	0.125
73	A	1	1	1.00	24	0.042
74	A	3	2	1.00	24	0.083
75	A	5	3	1.00	24	0.125
76	A	5	3	1.00	24	0.125
77	A	3	2	1.00	24	0.083
78	A	3	2	1.00	24	0.083
79	A	3	2	1.00	24	0.083
80	A	2	2	1.00	24	0.083
81	A	8	3	1.00	15	0.200
82	A	3	2	1.00	24	0.083
83	A	3	2	1.00	24	0.083
84	A	3	2	1.00	24	0.083
85	A	2	2	1.00	24	0.083
86	A	3	2	1.00	24	0.083
87	A	3	2	1.00	24	0.083
88	A	2	2	1.00	24	0.083
89	A	4	3	1.00	24	0.125
90	A	4	3	1.00	24	0.125
91	A	8	4	1.00	22	0.182
92	A	7	4	1.00	24	0.167
93	A	6	4	1.00	24	0.167
94	A	5	2	1.00	24	0.083

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	2	2	1.00	24	0.083
96	A	4	2	1.00	24	0.083
97	A	6	2	1.00	24	0.083
98	A	6	2	1.00	24	0.083
99	A	3	2	1.00	24	0.083
100	A	3	2	1.00	24	0.083
101	A	3	2	1.00	24	0.083
102	A	2	1	1.00	24	0.042
103	A	2	2	1.00	24	0.083
104	A	2	2	1.00	15	0.133
105	A	4	3	1.00	24	0.125
106	A	4	3	1.00	24	0.125
107	A	4	3	1.00	24	0.125
108	A	3	3	1.00	24	0.125
109	A	2	2	1.00	24	0.083
110	A	1	1	1.00	22	0.045
111	A	2	2	1.00	22	0.091
112	A	3	2	1.00	24	0.083
113	A	3	2	1.00	24	0.083
114	A	3	2	1.00	24	0.083
115	A	3	2	1.00	24	0.083
116	A	2	2	1.00	24	0.083
117	A	3	2	1.00	24	0.083
118	A	2	2	1.00	24	0.083
119	A	3	2	1.00	15	0.133
120	A	4	3	1.00	24	0.125
121	A	4	3	1.00	24	0.125
122	A	5	3	1.00	24	0.125
123	A	4	3	1.00	24	0.125
124	A	3	3	1.00	24	0.125
125	A	2	2	1.00	24	0.083
126	A	2	2	1.00	22	0.091
127	A	3	2	1.00	22	0.091
128	A	3	2	1.00	24	0.083
129	A	3	2	1.00	24	0.083

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	3	2	1.00	24	0.083
131	A	3	2	1.00	24	0.083
132	A	3	2	1.00	24	0.083
133	A	2	2	1.00	24	0.083
134	A	3	2	1.00	24	0.083
135	A	3	2	1.00	24	0.083
136	A	2	2	1.00	24	0.083
137	A	4	2	1.00	15	0.133
138	A	4	3	1.00	24	0.125
139	A	4	3	1.00	24	0.125
140	A	5	4	1.00	24	0.167
141	A	4	4	1.00	24	0.167
142	A	3	3	1.00	24	0.125
143	A	1	1	1.00	24	0.042
144	A	3	2	1.00	22	0.091
145	A	4	3	1.00	22	0.136
146	A	4	3	1.00	24	0.125
147	A	4	3	1.00	24	0.125
148	A	3	2	1.00	24	0.083
149	A	3	2	1.00	24	0.083
150	A	2	2	1.00	24	0.083
151	A	3	2	1.00	24	0.083
152	A	3	2	1.00	24	0.083
153	A	2	2	1.00	24	0.083
154	A	2	2	1.00	24	0.083
155	A	5	2	1.00	15	0.133
156	A	4	3	1.00	24	0.125
157	A	4	3	1.00	24	0.125
158	A	5	3	1.00	24	0.125
159	A	4	3	1.00	24	0.125
160	A	3	2	1.00	24	0.083
161	A	2	2	1.00	24	0.083
162	A	4	2	1.00	22	0.091
163	A	5	3	1.00	22	0.136
164	A	5	3	1.00	24	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
165	A	5	3	1.00	24	0.125
166	A	3	2	1.00	24	0.083
167	A	3	2	1.00	24	0.083
168	A	3	2	1.00	24	0.083
169	A	2	2	1.00	24	0.083
170	A	3	2	1.00	24	0.083
171	A	3	2	1.00	24	0.083
172	A	2	2	1.00	24	0.083
173	A	9	2	1.00	15	0.133
174	A	4	3	1.00	24	0.125
175	A	4	3	1.00	24	0.125
176	A	7	3	1.00	24	0.125
177	A	6	3	1.00	24	0.125
178	A	5	2	1.00	24	0.083
179	A	2	2	1.00	24	0.083
180	A	4	2	1.00	24	0.083
181	A	6	2	1.00	24	0.083
182	A	8	2	1.00	22	0.091
183	A	9	3	1.00	22	0.136
184	A	9	3	1.00	24	0.125
185	A	5	4	1.00	26	0.154
186	A	4	4	1.00	26	0.154
187	A	4	4	1.00	26	0.154
188	A	3	3	1.00	26	0.115
189	A	3	3	1.00	26	0.115
190	A	4	4	1.00	26	0.154
191	A	4	4	1.00	26	0.154
192	A	5	4	1.00	26	0.154
193	A	5	5	1.00	28	0.179
194	A	4	4	1.00	28	0.143
195	A	4	4	1.00	28	0.143
196	A	3	3	1.00	28	0.107
197	A	3	3	1.00	28	0.107
198	A	4	4	1.00	28	0.143
199	A	4	4	1.00	28	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
200	A	5	4	1.00	28	0.143
201	A	7	5	1.00	28	0.179
202	A	6	5	1.00	28	0.179
203	A	6	5	1.00	28	0.179
204	A	5	4	1.00	28	0.143
205	A	5	5	1.18	28	0.179
206	A	4	4	1.00	28	0.143
207	A	4	4	1.00	28	0.143
208	A	4	4	1.00	28	0.143
209	A	4	4	1.00	28	0.143
210	A	5	5	1.00	28	0.179
211	A	5	5	1.00	28	0.179
212	A	6	5	1.00	28	0.179
213	A	7	5	1.00	28	0.179
214	A	6	4	1.00	28	0.143
215	A	6	6	1.00	28	0.214
216	A	5	5	1.00	28	0.179
217	A	5	4	1.00	28	0.143
218	A	4	3	1.00	28	0.107
219	A	4	3	1.00	28	0.107
220	A	5	4	1.00	28	0.143
221	A	5	4	1.00	28	0.143
222	A	6	4	1.00	28	0.143
223	A	5	4	1.00	28	0.143
224	A	4	4	1.00	28	0.143
225	A	4	4	1.00	28	0.143
226	A	3	3	1.00	28	0.107
227	A	3	3	1.00	28	0.107
228	A	3	3	1.00	28	0.107
229	A	3	3	1.00	28	0.107
230	A	4	4	1.00	28	0.143
231	A	4	4	1.00	28	0.143
232	A	5	4	1.00	28	0.143
233	A	6	4	1.00	28	0.143
234	A	5	4	1.00	28	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
235	A	5	4	1.00	28	0.143
236	A	4	4	1.00	28	0.143
237	A	4	4	1.00	28	0.143
238	A	3	3	1.00	28	0.107
239	A	3	3	1.00	28	0.107
240	A	4	4	1.00	28	0.143
241	A	4	4	1.00	28	0.143
242	A	5	4	1.00	28	0.143
243	A	5	4	1.00	28	0.143
244	A	6	4	1.00	28	0.143
245	A	6	5	1.00	28	0.179
246	A	5	5	1.00	28	0.179
247	A	5	5	1.00	28	0.179
248	A	4	4	1.00	28	0.143
249	A	4	4	1.00	28	0.143
250	A	4	4	1.00	28	0.143
251	A	4	4	1.00	28	0.143
252	A	5	5	1.00	28	0.179
253	A	5	5	1.00	28	0.179
254	A	6	5	1.00	28	0.179
255	A	6	4	1.00	28	0.143
256	A	5	4	1.00	28	0.143
257	A	5	4	1.00	28	0.143
258	A	4	3	1.00	28	0.107
259	A	4	3	1.00	28	0.107
260	A	5	4	1.00	28	0.143
261	A	5	4	1.00	28	0.143
262	A	6	5	1.00	28	0.179
263	A	4	4	1.00	26	0.154
264	A	4	4	1.00	26	0.154
265	A	4	4	1.00	26	0.154
266	A	4	4	1.00	26	0.154
267	A	4	4	1.00	28	0.143
268	A	4	4	1.00	28	0.143
269	A	4	4	1.00	28	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
270	A	4	4	1.00	28	0.143
271	A	4	4	1.00	28	0.143
272	A	4	4	1.00	28	0.143
273	A	4	4	1.00	28	0.143
274	A	4	4	1.00	28	0.143
275	A	4	4	1.00	28	0.143
276	A	4	4	1.00	28	0.143
277	A	4	4	1.00	28	0.143
278	A	4	4	1.00	28	0.143
279	A	3	2	1.00	26	0.077
280	A	3	2	1.00	26	0.077
281	A	3	2	1.00	26	0.077
282	A	2	2	1.00	26	0.077
283	A	5	5	1.00	26	0.192
284	A	7	5	1.00	26	0.192
285	A	9	5	1.00	26	0.192
286	A	4	2	1.00	26	0.077
287	A	3	2	1.00	26	0.077
288	A	2	2	1.00	26	0.077
289	A	1	1	1.00	24	0.042
290	A	3	3	1.00	24	0.125
291	A	5	5	1.00	26	0.192
292	A	7	5	1.00	26	0.192
293	A	3	2	1.00	26	0.077
294	A	3	2	1.00	26	0.077
295	A	3	2	1.00	26	0.077
296	A	2	2	1.00	26	0.077
297	A	4	4	1.00	26	0.154
298	A	6	5	1.00	26	0.192
299	A	8	5	1.00	26	0.192
300	A	4	2	1.00	26	0.077
301	A	3	2	1.00	26	0.077
302	A	2	2	1.00	24	0.083
303	A	1	1	1.00	24	0.042
304	A	4	3	1.00	26	0.115

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
305	A	6	5	1.00	26	0.192
306	A	3	2	1.00	26	0.077
307	A	3	2	1.00	26	0.077
308	A	3	2	1.00	26	0.077
309	A	2	2	1.00	26	0.077
310	A	4	4	1.00	26	0.154
311	A	5	4	1.00	26	0.154
312	A	7	5	1.00	26	0.192
313	A	4	2	1.00	26	0.077
314	A	3	2	1.00	24	0.083
315	A	2	2	1.00	24	0.083
316	A	1	1	1.00	26	0.038
317	A	5	3	1.00	26	0.115
318	A	7	5	1.00	26	0.192
319	A	3	2	1.00	26	0.077
320	A	3	2	1.00	26	0.077
321	A	3	2	1.00	26	0.077
322	A	2	2	1.00	26	0.077
323	A	5	5	1.00	26	0.192
324	A	5	5	1.00	26	0.192
325	A	6	4	1.00	26	0.154
326	A	4	2	1.00	24	0.083
327	A	3	2	1.00	24	0.083
328	A	2	2	1.00	26	0.077
329	A	1	1	1.00	26	0.038
330	A	6	3	1.00	26	0.115
331	A	8	5	1.00	26	0.192
332	A	10	5	1.00	26	0.192
333	A	3	2	1.00	26	0.077
334	A	3	2	1.00	26	0.077
335	A	3	2	1.00	26	0.077
336	A	2	2	1.00	26	0.077
337	A	6	5	1.00	26	0.192
338	A	8	5	1.00	26	0.192
339	A	10	5	1.00	26	0.192

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
340	A	4	2	1.00	26	0.077
341	A	3	2	1.00	26	0.077
342	A	2	2	1.00	26	0.077
343	A	1	1	1.00	26	0.038
344	A	2	2	1.00	24	0.083
345	A	4	4	1.00	24	0.167
346	A	6	5	1.00	26	0.192
347	A	3	2	1.00	26	0.077
348	A	3	2	1.00	26	0.077
349	A	3	2	1.00	26	0.077
350	A	2	2	1.00	26	0.077
351	A	7	5	1.00	26	0.192
352	A	9	5	1.00	26	0.192
353	A	11	5	1.00	26	0.192
354	A	4	2	1.00	26	0.077
355	A	3	2	1.00	26	0.077
356	A	2	2	1.00	26	0.077
357	A	1	1	1.00	26	0.038
358	A	3	3	1.00	26	0.115
359	A	3	3	1.00	24	0.125
360	A	5	4	1.00	24	0.167
361	A	7	5	1.00	26	0.192
362	A	3	2	1.00	26	0.077
363	A	3	2	1.00	26	0.077
364	A	3	2	1.00	26	0.077
365	A	3	2	1.00	26	0.077
366	A	2	2	1.00	26	0.077
367	A	8	5	1.00	26	0.192
368	A	10	5	1.00	26	0.192
369	A	4	2	1.00	26	0.077
370	A	3	2	1.00	26	0.077
371	A	2	2	1.00	26	0.077
372	A	1	1	1.00	26	0.038
373	A	4	3	1.00	26	0.115
374	A	4	4	1.00	26	0.154

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
375	A	4	3	1.00	24	0.125
376	A	6	4	1.00	24	0.167
377	A	8	5	1.00	26	0.192
378	A	3	2	1.00	26	0.077
379	A	3	2	1.00	26	0.077
380	A	3	2	1.00	26	0.077
381	A	3	2	1.00	26	0.077
382	A	2	2	1.00	26	0.077
383	A	9	5	1.00	26	0.192
384	A	11	5	1.00	26	0.192
385	A	3	2	1.00	26	0.077
386	A	2	2	1.00	26	0.077
387	A	1	1	1.00	26	0.038
388	A	5	3	1.00	26	0.115
389	A	5	4	1.00	26	0.154
390	A	5	4	1.00	26	0.154
391	A	5	3	1.00	24	0.125
392	A	7	4	1.00	24	0.167
393	A	9	5	1.00	26	0.192
394	A	12	9	1.00	30	0.300
395	A	10	7	1.00	30	0.233
396	A	1	1	1.00	30	0.033
397	A	2	2	1.00	30	0.067
398	A	3	3	1.00	30	0.100
399	A	4	3	1.00	30	0.100
400	A	13	9	1.00	30	0.300
401	A	13	9	1.00	30	0.300
402	A	11	8	1.00	30	0.267
403	A	12	9	1.00	30	0.300
404	A	1	1	1.00	30	0.033
405	A	2	2	1.00	30	0.067
406	A	3	2	1.00	30	0.067
407	A	4	3	1.00	30	0.100
408	A	14	9	1.00	30	0.300
409	A	12	8	1.00	30	0.267

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
410	A	13	10	1.00	30	0.333
411	A	11	8	1.00	30	0.267
412	A	1	1	1.00	30	0.033
413	A	2	2	1.00	30	0.067
414	A	3	2	1.00	30	0.067
415	A	4	2	1.00	30	0.067
416	A	11	8	1.00	30	0.267
417	A	11	8	1.00	30	0.267
418	A	1	1	1.00	30	0.033
419	A	2	2	1.00	30	0.067
420	A	3	3	1.00	30	0.100
421	A	4	3	1.00	30	0.100
422	A	5	3	1.00	30	0.100
423	A	13	10	1.00	30	0.333
424	A	11	8	1.00	30	0.267
425	A	1	1	1.00	30	0.033
426	A	2	2	1.00	30	0.067
427	A	3	2	1.00	30	0.067
428	A	4	3	1.00	30	0.100
429	A	5	3	1.00	30	0.100
430	A	12	9	1.00	30	0.300
431	A	12	9	1.00	30	0.300
432	A	1	1	1.00	30	0.033
433	A	2	2	1.00	30	0.067
434	A	3	2	1.00	30	0.067
435	A	4	2	1.00	30	0.067
436	A	5	3	1.00	30	0.100
437	A	4	4	1.00	30	0.133
438	A	4	4	1.00	30	0.133
439	A	4	4	1.00	30	0.133
440	A	4	4	1.00	30	0.133
441	A	4	4	1.00	30	0.133
442	A	4	4	1.00	30	0.133
443	A	9	8	1.00	30	0.267
444	A	8	8	1.00	30	0.267

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
445	A	6	6	1.00	30	0.200
446	A	1	1	1.00	30	0.033
447	A	2	2	1.00	30	0.067
448	A	3	2	1.00	30	0.067
449	A	4	2	1.00	30	0.067
450	A	4	4	1.00	26	0.154
451	A	4	4	1.00	26	0.154
452	A	4	4	1.00	26	0.154
453	A	4	4	1.00	24	0.167
454	A	4	4	1.00	26	0.154
455	A	4	4	1.00	26	0.154
456	A	4	4	1.00	26	0.154
457	A	4	4	1.00	28	0.143
458	A	4	4	1.00	28	0.143
459	A	4	4	1.00	28	0.143
460	A	4	4	1.00	28	0.143
461	A	4	4	1.00	28	0.143
462	A	4	4	1.00	28	0.143
463	A	4	4	1.00	28	0.143
464	A	4	4	1.00	26	0.154
465	A	3	2	1.00	24	0.083
466	A	3	2	1.00	24	0.083
467	A	2	2	1.00	24	0.083
468	A	2	2	1.00	24	0.083
469	A	2	2	1.00	24	0.083
470	A	2	2	1.00	24	0.083
471	A	4	4	1.00	24	0.167
472	A	4	4	1.00	24	0.167
473	A	4	4	1.00	22	0.182
474	A	4	4	1.00	22	0.182
475	A	4	4	1.00	24	0.167
476	A	4	4	1.00	24	0.167
477	A	4	4	1.00	28	0.143
478	A	4	4	1.00	28	0.143
479	A	4	4	1.00	28	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
480	A	4	4	1.00	28	0.143
481	A	4	4	1.00	28	0.143
482	A	4	4	1.00	28	0.143
483	A	5	2	1.00	30	0.067
484	A	4	2	1.00	30	0.067
485	A	3	2	1.00	30	0.067
486	A	2	2	1.00	30	0.067
487	A	1	1	1.00	28	0.036
488	A	4	4	1.00	30	0.133
489	A	4	4	1.00	30	0.133
490	A	4	4	1.00	30	0.133
491	A	3	2	1.00	30	0.067
492	A	5	5	1.00	30	0.167
493	A	2	2	1.00	30	0.067
494	A	5	5	1.00	30	0.167
495	A	1	1	1.00	30	0.033
496	A	5	5	1.00	30	0.167
497	A	3	3	1.00	28	0.107
498	A	5	5	1.00	30	0.167
499	A	4	4	1.00	30	0.133
500	A	5	5	1.00	30	0.167
501	A	4	4	1.00	32	0.125
502	A	4	4	1.00	32	0.125
503	A	3	3	1.00	30	0.100
504	A	1	1	1.00	32	0.031
505	A	2	2	1.00	32	0.062
506	A	3	2	1.00	32	0.062
507	A	3	2	1.00	19	0.105
508	A	4	3	1.00	19	0.158
509	A	3	2	1.00	19	0.105
510	A	3	3	1.00	19	0.158
511	A	3	3	1.00	19	0.158
512	A	2	2	1.00	17	0.118
513	A	2	2	1.00	17	0.118
514	A	3	3	1.00	19	0.158

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
515	A	3	2	1.00	19	0.105
516	A	4	3	1.00	19	0.158
517	A	4	3	1.00	21	0.143
518	A	4	3	1.00	21	0.143
519	A	3	2	1.00	21	0.095
520	A	2	2	1.00	21	0.095
521	A	3	3	1.00	21	0.143
522	A	4	4	1.00	21	0.190
523	A	6	4	1.00	21	0.190
524	A	5	4	1.00	21	0.190
525	A	4	4	1.00	21	0.190
526	A	3	3	1.00	19	0.158
527	A	1	1	1.00	19	0.053
528	A	4	3	1.00	21	0.143
529	A	4	3	1.00	21	0.143
530	A	4	3	1.00	21	0.143
531	A	4	3	1.00	21	0.143
532	A	3	2	1.00	21	0.095
533	A	3	2	1.00	21	0.095
534	A	2	2	1.00	21	0.095
535	A	6	6	1.00	21	0.286
536	A	4	4	1.00	21	0.190
537	A	6	5	1.11	21	0.238
538	A	5	5	1.14	21	0.238
539	A	4	4	1.20	19	0.210
540	A	4	4	1.21	19	0.210
541	A	3	3	1.00	21	0.143
542	A	4	4	1.00	21	0.190
543	A	5	5	1.00	21	0.238
544	A	3	2	1.00	21	0.095
545	A	3	2	1.00	21	0.095
546	A	2	2	1.00	21	0.095
547	A	7	6	1.00	21	0.286
548	A	8	7	1.00	21	0.333
549	A	9	6	1.09	21	0.286

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
550	A	5	5	1.00	21	0.238
551	A	2	2	1.00	19	0.105
552	A	5	5	1.00	19	0.263
553	A	9	6	1.00	21	0.286
554	A	3	2	1.00	21	0.095
555	A	3	2	1.00	21	0.095
556	A	3	2	1.00	21	0.095
557	A	2	2	1.00	21	0.095
558	A	7	6	1.00	21	0.286
559	A	8	7	1.00	21	0.333
560	A	8	7	1.00	21	0.333
561	A	7	7	1.00	21	0.333
562	A	6	6	1.45	21	0.286
563	A	4	4	1.28	19	0.210
564	A	5	5	1.00	19	0.263
565	A	6	6	1.00	21	0.286
566	A	3	2	1.00	21	0.095
567	A	3	2	1.00	21	0.095
568	A	3	2	1.00	21	0.095
569	A	2	2	1.00	21	0.095
570	A	7	6	1.00	21	0.286
571	A	8	7	1.00	21	0.333
572	A	8	8	1.00	21	0.381
573	A	7	7	1.28	21	0.333
574	A	4	4	1.24	21	0.190
575	A	5	5	1.00	19	0.263
576	A	6	6	1.00	19	0.316
577	A	7	7	1.00	21	0.333
578	A	5	4	1.00	23	0.174
579	A	4	4	1.00	23	0.174
580	A	4	4	1.00	23	0.174
581	A	3	3	1.00	23	0.130
582	A	3	3	1.00	23	0.130
583	A	4	4	1.00	23	0.174
584	A	4	4	1.00	23	0.174

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
585	A	5	4	1.00	23	0.174
586	A	5	5	1.00	25	0.200
587	A	5	5	1.00	25	0.200
588	A	4	4	1.00	25	0.160
589	A	4	4	1.00	25	0.160
590	A	5	5	1.00	25	0.200
591	A	5	5	1.00	25	0.200
592	A	6	5	1.00	25	0.200
593	A	6	5	1.00	25	0.200
594	A	5	5	1.00	25	0.200
595	A	5	5	1.00	25	0.200
596	A	4	4	1.00	25	0.160
597	A	5	5	1.00	25	0.200
598	A	4	4	1.00	25	0.160
599	A	5	5	1.00	25	0.200
600	A	4	4	1.00	25	0.160
601	A	4	4	1.00	25	0.160
602	A	5	5	1.00	25	0.200
603	A	17	15	1.00	25	0.600
604	A	17	15	1.00	25	0.600
605	A	13	11	1.00	25	0.440
606	A	14	12	1.00	25	0.480
607	A	17	15	1.00	25	0.600
608	A	17	15	1.00	25	0.600
609	A	18	16	1.00	25	0.640
610	A	17	15	1.00	25	0.600
611	A	17	15	1.00	25	0.600
612	A	17	15	1.00	25	0.600
613	A	17	15	1.00	25	0.600
614	A	18	16	1.00	25	0.640
615	A	18	16	1.00	25	0.640
616	A	19	17	1.00	25	0.680
617	A	18	16	1.00	25	0.640
618	A	18	16	1.00	25	0.640
619	A	18	16	1.00	25	0.640

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
620	A	18	16	1.00	25	0.640
621	A	19	16	1.00	25	0.640
622	A	19	16	1.00	25	0.640
623	A	20	17	1.00	25	0.680
624	A	3	3	1.00	23	0.130
625	A	3	3	1.00	23	0.130
626	A	3	3	1.00	23	0.130
627	A	3	3	1.00	23	0.130
628	A	4	4	1.00	25	0.160
629	A	4	4	1.00	25	0.160
630	A	4	4	1.00	25	0.160
631	A	4	4	1.00	25	0.160
632	A	16	11	1.00	25	0.440
633	A	16	11	1.00	25	0.440
634	A	17	12	1.00	25	0.480
635	A	17	12	1.00	25	0.480
636	A	18	13	1.00	25	0.520
637	A	18	13	1.00	25	0.520
638	A	19	14	1.00	25	0.560
639	A	19	14	1.00	25	0.560
640	A	4	4	0.97	23	0.174
641	A	4	4	1.00	23	0.174
642	A	3	3	1.00	21	0.143
643	A	6	5	1.00	23	0.217
644	A	7	6	1.00	23	0.261
645	A	3	3	1.03	23	0.130
646	A	3	2	1.00	21	0.095
647	A	3	2	1.00	21	0.095
648	A	2	2	1.00	21	0.095
649	A	6	4	1.00	21	0.190
650	A	7	5	1.00	21	0.238
651	A	3	3	1.00	21	0.143
652	A	3	3	1.00	19	0.158
653	A	3	3	1.00	19	0.158
654	A	3	3	1.00	21	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
655	A	6	5	1.00	26	0.192
656	A	5	5	1.00	26	0.192
657	A	5	5	1.00	26	0.192
658	A	4	4	1.00	26	0.154
659	A	4	4	1.00	26	0.154
660	A	5	5	1.03	26	0.192
661	A	5	5	1.00	26	0.192
662	A	6	5	1.00	26	0.192
663	A	7	5	1.00	28	0.179
664	A	6	5	1.00	28	0.179
665	A	6	5	1.00	28	0.179
666	A	5	5	1.05	28	0.179
667	A	5	5	1.05	28	0.179
668	A	4	4	1.00	28	0.143
669	A	4	4	1.00	28	0.143
670	A	5	5	1.00	28	0.179
671	A	5	5	1.00	28	0.179
672	A	6	5	1.00	28	0.179
673	A	5	4	1.00	30	0.133
674	A	4	4	1.00	30	0.133
675	A	3	3	1.01	30	0.100
676	A	2	2	1.00	30	0.067
677	A	10	7	1.00	30	0.233
678	A	13	10	1.18	30	0.333
679	A	13	10	1.00	30	0.333
680	A	15	11	1.00	30	0.367
681	A	5	4	1.00	30	0.133
682	A	4	4	1.00	30	0.133
683	A	3	3	1.00	30	0.100
684	A	2	2	1.00	30	0.067
685	A	11	8	1.00	30	0.267
686	A	12	9	1.00	30	0.300
687	A	14	11	1.00	30	0.367
688	A	5	5	1.00	26	0.192
689	A	5	5	1.00	26	0.192

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
690	A	5	5	1.00	24	0.208
691	A	5	5	1.00	26	0.192
692	A	5	5	1.00	26	0.192
693	A	5	5	1.00	28	0.179
694	A	5	5	1.00	28	0.179
695	A	5	5	0.96	23	0.217
696	A	5	5	1.00	23	0.217
697	A	4	4	1.01	21	0.190
698	A	7	6	1.00	23	0.261
699	A	8	7	1.00	23	0.304
700	A	4	4	1.00	23	0.174

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \sec^{10}(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 94

$$\int \sec^{10}(c+dx)(a+ia\tan(c+dx))\,dx = \frac{ia\sec^{10}(c+dx)}{10d}+\frac{a\tan(c+dx)}{d}+\frac{4a\tan^3(c+dx)}{3d}+\frac{6a\tan^5(c+dx)}{5d}+\frac{4a\tan^7(c+dx)}{7d}+\frac{a\tan^9(c+dx)}{9d}$$

```
[Out] 1/10*I*a*sec(d*x+c)^10/d+a*tan(d*x+c)/d+4/3*a*tan(d*x+c)^3/d+6/5*a*tan(d*x+
c)^5/d+4/7*a*tan(d*x+c)^7/d+1/9*a*tan(d*x+c)^9/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3567, 3852}

$$\int \sec^{10}(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a\tan^9(c+dx)}{9d}+\frac{4a\tan^7(c+dx)}{7d}+\frac{6a\tan^5(c+dx)}{5d}+\frac{4a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{ia\sec^{10}(c+dx)}{10d}$$

```
[In] Int[Sec[c + d*x]^10*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((I/10)*a*Sec[c + d*x]^10)/d + (a*Tan[c + d*x])/d + (4*a*Tan[c + d*x]^3)/(3
*d) + (6*a*Tan[c + d*x]^5)/(5*d) + (4*a*Tan[c + d*x]^7)/(7*d) + (a*Tan[c +
d*x]^9)/(9*d)
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{ia\sec^{10}(c+dx)}{10d} + a\int \sec^{10}(c+dx)\,dx \\
&= \frac{ia\sec^{10}(c+dx)}{10d} - \frac{a\text{Subst}\left(\int \left(1+4x^2+6x^4+4x^6+x^8\right)\,dx, x, -\tan(c+dx)\right)}{d} \\
&= \frac{ia\sec^{10}(c+dx)}{10d} + \frac{a\tan(c+dx)}{d} + \frac{4a\tan^3(c+dx)}{3d} \\
&\quad + \frac{6a\tan^5(c+dx)}{5d} + \frac{4a\tan^7(c+dx)}{7d} + \frac{a\tan^9(c+dx)}{9d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.84

$$\begin{aligned}
&\int \sec^{10}(c+dx)(a+ia\tan(c+dx))\,dx \\
&= \frac{ia\sec^{10}(c+dx)}{10d} \\
&\quad + \frac{a\left(\tan(c+dx)+\frac{4}{3}\tan^3(c+dx)+\frac{6}{5}\tan^5(c+dx)+\frac{4}{7}\tan^7(c+dx)+\frac{1}{9}\tan^9(c+dx)\right)}{d}
\end{aligned}$$

```
[In] Integrate[Sec[c + d*x]^10*(a + I*a*Tan[c + d*x]),x]
[Out] ((I/10)*a*Sec[c + d*x]^10)/d + (a*(Tan[c + d*x] + (4*Tan[c + d*x]^3)/3 + (6
*Tan[c + d*x]^5)/5 + (4*Tan[c + d*x]^7)/7 + Tan[c + d*x]^9/9))/d
```

Maple [A] (verified)

Time = 176.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.83

method	result
risch	$\frac{256 i a \left(252 \,{\mathrm e}^{10 i \left(d x +c \right)}+210 \,{\mathrm e}^{8 i \left(d x +c \right)}+120 \,{\mathrm e}^{6 i \left(d x +c \right)}+45 \,{\mathrm e}^{4 i \left(d x +c \right)}+10 \,{\mathrm e}^{2 i \left(d x +c \right)}+1\right)}{315 d \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)^{10}}$
derivativedivides	$\frac{a \left(\tan \left(d x +c \right)+\frac{i \left(\tan^{10}\left(d x +c \right)\right)}{10}+\frac{\left(\tan^{9}\left(d x +c \right)\right)}{9}+\frac{i \left(\tan^{8}\left(d x +c \right)\right)}{2}+\frac{4 \left(\tan^{7}\left(d x +c \right)\right)}{7}+i \left(\tan^{6}\left(d x +c \right)\right)+\frac{6 \left(\tan^{5}\left(d x +c \right)\right)}{5}+i \left(\tan^{4}\right.}{d}$
default	$\frac{a \left(\tan \left(d x +c \right)+\frac{i \left(\tan^{10}\left(d x +c \right)\right)}{10}+\frac{\left(\tan^{9}\left(d x +c \right)\right)}{9}+\frac{i \left(\tan^{8}\left(d x +c \right)\right)}{2}+\frac{4 \left(\tan^{7}\left(d x +c \right)\right)}{7}+i \left(\tan^{6}\left(d x +c \right)\right)+\frac{6 \left(\tan^{5}\left(d x +c \right)\right)}{5}+i \left(\tan^{4}\right.}{d}$

```
[In] int(sec(d*x+c)^10*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 256/315*I*a*(252*exp(10*I*(d*x+c))+210*exp(8*I*(d*x+c))+120*exp(6*I*(d*x+c)
)+45*exp(4*I*(d*x+c))+10*exp(2*I*(d*x+c))+1)/d/(exp(2*I*(d*x+c))+1)^10
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(82) = 164.

Time = 0.24 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.01

$$\int \sec^{10}(c+dx)(a+ia\tan(c+dx))\,dx =$$

$$-\frac{256\left(-252 i\,ae^{(10 i\,dx+10 i\,c)}-210 i\,ae^{(8 i\,dx+8 i\,c)}-120 i\,ae^{(6 i\,dx+6 i}\right.}{315\left(de^{(20 i\,dx+20 i\,c)}+10\,de^{(18 i\,dx+18 i\,c)}+45\,de^{(16 i\,dx+16 i\,c)}+120\,de^{(14 i\,dx+14 i\,c)}+210\,de^{(12 i\,dx+12 i\,c)}+252\,de\right.}$$

```
[In] integrate(sec(d*x+c)^10*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -256/315*(-252*I*a*e^(10*I*d*x + 10*I*c) - 210*I*a*e^(8*I*d*x + 8*I*c) - 12
0*I*a*e^(6*I*d*x + 6*I*c) - 45*I*a*e^(4*I*d*x + 4*I*c) - 10*I*a*e^(2*I*d*x
+ 2*I*c) - I*a)/(d*e^(20*I*d*x + 20*I*c) + 10*d*e^(18*I*d*x + 18*I*c) + 45*
d*e^(16*I*d*x + 16*I*c) + 120*d*e^(14*I*d*x + 14*I*c) + 210*d*e^(12*I*d*x +
 12*I*c) + 252*d*e^(10*I*d*x + 10*I*c) + 210*d*e^(8*I*d*x + 8*I*c) + 120*d*
e^(6*I*d*x + 6*I*c) + 45*d*e^(4*I*d*x + 4*I*c) + 10*d*e^(2*I*d*x + 2*I*c) +
 d)
```

Sympy [A] (verification not implemented)

Time = 4.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.88

$$\int \sec^{10}(c+dx)(a+ia\tan(c+dx))\,dx$$
$$=\begin{cases} \frac{a\left(\frac{\tan^9(c+dx)}{9}+\frac{4\tan^7(c+dx)}{7}+\frac{6\tan^5(c+dx)}{5}+\frac{4\tan^3(c+dx)}{3}+\tan(c+dx)\right)+\frac{ia\sec^{10}(c+dx)}{10}}{d} & \text{for } d\neq 0 \\ x(ia\tan(c)+a)\sec^{10}(c) & \text{otherwise} \end{cases}$$

```
[In] integrate(sec(d*x+c)**10*(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise(((a*(tan(c + d*x)**9/9 + 4*tan(c + d*x)**7/7 + 6*tan(c + d*x)**5/
5 + 4*tan(c + d*x)**3/3 + tan(c + d*x)) + I*a*sec(c + d*x)**10/10)/d, Ne(d,
 0)), (x*(I*a*tan(c) + a)*sec(c)**10, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.21

$$\int \sec^{10}(c+dx)(a+ia\tan(c+dx))\,dx$$
$$=\frac{63i\,a\tan(dx+c)^{10}+70\,a\tan(dx+c)^9+315i\,a\tan(dx+c)^8+360\,a\tan(dx+c)^7+630i\,a\tan(dx+c)^6}{63}$$

```
[In] integrate(sec(d*x+c)^10*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/630*(63*I*a*tan(d*x + c)^10 + 70*a*tan(d*x + c)^9 + 315*I*a*tan(d*x + c)^
8 + 360*a*tan(d*x + c)^7 + 630*I*a*tan(d*x + c)^6 + 756*a*tan(d*x + c)^5 +
630*I*a*tan(d*x + c)^4 + 840*a*tan(d*x + c)^3 + 315*I*a*tan(d*x + c)^2 + 63
0*a*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.21

$$\int \sec^{10}(c+dx)(a+ia\tan(c+dx))\,dx =$$
$$-\frac{-63i\,a\tan(dx+c)^{10}-70\,a\tan(dx+c)^9-315i\,a\tan(dx+c)^8-360\,a\tan(dx+c)^7-630i\,a\tan(dx+}{}$$

```
[In] integrate(sec(d*x+c)^10*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/630*(-63*I*a*tan(d*x + c)^10 - 70*a*tan(d*x + c)^9 - 315*I*a*tan(d*x + c
)^8 - 360*a*tan(d*x + c)^7 - 630*I*a*tan(d*x + c)^6 - 756*a*tan(d*x + c)^5
- 630*I*a*tan(d*x + c)^4 - 840*a*tan(d*x + c)^3 - 315*I*a*tan(d*x + c)^2 -
630*a*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 4.02 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.13

$$\int \sec^{10}(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a\left(-\cos\left(c+d\,x\right)^{10}\,63\mathrm{i}+256\,\sin\left(c+d\,x\right)\,\cos\left(c+d\,x\right)^9+128\,\sin\left(c+d\,x\right)\,\cos\left(c+d\,x\right)^7+96\,\sin\left(c+d\,x\right.}{630\,d\cos\left(c+d\,x\right)^{10}}$$

```
[In] int((a + a*tan(c + d*x)*1i)/cos(c + d*x)^10,x)
```

```
[Out] (a*(70*cos(c + d*x)*sin(c + d*x) + 80*cos(c + d*x)^3*sin(c + d*x) + 96*cos(
c + d*x)^5*sin(c + d*x) + 128*cos(c + d*x)^7*sin(c + d*x) + 256*cos(c + d*x
)^9*sin(c + d*x) - cos(c + d*x)^10*63i + 63i))/(630*d*cos(c + d*x)^10)
```

3.2 $\int \sec^8(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 75

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))\,dx=\frac{ia\sec^8(c+dx)}{8d}+\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{d}+\frac{3a\tan^5(c+dx)}{5d}+\frac{a\tan^7(c+dx)}{7d}$$

```
[Out] 1/8*I*a*sec(d*x+c)^8/d+a*tan(d*x+c)/d+a*tan(d*x+c)^3/d+3/5*a*tan(d*x+c)^5/d
+1/7*a*tan(d*x+c)^7/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3567, 3852}

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))\,dx=\frac{a\tan^7(c+dx)}{7d}+\frac{3a\tan^5(c+dx)}{5d}+\frac{a\tan^3(c+dx)}{d}+\frac{a\tan(c+dx)}{d}+\frac{ia\sec^8(c+dx)}{8d}$$

```
[In] Int[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((I/8)*a*Sec[c + d*x]^8)/d + (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/d + (3
*a*Tan[c + d*x]^5)/(5*d) + (a*Tan[c + d*x]^7)/(7*d)
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
```

```
| NeQ[a^2 + b^2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{ia\sec^8(c+dx)}{8d} + a\int \sec^8(c+dx)\,dx \\ &= \frac{ia\sec^8(c+dx)}{8d} - \frac{a\text{Subst}\left(\int \left(1+3x^2+3x^4+x^6\right)\,dx, x, -\tan(c+dx)\right)}{d} \\ &= \frac{ia\sec^8(c+dx)}{8d} + \frac{a\tan(c+dx)}{d} + \frac{a\tan^3(c+dx)}{d} + \frac{3a\tan^5(c+dx)}{5d} + \frac{a\tan^7(c+dx)}{7d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.84

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))\,dx = \frac{ia\sec^8(c+dx)}{8d} + \frac{a\left(\tan(c+dx)+\tan^3(c+dx)+\frac{3}{5}\tan^5(c+dx)+\frac{1}{7}\tan^7(c+dx)\right)}{d}$$

```
[In] Integrate[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x]),x]
[Out] ((I/8)*a*Sec[c + d*x]^8)/d + (a*(Tan[c + d*x] + Tan[c + d*x]^3 + (3*Tan[c +
 d*x]^5)/5 + Tan[c + d*x]^7/7))/d
```

Maple [A] (verified)

Time = 63.45 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.89

method	result
risch	$\frac{32ia\left(70\,\mathrm{e}^{8i(dx+c)}+56\,\mathrm{e}^{6i(dx+c)}+28\,\mathrm{e}^{4i(dx+c)}+8\,\mathrm{e}^{2i(dx+c)}+1\right)}{35d\left(\mathrm{e}^{2i(dx+c)}+1\right)^8}$
derivativedivides	$\frac{a\left(\tan(dx+c)+\frac{i\left(\tan^8(dx+c)\right)}{8}+\frac{\left(\tan^7(dx+c)\right)}{7}+\frac{i\left(\tan^6(dx+c)\right)}{2}+\frac{3\left(\tan^5(dx+c)\right)}{5}+\frac{3i\left(\tan^4(dx+c)\right)}{4}+\tan^3(dx+c)+\frac{i\left(\tan^2(dx\right.}{2}\right.}{d}$
default	$\frac{a\left(\tan(dx+c)+\frac{i\left(\tan^8(dx+c)\right)}{8}+\frac{\left(\tan^7(dx+c)\right)}{7}+\frac{i\left(\tan^6(dx+c)\right)}{2}+\frac{3\left(\tan^5(dx+c)\right)}{5}+\frac{3i\left(\tan^4(dx+c)\right)}{4}+\tan^3(dx+c)+\frac{i\left(\tan^2(dx\right.}{2}\right.}{d}$

```
[In] int(sec(d*x+c)^8*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 32/35*I*a*(70*exp(8*I*(d*x+c))+56*exp(6*I*(d*x+c))+28*exp(4*I*(d*x+c))+8*ex
p(2*I*(d*x+c))+1)/d/(exp(2*I*(d*x+c))+1)^8
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(67) = 134$.

Time = 0.23 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.04

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{32\left(-70i\,ae^{(8i\,dx+8i\,c)} - 56i\,ae^{(6i\,dx+6i\,c)} - 28i\,ae^{(4i\,dx+4i\,c)} - 8i\,ae^{(2i\,dx+2i\,c)}\right.}{35\left(de^{(16i\,dx+16i\,c)} + 8\,de^{(14i\,dx+14i\,c)} + 28\,de^{(12i\,dx+12i\,c)} + 56\,de^{(10i\,dx+10i\,c)} + 70\,de^{(8i\,dx+8i\,c)} + 56\,de^{(6i\,dx+6i\,c)}\right.}$$

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -32/35*(-70*I*a*e^(8*I*d*x + 8*I*c) - 56*I*a*e^(6*I*d*x + 6*I*c) - 28*I*a*e
^(4*I*d*x + 4*I*c) - 8*I*a*e^(2*I*d*x + 2*I*c) - I*a)/(d*e^(16*I*d*x + 16*I
*c) + 8*d*e^(14*I*d*x + 14*I*c) + 28*d*e^(12*I*d*x + 12*I*c) + 56*d*e^(10*I
*d*x + 10*I*c) + 70*d*e^(8*I*d*x + 8*I*c) + 56*d*e^(6*I*d*x + 6*I*c) + 28*d
*e^(4*I*d*x + 4*I*c) + 8*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 2.40 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.91

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))\,dx = \begin{cases} \frac{a\left(\frac{\tan^7(c+dx)}{7} + \frac{3\tan^5(c+dx)}{5} + \tan^3(c+dx) + \tan(c+dx)\right) + \frac{ia\sec^8(c+dx)}{8}}{d} & \text{for } d \neq 0 \\ x(ia\tan(c) + a)\sec^8(c) & \text{otherwise} \end{cases}$$

```
[In] integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise(((a*(tan(c + d*x)**7/7 + 3*tan(c + d*x)**5/5 + tan(c + d*x)**3 +
tan(c + d*x)) + I*a*sec(c + d*x)**8/8)/d, Ne(d, 0)), (x*(I*a*tan(c) + a)*se
c(c)**8, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.23

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))\,dx$$
$$= \frac{35i\,a\tan(dx+c)^8 + 40\,a\tan(dx+c)^7 + 140i\,a\tan(dx+c)^6 + 168\,a\tan(dx+c)^5 + 210i\,a\tan(dx+c)^4}{280\,d}$$

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/280*(35*I*a*tan(d*x + c)^8 + 40*a*tan(d*x + c)^7 + 140*I*a*tan(d*x + c)^6
 + 168*a*tan(d*x + c)^5 + 210*I*a*tan(d*x + c)^4 + 280*a*tan(d*x + c)^3 + 1
40*I*a*tan(d*x + c)^2 + 280*a*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.23

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))\,dx =$$
$$-\frac{-35i\,a\tan(dx+c)^8 - 40\,a\tan(dx+c)^7 - 140i\,a\tan(dx+c)^6 - 168\,a\tan(dx+c)^5 - 210i\,a\tan(dx}{280\,d}$$

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/280*(-35*I*a*tan(d*x + c)^8 - 40*a*tan(d*x + c)^7 - 140*I*a*tan(d*x + c)
^6 - 168*a*tan(d*x + c)^5 - 210*I*a*tan(d*x + c)^4 - 280*a*tan(d*x + c)^3 -
 140*I*a*tan(d*x + c)^2 - 280*a*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 4.20 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.99

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))\,dx$$
$$= \frac{a\,\sin(c+d\,x)\,\left(280\cos(c+d\,x)^7 + \cos(c+d\,x)^6\,\sin(c+d\,x)\,140\mathrm{i} + 280\cos(c+d\,x)^5\sin(c+d\,x)^2 + \mathrm{co}\right.}{}$$

```
[In] int((a + a*tan(c + d*x)*1i)/cos(c + d*x)^8,x)
```

```
[Out] (a*sin(c + d*x)*(40*cos(c + d*x)*sin(c + d*x)^6 + cos(c + d*x)^6*sin(c + d*
x)*140i + 280*cos(c + d*x)^7 + sin(c + d*x)^7*35i + cos(c + d*x)^2*sin(c +
d*x)^5*140i + 168*cos(c + d*x)^3*sin(c + d*x)^4 + cos(c + d*x)^4*sin(c + d*
x)^3*210i + 280*cos(c + d*x)^5*sin(c + d*x)^2))/(280*d*cos(c + d*x)^8)
```

3.3 $\int \sec^6(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 62

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))\,dx = \frac{ia\sec^6(c+dx)}{6d}+\frac{a\tan(c+dx)}{d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan^5(c+dx)}{5d}$$

```
[Out] 1/6*I*a*sec(d*x+c)^6/d+a*tan(d*x+c)/d+2/3*a*tan(d*x+c)^3/d+1/5*a*tan(d*x+c)
^5/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3567, 3852}

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a\tan^5(c+dx)}{5d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{ia\sec^6(c+dx)}{6d}$$

```
[In] Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((I/6)*a*Sec[c + d*x]^6)/d + (a*Tan[c + d*x])/d + (2*a*Tan[c + d*x]^3)/(3*d
) + (a*Tan[c + d*x]^5)/(5*d)
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
```

```
| NeQ[a^2 + b^2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{ia\sec^6(c+dx)}{6d} + a\int \sec^6(c+dx)\,dx\\ &= \frac{ia\sec^6(c+dx)}{6d} - \frac{a\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x, -\tan(c+dx)\right)}{d}\\ &= \frac{ia\sec^6(c+dx)}{6d} + \frac{a\tan(c+dx)}{d} + \frac{2a\tan^3(c+dx)}{3d} + \frac{a\tan^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))\,dx = \frac{ia\sec^6(c+dx)}{6d} + \frac{a\left(\tan(c+dx)+\frac{2}{3}\tan^3(c+dx)+\frac{1}{5}\tan^5(c+dx)\right)}{d}$$

```
[In] Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x]),x]
[Out] ((I/6)*a*Sec[c + d*x]^6)/d + (a*(Tan[c + d*x] + (2*Tan[c + d*x]^3)/3 + Tan[
c + d*x]^5/5))/d
```

Maple [A] (verified)

Time = 18.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

method	result	size
risch	$\frac{16ia\left(20\,e^{6i(dx+c)}+15\,e^{4i(dx+c)}+6\,e^{2i(dx+c)}+1\right)}{15d\left(e^{2i(dx+c)}+1\right)^6}$	56
derivativedivides	$\frac{a\left(\tan(dx+c)+\frac{i\left(\tan^6(dx+c)\right)}{6}+\frac{\left(\tan^5(dx+c)\right)}{5}+\frac{i\left(\tan^4(dx+c)\right)}{2}+\frac{2\left(\tan^3(dx+c)\right)}{3}+\frac{i\left(\tan^2(dx+c)\right)}{2}\right)}{d}$	66
default	$\frac{a\left(\tan(dx+c)+\frac{i\left(\tan^6(dx+c)\right)}{6}+\frac{\left(\tan^5(dx+c)\right)}{5}+\frac{i\left(\tan^4(dx+c)\right)}{2}+\frac{2\left(\tan^3(dx+c)\right)}{3}+\frac{i\left(\tan^2(dx+c)\right)}{2}\right)}{d}$	66

```
[In] int(sec(d*x+c)^6*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 16/15*I*a*(20*exp(6*I*(d*x+c))+15*exp(4*I*(d*x+c))+6*exp(2*I*(d*x+c))+1)/d/
(exp(2*I*(d*x+c))+1)^6
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(54) = 108.

Time = 0.23 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.89

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{16\left(-20i\,ae^{(6i\,dx+6i\,c)}-15i\,ae^{(4i\,dx+4i\,c)}-6i\,ae^{(2i\,dx+2i\,c)}-i\,a\right)}{15\left(de^{(12i\,dx+12i\,c)}+6\,de^{(10i\,dx+10i\,c)}+15\,de^{(8i\,dx+8i\,c)}+20\,de^{(6i\,dx+6i\,c)}+15\,de^{(4i\,dx+4i\,c)}+6\,de^{(2i\,dx+2i\,c)}+d\right.}$$

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -16/15*(-20*I*a*e^(6*I*d*x + 6*I*c) - 15*I*a*e^(4*I*d*x + 4*I*c) - 6*I*a*e^
(2*I*d*x + 2*I*c) - I*a)/(d*e^(12*I*d*x + 12*I*c) + 6*d*e^(10*I*d*x + 10*I*
c) + 15*d*e^(8*I*d*x + 8*I*c) + 20*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*I*d*x
+ 4*I*c) + 6*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))\,dx = \begin{cases} \frac{a\left(\frac{\tan^5(c+dx)}{5}+\frac{2\tan^3(c+dx)}{3}+\tan(c+dx)\right)+\frac{ia\sec^6(c+dx)}{6}}{d} & \text{for } d\neq 0 \\ x(ia\tan(c)+a)\sec^6(c) & \text{otherwise} \end{cases}$$

```
[In] integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise(((a*(tan(c + d*x)**5/5 + 2*tan(c + d*x)**3/3 + tan(c + d*x)) + I*
a*sec(c + d*x)**6/6)/d, Ne(d, 0)), (x*(I*a*tan(c) + a)*sec(c)**6, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))\,dx = \frac{5i\,a\tan(dx+c)^6+6\,a\tan(dx+c)^5+15i\,a\tan(dx+c)^4+20\,a\tan(dx+c)^3+15i\,a\tan(dx+c)^2+30}{30\,d}$$

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
[Out] 1/30*(5*I*a*tan(d*x + c)^6 + 6*a*tan(d*x + c)^5 + 15*I*a*tan(d*x + c)^4 + 2
0*a*tan(d*x + c)^3 + 15*I*a*tan(d*x + c)^2 + 30*a*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{-5i\,a\tan(dx+c)^6-6\,a\tan(dx+c)^5-15i\,a\tan(dx+c)^4-20\,a\tan(dx+c)^3-15i\,a\tan(dx+c)^2-}{30\,d}$$

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] -1/30*(-5*I*a*tan(d*x + c)^6 - 6*a*tan(d*x + c)^5 - 15*I*a*tan(d*x + c)^4 -
 20*a*tan(d*x + c)^3 - 15*I*a*tan(d*x + c)^2 - 30*a*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 3.71 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.81

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a\sin(c+d\,x)\,\left(30\cos(c+d\,x)^5+\cos(c+d\,x)^4\,\sin(c+d\,x)\,15\mathrm{i}+20\cos(c+d\,x)^3\sin(c+d\,x)^2+\cos(c\right.}{30\,d\cos(c+d\,x)^6}$$

```
[In] int((a + a*tan(c + d*x)*1i)/cos(c + d*x)^6,x)
[Out] (a*sin(c + d*x)*(6*cos(c + d*x)*sin(c + d*x)^4 + cos(c + d*x)^4*sin(c + d*x
)*15i + 30*cos(c + d*x)^5 + sin(c + d*x)^5*5i + cos(c + d*x)^2*sin(c + d*x)
^3*15i + 20*cos(c + d*x)^3*sin(c + d*x)^2))/(30*d*cos(c + d*x)^6)
```

3.4 $\int \sec^4(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 46

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))\,dx = \frac{ia\sec^4(c+dx)}{4d}+\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}$$

```
[Out] 1/4*I*a*sec(d*x+c)^4/d+a*tan(d*x+c)/d+1/3*a*tan(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3567, 3852}

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{ia\sec^4(c+dx)}{4d}$$

```
[In] Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((I/4)*a*Sec[c + d*x]^4)/d + (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{ia\sec^4(c+dx)}{4d} + a\int \sec^4(c+dx)\,dx\\ &= \frac{ia\sec^4(c+dx)}{4d} - \frac{a\text{Subst}\left(\int (1+x^2)\ dx, x, -\tan(c+dx)\right)}{d}\\ &= \frac{ia\sec^4(c+dx)}{4d} + \frac{a\tan(c+dx)}{d} + \frac{a\tan^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))\,dx = \frac{ia\sec^4(c+dx)}{4d} + \frac{a\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d}$$

```
[In] Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x]),x]
[Out] ((I/4)*a*Sec[c + d*x]^4)/d + (a*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

method	result	size
derivativedivides	$\frac{a\left(\tan(dx+c)+\frac{i\left(\tan^4(dx+c)\right)}{4}+\frac{\left(\tan^3(dx+c)\right)}{3}+\frac{i\left(\tan^2(dx+c)\right)}{2}\right)}{d}$	45
default	$\frac{a\left(\tan(dx+c)+\frac{i\left(\tan^4(dx+c)\right)}{4}+\frac{\left(\tan^3(dx+c)\right)}{3}+\frac{i\left(\tan^2(dx+c)\right)}{2}\right)}{d}$	45
risch	$\frac{4ia\left(6\,\mathrm{e}^{4i(dx+c)}+4\,\mathrm{e}^{2i(dx+c)}+1\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^4}$	45

```
[In] int(sec(d*x+c)^4*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] a/d*(tan(d*x+c)+1/4*I*tan(d*x+c)^4+1/3*tan(d*x+c)^3+1/2*I*tan(d*x+c)^2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(40) = 80$.

Time = 0.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.76

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))\,dx$$
$$= -\frac{4\left(-6i\,ae^{(4i\,dx+4i\,c)}-4i\,ae^{(2i\,dx+2i\,c)}-i\,a\right)}{3\left(de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}+4\,de^{(2i\,dx+2i\,c)}+d\right)}$$

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -4/3*(-6*I*a*e^(4*I*d*x + 4*I*c) - 4*I*a*e^(2*I*d*x + 2*I*c) - I*a)/(d*e^(8
*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e
^(2*I*d*x + 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))\,dx = \begin{cases} \frac{a\left(\frac{\tan^3{(c+dx)}}{3}+\tan{(c+dx)}\right)+\frac{ia\sec^4{(c+dx)}}{4}}{d} & \text{for } d\neq 0 \\ x(ia\tan{(c)}+a)\sec^4{(c)} & \text{otherwise} \end{cases}$$

```
[In] integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise(((a*(tan(c + d*x)**3/3 + tan(c + d*x)) + I*a*sec(c + d*x)**4/4)/d
, Ne(d, 0)), (x*(I*a*tan(c) + a)*sec(c)**4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))\,dx$$
$$= \frac{3i\,a\tan{(dx+c)}^4+4\,a\tan{(dx+c)}^3+6i\,a\tan{(dx+c)}^2+12\,a\tan{(dx+c)}}{12\,d}$$

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*I*a*tan(d*x + c)^4 + 4*a*tan(d*x + c)^3 + 6*I*a*tan(d*x + c)^2 + 12
*a*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))\,dx$$
$$= -\frac{-3i\,a\tan\left(dx+c\right)^4 - 4\,a\tan\left(dx+c\right)^3 - 6i\,a\tan\left(dx+c\right)^2 - 12\,a\tan\left(dx+c\right)}{12\,d}$$

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(-3*I*a*tan(d*x + c)^4 - 4*a*tan(d*x + c)^3 - 6*I*a*tan(d*x + c)^2 -
12*a*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 4.00 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))\,dx$$
$$= \frac{\frac{1\mathrm{i}\,a\tan(c+d\,x)^4}{4} + \frac{a\tan(c+d\,x)^3}{3} + \frac{1\mathrm{i}\,a\tan(c+d\,x)^2}{2} + a\tan(c+d\,x)}{d}$$

```
[In] int((a + a*tan(c + d*x)*1i)/cos(c + d*x)^4,x)
```

```
[Out] (a*tan(c + d*x) + (a*tan(c + d*x)^2*1i)/2 + (a*tan(c + d*x)^3)/3 + (a*tan(c
 + d*x)^4*1i)/4)/d
```

3.5 $\int \sec^2(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 27

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{i(a+ia\tan(c+dx))^2}{2ad}$$

```
[Out] -1/2*I*(a+I*a*tan(d*x+c))^2/a/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3567, 3852, 8}

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a\tan(c+dx)}{d}+\frac{ia\sec^2(c+dx)}{2d}$$

```
[In] Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((I/2)*a*Sec[c + d*x]^2)/d + (a*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{ia\sec^2(c+dx)}{2d} + a\int \sec^2(c+dx)\,dx\\ &= \frac{ia\sec^2(c+dx)}{2d} - \frac{a\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{d}\\ &= \frac{ia\sec^2(c+dx)}{2d} + \frac{a\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))\,dx = \frac{ia\sec^2(c+dx)}{2d} + \frac{a\tan(c+dx)}{d}$$

```
[In] Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x]),x]
[Out] ((I/2)*a*Sec[c + d*x]^2)/d + (a*Tan[c + d*x])/d
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$-\frac{ia\left(-\frac{\left(\tan^2(dx+c)\right)}{2}+i\tan(dx+c)\right)}{d}$	28
default	$-\frac{ia\left(-\frac{\left(\tan^2(dx+c)\right)}{2}+i\tan(dx+c)\right)}{d}$	28
risch	$\frac{2ia\left(2\,\mathrm{e}^{2i(dx+c)}+1\right)}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}$	34

```
[In] int(sec(d*x+c)^2*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] -I*a/d*(-1/2*tan(d*x+c)^2+I*tan(d*x+c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(21) = 42.

Time = 0.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.67

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{2\left(-2i\,ae^{(2i\,dx+2i\,c)}-i\,a\right)}{de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d}$$

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
[Out] -2*(-2*I*a*e^(2*I*d*x + 2*I*c) - I*a)/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d
*x + 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))\,dx = \begin{cases} \frac{\frac{ia\tan^2(c+dx)}{2}+a\tan(c+dx)}{d} & \text{for } d\neq 0 \\ x(ia\tan(c)+a)\sec^2(c) & \text{otherwise} \end{cases}$$

```
[In] integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c)),x)
[Out] Piecewise((((I*a*tan(c + d*x)**2/2 + a*tan(c + d*x))/d, Ne(d, 0)), (x*(I*a*t
an(c) + a)*sec(c)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{i\left(i\,a\tan(dx+c)+a\right)^2}{2\,ad}$$

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
[Out] -1/2*I*(I*a*tan(d*x + c) + a)^2/(a*d)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{-i\,a\tan\left(dx+c\right)^2-2\,a\tan\left(dx+c\right)}{2\,d}$$

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] -1/2*(-I*a*tan(d*x + c)^2 - 2*a*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 3.87 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a\tan(c+d\,x)\,\left(2+\tan(c+d\,x)\,1\mathrm{i}\right)}{2\,d}$$

```
[In] int((a + a*tan(c + d*x)*1i)/cos(c + d*x)^2,x)
[Out] (a*tan(c + d*x)*(tan(c + d*x)*1i + 2))/(2*d)
```

3.6 $\int (a+ia\tan (c+dx)) \, dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int (a+ia\tan (c+dx)) \, dx=ax-\frac{ia\log (\cos (c+dx))}{d}$$

```
[Out] a*x-I*a*ln(cos(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3556}

$$\int (a+ia\tan (c+dx)) \, dx=ax-\frac{ia\log (\cos (c+dx))}{d}$$

```
[In] Int[a + I*a*Tan[c + d*x],x]
```

```
[Out] a*x - (I*a*Log[Cos[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= ax+(ia) \int \tan (c+dx) \, dx \\ &= ax-\frac{ia\log (\cos (c+dx))}{d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int (a+ia\tan(c+dx))\,dx = ax-\frac{ia\log(\cos(c+dx))}{d}$$

```
[In] Integrate[a + I*a*Tan[c + d*x],x]
[Out] a*x - (I*a*Log[Cos[c + d*x]])/d
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

method	result	size
default	$ax+\frac{ia\ln(1+\tan^2(dx+c))}{2d}$	23
norman	$ax+\frac{ia\ln(1+\tan^2(dx+c))}{2d}$	23
parallelrisch	$ax+\frac{ia\ln(1+\tan^2(dx+c))}{2d}$	23
parts	$ax+\frac{ia\ln(1+\tan^2(dx+c))}{2d}$	23
derivativedivides	$\frac{a\left(\frac{i\ln\left(1+\tan^2(dx+c)\right)}{2}+\arctan(\tan(dx+c))\right)}{d}$	28
risch	$-\frac{ia\ln(e^{2i(dx+c)}+1)}{d}-\frac{2ac}{d}$	28

```
[In] int(a+I*a*tan(d*x+c),x,method=_RETURNVERBOSE)
[Out] a*x+1/2*I*a/d*ln(1+tan(d*x+c)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int (a+ia\tan(c+dx))\,dx = -\frac{i\,a\log\left(e^{(2i\,dx+2i\,c)}+1\right)}{d}$$

```
[In] integrate(a+I*a*tan(d*x+c),x, algorithm="fricas")
[Out] -I*a*log(e^(2*I*d*x + 2*I*c) + 1)/d
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int (a + ia\tan(c + dx))\,dx = -\frac{ia\log\left(e^{2idx} + e^{-2ic}\right)}{d}$$

```
[In] integrate(a+I*a*tan(d*x+c),x)
[Out] -I*a*log(exp(2*I*d*x) + exp(-2*I*c))/d
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int (a + ia\tan(c + dx))\,dx = ax + \frac{i\,a\log\left(\sec\left(dx + c\right)\right)}{d}$$

```
[In] integrate(a+I*a*tan(d*x+c),x, algorithm="maxima")
[Out] a*x + I*a*log(sec(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int (a + ia\tan(c + dx))\,dx = ax - \frac{i\,a\log\left(|\cos\left(dx + c\right)|\right)}{d}$$

```
[In] integrate(a+I*a*tan(d*x+c),x, algorithm="giac")
[Out] a*x - I*a*log(abs(cos(d*x + c)))/d
```

Mupad [B] (verification not implemented)

Time = 3.72 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int (a + ia\tan(c + dx))\,dx = \frac{a\,\ln\left(\tan(c + d\,x) + 1\mathrm{i}\right)\,1\mathrm{i}}{d}$$

```
[In] int(a + a*tan(c + d*x)*1i,x)
[Out] (a*log(tan(c + d*x) + 1i)*1i)/d
```

3.7 $\int \cos^2(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 45

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))\,dx=\frac{ax}{2}-\frac{ia\cos^2(c+dx)}{2d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] 1/2*a*x-1/2*I*a*cos(d*x+c)^2/d+1/2*a*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3567, 2715, 8}

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))\,dx=-\frac{ia\cos^2(c+dx)}{2d}+\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}$$

```
[In] Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (a*x)/2 - ((I/2)*a*Cos[c + d*x]^2)/d + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{ia\cos^2(c+dx)}{2d}+a\int\cos^2(c+dx)\,dx\\ &= -\frac{ia\cos^2(c+dx)}{2d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}a\int 1\,dx\\ &= \frac{ax}{2}-\frac{ia\cos^2(c+dx)}{2d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.07

$$\int\cos^2(c+dx)(a+ia\tan(c+dx))\,dx=\frac{a(c+dx)}{2d}-\frac{ia\cos^2(c+dx)}{2d}+\frac{a\sin(2(c+dx))}{4d}$$

```
[In] Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x]),x]
[Out] (a*(c + d*x))/(2*d) - ((I/2)*a*Cos[c + d*x]^2)/d + (a*Sin[2*(c + d*x)])/(4*
d)
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.49

method	result	size
risch	$\frac{ax}{2}-\frac{ia\,e^{2i(dx+c)}}{4d}$	22
derivativedivides	$\frac{-\frac{ia\left(\cos^2(dx+c)\right)}{2}+a\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	42
default	$\frac{-\frac{ia\left(\cos^2(dx+c)\right)}{2}+a\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	42

```
[In] int(cos(d*x+c)^2*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/2*a*x-1/4*I/d*a*exp(2*I*(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.51

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))\,dx = \frac{2\,adx - i\,ae^{(2i\,dx+2i\,c)}}{4\,d}$$

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
[Out] 1/4*(2*a*d*x - I*a*e^(2*I*d*x + 2*I*c))/d
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))\,dx = \frac{ax}{2} + \begin{cases} -\frac{iae^{2ic}e^{2idx}}{4d} & \text{for } d \neq 0 \\ \frac{axe^{2ic}}{2} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c)),x)
[Out] a*x/2 + Piecewise((-I*a*exp(2*I*c)*exp(2*I*d*x)/(4*d), Ne(d, 0)), (a*x*exp(
2*I*c)/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))\,dx = \frac{(dx+c)a + \frac{a\tan(dx+c)-i\,a}{\tan(dx+c)^2+1}}{2\,d}$$

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
[Out] 1/2*((d*x + c)*a + (a*tan(d*x + c) - I*a)/(tan(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.51

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))\,dx = \frac{2\,adx - i\,ae^{(2i\,dx+2i\,c)}}{4\,d}$$

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] 1/4*(2*a*d*x - I*a*e^(2*I*d*x + 2*I*c))/d
```

Mupad [B] (verification not implemented)

Time = 4.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.49

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a\,x}{2} + \frac{a}{2\,d\,\left(\tan\left(c+d\,x\right)+1\mathrm{i}\right)}$$

```
[In] int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i),x)
[Out] (a*x)/2 + a/(2*d*(tan(c + d*x) + 1i))
```

3.8 $\int \cos^4(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 67

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))\,dx=\frac{3ax}{8}-\frac{ia\cos^4(c+dx)}{4d}+\frac{3a\cos(c+dx)\sin(c+dx)}{8d}+\frac{a\cos^3(c+dx)\sin(c+dx)}{4d}$$

```
[Out] 3/8*a*x-1/4*I*a*cos(d*x+c)^4/d+3/8*a*cos(d*x+c)*sin(d*x+c)/d+1/4*a*cos(d*x+
c)^3*sin(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3567, 2715, 8}

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))\,dx=-\frac{ia\cos^4(c+dx)}{4d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}$$

```
[In] Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (3*a*x)/8 - ((I/4)*a*Cos[c + d*x]^4)/d + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8
*d) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{ia\cos^4(c+dx)}{4d} + a\int \cos^4(c+dx)\,dx \\
&= -\frac{ia\cos^4(c+dx)}{4d} + \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}(3a)\int \cos^2(c+dx)\,dx \\
&= -\frac{ia\cos^4(c+dx)}{4d} + \frac{3a\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{8}(3a)\int 1\,dx \\
&= \frac{3ax}{8} - \frac{ia\cos^4(c+dx)}{4d} + \frac{3a\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos^3(c+dx)\sin(c+dx)}{4d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.69

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a(12c+12dx-8i\cos^4(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))}{32d}$$

```
[In] Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x]),x]
[Out] (a*(12*c + 12*d*x - (8*I)*Cos[c + d*x]^4 + 8*Sin[2*(c + d*x)] + Sin[4*(c +
d*x)]))/(32*d)
```

Maple [A] (verified)

Time = 3.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{-\frac{ia\left(\cos^4(dx+c)\right)}{4}+a\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$	53
default	$\frac{-\frac{ia\left(\cos^4(dx+c)\right)}{4}+a\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$	53
risch	$\frac{3ax}{8}-\frac{ia\,e^{4i(dx+c)}}{32d}-\frac{ia\cos(2dx+2c)}{8d}+\frac{a\sin(2dx+2c)}{4d}$	53

```
[In] int(cos(d*x+c)^4*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/4*I*a*cos(d*x+c)^4+a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+
3/8*d*x+3/8*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.84

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))\,dx = \frac{\left(12\,adxe^{(2i\,dx+2i\,c)}-i\,ae^{(6i\,dx+6i\,c)}-6i\,ae^{(4i\,dx+4i\,c)}+2i\,a\right)e^{(-2i\,dx-2i\,c)}}{32\,d}$$

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
[Out] 1/32*(12*a*d*x*e^(2*I*d*x + 2*I*c) - I*a*e^(6*I*d*x + 6*I*c) - 6*I*a*e^(4*I
*d*x + 4*I*c) + 2*I*a)*e^(-2*I*d*x - 2*I*c)/d
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.03

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))\,dx = \frac{3ax}{8}+\begin{cases}\frac{\left(-256iad^2e^{6ic}e^{4idx}-1536iad^2e^{4ic}e^{2idx}+512iad^2e^{-2idx}\right)e^{-2ic}}{8192d^3} & \text{for } d^3e^{2ic}\neq 0\\ x\left(-\frac{3a}{8}+\frac{\left(ae^{6ic}+3ae^{4ic}+3ae^{2ic}+a\right)e^{-2ic}}{8}\right) & \text{otherwise}\end{cases}$$

```
[In] integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c)),x)
```

```
[Out] 3*a*x/8 + Piecewise((((-256*I*a*d**2*exp(6*I*c)*exp(4*I*d*x) - 1536*I*a*d**2
*exp(4*I*c)*exp(2*I*d*x) + 512*I*a*d**2*exp(-2*I*d*x))*exp(-2*I*c)/(8192*d*
*3), Ne(d**3*exp(2*I*c), 0)), (x*(-3*a/8 + (a*exp(6*I*c) + 3*a*exp(4*I*c) +
 3*a*exp(2*I*c) + a)*exp(-2*I*c)/8), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))\,dx = \frac{3\,(dx+c)a+\frac{3\,a\tan(dx+c)^3+5\,a\tan(dx+c)-2i\,a}{\tan(dx+c)^4+2\,\tan(dx+c)^2+1}}{8\,d}$$

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/8*(3*(d*x + c)*a + (3*a*tan(d*x + c)^3 + 5*a*tan(d*x + c) - 2*I*a)/(tan(d
*x + c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.54

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))\,dx = \frac{\left(12\,adxe^{(2i\,dx+2i\,c)}+i\,ae^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-i\,ae^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx)}+e^{(-2i\,c)}\right)-i\,ae^{(6i\,dx+6i\,c)}-\right.}{32\,d}$$

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*(12*a*d*x*e^(2*I*d*x + 2*I*c) + I*a*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x
 + 2*I*c) + 1) - I*a*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) - I*
a*e^(6*I*d*x + 6*I*c) - 6*I*a*e^(4*I*d*x + 4*I*c) + 2*I*a)*e^(-2*I*d*x - 2*
I*c)/d
```

Mupad [B] (verification not implemented)

Time = 3.71 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))\,dx$$
$$=\frac{3\,a\,x}{8}+\frac{\frac{3\,a\,\tan(c+d\,x)^2}{8}+\frac{3\mathrm{i}\,a\,\tan(c+d\,x)}{8}+\frac{a}{4}}{d\,\left(\tan\left(c+d\,x\right)^3+\tan\left(c+d\,x\right)^2\,1\mathrm{i}+\tan\left(c+d\,x\right)+1\mathrm{i}\right)}$$

```
[In] int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i),x)
[Out] (3*a*x)/8 + (a/4 + (a*tan(c + d*x)*3i)/8 + (3*a*tan(c + d*x)^2)/8)/(d*(tan(
c + d*x) + tan(c + d*x)^2*1i + tan(c + d*x)^3 + 1i))
```

3.9 $\int \cos^6(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 89

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))\,dx = \frac{5ax}{16} - \frac{ia\cos^6(c+dx)}{6d} + \frac{5a\cos(c+dx)\sin(c+dx)}{16d} + \frac{5a\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{a\cos^5(c+dx)\sin(c+dx)}{6d}$$

```
[Out] 5/16*a*x-1/6*I*a*cos(d*x+c)^6/d+5/16*a*cos(d*x+c)*sin(d*x+c)/d+5/24*a*cos(d
*x+c)^3*sin(d*x+c)/d+1/6*a*cos(d*x+c)^5*sin(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3567, 2715, 8}

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{ia\cos^6(c+dx)}{6d} + \frac{a\sin(c+dx)\cos^5(c+dx)}{6d} + \frac{5a\sin(c+dx)\cos^3(c+dx)}{24d} + \frac{5a\sin(c+dx)\cos(c+dx)}{16d} + \frac{5ax}{16}$$

```
[In] Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (5*a*x)/16 - ((I/6)*a*Cos[c + d*x]^6)/d + (5*a*Cos[c + d*x]*Sin[c + d*x])/(
16*d) + (5*a*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a*Cos[c + d*x]^5*Sin[c
+ d*x])/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{ia\cos^6(c+dx)}{6d}+a\int\cos^6(c+dx)\,dx\\
&= -\frac{ia\cos^6(c+dx)}{6d}+\frac{a\cos^5(c+dx)\sin(c+dx)}{6d}+\frac{1}{6}(5a)\int\cos^4(c+dx)\,dx\\
&= -\frac{ia\cos^6(c+dx)}{6d}+\frac{5a\cos^3(c+dx)\sin(c+dx)}{24d}\\
&\quad+\frac{a\cos^5(c+dx)\sin(c+dx)}{6d}+\frac{1}{8}(5a)\int\cos^2(c+dx)\,dx\\
&= -\frac{ia\cos^6(c+dx)}{6d}+\frac{5a\cos(c+dx)\sin(c+dx)}{16d}+\frac{5a\cos^3(c+dx)\sin(c+dx)}{24d}\\
&\quad+\frac{a\cos^5(c+dx)\sin(c+dx)}{6d}+\frac{1}{16}(5a)\int 1\,dx\\
&= \frac{5ax}{16}-\frac{ia\cos^6(c+dx)}{6d}+\frac{5a\cos(c+dx)\sin(c+dx)}{16d}\\
&\quad+\frac{5a\cos^3(c+dx)\sin(c+dx)}{24d}+\frac{a\cos^5(c+dx)\sin(c+dx)}{6d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.63

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a(60c+60dx-32i\cos^6(c+dx)+45\sin(2(c+dx))+9\sin(4(c+dx))+\sin(6(c+dx)))}{192d}$$

```
[In] Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x]),x]
[Out] (a*(60*c + 60*d*x - (32*I)*Cos[c + d*x]^6 + 45*Sin[2*(c + d*x)] + 9*Sin[4*(
c + d*x)] + Sin[6*(c + d*x)]))/(192*d)
```

Maple [A] (verified)

Time = 14.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{-\frac{ia\left(\cos^6(dx+c)\right)}{6}+a\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}$	63
default	$\frac{-\frac{ia\left(\cos^6(dx+c)\right)}{6}+a\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}$	63
risch	$\frac{5ax}{16}-\frac{ia\,e^{6i(dx+c)}}{192d}-\frac{ia\cos(4dx+4c)}{32d}+\frac{3a\sin(4dx+4c)}{64d}-\frac{5ia\cos(2dx+2c)}{64d}+\frac{15a\sin(2dx+2c)}{64d}$	84

```
[In] int(cos(d*x+c)^6*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/6*I*a*cos(d*x+c)^6+a*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d
*x+c))*sin(d*x+c)+5/16*d*x+5/16*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.90

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))\,dx = \frac{\left(120\,adxe^{(4i\,dx+4i\,c)}-2i\,ae^{(10i\,dx+10i\,c)}-15i\,ae^{(8i\,dx+8i\,c)}-60i\,ae^{(6i\,dx+6i\,c)}+30i\,ae^{(2i\,dx+2i\,c)}+3i\,a\right)e^{(-4i\,dx-4i}}{384\,d}$$

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/384*(120*a*d*x*e^(4*I*d*x + 4*I*c) - 2*I*a*e^(10*I*d*x + 10*I*c) - 15*I*a
*e^(8*I*d*x + 8*I*c) - 60*I*a*e^(6*I*d*x + 6*I*c) + 30*I*a*e^(2*I*d*x + 2*I
*c) + 3*I*a)*e^(-4*I*d*x - 4*I*c)/d
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.37

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))\,dx = \frac{5ax}{16} + \begin{cases} \frac{(-33554432iad^4e^{12ic}e^{6idx}-251658240iad^4e^{10ic}e^{4idx}-1006632960iad^4e^{8ic}e^{2idx}+503316480iad^4e^{4ic}e^{-2idx}+50331648iad^4e^{2ic}e^{-4idx})e^{-}}{6442450944d^5} \\ x\left(-\frac{5a}{16}+\frac{(ae^{10ic}+5ae^{8ic}+10ae^{6ic}+10ae^{4ic}+5ae^{2ic}+a)e^{-4ic}}{32}\right) \end{cases}$$

```
[In] integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c)),x)
```

```
[Out] 5*a*x/16 + Piecewise(((-33554432*I*a*d**4*exp(12*I*c)*exp(6*I*d*x) - 251658
240*I*a*d**4*exp(10*I*c)*exp(4*I*d*x) - 1006632960*I*a*d**4*exp(8*I*c)*exp(
2*I*d*x) + 503316480*I*a*d**4*exp(4*I*c)*exp(-2*I*d*x) + 50331648*I*a*d**4*
exp(2*I*c)*exp(-4*I*d*x))*exp(-6*I*c)/(6442450944*d**5), Ne(d**5*exp(6*I*c)
, 0)), (x*(-5*a/16 + (a*exp(10*I*c) + 5*a*exp(8*I*c) + 10*a*exp(6*I*c) + 10
*a*exp(4*I*c) + 5*a*exp(2*I*c) + a)*exp(-4*I*c)/32), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.92

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))\,dx = \frac{15\,(dx+c)a + \frac{15\,a\tan(dx+c)^5+40\,a\tan(dx+c)^3+33\,a\tan(dx+c)-8i\,a}{\tan(dx+c)^6+3\,\tan(dx+c)^4+3\,\tan(dx+c)^2+1}}{48\,d}$$

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/48*(15*(d*x + c)*a + (15*a*tan(d*x + c)^5 + 40*a*tan(d*x + c)^3 + 33*a*ta
n(d*x + c) - 8*I*a)/(tan(d*x + c)^6 + 3*tan(d*x + c)^4 + 3*tan(d*x + c)^2 +
 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.43

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))\,dx = \frac{\left(120\,adxe^{(4i\,dx+2i\,c)}+12i\,ae^{(4i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-12i\,ae^{(4i\,dx+2i\,c)}\log\left(e^{(2i\,dx)}+e^{(-2i\,c)}\right)-2i\,ae^{(10i\,\cdots}\right.}{384\,d}$$

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/384*(120*a*d*x*e^(4*I*d*x + 2*I*c) + 12*I*a*e^(4*I*d*x + 2*I*c)*log(e^(2*
I*d*x + 2*I*c) + 1) - 12*I*a*e^(4*I*d*x + 2*I*c)*log(e^(2*I*d*x) + e^(-2*I*
c)) - 2*I*a*e^(10*I*d*x + 8*I*c) - 15*I*a*e^(8*I*d*x + 6*I*c) - 60*I*a*e^(6
*I*d*x + 4*I*c) + 30*I*a*e^(2*I*d*x) + 3*I*a*e^(-2*I*c))*e^(-4*I*d*x - 2*I*
c)/d
```

Mupad [B] (verification not implemented)

Time = 4.04 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.21

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))\,dx = \frac{5\,a\,x}{16} + \frac{\frac{5\,a\tan(c+d\,x)^4}{16}+\frac{5\mathrm{i}\,a\tan(c+d\,x)^3}{16}+\frac{25\,a\tan(c+d\,x)^2}{48}+\frac{25\mathrm{i}\,a\tan(c+d\,x)}{48}+\frac{a}{6}}{d\,\left(\tan\left(c+d\,x\right)^5+\tan\left(c+d\,x\right)^4\,1\mathrm{i}+2\tan\left(c+d\,x\right)^3+\tan\left(c+d\,x\right)^2\,2\mathrm{i}+\tan\left(c+d\,x\right)+1\mathrm{i}\right)}$$

```
[In] int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i),x)
```

```
[Out] (5*a*x)/16 + (a/6 + (a*tan(c + d*x)*25i)/48 + (25*a*tan(c + d*x)^2)/48 + (a
*tan(c + d*x)^3*5i)/16 + (5*a*tan(c + d*x)^4)/16)/(d*(tan(c + d*x) + tan(c
+ d*x)^2*2i + 2*tan(c + d*x)^3 + tan(c + d*x)^4*1i + tan(c + d*x)^5 + 1i))
```

3.10 $\int \cos^8(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 111

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))\,dx = \frac{35ax}{128} - \frac{ia\cos^8(c+dx)}{8d} + \frac{35a\cos(c+dx)\sin(c+dx)}{128d} + \frac{35a\cos^3(c+dx)\sin(c+dx)}{192d} + \frac{7a\cos^5(c+dx)\sin(c+dx)}{48d} + \frac{a\cos^7(c+dx)\sin(c+dx)}{8d}$$

```
[Out] 35/128*a*x-1/8*I*a*cos(d*x+c)^8/d+35/128*a*cos(d*x+c)*sin(d*x+c)/d+35/192*a
*cos(d*x+c)^3*sin(d*x+c)/d+7/48*a*cos(d*x+c)^5*sin(d*x+c)/d+1/8*a*cos(d*x+c
)^7*sin(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3567, 2715, 8}

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{ia\cos^8(c+dx)}{8d} + \frac{a\sin(c+dx)\cos^7(c+dx)}{8d} + \frac{7a\sin(c+dx)\cos^5(c+dx)}{48d} + \frac{35a\sin(c+dx)\cos^3(c+dx)}{192d} + \frac{35a\sin(c+dx)\cos(c+dx)}{128d} + \frac{35ax}{128}$$

```
[In] Int[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (35*a*x)/128 - ((I/8)*a*Cos[c + d*x]^8)/d + (35*a*Cos[c + d*x]*Sin[c + d*x]
)/(128*d) + (35*a*Cos[c + d*x]^3*Sin[c + d*x])/(192*d) + (7*a*Cos[c + d*x]^
5*Sin[c + d*x])/(48*d) + (a*Cos[c + d*x]^7*Sin[c + d*x])/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{ia\cos^8(c+dx)}{8d} + a\int \cos^8(c+dx)\,dx \\
&= -\frac{ia\cos^8(c+dx)}{8d} + \frac{a\cos^7(c+dx)\sin(c+dx)}{8d} + \frac{1}{8}(7a)\int \cos^6(c+dx)\,dx \\
&= -\frac{ia\cos^8(c+dx)}{8d} + \frac{7a\cos^5(c+dx)\sin(c+dx)}{48d} \\
&\quad + \frac{a\cos^7(c+dx)\sin(c+dx)}{8d} + \frac{1}{48}(35a)\int \cos^4(c+dx)\,dx \\
&= -\frac{ia\cos^8(c+dx)}{8d} + \frac{35a\cos^3(c+dx)\sin(c+dx)}{192d} + \frac{7a\cos^5(c+dx)\sin(c+dx)}{48d} \\
&\quad + \frac{a\cos^7(c+dx)\sin(c+dx)}{8d} + \frac{1}{64}(35a)\int \cos^2(c+dx)\,dx \\
&= -\frac{ia\cos^8(c+dx)}{8d} + \frac{35a\cos(c+dx)\sin(c+dx)}{128d} + \frac{35a\cos^3(c+dx)\sin(c+dx)}{192d} \\
&\quad + \frac{7a\cos^5(c+dx)\sin(c+dx)}{48d} + \frac{a\cos^7(c+dx)\sin(c+dx)}{8d} + \frac{1}{128}(35a)\int 1\,dx \\
&= \frac{35ax}{128} - \frac{ia\cos^8(c+dx)}{8d} + \frac{35a\cos(c+dx)\sin(c+dx)}{128d} + \frac{35a\cos^3(c+dx)\sin(c+dx)}{192d} \\
&\quad + \frac{7a\cos^5(c+dx)\sin(c+dx)}{48d} + \frac{a\cos^7(c+dx)\sin(c+dx)}{8d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.61

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))\,dx$$
$$= \frac{a(840c+840dx-384i\cos^8(c+dx)+672\sin(2(c+dx))+168\sin(4(c+dx))+32\sin(6(c+dx))+3\sin}{3072d}$$

```
[In] Integrate[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x]),x]
[Out] (a*(840*c + 840*d*x - (384*I)*Cos[c + d*x]^8 + 672*Sin[2*(c + d*x)] + 168*S
in[4*(c + d*x)] + 32*Sin[6*(c + d*x)] + 3*Sin[8*(c + d*x)]))/(3072*d)
```

Maple [A] (verified)

Time = 48.32 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.66

method	result
derivativedivides	$\frac{-\frac{ia\left(\cos^8(dx+c)\right)}{8}+a\left(\frac{\left(\cos^7(dx+c)+\frac{7\left(\cos^5(dx+c)\right)}{6}+\frac{35\left(\cos^3(dx+c)\right)}{24}+\frac{35\cos(dx+c)}{16}\right)\sin(dx+c)}{8}+\frac{35dx}{128}+\frac{35c}{128}\right)}{d}$
default	$\frac{-\frac{ia\left(\cos^8(dx+c)\right)}{8}+a\left(\frac{\left(\cos^7(dx+c)+\frac{7\left(\cos^5(dx+c)\right)}{6}+\frac{35\left(\cos^3(dx+c)\right)}{24}+\frac{35\cos(dx+c)}{16}\right)\sin(dx+c)}{8}+\frac{35dx}{128}+\frac{35c}{128}\right)}{d}$
risch	$\frac{35ax}{128}-\frac{ia\,e^{8i(dx+c)}}{1024d}-\frac{ia\cos(6dx+6c)}{128d}+\frac{a\sin(6dx+6c)}{96d}-\frac{7ia\cos(4dx+4c)}{256d}+\frac{7a\sin(4dx+4c)}{128d}-\frac{7ia\cos(2dx+}{128d}$

```
[In] int(cos(d*x+c)^8*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/8*I*a*cos(d*x+c)^8+a*(1/8*(cos(d*x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(
d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+35/128*d*x+35/128*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.94

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))\,dx$$
$$= \frac{(840\,adxe^{(6i\,dx+6i\,c)}-3i\,ae^{(14i\,dx+14i\,c)}-28i\,ae^{(12i\,dx+12i\,c)}-126i\,ae^{(10i\,dx+10i\,c)}-420i\,ae^{(8i\,dx+8i\,c)}+252i\,ae}{3072\,d}$$

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

[Out] 1/3072*(840*a*d*x*e^(6*I*d*x + 6*I*c) - 3*I*a*e^(14*I*d*x + 14*I*c) - 28*I*a*e^(12*I*d*x + 12*I*c) - 126*I*a*e^(10*I*d*x + 10*I*c) - 420*I*a*e^(8*I*d*x + 8*I*c) + 252*I*a*e^(4*I*d*x + 4*I*c) + 42*I*a*e^(2*I*d*x + 2*I*c) + 4*I*a)*e^(-6*I*d*x - 6*I*c)/d

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.51

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))\,dx = \frac{35ax}{128} + \begin{cases} \frac{(-1013309916158361 6iad^6e^{20ic}e^{8idx} - 94575592174780416iad^6e^{18ic}e^{6idx} - 425590164786511872iad^6e^{16ic}e^{4idx} - 1418633882621706240iad^6e \cdots}{103762935414616 \cdots} \\ x\left(-\frac{35a}{128} + \frac{(ae^{14ic}+7ae^{12ic}+21ae^{10ic}+35ae^{8ic}+35ae^{6ic}+21ae^{4ic}+7ae^{2ic}+a)e^{-6ic}}{128}\right) \end{cases}$$

[In] integrate(cos(d*x+c)**8*(a+I*a*tan(d*x+c)),x)

[Out] 35*a*x/128 + Piecewise(((-10133099161583616*I*a*d**6*exp(20*I*c)*exp(8*I*d*x) - 94575592174780416*I*a*d**6*exp(18*I*c)*exp(6*I*d*x) - 425590164786511872*I*a*d**6*exp(16*I*c)*exp(4*I*d*x) - 1418633882621706240*I*a*d**6*exp(14*I*c)*exp(2*I*d*x) + 851180329573023744*I*a*d**6*exp(10*I*c)*exp(-2*I*d*x) + 141863388262170624*I*a*d**6*exp(8*I*c)*exp(-4*I*d*x) + 13510798882111488*I*a*d**6*exp(6*I*c)*exp(-6*I*d*x))*exp(-12*I*c)/(103762935414616227 84*d**7), Ne(d**7*exp(12*I*c), 0)), (x*(-35*a/128 + (a*exp(14*I*c) + 7*a*exp(12*I*c) + 21*a*exp(10*I*c) + 35*a*exp(8*I*c) + 35*a*exp(6*I*c) + 21*a*exp(4*I*c) + 7*a*exp(2*I*c) + a)*exp(-6*I*c)/128), True))

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.93

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))\,dx = \frac{105\,(dx+c)a + \frac{105\,a\tan(dx+c)^7+385\,a\tan(dx+c)^5+511\,a\tan(dx+c)^3+279\,a\tan(dx+c)-48i\,a}{\tan(dx+c)^8+4\,\tan(dx+c)^6+6\,\tan(dx+c)^4+4\,\tan(dx+c)^2+1}}{384\,d}$$

[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c)),x, algorithm="maxima")

[Out] 1/384*(105*(d*x + c)*a + (105*a*tan(d*x + c)^7 + 385*a*tan(d*x + c)^5 + 511*a*tan(d*x + c)^3 + 279*a*tan(d*x + c) - 48*I*a)/(tan(d*x + c)^8 + 4*tan(d*x + c)^6 + 6*tan(d*x + c)^4 + 4*tan(d*x + c)^2 + 1))/d

Giac [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.36

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))\,dx = \frac{\left(840\,adxe^{(6i\,dx+2i\,c)} + 84i\,ae^{(6i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right) - 84i\,ae^{(6i\,dx+2i\,c)}\log\left(e^{(2i\,dx)}+e^{(-2i\,c)}\right) - 3i\,ae^{(14}\right.}{}$$

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/3072*(840*a*d*x*e^(6*I*d*x + 2*I*c) + 84*I*a*e^(6*I*d*x + 2*I*c)*log(e^(2
*I*d*x + 2*I*c) + 1) - 84*I*a*e^(6*I*d*x + 2*I*c)*log(e^(2*I*d*x) + e^(-2*I
*c)) - 3*I*a*e^(14*I*d*x + 10*I*c) - 28*I*a*e^(12*I*d*x + 8*I*c) - 126*I*a*
e^(10*I*d*x + 6*I*c) - 420*I*a*e^(8*I*d*x + 4*I*c) + 42*I*a*e^(2*I*d*x - 2*
I*c) + 252*I*a*e^(4*I*d*x) + 4*I*a*e^(-4*I*c))*e^(-6*I*d*x - 2*I*c)/d
```

Mupad [B] (verification not implemented)

Time = 5.55 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.37

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))\,dx = \frac{35\,a\,x}{128} + \frac{\frac{35\,a\tan(c+d\,x)^6}{128} + \frac{35\mathrm{i}\,a\tan(c+d\,x)^5}{128} + \frac{35\,a\tan(c+d\,x)^4}{48} + \frac{35\mathrm{i}\,a\tan(c+d\,x)^3}{48} + \frac{77\,a\tan(c+d\,x)^2}{128} + \frac{77\mathrm{i}\,a\tan(}{128}}{d\,\left(\tan\left(c+d\,x\right)^7 + \tan\left(c+d\,x\right)^6\,1\mathrm{i} + 3\tan\left(c+d\,x\right)^5 + \tan\left(c+d\,x\right)^4\,3\mathrm{i} + 3\tan\left(c+d\,x\right)^3 + \tan\left(c+d\right.\right.}$$

```
[In] int(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i),x)
```

```
[Out] (35*a*x)/128 + (a/8 + (a*tan(c + d*x)*77i)/128 + (77*a*tan(c + d*x)^2)/128
+ (a*tan(c + d*x)^3*35i)/48 + (35*a*tan(c + d*x)^4)/48 + (a*tan(c + d*x)^5*
35i)/128 + (35*a*tan(c + d*x)^6)/128)/(d*(tan(c + d*x) + tan(c + d*x)^2*3i
+ 3*tan(c + d*x)^3 + tan(c + d*x)^4*3i + 3*tan(c + d*x)^5 + tan(c + d*x)^6*
1i + tan(c + d*x)^7 + 1i))
```

3.11 $\int \sec^7(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 98

$$\int \sec^7(c+dx)(a+ia\tan(c+dx))\,dx = \frac{5a\text{arctanh}(\sin(c+dx))}{16d} + \frac{ia\sec^7(c+dx)}{7d} + \frac{5a\sec(c+dx)\tan(c+dx)}{16d} + \frac{5a\sec^3(c+dx)\tan(c+dx)}{24d} + \frac{a\sec^5(c+dx)\tan(c+dx)}{6d}$$

```
[Out] 5/16*a*arctanh(sin(d*x+c))/d+1/7*I*a*sec(d*x+c)^7/d+5/16*a*sec(d*x+c)*tan(d
*x+c)/d+5/24*a*sec(d*x+c)^3*tan(d*x+c)/d+1/6*a*sec(d*x+c)^5*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3567, 3853, 3855}

$$\int \sec^7(c+dx)(a+ia\tan(c+dx))\,dx = \frac{5a\text{arctanh}(\sin(c+dx))}{16d} + \frac{ia\sec^7(c+dx)}{7d} + \frac{a\tan(c+dx)\sec^5(c+dx)}{6d} + \frac{5a\tan(c+dx)\sec^3(c+dx)}{24d} + \frac{5a\tan(c+dx)\sec(c+dx)}{16d}$$

```
[In] Int[Sec[c + d*x]^7*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (5*a*ArcTanh[Sin[c + d*x]])/(16*d) + ((I/7)*a*Sec[c + d*x]^7)/d + (5*a*Sec[
c + d*x]*Tan[c + d*x])/(16*d) + (5*a*Sec[c + d*x]^3*Tan[c + d*x])/(24*d) +
(a*Sec[c + d*x]^5*Tan[c + d*x])/(6*d)
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{ia\sec^7(c+dx)}{7d} + a\int \sec^7(c+dx)\,dx \\
&= \frac{ia\sec^7(c+dx)}{7d} + \frac{a\sec^5(c+dx)\tan(c+dx)}{6d} + \frac{1}{6}(5a)\int \sec^5(c+dx)\,dx \\
&= \frac{ia\sec^7(c+dx)}{7d} + \frac{5a\sec^3(c+dx)\tan(c+dx)}{24d} \\
&\quad + \frac{a\sec^5(c+dx)\tan(c+dx)}{6d} + \frac{1}{8}(5a)\int \sec^3(c+dx)\,dx \\
&= \frac{ia\sec^7(c+dx)}{7d} + \frac{5a\sec(c+dx)\tan(c+dx)}{16d} + \frac{5a\sec^3(c+dx)\tan(c+dx)}{24d} \\
&\quad + \frac{a\sec^5(c+dx)\tan(c+dx)}{6d} + \frac{1}{16}(5a)\int \sec(c+dx)\,dx \\
&= \frac{5a\text{arctanh}(\sin(c+dx))}{16d} + \frac{ia\sec^7(c+dx)}{7d} + \frac{5a\sec(c+dx)\tan(c+dx)}{16d} \\
&\quad + \frac{5a\sec^3(c+dx)\tan(c+dx)}{24d} + \frac{a\sec^5(c+dx)\tan(c+dx)}{6d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int \sec^7(c+dx)(a+ia\tan(c+dx))\,dx = \frac{5a\text{arctanh}(\sin(c+dx))}{16d} + \frac{ia\sec^7(c+dx)}{7d} + \frac{5a\sec(c+dx)\tan(c+dx)}{16d} + \frac{5a\sec^3(c+dx)\tan(c+dx)}{24d} + \frac{a\sec^5(c+dx)\tan(c+dx)}{6d}$$

```
[In] Integrate[Sec[c + d*x]^7*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (5*a*ArcTanh[Sin[c + d*x]])/(16*d) + ((I/7)*a*Sec[c + d*x]^7)/d + (5*a*Sec[
c + d*x]*Tan[c + d*x])/(16*d) + (5*a*Sec[c + d*x]^3*Tan[c + d*x])/(24*d) +
(a*Sec[c + d*x]^5*Tan[c + d*x])/(6*d)
```

Maple [A] (verified)

Time = 36.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{\frac{ia}{7\cos(dx+c)^7}+a\left(-\left(-\frac{\left(\sec^5(dx+c)\right)}{6}-\frac{5\left(\sec^3(dx+c)\right)}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)}{d}$
default	$\frac{\frac{ia}{7\cos(dx+c)^7}+a\left(-\left(-\frac{\left(\sec^5(dx+c)\right)}{6}-\frac{5\left(\sec^3(dx+c)\right)}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)}{d}$
risch	$-\frac{ia\left(105\,\mathrm{e}^{13i(dx+c)}+700\,\mathrm{e}^{11i(dx+c)}+1981\,\mathrm{e}^{9i(dx+c)}-3072\,\mathrm{e}^{7i(dx+c)}-1981\,\mathrm{e}^{5i(dx+c)}-700\,\mathrm{e}^{3i(dx+c)}-105\,\mathrm{e}^{i(dx+c)}\right)}{168d\left(\mathrm{e}^{2i(dx+c)}+1\right)^7}+\frac{5a}{}$

```
[In] int(sec(d*x+c)^7*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/7*I*a/cos(d*x+c)^7+a*(-(-1/6*sec(d*x+c)^5-5/24*sec(d*x+c)^3-5/16*sec
(d*x+c))*tan(d*x+c)+5/16*ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 372 vs. 2(86) = 172.

Time = 0.25 (sec) , antiderivative size = 372, normalized size of antiderivative = 3.80

$$\int \sec^7(c+dx)(a+ia\tan(c+dx))\,dx$$
$$= \frac{-210i\,ae^{(13i\,dx+13i\,c)} - 1400i\,ae^{(11i\,dx+11i\,c)} - 3962i\,ae^{(9i\,dx+9i\,c)} + 6144i\,ae^{(7i\,dx+7i\,c)} + 3962i\,ae^{(5i\,dx+5i\,c)} + 1}{}$$

```
[In] integrate(sec(d*x+c)^7*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/336*(-210*I*a*e^(13*I*d*x + 13*I*c) - 1400*I*a*e^(11*I*d*x + 11*I*c) - 39
62*I*a*e^(9*I*d*x + 9*I*c) + 6144*I*a*e^(7*I*d*x + 7*I*c) + 3962*I*a*e^(5*I
*d*x + 5*I*c) + 1400*I*a*e^(3*I*d*x + 3*I*c) + 210*I*a*e^(I*d*x + I*c) + 10
5*(a*e^(14*I*d*x + 14*I*c) + 7*a*e^(12*I*d*x + 12*I*c) + 21*a*e^(10*I*d*x +
 10*I*c) + 35*a*e^(8*I*d*x + 8*I*c) + 35*a*e^(6*I*d*x + 6*I*c) + 21*a*e^(4*
I*d*x + 4*I*c) + 7*a*e^(2*I*d*x + 2*I*c) + a)*log(e^(I*d*x + I*c) + I) - 10
5*(a*e^(14*I*d*x + 14*I*c) + 7*a*e^(12*I*d*x + 12*I*c) + 21*a*e^(10*I*d*x +
 10*I*c) + 35*a*e^(8*I*d*x + 8*I*c) + 35*a*e^(6*I*d*x + 6*I*c) + 21*a*e^(4*
I*d*x + 4*I*c) + 7*a*e^(2*I*d*x + 2*I*c) + a)*log(e^(I*d*x + I*c) - I))/(d*
e^(14*I*d*x + 14*I*c) + 7*d*e^(12*I*d*x + 12*I*c) + 21*d*e^(10*I*d*x + 10*I
*c) + 35*d*e^(8*I*d*x + 8*I*c) + 35*d*e^(6*I*d*x + 6*I*c) + 21*d*e^(4*I*d*x
 + 4*I*c) + 7*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^7(c+dx)(a+ia\tan(c+dx))\,dx = ia\left(\int \left(-i\sec^7\left(c+dx\right)\right)\,dx + \int \tan\left(c+dx\right)\sec^7\left(c+dx\right)dx\right)$$

```
[In] integrate(sec(d*x+c)**7*(a+I*a*tan(d*x+c)),x)
```

```
[Out] I*a*(Integral(-I*sec(c + d*x)**7, x) + Integral(tan(c + d*x)*sec(c + d*x)**
7, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08

$$\int \sec^7(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{7\,a\left(\frac{2\left(15\,\sin(dx+c)^5-40\,\sin(dx+c)^3+33\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1} - 15\,\log\left(\sin\left(dx+c\right)+1\right) + 15\,\log\left(\sin\left(dx+c\right)-1\right)\right) - \frac{9}{\cos(}}{672\,d}$$

[In] integrate(sec(d*x+c)^7*(a+I*a*tan(d*x+c)),x, algorithm="maxima")

[Out] -1/672*(7*a*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 96*I*a/cos(d*x + c)^7)/d

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(86) = 172.

Time = 0.41 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.85

$$\int \sec^7(c+dx)(a+ia\tan(c+dx))\,dx = \frac{105\,a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right) - 105\,a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right) + \frac{2\left(231\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}-336i\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}\right.}{}}{}$$

[In] integrate(sec(d*x+c)^7*(a+I*a*tan(d*x+c)),x, algorithm="giac")

[Out] 1/336*(105*a*log(tan(1/2*d*x + 1/2*c) + 1) - 105*a*log(tan(1/2*d*x + 1/2*c) - 1) + 2*(231*a*tan(1/2*d*x + 1/2*c)^13 - 336*I*a*tan(1/2*d*x + 1/2*c)^12 - 196*a*tan(1/2*d*x + 1/2*c)^11 + 595*a*tan(1/2*d*x + 1/2*c)^9 - 1680*I*a*tan(1/2*d*x + 1/2*c)^8 - 595*a*tan(1/2*d*x + 1/2*c)^5 - 1008*I*a*tan(1/2*d*x + 1/2*c)^4 + 196*a*tan(1/2*d*x + 1/2*c)^3 - 231*a*tan(1/2*d*x + 1/2*c) - 48*I*a)/(tan(1/2*d*x + 1/2*c)^2 - 1)^7)/d

Mupad [B] (verification not implemented)

Time = 8.36 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.52

$$\int \sec^7(c+dx)(a+ia\tan(c+dx))\,dx = \frac{5\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}$$

$$-\frac{-\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{8}+2\mathrm{i}\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+\frac{7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{6}-\frac{85\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+10\mathrm{i}\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+\frac{85\,a\tan\left(\right.}{24}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+35\tan\left(\frac{c}{2}\right.\right.}$$

```
[In] int((a + a*tan(c + d*x)*1i)/cos(c + d*x)^7,x)
```

```
[Out] (5*a*atanh(tan(c/2 + (d*x)/2)))/(8*d) - ((a*2i)/7 + (11*a*tan(c/2 + (d*x)/2
))/8 - (7*a*tan(c/2 + (d*x)/2)^3)/6 + a*tan(c/2 + (d*x)/2)^4*6i + (85*a*tan
(c/2 + (d*x)/2)^5)/24 + a*tan(c/2 + (d*x)/2)^8*10i - (85*a*tan(c/2 + (d*x)/
2)^9)/24 + (7*a*tan(c/2 + (d*x)/2)^11)/6 + a*tan(c/2 + (d*x)/2)^12*2i - (11
*a*tan(c/2 + (d*x)/2)^13)/8)/(d*(7*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*x
)/2)^4 + 35*tan(c/2 + (d*x)/2)^6 - 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2 + (
d*x)/2)^10 - 7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 - 1))
```

3.12 $\int \sec^5(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 76

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))\,dx = \frac{3a\text{arctanh}(\sin(c+dx))}{8d}+\frac{ia\sec^5(c+dx)}{5d}+\frac{3a\sec(c+dx)\tan(c+dx)}{8d}+\frac{a\sec^3(c+dx)\tan(c+dx)}{4d}$$

```
[Out] 3/8*a*arctanh(sin(d*x+c))/d+1/5*I*a*sec(d*x+c)^5/d+3/8*a*sec(d*x+c)*tan(d*x
+c)/d+1/4*a*sec(d*x+c)^3*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3567, 3853, 3855}

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))\,dx = \frac{3a\text{arctanh}(\sin(c+dx))}{8d}+\frac{ia\sec^5(c+dx)}{5d}+\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a\tan(c+dx)\sec(c+dx)}{8d}$$

```
[In] Int[Sec[c + d*x]^5*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) + ((I/5)*a*Sec[c + d*x]^5)/d + (3*a*Sec[c
 + d*x]*Tan[c + d*x])/(8*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{ia\sec^5(c+dx)}{5d} + a\int \sec^5(c+dx)\,dx \\
&= \frac{ia\sec^5(c+dx)}{5d} + \frac{a\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}(3a)\int \sec^3(c+dx)\,dx \\
&= \frac{ia\sec^5(c+dx)}{5d} + \frac{3a\sec(c+dx)\tan(c+dx)}{8d} \\
&\quad + \frac{a\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{8}(3a)\int \sec(c+dx)\,dx \\
&= \frac{3a\operatorname{arctanh}(\sin(c+dx))}{8d} + \frac{ia\sec^5(c+dx)}{5d} \\
&\quad + \frac{3a\sec(c+dx)\tan(c+dx)}{8d} + \frac{a\sec^3(c+dx)\tan(c+dx)}{4d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\begin{aligned}
\int \sec^5(c+dx)(a+ia\tan(c+dx))\,dx &= \frac{3a\operatorname{arctanh}(\sin(c+dx))}{8d} + \frac{ia\sec^5(c+dx)}{5d} \\
&\quad + \frac{3a\sec(c+dx)\tan(c+dx)}{8d} \\
&\quad + \frac{a\sec^3(c+dx)\tan(c+dx)}{4d}
\end{aligned}$$

```
[In] Integrate[Sec[c + d*x]^5*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) + ((I/5)*a*Sec[c + d*x]^5)/d + (3*a*Sec[c
 + d*x]*Tan[c + d*x])/(8*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Maple [A] (verified)

Time = 8.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{\frac{ia}{5\cos(dx+c)^5}+a\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}$
default	$\frac{\frac{ia}{5\cos(dx+c)^5}+a\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}$
risch	$-\frac{ia\left(15\,e^{9i(dx+c)}+70\,e^{7i(dx+c)}-128\,e^{5i(dx+c)}-70\,e^{3i(dx+c)}-15\,e^{i(dx+c)}\right)}{20d\left(e^{2i(dx+c)}+1\right)^5}+\frac{3a\ln\left(e^{i(dx+c)}+i\right)}{8d}-\frac{3a\ln\left(e^{i(dx+c)}-i\right)}{8d}$

```
[In] int(sec(d*x+c)^5*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/5*I*a/cos(d*x+c)^5+a*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)
+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 276 vs. 2(66) = 132.

Time = 0.24 (sec) , antiderivative size = 276, normalized size of antiderivative = 3.63

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))\,dx$$
$$=\frac{-30i\,ae^{(9i\,dx+9i\,c)}-140i\,ae^{(7i\,dx+7i\,c)}+256i\,ae^{(5i\,dx+5i\,c)}+140i\,ae^{(3i\,dx+3i\,c)}+30i\,ae^{(i\,dx+i\,c)}+15\left(ae^{(10i\,dx+10i}\right.}{}$$

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/40*(-30*I*a*e^(9*I*d*x + 9*I*c) - 140*I*a*e^(7*I*d*x + 7*I*c) + 256*I*a*e
^(5*I*d*x + 5*I*c) + 140*I*a*e^(3*I*d*x + 3*I*c) + 30*I*a*e^(I*d*x + I*c) +
 15*(a*e^(10*I*d*x + 10*I*c) + 5*a*e^(8*I*d*x + 8*I*c) + 10*a*e^(6*I*d*x +
6*I*c) + 10*a*e^(4*I*d*x + 4*I*c) + 5*a*e^(2*I*d*x + 2*I*c) + a)*log(e^(I*d
*x + I*c) + I) - 15*(a*e^(10*I*d*x + 10*I*c) + 5*a*e^(8*I*d*x + 8*I*c) + 10
*a*e^(6*I*d*x + 6*I*c) + 10*a*e^(4*I*d*x + 4*I*c) + 5*a*e^(2*I*d*x + 2*I*c)
 + a)*log(e^(I*d*x + I*c) - I))/(d*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*I*d*x +
 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) + 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*
d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))\,dx = ia\left(\int \left(-i\sec^5\left(c+dx\right)\right)\,dx + \int \tan\left(c+dx\right)\sec^5\left(c+dx\right)dx\right)$$

```
[In] integrate(sec(d*x+c)**5*(a+I*a*tan(d*x+c)),x)
[Out] I*a*(Integral(-I*sec(c + d*x)**5, x) + Integral(tan(c + d*x)*sec(c + d*x)**
5, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.13

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{5\,a\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1} - 3\,\log\left(\sin\left(dx+c\right)+1\right) + 3\,\log\left(\sin\left(dx+c\right)-1\right)\right) - \frac{16i\,a}{\cos(dx+c)^5}}{80\,d}$$

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
[Out] -1/80*(5*a*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d
*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 16*I*
a/cos(d*x + c)^5)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(66) = 132.

Time = 0.37 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.83

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))\,dx = \frac{15\,a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right) - 15\,a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right) + \frac{2\left(25\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-40i\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-10\,a}{}}{40\,d}$$

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/40*(15*a*log(tan(1/2*d*x + 1/2*c) + 1) - 15*a*log(tan(1/2*d*x + 1/2*c) -
1) + 2*(25*a*tan(1/2*d*x + 1/2*c)^9 - 40*I*a*tan(1/2*d*x + 1/2*c)^8 - 10*a*
tan(1/2*d*x + 1/2*c)^7 - 80*I*a*tan(1/2*d*x + 1/2*c)^4 + 10*a*tan(1/2*d*x +
 1/2*c)^3 - 25*a*tan(1/2*d*x + 1/2*c) - 8*I*a)/(tan(1/2*d*x + 1/2*c)^2 - 1)
^5)/d
```

Mupad [B] (verification not implemented)

Time = 7.57 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.34

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))\,dx = \frac{3\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d} - \frac{-\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}+2\mathrm{i}\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2}+4\mathrm{i}\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}+\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+ a}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

```
[In] int((a + a*tan(c + d*x)*1i)/cos(c + d*x)^5,x)
```

```
[Out] (3*a*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((a*2i)/5 + (5*a*tan(c/2 + (d*x)/2)
)/4 - (a*tan(c/2 + (d*x)/2)^3)/2 + a*tan(c/2 + (d*x)/2)^4*4i + (a*tan(c/2 +
 (d*x)/2)^7)/2 + a*tan(c/2 + (d*x)/2)^8*2i - (5*a*tan(c/2 + (d*x)/2)^9)/4)/
(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2
)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.13 $\int \sec^3(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 54

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a\text{arctanh}(\sin(c+dx))}{2d} + \frac{ia\sec^3(c+dx)}{3d} + \frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

```
[Out] 1/2*a*arctanh(sin(d*x+c))/d+1/3*I*a*sec(d*x+c)^3/d+1/2*a*sec(d*x+c)*tan(d*x
+c)/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3567, 3853, 3855}

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a\text{arctanh}(\sin(c+dx))}{2d} + \frac{ia\sec^3(c+dx)}{3d} + \frac{a\tan(c+dx)\sec(c+dx)}{2d}$$

```
[In] Int[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + ((I/3)*a*Sec[c + d*x]^3)/d + (a*Sec[c + d
*x]*Tan[c + d*x])/(2*d)
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
```

```
| NeQ[a^2 + b^2, 0])
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{ia\sec^3(c+dx)}{3d} + a\int \sec^3(c+dx)\,dx \\
&= \frac{ia\sec^3(c+dx)}{3d} + \frac{a\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}a\int \sec(c+dx)\,dx \\
&= \frac{a\text{arctanh}(\sin(c+dx))}{2d} + \frac{ia\sec^3(c+dx)}{3d} + \frac{a\sec(c+dx)\tan(c+dx)}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a\text{arctanh}(\sin(c+dx))}{2d} + \frac{ia\sec^3(c+dx)}{3d} + \frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

```
[In] Integrate[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x]),x]
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + ((I/3)*a*Sec[c + d*x]^3)/d + (a*Sec[c + d
*x]*Tan[c + d*x])/(2*d)
```

Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{\frac{ia}{3\cos(dx+c)^3}+a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$	51
default	$\frac{\frac{ia}{3\cos(dx+c)^3}+a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$	51
risch	$-\frac{ia\left(3\,e^{5i(dx+c)}-8\,e^{3i(dx+c)}-3\,e^{i(dx+c)}\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}+\frac{a\ln\left(e^{i(dx+c)}+i\right)}{2d}-\frac{a\ln\left(e^{i(dx+c)}-i\right)}{2d}$	94

```
[In] int(sec(d*x+c)^3*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*I*a/cos(d*x+c)^3+a*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+ta
n(d*x+c))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(46) = 92.

Time = 0.24 (sec) , antiderivative size = 180, normalized size of antiderivative = 3.33

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))\,dx = \frac{-6i\,ae^{(5i\,dx+5i\,c)}+16i\,ae^{(3i\,dx+3i\,c)}+6i\,ae^{(i\,dx+i\,c)}+3\left(ae^{(6i\,dx+6i\,c)}+3\,ae^{(4i\,dx+4i\,c)}+3\,ae^{(2i\,dx+2i\,c)}+a\right)\log}{6\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,d\right.}$$

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(-6*I*a*e^(5*I*d*x + 5*I*c) + 16*I*a*e^(3*I*d*x + 3*I*c) + 6*I*a*e^(I*d
*x + I*c) + 3*(a*e^(6*I*d*x + 6*I*c) + 3*a*e^(4*I*d*x + 4*I*c) + 3*a*e^(2*I
*d*x + 2*I*c) + a)*log(e^(I*d*x + I*c) + I) - 3*(a*e^(6*I*d*x + 6*I*c) + 3*
a*e^(4*I*d*x + 4*I*c) + 3*a*e^(2*I*d*x + 2*I*c) + a)*log(e^(I*d*x + I*c) -
I))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I
*c) + d)
```

Sympy [F]

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))\,dx = ia\left(\int\left(-i\sec^3{(c+dx)}\right)\,dx + \int \tan{(c+dx)}\sec^3{(c+dx)}\,dx\right)$$

```
[In] integrate(sec(d*x+c)**3*(a+I*a*tan(d*x+c)),x)
[Out] I*a*(Integral(-I*sec(c + d*x)**3, x) + Integral(tan(c + d*x)*sec(c + d*x)**
3, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{3\,a\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx+c\right)+1\right) + \log\left(\sin\left(dx+c\right)-1\right)\right) - \frac{4i\,a}{\cos(dx+c)^3}}{12\,d}$$

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
[Out] -1/12*(3*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + l
og(sin(d*x + c) - 1)) - 4*I*a/cos(d*x + c)^3)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(46) = 92.

Time = 0.36 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.80

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))\,dx = \frac{3\,a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right) - 3\,a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right) + \frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 6i\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 - 3\,a\tan\left(\frac{1}{2}\,dx\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{6\,d}$$

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] 1/6*(3*a*log(tan(1/2*d*x + 1/2*c) + 1) - 3*a*log(tan(1/2*d*x + 1/2*c) - 1)
+ 2*(3*a*tan(1/2*d*x + 1/2*c)^5 - 6*I*a*tan(1/2*d*x + 1/2*c)^4 - 3*a*tan(1/
2*d*x + 1/2*c) - 2*I*a)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 5.96 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.98

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))\,dx$$
$$=\frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+2\mathrm{i}\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a\,2\mathrm{i}}{3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

```
[In] int((a + a*tan(c + d*x)*1i)/cos(c + d*x)^3,x)
```

```
[Out] (a*atanh(tan(c/2 + (d*x)/2)))/d - ((a*2i)/3 + a*tan(c/2 + (d*x)/2) + a*tan(
c/2 + (d*x)/2)^4*2i - a*tan(c/2 + (d*x)/2)^5)/(d*(3*tan(c/2 + (d*x)/2)^2 -
3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.14 $\int \sec(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 27

$$\int \sec(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a\text{arctanh}(\sin(c+dx))}{d} + \frac{ia\sec(c+dx)}{d}$$

```
[Out] a*arctanh(sin(d*x+c))/d+I*a*sec(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3567, 3855}

$$\int \sec(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a\text{arctanh}(\sin(c+dx))}{d} + \frac{ia\sec(c+dx)}{d}$$

```
[In] Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d + (I*a*Sec[c + d*x])/d
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{ia\sec(c+dx)}{d} + a\int \sec(c+dx)\,dx \\ &= \frac{a\text{arctanh}(\sin(c+dx))}{d} + \frac{ia\sec(c+dx)}{d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \sec(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a\text{arctanh}(\sin(c+dx))}{d} + \frac{ia\sec(c+dx)}{d}$$

```
[In] Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x]),x]
[Out] (a*ArcTanh[Sin[c + d*x]])/d + (I*a*Sec[c + d*x])/d
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

method	result	size
derivativedivides	$\frac{\frac{ia}{\cos(dx+c)}+a\ln(\sec(dx+c)+\tan(dx+c))}{d}$	34
default	$\frac{\frac{ia}{\cos(dx+c)}+a\ln(\sec(dx+c)+\tan(dx+c))}{d}$	34
risch	$\frac{2i\mathrm{e}^{i(dx+c)}a}{d(\mathrm{e}^{2i(dx+c)}+1)}+\frac{a\ln(\mathrm{e}^{i(dx+c)}+i)}{d}-\frac{a\ln(\mathrm{e}^{i(dx+c)}-i)}{d}$	68

```
[In] int(sec(d*x+c)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(I*a/cos(d*x+c)+a*ln(sec(d*x+c)+tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(25) = 50.

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 3.04

$$\int \sec(c+dx)(a+ia\tan(c+dx))\,dx = \frac{2i\,ae^{(i\,dx+i\,c)} + \left(ae^{(2i\,dx+2i\,c)}+a\right)\log\left(e^{(i\,dx+i\,c)}+i\right) - \left(ae^{(2i\,dx+2i\,c)}+a\right)\log\left(e^{(i\,dx+i\,c)}-i\right)}{de^{(2i\,dx+2i\,c)}+d}$$

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*I*a*e^(I*d*x + I*c) + (a*e^(2*I*d*x + 2*I*c) + a)*log(e^(I*d*x + I*c) +
I) - (a*e^(2*I*d*x + 2*I*c) + a)*log(e^(I*d*x + I*c) - I))/(d*e^(2*I*d*x +
2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int \sec(c+dx)(a+ia\tan(c+dx))\,dx = \begin{cases} \frac{a\log\left(\tan\left(c+dx\right)+\sec\left(c+dx\right)\right)+ia\sec\left(c+dx\right)}{d} & \text{for } d\neq 0 \\ x(ia\tan\left(c\right)+a)\sec\left(c\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise(((a*log(tan(c + d*x) + sec(c + d*x)) + I*a*sec(c + d*x))/d, Ne(d,
 0)), (x*(I*a*tan(c) + a)*sec(c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \sec(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+\frac{ia}{\cos(dx+c)}}{d}$$

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] (a*log(sec(d*x + c) + tan(d*x + c)) + I*a/cos(d*x + c))/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(25) = 50.

Time = 0.37 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.93

$$\int \sec(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-a\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)-\frac{2i\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}}{d}$$

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(tan(1/2*d*x + 1/2*c) + 1) - a*log(tan(1/2*d*x + 1/2*c) - 1) - 2*I*a/
(tan(1/2*d*x + 1/2*c)^2 - 1))/d
```

Mupad [B] (verification not implemented)

Time = 4.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int \sec(c+dx)(a+ia\tan(c+dx))\,dx = \frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{a\,2\mathrm{i}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

```
[In] int((a + a*tan(c + d*x)*1i)/cos(c + d*x),x)
[Out] (2*a*atanh(tan(c/2 + (d*x)/2)))/d - (a*2i)/(d*(tan(c/2 + (d*x)/2)^2 - 1))
```

3.15 $\int \cos(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 26

$$\int \cos(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{ia\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d}$$

```
[Out] -I*a*cos(d*x+c)/d+a*sin(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3567, 2717}

$$\int \cos(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a\sin(c+dx)}{d}-\frac{ia\cos(c+dx)}{d}$$

```
[In] Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((-I)*a*Cos[c + d*x])/d + (a*Sin[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rubi steps

$$\text{integral} = -\frac{ia\cos(c+dx)}{d} + a\int \cos(c+dx)\,dx$$
$$= -\frac{ia\cos(c+dx)}{d} + \frac{a\sin(c+dx)}{d}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.96

$$\int \cos(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{ia\cos(c)\cos(dx)}{d} + \frac{a\cos(dx)\sin(c)}{d} + \frac{a\cos(c)\sin(dx)}{d} + \frac{ia\sin(c)\sin(dx)}{d}$$

```
[In] Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x]),x]
[Out] ((-I)*a*Cos[c]*Cos[d*x])/d + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d
+ (I*a*Sin[c]*Sin[d*x])/d
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

method	result	size
risch	$-\frac{ia\,e^{i(dx+c)}}{d}$	17
derivativedivides	$\frac{-ia\cos(dx+c)+a\sin(dx+c)}{d}$	24
default	$\frac{-ia\cos(dx+c)+a\sin(dx+c)}{d}$	24

```
[In] int(cos(d*x+c)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] -I/d*a*exp(I*(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.58

$$\int \cos(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{i\,ae^{(i\,dx+i\,c)}}{d}$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
[Out] -I*a*e^(I*d*x + I*c)/d
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \cos(c+dx)(a+ia\tan(c+dx))\,dx = \begin{cases} -\frac{iae^{ic}e^{idx}}{d} & \text{for } d \neq 0 \\ axe^{ic} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c)),x)
[Out] Piecewise((-I*a*exp(I*c)*exp(I*d*x)/d, Ne(d, 0)), (a*x*exp(I*c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \cos(c+dx)(a+ia\tan(c+dx))\,dx = \frac{-i\,a\cos\left(dx+c\right)+a\sin\left(dx+c\right)}{d}$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
[Out] (-I*a*cos(d*x + c) + a*sin(d*x + c))/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(24) = 48.

Time = 0.36 (sec) , antiderivative size = 84, normalized size of antiderivative = 3.23

$$\int \cos(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{4i\,ae^{(i\,dx+i\,c)}+a\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+a\log\left(i\,e^{(i\,dx+i\,c)}-1\right)-a\log\left(-i\,e^{(i\,dx+i\,c)}+1\right)-a\log\left(-i\,e^{(i\,dx+i\,c}\right.}{4\,d}$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(4*I*a*e^(I*d*x + I*c) + a*log(I*e^(I*d*x + I*c) + 1) + a*log(I*e^(I*d
*x + I*c) - 1) - a*log(-I*e^(I*d*x + I*c) + 1) - a*log(-I*e^(I*d*x + I*c) -
 1))/d
```

Mupad [B] (verification not implemented)

Time = 3.95 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \cos(c+dx)(a+ia\tan(c+dx))\,dx = \frac{2\,a}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

```
[In] int(cos(c + d*x)*(a + a*tan(c + d*x)*1i),x)
```

```
[Out] (2*a)/(d*(tan(c/2 + (d*x)/2) + 1i))
```

3.16 $\int \cos^3(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 46

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{ia\cos^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}$$

```
[Out] -1/3*I*a*cos(d*x+c)^3/d+a*sin(d*x+c)/d-1/3*a*sin(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3567, 2713}

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{ia\cos^3(c+dx)}{3d}$$

```
[In] Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((-1/3*I)*a*Cos[c + d*x]^3)/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*
d)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
```

```
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{ia\cos^3(c+dx)}{3d} + a\int \cos^3(c+dx)\,dx \\ &= -\frac{ia\cos^3(c+dx)}{3d} - \frac{a\text{Subst}\left(\int (1-x^2)\,dx, x, -\sin(c+dx)\right)}{d} \\ &= -\frac{ia\cos^3(c+dx)}{3d} + \frac{a\sin(c+dx)}{d} - \frac{a\sin^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{ia\cos^3(c+dx)}{3d} + \frac{a\sin(c+dx)}{d} - \frac{a\sin^3(c+dx)}{3d}$$

```
[In] Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x]),x]
[Out] ((-1/3*I)*a*Cos[c + d*x]^3)/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*
d)
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{-\frac{ia\left(\cos^3(dx+c)\right)}{3}+\frac{a\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$	37
default	$\frac{-\frac{ia\left(\cos^3(dx+c)\right)}{3}+\frac{a\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$	37
risch	$-\frac{ia\,e^{3i(dx+c)}}{12d}-\frac{ia\cos(dx+c)}{4d}+\frac{3a\sin(dx+c)}{4d}$	43

```
[In] int(cos(d*x+c)^3*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/3*I*a*cos(d*x+c)^3+1/3*a*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))\,dx = \frac{\left(-i\,ae^{(4i\,dx+4i\,c)} - 6i\,ae^{(2i\,dx+2i\,c)} + 3i\,a\right)e^{(-i\,dx-i\,c)}}{12\,d}$$

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(-I*a*e^(4*I*d*x + 4*I*c) - 6*I*a*e^(2*I*d*x + 2*I*c) + 3*I*a)*e^(-I*d
*x - I*c)/d
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(37) = 74.

Time = 0.16 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.28

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))\,dx = \begin{cases} \frac{\left(-8iad^2e^{4ic}e^{3idx}-48iad^2e^{2ic}e^{idx}+24iad^2e^{-idx}\right)e^{-ic}}{96d^3} & \text{for } d^3e^{ic}\neq 0 \\ \frac{x\left(ae^{4ic}+2ae^{2ic}+a\right)e^{-ic}}{4} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise((((-8*I*a*d**2*exp(4*I*c)*exp(3*I*d*x) - 48*I*a*d**2*exp(2*I*c)*ex
p(I*d*x) + 24*I*a*d**2*exp(-I*d*x))*exp(-I*c)/(96*d**3), Ne(d**3*exp(I*c),
0)), (x*(a*exp(4*I*c) + 2*a*exp(2*I*c) + a)*exp(-I*c)/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{i\,a\cos\left(dx+c\right)^3 + \left(\sin\left(dx+c\right)^3 - 3\,\sin\left(dx+c\right)\right)a}{3\,d}$$

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/3*(I*a*cos(d*x + c)^3 + (sin(d*x + c)^3 - 3*sin(d*x + c))*a)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(40) = 80$.

Time = 0.40 (sec) , antiderivative size = 196, normalized size of antiderivative = 4.26

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{\left(9\,ae^{(i\,dx+i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+6\,ae^{(i\,dx+i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}-1\right)-9\,ae^{(i\,dx+i\,c)}\log\left(-i\,e^{(i\,dx+i\,c)}+1\right)-\right.}{}$$

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/48*(9*a*e^(I*d*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 6*a*e^(I*d*x + I*c)
*log(I*e^(I*d*x + I*c) - 1) - 9*a*e^(I*d*x + I*c)*log(-I*e^(I*d*x + I*c) +
1) - 6*a*e^(I*d*x + I*c)*log(-I*e^(I*d*x + I*c) - 1) - 3*a*e^(I*d*x + I*c)*
log(I*e^(I*d*x) + e^(-I*c)) + 3*a*e^(I*d*x + I*c)*log(-I*e^(I*d*x) + e^(-I*
c)) + 4*I*a*e^(4*I*d*x + 4*I*c) + 24*I*a*e^(2*I*d*x + 2*I*c) - 12*I*a)*e^(-
I*d*x - I*c)/d
```

Mupad [B] (verification not implemented)

Time = 3.71 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))\,dx = \frac{2\,a\left(-\frac{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,3\mathrm{i}}{4}-\frac{\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)^2\,1\mathrm{i}}{4}+\frac{9\,\sin(c+d\,x)}{8}+\frac{\sin(3\,c+3\,d\,x)}{8}\right)}{3\,d}$$

```
[In] int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i),x)
```

```
[Out] (2*a*((9*sin(c + d*x))/8 + sin(3*c + 3*d*x)/8 - (cos(c/2 + (d*x)/2)^2*3i)/4
 - (cos((3*c)/2 + (3*d*x)/2)^2*1i)/4))/(3*d)
```

3.17 $\int \cos^5(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 62

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{ia\cos^5(c+dx)}{5d}+\frac{a\sin(c+dx)}{d} - \frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin^5(c+dx)}{5d}$$

```
[Out] -1/5*I*a*cos(d*x+c)^5/d+a*sin(d*x+c)/d-2/3*a*sin(d*x+c)^3/d+1/5*a*sin(d*x+c
)^5/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3567, 2713}

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))\,dx = \frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^3(c+dx)}{3d} + \frac{a\sin(c+dx)}{d}-\frac{ia\cos^5(c+dx)}{5d}$$

```
[In] Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((-1/5*I)*a*Cos[c + d*x]^5)/d + (a*Sin[c + d*x])/d - (2*a*Sin[c + d*x]^3)/(
3*d) + (a*Sin[c + d*x]^5)/(5*d)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{ia\cos^5(c+dx)}{5d} + a\int \cos^5(c+dx)\,dx \\
&= -\frac{ia\cos^5(c+dx)}{5d} - \frac{a\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, -\sin(c+dx)\right)}{d} \\
&= -\frac{ia\cos^5(c+dx)}{5d} + \frac{a\sin(c+dx)}{d} - \frac{2a\sin^3(c+dx)}{3d} + \frac{a\sin^5(c+dx)}{5d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{ia\cos^5(c+dx)}{5d} + \frac{a\sin(c+dx)}{d} - \frac{2a\sin^3(c+dx)}{3d} + \frac{a\sin^5(c+dx)}{5d}$$

```
[In] Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x]),x]
[Out] ((-1/5*I)*a*Cos[c + d*x]^5)/d + (a*Sin[c + d*x])/d - (2*a*Sin[c + d*x]^3)/(
3*d) + (a*Sin[c + d*x]^5)/(5*d)
```

Maple [A] (verified)

Time = 6.79 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{-\frac{ia\left(\cos^{5}(dx+c)\right)}{5}+\frac{a\left(\frac{8}{3}+\cos^{4}(dx+c)+\frac{4\left(\cos^{2}(dx+c)\right)}{3}\right)\sin(dx+c)}{5}}{d}$	47
default	$\frac{-\frac{ia\left(\cos^{5}(dx+c)\right)}{5}+\frac{a\left(\frac{8}{3}+\cos^{4}(dx+c)+\frac{4\left(\cos^{2}(dx+c)\right)}{3}\right)\sin(dx+c)}{5}}{d}$	47
risch	$-\frac{ia\,\mathrm{e}^{5i(dx+c)}}{80d}-\frac{ia\cos(dx+c)}{8d}+\frac{5a\sin(dx+c)}{8d}-\frac{ia\cos(3dx+3c)}{16d}+\frac{5a\sin(3dx+3c)}{48d}$	74

```
[In] int(cos(d*x+c)^5*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/5*I*a*cos(d*x+c)^5+1/5*a*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*
x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))\,dx$$
$$=\frac{\left(-3i\,ae^{(8i\,dx+8i\,c)}-20i\,ae^{(6i\,dx+6i\,c)}-90i\,ae^{(4i\,dx+4i\,c)}+60i\,ae^{(2i\,dx+2i\,c)}+5i\,a\right)e^{(-3i\,dx-3i\,c)}}{240\,d}$$

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(-3*I*a*e^(8*I*d*x + 8*I*c) - 20*I*a*e^(6*I*d*x + 6*I*c) - 90*I*a*e^(
4*I*d*x + 4*I*c) + 60*I*a*e^(2*I*d*x + 2*I*c) + 5*I*a)*e^(-3*I*d*x - 3*I*c)
/d
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(53) = 106.

Time = 0.24 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.97

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))\,dx$$
$$=\begin{cases}\frac{\left(-18432iad^4e^{9ic}e^{5idx}-122880iad^4e^{7ic}e^{3idx}-552960iad^4e^{5ic}e^{idx}+368640iad^4e^{3ic}e^{-idx}+30720iad^4e^{ic}e^{-3idx}\right)e^{-4ic}}{1474560d^5} & \text{for } d^5e^{4ic}\neq 0\\ \frac{x\left(ae^{8ic}+4ae^{6ic}+6ae^{4ic}+4ae^{2ic}+a\right)e^{-3ic}}{16} & \text{otherwise}\end{cases}$$

```
[In] integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise(((-18432*I*a*d**4*exp(9*I*c)*exp(5*I*d*x) - 122880*I*a*d**4*exp(7
*I*c)*exp(3*I*d*x) - 552960*I*a*d**4*exp(5*I*c)*exp(I*d*x) + 368640*I*a*d**
4*exp(3*I*c)*exp(-I*d*x) + 30720*I*a*d**4*exp(I*c)*exp(-3*I*d*x))*exp(-4*I*
c)/(1474560*d**5), Ne(d**5*exp(4*I*c), 0)), (x*(a*exp(8*I*c) + 4*a*exp(6*I*
c) + 6*a*exp(4*I*c) + 4*a*exp(2*I*c) + a)*exp(-3*I*c)/16, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.79

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{3i\,a\cos\left(dx+c\right)^5-\left(3\,\sin\left(dx+c\right)^5-10\,\sin\left(dx+c\right)^3+15\,\sin\left(dx+c\right)\right)a}{15\,d}$$

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/15*(3*I*a*cos(d*x + c)^5 - (3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*si
n(d*x + c))*a)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 220 vs. 2(54) = 108.

Time = 0.53 (sec) , antiderivative size = 220, normalized size of antiderivative = 3.55

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{\left(135\,ae^{(3i\,dx+i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+90\,ae^{(3i\,dx+i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}-1\right)-135\,ae^{(3i\,dx+i\,c)}\log\left(-i\,e^{(i\,dx+i\,c)}\right.\right.}{}$$

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/960*(135*a*e^(3*I*d*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 90*a*e^(3*I*d*
x + I*c)*log(I*e^(I*d*x + I*c) - 1) - 135*a*e^(3*I*d*x + I*c)*log(-I*e^(I*d
*x + I*c) + 1) - 90*a*e^(3*I*d*x + I*c)*log(-I*e^(I*d*x + I*c) - 1) - 45*a*
e^(3*I*d*x + I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 45*a*e^(3*I*d*x + I*c)*log(
-I*e^(I*d*x) + e^(-I*c)) + 12*I*a*e^(8*I*d*x + 6*I*c) + 80*I*a*e^(6*I*d*x +
 4*I*c) + 360*I*a*e^(4*I*d*x + 2*I*c) - 240*I*a*e^(2*I*d*x) - 20*I*a*e^(-2*
I*c))*e^(-3*I*d*x - I*c)/d
```

Mupad [B] (verification not implemented)

Time = 6.00 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{2\,a\,\left(-\frac{75\,\sin(c+d\,x)}{16}-\frac{25\,\sin(3\,c+3\,d\,x)}{32}-\frac{3\,\sin(5\,c+5\,d\,x)}{32}+\frac{\cos(c+d\,x)\,15\mathrm{i}}{16}+\frac{\cos(3\,c+3\,d\,x)\,15\mathrm{i}}{32}+\frac{\cos(5\,c+5\,d\,x)\,3\mathrm{i}}{32}\right)}{15\,d}$$

```
[In] int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i),x)
```

```
[Out] -(2*a*((cos(c + d*x)*15i)/16 - (75*sin(c + d*x))/16 + (cos(3*c + 3*d*x)*15i
)/32 + (cos(5*c + 5*d*x)*3i)/32 - (25*sin(3*c + 3*d*x))/32 - (3*sin(5*c + 5
*d*x))/32))/(15*d)
```

3.18 $\int \cos^7(c+dx)(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 76

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{ia\cos^7(c+dx)}{7d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{d}+\frac{3a\sin^5(c+dx)}{5d}-\frac{a\sin^7(c+dx)}{7d}$$

```
[Out] -1/7*I*a*cos(d*x+c)^7/d+a*sin(d*x+c)/d-a*sin(d*x+c)^3/d+3/5*a*sin(d*x+c)^5/
d-1/7*a*sin(d*x+c)^7/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3567, 2713}

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{a\sin^7(c+dx)}{7d}+\frac{3a\sin^5(c+dx)}{5d}-\frac{a\sin^3(c+dx)}{d}+\frac{a\sin(c+dx)}{d}-\frac{ia\cos^7(c+dx)}{7d}$$

```
[In] Int[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((-1/7*I)*a*Cos[c + d*x]^7)/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/d +
 (3*a*Sin[c + d*x]^5)/(5*d) - (a*Sin[c + d*x]^7)/(7*d)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{ia\cos^7(c+dx)}{7d} + a\int \cos^7(c+dx)\,dx \\
&= -\frac{ia\cos^7(c+dx)}{7d} - \frac{a\text{Subst}\left(\int (1-3x^2+3x^4-x^6)\,dx, x, -\sin(c+dx)\right)}{d} \\
&= -\frac{ia\cos^7(c+dx)}{7d} + \frac{a\sin(c+dx)}{d} - \frac{a\sin^3(c+dx)}{d} + \frac{3a\sin^5(c+dx)}{5d} - \frac{a\sin^7(c+dx)}{7d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{ia\cos^7(c+dx)}{7d} + \frac{a\sin(c+dx)}{d} - \frac{a\sin^3(c+dx)}{d} + \frac{3a\sin^5(c+dx)}{5d} - \frac{a\sin^7(c+dx)}{7d}$$

```
[In] Integrate[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x]),x]
[Out] ((-1/7*I)*a*Cos[c + d*x]^7)/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/d +
 (3*a*Sin[c + d*x]^5)/(5*d) - (a*Sin[c + d*x]^7)/(7*d)
```

Maple [A] (verified)

Time = 27.38 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{-\frac{ia\left(\cos^7(dx+c)\right)}{7}+\frac{a\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{7}}{d}$
default	$\frac{-\frac{ia\left(\cos^7(dx+c)\right)}{7}+\frac{a\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{7}}{d}$
risch	$-\frac{ia\,e^{7i(dx+c)}}{448d}-\frac{5ia\cos(dx+c)}{64d}+\frac{35a\sin(dx+c)}{64d}-\frac{ia\cos(5dx+5c)}{64d}+\frac{7a\sin(5dx+5c)}{320d}-\frac{3ia\cos(3dx+3c)}{64d}+$

[In] int(cos(d*x+c)^7*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(-1/7*I*a*cos(d*x+c)^7+1/7*a*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*co
s(d*x+c)^2)*sin(d*x+c))

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.18

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))\,dx$$
$$=\frac{\left(-5i\,ae^{(12i\,dx+12i\,c)}-42i\,ae^{(10i\,dx+10i\,c)}-175i\,ae^{(8i\,dx+8i\,c)}-700i\,ae^{(6i\,dx+6i\,c)}+525i\,ae^{(4i\,dx+4i\,c)}+70i\,ae^{(2i}\right.}{2240\,d}$$

[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c)),x, algorithm="fricas")

[Out] 1/2240*(-5*I*a*e^(12*I*d*x + 12*I*c) - 42*I*a*e^(10*I*d*x + 10*I*c) - 175*I
*a*e^(8*I*d*x + 8*I*c) - 700*I*a*e^(6*I*d*x + 6*I*c) + 525*I*a*e^(4*I*d*x +
 4*I*c) + 70*I*a*e^(2*I*d*x + 2*I*c) + 7*I*a)*e^(-5*I*d*x - 5*I*c)/d

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(65) = 130.

Time = 0.32 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.33

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))\,dx$$
$$=\begin{cases}\frac{\left(-107374182400iad^6e^{16ic}e^{7idx}-901943132160iad^6e^{14ic}e^{5idx}-3758096384000iad^6e^{12ic}e^{3idx}-15032385536000iad^6e^{10ic}e^{idx}+11274289152\right.}{48103633715200d^7} \\ \frac{x\left(ae^{12ic}+6ae^{10ic}+15ae^{8ic}+20ae^{6ic}+15ae^{4ic}+6ae^{2ic}+a\right)e^{-5ic}}{64}\end{cases}$$

[In] integrate(cos(d*x+c)**7*(a+I*a*tan(d*x+c)),x)

```
[Out] Piecewise(((-107374182400*I*a*d**6*exp(16*I*c)*exp(7*I*d*x) - 901943132160*
I*a*d**6*exp(14*I*c)*exp(5*I*d*x) - 3758096384000*I*a*d**6*exp(12*I*c)*exp(
3*I*d*x) - 15032385536000*I*a*d**6*exp(10*I*c)*exp(I*d*x) + 11274289152000*
I*a*d**6*exp(8*I*c)*exp(-I*d*x) + 1503238553600*I*a*d**6*exp(6*I*c)*exp(-3*
I*d*x) + 150323855360*I*a*d**6*exp(4*I*c)*exp(-5*I*d*x))*exp(-9*I*c)/(48103
633715200*d**7), Ne(d**7*exp(9*I*c), 0)), (x*(a*exp(12*I*c) + 6*a*exp(10*I*
c) + 15*a*exp(8*I*c) + 20*a*exp(6*I*c) + 15*a*exp(4*I*c) + 6*a*exp(2*I*c) +
 a)*exp(-5*I*c)/64, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.76

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{5i\,a\cos{(dx+c)}^7 + \left(5\,\sin{(dx+c)}^7 - 21\,\sin{(dx+c)}^5 + 35\,\sin{(dx+c)}^3 - 35\,\sin{(dx+c)}\right)a}{35\,d}$$

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/35*(5*I*a*cos(d*x + c)^7 + (5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*si
n(d*x + c)^3 - 35*sin(d*x + c))*a)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(68) = 136.

Time = 0.52 (sec) , antiderivative size = 244, normalized size of antiderivative = 3.21

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{\left(1015\,ae^{(5i\,dx+i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right) + 700\,ae^{(5i\,dx+i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}-1\right) - 1015\,ae^{(5i\,dx+i\,c)}\log\left(-i\,e^{(i\,dx-}\right.\right.}{}$$

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/8960*(1015*a*e^(5*I*d*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 700*a*e^(5*I
*d*x + I*c)*log(I*e^(I*d*x + I*c) - 1) - 1015*a*e^(5*I*d*x + I*c)*log(-I*e^
(I*d*x + I*c) + 1) - 700*a*e^(5*I*d*x + I*c)*log(-I*e^(I*d*x + I*c) - 1) -
315*a*e^(5*I*d*x + I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 315*a*e^(5*I*d*x + I*
c)*log(-I*e^(I*d*x) + e^(-I*c)) + 20*I*a*e^(12*I*d*x + 8*I*c) + 168*I*a*e^(
10*I*d*x + 6*I*c) + 700*I*a*e^(8*I*d*x + 4*I*c) + 2800*I*a*e^(6*I*d*x + 2*I
*c) - 280*I*a*e^(2*I*d*x - 2*I*c) - 2100*I*a*e^(4*I*d*x) - 28*I*a*e^(-4*I*c
))*e^(-5*I*d*x - I*c)/d
```

Mupad [B] (verification not implemented)

Time = 6.66 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.22

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))\,dx = -\frac{2\,a\,\left(-\frac{1225\,\sin(c+d\,x)}{128}-\frac{245\,\sin(3\,c+3\,d\,x)}{128}-\frac{49\,\sin(5\,c+5\,d\,x)}{128}-\frac{5\,\sin(7\,c+7\,d\,x)}{128}+\frac{\cos(c+d\,x)\,175\mathrm{i}}{128}+\frac{\cos(3\,c+3\,d\,x)\,105\mathrm{i}}{128}+\frac{\cos(}{}\right.}{35\,d}$$

```
[In] int(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i),x)
```

```
[Out] -(2*a*((cos(c + d*x)*175i)/128 - (1225*sin(c + d*x))/128 + (cos(3*c + 3*d*x
)*105i)/128 + (cos(5*c + 5*d*x)*35i)/128 + (cos(7*c + 7*d*x)*5i)/128 - (245
*sin(3*c + 3*d*x))/128 - (49*sin(5*c + 5*d*x))/128 - (5*sin(7*c + 7*d*x))/1
28))/(35*d)
```

3.19 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 109

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{4i(a+ia\tan(c+dx))^6}{3a^4d} + \frac{12i(a+ia\tan(c+dx))^7}{7a^5d} - \frac{3i(a+ia\tan(c+dx))^8}{4a^6d} + \frac{i(a+ia\tan(c+dx))^9}{9a^7d}$$

```
[Out] -4/3*I*(a+I*a*tan(d*x+c))^6/a^4/d+12/7*I*(a+I*a*tan(d*x+c))^7/a^5/d-3/4*I*(
a+I*a*tan(d*x+c))^8/a^6/d+1/9*I*(a+I*a*tan(d*x+c))^9/a^7/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{i(a+ia\tan(c+dx))^9}{9a^7d} - \frac{3i(a+ia\tan(c+dx))^8}{4a^6d} + \frac{12i(a+ia\tan(c+dx))^7}{7a^5d} - \frac{4i(a+ia\tan(c+dx))^6}{3a^4d}$$

```
[In] Int[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (((-4*I)/3)*(a + I*a*Tan[c + d*x])^6)/(a^4*d) + (((12*I)/7)*(a + I*a*Tan[c
+ d*x])^7)/(a^5*d) - (((3*I)/4)*(a + I*a*Tan[c + d*x])^8)/(a^6*d) + ((I/9)*
(a + I*a*Tan[c + d*x])^9)/(a^7*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)^3(a+x)^5\,dx, x, ia\tan(c+dx)\right)}{a^7d} \\
&= -\frac{i\text{Subst}\left(\int \left(8a^3(a+x)^5-12a^2(a+x)^6+6a(a+x)^7-(a+x)^8\right)\,dx, x, ia\tan(c+dx)\right)}{a^7d} \\
&= -\frac{4i(a+ia\tan(c+dx))^6}{3a^4d}+\frac{12i(a+ia\tan(c+dx))^7}{7a^5d} \\
&\quad -\frac{3i(a+ia\tan(c+dx))^8}{4a^6d}+\frac{i(a+ia\tan(c+dx))^9}{9a^7d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.72

$$
\int \sec^8(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{a^2\sec^8(c+dx)(\cos(6(c+dx))+i\sin(6(c+dx)))(-40i+170i\cos(2(c+dx))+83\sec(c+dx)\sin(3(c
}{504d}
$$

```
[In] Integrate[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -1/504*(a^2*Sec[c + d*x]^8*(Cos[6*(c + d*x)] + I*Sin[6*(c + d*x)])*(-40*I +
 (170*I)*Cos[2*(c + d*x)] + 83*Sec[c + d*x]*Sin[3*(c + d*x)] + 27*Tan[c + d
*x]))/d
```

Maple [A] (verified)

Time = 114.00 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.73

method	result
risch	$\frac{64ia^2\left(126\,\mathrm{e}^{10i(dx+c)}+126\,\mathrm{e}^{8i(dx+c)}+84\,\mathrm{e}^{6i(dx+c)}+36\,\mathrm{e}^{4i(dx+c)}+9\,\mathrm{e}^{2i(dx+c)}+1\right)}{63d\left(\mathrm{e}^{2i(dx+c)}+1\right)^9}$
derivativedivides	$\frac{-a^2\left(\frac{\sin^3(dx+c)}{9\cos(dx+c)^9}+\frac{2\left(\sin^3(dx+c)\right)}{21\cos(dx+c)^7}+\frac{8\left(\sin^3(dx+c)\right)}{105\cos(dx+c)^5}+\frac{16\left(\sin^3(dx+c)\right)}{315\cos(dx+c)^3}\right)+\frac{ia^2}{4\cos(dx+c)^8}-a^2\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx+\right.}{35}\right.}{d}$
default	$\frac{-a^2\left(\frac{\sin^3(dx+c)}{9\cos(dx+c)^9}+\frac{2\left(\sin^3(dx+c)\right)}{21\cos(dx+c)^7}+\frac{8\left(\sin^3(dx+c)\right)}{105\cos(dx+c)^5}+\frac{16\left(\sin^3(dx+c)\right)}{315\cos(dx+c)^3}\right)+\frac{ia^2}{4\cos(dx+c)^8}-a^2\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx+\right.}{35}\right.}{d}$

```
[In] int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 64/63*I*a^2*(126*exp(10*I*(d*x+c))+126*exp(8*I*(d*x+c))+84*exp(6*I*(d*x+c))
+36*exp(4*I*(d*x+c))+9*exp(2*I*(d*x+c))+1)/d/(exp(2*I*(d*x+c))+1)^9
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(85) = 170.

Time = 0.23 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.73

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{64\left(-126i\,a^2e^{(10i\,dx+10i\,c)}-126i\,a^2e^{(8i\,dx+8i\,c)}-84i\,a^2e^{(6i\,dx+6i\,c)}-36i\,a^2e^{(4i\,d}\right.}{63\left(de^{(18i\,dx+18i\,c)}+9\,de^{(16i\,dx+16i\,c)}+36\,de^{(14i\,dx+14i\,c)}+84\,de^{(12i\,dx+12i\,c)}+126\,de^{(10i\,dx+10i\,c)}+126\,de^{(8i\,dx}\right.}$$

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -64/63*(-126*I*a^2*e^(10*I*d*x + 10*I*c) - 126*I*a^2*e^(8*I*d*x + 8*I*c) -
84*I*a^2*e^(6*I*d*x + 6*I*c) - 36*I*a^2*e^(4*I*d*x + 4*I*c) - 9*I*a^2*e^(2*
I*d*x + 2*I*c) - I*a^2)/(d*e^(18*I*d*x + 18*I*c) + 9*d*e^(16*I*d*x + 16*I*c
) + 36*d*e^(14*I*d*x + 14*I*c) + 84*d*e^(12*I*d*x + 12*I*c) + 126*d*e^(10*I
*d*x + 10*I*c) + 126*d*e^(8*I*d*x + 8*I*c) + 84*d*e^(6*I*d*x + 6*I*c) + 36*
d*e^(4*I*d*x + 4*I*c) + 9*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^2\,dx = -a^2\left(\int \tan^2(c+dx)\sec^8(c+dx)\,dx + \int \left(-2i\tan(c+dx)\sec^8(c+dx)\right)\,dx + \int \left(-\sec^8(c+dx)\right)\,dx\right)$$

```
[In] integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c))**2,x)
[Out] -a**2*(Integral(tan(c + d*x)**2*sec(c + d*x)**8, x) + Integral(-2*I*tan(c +
 d*x)*sec(c + d*x)**8, x) + Integral(-sec(c + d*x)**8, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.99

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{28\,a^2\tan(dx+c)^9 - 63i\,a^2\tan(dx+c)^8 + 72\,a^2\tan(dx+c)^7 - 252i\,a^2\tan(dx+c)^6 - 378i\,a^2\tan(dx}{252\,d}$$

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
[Out] -1/252*(28*a^2*tan(d*x + c)^9 - 63*I*a^2*tan(d*x + c)^8 + 72*a^2*tan(d*x +
c)^7 - 252*I*a^2*tan(d*x + c)^6 - 378*I*a^2*tan(d*x + c)^4 - 168*a^2*tan(d*
x + c)^3 - 252*I*a^2*tan(d*x + c)^2 - 252*a^2*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.56 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.99

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{28\,a^2\tan(dx+c)^9 - 63i\,a^2\tan(dx+c)^8 + 72\,a^2\tan(dx+c)^7 - 252i\,a^2\tan(dx+c)^6 - 378i\,a^2\tan(dx}{252\,d}$$

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
[Out] -1/252*(28*a^2*tan(d*x + c)^9 - 63*I*a^2*tan(d*x + c)^8 + 72*a^2*tan(d*x +
c)^7 - 252*I*a^2*tan(d*x + c)^6 - 378*I*a^2*tan(d*x + c)^4 - 168*a^2*tan(d*
x + c)^3 - 252*I*a^2*tan(d*x + c)^2 - 252*a^2*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 4.32 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.39

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$=\frac{a^2\,\sin\left(c+d\,x\right)\,\left(252\cos\left(c+d\,x\right)^8+\cos\left(c+d\,x\right)^7\,\sin\left(c+d\,x\right)\,252\mathrm{i}+168\cos\left(c+d\,x\right)^6\sin\left(c+d\,x\right)^2+\mathrm{co}\right.}{}$$

```
[In] int((a + a*tan(c + d*x)*1i)^2/cos(c + d*x)^8,x)
```

```
[Out] (a^2*sin(c + d*x)*(cos(c + d*x)*sin(c + d*x)^7*63i + cos(c + d*x)^7*sin(c +
 d*x)*252i + 252*cos(c + d*x)^8 - 28*sin(c + d*x)^8 - 72*cos(c + d*x)^2*sin
(c + d*x)^6 + cos(c + d*x)^3*sin(c + d*x)^5*252i + cos(c + d*x)^5*sin(c + d
*x)^3*378i + 168*cos(c + d*x)^6*sin(c + d*x)^2))/(252*d*cos(c + d*x)^9)
```

3.20 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{4i(a+ia\tan(c+dx))^5}{5a^3d} + \frac{2i(a+ia\tan(c+dx))^6}{3a^4d} - \frac{i(a+ia\tan(c+dx))^7}{7a^5d}$$

```
[Out] -4/5*I*(a+I*a*tan(d*x+c))^5/a^3/d+2/3*I*(a+I*a*tan(d*x+c))^6/a^4/d-1/7*I*(a
+I*a*tan(d*x+c))^7/a^5/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{i(a+ia\tan(c+dx))^7}{7a^5d} + \frac{2i(a+ia\tan(c+dx))^6}{3a^4d} - \frac{4i(a+ia\tan(c+dx))^5}{5a^3d}$$

```
[In] Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (((-4*I)/5)*(a + I*a*Tan[c + d*x])^5)/(a^3*d) + (((2*I)/3)*(a + I*a*Tan[c +
 d*x])^6)/(a^4*d) - ((I/7)*(a + I*a*Tan[c + d*x])^7)/(a^5*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)^2(a+x)^4\,dx, x, ia\tan(c+dx)\right)}{a^5 d}\\
&= -\frac{i\text{Subst}\left(\int \left(4a^2(a+x)^4-4a(a+x)^5+(a+x)^6\right)\,dx, x, ia\tan(c+dx)\right)}{a^5 d}\\
&= -\frac{4i(a+ia\tan(c+dx))^5}{5a^3d}+\frac{2i(a+ia\tan(c+dx))^6}{3a^4d}-\frac{i(a+ia\tan(c+dx))^7}{7a^5d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.77

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2\sec^7(c+dx)(7+22\cos(2(c+dx))-20i\sin(2(c+dx)))(-i\cos(5(c+dx))+\sin(5(c+dx)))}{105d}$$

```
[In] Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^2,x]
[Out] (a^2*Sec[c + d*x]^7*(7 + 22*Cos[2*(c + d*x)] - (20*I)*Sin[2*(c + d*x)])*((-
I)*Cos[5*(c + d*x)] + Sin[5*(c + d*x)]))/(105*d)
```

Maple [A] (verified)

Time = 34.89 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

method	result
risch	$\frac{128ia^2(35\,e^{8i(dx+c)}+35\,e^{6i(dx+c)}+21\,e^{4i(dx+c)}+7\,e^{2i(dx+c)}+1)}{105d(e^{2i(dx+c)}+1)^7}$
derivativedivides	$\frac{-a^2\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8\left(\sin^3(dx+c)\right)}{105\cos(dx+c)^3}\right)+\frac{ia^2}{3\cos(dx+c)^6}-a^2\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c}{d}$
default	$\frac{-a^2\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8\left(\sin^3(dx+c)\right)}{105\cos(dx+c)^3}\right)+\frac{ia^2}{3\cos(dx+c)^6}-a^2\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c}{d}$

```
[In] int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 128/105*I*a^2*(35*exp(8*I*(d*x+c))+35*exp(6*I*(d*x+c))+21*exp(4*I*(d*x+c))+
7*exp(2*I*(d*x+c))+1)/d/(exp(2*I*(d*x+c))+1)^7
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(64) = 128.

Time = 0.23 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.84

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{128\left(-35i\,a^2e^{(8i\,dx+8i\,c)}-35i\,a^2e^{(6i\,dx+6i\,c)}-21i\,a^2e^{(4i\,dx+4i\,c)}-7i\,a^2e^{(2i\,dx+2i\,c)}-i\,a\right.}{105\left(de^{(14i\,dx+14i\,c)}+7\,de^{(12i\,dx+12i\,c)}+21\,de^{(10i\,dx+10i\,c)}+35\,de^{(8i\,dx+8i\,c)}+35\,de^{(6i\,dx+6i\,c)}+21\,de^{(4i\,dx+4i\,c)}\right.}$$

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -128/105*(-35*I*a^2*e^(8*I*d*x + 8*I*c) - 35*I*a^2*e^(6*I*d*x + 6*I*c) - 21
*I*a^2*e^(4*I*d*x + 4*I*c) - 7*I*a^2*e^(2*I*d*x + 2*I*c) - I*a^2)/(d*e^(14*
I*d*x + 14*I*c) + 7*d*e^(12*I*d*x + 12*I*c) + 21*d*e^(10*I*d*x + 10*I*c) +
35*d*e^(8*I*d*x + 8*I*c) + 35*d*e^(6*I*d*x + 6*I*c) + 21*d*e^(4*I*d*x + 4*I
*c) + 7*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^2\,dx = -a^2\left(\int \tan^2(c+dx)\sec^6(c+dx)\,dx + \int \left(-2i\tan(c+dx)\sec^6(c+dx)\right)\,dx + \int \left(-\sec^6(c+dx)\right)\,dx\right)$$

```
[In] integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -a**2*(Integral(tan(c + d*x)**2*sec(c + d*x)**6, x) + Integral(-2*I*tan(c +
 d*x)*sec(c + d*x)**6, x) + Integral(-sec(c + d*x)**6, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.16

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{15\,a^2\tan\left(dx+c\right)^7 - 35i\,a^2\tan\left(dx+c\right)^6 + 21\,a^2\tan\left(dx+c\right)^5 - 105i\,a^2\tan\left(dx+c\right)^4 - 35\,a^2\tan\left(dx+\right.}{105\,d}$$

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/105*(15*a^2*tan(d*x + c)^7 - 35*I*a^2*tan(d*x + c)^6 + 21*a^2*tan(d*x +
c)^5 - 105*I*a^2*tan(d*x + c)^4 - 35*a^2*tan(d*x + c)^3 - 105*I*a^2*tan(d*x
 + c)^2 - 105*a^2*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.54 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.16

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{15\,a^2\tan\left(dx+c\right)^7 - 35i\,a^2\tan\left(dx+c\right)^6 + 21\,a^2\tan\left(dx+c\right)^5 - 105i\,a^2\tan\left(dx+c\right)^4 - 35\,a^2\tan\left(dx+\right.}{105\,d}$$

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/105*(15*a^2*tan(d*x + c)^7 - 35*I*a^2*tan(d*x + c)^6 + 21*a^2*tan(d*x +
c)^5 - 105*I*a^2*tan(d*x + c)^4 - 35*a^2*tan(d*x + c)^3 - 105*I*a^2*tan(d*x
 + c)^2 - 105*a^2*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 3.70 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.61

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2\,\sin\left(c+d\,x\right)\,\left(105\cos\left(c+d\,x\right)^6 + \cos\left(c+d\,x\right)^5\,\sin\left(c+d\,x\right)\,105\mathrm{i} + 35\cos\left(c+d\,x\right)^4\sin\left(c+d\,x\right)^2 + \cos\right.}{105\,d\,c}$$

```
[In] int((a + a*tan(c + d*x)*1i)^2/cos(c + d*x)^6,x)
```

```
[Out] (a^2*sin(c + d*x)*(cos(c + d*x)*sin(c + d*x)^5*35i + cos(c + d*x)^5*sin(c +
 d*x)*105i + 105*cos(c + d*x)^6 - 15*sin(c + d*x)^6 - 21*cos(c + d*x)^2*sin
(c + d*x)^4 + cos(c + d*x)^3*sin(c + d*x)^3*105i + 35*cos(c + d*x)^4*sin(c
+ d*x)^2))/(105*d*cos(c + d*x)^7)
```

3.21 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{i(a+ia\tan(c+dx))^4}{2a^2d}+\frac{i(a+ia\tan(c+dx))^5}{5a^3d}$$

```
[Out] -1/2*I*(a+I*a*tan(d*x+c))^4/a^2/d+1/5*I*(a+I*a*tan(d*x+c))^5/a^3/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{i(a+ia\tan(c+dx))^5}{5a^3d}-\frac{i(a+ia\tan(c+dx))^4}{2a^2d}$$

```
[In] Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((-1/2*I)*(a + I*a*Tan[c + d*x])^4)/(a^2*d) + ((I/5)*(a + I*a*Tan[c + d*x])
^5)/(a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
```

```
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)(a+x)^3\,dx, x, ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{i\text{Subst}\left(\int \left(2a(a+x)^3-(a+x)^4\right)\,dx, x, ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{i(a+ia\tan(c+dx))^4}{2a^2d}+\frac{i(a+ia\tan(c+dx))^5}{5a^3d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.62

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{a^2(-i+\tan(c+dx))^4(3i+2\tan(c+dx))}{10d}$$

```
[In] Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^2,x]
[Out] -1/10*(a^2*(-I + Tan[c + d*x])^4*(3*I + 2*Tan[c + d*x]))/d
```

Maple [A] (verified)

Time = 8.36 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05

method	result	size
risch	$\frac{8ia^2\left(10\,e^{6i(dx+c)}+10\,e^{4i(dx+c)}+5\,e^{2i(dx+c)}+1\right)}{5d\left(e^{2i(dx+c)}+1\right)^5}$	58
derivativedivides	$\frac{-a^2\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)+\frac{ia^2}{2\cos(dx+c)^4}-a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$	85
default	$\frac{-a^2\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)+\frac{ia^2}{2\cos(dx+c)^4}-a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$	85

```
[In] int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 8/5*I*a^2*(10*exp(6*I*(d*x+c))+10*exp(4*I*(d*x+c))+5*exp(2*I*(d*x+c))+1)/d/
(exp(2*I*(d*x+c))+1)^5
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(43) = 86.

Time = 0.23 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.05

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{8\left(-10i\,a^2e^{(6i\,dx+6i\,c)}-10i\,a^2e^{(4i\,dx+4i\,c)}-5i\,a^2e^{(2i\,dx+2i\,c)}-i\,a^2\right)}{5\left(de^{(10i\,dx+10i\,c)}+5\,de^{(8i\,dx+8i\,c)}+10\,de^{(6i\,dx+6i\,c)}+10\,de^{(4i\,dx+4i\,c)}+5\,de^{(2i\,dx+2i\,c)}+d\right)}$$

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -8/5*(-10*I*a^2*e^(6*I*d*x + 6*I*c) - 10*I*a^2*e^(4*I*d*x + 4*I*c) - 5*I*a^
2*e^(2*I*d*x + 2*I*c) - I*a^2)/(d*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*I*d*x +
8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) + 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*d
*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^2\,dx = -a^2\left(\int \tan^2(c+dx)\sec^4(c+dx)\,dx + \int \left(-2i\tan(c+dx)\sec^4(c+dx)\right)dx + \int \left(-\sec^4(c+dx)\right)dx\right)$$

```
[In] integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -a**2*(Integral(tan(c + d*x)**2*sec(c + d*x)**4, x) + Integral(-2*I*tan(c +
 d*x)*sec(c + d*x)**4, x) + Integral(-sec(c + d*x)**4, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{2\,a^2\tan(dx+c)^5-5i\,a^2\tan(dx+c)^4-10i\,a^2\tan(dx+c)^2-10\,a^2\tan(dx+c)}{10\,d}$$

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/10*(2*a^2*tan(d*x + c)^5 - 5*I*a^2*tan(d*x + c)^4 - 10*I*a^2*tan(d*x + c
)^2 - 10*a^2*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$= -\frac{2\,a^2\tan\left(dx+c\right)^5 - 5i\,a^2\tan\left(dx+c\right)^4 - 10i\,a^2\tan\left(dx+c\right)^2 - 10\,a^2\tan\left(dx+c\right)}{10\,d}$$

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/10*(2*a^2*tan(d*x + c)^5 - 5*I*a^2*tan(d*x + c)^4 - 10*I*a^2*tan(d*x + c
)^2 - 10*a^2*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 3.83 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$= \frac{-\frac{a^2\tan(c+d\,x)^5}{5} + \frac{a^2\tan(c+d\,x)^4\,1\mathrm{i}}{2} + a^2\tan(c+d\,x)^2\,1\mathrm{i} + a^2\tan(c+d\,x)}{d}$$

```
[In] int((a + a*tan(c + d*x)*1i)^2/cos(c + d*x)^4,x)
```

```
[Out] (a^2*tan(c + d*x) + a^2*tan(c + d*x)^2*1i + (a^2*tan(c + d*x)^4*1i)/2 - (a^
2*tan(c + d*x)^5)/5)/d
```

3.22 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^2\,dx=-\frac{i(a+ia\tan(c+dx))^3}{3ad}$$

```
[Out] -1/3*I*(a+I*a*tan(d*x+c))^3/a/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 32}

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^2\,dx=-\frac{i(a+ia\tan(c+dx))^3}{3ad}$$

```
[In] Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((-1/3*I)*(a + I*a*Tan[c + d*x])^3)/(a*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{i\text{Subst}\left(\int (a+x)^2\,dx, x, ia\tan(c+dx)\right)}{ad}$$
$$= -\frac{i(a+ia\tan(c+dx))^3}{3ad}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.85

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2\tan(c+dx)}{d} + \frac{ia^2\tan^2(c+dx)}{d} - \frac{a^2\tan^3(c+dx)}{3d}$$

```
[In] Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^2,x]
[Out] (a^2*Tan[c + d*x])/d + (I*a^2*Tan[c + d*x]^2)/d - (a^2*Tan[c + d*x]^3)/(3*d
)
```

Maple [A] (verified)

Time = 1.97 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.74

method	result	size
risch	$\frac{8ia^2\left(3\,\mathrm{e}^{4i(dx+c)}+3\,\mathrm{e}^{2i(dx+c)}+1\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}$	47
derivativedivides	$\frac{-\frac{a^2\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+\frac{ia^2}{\cos(dx+c)^2}+a^2\tan(dx+c)}{d}$	51
default	$\frac{-\frac{a^2\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+\frac{ia^2}{\cos(dx+c)^2}+a^2\tan(dx+c)}{d}$	51

```
[In] int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 8/3*I*a^2*(3*exp(4*I*(d*x+c))+3*exp(2*I*(d*x+c))+1)/d/(exp(2*I*(d*x+c))+1)^
3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(21) = 42.

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.78

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{8\left(-3i\,a^2e^{(4i\,dx+4i\,c)}-3i\,a^2e^{(2i\,dx+2i\,c)}-i\,a^2\right)}{3\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)}$$

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
[Out] -8/3*(-3*I*a^2*e^(4*I*d*x + 4*I*c) - 3*I*a^2*e^(2*I*d*x + 2*I*c) - I*a^2)/(
d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) +
 d)
```

Sympy [F]

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^2\,dx = -a^2\left(\int \tan^2(c+dx)\sec^2(c+dx)\,dx + \int \left(-2i\tan(c+dx)\sec^2(c+dx)\right)dx + \int \left(-\sec^2(c+dx)\right)dx\right)$$

```
[In] integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**2,x)
[Out] -a**2*(Integral(tan(c + d*x)**2*sec(c + d*x)**2, x) + Integral(-2*I*tan(c +
 d*x)*sec(c + d*x)**2, x) + Integral(-sec(c + d*x)**2, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{i\,(i\,a\tan(dx+c)+a)^3}{3\,ad}$$

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
[Out] -1/3*I*(I*a*tan(d*x + c) + a)^3/(a*d)
```

Giac [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.56

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{a^2\tan\left(dx+c\right)^3 - 3i\,a^2\tan\left(dx+c\right)^2 - 3\,a^2\tan\left(dx+c\right)}{3\,d}$$

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
[Out] -1/3*(a^2*tan(d*x + c)^3 - 3*I*a^2*tan(d*x + c)^2 - 3*a^2*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 4.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2\tan(c+d\,x)\,\left(-\tan(c+d\,x)^2+\tan(c+d\,x)\,3\mathrm{i}+3\right)}{3\,d}$$

```
[In] int((a + a*tan(c + d*x)*1i)^2/cos(c + d*x)^2,x)
[Out] (a^2*tan(c + d*x)*(tan(c + d*x)*3i - tan(c + d*x)^2 + 3))/(3*d)
```

3.23 $\int (a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int (a+ia\tan(c+dx))^2\,dx=2a^2x-\frac{2ia^2\log(\cos(c+dx))}{d}-\frac{a^2\tan(c+dx)}{d}$$

```
[Out] 2*a^2*x-2*I*a^2*ln(cos(d*x+c))/d-a^2*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3558, 3556}

$$\int (a+ia\tan(c+dx))^2\,dx=-\frac{a^2\tan(c+dx)}{d}-\frac{2ia^2\log(\cos(c+dx))}{d}+2a^2x$$

```
[In] Int[(a + I*a*Tan[c + d*x])^2,x]
[Out] 2*a^2*x - ((2*I)*a^2*Log[Cos[c + d*x]])/d - (a^2*Tan[c + d*x])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3558

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[b^2*(Tan[c + d*x]/d),
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2a^2x - \frac{a^2\tan(c+dx)}{d} + (2ia^2)\int \tan(c+dx)\,dx\\ &= 2a^2x - \frac{2ia^2\log(\cos(c+dx))}{d} - \frac{a^2\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int (a+ia\tan(c+dx))^2\,dx = -\frac{ia(-2a\log(i+\tan(c+dx))-ia\tan(c+dx))}{d}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^2,x]
[Out] ((-I)*a*(-2*a*Log[I + Tan[c + d*x]] - I*a*Tan[c + d*x]))/d
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{a^2(-\tan(dx+c)+i\ln(1+\tan^2(dx+c))+2\arctan(\tan(dx+c)))}{d}$	40
default	$\frac{a^2(-\tan(dx+c)+i\ln(1+\tan^2(dx+c))+2\arctan(\tan(dx+c)))}{d}$	40
parallelrisch	$\frac{ia^2\ln(1+\tan^2(dx+c))+2a^2dx-a^2\tan(dx+c)}{d}$	41
norman	$2a^2x-\frac{a^2\tan(dx+c)}{d}+\frac{ia^2\ln(1+\tan^2(dx+c))}{d}$	42
parts	$a^2x+\frac{ia^2\ln(1+\tan^2(dx+c))}{d}-\frac{a^2(\tan(dx+c)-\arctan(\tan(dx+c)))}{d}$	51
risch	$-\frac{4a^2c}{d}-\frac{2ia^2}{d(e^{2i(dx+c)}+1)}-\frac{2ia^2\ln(e^{2i(dx+c)}+1)}{d}$	54

```
[In] int((a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d*a^2*(-tan(d*x+c)+I*ln(1+tan(d*x+c)^2)+2*arctan(tan(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.47

$$\int (a+ia\tan(c+dx))^2\,dx = -\frac{2\left(i\,a^2+\left(i\,a^2e^{(2i\,dx+2i\,c)}+i\,a^2\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)\right)}{de^{(2i\,dx+2i\,c)}+d}$$

```
[In] integrate((a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
[Out] -2*(I*a^2 + (I*a^2*e^(2*I*d*x + 2*I*c) + I*a^2)*log(e^(2*I*d*x + 2*I*c) + 1
))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.39

$$\int (a+ia\tan(c+dx))^2\,dx = -\frac{2ia^2}{de^{2ic}e^{2idx}+d} - \frac{2ia^2\log\left(e^{2idx}+e^{-2ic}\right)}{d}$$

```
[In] integrate((a+I*a*tan(d*x+c))**2,x)
[Out] -2*I*a**2/(d*exp(2*I*c)*exp(2*I*d*x) + d) - 2*I*a**2*log(exp(2*I*d*x) + exp
(-2*I*c))/d
```

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

$$\int (a+ia\tan(c+dx))^2\,dx = a^2x + \frac{(dx+c-\tan(dx+c))a^2}{d} + \frac{2i\,a^2\log(\sec(dx+c))}{d}$$

```
[In] integrate((a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
[Out] a^2*x + (d*x + c - tan(d*x + c))*a^2/d + 2*I*a^2*log(sec(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.74

$$\int (a + ia\tan(c + dx))^2 \, dx = -\frac{2\left(i\,a^2e^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+i\,a^2\log\left(e^{(2i\,dx+2i\,c)}+1\right)+i\,a^2\right)}{de^{(2i\,dx+2i\,c)}+d}$$

```
[In] integrate((a+I*a*tan(d*x+c))^2,x, algorithm="giac")
[Out] -2*(I*a^2*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + I*a^2*log(e^(2
*I*d*x + 2*I*c) + 1) + I*a^2)/(d*e^(2*I*d*x + 2*I*c) + d)
```

Mupad [B] (verification not implemented)

Time = 3.63 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int (a + ia\tan(c + dx))^2 \, dx = \frac{a^2\left(-\tan(c + d\,x) + \ln\left(\tan(c + d\,x) + 1\mathrm{i}\right)\,2\mathrm{i}\right)}{d}$$

```
[In] int((a + a*tan(c + d*x)*1i)^2,x)
[Out] (a^2*(log(tan(c + d*x) + 1i)*2i - tan(c + d*x)))/d
```

3.24 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 25

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{ia^3}{d(a-ia\tan(c+dx))}$$

```
[Out] -I*a^3/d/(a-I*a*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 32}

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{ia^3}{d(a-ia\tan(c+dx))}$$

```
[In] Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((-I)*a^3)/(d*(a - I*a*Tan[c + d*x]))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{(ia^3)\,\text{Subst}\left(\int \frac{1}{(a-x)^2}\,dx, x, ia\tan(c+dx)\right)}{d}$$
$$= -\frac{ia^3}{d(a - ia\tan(c+dx))}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{ia^2(\cos(c+dx)+i\sin(c+dx))^2}{2d}$$

```
[In] Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^2,x]
[Out] ((-1/2*I)*a^2*(Cos[c + d*x] + I*Sin[c + d*x])^2)/d
```

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

method	result	size
risch	$-\frac{ia^2 e^{2i(dx+c)}}{2d}$	19
derivativedivides	$\frac{-a^2\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-ia^2\left(\cos^2(dx+c)\right)+a^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	73
default	$\frac{-a^2\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-ia^2\left(\cos^2(dx+c)\right)+a^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	73

```
[In] int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] -1/2*I/d*a^2*exp(2*I*(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{i\,a^2e^{(2i\,dx+2i\,c)}}{2\,d}$$

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
[Out] -1/2*I*a^2*e^(2*I*d*x + 2*I*c)/d
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^2\,dx = \begin{cases} -\frac{ia^2e^{2ic}e^{2idx}}{2d} & \text{for } d \neq 0 \\ a^2xe^{2ic} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**2,x)
[Out] Piecewise((-I*a**2*exp(2*I*c)*exp(2*I*d*x)/(2*d), Ne(d, 0)), (a**2*x*exp(2*
I*c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2\tan(dx+c) - i\,a^2}{\left(\tan(dx+c)^2+1\right)d}$$

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
[Out] (a^2*tan(d*x + c) - I*a^2)/((tan(d*x + c)^2 + 1)*d)
```

Giac [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{i\,a^2e^{(2i\,dx+2i\,c)}}{2\,d}$$

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
[Out] -1/2*I*a^2*e^(2*I*d*x + 2*I*c)/d
```

Mupad [B] (verification not implemented)

Time = 3.82 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2}{d\,(\tan(c+d\,x)+1\mathrm{i})}$$

```
[In] int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^2,x)
[Out] a^2/(d*(tan(c + d*x) + 1i))
```

3.25 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 63

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^2\,dx=\frac{a^2x}{4}-\frac{ia^4}{4d(a-ia\tan(c+dx))^2}-\frac{ia^3}{4d(a-ia\tan(c+dx))}$$

```
[Out] 1/4*a^2*x-1/4*I*a^4/d/(a-I*a*tan(d*x+c))^2-1/4*I*a^3/d/(a-I*a*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3568, 46, 212}

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^2\,dx=-\frac{ia^4}{4d(a-ia\tan(c+dx))^2}-\frac{ia^3}{4d(a-ia\tan(c+dx))}+\frac{a^2x}{4}$$

```
[In] Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*x)/4 - ((I/4)*a^4)/(d*(a - I*a*Tan[c + d*x])^2) - ((I/4)*a^3)/(d*(a -
I*a*Tan[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
```

```
n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(ia^5)\,\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)}\,dx, x, ia\tan(c+dx)\right)}{d}\\
&= -\frac{(ia^5)\,\text{Subst}\left(\int \left(\frac{1}{2a(a-x)^3}+\frac{1}{4a^2(a-x)^2}+\frac{1}{4a^2(a^2-x^2)}\right)dx, x, ia\tan(c+dx)\right)}{d}\\
&= -\frac{ia^4}{4d(a-ia\tan(c+dx))^2}-\frac{ia^3}{4d(a-ia\tan(c+dx))}\\
&\quad-\frac{(ia^3)\,\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, ia\tan(c+dx)\right)}{4d}\\
&= \frac{a^2x}{4}-\frac{ia^4}{4d(a-ia\tan(c+dx))^2}-\frac{ia^3}{4d(a-ia\tan(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\begin{aligned}
&\int \cos^4(c+dx)(a+ia\tan(c+dx))^2\,dx\\
&= \frac{a^2(2i+\tan(c+dx)+\arctan(\tan(c+dx))(i+\tan(c+dx))^2)}{4d(i+\tan(c+dx))^2}
\end{aligned}$$

```
[In] Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*(2*I + Tan[c + d*x] + ArcTan[Tan[c + d*x]]*(I + Tan[c + d*x])^2))/(4*d
*(I + Tan[c + d*x])^2)
```

Maple [A] (verified)

Time = 6.55 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

method	result
risch	$\frac{a^{2} x}{4}-\frac{i a^{2} {\mathrm e}^{4 i (d x +c)}}{16 d}-\frac{i a^{2} {\mathrm e}^{2 i (d x +c)}}{4 d}$
derivativedivides	$\frac{-a^{2} \left(-\frac{\left(\cos^{3}(d x +c)\right) \sin (d x +c)}{4}+\frac{\sin (d x +c) \cos (d x +c)}{8}+\frac{d x}{8}+\frac{c}{8}\right)-\frac{i a^{2} \left(\cos^{4}(d x +c)\right)}{2}+a^{2} \left(\frac{\left(\cos^{3}(d x +c)+\frac{3 \cos (d x +c)}{2}\right) \sin (d x +c)}{4}\right.}{d}$
default	$\frac{-a^{2} \left(-\frac{\left(\cos^{3}(d x +c)\right) \sin (d x +c)}{4}+\frac{\sin (d x +c) \cos (d x +c)}{8}+\frac{d x}{8}+\frac{c}{8}\right)-\frac{i a^{2} \left(\cos^{4}(d x +c)\right)}{2}+a^{2} \left(\frac{\left(\cos^{3}(d x +c)+\frac{3 \cos (d x +c)}{2}\right) \sin (d x +c)}{4}\right.}{d}$

```
[In] int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/4*a^2*x-1/16*I/d*a^2*exp(4*I*(d*x+c))-1/4*I/d*a^2*exp(2*I*(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.65

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{4\,a^2dx - i\,a^2e^{(4i\,dx+4i\,c)} - 4i\,a^2e^{(2i\,dx+2i\,c)}}{16\,d}$$

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
[Out] 1/16*(4*a^2*d*x - I*a^2*e^(4*I*d*x + 4*I*c) - 4*I*a^2*e^(2*I*d*x + 2*I*c))/
d
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.38

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2x}{4} + \begin{cases} \frac{-4ia^2de^{4ic}e^{4idx}-16ia^2de^{2ic}e^{2idx}}{64d^2} & \text{for } d^2 \neq 0 \\ x\left(\frac{a^2e^{4ic}}{4}+\frac{a^2e^{2ic}}{2}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**2,x)
[Out] a**2*x/4 + Piecewise(((-4*I*a**2*d*exp(4*I*c)*exp(4*I*d*x) - 16*I*a**2*d*ex
p(2*I*c)*exp(2*I*d*x))/(64*d**2), Ne(d**2, 0)), (x*(a**2*exp(4*I*c)/4 + a**
2*exp(2*I*c)/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{(dx+c)a^2 + \frac{a^2\tan(dx+c)^3 + 3\,a^2\tan(dx+c) - 2i\,a^2}{\tan(dx+c)^4 + 2\,\tan(dx+c)^2 + 1}}{4\,d}$$

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*((d*x + c)*a^2 + (a^2*tan(d*x + c)^3 + 3*a^2*tan(d*x + c) - 2*I*a^2)/(t
an(d*x + c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(49) = 98.

Time = 0.56 (sec) , antiderivative size = 257, normalized size of antiderivative = 4.08

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{8\,a^2dxe^{(4i\,dx+2i\,c)} + 16\,a^2dxe^{(2i\,dx)} + 8\,a^2dxe^{(-2i\,c)} - i\,a^2e^{(4i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right) - 2i\,a^2e^{(2i\,dx)}\log\left(e^{(\cdots}\right.}{}$$

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/32*(8*a^2*d*x*e^(4*I*d*x + 2*I*c) + 16*a^2*d*x*e^(2*I*d*x) + 8*a^2*d*x*e^
(-2*I*c) - I*a^2*e^(4*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 2*I*a^2
*e^(2*I*d*x)*log(e^(2*I*d*x + 2*I*c) + 1) - I*a^2*e^(-2*I*c)*log(e^(2*I*d*x
 + 2*I*c) + 1) + I*a^2*e^(4*I*d*x + 2*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) +
2*I*a^2*e^(2*I*d*x)*log(e^(2*I*d*x) + e^(-2*I*c)) + I*a^2*e^(-2*I*c)*log(e^
(2*I*d*x) + e^(-2*I*c)) - 2*I*a^2*e^(8*I*d*x + 6*I*c) - 12*I*a^2*e^(6*I*d*x
 + 4*I*c) - 18*I*a^2*e^(4*I*d*x + 2*I*c) - 8*I*a^2*e^(2*I*d*x))/(d*e^(4*I*d
*x + 2*I*c) + 2*d*e^(2*I*d*x) + d*e^(-2*I*c))
```

Mupad [B] (verification not implemented)

Time = 3.82 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2\,x}{4} + \frac{\frac{a^2\tan(c+d\,x)}{4} + \frac{a^2\,1\mathrm{i}}{2}}{d\,\left(\tan\left(c+d\,x\right)^2 + \tan\left(c+d\,x\right)\,2\mathrm{i} - 1\right)}$$

```
[In] int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] (a^2*x)/4 + ((a^2*tan(c + d*x))/4 + (a^2*1i)/2)/(d*(tan(c + d*x)*2i + tan(c
 + d*x)^2 - 1))
```

3.26 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 117

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2x}{4} - \frac{ia^5}{12d(a-ia\tan(c+dx))^3} - \frac{ia^4}{8d(a-ia\tan(c+dx))^2} - \frac{3ia^3}{16d(a-ia\tan(c+dx))} + \frac{ia^3}{16d(a+ia\tan(c+dx))}$$

```
[Out] 1/4*a^2*x-1/12*I*a^5/d/(a-I*a*tan(d*x+c))^3-1/8*I*a^4/d/(a-I*a*tan(d*x+c))^
2-3/16*I*a^3/d/(a-I*a*tan(d*x+c))+1/16*I*a^3/d/(a+I*a*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3568, 46, 212}

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{ia^5}{12d(a-ia\tan(c+dx))^3} - \frac{ia^4}{8d(a-ia\tan(c+dx))^2} - \frac{3ia^3}{16d(a-ia\tan(c+dx))} + \frac{ia^3}{16d(a+ia\tan(c+dx))} + \frac{a^2x}{4}$$

```
[In] Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*x)/4 - ((I/12)*a^5)/(d*(a - I*a*Tan[c + d*x])^3) - ((I/8)*a^4)/(d*(a -
 I*a*Tan[c + d*x])^2) - (((3*I)/16)*a^3)/(d*(a - I*a*Tan[c + d*x])) + ((I/1
6)*a^3)/(d*(a + I*a*Tan[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(ia^7)\operatorname{Subst}\left(\int \frac{1}{(a-x)^4(a+x)^2}\,dx, x, ia\tan(c+dx)\right)}{d}\\
&= -\frac{(ia^7)\operatorname{Subst}\left(\int \left(\frac{1}{4a^2(a-x)^4}+\frac{1}{4a^3(a-x)^3}+\frac{3}{16a^4(a-x)^2}+\frac{1}{16a^4(a+x)^2}+\frac{1}{4a^4(a^2-x^2)}\right)dx, x, ia\tan(c+dx)\right)}{d}\\
&= -\frac{ia^5}{12d(a-ia\tan(c+dx))^3}-\frac{ia^4}{8d(a-ia\tan(c+dx))^2}-\frac{3ia^3}{16d(a-ia\tan(c+dx))}\\
&\quad+\frac{ia^3}{16d(a+ia\tan(c+dx))}-\frac{(ia^3)\operatorname{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, ia\tan(c+dx)\right)}{4d}\\
&= \frac{a^2x}{4}-\frac{ia^5}{12d(a-ia\tan(c+dx))^3}-\frac{ia^4}{8d(a-ia\tan(c+dx))^2}\\
&\quad-\frac{3ia^3}{16d(a-ia\tan(c+dx))}+\frac{ia^3}{16d(a+ia\tan(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.85

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$= \frac{a^2(4i-\tan(c+dx)+6i\tan^2(c+dx)+3\tan^3(c+dx)+3\arctan(\tan(c+dx))(-i+\tan(c+dx))(i+\tan}{12d(-i+\tan(c+dx))(i+\tan(c+dx))^3}$$

[In] Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^2,x]

[Out] (a^2*(4*I - Tan[c + d*x] + (6*I)*Tan[c + d*x]^2 + 3*Tan[c + d*x]^3 + 3*ArcT
an[Tan[c + d*x]]*(-I + Tan[c + d*x])*(I + Tan[c + d*x])^3))/(12*d*(-I + Tan
[c + d*x])*(I + Tan[c + d*x])^3)

Maple [A] (verified)

Time = 26.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.68

method	result
risch	$\frac{a^2x}{4}-\frac{ia^2e^{6i(dx+c)}}{96d}-\frac{ia^2e^{4i(dx+c)}}{16d}-\frac{5ia^2\cos(2dx+2c)}{32d}+\frac{7a^2\sin(2dx+2c)}{32d}$
derivativedivides	$\frac{-a^2\left(-\frac{\left(\cos^5(dx+c)\right)\sin(dx+c)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)-\frac{ia^2\left(\cos^6(dx+c)\right)}{3}+a^2\left(\frac{\left(\cos^5(dx+c)+\frac{5(c}{}\right.}{}\right.}{d}$
default	$\frac{-a^2\left(-\frac{\left(\cos^5(dx+c)\right)\sin(dx+c)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)-\frac{ia^2\left(\cos^6(dx+c)\right)}{3}+a^2\left(\frac{\left(\cos^5(dx+c)+\frac{5(c}{}\right.}{}\right.}{d}$

[In] int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/4*a^2*x-1/96*I/d*a^2*exp(6*I*(d*x+c))-1/16*I/d*a^2*exp(4*I*(d*x+c))-5/32*
I/d*a^2*cos(2*d*x+2*c)+7/32/d*a^2*sin(2*d*x+2*c)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.67

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{\left(24\,a^2dxe^{(2i\,dx+2i\,c)}-i\,a^2e^{(8i\,dx+8i\,c)}-6i\,a^2e^{(6i\,dx+6i\,c)}-18i\,a^2e^{(4i\,dx+4i\,c)}+3i\,a^2\right)e^{(-2i\,dx-2i\,c)}}{96\,d}$$

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/96*(24*a^2*d*x*e^(2*I*d*x + 2*I*c) - I*a^2*e^(8*I*d*x + 8*I*c) - 6*I*a^2*
e^(6*I*d*x + 6*I*c) - 18*I*a^2*e^(4*I*d*x + 4*I*c) + 3*I*a^2)*e^(-2*I*d*x -
 2*I*c)/d
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.58

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2x}{4} + \begin{cases} \frac{\left(-8192ia^2d^3e^{8ic}e^{6idx}-49152ia^2d^3e^{6ic}e^{4idx}-147456ia^2d^3e^{4ic}e^{2idx}+24576ia^2d^3e^{-2idx}\right)e^{-2ic}}{786432d^4} & \text{for } d^4e^{2ic}\neq 0 \\ x\left(-\frac{a^2}{4}+\frac{\left(a^2e^{8ic}+4a^2e^{6ic}+6a^2e^{4ic}+4a^2e^{2ic}+a^2\right)e^{-2ic}}{16}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] a**2*x/4 + Piecewise(((-8192*I*a**2*d**3*exp(8*I*c)*exp(6*I*d*x) - 49152*I*
a**2*d**3*exp(6*I*c)*exp(4*I*d*x) - 147456*I*a**2*d**3*exp(4*I*c)*exp(2*I*d
*x) + 24576*I*a**2*d**3*exp(-2*I*d*x))*exp(-2*I*c)/(786432*d**4), Ne(d**4*e
xp(2*I*c), 0)), (x*(-a**2/4 + (a**2*exp(8*I*c) + 4*a**2*exp(6*I*c) + 6*a**2
*exp(4*I*c) + 4*a**2*exp(2*I*c) + a**2)*exp(-2*I*c)/16), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.79

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{3\,(dx+c)a^2 + \frac{3\,a^2\tan(dx+c)^5+8\,a^2\tan(dx+c)^3+9\,a^2\tan(dx+c)-4i\,a^2}{\tan(dx+c)^6+3\,\tan(dx+c)^4+3\,\tan(dx+c)^2+1}}{12\,d}$$

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/12*(3*(d*x + c)*a^2 + (3*a^2*tan(d*x + c)^5 + 8*a^2*tan(d*x + c)^3 + 9*a^
2*tan(d*x + c) - 4*I*a^2)/(tan(d*x + c)^6 + 3*tan(d*x + c)^4 + 3*tan(d*x +
c)^2 + 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.60 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.44

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{24\,a^2dxe^{(6i\,dx+4i\,c)} + 48\,a^2dxe^{(4i\,dx+2i\,c)} + 24\,a^2dxe^{(2i\,dx)} - i\,a^2e^{(12i\,dx+10i\,c)} - 8i\,a^2e^{(10i\,dx+8i\,c)} - 31i\,a^2e^{(8i\,dx+6}}{96\,(de^{(6i\,dx+4i\,c)} + 2\,de^{(4i\,dx+2i\,c)} + de^{(2i\,dx)}}$$

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/96*(24*a^2*d*x*e^(6*I*d*x + 4*I*c) + 48*a^2*d*x*e^(4*I*d*x + 2*I*c) + 24*
a^2*d*x*e^(2*I*d*x) - I*a^2*e^(12*I*d*x + 10*I*c) - 8*I*a^2*e^(10*I*d*x + 8
*I*c) - 31*I*a^2*e^(8*I*d*x + 6*I*c) - 42*I*a^2*e^(6*I*d*x + 4*I*c) - 15*I*
a^2*e^(4*I*d*x + 2*I*c) + 6*I*a^2*e^(2*I*d*x) + 3*I*a^2*e^(-2*I*c))/(d*e^(6
*I*d*x + 4*I*c) + 2*d*e^(4*I*d*x + 2*I*c) + d*e^(2*I*d*x))
```

Mupad [B] (verification not implemented)

Time = 3.64 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.75

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2\,x}{4} + \frac{\frac{a^2\,\tan(c+d\,x)^3}{4} + \frac{a^2\,\tan(c+d\,x)^2\,1{}\mathrm{i}}{2} - \frac{a^2\,\tan(c+d\,x)}{12} + \frac{a^2\,1{}\mathrm{i}}{3}}{d\,\left({\mathrm{tan}\left(c+d\,x\right)}^4 + {\mathrm{tan}\left(c+d\,x\right)}^3\,2{}\mathrm{i} + \mathrm{tan}\left(c+d\,x\right)\,2{}\mathrm{i} - 1\right)}$$

```
[In] int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] (a^2*x)/4 + ((a^2*1i)/3 - (a^2*tan(c + d*x))/12 + (a^2*tan(c + d*x)^2*1i)/2
 + (a^2*tan(c + d*x)^3)/4)/(d*(tan(c + d*x)*2i + tan(c + d*x)^3*2i + tan(c
+ d*x)^4 - 1))
```

3.27 $\int \cos^8(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 171

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{15a^2x}{64} - \frac{ia^6}{32d(a-ia\tan(c+dx))^4} - \frac{ia^5}{16d(a-ia\tan(c+dx))^3} - \frac{3ia^4}{32d(a-ia\tan(c+dx))^2} - \frac{5ia^3}{32d(a-ia\tan(c+dx))} + \frac{ia^4}{64d(a+ia\tan(c+dx))^2} + \frac{5ia^3}{64d(a+ia\tan(c+dx))}$$

```
[Out] 15/64*a^2*x-1/32*I*a^6/d/(a-I*a*tan(d*x+c))^4-1/16*I*a^5/d/(a-I*a*tan(d*x+c
))^3-3/32*I*a^4/d/(a-I*a*tan(d*x+c))^2-5/32*I*a^3/d/(a-I*a*tan(d*x+c))+1/64
*I*a^4/d/(a+I*a*tan(d*x+c))^2+5/64*I*a^3/d/(a+I*a*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3568, 46, 212}

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{ia^6}{32d(a-ia\tan(c+dx))^4} - \frac{ia^5}{16d(a-ia\tan(c+dx))^3} - \frac{3ia^4}{32d(a-ia\tan(c+dx))^2} + \frac{ia^4}{64d(a+ia\tan(c+dx))^2} - \frac{5ia^3}{32d(a-ia\tan(c+dx))} + \frac{5ia^3}{64d(a+ia\tan(c+dx))} + \frac{15a^2x}{64}$$

```
[In] Int[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (15*a^2*x)/64 - ((I/32)*a^6)/(d*(a - I*a*Tan[c + d*x])^4) - ((I/16)*a^5)/(d
*(a - I*a*Tan[c + d*x])^3) - (((3*I)/32)*a^4)/(d*(a - I*a*Tan[c + d*x])^2)
- (((5*I)/32)*a^3)/(d*(a - I*a*Tan[c + d*x])) + ((I/64)*a^4)/(d*(a + I*a*Ta
n[c + d*x])^2) + (((5*I)/64)*a^3)/(d*(a + I*a*Tan[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{(ia^9)\,\text{Subst}\left(\int \frac{1}{(a-x)^5(a+x)^3}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^9)\,\text{Subst}\left(\int \left(\frac{1}{8a^3(a-x)^5} + \frac{3}{16a^4(a-x)^4} + \frac{3}{16a^5(a-x)^3} + \frac{5}{32a^6(a-x)^2} + \frac{1}{32a^5(a+x)^3} + \frac{5}{64a^6(a+x)^2} + \frac{15}{64a^6(a^2-x^2)}\right)dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^6}{32d(a-ia\tan(c+dx))^4} - \frac{ia^5}{16d(a-ia\tan(c+dx))^3} - \frac{3ia^4}{32d(a-ia\tan(c+dx))^2} - \frac{5ia^3}{32d(a-ia\tan(c+dx))} + \frac{ia^4}{64d(a+ia\tan(c+dx))^2} + \frac{5ia^3}{64d(a+ia\tan(c+dx))} - \frac{(15ia^3)\,\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, ia\tan(c+dx)\right)}{64d}$$

$$= \frac{15a^2x}{64} - \frac{ia^6}{32d(a-ia\tan(c+dx))^4} - \frac{ia^5}{16d(a-ia\tan(c+dx))^3} - \frac{3ia^4}{32d(a-ia\tan(c+dx))^2} - \frac{5ia^3}{32d(a-ia\tan(c+dx))} + \frac{ia^4}{64d(a+ia\tan(c+dx))^2} + \frac{5ia^3}{64d(a+ia\tan(c+dx))}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.83

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{a^2\sec^6(c+dx)(-80i - 65i\cos(2(c+dx)) + 16i\cos(4(c+dx)) + i\cos(6(c+dx)) + 120\arctan(\tan(c+dx))(\cos(2(c+dx)) - i\sin(2(c+dx))) - 5\sin(2(c+dx)) + 32\sin(4(c+dx)) + 3\sin(6(c+dx)))}{512d(-i+\tan(c+dx))^2(i+\tan(c+dx))^4}$$

```
[In] Integrate[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -1/512*(a^2*Sec[c + d*x]^6*(-80*I - (65*I)*Cos[2*(c + d*x)] + (16*I)*Cos[4*
(c + d*x)] + I*Cos[6*(c + d*x)] + 120*ArcTan[Tan[c + d*x]]*(Cos[2*(c + d*x)
] - I*Sin[2*(c + d*x)]) - 5*Sin[2*(c + d*x)] + 32*Sin[4*(c + d*x)] + 3*Sin[
6*(c + d*x)]))/(d*(-I + Tan[c + d*x])^2*(I + Tan[c + d*x])^4)
```

Maple [A] (verified)

Time = 82.58 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.67

method	result
risch	$\frac{15a^2x}{64}-\frac{ia^2e^{8i(dx+c)}}{512d}-\frac{ia^2e^{6i(dx+c)}}{64d}-\frac{7ia^2\cos(4dx+4c)}{128d}+\frac{a^2\sin(4dx+4c)}{16d}-\frac{7ia^2\cos(2dx+2c)}{64d}+\frac{13a^2\sin(2}{64}$
derivativedivides	$\frac{-a^2\left(-\frac{\sin(dx+c)\left(\cos^7(dx+c)\right)}{8}+\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{48}+\frac{5dx}{128}+\frac{5c}{128}\right)-\frac{ia^2\left(\cos^8(dx+c)\right)}{4}+a}{d}$
default	$\frac{-a^2\left(-\frac{\sin(dx+c)\left(\cos^7(dx+c)\right)}{8}+\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{48}+\frac{5dx}{128}+\frac{5c}{128}\right)-\frac{ia^2\left(\cos^8(dx+c)\right)}{4}+a}{d}$

```
[In] int(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 15/64*a^2*x-1/512*I/d*a^2*exp(8*I*(d*x+c))-1/64*I/d*a^2*exp(6*I*(d*x+c))-7/
128*I/d*a^2*cos(4*d*x+4*c)+1/16/d*a^2*sin(4*d*x+4*c)-7/64*I/d*a^2*cos(2*d*x
+2*c)+13/64/d*a^2*sin(2*d*x+2*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.62

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$=\frac{\left(120\,a^2dxe^{(4i\,dx+4i\,c)}-i\,a^2e^{(12i\,dx+12i\,c)}-8i\,a^2e^{(10i\,dx+10i\,c)}-30i\,a^2e^{(8i\,dx+8i\,c)}-80i\,a^2e^{(6i\,dx+6i\,c)}+24i\,a^2e^{(2}}{512\,d}$$

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/512*(120*a^2*d*x*e^(4*I*d*x + 4*I*c) - I*a^2*e^(12*I*d*x + 12*I*c) - 8*I*
a^2*e^(10*I*d*x + 10*I*c) - 30*I*a^2*e^(8*I*d*x + 8*I*c) - 80*I*a^2*e^(6*I*
d*x + 6*I*c) + 24*I*a^2*e^(2*I*d*x + 2*I*c) + 2*I*a^2)*e^(-4*I*d*x - 4*I*c)
/d
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.58

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{15a^2x}{64} + \begin{cases} \frac{(-8589934592ia^2d^5e^{14ic}e^{8idx} - 68719476736ia^2d^5e^{12ic}e^{6idx} - 257698037760ia^2d^5e^{10ic}e^{4idx} - 687194767360ia^2d^5e^{8ic}e^{2idx} + 206158430208ia^2d^5 \ldots}{4398046511104d^6} \\ x\left(-\frac{15a^2}{64} + \frac{(a^2e^{12ic}+6a^2e^{10ic}+15a^2e^{8ic}+20a^2e^{6ic}+15a^2e^{4ic}+6a^2e^{2ic}+a^2)e^{-4ic}}{64}\right) \end{cases}$$

```
[In] integrate(cos(d*x+c)**8*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] 15*a**2*x/64 + Piecewise(((-8589934592*I*a**2*d**5*exp(14*I*c)*exp(8*I*d*x)
 - 68719476736*I*a**2*d**5*exp(12*I*c)*exp(6*I*d*x) - 257698037760*I*a**2*d
**5*exp(10*I*c)*exp(4*I*d*x) - 687194767360*I*a**2*d**5*exp(8*I*c)*exp(2*I*
d*x) + 206158430208*I*a**2*d**5*exp(4*I*c)*exp(-2*I*d*x) + 17179869184*I*a*
*2*d**5*exp(2*I*c)*exp(-4*I*d*x))*exp(-6*I*c)/(4398046511104*d**6), Ne(d**6
*exp(6*I*c), 0)), (x*(-15*a**2/64 + (a**2*exp(12*I*c) + 6*a**2*exp(10*I*c)
+ 15*a**2*exp(8*I*c) + 20*a**2*exp(6*I*c) + 15*a**2*exp(4*I*c) + 6*a**2*exp
(2*I*c) + a**2)*exp(-4*I*c)/64), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.67

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{15\,(dx+c)a^2 + \frac{15\,a^2\tan(dx+c)^7+55\,a^2\tan(dx+c)^5+73\,a^2\tan(dx+c)^3+49\,a^2\tan(dx+c)-16i\,a^2}{\tan(dx+c)^8+4\,\tan(dx+c)^6+6\,\tan(dx+c)^4+4\,\tan(dx+c)^2+1}}{64\,d}$$

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/64*(15*(d*x + c)*a^2 + (15*a^2*tan(d*x + c)^7 + 55*a^2*tan(d*x + c)^5 + 7
3*a^2*tan(d*x + c)^3 + 49*a^2*tan(d*x + c) - 16*I*a^2)/(tan(d*x + c)^8 + 4*
tan(d*x + c)^6 + 6*tan(d*x + c)^4 + 4*tan(d*x + c)^2 + 1))/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 342 vs. 2(133) = 266.

Time = 0.68 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.00

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$= \frac{120\,a^2dxe^{(8i\,dx+4i\,c)}+240\,a^2dxe^{(6i\,dx+2i\,c)}+120\,a^2dxe^{(4i\,dx)}+8i\,a^2e^{(8i\,dx+4i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+16i\,a^2e}{}$$

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/512*(120*a^2*d*x*e^(8*I*d*x + 4*I*c) + 240*a^2*d*x*e^(6*I*d*x + 2*I*c) +
120*a^2*d*x*e^(4*I*d*x) + 8*I*a^2*e^(8*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*I*
c) + 1) + 16*I*a^2*e^(6*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 8*I*a
^2*e^(4*I*d*x)*log(e^(2*I*d*x + 2*I*c) + 1) - 8*I*a^2*e^(8*I*d*x + 4*I*c)*l
og(e^(2*I*d*x) + e^(-2*I*c)) - 16*I*a^2*e^(6*I*d*x + 2*I*c)*log(e^(2*I*d*x)
 + e^(-2*I*c)) - 8*I*a^2*e^(4*I*d*x)*log(e^(2*I*d*x) + e^(-2*I*c)) - I*a^2*
e^(16*I*d*x + 12*I*c) - 10*I*a^2*e^(14*I*d*x + 10*I*c) - 47*I*a^2*e^(12*I*d
*x + 8*I*c) - 148*I*a^2*e^(10*I*d*x + 6*I*c) - 190*I*a^2*e^(8*I*d*x + 4*I*c
) - 56*I*a^2*e^(6*I*d*x + 2*I*c) + 28*I*a^2*e^(2*I*d*x - 2*I*c) + 50*I*a^2*
e^(4*I*d*x) + 2*I*a^2*e^(-4*I*c))/(d*e^(8*I*d*x + 4*I*c) + 2*d*e^(6*I*d*x +
 2*I*c) + d*e^(4*I*d*x))
```

Mupad [B] (verification not implemented)

Time = 4.95 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.84

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{15\,a^2\,x}{64}$$
$$+\frac{\frac{15\,a^2\tan(c+d\,x)^5}{64}+\frac{a^2\tan(c+d\,x)^4\,15\mathrm{i}}{32}+\frac{5\,a^2\tan(c+d\,x)^3}{32}+\frac{a^2\tan(c+d\,x)^2\,25\mathrm{i}}{32}-\frac{17\,a^2\tan(c+d\,x)}{64}+\frac{a^2\,1\mathrm{i}}{4}}{d\,\left(\tan(c+d\,x)^6+\tan(c+d\,x)^5\,2\mathrm{i}+\tan(c+d\,x)^4+\tan(c+d\,x)^3\,4\mathrm{i}-\tan(c+d\,x)^2+\tan(c+d\,x)\right.}$$

```
[In] int(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] (15*a^2*x)/64 + ((a^2*1i)/4 - (17*a^2*tan(c + d*x))/64 + (a^2*tan(c + d*x)^
2*25i)/32 + (5*a^2*tan(c + d*x)^3)/32 + (a^2*tan(c + d*x)^4*15i)/32 + (15*a
^2*tan(c + d*x)^5)/64)/(d*(tan(c + d*x)*2i - tan(c + d*x)^2 + tan(c + d*x)^
3*4i + tan(c + d*x)^4 + tan(c + d*x)^5*2i + tan(c + d*x)^6 - 1))
```

3.28 $\int \sec^5(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 118

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{7a^2\text{arctanh}(\sin(c+dx))}{16d} + \frac{7ia^2\sec^5(c+dx)}{30d} + \frac{7a^2\sec(c+dx)\tan(c+dx)}{16d} + \frac{7a^2\sec^3(c+dx)\tan(c+dx)}{24d} + \frac{i\sec^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{6d}$$

```
[Out] 7/16*a^2*arctanh(sin(d*x+c))/d+7/30*I*a^2*sec(d*x+c)^5/d+7/16*a^2*sec(d*x+c
)*tan(d*x+c)/d+7/24*a^2*sec(d*x+c)^3*tan(d*x+c)/d+1/6*I*sec(d*x+c)^5*(a^2+I
*a^2*tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3579, 3567, 3853, 3855}

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{7a^2\text{arctanh}(\sin(c+dx))}{16d} + \frac{7ia^2\sec^5(c+dx)}{30d} + \frac{i\sec^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{6d} + \frac{7a^2\tan(c+dx)\sec^3(c+dx)}{24d} + \frac{7a^2\tan(c+dx)\sec(c+dx)}{16d}$$

```
[In] Int[Sec[c + d*x]^5*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (7*a^2*ArcTanh[Sin[c + d*x]])/(16*d) + (((7*I)/30)*a^2*Sec[c + d*x]^5)/d +
(7*a^2*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (7*a^2*Sec[c + d*x]^3*Tan[c + d*
x])/(24*d) + ((I/6)*Sec[c + d*x]^5*(a^2 + I*a^2*Tan[c + d*x]))/d
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{i\sec^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{6d}+\frac{1}{6}(7a)\int\sec^5(c+dx)(a+ia\tan(c+dx))\,dx\\
&= \frac{7ia^2\sec^5(c+dx)}{30d}+\frac{i\sec^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{6d}+\frac{1}{6}\left(7a^2\right)\int\sec^5(c+dx)\,dx\\
&= \frac{7ia^2\sec^5(c+dx)}{30d}+\frac{7a^2\sec^3(c+dx)\tan(c+dx)}{24d}\\
&\quad+\frac{i\sec^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{6d}+\frac{1}{8}\left(7a^2\right)\int\sec^3(c+dx)\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{7ia^2\sec^5(c+dx)}{30d} + \frac{7a^2\sec(c+dx)\tan(c+dx)}{16d} + \frac{7a^2\sec^3(c+dx)\tan(c+dx)}{24d} \\
&\quad + \frac{i\sec^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{6d} + \frac{1}{16}\left(7a^2\right)\int \sec(c+dx)\,dx \\
&= \frac{7a^2\text{arctanh}(\sin(c+dx))}{16d} + \frac{7ia^2\sec^5(c+dx)}{30d} + \frac{7a^2\sec(c+dx)\tan(c+dx)}{16d} \\
&\quad + \frac{7a^2\sec^3(c+dx)\tan(c+dx)}{24d} + \frac{i\sec^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{6d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.92

$$
\begin{aligned}
\int \sec^5(c+dx)(a+ia\tan(c+dx))^2\,dx &= \frac{7a^2\text{arctanh}(\sin(c+dx))}{16d} + \frac{2ia^2\sec^5(c+dx)}{5d} \\
&\quad + \frac{7a^2\sec(c+dx)\tan(c+dx)}{16d} \\
&\quad + \frac{7a^2\sec^3(c+dx)\tan(c+dx)}{24d} \\
&\quad - \frac{a^2\sec^5(c+dx)\tan(c+dx)}{6d}
\end{aligned}
$$

```
[In] Integrate[Sec[c + d*x]^5*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (7*a^2*ArcTanh[Sin[c + d*x]])/(16*d) + (((2*I)/5)*a^2*Sec[c + d*x]^5)/d + (
7*a^2*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (7*a^2*Sec[c + d*x]^3*Tan[c + d*x
])/(24*d) - (a^2*Sec[c + d*x]^5*Tan[c + d*x])/(6*d)
```

Maple [A] (verified)

Time = 19.59 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.13

method	result
risch	$-\frac{ia^2\left(105\,\mathrm{e}^{11i(dx+c)}+595\,\mathrm{e}^{9i(dx+c)}-1686\,\mathrm{e}^{7i(dx+c)}-1386\,\mathrm{e}^{5i(dx+c)}-595\,\mathrm{e}^{3i(dx+c)}-105\,\mathrm{e}^{i(dx+c)}\right)}{120d\left(\mathrm{e}^{2i(dx+c)}+1\right)^6}-\frac{7a^2\ln\left(\mathrm{e}^{i(dx+c)}-i\right.}{16d}$
derivativedivides	$\frac{-a^2\left(\frac{\sin^3(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{16\cos(dx+c)^2}+\frac{\sin(dx+c)}{16}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+\frac{2ia^2}{5\cos(dx+c)^5}+a^2\left(-\left(-\frac{\left(\sec^3(dx+\right.}{4}\right.\right.}{d}$
default	$\frac{-a^2\left(\frac{\sin^3(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{16\cos(dx+c)^2}+\frac{\sin(dx+c)}{16}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+\frac{2ia^2}{5\cos(dx+c)^5}+a^2\left(-\left(-\frac{\left(\sec^3(dx+\right.}{4}\right.\right.}{d}$

```
[In] int(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/120*I*a^2/d/(exp(2*I*(d*x+c))+1)^6*(105*exp(11*I*(d*x+c))+595*exp(9*I*(d
*x+c))-1686*exp(7*I*(d*x+c))-1386*exp(5*I*(d*x+c))-595*exp(3*I*(d*x+c))-105
```

```
*exp(I*(d*x+c)))-7/16/d*a^2*ln(exp(I*(d*x+c))-I)+7/16/d*a^2*ln(exp(I*(d*x+c
))+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(102) = 204$.

Time = 0.25 (sec) , antiderivative size = 364, normalized size of antiderivative = 3.08

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{-210i\,a^2e^{(11i\,dx+11i\,c)} - 1190i\,a^2e^{(9i\,dx+9i\,c)} + 3372i\,a^2e^{(7i\,dx+7i\,c)} + 2772i\,a^2e^{(5i\,dx+5i\,c)} + 1190i\,a^2e^{(3i\,dx+3i\,c)}}{}$$

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/240*(-210*I*a^2*e^(11*I*d*x + 11*I*c) - 1190*I*a^2*e^(9*I*d*x + 9*I*c) +
3372*I*a^2*e^(7*I*d*x + 7*I*c) + 2772*I*a^2*e^(5*I*d*x + 5*I*c) + 1190*I*a^
2*e^(3*I*d*x + 3*I*c) + 210*I*a^2*e^(I*d*x + I*c) + 105*(a^2*e^(12*I*d*x +
12*I*c) + 6*a^2*e^(10*I*d*x + 10*I*c) + 15*a^2*e^(8*I*d*x + 8*I*c) + 20*a^2
*e^(6*I*d*x + 6*I*c) + 15*a^2*e^(4*I*d*x + 4*I*c) + 6*a^2*e^(2*I*d*x + 2*I*
c) + a^2)*log(e^(I*d*x + I*c) + I) - 105*(a^2*e^(12*I*d*x + 12*I*c) + 6*a^2
*e^(10*I*d*x + 10*I*c) + 15*a^2*e^(8*I*d*x + 8*I*c) + 20*a^2*e^(6*I*d*x + 6
*I*c) + 15*a^2*e^(4*I*d*x + 4*I*c) + 6*a^2*e^(2*I*d*x + 2*I*c) + a^2)*log(e
^(I*d*x + I*c) - I))/(d*e^(12*I*d*x + 12*I*c) + 6*d*e^(10*I*d*x + 10*I*c) +
 15*d*e^(8*I*d*x + 8*I*c) + 20*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*I*d*x + 4*
I*c) + 6*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^2\,dx = -a^2\left(\int \tan^2(c+dx)\sec^5(c+dx)\,dx + \int \left(-2i\tan(c+dx)\sec^5(c+dx)\right)dx + \int \left(-\sec^5(c+dx)\right)dx\right)$$

```
[In] integrate(sec(d*x+c)**5*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -a**2*(Integral(tan(c + d*x)**2*sec(c + d*x)**5, x) + Integral(-2*I*tan(c +
 d*x)*sec(c + d*x)**5, x) + Integral(-sec(c + d*x)**5, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.53

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^2\,dx =$$

$$-\frac{5\,a^2\left(\frac{2\left(3\,\sin(dx+c)^5-8\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)+30\,a^2\Bigg(}{480\,d}$$

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/480*(5*a^2*(2*(3*sin(d*x + c)^5 - 8*sin(d*x + c)^3 - 3*sin(d*x + c))/(si
n(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(d*x + c
) + 1) + 3*log(sin(d*x + c) - 1)) + 30*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x
 + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) +
3*log(sin(d*x + c) - 1)) - 192*I*a^2/cos(d*x + c)^5)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(102) = 204.

Time = 0.55 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.01

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$=\frac{105\,a^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-105\,a^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)+\frac{2\left(135\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-480i\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{}}{}$$

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/240*(105*a^2*log(tan(1/2*d*x + 1/2*c) + 1) - 105*a^2*log(tan(1/2*d*x + 1/
2*c) - 1) + 2*(135*a^2*tan(1/2*d*x + 1/2*c)^11 - 480*I*a^2*tan(1/2*d*x + 1/
2*c)^10 - 445*a^2*tan(1/2*d*x + 1/2*c)^9 + 480*I*a^2*tan(1/2*d*x + 1/2*c)^8
 - 330*a^2*tan(1/2*d*x + 1/2*c)^7 - 960*I*a^2*tan(1/2*d*x + 1/2*c)^6 - 330*
a^2*tan(1/2*d*x + 1/2*c)^5 + 960*I*a^2*tan(1/2*d*x + 1/2*c)^4 - 445*a^2*tan
(1/2*d*x + 1/2*c)^3 - 96*I*a^2*tan(1/2*d*x + 1/2*c)^2 + 135*a^2*tan(1/2*d*x
 + 1/2*c) + 96*I*a^2)/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

Mupad [B] (verification not implemented)

Time = 7.90 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.46

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{7\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}$$

$$-\frac{-\frac{9\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}4\mathrm{i}+\frac{89\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}-a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}4\mathrm{i}+\frac{11\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{4}+a^2\,\mathrm{ta}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}\right.}$$

```
[In] int((a + a*tan(c + d*x)*1i)^2/cos(c + d*x)^5,x)
```

```
[Out] (7*a^2*atanh(tan(c/2 + (d*x)/2)))/(8*d) - ((a^2*tan(c/2 + (d*x)/2)^2*4i)/5
+ (89*a^2*tan(c/2 + (d*x)/2)^3)/24 - a^2*tan(c/2 + (d*x)/2)^4*8i + (11*a^2*
tan(c/2 + (d*x)/2)^5)/4 + a^2*tan(c/2 + (d*x)/2)^6*8i + (11*a^2*tan(c/2 + (
d*x)/2)^7)/4 - a^2*tan(c/2 + (d*x)/2)^8*4i + (89*a^2*tan(c/2 + (d*x)/2)^9)/
24 + a^2*tan(c/2 + (d*x)/2)^10*4i - (9*a^2*tan(c/2 + (d*x)/2)^11)/8 - (a^2*
4i)/5 - (9*a^2*tan(c/2 + (d*x)/2))/8)/(d*(15*tan(c/2 + (d*x)/2)^4 - 6*tan(c
/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 - 6*tan
(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

3.29 $\int \sec^3(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 94

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{5a^2\text{arctanh}(\sin(c+dx))}{8d} + \frac{5ia^2\sec^3(c+dx)}{12d} + \frac{5a^2\sec(c+dx)\tan(c+dx)}{8d} + \frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}$$

```
[Out] 5/8*a^2*arctanh(sin(d*x+c))/d+5/12*I*a^2*sec(d*x+c)^3/d+5/8*a^2*sec(d*x+c)*
tan(d*x+c)/d+1/4*I*sec(d*x+c)^3*(a^2+I*a^2*tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3579, 3567, 3853, 3855}

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{5a^2\text{arctanh}(\sin(c+dx))}{8d} + \frac{5ia^2\sec^3(c+dx)}{12d} + \frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d} + \frac{5a^2\tan(c+dx)\sec(c+dx)}{8d}$$

```
[In] Int[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (5*a^2*ArcTanh[Sin[c + d*x]])/(8*d) + (((5*I)/12)*a^2*Sec[c + d*x]^3)/d + (
5*a^2*Sec[c + d*x]*Tan[c + d*x])/(8*d) + ((I/4)*Sec[c + d*x]^3*(a^2 + I*a^2
*Tan[c + d*x]))/d
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}+\frac{1}{4}(5a)\int\sec^3(c+dx)(a+ia\tan(c+dx))\,dx\\
&= \frac{5ia^2\sec^3(c+dx)}{12d}+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}+\frac{1}{4}\left(5a^2\right)\int\sec^3(c+dx)\,dx\\
&= \frac{5ia^2\sec^3(c+dx)}{12d}+\frac{5a^2\sec(c+dx)\tan(c+dx)}{8d}\\
&\quad+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}+\frac{1}{8}\left(5a^2\right)\int\sec(c+dx)\,dx\\
&= \frac{5a^2\operatorname{arctanh}(\sin(c+dx))}{8d}+\frac{5ia^2\sec^3(c+dx)}{12d}\\
&\quad+\frac{5a^2\sec(c+dx)\tan(c+dx)}{8d}+\frac{i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{4d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.89

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{5a^2\text{arctanh}(\sin(c+dx))}{8d} + \frac{2ia^2\sec^3(c+dx)}{3d} + \frac{5a^2\sec(c+dx)\tan(c+dx)}{8d} - \frac{a^2\sec^3(c+dx)\tan(c+dx)}{4d}$$

```
[In] Integrate[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (5*a^2*ArcTanh[Sin[c + d*x]])/(8*d) + (((2*I)/3)*a^2*Sec[c + d*x]^3)/d + (5
*a^2*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (a^2*Sec[c + d*x]^3*Tan[c + d*x])/(
4*d)
```

Maple [A] (verified)

Time = 3.99 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.18

method	result
risch	$-\frac{ia^2\left(15\,\mathrm{e}^{7i(dx+c)}-73\,\mathrm{e}^{5i(dx+c)}-55\,\mathrm{e}^{3i(dx+c)}-15\,\mathrm{e}^{i(dx+c)}\right)}{12d\left(\mathrm{e}^{2i(dx+c)}+1\right)^4}+\frac{5a^2\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{8d}-\frac{5a^2\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{8d}$
derivativedivides	$\frac{-a^2\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{2ia^2}{3\cos(dx+c)^3}+a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\ln(\sec(dx+\ldots\right.}{d}$
default	$\frac{-a^2\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{2ia^2}{3\cos(dx+c)^3}+a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\ln(\sec(dx+\ldots\right.}{d}$

```
[In] int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*I*a^2/d/(exp(2*I*(d*x+c))+1)^4*(15*exp(7*I*(d*x+c))-73*exp(5*I*(d*x+c
))-55*exp(3*I*(d*x+c))-15*exp(I*(d*x+c)))+5/8/d*a^2*ln(exp(I*(d*x+c))+I)-5/
8/d*a^2*ln(exp(I*(d*x+c))-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(80) = 160$.

Time = 0.25 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.72

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$= \frac{-30i\,a^2e^{(7i\,dx+7i\,c)} + 146i\,a^2e^{(5i\,dx+5i\,c)} + 110i\,a^2e^{(3i\,dx+3i\,c)} + 30i\,a^2e^{(i\,dx+i\,c)} + 15\left(a^2e^{(8i\,dx+8i\,c)} + 4\,a^2e^{(6i\,dx-}\right.}{24\left(de^{(8i\,c}\right.}$$

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(-30*I*a^2*e^(7*I*d*x + 7*I*c) + 146*I*a^2*e^(5*I*d*x + 5*I*c) + 110*I
*a^2*e^(3*I*d*x + 3*I*c) + 30*I*a^2*e^(I*d*x + I*c) + 15*(a^2*e^(8*I*d*x +
8*I*c) + 4*a^2*e^(6*I*d*x + 6*I*c) + 6*a^2*e^(4*I*d*x + 4*I*c) + 4*a^2*e^(2
*I*d*x + 2*I*c) + a^2)*log(e^(I*d*x + I*c) + I) - 15*(a^2*e^(8*I*d*x + 8*I*
c) + 4*a^2*e^(6*I*d*x + 6*I*c) + 6*a^2*e^(4*I*d*x + 4*I*c) + 4*a^2*e^(2*I*d
*x + 2*I*c) + a^2)*log(e^(I*d*x + I*c) - I))/(d*e^(8*I*d*x + 8*I*c) + 4*d*e
^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^2\,dx = -a^2\left(\int \tan^2(c+dx)\sec^3(c+dx)\,dx + \int \left(-2i\tan(c+dx)\sec^3(c+dx)\right)\,dx + \int \left(-\sec^3(c+dx)\right)\,dx\right)$$

```
[In] integrate(sec(d*x+c)**3*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -a**2*(Integral(tan(c + d*x)**2*sec(c + d*x)**3, x) + Integral(-2*I*tan(c +
 d*x)*sec(c + d*x)**3, x) + Integral(-sec(c + d*x)**3, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.38

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^2\,dx =$$
$$-\frac{3\,a^2\left(\frac{2\left(\sin(dx+c)^3+\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+12\,a^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\right.\right.}{48\,d}$$

[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")

```
[Out] -1/48*(3*a^2*(2*(sin(d*x + c)^3 + sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x
 + c)^2 + 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a^2*(2*s
in(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c)
 - 1)) - 32*I*a^2/cos(d*x + c)^3)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(80) = 160.

Time = 0.51 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.84

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$=\frac{15\,a^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-15\,a^2\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)+\frac{2\left(9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-48i\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-33\right.}{}}{24\,d}$$

[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")

```
[Out] 1/24*(15*a^2*log(tan(1/2*d*x + 1/2*c) + 1) - 15*a^2*log(tan(1/2*d*x + 1/2*c
) - 1) + 2*(9*a^2*tan(1/2*d*x + 1/2*c)^7 - 48*I*a^2*tan(1/2*d*x + 1/2*c)^6
- 33*a^2*tan(1/2*d*x + 1/2*c)^5 + 48*I*a^2*tan(1/2*d*x + 1/2*c)^4 - 33*a^2*
tan(1/2*d*x + 1/2*c)^3 - 16*I*a^2*tan(1/2*d*x + 1/2*c)^2 + 9*a^2*tan(1/2*d*
x + 1/2*c) + 16*I*a^2)/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 7.32 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.11

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{5\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d} - \frac{-\frac{3\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,4\mathrm{i}+\frac{11\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}-a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,4\mathrm{i}+\frac{11\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\frac{a^2\,\tan\left(\right.}{}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\right.}$$

```
[In] int((a + a*tan(c + d*x)*1i)^2/cos(c + d*x)^3,x)
```

```
[Out] (5*a^2*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((a^2*tan(c/2 + (d*x)/2)^2*4i)/3
+ (11*a^2*tan(c/2 + (d*x)/2)^3)/4 - a^2*tan(c/2 + (d*x)/2)^4*4i + (11*a^2*t
an(c/2 + (d*x)/2)^5)/4 + a^2*tan(c/2 + (d*x)/2)^6*4i - (3*a^2*tan(c/2 + (d*
x)/2)^7)/4 - (a^2*4i)/3 - (3*a^2*tan(c/2 + (d*x)/2))/4)/(d*(6*tan(c/2 + (d*
x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)
/2)^8 + 1))
```

3.30 $\int \sec(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 68

$$\int \sec(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{3a^2\text{arctanh}(\sin(c+dx))}{2d} + \frac{3ia^2\sec(c+dx)}{2d} + \frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}$$

```
[Out] 3/2*a^2*arctanh(sin(d*x+c))/d+3/2*I*a^2*sec(d*x+c)/d+1/2*I*sec(d*x+c)*(a^2+
I*a^2*tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3579, 3567, 3855}

$$\int \sec(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{3a^2\text{arctanh}(\sin(c+dx))}{2d} + \frac{3ia^2\sec(c+dx)}{2d} + \frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}$$

```
[In] Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (3*a^2*ArcTanh[Sin[c + d*x]])/(2*d) + (((3*I)/2)*a^2*Sec[c + d*x])/d + ((I/
2)*Sec[c + d*x]*(a^2 + I*a^2*Tan[c + d*x]))/d
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
```

```
| NeQ[a^2 + b^2, 0])
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}+\frac{1}{2}(3a)\int\sec(c+dx)(a+ia\tan(c+dx))\,dx\\ &= \frac{3ia^2\sec(c+dx)}{2d}+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}+\frac{1}{2}\left(3a^2\right)\int\sec(c+dx)\,dx\\ &= \frac{3a^2\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{3ia^2\sec(c+dx)}{2d}+\frac{i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{2d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

$$\int\sec(c+dx)(a+ia\tan(c+dx))^2\,dx=\frac{3a^2\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{2ia^2\sec(c+dx)}{d}-\frac{a^2\sec(c+dx)\tan(c+dx)}{2d}$$

```
[In] Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^2,x]
[Out] (3*a^2*ArcTanh[Sin[c + d*x]])/(2*d) + ((2*I)*a^2*Sec[c + d*x])/d - (a^2*Sec
[c + d*x]*Tan[c + d*x])/(2*d)
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.26

method	result	size
derivativedivides	$\frac{-a^{2}\left(\frac{\sin^{3}(dx+c)}{2\cos(dx+c)^{2}}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{2ia^{2}}{\cos(dx+c)}+a^{2}\ln(\sec(dx+c)+\tan(dx+c))}{d}$	86
default	$\frac{-a^{2}\left(\frac{\sin^{3}(dx+c)}{2\cos(dx+c)^{2}}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{2ia^{2}}{\cos(dx+c)}+a^{2}\ln(\sec(dx+c)+\tan(dx+c))}{d}$	86
risch	$\frac{ia^{2}\left(5\,{\rm e}^{3i(dx+c)}+3\,{\rm e}^{i(dx+c)}\right)}{d\left({\rm e}^{2i(dx+c)}+1\right)^{2}}+\frac{3a^{2}\ln\left({\rm e}^{i(dx+c)}+i\right)}{2d}-\frac{3a^{2}\ln\left({\rm e}^{i(dx+c)}-i\right)}{2d}$	89

```
[In] int(sec(d*x+c)*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+t
an(d*x+c)))+2*I*a^2/cos(d*x+c)+a^2*ln(sec(d*x+c)+tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(56) = 112.

Time = 0.25 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.18

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{2}\,dx = \frac{10i\,a^{2}e^{(3i\,dx+3i\,c)}+6i\,a^{2}e^{(i\,dx+i\,c)}+3\left(a^{2}e^{(4i\,dx+4i\,c)}+2\,a^{2}e^{(2i\,dx+2i\,c)}+a^{2}\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-3\left(a^{2}e^{(4i\,dx+4i\,c}\right.}{2\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(10*I*a^2*e^(3*I*d*x + 3*I*c) + 6*I*a^2*e^(I*d*x + I*c) + 3*(a^2*e^(4*I
*d*x + 4*I*c) + 2*a^2*e^(2*I*d*x + 2*I*c) + a^2)*log(e^(I*d*x + I*c) + I) -
 3*(a^2*e^(4*I*d*x + 4*I*c) + 2*a^2*e^(2*I*d*x + 2*I*c) + a^2)*log(e^(I*d*x
 + I*c) - I))/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{2}\,dx = -a^{2}\left(\int \tan^{2}\left(c+dx\right)\sec\left(c+dx\right)dx + \int\left(-2i\tan\left(c+dx\right)\sec\left(c+dx\right)\right)dx + \int\left(-\sec\left(c+dx\right)\right)dx\right)$$

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -a**2*(Integral(tan(c + d*x)**2*sec(c + d*x), x) + Integral(-2*I*tan(c + d*
x)*sec(c + d*x), x) + Integral(-sec(c + d*x), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.22

$$\int \sec(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$= \frac{a^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}+\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)\right)+4a^2\log(\sec(dx+c)+\tan(dx+c))+}{4d}$$

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log
(sin(d*x + c) - 1)) + 4*a^2*log(sec(d*x + c) + tan(d*x + c)) + 8*I*a^2/cos(
d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.57

$$\int \sec(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$= \frac{3a^2\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)-3a^2\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)-\frac{2\left(a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+4i\,a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a^2\tan\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2}}{2d}$$

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*a^2*log(tan(1/2*d*x + 1/2*c) + 1) - 3*a^2*log(tan(1/2*d*x + 1/2*c) -
 1) - 2*(a^2*tan(1/2*d*x + 1/2*c)^3 + 4*I*a^2*tan(1/2*d*x + 1/2*c)^2 + a^2*
tan(1/2*d*x + 1/2*c) - 4*I*a^2)/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 4.63 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.53

$$\int \sec(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$= \frac{3\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,4\mathrm{i}+a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a^2\,4\mathrm{i}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

```
[In] int((a + a*tan(c + d*x)*1i)^2/cos(c + d*x),x)
```

```
[Out] (3*a^2*atanh(tan(c/2 + (d*x)/2)))/d - (a^2*tan(c/2 + (d*x)/2)^2*4i + a^2*ta
n(c/2 + (d*x)/2)^3 - a^2*4i + a^2*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2
)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1))
```

3.31 $\int \cos(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 46

$$\int \cos(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{a^2\text{arctanh}(\sin(c+dx))}{d} - \frac{2i\cos(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}$$

```
[Out] -a^2*arctanh(sin(d*x+c))/d-2*I*cos(d*x+c)*(a^2+I*a^2*tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3577, 3855}

$$\int \cos(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{a^2\text{arctanh}(\sin(c+dx))}{d} - \frac{2i\cos(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}$$

```
[In] Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -((a^2*ArcTanh[Sin[c + d*x]])/d) - ((2*I)*Cos[c + d*x]*(a^2 + I*a^2*Tan[c +
 d*x]))/d
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
```

```
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2i\cos(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d} - a^2\int\sec(c+dx)\,dx\\ &= -\frac{a^2\text{arctanh}(\sin(c+dx))}{d} - \frac{2i\cos(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{d}\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 180 vs. 2(46) = 92.

Time = 0.52 (sec) , antiderivative size = 180, normalized size of antiderivative = 3.91

$$\int\cos(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$= \frac{a^2\left(\cos\left(\frac{1}{2}(c+dx)\right)\left(-2i+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right.\right.\right.}{}$$

```
[In] Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*(Cos[(c + d*x)/2]*(-2*I + Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - L
og[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (2 - I*Log[Cos[(c + d*x)/2] - Si
n[(c + d*x)/2]] + I*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])*Sin[(c + d*x)
/2])*(Cos[(c + 5*d*x)/2] + I*Sin[(c + 5*d*x)/2]))/(d*(Cos[d*x] + I*Sin[d*x]
)^2)
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.22

method	result	size
derivativedivides	$\frac{-a^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))-2ia^2\cos(dx+c)+a^2\sin(dx+c)}{d}$	56
default	$\frac{-a^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))-2ia^2\cos(dx+c)+a^2\sin(dx+c)}{d}$	56
risch	$-\frac{2ia^2\mathrm{e}^{i(dx+c)}}{d}+\frac{a^2\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{d}-\frac{a^2\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d}$	61

```
[In] int(cos(d*x+c)*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(-a^2*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))-2*I*a^2*cos(d*x+c)+a^2*si
n(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int \cos(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{-2i\,a^2e^{(i\,dx+i\,c)}-a^2\log\left(e^{(i\,dx+i\,c)}+i\right)+a^2\log\left(e^{(i\,dx+i\,c)}-i\right)}{d}$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
[Out] (-2*I*a^2*e^(I*d*x + I*c) - a^2*log(e^(I*d*x + I*c) + I) + a^2*log(e^(I*d*x
 + I*c) - I))/d
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.48

$$\int \cos(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2\left(\log\left(e^{idx}-ie^{-ic}\right)-\log\left(e^{idx}+ie^{-ic}\right)\right)}{d} + \begin{cases} -\frac{2ia^2e^{ic}e^{idx}}{d} & \text{for } d\neq 0 \\ 2a^2xe^{ic} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**2,x)
[Out] a**2*(log(exp(I*d*x) - I*exp(-I*c)) - log(exp(I*d*x) + I*exp(-I*c)))/d + Pi
ecewise((-2*I*a**2*exp(I*c)*exp(I*d*x)/d, Ne(d, 0)), (2*a**2*x*exp(I*c), Tr
ue))
```

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.33

$$\int \cos(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{a^2(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)-2\sin(dx+c))+4i\,a^2\cos(dx+c)-2\,a^2\sin(dx+c)}{2\,d}$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c))
+ 4*I*a^2*cos(d*x + c) - 2*a^2*sin(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.22

$$\int \cos(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{-2i\,a^2e^{(i\,dx+i\,c)}-a^2\log\left(i\,e^{(i\,dx+i\,c)}-1\right)+a^2\log\left(-i\,e^{(i\,dx+i\,c)}-1\right)}{d}$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (-2*I*a^2*e^(I*d*x + I*c) - a^2*log(I*e^(I*d*x + I*c) - 1) + a^2*log(-I*e^(
I*d*x + I*c) - 1))/d
```

Mupad [B] (verification not implemented)

Time = 3.75 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \cos(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{4\,a^2}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

```
[In] int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] (4*a^2)/(d*(tan(c/2 + (d*x)/2) + 1i)) - (2*a^2*atanh(tan(c/2 + (d*x)/2)))/d
```

3.32 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 51

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2\sin(c+dx)}{3d} - \frac{2i\cos^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{3d}$$

```
[Out] 1/3*a^2*sin(d*x+c)/d-2/3*I*cos(d*x+c)^3*(a^2+I*a^2*tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3577, 2717}

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{a^2\sin(c+dx)}{3d} - \frac{2i\cos^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{3d}$$

```
[In] Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x])/(3*d) - (((2*I)/3)*Cos[c + d*x]^3*(a^2 + I*a^2*Tan[c + d
*x]))/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2i\cos^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{3d}+\frac{1}{3}a^2\int\cos(c+dx)\,dx\\&=\frac{a^2\sin(c+dx)}{3d}-\frac{2i\cos^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{3d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02

$$\int\cos^3(c+dx)(a+ia\tan(c+dx))^2\,dx=-\frac{2ia^2\cos^3(c+dx)}{3d}+\frac{a^2\sin(c+dx)}{d}-\frac{2a^2\sin^3(c+dx)}{3d}$$

```
[In] Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^2,x]
[Out] (((-2*I)/3)*a^2*Cos[c + d*x]^3)/d + (a^2*Sin[c + d*x])/d - (2*a^2*Sin[c + d
*x]^3)/(3*d)
```

Maple [A] (verified)

Time = 3.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.75

method	result	size
risch	$-\frac{ia^2e^{3i(dx+c)}}{6d}-\frac{ia^2e^{i(dx+c)}}{2d}$	38
derivativedivides	$\frac{-\frac{a^2\left(\sin^3(dx+c)\right)}{3}-\frac{2ia^2\left(\cos^3(dx+c)\right)}{3}+\frac{a^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$	54
default	$\frac{-\frac{a^2\left(\sin^3(dx+c)\right)}{3}-\frac{2ia^2\left(\cos^3(dx+c)\right)}{3}+\frac{a^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$	54

```
[In] int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*I/d*a^2*exp(3*I*(d*x+c))-1/2*I/d*a^2*exp(I*(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{-i\,a^2e^{(3i\,dx+3i\,c)} - 3i\,a^2e^{(i\,dx+i\,c)}}{6\,d}$$

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
[Out] 1/6*(-I*a^2*e^(3*I*d*x + 3*I*c) - 3*I*a^2*e^(I*d*x + I*c))/d
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.47

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^2\,dx = \begin{cases} \frac{-2ia^2de^{3ic}e^{3idx}-6ia^2de^{ic}e^{idx}}{12d^2} & \text{for } d^2 \neq 0 \\ x\left(\frac{a^2e^{3ic}}{2}+\frac{a^2e^{ic}}{2}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**2,x)
[Out] Piecewise(((-2*I*a**2*d*exp(3*I*c)*exp(3*I*d*x) - 6*I*a**2*d*exp(I*c)*exp(I
*d*x))/(12*d**2), Ne(d**2, 0)), (x*(a**2*exp(3*I*c)/2 + a**2*exp(I*c)/2), T
rue))
```

Maxima [A] (verification not implemented)

none

Time = 0.65 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{2i\,a^2\cos\left(dx+c\right)^3 + a^2\sin\left(dx+c\right)^3 + \left(\sin\left(dx+c\right)^3 - 3\,\sin\left(dx+c\right)\right)a^2}{3\,d}$$

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
[Out] -1/3*(2*I*a^2*cos(d*x + c)^3 + a^2*sin(d*x + c)^3 + (sin(d*x + c)^3 - 3*sin
(d*x + c))*a^2)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 531 vs. $2(43) = 86$.

Time = 0.52 (sec) , antiderivative size = 531, normalized size of antiderivative = 10.41

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{24\,a^2e^{(4i\,dx+2i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+48\,a^2e^{(2i\,dx)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+24\,a^2e^{(-2i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+}{}$$

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/96*(24*a^2*e^(4*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 48*a^2*e^(2*
I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 24*a^2*e^(-2*I*c)*log(I*e^(I*d*x + I*c)
 + 1) + 27*a^2*e^(4*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 54*a^2*e^(2
*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 27*a^2*e^(-2*I*c)*log(I*e^(I*d*x + I*c
) - 1) - 24*a^2*e^(4*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 48*a^2*e^
(2*I*d*x)*log(-I*e^(I*d*x + I*c) + 1) - 24*a^2*e^(-2*I*c)*log(-I*e^(I*d*x +
 I*c) + 1) - 27*a^2*e^(4*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 54*a^
2*e^(2*I*d*x)*log(-I*e^(I*d*x + I*c) - 1) - 27*a^2*e^(-2*I*c)*log(-I*e^(I*d
*x + I*c) - 1) + 3*a^2*e^(4*I*d*x + 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 6*
a^2*e^(2*I*d*x)*log(I*e^(I*d*x) + e^(-I*c)) + 3*a^2*e^(-2*I*c)*log(I*e^(I*d
*x) + e^(-I*c)) - 3*a^2*e^(4*I*d*x + 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) -
6*a^2*e^(2*I*d*x)*log(-I*e^(I*d*x) + e^(-I*c)) - 3*a^2*e^(-2*I*c)*log(-I*e^
(I*d*x) + e^(-I*c)) + 16*I*a^2*e^(7*I*d*x + 5*I*c) + 80*I*a^2*e^(5*I*d*x +
3*I*c) + 112*I*a^2*e^(3*I*d*x + I*c) + 48*I*a^2*e^(I*d*x - I*c))/(d*e^(4*I*
d*x + 2*I*c) + 2*d*e^(2*I*d*x) + d*e^(-2*I*c))
```

Mupad [B] (verification not implemented)

Time = 3.71 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.53

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{2\,a^2\,\left(3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,3{}\mathrm{i}-2\right)}{3\,d\,\left(-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,3{}\mathrm{i}+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1{}\mathrm{i}\right)}$$

```
[In] int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] -(2*a^2*(tan(c/2 + (d*x)/2)*3i + 3*tan(c/2 + (d*x)/2)^2 - 2))/(3*d*(3*tan(c
/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2*3i - tan(c/2 + (d*x)/2)^3 + 1i))
```

3.33 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 69

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^2\,dx=\frac{3a^2\sin(c+dx)}{5d}-\frac{a^2\sin^3(c+dx)}{5d}-\frac{2i\cos^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}$$

```
[Out] 3/5*a^2*sin(d*x+c)/d-1/5*a^2*sin(d*x+c)^3/d-2/5*I*cos(d*x+c)^5*(a^2+I*a^2*t
an(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3577, 2713}

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^2\,dx=-\frac{a^2\sin^3(c+dx)}{5d}+\frac{3a^2\sin(c+dx)}{5d}-\frac{2i\cos^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}$$

```
[In] Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (3*a^2*Sin[c + d*x])/(5*d) - (a^2*Sin[c + d*x]^3)/(5*d) - (((2*I)/5)*Cos[c
+ d*x]^5*(a^2 + I*a^2*Tan[c + d*x]))/d
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2i\cos^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}+\frac{1}{5}\left(3a^2\right)\int\cos^3(c+dx)\,dx\\ &= -\frac{2i\cos^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}-\frac{\left(3a^2\right)\operatorname{Subst}\left(\int\left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{5d}\\ &= \frac{3a^2\sin(c+dx)}{5d}-\frac{a^2\sin^3(c+dx)}{5d}-\frac{2i\cos^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{5d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int\cos^5(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{2ia^2\cos^5(c+dx)}{5d}+\frac{a^2\sin(c+dx)}{d}-\frac{a^2\sin^3(c+dx)}{d}+\frac{2a^2\sin^5(c+dx)}{5d}$$

```
[In] Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (((-2*I)/5)*a^2*Cos[c + d*x]^5)/d + (a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*x
]^3)/d + (2*a^2*Sin[c + d*x]^5)/(5*d)
```

Maple [A] (verified)

Time = 14.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

method	result
risch	$-\frac{i a^2 e^{5i(dx+c)}}{40d}-\frac{i a^2 e^{3i(dx+c)}}{8d}-\frac{i a^2\cos(dx+c)}{4d}+\frac{a^2\sin(dx+c)}{2d}$
derivativedivides	$\frac{-a^2\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)-\frac{2ia^2\left(\cos^5(dx+c)\right)}{5}+\frac{a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(a}{5}}{d}$
default	$\frac{-a^2\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)-\frac{2ia^2\left(\cos^5(dx+c)\right)}{5}+\frac{a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(a}{5}}{d}$

```
[In] int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] -1/40*I/d*a^2*exp(5*I*(d*x+c))-1/8*I/d*a^2*exp(3*I*(d*x+c))-1/4*I/d*a^2*cos
(d*x+c)+1/2*a^2*sin(d*x+c)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.90

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{\left(-i\,a^2e^{(6i\,dx+6i\,c)}-5i\,a^2e^{(4i\,dx+4i\,c)}-15i\,a^2e^{(2i\,dx+2i\,c)}+5i\,a^2\right)e^{(-i\,dx-i\,c)}}{40\,d}$$

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
[Out] 1/40*(-I*a^2*e^(6*I*d*x + 6*I*c) - 5*I*a^2*e^(4*I*d*x + 4*I*c) - 15*I*a^2*e
^(2*I*d*x + 2*I*c) + 5*I*a^2)*e^(-I*d*x - I*c)/d
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(60) = 120.

Time = 0.23 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.22

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^2\,dx = \begin{cases} \frac{\left(-512ia^2d^3e^{6ic}e^{5idx}-2560ia^2d^3e^{4ic}e^{3idx}-7680ia^2d^3e^{2ic}e^{idx}+2560ia^2d^3e^{-idx}\right)e^{-ic}}{20480d^4} & \text{for } d^4e^{ic}\neq 0 \\ \frac{x\left(a^2e^{6ic}+3a^2e^{4ic}+3a^2e^{2ic}+a^2\right)e^{-ic}}{8} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise(((-512*I*a**2*d**3*exp(6*I*c)*exp(5*I*d*x) - 2560*I*a**2*d**3*exp
(4*I*c)*exp(3*I*d*x) - 7680*I*a**2*d**3*exp(2*I*c)*exp(I*d*x) + 2560*I*a**2
*d**3*exp(-I*d*x))*exp(-I*c)/(20480*d**4), Ne(d**4*exp(I*c), 0)), (x*(a**2*
exp(6*I*c) + 3*a**2*exp(4*I*c) + 3*a**2*exp(2*I*c) + a**2)*exp(-I*c)/8, Tru
e))
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.14

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{6i\,a^2\cos{(dx+c)}^5 - \left(3\,\sin{(dx+c)}^5 - 5\,\sin{(dx+c)}^3\right)a^2 - \left(3\,\sin{(dx+c)}^5 - 10\,\sin{(dx+c)}^3 + 15\,\sin\right.}{15\,d}$$

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/15*(6*I*a^2*cos(d*x + c)^5 - (3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*a^2 -
 (3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^2)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 613 vs. 2(59) = 118.

Time = 0.60 (sec) , antiderivative size = 613, normalized size of antiderivative = 8.88

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{45\,a^2e^{(5i\,dx+3i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right) + 90\,a^2e^{(3i\,dx+i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right) + 45\,a^2e^{(i\,dx-i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+\right.}{}$$

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/160*(45*a^2*e^(5*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) + 1) + 90*a^2*e^(3
*I*d*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 45*a^2*e^(I*d*x - I*c)*log(I*e^(
I*d*x + I*c) + 1) + 40*a^2*e^(5*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) - 1) +
 80*a^2*e^(3*I*d*x + I*c)*log(I*e^(I*d*x + I*c) - 1) + 40*a^2*e^(I*d*x - I*
c)*log(I*e^(I*d*x + I*c) - 1) - 45*a^2*e^(5*I*d*x + 3*I*c)*log(-I*e^(I*d*x
+ I*c) + 1) - 90*a^2*e^(3*I*d*x + I*c)*log(-I*e^(I*d*x + I*c) + 1) - 45*a^2
*e^(I*d*x - I*c)*log(-I*e^(I*d*x + I*c) + 1) - 40*a^2*e^(5*I*d*x + 3*I*c)*l
og(-I*e^(I*d*x + I*c) - 1) - 80*a^2*e^(3*I*d*x + I*c)*log(-I*e^(I*d*x + I*c
```

```
) - 1) - 40*a^2*e^(I*d*x - I*c)*log(-I*e^(I*d*x + I*c) - 1) - 5*a^2*e^(5*I*
d*x + 3*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 10*a^2*e^(3*I*d*x + I*c)*log(I*e
^(I*d*x) + e^(-I*c)) - 5*a^2*e^(I*d*x - I*c)*log(I*e^(I*d*x) + e^(-I*c)) +
5*a^2*e^(5*I*d*x + 3*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 10*a^2*e^(3*I*d*x
+ I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 5*a^2*e^(I*d*x - I*c)*log(-I*e^(I*d*x
) + e^(-I*c)) + 4*I*a^2*e^(10*I*d*x + 8*I*c) + 28*I*a^2*e^(8*I*d*x + 6*I*c)
 + 104*I*a^2*e^(6*I*d*x + 4*I*c) + 120*I*a^2*e^(4*I*d*x + 2*I*c) + 20*I*a^2
*e^(2*I*d*x) - 20*I*a^2*e^(-2*I*c))/(d*e^(5*I*d*x + 3*I*c) + 2*d*e^(3*I*d*x
 + I*c) + d*e^(I*d*x - I*c))
```

Mupad [B] (verification not implemented)

Time = 5.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{2\,a^2\,\left(\frac{5\,\sin(3\,c+3\,d\,x)}{16}-\frac{\cos(5\,c+5\,d\,x)\,1\mathrm{i}}{16}-\frac{\cos(3\,c+3\,d\,x)\,5\mathrm{i}}{16}+\frac{\sin(5\,c+5\,d\,x)}{16}+\frac{5\,\sqrt{3}\,\sin\left(c+d\,x-\frac{\ln(3)\,1\mathrm{i}}{2}\right)}{8}\right)}{5\,d}$$

```
[In] int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] (2*a^2*((5*sin(3*c + 3*d*x))/16 - (cos(5*c + 5*d*x)*1i)/16 - (cos(3*c + 3*d
*x)*5i)/16 + sin(5*c + 5*d*x)/16 + (5*3^(1/2)*sin(c - (log(3)*1i)/2 + d*x))
/8))/(5*d)
```

3.34 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^2\,dx=\frac{5a^2\sin(c+dx)}{7d}-\frac{10a^2\sin^3(c+dx)}{21d}+\frac{a^2\sin^5(c+dx)}{7d}-\frac{2i\cos^7(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}$$

```
[Out] 5/7*a^2*sin(d*x+c)/d-10/21*a^2*sin(d*x+c)^3/d+1/7*a^2*sin(d*x+c)^5/d-2/7*I*
cos(d*x+c)^7*(a^2+I*a^2*tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3577, 2713}

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^2\,dx=\frac{a^2\sin^5(c+dx)}{7d}-\frac{10a^2\sin^3(c+dx)}{21d}+\frac{5a^2\sin(c+dx)}{7d}-\frac{2i\cos^7(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}$$

```
[In] Int[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (5*a^2*Sin[c + d*x])/(7*d) - (10*a^2*Sin[c + d*x]^3)/(21*d) + (a^2*Sin[c +
d*x]^5)/(7*d) - (((2*I)/7)*Cos[c + d*x]^7*(a^2 + I*a^2*Tan[c + d*x]))/d
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2i\cos^7(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}+\frac{1}{7}\left(5a^2\right)\int\cos^5(c+dx)\,dx\\ &= -\frac{2i\cos^7(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}-\frac{\left(5a^2\right)\text{Subst}\left(\int\left(1-2x^2+x^4\right)\,dx,x,-\sin(c+dx)\right)}{7d}\\ &= \frac{5a^2\sin(c+dx)}{7d}-\frac{10a^2\sin^3(c+dx)}{21d}+\frac{a^2\sin^5(c+dx)}{7d}-\frac{2i\cos^7(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{7d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

$$\begin{aligned}\int\cos^7(c+dx)(a+ia\tan(c+dx))^2\,dx = &-\frac{2ia^2\cos^7(c+dx)}{7d}\\ &+\frac{a^2\sin(c+dx)}{d}-\frac{4a^2\sin^3(c+dx)}{3d}\\ &+\frac{a^2\sin^5(c+dx)}{d}-\frac{2a^2\sin^7(c+dx)}{7d}\end{aligned}$$

```
[In] Integrate[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (((-2*I)/7)*a^2*Cos[c + d*x]^7)/d + (a^2*Sin[c + d*x])/d - (4*a^2*Sin[c + d
*x]^3)/(3*d) + (a^2*Sin[c + d*x]^5)/d - (2*a^2*Sin[c + d*x]^7)/(7*d)
```

Maple [A] (verified)

Time = 49.18 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.17

method	result
risch	$-\frac{ia^2 e^{7i(dx+c)}}{224d}-\frac{ia^2 e^{5i(dx+c)}}{32d}-\frac{5ia^2\cos(dx+c)}{32d}+\frac{15a^2\sin(dx+c)}{32d}-\frac{3ia^2\cos(3dx+3c)}{32d}+\frac{11a^2\sin(3dx+3c)}{96d}$
derivativedivides	$\frac{-a^2\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)-\frac{2ia^2\left(\cos^7(dx+c)\right)}{7}+\frac{a^2\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6(c}{}\right.}{}}{d}$
default	$\frac{-a^2\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)-\frac{2ia^2\left(\cos^7(dx+c)\right)}{7}+\frac{a^2\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6(c}{}\right.}{}}{d}$

```
[In] int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/224*I/d*a^2*exp(7*I*(d*x+c))-1/32*I/d*a^2*exp(5*I*(d*x+c))-5/32*I/d*a^2*
cos(d*x+c)+15/32*a^2*sin(d*x+c)/d-3/32*I/d*a^2*cos(3*d*x+3*c)+11/96/d*a^2*s
in(3*d*x+3*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{\left(-3i\,a^2e^{(10i\,dx+10i\,c)}-21i\,a^2e^{(8i\,dx+8i\,c)}-70i\,a^2e^{(6i\,dx+6i\,c)}-210i\,a^2e^{(4i\,dx+4i\,c)}+105i\,a^2e^{(2i\,dx+2i\,c)}+7i\,a^2\right)e^{(\cdot}}{672\,d}$$

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/672*(-3*I*a^2*e^(10*I*d*x + 10*I*c) - 21*I*a^2*e^(8*I*d*x + 8*I*c) - 70*I
*a^2*e^(6*I*d*x + 6*I*c) - 210*I*a^2*e^(4*I*d*x + 4*I*c) + 105*I*a^2*e^(2*I
*d*x + 2*I*c) + 7*I*a^2)*e^(-3*I*d*x - 3*I*c)/d
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(76) = 152.

Time = 0.31 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.74

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$= \begin{cases} \frac{(-75497472ia^2d^5e^{11ic}e^{7idx}-528482304ia^2d^5e^{9ic}e^{5idx}-1761607680ia^2d^5e^{7ic}e^{3idx}-5284823040ia^2d^5e^{5ic}e^{idx}+2642411520ia^2d^5e^{3ic}e^{-idx}+}{16911433728d^6} \\ \frac{x(a^2e^{10ic}+5a^2e^{8ic}+10a^2e^{6ic}+10a^2e^{4ic}+5a^2e^{2ic}+a^2)e^{-3ic}}{32} \end{cases}$$

```
[In] integrate(cos(d*x+c)**7*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise(((-75497472*I*a**2*d**5*exp(11*I*c)*exp(7*I*d*x) - 528482304*I*a*
*2*d**5*exp(9*I*c)*exp(5*I*d*x) - 1761607680*I*a**2*d**5*exp(7*I*c)*exp(3*I
*d*x) - 5284823040*I*a**2*d**5*exp(5*I*c)*exp(I*d*x) + 2642411520*I*a**2*d*
*5*exp(3*I*c)*exp(-I*d*x) + 176160768*I*a**2*d**5*exp(I*c)*exp(-3*I*d*x))*e
xp(-4*I*c)/(16911433728*d**6), Ne(d**6*exp(4*I*c), 0)), (x*(a**2*exp(10*I*c
) + 5*a**2*exp(8*I*c) + 10*a**2*exp(6*I*c) + 10*a**2*exp(4*I*c) + 5*a**2*ex
p(2*I*c) + a**2)*exp(-3*I*c)/32, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.13

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^2\,dx =$$
$$-\frac{30i\,a^2\cos(dx+c)^7+\left(15\sin(dx+c)^7-42\sin(dx+c)^5+35\sin(dx+c)^3\right)a^2+3\left(5\sin(dx+c)^7-\right.}{105\,d}$$

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/105*(30*I*a^2*cos(d*x + c)^7 + (15*sin(d*x + c)^7 - 42*sin(d*x + c)^5 +
35*sin(d*x + c)^3)*a^2 + 3*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin(d
*x + c)^3 - 35*sin(d*x + c))*a^2)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 641 vs. 2(75) = 150.

Time = 0.67 (sec) , antiderivative size = 641, normalized size of antiderivative = 7.37

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{2583\,a^2e^{(7i\,dx+3i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+5166\,a^2e^{(5i\,dx+i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+2583\,a^2e^{(3i\,dx-i\,c)}\log\left(i\,e^{(i\,c}\right.}{}$$

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/10752*(2583*a^2*e^(7*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) + 1) + 5166*a^
2*e^(5*I*d*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 2583*a^2*e^(3*I*d*x - I*c)
*log(I*e^(I*d*x + I*c) + 1) + 2121*a^2*e^(7*I*d*x + 3*I*c)*log(I*e^(I*d*x +
 I*c) - 1) + 4242*a^2*e^(5*I*d*x + I*c)*log(I*e^(I*d*x + I*c) - 1) + 2121*a
^2*e^(3*I*d*x - I*c)*log(I*e^(I*d*x + I*c) - 1) - 2583*a^2*e^(7*I*d*x + 3*I
*c)*log(-I*e^(I*d*x + I*c) + 1) - 5166*a^2*e^(5*I*d*x + I*c)*log(-I*e^(I*d*
x + I*c) + 1) - 2583*a^2*e^(3*I*d*x - I*c)*log(-I*e^(I*d*x + I*c) + 1) - 21
21*a^2*e^(7*I*d*x + 3*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 4242*a^2*e^(5*I*d*
x + I*c)*log(-I*e^(I*d*x + I*c) - 1) - 2121*a^2*e^(3*I*d*x - I*c)*log(-I*e^
(I*d*x + I*c) - 1) - 462*a^2*e^(7*I*d*x + 3*I*c)*log(I*e^(I*d*x) + e^(-I*c)
) - 924*a^2*e^(5*I*d*x + I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 462*a^2*e^(3*I*
d*x - I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 462*a^2*e^(7*I*d*x + 3*I*c)*log(-I
*e^(I*d*x) + e^(-I*c)) + 924*a^2*e^(5*I*d*x + I*c)*log(-I*e^(I*d*x) + e^(-I
*c)) + 462*a^2*e^(3*I*d*x - I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 48*I*a^2*e^
(14*I*d*x + 10*I*c) + 432*I*a^2*e^(12*I*d*x + 8*I*c) + 1840*I*a^2*e^(10*I*d
*x + 6*I*c) + 5936*I*a^2*e^(8*I*d*x + 4*I*c) + 6160*I*a^2*e^(6*I*d*x + 2*I*
c) - 1904*I*a^2*e^(2*I*d*x - 2*I*c) - 112*I*a^2*e^(4*I*d*x) - 112*I*a^2*e^(
-4*I*c))/(d*e^(7*I*d*x + 3*I*c) + 2*d*e^(5*I*d*x + I*c) + d*e^(3*I*d*x - I*
c))
```

Mupad [B] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.94

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{2\,a^2\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2{}\mathrm{i}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)} + \frac{256\,a^2\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-{}\mathrm{i}\right)}{7\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^7} - \frac{8\,a^2\,\left(4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-9{}\mathrm{i}\right)}{3\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^2} - \frac{128\,a^2\,\left(6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-7{}\mathrm{i}\right)}{7\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^6} + \frac{16\,a^2\,\left(8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-15{}\mathrm{i}\right)}{3\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^3} - \frac{32\,a^2\,\left(22\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-35{}\mathrm{i}\right)}{7\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^4} + \frac{32\,a^2\,\left(31\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-42{}\mathrm{i}\right)}{7\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^5}$$

```
[In] int(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] (2*a^2*(tan(c/2 + (d*x)/2) - 2i))/(d*(tan(c/2 + (d*x)/2)^2 + 1)) + (256*a^2
*(tan(c/2 + (d*x)/2) - 1i))/(7*d*(tan(c/2 + (d*x)/2)^2 + 1)^7) - (8*a^2*(4*
tan(c/2 + (d*x)/2) - 9i))/(3*d*(tan(c/2 + (d*x)/2)^2 + 1)^2) - (128*a^2*(6*
tan(c/2 + (d*x)/2) - 7i))/(7*d*(tan(c/2 + (d*x)/2)^2 + 1)^6) + (16*a^2*(8*t
an(c/2 + (d*x)/2) - 15i))/(3*d*(tan(c/2 + (d*x)/2)^2 + 1)^3) - (32*a^2*(22*
tan(c/2 + (d*x)/2) - 35i))/(7*d*(tan(c/2 + (d*x)/2)^2 + 1)^4) + (32*a^2*(31
*tan(c/2 + (d*x)/2) - 42i))/(7*d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

3.35 $\int \cos^9(c+dx)(a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 105

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{7a^2\sin(c+dx)}{9d} - \frac{7a^2\sin^3(c+dx)}{9d} + \frac{7a^2\sin^5(c+dx)}{15d} - \frac{a^2\sin^7(c+dx)}{9d} - \frac{2i\cos^9(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}$$

```
[Out] 7/9*a^2*sin(d*x+c)/d-7/9*a^2*sin(d*x+c)^3/d+7/15*a^2*sin(d*x+c)^5/d-1/9*a^2
*sin(d*x+c)^7/d-2/9*I*cos(d*x+c)^9*(a^2+I*a^2*tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3577, 2713}

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^2\,dx = -\frac{a^2\sin^7(c+dx)}{9d} + \frac{7a^2\sin^5(c+dx)}{15d} - \frac{7a^2\sin^3(c+dx)}{9d} + \frac{7a^2\sin(c+dx)}{9d} - \frac{2i\cos^9(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}$$

```
[In] Int[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (7*a^2*Sin[c + d*x])/(9*d) - (7*a^2*Sin[c + d*x]^3)/(9*d) + (7*a^2*Sin[c +
d*x]^5)/(15*d) - (a^2*Sin[c + d*x]^7)/(9*d) - (((2*I)/9)*Cos[c + d*x]^9*(a^
2 + I*a^2*Tan[c + d*x]))/d
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2i\cos^9(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}+\frac{1}{9}\left(7a^2\right)\int\cos^7(c+dx)\,dx\\
&= -\frac{2i\cos^9(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}\\
&\quad-\frac{(7a^2)\,\text{Subst}\left(\int\left(1-3x^2+3x^4-x^6\right)\,dx,x,-\sin(c+dx)\right)}{9d}\\
&= \frac{7a^2\sin(c+dx)}{9d}-\frac{7a^2\sin^3(c+dx)}{9d}+\frac{7a^2\sin^5(c+dx)}{15d}\\
&\quad-\frac{a^2\sin^7(c+dx)}{9d}-\frac{2i\cos^9(c+dx)\left(a^2+ia^2\tan(c+dx)\right)}{9d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.99

$$\begin{aligned}
\int\cos^9(c+dx)(a+ia\tan(c+dx))^2\,dx &= -\frac{2ia^2\cos^9(c+dx)}{9d}+\frac{a^2\sin(c+dx)}{d}\\
&\quad-\frac{5a^2\sin^3(c+dx)}{3d}+\frac{9a^2\sin^5(c+dx)}{5d}\\
&\quad-\frac{a^2\sin^7(c+dx)}{d}+\frac{2a^2\sin^9(c+dx)}{9d}
\end{aligned}$$

```
[In] Integrate[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (((-2*I)/9)*a^2*Cos[c + d*x]^9)/d + (a^2*Sin[c + d*x])/d - (5*a^2*Sin[c + d
*x]^3)/(3*d) + (9*a^2*Sin[c + d*x]^5)/(5*d) - (a^2*Sin[c + d*x]^7)/d + (2*a
^2*Sin[c + d*x]^9)/(9*d)
```

Maple [A] (verified)

Time = 135.74 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{-a^2\left(-\frac{\left(\cos^8(dx+c)\right)\sin(dx+c)}{9}+\frac{\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{63}\right)-\frac{2ia^2\left(\cos^9(dx+c)\right)}{9}+\frac{a^2\left(\frac{12}{35}\right.}{}}{d}$
default	$\frac{-a^2\left(-\frac{\left(\cos^8(dx+c)\right)\sin(dx+c)}{9}+\frac{\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{63}\right)-\frac{2ia^2\left(\cos^9(dx+c)\right)}{9}+\frac{a^2\left(\frac{12}{35}\right.}{}}{d}$
risch	$-\frac{ia^2 e^{9i(dx+c)}}{1152d}-\frac{ia^2 e^{7i(dx+c)}}{128d}-\frac{7ia^2\cos(dx+c)}{64d}+\frac{7a^2\sin(dx+c)}{16d}-\frac{ia^2\cos(5dx+5c)}{32d}+\frac{11a^2\sin(5dx+5c)}{320d}-\frac{7i}{}$

```
[In] int(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*(-1/9*cos(d*x+c)^8*sin(d*x+c)+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x
+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))-2/9*I*a^2*cos(d*x+c)^9+1/9*a^2*(128/35+
cos(d*x+c)^8+8/7*cos(d*x+c)^6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*
x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.12

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$=\frac{\left(-5i\,a^2e^{(14i\,dx+14i\,c)}-45i\,a^2e^{(12i\,dx+12i\,c)}-189i\,a^2e^{(10i\,dx+10i\,c)}-525i\,a^2e^{(8i\,dx+8i\,c)}-1575i\,a^2e^{(6i\,dx+6i\,c)}+94\right.}{5760\,d}$$

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/5760*(-5*I*a^2*e^(14*I*d*x + 14*I*c) - 45*I*a^2*e^(12*I*d*x + 12*I*c) - 1
89*I*a^2*e^(10*I*d*x + 10*I*c) - 525*I*a^2*e^(8*I*d*x + 8*I*c) - 1575*I*a^2
*e^(6*I*d*x + 6*I*c) + 945*I*a^2*e^(4*I*d*x + 4*I*c) + 105*I*a^2*e^(2*I*d*x
 + 2*I*c) + 9*I*a^2)*e^(-5*I*d*x - 5*I*c)/d
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(94) = 188.

Time = 0.39 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.99

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^2\,dx$$
$$= \begin{cases} \frac{(-126663739519795200ia^2d^7e^{18ic}e^{9idx}-113997365567815680 0ia^2d^7e^{16ic}e^{7idx}-4787889353848258560ia^2d^7e^{14ic}e^{5idx}-13299692649578496 \cdots}{} \\ \frac{x(a^2e^{14ic}+7a^2e^{12ic}+21a^2e^{10ic}+35a^2e^{8ic}+35a^2e^{6ic}+21a^2e^{4ic}+7a^2e^{2ic}+a^2)e^{-5ic}}{128} \end{cases}$$

```
[In] integrate(cos(d*x+c)**9*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise(((-126663739519795200*I*a**2*d**7*exp(18*I*c)*exp(9*I*d*x) - 1139
973655678156800*I*a**2*d**7*exp(16*I*c)*exp(7*I*d*x) - 4787889353848258560*
I*a**2*d**7*exp(14*I*c)*exp(5*I*d*x) - 13299692649578496000*I*a**2*d**7*exp
(12*I*c)*exp(3*I*d*x) - 39899077948735488000*I*a**2*d**7*exp(10*I*c)*exp(I*
d*x) + 23939446769241292800*I*a**2*d**7*exp(8*I*c)*exp(-I*d*x) + 2659938529
915699200*I*a**2*d**7*exp(6*I*c)*exp(-3*I*d*x) + 227994731135631360*I*a**2*
d**7*exp(4*I*c)*exp(-5*I*d*x))*exp(-9*I*c)/(1459166279268040704 00*d**8), Ne
(d**8*exp(9*I*c), 0)), (x*(a**2*exp(14*I*c) + 7*a**2*exp(12*I*c) + 21*a**2*
exp(10*I*c) + 35*a**2*exp(8*I*c) + 35*a**2*exp(6*I*c) + 21*a**2*exp(4*I*c)
+ 7*a**2*exp(2*I*c) + a**2)*exp(-5*I*c)/128, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.13

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^2\,dx =$$
$$-\frac{70i\,a^2\cos{(dx+c)}^9-\left(35\,\sin{(dx+c)}^9-135\,\sin{(dx+c)}^7+189\,\sin{(dx+c)}^5-105\,\sin{(dx+c)}^3\right)a^2 \cdots}{315 \cdots}$$

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/315*(70*I*a^2*cos(d*x + c)^9 - (35*sin(d*x + c)^9 - 135*sin(d*x + c)^7 +
 189*sin(d*x + c)^5 - 105*sin(d*x + c)^3)*a^2 - (35*sin(d*x + c)^9 - 180*si
n(d*x + c)^7 + 378*sin(d*x + c)^5 - 420*sin(d*x + c)^3 + 315*sin(d*x + c))*
a^2)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 669 vs. $2(91) = 182$.

Time = 0.72 (sec) , antiderivative size = 669, normalized size of antiderivative = 6.37

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^2\,dx =$$

$$-\frac{18585\,a^2e^{(9i\,dx+3i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+37170\,a^2e^{(7i\,dx+i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+18585\,a^2e^{(5i\,dx-i\,c)}\log\left(i\right.}{}$$

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/92160*(18585*a^2*e^(9*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) + 1) + 37170*
a^2*e^(7*I*d*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 18585*a^2*e^(5*I*d*x - I
*c)*log(I*e^(I*d*x + I*c) + 1) + 14625*a^2*e^(9*I*d*x + 3*I*c)*log(I*e^(I*d
*x + I*c) - 1) + 29250*a^2*e^(7*I*d*x + I*c)*log(I*e^(I*d*x + I*c) - 1) + 1
4625*a^2*e^(5*I*d*x - I*c)*log(I*e^(I*d*x + I*c) - 1) - 18585*a^2*e^(9*I*d*
x + 3*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 37170*a^2*e^(7*I*d*x + I*c)*log(-I
*e^(I*d*x + I*c) + 1) - 18585*a^2*e^(5*I*d*x - I*c)*log(-I*e^(I*d*x + I*c)
+ 1) - 14625*a^2*e^(9*I*d*x + 3*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 29250*a^
2*e^(7*I*d*x + I*c)*log(-I*e^(I*d*x + I*c) - 1) - 14625*a^2*e^(5*I*d*x - I*
c)*log(-I*e^(I*d*x + I*c) - 1) - 3960*a^2*e^(9*I*d*x + 3*I*c)*log(I*e^(I*d*
x) + e^(-I*c)) - 7920*a^2*e^(7*I*d*x + I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 3
960*a^2*e^(5*I*d*x - I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 3960*a^2*e^(9*I*d*x
 + 3*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 7920*a^2*e^(7*I*d*x + I*c)*log(-I*
e^(I*d*x) + e^(-I*c)) + 3960*a^2*e^(5*I*d*x - I*c)*log(-I*e^(I*d*x) + e^(-I
*c)) + 80*I*a^2*e^(18*I*d*x + 12*I*c) + 880*I*a^2*e^(16*I*d*x + 10*I*c) + 4
544*I*a^2*e^(14*I*d*x + 8*I*c) + 15168*I*a^2*e^(12*I*d*x + 6*I*c) + 45024*I
*a^2*e^(10*I*d*x + 4*I*c) + 43680*I*a^2*e^(8*I*d*x + 2*I*c) - 18624*I*a^2*e
^(4*I*d*x - 2*I*c) - 1968*I*a^2*e^(2*I*d*x - 4*I*c) - 6720*I*a^2*e^(6*I*d*x
) - 144*I*a^2*e^(-6*I*c))/(d*e^(9*I*d*x + 3*I*c) + 2*d*e^(7*I*d*x + I*c) +
d*e^(5*I*d*x - I*c))
```

Mupad [B] (verification not implemented)

Time = 6.13 (sec) , antiderivative size = 330, normalized size of antiderivative = 3.14

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^2\,dx = \frac{2\,a^2\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2{}\mathrm{i}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}+\frac{1024\,a^2\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\mathrm{i}\right)}{9\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^9}-\frac{8\,a^2\,\left(5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-12{}\mathrm{i}\right)}{3\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^2}-\frac{512\,a^2\,\left(8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-9{}\mathrm{i}\right)}{9\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^8}+\frac{128\,a^2\,\left(19\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-24{}\mathrm{i}\right)}{3\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^7}-\frac{64\,a^2\,\left(19\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-35{}\mathrm{i}\right)}{5\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^4}+\frac{56\,a^2\,\left(19\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-40{}\mathrm{i}\right)}{15\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^3}-\frac{128\,a^2\,\left(59\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-84{}\mathrm{i}\right)}{9\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^6}+\frac{32\,a^2\,\left(781\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1260{}\mathrm{i}\right)}{45\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^5}$$

```
[In] int(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] (2*a^2*(tan(c/2 + (d*x)/2) - 2i))/(d*(tan(c/2 + (d*x)/2)^2 + 1)) + (1024*a^
2*(tan(c/2 + (d*x)/2) - 1i))/(9*d*(tan(c/2 + (d*x)/2)^2 + 1)^9) - (8*a^2*(5
*tan(c/2 + (d*x)/2) - 12i))/(3*d*(tan(c/2 + (d*x)/2)^2 + 1)^2) - (512*a^2*(
8*tan(c/2 + (d*x)/2) - 9i))/(9*d*(tan(c/2 + (d*x)/2)^2 + 1)^8) + (128*a^2*(
19*tan(c/2 + (d*x)/2) - 24i))/(3*d*(tan(c/2 + (d*x)/2)^2 + 1)^7) - (64*a^2*
(19*tan(c/2 + (d*x)/2) - 35i))/(5*d*(tan(c/2 + (d*x)/2)^2 + 1)^4) + (56*a^2
*(19*tan(c/2 + (d*x)/2) - 40i))/(15*d*(tan(c/2 + (d*x)/2)^2 + 1)^3) - (128*
a^2*(59*tan(c/2 + (d*x)/2) - 84i))/(9*d*(tan(c/2 + (d*x)/2)^2 + 1)^6) + (32
*a^2*(781*tan(c/2 + (d*x)/2) - 1260i))/(45*d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

3.36 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 109

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^3\,dx=-\frac{8i(a+ia\tan(c+dx))^7}{7a^4d}+\frac{3i(a+ia\tan(c+dx))^8}{2a^5d}-\frac{2i(a+ia\tan(c+dx))^9}{3a^6d}+\frac{i(a+ia\tan(c+dx))^{10}}{10a^7d}$$

```
[Out] -8/7*I*(a+I*a*tan(d*x+c))^7/a^4/d+3/2*I*(a+I*a*tan(d*x+c))^8/a^5/d-2/3*I*(a
+I*a*tan(d*x+c))^9/a^6/d+1/10*I*(a+I*a*tan(d*x+c))^10/a^7/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^3\,dx=\frac{i(a+ia\tan(c+dx))^{10}}{10a^7d}-\frac{2i(a+ia\tan(c+dx))^9}{3a^6d}+\frac{3i(a+ia\tan(c+dx))^8}{2a^5d}-\frac{8i(a+ia\tan(c+dx))^7}{7a^4d}$$

```
[In] Int[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (((-8*I)/7)*(a + I*a*Tan[c + d*x])^7)/(a^4*d) + (((3*I)/2)*(a + I*a*Tan[c +
 d*x])^8)/(a^5*d) - (((2*I)/3)*(a + I*a*Tan[c + d*x])^9)/(a^6*d) + ((I/10)*
(a + I*a*Tan[c + d*x])^10)/(a^7*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)^3(a+x)^6\,dx, x, ia\tan(c+dx)\right)}{a^7d} \\
&= -\frac{i\text{Subst}\left(\int \left(8a^3(a+x)^6-12a^2(a+x)^7+6a(a+x)^8-(a+x)^9\right)\,dx, x, ia\tan(c+dx)\right)}{a^7d} \\
&= -\frac{8i(a+ia\tan(c+dx))^7}{7a^4d}+\frac{3i(a+ia\tan(c+dx))^8}{2a^5d} \\
&\quad -\frac{2i(a+ia\tan(c+dx))^9}{3a^6d}+\frac{i(a+ia\tan(c+dx))^{10}}{10a^7d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.72

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{a^3\sec^9(c+dx)(\cos(7(c+dx))+i\sin(7(c+dx)))(-66i+242i\cos(2(c+dx))+119\sec(c+dx)\sin(3(c+dx))}{840d}$$

```
[In] Integrate[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] -1/840*(a^3*Sec[c + d*x]^9*(Cos[7*(c + d*x)] + I*Sin[7*(c + d*x)])*(-66*I +
 (242*I)*Cos[2*(c + d*x)] + 119*Sec[c + d*x]*Sin[3*(c + d*x)] + 35*Tan[c +
d*x]))/d
```

Maple [A] (verified)

Time = 182.42 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.83

method	result
risch	$\frac{128 i a^{3} \left(210 \,{\mathrm e}^{12 i \left(d x +c \right)}+252 \,{\mathrm e}^{10 i \left(d x +c \right)}+210 \,{\mathrm e}^{8 i \left(d x +c \right)}+120 \,{\mathrm e}^{6 i \left(d x +c \right)}+45 \,{\mathrm e}^{4 i \left(d x +c \right)}+10 \,{\mathrm e}^{2 i \left(d x +c \right)}+1\right)}{105 d \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)^{10}}$
derivativedivides	$\frac{-i a^{3} \left(\frac{\sin^{4}\left(d x +c \right)}{10 \cos \left(d x +c \right)^{10}}+\frac{3 \left(\sin^{4}\left(d x +c \right)\right)}{40 \cos \left(d x +c \right)^{8}}+\frac{\sin^{4}\left(d x +c \right)}{20 \cos \left(d x +c \right)^{6}}+\frac{\sin^{4}\left(d x +c \right)}{40 \cos \left(d x +c \right)^{4}}\right)-3 a^{3} \left(\frac{\sin^{3}\left(d x +c \right)}{9 \cos \left(d x +c \right)^{9}}+\frac{2 \left(\sin^{3}\left(d x +c \right)\right)}{21 \cos \left(d x +c \right)^{7}}+\frac{8 \left(\sin^{3}\left(d x +c \right)\right)}{105 \cos \left(d x +c \right)^{5}}\right.}{d}$
default	$\frac{-i a^{3} \left(\frac{\sin^{4}\left(d x +c \right)}{10 \cos \left(d x +c \right)^{10}}+\frac{3 \left(\sin^{4}\left(d x +c \right)\right)}{40 \cos \left(d x +c \right)^{8}}+\frac{\sin^{4}\left(d x +c \right)}{20 \cos \left(d x +c \right)^{6}}+\frac{\sin^{4}\left(d x +c \right)}{40 \cos \left(d x +c \right)^{4}}\right)-3 a^{3} \left(\frac{\sin^{3}\left(d x +c \right)}{9 \cos \left(d x +c \right)^{9}}+\frac{2 \left(\sin^{3}\left(d x +c \right)\right)}{21 \cos \left(d x +c \right)^{7}}+\frac{8 \left(\sin^{3}\left(d x +c \right)\right)}{105 \cos \left(d x +c \right)^{5}}\right.}{d}$

```
[In] int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 128/105*I*a^3*(210*exp(12*I*(d*x+c))+252*exp(10*I*(d*x+c))+210*exp(8*I*(d*x
+c))+120*exp(6*I*(d*x+c))+45*exp(4*I*(d*x+c))+10*exp(2*I*(d*x+c))+1)/d/(exp
(2*I*(d*x+c))+1)^10
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(85) = 170.

Time = 0.23 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.97

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{128\left(-210i\,a^3e^{(12i\,dx+12i\,c)}-252i\,a^3e^{(10i\,dx+10i\,c)}-210i\,a^3e^{(8i\,dx+8i\,c)}-120i\,a\right.}{105\left(de^{(20i\,dx+20i\,c)}+10\,de^{(18i\,dx+18i\,c)}+45\,de^{(16i\,dx+16i\,c)}+120\,de^{(14i\,dx+14i\,c)}+210\,de^{(12i\,dx+12i\,c)}+252\,de^{(}\right.}$$

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -128/105*(-210*I*a^3*e^(12*I*d*x + 12*I*c) - 252*I*a^3*e^(10*I*d*x + 10*I*c
) - 210*I*a^3*e^(8*I*d*x + 8*I*c) - 120*I*a^3*e^(6*I*d*x + 6*I*c) - 45*I*a^
3*e^(4*I*d*x + 4*I*c) - 10*I*a^3*e^(2*I*d*x + 2*I*c) - I*a^3)/(d*e^(20*I*d*
x + 20*I*c) + 10*d*e^(18*I*d*x + 18*I*c) + 45*d*e^(16*I*d*x + 16*I*c) + 120
*d*e^(14*I*d*x + 14*I*c) + 210*d*e^(12*I*d*x + 12*I*c) + 252*d*e^(10*I*d*x
+ 10*I*c) + 210*d*e^(8*I*d*x + 8*I*c) + 120*d*e^(6*I*d*x + 6*I*c) + 45*d*e^
(4*I*d*x + 4*I*c) + 10*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^3\,dx = -ia^3\left(\int i\sec^8{(c+dx)}\,dx + \int \left(-3\tan{(c+dx)}\sec^8{(c+dx)}\right)\,dx + \int \tan^3{(c+dx)}\sec^8{(c+dx)}\,dx + \int \left(-3i\tan^2{(c+dx)}\sec^8{(c+dx)}\right)\,dx\right)$$

```
[In] integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] -I*a**3*(Integral(I*sec(c + d*x)**8, x) + Integral(-3*tan(c + d*x)*sec(c +
d*x)**8, x) + Integral(tan(c + d*x)**3*sec(c + d*x)**8, x) + Integral(-3*I*
tan(c + d*x)**2*sec(c + d*x)**8, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.99

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{21i\,a^3\tan{(dx+c)}^{10} + 70\,a^3\tan{(dx+c)}^9 + 240\,a^3\tan{(dx+c)}^7 - 210i\,a^3\tan{(dx+c)}^6 + 252\,a^3\tan{(d}}{210\,d}$$

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/210*(21*I*a^3*tan(d*x + c)^10 + 70*a^3*tan(d*x + c)^9 + 240*a^3*tan(d*x
+ c)^7 - 210*I*a^3*tan(d*x + c)^6 + 252*a^3*tan(d*x + c)^5 - 420*I*a^3*tan(
d*x + c)^4 - 315*I*a^3*tan(d*x + c)^2 - 210*a^3*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.60 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.99

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{21i\,a^3\tan{(dx+c)}^{10} + 70\,a^3\tan{(dx+c)}^9 + 240\,a^3\tan{(dx+c)}^7 - 210i\,a^3\tan{(dx+c)}^6 + 252\,a^3\tan{(d}}{210\,d}$$

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/210*(21*I*a^3*tan(d*x + c)^10 + 70*a^3*tan(d*x + c)^9 + 240*a^3*tan(d*x
+ c)^7 - 210*I*a^3*tan(d*x + c)^6 + 252*a^3*tan(d*x + c)^5 - 420*I*a^3*tan(
d*x + c)^4 - 315*I*a^3*tan(d*x + c)^2 - 210*a^3*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 3.80 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.39

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^3\,dx =$$

$$-\frac{a^3\,\sin\left(c+d\,x\right)\,\left(-210\cos\left(c+d\,x\right)^9-\cos\left(c+d\,x\right)^8\,\sin\left(c+d\,x\right)\,315\mathrm{i}-\cos\left(c+d\,x\right)^6\sin\left(c+d\,x\right)^3\,420\mathrm{i}\right.}{}$$

```
[In] int((a + a*tan(c + d*x)*1i)^3/cos(c + d*x)^8,x)
```

```
[Out] -(a^3*sin(c + d*x)*(70*cos(c + d*x)*sin(c + d*x)^8 - cos(c + d*x)^8*sin(c +
 d*x)*315i - 210*cos(c + d*x)^9 + sin(c + d*x)^9*21i + 240*cos(c + d*x)^3*s
in(c + d*x)^6 - cos(c + d*x)^4*sin(c + d*x)^5*210i + 252*cos(c + d*x)^5*sin
(c + d*x)^4 - cos(c + d*x)^6*sin(c + d*x)^3*420i))/(210*d*cos(c + d*x)^10)
```

3.37 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{2i(a+ia\tan(c+dx))^6}{3a^3d} + \frac{4i(a+ia\tan(c+dx))^7}{7a^4d} - \frac{i(a+ia\tan(c+dx))^8}{8a^5d}$$

```
[Out] -2/3*I*(a+I*a*tan(d*x+c))^6/a^3/d+4/7*I*(a+I*a*tan(d*x+c))^7/a^4/d-1/8*I*(a
+I*a*tan(d*x+c))^8/a^5/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{i(a+ia\tan(c+dx))^8}{8a^5d} + \frac{4i(a+ia\tan(c+dx))^7}{7a^4d} - \frac{2i(a+ia\tan(c+dx))^6}{3a^3d}$$

```
[In] Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (((-2*I)/3)*(a + I*a*Tan[c + d*x])^6)/(a^3*d) + (((4*I)/7)*(a + I*a*Tan[c +
 d*x])^7)/(a^4*d) - ((I/8)*(a + I*a*Tan[c + d*x])^8)/(a^5*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)^2(a+x)^5\,dx, x, ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{i\text{Subst}\left(\int \left(4a^2(a+x)^5-4a(a+x)^6+(a+x)^7\right)\,dx, x, ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{2i(a+ia\tan(c+dx))^6}{3a^3d}+\frac{4i(a+ia\tan(c+dx))^7}{7a^4d}-\frac{i(a+ia\tan(c+dx))^8}{8a^5d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.77

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{a^3\sec^8(c+dx)(8+29\cos(2(c+dx))-27i\sin(2(c+dx)))(-i\cos(6(c+dx))+\sin(6(c+dx)))}{168d}$$

```
[In] Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^3,x]
[Out] (a^3*Sec[c + d*x]^8*(8 + 29*Cos[2*(c + d*x)] - (27*I)*Sin[2*(c + d*x)])*((-
I)*Cos[6*(c + d*x)] + Sin[6*(c + d*x)]))/(168*d)
```

Maple [A] (verified)

Time = 64.76 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

method	result
risch	$\frac{32ia^3\left(56\,\mathrm{e}^{10i(dx+c)}+70\,\mathrm{e}^{8i(dx+c)}+56\,\mathrm{e}^{6i(dx+c)}+28\,\mathrm{e}^{4i(dx+c)}+8\,\mathrm{e}^{2i(dx+c)}+1\right)}{21d\left(\mathrm{e}^{2i(dx+c)}+1\right)^8}$
derivativedivides	$\frac{-ia^3\left(\frac{\sin^4(dx+c)}{8\cos(dx+c)^8}+\frac{\sin^4(dx+c)}{12\cos(dx+c)^6}+\frac{\sin^4(dx+c)}{24\cos(dx+c)^4}\right)-3a^3\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8\left(\sin^3(dx+c)\right)}{105\cos(dx+c)^3}\right)+\frac{ia^3}{2\cos(dx+c)^6}}{d}$
default	$\frac{-ia^3\left(\frac{\sin^4(dx+c)}{8\cos(dx+c)^8}+\frac{\sin^4(dx+c)}{12\cos(dx+c)^6}+\frac{\sin^4(dx+c)}{24\cos(dx+c)^4}\right)-3a^3\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8\left(\sin^3(dx+c)\right)}{105\cos(dx+c)^3}\right)+\frac{ia^3}{2\cos(dx+c)^6}}{d}$

```
[In] int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 32/21*I*a^3*(56*exp(10*I*(d*x+c))+70*exp(8*I*(d*x+c))+56*exp(6*I*(d*x+c))+2
8*exp(4*I*(d*x+c))+8*exp(2*I*(d*x+c))+1)/d/(exp(2*I*(d*x+c))+1)^8
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(64) = 128$.

Time = 0.24 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.16

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{32\left(-56i\,a^3e^{(10i\,dx+10i\,c)}-70i\,a^3e^{(8i\,dx+8i\,c)}-56i\,a^3e^{(6i\,dx+6i\,c)}-28i\,a^3e^{(4i\,dx+4i\,c)}-8i\right.}{21\left(de^{(16i\,dx+16i\,c)}+8\,de^{(14i\,dx+14i\,c)}+28\,de^{(12i\,dx+12i\,c)}+56\,de^{(10i\,dx+10i\,c)}+70\,de^{(8i\,dx+8i\,c)}+56\,de^{(6i\,dx+6i}\right.}$$

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -32/21*(-56*I*a^3*e^(10*I*d*x + 10*I*c) - 70*I*a^3*e^(8*I*d*x + 8*I*c) - 56
*I*a^3*e^(6*I*d*x + 6*I*c) - 28*I*a^3*e^(4*I*d*x + 4*I*c) - 8*I*a^3*e^(2*I*
d*x + 2*I*c) - I*a^3)/(d*e^(16*I*d*x + 16*I*c) + 8*d*e^(14*I*d*x + 14*I*c)
+ 28*d*e^(12*I*d*x + 12*I*c) + 56*d*e^(10*I*d*x + 10*I*c) + 70*d*e^(8*I*d*x
 + 8*I*c) + 56*d*e^(6*I*d*x + 6*I*c) + 28*d*e^(4*I*d*x + 4*I*c) + 8*d*e^(2*
I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^3\,dx = -ia^3\left(\int i\sec^6(c+dx)\,dx + \int \left(-3\tan(c+dx)\sec^6(c+dx)\right)\,dx + \int \tan^3(c+dx)\sec^6(c+dx)\,dx + \int \left(-3i\tan^2(c+dx)\sec^6(c+dx)\right)\,dx\right)$$

```
[In] integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] -I*a**3*(Integral(I*sec(c + d*x)**6, x) + Integral(-3*tan(c + d*x)*sec(c +
d*x)**6, x) + Integral(tan(c + d*x)**3*sec(c + d*x)**6, x) + Integral(-3*I*
tan(c + d*x)**2*sec(c + d*x)**6, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.32

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{21i\,a^3\tan(dx+c)^8 + 72\,a^3\tan(dx+c)^7 - 28i\,a^3\tan(dx+c)^6 + 168\,a^3\tan(dx+c)^5 - 210i\,a^3\tan(dx}{168\,d}$$

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/168*(21*I*a^3*tan(d*x + c)^8 + 72*a^3*tan(d*x + c)^7 - 28*I*a^3*tan(d*x
+ c)^6 + 168*a^3*tan(d*x + c)^5 - 210*I*a^3*tan(d*x + c)^4 + 56*a^3*tan(d*x
 + c)^3 - 252*I*a^3*tan(d*x + c)^2 - 168*a^3*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.54 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.32

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{21i\,a^3\tan(dx+c)^8 + 72\,a^3\tan(dx+c)^7 - 28i\,a^3\tan(dx+c)^6 + 168\,a^3\tan(dx+c)^5 - 210i\,a^3\tan(dx}{168\,d}$$

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/168*(21*I*a^3*tan(d*x + c)^8 + 72*a^3*tan(d*x + c)^7 - 28*I*a^3*tan(d*x
+ c)^6 + 168*a^3*tan(d*x + c)^5 - 210*I*a^3*tan(d*x + c)^4 + 56*a^3*tan(d*x
 + c)^3 - 252*I*a^3*tan(d*x + c)^2 - 168*a^3*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 4.17 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.84

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{a^3\sin(c+d\,x)\,\left(-168\cos(c+d\,x)^7-\cos(c+d\,x)^6\,\sin(c+d\,x)\,252\mathrm{i}+56\cos(c+d\,x)^5\sin(c+d\,x)^2\right.}{}$$

```
[In] int((a + a*tan(c + d*x)*1i)^3/cos(c + d*x)^6,x)
```

```
[Out] -(a^3*sin(c + d*x)*(72*cos(c + d*x)*sin(c + d*x)^6 - cos(c + d*x)^6*sin(c +
 d*x)*252i - 168*cos(c + d*x)^7 + sin(c + d*x)^7*21i - cos(c + d*x)^2*sin(c
 + d*x)^5*28i + 168*cos(c + d*x)^3*sin(c + d*x)^4 - cos(c + d*x)^4*sin(c +
d*x)^3*210i + 56*cos(c + d*x)^5*sin(c + d*x)^2))/(168*d*cos(c + d*x)^8)
```

3.38 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^3\,dx=-\frac{2i(a+ia\tan(c+dx))^5}{5a^2d}+\frac{i(a+ia\tan(c+dx))^6}{6a^3d}$$

```
[Out] -2/5*I*(a+I*a*tan(d*x+c))^5/a^2/d+1/6*I*(a+I*a*tan(d*x+c))^6/a^3/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^3\,dx=\frac{i(a+ia\tan(c+dx))^6}{6a^3d}-\frac{2i(a+ia\tan(c+dx))^5}{5a^2d}$$

```
[In] Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (((-2*I)/5)*(a + I*a*Tan[c + d*x])^5)/(a^2*d) + ((I/6)*(a + I*a*Tan[c + d*x
])^6)/(a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
```

```
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)(a+x)^4\,dx, x, ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{i\text{Subst}\left(\int \left(2a(a+x)^4-(a+x)^5\right)\,dx, x, ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{2i(a+ia\tan(c+dx))^5}{5a^2d}+\frac{i(a+ia\tan(c+dx))^6}{6a^3d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.62

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{a^3(7-5i\tan(c+dx))(-i+\tan(c+dx))^5}{30d}$$

```
[In] Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^3,x]
[Out] (a^3*(7 - (5*I)*Tan[c + d*x])*(-I + Tan[c + d*x])^5)/(30*d)
```

Maple [A] (verified)

Time = 19.53 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.25

method	result
risch	$\frac{32ia^3\left(15\,e^{8i(dx+c)}+20\,e^{6i(dx+c)}+15\,e^{4i(dx+c)}+6\,e^{2i(dx+c)}+1\right)}{15d\left(e^{2i(dx+c)}+1\right)^6}$
derivativedivides	$\frac{-ia^3\left(\frac{\sin^4(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^4(dx+c)}{12\cos(dx+c)^4}\right)-3a^3\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)+\frac{3ia^3}{4\cos(dx+c)^4}-a^3\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(}{d}$
default	$\frac{-ia^3\left(\frac{\sin^4(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^4(dx+c)}{12\cos(dx+c)^4}\right)-3a^3\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)+\frac{3ia^3}{4\cos(dx+c)^4}-a^3\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(}{d}$

```
[In] int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 32/15*I*a^3*(15*exp(8*I*(d*x+c))+20*exp(6*I*(d*x+c))+15*exp(4*I*(d*x+c))+6*
exp(2*I*(d*x+c))+1)/d/(exp(2*I*(d*x+c))+1)^6
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(43) = 86.

Time = 0.23 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.53

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{32\left(-15i\,a^3e^{(8i\,dx+8i\,c)}-20i\,a^3e^{(6i\,dx+6i\,c)}-15i\,a^3e^{(4i\,dx+4i\,c)}-6i\,a^3e^{(2i\,dx+2i\,c)}-i\,a^3\right)}{15\left(de^{(12i\,dx+12i\,c)}+6\,de^{(10i\,dx+10i\,c)}+15\,de^{(8i\,dx+8i\,c)}+20\,de^{(6i\,dx+6i\,c)}+15\,de^{(4i\,dx+4i\,c)}+6\,de^{(2i\,dx+2i\,c)}+d\right)}$$

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -32/15*(-15*I*a^3*e^(8*I*d*x + 8*I*c) - 20*I*a^3*e^(6*I*d*x + 6*I*c) - 15*I
*a^3*e^(4*I*d*x + 4*I*c) - 6*I*a^3*e^(2*I*d*x + 2*I*c) - I*a^3)/(d*e^(12*I*
d*x + 12*I*c) + 6*d*e^(10*I*d*x + 10*I*c) + 15*d*e^(8*I*d*x + 8*I*c) + 20*d
*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*I*d*x + 4*I*c) + 6*d*e^(2*I*d*x + 2*I*c) +
 d)
```

Sympy [F]

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^3\,dx = -ia^3\left(\int i\sec^4(c+dx)\,dx + \int\left(-3\tan(c+dx)\sec^4(c+dx)\right)dx + \int\tan^3(c+dx)\sec^4(c+dx)\,dx + \int\left(-3i\tan^2(c+dx)\sec^4(c+dx)\right)dx\right)$$

```
[In] integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] -I*a**3*(Integral(I*sec(c + d*x)**4, x) + Integral(-3*tan(c + d*x)*sec(c +
d*x)**4, x) + Integral(tan(c + d*x)**3*sec(c + d*x)**4, x) + Integral(-3*I*
tan(c + d*x)**2*sec(c + d*x)**4, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.51 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.49

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^3\,dx =$$
$$-\frac{5i\,a^3\tan\left(dx+c\right)^6+18\,a^3\tan\left(dx+c\right)^5-15i\,a^3\tan\left(dx+c\right)^4+20\,a^3\tan\left(dx+c\right)^3-45i\,a^3\tan\left(dx+\right.}{30\,d}$$

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/30*(5*I*a^3*tan(d*x + c)^6 + 18*a^3*tan(d*x + c)^5 - 15*I*a^3*tan(d*x +
c)^4 + 20*a^3*tan(d*x + c)^3 - 45*I*a^3*tan(d*x + c)^2 - 30*a^3*tan(d*x + c
))/d
```

Giac [A] (verification not implemented)

none

Time = 0.54 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.49

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^3\,dx =$$
$$-\frac{5i\,a^3\tan\left(dx+c\right)^6+18\,a^3\tan\left(dx+c\right)^5-15i\,a^3\tan\left(dx+c\right)^4+20\,a^3\tan\left(dx+c\right)^3-45i\,a^3\tan\left(dx+\right.}{30\,d}$$

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/30*(5*I*a^3*tan(d*x + c)^6 + 18*a^3*tan(d*x + c)^5 - 15*I*a^3*tan(d*x +
c)^4 + 20*a^3*tan(d*x + c)^3 - 45*I*a^3*tan(d*x + c)^2 - 30*a^3*tan(d*x + c
))/d
```

Mupad [B] (verification not implemented)

Time = 3.87 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.07

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^3\,dx =$$
$$-\frac{a^3\,\sin\left(c+d\,x\right)\,\left(-30\cos\left(c+d\,x\right)^5-\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)\,45\mathrm{i}+20\cos\left(c+d\,x\right)^3\sin\left(c+d\,x\right)^2-\right.}{30\,d\cos\left(c+d\,x\right)^6}$$

```
[In] int((a + a*tan(c + d*x)*1i)^3/cos(c + d*x)^4,x)
```

```
[Out] -(a^3*sin(c + d*x)*(18*cos(c + d*x)*sin(c + d*x)^4 - cos(c + d*x)^4*sin(c +
 d*x)*45i - 30*cos(c + d*x)^5 + sin(c + d*x)^5*5i - cos(c + d*x)^2*sin(c +
d*x)^3*15i + 20*cos(c + d*x)^3*sin(c + d*x)^2))/(30*d*cos(c + d*x)^6)
```

3.39 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^3\,dx=-\frac{i(a+ia\tan(c+dx))^4}{4ad}$$

```
[Out] -1/4*I*(a+I*a*tan(d*x+c))^4/a/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 32}

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^3\,dx=-\frac{i(a+ia\tan(c+dx))^4}{4ad}$$

```
[In] Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((-1/4*I)*(a + I*a*Tan[c + d*x])^4)/(a*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{i\text{Subst}\left(\int (a+x)^3\,dx, x, ia\tan(c+dx)\right)}{ad}$$
$$= -\frac{i(a+ia\tan(c+dx))^4}{4ad}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.85

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$= \frac{a^3\tan(c+dx)\left(4+6i\tan(c+dx)-4\tan^2(c+dx)-i\tan^3(c+dx)\right)}{4d}$$

```
[In] Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^3,x]
[Out] (a^3*Tan[c + d*x]*(4 + (6*I)*Tan[c + d*x] - 4*Tan[c + d*x]^2 - I*Tan[c + d*
x]^3))/(4*d)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(23) = 46.

Time = 4.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

method	result	size
risch	$\frac{4ia^3\left(4\,\mathrm{e}^{6i(dx+c)}+6\,\mathrm{e}^{4i(dx+c)}+4\,\mathrm{e}^{2i(dx+c)}+1\right)}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^4}$	58
derivativedivides	$\frac{-\frac{ia^3\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}-\frac{a^3\left(\sin^3(dx+c)\right)}{\cos(dx+c)^3}+\frac{3ia^3}{2\cos(dx+c)^2}+a^3\tan(dx+c)}{d}$	73
default	$\frac{-\frac{ia^3\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}-\frac{a^3\left(\sin^3(dx+c)\right)}{\cos(dx+c)^3}+\frac{3ia^3}{2\cos(dx+c)^2}+a^3\tan(dx+c)}{d}$	73

```
[In] int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 4*I*a^3*(4*exp(6*I*(d*x+c))+6*exp(4*I*(d*x+c))+4*exp(2*I*(d*x+c))+1)/d/(exp
(2*I*(d*x+c))+1)^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(21) = 42.

Time = 0.23 (sec) , antiderivative size = 101, normalized size of antiderivative = 3.74

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{4\left(-4i\,a^3e^{(6i\,dx+6i\,c)}-6i\,a^3e^{(4i\,dx+4i\,c)}-4i\,a^3e^{(2i\,dx+2i\,c)}-i\,a^3\right)}{de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}+4\,de^{(2i\,dx+2i\,c)}+d}$$

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -4*(-4*I*a^3*e^(6*I*d*x + 6*I*c) - 6*I*a^3*e^(4*I*d*x + 4*I*c) - 4*I*a^3*e^
(2*I*d*x + 2*I*c) - I*a^3)/(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c)
 + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^3\,dx = -ia^3\left(\int i\sec^2(c+dx)\,dx + \int \left(-3\tan(c+dx)\sec^2(c+dx)\right)\,dx + \int \tan^3(c+dx)\sec^2(c+dx)\,dx + \int \left(-3i\tan^2(c+dx)\sec^2(c+dx)\right)\,dx\right)$$

```
[In] integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] -I*a**3*(Integral(I*sec(c + d*x)**2, x) + Integral(-3*tan(c + d*x)*sec(c +
d*x)**2, x) + Integral(tan(c + d*x)**3*sec(c + d*x)**2, x) + Integral(-3*I*
tan(c + d*x)**2*sec(c + d*x)**2, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{i\left(i\,a\tan(dx+c)+a\right)^4}{4\,ad}$$

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*I*(I*a*tan(d*x + c) + a)^4/(a*d)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(21) = 42$.

Time = 0.49 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.07

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{i\,a^3\tan\left(dx+c\right)^4+4\,a^3\tan\left(dx+c\right)^3-6i\,a^3\tan\left(dx+c\right)^2-4\,a^3\tan\left(dx+c\right)}{4\,d}$$

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/4*(I*a^3*tan(d*x + c)^4 + 4*a^3*tan(d*x + c)^3 - 6*I*a^3*tan(d*x + c)^2
- 4*a^3*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 3.85 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.07

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{-\frac{a^3\tan(c+d\,x)^4\,1\mathrm{i}}{4}-a^3\tan(c+d\,x)^3+\frac{a^3\tan(c+d\,x)^2\,3\mathrm{i}}{2}+a^3\tan(c+d\,x)}{d}$$

```
[In] int((a + a*tan(c + d*x)*1i)^3/cos(c + d*x)^2,x)
```

```
[Out] (a^3*tan(c + d*x) + (a^3*tan(c + d*x)^2*3i)/2 - a^3*tan(c + d*x)^3 - (a^3*t
an(c + d*x)^4*1i)/4)/d
```

3.40 $\int (a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 15, antiderivative size = 63

$$\int (a+ia\tan(c+dx))^3\,dx = 4a^3x - \frac{4ia^3\log(\cos(c+dx))}{d} - \frac{2a^3\tan(c+dx)}{d} + \frac{ia(a+ia\tan(c+dx))^2}{2d}$$

```
[Out] 4*a^3*x-4*I*a^3*ln(cos(d*x+c))/d-2*a^3*tan(d*x+c)/d+1/2*I*a*(a+I*a*tan(d*x+
c))^2/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3559, 3558, 3556}

$$\int (a+ia\tan(c+dx))^3\,dx = -\frac{2a^3\tan(c+dx)}{d} - \frac{4ia^3\log(\cos(c+dx))}{d} + 4a^3x + \frac{ia(a+ia\tan(c+dx))^2}{2d}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] 4*a^3*x - ((4*I)*a^3*Log[Cos[c + d*x]])/d - (2*a^3*Tan[c + d*x])/d + ((I/2)
*a*(a + I*a*Tan[c + d*x])^2)/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3558

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[b^2*(Tan[c + d*x]/d),
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{ia(a+ia\tan(c+dx))^2}{2d} + (2a)\int (a+ia\tan(c+dx))^2\,dx \\
&= 4a^3x - \frac{2a^3\tan(c+dx)}{d} + \frac{ia(a+ia\tan(c+dx))^2}{2d} + \left(4ia^3\right)\int \tan(c+dx)\,dx \\
&= 4a^3x - \frac{4ia^3\log(\cos(c+dx))}{d} - \frac{2a^3\tan(c+dx)}{d} + \frac{ia(a+ia\tan(c+dx))^2}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.73

$$\int (a+ia\tan(c+dx))^3\,dx = \frac{ia^3(8\log(i+\tan(c+dx)) + 6i\tan(c+dx) - \tan^2(c+dx))}{2d}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^3,x]
[Out] ((I/2)*a^3*(8*Log[I + Tan[c + d*x]] + (6*I)*Tan[c + d*x] - Tan[c + d*x]^2))
/d
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{a^3\left(-3\tan(dx+c)-\frac{i\left(\tan^2(dx+c)\right)}{2}+2i\ln\left(1+\tan^2(dx+c)\right)+4\arctan(\tan(dx+c))\right)}{d}$
default	$\frac{a^3\left(-3\tan(dx+c)-\frac{i\left(\tan^2(dx+c)\right)}{2}+2i\ln\left(1+\tan^2(dx+c)\right)+4\arctan(\tan(dx+c))\right)}{d}$
parallelrisch	$\frac{-ia^3(\tan^2(dx+c))+4ia^3\ln(1+\tan^2(dx+c))+8a^3dx-6a^3\tan(dx+c)}{2d}$
norman	$4a^3x-\frac{3a^3\tan(dx+c)}{d}-\frac{ia^3(\tan^2(dx+c))}{2d}+\frac{2ia^3\ln(1+\tan^2(dx+c))}{d}$
risch	$-\frac{8a^3c}{d}-\frac{2ia^3(4\,e^{2i(dx+c)}+3)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{4ia^3\ln(e^{2i(dx+c)}+1)}{d}$
parts	$a^3x-\frac{ia^3\left(\frac{\left(\tan^2(dx+c)\right)}{2}-\frac{\ln\left(1+\tan^2(dx+c)\right)}{2}\right)}{d}+\frac{3ia^3\ln(1+\tan^2(dx+c))}{2d}-\frac{3a^3(\tan(dx+c)-\arctan(\tan(dx+c)))}{d}$

```
[In] int((a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a^3*(-3*tan(d*x+c)-1/2*I*tan(d*x+c)^2+2*I*ln(1+tan(d*x+c)^2)+4*arctan(t
an(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.54

$$\int (a+ia\tan(c+dx))^3\,dx = -\frac{2\left(4i\,a^3e^{(2i\,dx+2i\,c)}+3i\,a^3+2\left(i\,a^3e^{(4i\,dx+4i\,c)}+2i\,a^3e^{(2i\,dx+2i\,c)}+i\,a^3\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)\right)}{de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d}$$

```
[In] integrate((a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -2*(4*I*a^3*e^(2*I*d*x + 2*I*c) + 3*I*a^3 + 2*(I*a^3*e^(4*I*d*x + 4*I*c) +
2*I*a^3*e^(2*I*d*x + 2*I*c) + I*a^3)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(4*
I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.49

$$\int (a+ia\tan(c+dx))^3\,dx = -\frac{4ia^3\log\left(e^{2idx}+e^{-2ic}\right)}{d}+\frac{-8ia^3e^{2ic}e^{2idx}-6ia^3}{de^{4ic}e^{4idx}+2de^{2ic}e^{2idx}+d}$$

```
[In] integrate((a+I*a*tan(d*x+c))**3,x)
[Out] -4*I*a**3*log(exp(2*I*d*x) + exp(-2*I*c))/d + (-8*I*a**3*exp(2*I*c)*exp(2*I
*d*x) - 6*I*a**3)/(d*exp(4*I*c)*exp(4*I*d*x) + 2*d*exp(2*I*c)*exp(2*I*d*x)
+ d)
```

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.21

$$\int (a+ia\tan(c+dx))^3\,dx = a^3x+\frac{3\,(dx+c-\tan{(dx+c)})a^3}{d} + \frac{i\,a^3\left(\frac{1}{\sin(dx+c)^2-1}-\log\left(\sin{(dx+c)}^2-1\right)\right)}{2\,d} + \frac{3i\,a^3\log{(\sec{(dx+c)})}}{d}$$

```
[In] integrate((a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
[Out] a^3*x + 3*(d*x + c - tan(d*x + c))*a^3/d + 1/2*I*a^3*(1/(sin(d*x + c)^2 - 1
) - log(sin(d*x + c)^2 - 1))/d + 3*I*a^3*log(sec(d*x + c))/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(55) = 110.

Time = 0.40 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.87

$$\int (a+ia\tan(c+dx))^3\,dx = -\frac{2\left(2i\,a^3e^{(4i\,dx+4i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+4i\,a^3e^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+4i\,a^3e^{(2i\,dx+2i\,c)}+2i\,a^3\log\right.}{de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d}$$

```
[In] integrate((a+I*a*tan(d*x+c))^3,x, algorithm="giac")
[Out] -2*(2*I*a^3*e^(4*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 4*I*a^3*e^(2
*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 4*I*a^3*e^(2*I*d*x + 2*I*c)
+ 2*I*a^3*log(e^(2*I*d*x + 2*I*c) + 1) + 3*I*a^3)/(d*e^(4*I*d*x + 4*I*c) +
2*d*e^(2*I*d*x + 2*I*c) + d)
```

Mupad [B] (verification not implemented)

Time = 4.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.65

$$\int (a+ia\tan(c+dx))^3\,dx = -\frac{a^3\left(6\tan(c+d\,x)-\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,8\mathrm{i}+\tan(c+d\,x)^2\,1\mathrm{i}\right)}{2\,d}$$

```
[In] int((a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] -(a^3*(6*tan(c + d*x) - log(tan(c + d*x) + 1i)*8i + tan(c + d*x)^2*1i))/(2*
d)
```

3.41 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 49

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^3\,dx=-a^3x+\frac{ia^3\log(\cos(c+dx))}{d}-\frac{2ia^4}{d(a-ia\tan(c+dx))}$$

```
[Out] -a^3*x+I*a^3*ln(cos(d*x+c))/d-2*I*a^4/d/(a-I*a*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^3\,dx=-\frac{2ia^4}{d(a-ia\tan(c+dx))}+\frac{ia^3\log(\cos(c+dx))}{d}-a^3x$$

```
[In] Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] -(a^3*x) + (I*a^3*Log[Cos[c + d*x]])/d - ((2*I)*a^4)/(d*(a - I*a*Tan[c + d*
x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
```

```
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(ia^3)\,\text{Subst}\left(\int \frac{a+x}{(a-x)^2}\,dx, x, ia\tan(c+dx)\right)}{d}\\ &= -\frac{(ia^3)\,\text{Subst}\left(\int \left(\frac{2a}{(a-x)^2}+\frac{1}{-a+x}\right)\,dx, x, ia\tan(c+dx)\right)}{d}\\ &= -a^3x+\frac{ia^3\log(\cos(c+dx))}{d}-\frac{2ia^4}{d(a-ia\tan(c+dx))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{ia^3\left(\log(i+\tan(c+dx))+\frac{2a}{a-ia\tan(c+dx)}\right)}{d}$$

```
[In] Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^3,x]
[Out] ((-I)*a^3*(Log[I + Tan[c + d*x]] + (2*a)/(a - I*a*Tan[c + d*x])))/d
```

Maple [A] (verified)

Time = 2.85 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

method	result
risch	$-\frac{ia^3 e^{2i(dx+c)}}{d}+\frac{2a^3c}{d}+\frac{ia^3\ln\left(e^{2i(dx+c)}+1\right)}{d}$
derivativedivides	$\frac{-ia^3\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)-3a^3\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\frac{3ia^3\left(\cos^2(dx+c)\right)}{2}+a^3\left(\frac{\sin(dx+c)\cos(dx+c)}{2}\right.}{d}$
default	$\frac{-ia^3\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)-3a^3\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\frac{3ia^3\left(\cos^2(dx+c)\right)}{2}+a^3\left(\frac{\sin(dx+c)\cos(dx+c)}{2}\right.}{d}$

```
[In] int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] -I/d*a^3*exp(2*I*(d*x+c))+2/d*a^3*c+I/d*a^3*ln(exp(2*I*(d*x+c))+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{-i\,a^3e^{(2i\,dx+2i\,c)}+i\,a^3\log\left(e^{(2i\,dx+2i\,c)}+1\right)}{d}$$

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
[Out] (-I*a^3*e^(2*I*d*x + 2*I*c) + I*a^3*log(e^(2*I*d*x + 2*I*c) + 1))/d
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.24

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{ia^3\log\left(e^{2idx}+e^{-2ic}\right)}{d} + \begin{cases} -\frac{ia^3e^{2ic}e^{2idx}}{d} & \text{for } d\neq 0 \\ 2a^3xe^{2ic} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**3,x)
[Out] I*a**3*log(exp(2*I*d*x) + exp(-2*I*c))/d + Piecewise((-I*a**3*exp(2*I*c)*ex
p(2*I*d*x)/d, Ne(d, 0)), (2*a**3*x*exp(2*I*c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.27

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{2\,(dx+c)a^3+i\,a^3\log\left(\tan\left(dx+c\right)^2+1\right)-\frac{4\,(a^3\tan(dx+c)-i\,a^3)}{\tan(dx+c)^2+1}}{2\,d}$$

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
[Out] -1/2*(2*(d*x + c)*a^3 + I*a^3*log(tan(d*x + c)^2 + 1) - 4*(a^3*tan(d*x + c)
 - I*a^3)/(tan(d*x + c)^2 + 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.61 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{-i\,a^3e^{(2i\,dx+2i\,c)}+i\,a^3\log\left(e^{(2i\,dx+2i\,c)}+1\right)}{d}$$

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
[Out] (-I*a^3*e^(2*I*d*x + 2*I*c) + I*a^3*log(e^(2*I*d*x + 2*I*c) + 1))/d
```

Mupad [B] (verification not implemented)

Time = 3.64 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{2\,a^3}{d\,\left(\tan\left(c+d\,x\right)+1\mathrm{i}\right)}-\frac{a^3\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{d}$$

```
[In] int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^3,x)
[Out] (2*a^3)/(d*(tan(c + d*x) + 1i)) - (a^3*log(tan(c + d*x) + 1i)*1i)/d
```

3.42 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^3\,dx=-\frac{ia^5}{2d(a-ia\tan(c+dx))^2}$$

```
[Out] -1/2*I*a^5/d/(a-I*a*tan(d*x+c))^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 32}

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^3\,dx=-\frac{ia^5}{2d(a-ia\tan(c+dx))^2}$$

```
[In] Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^3,x]
[Out] ((-1/2*I)*a^5)/(d*(a - I*a*Tan[c + d*x])^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && 
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{(ia^5)\,\text{Subst}\left(\int \frac{1}{(a-x)^3}\,dx, x, ia\tan(c+dx)\right)}{d}$$
$$= -\frac{ia^5}{2d(a-ia\tan(c+dx))^2}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{ia^3}{2d(i+\tan(c+dx))^2}$$

[In] Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^3,x]

[Out] ((I/2)*a^3)/(d*(I + Tan[c + d*x])^2)

Maple [A] (verified)

Time = 13.76 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

method	result
risch	$-\frac{ia^3 e^{4i(dx+c)}}{8d}-\frac{ia^3 e^{2i(dx+c)}}{4d}$
derivativedivides	$\frac{-\frac{ia^3\left(\sin^4(dx+c)\right)}{4}-3a^3\left(-\frac{\left(\cos^3(dx+c)\right)\sin(dx+c)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{3ia^3\left(\cos^4(dx+c)\right)}{4}+a^3\left(\frac{\left(\cos^3(dx+c)+\frac{3}{}\right.}{}\right.}{d}$
default	$\frac{-\frac{ia^3\left(\sin^4(dx+c)\right)}{4}-3a^3\left(-\frac{\left(\cos^3(dx+c)\right)\sin(dx+c)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{3ia^3\left(\cos^4(dx+c)\right)}{4}+a^3\left(\frac{\left(\cos^3(dx+c)+\frac{3}{}\right.}{}\right.}{d}$

[In] int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] -1/8*I/d*a^3*exp(4*I*(d*x+c))-1/4*I/d*a^3*exp(2*I*(d*x+c))

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{-i\,a^3e^{(4i\,dx+4i\,c)} - 2i\,a^3e^{(2i\,dx+2i\,c)}}{8\,d}$$

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(-I*a^3*e^(4*I*d*x + 4*I*c) - 2*I*a^3*e^(2*I*d*x + 2*I*c))/d
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(22) = 44.

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.96

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^3\,dx = \begin{cases} \frac{-4ia^3de^{4ic}e^{4idx}-8ia^3de^{2ic}e^{2idx}}{32d^2} & \text{for } d^2 \neq 0 \\ x\left(\frac{a^3e^{4ic}}{2}+\frac{a^3e^{2ic}}{2}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise(((-4*I*a**3*d*exp(4*I*c)*exp(4*I*d*x) - 8*I*a**3*d*exp(2*I*c)*exp
(2*I*d*x))/(32*d**2), Ne(d**2, 0)), (x*(a**3*exp(4*I*c)/2 + a**3*exp(2*I*c)
/2), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(21) = 42.

Time = 0.37 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.11

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{-i\,a^3\tan\left(dx+c\right)^2 - 2\,a^3\tan\left(dx+c\right) + i\,a^3}{2\,\left(\tan\left(dx+c\right)^4 + 2\,\tan\left(dx+c\right)^2 + 1\right)d}$$

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(-I*a^3*tan(d*x + c)^2 - 2*a^3*tan(d*x + c) + I*a^3)/((tan(d*x + c)^4
+ 2*tan(d*x + c)^2 + 1)*d)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(21) = 42$.

Time = 0.71 (sec) , antiderivative size = 135, normalized size of antiderivative = 5.00

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{i\,a^3e^{(12i\,dx+8i\,c)}+6i\,a^3e^{(10i\,dx+6i\,c)}+14i\,a^3e^{(8i\,dx+4i\,c)}+16i\,a^3e^{(6i\,dx+2i\,c)}+2i\,a^3e^{(2i\,dx-2i\,c)}+9i\,a^3e^{(4i\,dx)}}{8\left(de^{(8i\,dx+4i\,c)}+4\,de^{(6i\,dx+2i\,c)}+4\,de^{(2i\,dx-2i\,c)}+6\,de^{(4i\,dx)}+de^{(-4i\,c)}\right)}$$

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(I*a^3*e^(12*I*d*x + 8*I*c) + 6*I*a^3*e^(10*I*d*x + 6*I*c) + 14*I*a^3*
e^(8*I*d*x + 4*I*c) + 16*I*a^3*e^(6*I*d*x + 2*I*c) + 2*I*a^3*e^(2*I*d*x - 2
*I*c) + 9*I*a^3*e^(4*I*d*x))/(d*e^(8*I*d*x + 4*I*c) + 4*d*e^(6*I*d*x + 2*I*
c) + 4*d*e^(2*I*d*x - 2*I*c) + 6*d*e^(4*I*d*x) + d*e^(-4*I*c))
```

Mupad [B] (verification not implemented)

Time = 3.94 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{a^3\left(\frac{e^{c\,2i+d\,x\,2i}}{2}+\frac{e^{c\,4i+d\,x\,4i}}{4}\right)\,1i}{2\,d}$$

```
[In] int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] -(a^3*(exp(c*2i + d*x*2i)/2 + exp(c*4i + d*x*4i)/4)*1i)/(2*d)
```

3.43 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 90

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^3\,dx=\frac{a^3x}{8}-\frac{ia^6}{6d(a-ia\tan(c+dx))^3}-\frac{ia^5}{8d(a-ia\tan(c+dx))^2}-\frac{ia^4}{8d(a-ia\tan(c+dx))}$$

```
[Out] 1/8*a^3*x-1/6*I*a^6/d/(a-I*a*tan(d*x+c))^3-1/8*I*a^5/d/(a-I*a*tan(d*x+c))^2
-1/8*I*a^4/d/(a-I*a*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3568, 46, 212}

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^3\,dx=-\frac{ia^6}{6d(a-ia\tan(c+dx))^3}-\frac{ia^5}{8d(a-ia\tan(c+dx))^2}-\frac{ia^4}{8d(a-ia\tan(c+dx))}+\frac{a^3x}{8}$$

```
[In] Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*x)/8 - ((I/6)*a^6)/(d*(a - I*a*Tan[c + d*x])^3) - ((I/8)*a^5)/(d*(a -
I*a*Tan[c + d*x])^2) - ((I/8)*a^4)/(d*(a - I*a*Tan[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(ia^7)\,\text{Subst}\left(\int \frac{1}{(a-x)^4(a+x)}\,dx, x, ia\tan(c+dx)\right)}{d}\\
&= -\frac{(ia^7)\,\text{Subst}\left(\int \left(\frac{1}{2a(a-x)^4}+\frac{1}{4a^2(a-x)^3}+\frac{1}{8a^3(a-x)^2}+\frac{1}{8a^3(a^2-x^2)}\right)\,dx, x, ia\tan(c+dx)\right)}{d}\\
&= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3}-\frac{ia^5}{8d(a-ia\tan(c+dx))^2}\\
&\quad -\frac{ia^4}{8d(a-ia\tan(c+dx))}-\frac{(ia^4)\,\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, ia\tan(c+dx)\right)}{8d}\\
&= \frac{a^3x}{8}-\frac{ia^6}{6d(a-ia\tan(c+dx))^3}-\frac{ia^5}{8d(a-ia\tan(c+dx))^2}-\frac{ia^4}{8d(a-ia\tan(c+dx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.72

$$
\begin{aligned}
&\int \cos^6(c+dx)(a+ia\tan(c+dx))^3\,dx\\
&= \frac{a^3(-10+9i\tan(c+dx)+3\tan^2(c+dx)+3\arctan(\tan(c+dx))(i+\tan(c+dx))^3)}{24d(i+\tan(c+dx))^3}
\end{aligned}
$$

```
[In] Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^3,x]
```

[Out] (a^3*(-10 + (9*I)*Tan[c + d*x] + 3*Tan[c + d*x]^2 + 3*ArcTan[Tan[c + d*x]]*(I + Tan[c + d*x])^3))/(24*d*(I + Tan[c + d*x])^3)

Maple [A] (verified)

Time = 47.77 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.69

method	result
risch	$\frac{a^3x}{8} - \frac{ia^3 e^{6i(dx+c)}}{48d} - \frac{3ia^3 e^{4i(dx+c)}}{32d} - \frac{3ia^3 e^{2i(dx+c)}}{16d}$
derivativedivides	$\frac{-ia^3\left(-\frac{\left(\cos^4(dx+c)\right)\left(\sin^2(dx+c)\right)}{6}-\frac{\left(\cos^4(dx+c)\right)}{12}\right)-3a^3\left(-\frac{\left(\cos^5(dx+c)\right)\sin(dx+c)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c}{24}\right.}{d}$
default	$\frac{-ia^3\left(-\frac{\left(\cos^4(dx+c)\right)\left(\sin^2(dx+c)\right)}{6}-\frac{\left(\cos^4(dx+c)\right)}{12}\right)-3a^3\left(-\frac{\left(\cos^5(dx+c)\right)\sin(dx+c)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c}{24}\right.}{d}$

[In] int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/8*a^3*x-1/48*I/d*a^3*exp(6*I*(d*x+c))-3/32*I/d*a^3*exp(4*I*(d*x+c))-3/16*I/d*a^3*exp(2*I*(d*x+c))

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.61

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{12\,a^3dx - 2i\,a^3e^{(6i\,dx+6i\,c)} - 9i\,a^3e^{(4i\,dx+4i\,c)} - 18i\,a^3e^{(2i\,dx+2i\,c)}}{96\,d}$$

[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")

[Out] 1/96*(12*a^3*d*x - 2*I*a^3*e^(6*I*d*x + 6*I*c) - 9*I*a^3*e^(4*I*d*x + 4*I*c) - 18*I*a^3*e^(2*I*d*x + 2*I*c))/d

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.46

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$=\frac{a^3x}{8}+\begin{cases}\frac{-512ia^3d^2e^{6ic}e^{6idx}-2304ia^3d^2e^{4ic}e^{4idx}-4608ia^3d^2e^{2ic}e^{2idx}}{24576d^3} & \text{for } d^3\neq 0\\ x\left(\frac{a^3e^{6ic}}{8}+\frac{3a^3e^{4ic}}{8}+\frac{3a^3e^{2ic}}{8}\right) & \text{otherwise}\end{cases}$$

```
[In] integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] a**3*x/8 + Piecewise(((-512*I*a**3*d**2*exp(6*I*c)*exp(6*I*d*x) - 2304*I*a*
*3*d**2*exp(4*I*c)*exp(4*I*d*x) - 4608*I*a**3*d**2*exp(2*I*c)*exp(2*I*d*x))
/(24576*d**3), Ne(d**3, 0)), (x*(a**3*exp(6*I*c)/8 + 3*a**3*exp(4*I*c)/8 +
3*a**3*exp(2*I*c)/8), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.17

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$=\frac{3\,(dx+c)a^3+\frac{3\,a^3\tan(dx+c)^5+8\,a^3\tan(dx+c)^3+6i\,a^3\tan(dx+c)^2+21\,a^3\tan(dx+c)-10i\,a^3}{\tan(dx+c)^6+3\,\tan(dx+c)^4+3\,\tan(dx+c)^2+1}}{24\,d}$$

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/24*(3*(d*x + c)*a^3 + (3*a^3*tan(d*x + c)^5 + 8*a^3*tan(d*x + c)^3 + 6*I*
a^3*tan(d*x + c)^2 + 21*a^3*tan(d*x + c) - 10*I*a^3)/(tan(d*x + c)^6 + 3*ta
n(d*x + c)^4 + 3*tan(d*x + c)^2 + 1))/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 457 vs. 2(70) = 140.

Time = 0.81 (sec) , antiderivative size = 457, normalized size of antiderivative = 5.08

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$=\frac{12\,a^3dxe^{(8i\,dx+4i\,c)}+48\,a^3dxe^{(6i\,dx+2i\,c)}+48\,a^3dxe^{(2i\,dx-2i\,c)}+72\,a^3dxe^{(4i\,dx)}+12\,a^3dxe^{(-4i\,c)}-3i\,a^3e^{(8i\,dx+4}}{}$$

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/96*(12*a^3*d*x*e^(8*I*d*x + 4*I*c) + 48*a^3*d*x*e^(6*I*d*x + 2*I*c) + 48*
a^3*d*x*e^(2*I*d*x - 2*I*c) + 72*a^3*d*x*e^(4*I*d*x) + 12*a^3*d*x*e^(-4*I*c
) - 3*I*a^3*e^(8*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 12*I*a^3*e^(
6*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 12*I*a^3*e^(2*I*d*x - 2*I*c
)*log(e^(2*I*d*x + 2*I*c) + 1) - 18*I*a^3*e^(4*I*d*x)*log(e^(2*I*d*x + 2*I*
c) + 1) - 3*I*a^3*e^(-4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 3*I*a^3*e^(8*I*
d*x + 4*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 12*I*a^3*e^(6*I*d*x + 2*I*c)*l
og(e^(2*I*d*x) + e^(-2*I*c)) + 12*I*a^3*e^(2*I*d*x - 2*I*c)*log(e^(2*I*d*x)
 + e^(-2*I*c)) + 18*I*a^3*e^(4*I*d*x)*log(e^(2*I*d*x) + e^(-2*I*c)) + 3*I*a
^3*e^(-4*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) - 2*I*a^3*e^(14*I*d*x + 10*I*c)
 - 17*I*a^3*e^(12*I*d*x + 8*I*c) - 66*I*a^3*e^(10*I*d*x + 6*I*c) - 134*I*a^
3*e^(8*I*d*x + 4*I*c) - 146*I*a^3*e^(6*I*d*x + 2*I*c) - 18*I*a^3*e^(2*I*d*x
 - 2*I*c) - 81*I*a^3*e^(4*I*d*x))/(d*e^(8*I*d*x + 4*I*c) + 4*d*e^(6*I*d*x +
 2*I*c) + 4*d*e^(2*I*d*x - 2*I*c) + 6*d*e^(4*I*d*x) + d*e^(-4*I*c))
```

Mupad [B] (verification not implemented)

Time = 4.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.86

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{a^3\,x}{8}-\frac{\frac{a^3\,\tan\left(c+d\,x\right)^2}{8}+\frac{a^3\,\tan\left(c+d\,x\right)\,3{}\mathrm{i}}{8}-\frac{5\,a^3}{12}}{d\,\left(-{\tan\left(c+d\,x\right)}^3-{\tan\left(c+d\,x\right)}^2\,3{}\mathrm{i}+3\,\tan\left(c+d\,x\right)+1{}\mathrm{i}\right)}$$

```
[In] int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] (a^3*x)/8 - ((a^3*tan(c + d*x)*3i)/8 - (5*a^3)/12 + (a^3*tan(c + d*x)^2)/8)
/(d*(3*tan(c + d*x) - tan(c + d*x)^2*3i - tan(c + d*x)^3 + 1i))
```

3.44 $\int \cos^8(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 144

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{5a^3x}{32} - \frac{ia^7}{16d(a-ia\tan(c+dx))^4} - \frac{ia^6}{12d(a-ia\tan(c+dx))^3} - \frac{3ia^5}{32d(a-ia\tan(c+dx))^2} - \frac{ia^4}{8d(a-ia\tan(c+dx))} + \frac{ia^4}{32d(a+ia\tan(c+dx))}$$

```
[Out] 5/32*a^3*x-1/16*I*a^7/d/(a-I*a*tan(d*x+c))^4-1/12*I*a^6/d/(a-I*a*tan(d*x+c)
)^3-3/32*I*a^5/d/(a-I*a*tan(d*x+c))^2-1/8*I*a^4/d/(a-I*a*tan(d*x+c))+1/32*I
*a^4/d/(a+I*a*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used

$= \{3568, 46, 212\}$

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{ia^7}{16d(a-ia\tan(c+dx))^4} - \frac{ia^6}{12d(a-ia\tan(c+dx))^3} - \frac{3ia^5}{32d(a-ia\tan(c+dx))^2} - \frac{ia^4}{8d(a-ia\tan(c+dx))} + \frac{ia^4}{32d(a+ia\tan(c+dx))} + \frac{5a^3x}{32}$$

```
[In] Int[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (5*a^3*x)/32 - ((I/16)*a^7)/(d*(a - I*a*Tan[c + d*x])^4) - ((I/12)*a^6)/(d*
(a - I*a*Tan[c + d*x])^3) - (((3*I)/32)*a^5)/(d*(a - I*a*Tan[c + d*x])^2) -
 ((I/8)*a^4)/(d*(a - I*a*Tan[c + d*x])) + ((I/32)*a^4)/(d*(a + I*a*Tan[c +
d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{(ia^9)\,\text{Subst}\left(\int \frac{1}{(a-x)^5(a+x)^2}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^9)\,\text{Subst}\left(\int \left(\frac{1}{4a^2(a-x)^5}+\frac{1}{4a^3(a-x)^4}+\frac{3}{16a^4(a-x)^3}+\frac{1}{8a^5(a-x)^2}+\frac{1}{32a^5(a+x)^2}+\frac{5}{32a^5(a^2-x^2)}\right)dx, x, ia\,\text{t}\right.}{d}$$

$$= -\frac{ia^7}{16d(a-ia\tan(c+dx))^4}-\frac{ia^6}{12d(a-ia\tan(c+dx))^3} - \frac{3ia^5}{32d(a-ia\tan(c+dx))^2}-\frac{ia^4}{8d(a-ia\tan(c+dx))} + \frac{ia^4}{32d(a+ia\tan(c+dx))}-\frac{(5ia^4)\,\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, ia\tan(c+dx)\right)}{32d}$$

$$= \frac{5a^3x}{32}-\frac{ia^7}{16d(a-ia\tan(c+dx))^4}-\frac{ia^6}{12d(a-ia\tan(c+dx))^3} - \frac{3ia^5}{32d(a-ia\tan(c+dx))^2}-\frac{ia^4}{8d(a-ia\tan(c+dx))}+\frac{ia^4}{32d(a+ia\tan(c+dx))}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.95

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{ia^9\left(\frac{5i\arctan(\tan(c+dx))}{32a^6}+\frac{1}{16a^2(a-ia\tan(c+dx))^4}+\frac{1}{12a^3(a-ia\tan(c+dx))^3}+\frac{3}{32a^4(a-ia\tan(c+dx))^2}+\frac{1}{8a^5(a-ia\tan(c+dx))}-\right.}{d}$$

```
[In] Integrate[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((-I)*a^9*((((5*I)/32)*ArcTan[Tan[c + d*x]])/a^6 + 1/(16*a^2*(a - I*a*Tan[c
 + d*x])^4) + 1/(12*a^3*(a - I*a*Tan[c + d*x])^3) + 3/(32*a^4*(a - I*a*Tan[
c + d*x])^2) + 1/(8*a^5*(a - I*a*Tan[c + d*x])) - 1/(32*a^5*(a + I*a*Tan[c
+ d*x]))))/d
```

Maple [A] (verified)

Time = 133.41 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.67

method	result
risch	$\frac{5a^3x}{32} - \frac{ia^3e^{8i(dx+c)}}{256d} - \frac{5ia^3e^{6i(dx+c)}}{192d} - \frac{5ia^3e^{4i(dx+c)}}{64d} - \frac{9ia^3\cos(2dx+2c)}{64d} + \frac{11a^3\sin(2dx+2c)}{64d}$
derivativedivides	$-ia^3\left(-\frac{\left(\cos^6(dx+c)\right)\left(\sin^2(dx+c)\right)}{8} - \frac{\left(\cos^6(dx+c)\right)}{24}\right) - 3a^3\left(-\frac{\sin(dx+c)\left(\cos^7(dx+c)\right)}{8} + \frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos}{}\right.}{48}\right.$
default	$-ia^3\left(-\frac{\left(\cos^6(dx+c)\right)\left(\sin^2(dx+c)\right)}{8} - \frac{\left(\cos^6(dx+c)\right)}{24}\right) - 3a^3\left(-\frac{\sin(dx+c)\left(\cos^7(dx+c)\right)}{8} + \frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos}{}\right.}{48}\right.$

```
[In] int(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 5/32*a^3*x-1/256*I/d*a^3*exp(8*I*(d*x+c))-5/192*I/d*a^3*exp(6*I*(d*x+c))-5/
64*I/d*a^3*exp(4*I*(d*x+c))-9/64*I/d*a^3*cos(2*d*x+2*c)+11/64/d*a^3*sin(2*d
*x+2*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.64

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{\left(120\,a^3dxe^{(2i\,dx+2i\,c)} - 3i\,a^3e^{(10i\,dx+10i\,c)} - 20i\,a^3e^{(8i\,dx+8i\,c)} - 60i\,a^3e^{(6i\,dx+6i\,c)} - 120i\,a^3e^{(4i\,dx+4i\,c)} + 12i\,a^3\right)}{768\,d}$$

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/768*(120*a^3*d*x*e^(2*I*d*x + 2*I*c) - 3*I*a^3*e^(10*I*d*x + 10*I*c) - 20
*I*a^3*e^(8*I*d*x + 8*I*c) - 60*I*a^3*e^(6*I*d*x + 6*I*c) - 120*I*a^3*e^(4*
I*d*x + 4*I*c) + 12*I*a^3)*e^(-2*I*d*x - 2*I*c)/d
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.57

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{5a^3x}{32} + \begin{cases} \frac{(-25165824ia^3d^4e^{10ic}e^{8idx} - 167772160ia^3d^4e^{8ic}e^{6idx} - 503316480ia^3d^4e^{6ic}e^{4idx} - 1006632960ia^3d^4e^{4ic}e^{2idx} + 100663296ia^3d^4e^{-2idx})e^{-}}{6442450944d^5} \\ x\left(-\frac{5a^3}{32} + \frac{(a^3e^{10ic}+5a^3e^{8ic}+10a^3e^{6ic}+10a^3e^{4ic}+5a^3e^{2ic}+a^3)e^{-2ic}}{32}\right) \end{cases}$$

```
[In] integrate(cos(d*x+c)**8*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] 5*a**3*x/32 + Piecewise(((-25165824*I*a**3*d**4*exp(10*I*c)*exp(8*I*d*x) -
167772160*I*a**3*d**4*exp(8*I*c)*exp(6*I*d*x) - 503316480*I*a**3*d**4*exp(6
*I*c)*exp(4*I*d*x) - 1006632960*I*a**3*d**4*exp(4*I*c)*exp(2*I*d*x) + 10066
3296*I*a**3*d**4*exp(-2*I*d*x))*exp(-2*I*c)/(6442450944*d**5), Ne(d**5*exp(
2*I*c), 0)), (x*(-5*a**3/32 + (a**3*exp(10*I*c) + 5*a**3*exp(8*I*c) + 10*a*
*3*exp(6*I*c) + 10*a**3*exp(4*I*c) + 5*a**3*exp(2*I*c) + a**3)*exp(-2*I*c)/
32), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.89

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{15\,(dx+c)a^3 + \frac{15\,a^3\tan(dx+c)^7+55\,a^3\tan(dx+c)^5+73\,a^3\tan(dx+c)^3+16i\,a^3\tan(dx+c)^2+81\,a^3\tan(dx+c)-32i\,a^3}{\tan(dx+c)^8+4\,\tan(dx+c)^6+6\,\tan(dx+c)^4+4\,\tan(dx+c)^2+1}}{96\,d}$$

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/96*(15*(d*x + c)*a^3 + (15*a^3*tan(d*x + c)^7 + 55*a^3*tan(d*x + c)^5 + 7
3*a^3*tan(d*x + c)^3 + 16*I*a^3*tan(d*x + c)^2 + 81*a^3*tan(d*x + c) - 32*I
*a^3)/(tan(d*x + c)^8 + 4*tan(d*x + c)^6 + 6*tan(d*x + c)^4 + 4*tan(d*x + c
)^2 + 1))/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 514 vs. 2(112) = 224.

Time = 0.92 (sec) , antiderivative size = 514, normalized size of antiderivative = 3.57

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{240\,a^3dxe^{(10i\,dx+6i\,c)}+960\,a^3dxe^{(8i\,dx+4i\,c)}+1440\,a^3dxe^{(6i\,dx+2i\,c)}+240\,a^3dxe^{(2i\,dx-2i\,c)}+960\,a^3dxe^{(4i\,dx)}-3}{}$$

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/1536*(240*a^3*d*x*e^(10*I*d*x + 6*I*c) + 960*a^3*d*x*e^(8*I*d*x + 4*I*c)
+ 1440*a^3*d*x*e^(6*I*d*x + 2*I*c) + 240*a^3*d*x*e^(2*I*d*x - 2*I*c) + 960*
a^3*d*x*e^(4*I*d*x) - 33*I*a^3*e^(10*I*d*x + 6*I*c)*log(e^(2*I*d*x + 2*I*c)
 + 1) - 132*I*a^3*e^(8*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 198*I*
```

```
a^3*e^(6*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 33*I*a^3*e^(2*I*d*x
- 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 132*I*a^3*e^(4*I*d*x)*log(e^(2*I*d*
x + 2*I*c) + 1) + 33*I*a^3*e^(10*I*d*x + 6*I*c)*log(e^(2*I*d*x) + e^(-2*I*c
)) + 132*I*a^3*e^(8*I*d*x + 4*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 198*I*a^
3*e^(6*I*d*x + 2*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 33*I*a^3*e^(2*I*d*x -
 2*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 132*I*a^3*e^(4*I*d*x)*log(e^(2*I*d*
x) + e^(-2*I*c)) - 6*I*a^3*e^(18*I*d*x + 14*I*c) - 64*I*a^3*e^(16*I*d*x + 1
2*I*c) - 316*I*a^3*e^(14*I*d*x + 10*I*c) - 984*I*a^3*e^(12*I*d*x + 8*I*c) -
 1846*I*a^3*e^(10*I*d*x + 6*I*c) - 1936*I*a^3*e^(8*I*d*x + 4*I*c) - 984*I*a
^3*e^(6*I*d*x + 2*I*c) + 96*I*a^3*e^(2*I*d*x - 2*I*c) - 96*I*a^3*e^(4*I*d*x
) + 24*I*a^3*e^(-4*I*c))/(d*e^(10*I*d*x + 6*I*c) + 4*d*e^(8*I*d*x + 4*I*c)
+ 6*d*e^(6*I*d*x + 2*I*c) + d*e^(2*I*d*x - 2*I*c) + 4*d*e^(4*I*d*x))
```

Mupad [B] (verification not implemented)

Time = 4.01 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.87

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^3\,dx=\frac{5\,a^3\,x}{32}-\frac{\frac{5\,a^3\,\tan(c+d\,x)^4}{32}+\frac{a^3\,\tan(c+d\,x)^3\,15\mathrm{i}}{32}-\frac{35\,a^3\,\tan(c+d\,x)^2}{96}+\frac{a^3\,\tan(c+d\,x)\,5\mathrm{i}}{32}-\frac{a^3}{3}}{d\,\left(-{\tan\left(c+d\,x\right)}^5-{\tan\left(c+d\,x\right)}^4\,3\mathrm{i}+2\,{\tan\left(c+d\,x\right)}^3-{\tan\left(c+d\,x\right)}^2\,2\mathrm{i}+3\,\tan\left(c+d\,x\right)+1\mathrm{i}\right)}$$

```
[In] int(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] (5*a^3*x)/32 - ((a^3*tan(c + d*x)*5i)/32 - a^3/3 - (35*a^3*tan(c + d*x)^2)/
96 + (a^3*tan(c + d*x)^3*15i)/32 + (5*a^3*tan(c + d*x)^4)/32)/(d*(3*tan(c +
 d*x) - tan(c + d*x)^2*2i + 2*tan(c + d*x)^3 - tan(c + d*x)^4*3i - tan(c +
d*x)^5 + 1i))
```

3.45 $\int \sec^3(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 127

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^3\,dx=\frac{7a^3\text{arctanh}(\sin(c+dx))}{8d}+\frac{7ia^3\sec^3(c+dx)}{12d}+\frac{7a^3\sec(c+dx)\tan(c+dx)}{8d}+\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^2}{5d}+\frac{7i\sec^3(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{20d}$$

```
[Out] 7/8*a^3*arctanh(sin(d*x+c))/d+7/12*I*a^3*sec(d*x+c)^3/d+7/8*a^3*sec(d*x+c)*
tan(d*x+c)/d+1/5*I*a*sec(d*x+c)^3*(a+I*a*tan(d*x+c))^2/d+7/20*I*sec(d*x+c)^
3*(a^3+I*a^3*tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3579, 3567, 3853, 3855}

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^3\,dx=\frac{7a^3\text{arctanh}(\sin(c+dx))}{8d}+\frac{7ia^3\sec^3(c+dx)}{12d}+\frac{7i\sec^3(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{20d}+\frac{7a^3\tan(c+dx)\sec(c+dx)}{8d}+\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

```
[In] Int[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (7*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (((7*I)/12)*a^3*Sec[c + d*x]^3)/d + (
7*a^3*Sec[c + d*x]*Tan[c + d*x])/(8*d) + ((I/5)*a*Sec[c + d*x]^3*(a + I*a*T
an[c + d*x])^2)/d + (((7*I)/20)*Sec[c + d*x]^3*(a^3 + I*a^3*Tan[c + d*x]))/
d
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^2}{5d}+\frac{1}{5}(7a)\int \sec^3(c+dx)(a+ia\tan(c+dx))^2\,dx\\
&= \frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^2}{5d}+\frac{7i\sec^3(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{20d}\\
&\quad+\frac{1}{4}\left(7a^2\right)\int \sec^3(c+dx)(a+ia\tan(c+dx))\,dx\\
&= \frac{7ia^3\sec^3(c+dx)}{12d}+\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^2}{5d}\\
&\quad+\frac{7i\sec^3(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{20d}+\frac{1}{4}\left(7a^3\right)\int \sec^3(c+dx)\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{7ia^3\sec^3(c+dx)}{12d} + \frac{7a^3\sec(c+dx)\tan(c+dx)}{8d} \\
&\quad + \frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^2}{5d} \\
&\quad + \frac{7i\sec^3(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{20d} + \frac{1}{8}\left(7a^3\right)\int\sec(c+dx)\,dx \\
&= \frac{7a^3\text{arctanh}(\sin(c+dx))}{8d} + \frac{7ia^3\sec^3(c+dx)}{12d} + \frac{7a^3\sec(c+dx)\tan(c+dx)}{8d} \\
&\quad + \frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^2}{5d} + \frac{7i\sec^3(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{20d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.72 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.80

$$
\int \sec^3(c+dx)(a+ia\tan(c+dx))^3\,dx
= \frac{a^3(\cos(3dx)+i\sin(3dx))\left(1680\text{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)+\sec^5(c+dx)(448i+640i\cos(2(c+dx)\right.}{960d(\cos(dx)+i\sin(dx))^3}
$$

```
[In] Integrate[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*(Cos[3*d*x] + I*Sin[3*d*x])*(1680*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]
] + Sec[c + d*x]^5*(448*I + (640*I)*Cos[2*(c + d*x)] - 150*Sin[2*(c + d*x)]
 + 105*Sin[4*(c + d*x)])))/(960*d*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [A] (verified)

Time = 8.87 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{ia^3\left(105\,\mathrm{e}^{9i(dx+c)}-790\,\mathrm{e}^{7i(dx+c)}-896\,\mathrm{e}^{5i(dx+c)}-490\,\mathrm{e}^{3i(dx+c)}-105\,\mathrm{e}^{i(dx+c)}\right)}{60d\left(\mathrm{e}^{2i(dx+c)}+1\right)^5}+\frac{7a^3\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{8d}-\frac{7a^3\ln\left(\mathrm{e}^{i(dx-\right.}}{8d}$
derivativedivides	$\frac{-ia^3\left(\frac{\sin^4(dx+c)}{5\cos(dx+c)^5}+\frac{\sin^4(dx+c)}{15\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{15\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{15}\right)-3a^3\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}\right.}{d}$
default	$\frac{-ia^3\left(\frac{\sin^4(dx+c)}{5\cos(dx+c)^5}+\frac{\sin^4(dx+c)}{15\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{15\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{15}\right)-3a^3\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}\right.}{d}$

```
[In] int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/60*I*a^3/d/(exp(2*I*(d*x+c))+1)^5*(105*exp(9*I*(d*x+c))-790*exp(7*I*(d*x
+c))-896*exp(5*I*(d*x+c))-490*exp(3*I*(d*x+c))-105*exp(I*(d*x+c)))+7/8/d*a^
3*ln(exp(I*(d*x+c))+I)-7/8/d*a^3*ln(exp(I*(d*x+c))-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(107) = 214.

Time = 0.24 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.44

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{-210i\,a^3e^{(9i\,dx+9i\,c)}+1580i\,a^3e^{(7i\,dx+7i\,c)}+1792i\,a^3e^{(5i\,dx+5i\,c)}+980i\,a^3e^{(3i\,dx+3i\,c)}+210i\,a^3e^{(i\,dx+i\,c)}+105}{}$$

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/120*(-210*I*a^3*e^(9*I*d*x + 9*I*c) + 1580*I*a^3*e^(7*I*d*x + 7*I*c) + 17
92*I*a^3*e^(5*I*d*x + 5*I*c) + 980*I*a^3*e^(3*I*d*x + 3*I*c) + 210*I*a^3*e^
(I*d*x + I*c) + 105*(a^3*e^(10*I*d*x + 10*I*c) + 5*a^3*e^(8*I*d*x + 8*I*c)
+ 10*a^3*e^(6*I*d*x + 6*I*c) + 10*a^3*e^(4*I*d*x + 4*I*c) + 5*a^3*e^(2*I*d*
x + 2*I*c) + a^3)*log(e^(I*d*x + I*c) + I) - 105*(a^3*e^(10*I*d*x + 10*I*c)
 + 5*a^3*e^(8*I*d*x + 8*I*c) + 10*a^3*e^(6*I*d*x + 6*I*c) + 10*a^3*e^(4*I*d
*x + 4*I*c) + 5*a^3*e^(2*I*d*x + 2*I*c) + a^3)*log(e^(I*d*x + I*c) - I))/(d
*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c)
 + 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^3\,dx = -ia^3\left(\int i\sec^3(c+dx)\,dx + \int \left(-3\tan(c+dx)\sec^3(c+dx)\right)\,dx + \int \tan^3(c+dx)\sec^3(c+dx)\,dx + \int \left(-3i\tan^2(c+dx)\sec^3(c+dx)\right)\,dx\right)$$

```
[In] integrate(sec(d*x+c)**3*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] -I*a**3*(Integral(I*sec(c + d*x)**3, x) + Integral(-3*tan(c + d*x)*sec(c +
d*x)**3, x) + Integral(tan(c + d*x)**3*sec(c + d*x)**3, x) + Integral(-3*I*
tan(c + d*x)**2*sec(c + d*x)**3, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.22

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^3\,dx =$$

$$-\frac{45\,a^3\left(\frac{2\left(\sin(dx+c)^3+\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+60\,a^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\right.}{240\,d}$$

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/240*(45*a^3*(2*(sin(d*x + c)^3 + sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d
*x + c)^2 + 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 60*a^3*(2
*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
c) - 1)) - 240*I*a^3/cos(d*x + c)^3 - 16*I*(5*cos(d*x + c)^2 - 3)*a^3/cos(d
*x + c)^5)/d
```

Giac [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.49

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^3\,dx$$

$$=\frac{105\,a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-105\,a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)+\frac{2\left(15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-360i\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}{}}{}$$

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/120*(105*a^3*log(tan(1/2*d*x + 1/2*c) + 1) - 105*a^3*log(tan(1/2*d*x + 1/
2*c) - 1) + 2*(15*a^3*tan(1/2*d*x + 1/2*c)^9 - 360*I*a^3*tan(1/2*d*x + 1/2*
c)^8 - 390*a^3*tan(1/2*d*x + 1/2*c)^7 + 960*I*a^3*tan(1/2*d*x + 1/2*c)^6 -
400*I*a^3*tan(1/2*d*x + 1/2*c)^4 + 390*a^3*tan(1/2*d*x + 1/2*c)^3 + 320*I*a
^3*tan(1/2*d*x + 1/2*c)^2 - 15*a^3*tan(1/2*d*x + 1/2*c) - 136*I*a^3)/(tan(1
/2*d*x + 1/2*c)^2 - 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 7.62 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.80

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{7\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}$$
$$-\frac{-\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}+a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8 6\mathrm{i}+\frac{13\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2}-a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 16\mathrm{i}+\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 20\mathrm{i}}{3}-\frac{13\,a^3\,\mathrm{ta}}{}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right.}$$

```
[In] int((a + a*tan(c + d*x)*1i)^3/cos(c + d*x)^3,x)
```

```
[Out] (7*a^3*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((a^3*tan(c/2 + (d*x)/2)^4*20i)/3
 - (13*a^3*tan(c/2 + (d*x)/2)^3)/2 - (a^3*tan(c/2 + (d*x)/2)^2*16i)/3 - a^3
*tan(c/2 + (d*x)/2)^6*16i + (13*a^3*tan(c/2 + (d*x)/2)^7)/2 + a^3*tan(c/2 +
 (d*x)/2)^8*6i - (a^3*tan(c/2 + (d*x)/2)^9)/4 + (a^3*34i)/15 + (a^3*tan(c/2
 + (d*x)/2))/4)/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*t
an(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.46 $\int \sec(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 99

$$\int \sec(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{5a^3\text{arctanh}(\sin(c+dx))}{2d}+\frac{5ia^3\sec(c+dx)}{2d}+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}+\frac{5i\sec(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{6d}$$

```
[Out] 5/2*a^3*arctanh(sin(d*x+c))/d+5/2*I*a^3*sec(d*x+c)/d+1/3*I*a*sec(d*x+c)*(a+
I*a*tan(d*x+c))^2/d+5/6*I*sec(d*x+c)*(a^3+I*a^3*tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3579, 3567, 3855}

$$\int \sec(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{5a^3\text{arctanh}(\sin(c+dx))}{2d}+\frac{5ia^3\sec(c+dx)}{2d}+\frac{5i\sec(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{6d}+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}$$

```
[In] Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (5*a^3*ArcTanh[Sin[c + d*x]])/(2*d) + (((5*I)/2)*a^3*Sec[c + d*x])/d + ((I/
3)*a*Sec[c + d*x]*(a + I*a*Tan[c + d*x])^2)/d + (((5*I)/6)*Sec[c + d*x]*(a^
3 + I*a^3*Tan[c + d*x]))/d
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d} + \frac{1}{3}(5a)\int \sec(c+dx)(a+ia\tan(c+dx))^2\,dx \\
&= \frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d} + \frac{5i\sec(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{6d} \\
&\quad + \frac{1}{2}\left(5a^2\right)\int \sec(c+dx)(a+ia\tan(c+dx))\,dx \\
&= \frac{5ia^3\sec(c+dx)}{2d} + \frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d} \\
&\quad + \frac{5i\sec(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{6d} + \frac{1}{2}\left(5a^3\right)\int \sec(c+dx)\,dx \\
&= \frac{5a^3\operatorname{arctanh}(\sin(c+dx))}{2d} + \frac{5ia^3\sec(c+dx)}{2d} \\
&\quad + \frac{ia\sec(c+dx)(a+ia\tan(c+dx))^2}{3d} + \frac{5i\sec(c+dx)\left(a^3+ia^3\tan(c+dx)\right)}{6d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.94

$$\int \sec(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$= \frac{a^3(\cos(3dx)+i\sin(3dx))\left(60\text{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)+i\sec^3(c+dx)(20+24\cos(2(c+dx))+9\right.}{12d(\cos(dx)+i\sin(dx))^3}$$

```
[In] Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*(Cos[3*d*x] + I*Sin[3*d*x])*(60*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]]
+ I*Sec[c + d*x]^3*(20 + 24*Cos[2*(c + d*x)] + (9*I)*Sin[2*(c + d*x)])))/(1
2*d*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01

method	result
risch	$\frac{ia^3\left(33\,\mathrm{e}^{5i(dx+c)}+40\,\mathrm{e}^{3i(dx+c)}+15\,\mathrm{e}^{i(dx+c)}\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}+\frac{5a^3\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2d}-\frac{5a^3\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2d}$
derivativedivides	$\frac{-ia^3\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right)-3a^3\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+}{d}$
default	$\frac{-ia^3\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right)-3a^3\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+}{d}$

```
[In] int(sec(d*x+c)*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*I*a^3/d/(exp(2*I*(d*x+c))+1)^3*(33*exp(5*I*(d*x+c))+40*exp(3*I*(d*x+c))
+15*exp(I*(d*x+c)))+5/2/d*a^3*ln(exp(I*(d*x+c))+I)-5/2/d*a^3*ln(exp(I*(d*x+
c))-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(81) = 162.

Time = 0.24 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.04

$$\int \sec(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$= \frac{66i\,a^3e^{(5i\,dx+5i\,c)}+80i\,a^3e^{(3i\,dx+3i\,c)}+30i\,a^3e^{(i\,dx+i\,c)}+15\left(a^3e^{(6i\,dx+6i\,c)}+3\,a^3e^{(4i\,dx+4i\,c)}+3\,a^3e^{(2i\,dx+2i\,c)}+a\right.}{6\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}\right.}$$

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(66*I*a^3*e^(5*I*d*x + 5*I*c) + 80*I*a^3*e^(3*I*d*x + 3*I*c) + 30*I*a^3
*e^(I*d*x + I*c) + 15*(a^3*e^(6*I*d*x + 6*I*c) + 3*a^3*e^(4*I*d*x + 4*I*c)
+ 3*a^3*e^(2*I*d*x + 2*I*c) + a^3)*log(e^(I*d*x + I*c) + I) - 15*(a^3*e^(6*
I*d*x + 6*I*c) + 3*a^3*e^(4*I*d*x + 4*I*c) + 3*a^3*e^(2*I*d*x + 2*I*c) + a^
3)*log(e^(I*d*x + I*c) - I))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*
c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^3\,dx = -ia^3\left(\int i\sec(c+dx)\,dx + \int(-3\tan(c+dx)\sec(c+dx))\,dx + \int\tan^3(c+dx)\sec(c+dx)\,dx + \int(-3i\tan^2(c+dx)\sec(c+dx))\,dx\right)$$

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] -I*a**3*(Integral(I*sec(c + d*x), x) + Integral(-3*tan(c + d*x)*sec(c + d*x
), x) + Integral(tan(c + d*x)**3*sec(c + d*x), x) + Integral(-3*I*tan(c + d
*x)**2*sec(c + d*x), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.10

$$\int \sec(c+dx)(a+ia\tan(c+dx))^3\,dx$$

$$= \frac{9\,a^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} + \log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)\right) + 12\,a^3\log(\sec(dx+c)+\tan(dx+c)}{12\,d}$$

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/12*(9*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) -
log(sin(d*x + c) - 1)) + 12*a^3*log(sec(d*x + c) + tan(d*x + c)) + 36*I*a^3
/cos(d*x + c) + 4*I*(3*cos(d*x + c)^2 - 1)*a^3/cos(d*x + c)^3)/d
```

Giac [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.26

$$\int \sec(c+dx)(a+ia\tan(c+dx))^3\,dx$$

$$= \frac{15\,a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-15\,a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)-\frac{2\left(9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+18i\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-48\cdots\right.}{\left(\tan\left(\frac{1}{2}\,dx+\cdots\right.\right.}}{6\,d}$$

[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")

[Out] 1/6*(15*a^3*log(tan(1/2*d*x + 1/2*c) + 1) - 15*a^3*log(tan(1/2*d*x + 1/2*c) - 1) - 2*(9*a^3*tan(1/2*d*x + 1/2*c)^5 + 18*I*a^3*tan(1/2*d*x + 1/2*c)^4 - 48*I*a^3*tan(1/2*d*x + 1/2*c)^2 - 9*a^3*tan(1/2*d*x + 1/2*c) + 22*I*a^3)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d

Mupad [B] (verification not implemented)

Time = 6.25 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.37

$$\int \sec(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{5\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$
$$-\frac{3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 6\mathrm{i}-a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 16\mathrm{i}-3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a^3\,22\mathrm{i}}{3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

[In] int((a + a*tan(c + d*x)*1i)^3/cos(c + d*x),x)

[Out] (5*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (a^3*tan(c/2 + (d*x)/2)^4*6i - a^3*tan(c/2 + (d*x)/2)^2*16i + 3*a^3*tan(c/2 + (d*x)/2)^5 + (a^3*22i)/3 - 3*a^3*tan(c/2 + (d*x)/2))/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))

3.47 $\int \cos(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 61

$$\int \cos(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{3a^3\text{arctanh}(\sin(c+dx))}{d}-\frac{3ia^3\sec(c+dx)}{d}-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^2}{d}$$

```
[Out] -3*a^3*arctanh(sin(d*x+c))/d-3*I*a^3*sec(d*x+c)/d-2*I*a*cos(d*x+c)*(a+I*a*t
an(d*x+c))^2/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3577, 3567, 3855}

$$\int \cos(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{3a^3\text{arctanh}(\sin(c+dx))}{d}-\frac{3ia^3\sec(c+dx)}{d}-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^2}{d}$$

```
[In] Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (-3*a^3*ArcTanh[Sin[c + d*x]])/d - ((3*I)*a^3*Sec[c + d*x])/d - ((2*I)*a*Co
s[c + d*x]*(a + I*a*Tan[c + d*x])^2)/d
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
```

```
| NeQ[a^2 + b^2, 0])
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^2}{d} - (3a^2)\int \sec(c+dx)(a+ia\tan(c+dx))\,dx \\
&= -\frac{3ia^3\sec(c+dx)}{d} - \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^2}{d} - (3a^3)\int \sec(c+dx)\,dx \\
&= -\frac{3a^3\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{3ia^3\sec(c+dx)}{d} - \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^2}{d}
\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 123 vs. 2(61) = 122.

Time = 1.56 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.02

$$\int \cos(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$= \frac{a^3\cos^2(c+dx)\left(6\operatorname{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)\cos(c+dx)(i\cos(3c)+\sin(3c))+(-\cos(2c-dx)+i\,\mathrm{s}\right.}{d(\cos(dx)+i\sin(dx))^3}$$

```
[In] Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*Cos[c + d*x]^2*(6*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]]*Cos[c + d*x]*(
I*Cos[3*c] + Sin[3*c]) + (-Cos[2*c - d*x] + I*Sin[2*c - d*x])*(5*Cos[c + d*
x] - I*Sin[c + d*x]))*(-I + Tan[c + d*x])^3)/(d*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.52

method	result
risch	$-\frac{4 i a^{3} {\mathrm e}^{i (d x +c)}}{d}-\frac{2 i {\mathrm e}^{i (d x +c)} a^{3}}{d \left({\mathrm e}^{2 i (d x +c)}+1\right)}-\frac{3 a^{3} \ln \left({\mathrm e}^{i (d x +c)}+i\right)}{d}+\frac{3 a^{3} \ln \left({\mathrm e}^{i (d x +c)}-i\right)}{d}$
derivativedivides	$\frac{-i a^{3} \left(\frac{\sin^{4}(d x +c)}{\cos (d x +c)}+\left(2+\sin^{2}(d x +c)\right) \cos (d x +c)\right)-3 a^{3} \left(-\sin (d x +c)+\ln (\sec (d x +c)+\tan (d x +c))\right)-3 i a^{3} \cos (d x +c)+a^{3} \text{si}}{d}$
default	$\frac{-i a^{3} \left(\frac{\sin^{4}(d x +c)}{\cos (d x +c)}+\left(2+\sin^{2}(d x +c)\right) \cos (d x +c)\right)-3 a^{3} \left(-\sin (d x +c)+\ln (\sec (d x +c)+\tan (d x +c))\right)-3 i a^{3} \cos (d x +c)+a^{3} \text{si}}{d}$

```
[In] int(cos(d*x+c)*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -4*I/d*a^3*exp(I*(d*x+c))-2*I*exp(I*(d*x+c))*a^3/d/(exp(2*I*(d*x+c))+1)-3/d
*a^3*ln(exp(I*(d*x+c))+I)+3/d*a^3*ln(exp(I*(d*x+c))-I)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.75

$$\int \cos(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{-4i\,a^3e^{(3i\,dx+3i\,c)}-6i\,a^3e^{(i\,dx+i\,c)}-3\left(a^3e^{(2i\,dx+2i\,c)}+a^3\right)\log\left(e^{(i\,dx+i\,c)}+i\right)+3\left(a^3e^{(2i\,dx+2i\,c)}+a^3\right)\log\left(e^{(i}\right.}{de^{(2i\,dx+2i\,c)}+d}$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] (-4*I*a^3*e^(3*I*d*x + 3*I*c) - 6*I*a^3*e^(I*d*x + I*c) - 3*(a^3*e^(2*I*d*x
 + 2*I*c) + a^3)*log(e^(I*d*x + I*c) + I) + 3*(a^3*e^(2*I*d*x + 2*I*c) + a^
3)*log(e^(I*d*x + I*c) - I))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.75

$$\int \cos(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{2ia^3e^{ic}e^{idx}}{de^{2ic}e^{2idx}+d} + \frac{3a^3\left(\log\left(e^{idx}-ie^{-ic}\right)-\log\left(e^{idx}+ie^{-ic}\right)\right)}{d} + \begin{cases} -\frac{4ia^3e^{ic}e^{idx}}{d} & \text{for } d \neq 0 \\ 4a^3xe^{ic} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] -2*I*a**3*exp(I*c)*exp(I*d*x)/(d*exp(2*I*c)*exp(2*I*d*x) + d) + 3*a**3*(log
(exp(I*d*x) - I*exp(-I*c)) - log(exp(I*d*x) + I*exp(-I*c)))/d + Piecewise((
-4*I*a**3*exp(I*c)*exp(I*d*x)/d, Ne(d, 0)), (4*a**3*x*exp(I*c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.34

$$\int \cos(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{2i\,a^3\left(\frac{1}{\cos(dx+c)}+\cos\left(dx+c\right)\right)+3\,a^3(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)-2\,\sin\left(dx+c\right))+6}{2\,d}$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*I*a^3*(1/cos(d*x + c) + cos(d*x + c)) + 3*a^3*(log(sin(d*x + c) + 1
) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)) + 6*I*a^3*cos(d*x + c) - 2*a^3*
sin(d*x + c))/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 234 vs. 2(55) = 110.

Time = 0.60 (sec) , antiderivative size = 234, normalized size of antiderivative = 3.84

$$\int \cos(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{63\,a^3e^{(2i\,dx+2i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)-33\,a^3e^{(2i\,dx+2i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}-1\right)-63\,a^3e^{(2i\,dx+2i\,c)}\log\left(-i\,e^{(i\,dx+i\,c)}\right.}{}$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/32*(63*a^3*e^(2*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) - 33*a^3*e^(2*I
*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) - 63*a^3*e^(2*I*d*x + 2*I*c)*log(-
I*e^(I*d*x + I*c) + 1) + 33*a^3*e^(2*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c)
- 1) - 128*I*a^3*e^(3*I*d*x + 3*I*c) - 192*I*a^3*e^(I*d*x + I*c) + 63*a^3*l
og(I*e^(I*d*x + I*c) + 1) - 33*a^3*log(I*e^(I*d*x + I*c) - 1) - 63*a^3*log(
-I*e^(I*d*x + I*c) + 1) + 33*a^3*log(-I*e^(I*d*x + I*c) - 1))/(d*e^(2*I*d*x
 + 2*I*c) + d)
```

Mupad [B] (verification not implemented)

Time = 4.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.67

$$\int \cos(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$= -\frac{6\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{8\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,2\mathrm{i}-10\,a^3}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,1\mathrm{i}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

```
[In] int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] - (6*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (8*a^3*tan(c/2 + (d*x)/2)^2 - 10*a^
3 + a^3*tan(c/2 + (d*x)/2)*2i)/(d*(tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^
2*1i - tan(c/2 + (d*x)/2)^3 + 1i))
```

3.48 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 32

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^3\,dx=-\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

```
[Out] -1/3*I*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^3/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {3569}

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^3\,dx=-\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

```
[In] Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((-1/3*I)*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^3)/d
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\text{integral}=-\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{ia^3(\cos(c+dx)+i\sin(c+dx))^3}{3d}$$

```
[In] Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^3,x]
[Out] ((-1/3*I)*a^3*(Cos[c + d*x] + I*Sin[c + d*x])^3)/d
```

Maple [A] (verified)

Time = 6.81 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.59

method	result	size
risch	$-\frac{ia^3 e^{3i(dx+c)}}{3d}$	19
derivativedivides	$\frac{\frac{ia^3\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}-a^3\left(\sin^3(dx+c)\right)-ia^3\left(\cos^3(dx+c)\right)+\frac{a^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$	76
default	$\frac{\frac{ia^3\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}-a^3\left(\sin^3(dx+c)\right)-ia^3\left(\cos^3(dx+c)\right)+\frac{a^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$	76

```
[In] int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] -1/3*I/d*a^3*exp(3*I*(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.53

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{i\,a^3 e^{(3i\,dx+3i\,c)}}{3\,d}$$

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
[Out] -1/3*I*a^3*e^(3*I*d*x + 3*I*c)/d
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^3\,dx = \begin{cases} -\frac{ia^3e^{3ic}e^{3idx}}{3d} & \text{for } d \neq 0 \\ a^3xe^{3ic} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise((-I*a**3*exp(3*I*c)*exp(3*I*d*x)/(3*d), Ne(d, 0)), (a**3*x*exp(3*
I*c), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(26) = 52.

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.34

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{3i\,a^3\cos(dx+c)^3 + 3\,a^3\sin(dx+c)^3 + i\left(\cos(dx+c)^3 - 3\cos(dx+c)\right)a^3 + \left(\sin(dx+c)^3 - 3\sin(dx$$
$$3\,d$$

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/3*(3*I*a^3*cos(d*x + c)^3 + 3*a^3*sin(d*x + c)^3 + I*(cos(d*x + c)^3 - 3
*cos(d*x + c))*a^3 + (sin(d*x + c)^3 - 3*sin(d*x + c))*a^3)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 901 vs. 2(26) = 52.

Time = 0.71 (sec) , antiderivative size = 901, normalized size of antiderivative = 28.16

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^3\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/384*(108*a^3*e^(8*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 432*a^3*e^
(6*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 432*a^3*e^(2*I*d*x - 2*I*c)*
log(I*e^(I*d*x + I*c) + 1) + 648*a^3*e^(4*I*d*x)*log(I*e^(I*d*x + I*c) + 1)
 + 108*a^3*e^(-4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 111*a^3*e^(8*I*d*x + 4*I
*c)*log(I*e^(I*d*x + I*c) - 1) + 444*a^3*e^(6*I*d*x + 2*I*c)*log(I*e^(I*d*x
```

```
 + I*c) - 1) + 444*a^3*e^(2*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 666
*a^3*e^(4*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 111*a^3*e^(-4*I*c)*log(I*e^(I
*d*x + I*c) - 1) - 108*a^3*e^(8*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1)
- 432*a^3*e^(6*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 432*a^3*e^(2*I*
d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 648*a^3*e^(4*I*d*x)*log(-I*e^(I*
d*x + I*c) + 1) - 108*a^3*e^(-4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 111*a^3*
e^(8*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 444*a^3*e^(6*I*d*x + 2*I*
c)*log(-I*e^(I*d*x + I*c) - 1) - 444*a^3*e^(2*I*d*x - 2*I*c)*log(-I*e^(I*d*
x + I*c) - 1) - 666*a^3*e^(4*I*d*x)*log(-I*e^(I*d*x + I*c) - 1) - 111*a^3*e
^(-4*I*c)*log(-I*e^(I*d*x + I*c) - 1) + 3*a^3*e^(8*I*d*x + 4*I*c)*log(I*e^(
I*d*x) + e^(-I*c)) + 12*a^3*e^(6*I*d*x + 2*I*c)*log(I*e^(I*d*x) + e^(-I*c))
 + 12*a^3*e^(2*I*d*x - 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 18*a^3*e^(4*I*d
*x)*log(I*e^(I*d*x) + e^(-I*c)) + 3*a^3*e^(-4*I*c)*log(I*e^(I*d*x) + e^(-I*
c)) - 3*a^3*e^(8*I*d*x + 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 12*a^3*e^(6*
I*d*x + 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 12*a^3*e^(2*I*d*x - 2*I*c)*lo
g(-I*e^(I*d*x) + e^(-I*c)) - 18*a^3*e^(4*I*d*x)*log(-I*e^(I*d*x) + e^(-I*c)
) - 3*a^3*e^(-4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 128*I*a^3*e^(11*I*d*x +
 7*I*c) + 512*I*a^3*e^(9*I*d*x + 5*I*c) + 768*I*a^3*e^(7*I*d*x + 3*I*c) + 5
12*I*a^3*e^(5*I*d*x + I*c) + 128*I*a^3*e^(3*I*d*x - I*c))/(d*e^(8*I*d*x + 4
*I*c) + 4*d*e^(6*I*d*x + 2*I*c) + 4*d*e^(2*I*d*x - 2*I*c) + 6*d*e^(4*I*d*x)
 + d*e^(-4*I*c))
```

Mupad [B] (verification not implemented)

Time = 3.98 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.06

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$= -\frac{2\,a^3\left(3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}{3\,d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,3\mathrm{i}+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

```
[In] int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] -(2*a^3*(3*tan(c/2 + (d*x)/2)^2 - 1))/(3*d*(3*tan(c/2 + (d*x)/2) - tan(c/2
+ (d*x)/2)^2*3i - tan(c/2 + (d*x)/2)^3 + 1i))
```

3.49 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 88

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^3\,dx=-\frac{ia^3\cos^3(c+dx)}{15d}+\frac{a^3\sin(c+dx)}{5d}-\frac{a^3\sin^3(c+dx)}{15d}-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

```
[Out] -1/15*I*a^3*cos(d*x+c)^3/d+1/5*a^3*sin(d*x+c)/d-1/15*a^3*sin(d*x+c)^3/d-2/5
*I*a*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^2/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3577, 3567, 2713}

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^3\,dx=-\frac{a^3\sin^3(c+dx)}{15d}+\frac{a^3\sin(c+dx)}{5d}-\frac{ia^3\cos^3(c+dx)}{15d}-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^2}{5d}$$

```
[In] Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((-1/15*I)*a^3*Cos[c + d*x]^3)/d + (a^3*Sin[c + d*x])/(5*d) - (a^3*Sin[c +
d*x]^3)/(15*d) - (((2*I)/5)*a*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^2)/d
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^2}{5d}+\frac{1}{5}a^2\int\cos^3(c+dx)(a+ia\tan(c+dx))\,dx\\
&= -\frac{ia^3\cos^3(c+dx)}{15d}-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^2}{5d}+\frac{1}{5}a^3\int\cos^3(c+dx)\,dx\\
&= -\frac{ia^3\cos^3(c+dx)}{15d}-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^2}{5d}\\
&\quad-\frac{a^3\text{Subst}\left(\int\left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{5d}\\
&= -\frac{ia^3\cos^3(c+dx)}{15d}+\frac{a^3\sin(c+dx)}{5d}-\frac{a^3\sin^3(c+dx)}{15d}-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^2}{5d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.65

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$=\frac{a^3(-i\cos(2(c+dx))+\sin(2(c+dx)))\left(\cos(c+dx)\left(4+25\sqrt{\cos^2(c+dx)}\right)+\left(4+3\sqrt{\cos^2(c+dx)}\right)\cos\right.}{60d\sqrt{\cos^2(c+}}$$

```
[In] Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*((-I)*Cos[2*(c + d*x)] + Sin[2*(c + d*x)])*(Cos[c + d*x]*(4 + 25*Sqrt[
Cos[c + d*x]^2]) + (4 + 3*Sqrt[Cos[c + d*x]^2])*Cos[3*(c + d*x)] - I*((4 +
5*Sqrt[Cos[c + d*x]^2])*Sin[c + d*x] + (4 - 3*Sqrt[Cos[c + d*x]^2])*Sin[3*(
c + d*x)])))/(60*d*Sqrt[Cos[c + d*x]^2])
```

Maple [A] (verified)

Time = 26.52 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

method	result
risch	$-\frac{i a^3 e^{5i(dx+c)}}{20d}-\frac{i a^3 e^{3i(dx+c)}}{6d}-\frac{i a^3 e^{i(dx+c)}}{4d}$
derivativedivides	$\frac{-i a^3\left(-\frac{\left(\cos^3(dx+c)\right)\left(\sin^2(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)-3a^3\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)-\frac{3ia^3\left(\cos\right.}{}}{d}$
default	$\frac{-i a^3\left(-\frac{\left(\cos^3(dx+c)\right)\left(\sin^2(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)-3a^3\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)-\frac{3ia^3\left(\cos\right.}{}}{d}$

```
[In] int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/20*I/d*a^3*exp(5*I*(d*x+c))-1/6*I/d*a^3*exp(3*I*(d*x+c))-1/4*I/d*a^3*exp
(I*(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.55

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{-3i\,a^3e^{(5i\,dx+5i\,c)} - 10i\,a^3e^{(3i\,dx+3i\,c)} - 15i\,a^3e^{(i\,dx+i\,c)}}{60\,d}$$

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
[Out] 1/60*(-3*I*a^3*e^(5*I*d*x + 5*I*c) - 10*I*a^3*e^(3*I*d*x + 3*I*c) - 15*I*a^
3*e^(I*d*x + I*c))/d
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.32

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^3\,dx = \begin{cases} \frac{-24ia^3d^2e^{5ic}e^{5idx}-80ia^3d^2e^{3ic}e^{3idx}-120ia^3d^2e^{ic}e^{idx}}{480d^3} & \text{for } d^3\neq 0 \\ x\left(\frac{a^3e^{5ic}}{4}+\frac{a^3e^{3ic}}{2}+\frac{a^3e^{ic}}{4}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**3,x)
[Out] Piecewise(((-24*I*a**3*d**2*exp(5*I*c)*exp(5*I*d*x) - 80*I*a**3*d**2*exp(3*
I*c)*exp(3*I*d*x) - 120*I*a**3*d**2*exp(I*c)*exp(I*d*x))/(480*d**3), Ne(d**
3, 0)), (x*(a**3*exp(5*I*c)/4 + a**3*exp(3*I*c)/2 + a**3*exp(I*c)/4), True)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.19

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{9i\,a^3\cos(dx+c)^5 + i\left(3\cos(dx+c)^5 - 5\cos(dx+c)^3\right)a^3 - 3\left(3\sin(dx+c)^5 - 5\sin(dx+c)^3\right)a^3 -}{15\,d}$$

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
[Out] -1/15*(9*I*a^3*cos(d*x + c)^5 + I*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a^3
 - 3*(3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*a^3 - (3*sin(d*x + c)^5 - 10*sin
(d*x + c)^3 + 15*sin(d*x + c))*a^3)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 929 vs. 2(74) = 148.

Time = 0.85 (sec) , antiderivative size = 929, normalized size of antiderivative = 10.56

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^3\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/7680*(1785*a^3*e^(8*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 7140*a^3*
e^(6*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 7140*a^3*e^(2*I*d*x - 2*I*
c)*log(I*e^(I*d*x + I*c) + 1) + 10710*a^3*e^(4*I*d*x)*log(I*e^(I*d*x + I*c)
 + 1) + 1785*a^3*e^(-4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1530*a^3*e^(8*I*d*
x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 6120*a^3*e^(6*I*d*x + 2*I*c)*log(I*
e^(I*d*x + I*c) - 1) + 6120*a^3*e^(2*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) -
 1) + 9180*a^3*e^(4*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 1530*a^3*e^(-4*I*c)
*log(I*e^(I*d*x + I*c) - 1) - 1785*a^3*e^(8*I*d*x + 4*I*c)*log(-I*e^(I*d*x
+ I*c) + 1) - 7140*a^3*e^(6*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 71
40*a^3*e^(2*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 10710*a^3*e^(4*I*d
*x)*log(-I*e^(I*d*x + I*c) + 1) - 1785*a^3*e^(-4*I*c)*log(-I*e^(I*d*x + I*c
) + 1) - 1530*a^3*e^(8*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 6120*a^
3*e^(6*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 6120*a^3*e^(2*I*d*x - 2
*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 9180*a^3*e^(4*I*d*x)*log(-I*e^(I*d*x +
I*c) - 1) - 1530*a^3*e^(-4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 255*a^3*e^(8*
I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 1020*a^3*e^(6*I*d*x + 2*I*c)*l
og(I*e^(I*d*x) + e^(-I*c)) - 1020*a^3*e^(2*I*d*x - 2*I*c)*log(I*e^(I*d*x) +
 e^(-I*c)) - 1530*a^3*e^(4*I*d*x)*log(I*e^(I*d*x) + e^(-I*c)) - 255*a^3*e^(
-4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 255*a^3*e^(8*I*d*x + 4*I*c)*log(-I*e^
(I*d*x) + e^(-I*c)) + 1020*a^3*e^(6*I*d*x + 2*I*c)*log(-I*e^(I*d*x) + e^(-I
*c)) + 1020*a^3*e^(2*I*d*x - 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 1530*a^3
*e^(4*I*d*x)*log(-I*e^(I*d*x) + e^(-I*c)) + 255*a^3*e^(-4*I*c)*log(-I*e^(I*
d*x) + e^(-I*c)) - 384*I*a^3*e^(13*I*d*x + 9*I*c) - 2816*I*a^3*e^(11*I*d*x
+ 7*I*c) - 9344*I*a^3*e^(9*I*d*x + 5*I*c) - 16896*I*a^3*e^(7*I*d*x + 3*I*c)
 - 17024*I*a^3*e^(5*I*d*x + I*c) - 8960*I*a^3*e^(3*I*d*x - I*c) - 1920*I*a^
3*e^(I*d*x - 3*I*c))/(d*e^(8*I*d*x + 4*I*c) + 4*d*e^(6*I*d*x + 2*I*c) + 4*d
*e^(2*I*d*x - 2*I*c) + 6*d*e^(4*I*d*x) + d*e^(-4*I*c))
```

Mupad [B] (verification not implemented)

Time = 4.79 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.48

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$=\frac{2\,a^3\,\left(15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,30\mathrm{i}-40\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,20\mathrm{i}+7\right)}{15\,d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,5\mathrm{i}-10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,10\mathrm{i}+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

```
[In] int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] (2*a^3*(tan(c/2 + (d*x)/2)^3*30i - 40*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x
)/2)*20i + 15*tan(c/2 + (d*x)/2)^4 + 7))/(15*d*(5*tan(c/2 + (d*x)/2) - tan(
c/2 + (d*x)/2)^2*10i - 10*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4*5i +
tan(c/2 + (d*x)/2)^5 + 1i))
```

3.50 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 106

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{3ia^3\cos^5(c+dx)}{35d}+\frac{3a^3\sin(c+dx)}{7d}-\frac{2a^3\sin^3(c+dx)}{7d}+\frac{3a^3\sin^5(c+dx)}{35d}-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^2}{7d}$$

```
[Out] -3/35*I*a^3*cos(d*x+c)^5/d+3/7*a^3*sin(d*x+c)/d-2/7*a^3*sin(d*x+c)^3/d+3/35
*a^3*sin(d*x+c)^5/d-2/7*I*a*cos(d*x+c)^7*(a+I*a*tan(d*x+c))^2/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3577, 3567, 2713}

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{3a^3\sin^5(c+dx)}{35d}-\frac{2a^3\sin^3(c+dx)}{7d}+\frac{3a^3\sin(c+dx)}{7d}-\frac{3ia^3\cos^5(c+dx)}{35d}-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^2}{7d}$$

```
[In] Int[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (((-3*I)/35)*a^3*Cos[c + d*x]^5)/d + (3*a^3*Sin[c + d*x])/(7*d) - (2*a^3*Si
n[c + d*x]^3)/(7*d) + (3*a^3*Sin[c + d*x]^5)/(35*d) - (((2*I)/7)*a*Cos[c +
d*x]^7*(a + I*a*Tan[c + d*x])^2)/d
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^2}{7d} \\
&\quad + \frac{1}{7}\left(3a^2\right)\int \cos^5(c+dx)(a+ia\tan(c+dx))\,dx \\
&= -\frac{3ia^3\cos^5(c+dx)}{35d} - \frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^2}{7d} + \frac{1}{7}\left(3a^3\right)\int \cos^5(c+dx)\,dx \\
&= -\frac{3ia^3\cos^5(c+dx)}{35d} - \frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^2}{7d} \\
&\quad - \frac{\left(3a^3\right)\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, -\sin(c+dx)\right)}{7d} \\
&= -\frac{3ia^3\cos^5(c+dx)}{35d} + \frac{3a^3\sin(c+dx)}{7d} - \frac{2a^3\sin^3(c+dx)}{7d} \\
&\quad + \frac{3a^3\sin^5(c+dx)}{35d} - \frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^2}{7d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.70

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$=\frac{a^3(-i\cos(3(c+dx))+\sin(3(c+dx)))\left(35\sqrt{\cos^2(c+dx)}+\left(8+84\sqrt{\cos^2(c+dx)}\right)\cos(2(c+dx))+\left(8-\right.\right.}{}$$

```
[In] Integrate[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*((-I)*Cos[3*(c + d*x)] + Sin[3*(c + d*x)])*(35*Sqrt[Cos[c + d*x]^2] +
(8 + 84*Sqrt[Cos[c + d*x]^2])*Cos[2*(c + d*x)] + (8 - 15*Sqrt[Cos[c + d*x]^
2])*Cos[4*(c + d*x)] - (8*I)*Sin[2*(c + d*x)] - (56*I)*Sqrt[Cos[c + d*x]^2]
*Sin[2*(c + d*x)] - (8*I)*Sin[4*(c + d*x)] + (20*I)*Sqrt[Cos[c + d*x]^2]*Si
n[4*(c + d*x)]))/(280*d*Sqrt[Cos[c + d*x]^2])
```

Maple [A] (verified)

Time = 81.92 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.80

method	result
risch	$-\frac{ia^3 e^{7i(dx+c)}}{112d}-\frac{ia^3 e^{5i(dx+c)}}{20d}-\frac{ia^3 e^{3i(dx+c)}}{8d}-\frac{3ia^3\cos(dx+c)}{16d}+\frac{5a^3\sin(dx+c)}{16d}$
derivativedivides	$\frac{-ia^3\left(-\frac{\left(\cos^5(dx+c)\right)\left(\sin^2(dx+c)\right)}{7}-\frac{2\left(\cos^5(dx+c)\right)}{35}\right)-3a^3\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin}{35}\right.}{d}$
default	$\frac{-ia^3\left(-\frac{\left(\cos^5(dx+c)\right)\left(\sin^2(dx+c)\right)}{7}-\frac{2\left(\cos^5(dx+c)\right)}{35}\right)-3a^3\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin}{35}\right.}{d}$

```
[In] int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/112*I/d*a^3*exp(7*I*(d*x+c))-1/20*I/d*a^3*exp(5*I*(d*x+c))-1/8*I/d*a^3*e
xp(3*I*(d*x+c))-3/16*I/d*a^3*cos(d*x+c)+5/16*a^3*sin(d*x+c)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.72

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$=\frac{\left(-5i\,a^3e^{(8i\,dx+8i\,c)}-28i\,a^3e^{(6i\,dx+6i\,c)}-70i\,a^3e^{(4i\,dx+4i\,c)}-140i\,a^3e^{(2i\,dx+2i\,c)}+35i\,a^3\right)e^{(-i\,dx-i\,c)}}{560\,d}$$

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/560*(-5*I*a^3*e^(8*I*d*x + 8*I*c) - 28*I*a^3*e^(6*I*d*x + 6*I*c) - 70*I*a
^3*e^(4*I*d*x + 4*I*c) - 140*I*a^3*e^(2*I*d*x + 2*I*c) + 35*I*a^3)*e^(-I*d*
x - I*c)/d
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.79

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$=\begin{cases}\frac{\left(-10240ia^3d^4e^{8ic}e^{7idx}-57344ia^3d^4e^{6ic}e^{5idx}-143360ia^3d^4e^{4ic}e^{3idx}-286720ia^3d^4e^{2ic}e^{idx}+71680ia^3d^4e^{-idx}\right)e^{-ic}}{1146880d^5} & \text{for } d^5e^{ic}\neq 0\\ \frac{x\left(a^3e^{8ic}+4a^3e^{6ic}+6a^3e^{4ic}+4a^3e^{2ic}+a^3\right)e^{-ic}}{16} & \text{otherwise}\end{cases}$$

```
[In] integrate(cos(d*x+c)**7*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise(((-10240*I*a**3*d**4*exp(8*I*c)*exp(7*I*d*x) - 57344*I*a**3*d**4*
exp(6*I*c)*exp(5*I*d*x) - 143360*I*a**3*d**4*exp(4*I*c)*exp(3*I*d*x) - 2867
20*I*a**3*d**4*exp(2*I*c)*exp(I*d*x) + 71680*I*a**3*d**4*exp(-I*d*x))*exp(-
I*c)/(1146880*d**5), Ne(d**5*exp(I*c), 0)), (x*(a**3*exp(8*I*c) + 4*a**3*ex
p(6*I*c) + 6*a**3*exp(4*I*c) + 4*a**3*exp(2*I*c) + a**3)*exp(-I*c)/16, True
))
```

Maxima [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.16

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^3\,dx=$$
$$-\frac{15i\,a^3\cos\left(dx+c\right)^7+i\left(5\,\cos\left(dx+c\right)^7-7\,\cos\left(dx+c\right)^5\right)a^3+\left(15\,\sin\left(dx+c\right)^7-42\,\sin\left(dx+c\right)^5+3\right.}{35\,d}$$

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/35*(15*I*a^3*cos(d*x + c)^7 + I*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a^
3 + (15*sin(d*x + c)^7 - 42*sin(d*x + c)^5 + 35*sin(d*x + c)^3)*a^3 + (5*si
n(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin(d*x + c)^3 - 35*sin(d*x + c))*a^3
)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(90) = 180$.

Time = 0.94 (sec) , antiderivative size = 465, normalized size of antiderivative = 4.39

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{19635\,a^3e^{(5i\,dx+3i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+39270\,a^3e^{(3i\,dx+i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+19635\,a^3e^{(i\,dx-i\,c)}\log\left(i\,e^{(i\,\ldots}\right.}{}$$

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/71680*(19635*a^3*e^(5*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) + 1) + 39270*a
^3*e^(3*I*d*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 19635*a^3*e^(I*d*x - I*c)
*log(I*e^(I*d*x + I*c) + 1) + 19635*a^3*e^(5*I*d*x + 3*I*c)*log(I*e^(I*d*x
+ I*c) - 1) + 39270*a^3*e^(3*I*d*x + I*c)*log(I*e^(I*d*x + I*c) - 1) + 1963
5*a^3*e^(I*d*x - I*c)*log(I*e^(I*d*x + I*c) - 1) - 19635*a^3*e^(5*I*d*x + 3
*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 39270*a^3*e^(3*I*d*x + I*c)*log(-I*e^(I
*d*x + I*c) + 1) - 19635*a^3*e^(I*d*x - I*c)*log(-I*e^(I*d*x + I*c) + 1) -
19635*a^3*e^(5*I*d*x + 3*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 39270*a^3*e^(3*
I*d*x + I*c)*log(-I*e^(I*d*x + I*c) - 1) - 19635*a^3*e^(I*d*x - I*c)*log(-I
*e^(I*d*x + I*c) - 1) - 640*I*a^3*e^(12*I*d*x + 10*I*c) - 4864*I*a^3*e^(10*
I*d*x + 8*I*c) - 16768*I*a^3*e^(8*I*d*x + 6*I*c) - 39424*I*a^3*e^(6*I*d*x +
 4*I*c) - 40320*I*a^3*e^(4*I*d*x + 2*I*c) - 8960*I*a^3*e^(2*I*d*x) + 4480*I
*a^3*e^(-2*I*c))/(d*e^(5*I*d*x + 3*I*c) + 2*d*e^(3*I*d*x + I*c) + d*e^(I*d*
x - I*c))
```

Mupad [B] (verification not implemented)

Time = 5.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.26

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{2\,a^3\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{17\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}-\frac{17\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{2}+\frac{31\,\sin\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{2}-\frac{5\,\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{2}+\frac{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,35\mathrm{i}}{8}-\frac{\cos\left(\frac{3\,c}{2}\ldots\right.}{}\right.}{35\,d\,\left(\cos\left(3\,c+3\,d\,x\right)-\sin\left(3\,c+3\,d\,x\right)\,1\mathrm{i}\right)}$$

```
[In] int(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] -(2*a^3*cos(c/2 + (d*x)/2)*((cos(c/2 + (d*x)/2)*35i)/8 - (cos((3*c)/2 + (3*
d*x)/2)*35i)/8 + (cos((5*c)/2 + (5*d*x)/2)*119i)/8 - (cos((7*c)/2 + (7*d*x)
/2)*15i)/8 + (17*sin(c/2 + (d*x)/2))/2 - (17*sin((3*c)/2 + (3*d*x)/2))/2 +
(31*sin((5*c)/2 + (5*d*x)/2))/2 - (5*sin((7*c)/2 + (7*d*x)/2))/2))/(35*d*(c
os(3*c + 3*d*x) - sin(3*c + 3*d*x)*1i))
```

3.51 $\int \cos^9(c+dx)(a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 24, antiderivative size = 124

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^3\,dx=-\frac{5ia^3\cos^7(c+dx)}{63d}+\frac{5a^3\sin(c+dx)}{9d}-\frac{5a^3\sin^3(c+dx)}{9d}+\frac{a^3\sin^5(c+dx)}{3d}-\frac{5a^3\sin^7(c+dx)}{63d}-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^2}{9d}$$

```
[Out] -5/63*I*a^3*cos(d*x+c)^7/d+5/9*a^3*sin(d*x+c)/d-5/9*a^3*sin(d*x+c)^3/d+1/3*
a^3*sin(d*x+c)^5/d-5/63*a^3*sin(d*x+c)^7/d-2/9*I*a*cos(d*x+c)^9*(a+I*a*tan(
d*x+c))^2/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3577, 3567, 2713}

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^3\,dx=-\frac{5a^3\sin^7(c+dx)}{63d}+\frac{a^3\sin^5(c+dx)}{3d}-\frac{5a^3\sin^3(c+dx)}{9d}+\frac{5a^3\sin(c+dx)}{9d}-\frac{5ia^3\cos^7(c+dx)}{63d}-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^2}{9d}$$

```
[In] Int[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (((-5*I)/63)*a^3*Cos[c + d*x]^7)/d + (5*a^3*Sin[c + d*x])/(9*d) - (5*a^3*Si
n[c + d*x]^3)/(9*d) + (a^3*Sin[c + d*x]^5)/(3*d) - (5*a^3*Sin[c + d*x]^7)/(
63*d) - (((2*I)/9)*a*Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^2)/d
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^2}{9d} \\
&\quad +\frac{1}{9}\left(5a^2\right)\int\cos^7(c+dx)(a+ia\tan(c+dx))\,dx \\
&= -\frac{5ia^3\cos^7(c+dx)}{63d}-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^2}{9d}+\frac{1}{9}\left(5a^3\right)\int\cos^7(c+dx)\,dx \\
&= -\frac{5ia^3\cos^7(c+dx)}{63d}-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^2}{9d} \\
&\quad -\frac{\left(5a^3\right)\text{Subst}\left(\int\left(1-3x^2+3x^4-x^6\right)\,dx,x,-\sin(c+dx)\right)}{9d} \\
&= -\frac{5ia^3\cos^7(c+dx)}{63d}+\frac{5a^3\sin(c+dx)}{9d}-\frac{5a^3\sin^3(c+dx)}{9d}+\frac{a^3\sin^5(c+dx)}{3d} \\
&\quad -\frac{5a^3\sin^7(c+dx)}{63d}-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^2}{9d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.82

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^3\,dx$$
$$= \frac{a^3(-i\cos(3(c+dx))+\sin(3(c+dx)))\left(210\sqrt{\cos^2(c+dx)}+\left(32+567\sqrt{\cos^2(c+dx)}\right)\cos(2(c+dx))+\right.}{}$$

```
[In] Integrate[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*((-I)*Cos[3*(c + d*x)] + Sin[3*(c + d*x)])*(210*Sqrt[Cos[c + d*x]^2] +
 (32 + 567*Sqrt[Cos[c + d*x]^2])*Cos[2*(c + d*x)] + (32 - 162*Sqrt[Cos[c +
d*x]^2])*Cos[4*(c + d*x)] - 7*Sqrt[Cos[c + d*x]^2]*Cos[6*(c + d*x)] - (32*I
)*Sin[2*(c + d*x)] - (378*I)*Sqrt[Cos[c + d*x]^2]*Sin[2*(c + d*x)] - (32*I)
*Sin[4*(c + d*x)] + (216*I)*Sqrt[Cos[c + d*x]^2]*Sin[4*(c + d*x)] + (14*I)*
Sqrt[Cos[c + d*x]^2]*Sin[6*(c + d*x)]))/(2016*d*Sqrt[Cos[c + d*x]^2])
```

Maple [A] (verified)

Time = 215.94 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.97

method	result
risch	$-\frac{i a^3 e^{9i(dx+c)}}{576d}-\frac{3i a^3 e^{7i(dx+c)}}{224d}-\frac{3i a^3 e^{5i(dx+c)}}{64d}-\frac{9i a^3\cos(dx+c)}{64d}+\frac{21a^3\sin(dx+c)}{64d}-\frac{19i a^3\cos(3dx+3c)}{192d}+\frac{7}{}$
derivativedivides	$\frac{-i a^3\left(-\frac{(\cos^7(dx+c))(\sin^2(dx+c))}{9}-\frac{2(\cos^7(dx+c))}{63}\right)-3a^3\left(-\frac{(\cos^8(dx+c))\sin(dx+c)}{9}+\frac{\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6(\cos^4(dx+c))}{5}+\frac{8}{}\right.}{63}\right.}{}$
default	$\frac{-i a^3\left(-\frac{(\cos^7(dx+c))(\sin^2(dx+c))}{9}-\frac{2(\cos^7(dx+c))}{63}\right)-3a^3\left(-\frac{(\cos^8(dx+c))\sin(dx+c)}{9}+\frac{\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6(\cos^4(dx+c))}{5}+\frac{8}{}\right.}{63}\right.}{}$

```
[In] int(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/576*I/d*a^3*exp(9*I*(d*x+c))-3/224*I/d*a^3*exp(7*I*(d*x+c))-3/64*I/d*a^3
*exp(5*I*(d*x+c))-9/64*I/d*a^3*cos(d*x+c)+21/64*a^3*sin(d*x+c)/d-19/192*I/d
*a^3*cos(3*d*x+3*c)+7/64/d*a^3*sin(3*d*x+3*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.84

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{\left(-7i\,a^3e^{(12i\,dx+12i\,c)}-54i\,a^3e^{(10i\,dx+10i\,c)}-189i\,a^3e^{(8i\,dx+8i\,c)}-420i\,a^3e^{(6i\,dx+6i\,c)}-945i\,a^3e^{(4i\,dx+4i\,c)}+378\right.}{4032\,d}$$

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4032*(-7*I*a^3*e^(12*I*d*x + 12*I*c) - 54*I*a^3*e^(10*I*d*x + 10*I*c) - 1
89*I*a^3*e^(8*I*d*x + 8*I*c) - 420*I*a^3*e^(6*I*d*x + 6*I*c) - 945*I*a^3*e^
(4*I*d*x + 4*I*c) + 378*I*a^3*e^(2*I*d*x + 2*I*c) + 21*I*a^3)*e^(-3*I*d*x -
 3*I*c)/d
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(112) = 224$.

Time = 0.42 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.22

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^3\,dx = \begin{cases} \frac{\left(-270582939648ia^3d^6e^{13ic}e^{9idx}-2087354105856ia^3d^6e^{11ic}e^{7idx}-7305739370496ia^3d^6e^{9ic}e^{5idx}-16234976378880ia^3d^6e^{7ic}e^{3idx}-36528696\right.}{155855773237248d^7} \\ \frac{x\left(a^3e^{12ic}+6a^3e^{10ic}+15a^3e^{8ic}+20a^3e^{6ic}+15a^3e^{4ic}+6a^3e^{2ic}+a^3\right)e^{-3ic}}{64} \end{cases}$$

```
[In] integrate(cos(d*x+c)**9*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise(((-270582939648*I*a**3*d**6*exp(13*I*c)*exp(9*I*d*x) - 2087354105
856*I*a**3*d**6*exp(11*I*c)*exp(7*I*d*x) - 7305739370496*I*a**3*d**6*exp(9*
I*c)*exp(5*I*d*x) - 16234976378880*I*a**3*d**6*exp(7*I*c)*exp(3*I*d*x) - 36
528696852480*I*a**3*d**6*exp(5*I*c)*exp(I*d*x) + 14611478740992*I*a**3*d**6
*exp(3*I*c)*exp(-I*d*x) + 811748818944*I*a**3*d**6*exp(I*c)*exp(-3*I*d*x))*
exp(-4*I*c)/(155855773237248*d**7), Ne(d**7*exp(4*I*c), 0)), (x*(a**3*exp(1
2*I*c) + 6*a**3*exp(10*I*c) + 15*a**3*exp(8*I*c) + 20*a**3*exp(6*I*c) + 15*
a**3*exp(4*I*c) + 6*a**3*exp(2*I*c) + a**3)*exp(-3*I*c)/64, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.17

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^3\,dx = -\frac{105i\,a^3\cos{(dx+c)}^9+5i\left(7\,\cos{(dx+c)}^9-9\,\cos{(dx+c)}^7\right)a^3-3\left(35\,\sin{(dx+c)}^9-135\,\sin{(dx+c)}^7\right.}{}$$

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/315*(105*I*a^3*cos(d*x + c)^9 + 5*I*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7
)*a^3 - 3*(35*sin(d*x + c)^9 - 135*sin(d*x + c)^7 + 189*sin(d*x + c)^5 - 10
5*sin(d*x + c)^3)*a^3 - (35*sin(d*x + c)^9 - 180*sin(d*x + c)^7 + 378*sin(d
*x + c)^5 - 420*sin(d*x + c)^3 + 315*sin(d*x + c))*a^3)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1039 vs. 2(106) = 212.

Time = 0.78 (sec) , antiderivative size = 1039, normalized size of antiderivative = 8.38

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^3\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/516096*(119511*a^3*e^(11*I*d*x + 5*I*c)*log(I*e^(I*d*x + I*c) + 1) + 4780
44*a^3*e^(9*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) + 1) + 717066*a^3*e^(7*I*d
*x + I*c)*log(I*e^(I*d*x + I*c) + 1) + 478044*a^3*e^(5*I*d*x - I*c)*log(I*e
^(I*d*x + I*c) + 1) + 119511*a^3*e^(3*I*d*x - 3*I*c)*log(I*e^(I*d*x + I*c)
+ 1) + 128898*a^3*e^(11*I*d*x + 5*I*c)*log(I*e^(I*d*x + I*c) - 1) + 515592*
a^3*e^(9*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) - 1) + 773388*a^3*e^(7*I*d*x
+ I*c)*log(I*e^(I*d*x + I*c) - 1) + 515592*a^3*e^(5*I*d*x - I*c)*log(I*e^(I
*d*x + I*c) - 1) + 128898*a^3*e^(3*I*d*x - 3*I*c)*log(I*e^(I*d*x + I*c) - 1
) - 119511*a^3*e^(11*I*d*x + 5*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 478044*a^
3*e^(9*I*d*x + 3*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 717066*a^3*e^(7*I*d*x +
 I*c)*log(-I*e^(I*d*x + I*c) + 1) - 478044*a^3*e^(5*I*d*x - I*c)*log(-I*e^(
I*d*x + I*c) + 1) - 119511*a^3*e^(3*I*d*x - 3*I*c)*log(-I*e^(I*d*x + I*c) +
 1) - 128898*a^3*e^(11*I*d*x + 5*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 515592*
a^3*e^(9*I*d*x + 3*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 773388*a^3*e^(7*I*d*x
 + I*c)*log(-I*e^(I*d*x + I*c) - 1) - 515592*a^3*e^(5*I*d*x - I*c)*log(-I*e
^(I*d*x + I*c) - 1) - 128898*a^3*e^(3*I*d*x - 3*I*c)*log(-I*e^(I*d*x + I*c)
```

```
 - 1) + 9387*a^3*e^(11*I*d*x + 5*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 37548*a
^3*e^(9*I*d*x + 3*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 56322*a^3*e^(7*I*d*x +
 I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 37548*a^3*e^(5*I*d*x - I*c)*log(I*e^(I*
d*x) + e^(-I*c)) + 9387*a^3*e^(3*I*d*x - 3*I*c)*log(I*e^(I*d*x) + e^(-I*c))
 - 9387*a^3*e^(11*I*d*x + 5*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 37548*a^3*e
^(9*I*d*x + 3*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 56322*a^3*e^(7*I*d*x + I*
c)*log(-I*e^(I*d*x) + e^(-I*c)) - 37548*a^3*e^(5*I*d*x - I*c)*log(-I*e^(I*d
*x) + e^(-I*c)) - 9387*a^3*e^(3*I*d*x - 3*I*c)*log(-I*e^(I*d*x) + e^(-I*c))
 - 896*I*a^3*e^(20*I*d*x + 14*I*c) - 10496*I*a^3*e^(18*I*d*x + 12*I*c) - 57
216*I*a^3*e^(16*I*d*x + 10*I*c) - 195584*I*a^3*e^(14*I*d*x + 8*I*c) - 50969
6*I*a^3*e^(12*I*d*x + 6*I*c) - 861696*I*a^3*e^(10*I*d*x + 4*I*c) - 768768*I
*a^3*e^(8*I*d*x + 2*I*c) + 88704*I*a^3*e^(4*I*d*x - 2*I*c) + 59136*I*a^3*e^
(2*I*d*x - 4*I*c) - 236544*I*a^3*e^(6*I*d*x) + 2688*I*a^3*e^(-6*I*c))/(d*e^
(11*I*d*x + 5*I*c) + 4*d*e^(9*I*d*x + 3*I*c) + 6*d*e^(7*I*d*x + I*c) + 4*d*
e^(5*I*d*x - I*c) + d*e^(3*I*d*x - 3*I*c))
```

Mupad [B] (verification not implemented)

Time = 5.73 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.66

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^3\,dx = \frac{2a^3\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-3\mathrm{i}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)} + \frac{2048a^3\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\mathrm{i}\right)}{9d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^9} - \frac{1024a^3\left(8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-9\mathrm{i}\right)}{9d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^8} - \frac{4a^3\left(14\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-39\mathrm{i}\right)}{3d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^2} + \frac{8a^3\left(43\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-97\mathrm{i}\right)}{3d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^3} - \frac{16a^3\left(188\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-357\mathrm{i}\right)}{7d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^4} + \frac{128a^3\left(263\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-333\mathrm{i}\right)}{21d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^7} - \frac{64a^3\left(1598\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-2289\mathrm{i}\right)}{63d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^6} + \frac{32a^3\left(2041\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-3339\mathrm{i}\right)}{63d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^5}$$

```
[In] int(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] (2*a^3*(tan(c/2 + (d*x)/2) - 3i))/(d*(tan(c/2 + (d*x)/2)^2 + 1)) + (2048*a^
3*(tan(c/2 + (d*x)/2) - 1i))/(9*d*(tan(c/2 + (d*x)/2)^2 + 1)^9) - (1024*a^3
*(8*tan(c/2 + (d*x)/2) - 9i))/(9*d*(tan(c/2 + (d*x)/2)^2 + 1)^8) - (4*a^3*(
14*tan(c/2 + (d*x)/2) - 39i))/(3*d*(tan(c/2 + (d*x)/2)^2 + 1)^2) + (8*a^3*(
43*tan(c/2 + (d*x)/2) - 97i))/(3*d*(tan(c/2 + (d*x)/2)^2 + 1)^3) - (16*a^3*
(188*tan(c/2 + (d*x)/2) - 357i))/(7*d*(tan(c/2 + (d*x)/2)^2 + 1)^4) + (128*
a^3*(263*tan(c/2 + (d*x)/2) - 333i))/(21*d*(tan(c/2 + (d*x)/2)^2 + 1)^7) -
(64*a^3*(1598*tan(c/2 + (d*x)/2) - 2289i))/(63*d*(tan(c/2 + (d*x)/2)^2 + 1)
^6) + (32*a^3*(2041*tan(c/2 + (d*x)/2) - 3339i))/(63*d*(tan(c/2 + (d*x)/2)^
2 + 1)^5)
```

3.52 $\int \sec^3(c+dx)(a+ia\tan(c+dx))^4\,dx$

Optimal result

Integrand size = 24, antiderivative size = 163

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{21a^4\text{arctanh}(\sin(c+dx))}{16d}+\frac{7ia^4\sec^3(c+dx)}{8d}+\frac{21a^4\sec(c+dx)\tan(c+dx)}{16d}+\frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d}+\frac{3i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{10d}+\frac{21i\sec^3(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{40d}$$

```
[Out] 21/16*a^4*arctanh(sin(d*x+c))/d+7/8*I*a^4*sec(d*x+c)^3/d+21/16*a^4*sec(d*x+
c)*tan(d*x+c)/d+1/6*I*a*sec(d*x+c)^3*(a+I*a*tan(d*x+c))^3/d+3/10*I*sec(d*x+
c)^3*(a^2+I*a^2*tan(d*x+c))^2/d+21/40*I*sec(d*x+c)^3*(a^4+I*a^4*tan(d*x+c))
/d
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used

$= \{3579, 3567, 3853, 3855\}$

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{21a^4\text{arctanh}(\sin(c+dx))}{16d} + \frac{7ia^4\sec^3(c+dx)}{8d} + \frac{21i\sec^3(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{40d} + \frac{21a^4\tan(c+dx)\sec(c+dx)}{16d} + \frac{3i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{10d} + \frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d}$$

```
[In] Int[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (21*a^4*ArcTanh[Sin[c + d*x]])/(16*d) + (((7*I)/8)*a^4*Sec[c + d*x]^3)/d +
(21*a^4*Sec[c + d*x]*Tan[c + d*x])/(16*d) + ((I/6)*a*Sec[c + d*x]^3*(a + I*
a*Tan[c + d*x])^3)/d + (((3*I)/10)*Sec[c + d*x]^3*(a^2 + I*a^2*Tan[c + d*x]
)^2)/d + (((21*I)/40)*Sec[c + d*x]^3*(a^4 + I*a^4*Tan[c + d*x]))/d
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d} + \frac{1}{2}(3a)\int \sec^3(c+dx)(a+ia\tan(c+dx))^3\,dx$$

$$= \frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d} + \frac{3i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{10d} + \frac{1}{10}\left(21a^2\right)\int \sec^3(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$= \frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d} + \frac{3i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{10d} + \frac{21i\sec^3(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{40d} + \frac{1}{8}\left(21a^3\right)\int \sec^3(c+dx)(a+ia\tan(c+dx))\,dx$$

$$= \frac{7ia^4\sec^3(c+dx)}{8d} + \frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d} + \frac{3i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{10d} + \frac{21i\sec^3(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{40d} + \frac{1}{8}\left(21a^4\right)\int \sec^3(c+dx)\,dx$$

$$= \frac{7ia^4\sec^3(c+dx)}{8d} + \frac{21a^4\sec(c+dx)\tan(c+dx)}{16d} + \frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d} + \frac{3i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{10d} + \frac{21i\sec^3(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{40d} + \frac{1}{16}\left(21a^4\right)\int \sec(c+dx)\,dx$$

$$= \frac{21a^4\text{arctanh}(\sin(c+dx))}{16d} + \frac{7ia^4\sec^3(c+dx)}{8d} + \frac{21a^4\sec(c+dx)\tan(c+dx)}{16d} + \frac{ia\sec^3(c+dx)(a+ia\tan(c+dx))^3}{6d} + \frac{3i\sec^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{10d} + \frac{21i\sec^3(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{40d}$$

Mathematica [A] (verified)

Time = 2.05 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.05

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^4\,dx = -\frac{a^4\sec^2(c+dx)(\cos(4c)-i\sin(4c))\left(-4608i\cos(c+dx)+5040\cos^6(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\right.\right.\right.}{}$$

```
[In] Integrate[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] -1/3840*(a^4*Sec[c + d*x]^2*(Cos[4*c] - I*Sin[4*c])*((-4608*I)*Cos[c + d*x]
 + 5040*Cos[c + d*x]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]]) + 5*((-512*I)*Cos[3*(c + d*x)] + 90*Sin[c
 + d*x] + 155*Sin[3*(c + d*x)] - 63*Sin[5*(c + d*x)]))*(-I + Tan[c + d*x])^
4)/(d*(Cos[d*x] + I*Sin[d*x])^4)
```

Maple [A] (verified)

Time = 20.18 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.82

method	result
risch	$-\frac{i a^4\left(315\,e^{11i(dx+c)}-3335\,e^{9i(dx+c)}-5058\,e^{7i(dx+c)}-4158\,e^{5i(dx+c)}-1785\,e^{3i(dx+c)}-315\,e^{i(dx+c)}\right)}{120d\left(e^{2i(dx+c)}+1\right)^6}+\frac{21a^4\ln\left(e^{i(dx+c}\right.}{16d}$
derivativedivides	$\frac{a^4\left(\frac{\sin^5(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^5(dx+c)}{24\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{48\cos(dx+c)^2}-\frac{\left(\sin^3(dx+c)\right)}{48}-\frac{\sin(dx+c)}{16}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)-4ia^4\left(\frac{\sin^4(dx+c)}{5\cos(dx+c)^5}\right.}{}$
default	$\frac{a^4\left(\frac{\sin^5(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^5(dx+c)}{24\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{48\cos(dx+c)^2}-\frac{\left(\sin^3(dx+c)\right)}{48}-\frac{\sin(dx+c)}{16}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)-4ia^4\left(\frac{\sin^4(dx+c)}{5\cos(dx+c)^5}\right.}{}$

```
[In] int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/120*I*a^4/d/(exp(2*I*(d*x+c))+1)^6*(315*exp(11*I*(d*x+c))-3335*exp(9*I*(
d*x+c))-5058*exp(7*I*(d*x+c))-4158*exp(5*I*(d*x+c))-1785*exp(3*I*(d*x+c))-3
15*exp(I*(d*x+c)))+21/16*a^4/d*ln(exp(I*(d*x+c))+I)-21/16*a^4/d*ln(exp(I*(d
*x+c))-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 364 vs. 2(137) = 274.

Time = 0.24 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.23

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{-630i\,a^4e^{(11i\,dx+11i\,c)}+6670i\,a^4e^{(9i\,dx+9i\,c)}+10116i\,a^4e^{(7i\,dx+7i\,c)}+8316i\,a^4e^{(5i\,dx+5i\,c)}+3570i\,a^4e^{(3i\,dx+3i\,c)}}{}$$

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/240*(-630*I*a^4*e^(11*I*d*x + 11*I*c) + 6670*I*a^4*e^(9*I*d*x + 9*I*c) +
10116*I*a^4*e^(7*I*d*x + 7*I*c) + 8316*I*a^4*e^(5*I*d*x + 5*I*c) + 3570*I*a
^4*e^(3*I*d*x + 3*I*c) + 630*I*a^4*e^(I*d*x + I*c) + 315*(a^4*e^(12*I*d*x +
 12*I*c) + 6*a^4*e^(10*I*d*x + 10*I*c) + 15*a^4*e^(8*I*d*x + 8*I*c) + 20*a^
4*e^(6*I*d*x + 6*I*c) + 15*a^4*e^(4*I*d*x + 4*I*c) + 6*a^4*e^(2*I*d*x + 2*I
*c) + a^4)*log(e^(I*d*x + I*c) + I) - 315*(a^4*e^(12*I*d*x + 12*I*c) + 6*a^
4*e^(10*I*d*x + 10*I*c) + 15*a^4*e^(8*I*d*x + 8*I*c) + 20*a^4*e^(6*I*d*x +
6*I*c) + 15*a^4*e^(4*I*d*x + 4*I*c) + 6*a^4*e^(2*I*d*x + 2*I*c) + a^4)*log(
e^(I*d*x + I*c) - I))/(d*e^(12*I*d*x + 12*I*c) + 6*d*e^(10*I*d*x + 10*I*c)
+ 15*d*e^(8*I*d*x + 8*I*c) + 20*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*I*d*x + 4
*I*c) + 6*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^4\,dx = a^4\left(\int\left(-6\tan^2(c+dx)\sec^3(c+dx)\right)dx + \int\tan^4(c+dx)\sec^3(c+dx)\,dx + \int 4i\tan(c+dx)\sec^3(c+dx)\,dx + \int\left(-4i\tan^3(c+dx)\sec^3(c+dx)\right)dx + \int\sec^3(c+dx)\,dx\right)$$

```
[In] integrate(sec(d*x+c)**3*(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] a**4*(Integral(-6*tan(c + d*x)**2*sec(c + d*x)**3, x) + Integral(tan(c + d*
x)**4*sec(c + d*x)**3, x) + Integral(4*I*tan(c + d*x)*sec(c + d*x)**3, x) +
 Integral(-4*I*tan(c + d*x)**3*sec(c + d*x)**3, x) + Integral(sec(c + d*x)*
*3, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.51

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^4\,dx = -\frac{5\,a^4\left(\frac{2\left(3\,\sin(dx+c)^5+8\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)+180\,a^4\left(\right.}{}$$

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/480*(5*a^4*(2*(3*sin(d*x + c)^5 + 8*sin(d*x + c)^3 - 3*sin(d*x + c))/(si
n(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(d*x + c
) + 1) + 3*log(sin(d*x + c) - 1)) + 180*a^4*(2*(sin(d*x + c)^3 + sin(d*x +
c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - log(sin(d*x + c) + 1) + log(s
in(d*x + c) - 1)) + 120*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(
d*x + c) + 1) + log(sin(d*x + c) - 1)) - 640*I*a^4/cos(d*x + c)^3 - 128*I*(
5*cos(d*x + c)^2 - 3)*a^4/cos(d*x + c)^5)/d
```

Giac [A] (verification not implemented)

none

Time = 0.60 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.45

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{315\,a^4\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-315\,a^4\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)-\frac{2\left(75\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+960i\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}{}}{}$$

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/240*(315*a^4*log(tan(1/2*d*x + 1/2*c) + 1) - 315*a^4*log(tan(1/2*d*x + 1/
2*c) - 1) - 2*(75*a^4*tan(1/2*d*x + 1/2*c)^11 + 960*I*a^4*tan(1/2*d*x + 1/2
*c)^10 + 1175*a^4*tan(1/2*d*x + 1/2*c)^9 - 4800*I*a^4*tan(1/2*d*x + 1/2*c)^
8 - 1890*a^4*tan(1/2*d*x + 1/2*c)^7 + 4480*I*a^4*tan(1/2*d*x + 1/2*c)^6 - 1
890*a^4*tan(1/2*d*x + 1/2*c)^5 - 1920*I*a^4*tan(1/2*d*x + 1/2*c)^4 + 1175*a
^4*tan(1/2*d*x + 1/2*c)^3 + 1728*I*a^4*tan(1/2*d*x + 1/2*c)^2 + 75*a^4*tan(
1/2*d*x + 1/2*c) - 448*I*a^4)/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

Mupad [B] (verification not implemented)

Time = 8.39 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.78

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{21\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}$$
$$-\frac{\frac{5\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\,8\mathrm{i}+\frac{235\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}-a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}\,40\mathrm{i}-\frac{63\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{4}+\frac{a^4\,\tan}{}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.\right.}$$

```
[In] int((a + a*tan(c + d*x)*1i)^4/cos(c + d*x)^3,x)
```

```
[Out] (21*a^4*atanh(tan(c/2 + (d*x)/2)))/(8*d) - ((a^4*tan(c/2 + (d*x)/2)^2*72i)/
5 + (235*a^4*tan(c/2 + (d*x)/2)^3)/24 - a^4*tan(c/2 + (d*x)/2)^4*16i - (63*
a^4*tan(c/2 + (d*x)/2)^5)/4 + (a^4*tan(c/2 + (d*x)/2)^6*112i)/3 - (63*a^4*t
an(c/2 + (d*x)/2)^7)/4 - a^4*tan(c/2 + (d*x)/2)^8*40i + (235*a^4*tan(c/2 +
(d*x)/2)^9)/24 + a^4*tan(c/2 + (d*x)/2)^10*8i + (5*a^4*tan(c/2 + (d*x)/2)^1
1)/8 - (a^4*56i)/15 + (5*a^4*tan(c/2 + (d*x)/2))/8)/(d*(15*tan(c/2 + (d*x)/
2)^4 - 6*tan(c/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x
)/2)^8 - 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

3.53 $\int \sec(c+dx)(a+ia\tan(c+dx))^4\,dx$

Optimal result

Integrand size = 22, antiderivative size = 133

$$\int \sec(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{35a^4\text{arctanh}(\sin(c+dx))}{8d}+\frac{35ia^4\sec(c+dx)}{8d}+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d}+\frac{7i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{12d}+\frac{35i\sec(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{24d}$$

```
[Out] 35/8*a^4*arctanh(sin(d*x+c))/d+35/8*I*a^4*sec(d*x+c)/d+1/4*I*a*sec(d*x+c)*(
a+I*a*tan(d*x+c))^3/d+7/12*I*sec(d*x+c)*(a^2+I*a^2*tan(d*x+c))^2/d+35/24*I*
sec(d*x+c)*(a^4+I*a^4*tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3579, 3567, 3855}

$$\int \sec(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{35a^4\text{arctanh}(\sin(c+dx))}{8d}+\frac{35ia^4\sec(c+dx)}{8d}+\frac{35i\sec(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{24d}+\frac{7i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{12d}+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d}$$

```
[In] Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (35*a^4*ArcTanh[Sin[c + d*x]])/(8*d) + (((35*I)/8)*a^4*Sec[c + d*x])/d + ((
I/4)*a*Sec[c + d*x]*(a + I*a*Tan[c + d*x])^3)/d + (((7*I)/12)*Sec[c + d*x]*
(a^2 + I*a^2*Tan[c + d*x])^2)/d + (((35*I)/24)*Sec[c + d*x]*(a^4 + I*a^4*Ta
n[c + d*x]))/d
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d}+\frac{1}{4}(7a)\int \sec(c+dx)(a+ia\tan(c+dx))^3\,dx\\
&= \frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d}+\frac{7i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{12d}\\
&\quad+\frac{1}{12}\left(35a^2\right)\int \sec(c+dx)(a+ia\tan(c+dx))^2\,dx\\
&= \frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d}+\frac{7i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{12d}\\
&\quad+\frac{35i\sec(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{24d}+\frac{1}{8}\left(35a^3\right)\int \sec(c+dx)(a\\
&\quad+ia\tan(c+dx))\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{35ia^4\sec(c+dx)}{8d} + \frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d} \\
&\quad + \frac{7i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{12d} \\
&\quad + \frac{35i\sec(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{24d} + \frac{1}{8}\left(35a^4\right)\int \sec(c+dx)\,dx \\
&= \frac{35a^4\text{arctanh}(\sin(c+dx))}{8d} + \frac{35ia^4\sec(c+dx)}{8d} + \frac{ia\sec(c+dx)(a+ia\tan(c+dx))^3}{4d} \\
&\quad + \frac{7i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{12d} + \frac{35i\sec(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{24d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.53 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.78

$$
\int \sec(c+dx)(a+ia\tan(c+dx))^4\,dx = -\frac{a^4\sec^4(c+dx)\left(-896i\cos(c+dx)+3\left(-128i\cos(3(c+dx))+105\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx\right.\right.\right.\right.}{}
$$

```
[In] Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] -1/192*(a^4*Sec[c + d*x]^4*((-896*I)*Cos[c + d*x] + 3*((-128*I)*Cos[3*(c +
d*x)] + 105*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 35*Cos[4*(c + d*x)]*
Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 140*Cos[2*(c + d*x)]*(Log[Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])
- 105*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 35*Cos[4*(c + d*x)]*Log[Co
s[(c + d*x)/2] + Sin[(c + d*x)/2]] + 42*Sin[c + d*x] + 58*Sin[3*(c + d*x)])
))/d
```

Maple [A] (verified)

Time = 3.81 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.83

method	result
risch	$\frac{ia^4\left(279\,\mathrm{e}^{7i(dx+c)}+511\,\mathrm{e}^{5i(dx+c)}+385\,\mathrm{e}^{3i(dx+c)}+105\,\mathrm{e}^{i(dx+c)}\right)}{12d\left(\mathrm{e}^{2i(dx+c)}+1\right)^4}-\frac{35a^4\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{8d}+\frac{35a^4\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{8d}$
derivativedivides	$\frac{a^4\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{8\cos(dx+c)^2}-\frac{\left(\sin^3(dx+c)\right)}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-4ia^4\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\right.}{}$
default	$\frac{a^4\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{8\cos(dx+c)^2}-\frac{\left(\sin^3(dx+c)\right)}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-4ia^4\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\right.}{}$

```
[In] int(sec(d*x+c)*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*I*a^4/d/(exp(2*I*(d*x+c))+1)^4*(279*exp(7*I*(d*x+c))+511*exp(5*I*(d*x+
c))+385*exp(3*I*(d*x+c))+105*exp(I*(d*x+c)))-35/8*a^4/d*ln(exp(I*(d*x+c))-I
)+35/8*a^4/d*ln(exp(I*(d*x+c))+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 256 vs. 2(109) = 218.

Time = 0.25 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.92

$$\int \sec(c+dx)(a+ia\tan(c+dx))^4\,dx$$
$$= \frac{558i\,a^4e^{(7i\,dx+7i\,c)} + 1022i\,a^4e^{(5i\,dx+5i\,c)} + 770i\,a^4e^{(3i\,dx+3i\,c)} + 210i\,a^4e^{(i\,dx+i\,c)} + 105\left(a^4e^{(8i\,dx+8i\,c)} + 4\,a^4e^{(6i\ldots}\right.}{24\,(de^{(\ldots}}$$

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/24*(558*I*a^4*e^(7*I*d*x + 7*I*c) + 1022*I*a^4*e^(5*I*d*x + 5*I*c) + 770*
I*a^4*e^(3*I*d*x + 3*I*c) + 210*I*a^4*e^(I*d*x + I*c) + 105*(a^4*e^(8*I*d*x
 + 8*I*c) + 4*a^4*e^(6*I*d*x + 6*I*c) + 6*a^4*e^(4*I*d*x + 4*I*c) + 4*a^4*e
^(2*I*d*x + 2*I*c) + a^4)*log(e^(I*d*x + I*c) + I) - 105*(a^4*e^(8*I*d*x +
8*I*c) + 4*a^4*e^(6*I*d*x + 6*I*c) + 6*a^4*e^(4*I*d*x + 4*I*c) + 4*a^4*e^(2
*I*d*x + 2*I*c) + a^4)*log(e^(I*d*x + I*c) - I))/(d*e^(8*I*d*x + 8*I*c) + 4
*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c)
+ d)
```

Sympy [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^4\,dx = a^4\left(\int\left(-6\tan^2(c+dx)\sec(c+dx)\right)\,dx + \int\tan^4(c+dx)\sec(c+dx)\,dx + \int 4i\tan(c+dx)\sec(c+dx)\,dx + \int\left(-4i\tan^3(c+dx)\sec(c+dx)\right)\,dx + \int\sec(c+dx)\,dx\right)$$

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] a**4*(Integral(-6*tan(c + d*x)**2*sec(c + d*x), x) + Integral(tan(c + d*x)*
*4*sec(c + d*x), x) + Integral(4*I*tan(c + d*x)*sec(c + d*x), x) + Integral
(-4*I*tan(c + d*x)**3*sec(c + d*x), x) + Integral(sec(c + d*x), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.35

$$\int \sec(c+dx)(a+ia\tan(c+dx))^4\,dx$$
$$= \frac{3\,a^4\left(\frac{2\left(5\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}+3\,\log\left(\sin\left(dx+c\right)+1\right)-3\,\log\left(\sin\left(dx+c\right)-1\right)\right)+72\,a^4\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\mathrm{l}\right.}{}$$

[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")

[Out] 1/48*(3*a^4*(2*(5*sin(d*x + c)^3 - 3*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) + 3*log(sin(d*x + c) + 1) - 3*log(sin(d*x + c) - 1)) + 72*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 48*a^4*log(sec(d*x + c) + tan(d*x + c)) + 192*I*a^4/cos(d*x + c) + 64*I*(3*cos(d*x + c)^2 - 1)*a^4/cos(d*x + c)^3)/d

Giac [A] (verification not implemented)

none

Time = 0.56 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.30

$$\int \sec(c+dx)(a+ia\tan(c+dx))^4\,dx$$
$$= \frac{105\,a^4\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-105\,a^4\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)-\frac{2\left(81\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+96i\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6\right.}{}}{24\,d}$$

[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")

[Out] 1/24*(105*a^4*log(tan(1/2*d*x + 1/2*c) + 1) - 105*a^4*log(tan(1/2*d*x + 1/2*c) - 1) - 2*(81*a^4*tan(1/2*d*x + 1/2*c)^7 + 96*I*a^4*tan(1/2*d*x + 1/2*c)^6 - 105*a^4*tan(1/2*d*x + 1/2*c)^5 - 480*I*a^4*tan(1/2*d*x + 1/2*c)^4 - 105*a^4*tan(1/2*d*x + 1/2*c)^3 + 544*I*a^4*tan(1/2*d*x + 1/2*c)^2 + 81*a^4*tan(1/2*d*x + 1/2*c) - 160*I*a^4)/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d

Mupad [B] (verification not implemented)

Time = 7.60 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.49

$$\int \sec(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{35\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d} - \frac{\frac{27\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,8\mathrm{i}-\frac{35\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}-a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,40\mathrm{i}-\frac{35\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\frac{a^4\,\tan\left(\right.}{}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right.}$$

```
[In] int((a + a*tan(c + d*x)*1i)^4/cos(c + d*x),x)
```

```
[Out] (35*a^4*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((a^4*tan(c/2 + (d*x)/2)^2*136i)
/3 - (35*a^4*tan(c/2 + (d*x)/2)^3)/4 - a^4*tan(c/2 + (d*x)/2)^4*40i - (35*a
^4*tan(c/2 + (d*x)/2)^5)/4 + a^4*tan(c/2 + (d*x)/2)^6*8i + (27*a^4*tan(c/2
+ (d*x)/2)^7)/4 - (a^4*40i)/3 + (27*a^4*tan(c/2 + (d*x)/2))/4)/(d*(6*tan(c/
2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2
+ (d*x)/2)^8 + 1))
```

3.54 $\int \cos(c+dx)(a+ia\tan(c+dx))^4\,dx$

Optimal result

Integrand size = 22, antiderivative size = 97

$$\int \cos(c+dx)(a+ia\tan(c+dx))^4\,dx=-\frac{15a^4\text{arctanh}(\sin(c+dx))}{2d}-\frac{15ia^4\sec(c+dx)}{2d}-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^3}{d}-\frac{5i\sec(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{2d}$$

```
[Out] -15/2*a^4*arctanh(sin(d*x+c))/d-15/2*I*a^4*sec(d*x+c)/d-2*I*a*cos(d*x+c)*(a
+I*a*tan(d*x+c))^3/d-5/2*I*sec(d*x+c)*(a^4+I*a^4*tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3577, 3579, 3567, 3855}

$$\int \cos(c+dx)(a+ia\tan(c+dx))^4\,dx=-\frac{15a^4\text{arctanh}(\sin(c+dx))}{2d}-\frac{15ia^4\sec(c+dx)}{2d}-\frac{5i\sec(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{2d}-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^3}{d}$$

```
[In] Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (-15*a^4*ArcTanh[Sin[c + d*x]])/(2*d) - (((15*I)/2)*a^4*Sec[c + d*x])/d - (
(2*I)*a*Cos[c + d*x]*(a + I*a*Tan[c + d*x])^3)/d - (((5*I)/2)*Sec[c + d*x]*
(a^4 + I*a^4*Tan[c + d*x]))/d
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^3}{d} - (5a^2)\int \sec(c+dx)(a+ia\tan(c+dx))^2\,dx \\
&= -\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^3}{d} - \frac{5i\sec(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{2d} \\
&\quad -\frac{1}{2}(15a^3)\int \sec(c+dx)(a+ia\tan(c+dx))\,dx \\
&= -\frac{15ia^4\sec(c+dx)}{2d} - \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^3}{d} \\
&\quad -\frac{5i\sec(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{2d} - \frac{1}{2}(15a^4)\int \sec(c+dx)\,dx
\end{aligned}$$

$$= -\frac{15a^4\text{arctanh}(\sin(c+dx))}{2d} - \frac{15ia^4\sec(c+dx)}{2d} - \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^3}{d} - \frac{5i\sec(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{2d}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 906 vs. 2(97) = 194.

Time = 6.77 (sec) , antiderivative size = 906, normalized size of antiderivative = 9.34

$$\int \cos(c+dx)(a+ia\tan(c+dx))^4\,dx$$
$$= \frac{15\cos(4c)\cos^4(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(a+ia\tan(c+dx))^4}{2d(\cos(dx)+i\sin(dx))^4}$$
$$- \frac{15\cos(4c)\cos^4(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(a+ia\tan(c+dx))^4}{2d(\cos(dx)+i\sin(dx))^4}$$
$$+ \frac{\cos(dx)\cos^4(c+dx)(-8i\cos(3c)-8\sin(3c))(a+ia\tan(c+dx))^4}{d(\cos(dx)+i\sin(dx))^4}$$
$$+ \frac{\cos^4(c+dx)\sec(c)(-4i\cos(4c)-4\sin(4c))(a+ia\tan(c+dx))^4}{d(\cos(dx)+i\sin(dx))^4}$$
$$- \frac{15i\cos^4(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sin(4c)(a+ia\tan(c+dx))^4}{2d(\cos(dx)+i\sin(dx))^4}$$
$$+ \frac{15i\cos^4(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sin(4c)(a+ia\tan(c+dx))^4}{2d(\cos(dx)+i\sin(dx))^4}$$
$$+ \frac{\cos^4(c+dx)(8\cos(3c)-8i\sin(3c))\sin(dx)(a+ia\tan(c+dx))^4}{d(\cos(dx)+i\sin(dx))^4}$$
$$+ \frac{\cos^4(c+dx)\left(\frac{1}{4}\cos(4c)-\frac{1}{4}i\sin(4c)\right)(a+ia\tan(c+dx))^4}{d(\cos(dx)+i\sin(dx))^4\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^2}$$
$$- \frac{i\cos^4(c+dx)(4\cos(4c)-4i\sin(4c))\sin\left(\frac{dx}{2}\right)(a+ia\tan(c+dx))^4}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)(\cos(dx)+i\sin(dx))^4\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}$$
$$+ \frac{\cos^4(c+dx)\left(-\frac{1}{4}\cos(4c)+\frac{1}{4}i\sin(4c)\right)(a+ia\tan(c+dx))^4}{d(\cos(dx)+i\sin(dx))^4\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^2}$$
$$+ \frac{i\cos^4(c+dx)(4\cos(4c)-4i\sin(4c))\sin\left(\frac{dx}{2}\right)(a+ia\tan(c+dx))^4}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)(\cos(dx)+i\sin(dx))^4\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}$$

```
[In] Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (15*Cos[4*c]*Cos[c + d*x]^4*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*(a
 + I*a*Tan[c + d*x])^4)/(2*d*(Cos[d*x] + I*Sin[d*x])^4) - (15*Cos[4*c]*Cos[
c + d*x]^4*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*(a + I*a*Tan[c + d*
```

```
x])^4)/(2*d*(Cos[d*x] + I*Sin[d*x])^4) + (Cos[d*x]*Cos[c + d*x]^4*((-8*I)*C
os[3*c] - 8*Sin[3*c])*(a + I*a*Tan[c + d*x])^4)/(d*(Cos[d*x] + I*Sin[d*x])^
4) + (Cos[c + d*x]^4*Sec[c]*((-4*I)*Cos[4*c] - 4*Sin[4*c])*(a + I*a*Tan[c +
 d*x])^4)/(d*(Cos[d*x] + I*Sin[d*x])^4) - (((15*I)/2)*Cos[c + d*x]^4*Log[Co
s[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sin[4*c]*(a + I*a*Tan[c + d*x])^4)/(
d*(Cos[d*x] + I*Sin[d*x])^4) + (((15*I)/2)*Cos[c + d*x]^4*Log[Cos[c/2 + (d*
x)/2] + Sin[c/2 + (d*x)/2]]*Sin[4*c]*(a + I*a*Tan[c + d*x])^4)/(d*(Cos[d*x]
 + I*Sin[d*x])^4) + (Cos[c + d*x]^4*(8*Cos[3*c] - (8*I)*Sin[3*c])*Sin[d*x]*
(a + I*a*Tan[c + d*x])^4)/(d*(Cos[d*x] + I*Sin[d*x])^4) + (Cos[c + d*x]^4*(
Cos[4*c]/4 - (I/4)*Sin[4*c])*(a + I*a*Tan[c + d*x])^4)/(d*(Cos[d*x] + I*Sin
[d*x])^4*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) - (I*Cos[c + d*x]^4*(
4*Cos[4*c] - (4*I)*Sin[4*c])*Sin[(d*x)/2]*(a + I*a*Tan[c + d*x])^4)/(d*(Cos
[c/2] - Sin[c/2])*(Cos[d*x] + I*Sin[d*x])^4*(Cos[c/2 + (d*x)/2] - Sin[c/2 +
 (d*x)/2])) + (Cos[c + d*x]^4*(-1/4*Cos[4*c] + (I/4)*Sin[4*c])*(a + I*a*Tan
[c + d*x])^4)/(d*(Cos[d*x] + I*Sin[d*x])^4*(Cos[c/2 + (d*x)/2] + Sin[c/2 +
(d*x)/2])^2) + (I*Cos[c + d*x]^4*(4*Cos[4*c] - (4*I)*Sin[4*c])*Sin[(d*x)/2]
*(a + I*a*Tan[c + d*x])^4)/(d*(Cos[c/2] + Sin[c/2])*(Cos[d*x] + I*Sin[d*x])
^4*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [A] (verified)

Time = 3.19 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.10

method	result
risch	$-\frac{8 i a^{4} {\mathrm e}^{i (d x +c)}}{d}-\frac{i a^{4} \left(9 \,{\mathrm e}^{3 i (d x +c)}+7 \,{\mathrm e}^{i (d x +c)}\right)}{d \left({\mathrm e}^{2 i (d x +c)}+1\right)^{2}}-\frac{15 a^{4} \ln \left({\mathrm e}^{i (d x +c)}+i\right)}{2 d}+\frac{15 a^{4} \ln \left({\mathrm e}^{i (d x +c)}-i\right)}{2 d}$
derivativedivides	$\frac{a^{4} \left(\frac{\sin^{5}(d x +c)}{2 \cos \left(d x +c \right)^{2}}+\frac{\left(\sin^{3}(d x +c)\right)}{2}+\frac{3 \sin \left(d x +c \right)}{2}-\frac{3 \ln \left(\sec \left(d x +c \right)+\tan \left(d x +c \right)\right)}{2}\right)-4 i a^{4} \left(\frac{\sin^{4}(d x +c)}{\cos \left(d x +c \right)}+\left(2+\sin^{2}(d x +c)\right) \cos (d x +c}{d}$
default	$\frac{a^{4} \left(\frac{\sin^{5}(d x +c)}{2 \cos \left(d x +c \right)^{2}}+\frac{\left(\sin^{3}(d x +c)\right)}{2}+\frac{3 \sin \left(d x +c \right)}{2}-\frac{3 \ln \left(\sec \left(d x +c \right)+\tan \left(d x +c \right)\right)}{2}\right)-4 i a^{4} \left(\frac{\sin^{4}(d x +c)}{\cos \left(d x +c \right)}+\left(2+\sin^{2}(d x +c)\right) \cos (d x +c}{d}$

```
[In] int(cos(d*x+c)*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -8*I*a^4/d*exp(I*(d*x+c))-I*a^4/d/(exp(2*I*(d*x+c))+1)^2*(9*exp(3*I*(d*x+c)
)+7*exp(I*(d*x+c)))-15/2*a^4/d*ln(exp(I*(d*x+c))+I)+15/2*a^4/d*ln(exp(I*(d*
x+c))-I)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.67

$$\int \cos(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{-16i\,a^4e^{(5i\,dx+5i\,c)}-50i\,a^4e^{(3i\,dx+3i\,c)}-30i\,a^4e^{(i\,dx+i\,c)}-15\left(a^4e^{(4i\,dx+4i\,c)}+2\,a^4e^{(2i\,dx+2i\,c)}+a^4\right)\log\left(e^{(i\,dx+i\,c)}\right.}{2\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/2*(-16*I*a^4*e^(5*I*d*x + 5*I*c) - 50*I*a^4*e^(3*I*d*x + 3*I*c) - 30*I*a^
4*e^(I*d*x + I*c) - 15*(a^4*e^(4*I*d*x + 4*I*c) + 2*a^4*e^(2*I*d*x + 2*I*c)
 + a^4)*log(e^(I*d*x + I*c) + I) + 15*(a^4*e^(4*I*d*x + 4*I*c) + 2*a^4*e^(2
*I*d*x + 2*I*c) + a^4)*log(e^(I*d*x + I*c) - I))/(d*e^(4*I*d*x + 4*I*c) + 2
*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.58

$$\int \cos(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{15a^4\left(\frac{\log\left(e^{idx}-ie^{-ic}\right)}{2}-\frac{\log\left(e^{idx}+ie^{-ic}\right)}{2}\right)}{d} + \frac{-9ia^4e^{3ic}e^{3idx}-7ia^4e^{ic}e^{idx}}{de^{4ic}e^{4idx}+2de^{2ic}e^{2idx}+d} + \begin{cases} -\frac{8ia^4e^{ic}e^{idx}}{d} & \text{for } d\neq 0 \\ 8a^4xe^{ic} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] 15*a**4*(log(exp(I*d*x) - I*exp(-I*c))/2 - log(exp(I*d*x) + I*exp(-I*c))/2)
/d + (-9*I*a**4*exp(3*I*c)*exp(3*I*d*x) - 7*I*a**4*exp(I*c)*exp(I*d*x))/(d*
exp(4*I*c)*exp(4*I*d*x) + 2*d*exp(2*I*c)*exp(2*I*d*x) + d) + Piecewise((-8*
I*a**4*exp(I*c)*exp(I*d*x)/d, Ne(d, 0)), (8*a**4*x*exp(I*c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.41

$$\int \cos(c+dx)(a+ia\tan(c+dx))^4\,dx =$$
$$-\frac{a^4\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+3\,\log\left(\sin\left(dx+c\right)+1\right)-3\,\log\left(\sin\left(dx+c\right)-1\right)-4\,\sin\left(dx+c\right)\right)+16i\,a^4\left(\frac{1}{\cos(dx+c)}\right.}{}$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/4*(a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x + c) + 1) -
3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)) + 16*I*a^4*(1/cos(d*x + c) + cos(
d*x + c)) + 12*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d
*x + c)) + 16*I*a^4*cos(d*x + c) - 4*a^4*sin(d*x + c))/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 372 vs. 2(81) = 162.

Time = 0.67 (sec) , antiderivative size = 372, normalized size of antiderivative = 3.84

$$\int \cos(c+dx)(a+ia\tan(c+dx))^4\,dx$$
$$=\frac{235\,a^4e^{(4i\,dx+4i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+470\,a^4e^{(2i\,dx+2i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)-5\,a^4e^{(4i\,dx+4i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}\right.}{}$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/32*(235*a^4*e^(4*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 470*a^4*e^(2
*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) - 5*a^4*e^(4*I*d*x + 4*I*c)*log(
I*e^(I*d*x + I*c) - 1) - 10*a^4*e^(2*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) -
 1) - 235*a^4*e^(4*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 470*a^4*e^(
2*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) + 5*a^4*e^(4*I*d*x + 4*I*c)*lo
g(-I*e^(I*d*x + I*c) - 1) + 10*a^4*e^(2*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*
c) - 1) - 256*I*a^4*e^(5*I*d*x + 5*I*c) - 800*I*a^4*e^(3*I*d*x + 3*I*c) - 4
80*I*a^4*e^(I*d*x + I*c) + 235*a^4*log(I*e^(I*d*x + I*c) + 1) - 5*a^4*log(I
*e^(I*d*x + I*c) - 1) - 235*a^4*log(-I*e^(I*d*x + I*c) + 1) + 5*a^4*log(-I*
e^(I*d*x + I*c) - 1))/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Mupad [B] (verification not implemented)

Time = 6.34 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.64

$$\int \cos(c+dx)(a+ia\tan(c+dx))^4\,dx = -\frac{15\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} + \frac{17\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,9\mathrm{i}-39\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,7\mathrm{i}+24\,a^4}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,1\mathrm{i}-2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,2\mathrm{i}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

```
[In] int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] (a^4*tan(c/2 + (d*x)/2)^3*9i - 39*a^4*tan(c/2 + (d*x)/2)^2 + 17*a^4*tan(c/2
 + (d*x)/2)^4 + 24*a^4 - a^4*tan(c/2 + (d*x)/2)*7i)/(d*(tan(c/2 + (d*x)/2)
- tan(c/2 + (d*x)/2)^2*2i - 2*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4*1
i + tan(c/2 + (d*x)/2)^5 + 1i)) - (15*a^4*atanh(tan(c/2 + (d*x)/2)))/d
```

3.55 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^4\,dx$

Optimal result

Integrand size = 24, antiderivative size = 78

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{a^4\text{arctanh}(\sin(c+dx))}{d} - \frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d} + \frac{2i\cos(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{d}$$

```
[Out] a^4*arctanh(sin(d*x+c))/d-2/3*I*a*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^3/d+2*I*c
os(d*x+c)*(a^4+I*a^4*tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3577, 3855}

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{a^4\text{arctanh}(\sin(c+dx))}{d} + \frac{2i\cos(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{d} - \frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d}$$

```
[In] Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (a^4*ArcTanh[Sin[c + d*x]])/d - (((2*I)/3)*a*Cos[c + d*x]^3*(a + I*a*Tan[c
+ d*x])^3)/d + ((2*I)*Cos[c + d*x]*(a^4 + I*a^4*Tan[c + d*x]))/d
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d} - a^2\int \cos(c+dx)(a+ia\tan(c+dx))^2\,dx \\
&= -\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d} \\
&\quad + \frac{2i\cos(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{d} + a^4\int \sec(c+dx)\,dx \\
&= \frac{a^4\text{arctanh}(\sin(c+dx))}{d} - \frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d} \\
&\quad + \frac{2i\cos(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{d}
\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 246 vs. 2(78) = 156.

Time = 0.94 (sec) , antiderivative size = 246, normalized size of antiderivative = 3.15

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^4\,dx$$
$$= \frac{a^4\left(-3\cos(4c)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+3\cos(4c)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right) -$$

```
[In] Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (a^4*(-3*Cos[4*c]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 3*Cos[4*c]*Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 2*Cos[3*d*x]*Sin[c] + 6*Cos[d*x]*Si
```

```
n[3*c] + (3*I)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[4*c] - (3*I)*Lo
g[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[4*c] + Cos[3*c]*((6*I)*Cos[d*x]
 - 6*Sin[d*x]) + (6*I)*Sin[3*c]*Sin[d*x] - (2*I)*Sin[c]*Sin[3*d*x] + 2*Cos[c
]*((-I)*Cos[3*d*x] + Sin[3*d*x]))*(Cos[c + d*x] + I*Sin[c + d*x])^4)/(3*d*(
Cos[d*x] + I*Sin[d*x])^4)
```

Maple [A] (verified)

Time = 13.92 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01

method	result
risch	$-\frac{2ia^4 e^{3i(dx+c)}}{3d}+\frac{2ia^4 e^{i(dx+c)}}{d}+\frac{a^4\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{a^4\ln\left(e^{i(dx+c)}-i\right)}{d}$
derivativedivides	$\frac{a^4\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+\frac{4ia^4\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}-2a^4\left(\sin^3(dx+c)\right)-\frac{4ia^4(c\ldots}{}}{d}$
default	$\frac{a^4\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+\frac{4ia^4\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}-2a^4\left(\sin^3(dx+c)\right)-\frac{4ia^4(c\ldots}{}}{d}$

```
[In] int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*a^4/d*exp(3*I*(d*x+c))+2*I*a^4/d*exp(I*(d*x+c))+a^4/d*ln(exp(I*(d*x+
c))+I)-a^4/d*ln(exp(I*(d*x+c))-I)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.87

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{-2i\,a^4e^{(3i\,dx+3i\,c)}+6i\,a^4e^{(i\,dx+i\,c)}+3\,a^4\log\left(e^{(i\,dx+i\,c)}+i\right)-3\,a^4\log\left(e^{(i\,dx+i\,c)}-i\right)}{3\,d}$$

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(-2*I*a^4*e^(3*I*d*x + 3*I*c) + 6*I*a^4*e^(I*d*x + I*c) + 3*a^4*log(e^(
I*d*x + I*c) + I) - 3*a^4*log(e^(I*d*x + I*c) - I))/d
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.40

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{a^4\left(-\log\left(e^{idx}-ie^{-ic}\right)+\log\left(e^{idx}+ie^{-ic}\right)\right)}{d} + \begin{cases} \frac{-2ia^4de^{3ic}e^{3idx}+6ia^4de^{ic}e^{idx}}{3d^2} & \text{for } d^2 \neq 0 \\ x\left(2a^4e^{3ic}-2a^4e^{ic}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] a**4*(-log(exp(I*d*x) - I*exp(-I*c)) + log(exp(I*d*x) + I*exp(-I*c)))/d + P
iecewise(((-2*I*a**4*d*exp(3*I*c)*exp(3*I*d*x) + 6*I*a**4*d*exp(I*c)*exp(I*
d*x))/(3*d**2), Ne(d**2, 0)), (x*(2*a**4*exp(3*I*c) - 2*a**4*exp(I*c)), Tru
e))
```

Maxima [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.55

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^4\,dx = -\frac{8i\,a^4\cos\left(dx+c\right)^3+12\,a^4\sin\left(dx+c\right)^3+8i\left(\cos\left(dx+c\right)^3-3\,\cos\left(dx+c\right)\right)a^4+\left(2\,\sin\left(dx+c\right)^3-3\,\log\right.}{6\,d}$$

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/6*(8*I*a^4*cos(d*x + c)^3 + 12*a^4*sin(d*x + c)^3 + 8*I*(cos(d*x + c)^3
- 3*cos(d*x + c))*a^4 + (2*sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log
(sin(d*x + c) - 1) + 6*sin(d*x + c))*a^4 + 2*(sin(d*x + c)^3 - 3*sin(d*x +
c))*a^4)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1299 vs. 2(68) = 136.

Time = 0.97 (sec) , antiderivative size = 1299, normalized size of antiderivative = 16.65

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^4\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/768*(1110*a^4*e^(12*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 6660*a^4*
e^(10*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 16650*a^4*e^(8*I*d*x + 2*
I*c)*log(I*e^(I*d*x + I*c) + 1) + 16650*a^4*e^(4*I*d*x - 2*I*c)*log(I*e^(I*
d*x + I*c) + 1) + 6660*a^4*e^(2*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) + 1) +
 22200*a^4*e^(6*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 1110*a^4*e^(-6*I*c)*log
(I*e^(I*d*x + I*c) + 1) + 1875*a^4*e^(12*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*
c) - 1) + 11250*a^4*e^(10*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 28125
*a^4*e^(8*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 28125*a^4*e^(4*I*d*x
- 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 11250*a^4*e^(2*I*d*x - 4*I*c)*log(I*e
^(I*d*x + I*c) - 1) + 37500*a^4*e^(6*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 18
75*a^4*e^(-6*I*c)*log(I*e^(I*d*x + I*c) - 1) - 1110*a^4*e^(12*I*d*x + 6*I*c
)*log(-I*e^(I*d*x + I*c) + 1) - 6660*a^4*e^(10*I*d*x + 4*I*c)*log(-I*e^(I*d
*x + I*c) + 1) - 16650*a^4*e^(8*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1)
- 16650*a^4*e^(4*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 6660*a^4*e^(2
*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 22200*a^4*e^(6*I*d*x)*log(-I*
e^(I*d*x + I*c) + 1) - 1110*a^4*e^(-6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 18
75*a^4*e^(12*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 11250*a^4*e^(10*I
*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 28125*a^4*e^(8*I*d*x + 2*I*c)*l
og(-I*e^(I*d*x + I*c) - 1) - 28125*a^4*e^(4*I*d*x - 2*I*c)*log(-I*e^(I*d*x
+ I*c) - 1) - 11250*a^4*e^(2*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 3
7500*a^4*e^(6*I*d*x)*log(-I*e^(I*d*x + I*c) - 1) - 1875*a^4*e^(-6*I*c)*log(
-I*e^(I*d*x + I*c) - 1) - 3*a^4*e^(12*I*d*x + 6*I*c)*log(I*e^(I*d*x) + e^(-
I*c)) - 18*a^4*e^(10*I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 45*a^4*e^
(8*I*d*x + 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 45*a^4*e^(4*I*d*x - 2*I*c)*
log(I*e^(I*d*x) + e^(-I*c)) - 18*a^4*e^(2*I*d*x - 4*I*c)*log(I*e^(I*d*x) +
e^(-I*c)) - 60*a^4*e^(6*I*d*x)*log(I*e^(I*d*x) + e^(-I*c)) - 3*a^4*e^(-6*I*
c)*log(I*e^(I*d*x) + e^(-I*c)) + 3*a^4*e^(12*I*d*x + 6*I*c)*log(-I*e^(I*d*x
) + e^(-I*c)) + 18*a^4*e^(10*I*d*x + 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) +
45*a^4*e^(8*I*d*x + 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 45*a^4*e^(4*I*d*x
 - 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 18*a^4*e^(2*I*d*x - 4*I*c)*log(-I*
e^(I*d*x) + e^(-I*c)) + 60*a^4*e^(6*I*d*x)*log(-I*e^(I*d*x) + e^(-I*c)) + 3
*a^4*e^(-6*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 512*I*a^4*e^(15*I*d*x + 9*I*
c) - 1536*I*a^4*e^(13*I*d*x + 7*I*c) + 1536*I*a^4*e^(11*I*d*x + 5*I*c) + 12
800*I*a^4*e^(9*I*d*x + 3*I*c) + 23040*I*a^4*e^(7*I*d*x + I*c) + 19968*I*a^4
*e^(5*I*d*x - I*c) + 8704*I*a^4*e^(3*I*d*x - 3*I*c) + 1536*I*a^4*e^(I*d*x -
 5*I*c))/(d*e^(12*I*d*x + 6*I*c) + 6*d*e^(10*I*d*x + 4*I*c) + 15*d*e^(8*I*d
*x + 2*I*c) + 15*d*e^(4*I*d*x - 2*I*c) + 6*d*e^(2*I*d*x - 4*I*c) + 20*d*e^(
6*I*d*x) + d*e^(-6*I*c))
```

Mupad [B] (verification not implemented)

Time = 4.15 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.13

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^4\,dx$$
$$=\frac{2\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$
$$-\frac{\frac{8\,a^4}{3}-a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,8\mathrm{i}}{d\,\left(-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,3\mathrm{i}+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

```
[In] int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] (2*a^4*atanh(tan(c/2 + (d*x)/2)))/d - ((8*a^4)/3 - a^4*tan(c/2 + (d*x)/2)*8
i)/(d*(3*tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2*3i - tan(c/2 + (d*x)/2)^
3 + 1i))
```

3.56 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^4\,dx$

Optimal result

Integrand size = 24, antiderivative size = 66

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^4\,dx=-\frac{ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{15d}-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^4}{5d}$$

```
[Out] -1/15*I*a*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^3/d-1/5*I*cos(d*x+c)^5*(a+I*a*tan
(d*x+c))^4/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3578, 3569}

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^4\,dx=-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^4}{5d}-\frac{ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{15d}$$

```
[In] Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((-1/15*I)*a*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^3)/d - ((I/5)*Cos[c + d*
x]^5*(a + I*a*Tan[c + d*x])^4)/d
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
```

```
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^4}{5d}+\frac{1}{5}a\int\cos^3(c+dx)(a+ia\tan(c+dx))^3\,dx\\ &= -\frac{ia\cos^3(c+dx)(a+ia\tan(c+dx))^3}{15d}-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^4}{5d}\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 145 vs. 2(66) = 132.

Time = 0.55 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.20

$$\int\cos^5(c+dx)(a+ia\tan(c+dx))^4\,dx$$
$$=\frac{a^4(-i\cos(2(c+dx))+\sin(2(c+dx)))\left(\cos(c+dx)\left(8+5\sqrt{\cos^2(c+dx)}\right)+\left(8+3\sqrt{\cos^2(c+dx)}\right)\cos(3\cdots}{30d\sqrt{\cos^2(c+\cdots}}$$

```
[In] Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (a^4*((-I)*Cos[2*(c + d*x)] + Sin[2*(c + d*x)])*(Cos[c + d*x]*(8 + 5*Sqrt[C
os[c + d*x]^2]) + (8 + 3*Sqrt[Cos[c + d*x]^2])*Cos[3*(c + d*x)] + I*((-8 +
5*Sqrt[Cos[c + d*x]^2])*Sin[c + d*x] + (-8 + 3*Sqrt[Cos[c + d*x]^2])*Sin[3*
(c + d*x)])))/(30*d*Sqrt[Cos[c + d*x]^2])
```

Maple [A] (verified)

Time = 47.41 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.58

method	result
risch	$-\frac{ia^4 e^{5i(dx+c)}}{10d}-\frac{ia^4 e^{3i(dx+c)}}{6d}$
derivativedivides	$\frac{\frac{a^4\left(\sin^5(dx+c)\right)}{5}-4ia^4\left(-\frac{\left(\cos^3(dx+c)\right)\left(\sin^2(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)-6a^4\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin}{15}\right.}{d}$
default	$\frac{\frac{a^4\left(\sin^5(dx+c)\right)}{5}-4ia^4\left(-\frac{\left(\cos^3(dx+c)\right)\left(\sin^2(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)-6a^4\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin}{15}\right.}{d}$

[In] int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)

[Out] -1/10*I*a^4/d*exp(5*I*(d*x+c))-1/6*I*a^4/d*exp(3*I*(d*x+c))

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.52

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^4\,dx=\frac{-3i\,a^4e^{(5i\,dx+5i\,c)}-5i\,a^4e^{(3i\,dx+3i\,c)}}{30\,d}$$

[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")

[Out] 1/30*(-3*I*a^4*e^(5*I*d*x + 5*I*c) - 5*I*a^4*e^(3*I*d*x + 3*I*c))/d

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.21

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^4\,dx=\begin{cases}\frac{-6ia^4de^{5ic}e^{5idx}-10ia^4de^{3ic}e^{3idx}}{60d^2} & \text{for } d^2\neq 0\\ x\left(\frac{a^4e^{5ic}}{2}+\frac{a^4e^{3ic}}{2}\right) & \text{otherwise}\end{cases}$$

[In] integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**4,x)

[Out] Piecewise(((-6*I*a**4*d*exp(5*I*c)*exp(5*I*d*x) - 10*I*a**4*d*exp(3*I*c)*ex
p(3*I*d*x))/(60*d**2), Ne(d**2, 0)), (x*(a**4*exp(5*I*c)/2 + a**4*exp(3*I*c
)/2), True))

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(54) = 108.

Time = 0.42 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.79

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^4\,dx =$$
$$-\frac{12i\,a^4\cos\left(dx+c\right)^5-3\,a^4\sin\left(dx+c\right)^5+4i\left(3\,\cos\left(dx+c\right)^5-5\,\cos\left(dx+c\right)^3\right)a^4-6\left(3\,\sin\left(dx+c\right)^5-\right.}{15\,d}$$

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/15*(12*I*a^4*cos(d*x + c)^5 - 3*a^4*sin(d*x + c)^5 + 4*I*(3*cos(d*x + c)
^5 - 5*cos(d*x + c)^3)*a^4 - 6*(3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*a^4 -
(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^4)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 915 vs. 2(54) = 108.

Time = 1.08 (sec) , antiderivative size = 915, normalized size of antiderivative = 13.86

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^4\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/7680*(9075*a^4*e^(8*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 36300*a^4
*e^(6*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 36300*a^4*e^(2*I*d*x - 2*
I*c)*log(I*e^(I*d*x + I*c) + 1) + 54450*a^4*e^(4*I*d*x)*log(I*e^(I*d*x + I*
c) + 1) + 9075*a^4*e^(-4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 9000*a^4*e^(8*I*
d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 36000*a^4*e^(6*I*d*x + 2*I*c)*log
(I*e^(I*d*x + I*c) - 1) + 36000*a^4*e^(2*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*
c) - 1) + 54000*a^4*e^(4*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 9000*a^4*e^(-4
*I*c)*log(I*e^(I*d*x + I*c) - 1) - 9075*a^4*e^(8*I*d*x + 4*I*c)*log(-I*e^(I
*d*x + I*c) + 1) - 36300*a^4*e^(6*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1
) - 36300*a^4*e^(2*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 54450*a^4*e
^(4*I*d*x)*log(-I*e^(I*d*x + I*c) + 1) - 9075*a^4*e^(-4*I*c)*log(-I*e^(I*d*
x + I*c) + 1) - 9000*a^4*e^(8*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) -
36000*a^4*e^(6*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 36000*a^4*e^(2*
I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 54000*a^4*e^(4*I*d*x)*log(-I*e
^(I*d*x + I*c) - 1) - 9000*a^4*e^(-4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 75*
a^4*e^(8*I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 300*a^4*e^(6*I*d*x +
```

```
2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 300*a^4*e^(2*I*d*x - 2*I*c)*log(I*e^(I
*d*x) + e^(-I*c)) - 450*a^4*e^(4*I*d*x)*log(I*e^(I*d*x) + e^(-I*c)) - 75*a^
4*e^(-4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 75*a^4*e^(8*I*d*x + 4*I*c)*log(-
I*e^(I*d*x) + e^(-I*c)) + 300*a^4*e^(6*I*d*x + 2*I*c)*log(-I*e^(I*d*x) + e^
(-I*c)) + 300*a^4*e^(2*I*d*x - 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 450*a^
4*e^(4*I*d*x)*log(-I*e^(I*d*x) + e^(-I*c)) + 75*a^4*e^(-4*I*c)*log(-I*e^(I*
d*x) + e^(-I*c)) - 768*I*a^4*e^(13*I*d*x + 9*I*c) - 4352*I*a^4*e^(11*I*d*x
+ 7*I*c) - 9728*I*a^4*e^(9*I*d*x + 5*I*c) - 10752*I*a^4*e^(7*I*d*x + 3*I*c)
 - 5888*I*a^4*e^(5*I*d*x + I*c) - 1280*I*a^4*e^(3*I*d*x - I*c))/(d*e^(8*I*d
*x + 4*I*c) + 4*d*e^(6*I*d*x + 2*I*c) + 4*d*e^(2*I*d*x - 2*I*c) + 6*d*e^(4*
I*d*x) + d*e^(-4*I*c))
```

Mupad [B] (verification not implemented)

Time = 4.88 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.97

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{2\,a^4\,\left(15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,15\mathrm{i}-25\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,5\mathrm{i}+4\right)}{15\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,5\mathrm{i}-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,10\mathrm{i}+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

```
[In] int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] (2*a^4*(tan(c/2 + (d*x)/2)^3*15i - 25*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x
)/2)*5i + 15*tan(c/2 + (d*x)/2)^4 + 4))/(15*d*(5*tan(c/2 + (d*x)/2) - tan(c
/2 + (d*x)/2)^2*10i - 10*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4*5i + t
an(c/2 + (d*x)/2)^5 + 1i))
```

3.57 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^4\,dx$

Optimal result

Integrand size = 24, antiderivative size = 102

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{3a^4\sin(c+dx)}{35d} - \frac{a^4\sin^3(c+dx)}{35d} - \frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^3}{7d} - \frac{2i\cos^5(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{35d}$$

```
[Out] 3/35*a^4*sin(d*x+c)/d-1/35*a^4*sin(d*x+c)^3/d-2/7*I*a*cos(d*x+c)^7*(a+I*a*t
an(d*x+c))^3/d-2/35*I*cos(d*x+c)^5*(a^4+I*a^4*tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3577, 2713}

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^4\,dx = -\frac{a^4\sin^3(c+dx)}{35d} + \frac{3a^4\sin(c+dx)}{35d} - \frac{2i\cos^5(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{35d} - \frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^3}{7d}$$

```
[In] Int[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (3*a^4*Sin[c + d*x])/(35*d) - (a^4*Sin[c + d*x]^3)/(35*d) - (((2*I)/7)*a*Co
s[c + d*x]^7*(a + I*a*Tan[c + d*x])^3)/d - (((2*I)/35)*Cos[c + d*x]^5*(a^4
+ I*a^4*Tan[c + d*x]))/d
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^3}{7d}+\frac{1}{7}a^2\int\cos^5(c+dx)(a+ia\tan(c+dx))^2\,dx\\
&= -\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^3}{7d}\\
&\quad-\frac{2i\cos^5(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{35d}+\frac{1}{35}\left(3a^4\right)\int\cos^3(c+dx)\,dx\\
&= -\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^3}{7d}-\frac{2i\cos^5(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{35d}\\
&\quad-\frac{\left(3a^4\right)\text{Subst}\left(\int\left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{35d}\\
&= \frac{3a^4\sin(c+dx)}{35d}-\frac{a^4\sin^3(c+dx)}{35d}-\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^3}{7d}\\
&\quad-\frac{2i\cos^5(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{35d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.77

$$\int\cos^7(c+dx)(a+ia\tan(c+dx))^4\,dx$$

$$=\frac{a^4(-i\cos(3(c+dx))+\sin(3(c+dx)))\left(35\sqrt{\cos^2(c+dx)}+8\left(4+7\sqrt{\cos^2(c+dx)}\right)\cos(2(c+dx))+\left(3\right.\right.}{}$$

[In] Integrate[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^4,x]

[Out] (a^4*((-I)*Cos[3*(c + d*x)] + Sin[3*(c + d*x)])*(35*Sqrt[Cos[c + d*x]^2] + 8*(4 + 7*Sqrt[Cos[c + d*x]^2])*Cos[2*(c + d*x)] + (32 + 5*Sqrt[Cos[c + d*x]^2])*Cos[4*(c + d*x)] - (32*I)*Sin[2*(c + d*x)] - (14*I)*Sqrt[Cos[c + d*x]^2]*Sin[2*(c + d*x)] - (32*I)*Sin[4*(c + d*x)] + (5*I)*Sqrt[Cos[c + d*x]^2]*Sin[4*(c + d*x)]))/(280*d*Sqrt[Cos[c + d*x]^2])

Maple [A] (verified)

Time = 131.89 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{i a^4 e^{7i(dx+c)}}{56d}-\frac{3i a^4 e^{5i(dx+c)}}{40d}-\frac{i a^4 e^{3i(dx+c)}}{8d}-\frac{i a^4 e^{i(dx+c)}}{8d}$
derivativedivides	$\frac{a^4\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^4(dx+c)\right)}{7}-\frac{3\sin(dx+c)\left(\cos^4(dx+c)\right)}{35}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{35}\right)-4ia^4\left(-\frac{\left(\cos^5(dx+c)\right)\left(\sin^2(dx+c)\right)}{7}\right.}{}$
default	$\frac{a^4\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^4(dx+c)\right)}{7}-\frac{3\sin(dx+c)\left(\cos^4(dx+c)\right)}{35}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{35}\right)-4ia^4\left(-\frac{\left(\cos^5(dx+c)\right)\left(\sin^2(dx+c)\right)}{7}\right.}{}$

[In] int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)

[Out] -1/56*I*a^4/d*exp(7*I*(d*x+c))-3/40*I*a^4/d*exp(5*I*(d*x+c))-1/8*I*a^4/d*exp(3*I*(d*x+c))-1/8*I*a^4/d*exp(I*(d*x+c))

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.61

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{-5i\,a^4e^{(7i\,dx+7i\,c)}-21i\,a^4e^{(5i\,dx+5i\,c)}-35i\,a^4e^{(3i\,dx+3i\,c)}-35i\,a^4e^{(i\,dx+i\,c)}}{280\,d}$$

[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")

[Out] 1/280*(-5*I*a^4*e^(7*I*d*x + 7*I*c) - 21*I*a^4*e^(5*I*d*x + 5*I*c) - 35*I*a^4*e^(3*I*d*x + 3*I*c) - 35*I*a^4*e^(I*d*x + I*c))/d

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.53

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^4\,dx = \begin{cases} \frac{-2560ia^4d^3e^{7ic}e^{7idx}-10752ia^4d^3e^{5ic}e^{5idx}-17920ia^4d^3e^{3ic}e^{3idx}-17920ia^4d^3e^{ic}e^{idx}}{143360d^4} & \text{for } d^4\neq 0 \\ x\left(\frac{a^4e^{7ic}}{8}+\frac{3a^4e^{5ic}}{8}+\frac{3a^4e^{3ic}}{8}+\frac{a^4e^{ic}}{8}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)**7*(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise(((-2560*I*a**4*d**3*exp(7*I*c)*exp(7*I*d*x) - 10752*I*a**4*d**3*e
xp(5*I*c)*exp(5*I*d*x) - 17920*I*a**4*d**3*exp(3*I*c)*exp(3*I*d*x) - 17920*
I*a**4*d**3*exp(I*c)*exp(I*d*x))/(143360*d**4), Ne(d**4, 0)), (x*(a**4*exp(
7*I*c)/8 + 3*a**4*exp(5*I*c)/8 + 3*a**4*exp(3*I*c)/8 + a**4*exp(I*c)/8), Tr
ue))
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.46

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^4\,dx = -\frac{20i\,a^4\cos(dx+c)^7+4i\left(5\cos(dx+c)^7-7\cos(dx+c)^5\right)a^4+2\left(15\sin(dx+c)^7-42\sin(dx+c)^5\right.}{}$$

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/35*(20*I*a^4*cos(d*x + c)^7 + 4*I*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*
a^4 + 2*(15*sin(d*x + c)^7 - 42*sin(d*x + c)^5 + 35*sin(d*x + c)^3)*a^4 + (
5*sin(d*x + c)^7 - 7*sin(d*x + c)^5)*a^4 + (5*sin(d*x + c)^7 - 21*sin(d*x +
 c)^5 + 35*sin(d*x + c)^3 - 35*sin(d*x + c))*a^4)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1327 vs. 2(86) = 172.

Time = 0.76 (sec) , antiderivative size = 1327, normalized size of antiderivative = 13.01

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^4\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/143360*(89950*a^4*e^(12*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 53970
0*a^4*e^(10*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1349250*a^4*e^(8*I*
d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1349250*a^4*e^(4*I*d*x - 2*I*c)*l
og(I*e^(I*d*x + I*c) + 1) + 539700*a^4*e^(2*I*d*x - 4*I*c)*log(I*e^(I*d*x +
 I*c) + 1) + 1799000*a^4*e^(6*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 89950*a^4
*e^(-6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 86065*a^4*e^(12*I*d*x + 6*I*c)*log
(I*e^(I*d*x + I*c) - 1) + 516390*a^4*e^(10*I*d*x + 4*I*c)*log(I*e^(I*d*x +
I*c) - 1) + 1290975*a^4*e^(8*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 12
90975*a^4*e^(4*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 516390*a^4*e^(2*
I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1721300*a^4*e^(6*I*d*x)*log(I*e
^(I*d*x + I*c) - 1) + 86065*a^4*e^(-6*I*c)*log(I*e^(I*d*x + I*c) - 1) - 899
50*a^4*e^(12*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 539700*a^4*e^(10*
I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1349250*a^4*e^(8*I*d*x + 2*I*c
)*log(-I*e^(I*d*x + I*c) + 1) - 1349250*a^4*e^(4*I*d*x - 2*I*c)*log(-I*e^(I
*d*x + I*c) + 1) - 539700*a^4*e^(2*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) +
1) - 1799000*a^4*e^(6*I*d*x)*log(-I*e^(I*d*x + I*c) + 1) - 89950*a^4*e^(-6*
I*c)*log(-I*e^(I*d*x + I*c) + 1) - 86065*a^4*e^(12*I*d*x + 6*I*c)*log(-I*e^
(I*d*x + I*c) - 1) - 516390*a^4*e^(10*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c)
 - 1) - 1290975*a^4*e^(8*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 12909
75*a^4*e^(4*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 516390*a^4*e^(2*I*
d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 1721300*a^4*e^(6*I*d*x)*log(-I*e
^(I*d*x + I*c) - 1) - 86065*a^4*e^(-6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 38
85*a^4*e^(12*I*d*x + 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 23310*a^4*e^(10*I
*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 58275*a^4*e^(8*I*d*x + 2*I*c)*l
og(I*e^(I*d*x) + e^(-I*c)) - 58275*a^4*e^(4*I*d*x - 2*I*c)*log(I*e^(I*d*x)
+ e^(-I*c)) - 23310*a^4*e^(2*I*d*x - 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 7
7700*a^4*e^(6*I*d*x)*log(I*e^(I*d*x) + e^(-I*c)) - 3885*a^4*e^(-6*I*c)*log(
I*e^(I*d*x) + e^(-I*c)) + 3885*a^4*e^(12*I*d*x + 6*I*c)*log(-I*e^(I*d*x) +
e^(-I*c)) + 23310*a^4*e^(10*I*d*x + 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 5
8275*a^4*e^(8*I*d*x + 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 58275*a^4*e^(4*
I*d*x - 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 23310*a^4*e^(2*I*d*x - 4*I*c)
*log(-I*e^(I*d*x) + e^(-I*c)) + 77700*a^4*e^(6*I*d*x)*log(-I*e^(I*d*x) + e^
(-I*c)) + 3885*a^4*e^(-6*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 2560*I*a^4*e^(
19*I*d*x + 13*I*c) - 26112*I*a^4*e^(17*I*d*x + 11*I*c) - 120832*I*a^4*e^(15
*I*d*x + 9*I*c) - 337920*I*a^4*e^(13*I*d*x + 7*I*c) - 629760*I*a^4*e^(11*I*
d*x + 5*I*c) - 803840*I*a^4*e^(9*I*d*x + 3*I*c) - 694272*I*a^4*e^(7*I*d*x +
 I*c) - 387072*I*a^4*e^(5*I*d*x - I*c) - 125440*I*a^4*e^(3*I*d*x - 3*I*c) -
 17920*I*a^4*e^(I*d*x - 5*I*c))/(d*e^(12*I*d*x + 6*I*c) + 6*d*e^(10*I*d*x +
 4*I*c) + 15*d*e^(8*I*d*x + 2*I*c) + 15*d*e^(4*I*d*x - 2*I*c) + 6*d*e^(2*I*
d*x - 4*I*c) + 20*d*e^(6*I*d*x) + d*e^(-6*I*c))
```

Mupad [B] (verification not implemented)

Time = 4.93 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.82

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^4\,dx =$$

$$-\frac{2\,a^4\left(35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5 105\mathrm{i}-210\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 210\mathrm{i}+147\tan(\right.}{35\,d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 7\mathrm{i}+21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 35\mathrm{i}-35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\right.}$$

```
[In] int(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] -(2*a^4*(tan(c/2 + (d*x)/2)*49i + 147*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x
)/2)^3*210i - 210*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^5*105i + 35*tan
(c/2 + (d*x)/2)^6 - 12))/(35*d*(7*tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2
*21i - 35*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4*35i + 21*tan(c/2 + (d
*x)/2)^5 - tan(c/2 + (d*x)/2)^6*7i - tan(c/2 + (d*x)/2)^7 + 1i))
```

3.58 $\int \cos^9(c+dx)(a+ia\tan(c+dx))^4\,dx$

Optimal result

Integrand size = 24, antiderivative size = 120

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{5a^4\sin(c+dx)}{21d} - \frac{10a^4\sin^3(c+dx)}{63d} + \frac{a^4\sin^5(c+dx)}{21d} - \frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^3}{9d} - \frac{2i\cos^7(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{21d}$$

```
[Out] 5/21*a^4*sin(d*x+c)/d-10/63*a^4*sin(d*x+c)^3/d+1/21*a^4*sin(d*x+c)^5/d-2/9*
I*a*cos(d*x+c)^9*(a+I*a*tan(d*x+c))^3/d-2/21*I*cos(d*x+c)^7*(a^4+I*a^4*tan(
d*x+c))/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3577, 2713}

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{a^4\sin^5(c+dx)}{21d} - \frac{10a^4\sin^3(c+dx)}{63d} + \frac{5a^4\sin(c+dx)}{21d} - \frac{2i\cos^7(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{21d} - \frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^3}{9d}$$

```
[In] Int[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (5*a^4*Sin[c + d*x])/(21*d) - (10*a^4*Sin[c + d*x]^3)/(63*d) + (a^4*Sin[c +
 d*x]^5)/(21*d) - (((2*I)/9)*a*Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^3)/d -
 (((2*I)/21)*Cos[c + d*x]^7*(a^4 + I*a^4*Tan[c + d*x]))/d
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^3}{9d}+\frac{1}{3}a^2\int\cos^7(c+dx)(a+ia\tan(c+dx))^2\,dx\\
&= -\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^3}{9d}\\
&\quad-\frac{2i\cos^7(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{21d}+\frac{1}{21}\left(5a^4\right)\int\cos^5(c+dx)\,dx\\
&= -\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^3}{9d}-\frac{2i\cos^7(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{21d}\\
&\quad-\frac{\left(5a^4\right)\text{Subst}\left(\int\left(1-2x^2+x^4\right)\,dx,x,-\sin(c+dx)\right)}{21d}\\
&= \frac{5a^4\sin(c+dx)}{21d}-\frac{10a^4\sin^3(c+dx)}{63d}+\frac{a^4\sin^5(c+dx)}{21d}\\
&\quad-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^3}{9d}-\frac{2i\cos^7(c+dx)\left(a^4+ia^4\tan(c+dx)\right)}{21d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.80

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^4\,dx$$
$$= \frac{a^4(-i\cos(4(c+dx))+\sin(4(c+dx)))\left(168\cos(c+dx)\sqrt{\cos^2(c+dx)}+4\left(16+45\sqrt{\cos^2(c+dx)}\right)\cos(3\right.}{}$$

```
[In] Integrate[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (a^4*((-I)*Cos[4*(c + d*x)] + Sin[4*(c + d*x)])*(168*Cos[c + d*x]*Sqrt[Cos[
c + d*x]^2] + 4*(16 + 45*Sqrt[Cos[c + d*x]^2])*Cos[3*(c + d*x)] + 64*Cos[5*
(c + d*x)] - 28*Sqrt[Cos[c + d*x]^2]*Cos[5*(c + d*x)] - (42*I)*Sqrt[Cos[c +
 d*x]^2]*Sin[c + d*x] - (64*I)*Sin[3*(c + d*x)] - (135*I)*Sqrt[Cos[c + d*x]
^2]*Sin[3*(c + d*x)] - (64*I)*Sin[5*(c + d*x)] + (35*I)*Sqrt[Cos[c + d*x]^2
]*Sin[5*(c + d*x)]))/(1008*d*Sqrt[Cos[c + d*x]^2])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(106) = 212.

Time = 0.82 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.94

$$\frac{a^4\left(-\frac{(\sin^3(dx+c))(\cos^6(dx+c))}{9}-\frac{\sin(dx+c)(\cos^6(dx+c))}{21}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{105}\right)-4ia^4\left(-\frac{(\cos^7(dx+c))(\sin\right.}{9}}{}$$

```
[In] int(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^4,x)
```

```
[Out] 1/d*(a^4*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(d*x+c)^6+1/105
*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-4*I*a^4*(-1/9*cos(d*x+c)^7
*sin(d*x+c)^2-2/63*cos(d*x+c)^7)-6*a^4*(-1/9*cos(d*x+c)^8*sin(d*x+c)+1/63*(
16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))-4/9*I*a^4*
cos(d*x+c)^9+1/9*a^4*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c)^6+48/35*cos(d*x+c)
^4+64/35*cos(d*x+c)^2)*sin(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.75

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{\left(-7i\,a^4e^{(10i\,dx+10i\,c)}-45i\,a^4e^{(8i\,dx+8i\,c)}-126i\,a^4e^{(6i\,dx+6i\,c)}-210i\,a^4e^{(4i\,dx+4i\,c)}-315i\,a^4e^{(2i\,dx+2i\,c)}+63i\,a^4\right.}{2016\,d}$$

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/2016*(-7*I*a^4*e^(10*I*d*x + 10*I*c) - 45*I*a^4*e^(8*I*d*x + 8*I*c) - 126
*I*a^4*e^(6*I*d*x + 6*I*c) - 210*I*a^4*e^(4*I*d*x + 4*I*c) - 315*I*a^4*e^(2
*I*d*x + 2*I*c) + 63*I*a^4)*e^(-I*d*x - I*c)/d
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(107) = 214.

Time = 0.39 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.90

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^4\,dx = \begin{cases} \frac{(-176160768ia^4d^5e^{10ic}e^{9idx}-1132462080ia^4d^5e^{8ic}e^{7idx}-3170893824ia^4d^5e^{6ic}e^{5idx}-5284823040ia^4d^5e^{4ic}e^{3idx}-7927234560ia^4d^5e^{2ic}e^{idx}}{50734301184d^6} \\ \frac{x(a^4e^{10ic}+5a^4e^{8ic}+10a^4e^{6ic}+10a^4e^{4ic}+5a^4e^{2ic}+a^4)e^{-ic}}{32} \end{cases}$$

```
[In] integrate(cos(d*x+c)**9*(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise(((-176160768*I*a**4*d**5*exp(10*I*c)*exp(9*I*d*x) - 1132462080*I*
a**4*d**5*exp(8*I*c)*exp(7*I*d*x) - 3170893824*I*a**4*d**5*exp(6*I*c)*exp(5
*I*d*x) - 5284823040*I*a**4*d**5*exp(4*I*c)*exp(3*I*d*x) - 7927234560*I*a**
4*d**5*exp(2*I*c)*exp(I*d*x) + 1585446912*I*a**4*d**5*exp(-I*d*x))*exp(-I*c
)/(50734301184*d**6), Ne(d**6*exp(I*c), 0)), (x*(a**4*exp(10*I*c) + 5*a**4*
exp(8*I*c) + 10*a**4*exp(6*I*c) + 10*a**4*exp(4*I*c) + 5*a**4*exp(2*I*c) +
a**4)*exp(-I*c)/32, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.51

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^4\,dx =$$
$$-\frac{140i\,a^4\cos{(dx+c)}^9+20i\,\left(7\,\cos{(dx+c)}^9-9\,\cos{(dx+c)}^7\right)a^4-\left(35\,\sin{(dx+c)}^9-90\,\sin{(dx+c)}^7+\right.}{}$$

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/315*(140*I*a^4*cos(d*x + c)^9 + 20*I*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^
7)*a^4 - (35*sin(d*x + c)^9 - 90*sin(d*x + c)^7 + 63*sin(d*x + c)^5)*a^4 -
6*(35*sin(d*x + c)^9 - 135*sin(d*x + c)^7 + 189*sin(d*x + c)^5 - 105*sin(d*
x + c)^3)*a^4 - (35*sin(d*x + c)^9 - 180*sin(d*x + c)^7 + 378*sin(d*x + c)^
5 - 420*sin(d*x + c)^3 + 315*sin(d*x + c))*a^4)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1409 vs. 2(102) = 204.

Time = 0.81 (sec) , antiderivative size = 1409, normalized size of antiderivative = 11.74

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^4\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/516096*(435267*a^4*e^(13*I*d*x + 7*I*c)*log(I*e^(I*d*x + I*c) + 1) + 2611
602*a^4*e^(11*I*d*x + 5*I*c)*log(I*e^(I*d*x + I*c) + 1) + 6529005*a^4*e^(9*
I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) + 1) + 8705340*a^4*e^(7*I*d*x + I*c)*l
og(I*e^(I*d*x + I*c) + 1) + 6529005*a^4*e^(5*I*d*x - I*c)*log(I*e^(I*d*x +
I*c) + 1) + 2611602*a^4*e^(3*I*d*x - 3*I*c)*log(I*e^(I*d*x + I*c) + 1) + 43
5267*a^4*e^(I*d*x - 5*I*c)*log(I*e^(I*d*x + I*c) + 1) + 427896*a^4*e^(13*I*
d*x + 7*I*c)*log(I*e^(I*d*x + I*c) - 1) + 2567376*a^4*e^(11*I*d*x + 5*I*c)*
log(I*e^(I*d*x + I*c) - 1) + 6418440*a^4*e^(9*I*d*x + 3*I*c)*log(I*e^(I*d*x
 + I*c) - 1) + 8557920*a^4*e^(7*I*d*x + I*c)*log(I*e^(I*d*x + I*c) - 1) + 6
418440*a^4*e^(5*I*d*x - I*c)*log(I*e^(I*d*x + I*c) - 1) + 2567376*a^4*e^(3*
I*d*x - 3*I*c)*log(I*e^(I*d*x + I*c) - 1) + 427896*a^4*e^(I*d*x - 5*I*c)*lo
g(I*e^(I*d*x + I*c) - 1) - 435267*a^4*e^(13*I*d*x + 7*I*c)*log(-I*e^(I*d*x
+ I*c) + 1) - 2611602*a^4*e^(11*I*d*x + 5*I*c)*log(-I*e^(I*d*x + I*c) + 1)
- 6529005*a^4*e^(9*I*d*x + 3*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 8705340*a^4
*e^(7*I*d*x + I*c)*log(-I*e^(I*d*x + I*c) + 1) - 6529005*a^4*e^(5*I*d*x - I
```

```
*c)*log(-I*e^(I*d*x + I*c) + 1) - 2611602*a^4*e^(3*I*d*x - 3*I*c)*log(-I*e^
(I*d*x + I*c) + 1) - 435267*a^4*e^(I*d*x - 5*I*c)*log(-I*e^(I*d*x + I*c) +
1) - 427896*a^4*e^(13*I*d*x + 7*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 2567376*
a^4*e^(11*I*d*x + 5*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 6418440*a^4*e^(9*I*d
*x + 3*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 8557920*a^4*e^(7*I*d*x + I*c)*log
(-I*e^(I*d*x + I*c) - 1) - 6418440*a^4*e^(5*I*d*x - I*c)*log(-I*e^(I*d*x +
I*c) - 1) - 2567376*a^4*e^(3*I*d*x - 3*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 4
27896*a^4*e^(I*d*x - 5*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 7371*a^4*e^(13*I*
d*x + 7*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 44226*a^4*e^(11*I*d*x + 5*I*c)*l
og(I*e^(I*d*x) + e^(-I*c)) - 110565*a^4*e^(9*I*d*x + 3*I*c)*log(I*e^(I*d*x)
 + e^(-I*c)) - 147420*a^4*e^(7*I*d*x + I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 1
10565*a^4*e^(5*I*d*x - I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 44226*a^4*e^(3*I*
d*x - 3*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 7371*a^4*e^(I*d*x - 5*I*c)*log(I
*e^(I*d*x) + e^(-I*c)) + 7371*a^4*e^(13*I*d*x + 7*I*c)*log(-I*e^(I*d*x) + e
^(-I*c)) + 44226*a^4*e^(11*I*d*x + 5*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 11
0565*a^4*e^(9*I*d*x + 3*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 147420*a^4*e^(7
*I*d*x + I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 110565*a^4*e^(5*I*d*x - I*c)*l
og(-I*e^(I*d*x) + e^(-I*c)) + 44226*a^4*e^(3*I*d*x - 3*I*c)*log(-I*e^(I*d*x
) + e^(-I*c)) + 7371*a^4*e^(I*d*x - 5*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 1
792*I*a^4*e^(22*I*d*x + 16*I*c) - 22272*I*a^4*e^(20*I*d*x + 14*I*c) - 12825
6*I*a^4*e^(18*I*d*x + 12*I*c) - 455936*I*a^4*e^(16*I*d*x + 10*I*c) - 114432
0*I*a^4*e^(14*I*d*x + 8*I*c) - 2102784*I*a^4*e^(12*I*d*x + 6*I*c) - 2742784
*I*a^4*e^(10*I*d*x + 4*I*c) - 2382336*I*a^4*e^(8*I*d*x + 2*I*c) - 295680*I*
a^4*e^(4*I*d*x - 2*I*c) + 16128*I*a^4*e^(2*I*d*x - 4*I*c) - 1241856*I*a^4*e
^(6*I*d*x) + 16128*I*a^4*e^(-6*I*c))/(d*e^(13*I*d*x + 7*I*c) + 6*d*e^(11*I*
d*x + 5*I*c) + 15*d*e^(9*I*d*x + 3*I*c) + 20*d*e^(7*I*d*x + I*c) + 15*d*e^(
5*I*d*x - I*c) + 6*d*e^(3*I*d*x - 3*I*c) + d*e^(I*d*x - 5*I*c))
```

Mupad [B] (verification not implemented)

Time = 6.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.21

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^4\,dx = \frac{2\,a^4\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{89\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}-\frac{55\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{4}+\frac{55\,\sin\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{4}-\frac{355\,\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{16}+\frac{35\,\sin\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)}{16}-\cos(\cdots\right.}{63\,d\,\left(\cos\left(4\,c+4\,d\,x\right)-\sin\left(4\,c+4\,d\,x\right)\,1\mathrm{i}\right)}$$

```
[In] int(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] (2*a^4*cos(c/2 + (d*x)/2)*((cos((5*c)/2 + (5*d*x)/2)*21i)/2 - (cos((3*c)/2
+ (3*d*x)/2)*21i)/2 - (cos((7*c)/2 + (7*d*x)/2)*87i)/4 + (cos((9*c)/2 + (9*
d*x)/2)*7i)/4 + (89*sin(c/2 + (d*x)/2))/8 - (55*sin((3*c)/2 + (3*d*x)/2))/4
 + (55*sin((5*c)/2 + (5*d*x)/2))/4 - (355*sin((7*c)/2 + (7*d*x)/2))/16 + (3
5*sin((9*c)/2 + (9*d*x)/2))/16))/(63*d*(cos(4*c + 4*d*x) - sin(4*c + 4*d*x)
*1i))
```

3.59 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal result

Integrand size = 24, antiderivative size = 109

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{8i(a+ia\tan(c+dx))^9}{9a^4d} + \frac{6i(a+ia\tan(c+dx))^{10}}{5a^5d} - \frac{6i(a+ia\tan(c+dx))^{11}}{11a^6d} + \frac{i(a+ia\tan(c+dx))^{12}}{12a^7d}$$

```
[Out] -8/9*I*(a+I*a*tan(d*x+c))^9/a^4/d+6/5*I*(a+I*a*tan(d*x+c))^10/a^5/d-6/11*I*
(a+I*a*tan(d*x+c))^11/a^6/d+1/12*I*(a+I*a*tan(d*x+c))^12/a^7/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{i(a+ia\tan(c+dx))^{12}}{12a^7d} - \frac{6i(a+ia\tan(c+dx))^{11}}{11a^6d} + \frac{6i(a+ia\tan(c+dx))^{10}}{5a^5d} - \frac{8i(a+ia\tan(c+dx))^9}{9a^4d}$$

```
[In] Int[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (((-8*I)/9)*(a + I*a*Tan[c + d*x])^9)/(a^4*d) + (((6*I)/5)*(a + I*a*Tan[c +
 d*x])^10)/(a^5*d) - (((6*I)/11)*(a + I*a*Tan[c + d*x])^11)/(a^6*d) + ((I/1
2)*(a + I*a*Tan[c + d*x])^12)/(a^7*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)^3(a+x)^8\,dx, x, ia\tan(c+dx)\right)}{a^7d}\\ &= -\frac{i\text{Subst}\left(\int \left(8a^3(a+x)^8-12a^2(a+x)^9+6a(a+x)^{10}-(a+x)^{11}\right)\,dx, x, ia\tan(c+dx)\right)}{a^7d}\\ &= -\frac{8i(a+ia\tan(c+dx))^9}{9a^4d}+\frac{6i(a+ia\tan(c+dx))^{10}}{5a^5d}\\ &\quad -\frac{6i(a+ia\tan(c+dx))^{11}}{11a^6d}+\frac{i(a+ia\tan(c+dx))^{12}}{12a^7d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.74

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{a^5\sec^{12}(c+dx)(78\cos(c+dx)+221\cos(3(c+dx))-3i(18\sin(c+dx)+73\sin(3(c+dx))))(-i\cos(9(c}{1980d}$$

```
[In] Integrate[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*Sec[c + d*x]^12*(78*Cos[c + d*x] + 221*Cos[3*(c + d*x)] - (3*I)*(18*Si
n[c + d*x] + 73*Sin[3*(c + d*x)]))*((-I)*Cos[9*(c + d*x)] + Sin[9*(c + d*x)
]))/(1980*d)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 376 vs. 2(93) = 186.

Time = 1.15 (sec) , antiderivative size = 377, normalized size of antiderivative = 3.46

$$ia^5\left(\frac{\sin^6(dx+c)}{12\cos(dx+c)^{12}}+\frac{\sin^6(dx+c)}{20\cos(dx+c)^{10}}+\frac{\sin^6(dx+c)}{40\cos(dx+c)^8}+\frac{\sin^6(dx+c)}{120\cos(dx+c)^6}\right)+5a^5\left(\frac{\sin^5(dx+c)}{11\cos(dx+c)^{11}}+\frac{2(\sin^5(dx+c))}{33\cos(dx+c)^9}+\frac{8(\sin^5(dx+c))}{231\cos(dx+c)^7}\right.$$

```
[In] int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x)
```

```
[Out] 1/d*(I*a^5*(1/12*sin(d*x+c)^6/cos(d*x+c)^12+1/20*sin(d*x+c)^6/cos(d*x+c)^10
+1/40*sin(d*x+c)^6/cos(d*x+c)^8+1/120*sin(d*x+c)^6/cos(d*x+c)^6)+5*a^5*(1/1
1*sin(d*x+c)^5/cos(d*x+c)^11+2/33*sin(d*x+c)^5/cos(d*x+c)^9+8/231*sin(d*x+c
)^5/cos(d*x+c)^7+16/1155*sin(d*x+c)^5/cos(d*x+c)^5)-10*I*a^5*(1/10*sin(d*x+
c)^4/cos(d*x+c)^10+3/40*sin(d*x+c)^4/cos(d*x+c)^8+1/20*sin(d*x+c)^4/cos(d*x
+c)^6+1/40*sin(d*x+c)^4/cos(d*x+c)^4)-10*a^5*(1/9*sin(d*x+c)^3/cos(d*x+c)^9
+2/21*sin(d*x+c)^3/cos(d*x+c)^7+8/105*sin(d*x+c)^3/cos(d*x+c)^5+16/315*sin(
d*x+c)^3/cos(d*x+c)^3)+5/8*I*a^5/cos(d*x+c)^8-a^5*(-16/35-1/7*sec(d*x+c)^6-
6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan(d*x+c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(85) = 170.

Time = 0.23 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.45

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{1024\left(-495i\,a^5e^{(16i\,dx+16i\,c)}-792i\,a^5e^{(14i\,dx+14i\,c)}-924i\,a^5e^{(12i\,dx+12i\,c)}-792i\,a\right.}{495\left(de^{(24i\,dx+24i\,c)}+12\,de^{(22i\,dx+22i\,c)}+66\,de^{(20i\,dx+20i\,c)}+220\,de^{(18i\,dx+18i\,c)}+495\,de^{(16i\,dx+16i\,c)}+792\,de^{(}\right.}$$

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1024/495*(-495*I*a^5*e^(16*I*d*x + 16*I*c) - 792*I*a^5*e^(14*I*d*x + 14*I*
c) - 924*I*a^5*e^(12*I*d*x + 12*I*c) - 792*I*a^5*e^(10*I*d*x + 10*I*c) - 49
5*I*a^5*e^(8*I*d*x + 8*I*c) - 220*I*a^5*e^(6*I*d*x + 6*I*c) - 66*I*a^5*e^(4
*I*d*x + 4*I*c) - 12*I*a^5*e^(2*I*d*x + 2*I*c) - I*a^5)/(d*e^(24*I*d*x + 24
*I*c) + 12*d*e^(22*I*d*x + 22*I*c) + 66*d*e^(20*I*d*x + 20*I*c) + 220*d*e^(
18*I*d*x + 18*I*c) + 495*d*e^(16*I*d*x + 16*I*c) + 792*d*e^(14*I*d*x + 14*I
*c) + 924*d*e^(12*I*d*x + 12*I*c) + 792*d*e^(10*I*d*x + 10*I*c) + 495*d*e^(
8*I*d*x + 8*I*c) + 220*d*e^(6*I*d*x + 6*I*c) + 66*d*e^(4*I*d*x + 4*I*c) + 1
2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^5\,dx = ia^5\left(\int \left(-i\sec^8(c+dx)\right)\,dx + \int 5\tan(c+dx)\sec^8(c+dx)\,dx + \int \left(-10\tan^3(c+dx)\sec^8(c+dx)\right)\,dx + \int \tan^5(c+dx)\sec^8(c+dx)\,dx + \int 10i\tan^2(c+dx)\sec^8(c+dx)\,dx + \int \left(-5i\tan^4(c+dx)\sec^8(c+dx)\right)\,dx\right)$$

```
[In] integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] I*a**5*(Integral(-I*sec(c + d*x)**8, x) + Integral(5*tan(c + d*x)*sec(c + d
*x)**8, x) + Integral(-10*tan(c + d*x)**3*sec(c + d*x)**8, x) + Integral(ta
n(c + d*x)**5*sec(c + d*x)**8, x) + Integral(10*I*tan(c + d*x)**2*sec(c + d
*x)**8, x) + Integral(-5*I*tan(c + d*x)**4*sec(c + d*x)**8, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.47

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{-165i\,a^5\tan(dx+c)^{12} - 900\,a^5\tan(dx+c)^{11} + 1386i\,a^5\tan(dx+c)^{10} - 1100\,a^5\tan(dx+c)^9 + 5445$$

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/1980*(-165*I*a^5*tan(d*x + c)^12 - 900*a^5*tan(d*x + c)^11 + 1386*I*a^5*
tan(d*x + c)^10 - 1100*a^5*tan(d*x + c)^9 + 5445*I*a^5*tan(d*x + c)^8 + 396
0*a^5*tan(d*x + c)^7 + 4620*I*a^5*tan(d*x + c)^6 + 8712*a^5*tan(d*x + c)^5
- 2475*I*a^5*tan(d*x + c)^4 + 4620*a^5*tan(d*x + c)^3 - 4950*I*a^5*tan(d*x
+ c)^2 - 1980*a^5*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.86 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.47

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^5\,dx =$$

$$-\frac{-165i\,a^5\tan(dx+c)^{12} - 900\,a^5\tan(dx+c)^{11} + 1386i\,a^5\tan(dx+c)^{10} - 1100\,a^5\tan(dx+c)^9 + 5445i}{}$$

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/1980*(-165*I*a^5*tan(d*x + c)^12 - 900*a^5*tan(d*x + c)^11 + 1386*I*a^5*
tan(d*x + c)^10 - 1100*a^5*tan(d*x + c)^9 + 5445*I*a^5*tan(d*x + c)^8 + 396
0*a^5*tan(d*x + c)^7 + 4620*I*a^5*tan(d*x + c)^6 + 8712*a^5*tan(d*x + c)^5
- 2475*I*a^5*tan(d*x + c)^4 + 4620*a^5*tan(d*x + c)^3 - 4950*I*a^5*tan(d*x
+ c)^2 - 1980*a^5*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 4.23 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.34

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^5\,dx$$

$$= \frac{a^5\left(-\cos(c+d\,x)^{12}\,1749\mathrm{i} + 2048\,\sin(c+d\,x)\,\cos(c+d\,x)^{11} + 1024\,\sin(c+d\,x)\,\cos(c+d\,x)^9 + 768\,\sin(\right.}{}$$

```
[In] int((a + a*tan(c + d*x)*1i)^5/cos(c + d*x)^8,x)
```

```
[Out] (a^5*(900*cos(c + d*x)*sin(c + d*x) - 3400*cos(c + d*x)^3*sin(c + d*x) + 64
0*cos(c + d*x)^5*sin(c + d*x) + 768*cos(c + d*x)^7*sin(c + d*x) + 1024*cos(
c + d*x)^9*sin(c + d*x) + 2048*cos(c + d*x)^11*sin(c + d*x) - cos(c + d*x)^
2*2376i + cos(c + d*x)^4*3960i - cos(c + d*x)^12*1749i + 165i))/(1980*d*cos
(c + d*x)^12)
```

3.60 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{i(a+ia\tan(c+dx))^8}{2a^3d}+\frac{4i(a+ia\tan(c+dx))^9}{9a^4d} - \frac{i(a+ia\tan(c+dx))^{10}}{10a^5d}$$

```
[Out] -1/2*I*(a+I*a*tan(d*x+c))^8/a^3/d+4/9*I*(a+I*a*tan(d*x+c))^9/a^4/d-1/10*I*(
a+I*a*tan(d*x+c))^10/a^5/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{i(a+ia\tan(c+dx))^{10}}{10a^5d} + \frac{4i(a+ia\tan(c+dx))^9}{9a^4d}-\frac{i(a+ia\tan(c+dx))^8}{2a^3d}$$

```
[In] Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] ((-1/2*I)*(a + I*a*Tan[c + d*x])^8)/(a^3*d) + (((4*I)/9)*(a + I*a*Tan[c + d
*x])^9)/(a^4*d) - ((I/10)*(a + I*a*Tan[c + d*x])^10)/(a^5*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)^2(a+x)^7\,dx, x, ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{i\text{Subst}\left(\int \left(4a^2(a+x)^7-4a(a+x)^8+(a+x)^9\right)\,dx, x, ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{i(a+ia\tan(c+dx))^8}{2a^3d}+\frac{4i(a+ia\tan(c+dx))^9}{9a^4d}-\frac{i(a+ia\tan(c+dx))^{10}}{10a^5d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.77

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{a^5\sec^{10}(c+dx)(5+23\cos(2(c+dx))-22i\sin(2(c+dx)))(-i\cos(8(c+dx))+\sin(8(c+dx)))}{180d}$$

```
[In] Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^5,x]
[Out] (a^5*Sec[c + d*x]^10*(5 + 23*Cos[2*(c + d*x)] - (22*I)*Sin[2*(c + d*x)])*((
-I)*Cos[8*(c + d*x)] + Sin[8*(c + d*x)]))/(180*d)
```

Maple [A] (verified)

Time = 181.82 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.24

method	result
risch	$\frac{128ia^5(120\,\mathrm{e}^{14i(dx+c)}+210\,\mathrm{e}^{12i(dx+c)}+252\,\mathrm{e}^{10i(dx+c)}+210\,\mathrm{e}^{8i(dx+c)}+120\,\mathrm{e}^{6i(dx+c)}+45\,\mathrm{e}^{4i(dx+c)}+10\,\mathrm{e}^{2i(dx+c)}+1)}{45d(\mathrm{e}^{2i(dx+c)}+1)^{10}}$
derivativedivides	$ia^5\left(\frac{\sin^6(dx+c)}{10\cos(dx+c)^{10}}+\frac{\sin^6(dx+c)}{20\cos(dx+c)^8}+\frac{\sin^6(dx+c)}{60\cos(dx+c)^6}\right)+5a^5\left(\frac{\sin^5(dx+c)}{9\cos(dx+c)^9}+\frac{4(\sin^5(dx+c))}{63\cos(dx+c)^7}+\frac{8(\sin^5(dx+c))}{315\cos(dx+c)^5}\right)-10ia^5\left(\frac{\sin^4(}{8\cos(}\right.$
default	$ia^5\left(\frac{\sin^6(dx+c)}{10\cos(dx+c)^{10}}+\frac{\sin^6(dx+c)}{20\cos(dx+c)^8}+\frac{\sin^6(dx+c)}{60\cos(dx+c)^6}\right)+5a^5\left(\frac{\sin^5(dx+c)}{9\cos(dx+c)^9}+\frac{4(\sin^5(dx+c))}{63\cos(dx+c)^7}+\frac{8(\sin^5(dx+c))}{315\cos(dx+c)^5}\right)-10ia^5\left(\frac{\sin^4(}{8\cos(}\right.$

```
[In] int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 128/45*I*a^5*(120*exp(14*I*(d*x+c))+210*exp(12*I*(d*x+c))+252*exp(10*I*(d*x
+c))+210*exp(8*I*(d*x+c))+120*exp(6*I*(d*x+c))+45*exp(4*I*(d*x+c))+10*exp(2
*I*(d*x+c))+1)/d/(exp(2*I*(d*x+c))+1)^10
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(64) = 128$.

Time = 0.24 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.79

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^5\,dx =$$

$$-\frac{128\left(-120i\,a^5e^{(14i\,dx+14i\,c)}-210i\,a^5e^{(12i\,dx+12i\,c)}-252i\,a^5e^{(10i\,dx+10i\,c)}-210i\,a^5e^{(8i\,dx+8}\right.}{45\left(de^{(20i\,dx+20i\,c)}+10\,de^{(18i\,dx+18i\,c)}+45\,de^{(16i\,dx+16i\,c)}+120\,de^{(14i\,dx+14i\,c)}+210\,de^{(12i\,dx+12i\,c)}+252\,de^{(}\right.}$$

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -128/45*(-120*I*a^5*e^(14*I*d*x + 14*I*c) - 210*I*a^5*e^(12*I*d*x + 12*I*c)
 - 252*I*a^5*e^(10*I*d*x + 10*I*c) - 210*I*a^5*e^(8*I*d*x + 8*I*c) - 120*I*
a^5*e^(6*I*d*x + 6*I*c) - 45*I*a^5*e^(4*I*d*x + 4*I*c) - 10*I*a^5*e^(2*I*d*
x + 2*I*c) - I*a^5)/(d*e^(20*I*d*x + 20*I*c) + 10*d*e^(18*I*d*x + 18*I*c) +
 45*d*e^(16*I*d*x + 16*I*c) + 120*d*e^(14*I*d*x + 14*I*c) + 210*d*e^(12*I*d
*x + 12*I*c) + 252*d*e^(10*I*d*x + 10*I*c) + 210*d*e^(8*I*d*x + 8*I*c) + 12
0*d*e^(6*I*d*x + 6*I*c) + 45*d*e^(4*I*d*x + 4*I*c) + 10*d*e^(2*I*d*x + 2*I*
c) + d)
```

Sympy [F]

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^5\,dx = ia^5\left(\int \left(-i\sec^6\left(c+dx\right)\right)\,dx + \int 5\tan\left(c+dx\right)\sec^6\left(c+dx\right)dx + \int \left(-10\tan^3\left(c+dx\right)\sec^6\left(c+dx\right)\right)\,dx + \int \tan^5\left(c+dx\right)\sec^6\left(c+dx\right)dx + \int 10i\tan^2\left(c+dx\right)\sec^6\left(c+dx\right)dx + \int \left(-5i\tan^4\left(c+dx\right)\sec^6\left(c+dx\right)\right)\,dx\right)$$

```
[In] integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] I*a**5*(Integral(-I*sec(c + d*x)**6, x) + Integral(5*tan(c + d*x)*sec(c + d
*x)**6, x) + Integral(-10*tan(c + d*x)**3*sec(c + d*x)**6, x) + Integral(ta
n(c + d*x)**5*sec(c + d*x)**6, x) + Integral(10*I*tan(c + d*x)**2*sec(c + d
*x)**6, x) + Integral(-5*I*tan(c + d*x)**4*sec(c + d*x)**6, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.32

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{-9i\,a^5\tan\left(dx+c\right)^{10} - 50\,a^5\tan\left(dx+c\right)^9 + 90i\,a^5\tan\left(dx+c\right)^8 + 210i\,a^5\tan\left(dx+c\right)^6 + 252\,a^5\tan\left(dx\right.}{90\,d}$$

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/90*(-9*I*a^5*tan(d*x + c)^10 - 50*a^5*tan(d*x + c)^9 + 90*I*a^5*tan(d*x
+ c)^8 + 210*I*a^5*tan(d*x + c)^6 + 252*a^5*tan(d*x + c)^5 + 240*a^5*tan(d*
x + c)^3 - 225*I*a^5*tan(d*x + c)^2 - 90*a^5*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.81 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.32

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{-9i\,a^5\tan\left(dx+c\right)^{10} - 50\,a^5\tan\left(dx+c\right)^9 + 90i\,a^5\tan\left(dx+c\right)^8 + 210i\,a^5\tan\left(dx+c\right)^6 + 252\,a^5\tan\left(d\right.}{90\,d}$$

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/90*(-9*I*a^5*tan(d*x + c)^10 - 50*a^5*tan(d*x + c)^9 + 90*I*a^5*tan(d*x
+ c)^8 + 210*I*a^5*tan(d*x + c)^6 + 252*a^5*tan(d*x + c)^5 + 240*a^5*tan(d*
x + c)^3 - 225*I*a^5*tan(d*x + c)^2 - 90*a^5*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 4.05 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.84

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{a^5\,\sin\left(c+d\,x\right)\,\left(90\cos\left(c+d\,x\right)^9 + \cos\left(c+d\,x\right)^8\,\sin\left(c+d\,x\right)\,225\mathrm{i} - 240\cos\left(c+d\,x\right)^7\sin\left(c+d\,x\right)^2 - 2\right.}{}$$

```
[In] int((a + a*tan(c + d*x)*1i)^5/cos(c + d*x)^6,x)
```

```
[Out] (a^5*sin(c + d*x)*(50*cos(c + d*x)*sin(c + d*x)^8 + cos(c + d*x)^8*sin(c +
d*x)*225i + 90*cos(c + d*x)^9 + sin(c + d*x)^9*9i - cos(c + d*x)^2*sin(c +
d*x)^7*90i - cos(c + d*x)^4*sin(c + d*x)^5*210i - 252*cos(c + d*x)^5*sin(c
+ d*x)^4 - 240*cos(c + d*x)^7*sin(c + d*x)^2))/(90*d*cos(c + d*x)^10)
```

3.61 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{2i(a+ia\tan(c+dx))^7}{7a^2d}+\frac{i(a+ia\tan(c+dx))^8}{8a^3d}$$

```
[Out] -2/7*I*(a+I*a*tan(d*x+c))^7/a^2/d+1/8*I*(a+I*a*tan(d*x+c))^8/a^3/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{i(a+ia\tan(c+dx))^8}{8a^3d}-\frac{2i(a+ia\tan(c+dx))^7}{7a^2d}$$

```
[In] Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (((-2*I)/7)*(a + I*a*Tan[c + d*x])^7)/(a^2*d) + ((I/8)*(a + I*a*Tan[c + d*x
])^8)/(a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
```

```
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)(a+x)^6\,dx, x, ia\tan(c+dx)\right)}{a^3d} \\
&= -\frac{i\text{Subst}\left(\int \left(2a(a+x)^6-(a+x)^7\right)\,dx, x, ia\tan(c+dx)\right)}{a^3d} \\
&= -\frac{2i(a+ia\tan(c+dx))^7}{7a^2d}+\frac{i(a+ia\tan(c+dx))^8}{8a^3d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

$$
\int \sec^4(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{ia^5(-i+\tan(c+dx))^7(9i+7\tan(c+dx))}{56d}
$$

```
[In] Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^5,x]
[Out] ((I/56)*a^5*(-I + Tan[c + d*x])^7*(9*I + 7*Tan[c + d*x]))/d
```

Maple [A] (verified)

Time = 66.86 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.65

method	result
risch	$\frac{32ia^5\left(28\,e^{12i(dx+c)}+56\,e^{10i(dx+c)}+70\,e^{8i(dx+c)}+56\,e^{6i(dx+c)}+28\,e^{4i(dx+c)}+8\,e^{2i(dx+c)}+1\right)}{7d\left(e^{2i(dx+c)}+1\right)^8}$
derivativedivides	$\frac{ia^5\left(\frac{\sin^6(dx+c)}{8\cos(dx+c)^8}+\frac{\sin^6(dx+c)}{24\cos(dx+c)^6}\right)+5a^5\left(\frac{\sin^5(dx+c)}{7\cos(dx+c)^7}+\frac{2\left(\sin^5(dx+c)\right)}{35\cos(dx+c)^5}\right)-10ia^5\left(\frac{\sin^4(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^4(dx+c)}{12\cos(dx+c)^4}\right)-10a^5\left(\cdots\right.}{d}$
default	$\frac{ia^5\left(\frac{\sin^6(dx+c)}{8\cos(dx+c)^8}+\frac{\sin^6(dx+c)}{24\cos(dx+c)^6}\right)+5a^5\left(\frac{\sin^5(dx+c)}{7\cos(dx+c)^7}+\frac{2\left(\sin^5(dx+c)\right)}{35\cos(dx+c)^5}\right)-10ia^5\left(\frac{\sin^4(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^4(dx+c)}{12\cos(dx+c)^4}\right)-10a^5\left(\cdots\right.}{d}$

```
[In] int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
[Out] 32/7*I*a^5*(28*exp(12*I*(d*x+c))+56*exp(10*I*(d*x+c))+70*exp(8*I*(d*x+c))+5
6*exp(6*I*(d*x+c))+28*exp(4*I*(d*x+c))+8*exp(2*I*(d*x+c))+1)/d/(exp(2*I*(d*
x+c))+1)^8
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(43) = 86$.

Time = 0.24 (sec) , antiderivative size = 191, normalized size of antiderivative = 3.47

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{32\left(-28i\,a^5e^{(12i\,dx+12i\,c)}-56i\,a^5e^{(10i\,dx+10i\,c)}-70i\,a^5e^{(8i\,dx+8i\,c)}-56i\,a^5e^{(6i\,dx+6i\,c)}-28i\,a^5e^{(4i\,dx+}\right)}{7\left(de^{(16i\,dx+16i\,c)}+8\,de^{(14i\,dx+14i\,c)}+28\,de^{(12i\,dx+12i\,c)}+56\,de^{(10i\,dx+10i\,c)}+70\,de^{(8i\,dx+8i\,c)}+56\,de^{(6i\,dx+6i\,c)}\right)}$$

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -32/7*(-28*I*a^5*e^(12*I*d*x + 12*I*c) - 56*I*a^5*e^(10*I*d*x + 10*I*c) - 7
0*I*a^5*e^(8*I*d*x + 8*I*c) - 56*I*a^5*e^(6*I*d*x + 6*I*c) - 28*I*a^5*e^(4*
I*d*x + 4*I*c) - 8*I*a^5*e^(2*I*d*x + 2*I*c) - I*a^5)/(d*e^(16*I*d*x + 16*I
*c) + 8*d*e^(14*I*d*x + 14*I*c) + 28*d*e^(12*I*d*x + 12*I*c) + 56*d*e^(10*I
*d*x + 10*I*c) + 70*d*e^(8*I*d*x + 8*I*c) + 56*d*e^(6*I*d*x + 6*I*c) + 28*d
*e^(4*I*d*x + 4*I*c) + 8*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^5\,dx = ia^5\left(\int\left(-i\sec^4\left(c+dx\right)\right)\,dx + \int 5\tan\left(c+dx\right)\sec^4\left(c+dx\right)dx + \int\left(-10\tan^3\left(c+dx\right)\sec^4\left(c+dx\right)\right)\,dx + \int\tan^5\left(c+dx\right)\sec^4\left(c+dx\right)dx + \int 10i\tan^2\left(c+dx\right)\sec^4\left(c+dx\right)dx + \int\left(-5i\tan^4\left(c+dx\right)\sec^4\left(c+dx\right)\right)\,dx\right)$$

```
[In] integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] I*a**5*(Integral(-I*sec(c + d*x)**4, x) + Integral(5*tan(c + d*x)*sec(c + d
*x)**4, x) + Integral(-10*tan(c + d*x)**3*sec(c + d*x)**4, x) + Integral(ta
n(c + d*x)**5*sec(c + d*x)**4, x) + Integral(10*I*tan(c + d*x)**2*sec(c + d
*x)**4, x) + Integral(-5*I*tan(c + d*x)**4*sec(c + d*x)**4, x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(43) = 86.

Time = 0.33 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.96

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^5\,dx =$$
$$-\frac{-7i\,a^5\tan\left(dx+c\right)^8-40\,a^5\tan\left(dx+c\right)^7+84i\,a^5\tan\left(dx+c\right)^6+56\,a^5\tan\left(dx+c\right)^5+70i\,a^5\tan\left(dx-\right.}{56\,d}$$

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/56*(-7*I*a^5*tan(d*x + c)^8 - 40*a^5*tan(d*x + c)^7 + 84*I*a^5*tan(d*x +
 c)^6 + 56*a^5*tan(d*x + c)^5 + 70*I*a^5*tan(d*x + c)^4 + 168*a^5*tan(d*x +
 c)^3 - 140*I*a^5*tan(d*x + c)^2 - 56*a^5*tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(43) = 86.

Time = 0.78 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.96

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^5\,dx =$$
$$-\frac{-7i\,a^5\tan\left(dx+c\right)^8-40\,a^5\tan\left(dx+c\right)^7+84i\,a^5\tan\left(dx+c\right)^6+56\,a^5\tan\left(dx+c\right)^5+70i\,a^5\tan\left(dx-\right.}{56\,d}$$

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/56*(-7*I*a^5*tan(d*x + c)^8 - 40*a^5*tan(d*x + c)^7 + 84*I*a^5*tan(d*x +
 c)^6 + 56*a^5*tan(d*x + c)^5 + 70*I*a^5*tan(d*x + c)^4 + 168*a^5*tan(d*x +
 c)^3 - 140*I*a^5*tan(d*x + c)^2 - 56*a^5*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 3.89 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.75

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$=\frac{a^5\,\sin\left(c+d\,x\right)\,\left(56\cos\left(c+d\,x\right)^7+\cos\left(c+d\,x\right)^6\,\sin\left(c+d\,x\right)\,140\mathrm{i}-168\cos\left(c+d\,x\right)^5\sin\left(c+d\,x\right)^2-\mathrm{co}\right.}{}$$

```
[In] int((a + a*tan(c + d*x)*1i)^5/cos(c + d*x)^4,x)
```

```
[Out] (a^5*sin(c + d*x)*(40*cos(c + d*x)*sin(c + d*x)^6 + cos(c + d*x)^6*sin(c +
d*x)*140i + 56*cos(c + d*x)^7 + sin(c + d*x)^7*7i - cos(c + d*x)^2*sin(c +
d*x)^5*84i - 56*cos(c + d*x)^3*sin(c + d*x)^4 - cos(c + d*x)^4*sin(c + d*x)
^3*70i - 168*cos(c + d*x)^5*sin(c + d*x)^2))/(56*d*cos(c + d*x)^8)
```

3.62 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{i(a+ia\tan(c+dx))^6}{6ad}$$

```
[Out] -1/6*I*(a+I*a*tan(d*x+c))^6/a/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 32}

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{i(a+ia\tan(c+dx))^6}{6ad}$$

```
[In] Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^5,x]
[Out] ((-1/6*I)*(a + I*a*Tan[c + d*x])^6)/(a*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{i\text{Subst}\left(\int (a+x)^5\,dx, x, ia\tan(c+dx)\right)}{ad}$$
$$= -\frac{i(a+ia\tan(c+dx))^6}{6ad}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 72 vs. $2(27) = 54$.

Time = 0.34 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.67

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \frac{a^5\tan(c+dx)\left(6+15i\tan(c+dx)-20\tan^2(c+dx)-15i\tan^3(c+dx)+6\tan^4(c+dx)+i\tan^5(c+dx)\right)}{6d}$$

```
[In] Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*Tan[c + d*x]*(6 + (15*I)*Tan[c + d*x] - 20*Tan[c + d*x]^2 - (15*I)*Tan
[c + d*x]^3 + 6*Tan[c + d*x]^4 + I*Tan[c + d*x]^5))/(6*d)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(23) = 46$.

Time = 20.70 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.96

method	result	size
risch	$\frac{32ia^5\left(6\,\mathrm{e}^{10i(dx+c)}+15\,\mathrm{e}^{8i(dx+c)}+20\,\mathrm{e}^{6i(dx+c)}+15\,\mathrm{e}^{4i(dx+c)}+6\,\mathrm{e}^{2i(dx+c)}+1\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^6}$	80
derivativedivides	$\frac{\frac{ia^5\left(\sin^6(dx+c)\right)}{6\cos(dx+c)^6}+\frac{a^5\left(\sin^5(dx+c)\right)}{\cos(dx+c)^5}-\frac{5ia^5\left(\sin^4(dx+c)\right)}{2\cos(dx+c)^4}-\frac{10a^5\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+\frac{5ia^5}{2\cos(dx+c)^2}+a^5\tan(dx+c)}{d}$	115
default	$\frac{\frac{ia^5\left(\sin^6(dx+c)\right)}{6\cos(dx+c)^6}+\frac{a^5\left(\sin^5(dx+c)\right)}{\cos(dx+c)^5}-\frac{5ia^5\left(\sin^4(dx+c)\right)}{2\cos(dx+c)^4}-\frac{10a^5\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+\frac{5ia^5}{2\cos(dx+c)^2}+a^5\tan(dx+c)}{d}$	115

```
[In] int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 32/3*I*a^5*(6*exp(10*I*(d*x+c))+15*exp(8*I*(d*x+c))+20*exp(6*I*(d*x+c))+15*
exp(4*I*(d*x+c))+6*exp(2*I*(d*x+c))+1)/d/(exp(2*I*(d*x+c))+1)^6
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(21) = 42.

Time = 0.23 (sec) , antiderivative size = 153, normalized size of antiderivative = 5.67

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{32\left(-6i\,a^5e^{(10i\,dx+10i\,c)}-15i\,a^5e^{(8i\,dx+8i\,c)}-20i\,a^5e^{(6i\,dx+6i\,c)}-15i\,a^5e^{(4i\,dx+4i\,c)}-6i\,a^5e^{(2i\,dx+2i\,c)}-i\,a^5\right)}{3\left(de^{(12i\,dx+12i\,c)}+6\,de^{(10i\,dx+10i\,c)}+15\,de^{(8i\,dx+8i\,c)}+20\,de^{(6i\,dx+6i\,c)}+15\,de^{(4i\,dx+4i\,c)}+6\,de^{(2i\,dx+2i\,c)}+d\right)}$$

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -32/3*(-6*I*a^5*e^(10*I*d*x + 10*I*c) - 15*I*a^5*e^(8*I*d*x + 8*I*c) - 20*I
*a^5*e^(6*I*d*x + 6*I*c) - 15*I*a^5*e^(4*I*d*x + 4*I*c) - 6*I*a^5*e^(2*I*d*
x + 2*I*c) - I*a^5)/(d*e^(12*I*d*x + 12*I*c) + 6*d*e^(10*I*d*x + 10*I*c) +
15*d*e^(8*I*d*x + 8*I*c) + 20*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*I*d*x + 4*I
*c) + 6*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^5\,dx = ia^5\left(\int\left(-i\sec^2(c+dx)\right)dx + \int 5\tan(c+dx)\sec^2(c+dx)\,dx + \int\left(-10\tan^3(c+dx)\sec^2(c+dx)\right)dx + \int \tan^5(c+dx)\sec^2(c+dx)\,dx + \int 10i\tan^2(c+dx)\sec^2(c+dx)\,dx + \int\left(-5i\tan^4(c+dx)\sec^2(c+dx)\right)dx\right)$$

```
[In] integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] I*a**5*(Integral(-I*sec(c + d*x)**2, x) + Integral(5*tan(c + d*x)*sec(c + d
*x)**2, x) + Integral(-10*tan(c + d*x)**3*sec(c + d*x)**2, x) + Integral(ta
n(c + d*x)**5*sec(c + d*x)**2, x) + Integral(10*I*tan(c + d*x)**2*sec(c + d
*x)**2, x) + Integral(-5*I*tan(c + d*x)**4*sec(c + d*x)**2, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{i\,(i\,a\tan(dx+c)+a)^6}{6\,ad}$$

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/6*I*(I*a*tan(d*x + c) + a)^6/(a*d)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(21) = 42.

Time = 0.75 (sec) , antiderivative size = 82, normalized size of antiderivative = 3.04

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{-i\,a^5\tan(dx+c)^6 - 6\,a^5\tan(dx+c)^5 + 15i\,a^5\tan(dx+c)^4 + 20\,a^5\tan(dx+c)^3 - 15i\,a^5\tan(dx+c)^2}{6\,d}$$

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/6*(-I*a^5*tan(d*x + c)^6 - 6*a^5*tan(d*x + c)^5 + 15*I*a^5*tan(d*x + c)^
4 + 20*a^5*tan(d*x + c)^3 - 15*I*a^5*tan(d*x + c)^2 - 6*a^5*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 3.75 (sec) , antiderivative size = 114, normalized size of antiderivative = 4.22

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{a^5\sin(c+d\,x)\,\left(6\cos(c+d\,x)^5 + \cos(c+d\,x)^4\sin(c+d\,x)\,15\mathrm{i} - 20\cos(c+d\,x)^3\sin(c+d\,x)^2 - \cos(c\right.}{6\,d\cos(c+d\,x)^6}$$

```
[In] int((a + a*tan(c + d*x)*1i)^5/cos(c + d*x)^2,x)
```

```
[Out] (a^5*sin(c + d*x)*(6*cos(c + d*x)*sin(c + d*x)^4 + cos(c + d*x)^4*sin(c + d
*x)*15i + 6*cos(c + d*x)^5 + sin(c + d*x)^5*1i - cos(c + d*x)^2*sin(c + d*x
)^3*15i - 20*cos(c + d*x)^3*sin(c + d*x)^2))/(6*d*cos(c + d*x)^6)
```

3.63 $\int (a+ia\tan(c+dx))^5\,dx$

Optimal result

Integrand size = 15, antiderivative size = 117

$$\int (a+ia\tan(c+dx))^5\,dx=16a^5x-\frac{16ia^5\log(\cos(c+dx))}{d}-\frac{8a^5\tan(c+dx)}{d}+\frac{2ia^2(a+ia\tan(c+dx))^3}{3d}+\frac{ia(a+ia\tan(c+dx))^4}{4d}+\frac{2ia(a^2+ia^2\tan(c+dx))^2}{d}$$

```
[Out] 16*a^5*x-16*I*a^5*ln(cos(d*x+c))/d-8*a^5*tan(d*x+c)/d+2/3*I*a^2*(a+I*a*tan(
d*x+c))^3/d+1/4*I*a*(a+I*a*tan(d*x+c))^4/d+2*I*a*(a^2+I*a^2*tan(d*x+c))^2/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3559, 3558, 3556}

$$\int (a+ia\tan(c+dx))^5\,dx=-\frac{8a^5\tan(c+dx)}{d}-\frac{16ia^5\log(\cos(c+dx))}{d}+16a^5x+\frac{2ia^2(a+ia\tan(c+dx))^3}{3d}+\frac{2ia(a^2+ia^2\tan(c+dx))^2}{d}+\frac{ia(a+ia\tan(c+dx))^4}{4d}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] 16*a^5*x - ((16*I)*a^5*Log[Cos[c + d*x]])/d - (8*a^5*Tan[c + d*x])/d + (((2
*I)/3)*a^2*(a + I*a*Tan[c + d*x])^3)/d + ((I/4)*a*(a + I*a*Tan[c + d*x])^4)
/d + ((2*I)*a*(a^2 + I*a^2*Tan[c + d*x])^2)/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3558

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[b^2*(Tan[c + d*x]/d),
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{ia(a+ia\tan(c+dx))^4}{4d}+(2a)\int(a+ia\tan(c+dx))^4\,dx\\
&= \frac{2ia^2(a+ia\tan(c+dx))^3}{3d}+\frac{ia(a+ia\tan(c+dx))^4}{4d}+\left(4a^2\right)\int(a+ia\tan(c+dx))^3\,dx\\
&= \frac{2ia^3(a+ia\tan(c+dx))^2}{d}+\frac{2ia^2(a+ia\tan(c+dx))^3}{3d}\\
&\quad+\frac{ia(a+ia\tan(c+dx))^4}{4d}+\left(8a^3\right)\int(a+ia\tan(c+dx))^2\,dx\\
&= 16a^5x-\frac{8a^5\tan(c+dx)}{d}+\frac{2ia^3(a+ia\tan(c+dx))^2}{d}+\frac{2ia^2(a+ia\tan(c+dx))^3}{3d}\\
&\quad+\frac{ia(a+ia\tan(c+dx))^4}{4d}+\left(16ia^5\right)\int\tan(c+dx)\,dx\\
&= 16a^5x-\frac{16ia^5\log(\cos(c+dx))}{d}-\frac{8a^5\tan(c+dx)}{d}+\frac{2ia^3(a+ia\tan(c+dx))^2}{d}\\
&\quad+\frac{2ia^2(a+ia\tan(c+dx))^3}{3d}+\frac{ia(a+ia\tan(c+dx))^4}{4d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.61

$$\int (a+ia\tan(c+dx))^5\,dx = \frac{a^5(35i+192i\log(i+\tan(c+dx))-180\tan(c+dx)-66i\tan^2(c+dx)+20\tan^3(c+dx)+3i\tan^4(c+$$
$$12d$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*(35*I + (192*I)*Log[I + Tan[c + d*x]] - 180*Tan[c + d*x] - (66*I)*Tan[
c + d*x]^2 + 20*Tan[c + d*x]^3 + (3*I)*Tan[c + d*x]^4))/(12*d)
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.62

method	result
derivativedivides	$\frac{a^5\left(-15\tan(dx+c)+\frac{i\left(\tan^4(dx+c)\right)}{4}+\frac{5\left(\tan^3(dx+c)\right)}{3}-\frac{11i\left(\tan^2(dx+c)\right)}{2}+8i\ln(1+\tan^2(dx+c))+16\arctan(\tan(dx+c))\right)}{d}$
default	$\frac{a^5\left(-15\tan(dx+c)+\frac{i\left(\tan^4(dx+c)\right)}{4}+\frac{5\left(\tan^3(dx+c)\right)}{3}-\frac{11i\left(\tan^2(dx+c)\right)}{2}+8i\ln(1+\tan^2(dx+c))+16\arctan(\tan(dx+c))\right)}{d}$
parallelrisch	$\frac{3ia^5\left(\tan^4(dx+c)\right)-66ia^5\left(\tan^2(dx+c)\right)+20\left(\tan^3(dx+c)\right)a^5+96ia^5\ln(1+\tan^2(dx+c))+192a^5xd-180a^5\tan(dx+c)}{12d}$
risch	$-\frac{32a^5c}{d}-\frac{4ia^5\left(48\,\mathrm{e}^{6i(dx+c)}+108\,\mathrm{e}^{4i(dx+c)}+88\,\mathrm{e}^{2i(dx+c)}+25\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^4}-\frac{16ia^5\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{d}$
norman	$16a^5x-\frac{15a^5\tan(dx+c)}{d}+\frac{5a^5\left(\tan^3(dx+c)\right)}{3d}-\frac{11ia^5\left(\tan^2(dx+c)\right)}{2d}+\frac{ia^5\left(\tan^4(dx+c)\right)}{4d}+\frac{8ia^5\ln(1+\tan^2(dx$
parts	$a^5x-\frac{10ia^5\left(\frac{\left(\tan^2(dx+c)\right)}{2}-\frac{\ln\left(1+\tan^2(dx+c)\right)}{2}\right)}{d}+\frac{ia^5\left(\frac{\left(\tan^4(dx+c)\right)}{4}-\frac{\left(\tan^2(dx+c)\right)}{2}+\frac{\ln\left(1+\tan^2(dx+c)\right)}{2}\right)}{d}+\frac{5ia^5}{}$

```
[In] int((a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a^5*(-15*tan(d*x+c)+1/4*I*tan(d*x+c)^4+5/3*tan(d*x+c)^3-11/2*I*tan(d*x+
c)^2+8*I*ln(1+tan(d*x+c)^2)+16*arctan(tan(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.51

$$\int (a + ia\tan(c + dx))^5 \, dx = -\frac{4\left(48i\,a^5e^{(6i\,dx+6i\,c)} + 108i\,a^5e^{(4i\,dx+4i\,c)} + 88i\,a^5e^{(2i\,dx+2i\,c)} + 25i\,a^5 + 12\left(i\,a^5e^{(8i\,dx+8i\,c)} + 4i\,a^5e^{(6i\,dx+6i\,c)} \right.\right.}{3\left(de^{(8i\,dx+8i\,c)} + 4\,de^{(6i\,dx+6i\,c)} + 6\,de^{(4i\,dx+4i\,c)} + 4\,de^{(2i}\right.}$$

```
[In] integrate((a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -4/3*(48*I*a^5*e^(6*I*d*x + 6*I*c) + 108*I*a^5*e^(4*I*d*x + 4*I*c) + 88*I*a
^5*e^(2*I*d*x + 2*I*c) + 25*I*a^5 + 12*(I*a^5*e^(8*I*d*x + 8*I*c) + 4*I*a^5
*e^(6*I*d*x + 6*I*c) + 6*I*a^5*e^(4*I*d*x + 4*I*c) + 4*I*a^5*e^(2*I*d*x + 2
*I*c) + I*a^5)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(8*I*d*x + 8*I*c) + 4*d*e
^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.52

$$\int (a + ia\tan(c + dx))^5 \, dx = -\frac{16ia^5\log\left(e^{2idx} + e^{-2ic}\right)}{d} + \frac{-192ia^5e^{6ic}e^{6idx} - 432ia^5e^{4ic}e^{4idx} - 352ia^5e^{2ic}e^{2idx} - 100ia^5}{3de^{8ic}e^{8idx} + 12de^{6ic}e^{6idx} + 18de^{4ic}e^{4idx} + 12de^{2ic}e^{2idx} + 3d}$$

```
[In] integrate((a+I*a*tan(d*x+c))**5,x)
```

```
[Out] -16*I*a**5*log(exp(2*I*d*x) + exp(-2*I*c))/d + (-192*I*a**5*exp(6*I*c)*exp(
6*I*d*x) - 432*I*a**5*exp(4*I*c)*exp(4*I*d*x) - 352*I*a**5*exp(2*I*c)*exp(2
*I*d*x) - 100*I*a**5)/(3*d*exp(8*I*c)*exp(8*I*d*x) + 12*d*exp(6*I*c)*exp(6*
I*d*x) + 18*d*exp(4*I*c)*exp(4*I*d*x) + 12*d*exp(2*I*c)*exp(2*I*d*x) + 3*d)
```

Maxima [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.41

$$\int (a+ia\tan(c+dx))^5\,dx = a^5x + \frac{5\left(\tan(dx+c)^3+3\,dx+3\,c-3\,\tan(dx+c)\right)a^5}{3\,d} + \frac{10\,(dx+c-\tan(dx+c))a^5}{d} + \frac{i\,a^5\left(\frac{4\,\sin(dx+c)^2-3}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-2\,\log\left(\sin(dx+c)^2-1\right)\right)}{4\,d} + \frac{5i\,a^5\left(\frac{1}{\sin(dx+c)^2-1}-\log\left(\sin(dx+c)^2-1\right)\right)}{d} + \frac{5i\,a^5\log(\sec(dx+c))}{d}$$

```
[In] integrate((a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] a^5*x + 5/3*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^5/d + 10*(d*x
 + c - tan(d*x + c))*a^5/d + 1/4*I*a^5*((4*sin(d*x + c)^2 - 3)/(sin(d*x + c
)^4 - 2*sin(d*x + c)^2 + 1) - 2*log(sin(d*x + c)^2 - 1))/d + 5*I*a^5*(1/(si
n(d*x + c)^2 - 1) - log(sin(d*x + c)^2 - 1))/d + 5*I*a^5*log(sec(d*x + c))/
d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(99) = 198.

Time = 0.40 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.90

$$\int (a+ia\tan(c+dx))^5\,dx = -\frac{4\left(12i\,a^5e^{(8i\,dx+8i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+48i\,a^5e^{(6i\,dx+6i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+72i\,a^5e^{(4i\,dx+4i\,c)}\log\left(e^{(2}\right.\right.}{3\left(de^{(8i\,dx+}\right.}$$

```
[In] integrate((a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -4/3*(12*I*a^5*e^(8*I*d*x + 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 48*I*a^5*
e^(6*I*d*x + 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 72*I*a^5*e^(4*I*d*x + 4*
I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 48*I*a^5*e^(2*I*d*x + 2*I*c)*log(e^(2*I
*d*x + 2*I*c) + 1) + 48*I*a^5*e^(6*I*d*x + 6*I*c) + 108*I*a^5*e^(4*I*d*x +
4*I*c) + 88*I*a^5*e^(2*I*d*x + 2*I*c) + 12*I*a^5*log(e^(2*I*d*x + 2*I*c) +
1) + 25*I*a^5)/(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*
I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Mupad [B] (verification not implemented)

Time = 4.40 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.62

$$\int (a + ia\tan(c + dx))^5 \, dx$$
$$= \frac{a^5 \ln(\tan(c + d\,x) + 1\mathrm{i})\ 16\mathrm{i} - 15\,a^5\tan(c + d\,x) - \frac{a^5\tan(c+d\,x)^2\,11\mathrm{i}}{2} + \frac{5\,a^5\tan(c+d\,x)^3}{3} + \frac{a^5\tan(c+d\,x)^4\,1\mathrm{i}}{4}}{d}$$

```
[In] int((a + a*tan(c + d*x)*1i)^5,x)
```

```
[Out] (a^5*log(tan(c + d*x) + 1i)*16i - 15*a^5*tan(c + d*x) - (a^5*tan(c + d*x)^2
*11i)/2 + (5*a^5*tan(c + d*x)^3)/3 + (a^5*tan(c + d*x)^4*1i)/4)/d
```

3.64 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal result

Integrand size = 24, antiderivative size = 83

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^5\,dx = -12a^5x+\frac{12ia^5\log(\cos(c+dx))}{d}+\frac{5a^5\tan(c+dx)}{d}+\frac{ia^5\tan^2(c+dx)}{2d}-\frac{8ia^6}{d(a-ia\tan(c+dx))}$$

```
[Out] -12*a^5*x+12*I*a^5*ln(cos(d*x+c))/d+5*a^5*tan(d*x+c)/d+1/2*I*a^5*tan(d*x+c)
^2/d-8*I*a^6/d/(a-I*a*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{8ia^6}{d(a-ia\tan(c+dx))}+\frac{ia^5\tan^2(c+dx)}{2d}+\frac{5a^5\tan(c+dx)}{d}+\frac{12ia^5\log(\cos(c+dx))}{d}-12a^5x$$

```
[In] Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] -12*a^5*x + ((12*I)*a^5*Log[Cos[c + d*x]])/d + (5*a^5*Tan[c + d*x])/d + ((I
/2)*a^5*Tan[c + d*x]^2)/d - ((8*I)*a^6)/(d*(a - I*a*Tan[c + d*x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(ia^3)\,\text{Subst}\left(\int \frac{(a+x)^3}{(a-x)^2}\,dx, x, ia\tan(c+dx)\right)}{d}\\ &= -\frac{(ia^3)\,\text{Subst}\left(\int \left(5a+\frac{8a^3}{(a-x)^2}-\frac{12a^2}{a-x}+x\right)\,dx, x, ia\tan(c+dx)\right)}{d}\\ &= -12a^5x+\frac{12ia^5\log(\cos(c+dx))}{d}+\frac{5a^5\tan(c+dx)}{d}\\ &\quad+\frac{ia^5\tan^2(c+dx)}{2d}-\frac{8ia^6}{d(a-ia\tan(c+dx))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.75

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{ia^5\left(24\log(i+\tan(c+dx))+10i\tan(c+dx)-\tan^2(c+dx)+\frac{16i}{i+\tan(c+dx)}\right)}{2d}$$

```
[In] Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^5,x]
[Out] ((-1/2*I)*a^5*(24*Log[I + Tan[c + d*x]] + (10*I)*Tan[c + d*x] - Tan[c + d*x
]^2 + (16*I)/(I + Tan[c + d*x])))/d
```

Maple [A] (verified)

Time = 14.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02

method	result
risch	$-\frac{4ia^5 e^{2i(dx+c)}}{d}+\frac{24a^5c}{d}+\frac{2ia^5(6\,e^{2i(dx+c)}+5)}{d(e^{2i(dx+c)}+1)^2}+\frac{12ia^5\ln(e^{2i(dx+c)}+1)}{d}$
derivativedivides	$\frac{ia^5\left(\frac{\sin^6(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^4(dx+c)\right)}{2}+\sin^2(dx+c)+2\ln(\cos(dx+c))\right)+5a^5\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)\right.}{}$
default	$\frac{ia^5\left(\frac{\sin^6(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^4(dx+c)\right)}{2}+\sin^2(dx+c)+2\ln(\cos(dx+c))\right)+5a^5\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)\right.}{}$

```
[In] int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
[Out] -4*I/d*a^5*exp(2*I*(d*x+c))+24/d*a^5*c+2*I*a^5*(6*exp(2*I*(d*x+c))+5)/d/(ex
p(2*I*(d*x+c))+1)^2+12*I/d*a^5*ln(exp(2*I*(d*x+c))+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.51

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{2\left(2i\,a^5e^{(6i\,dx+6i\,c)}+4i\,a^5e^{(4i\,dx+4i\,c)}-4i\,a^5e^{(2i\,dx+2i\,c)}-5i\,a^5+6\left(-i\,a^5e^{(4i\,dx+4i\,c)}-2i\,a^5e^{(2i\,dx+2i\,c)}-i\,a\right.}{de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d}$$

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
[Out] -2*(2*I*a^5*e^(6*I*d*x + 6*I*c) + 4*I*a^5*e^(4*I*d*x + 4*I*c) - 4*I*a^5*e^(
2*I*d*x + 2*I*c) - 5*I*a^5 + 6*(-I*a^5*e^(4*I*d*x + 4*I*c) - 2*I*a^5*e^(2*I
*d*x + 2*I*c) - I*a^5)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(4*I*d*x + 4*I*c)
 + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.58

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{12ia^5\log\left(e^{2idx}+e^{-2ic}\right)}{d} + \frac{12ia^5e^{2ic}e^{2idx}+10ia^5}{de^{4ic}e^{4idx}+2de^{2ic}e^{2idx}+d} + \begin{cases} -\frac{4ia^5e^{2ic}e^{2idx}}{d} & \text{for } d\neq 0 \\ 8a^5xe^{2ic} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] 12*I*a**5*log(exp(2*I*d*x) + exp(-2*I*c))/d + (12*I*a**5*exp(2*I*c)*exp(2*I
*d*x) + 10*I*a**5)/(d*exp(4*I*c)*exp(4*I*d*x) + 2*d*exp(2*I*c)*exp(2*I*d*x)
 + d) + Piecewise((-4*I*a**5*exp(2*I*c)*exp(2*I*d*x)/d, Ne(d, 0)), (8*a**5*
x*exp(2*I*c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{-i\,a^5\tan\left(dx+c\right)^2+24\left(dx+c\right)a^5+12i\,a^5\log\left(\tan\left(dx+c\right)^2+1\right)-10\,a^5\tan\left(dx+c\right)-\frac{16\left(a^5\tan(dx+c)-\right.}{\tan(dx+c)^2+}}{2\,d}$$

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/2*(-I*a^5*tan(d*x + c)^2 + 24*(d*x + c)*a^5 + 12*I*a^5*log(tan(d*x + c)^
2 + 1) - 10*a^5*tan(d*x + c) - 16*(a^5*tan(d*x + c) - I*a^5)/(tan(d*x + c)^
2 + 1))/d
```

Giac [A] (verification not implemented)

none

Time = 0.94 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.76

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{2\left(-6i\,a^5e^{(4i\,dx+4i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-12i\,a^5e^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+2i\,a^5e^{(6i\,dx+6i\,c)}+4i\,a^5e^{(}\right.}{de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d}$$

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -2*(-6*I*a^5*e^(4*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 12*I*a^5*e^
(2*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 2*I*a^5*e^(6*I*d*x + 6*I*c
) + 4*I*a^5*e^(4*I*d*x + 4*I*c) - 4*I*a^5*e^(2*I*d*x + 2*I*c) - 6*I*a^5*log
(e^(2*I*d*x + 2*I*c) + 1) - 5*I*a^5)/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*
x + 2*I*c) + d)
```

Mupad [B] (verification not implemented)

Time = 3.72 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.84

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{8\,a^5}{d\,\left(\tan\left(c+d\,x\right)+1\mathrm{i}\right)} - \frac{a^5\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,12\mathrm{i}}{d} + \frac{5\,a^5\tan(c+d\,x)}{d} + \frac{a^5\tan(c+d\,x)^2\,1\mathrm{i}}{2\,d}$$

```
[In] int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^5,x)
```

```
[Out] (8*a^5)/(d*(tan(c + d*x) + 1i)) - (a^5*log(tan(c + d*x) + 1i)*12i)/d + (5*a
^5*tan(c + d*x))/d + (a^5*tan(c + d*x)^2*1i)/(2*d)
```

3.65 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal result

Integrand size = 24, antiderivative size = 73

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^5\,dx = a^5x-\frac{ia^5\log(\cos(c+dx))}{d}-\frac{2ia^7}{d(a-ia\tan(c+dx))^2}+\frac{4ia^6}{d(a-ia\tan(c+dx))}$$

```
[Out] a^5*x-I*a^5*ln(cos(d*x+c))/d-2*I*a^7/d/(a-I*a*tan(d*x+c))^2+4*I*a^6/d/(a-I*
a*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{2ia^7}{d(a-ia\tan(c+dx))^2}+\frac{4ia^6}{d(a-ia\tan(c+dx))}-\frac{ia^5\log(\cos(c+dx))}{d}+a^5x$$

```
[In] Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] a^5*x - (I*a^5*Log[Cos[c + d*x]])/d - ((2*I)*a^7)/(d*(a - I*a*Tan[c + d*x])
^2) + ((4*I)*a^6)/(d*(a - I*a*Tan[c + d*x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(ia^5)\operatorname{Subst}\left(\int \frac{(a+x)^2}{(a-x)^3}\,dx, x, ia\tan(c+dx)\right)}{d}\\ &= -\frac{(ia^5)\operatorname{Subst}\left(\int \left(\frac{4a^2}{(a-x)^3}-\frac{4a}{(a-x)^2}+\frac{1}{a-x}\right)dx, x, ia\tan(c+dx)\right)}{d}\\ &= a^5x-\frac{ia^5\log(\cos(c+dx))}{d}-\frac{2ia^7}{d(a-ia\tan(c+dx))^2}+\frac{4ia^6}{d(a-ia\tan(c+dx))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{ia^5\left(-\log(i+\tan(c+dx))+\frac{2-4i\tan(c+dx)}{(i+\tan(c+dx))^2}\right)}{d}$$

```
[In] Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^5,x]
[Out] ((-I)*a^5*(-Log[I + Tan[c + d*x]] + (2 - (4*I)*Tan[c + d*x])/(I + Tan[c + d
*x])^2))/d
```

Maple [A] (verified)

Time = 48.94 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.93

method	result
risch	$-\frac{ia^5 e^{4i(dx+c)}}{2d}+\frac{ia^5 e^{2i(dx+c)}}{d}-\frac{2a^5 c}{d}-\frac{ia^5 \ln(e^{2i(dx+c)}+1)}{d}$
derivativedivides	$\frac{ia^5\left(-\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+5a^5\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{5ia^5\left(\sin^4(\right.}{2}}{}$
default	$\frac{ia^5\left(-\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+5a^5\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{5ia^5\left(\sin^4(\right.}{2}}{}$

```
[In] int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I/d*a^5*exp(4*I*(d*x+c))+I/d*a^5*exp(2*I*(d*x+c))-2/d*a^5*c-I/d*a^5*ln
(exp(2*I*(d*x+c))+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.70

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{-i\,a^5e^{(4i\,dx+4i\,c)}+2i\,a^5e^{(2i\,dx+2i\,c)}-2i\,a^5\log\left(e^{(2i\,dx+2i\,c)}+1\right)}{2\,d}$$

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/2*(-I*a^5*e^(4*I*d*x + 4*I*c) + 2*I*a^5*e^(2*I*d*x + 2*I*c) - 2*I*a^5*log
(e^(2*I*d*x + 2*I*c) + 1))/d
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.40

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{ia^5\log\left(e^{2idx}+e^{-2ic}\right)}{d} + \begin{cases}\frac{-ia^5de^{4ic}e^{4idx}+2ia^5de^{2ic}e^{2idx}}{2d^2} & \text{for } d^2\neq 0\\ x(2a^5e^{4ic}-2a^5e^{2ic}) & \text{otherwise}\end{cases}$$

```
[In] integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] -I*a**5*log(exp(2*I*d*x) + exp(-2*I*c))/d + Piecewise(((-I*a**5*d*exp(4*I*c
)*exp(4*I*d*x) + 2*I*a**5*d*exp(2*I*c)*exp(2*I*d*x))/(2*d**2), Ne(d**2, 0))
, (x*(2*a**5*exp(4*I*c) - 2*a**5*exp(2*I*c)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.21

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{2\,(dx+c)a^5 + i\,a^5\log\left(\tan\left(dx+c\right)^2+1\right) - \frac{4\left(2\,a^5\tan(dx+c)^3 - 3i\,a^5\tan(dx+c)^2 - i\,a^5\right)}{\tan(dx+c)^4+2\,\tan(dx+c)^2+1}}{2\,d}$$

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(d*x + c)*a^5 + I*a^5*log(tan(d*x + c)^2 + 1) - 4*(2*a^5*tan(d*x + c
)^3 - 3*I*a^5*tan(d*x + c)^2 - I*a^5)/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 +
1))/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 450 vs. 2(63) = 126.

Time = 0.70 (sec) , antiderivative size = 450, normalized size of antiderivative = 6.16

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{2i\,a^5e^{(16i\,dx+8i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+16i\,a^5e^{(14i\,dx+6i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+56i\,a^5e^{(12i\,dx+4i\,c)}\log\left(e^{(2i}\right.}{}$$

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/2*(2*I*a^5*e^(16*I*d*x + 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 16*I*a^5*
e^(14*I*d*x + 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 56*I*a^5*e^(12*I*d*x +
4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 112*I*a^5*e^(10*I*d*x + 2*I*c)*log(e^
(2*I*d*x + 2*I*c) + 1) + 112*I*a^5*e^(6*I*d*x - 2*I*c)*log(e^(2*I*d*x + 2*I
*c) + 1) + 56*I*a^5*e^(4*I*d*x - 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 16*I
*a^5*e^(2*I*d*x - 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 140*I*a^5*e^(8*I*d*
x)*log(e^(2*I*d*x + 2*I*c) + 1) + 2*I*a^5*e^(-8*I*c)*log(e^(2*I*d*x + 2*I*c
) + 1) + I*a^5*e^(20*I*d*x + 12*I*c) + 6*I*a^5*e^(18*I*d*x + 10*I*c) + 12*I
*a^5*e^(16*I*d*x + 8*I*c) - 42*I*a^5*e^(12*I*d*x + 4*I*c) - 84*I*a^5*e^(10*
I*d*x + 2*I*c) - 48*I*a^5*e^(6*I*d*x - 2*I*c) - 15*I*a^5*e^(4*I*d*x - 4*I*c
) - 2*I*a^5*e^(2*I*d*x - 6*I*c) - 84*I*a^5*e^(8*I*d*x))/(d*e^(16*I*d*x + 8*
I*c) + 8*d*e^(14*I*d*x + 6*I*c) + 28*d*e^(12*I*d*x + 4*I*c) + 56*d*e^(10*I*
d*x + 2*I*c) + 56*d*e^(6*I*d*x - 2*I*c) + 28*d*e^(4*I*d*x - 4*I*c) + 8*d*e^
(2*I*d*x - 6*I*c) + 70*d*e^(8*I*d*x) + d*e^(-8*I*c))
```

Mupad [B] (verification not implemented)

Time = 4.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{a^5\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,1\mathrm{i}}{d} - \frac{4\,a^5\tan(c+d\,x)+a^5\,2\mathrm{i}}{d\,\left(\tan\left(c+d\,x\right)^2+\tan\left(c+d\,x\right)\,2\mathrm{i}-1\right)}$$

```
[In] int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^5,x)
[Out] (a^5*log(tan(c + d*x) + 1i)*1i)/d - (4*a^5*tan(c + d*x) + a^5*2i)/(d*(tan(c
 + d*x)*2i + tan(c + d*x)^2 - 1))
```

3.66 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{2ia^8}{3d(a-ia\tan(c+dx))^3}+\frac{ia^7}{2d(a-ia\tan(c+dx))^2}$$

```
[Out] -2/3*I*a^8/d/(a-I*a*tan(d*x+c))^3+1/2*I*a^7/d/(a-I*a*tan(d*x+c))^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{ia^7}{2d(a-ia\tan(c+dx))^2}-\frac{2ia^8}{3d(a-ia\tan(c+dx))^3}$$

```
[In] Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (((-2*I)/3)*a^8)/(d*(a - I*a*Tan[c + d*x])^3) + ((I/2)*a^7)/(d*(a - I*a*Tan
[c + d*x])^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
```

```
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(ia^7)\,\text{Subst}\left(\int \frac{a+x}{(a-x)^4}\,dx, x, ia\tan(c+dx)\right)}{d} \\
&= -\frac{(ia^7)\,\text{Subst}\left(\int \left(\frac{2a}{(a-x)^4} - \frac{1}{(a-x)^3}\right)dx, x, ia\tan(c+dx)\right)}{d} \\
&= -\frac{2ia^8}{3d(a-ia\tan(c+dx))^3} + \frac{ia^7}{2d(a-ia\tan(c+dx))^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{ia^5(-i+3\tan(c+dx))}{6d(i+\tan(c+dx))^3}$$

```
[In] Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^5,x]
[Out] ((-1/6*I)*a^5*(-I + 3*Tan[c + d*x]))/(d*(I + Tan[c + d*x])^3)
```

Maple [A] (verified)

Time = 135.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{i a^5 e^{6i(dx+c)}}{12d} - \frac{i a^5 e^{4i(dx+c)}}{8d}$
derivativedivides	$\frac{i a^5 \left(\sin^6(dx+c)\right)}{6} + 5a^5\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^3(dx+c)\right)}{6} - \frac{\left(\cos^3(dx+c)\right)\sin(dx+c)}{8} + \frac{\sin(dx+c)\cos(dx+c)}{16} + \frac{dx}{16} + \frac{c}{16}\right) - 10ia^5\left(-\frac{(\cos \cdots\right.$
default	$\frac{i a^5 \left(\sin^6(dx+c)\right)}{6} + 5a^5\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^3(dx+c)\right)}{6} - \frac{\left(\cos^3(dx+c)\right)\sin(dx+c)}{8} + \frac{\sin(dx+c)\cos(dx+c)}{16} + \frac{dx}{16} + \frac{c}{16}\right) - 10ia^5\left(-\frac{(\cos \cdots\right.$

```
[In] int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
[Out] -1/12*I/d*a^5*exp(6*I*(d*x+c))-1/8*I/d*a^5*exp(4*I*(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.62

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^5\,dx=\frac{-2i\,a^5e^{(6i\,dx+6i\,c)}-3i\,a^5e^{(4i\,dx+4i\,c)}}{24\,d}$$

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/24*(-2*I*a^5*e^(6*I*d*x + 6*I*c) - 3*I*a^5*e^(4*I*d*x + 4*I*c))/d
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.45

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^5\,dx=\begin{cases}\frac{-8ia^5de^{6ic}e^{6idx}-12ia^5de^{4ic}e^{4idx}}{96d^2} & \text{for } d^2\neq 0\\ x\left(\frac{a^5e^{6ic}}{2}+\frac{a^5e^{4ic}}{2}\right) & \text{otherwise}\end{cases}$$

```
[In] integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] Piecewise(((-8*I*a**5*d*exp(6*I*c)*exp(6*I*d*x) - 12*I*a**5*d*exp(4*I*c)*ex
p(4*I*d*x))/(96*d**2), Ne(d**2, 0)), (x*(a**5*exp(6*I*c)/2 + a**5*exp(4*I*c
)/2), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(43) = 86.

Time = 0.51 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.69

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{3i\,a^5\tan\left(dx+c\right)^4+10\,a^5\tan\left(dx+c\right)^3-12i\,a^5\tan\left(dx+c\right)^2-6\,a^5\tan\left(dx+c\right)+i\,a^5}{6\left(\tan\left(dx+c\right)^6+3\,\tan\left(dx+c\right)^4+3\,\tan\left(dx+c\right)^2+1\right)d}$$

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/6*(3*I*a^5*tan(d*x + c)^4 + 10*a^5*tan(d*x + c)^3 - 12*I*a^5*tan(d*x + c
)^2 - 6*a^5*tan(d*x + c) + I*a^5)/((tan(d*x + c)^6 + 3*tan(d*x + c)^4 + 3*t
an(d*x + c)^2 + 1)*d)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(43) = 86$.

Time = 0.70 (sec) , antiderivative size = 187, normalized size of antiderivative = 3.40

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{2i\,a^5e^{(18i\,dx+12i\,c)} + 15i\,a^5e^{(16i\,dx+10i\,c)} + 48i\,a^5e^{(14i\,dx+8i\,c)} + 85i\,a^5e^{(12i\,dx+6i\,c)} + 90i\,a^5e^{(10i\,dx+4i\,c)} + 57i\,a^5e}{24\left(de^{(12i\,dx+6i\,c)} + 6\,de^{(10i\,dx+4i\,c)} + 15\,de^{(8i\,dx+2i\,c)} + 15\,de^{(4i\,dx-2i\,c)} + 6\,de^{(2i\,dx-4i\,c)} + \right.}$$

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/24*(2*I*a^5*e^(18*I*d*x + 12*I*c) + 15*I*a^5*e^(16*I*d*x + 10*I*c) + 48*
I*a^5*e^(14*I*d*x + 8*I*c) + 85*I*a^5*e^(12*I*d*x + 6*I*c) + 90*I*a^5*e^(10
*I*d*x + 4*I*c) + 57*I*a^5*e^(8*I*d*x + 2*I*c) + 3*I*a^5*e^(4*I*d*x - 2*I*c
) + 20*I*a^5*e^(6*I*d*x))/(d*e^(12*I*d*x + 6*I*c) + 6*d*e^(10*I*d*x + 4*I*c
) + 15*d*e^(8*I*d*x + 2*I*c) + 15*d*e^(4*I*d*x - 2*I*c) + 6*d*e^(2*I*d*x -
4*I*c) + 20*d*e^(6*I*d*x) + d*e^(-6*I*c))
```

Mupad [B] (verification not implemented)

Time = 4.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{a^5\left(1+\tan(c+d\,x)\;3\mathrm{i}\right)}{6\,d\,\left(-{\tan\left(c+d\,x\right)}^3-{\tan\left(c+d\,x\right)}^2\,3\mathrm{i}+3\,\tan\left(c+d\,x\right)+1\mathrm{i}\right)}$$

```
[In] int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^5,x)
```

```
[Out] (a^5*(tan(c + d*x)*3i + 1))/(6*d*(3*tan(c + d*x) - tan(c + d*x)^2*3i - tan(
c + d*x)^3 + 1i))
```

3.67 $\int \cos^8(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{ia^9}{4d(a-ia\tan(c+dx))^4}$$

```
[Out] -1/4*I*a^9/d/(a-I*a*tan(d*x+c))^4
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 32}

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{ia^9}{4d(a-ia\tan(c+dx))^4}$$

```
[In] Int[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x])^5,x]
[Out] ((-1/4*I)*a^9)/(d*(a - I*a*Tan[c + d*x])^4)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{(ia^9)\,\text{Subst}\left(\int \frac{1}{(a-x)^5}\,dx, x, ia\tan(c+dx)\right)}{d}$$
$$= -\frac{ia^9}{4d(a-ia\tan(c+dx))^4}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{ia^5}{4d(i+\tan(c+dx))^4}$$

```
[In] Integrate[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x])^5,x]
[Out] ((-1/4*I)*a^5)/(d*(I + Tan[c + d*x])^4)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 300 vs. 2(23) = 46.

Time = 1.38 (sec) , antiderivative size = 301, normalized size of antiderivative = 11.15

$$ia^5\left(-\frac{(\sin^4(dx+c))(\cos^4(dx+c))}{8} - \frac{(\cos^4(dx+c))(\sin^2(dx+c))}{12} - \frac{(\cos^4(dx+c))}{24}\right) + 5a^5\left(-\frac{(\sin^3(dx+c))(\cos^5(dx+c))}{8} - \frac{(\cos^5(dx+c))\,\mathrm{si}}{16}\right.$$

```
[In] int(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x)
[Out] 1/d*(I*a^5*(-1/8*sin(d*x+c)^4*cos(d*x+c)^4-1/12*cos(d*x+c)^4*sin(d*x+c)^2-1
/24*cos(d*x+c)^4)+5*a^5*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*cos(d*x+c)^5*s
in(d*x+c)+1/64*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)-
10*I*a^5*(-1/8*cos(d*x+c)^6*sin(d*x+c)^2-1/24*cos(d*x+c)^6)-10*a^5*(-1/8*si
n(d*x+c)*cos(d*x+c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*
sin(d*x+c)+5/128*d*x+5/128*c)-5/8*I*a^5*cos(d*x+c)^8+a^5*(1/8*(cos(d*x+c)^7
+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+35/128*d*
x+35/128*c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(21) = 42$.

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.30

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{-i\,a^5e^{(8i\,dx+8i\,c)} - 4i\,a^5e^{(6i\,dx+6i\,c)} - 6i\,a^5e^{(4i\,dx+4i\,c)} - 4i\,a^5e^{(2i\,dx+2i\,c)}}{64\,d}$$

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/64*(-I*a^5*e^(8*I*d*x + 8*I*c) - 4*I*a^5*e^(6*I*d*x + 6*I*c) - 6*I*a^5*e^
(4*I*d*x + 4*I*c) - 4*I*a^5*e^(2*I*d*x + 2*I*c))/d
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(22) = 44$.

Time = 0.31 (sec) , antiderivative size = 162, normalized size of antiderivative = 6.00

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^5\,dx = \begin{cases} \frac{-8192ia^5d^3e^{8ic}e^{8idx}-32768ia^5d^3e^{6ic}e^{6idx}-49152ia^5d^3e^{4ic}e^{4idx}-32768ia^5d^3e^{2ic}e^{2idx}}{524288d^4} & \text{for } d^4 \neq 0 \\ x\left(\frac{a^5e^{8ic}}{8}+\frac{3a^5e^{6ic}}{8}+\frac{3a^5e^{4ic}}{8}+\frac{a^5e^{2ic}}{8}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)**8*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] Piecewise(((-8192*I*a**5*d**3*exp(8*I*c)*exp(8*I*d*x) - 32768*I*a**5*d**3*e
xp(6*I*c)*exp(6*I*d*x) - 49152*I*a**5*d**3*exp(4*I*c)*exp(4*I*d*x) - 32768*
I*a**5*d**3*exp(2*I*c)*exp(2*I*d*x))/(524288*d**4), Ne(d**4, 0)), (x*(a**5*
exp(8*I*c)/8 + 3*a**5*exp(6*I*c)/8 + 3*a**5*exp(4*I*c)/8 + a**5*exp(2*I*c)/
8), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(21) = 42.

Time = 0.59 (sec) , antiderivative size = 103, normalized size of antiderivative = 3.81

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{i\,a^5\tan\left(dx+c\right)^4+4\,a^5\tan\left(dx+c\right)^3-6i\,a^5\tan\left(dx+c\right)^2-4\,a^5\tan\left(dx+c\right)+i\,a^5}{4\left(\tan\left(dx+c\right)^8+4\,\tan\left(dx+c\right)^6+6\,\tan\left(dx+c\right)^4+4\,\tan\left(dx+c\right)^2+1\right)d}$$

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/4*(I*a^5*tan(d*x + c)^4 + 4*a^5*tan(d*x + c)^3 - 6*I*a^5*tan(d*x + c)^2
- 4*a^5*tan(d*x + c) + I*a^5)/((tan(d*x + c)^8 + 4*tan(d*x + c)^6 + 6*tan(d
*x + c)^4 + 4*tan(d*x + c)^2 + 1)*d)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(21) = 42.

Time = 0.78 (sec) , antiderivative size = 267, normalized size of antiderivative = 9.89

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{i\,a^5e^{(24i\,dx+16i\,c)}+12i\,a^5e^{(22i\,dx+14i\,c)}+66i\,a^5e^{(20i\,dx+12i\,c)}+220i\,a^5e^{(18i\,dx+10i\,c)}+494i\,a^5e^{(16i\,dx+8i\,c)}+784}{64\left(de^{(16i\,dx+8i\,c)}+8\,de^{(14i\,dx+6i\,c)}+28\,de^{(12i\,dx+4i\,c)}+56\,de^{(1}\right.}$$

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/64*(I*a^5*e^(24*I*d*x + 16*I*c) + 12*I*a^5*e^(22*I*d*x + 14*I*c) + 66*I*
a^5*e^(20*I*d*x + 12*I*c) + 220*I*a^5*e^(18*I*d*x + 10*I*c) + 494*I*a^5*e^(
16*I*d*x + 8*I*c) + 784*I*a^5*e^(14*I*d*x + 6*I*c) + 896*I*a^5*e^(12*I*d*x
+ 4*I*c) + 736*I*a^5*e^(10*I*d*x + 2*I*c) + 164*I*a^5*e^(6*I*d*x - 2*I*c) +
 38*I*a^5*e^(4*I*d*x - 4*I*c) + 4*I*a^5*e^(2*I*d*x - 6*I*c) + 425*I*a^5*e^(
8*I*d*x))/(d*e^(16*I*d*x + 8*I*c) + 8*d*e^(14*I*d*x + 6*I*c) + 28*d*e^(12*I
*d*x + 4*I*c) + 56*d*e^(10*I*d*x + 2*I*c) + 56*d*e^(6*I*d*x - 2*I*c) + 28*d
*e^(4*I*d*x - 4*I*c) + 8*d*e^(2*I*d*x - 6*I*c) + 70*d*e^(8*I*d*x) + d*e^(-8
*I*c))
```

Mupad [B] (verification not implemented)

Time = 3.74 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.33

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{\frac{a^5\cos(c+d\,x)^4\,1\mathrm{i}}{4}+a^5\cos\left(c+d\,x\right)^6\left(\tan(c+d\,x)-2\mathrm{i}\right)-2\,a^5\cos\left(c+d\,x\right)^8\left(\tan(c+d\,x)-\mathrm{i}\right)}{d}$$

```
[In] int(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^5,x)
```

```
[Out] -((a^5*cos(c + d*x)^4*1i)/4 + a^5*cos(c + d*x)^6*(tan(c + d*x) - 2i) - 2*a^
5*cos(c + d*x)^8*(tan(c + d*x) - 1i))/d
```

3.68 $\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal result

Integrand size = 24, antiderivative size = 144

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{a^5x}{32} - \frac{ia^{10}}{10d(a-ia\tan(c+dx))^5} - \frac{ia^9}{16d(a-ia\tan(c+dx))^4} - \frac{ia^8}{24d(a-ia\tan(c+dx))^3} - \frac{ia^7}{32d(a-ia\tan(c+dx))^2} - \frac{ia^6}{32d(a-ia\tan(c+dx))}$$

```
[Out] 1/32*a^5*x-1/10*I*a^10/d/(a-I*a*tan(d*x+c))^5-1/16*I*a^9/d/(a-I*a*tan(d*x+c
))^4-1/24*I*a^8/d/(a-I*a*tan(d*x+c))^3-1/32*I*a^7/d/(a-I*a*tan(d*x+c))^2-1/
32*I*a^6/d/(a-I*a*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used

$= \{3568, 46, 212\}$

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{ia^{10}}{10d(a-ia\tan(c+dx))^5} - \frac{ia^9}{16d(a-ia\tan(c+dx))^4} - \frac{ia^8}{24d(a-ia\tan(c+dx))^3} - \frac{ia^7}{32d(a-ia\tan(c+dx))^2} - \frac{ia^6}{32d(a-ia\tan(c+dx))} + \frac{a^5x}{32}$$

```
[In] Int[Cos[c + d*x]^10*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*x)/32 - ((I/10)*a^10)/(d*(a - I*a*Tan[c + d*x])^5) - ((I/16)*a^9)/(d*(
a - I*a*Tan[c + d*x])^4) - ((I/24)*a^8)/(d*(a - I*a*Tan[c + d*x])^3) - ((I/
32)*a^7)/(d*(a - I*a*Tan[c + d*x])^2) - ((I/32)*a^6)/(d*(a - I*a*Tan[c + d*
x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{(ia^{11})\,\text{Subst}\left(\int \frac{1}{(a-x)^6(a+x)}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^{11})\,\text{Subst}\left(\int\left(\frac{1}{2a(a-x)^6}+\frac{1}{4a^2(a-x)^5}+\frac{1}{8a^3(a-x)^4}+\frac{1}{16a^4(a-x)^3}+\frac{1}{32a^5(a-x)^2}+\frac{1}{32a^5(a^2-x^2)}\right)dx,x,ia\,\text{t}\right.}{d}$$

$$= -\frac{ia^{10}}{10d(a-ia\tan(c+dx))^5}-\frac{ia^9}{16d(a-ia\tan(c+dx))^4}-\frac{ia^8}{24d(a-ia\tan(c+dx))^3}-\frac{ia^7}{32d(a-ia\tan(c+dx))^2}-\frac{ia^6}{32d(a-ia\tan(c+dx))}-\frac{(ia^6)\,\text{Subst}\left(\int\frac{1}{a^2-x^2}\,dx,x,ia\tan(c+dx)\right)}{32d}$$

$$= \frac{a^5x}{32}-\frac{ia^{10}}{10d(a-ia\tan(c+dx))^5}-\frac{ia^9}{16d(a-ia\tan(c+dx))^4}-\frac{ia^8}{24d(a-ia\tan(c+dx))^3}-\frac{ia^7}{32d(a-ia\tan(c+dx))^2}-\frac{ia^6}{32d(a-ia\tan(c+dx))}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.85

$$\int\cos^{10}(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{a^5\sec^5(c+dx)(500\cos(c+dx)+375\cos(3(c+dx))+149\cos(5(c+dx))-100i\sin(c+dx)-225i\sin(3(\cdots}{3840d(i+\tan(c+\cdots}$$

```
[In] Integrate[Cos[c + d*x]^10*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*Sec[c + d*x]^5*(500*Cos[c + d*x] + 375*Cos[3*(c + d*x)] + 149*Cos[5*(c
 + d*x)] - (100*I)*Sin[c + d*x] - (225*I)*Sin[3*(c + d*x)] - (125*I)*Sin[5*
(c + d*x)] + 120*ArcTan[Tan[c + d*x]]*(I*Cos[5*(c + d*x)] + Sin[5*(c + d*x)
])))/(3840*d*(I + Tan[c + d*x])^5)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 330 vs. 2(122) = 244.

Time = 0.70 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.30

$$\frac{ia^5\left(-\frac{(\sin^4(dx+c))(\cos^6(dx+c))}{10}-\frac{(\cos^6(dx+c))(\sin^2(dx+c))}{20}-\frac{(\cos^6(dx+c))}{60}\right)+5a^5\left(-\frac{(\sin^3(dx+c))(\cos^7(dx+c))}{10}-\frac{3\sin(dx+c)(\text{co}\cdots}{80}\right.}{}$$

```
[In] int(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^5,x)
```

```
[Out] 1/d*(I*a^5*(-1/10*sin(d*x+c)^4*cos(d*x+c)^6-1/20*cos(d*x+c)^6*sin(d*x+c)^2-
1/60*cos(d*x+c)^6)+5*a^5*(-1/10*sin(d*x+c)^3*cos(d*x+c)^7-3/80*sin(d*x+c)*c
os(d*x+c)^7+1/160*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c
)+3/256*d*x+3/256*c)-10*I*a^5*(-1/10*cos(d*x+c)^8*sin(d*x+c)^2-1/40*cos(d*x
+c)^8)-10*a^5*(-1/10*sin(d*x+c)*cos(d*x+c)^9+1/80*(cos(d*x+c)^7+7/6*cos(d*x
+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+7/256*d*x+7/256*c)-1/
2*I*a^5*cos(d*x+c)^10+a^5*(1/10*(cos(d*x+c)^9+9/8*cos(d*x+c)^7+21/16*cos(d*
x+c)^5+105/64*cos(d*x+c)^3+315/128*cos(d*x+c))*sin(d*x+c)+63/256*d*x+63/256
*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.58

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$=\frac{120\,a^5dx-12i\,a^5e^{(10i\,dx+10i\,c)}-75i\,a^5e^{(8i\,dx+8i\,c)}-200i\,a^5e^{(6i\,dx+6i\,c)}-300i\,a^5e^{(4i\,dx+4i\,c)}-300i\,a^5e^{(2i\,dx+2i}}{3840\,d}$$

```
[In] integrate(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/3840*(120*a^5*d*x - 12*I*a^5*e^(10*I*d*x + 10*I*c) - 75*I*a^5*e^(8*I*d*x
+ 8*I*c) - 200*I*a^5*e^(6*I*d*x + 6*I*c) - 300*I*a^5*e^(4*I*d*x + 4*I*c) -
300*I*a^5*e^(2*I*d*x + 2*I*c))/d
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.45

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^5\,dx=\frac{a^5x}{32}$$
$$+\begin{cases}\frac{-100663296ia^5d^4e^{10ic}e^{10idx}-629145600ia^5d^4e^{8ic}e^{8idx}-1677721600ia^5d^4e^{6ic}e^{6idx}-2516582400ia^5d^4e^{4ic}e^{4idx}-2516582400ia^5d^4e^{2ic}e^{2i}}{32212254720d^5} \\ x\left(\frac{a^5e^{10ic}}{32}+\frac{5a^5e^{8ic}}{32}+\frac{5a^5e^{6ic}}{16}+\frac{5a^5e^{4ic}}{16}+\frac{5a^5e^{2ic}}{32}\right)\end{cases}$$

```
[In] integrate(cos(d*x+c)**10*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] a**5*x/32 + Piecewise(((-100663296*I*a**5*d**4*exp(10*I*c)*exp(10*I*d*x) -
629145600*I*a**5*d**4*exp(8*I*c)*exp(8*I*d*x) - 1677721600*I*a**5*d**4*exp(
6*I*c)*exp(6*I*d*x) - 2516582400*I*a**5*d**4*exp(4*I*c)*exp(4*I*d*x) - 2516
582400*I*a**5*d**4*exp(2*I*c)*exp(2*I*d*x))/(32212254720*d**5), Ne(d**5, 0)
), (x*(a**5*exp(10*I*c)/32 + 5*a**5*exp(8*I*c)/32 + 5*a**5*exp(6*I*c)/16 +
5*a**5*exp(4*I*c)/16 + 5*a**5*exp(2*I*c)/32), True))
```

Maxima [A] (verification not implemented)

none

Time = 1.33 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.14

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \frac{15\,(dx+c)a^5 + \frac{15\,a^5\tan(dx+c)^9+70\,a^5\tan(dx+c)^7+128\,a^5\tan(dx+c)^5-80i\,a^5\tan(dx+c)^4-230\,a^5\tan(dx+c)^3+560i\,a^5\tan(dx+c)^2+465}{\tan(dx+c)^{10}+5\,\tan(dx+c)^8+10\,\tan(dx+c)^6+10\,\tan(dx+c)^4+5\,\tan(dx+c)^2+1}}{480\,d}$$

```
[In] integrate(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/480*(15*(d*x + c)*a^5 + (15*a^5*tan(d*x + c)^9 + 70*a^5*tan(d*x + c)^7 +
128*a^5*tan(d*x + c)^5 - 80*I*a^5*tan(d*x + c)^4 - 230*a^5*tan(d*x + c)^3 +
 560*I*a^5*tan(d*x + c)^2 + 465*a^5*tan(d*x + c) - 128*I*a^5)/(tan(d*x + c)
^10 + 5*tan(d*x + c)^8 + 10*tan(d*x + c)^6 + 10*tan(d*x + c)^4 + 5*tan(d*x
+ c)^2 + 1))/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 857 vs. 2(112) = 224.

Time = 0.90 (sec) , antiderivative size = 857, normalized size of antiderivative = 5.95

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^5\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/15360*(480*a^5*d*x*e^(16*I*d*x + 8*I*c) + 3840*a^5*d*x*e^(14*I*d*x + 6*I*
c) + 13440*a^5*d*x*e^(12*I*d*x + 4*I*c) + 26880*a^5*d*x*e^(10*I*d*x + 2*I*c
) + 26880*a^5*d*x*e^(6*I*d*x - 2*I*c) + 13440*a^5*d*x*e^(4*I*d*x - 4*I*c) +
 3840*a^5*d*x*e^(2*I*d*x - 6*I*c) + 33600*a^5*d*x*e^(8*I*d*x) + 480*a^5*d*x
*e^(-8*I*c) - 195*I*a^5*e^(16*I*d*x + 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) -
 1560*I*a^5*e^(14*I*d*x + 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 5460*I*a^5*
e^(12*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 10920*I*a^5*e^(10*I*d*x
 + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 10920*I*a^5*e^(6*I*d*x - 2*I*c)*lo
g(e^(2*I*d*x + 2*I*c) + 1) - 5460*I*a^5*e^(4*I*d*x - 4*I*c)*log(e^(2*I*d*x
+ 2*I*c) + 1) - 1560*I*a^5*e^(2*I*d*x - 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1)
 - 13650*I*a^5*e^(8*I*d*x)*log(e^(2*I*d*x + 2*I*c) + 1) - 195*I*a^5*e^(-8*I
*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 195*I*a^5*e^(16*I*d*x + 8*I*c)*log(e^(2*
I*d*x) + e^(-2*I*c)) + 1560*I*a^5*e^(14*I*d*x + 6*I*c)*log(e^(2*I*d*x) + e^
(-2*I*c)) + 5460*I*a^5*e^(12*I*d*x + 4*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) +
 10920*I*a^5*e^(10*I*d*x + 2*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 10920*I*a
```

```
^5*e^(6*I*d*x - 2*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 5460*I*a^5*e^(4*I*d*
x - 4*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 1560*I*a^5*e^(2*I*d*x - 6*I*c)*l
og(e^(2*I*d*x) + e^(-2*I*c)) + 13650*I*a^5*e^(8*I*d*x)*log(e^(2*I*d*x) + e^
(-2*I*c)) + 195*I*a^5*e^(-8*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) - 48*I*a^5*e
^(26*I*d*x + 18*I*c) - 684*I*a^5*e^(24*I*d*x + 16*I*c) - 4544*I*a^5*e^(22*I
*d*x + 14*I*c) - 18688*I*a^5*e^(20*I*d*x + 12*I*c) - 53360*I*a^5*e^(18*I*d*
x + 10*I*c) - 111688*I*a^5*e^(16*I*d*x + 8*I*c) - 174944*I*a^5*e^(14*I*d*x
+ 6*I*c) - 204784*I*a^5*e^(12*I*d*x + 4*I*c) - 176048*I*a^5*e^(10*I*d*x + 2
*I*c) - 44000*I*a^5*e^(6*I*d*x - 2*I*c) - 10800*I*a^5*e^(4*I*d*x - 4*I*c) -
 1200*I*a^5*e^(2*I*d*x - 6*I*c) - 107500*I*a^5*e^(8*I*d*x))/(d*e^(16*I*d*x
+ 8*I*c) + 8*d*e^(14*I*d*x + 6*I*c) + 28*d*e^(12*I*d*x + 4*I*c) + 56*d*e^(1
0*I*d*x + 2*I*c) + 56*d*e^(6*I*d*x - 2*I*c) + 28*d*e^(4*I*d*x - 4*I*c) + 8*
d*e^(2*I*d*x - 6*I*c) + 70*d*e^(8*I*d*x) + d*e^(-8*I*c))
```

Mupad [B] (verification not implemented)

Time = 4.74 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.85

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{a^5\,x}{32} + \frac{\frac{a^5\tan(c+d\,x)^4}{32}+\frac{a^5\tan(c+d\,x)^3\,5{}\mathrm{i}}{32}-\frac{31\,a^5\tan(c+d\,x)^2}{96}-\frac{a^5\tan(c+d\,x)\,35{}\mathrm{i}}{96}+\frac{4\,a^5}{15}}{d\,\left({\tan\left(c+d\,x\right)}^5+{\tan\left(c+d\,x\right)}^4\,5{}\mathrm{i}-10\,{\tan\left(c+d\,x\right)}^3-{\tan\left(c+d\,x\right)}^2\,10{}\mathrm{i}+5\,\tan\left(c+d\,x\right)+1{}\mathrm{i}\right)}$$

```
[In] int(cos(c + d*x)^10*(a + a*tan(c + d*x)*1i)^5,x)
```

```
[Out] (a^5*x)/32 + ((4*a^5)/15 - (a^5*tan(c + d*x)*35i)/96 - (31*a^5*tan(c + d*x)
^2)/96 + (a^5*tan(c + d*x)^3*5i)/32 + (a^5*tan(c + d*x)^4)/32)/(d*(5*tan(c
+ d*x) - tan(c + d*x)^2*10i - 10*tan(c + d*x)^3 + tan(c + d*x)^4*5i + tan(c
 + d*x)^5 + 1i))
```

3.69 $\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal result

Integrand size = 24, antiderivative size = 198

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{7a^5x}{128} - \frac{ia^{11}}{24d(a-ia\tan(c+dx))^6} - \frac{ia^{10}}{20d(a-ia\tan(c+dx))^5} - \frac{3ia^9}{64d(a-ia\tan(c+dx))^4} - \frac{ia^8}{24d(a-ia\tan(c+dx))^3} - \frac{5ia^7}{128d(a-ia\tan(c+dx))^2} - \frac{3ia^6}{64d(a-ia\tan(c+dx))} + \frac{ia^6}{128d(a+ia\tan(c+dx))}$$

```
[Out] 7/128*a^5*x-1/24*I*a^11/d/(a-I*a*tan(d*x+c))^6-1/20*I*a^10/d/(a-I*a*tan(d*x
+c))^5-3/64*I*a^9/d/(a-I*a*tan(d*x+c))^4-1/24*I*a^8/d/(a-I*a*tan(d*x+c))^3-
5/128*I*a^7/d/(a-I*a*tan(d*x+c))^2-3/64*I*a^6/d/(a-I*a*tan(d*x+c))+1/128*I*
a^6/d/(a+I*a*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3568, 46, 212}

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= -\frac{ia^{11}}{24d(a-ia\tan(c+dx))^6} - \frac{ia^{10}}{20d(a-ia\tan(c+dx))^5} - \frac{3ia^9}{64d(a-ia\tan(c+dx))^4} - \frac{ia^8}{24d(a-ia\tan(c+dx))^3} - \frac{5ia^7}{128d(a-ia\tan(c+dx))^2} - \frac{3ia^6}{64d(a-ia\tan(c+dx))} + \frac{ia^6}{128d(a+ia\tan(c+dx))} + \frac{7a^5x}{128}$$

```
[In] Int[Cos[c + d*x]^12*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (7*a^5*x)/128 - ((I/24)*a^11)/(d*(a - I*a*Tan[c + d*x])^6) - ((I/20)*a^10)/
(d*(a - I*a*Tan[c + d*x])^5) - (((3*I)/64)*a^9)/(d*(a - I*a*Tan[c + d*x])^4
) - ((I/24)*a^8)/(d*(a - I*a*Tan[c + d*x])^3) - (((5*I)/128)*a^7)/(d*(a - I
*a*Tan[c + d*x])^2) - (((3*I)/64)*a^6)/(d*(a - I*a*Tan[c + d*x])) + ((I/128
)*a^6)/(d*(a + I*a*Tan[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{(ia^{13})\,\text{Subst}\left(\int \frac{1}{(a-x)^7(a+x)^2}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^{13})\,\text{Subst}\left(\int \left(\frac{1}{4a^2(a-x)^7} + \frac{1}{4a^3(a-x)^6} + \frac{3}{16a^4(a-x)^5} + \frac{1}{8a^5(a-x)^4} + \frac{5}{64a^6(a-x)^3} + \frac{3}{64a^7(a-x)^2} + \frac{1}{128a^7(a+x)}\right.\right.}{d}$$

$$= -\frac{ia^{11}}{24d(a-ia\tan(c+dx))^6} - \frac{ia^{10}}{20d(a-ia\tan(c+dx))^5} - \frac{3ia^9}{64d(a-ia\tan(c+dx))^4} - \frac{ia^8}{24d(a-ia\tan(c+dx))^3} - \frac{5ia^7}{128d(a-ia\tan(c+dx))^2} - \frac{3ia^6}{64d(a-ia\tan(c+dx))} + \frac{ia^6}{128d(a+ia\tan(c+dx))} - \frac{(7ia^6)\,\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, ia\tan(c+dx)\right)}{128d}$$

$$= \frac{7a^5x}{128} - \frac{ia^{11}}{24d(a-ia\tan(c+dx))^6} - \frac{ia^{10}}{20d(a-ia\tan(c+dx))^5} - \frac{3ia^9}{64d(a-ia\tan(c+dx))^4} - \frac{ia^8}{24d(a-ia\tan(c+dx))^3} - \frac{5ia^7}{128d(a-ia\tan(c+dx))^2} - \frac{3ia^6}{64d(a-ia\tan(c+dx))} + \frac{ia^6}{128d(a+ia\tan(c+dx))}$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.80

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{a^5\sec^7(c+dx)(-1750\cos(c+dx) - 1575\cos(3(c+dx)) - 693\cos(5(c+dx)) + 50\cos(7(c+dx)) + 350}{153}$$

```
[In] Integrate[Cos[c + d*x]^12*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] -1/15360*(a^5*Sec[c + d*x]^7*(-1750*Cos[c + d*x] - 1575*Cos[3*(c + d*x)] -
693*Cos[5*(c + d*x)] + 50*Cos[7*(c + d*x)] + (350*I)*Sin[c + d*x] + (945*I)
*Sin[3*(c + d*x)] - (840*I)*ArcTan[Tan[c + d*x]]*(Cos[5*(c + d*x)] - I*Sin[
5*(c + d*x)]) + (525*I)*Sin[5*(c + d*x)] - (70*I)*Sin[7*(c + d*x)]))/(d*(-I
 + Tan[c + d*x])*(I + Tan[c + d*x])^6)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(168) = 336.

Time = 0.79 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.82

$$\frac{ia^5\left(-\frac{(\cos^8(dx+c))(\sin^4(dx+c))}{12}-\frac{(\cos^8(dx+c))(\sin^2(dx+c))}{30}-\frac{(\cos^8(dx+c))}{120}\right)+5a^5\left(-\frac{(\sin^3(dx+c))(\cos^9(dx+c))}{12}-\frac{\sin(dx+c)(\cos}{40}\right.}{}$$

```
[In] int(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^5,x)
```

```
[Out] 1/d*(I*a^5*(-1/12*cos(d*x+c)^8*sin(d*x+c)^4-1/30*cos(d*x+c)^8*sin(d*x+c)^2-
1/120*cos(d*x+c)^8)+5*a^5*(-1/12*sin(d*x+c)^3*cos(d*x+c)^9-1/40*sin(d*x+c)*
cos(d*x+c)^9+1/320*(cos(d*x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*
cos(d*x+c))*sin(d*x+c)+7/1024*d*x+7/1024*c)-10*I*a^5*(-1/12*cos(d*x+c)^10*s
in(d*x+c)^2-1/60*cos(d*x+c)^10)-10*a^5*(-1/12*sin(d*x+c)*cos(d*x+c)^11+1/12
0*(cos(d*x+c)^9+9/8*cos(d*x+c)^7+21/16*cos(d*x+c)^5+105/64*cos(d*x+c)^3+315
/128*cos(d*x+c))*sin(d*x+c)+21/1024*d*x+21/1024*c)-5/12*I*a^5*cos(d*x+c)^12
+a^5*(1/12*(cos(d*x+c)^11+11/10*cos(d*x+c)^9+99/80*cos(d*x+c)^7+231/160*cos
(d*x+c)^5+231/128*cos(d*x+c)^3+693/256*cos(d*x+c))*sin(d*x+c)+231/1024*d*x+
231/1024*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.61

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{\left(840\,a^5dxe^{(2i\,dx+2i\,c)}-10i\,a^5e^{(14i\,dx+14i\,c)}-84i\,a^5e^{(12i\,dx+12i\,c)}-315i\,a^5e^{(10i\,dx+10i\,c)}-700i\,a^5e^{(8i\,dx+8i\,c)}-1\right.}{15360\,d}$$

```
[In] integrate(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/15360*(840*a^5*d*x*e^(2*I*d*x + 2*I*c) - 10*I*a^5*e^(14*I*d*x + 14*I*c) -
 84*I*a^5*e^(12*I*d*x + 12*I*c) - 315*I*a^5*e^(10*I*d*x + 10*I*c) - 700*I*a
^5*e^(8*I*d*x + 8*I*c) - 1050*I*a^5*e^(6*I*d*x + 6*I*c) - 1260*I*a^5*e^(4*I
*d*x + 4*I*c) + 60*I*a^5)*e^(-2*I*d*x - 2*I*c)/d
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.52

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{7a^5x}{128} + \begin{cases} \frac{(-33776997205278720ia^5d^6e^{14ic}e^{12idx}-283726776524341248ia^5d^6e^{12ic}e^{10idx}-1063975411966279680ia^5d^6e^{10ic}e^{8idx}-2364389804369510400 \cdots}{51881467707308 \cdots} \\ x\left(-\frac{7a^5}{128}+\frac{(a^5e^{14ic}+7a^5e^{12ic}+21a^5e^{10ic}+35a^5e^{8ic}+35a^5e^{6ic}+21a^5e^{4ic}+7a^5e^{2ic}+a^5)e^{-2ic}}{128}\right) \end{cases}$$

```
[In] integrate(cos(d*x+c)**12*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] 7*a**5*x/128 + Piecewise(((-33776997205278720*I*a**5*d**6*exp(14*I*c)*exp(1
2*I*d*x) - 283726776524341248*I*a**5*d**6*exp(12*I*c)*exp(10*I*d*x) - 10639
75411966279680*I*a**5*d**6*exp(10*I*c)*exp(8*I*d*x) - 2364389804369510400*I
*a**5*d**6*exp(8*I*c)*exp(6*I*d*x) - 3546584706554265600*I*a**5*d**6*exp(6*
I*c)*exp(4*I*d*x) - 4255901647865118720*I*a**5*d**6*exp(4*I*c)*exp(2*I*d*x)
 + 20266198323167232O*I*a**5*d**6*exp(-2*I*d*x))*exp(-2*I*c)/(5188146770730
8113920*d**7), Ne(d**7*exp(2*I*c), 0)), (x*(-7*a**5/128 + (a**5*exp(14*I*c)
 + 7*a**5*exp(12*I*c) + 21*a**5*exp(10*I*c) + 35*a**5*exp(8*I*c) + 35*a**5*
exp(6*I*c) + 21*a**5*exp(4*I*c) + 7*a**5*exp(2*I*c) + a**5)*exp(-2*I*c)/128
), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.94

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{105\,(dx+c)a^5 + \frac{105\,a^5\tan(dx+c)^{11}+595\,a^5\tan(dx+c)^9+1386\,a^5\tan(dx+c)^7+1686\,a^5\tan(dx+c)^5-240i\,a^5\tan(dx+c)^4+45\,a^5\tan(dx+c)^3 \cdots}{\tan(dx+c)^{12}+6\,\tan(dx+c)^{10}+15\,\tan(dx+c)^8+20\,\tan(dx+c)^6+15\,\tan(dx+c)^4+6\,\mathrm{t} \cdots}}{1920\,d}$$

```
[In] integrate(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/1920*(105*(d*x + c)*a^5 + (105*a^5*tan(d*x + c)^11 + 595*a^5*tan(d*x + c)
^9 + 1386*a^5*tan(d*x + c)^7 + 1686*a^5*tan(d*x + c)^5 - 240*I*a^5*tan(d*x
+ c)^4 + 45*a^5*tan(d*x + c)^3 + 1824*I*a^5*tan(d*x + c)^2 + 1815*a^5*tan(d
*x + c) - 496*I*a^5)/(tan(d*x + c)^12 + 6*tan(d*x + c)^10 + 15*tan(d*x + c)
^8 + 20*tan(d*x + c)^6 + 15*tan(d*x + c)^4 + 6*tan(d*x + c)^2 + 1))/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 914 vs. 2(154) = 308.

Time = 0.88 (sec) , antiderivative size = 914, normalized size of antiderivative = 4.62

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^5\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/122880*(6720*a^5*d*x*e^(18*I*d*x + 10*I*c) + 53760*a^5*d*x*e^(16*I*d*x +
8*I*c) + 188160*a^5*d*x*e^(14*I*d*x + 6*I*c) + 376320*a^5*d*x*e^(12*I*d*x +
 4*I*c) + 470400*a^5*d*x*e^(10*I*d*x + 2*I*c) + 188160*a^5*d*x*e^(6*I*d*x -
 2*I*c) + 53760*a^5*d*x*e^(4*I*d*x - 4*I*c) + 6720*a^5*d*x*e^(2*I*d*x - 6*I
*c) + 376320*a^5*d*x*e^(8*I*d*x) - 2355*I*a^5*e^(18*I*d*x + 10*I*c)*log(e^(
2*I*d*x + 2*I*c) + 1) - 18840*I*a^5*e^(16*I*d*x + 8*I*c)*log(e^(2*I*d*x + 2
*I*c) + 1) - 65940*I*a^5*e^(14*I*d*x + 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1)
- 131880*I*a^5*e^(12*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 164850*I
*a^5*e^(10*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 65940*I*a^5*e^(6*I
*d*x - 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 18840*I*a^5*e^(4*I*d*x - 4*I*c
)*log(e^(2*I*d*x + 2*I*c) + 1) - 2355*I*a^5*e^(2*I*d*x - 6*I*c)*log(e^(2*I*
d*x + 2*I*c) + 1) - 131880*I*a^5*e^(8*I*d*x)*log(e^(2*I*d*x + 2*I*c) + 1) +
 2355*I*a^5*e^(18*I*d*x + 10*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 18840*I*a
^5*e^(16*I*d*x + 8*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 65940*I*a^5*e^(14*I
*d*x + 6*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 131880*I*a^5*e^(12*I*d*x + 4*
I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 164850*I*a^5*e^(10*I*d*x + 2*I*c)*log(
e^(2*I*d*x) + e^(-2*I*c)) + 65940*I*a^5*e^(6*I*d*x - 2*I*c)*log(e^(2*I*d*x)
 + e^(-2*I*c)) + 18840*I*a^5*e^(4*I*d*x - 4*I*c)*log(e^(2*I*d*x) + e^(-2*I*
c)) + 2355*I*a^5*e^(2*I*d*x - 6*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 131880
*I*a^5*e^(8*I*d*x)*log(e^(2*I*d*x) + e^(-2*I*c)) - 80*I*a^5*e^(30*I*d*x + 2
2*I*c) - 1312*I*a^5*e^(28*I*d*x + 20*I*c) - 10136*I*a^5*e^(26*I*d*x + 18*I*
c) - 49056*I*a^5*e^(24*I*d*x + 16*I*c) - 166992*I*a^5*e^(22*I*d*x + 14*I*c)
 - 426720*I*a^5*e^(20*I*d*x + 12*I*c) - 845712*I*a^5*e^(18*I*d*x + 10*I*c)
- 1304736*I*a^5*e^(16*I*d*x + 8*I*c) - 1538256*I*a^5*e^(14*I*d*x + 6*I*c) -
 1340192*I*a^5*e^(12*I*d*x + 4*I*c) - 820120*I*a^5*e^(10*I*d*x + 2*I*c) - 6
2160*I*a^5*e^(6*I*d*x - 2*I*c) + 3360*I*a^5*e^(4*I*d*x - 4*I*c) + 3840*I*a^
5*e^(2*I*d*x - 6*I*c) - 321440*I*a^5*e^(8*I*d*x) + 480*I*a^5*e^(-8*I*c))/(d
*e^(18*I*d*x + 10*I*c) + 8*d*e^(16*I*d*x + 8*I*c) + 28*d*e^(14*I*d*x + 6*I*
c) + 56*d*e^(12*I*d*x + 4*I*c) + 70*d*e^(10*I*d*x + 2*I*c) + 28*d*e^(6*I*d*
x - 2*I*c) + 8*d*e^(4*I*d*x - 4*I*c) + d*e^(2*I*d*x - 6*I*c) + 56*d*e^(8*I*
d*x))
```

Mupad [B] (verification not implemented)

Time = 5.56 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.86

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{7\,a^5\,x}{128}$$
$$-\frac{-\frac{7\,a^5\,\tan(c+d\,x)^6}{128}-\frac{a^5\,\tan(c+d\,x)^5\,35\mathrm{i}}{128}+\frac{49\,a^5\,\tan(c+d\,x)^4}{96}+\frac{a^5\,\tan(c+d\,x)^3\,35\mathrm{i}}{96}+\frac{63\,a^5\,\tan(c+d\,x)^2}{640}+\frac{a^5\,\tan(c+\cdots}{38\cdots}}{d\,\left(\tan\left(c+d\,x\right)^7+\tan\left(c+d\,x\right)^6\,5\mathrm{i}-9\tan\left(c+d\,x\right)^5-\tan\left(c+d\,x\right)^4\,5\mathrm{i}-5\tan\left(c+d\,x\right)^3-\tan\left(c+d\,x\cdots\right.\right.}$$

```
[In] int(cos(c + d*x)^12*(a + a*tan(c + d*x)*1i)^5,x)
```

```
[Out] (7*a^5*x)/128 - ((a^5*tan(c + d*x)*133i)/384 - (31*a^5)/120 + (63*a^5*tan(c
 + d*x)^2)/640 + (a^5*tan(c + d*x)^3*35i)/96 + (49*a^5*tan(c + d*x)^4)/96 -
 (a^5*tan(c + d*x)^5*35i)/128 - (7*a^5*tan(c + d*x)^6)/128)/(d*(5*tan(c + d
*x) - tan(c + d*x)^2*9i - 5*tan(c + d*x)^3 - tan(c + d*x)^4*5i - 9*tan(c +
d*x)^5 + tan(c + d*x)^6*5i + tan(c + d*x)^7 + 1i))
```

3.70 $\int \sec(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal result

Integrand size = 22, antiderivative size = 167

$$\int \sec(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{63a^5\text{arctanh}(\sin(c+dx))}{8d}+\frac{63ia^5\sec(c+dx)}{8d}+\frac{9ia^2\sec(c+dx)(a+ia\tan(c+dx))^3}{20d}+\frac{ia\sec(c+dx)(a+ia\tan(c+dx))^4}{5d}+\frac{21ia\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{20d}+\frac{21i\sec(c+dx)\left(a^5+ia^5\tan(c+dx)\right)}{8d}$$

```
[Out] 63/8*a^5*arctanh(sin(d*x+c))/d+63/8*I*a^5*sec(d*x+c)/d+9/20*I*a^2*sec(d*x+c
)*(a+I*a*tan(d*x+c))^3/d+1/5*I*a*sec(d*x+c)*(a+I*a*tan(d*x+c))^4/d+21/20*I*
a*sec(d*x+c)*(a^2+I*a^2*tan(d*x+c))^2/d+21/8*I*sec(d*x+c)*(a^5+I*a^5*tan(d*
x+c))/d
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used

$= \{3579, 3567, 3855\}$

$$\int \sec(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{63a^5\operatorname{arctanh}(\sin(c+dx))}{8d} + \frac{63ia^5\sec(c+dx)}{8d} + \frac{21i\sec(c+dx)\left(a^5+ia^5\tan(c+dx)\right)}{8d} + \frac{9ia^2\sec(c+dx)(a+ia\tan(c+dx))^3}{20d} + \frac{21ia\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^2}{20d} + \frac{ia\sec(c+dx)(a+ia\tan(c+dx))^4}{5d}$$

```
[In] Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (63*a^5*ArcTanh[Sin[c + d*x]])/(8*d) + (((63*I)/8)*a^5*Sec[c + d*x])/d + ((
(9*I)/20)*a^2*Sec[c + d*x]*(a + I*a*Tan[c + d*x])^3)/d + ((I/5)*a*Sec[c + d
*x]*(a + I*a*Tan[c + d*x])^4)/d + (((21*I)/20)*a*Sec[c + d*x]*(a^2 + I*a^2*
Tan[c + d*x])^2)/d + (((21*I)/8)*Sec[c + d*x]*(a^5 + I*a^5*Tan[c + d*x]))/d
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{ia\sec(c+dx)(a+ia\tan(c+dx))^4}{5d} + \frac{1}{5}(9a)\int \sec(c+dx)(a+ia\tan(c+dx))^4\,dx$$

$$
\begin{aligned}
&= \frac{9ia^2\sec(c+dx)(a+ia\tan(c+dx))^3}{20d} + \frac{ia\sec(c+dx)(a+ia\tan(c+dx))^4}{5d} \\
&\quad + \frac{1}{20}(63a^2)\int \sec(c+dx)(a+ia\tan(c+dx))^3\,dx \\
&= \frac{21ia^3\sec(c+dx)(a+ia\tan(c+dx))^2}{20d} + \frac{9ia^2\sec(c+dx)(a+ia\tan(c+dx))^3}{20d} \\
&\quad + \frac{ia\sec(c+dx)(a+ia\tan(c+dx))^4}{5d} + \frac{1}{4}(21a^3)\int \sec(c+dx)(a+ia\tan(c \\
&\qquad + dx))^2\,dx \\
&= \frac{21ia^3\sec(c+dx)(a+ia\tan(c+dx))^2}{20d} + \frac{9ia^2\sec(c+dx)(a+ia\tan(c+dx))^3}{20d} \\
&\quad + \frac{ia\sec(c+dx)(a+ia\tan(c+dx))^4}{5d} + \frac{21i\sec(c+dx)\left(a^5+ia^5\tan(c+dx)\right)}{8d} \\
&\quad + \frac{1}{8}(63a^4)\int \sec(c+dx)(a+ia\tan(c+dx))\,dx \\
&= \frac{63ia^5\sec(c+dx)}{8d} + \frac{21ia^3\sec(c+dx)(a+ia\tan(c+dx))^2}{20d} \\
&\quad + \frac{9ia^2\sec(c+dx)(a+ia\tan(c+dx))^3}{20d} + \frac{ia\sec(c+dx)(a+ia\tan(c+dx))^4}{5d} \\
&\quad + \frac{21i\sec(c+dx)\left(a^5+ia^5\tan(c+dx)\right)}{8d} + \frac{1}{8}(63a^5)\int \sec(c+dx)\,dx \\
&= \frac{63a^5\text{arctanh}(\sin(c+dx))}{8d} + \frac{63ia^5\sec(c+dx)}{8d} \\
&\quad + \frac{21ia^3\sec(c+dx)(a+ia\tan(c+dx))^2}{20d} + \frac{9ia^2\sec(c+dx)(a+ia\tan(c+dx))^3}{20d} \\
&\quad + \frac{ia\sec(c+dx)(a+ia\tan(c+dx))^4}{5d} + \frac{21i\sec(c+dx)\left(a^5+ia^5\tan(c+dx)\right)}{8d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.11 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.69

$$
\int \sec(c+dx)(a+ia\tan(c+dx))^5\,dx
= \frac{a^5(\cos(5dx)+i\sin(5dx))\left(5040\text{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)+i\sec^5(c+dx)(1344+1920\cos(2(c+\right.}{320d(\cos(dx)+i\sin(dx))^5}
$$

```
[In] Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*(Cos[5*d*x] + I*Sin[5*d*x])*(5040*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]
] + I*Sec[c + d*x]^5*(1344 + 1920*Cos[2*(c + d*x)] + 640*Cos[4*(c + d*x)] +
 (450*I)*Sin[2*(c + d*x)] + (325*I)*Sin[4*(c + d*x)])))/(320*d*(Cos[d*x] +
I*Sin[d*x])^5)
```

Maple [A] (verified)

Time = 8.78 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.73

method	result
risch	$\frac{i a^5 \left(965\,\mathrm{e}^{9 i (d x+c)}+2370\,\mathrm{e}^{7 i (d x+c)}+2688\,\mathrm{e}^{5 i (d x+c)}+1470\,\mathrm{e}^{3 i (d x+c)}+315\,\mathrm{e}^{i (d x+c)}\right)}{20 d \left(\mathrm{e}^{2 i (d x+c)}+1\right)^5}+\frac{63 a^5 \ln\left(\mathrm{e}^{i (d x+c)}+i\right)}{8 d}-\frac{63 a^5 \ln(\mathrm{e}^{i}}{8}$
derivativedivides	$i a^5 \left(\frac{\sin^6(d x+c)}{5\cos(d x+c)^5}-\frac{\sin^6(d x+c)}{15\cos(d x+c)^3}+\frac{\sin^6(d x+c)}{5\cos(d x+c)}+\frac{\left(\frac{8}{3}+\sin^4(d x+c)+\frac{4\left(\sin^2(d x+c)\right)}{3}\right)\cos(d x+c)}{5}\right)+5 a^5\left(\frac{\sin^5(d x+c)}{4\cos(d x+c)^4}-\frac{\sin^5(d x}{8\cos(d x}\right.$
default	$i a^5 \left(\frac{\sin^6(d x+c)}{5\cos(d x+c)^5}-\frac{\sin^6(d x+c)}{15\cos(d x+c)^3}+\frac{\sin^6(d x+c)}{5\cos(d x+c)}+\frac{\left(\frac{8}{3}+\sin^4(d x+c)+\frac{4\left(\sin^2(d x+c)\right)}{3}\right)\cos(d x+c)}{5}\right)+5 a^5\left(\frac{\sin^5(d x+c)}{4\cos(d x+c)^4}-\frac{\sin^5(d x}{8\cos(d x}\right.$

```
[In] int(sec(d*x+c)*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/20*I*a^5/d/(exp(2*I*(d*x+c))+1)^5*(965*exp(9*I*(d*x+c))+2370*exp(7*I*(d*x
+c))+2688*exp(5*I*(d*x+c))+1470*exp(3*I*(d*x+c))+315*exp(I*(d*x+c)))+63/8/d
*a^5*ln(exp(I*(d*x+c))+I)-63/8/d*a^5*ln(exp(I*(d*x+c))-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(137) = 274.

Time = 0.25 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.86

$$\int \sec(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$=\frac{1930i\,a^5e^{(9i\,dx+9i\,c)}+4740i\,a^5e^{(7i\,dx+7i\,c)}+5376i\,a^5e^{(5i\,dx+5i\,c)}+2940i\,a^5e^{(3i\,dx+3i\,c)}+630i\,a^5e^{(i\,dx+i\,c)}+315\,(}{}$$

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/40*(1930*I*a^5*e^(9*I*d*x + 9*I*c) + 4740*I*a^5*e^(7*I*d*x + 7*I*c) + 537
6*I*a^5*e^(5*I*d*x + 5*I*c) + 2940*I*a^5*e^(3*I*d*x + 3*I*c) + 630*I*a^5*e^
(I*d*x + I*c) + 315*(a^5*e^(10*I*d*x + 10*I*c) + 5*a^5*e^(8*I*d*x + 8*I*c)
+ 10*a^5*e^(6*I*d*x + 6*I*c) + 10*a^5*e^(4*I*d*x + 4*I*c) + 5*a^5*e^(2*I*d*
x + 2*I*c) + a^5)*log(e^(I*d*x + I*c) + I) - 315*(a^5*e^(10*I*d*x + 10*I*c)
 + 5*a^5*e^(8*I*d*x + 8*I*c) + 10*a^5*e^(6*I*d*x + 6*I*c) + 10*a^5*e^(4*I*d
*x + 4*I*c) + 5*a^5*e^(2*I*d*x + 2*I*c) + a^5)*log(e^(I*d*x + I*c) - I))/(d
*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c)
 + 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^5\,dx = ia^5\left(\int(-i\sec(c+dx))\,dx + \int 5\tan(c+dx)\sec(c+dx)\,dx + \int\left(-10\tan^3(c+dx)\sec(c+dx)\right)\,dx + \int \tan^5(c+dx)\sec(c+dx)\,dx + \int 10i\tan^2(c+dx)\sec(c+dx)\,dx + \int\left(-5i\tan^4(c+dx)\sec(c+dx)\right)\,dx\right)$$

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] I*a**5*(Integral(-I*sec(c + d*x), x) + Integral(5*tan(c + d*x)*sec(c + d*x)
, x) + Integral(-10*tan(c + d*x)**3*sec(c + d*x), x) + Integral(tan(c + d*x
)**5*sec(c + d*x), x) + Integral(10*I*tan(c + d*x)**2*sec(c + d*x), x) + In
tegral(-5*I*tan(c + d*x)**4*sec(c + d*x), x))
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.29

$$\int \sec(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{75\,a^5\left(\frac{2\left(5\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1} + 3\,\log\left(\sin\left(dx+c\right)+1\right) - 3\,\log\left(\sin\left(dx+c\right)-1\right)\right) + 600\,a^5\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}\right.}{}$$

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/240*(75*a^5*(2*(5*sin(d*x + c)^3 - 3*sin(d*x + c))/(sin(d*x + c)^4 - 2*si
n(d*x + c)^2 + 1) + 3*log(sin(d*x + c) + 1) - 3*log(sin(d*x + c) - 1)) + 60
0*a^5*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log(si
n(d*x + c) - 1)) + 240*a^5*log(sec(d*x + c) + tan(d*x + c)) + 1200*I*a^5/co
s(d*x + c) + 800*I*(3*cos(d*x + c)^2 - 1)*a^5/cos(d*x + c)^3 + 16*I*(15*cos
(d*x + c)^4 - 10*cos(d*x + c)^2 + 3)*a^5/cos(d*x + c)^5)/d
```

Giac [A] (verification not implemented)

none

Time = 0.72 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.13

$$\int \sec(c+dx)(a+ia\tan(c+dx))^5\,dx$$

$$= \frac{315\,a^5\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-315\,a^5\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)-\frac{2\left(275\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+200i\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{}}{}$$

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/40*(315*a^5*log(tan(1/2*d*x + 1/2*c) + 1) - 315*a^5*log(tan(1/2*d*x + 1/2
*c) - 1) - 2*(275*a^5*tan(1/2*d*x + 1/2*c)^9 + 200*I*a^5*tan(1/2*d*x + 1/2*
c)^8 - 750*a^5*tan(1/2*d*x + 1/2*c)^7 - 1600*I*a^5*tan(1/2*d*x + 1/2*c)^6 +
 3280*I*a^5*tan(1/2*d*x + 1/2*c)^4 + 750*a^5*tan(1/2*d*x + 1/2*c)^3 - 2240*
I*a^5*tan(1/2*d*x + 1/2*c)^2 - 275*a^5*tan(1/2*d*x + 1/2*c) + 488*I*a^5)/(t
an(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 7.80 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.37

$$\int \sec(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{63\,a^5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}$$

$$-\frac{\frac{55\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}+a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8 10\mathrm{i}-\frac{75\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2}-a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 80\mathrm{i}+a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 164\mathrm{i}+}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\tan\left(\frac{c}{2}+\right.\right.}$$

```
[In] int((a + a*tan(c + d*x)*1i)^5/cos(c + d*x),x)
```

```
[Out] (63*a^5*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((75*a^5*tan(c/2 + (d*x)/2)^3)/2
 - a^5*tan(c/2 + (d*x)/2)^2*112i + a^5*tan(c/2 + (d*x)/2)^4*164i - a^5*tan(
c/2 + (d*x)/2)^6*80i - (75*a^5*tan(c/2 + (d*x)/2)^7)/2 + a^5*tan(c/2 + (d*x
)/2)^8*10i + (55*a^5*tan(c/2 + (d*x)/2)^9)/4 + (a^5*122i)/5 - (55*a^5*tan(c
/2 + (d*x)/2))/4)/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10
*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1)
)
```

3.71 $\int \cos(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal result

Integrand size = 22, antiderivative size = 130

$$\int \cos(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{35a^5\text{arctanh}(\sin(c+dx))}{2d} - \frac{35ia^5\sec(c+dx)}{2d} - \frac{7ia^3\sec(c+dx)(a+ia\tan(c+dx))^2}{3d} - \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^4}{d} - \frac{35i\sec(c+dx)\left(a^5+ia^5\tan(c+dx)\right)}{6d}$$

```
[Out] -35/2*a^5*arctanh(sin(d*x+c))/d-35/2*I*a^5*sec(d*x+c)/d-7/3*I*a^3*sec(d*x+c
)*(a+I*a*tan(d*x+c))^2/d-2*I*a*cos(d*x+c)*(a+I*a*tan(d*x+c))^4/d-35/6*I*sec
(d*x+c)*(a^5+I*a^5*tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3577, 3579, 3567, 3855}

$$\int \cos(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{35a^5\text{arctanh}(\sin(c+dx))}{2d} - \frac{35ia^5\sec(c+dx)}{2d} - \frac{35i\sec(c+dx)\left(a^5+ia^5\tan(c+dx)\right)}{6d} - \frac{7ia^3\sec(c+dx)(a+ia\tan(c+dx))^2}{3d} - \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^4}{d}$$

```
[In] Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (-35*a^5*ArcTanh[Sin[c + d*x]])/(2*d) - (((35*I)/2)*a^5*Sec[c + d*x])/d - (
((7*I)/3)*a^3*Sec[c + d*x]*(a + I*a*Tan[c + d*x])^2)/d - ((2*I)*a*Cos[c + d
*x]*(a + I*a*Tan[c + d*x])^4)/d - (((35*I)/6)*Sec[c + d*x]*(a^5 + I*a^5*Tan
[c + d*x]))/d
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^4}{d}-\left(7a^2\right)\int\sec(c+dx)(a+ia\tan(c+dx))^3\,dx\\ &= -\frac{7ia^3\sec(c+dx)(a+ia\tan(c+dx))^2}{3d}-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^4}{d}\\ &\quad-\frac{1}{3}\left(35a^3\right)\int\sec(c+dx)(a+ia\tan(c+dx))^2\,dx\end{aligned}$$

$$= -\frac{7ia^3\sec(c+dx)(a+ia\tan(c+dx))^2}{3d} - \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^4}{d} - \frac{35i\sec(c+dx)\left(a^5+ia^5\tan(c+dx)\right)}{6d} - \frac{1}{2}(35a^4)\int \sec(c+dx)(a + ia\tan(c+dx))\,dx$$

$$= -\frac{35ia^5\sec(c+dx)}{2d} - \frac{7ia^3\sec(c+dx)(a+ia\tan(c+dx))^2}{3d} - \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^4}{d} - \frac{35i\sec(c+dx)\left(a^5+ia^5\tan(c+dx)\right)}{6d} - \frac{1}{2}(35a^5)\int \sec(c+dx)\,dx$$

$$= -\frac{35a^5\text{arctanh}(\sin(c+dx))}{2d} - \frac{35ia^5\sec(c+dx)}{2d} - \frac{7ia^3\sec(c+dx)(a+ia\tan(c+dx))^2}{3d} - \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^4}{d} - \frac{35i\sec(c+dx)\left(a^5+ia^5\tan(c+dx)\right)}{6d}$$

Mathematica [A] (verified)

Time = 1.78 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.16

$$\int \cos(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{a^5\cos^2(c+dx)\left(-840i\text{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)\cos^3(c+dx)(\cos(5c)-i\sin(5c))+(\cos(4c-dx\right.}{24d(\text{co}}$$

```
[In] Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*Cos[c + d*x]^2*((-840*I)*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]]*Cos[c +
 d*x]^3*(Cos[5*c] - I*Sin[5*c]) + (Cos[4*c - d*x] - I*Sin[4*c - d*x])*(511*
Cos[c + d*x] + 153*Cos[3*(c + d*x)] - I*(49*Sin[c + d*x] + 57*Sin[3*(c + d*
x)])))*(-I + Tan[c + d*x])^5)/(24*d*(Cos[d*x] + I*Sin[d*x])^5)
```

Maple [A] (verified)

Time = 7.33 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{16ia^5 e^{i(dx+c)}}{d}-\frac{ia^5(87\,e^{5i(dx+c)}+136\,e^{3i(dx+c)}+57\,e^{i(dx+c)})}{3d(e^{2i(dx+c)}+1)^3}+\frac{35a^5\ln(e^{i(dx+c)}-i)}{2d}-\frac{35a^5\ln(e^{i(dx+c)}+i)}{2d}$
derivativedivides	$\frac{ia^5\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}-\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right)+5a^5\left(\frac{\sin^5(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^3(dx+c)\right)}{2}+\frac{3\sin(d}{2}\right.}{}$
default	$\frac{ia^5\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}-\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right)+5a^5\left(\frac{\sin^5(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^3(dx+c)\right)}{2}+\frac{3\sin(d}{2}\right.}{}$

```
[In] int(cos(d*x+c)*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] -16*I/d*a^5*exp(I*(d*x+c))-1/3*I*a^5/d/(exp(2*I*(d*x+c))+1)^3*(87*exp(5*I*(
d*x+c))+136*exp(3*I*(d*x+c))+57*exp(I*(d*x+c)))+35/2/d*a^5*ln(exp(I*(d*x+c)
)-I)-35/2/d*a^5*ln(exp(I*(d*x+c))+I)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.66

$$\int \cos(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$=\frac{-96i\,a^5e^{(7i\,dx+7i\,c)}-462i\,a^5e^{(5i\,dx+5i\,c)}-560i\,a^5e^{(3i\,dx+3i\,c)}-210i\,a^5e^{(i\,dx+i\,c)}-105\left(a^5e^{(6i\,dx+6i\,c)}+3\,a^5e^{(4i\,d}\right.}{6\left(de^{(6i\,dx+6i\,c)}+\right.}$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/6*(-96*I*a^5*e^(7*I*d*x + 7*I*c) - 462*I*a^5*e^(5*I*d*x + 5*I*c) - 560*I*
a^5*e^(3*I*d*x + 3*I*c) - 210*I*a^5*e^(I*d*x + I*c) - 105*(a^5*e^(6*I*d*x +
 6*I*c) + 3*a^5*e^(4*I*d*x + 4*I*c) + 3*a^5*e^(2*I*d*x + 2*I*c) + a^5)*log(
e^(I*d*x + I*c) + I) + 105*(a^5*e^(6*I*d*x + 6*I*c) + 3*a^5*e^(4*I*d*x + 4*
I*c) + 3*a^5*e^(2*I*d*x + 2*I*c) + a^5)*log(e^(I*d*x + I*c) - I))/(d*e^(6*I
*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.52

$$\int \cos(c+dx)(a+ia\tan(c+dx))^5\,dx=\frac{35a^5\left(\frac{\log\left(e^{idx}-ie^{-ic}\right)}{2}-\frac{\log\left(e^{idx}+ie^{-ic}\right)}{2}\right)}{d}+\frac{-87ia^5e^{5ic}e^{5idx}-136ia^5e^{3ic}e^{3idx}-57ia^5e^{ic}e^{idx}}{3de^{6ic}e^{6idx}+9de^{4ic}e^{4idx}+9de^{2ic}e^{2idx}+3d}+\begin{cases}-\frac{16ia^5e^{ic}e^{idx}}{d} & \text{for}\ d\neq 0\\ 16a^5xe^{ic} & \text{otherwise}\end{cases}$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] 35*a**5*(log(exp(I*d*x) - I*exp(-I*c))/2 - log(exp(I*d*x) + I*exp(-I*c))/2)
/d + (-87*I*a**5*exp(5*I*c)*exp(5*I*d*x) - 136*I*a**5*exp(3*I*c)*exp(3*I*d*
x) - 57*I*a**5*exp(I*c)*exp(I*d*x))/(3*d*exp(6*I*c)*exp(6*I*d*x) + 9*d*exp(
4*I*c)*exp(4*I*d*x) + 9*d*exp(2*I*c)*exp(2*I*d*x) + 3*d) + Piecewise((-16*I
*a**5*exp(I*c)*exp(I*d*x)/d, Ne(d, 0)), (16*a**5*x*exp(I*c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.33

$$\int \cos(c+dx)(a+ia\tan(c+dx))^5\,dx =$$

$$-\frac{15\,a^5\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+3\,\log\left(\sin\left(dx+c\right)+1\right)-3\,\log\left(\sin\left(dx+c\right)-1\right)-4\,\sin\left(dx+c\right)\right)+120i\,a^5\left(\frac{1}{\cos(d}\right.}{}$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/12*(15*a^5*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x + c) + 1
) - 3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)) + 120*I*a^5*(1/cos(d*x + c) +
 cos(d*x + c)) + 4*I*a^5*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x
 + c)) + 60*a^5*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x
+ c)) + 60*I*a^5*cos(d*x + c) - 12*a^5*sin(d*x + c))/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 510 vs. 2(108) = 216.

Time = 0.81 (sec) , antiderivative size = 510, normalized size of antiderivative = 3.92

$$\int \cos(c+dx)(a+ia\tan(c+dx))^5\,dx$$

$$=\frac{8295\,a^5e^{(6i\,dx+6i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+24885\,a^5e^{(4i\,dx+4i\,c)}\log\left(i\,e^{(i\,dx+i\,c)}+1\right)+24885\,a^5e^{(2i\,dx+2i\,c)}\log\left(i\right.}{}$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/1536*(8295*a^5*e^(6*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 24885*a^5
*e^(4*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 24885*a^5*e^(2*I*d*x + 2*
I*c)*log(I*e^(I*d*x + I*c) + 1) - 18585*a^5*e^(6*I*d*x + 6*I*c)*log(I*e^(I*
d*x + I*c) - 1) - 55755*a^5*e^(4*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1)
- 55755*a^5*e^(2*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) - 8295*a^5*e^(6*
```

```
I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 24885*a^5*e^(4*I*d*x + 4*I*c)*
log(-I*e^(I*d*x + I*c) + 1) - 24885*a^5*e^(2*I*d*x + 2*I*c)*log(-I*e^(I*d*x
 + I*c) + 1) + 18585*a^5*e^(6*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) +
55755*a^5*e^(4*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) + 55755*a^5*e^(2*
I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 24576*I*a^5*e^(7*I*d*x + 7*I*c
) - 118272*I*a^5*e^(5*I*d*x + 5*I*c) - 143360*I*a^5*e^(3*I*d*x + 3*I*c) - 5
3760*I*a^5*e^(I*d*x + I*c) + 8295*a^5*log(I*e^(I*d*x + I*c) + 1) - 18585*a^
5*log(I*e^(I*d*x + I*c) - 1) - 8295*a^5*log(-I*e^(I*d*x + I*c) + 1) + 18585
*a^5*log(-I*e^(I*d*x + I*c) - 1))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x +
 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Mupad [B] (verification not implemented)

Time = 8.25 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.71

$$\int \cos(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{35\,a^5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{37\,a^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 + a^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\,27\mathrm{i} - 118\,a^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 - a^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\,48\mathrm{i} + 139\,a^5\tan\left(\frac{c}{2}+\right.}{d\left(-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7 - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\,1\mathrm{i} + 3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\,3\mathrm{i} - 3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 - \tan\left(\frac{c}{2}\right.\right.}$$

```
[In] int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^5,x)
```

```
[Out] - (35*a^5*atanh(tan(c/2 + (d*x)/2)))/d - (139*a^5*tan(c/2 + (d*x)/2)^2 - a^
5*tan(c/2 + (d*x)/2)^3*48i - 118*a^5*tan(c/2 + (d*x)/2)^4 + a^5*tan(c/2 + (
d*x)/2)^5*27i + 37*a^5*tan(c/2 + (d*x)/2)^6 - (166*a^5)/3 + (a^5*tan(c/2 +
(d*x)/2)*55i)/3)/(d*(tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2*3i - 3*tan(c
/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4*3i + 3*tan(c/2 + (d*x)/2)^5 - tan(c/
2 + (d*x)/2)^6*1i - tan(c/2 + (d*x)/2)^7 + 1i))
```

3.72 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal result

Integrand size = 24, antiderivative size = 98

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^5\,dx=\frac{5a^5\text{arctanh}(\sin(c+dx))}{d}+\frac{5ia^5\sec(c+dx)}{d}+\frac{10ia^3\cos(c+dx)(a+ia\tan(c+dx))^2}{3d}-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^4}{3d}$$

```
[Out] 5*a^5*arctanh(sin(d*x+c))/d+5*I*a^5*sec(d*x+c)/d+10/3*I*a^3*cos(d*x+c)*(a+I
*a*tan(d*x+c))^2/d-2/3*I*a*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^4/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3577, 3567, 3855}

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^5\,dx=\frac{5a^5\text{arctanh}(\sin(c+dx))}{d}+\frac{5ia^5\sec(c+dx)}{d}+\frac{10ia^3\cos(c+dx)(a+ia\tan(c+dx))^2}{3d}-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^4}{3d}$$

```
[In] Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (5*a^5*ArcTanh[Sin[c + d*x]])/d + ((5*I)*a^5*Sec[c + d*x])/d + (((10*I)/3)*
a^3*Cos[c + d*x]*(a + I*a*Tan[c + d*x])^2)/d - (((2*I)/3)*a*Cos[c + d*x]^3*
(a + I*a*Tan[c + d*x])^4)/d
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^4}{3d} \\
&\quad -\frac{1}{3}\left(5a^2\right)\int\cos(c+dx)(a+ia\tan(c+dx))^3\,dx \\
&= \frac{10ia^3\cos(c+dx)(a+ia\tan(c+dx))^2}{3d}-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^4}{3d} \\
&\quad +\left(5a^4\right)\int\sec(c+dx)(a+ia\tan(c+dx))\,dx \\
&= \frac{5ia^5\sec(c+dx)}{d}+\frac{10ia^3\cos(c+dx)(a+ia\tan(c+dx))^2}{3d} \\
&\quad -\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^4}{3d}+\left(5a^5\right)\int\sec(c+dx)\,dx \\
&= \frac{5a^5\operatorname{arctanh}(\sin(c+dx))}{d}+\frac{5ia^5\sec(c+dx)}{d} \\
&\quad +\frac{10ia^3\cos(c+dx)(a+ia\tan(c+dx))^2}{3d}-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^4}{3d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.97 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.33

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$=\frac{a^5\cos^4(c+dx)\left(30\text{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)\cos(c+dx)(i\cos(5c)+\sin(5c))-(\cos(3c-2dx)-\right.}{3d(\cos(dx)+i\sin(dx))}$$

```
[In] Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*Cos[c + d*x]^4*(30*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]]*Cos[c + d*x]*
(I*Cos[5*c] + Sin[5*c]) - (Cos[3*c - 2*d*x] - I*Sin[3*c - 2*d*x])*(10 + 13*
Cos[2*(c + d*x)] - (17*I)*Sin[2*(c + d*x)]))*(-I + Tan[c + d*x])^5)/(3*d*(C
os[d*x] + I*Sin[d*x])^5)
```

Maple [A] (verified)

Time = 26.35 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.13

method	result
risch	$-\frac{4ia^5e^{3i(dx+c)}}{3d}+\frac{8ia^5e^{i(dx+c)}}{d}+\frac{2ia^5e^{i(dx+c)}}{d\left(e^{2i(dx+c)}+1\right)}+\frac{5a^5\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{5a^5\ln\left(e^{i(dx+c)}-i\right)}{d}$
derivativedivides	$\frac{ia^5\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right)+5a^5\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan\right.}{d}$
default	$\frac{ia^5\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right)+5a^5\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan\right.}{d}$

```
[In] int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] -4/3*I/d*a^5*exp(3*I*(d*x+c))+8*I/d*a^5*exp(I*(d*x+c))+2*I*a^5*exp(I*(d*x+c
))/d/(exp(2*I*(d*x+c))+1)+5/d*a^5*ln(exp(I*(d*x+c))+I)-5/d*a^5*ln(exp(I*(d*
x+c))-I)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.24

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \frac{-4i\,a^5e^{(5i\,dx+5i\,c)}+20i\,a^5e^{(3i\,dx+3i\,c)}+30i\,a^5e^{(i\,dx+i\,c)}+15\left(a^5e^{(2i\,dx+2i\,c)}+a^5\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-15\left(a^5e^{(2\cdots}\right.}{3\left(de^{(2i\,dx+2i\,c)}+d\right)}$$

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/3*(-4*I*a^5*e^(5*I*d*x + 5*I*c) + 20*I*a^5*e^(3*I*d*x + 3*I*c) + 30*I*a^5
*e^(I*d*x + I*c) + 15*(a^5*e^(2*I*d*x + 2*I*c) + a^5)*log(e^(I*d*x + I*c) +
 I) - 15*(a^5*e^(2*I*d*x + 2*I*c) + a^5)*log(e^(I*d*x + I*c) - I))/(d*e^(2*
I*d*x + 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.51

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{2ia^5e^{ic}e^{idx}}{de^{2ic}e^{2idx}+d} + \frac{5a^5\left(-\log\left(e^{idx}-ie^{-ic}\right)+\log\left(e^{idx}+ie^{-ic}\right)\right)}{d} + \begin{cases} \frac{-4ia^5de^{3ic}e^{3idx}+24ia^5de^{ic}e^{idx}}{3d^2} & \text{for } d^2\neq 0 \\ x(4a^5e^{3ic}-8a^5e^{ic}) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] 2*I*a**5*exp(I*c)*exp(I*d*x)/(d*exp(2*I*c)*exp(2*I*d*x) + d) + 5*a**5*(-log
(exp(I*d*x) - I*exp(-I*c)) + log(exp(I*d*x) + I*exp(-I*c)))/d + Piecewise((
(-4*I*a**5*d*exp(3*I*c)*exp(3*I*d*x) + 24*I*a**5*d*exp(I*c)*exp(I*d*x))/(3*
d**2), Ne(d**2, 0)), (x*(4*a**5*exp(3*I*c) - 8*a**5*exp(I*c)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.57

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^5\,dx =$$
$$-\frac{10i\,a^5\cos\left(dx+c\right)^3+20\,a^5\sin\left(dx+c\right)^3+2i\left(\cos\left(dx+c\right)^3-\frac{3}{\cos(dx+c)}-6\,\cos\left(dx+c\right)\right)a^5+20i\left(\cos\left(\right.\right.}{}$$

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/6*(10*I*a^5*cos(d*x + c)^3 + 20*a^5*sin(d*x + c)^3 + 2*I*(cos(d*x + c)^3
 - 3/cos(d*x + c) - 6*cos(d*x + c))*a^5 + 20*I*(cos(d*x + c)^3 - 3*cos(d*x
+ c))*a^5 + 5*(2*sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x +
 c) - 1) + 6*sin(d*x + c))*a^5 + 2*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^5)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1683 vs. 2(84) = 168.

Time = 1.18 (sec) , antiderivative size = 1683, normalized size of antiderivative = 17.17

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^5\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/6144*(39225*a^5*e^(16*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 313800
*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1098300*a^5*e^(12*I*
d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 2196600*a^5*e^(10*I*d*x + 2*I*c)*
log(I*e^(I*d*x + I*c) + 1) + 2196600*a^5*e^(6*I*d*x - 2*I*c)*log(I*e^(I*d*x
 + I*c) + 1) + 1098300*a^5*e^(4*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) + 1) +
 313800*a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 2745750*a^5*e^
(8*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 39225*a^5*e^(-8*I*c)*log(I*e^(I*d*x
+ I*c) + 1) + 8520*a^5*e^(16*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) - 1) + 68
160*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 238560*a^5*e^(12*
I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 477120*a^5*e^(10*I*d*x + 2*I*c)
*log(I*e^(I*d*x + I*c) - 1) + 477120*a^5*e^(6*I*d*x - 2*I*c)*log(I*e^(I*d*x
 + I*c) - 1) + 238560*a^5*e^(4*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) - 1) +
68160*a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 596400*a^5*e^(8*
I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 8520*a^5*e^(-8*I*c)*log(I*e^(I*d*x + I*
c) - 1) - 39225*a^5*e^(16*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 3138
00*a^5*e^(14*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1098300*a^5*e^(12
```

```
*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 2196600*a^5*e^(10*I*d*x + 2*I
*c)*log(-I*e^(I*d*x + I*c) + 1) - 2196600*a^5*e^(6*I*d*x - 2*I*c)*log(-I*e^
(I*d*x + I*c) + 1) - 1098300*a^5*e^(4*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c)
 + 1) - 313800*a^5*e^(2*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 274575
0*a^5*e^(8*I*d*x)*log(-I*e^(I*d*x + I*c) + 1) - 39225*a^5*e^(-8*I*c)*log(-I
*e^(I*d*x + I*c) + 1) - 8520*a^5*e^(16*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c
) - 1) - 68160*a^5*e^(14*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 23856
0*a^5*e^(12*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 477120*a^5*e^(10*I
*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 477120*a^5*e^(6*I*d*x - 2*I*c)*
log(-I*e^(I*d*x + I*c) - 1) - 238560*a^5*e^(4*I*d*x - 4*I*c)*log(-I*e^(I*d*
x + I*c) - 1) - 68160*a^5*e^(2*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) -
 596400*a^5*e^(8*I*d*x)*log(-I*e^(I*d*x + I*c) - 1) - 8520*a^5*e^(-8*I*c)*l
og(-I*e^(I*d*x + I*c) - 1) + 15*a^5*e^(16*I*d*x + 8*I*c)*log(I*e^(I*d*x) +
e^(-I*c)) + 120*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 420*
a^5*e^(12*I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 840*a^5*e^(10*I*d*x
+ 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 840*a^5*e^(6*I*d*x - 2*I*c)*log(I*e^
(I*d*x) + e^(-I*c)) + 420*a^5*e^(4*I*d*x - 4*I*c)*log(I*e^(I*d*x) + e^(-I*c
)) + 120*a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 1050*a^5*e^(
8*I*d*x)*log(I*e^(I*d*x) + e^(-I*c)) + 15*a^5*e^(-8*I*c)*log(I*e^(I*d*x) +
e^(-I*c)) - 15*a^5*e^(16*I*d*x + 8*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 120*
a^5*e^(14*I*d*x + 6*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 420*a^5*e^(12*I*d*x
 + 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 840*a^5*e^(10*I*d*x + 2*I*c)*log(-
I*e^(I*d*x) + e^(-I*c)) - 840*a^5*e^(6*I*d*x - 2*I*c)*log(-I*e^(I*d*x) + e^
(-I*c)) - 420*a^5*e^(4*I*d*x - 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 120*a^
5*e^(2*I*d*x - 6*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 1050*a^5*e^(8*I*d*x)*l
og(-I*e^(I*d*x) + e^(-I*c)) - 15*a^5*e^(-8*I*c)*log(-I*e^(I*d*x) + e^(-I*c)
) + 8192*I*a^5*e^(19*I*d*x + 11*I*c) + 16384*I*a^5*e^(17*I*d*x + 9*I*c) - 1
76128*I*a^5*e^(15*I*d*x + 7*I*c) - 1003520*I*a^5*e^(13*I*d*x + 5*I*c) - 243
7120*I*a^5*e^(11*I*d*x + 3*I*c) - 3411968*I*a^5*e^(9*I*d*x + I*c) - 2953216
*I*a^5*e^(7*I*d*x - I*c) - 1568768*I*a^5*e^(5*I*d*x - 3*I*c) - 471040*I*a^5
*e^(3*I*d*x - 5*I*c) - 61440*I*a^5*e^(I*d*x - 7*I*c))/(d*e^(16*I*d*x + 8*I*
c) + 8*d*e^(14*I*d*x + 6*I*c) + 28*d*e^(12*I*d*x + 4*I*c) + 56*d*e^(10*I*d*
x + 2*I*c) + 56*d*e^(6*I*d*x - 2*I*c) + 28*d*e^(4*I*d*x - 4*I*c) + 8*d*e^(2
*I*d*x - 6*I*c) + 70*d*e^(8*I*d*x) + d*e^(-8*I*c))
```

Mupad [B] (verification not implemented)

Time = 6.14 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.65

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{10\,a^5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{8\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,34\mathrm{i}-\frac{82\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}-a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,38\mathrm{i}+\frac{46\,a^5}{3}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,3\mathrm{i}-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,4\mathrm{i}+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

```
[In] int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^5,x)
```

```
[Out] (10*a^5*atanh(tan(c/2 + (d*x)/2)))/d - (a^5*tan(c/2 + (d*x)/2)^3*34i - (82*
a^5*tan(c/2 + (d*x)/2)^2)/3 + 8*a^5*tan(c/2 + (d*x)/2)^4 + (46*a^5)/3 - a^5
*tan(c/2 + (d*x)/2)*38i)/(d*(3*tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2*4i
 - 4*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4*3i + tan(c/2 + (d*x)/2)^5
+ 1i))
```

3.73 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal result

Integrand size = 24, antiderivative size = 32

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^5}{5d}$$

```
[Out] -1/5*I*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^5/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {3569}

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^5}{5d}$$

```
[In] Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] ((-1/5*I)*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^5)/d
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\text{integral} = -\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^5}{5d}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{ia^5(\cos(c+dx)+i\sin(c+dx))^5}{5d}$$

```
[In] Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^5,x]
[Out] ((-1/5*I)*a^5*(Cos[c + d*x] + I*Sin[c + d*x])^5)/d
```

Maple [A] (verified)

Time = 81.45 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.59

method	result
risch	$-\frac{ia^5 e^{5i(dx+c)}}{5d}$
derivativedivides	$-\frac{ia^5\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}+a^5\left(\sin^5(dx+c)\right)-10ia^5\left(-\frac{\left(\cos^3(dx+c)\right)\left(\sin^2(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)$
default	$-\frac{ia^5\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}+a^5\left(\sin^5(dx+c)\right)-10ia^5\left(-\frac{\left(\cos^3(dx+c)\right)\left(\sin^2(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)$

```
[In] int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
[Out] -1/5*I/d*a^5*exp(5*I*(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.53

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{i\,a^5 e^{(5i\,dx+5i\,c)}}{5\,d}$$

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
[Out] -1/5*I*a^5*e^(5*I*d*x + 5*I*c)/d
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^5\,dx = \begin{cases} -\frac{ia^5e^{5ic}e^{5idx}}{5d} & \text{for } d \neq 0 \\ a^5xe^{5ic} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] Piecewise((-I*a**5*exp(5*I*c)*exp(5*I*d*x)/(5*d), Ne(d, 0)), (a**5*x*exp(5*
I*c), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(26) = 52.

Time = 0.63 (sec) , antiderivative size = 152, normalized size of antiderivative = 4.75

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^5\,dx =$$

$$-\frac{15i\,a^5\cos(dx+c)^5 - 15\,a^5\sin(dx+c)^5 + 10i\left(3\cos(dx+c)^5 - 5\cos(dx+c)^3\right)a^5 + i\left(3\cos(dx+c)^5\right.}{}$$

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/15*(15*I*a^5*cos(d*x + c)^5 - 15*a^5*sin(d*x + c)^5 + 10*I*(3*cos(d*x +
c)^5 - 5*cos(d*x + c)^3)*a^5 + I*(3*cos(d*x + c)^5 - 10*cos(d*x + c)^3 + 15
*cos(d*x + c))*a^5 - 10*(3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*a^5 - (3*sin(
d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^5)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1669 vs. 2(26) = 52.

Time = 0.89 (sec) , antiderivative size = 1669, normalized size of antiderivative = 52.16

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^5\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/40960*(11375*a^5*e^(16*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 91000
*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 318500*a^5*e^(12*I*d
*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 637000*a^5*e^(10*I*d*x + 2*I*c)*lo
```

g(I*e^(I*d*x + I*c) + 1) + 637000*a^5*e^(6*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 318500*a^5*e^(4*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 910 00*a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 796250*a^5*e^(8*I*d *x)*log(I*e^(I*d*x + I*c) + 1) + 11375*a^5*e^(-8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 11590*a^5*e^(16*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) - 1) + 92720*a ^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 324520*a^5*e^(12*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 649040*a^5*e^(10*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 649040*a^5*e^(6*I*d*x - 2*I*c)*log(I*e^(I*d*x + I* c) - 1) + 324520*a^5*e^(4*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 92720 *a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 811300*a^5*e^(8*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 11590*a^5*e^(-8*I*c)*log(I*e^(I*d*x + I*c) - 1) - 11375*a^5*e^(16*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 91000*a^ 5*e^(14*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 318500*a^5*e^(12*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 637000*a^5*e^(10*I*d*x + 2*I*c)*log (-I*e^(I*d*x + I*c) + 1) - 637000*a^5*e^(6*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 318500*a^5*e^(4*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 9 1000*a^5*e^(2*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 796250*a^5*e^(8* I*d*x)*log(-I*e^(I*d*x + I*c) + 1) - 11375*a^5*e^(-8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 11590*a^5*e^(16*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 9 2720*a^5*e^(14*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 324520*a^5*e^(1 2*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 649040*a^5*e^(10*I*d*x + 2*I *c)*log(-I*e^(I*d*x + I*c) - 1) - 649040*a^5*e^(6*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 324520*a^5*e^(4*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 92720*a^5*e^(2*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 811300*a^ 5*e^(8*I*d*x)*log(-I*e^(I*d*x + I*c) - 1) - 11590*a^5*e^(-8*I*c)*log(-I*e^(I*d*x + I*c) - 1) + 215*a^5*e^(16*I*d*x + 8*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 1720*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 6020*a^5*e^ (12*I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 12040*a^5*e^(10*I*d*x + 2* I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 12040*a^5*e^(6*I*d*x - 2*I*c)*log(I*e^(I *d*x) + e^(-I*c)) + 6020*a^5*e^(4*I*d*x - 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 1720*a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 15050*a^5*e^ (8*I*d*x)*log(I*e^(I*d*x) + e^(-I*c)) + 215*a^5*e^(-8*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 215*a^5*e^(16*I*d*x + 8*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 1 720*a^5*e^(14*I*d*x + 6*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 6020*a^5*e^(12* I*d*x + 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 12040*a^5*e^(10*I*d*x + 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 12040*a^5*e^(6*I*d*x - 2*I*c)*log(-I*e^(I* d*x) + e^(-I*c)) - 6020*a^5*e^(4*I*d*x - 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 1720*a^5*e^(2*I*d*x - 6*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 15050*a^5*e ^(8*I*d*x)*log(-I*e^(I*d*x) + e^(-I*c)) - 215*a^5*e^(-8*I*c)*log(-I*e^(I*d* x) + e^(-I*c)) + 8192*I*a^5*e^(21*I*d*x + 13*I*c) + 65536*I*a^5*e^(19*I*d*x + 11*I*c) + 229376*I*a^5*e^(17*I*d*x + 9*I*c) + 458752*I*a^5*e^(15*I*d*x + 7*I*c) + 573440*I*a^5*e^(13*I*d*x + 5*I*c) + 458752*I*a^5*e^(11*I*d*x + 3* I*c) + 229376*I*a^5*e^(9*I*d*x + I*c) + 65536*I*a^5*e^(7*I*d*x - I*c) + 819 2*I*a^5*e^(5*I*d*x - 3*I*c))/(d*e^(16*I*d*x + 8*I*c) + 8*d*e^(14*I*d*x + 6* I*c) + 28*d*e^(12*I*d*x + 4*I*c) + 56*d*e^(10*I*d*x + 2*I*c) + 56*d*e^(6*I*

```
d*x - 2*I*c) + 28*d*e^(4*I*d*x - 4*I*c) + 8*d*e^(2*I*d*x - 6*I*c) + 70*d*e^
(8*I*d*x) + d*e^(-8*I*c))
```

Mupad [B] (verification not implemented)

Time = 4.31 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.25

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$=\frac{2\,a^5\left(5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{5\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 5\mathrm{i}-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 10\mathrm{i}+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

```
[In] int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^5,x)
```

```
[Out] (2*a^5*(5*tan(c/2 + (d*x)/2)^4 - 10*tan(c/2 + (d*x)/2)^2 + 1))/(5*d*(5*tan(
c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2*10i - 10*tan(c/2 + (d*x)/2)^3 + tan(c
/2 + (d*x)/2)^4*5i + tan(c/2 + (d*x)/2)^5 + 1i))
```

3.74 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal result

Integrand size = 24, antiderivative size = 101

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{2ia^2\cos^3(c+dx)(a+ia\tan(c+dx))^3}{105d} - \frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^4}{35d} - \frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^5}{7d}$$

```
[Out] -2/105*I*a^2*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^3/d-2/35*I*a*cos(d*x+c)^5*(a+I
*a*tan(d*x+c))^4/d-1/7*I*cos(d*x+c)^7*(a+I*a*tan(d*x+c))^5/d
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3578, 3569}

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{2ia^2\cos^3(c+dx)(a+ia\tan(c+dx))^3}{105d} - \frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^5}{7d} - \frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^4}{35d}$$

```
[In] Int[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (((-2*I)/105)*a^2*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^3)/d - (((2*I)/35)*
a*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^4)/d - ((I/7)*Cos[c + d*x]^7*(a + I
*a*Tan[c + d*x])^5)/d
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^5}{7d}+\frac{1}{7}(2a)\int\cos^5(c+dx)(a+ia\tan(c+dx))^4\,dx\\ &= -\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^4}{35d}-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^5}{7d}\\ &\quad+\frac{1}{35}(2a^2)\int\cos^3(c+dx)(a+ia\tan(c+dx))^3\,dx\\ &= -\frac{2ia^2\cos^3(c+dx)(a+ia\tan(c+dx))^3}{105d}\\ &\quad-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^4}{35d}-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^5}{7d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.21

$$\int\cos^7(c+dx)(a+ia\tan(c+dx))^5\,dx$$

$$=\frac{a^5\sec(c+dx)(-i\cos(4(c+dx))+\sin(4(c+dx)))\left(77+92\cos(2(c+dx))+\left(15+416\sqrt{\cos^2(c+dx)}\right)\text{co}\right.}{840d}$$

```
[In] Integrate[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*Sec[c + d*x]*((-I)*Cos[4*(c + d*x)] + Sin[4*(c + d*x)])*(77 + 92*Cos[2
*(c + d*x)] + (15 + 416*Sqrt[Cos[c + d*x]^2])*Cos[4*(c + d*x)] + (22*I)*Sin
[2*(c + d*x)] + (15*I)*Sin[4*(c + d*x)] - (416*I)*Sqrt[Cos[c + d*x]^2]*Sin[
4*(c + d*x)]))/(840*d)
```

Maple [A] (verified)

Time = 212.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.55

method	result
risch	$-\frac{i a^{5} {\mathrm e}^{7 i \left(d x +c \right)}}{28 d}-\frac{i a^{5} {\mathrm e}^{5 i \left(d x +c \right)}}{10 d}-\frac{i a^{5} {\mathrm e}^{3 i \left(d x +c \right)}}{12 d}$
derivativedivides	$\frac{i a^{5} \left(-\frac{\left(\cos^{3}\left(d x +c \right)\right) \left(\sin^{4}\left(d x +c \right)\right)}{7}-\frac{4 \left(\cos^{3}\left(d x +c \right)\right) \left(\sin^{2}\left(d x +c \right)\right)}{35}-\frac{8 \left(\cos^{3}\left(d x +c \right)\right)}{105}\right)+5 a^{5} \left(-\frac{\left(\sin^{3}\left(d x +c \right)\right) \left(\cos^{4}\left(d x +c \right)\right)}{7}-\frac{3 \sin}{}\right.}{}$
default	$\frac{i a^{5} \left(-\frac{\left(\cos^{3}\left(d x +c \right)\right) \left(\sin^{4}\left(d x +c \right)\right)}{7}-\frac{4 \left(\cos^{3}\left(d x +c \right)\right) \left(\sin^{2}\left(d x +c \right)\right)}{35}-\frac{8 \left(\cos^{3}\left(d x +c \right)\right)}{105}\right)+5 a^{5} \left(-\frac{\left(\sin^{3}\left(d x +c \right)\right) \left(\cos^{4}\left(d x +c \right)\right)}{7}-\frac{3 \sin}{}\right.}{}$

```
[In] int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/28*I/d*a^5*exp(7*I*(d*x+c))-1/10*I/d*a^5*exp(5*I*(d*x+c))-1/12*I/d*a^5*e
xp(3*I*(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.48

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{-15i\,a^5e^{(7i\,dx+7i\,c)}-42i\,a^5e^{(5i\,dx+5i\,c)}-35i\,a^5e^{(3i\,dx+3i\,c)}}{420\,d}$$

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/420*(-15*I*a^5*e^(7*I*d*x + 7*I*c) - 42*I*a^5*e^(5*I*d*x + 5*I*c) - 35*I*
a^5*e^(3*I*d*x + 3*I*c))/d
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.20

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^5\,dx = \begin{cases} \frac{-120ia^5d^2e^{7ic}e^{7idx}-336ia^5d^2e^{5ic}e^{5idx}-280ia^5d^2e^{3ic}e^{3idx}}{3360d^3} & \text{for } d^3\neq 0 \\ x\left(\frac{a^5e^{7ic}}{4}+\frac{a^5e^{5ic}}{2}+\frac{a^5e^{3ic}}{4}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)**7*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] Piecewise(((-120*I*a**5*d**2*exp(7*I*c)*exp(7*I*d*x) - 336*I*a**5*d**2*exp(
5*I*c)*exp(5*I*d*x) - 280*I*a**5*d**2*exp(3*I*c)*exp(3*I*d*x))/(3360*d**3),
 Ne(d**3, 0)), (x*(a**5*exp(7*I*c)/4 + a**5*exp(5*I*c)/2 + a**5*exp(3*I*c)/
4), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(83) = 166.

Time = 0.41 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.85

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{75i\,a^5\cos{(dx+c)}^7 + i\left(15\,\cos{(dx+c)}^7 - 42\,\cos{(dx+c)}^5 + 35\,\cos{(dx+c)}^3\right)a^5 + 30i\left(5\,\cos{(dx+c)}^7\right.}{}$$

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/105*(75*I*a^5*cos(d*x + c)^7 + I*(15*cos(d*x + c)^7 - 42*cos(d*x + c)^5
+ 35*cos(d*x + c)^3)*a^5 + 30*I*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a^5 +
 10*(15*sin(d*x + c)^7 - 42*sin(d*x + c)^5 + 35*sin(d*x + c)^3)*a^5 + 15*(5
*sin(d*x + c)^7 - 7*sin(d*x + c)^5)*a^5 + 3*(5*sin(d*x + c)^7 - 21*sin(d*x
+ c)^5 + 35*sin(d*x + c)^3 - 35*sin(d*x + c))*a^5)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1697 vs. 2(83) = 166.

Time = 0.94 (sec) , antiderivative size = 1697, normalized size of antiderivative = 16.80

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^5\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/3440640*(7357770*a^5*e^(16*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 5
8862160*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 206017560*a^5
*e^(12*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 412035120*a^5*e^(10*I*d*
x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 412035120*a^5*e^(6*I*d*x - 2*I*c)*l
og(I*e^(I*d*x + I*c) + 1) + 206017560*a^5*e^(4*I*d*x - 4*I*c)*log(I*e^(I*d*
x + I*c) + 1) + 58862160*a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) + 1)
 + 515043900*a^5*e^(8*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 7357770*a^5*e^(-8
*I*c)*log(I*e^(I*d*x + I*c) + 1) + 7390425*a^5*e^(16*I*d*x + 8*I*c)*log(I*e
```

```
^(I*d*x + I*c) - 1) + 59123400*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*
c) - 1) + 206931900*a^5*e^(12*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 4
13863800*a^5*e^(10*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 413863800*a^
5*e^(6*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 206931900*a^5*e^(4*I*d*x
 - 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 59123400*a^5*e^(2*I*d*x - 6*I*c)*log
(I*e^(I*d*x + I*c) - 1) + 517329750*a^5*e^(8*I*d*x)*log(I*e^(I*d*x + I*c) -
 1) + 7390425*a^5*e^(-8*I*c)*log(I*e^(I*d*x + I*c) - 1) - 7357770*a^5*e^(16
*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 58862160*a^5*e^(14*I*d*x + 6*
I*c)*log(-I*e^(I*d*x + I*c) + 1) - 206017560*a^5*e^(12*I*d*x + 4*I*c)*log(-
I*e^(I*d*x + I*c) + 1) - 412035120*a^5*e^(10*I*d*x + 2*I*c)*log(-I*e^(I*d*x
 + I*c) + 1) - 412035120*a^5*e^(6*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1
) - 206017560*a^5*e^(4*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 5886216
0*a^5*e^(2*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 515043900*a^5*e^(8*
I*d*x)*log(-I*e^(I*d*x + I*c) + 1) - 7357770*a^5*e^(-8*I*c)*log(-I*e^(I*d*x
 + I*c) + 1) - 7390425*a^5*e^(16*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) - 1)
 - 59123400*a^5*e^(14*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 20693190
0*a^5*e^(12*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 413863800*a^5*e^(1
0*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 413863800*a^5*e^(6*I*d*x - 2
*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 206931900*a^5*e^(4*I*d*x - 4*I*c)*log(-
I*e^(I*d*x + I*c) - 1) - 59123400*a^5*e^(2*I*d*x - 6*I*c)*log(-I*e^(I*d*x +
 I*c) - 1) - 517329750*a^5*e^(8*I*d*x)*log(-I*e^(I*d*x + I*c) - 1) - 739042
5*a^5*e^(-8*I*c)*log(-I*e^(I*d*x + I*c) - 1) + 32655*a^5*e^(16*I*d*x + 8*I*
c)*log(I*e^(I*d*x) + e^(-I*c)) + 261240*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I
*d*x) + e^(-I*c)) + 914340*a^5*e^(12*I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I
*c)) + 1828680*a^5*e^(10*I*d*x + 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 18286
80*a^5*e^(6*I*d*x - 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 914340*a^5*e^(4*I*
d*x - 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 261240*a^5*e^(2*I*d*x - 6*I*c)*l
og(I*e^(I*d*x) + e^(-I*c)) + 2285850*a^5*e^(8*I*d*x)*log(I*e^(I*d*x) + e^(-
I*c)) + 32655*a^5*e^(-8*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 32655*a^5*e^(16*
I*d*x + 8*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 261240*a^5*e^(14*I*d*x + 6*I*
c)*log(-I*e^(I*d*x) + e^(-I*c)) - 914340*a^5*e^(12*I*d*x + 4*I*c)*log(-I*e^
(I*d*x) + e^(-I*c)) - 1828680*a^5*e^(10*I*d*x + 2*I*c)*log(-I*e^(I*d*x) + e
^(-I*c)) - 1828680*a^5*e^(6*I*d*x - 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 9
14340*a^5*e^(4*I*d*x - 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 261240*a^5*e^(
2*I*d*x - 6*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 2285850*a^5*e^(8*I*d*x)*log
(-I*e^(I*d*x) + e^(-I*c)) - 32655*a^5*e^(-8*I*c)*log(-I*e^(I*d*x) + e^(-I*c
)) + 122880*I*a^5*e^(23*I*d*x + 15*I*c) + 1327104*I*a^5*e^(21*I*d*x + 13*I*
c) + 6479872*I*a^5*e^(19*I*d*x + 11*I*c) + 18808832*I*a^5*e^(17*I*d*x + 9*I
*c) + 35897344*I*a^5*e^(15*I*d*x + 7*I*c) + 47022080*I*a^5*e^(13*I*d*x + 5*
I*c) + 42778624*I*a^5*e^(11*I*d*x + 3*I*c) + 26673152*I*a^5*e^(9*I*d*x + I*
c) + 10903552*I*a^5*e^(7*I*d*x - I*c) + 2637824*I*a^5*e^(5*I*d*x - 3*I*c) +
 286720*I*a^5*e^(3*I*d*x - 5*I*c))/(d*e^(16*I*d*x + 8*I*c) + 8*d*e^(14*I*d*
x + 6*I*c) + 28*d*e^(12*I*d*x + 4*I*c) + 56*d*e^(10*I*d*x + 2*I*c) + 56*d*e
^(6*I*d*x - 2*I*c) + 28*d*e^(4*I*d*x - 4*I*c) + 8*d*e^(2*I*d*x - 6*I*c) + 7
0*d*e^(8*I*d*x) + d*e^(-8*I*c))
```

Mupad [B] (verification not implemented)

Time = 4.71 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.84

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{2\,a^5\,\left(105\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,210\mathrm{i}-455\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,350\mathrm{i}+273\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 \ldots\right)}{105\,d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,7\mathrm{i}+21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,35\mathrm{i}-35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3- \ldots\right)}$$

```
[In] int(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^5,x)
```

```
[Out] -(2*a^5*(tan(c/2 + (d*x)/2)*56i + 273*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x
)/2)^3*350i - 455*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^5*210i + 105*ta
n(c/2 + (d*x)/2)^6 - 23))/(105*d*(7*tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)
^2*21i - 35*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4*35i + 21*tan(c/2 +
(d*x)/2)^5 - tan(c/2 + (d*x)/2)^6*7i - tan(c/2 + (d*x)/2)^7 + 1i))
```

3.75 $\int \cos^9(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal result

Integrand size = 24, antiderivative size = 141

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{ia^5\cos^5(c+dx)}{105d}+\frac{a^5\sin(c+dx)}{21d}-\frac{2a^5\sin^3(c+dx)}{63d}+\frac{a^5\sin^5(c+dx)}{105d}-\frac{2ia^3\cos^7(c+dx)(a+ia\tan(c+dx))^2}{63d}-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^4}{9d}$$

```
[Out] -1/105*I*a^5*cos(d*x+c)^5/d+1/21*a^5*sin(d*x+c)/d-2/63*a^5*sin(d*x+c)^3/d+1
/105*a^5*sin(d*x+c)^5/d-2/63*I*a^3*cos(d*x+c)^7*(a+I*a*tan(d*x+c))^2/d-2/9*
I*a*cos(d*x+c)^9*(a+I*a*tan(d*x+c))^4/d
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3577, 3567, 2713}

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{a^5\sin^5(c+dx)}{105d}-\frac{2a^5\sin^3(c+dx)}{63d}+\frac{a^5\sin(c+dx)}{21d}-\frac{ia^5\cos^5(c+dx)}{105d}-\frac{2ia^3\cos^7(c+dx)(a+ia\tan(c+dx))^2}{63d}-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^4}{9d}$$

```
[In] Int[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] ((-1/105*I)*a^5*Cos[c + d*x]^5)/d + (a^5*Sin[c + d*x])/(21*d) - (2*a^5*Sin[
c + d*x]^3)/(63*d) + (a^5*Sin[c + d*x]^5)/(105*d) - (((2*I)/63)*a^3*Cos[c +
 d*x]^7*(a + I*a*Tan[c + d*x])^2)/d - (((2*I)/9)*a*Cos[c + d*x]^9*(a + I*a*
Tan[c + d*x])^4)/d
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^4}{9d}+\frac{1}{9}a^2\int\cos^7(c+dx)(a+ia\tan(c+dx))^3\,dx\\
&= -\frac{2ia^3\cos^7(c+dx)(a+ia\tan(c+dx))^2}{63d}-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^4}{9d}\\
&\quad+\frac{1}{21}a^4\int\cos^5(c+dx)(a+ia\tan(c+dx))\,dx\\
&= -\frac{ia^5\cos^5(c+dx)}{105d}-\frac{2ia^3\cos^7(c+dx)(a+ia\tan(c+dx))^2}{63d}\\
&\quad-\frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^4}{9d}+\frac{1}{21}a^5\int\cos^5(c+dx)\,dx
\end{aligned}
$$

$$= -\frac{ia^5\cos^5(c+dx)}{105d} - \frac{2ia^3\cos^7(c+dx)(a+ia\tan(c+dx))^2}{63d} - \frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^4}{9d} - \frac{a^5\text{Subst}\left(\int(1-2x^2+x^4)\,dx, x, -\sin(c+dx)\right)}{21d}$$

$$= -\frac{ia^5\cos^5(c+dx)}{105d} + \frac{a^5\sin(c+dx)}{21d} - \frac{2a^5\sin^3(c+dx)}{63d} + \frac{a^5\sin^5(c+dx)}{105d} - \frac{2ia^3\cos^7(c+dx)(a+ia\tan(c+dx))^2}{63d} - \frac{2ia\cos^9(c+dx)(a+ia\tan(c+dx))^4}{9d}$$

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.04

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \frac{a^5\sec(c+dx)(-i\cos(5(c+dx))+\sin(5(c+dx)))\left(678\cos(c+dx)+475\cos(3(c+dx))+175\cos(5(c+\right.}{}$$

```
[In] Integrate[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*Sec[c + d*x]*((-I)*Cos[5*(c + d*x)] + Sin[5*(c + d*x)])*(678*Cos[c + d
*x] + 475*Cos[3*(c + d*x)] + 175*Cos[5*(c + d*x)] + 1472*Sqrt[Cos[c + d*x]^
2]*Cos[5*(c + d*x)] - (120*I)*Sin[c + d*x] - (260*I)*Sin[3*(c + d*x)] - (14
0*I)*Sin[5*(c + d*x)] - (1472*I)*Sqrt[Cos[c + d*x]^2]*Sin[5*(c + d*x)]))/(5
040*d)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(124) = 248.

Time = 0.79 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.04

$$\frac{ia^5\left(-\frac{(\sin^4(dx+c))(\cos^5(dx+c))}{9} - \frac{4(\cos^5(dx+c))(\sin^2(dx+c))}{63} - \frac{8(\cos^5(dx+c))}{315}\right) + 5a^5\left(-\frac{(\sin^3(dx+c))(\cos^6(dx+c))}{9} - \frac{\sin(dx+c)}{}\right.}{}$$

```
[In] int(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^5,x)
```

```
[Out] 1/d*(I*a^5*(-1/9*sin(d*x+c)^4*cos(d*x+c)^5-4/63*cos(d*x+c)^5*sin(d*x+c)^2-8
/315*cos(d*x+c)^5)+5*a^5*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*co
s(d*x+c)^6+1/105*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-10*I*a^5*(
```

```
-1/9*cos(d*x+c)^7*sin(d*x+c)^2-2/63*cos(d*x+c)^7)-10*a^5*(-1/9*cos(d*x+c)^8
*sin(d*x+c)+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(
d*x+c))-5/9*I*a^5*cos(d*x+c)^9+1/9*a^5*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c)^
6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.54

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{-35i\,a^5e^{(9i\,dx+9i\,c)}-180i\,a^5e^{(7i\,dx+7i\,c)}-378i\,a^5e^{(5i\,dx+5i\,c)}-420i\,a^5e^{(3i\,dx+3i\,c)}-315i\,a^5e^{(i\,dx+i\,c)}}{5040\,d}$$

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/5040*(-35*I*a^5*e^(9*I*d*x + 9*I*c) - 180*I*a^5*e^(7*I*d*x + 7*I*c) - 378
*I*a^5*e^(5*I*d*x + 5*I*c) - 420*I*a^5*e^(3*I*d*x + 3*I*c) - 315*I*a^5*e^(I
*d*x + I*c))/d
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.36

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^5\,dx = \begin{cases} \frac{-215040ia^5d^4e^{9ic}e^{9idx}-1105920ia^5d^4e^{7ic}e^{7idx}-2322432ia^5d^4e^{5ic}e^{5idx}-2580480ia^5d^4e^{3ic}e^{3idx}-1935360ia^5d^4e^{ic}e^{idx}}{30965760d^5} & \text{for } d^5\neq 0 \\ x\left(\frac{a^5e^{9ic}}{16}+\frac{a^5e^{7ic}}{4}+\frac{3a^5e^{5ic}}{8}+\frac{a^5e^{3ic}}{4}+\frac{a^5e^{ic}}{16}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)**9*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] Piecewise(((-215040*I*a**5*d**4*exp(9*I*c)*exp(9*I*d*x) - 1105920*I*a**5*d*
*4*exp(7*I*c)*exp(7*I*d*x) - 2322432*I*a**5*d**4*exp(5*I*c)*exp(5*I*d*x) -
2580480*I*a**5*d**4*exp(3*I*c)*exp(3*I*d*x) - 1935360*I*a**5*d**4*exp(I*c)*
exp(I*d*x))/(30965760*d**5), Ne(d**5, 0)), (x*(a**5*exp(9*I*c)/16 + a**5*ex
p(7*I*c)/4 + 3*a**5*exp(5*I*c)/8 + a**5*exp(3*I*c)/4 + a**5*exp(I*c)/16), T
rue))
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.54

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{175i\,a^5\cos(dx+c)^9 + i\left(35\cos(dx+c)^9 - 90\cos(dx+c)^7 + 63\cos(dx+c)^5\right)a^5 + 50i\left(7\cos(dx+c\right.}{}$$

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/315*(175*I*a^5*cos(d*x + c)^9 + I*(35*cos(d*x + c)^9 - 90*cos(d*x + c)^7
 + 63*cos(d*x + c)^5)*a^5 + 50*I*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a^5
- 5*(35*sin(d*x + c)^9 - 90*sin(d*x + c)^7 + 63*sin(d*x + c)^5)*a^5 - 10*(3
5*sin(d*x + c)^9 - 135*sin(d*x + c)^7 + 189*sin(d*x + c)^5 - 105*sin(d*x +
c)^3)*a^5 - (35*sin(d*x + c)^9 - 180*sin(d*x + c)^7 + 378*sin(d*x + c)^5 -
420*sin(d*x + c)^3 + 315*sin(d*x + c))*a^5)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1725 vs. 2(119) = 238.

Time = 0.95 (sec) , antiderivative size = 1725, normalized size of antiderivative = 12.23

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^5\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/41287680*(69853455*a^5*e^(16*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) +
 558827640*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1955896740
*a^5*e^(12*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 3911793480*a^5*e^(10
*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 3911793480*a^5*e^(6*I*d*x - 2*
I*c)*log(I*e^(I*d*x + I*c) + 1) + 1955896740*a^5*e^(4*I*d*x - 4*I*c)*log(I*
e^(I*d*x + I*c) + 1) + 558827640*a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d*x + I
*c) + 1) + 4889741850*a^5*e^(8*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 69853455
*a^5*e^(-8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 70703325*a^5*e^(16*I*d*x + 8*I
*c)*log(I*e^(I*d*x + I*c) - 1) + 565626600*a^5*e^(14*I*d*x + 6*I*c)*log(I*e
^(I*d*x + I*c) - 1) + 1979693100*a^5*e^(12*I*d*x + 4*I*c)*log(I*e^(I*d*x +
I*c) - 1) + 3959386200*a^5*e^(10*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1)
+ 3959386200*a^5*e^(6*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 197969310
0*a^5*e^(4*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 565626600*a^5*e^(2*I
*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 4949232750*a^5*e^(8*I*d*x)*log(I
```

```
*e^(I*d*x + I*c) - 1) + 70703325*a^5*e^(-8*I*c)*log(I*e^(I*d*x + I*c) - 1)
- 69853455*a^5*e^(16*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 558827640
*a^5*e^(14*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1955896740*a^5*e^(1
2*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 3911793480*a^5*e^(10*I*d*x +
 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 3911793480*a^5*e^(6*I*d*x - 2*I*c)*lo
g(-I*e^(I*d*x + I*c) + 1) - 1955896740*a^5*e^(4*I*d*x - 4*I*c)*log(-I*e^(I*
d*x + I*c) + 1) - 558827640*a^5*e^(2*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c)
+ 1) - 4889741850*a^5*e^(8*I*d*x)*log(-I*e^(I*d*x + I*c) + 1) - 69853455*a^
5*e^(-8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 70703325*a^5*e^(16*I*d*x + 8*I*c
)*log(-I*e^(I*d*x + I*c) - 1) - 565626600*a^5*e^(14*I*d*x + 6*I*c)*log(-I*e
^(I*d*x + I*c) - 1) - 1979693100*a^5*e^(12*I*d*x + 4*I*c)*log(-I*e^(I*d*x +
 I*c) - 1) - 3959386200*a^5*e^(10*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1
) - 3959386200*a^5*e^(6*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 197969
3100*a^5*e^(4*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 565626600*a^5*e^
(2*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 4949232750*a^5*e^(8*I*d*x)*
log(-I*e^(I*d*x + I*c) - 1) - 70703325*a^5*e^(-8*I*c)*log(-I*e^(I*d*x + I*c
) - 1) + 849870*a^5*e^(16*I*d*x + 8*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 6798
960*a^5*e^(14*I*d*x + 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 23796360*a^5*e^(
12*I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 47592720*a^5*e^(10*I*d*x +
2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 47592720*a^5*e^(6*I*d*x - 2*I*c)*log(I
*e^(I*d*x) + e^(-I*c)) + 23796360*a^5*e^(4*I*d*x - 4*I*c)*log(I*e^(I*d*x) +
 e^(-I*c)) + 6798960*a^5*e^(2*I*d*x - 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) +
59490900*a^5*e^(8*I*d*x)*log(I*e^(I*d*x) + e^(-I*c)) + 849870*a^5*e^(-8*I*c
)*log(I*e^(I*d*x) + e^(-I*c)) - 849870*a^5*e^(16*I*d*x + 8*I*c)*log(-I*e^(I
*d*x) + e^(-I*c)) - 6798960*a^5*e^(14*I*d*x + 6*I*c)*log(-I*e^(I*d*x) + e^(
-I*c)) - 23796360*a^5*e^(12*I*d*x + 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 4
7592720*a^5*e^(10*I*d*x + 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 47592720*a^
5*e^(6*I*d*x - 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 23796360*a^5*e^(4*I*d*
x - 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 6798960*a^5*e^(2*I*d*x - 6*I*c)*l
og(-I*e^(I*d*x) + e^(-I*c)) - 59490900*a^5*e^(8*I*d*x)*log(-I*e^(I*d*x) + e
^(-I*c)) - 849870*a^5*e^(-8*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 286720*I*a^
5*e^(25*I*d*x + 17*I*c) + 3768320*I*a^5*e^(23*I*d*x + 15*I*c) + 22921216*I*
a^5*e^(21*I*d*x + 13*I*c) + 85557248*I*a^5*e^(19*I*d*x + 11*I*c) + 21945548
8*I*a^5*e^(17*I*d*x + 9*I*c) + 409665536*I*a^5*e^(15*I*d*x + 7*I*c) + 57229
3120*I*a^5*e^(13*I*d*x + 5*I*c) + 602341376*I*a^5*e^(11*I*d*x + 3*I*c) + 47
2096768*I*a^5*e^(9*I*d*x + I*c) + 267091968*I*a^5*e^(7*I*d*x - I*c) + 10287
5136*I*a^5*e^(5*I*d*x - 3*I*c) + 24084480*I*a^5*e^(3*I*d*x - 5*I*c) + 25804
80*I*a^5*e^(I*d*x - 7*I*c))/(d*e^(16*I*d*x + 8*I*c) + 8*d*e^(14*I*d*x + 6*I
*c) + 28*d*e^(12*I*d*x + 4*I*c) + 56*d*e^(10*I*d*x + 2*I*c) + 56*d*e^(6*I*d
*x - 2*I*c) + 28*d*e^(4*I*d*x - 4*I*c) + 8*d*e^(2*I*d*x - 6*I*c) + 70*d*e^(
8*I*d*x) + d*e^(-8*I*c))
```

Mupad [B] (verification not implemented)

Time = 4.86 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.56

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$=-\frac{a^5\left(\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,1\mathrm{i}}{16}+\frac{\mathrm{e}^{c\,3\mathrm{i}+d\,x\,3\mathrm{i}}\,1\mathrm{i}}{12}+\frac{\mathrm{e}^{c\,5\mathrm{i}+d\,x\,5\mathrm{i}}\,3\mathrm{i}}{40}+\frac{\mathrm{e}^{c\,7\mathrm{i}+d\,x\,7\mathrm{i}}\,1\mathrm{i}}{28}+\frac{\mathrm{e}^{c\,9\mathrm{i}+d\,x\,9\mathrm{i}}\,1\mathrm{i}}{144}\right)}{d}$$

```
[In] int(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^5,x)
```

```
[Out] -(a^5*((exp(c*1i + d*x*1i)*1i)/16 + (exp(c*3i + d*x*3i)*1i)/12 + (exp(c*5i
+ d*x*5i)*3i)/40 + (exp(c*7i + d*x*7i)*1i)/28 + (exp(c*9i + d*x*9i)*1i)/144
))/d
```

3.76 $\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^5\,dx$

Optimal result

Integrand size = 24, antiderivative size = 159

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{5ia^5\cos^7(c+dx)}{231d} + \frac{5a^5\sin(c+dx)}{33d} - \frac{5a^5\sin^3(c+dx)}{33d} + \frac{a^5\sin^5(c+dx)}{11d} - \frac{5a^5\sin^7(c+dx)}{231d} - \frac{2ia^3\cos^9(c+dx)(a+ia\tan(c+dx))^2}{33d} - \frac{2ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^4}{11d}$$

```
[Out] -5/231*I*a^5*cos(d*x+c)^7/d+5/33*a^5*sin(d*x+c)/d-5/33*a^5*sin(d*x+c)^3/d+1
/11*a^5*sin(d*x+c)^5/d-5/231*a^5*sin(d*x+c)^7/d-2/33*I*a^3*cos(d*x+c)^9*(a+
I*a*tan(d*x+c))^2/d-2/11*I*a*cos(d*x+c)^11*(a+I*a*tan(d*x+c))^4/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used

$= \{3577, 3567, 2713\}$

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^5\,dx = -\frac{5a^5\sin^7(c+dx)}{231d} + \frac{a^5\sin^5(c+dx)}{11d} - \frac{5a^5\sin^3(c+dx)}{33d} + \frac{5a^5\sin(c+dx)}{33d} - \frac{5ia^5\cos^7(c+dx)}{231d} - \frac{2ia^3\cos^9(c+dx)(a+ia\tan(c+dx))^2}{33d} - \frac{2ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^4}{11d}$$

```
[In] Int[Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (((-5*I)/231)*a^5*Cos[c + d*x]^7)/d + (5*a^5*Sin[c + d*x])/(33*d) - (5*a^5*
Sin[c + d*x]^3)/(33*d) + (a^5*Sin[c + d*x]^5)/(11*d) - (5*a^5*Sin[c + d*x]^
7)/(231*d) - (((2*I)/33)*a^3*Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^2)/d - (
((2*I)/11)*a*Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^4)/d
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^4}{11d} \\
&\quad + \frac{1}{11}(3a^2)\int\cos^9(c+dx)(a+ia\tan(c+dx))^3\,dx \\
&= -\frac{2ia^3\cos^9(c+dx)(a+ia\tan(c+dx))^2}{33d} - \frac{2ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^4}{11d} \\
&\quad + \frac{1}{33}(5a^4)\int\cos^7(c+dx)(a+ia\tan(c+dx))\,dx \\
&= -\frac{5ia^5\cos^7(c+dx)}{231d} - \frac{2ia^3\cos^9(c+dx)(a+ia\tan(c+dx))^2}{33d} \\
&\quad - \frac{2ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^4}{11d} + \frac{1}{33}(5a^5)\int\cos^7(c+dx)\,dx \\
&= -\frac{5ia^5\cos^7(c+dx)}{231d} - \frac{2ia^3\cos^9(c+dx)(a+ia\tan(c+dx))^2}{33d} \\
&\quad - \frac{2ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^4}{11d} \\
&\quad - \frac{(5a^5)\,\text{Subst}\left(\int(1-3x^2+3x^4-x^6)\,dx, x, -\sin(c+dx)\right)}{33d} \\
&= -\frac{5ia^5\cos^7(c+dx)}{231d} + \frac{5a^5\sin(c+dx)}{33d} - \frac{5a^5\sin^3(c+dx)}{33d} + \frac{a^5\sin^5(c+dx)}{11d} - \frac{5a^5\sin^7(c+dx)}{231d} \\
&\quad - \frac{2ia^3\cos^9(c+dx)(a+ia\tan(c+dx))^2}{33d} - \frac{2ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^4}{11d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.07

$$
\int\cos^{11}(c+dx)(a+ia\tan(c+dx))^5\,dx
$$

$$
= \frac{ia^5\sec(c+dx)(\cos(5(c+dx))+i\sin(5(c+dx)))\left(-1749\cos(c+dx)-1595\cos(3(c+dx))-665\cos(5(c\right.}{}
$$

```
[In] Integrate[Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] ((I/14784)*a^5*Sec[c + d*x]*(Cos[5*(c + d*x)] + I*Sin[5*(c + d*x)])*(-1749*
Cos[c + d*x] - 1595*Cos[3*(c + d*x)] - 665*Cos[5*(c + d*x)] - 2816*Sqrt[Cos
[c + d*x]^2]*Cos[5*(c + d*x)] + 105*Cos[7*(c + d*x)] + (330*I)*Sin[c + d*x]
 + (946*I)*Sin[3*(c + d*x)] + (490*I)*Sin[5*(c + d*x)] + (2816*I)*Sqrt[Cos[
c + d*x]^2]*Sin[5*(c + d*x)] - (126*I)*Sin[7*(c + d*x)]))/d
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(140) = 280.

Time = 0.67 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.99

$$\frac{ia^5\left(-\frac{(\cos^7(dx+c))(\sin^4(dx+c))}{11} - \frac{4(\cos^7(dx+c))(\sin^2(dx+c))}{99} - \frac{8(\cos^7(dx+c))}{693}\right) + 5a^5\left(-\frac{(\sin^3(dx+c))(\cos^8(dx+c))}{11} - \frac{(\cos^8(dx+ \cdots}{}\right.}{}$$

[In] int(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^5,x)

```
[Out] 1/d*(I*a^5*(-1/11*cos(d*x+c)^7*sin(d*x+c)^4-4/99*cos(d*x+c)^7*sin(d*x+c)^2-
8/693*cos(d*x+c)^7)+5*a^5*(-1/11*sin(d*x+c)^3*cos(d*x+c)^8-1/33*cos(d*x+c)^
8*sin(d*x+c)+1/231*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*si
n(d*x+c))-10*I*a^5*(-1/11*cos(d*x+c)^9*sin(d*x+c)^2-2/99*cos(d*x+c)^9)-10*a
^5*(-1/11*sin(d*x+c)*cos(d*x+c)^10+1/99*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c)
^6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*x+c))-5/11*I*a^5*cos(d*x+c)
^11+1/11*a^5*(256/63+cos(d*x+c)^10+10/9*cos(d*x+c)^8+80/63*cos(d*x+c)^6+32/
21*cos(d*x+c)^4+128/63*cos(d*x+c)^2)*sin(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.65

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^5\,dx = \frac{(-21i\,a^5e^{(12i\,dx+12i\,c)} - 154i\,a^5e^{(10i\,dx+10i\,c)} - 495i\,a^5e^{(8i\,dx+8i\,c)} - 924i\,a^5e^{(6i\,dx+6i\,c)} - 1155i\,a^5e^{(4i\,dx+4i\,c)} - \cdots}{14784\,d}$$

```
[In] integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/14784*(-21*I*a^5*e^(12*I*d*x + 12*I*c) - 154*I*a^5*e^(10*I*d*x + 10*I*c)
- 495*I*a^5*e^(8*I*d*x + 8*I*c) - 924*I*a^5*e^(6*I*d*x + 6*I*c) - 1155*I*a^
5*e^(4*I*d*x + 4*I*c) - 1386*I*a^5*e^(2*I*d*x + 2*I*c) + 231*I*a^5)*e^(-I*d
*x - I*c)/d
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.67

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$=\begin{cases}\frac{(-90194313216ia^5d^6e^{12ic}e^{11idx}-661424963584ia^5d^6e^{10ic}e^{9idx}-2126008811520ia^5d^6e^{8ic}e^{7idx}-3968549781504ia^5d^6e^{6ic}e^{5idx}-496068722688}{63496796504064d^7}\\ \frac{x(a^5e^{12ic}+6a^5e^{10ic}+15a^5e^{8ic}+20a^5e^{6ic}+15a^5e^{4ic}+6a^5e^{2ic}+a^5)e^{-ic}}{64}\end{cases}$$

```
[In] integrate(cos(d*x+c)**11*(a+I*a*tan(d*x+c))**5,x)
```

```
[Out] Piecewise(((-90194313216*I*a**5*d**6*exp(12*I*c)*exp(11*I*d*x) - 6614249635
84*I*a**5*d**6*exp(10*I*c)*exp(9*I*d*x) - 2126008811520*I*a**5*d**6*exp(8*I
*c)*exp(7*I*d*x) - 3968549781504*I*a**5*d**6*exp(6*I*c)*exp(5*I*d*x) - 4960
687226880*I*a**5*d**6*exp(4*I*c)*exp(3*I*d*x) - 5952824672256*I*a**5*d**6*e
xp(2*I*c)*exp(I*d*x) + 992137445376*I*a**5*d**6*exp(-I*d*x))*exp(-I*c)/(634
96796504064*d**7), Ne(d**7*exp(I*c), 0)), (x*(a**5*exp(12*I*c) + 6*a**5*exp
(10*I*c) + 15*a**5*exp(8*I*c) + 20*a**5*exp(6*I*c) + 15*a**5*exp(4*I*c) + 6
*a**5*exp(2*I*c) + a**5)*exp(-I*c)/64, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.55

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^5\,dx =$$
$$-\frac{315i\,a^5\cos{(dx+c)}^{11}+i\left(63\,\cos{(dx+c)}^{11}-154\,\cos{(dx+c)}^{9}+99\,\cos{(dx+c)}^{7}\right)a^5+70i\left(9\,\cos{(dx+\right.}{}}{}$$

```
[In] integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/693*(315*I*a^5*cos(d*x + c)^11 + I*(63*cos(d*x + c)^11 - 154*cos(d*x + c
)^9 + 99*cos(d*x + c)^7)*a^5 + 70*I*(9*cos(d*x + c)^11 - 11*cos(d*x + c)^9)
*a^5 + 2*(315*sin(d*x + c)^11 - 1540*sin(d*x + c)^9 + 2970*sin(d*x + c)^7 -
 2772*sin(d*x + c)^5 + 1155*sin(d*x + c)^3)*a^5 + 3*(105*sin(d*x + c)^11 -
385*sin(d*x + c)^9 + 495*sin(d*x + c)^7 - 231*sin(d*x + c)^5)*a^5 + (63*sin
(d*x + c)^11 - 385*sin(d*x + c)^9 + 990*sin(d*x + c)^7 - 1386*sin(d*x + c)^
5 + 1155*sin(d*x + c)^3 - 693*sin(d*x + c))*a^5)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1807 vs. 2(135) = 270.

Time = 1.03 (sec) , antiderivative size = 1807, normalized size of antiderivative = 11.36

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^5\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/121110528*(168111405*a^5*e^(17*I*d*x + 9*I*c)*log(I*e^(I*d*x + I*c) + 1)
 + 1344891240*a^5*e^(15*I*d*x + 7*I*c)*log(I*e^(I*d*x + I*c) + 1) + 4707119
340*a^5*e^(13*I*d*x + 5*I*c)*log(I*e^(I*d*x + I*c) + 1) + 9414238680*a^5*e^
(11*I*d*x + 3*I*c)*log(I*e^(I*d*x + I*c) + 1) + 11767798350*a^5*e^(9*I*d*x
+ I*c)*log(I*e^(I*d*x + I*c) + 1) + 9414238680*a^5*e^(7*I*d*x - I*c)*log(I*
e^(I*d*x + I*c) + 1) + 4707119340*a^5*e^(5*I*d*x - 3*I*c)*log(I*e^(I*d*x +
I*c) + 1) + 1344891240*a^5*e^(3*I*d*x - 5*I*c)*log(I*e^(I*d*x + I*c) + 1) +
 168111405*a^5*e^(I*d*x - 7*I*c)*log(I*e^(I*d*x + I*c) + 1) + 170251620*a^5
*e^(17*I*d*x + 9*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1362012960*a^5*e^(15*I*d
*x + 7*I*c)*log(I*e^(I*d*x + I*c) - 1) + 4767045360*a^5*e^(13*I*d*x + 5*I*c
)*log(I*e^(I*d*x + I*c) - 1) + 9534090720*a^5*e^(11*I*d*x + 3*I*c)*log(I*e^
(I*d*x + I*c) - 1) + 11917613400*a^5*e^(9*I*d*x + I*c)*log(I*e^(I*d*x + I*c
) - 1) + 9534090720*a^5*e^(7*I*d*x - I*c)*log(I*e^(I*d*x + I*c) - 1) + 4767
045360*a^5*e^(5*I*d*x - 3*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1362012960*a^5*
e^(3*I*d*x - 5*I*c)*log(I*e^(I*d*x + I*c) - 1) + 170251620*a^5*e^(I*d*x - 7
*I*c)*log(I*e^(I*d*x + I*c) - 1) - 168111405*a^5*e^(17*I*d*x + 9*I*c)*log(-
I*e^(I*d*x + I*c) + 1) - 1344891240*a^5*e^(15*I*d*x + 7*I*c)*log(-I*e^(I*d*
x + I*c) + 1) - 4707119340*a^5*e^(13*I*d*x + 5*I*c)*log(-I*e^(I*d*x + I*c)
+ 1) - 9414238680*a^5*e^(11*I*d*x + 3*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 11
767798350*a^5*e^(9*I*d*x + I*c)*log(-I*e^(I*d*x + I*c) + 1) - 9414238680*a^
5*e^(7*I*d*x - I*c)*log(-I*e^(I*d*x + I*c) + 1) - 4707119340*a^5*e^(5*I*d*x
 - 3*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1344891240*a^5*e^(3*I*d*x - 5*I*c)*
log(-I*e^(I*d*x + I*c) + 1) - 168111405*a^5*e^(I*d*x - 7*I*c)*log(-I*e^(I*d
*x + I*c) + 1) - 170251620*a^5*e^(17*I*d*x + 9*I*c)*log(-I*e^(I*d*x + I*c)
- 1) - 1362012960*a^5*e^(15*I*d*x + 7*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 47
67045360*a^5*e^(13*I*d*x + 5*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 9534090720*
a^5*e^(11*I*d*x + 3*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 11917613400*a^5*e^(9
*I*d*x + I*c)*log(-I*e^(I*d*x + I*c) - 1) - 9534090720*a^5*e^(7*I*d*x - I*c
)*log(-I*e^(I*d*x + I*c) - 1) - 4767045360*a^5*e^(5*I*d*x - 3*I*c)*log(-I*e
^(I*d*x + I*c) - 1) - 1362012960*a^5*e^(3*I*d*x - 5*I*c)*log(-I*e^(I*d*x +
I*c) - 1) - 170251620*a^5*e^(I*d*x - 7*I*c)*log(-I*e^(I*d*x + I*c) - 1) + 2
140215*a^5*e^(17*I*d*x + 9*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 17121720*a^5*
e^(15*I*d*x + 7*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 59926020*a^5*e^(13*I*d*x
 + 5*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 119852040*a^5*e^(11*I*d*x + 3*I*c)*
```

```
log(I*e^(I*d*x) + e^(-I*c)) + 149815050*a^5*e^(9*I*d*x + I*c)*log(I*e^(I*d*
x) + e^(-I*c)) + 119852040*a^5*e^(7*I*d*x - I*c)*log(I*e^(I*d*x) + e^(-I*c)
) + 59926020*a^5*e^(5*I*d*x - 3*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 17121720
*a^5*e^(3*I*d*x - 5*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 2140215*a^5*e^(I*d*x
 - 7*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 2140215*a^5*e^(17*I*d*x + 9*I*c)*lo
g(-I*e^(I*d*x) + e^(-I*c)) - 17121720*a^5*e^(15*I*d*x + 7*I*c)*log(-I*e^(I*
d*x) + e^(-I*c)) - 59926020*a^5*e^(13*I*d*x + 5*I*c)*log(-I*e^(I*d*x) + e^(
-I*c)) - 119852040*a^5*e^(11*I*d*x + 3*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) -
149815050*a^5*e^(9*I*d*x + I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 119852040*a^
5*e^(7*I*d*x - I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 59926020*a^5*e^(5*I*d*x
- 3*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 17121720*a^5*e^(3*I*d*x - 5*I*c)*lo
g(-I*e^(I*d*x) + e^(-I*c)) - 2140215*a^5*e^(I*d*x - 7*I*c)*log(-I*e^(I*d*x)
 + e^(-I*c)) + 172032*I*a^5*e^(28*I*d*x + 20*I*c) + 2637824*I*a^5*e^(26*I*d
*x + 18*I*c) + 18964480*I*a^5*e^(24*I*d*x + 16*I*c) + 84967424*I*a^5*e^(22*
I*d*x + 14*I*c) + 266248192*I*a^5*e^(20*I*d*x + 12*I*c) + 624017408*I*a^5*e
^(18*I*d*x + 10*I*c) + 1137074176*I*a^5*e^(16*I*d*x + 8*I*c) + 1626275840*I
*a^5*e^(14*I*d*x + 6*I*c) + 1792860160*I*a^5*e^(12*I*d*x + 4*I*c) + 1464320
000*I*a^5*e^(10*I*d*x + 2*I*c) + 295206912*I*a^5*e^(6*I*d*x - 2*I*c) + 4730
8800*I*a^5*e^(4*I*d*x - 4*I*c) - 3784704*I*a^5*e^(2*I*d*x - 6*I*c) + 832905
216*I*a^5*e^(8*I*d*x) - 1892352*I*a^5*e^(-8*I*c))/(d*e^(17*I*d*x + 9*I*c) +
 8*d*e^(15*I*d*x + 7*I*c) + 28*d*e^(13*I*d*x + 5*I*c) + 56*d*e^(11*I*d*x +
3*I*c) + 70*d*e^(9*I*d*x + I*c) + 56*d*e^(7*I*d*x - I*c) + 28*d*e^(5*I*d*x
- 3*I*c) + 8*d*e^(3*I*d*x - 5*I*c) + d*e^(I*d*x - 7*I*c))
```

Mupad [B] (verification not implemented)

Time = 5.81 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.87

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^5\,dx$$
$$= \frac{a^5\left(\frac{5\sin(3c+3dx)}{64} - \frac{\cos(5c+5dx)\,1i}{16} - \frac{\cos(7c+7dx)\,15i}{448} - \frac{\cos(9c+9dx)\,1i}{96} - \frac{\cos(11c+11dx)\,1i}{704} - \frac{\cos(3c+3dx)\,5i}{64} + \frac{\sin(5c+5dx}{16}\right.}{d}$$

```
[In] int(cos(c + d*x)^11*(a + a*tan(c + d*x)*1i)^5,x)
```

```
[Out] (a^5*((5*sin(3*c + 3*d*x))/64 - (cos(5*c + 5*d*x)*1i)/16 - (cos(7*c + 7*d*x
)*15i)/448 - (cos(9*c + 9*d*x)*1i)/96 - (cos(11*c + 11*d*x)*1i)/704 - (cos(
3*c + 3*d*x)*5i)/64 + sin(5*c + 5*d*x)/16 + (15*sin(7*c + 7*d*x))/448 + sin
(9*c + 9*d*x)/96 + sin(11*c + 11*d*x)/704 + (24^(1/2)*cos(c - atanh(7/5)*1i
 + d*x))/64))/d
```

3.77 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 24, antiderivative size = 109

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^8\,dx=-\frac{2i(a+ia\tan(c+dx))^{12}}{3a^4d}+\frac{12i(a+ia\tan(c+dx))^{13}}{13a^5d}-\frac{3i(a+ia\tan(c+dx))^{14}}{7a^6d}+\frac{i(a+ia\tan(c+dx))^{15}}{15a^7d}$$

```
[Out] -2/3*I*(a+I*a*tan(d*x+c))^12/a^4/d+12/13*I*(a+I*a*tan(d*x+c))^13/a^5/d-3/7*
I*(a+I*a*tan(d*x+c))^14/a^6/d+1/15*I*(a+I*a*tan(d*x+c))^15/a^7/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^8\,dx=\frac{i(a+ia\tan(c+dx))^{15}}{15a^7d}-\frac{3i(a+ia\tan(c+dx))^{14}}{7a^6d}+\frac{12i(a+ia\tan(c+dx))^{13}}{13a^5d}-\frac{2i(a+ia\tan(c+dx))^{12}}{3a^4d}$$

```
[In] Int[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (((-2*I)/3)*(a + I*a*Tan[c + d*x])^12)/(a^4*d) + (((12*I)/13)*(a + I*a*Tan[
c + d*x])^13)/(a^5*d) - (((3*I)/7)*(a + I*a*Tan[c + d*x])^14)/(a^6*d) + ((I
/15)*(a + I*a*Tan[c + d*x])^15)/(a^7*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)^3(a+x)^{11}\,dx, x, ia\tan(c+dx)\right)}{a^7d}\\
&= -\frac{i\text{Subst}\left(\int \left(8a^3(a+x)^{11}-12a^2(a+x)^{12}+6a(a+x)^{13}-(a+x)^{14}\right)\,dx, x, ia\tan(c+dx)\right)}{a^7d}\\
&= -\frac{2i(a+ia\tan(c+dx))^{12}}{3a^4d}+\frac{12i(a+ia\tan(c+dx))^{13}}{13a^5d}\\
&\quad -\frac{3i(a+ia\tan(c+dx))^{14}}{7a^6d}+\frac{i(a+ia\tan(c+dx))^{15}}{15a^7d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.93 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.51

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{a^8(-i+\tan(c+dx))^{12}\left(-144i-363\tan(c+dx)+312i\tan^2(c+dx)+91\tan^3(c+dx)\right)}{1365d}$$

```
[In] Integrate[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*(-I + Tan[c + d*x])^12*(-144*I - 363*Tan[c + d*x] + (312*I)*Tan[c + d*
x]^2 + 91*Tan[c + d*x]^3))/(1365*d)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 610 vs. 2(93) = 186.

Time = 0.59 (sec) , antiderivative size = 611, normalized size of antiderivative = 5.61

$$a^8\left(\frac{\sin^9(dx+c)}{15\cos(dx+c)^{15}}+\frac{2(\sin^9(dx+c))}{65\cos(dx+c)^{13}}+\frac{8(\sin^9(dx+c))}{715\cos(dx+c)^{11}}+\frac{16(\sin^9(dx+c))}{6435\cos(dx+c)^9}\right)+56ia^8\left(\frac{\sin^6(dx+c)}{12\cos(dx+c)^{12}}+\frac{\sin^6(dx+c)}{20\cos(dx+c)^{10}}+\frac{\sin^6(dx}{40\cos(dx}\right.$$

```
[In] int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 1/d*(a^8*(1/15*sin(d*x+c)^9/cos(d*x+c)^15+2/65*sin(d*x+c)^9/cos(d*x+c)^13+8
/715*sin(d*x+c)^9/cos(d*x+c)^11+16/6435*sin(d*x+c)^9/cos(d*x+c)^9)+56*I*a^8
*(1/12*sin(d*x+c)^6/cos(d*x+c)^12+1/20*sin(d*x+c)^6/cos(d*x+c)^10+1/40*sin(
d*x+c)^6/cos(d*x+c)^8+1/120*sin(d*x+c)^6/cos(d*x+c)^6)-28*a^8*(1/13*sin(d*x
+c)^7/cos(d*x+c)^13+6/143*sin(d*x+c)^7/cos(d*x+c)^11+8/429*sin(d*x+c)^7/cos
(d*x+c)^9+16/3003*sin(d*x+c)^7/cos(d*x+c)^7)-8*I*a^8*(1/14*sin(d*x+c)^8/cos
(d*x+c)^14+1/28*sin(d*x+c)^8/cos(d*x+c)^12+1/70*sin(d*x+c)^8/cos(d*x+c)^10+
1/280*sin(d*x+c)^8/cos(d*x+c)^8)+70*a^8*(1/11*sin(d*x+c)^5/cos(d*x+c)^11+2/
33*sin(d*x+c)^5/cos(d*x+c)^9+8/231*sin(d*x+c)^5/cos(d*x+c)^7+16/1155*sin(d*
x+c)^5/cos(d*x+c)^5)+I*a^8/cos(d*x+c)^8-28*a^8*(1/9*sin(d*x+c)^3/cos(d*x+c)
^9+2/21*sin(d*x+c)^3/cos(d*x+c)^7+8/105*sin(d*x+c)^3/cos(d*x+c)^5+16/315*si
n(d*x+c)^3/cos(d*x+c)^3)-56*I*a^8*(1/10*sin(d*x+c)^4/cos(d*x+c)^10+3/40*sin
(d*x+c)^4/cos(d*x+c)^8+1/20*sin(d*x+c)^4/cos(d*x+c)^6+1/40*sin(d*x+c)^4/cos
(d*x+c)^4)-a^8*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2
)*tan(d*x+c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(85) = 170.

Time = 0.24 (sec) , antiderivative size = 345, normalized size of antiderivative = 3.17

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{8192\left(-1365i\,a^8e^{(22i\,dx+22i\,c)}-3003i\,a^8e^{(20i\,dx+20i\,c)}-5005i\,a^8e^{(18i\,dx+18i\,c)}-64\right.}{1365\left(de^{(30i\,dx+30i\,c)}+15\,de^{(28i\,dx+28i\,c)}+105\,de^{(26i\,dx+26i\,c)}+455\,de^{(24i\,dx+24i\,c)}+1365\,de^{(22i\,dx+22i\,c)}+30\right.}$$

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -8192/1365*(-1365*I*a^8*e^(22*I*d*x + 22*I*c) - 3003*I*a^8*e^(20*I*d*x + 20
*I*c) - 5005*I*a^8*e^(18*I*d*x + 18*I*c) - 6435*I*a^8*e^(16*I*d*x + 16*I*c)
 - 6435*I*a^8*e^(14*I*d*x + 14*I*c) - 5005*I*a^8*e^(12*I*d*x + 12*I*c) - 30
03*I*a^8*e^(10*I*d*x + 10*I*c) - 1365*I*a^8*e^(8*I*d*x + 8*I*c) - 455*I*a^8
```

```
*e^(6*I*d*x + 6*I*c) - 105*I*a^8*e^(4*I*d*x + 4*I*c) - 15*I*a^8*e^(2*I*d*x
+ 2*I*c) - I*a^8)/(d*e^(30*I*d*x + 30*I*c) + 15*d*e^(28*I*d*x + 28*I*c) + 1
05*d*e^(26*I*d*x + 26*I*c) + 455*d*e^(24*I*d*x + 24*I*c) + 1365*d*e^(22*I*d
*x + 22*I*c) + 3003*d*e^(20*I*d*x + 20*I*c) + 5005*d*e^(18*I*d*x + 18*I*c)
+ 6435*d*e^(16*I*d*x + 16*I*c) + 6435*d*e^(14*I*d*x + 14*I*c) + 5005*d*e^(1
2*I*d*x + 12*I*c) + 3003*d*e^(10*I*d*x + 10*I*c) + 1365*d*e^(8*I*d*x + 8*I*
c) + 455*d*e^(6*I*d*x + 6*I*c) + 105*d*e^(4*I*d*x + 4*I*c) + 15*d*e^(2*I*d*
x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^8\,dx = a^8\left(\int \left(-28\tan^2(c+dx)\sec^8(c+dx)\right)\,dx + \int 70\tan^4(c+dx)\sec^8(c+dx)\,dx + \int \left(-28\tan^6(c+dx)\sec^8(c+dx)\right)\,dx + \int \tan^8(c+dx)\sec^8(c+dx)\,dx + \int 8i\tan(c+dx)\sec^8(c+dx)\,dx + \int \left(-56i\tan^3(c+dx)\sec^8(c+dx)\right)\,dx + \int 56i\tan^5(c+dx)\sec^8(c+dx)\,dx + \int \left(-8i\tan^7(c+dx)\sec^8(c+dx)\right)\,dx + \int \sec^8(c+dx)\,dx\right)$$

```
[In] integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] a**8*(Integral(-28*tan(c + d*x)**2*sec(c + d*x)**8, x) + Integral(70*tan(c
+ d*x)**4*sec(c + d*x)**8, x) + Integral(-28*tan(c + d*x)**6*sec(c + d*x)**
8, x) + Integral(tan(c + d*x)**8*sec(c + d*x)**8, x) + Integral(8*I*tan(c +
 d*x)*sec(c + d*x)**8, x) + Integral(-56*I*tan(c + d*x)**3*sec(c + d*x)**8,
 x) + Integral(56*I*tan(c + d*x)**5*sec(c + d*x)**8, x) + Integral(-8*I*tan
(c + d*x)**7*sec(c + d*x)**8, x) + Integral(sec(c + d*x)**8, x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(85) = 170.

Time = 0.28 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.71

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{91\,a^8\tan\left(dx+c\right)^{15} - 780i\,a^8\tan\left(dx+c\right)^{14} - 2625\,a^8\tan\left(dx+c\right)^{13} + 3640i\,a^8\tan\left(dx+c\right)^{12} - 1365\,a^8\,\mathrm{t}}{}$$

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/1365*(91*a^8*tan(d*x + c)^15 - 780*I*a^8*tan(d*x + c)^14 - 2625*a^8*tan(d
*x + c)^13 + 3640*I*a^8*tan(d*x + c)^12 - 1365*a^8*tan(d*x + c)^11 + 12012*
I*a^8*tan(d*x + c)^10 + 15015*a^8*tan(d*x + c)^9 + 19305*a^8*tan(d*x + c)^7
 - 20020*I*a^8*tan(d*x + c)^6 - 3003*a^8*tan(d*x + c)^5 - 10920*I*a^8*tan(d
*x + c)^4 - 11375*a^8*tan(d*x + c)^3 + 5460*I*a^8*tan(d*x + c)^2 + 1365*a^8
*tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(85) = 170.

Time = 1.20 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.71

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{91\,a^8\tan\left(dx+c\right)^{15} - 780i\,a^8\tan\left(dx+c\right)^{14} - 2625\,a^8\tan\left(dx+c\right)^{13} + 3640i\,a^8\tan\left(dx+c\right)^{12} - 1365\,a^8\,\mathrm{t}}{}$$

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/1365*(91*a^8*tan(d*x + c)^15 - 780*I*a^8*tan(d*x + c)^14 - 2625*a^8*tan(d
*x + c)^13 + 3640*I*a^8*tan(d*x + c)^12 - 1365*a^8*tan(d*x + c)^11 + 12012*
I*a^8*tan(d*x + c)^10 + 15015*a^8*tan(d*x + c)^9 + 19305*a^8*tan(d*x + c)^7
 - 20020*I*a^8*tan(d*x + c)^6 - 3003*a^8*tan(d*x + c)^5 - 10920*I*a^8*tan(d
*x + c)^4 - 11375*a^8*tan(d*x + c)^3 + 5460*I*a^8*tan(d*x + c)^2 + 1365*a^8
*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 5.60 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.40

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{a^8\left(\frac{\sin(9\,c+9\,d\,x)}{12}+\frac{\sin(11\,c+11\,d\,x)}{52}+\frac{\sin(13\,c+13\,d\,x)}{364}+\frac{\sin(15\,c+15\,d\,x)}{5460}+\frac{\cos(c+d\,x)\,297\mathrm{i}}{7168}+\frac{\cos(3\,c+3\,d\,x)\,33\mathrm{i}}{1024}+\frac{\cos(5\,c+5\,d\,x)\,99\mathrm{i}}{5120}\right.}{d\cos\left(c+d\,x\right)^{15}}$$

```
[In] int((a + a*tan(c + d*x)*1i)^8/cos(c + d*x)^8,x)
```

```
[Out] (a^8*((cos(c + d*x)*297i)/7168 + (cos(3*c + 3*d*x)*33i)/1024 + (cos(5*c + 5
*d*x)*99i)/5120 + (cos(7*c + 7*d*x)*9i)/1024 - (cos(9*c + 9*d*x)*247i)/3072
 - (cos(11*c + 11*d*x)*19i)/1024 - (cos(13*c + 13*d*x)*19i)/7168 - (cos(15*
c + 15*d*x)*19i)/107520 + sin(9*c + 9*d*x)/12 + sin(11*c + 11*d*x)/52 + sin
(13*c + 13*d*x)/364 + sin(15*c + 15*d*x)/5460))/(d*cos(c + d*x)^15)
```

3.78 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{4i(a+ia\tan(c+dx))^{11}}{11a^3d} + \frac{i(a+ia\tan(c+dx))^{12}}{3a^4d} - \frac{i(a+ia\tan(c+dx))^{13}}{13a^5d}$$

```
[Out] -4/11*I*(a+I*a*tan(d*x+c))^11/a^3/d+1/3*I*(a+I*a*tan(d*x+c))^12/a^4/d-1/13*
I*(a+I*a*tan(d*x+c))^13/a^5/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{i(a+ia\tan(c+dx))^{13}}{13a^5d} + \frac{i(a+ia\tan(c+dx))^{12}}{3a^4d} - \frac{4i(a+ia\tan(c+dx))^{11}}{11a^3d}$$

```
[In] Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (((-4*I)/11)*(a + I*a*Tan[c + d*x])^11)/(a^3*d) + ((I/3)*(a + I*a*Tan[c + d
*x])^12)/(a^4*d) - ((I/13)*(a + I*a*Tan[c + d*x])^13)/(a^5*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)^2(a+x)^{10}\,dx, x, ia\tan(c+dx)\right)}{a^5 d}\\ &= -\frac{i\text{Subst}\left(\int \left(4a^2(a+x)^{10}-4a(a+x)^{11}+(a+x)^{12}\right)\,dx, x, ia\tan(c+dx)\right)}{a^5 d}\\ &= -\frac{4i(a+ia\tan(c+dx))^{11}}{11a^3 d}+\frac{i(a+ia\tan(c+dx))^{12}}{3a^4 d}-\frac{i(a+ia\tan(c+dx))^{13}}{13a^5 d}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.54

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{a^8(-i+\tan(c+dx))^{11}\left(-46+77i\tan(c+dx)+33\tan^2(c+dx)\right)}{429d}$$

```
[In] Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^8,x]
[Out] (a^8*(-I + Tan[c + d*x])^11*(-46 + (77*I)*Tan[c + d*x] + 33*Tan[c + d*x]^2)
)/(429*d)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 474 vs. 2(70) = 140.

Time = 0.58 (sec) , antiderivative size = 475, normalized size of antiderivative = 5.79

$$\frac{a^8\left(\frac{\sin^9(dx+c)}{13\cos(dx+c)^{13}}+\frac{4(\sin^9(dx+c))}{143\cos(dx+c)^{11}}+\frac{8(\sin^9(dx+c))}{1287\cos(dx+c)^9}\right)+56ia^8\left(\frac{\sin^6(dx+c)}{10\cos(dx+c)^{10}}+\frac{\sin^6(dx+c)}{20\cos(dx+c)^8}+\frac{\sin^6(dx+c)}{60\cos(dx+c)^6}\right)-28a^8\left(\frac{\text{s}}{11}\right.}{}$$

```
[In] int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 1/d*(a^8*(1/13*sin(d*x+c)^9/cos(d*x+c)^13+4/143*sin(d*x+c)^9/cos(d*x+c)^11+
8/1287*sin(d*x+c)^9/cos(d*x+c)^9)+56*I*a^8*(1/10*sin(d*x+c)^6/cos(d*x+c)^10
+1/20*sin(d*x+c)^6/cos(d*x+c)^8+1/60*sin(d*x+c)^6/cos(d*x+c)^6)-28*a^8*(1/1
1*sin(d*x+c)^7/cos(d*x+c)^11+4/99*sin(d*x+c)^7/cos(d*x+c)^9+8/693*sin(d*x+c
)^7/cos(d*x+c)^7)-8*I*a^8*(1/12*sin(d*x+c)^8/cos(d*x+c)^12+1/30*sin(d*x+c)^
8/cos(d*x+c)^10+1/120*sin(d*x+c)^8/cos(d*x+c)^8)+70*a^8*(1/9*sin(d*x+c)^5/c
os(d*x+c)^9+4/63*sin(d*x+c)^5/cos(d*x+c)^7+8/315*sin(d*x+c)^5/cos(d*x+c)^5)
-56*I*a^8*(1/8*sin(d*x+c)^4/cos(d*x+c)^8+1/12*sin(d*x+c)^4/cos(d*x+c)^6+1/2
4*sin(d*x+c)^4/cos(d*x+c)^4)-28*a^8*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin
(d*x+c)^3/cos(d*x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)+4/3*I*a^8/cos(d*x+c
)^6-a^8*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 307 vs. 2(64) = 128.

Time = 0.23 (sec) , antiderivative size = 307, normalized size of antiderivative = 3.74

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{4096\left(-286i\,a^8e^{(20i\,dx+20i\,c)}-715i\,a^8e^{(18i\,dx+18i\,c)}-1287i\,a^8e^{(16i\,dx+16i\,c)}-1716i\,a^8e^{(14i\,dx}\right.}{429\left(de^{(26i\,dx+26i\,c)}+13\,de^{(24i\,dx+24i\,c)}+78\,de^{(22i\,dx+22i\,c)}+286\,de^{(20i\,dx+20i\,c)}+715\,de^{(18i\,dx+18i\,c)}+1287\,d\right.}$$

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -4096/429*(-286*I*a^8*e^(20*I*d*x + 20*I*c) - 715*I*a^8*e^(18*I*d*x + 18*I*
c) - 1287*I*a^8*e^(16*I*d*x + 16*I*c) - 1716*I*a^8*e^(14*I*d*x + 14*I*c) -
1716*I*a^8*e^(12*I*d*x + 12*I*c) - 1287*I*a^8*e^(10*I*d*x + 10*I*c) - 715*I
*a^8*e^(8*I*d*x + 8*I*c) - 286*I*a^8*e^(6*I*d*x + 6*I*c) - 78*I*a^8*e^(4*I*
d*x + 4*I*c) - 13*I*a^8*e^(2*I*d*x + 2*I*c) - I*a^8)/(d*e^(26*I*d*x + 26*I*
c) + 13*d*e^(24*I*d*x + 24*I*c) + 78*d*e^(22*I*d*x + 22*I*c) + 286*d*e^(20*
I*d*x + 20*I*c) + 715*d*e^(18*I*d*x + 18*I*c) + 1287*d*e^(16*I*d*x + 16*I*c
) + 1716*d*e^(14*I*d*x + 14*I*c) + 1716*d*e^(12*I*d*x + 12*I*c) + 1287*d*e^
(10*I*d*x + 10*I*c) + 715*d*e^(8*I*d*x + 8*I*c) + 286*d*e^(6*I*d*x + 6*I*c)
 + 78*d*e^(4*I*d*x + 4*I*c) + 13*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^8\,dx = a^8\left(\int \left(-28\tan^2{(c+dx)}\sec^6{(c+dx)}\right)\,dx + \int 70\tan^4{(c+dx)}\sec^6{(c+dx)}\,dx + \int \left(-28\tan^6{(c+dx)}\sec^6{(c+dx)}\right)\,dx + \int \tan^8{(c+dx)}\sec^6{(c+dx)}\,dx + \int 8i\tan{(c+dx)}\sec^6{(c+dx)}\,dx + \int \left(-56i\tan^3{(c+dx)}\sec^6{(c+dx)}\right)\,dx + \int 56i\tan^5{(c+dx)}\sec^6{(c+dx)}\,dx + \int \left(-8i\tan^7{(c+dx)}\sec^6{(c+dx)}\right)\,dx + \int \sec^6{(c+dx)}\,dx\right)$$

```
[In] integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] a**8*(Integral(-28*tan(c + d*x)**2*sec(c + d*x)**6, x) + Integral(70*tan(c
+ d*x)**4*sec(c + d*x)**6, x) + Integral(-28*tan(c + d*x)**6*sec(c + d*x)**
6, x) + Integral(tan(c + d*x)**8*sec(c + d*x)**6, x) + Integral(8*I*tan(c +
 d*x)*sec(c + d*x)**6, x) + Integral(-56*I*tan(c + d*x)**3*sec(c + d*x)**6,
 x) + Integral(56*I*tan(c + d*x)**5*sec(c + d*x)**6, x) + Integral(-8*I*tan
(c + d*x)**7*sec(c + d*x)**6, x) + Integral(sec(c + d*x)**6, x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(64) = 128$.

Time = 0.34 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.11

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{33\,a^8\tan{(dx+c)}^{13} - 286i\,a^8\tan{(dx+c)}^{12} - 1014\,a^8\tan{(dx+c)}^{11} + 1716i\,a^8\tan{(dx+c)}^{10} + 715\,a^8\tan}{}$$

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/429*(33*a^8*tan(d*x + c)^13 - 286*I*a^8*tan(d*x + c)^12 - 1014*a^8*tan(d*
x + c)^11 + 1716*I*a^8*tan(d*x + c)^10 + 715*a^8*tan(d*x + c)^9 + 2574*I*a^
8*tan(d*x + c)^8 + 5148*a^8*tan(d*x + c)^7 - 3432*I*a^8*tan(d*x + c)^6 + 12
87*a^8*tan(d*x + c)^5 - 4290*I*a^8*tan(d*x + c)^4 - 3718*a^8*tan(d*x + c)^3
 + 1716*I*a^8*tan(d*x + c)^2 + 429*a^8*tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(64) = 128$.

Time = 1.11 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.11

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{33\,a^8\tan\left(dx+c\right)^{13} - 286i\,a^8\tan\left(dx+c\right)^{12} - 1014\,a^8\tan\left(dx+c\right)^{11} + 1716i\,a^8\tan\left(dx+c\right)^{10} + 715\,a^8\,\mathrm{ta}}{}$$

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/429*(33*a^8*tan(d*x + c)^13 - 286*I*a^8*tan(d*x + c)^12 - 1014*a^8*tan(d*
x + c)^11 + 1716*I*a^8*tan(d*x + c)^10 + 715*a^8*tan(d*x + c)^9 + 2574*I*a^
8*tan(d*x + c)^8 + 5148*a^8*tan(d*x + c)^7 - 3432*I*a^8*tan(d*x + c)^6 + 12
87*a^8*tan(d*x + c)^5 - 4290*I*a^8*tan(d*x + c)^4 - 3718*a^8*tan(d*x + c)^3
 + 1716*I*a^8*tan(d*x + c)^2 + 429*a^8*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 5.87 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.32

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8\,\sin\left(c+d\,x\right)\,\left(2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)\,\left(-184\sin\left(c+d\,x\right)^2 - 184\sin\left(2\,c+2\,d\,x\right)^2 + \frac{\sin(2\,c+2\,d\,x)\,9867\mathrm{i}}{256} - 18}{}$$

```
[In] int((a + a*tan(c + d*x)*1i)^8/cos(c + d*x)^6,x)
```

```
[Out] (a^8*sin(c + d*x)*(2*sin(c/2 + (d*x)/2)^2 - 1)*((sin(2*c + 2*d*x)*9867i)/25
6 + (sin(4*c + 4*d*x)*69069i)/1024 + (sin(6*c + 6*d*x)*42757i)/512 + (sin(8
*c + 8*d*x)*23023i)/256 + (sin(10*c + 10*d*x)*7007i)/512 + (sin(12*c + 12*d
*x)*1001i)/1024 - 184*sin(2*c + 2*d*x)^2 - 184*sin(3*c + 3*d*x)^2 - 184*sin
(4*c + 4*d*x)^2 - 28*sin(5*c + 5*d*x)^2 - 2*sin(6*c + 6*d*x)^2 - 184*sin(c
+ d*x)^2 + 429))/(429*d*(sin(c + d*x)^2 - 1)^7)
```

3.79 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{i(a+ia\tan(c+dx))^{10}}{5a^2d}+\frac{i(a+ia\tan(c+dx))^{11}}{11a^3d}$$

```
[Out] -1/5*I*(a+I*a*tan(d*x+c))^10/a^2/d+1/11*I*(a+I*a*tan(d*x+c))^11/a^3/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{i(a+ia\tan(c+dx))^{11}}{11a^3d}-\frac{i(a+ia\tan(c+dx))^{10}}{5a^2d}$$

```
[In] Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((-1/5*I)*(a + I*a*Tan[c + d*x])^10)/(a^2*d) + ((I/11)*(a + I*a*Tan[c + d*x
])^11)/(a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
```

```
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)(a+x)^9\,dx, x, ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{i\text{Subst}\left(\int \left(2a(a+x)^9-(a+x)^{10}\right)\,dx, x, ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{i(a+ia\tan(c+dx))^{10}}{5a^2d}+\frac{i(a+ia\tan(c+dx))^{11}}{11a^3d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.62

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{a^8(-i+\tan(c+dx))^{10}(6i+5\tan(c+dx))}{55d}$$

```
[In] Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^8,x]
[Out] (a^8*(-I + Tan[c + d*x])^10*(6*I + 5*Tan[c + d*x]))/(55*d)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(47) = 94.

Time = 286.48 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.25

method	result
risch	$\frac{1024ia^8\left(55\,e^{18i(dx+c)}+165\,e^{16i(dx+c)}+330\,e^{14i(dx+c)}+462\,e^{12i(dx+c)}+462\,e^{10i(dx+c)}+330\,e^{8i(dx+c)}+165\,e^{6i(dx+c)}+5\right.}{55d\left(e^{2i(dx+c)}+1\right)^{11}}$
derivativedivides	$a^8\left(\frac{\sin^9(dx+c)}{11\cos(dx+c)^{11}}+\frac{2\left(\sin^9(dx+c)\right)}{99\cos(dx+c)^9}\right)-56ia^8\left(\frac{\sin^4(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^4(dx+c)}{12\cos(dx+c)^4}\right)-28a^8\left(\frac{\sin^7(dx+c)}{9\cos(dx+c)^9}+\frac{2\left(\sin^7(dx+c)\right)}{63\cos(dx+c)^7}\right)+\frac{2}{\cos(}$
default	$a^8\left(\frac{\sin^9(dx+c)}{11\cos(dx+c)^{11}}+\frac{2\left(\sin^9(dx+c)\right)}{99\cos(dx+c)^9}\right)-56ia^8\left(\frac{\sin^4(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^4(dx+c)}{12\cos(dx+c)^4}\right)-28a^8\left(\frac{\sin^7(dx+c)}{9\cos(dx+c)^9}+\frac{2\left(\sin^7(dx+c)\right)}{63\cos(dx+c)^7}\right)+\frac{2}{\cos(}$

```
[In] int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
[Out] 1024/55*I*a^8*(55*exp(18*I*(d*x+c))+165*exp(16*I*(d*x+c))+330*exp(14*I*(d*x
+c))+462*exp(12*I*(d*x+c))+462*exp(10*I*(d*x+c))+330*exp(8*I*(d*x+c))+165*e
```

```
xp(6*I*(d*x+c))+55*exp(4*I*(d*x+c))+11*exp(2*I*(d*x+c))+1)/d/(exp(2*I*(d*x+
c))+1)^11
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(43) = 86.

Time = 0.24 (sec) , antiderivative size = 269, normalized size of antiderivative = 4.89

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{1024\left(-55i\,a^8e^{(18i\,dx+18i\,c)} - 165i\,a^8e^{(16i\,dx+16i\,c)} - 330i\,a^8e^{(14i\,dx+14i\,c)} - 462i\,a^8e^{(12i\,dx+12i\,c)} - 462i\,a\right.}{55\left(de^{(22i\,dx+22i\,c)} + 11\,de^{(20i\,dx+20i\,c)} + 55\,de^{(18i\,dx+18i\,c)} + 165\,de^{(16i\,dx+16i\,c)} + 330\,de^{(14i\,dx+14i\,c)} + 462\,de^{(12\right.}}$$

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -1024/55*(-55*I*a^8*e^(18*I*d*x + 18*I*c) - 165*I*a^8*e^(16*I*d*x + 16*I*c)
 - 330*I*a^8*e^(14*I*d*x + 14*I*c) - 462*I*a^8*e^(12*I*d*x + 12*I*c) - 462*
I*a^8*e^(10*I*d*x + 10*I*c) - 330*I*a^8*e^(8*I*d*x + 8*I*c) - 165*I*a^8*e^(
6*I*d*x + 6*I*c) - 55*I*a^8*e^(4*I*d*x + 4*I*c) - 11*I*a^8*e^(2*I*d*x + 2*I
*c) - I*a^8)/(d*e^(22*I*d*x + 22*I*c) + 11*d*e^(20*I*d*x + 20*I*c) + 55*d*e
^(18*I*d*x + 18*I*c) + 165*d*e^(16*I*d*x + 16*I*c) + 330*d*e^(14*I*d*x + 14
*I*c) + 462*d*e^(12*I*d*x + 12*I*c) + 462*d*e^(10*I*d*x + 10*I*c) + 330*d*e
^(8*I*d*x + 8*I*c) + 165*d*e^(6*I*d*x + 6*I*c) + 55*d*e^(4*I*d*x + 4*I*c) +
 11*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^8\,dx = a^8\Bigg(\int \left(-28\tan^2(c+dx)\sec^4(c+dx)\right)\,dx + \int 70\tan^4(c+dx)\sec^4(c+dx)\,dx + \int \left(-28\tan^6(c+dx)\sec^4(c+dx)\right)\,dx + \int \tan^8(c+dx)\sec^4(c+dx)\,dx + \int 8i\tan(c+dx)\sec^4(c+dx)\,dx + \int \left(-56i\tan^3(c+dx)\sec^4(c+dx)\right)\,dx + \int 56i\tan^5(c+dx)\sec^4(c+dx)\,dx + \int \left(-8i\tan^7(c+dx)\sec^4(c+dx)\right)\,dx + \int \sec^4(c+dx)\,dx\Bigg)$$

```
[In] integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] a**8*(Integral(-28*tan(c + d*x)**2*sec(c + d*x)**4, x) + Integral(70*tan(c
+ d*x)**4*sec(c + d*x)**4, x) + Integral(-28*tan(c + d*x)**6*sec(c + d*x)**
4, x) + Integral(tan(c + d*x)**8*sec(c + d*x)**4, x) + Integral(8*I*tan(c +
 d*x)*sec(c + d*x)**4, x) + Integral(-56*I*tan(c + d*x)**3*sec(c + d*x)**4,
 x) + Integral(56*I*tan(c + d*x)**5*sec(c + d*x)**4, x) + Integral(-8*I*tan
(c + d*x)**7*sec(c + d*x)**4, x) + Integral(sec(c + d*x)**4, x))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(43) = 86.

Time = 0.35 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.44

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{5\,a^8\tan(dx+c)^{11} - 44i\,a^8\tan(dx+c)^{10} - 165\,a^8\tan(dx+c)^9 + 330i\,a^8\tan(dx+c)^8 + 330\,a^8\tan(dx+\ldots}{\ldots}$$

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/55*(5*a^8*tan(d*x + c)^11 - 44*I*a^8*tan(d*x + c)^10 - 165*a^8*tan(d*x +
c)^9 + 330*I*a^8*tan(d*x + c)^8 + 330*a^8*tan(d*x + c)^7 + 462*a^8*tan(d*x
+ c)^5 - 660*I*a^8*tan(d*x + c)^4 - 495*a^8*tan(d*x + c)^3 + 220*I*a^8*tan(
d*x + c)^2 + 55*a^8*tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(43) = 86$.

Time = 1.13 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.44

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{5\,a^8\tan\left(dx+c\right)^{11} - 44i\,a^8\tan\left(dx+c\right)^{10} - 165\,a^8\tan\left(dx+c\right)^9 + 330i\,a^8\tan\left(dx+c\right)^8 + 330\,a^8\tan\left(dx+\right.}{}$$

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```



```
[Out] 1/55*(5*a^8*tan(d*x + c)^11 - 44*I*a^8*tan(d*x + c)^10 - 165*a^8*tan(d*x +
c)^9 + 330*I*a^8*tan(d*x + c)^8 + 330*a^8*tan(d*x + c)^7 + 462*a^8*tan(d*x
+ c)^5 - 660*I*a^8*tan(d*x + c)^4 - 495*a^8*tan(d*x + c)^3 + 220*I*a^8*tan(
d*x + c)^2 + 55*a^8*tan(d*x + c))/d
```


Mupad [B] (verification not implemented)

Time = 4.73 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.95

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{a^8\left(\frac{\sin(9\,c+9\,d\,x)}{10} + \frac{\sin(11\,c+11\,d\,x)}{110} + \frac{\cos(c+d\,x)\,63\mathrm{i}}{1280} + \frac{\cos(3\,c+3\,d\,x)\,9\mathrm{i}}{256} + \frac{\cos(5\,c+5\,d\,x)\,9\mathrm{i}}{512} + \frac{\cos(7\,c+7\,d\,x)\,3\mathrm{i}}{512} - \frac{\cos(9\,c+9\,d\,x)\,253\mathrm{i}}{2560} -\right.}{d\cos\left(c+d\,x\right)^{11}}$$

```
[In] int((a + a*tan(c + d*x)*1i)^8/cos(c + d*x)^4,x)
```

```
[Out] (a^8*((cos(c + d*x)*63i)/1280 + (cos(3*c + 3*d*x)*9i)/256 + (cos(5*c + 5*d*
x)*9i)/512 + (cos(7*c + 7*d*x)*3i)/512 - (cos(9*c + 9*d*x)*253i)/2560 - (co
s(11*c + 11*d*x)*23i)/2560 + sin(9*c + 9*d*x)/10 + sin(11*c + 11*d*x)/110))
/(d*cos(c + d*x)^11)
```

3.80 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{i(a+ia\tan(c+dx))^9}{9ad}$$

```
[Out] -1/9*I*(a+I*a*tan(d*x+c))^9/a/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 32}

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{i(a+ia\tan(c+dx))^9}{9ad}$$

```
[In] Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((-1/9*I)*(a + I*a*Tan[c + d*x])^9)/(a*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{i\text{Subst}\left(\int (a+x)^8\,dx, x, ia\tan(c+dx)\right)}{ad}$$
$$= -\frac{i(a+ia\tan(c+dx))^9}{9ad}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 102 vs. 2(27) = 54.

Time = 0.67 (sec) , antiderivative size = 102, normalized size of antiderivative = 3.78

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8\tan(c+dx)\left(9+36i\tan(c+dx)-84\tan^2(c+dx)-126i\tan^3(c+dx)+126\tan^4(c+dx)+84i\tan^5(c\right.}{9d}$$

```
[In] Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*Tan[c + d*x]*(9 + (36*I)*Tan[c + d*x] - 84*Tan[c + d*x]^2 - (126*I)*Ta
n[c + d*x]^3 + 126*Tan[c + d*x]^4 + (84*I)*Tan[c + d*x]^5 - 36*Tan[c + d*x]
^6 - (9*I)*Tan[c + d*x]^7 + Tan[c + d*x]^8))/(9*d)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(23) = 46.

Time = 111.63 (sec) , antiderivative size = 113, normalized size of antiderivative = 4.19

method	result
risch	$\frac{512ia^8\left(9\,e^{16i(dx+c)}+36\,e^{14i(dx+c)}+84\,e^{12i(dx+c)}+126\,e^{10i(dx+c)}+126\,e^{8i(dx+c)}+84\,e^{6i(dx+c)}+36\,e^{4i(dx+c)}+9\,e^{2i(dx+c)}\right.}{9d\left(e^{2i(dx+c)}+1\right)^9}$
derivativedivides	$\frac{\frac{a^8\left(\sin^9(dx+c)\right)}{9\cos(dx+c)^9}+\frac{28ia^8\left(\sin^6(dx+c)\right)}{3\cos(dx+c)^6}-\frac{4a^8\left(\sin^7(dx+c)\right)}{\cos(dx+c)^7}+\frac{4ia^8}{\cos(dx+c)^2}+\frac{14a^8\left(\sin^5(dx+c)\right)}{\cos(dx+c)^5}-\frac{14ia^8\left(\sin^4(dx+c)\right)}{\cos(dx+c)^4}-\frac{28a^8\left(\sin^3(dx+c\right.}{3\cos(dx+c)^3}}{d}$
default	$\frac{\frac{a^8\left(\sin^9(dx+c)\right)}{9\cos(dx+c)^9}+\frac{28ia^8\left(\sin^6(dx+c)\right)}{3\cos(dx+c)^6}-\frac{4a^8\left(\sin^7(dx+c)\right)}{\cos(dx+c)^7}+\frac{4ia^8}{\cos(dx+c)^2}+\frac{14a^8\left(\sin^5(dx+c)\right)}{\cos(dx+c)^5}-\frac{14ia^8\left(\sin^4(dx+c)\right)}{\cos(dx+c)^4}-\frac{28a^8\left(\sin^3(dx+c\right.}{3\cos(dx+c)^3}}{d}$

```
[In] int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 512/9*I*a^8*(9*exp(16*I*(d*x+c))+36*exp(14*I*(d*x+c))+84*exp(12*I*(d*x+c))+
126*exp(10*I*(d*x+c))+126*exp(8*I*(d*x+c))+84*exp(6*I*(d*x+c))+36*exp(4*I*(
d*x+c))+9*exp(2*I*(d*x+c))+1)/d/(exp(2*I*(d*x+c))+1)^9
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(21) = 42$.

Time = 0.24 (sec) , antiderivative size = 231, normalized size of antiderivative = 8.56

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{512\left(-9i\,a^8e^{(16i\,dx+16i\,c)}-36i\,a^8e^{(14i\,dx+14i\,c)}-84i\,a^8e^{(12i\,dx+12i\,c)}-126i\,a^8e^{(10i\,dx+10i\,c)}-126i\,a^8e^{(8i\,dx+8}\right.}{9\left(de^{(18i\,dx+18i\,c)}+9\,de^{(16i\,dx+16i\,c)}+36\,de^{(14i\,dx+14i\,c)}+84\,de^{(12i\,dx+12i\,c)}+126\,de^{(10i\,dx+10i\,c)}+126\,de^{(8}\right.}$$

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -512/9*(-9*I*a^8*e^(16*I*d*x + 16*I*c) - 36*I*a^8*e^(14*I*d*x + 14*I*c) - 8
4*I*a^8*e^(12*I*d*x + 12*I*c) - 126*I*a^8*e^(10*I*d*x + 10*I*c) - 126*I*a^8
*e^(8*I*d*x + 8*I*c) - 84*I*a^8*e^(6*I*d*x + 6*I*c) - 36*I*a^8*e^(4*I*d*x +
 4*I*c) - 9*I*a^8*e^(2*I*d*x + 2*I*c) - I*a^8)/(d*e^(18*I*d*x + 18*I*c) + 9
*d*e^(16*I*d*x + 16*I*c) + 36*d*e^(14*I*d*x + 14*I*c) + 84*d*e^(12*I*d*x +
12*I*c) + 126*d*e^(10*I*d*x + 10*I*c) + 126*d*e^(8*I*d*x + 8*I*c) + 84*d*e^
(6*I*d*x + 6*I*c) + 36*d*e^(4*I*d*x + 4*I*c) + 9*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^8\,dx = a^8\left(\int\left(-28\tan^2(c+dx)\sec^2(c+dx)\right)dx + \int 70\tan^4(c+dx)\sec^2(c+dx)\,dx + \int\left(-28\tan^6(c+dx)\sec^2(c+dx)\right)dx + \int\tan^8(c+dx)\sec^2(c+dx)\,dx + \int 8i\tan(c+dx)\sec^2(c+dx)\,dx + \int\left(-56i\tan^3(c+dx)\sec^2(c+dx)\right)dx + \int 56i\tan^5(c+dx)\sec^2(c+dx)\,dx + \int\left(-8i\tan^7(c+dx)\sec^2(c+dx)\right)dx + \int\sec^2(c+dx)\,dx\right)$$

```
[In] integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] a**8*(Integral(-28*tan(c + d*x)**2*sec(c + d*x)**2, x) + Integral(70*tan(c
+ d*x)**4*sec(c + d*x)**2, x) + Integral(-28*tan(c + d*x)**6*sec(c + d*x)**
2, x) + Integral(tan(c + d*x)**8*sec(c + d*x)**2, x) + Integral(8*I*tan(c +
 d*x)*sec(c + d*x)**2, x) + Integral(-56*I*tan(c + d*x)**3*sec(c + d*x)**2,
 x) + Integral(56*I*tan(c + d*x)**5*sec(c + d*x)**2, x) + Integral(-8*I*tan
(c + d*x)**7*sec(c + d*x)**2, x) + Integral(sec(c + d*x)**2, x))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{i\,(i\,a\tan{(dx+c)}+a)^9}{9\,ad}$$

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/9*I*(I*a*tan(d*x + c) + a)^9/(a*d)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(21) = 42.

Time = 1.06 (sec) , antiderivative size = 120, normalized size of antiderivative = 4.44

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8\tan{(dx+c)}^9 - 9i\,a^8\tan{(dx+c)}^8 - 36\,a^8\tan{(dx+c)}^7 + 84i\,a^8\tan{(dx+c)}^6 + 126\,a^8\tan{(dx+c)}^5 - 1}{9\,d}$$

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/9*(a^8*tan(d*x + c)^9 - 9*I*a^8*tan(d*x + c)^8 - 36*a^8*tan(d*x + c)^7 +
84*I*a^8*tan(d*x + c)^6 + 126*a^8*tan(d*x + c)^5 - 126*I*a^8*tan(d*x + c)^4
 - 84*a^8*tan(d*x + c)^3 + 36*I*a^8*tan(d*x + c)^2 + 9*a^8*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 4.38 (sec) , antiderivative size = 83, normalized size of antiderivative = 3.07

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{a^8\left(\sin\left(9\,c+9\,d\,x\right)+\frac{\cos(c+d\,x)\,63\mathrm{i}}{128}+\frac{\cos(3\,c+3\,d\,x)\,21\mathrm{i}}{64}+\frac{\cos(5\,c+5\,d\,x)\,9\mathrm{i}}{64}+\frac{\cos(7\,c+7\,d\,x)\,9\mathrm{i}}{256}-\frac{\cos(9\,c+9\,d\,x)\,255\mathrm{i}}{256}\right)}{9\,d\cos\left(c+d\,x\right)^9}$$

```
[In] int((a + a*tan(c + d*x)*1i)^8/cos(c + d*x)^2,x)
```

```
[Out] (a^8*((cos(c + d*x)*63i)/128 + (cos(3*c + 3*d*x)*21i)/64 + (cos(5*c + 5*d*x
)*9i)/64 + (cos(7*c + 7*d*x)*9i)/256 - (cos(9*c + 9*d*x)*255i)/256 + sin(9*
c + 9*d*x)))/(9*d*cos(c + d*x)^9)
```

3.81 $\int (a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 15, antiderivative size = 200

$$\int (a+ia\tan(c+dx))^8\,dx = 128a^8x - \frac{128ia^8\log(\cos(c+dx))}{d} - \frac{64a^8\tan(c+dx)}{d} + \frac{4ia^3(a+ia\tan(c+dx))^5}{5d} + \frac{ia^2(a+ia\tan(c+dx))^6}{3d} + \frac{ia(a+ia\tan(c+dx))^7}{7d} + \frac{16ia^2(a^2+ia^2\tan(c+dx))^3}{3d} + \frac{2i(a^2+ia^2\tan(c+dx))^4}{d} + \frac{16i(a^4+ia^4\tan(c+dx))^2}{d}$$

```
[Out] 128*a^8*x-128*I*a^8*ln(cos(d*x+c))/d-64*a^8*tan(d*x+c)/d+4/5*I*a^3*(a+I*a*t
an(d*x+c))^5/d+1/3*I*a^2*(a+I*a*tan(d*x+c))^6/d+1/7*I*a*(a+I*a*tan(d*x+c))^
7/d+16/3*I*a^2*(a^2+I*a^2*tan(d*x+c))^3/d+2*I*(a^2+I*a^2*tan(d*x+c))^4/d+16
*I*(a^4+I*a^4*tan(d*x+c))^2/d
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3559, 3558, 3556}

$$\int (a+ia\tan(c+dx))^8\,dx = -\frac{64a^8\tan(c+dx)}{d} - \frac{128ia^8\log(\cos(c+dx))}{d} + 128a^8x + \frac{16i(a^4+ia^4\tan(c+dx))^2}{d} + \frac{4ia^3(a+ia\tan(c+dx))^5}{5d} + \frac{ia^2(a+ia\tan(c+dx))^6}{3d} + \frac{16ia^2(a^2+ia^2\tan(c+dx))^3}{3d} + \frac{2i(a^2+ia^2\tan(c+dx))^4}{d} + \frac{ia(a+ia\tan(c+dx))^7}{7d}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] 128*a^8*x - ((128*I)*a^8*Log[Cos[c + d*x]])/d - (64*a^8*Tan[c + d*x])/d + (
((4*I)/5)*a^3*(a + I*a*Tan[c + d*x])^5)/d + ((I/3)*a^2*(a + I*a*Tan[c + d*x
])^6)/d + ((I/7)*a*(a + I*a*Tan[c + d*x])^7)/d + (((16*I)/3)*a^2*(a^2 + I*a
^2*Tan[c + d*x])^3)/d + ((2*I)*(a^2 + I*a^2*Tan[c + d*x])^4)/d + ((16*I)*(a
^4 + I*a^4*Tan[c + d*x])^2)/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3558

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[b^2*(Tan[c + d*x]/d),
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{ia(a+ia\tan(c+dx))^7}{7d} + (2a)\int (a+ia\tan(c+dx))^7\,dx \\
&= \frac{ia^2(a+ia\tan(c+dx))^6}{3d} + \frac{ia(a+ia\tan(c+dx))^7}{7d} + \left(4a^2\right)\int (a+ia\tan(c+dx))^6\,dx \\
&= \frac{4ia^3(a+ia\tan(c+dx))^5}{5d} + \frac{ia^2(a+ia\tan(c+dx))^6}{3d} \\
&\quad + \frac{ia(a+ia\tan(c+dx))^7}{7d} + \left(8a^3\right)\int (a+ia\tan(c+dx))^5\,dx \\
&= \frac{4ia^3(a+ia\tan(c+dx))^5}{5d} + \frac{ia^2(a+ia\tan(c+dx))^6}{3d} + \frac{ia(a+ia\tan(c+dx))^7}{7d} \\
&\quad + \frac{2i\left(a^2+ia^2\tan(c+dx)\right)^4}{d} + \left(16a^4\right)\int (a+ia\tan(c+dx))^4\,dx \\
&= \frac{16ia^5(a+ia\tan(c+dx))^3}{3d} + \frac{4ia^3(a+ia\tan(c+dx))^5}{5d} + \frac{ia^2(a+ia\tan(c+dx))^6}{3d} \\
&\quad + \frac{ia(a+ia\tan(c+dx))^7}{7d} + \frac{2i\left(a^2+ia^2\tan(c+dx)\right)^4}{d} + \left(32a^5\right)\int (a+ia\tan(c+dx))^3\,dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{16ia^5(a+ia\tan(c+dx))^3}{3d} + \frac{4ia^3(a+ia\tan(c+dx))^5}{5d} \\
&\quad + \frac{ia^2(a+ia\tan(c+dx))^6}{3d} + \frac{ia(a+ia\tan(c+dx))^7}{7d} + \frac{2i(a^2+ia^2\tan(c+dx))^4}{d} \\
&\quad + \frac{16i(a^4+ia^4\tan(c+dx))^2}{d} + (64a^6)\int (a+ia\tan(c+dx))^2\,dx \\
&= 128a^8x - \frac{64a^8\tan(c+dx)}{d} + \frac{16ia^5(a+ia\tan(c+dx))^3}{3d} + \frac{4ia^3(a+ia\tan(c+dx))^5}{5d} \\
&\quad + \frac{ia^2(a+ia\tan(c+dx))^6}{3d} + \frac{ia(a+ia\tan(c+dx))^7}{7d} + \frac{2i(a^2+ia^2\tan(c+dx))^4}{d} \\
&\quad + \frac{16i(a^4+ia^4\tan(c+dx))^2}{d} + (128ia^8)\int \tan(c+dx)\,dx \\
&= 128a^8x - \frac{128ia^8\log(\cos(c+dx))}{d} - \frac{64a^8\tan(c+dx)}{d} + \frac{16ia^5(a+ia\tan(c+dx))^3}{3d} \\
&\quad + \frac{4ia^3(a+ia\tan(c+dx))^5}{5d} + \frac{ia^2(a+ia\tan(c+dx))^6}{3d} + \frac{ia(a+ia\tan(c+dx))^7}{7d} \\
&\quad + \frac{2i(a^2+ia^2\tan(c+dx))^4}{d} + \frac{16i(a^4+ia^4\tan(c+dx))^2}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.50

$$
\int (a+ia\tan(c+dx))^8\,dx
$$
$$
= \frac{a^8(13440i\log(i+\tan(c+dx)) - 13335\tan(c+dx) - 6300i\tan^2(c+dx) + 3465\tan^3(c+dx) + 1680i\tan}{105d}
$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*((13440*I)*Log[I + Tan[c + d*x]] - 13335*Tan[c + d*x] - (6300*I)*Tan[c
 + d*x]^2 + 3465*Tan[c + d*x]^3 + (1680*I)*Tan[c + d*x]^4 - 609*Tan[c + d*x
]^5 - (140*I)*Tan[c + d*x]^6 + 15*Tan[c + d*x]^7))/(105*d)
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.52

method	result
derivativedivides	$\frac{a^8\left(-127\tan(dx+c)+\frac{\left(\tan^7(dx+c)\right)}{7}-\frac{4i\left(\tan^6(dx+c)\right)}{3}-\frac{29\left(\tan^5(dx+c)\right)}{5}+16i\left(\tan^4(dx+c)\right)+33\left(\tan^3(dx+c)\right)-60i\left(\tan^2(\right.\right.}{d}$
default	$\frac{a^8\left(-127\tan(dx+c)+\frac{\left(\tan^7(dx+c)\right)}{7}-\frac{4i\left(\tan^6(dx+c)\right)}{3}-\frac{29\left(\tan^5(dx+c)\right)}{5}+16i\left(\tan^4(dx+c)\right)+33\left(\tan^3(dx+c)\right)-60i\left(\tan^2(\right.\right.}{d}$
risch	$-\frac{256a^8c}{d}-\frac{32ia^8\left(2940\,e^{12i(dx+c)}+13230\,e^{10i(dx+c)}+26950\,e^{8i(dx+c)}+30625\,e^{6i(dx+c)}+20139\,e^{4i(dx+c)}+7203\,e^{2i(dx\right.}{105d\left(e^{2i(dx+c)}+1\right)^7}$
parallelrisch	$\frac{-140ia^8\left(\tan^6(dx+c)\right)+15\left(\tan^7(dx+c)\right)a^8+1680ia^8\left(\tan^4(dx+c)\right)-609\left(\tan^5(dx+c)\right)a^8-6300ia^8\left(\tan^2(dx+c)\right)+3465}{105d}$
norman	$128a^8x-\frac{127a^8\tan(dx+c)}{d}+\frac{33a^8\left(\tan^3(dx+c)\right)}{d}-\frac{29a^8\left(\tan^5(dx+c)\right)}{5d}+\frac{a^8\left(\tan^7(dx+c)\right)}{7d}-\frac{60ia^8\left(\tan^2(dx+\right.}{d}$
parts	$a^8x+\frac{a^8\left(\frac{\left(\tan^7(dx+c)\right)}{7}-\frac{\left(\tan^5(dx+c)\right)}{5}+\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+\arctan(\tan(dx+c))\right)}{d}-\frac{56ia^8\left(\frac{\left(\tan^2(dx+c)\right)}{2}-\right.}{d}$

[In] int((a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)

[Out] 1/d*a^8*(-127*tan(d*x+c)+1/7*tan(d*x+c)^7-4/3*I*tan(d*x+c)^6-29/5*tan(d*x+c)^5+16*I*tan(d*x+c)^4+33*tan(d*x+c)^3-60*I*tan(d*x+c)^2+64*I*ln(1+tan(d*x+c)^2)+128*arctan(tan(d*x+c)))

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.48

$$\int(a+ia\tan(c+dx))^8\,dx = -\frac{32\left(2940i\,a^8e^{(12i\,dx+12i\,c)}+13230i\,a^8e^{(10i\,dx+10i\,c)}+26950i\,a^8e^{(8i\,dx+8i\,c)}+30625i\,a^8e^{(6i\,dx+6i\,c)}+20139i\,a\right.}{105\left(de^{(14i\right.}}$$

[In] integrate((a+I*a*tan(d*x+c))^8,x, algorithm="fricas")

[Out] -32/105*(2940*I*a^8*e^(12*I*d*x + 12*I*c) + 13230*I*a^8*e^(10*I*d*x + 10*I*c) + 26950*I*a^8*e^(8*I*d*x + 8*I*c) + 30625*I*a^8*e^(6*I*d*x + 6*I*c) + 20139*I*a^8*e^(4*I*d*x + 4*I*c) + 7203*I*a^8*e^(2*I*d*x + 2*I*c) + 1089*I*a^8 + 420*(I*a^8*e^(14*I*d*x + 14*I*c) + 7*I*a^8*e^(12*I*d*x + 12*I*c) + 21*I*a^8*e^(10*I*d*x + 10*I*c) + 35*I*a^8*e^(8*I*d*x + 8*I*c) + 35*I*a^8*e^(6*I*d*x + 6*I*c) + 21*I*a^8*e^(4*I*d*x + 4*I*c) + 7*I*a^8*e^(2*I*d*x + 2*I*c) + I*a^8)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(14*I*d*x + 14*I*c) + 7*d*e^(12*I*d*x + 12*I*c) + 21*d*e^(10*I*d*x + 10*I*c) + 35*d*e^(8*I*d*x + 8*I*c) + 35*d*e^(6*I*d*x + 6*I*c) + 21*d*e^(4*I*d*x + 4*I*c) + 7*d*e^(2*I*d*x + 2*I*c) + d)

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.50

$$\int (a+ia\tan(c+dx))^8\,dx = -\frac{128ia^8\log\left(e^{2idx}+e^{-2ic}\right)}{d} + \frac{-94080ia^8e^{12ic}e^{12idx}-423360ia^8e^{10ic}e^{10idx}-862400ia^8e^{8ic}e^{8idx}-980000ia^8e^{6ic}e^{6idx}-644448ia^8e^{4ic}e^{4ic}}{105de^{14ic}e^{14idx}+735de^{12ic}e^{12idx}+2205de^{10ic}e^{10idx}+3675de^{8ic}e^{8idx}+3675de^{6ic}e^{6idx}+2205de^{4ic}e}$$

```
[In] integrate((a+I*a*tan(d*x+c))**8,x)
```

```
[Out] -128*I*a**8*log(exp(2*I*d*x) + exp(-2*I*c))/d + (-94080*I*a**8*exp(12*I*c)*
exp(12*I*d*x) - 423360*I*a**8*exp(10*I*c)*exp(10*I*d*x) - 862400*I*a**8*exp
(8*I*c)*exp(8*I*d*x) - 980000*I*a**8*exp(6*I*c)*exp(6*I*d*x) - 644448*I*a**
8*exp(4*I*c)*exp(4*I*d*x) - 230496*I*a**8*exp(2*I*c)*exp(2*I*d*x) - 34848*I
*a**8)/(105*d*exp(14*I*c)*exp(14*I*d*x) + 735*d*exp(12*I*c)*exp(12*I*d*x) +
 2205*d*exp(10*I*c)*exp(10*I*d*x) + 3675*d*exp(8*I*c)*exp(8*I*d*x) + 3675*d
*exp(6*I*c)*exp(6*I*d*x) + 2205*d*exp(4*I*c)*exp(4*I*d*x) + 735*d*exp(2*I*c
)*exp(2*I*d*x) + 105*d)
```

Maxima [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.60

$$\int (a+ia\tan(c+dx))^8\,dx = \frac{15\,a^8\tan\left(dx+c\right)^7-140i\,a^8\tan\left(dx+c\right)^6-609\,a^8\tan\left(dx+c\right)^5+1680i\,a^8\tan\left(dx+c\right)^4+3465\,a^8\tan\left(dx\right.}{1}$$

```
[In] integrate((a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/105*(15*a^8*tan(d*x + c)^7 - 140*I*a^8*tan(d*x + c)^6 - 609*a^8*tan(d*x +
 c)^5 + 1680*I*a^8*tan(d*x + c)^4 + 3465*a^8*tan(d*x + c)^3 - 6300*I*a^8*ta
n(d*x + c)^2 + 13440*(d*x + c)*a^8 + 6720*I*a^8*log(tan(d*x + c)^2 + 1) - 1
3335*a^8*tan(d*x + c))/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 378 vs. 2(166) = 332.

Time = 0.54 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.89

$$\int (a+ia\tan(c+dx))^8\,dx = -\frac{32\left(420i\,a^8e^{(14i\,dx+14i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+2940i\,a^8e^{(12i\,dx+12i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)+8820i\,a^8e^{(10i\,dx}\right.}{}$$

```
[In] integrate((a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -32/105*(420*I*a^8*e^(14*I*d*x + 14*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 294
0*I*a^8*e^(12*I*d*x + 12*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 8820*I*a^8*e^(
10*I*d*x + 10*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 14700*I*a^8*e^(8*I*d*x +
8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 14700*I*a^8*e^(6*I*d*x + 6*I*c)*log(e
^(2*I*d*x + 2*I*c) + 1) + 8820*I*a^8*e^(4*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2
*I*c) + 1) + 2940*I*a^8*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) +
2940*I*a^8*e^(12*I*d*x + 12*I*c) + 13230*I*a^8*e^(10*I*d*x + 10*I*c) + 2695
0*I*a^8*e^(8*I*d*x + 8*I*c) + 30625*I*a^8*e^(6*I*d*x + 6*I*c) + 20139*I*a^8
*e^(4*I*d*x + 4*I*c) + 7203*I*a^8*e^(2*I*d*x + 2*I*c) + 420*I*a^8*log(e^(2*
I*d*x + 2*I*c) + 1) + 1089*I*a^8)/(d*e^(14*I*d*x + 14*I*c) + 7*d*e^(12*I*d*
x + 12*I*c) + 21*d*e^(10*I*d*x + 10*I*c) + 35*d*e^(8*I*d*x + 8*I*c) + 35*d*
e^(6*I*d*x + 6*I*c) + 21*d*e^(4*I*d*x + 4*I*c) + 7*d*e^(2*I*d*x + 2*I*c) +
d)
```

Mupad [B] (verification not implemented)

Time = 4.29 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.56

$$\int (a+ia\tan(c+dx))^8\,dx = \frac{33\,a^8\tan(c+d\,x)^3-127\,a^8\tan(c+d\,x)-\frac{29\,a^8\tan(c+d\,x)^5}{5}+\frac{a^8\tan(c+d\,x)^7}{7}+a^8\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,128\mathrm{i}-}{d}$$

```
[In] int((a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] (a^8*log(tan(c + d*x) + 1i)*128i - 127*a^8*tan(c + d*x) - a^8*tan(c + d*x)^
2*60i + 33*a^8*tan(c + d*x)^3 + a^8*tan(c + d*x)^4*16i - (29*a^8*tan(c + d*
x)^5)/5 - (a^8*tan(c + d*x)^6*4i)/3 + (a^8*tan(c + d*x)^7)/7)/d
```

3.82 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 24, antiderivative size = 133

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^8\,dx = -192a^8x+\frac{192ia^8\log(\cos(c+dx))}{d}+\frac{129a^8\tan(c+dx)}{d}+\frac{36ia^8\tan^2(c+dx)}{d}-\frac{10a^8\tan^3(c+dx)}{d}-\frac{2ia^8\tan^4(c+dx)}{d}+\frac{a^8\tan^5(c+dx)}{5d}-\frac{64ia^9}{d(a-ia\tan(c+dx))}$$

```
[Out] -192*a^8*x+192*I*a^8*ln(cos(d*x+c))/d+129*a^8*tan(d*x+c)/d+36*I*a^8*tan(d*x
+c)^2/d-10*a^8*tan(d*x+c)^3/d-2*I*a^8*tan(d*x+c)^4/d+1/5*a^8*tan(d*x+c)^5/d
-64*I*a^9/d/(a-I*a*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{64ia^9}{d(a-ia\tan(c+dx))}+\frac{a^8\tan^5(c+dx)}{5d}-\frac{2ia^8\tan^4(c+dx)}{d}-\frac{10a^8\tan^3(c+dx)}{d}+\frac{36ia^8\tan^2(c+dx)}{d}+\frac{129a^8\tan(c+dx)}{d}+\frac{192ia^8\log(\cos(c+dx))}{d}-192a^8x$$

```
[In] Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] -192*a^8*x + ((192*I)*a^8*Log[Cos[c + d*x]])/d + (129*a^8*Tan[c + d*x])/d +
 ((36*I)*a^8*Tan[c + d*x]^2)/d - (10*a^8*Tan[c + d*x]^3)/d - ((2*I)*a^8*Tan
[c + d*x]^4)/d + (a^8*Tan[c + d*x]^5)/(5*d) - ((64*I)*a^9)/(d*(a - I*a*Tan[
c + d*x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(ia^3)\,\text{Subst}\left(\int \frac{(a+x)^6}{(a-x)^2}\,dx, x, ia\tan(c+dx)\right)}{d}\\ &= -\frac{(ia^3)\,\text{Subst}\left(\int \left(129a^4+\frac{64a^6}{(a-x)^2}-\frac{192a^5}{a-x}+72a^3x+30a^2x^2+8ax^3+x^4\right)\,dx, x, ia\tan(c+dx)\right)}{d}\\ &= -192a^8x+\frac{192ia^8\log(\cos(c+dx))}{d}+\frac{129a^8\tan(c+dx)}{d}+\frac{36ia^8\tan^2(c+dx)}{d}\\ &\quad-\frac{10a^8\tan^3(c+dx)}{d}-\frac{2ia^8\tan^4(c+dx)}{d}+\frac{a^8\tan^5(c+dx)}{5d}-\frac{64ia^9}{d(a-ia\tan(c+dx))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.72

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{ia^8\left(960\log(i+\tan(c+dx))+645i\tan(c+dx)-180\tan^2(c+dx)-50i\tan^3(c+dx)+10\tan^4(c+d\right.}{5d}$$

```
[In] Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((-1/5*I)*a^8*(960*Log[I + Tan[c + d*x]] + (645*I)*Tan[c + d*x] - 180*Tan[c
 + d*x]^2 - (50*I)*Tan[c + d*x]^3 + 10*Tan[c + d*x]^4 + I*Tan[c + d*x]^5 +
(320*I)/(I + Tan[c + d*x])))/d
```

Maple [A] (verified)

Time = 84.15 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.89

method	result
risch	$-\frac{32 i a^8 e^{2 i (dx+c)}}{d}+\frac{384 a^8 c}{d}+\frac{16 i a^8 \left(150 e^{8 i (dx+c)}+500 e^{6 i (dx+c)}+650 e^{4 i (dx+c)}+385 e^{2 i (dx+c)}+87\right)}{5 d \left(e^{2 i (dx+c)}+1\right)^5}+\frac{192 i a^8 \ln(e^{2}}{d}$
derivativedivides	$\frac{a^8 \left(\frac{\sin^9(dx+c)}{5 \cos(dx+c)^5}-\frac{4 \left(\sin^9(dx+c)\right)}{15 \cos(dx+c)^3}+\frac{8 \left(\sin^9(dx+c)\right)}{5 \cos(dx+c)}+\frac{8 \left(\sin^7(dx+c)+\frac{7 \left(\sin^5(dx+c)\right)}{6}+\frac{35 \left(\sin^3(dx+c)\right)}{24}+\frac{35 \sin(dx+c)}{16}\right) \cos(dx+c)}{5}-\right.}{}$
default	$\frac{a^8 \left(\frac{\sin^9(dx+c)}{5 \cos(dx+c)^5}-\frac{4 \left(\sin^9(dx+c)\right)}{15 \cos(dx+c)^3}+\frac{8 \left(\sin^9(dx+c)\right)}{5 \cos(dx+c)}+\frac{8 \left(\sin^7(dx+c)+\frac{7 \left(\sin^5(dx+c)\right)}{6}+\frac{35 \left(\sin^3(dx+c)\right)}{24}+\frac{35 \sin(dx+c)}{16}\right) \cos(dx+c)}{5}-\right.}{}$

```
[In] int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] -32*I/d*a^8*exp(2*I*(d*x+c))+384/d*a^8*c+16/5*I*a^8*(150*exp(8*I*(d*x+c))+5
00*exp(6*I*(d*x+c))+650*exp(4*I*(d*x+c))+385*exp(2*I*(d*x+c))+87)/d/(exp(2*
I*(d*x+c))+1)^5+192*I/d*a^8*ln(exp(2*I*(d*x+c))+1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(121) = 242.

Time = 0.26 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.84

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{16 \left(10 i a^8 e^{(12 i dx+12 i c)}+50 i a^8 e^{(10 i dx+10 i c)}-50 i a^8 e^{(8 i dx+8 i c)}-400 i a^8 e^{(6 i dx+6 i c)}-600 i a^8 e^{(4 i dx+4 i c)}-3\right.}{5 \left(d e^{(10 i dx+10 i c)}+5 d\right.}$$

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -16/5*(10*I*a^8*e^(12*I*d*x + 12*I*c) + 50*I*a^8*e^(10*I*d*x + 10*I*c) - 50
*I*a^8*e^(8*I*d*x + 8*I*c) - 400*I*a^8*e^(6*I*d*x + 6*I*c) - 600*I*a^8*e^(4
*I*d*x + 4*I*c) - 375*I*a^8*e^(2*I*d*x + 2*I*c) - 87*I*a^8 + 60*(-I*a^8*e^(
10*I*d*x + 10*I*c) - 5*I*a^8*e^(8*I*d*x + 8*I*c) - 10*I*a^8*e^(6*I*d*x + 6*
I*c) - 10*I*a^8*e^(4*I*d*x + 4*I*c) - 5*I*a^8*e^(2*I*d*x + 2*I*c) - I*a^8)*
log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*I*d*x + 8
*I*c) + 10*d*e^(6*I*d*x + 6*I*c) + 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*d*
x + 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.93

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{192ia^8\log\left(e^{2idx}+e^{-2ic}\right)}{d} + \frac{2400ia^8e^{8ic}e^{8idx}+8000ia^8e^{6ic}e^{6idx}+10400ia^8e^{4ic}e^{4idx}+6160ia^8e^{2ic}e^{2idx}+1392ia^8}{5de^{10ic}e^{10idx}+25de^{8ic}e^{8idx}+50de^{6ic}e^{6idx}+50de^{4ic}e^{4idx}+25de^{2ic}e^{2idx}+5d} + \begin{cases} -\frac{32ia^8e^{2ic}e^{2idx}}{d} & \text{for } d\neq 0 \\ 64a^8xe^{2ic} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] 192*I*a**8*log(exp(2*I*d*x) + exp(-2*I*c))/d + (2400*I*a**8*exp(8*I*c)*exp(
8*I*d*x) + 8000*I*a**8*exp(6*I*c)*exp(6*I*d*x) + 10400*I*a**8*exp(4*I*c)*ex
p(4*I*d*x) + 6160*I*a**8*exp(2*I*c)*exp(2*I*d*x) + 1392*I*a**8)/(5*d*exp(10
*I*c)*exp(10*I*d*x) + 25*d*exp(8*I*c)*exp(8*I*d*x) + 50*d*exp(6*I*c)*exp(6*
I*d*x) + 50*d*exp(4*I*c)*exp(4*I*d*x) + 25*d*exp(2*I*c)*exp(2*I*d*x) + 5*d)
 + Piecewise((-32*I*a**8*exp(2*I*c)*exp(2*I*d*x)/d, Ne(d, 0)), (64*a**8*x*e
xp(2*I*c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.93

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8\tan(dx+c)^5 - 10i\,a^8\tan(dx+c)^4 - 50\,a^8\tan(dx+c)^3 + 180i\,a^8\tan(dx+c)^2 - 960\,(dx+c)a^8 - 48}{5\,d}$$

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/5*(a^8*tan(d*x + c)^5 - 10*I*a^8*tan(d*x + c)^4 - 50*a^8*tan(d*x + c)^3 +
 180*I*a^8*tan(d*x + c)^2 - 960*(d*x + c)*a^8 - 480*I*a^8*log(tan(d*x + c)^
2 + 1) + 645*a^8*tan(d*x + c) + 320*(a^8*tan(d*x + c) - I*a^8)/(tan(d*x + c
)^2 + 1))/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(121) = 242$.

Time = 0.91 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.27

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^8\,dx =$$

$$-\frac{16\left(-60i\,a^8e^{(10i\,dx+10i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-300i\,a^8e^{(8i\,dx+8i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-600i\,a^8e^{(6i\,dx+6i\,c)}\,\mathrm{l}\ldots}{\ldots}$$

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -16/5*(-60*I*a^8*e^(10*I*d*x + 10*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 300*I
*a^8*e^(8*I*d*x + 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 600*I*a^8*e^(6*I*d*
x + 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 600*I*a^8*e^(4*I*d*x + 4*I*c)*log
(e^(2*I*d*x + 2*I*c) + 1) - 300*I*a^8*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x +
2*I*c) + 1) + 10*I*a^8*e^(12*I*d*x + 12*I*c) + 50*I*a^8*e^(10*I*d*x + 10*I*
c) - 50*I*a^8*e^(8*I*d*x + 8*I*c) - 400*I*a^8*e^(6*I*d*x + 6*I*c) - 600*I*a
^8*e^(4*I*d*x + 4*I*c) - 375*I*a^8*e^(2*I*d*x + 2*I*c) - 60*I*a^8*log(e^(2*
I*d*x + 2*I*c) + 1) - 87*I*a^8)/(d*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*I*d*x +
 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) + 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*
d*x + 2*I*c) + d)
```

Mupad [B] (verification not implemented)

Time = 3.80 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.77

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$=\frac{\frac{64\,a^8}{\tan(c+d\,x)+1\mathrm{i}}+129\,a^8\tan(c+d\,x)-10\,a^8\tan(c+d\,x)^3+\frac{a^8\tan(c+d\,x)^5}{5}-a^8\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,192\mathrm{i}+a^8\,\mathrm{t}\ldots}{d}$$

```
[In] int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] ((64*a^8)/(tan(c + d*x) + 1i) - a^8*log(tan(c + d*x) + 1i)*192i + 129*a^8*t
an(c + d*x) + a^8*tan(c + d*x)^2*36i - 10*a^8*tan(c + d*x)^3 - a^8*tan(c +
d*x)^4*2i + (a^8*tan(c + d*x)^5)/5)/d
```

3.83 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 24, antiderivative size = 124

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^8\,dx = 80a^8x-\frac{80ia^8\log(\cos(c+dx))}{d}-\frac{31a^8\tan(c+dx)}{d}-\frac{4ia^8\tan^2(c+dx)}{d}+\frac{a^8\tan^3(c+dx)}{3d}-\frac{16ia^{10}}{d(a-ia\tan(c+dx))^2}+\frac{80ia^9}{d(a-ia\tan(c+dx))}$$

```
[Out] 80*a^8*x-80*I*a^8*ln(cos(d*x+c))/d-31*a^8*tan(d*x+c)/d-4*I*a^8*tan(d*x+c)^2
/d+1/3*a^8*tan(d*x+c)^3/d-16*I*a^10/d/(a-I*a*tan(d*x+c))^2+80*I*a^9/d/(a-I*
a*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{16ia^{10}}{d(a-ia\tan(c+dx))^2}+\frac{80ia^9}{d(a-ia\tan(c+dx))}+\frac{a^8\tan^3(c+dx)}{3d}-\frac{4ia^8\tan^2(c+dx)}{d}-\frac{31a^8\tan(c+dx)}{d}-\frac{80ia^8\log(\cos(c+dx))}{d}+80a^8x$$

```
[In] Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] 80*a^8*x - ((80*I)*a^8*Log[Cos[c + d*x]])/d - (31*a^8*Tan[c + d*x])/d - ((4
*I)*a^8*Tan[c + d*x]^2)/d + (a^8*Tan[c + d*x]^3)/(3*d) - ((16*I)*a^10)/(d*(
a - I*a*Tan[c + d*x])^2) + ((80*I)*a^9)/(d*(a - I*a*Tan[c + d*x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(ia^5)\,\text{Subst}\left(\int \frac{(a+x)^5}{(a-x)^3}\,dx, x, ia\tan(c+dx)\right)}{d}\\
&= -\frac{(ia^5)\,\text{Subst}\left(\int \left(-31a^2+\frac{32a^5}{(a-x)^3}-\frac{80a^4}{(a-x)^2}+\frac{80a^3}{a-x}-8ax-x^2\right)dx, x, ia\tan(c+dx)\right)}{d}\\
&= 80a^8x-\frac{80ia^8\log(\cos(c+dx))}{d}-\frac{31a^8\tan(c+dx)}{d}-\frac{4ia^8\tan^2(c+dx)}{d}\\
&\quad+\frac{a^8\tan^3(c+dx)}{3d}-\frac{16ia^{10}}{d(a-ia\tan(c+dx))^2}+\frac{80ia^9}{d(a-ia\tan(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.69

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{ia^8\left(-93i\tan(c+dx)+12\tan^2(c+dx)+i\tan^3(c+dx)+48\left(-5\log(i+\tan(c+dx))+\frac{4-5i\tan(c+dx)}{(i+\tan(c+dx))^2}\right)\right)}{3d}$$

```
[In] Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((-1/3*I)*a^8*((-93*I)*Tan[c + d*x] + 12*Tan[c + d*x]^2 + I*Tan[c + d*x]^3
+ 48*(-5*Log[I + Tan[c + d*x]] + (4 - (5*I)*Tan[c + d*x])/(I + Tan[c + d*x]
)^2)))/d
```

Maple [A] (verified)

Time = 219.14 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.92

method	result
risch	$-\frac{4 i a^{8} {\mathrm e}^{4 i \left(d x +c \right)}}{d}+\frac{32 i a^{8} {\mathrm e}^{2 i \left(d x +c \right)}}{d}-\frac{160 a^{8} c}{d}-\frac{4 i a^{8} \left(60 \,{\mathrm e}^{4 i \left(d x +c \right)}+105 \,{\mathrm e}^{2 i \left(d x +c \right)}+47\right)}{3 d \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)^{3}}-\frac{80 i a^{8} \ln \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)}{d}$
derivativedivides	$a^{8} \left(\frac{\sin^{9}\left(d x +c \right)}{3 \cos \left(d x +c \right)^{3}}-\frac{2 \left(\sin^{9}\left(d x +c \right)\right)}{\cos \left(d x +c \right)}-2 \left(\sin^{7}\left(d x +c \right)+\frac{7 \left(\sin^{5}\left(d x +c \right)\right)}{6}+\frac{35 \left(\sin^{3}\left(d x +c \right)\right)}{24}+\frac{35 \sin \left(d x +c \right)}{16}\right) \cos \left(d x +c \right)+\frac{35 d x}{8}+\frac{35c}{8}\right.$
default	$a^{8} \left(\frac{\sin^{9}\left(d x +c \right)}{3 \cos \left(d x +c \right)^{3}}-\frac{2 \left(\sin^{9}\left(d x +c \right)\right)}{\cos \left(d x +c \right)}-2 \left(\sin^{7}\left(d x +c \right)+\frac{7 \left(\sin^{5}\left(d x +c \right)\right)}{6}+\frac{35 \left(\sin^{3}\left(d x +c \right)\right)}{24}+\frac{35 \sin \left(d x +c \right)}{16}\right) \cos \left(d x +c \right)+\frac{35 d x}{8}+\frac{35c}{8}\right.$

```
[In] int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] -4*I/d*a^8*exp(4*I*(d*x+c))+32*I/d*a^8*exp(2*I*(d*x+c))-160/d*a^8*c-4/3*I*a
^8*(60*exp(4*I*(d*x+c))+105*exp(2*I*(d*x+c))+47)/d/(exp(2*I*(d*x+c))+1)^3-8
0*I/d*a^8*ln(exp(2*I*(d*x+c))+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.44

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{4\left(3i\,a^8e^{(10i\,dx+10i\,c)}-15i\,a^8e^{(8i\,dx+8i\,c)}-63i\,a^8e^{(6i\,dx+6i\,c)}-9i\,a^8e^{(4i\,dx+4i\,c)}+81i\,a^8e^{(2i\,dx+2i\,c)}+47i\,a^8+\right.}{3\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de\right.}$$

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -4/3*(3*I*a^8*e^(10*I*d*x + 10*I*c) - 15*I*a^8*e^(8*I*d*x + 8*I*c) - 63*I*a
^8*e^(6*I*d*x + 6*I*c) - 9*I*a^8*e^(4*I*d*x + 4*I*c) + 81*I*a^8*e^(2*I*d*x
+ 2*I*c) + 47*I*a^8 + 60*(I*a^8*e^(6*I*d*x + 6*I*c) + 3*I*a^8*e^(4*I*d*x +
4*I*c) + 3*I*a^8*e^(2*I*d*x + 2*I*c) + I*a^8)*log(e^(2*I*d*x + 2*I*c) + 1))
/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c)
 + d)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.74

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{80ia^8\log\left(e^{2idx}+e^{-2ic}\right)}{d} + \frac{-240ia^8e^{4ic}e^{4idx}-420ia^8e^{2ic}e^{2idx}-188ia^8}{3de^{6ic}e^{6idx}+9de^{4ic}e^{4idx}+9de^{2ic}e^{2idx}+3d} + \begin{cases} \frac{-4ia^8de^{4ic}e^{4idx}+32ia^8de^{2ic}e^{2idx}}{d^2} & \text{for } d^2\neq 0 \\ x(16a^8e^{4ic}-64a^8e^{2ic}) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] -80*I*a**8*log(exp(2*I*d*x) + exp(-2*I*c))/d + (-240*I*a**8*exp(4*I*c)*exp(
4*I*d*x) - 420*I*a**8*exp(2*I*c)*exp(2*I*d*x) - 188*I*a**8)/(3*d*exp(6*I*c)
*exp(6*I*d*x) + 9*d*exp(4*I*c)*exp(4*I*d*x) + 9*d*exp(2*I*c)*exp(2*I*d*x) +
 3*d) + Piecewise(((-4*I*a**8*d*exp(4*I*c)*exp(4*I*d*x) + 32*I*a**8*d*exp(2
*I*c)*exp(2*I*d*x))/d**2, Ne(d**2, 0)), (x*(16*a**8*exp(4*I*c) - 64*a**8*ex
p(2*I*c)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.09

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$= \frac{a^8\tan(dx+c)^3 - 12i\,a^8\tan(dx+c)^2 + 240\,(dx+c)a^8 + 120i\,a^8\log\left(\tan(dx+c)^2+1\right) - 93\,a^8\tan(dx+}{3\,d}$$

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/3*(a^8*tan(d*x + c)^3 - 12*I*a^8*tan(d*x + c)^2 + 240*(d*x + c)*a^8 + 120
*I*a^8*log(tan(d*x + c)^2 + 1) - 93*a^8*tan(d*x + c) - 48*(5*a^8*tan(d*x +
c)^3 - 6*I*a^8*tan(d*x + c)^2 + 3*a^8*tan(d*x + c) - 4*I*a^8)/(tan(d*x + c)
^4 + 2*tan(d*x + c)^2 + 1))/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 785 vs. $2(110) = 220$.

Time = 1.04 (sec) , antiderivative size = 785, normalized size of antiderivative = 6.33

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^8\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -4/3*(60*I*a^8*e^(28*I*d*x + 14*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 840*I*a
^8*e^(26*I*d*x + 12*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 5460*I*a^8*e^(24*I*
d*x + 10*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 21840*I*a^8*e^(22*I*d*x + 8*I*
c)*log(e^(2*I*d*x + 2*I*c) + 1) + 60060*I*a^8*e^(20*I*d*x + 6*I*c)*log(e^(2
*I*d*x + 2*I*c) + 1) + 120120*I*a^8*e^(18*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2
*I*c) + 1) + 180180*I*a^8*e^(16*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1)
 + 180180*I*a^8*e^(12*I*d*x - 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 120120*
I*a^8*e^(10*I*d*x - 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 60060*I*a^8*e^(8*
I*d*x - 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 21840*I*a^8*e^(6*I*d*x - 8*I*
c)*log(e^(2*I*d*x + 2*I*c) + 1) + 5460*I*a^8*e^(4*I*d*x - 10*I*c)*log(e^(2*
I*d*x + 2*I*c) + 1) + 840*I*a^8*e^(2*I*d*x - 12*I*c)*log(e^(2*I*d*x + 2*I*c
) + 1) + 205920*I*a^8*e^(14*I*d*x)*log(e^(2*I*d*x + 2*I*c) + 1) + 60*I*a^8*
e^(-14*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 3*I*a^8*e^(32*I*d*x + 18*I*c) +
18*I*a^8*e^(30*I*d*x + 16*I*c) - 63*I*a^8*e^(28*I*d*x + 14*I*c) - 1032*I*a^
8*e^(26*I*d*x + 12*I*c) - 4968*I*a^8*e^(24*I*d*x + 10*I*c) - 13516*I*a^8*e^
(22*I*d*x + 8*I*c) - 22847*I*a^8*e^(20*I*d*x + 6*I*c) - 22066*I*a^8*e^(18*I
*d*x + 4*I*c) - 3234*I*a^8*e^(16*I*d*x + 2*I*c) + 44979*I*a^8*e^(12*I*d*x -
 2*I*c) + 43332*I*a^8*e^(10*I*d*x - 4*I*c) + 27672*I*a^8*e^(8*I*d*x - 6*I*c
) + 12048*I*a^8*e^(6*I*d*x - 8*I*c) + 3467*I*a^8*e^(4*I*d*x - 10*I*c) + 598
*I*a^8*e^(2*I*d*x - 12*I*c) + 25674*I*a^8*e^(14*I*d*x) + 47*I*a^8*e^(-14*I*
c))/(d*e^(28*I*d*x + 14*I*c) + 14*d*e^(26*I*d*x + 12*I*c) + 91*d*e^(24*I*d*
x + 10*I*c) + 364*d*e^(22*I*d*x + 8*I*c) + 1001*d*e^(20*I*d*x + 6*I*c) + 20
02*d*e^(18*I*d*x + 4*I*c) + 3003*d*e^(16*I*d*x + 2*I*c) + 3003*d*e^(12*I*d*
x - 2*I*c) + 2002*d*e^(10*I*d*x - 4*I*c) + 1001*d*e^(8*I*d*x - 6*I*c) + 364
*d*e^(6*I*d*x - 8*I*c) + 91*d*e^(4*I*d*x - 10*I*c) + 14*d*e^(2*I*d*x - 12*I
*c) + 3432*d*e^(14*I*d*x) + d*e^(-14*I*c))
```

Mupad [B] (verification not implemented)

Time = 4.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.90

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{a^8\tan(c+d\,x)^3}{3\,d} - \frac{80\,a^8\tan(c+d\,x)+a^8\,64\mathrm{i}}{d\,\left(\tan\left(c+d\,x\right)^2+\tan\left(c+d\,x\right)\,2\mathrm{i}-1\right)} - \frac{31\,a^8\tan(c+d\,x)}{d} + \frac{a^8\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,80\mathrm{i}}{d} - \frac{a^8\tan(c+d\,x)^2\,4\mathrm{i}}{d}$$

```
[In] int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] (a^8*log(tan(c + d*x) + 1i)*80i)/d - (31*a^8*tan(c + d*x))/d - (80*a^8*tan(
c + d*x) + a^8*64i)/(d*(tan(c + d*x)*2i + tan(c + d*x)^2 - 1)) - (a^8*tan(c
 + d*x)^2*4i)/d + (a^8*tan(c + d*x)^3)/(3*d)
```

3.84 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 24, antiderivative size = 114

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^8\,dx = -8a^8x+\frac{8ia^8\log(\cos(c+dx))}{d}+\frac{a^8\tan(c+dx)}{d}-\frac{16ia^{11}}{3d(a-ia\tan(c+dx))^3}+\frac{16ia^{10}}{d(a-ia\tan(c+dx))^2}-\frac{24ia^9}{d(a-ia\tan(c+dx))}$$

```
[Out] -8*a^8*x+8*I*a^8*ln(cos(d*x+c))/d+a^8*tan(d*x+c)/d-16/3*I*a^11/d/(a-I*a*tan
(d*x+c))^3+16*I*a^10/d/(a-I*a*tan(d*x+c))^2-24*I*a^9/d/(a-I*a*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{16ia^{11}}{3d(a-ia\tan(c+dx))^3}+\frac{16ia^{10}}{d(a-ia\tan(c+dx))^2}-\frac{24ia^9}{d(a-ia\tan(c+dx))}+\frac{a^8\tan(c+dx)}{d}+\frac{8ia^8\log(\cos(c+dx))}{d}-8a^8x$$

```
[In] Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] -8*a^8*x + ((8*I)*a^8*Log[Cos[c + d*x]])/d + (a^8*Tan[c + d*x])/d - (((16*I
)/3)*a^11)/(d*(a - I*a*Tan[c + d*x])^3) + ((16*I)*a^10)/(d*(a - I*a*Tan[c +
 d*x])^2) - ((24*I)*a^9)/(d*(a - I*a*Tan[c + d*x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(ia^7)\,\text{Subst}\left(\int \frac{(a+x)^4}{(a-x)^4}\,dx, x, ia\tan(c+dx)\right)}{d}\\
&= -\frac{(ia^7)\,\text{Subst}\left(\int \left(1+\frac{16a^4}{(a-x)^4}-\frac{32a^3}{(a-x)^3}+\frac{24a^2}{(a-x)^2}-\frac{8a}{a-x}\right)\,dx, x, ia\tan(c+dx)\right)}{d}\\
&= -8a^8x+\frac{8ia^8\log(\cos(c+dx))}{d}+\frac{a^8\tan(c+dx)}{d}-\frac{16ia^{11}}{3d(a-ia\tan(c+dx))^3}\\
&\quad+\frac{16ia^{10}}{d(a-ia\tan(c+dx))^2}-\frac{24ia^9}{d(a-ia\tan(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.68

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{ia^7\left(8a\log(i+\tan(c+dx))+ia\tan(c+dx)+\frac{8ia(-5+12i\tan(c+dx)+9\tan^2(c+dx))}{3(i+\tan(c+dx))^3}\right)}{d}$$

```
[In] Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((-I)*a^7*(8*a*Log[I + Tan[c + d*x]] + I*a*Tan[c + d*x] + (((8*I)/3)*a*(-5
+ (12*I)*Tan[c + d*x] + 9*Tan[c + d*x]^2))/(I + Tan[c + d*x])^3))/d
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 318 vs. 2(105) = 210.

Time = 0.84 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.80

$$\frac{32ia^8(\sin^6(dx+c))}{3d}+\frac{14ia^8(\cos^4(dx+c))}{3d}+\frac{28ia^8(\sin^2(dx+c))(\cos^4(dx+c))}{3d}+\frac{8ia^8\ln(\cos(dx+c))}{d}+\frac{a^8(}{}$$

```
[In] int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 32/3*I/d*a^8*sin(d*x+c)^6+14/3*I/d*a^8*cos(d*x+c)^4+28/3*I/d*a^8*sin(d*x+c)
^2*cos(d*x+c)^4+8*I*a^8*ln(cos(d*x+c))/d+1/d*a^8*sin(d*x+c)^7*cos(d*x+c)+2*
I/d*a^8*sin(d*x+c)^4-4/3*I/d*a^8*cos(d*x+c)^6-35/3/d*a^8*sin(d*x+c)^3*cos(d
*x+c)^3-233/24/d*a^8*sin(d*x+c)*cos(d*x+c)^3+29/6/d*a^8*cos(d*x+c)^5*sin(d*
x+c)+4*I/d*a^8*sin(d*x+c)^2-8*a^8*x+1/d*a^8*sin(d*x+c)^9/cos(d*x+c)+35/6/d*
a^8*cos(d*x+c)*sin(d*x+c)^5+175/24/d*a^8*cos(d*x+c)*sin(d*x+c)^3+111/8/d*a^
8*sin(d*x+c)*cos(d*x+c)-8/d*a^8*c
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.99

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{2\left(i\,a^8e^{(8i\,dx+8i\,c)}-2i\,a^8e^{(6i\,dx+6i\,c)}+6i\,a^8e^{(4i\,dx+4i\,c)}+9i\,a^8e^{(2i\,dx+2i\,c)}-3i\,a^8+12\left(-i\,a^8e^{(2i\,dx+2i\,c)}-i\,a\right.\right.}{3\left(de^{(2i\,dx+2i\,c)}+d\right)}$$

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -2/3*(I*a^8*e^(8*I*d*x + 8*I*c) - 2*I*a^8*e^(6*I*d*x + 6*I*c) + 6*I*a^8*e^(
4*I*d*x + 4*I*c) + 9*I*a^8*e^(2*I*d*x + 2*I*c) - 3*I*a^8 + 12*(-I*a^8*e^(2*
I*d*x + 2*I*c) - I*a^8)*log(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(2*I*d*x + 2*I*c
) + d)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.51

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{2ia^8}{de^{2ic}e^{2idx}+d} + \frac{8ia^8\log\left(e^{2idx}+e^{-2ic}\right)}{d} + \begin{cases} \frac{-2ia^8d^2e^{6ic}e^{6idx}+6ia^8d^2e^{4ic}e^{4idx}-18ia^8d^2e^{2ic}e^{2idx}}{3d^3} & \text{for } d^3 \neq 0 \\ x\left(4a^8e^{6ic}-8a^8e^{4ic}+12a^8e^{2ic}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] 2*I*a**8/(d*exp(2*I*c)*exp(2*I*d*x) + d) + 8*I*a**8*log(exp(2*I*d*x) + exp(
-2*I*c))/d + Piecewise(((-2*I*a**8*d**2*exp(6*I*c)*exp(6*I*d*x) + 6*I*a**8*
d**2*exp(4*I*c)*exp(4*I*d*x) - 18*I*a**8*d**2*exp(2*I*c)*exp(2*I*d*x))/(3*d
**3), Ne(d**3, 0)), (x*(4*a**8*exp(6*I*c) - 8*a**8*exp(4*I*c) + 12*a**8*exp
(2*I*c)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.28

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^8\,dx =$$

$$-\frac{24\,(dx+c)a^8 + 12i\,a^8\log\left(\tan\left(dx+c\right)^2+1\right) - 3\,a^8\tan\left(dx+c\right) - \frac{8\left(9\,a^8\tan(dx+c)^5-15i\,a^8\tan(dx+c)^4+4\,a^8\tan(\cdots\right.}{\tan(dx+c)^6+3\tan(\cdots}}{3\,d}$$

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/3*(24*(d*x + c)*a^8 + 12*I*a^8*log(tan(d*x + c)^2 + 1) - 3*a^8*tan(d*x +
 c) - 8*(9*a^8*tan(d*x + c)^5 - 15*I*a^8*tan(d*x + c)^4 + 4*a^8*tan(d*x + c
)^3 - 12*I*a^8*tan(d*x + c)^2 + 3*a^8*tan(d*x + c) - 5*I*a^8)/(tan(d*x + c)
^6 + 3*tan(d*x + c)^4 + 3*tan(d*x + c)^2 + 1))/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 799 vs. $2(98) = 196$.

Time = 1.09 (sec) , antiderivative size = 799, normalized size of antiderivative = 7.01

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^8\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -2/3*(-12*I*a^8*e^(28*I*d*x + 14*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 168*I*
a^8*e^(26*I*d*x + 12*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 1092*I*a^8*e^(24*I
*d*x + 10*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 4368*I*a^8*e^(22*I*d*x + 8*I*
c)*log(e^(2*I*d*x + 2*I*c) + 1) - 12012*I*a^8*e^(20*I*d*x + 6*I*c)*log(e^(2
*I*d*x + 2*I*c) + 1) - 24024*I*a^8*e^(18*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*
I*c) + 1) - 36036*I*a^8*e^(16*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) -
 36036*I*a^8*e^(12*I*d*x - 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 24024*I*a^
8*e^(10*I*d*x - 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 12012*I*a^8*e^(8*I*d*
x - 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 4368*I*a^8*e^(6*I*d*x - 8*I*c)*lo
g(e^(2*I*d*x + 2*I*c) + 1) - 1092*I*a^8*e^(4*I*d*x - 10*I*c)*log(e^(2*I*d*x
 + 2*I*c) + 1) - 168*I*a^8*e^(2*I*d*x - 12*I*c)*log(e^(2*I*d*x + 2*I*c) + 1
) - 41184*I*a^8*e^(14*I*d*x)*log(e^(2*I*d*x + 2*I*c) + 1) - 12*I*a^8*e^(-14
*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + I*a^8*e^(34*I*d*x + 20*I*c) + 11*I*a^8
*e^(32*I*d*x + 18*I*c) + 58*I*a^8*e^(30*I*d*x + 16*I*c) + 217*I*a^8*e^(28*I
*d*x + 14*I*c) + 725*I*a^8*e^(26*I*d*x + 12*I*c) + 2236*I*a^8*e^(24*I*d*x +
 10*I*c) + 5772*I*a^8*e^(22*I*d*x + 8*I*c) + 11583*I*a^8*e^(20*I*d*x + 6*I*
c) + 17589*I*a^8*e^(18*I*d*x + 4*I*c) + 20020*I*a^8*e^(16*I*d*x + 2*I*c) +
10231*I*a^8*e^(12*I*d*x - 2*I*c) + 4147*I*a^8*e^(10*I*d*x - 4*I*c) + 872*I*
a^8*e^(8*I*d*x - 6*I*c) - 80*I*a^8*e^(6*I*d*x - 8*I*c) - 111*I*a^8*e^(4*I*d
*x - 10*I*c) - 30*I*a^8*e^(2*I*d*x - 12*I*c) + 16874*I*a^8*e^(14*I*d*x) - 3
*I*a^8*e^(-14*I*c))/(d*e^(28*I*d*x + 14*I*c) + 14*d*e^(26*I*d*x + 12*I*c) +
 91*d*e^(24*I*d*x + 10*I*c) + 364*d*e^(22*I*d*x + 8*I*c) + 1001*d*e^(20*I*d
*x + 6*I*c) + 2002*d*e^(18*I*d*x + 4*I*c) + 3003*d*e^(16*I*d*x + 2*I*c) + 3
003*d*e^(12*I*d*x - 2*I*c) + 2002*d*e^(10*I*d*x - 4*I*c) + 1001*d*e^(8*I*d*
x - 6*I*c) + 364*d*e^(6*I*d*x - 8*I*c) + 91*d*e^(4*I*d*x - 10*I*c) + 14*d*e
^(2*I*d*x - 12*I*c) + 3432*d*e^(14*I*d*x) + d*e^(-14*I*c))
```

Mupad [B] (verification not implemented)

Time = 4.22 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$=\frac{a^8\tan(c+d\,x)}{d}-\frac{24\,a^8\tan(c+d\,x)^2+a^8\tan(c+d\,x)\;32\mathrm{i}-\frac{40\,a^8}{3}}{d\,\left(-\tan\left(c+d\,x\right)^3-\tan\left(c+d\,x\right)^2 3\mathrm{i}+3\tan\left(c+d\,x\right)+1\mathrm{i}\right)}$$
$$-\frac{a^8\;\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\;8\mathrm{i}}{d}$$

```
[In] int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] (a^8*tan(c + d*x))/d - (a^8*log(tan(c + d*x) + 1i)*8i)/d - (a^8*tan(c + d*x
)*32i - (40*a^8)/3 + 24*a^8*tan(c + d*x)^2)/(d*(3*tan(c + d*x) - tan(c + d*
x)^2*3i - tan(c + d*x)^3 + 1i))
```

3.85 $\int \cos^8(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 24, antiderivative size = 43

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{i(a^3+ia^3\tan(c+dx))^4}{8d(a-ia\tan(c+dx))^4}$$

```
[Out] -1/8*I*(a^3+I*a^3*tan(d*x+c))^4/d/(a-I*a*tan(d*x+c))^4
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 37}

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{i(a^3+ia^3\tan(c+dx))^4}{8d(a-ia\tan(c+dx))^4}$$

```
[In] Int[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x])^8,x]
[Out] ((-1/8*I)*(a^3 + I*a^3*Tan[c + d*x])^4)/(d*(a - I*a*Tan[c + d*x])^4)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
```

```
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{(ia^9)\,\text{Subst}\left(\int \frac{(a+x)^3}{(a-x)^5}\,dx, x, ia\tan(c+dx)\right)}{d}$$
$$= -\frac{i(a^3 + ia^3\tan(c+dx))^4}{8d(a - ia\tan(c+dx))^4}$$

Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{ia^8(\cos(c+dx)+i\sin(c+dx))^8}{8d}$$

```
[In] Integrate[Cos[c + d*x]^8*(a + I*a*Tan[c + d*x])^8,x]
[Out] ((-1/8*I)*a^8*(Cos[c + d*x] + I*Sin[c + d*x])^8)/d
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 450 vs. 2(38) = 76.

Time = 1.40 (sec) , antiderivative size = 451, normalized size of antiderivative = 10.49

$$\frac{a^8\left(-\frac{\left(\sin^7(dx+c)+\frac{7\left(\sin^5(dx+c)\right)}{6}+\frac{35\left(\sin^3(dx+c)\right)}{24}+\frac{35\sin(dx+c)}{16}\right)\cos(dx+c)}{8}+\frac{35dx}{128}+\frac{35c}{128}\right)-ia^8(\cos^8(dx+c))-28a^8\left(-\frac{(\sin}{}\right.}{}$$

```
[In] int(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x)
[Out] 1/d*(a^8*(-1/8*(sin(d*x+c)^7+7/6*sin(d*x+c)^5+35/24*sin(d*x+c)^3+35/16*sin(
d*x+c))*cos(d*x+c)+35/128*d*x+35/128*c)-I*a^8*cos(d*x+c)^8-28*a^8*(-1/8*sin
(d*x+c)^5*cos(d*x+c)^3-5/48*sin(d*x+c)^3*cos(d*x+c)^3-5/64*cos(d*x+c)^3*sin
(d*x+c)+5/128*sin(d*x+c)*cos(d*x+c)+5/128*d*x+5/128*c)-56*I*a^8*(-1/8*cos(d
*x+c)^6*sin(d*x+c)^2-1/24*cos(d*x+c)^6)+70*a^8*(-1/8*sin(d*x+c)^3*cos(d*x+c
)^5-1/16*cos(d*x+c)^5*sin(d*x+c)+1/64*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x
+c)+3/128*d*x+3/128*c)-I*a^8*sin(d*x+c)^8-28*a^8*(-1/8*sin(d*x+c)*cos(d*x+c
)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/128*d
*x+5/128*c)+56*I*a^8*(-1/8*sin(d*x+c)^4*cos(d*x+c)^4-1/12*cos(d*x+c)^4*sin(
d*x+c)^2-1/24*cos(d*x+c)^4)+a^8*(1/8*(cos(d*x+c)^7+7/6*cos(d*x+c)^5+35/24*c
os(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+35/128*d*x+35/128*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.40

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{i\,a^8e^{(8i\,dx+8i\,c)}}{8\,d}$$

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -1/8*I*a^8*e^(8*I*d*x + 8*I*c)/d
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^8\,dx = \begin{cases} -\frac{ia^8e^{8ic}e^{8idx}}{8d} & \text{for}\ d \neq 0 \\ a^8xe^{8ic} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)**8*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise((-I*a**8*exp(8*I*c)*exp(8*I*d*x)/(8*d), Ne(d, 0)), (a**8*x*exp(8*
I*c), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(35) = 70.

Time = 0.33 (sec) , antiderivative size = 136, normalized size of antiderivative = 3.16

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{a^8\tan(dx+c)^7 - 4i\,a^8\tan(dx+c)^6 - 7\,a^8\tan(dx+c)^5 + 8i\,a^8\tan(dx+c)^4 + 7\,a^8\tan(dx+c)^3 - 4}{(\tan(dx+c)^8 + 4\,\tan(dx+c)^6 + 6\,\tan(dx+c)^4 + 4\,\tan(dx+c)^2 +}$$

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -(a^8*tan(d*x + c)^7 - 4*I*a^8*tan(d*x + c)^6 - 7*a^8*tan(d*x + c)^5 + 8*I*
a^8*tan(d*x + c)^4 + 7*a^8*tan(d*x + c)^3 - 4*I*a^8*tan(d*x + c)^2 - a^8*ta
n(d*x + c))/((tan(d*x + c)^8 + 4*tan(d*x + c)^6 + 6*tan(d*x + c)^4 + 4*tan(
d*x + c)^2 + 1)*d)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 381 vs. 2(35) = 70.

Time = 1.12 (sec) , antiderivative size = 381, normalized size of antiderivative = 8.86

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{i\,a^8e^{(36i\,dx+22i\,c)}+14i\,a^8e^{(34i\,dx+20i\,c)}+91i\,a^8e^{(32i\,dx+18i\,c)}+364i\,a^8e^{(30i\,dx+16i\,c)}+1001i\,a^8e^{(28i\,dx+14i\,c)}+20}{8\left(de^{(28i\,dx+14i\,c)}+14\,de^{(26i\,dx+12i\,c)}+91\,de^{(24i\,dx+10i\,c)}+364\,de^{(22i\,dx+8i\,c)}+1001\,de^{(20i\,dx+}\right.}$$

```
[In] integrate(cos(d*x+c)^8*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/8*(I*a^8*e^(36*I*d*x + 22*I*c) + 14*I*a^8*e^(34*I*d*x + 20*I*c) + 91*I*a
^8*e^(32*I*d*x + 18*I*c) + 364*I*a^8*e^(30*I*d*x + 16*I*c) + 1001*I*a^8*e^(
28*I*d*x + 14*I*c) + 2002*I*a^8*e^(26*I*d*x + 12*I*c) + 3003*I*a^8*e^(24*I*
d*x + 10*I*c) + 3432*I*a^8*e^(22*I*d*x + 8*I*c) + 3003*I*a^8*e^(20*I*d*x +
6*I*c) + 2002*I*a^8*e^(18*I*d*x + 4*I*c) + 1001*I*a^8*e^(16*I*d*x + 2*I*c)
+ 91*I*a^8*e^(12*I*d*x - 2*I*c) + 14*I*a^8*e^(10*I*d*x - 4*I*c) + I*a^8*e^(
8*I*d*x - 6*I*c) + 364*I*a^8*e^(14*I*d*x))/(d*e^(28*I*d*x + 14*I*c) + 14*d*
e^(26*I*d*x + 12*I*c) + 91*d*e^(24*I*d*x + 10*I*c) + 364*d*e^(22*I*d*x + 8*
I*c) + 1001*d*e^(20*I*d*x + 6*I*c) + 2002*d*e^(18*I*d*x + 4*I*c) + 3003*d*e
^(16*I*d*x + 2*I*c) + 3003*d*e^(12*I*d*x - 2*I*c) + 2002*d*e^(10*I*d*x - 4*
I*c) + 1001*d*e^(8*I*d*x - 6*I*c) + 364*d*e^(6*I*d*x - 8*I*c) + 91*d*e^(4*I
*d*x - 10*I*c) + 14*d*e^(2*I*d*x - 12*I*c) + 3432*d*e^(14*I*d*x) + d*e^(-14
*I*c))
```

Mupad [B] (verification not implemented)

Time = 3.84 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.53

$$\int \cos^8(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{a^8\tan(c+d\,x)\,\left(\tan(c+d\,x)^2-1\right)}{d\,\left(\tan\left(c+d\,x\right)^4+\tan\left(c+d\,x\right)^3\,4\mathrm{i}-6\tan\left(c+d\,x\right)^2-\tan\left(c+d\,x\right)\,4\mathrm{i}+1\right)}$$

```
[In] int(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] -(a^8*tan(c + d*x)*(tan(c + d*x)^2 - 1))/(d*(tan(c + d*x)^3*4i - 6*tan(c +
d*x)^2 - tan(c + d*x)*4i + tan(c + d*x)^4 + 1))
```

3.86 $\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 24, antiderivative size = 80

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{4ia^{13}}{5d(a-ia\tan(c+dx))^5} + \frac{ia^{12}}{d(a-ia\tan(c+dx))^4} - \frac{ia^{11}}{3d(a-ia\tan(c+dx))^3}$$

```
[Out] -4/5*I*a^13/d/(a-I*a*tan(d*x+c))^5+I*a^12/d/(a-I*a*tan(d*x+c))^4-1/3*I*a^11
/d/(a-I*a*tan(d*x+c))^3
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{4ia^{13}}{5d(a-ia\tan(c+dx))^5} + \frac{ia^{12}}{d(a-ia\tan(c+dx))^4} - \frac{ia^{11}}{3d(a-ia\tan(c+dx))^3}$$

```
[In] Int[Cos[c + d*x]^10*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (((-4*I)/5)*a^13)/(d*(a - I*a*Tan[c + d*x])^5) + (I*a^12)/(d*(a - I*a*Tan[c
 + d*x])^4) - ((I/3)*a^11)/(d*(a - I*a*Tan[c + d*x])^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(ia^{11})\,\text{Subst}\left(\int \frac{(a+x)^2}{(a-x)^6}\,dx, x, ia\tan(c+dx)\right)}{d}\\ &= -\frac{(ia^{11})\,\text{Subst}\left(\int \left(\frac{4a^2}{(a-x)^6}-\frac{4a}{(a-x)^5}+\frac{1}{(a-x)^4}\right)\,dx, x, ia\tan(c+dx)\right)}{d}\\ &= -\frac{4ia^{13}}{5d(a-ia\tan(c+dx))^5}+\frac{ia^{12}}{d(a-ia\tan(c+dx))^4}-\frac{ia^{11}}{3d(a-ia\tan(c+dx))^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.55

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{a^8(-2-5i\tan(c+dx)+5\tan^2(c+dx))}{15d(i+\tan(c+dx))^5}$$

```
[In] Integrate[Cos[c + d*x]^10*(a + I*a*Tan[c + d*x])^8,x]
[Out] -1/15*(a^8*(-2 - (5*I)*Tan[c + d*x] + 5*Tan[c + d*x]^2))/(d*(I + Tan[c + d*
x])^5)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 587 vs. $2(70) = 140$.

Time = 1.18 (sec) , antiderivative size = 588, normalized size of antiderivative = 7.35

$$a^8\left(-\frac{(\sin^7(dx+c))(\cos^3(dx+c))}{10}-\frac{7(\sin^5(dx+c))(\cos^3(dx+c))}{80}-\frac{7(\sin^3(dx+c))(\cos^3(dx+c))}{96}-\frac{7(\cos^3(dx+c))\sin(dx+c)}{128}+\frac{7\sin(dx+c)}{25\ldots}\right.$$

```
[In] int(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 1/d*(a^8*(-1/10*sin(d*x+c)^7*cos(d*x+c)^3-7/80*sin(d*x+c)^5*cos(d*x+c)^3-7/
96*sin(d*x+c)^3*cos(d*x+c)^3-7/128*cos(d*x+c)^3*sin(d*x+c)+7/256*sin(d*x+c)
*cos(d*x+c)+7/256*d*x+7/256*c)-8*I*a^8*(-1/10*sin(d*x+c)^6*cos(d*x+c)^4-3/4
0*sin(d*x+c)^4*cos(d*x+c)^4-1/20*cos(d*x+c)^4*sin(d*x+c)^2-1/40*cos(d*x+c)^
4)-28*a^8*(-1/10*sin(d*x+c)^5*cos(d*x+c)^5-1/16*sin(d*x+c)^3*cos(d*x+c)^5-1
/32*cos(d*x+c)^5*sin(d*x+c)+1/128*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+
3/256*d*x+3/256*c)-56*I*a^8*(-1/10*cos(d*x+c)^8*sin(d*x+c)^2-1/40*cos(d*x+c
)^8)+70*a^8*(-1/10*sin(d*x+c)^3*cos(d*x+c)^7-3/80*sin(d*x+c)*cos(d*x+c)^7+1
/160*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+3/256*d*x+3
/256*c)-4/5*I*a^8*cos(d*x+c)^10-28*a^8*(-1/10*sin(d*x+c)*cos(d*x+c)^9+1/80*
(cos(d*x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x
+c)+7/256*d*x+7/256*c)+56*I*a^8*(-1/10*sin(d*x+c)^4*cos(d*x+c)^6-1/20*cos(d
*x+c)^6*sin(d*x+c)^2-1/60*cos(d*x+c)^6)+a^8*(1/10*(cos(d*x+c)^9+9/8*cos(d*x
+c)^7+21/16*cos(d*x+c)^5+105/64*cos(d*x+c)^3+315/128*cos(d*x+c))*sin(d*x+c)
+63/256*d*x+63/256*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.60

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{-6i\,a^8e^{(10i\,dx+10i\,c)}-15i\,a^8e^{(8i\,dx+8i\,c)}-10i\,a^8e^{(6i\,dx+6i\,c)}}{240\,d}$$

```
[In] integrate(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/240*(-6*I*a^8*e^(10*I*d*x + 10*I*c) - 15*I*a^8*e^(8*I*d*x + 8*I*c) - 10*I
*a^8*e^(6*I*d*x + 6*I*c))/d
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.51

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^8\,dx = \begin{cases} \frac{-384ia^8d^2e^{10ic}e^{10idx}-960ia^8d^2e^{8ic}e^{8idx}-640ia^8d^2e^{6ic}e^{6idx}}{15360d^3} & \text{for } d^3\neq 0 \\ x\left(\frac{a^8e^{10ic}}{4}+\frac{a^8e^{8ic}}{2}+\frac{a^8e^{6ic}}{4}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)**10*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise(((-384*I*a**8*d**2*exp(10*I*c)*exp(10*I*d*x) - 960*I*a**8*d**2*ex
p(8*I*c)*exp(8*I*d*x) - 640*I*a**8*d**2*exp(6*I*c)*exp(6*I*d*x))/(15360*d**
3), Ne(d**3, 0)), (x*(a**8*exp(10*I*c)/4 + a**8*exp(8*I*c)/2 + a**8*exp(6*I
*c)/4), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(64) = 128.

Time = 0.35 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.90

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{5\,a^8\tan\left(dx+c\right)^7-30i\,a^8\tan\left(dx+c\right)^6-77\,a^8\tan\left(dx+c\right)^5+110i\,a^8\tan\left(dx+c\right)^4+95\,a^8\tan\left(dx+c\right.}{15\left(\tan\left(dx+c\right)^{10}+5\,\tan\left(dx+c\right)^8+10\,\tan\left(dx+c\right)^6+10\,\tan\left(dx+c\right)^4\right.}$$

```
[In] integrate(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/15*(5*a^8*tan(d*x + c)^7 - 30*I*a^8*tan(d*x + c)^6 - 77*a^8*tan(d*x + c)
^5 + 110*I*a^8*tan(d*x + c)^4 + 95*a^8*tan(d*x + c)^3 - 50*I*a^8*tan(d*x +
c)^2 - 15*a^8*tan(d*x + c) + 2*I*a^8)/((tan(d*x + c)^10 + 5*tan(d*x + c)^8
+ 10*tan(d*x + c)^6 + 10*tan(d*x + c)^4 + 5*tan(d*x + c)^2 + 1)*d)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 409 vs. 2(64) = 128.

Time = 1.23 (sec) , antiderivative size = 409, normalized size of antiderivative = 5.11

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{6i\,a^8e^{(38i\,dx+24i\,c)}+99i\,a^8e^{(36i\,dx+22i\,c)}+766i\,a^8e^{(34i\,dx+20i\,c)}+3689i\,a^8e^{(32i\,dx+18i\,c)}+12376i\,a^8e^{(30i\,dx+16i\,c)}}{240\left(de^{(28i\,dx+14i\,c)}+14\,de^{(26i\,dx+12i\,c)}+91\,de^{(24i\,dx+10i\,c)}+\right.}$$

```
[In] integrate(cos(d*x+c)^10*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/240*(6*I*a^8*e^(38*I*d*x + 24*I*c) + 99*I*a^8*e^(36*I*d*x + 22*I*c) + 76
6*I*a^8*e^(34*I*d*x + 20*I*c) + 3689*I*a^8*e^(32*I*d*x + 18*I*c) + 12376*I*
a^8*e^(30*I*d*x + 16*I*c) + 30667*I*a^8*e^(28*I*d*x + 14*I*c) + 58058*I*a^8
*e^(26*I*d*x + 12*I*c) + 85657*I*a^8*e^(24*I*d*x + 10*I*c) + 99528*I*a^8*e^
(22*I*d*x + 8*I*c) + 91377*I*a^8*e^(20*I*d*x + 6*I*c) + 66066*I*a^8*e^(18*I
*d*x + 4*I*c) + 37219*I*a^8*e^(16*I*d*x + 2*I*c) + 5089*I*a^8*e^(12*I*d*x -
 2*I*c) + 1126*I*a^8*e^(10*I*d*x - 4*I*c) + 155*I*a^8*e^(8*I*d*x - 6*I*c) +
 10*I*a^8*e^(6*I*d*x - 8*I*c) + 16016*I*a^8*e^(14*I*d*x))/(d*e^(28*I*d*x +
14*I*c) + 14*d*e^(26*I*d*x + 12*I*c) + 91*d*e^(24*I*d*x + 10*I*c) + 364*d*e
^(22*I*d*x + 8*I*c) + 1001*d*e^(20*I*d*x + 6*I*c) + 2002*d*e^(18*I*d*x + 4*
I*c) + 3003*d*e^(16*I*d*x + 2*I*c) + 3003*d*e^(12*I*d*x - 2*I*c) + 2002*d*e
^(10*I*d*x - 4*I*c) + 1001*d*e^(8*I*d*x - 6*I*c) + 364*d*e^(6*I*d*x - 8*I*c
) + 91*d*e^(4*I*d*x - 10*I*c) + 14*d*e^(2*I*d*x - 12*I*c) + 3432*d*e^(14*I*
d*x) + d*e^(-14*I*c))
```

Mupad [B] (verification not implemented)

Time = 4.49 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02

$$\int \cos^{10}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$=\frac{a^8\left(-5\tan(c+d\,x)^2+\tan(c+d\,x)\ 5\mathrm{i}+2\right)}{15\,d\ \left(\tan\left(c+d\,x\right)^5+\tan\left(c+d\,x\right)^4 5\mathrm{i}-10\tan\left(c+d\,x\right)^3-\tan\left(c+d\,x\right)^2 10\mathrm{i}+5\tan\left(c+d\,x\right)+1\mathrm{i}\right)}$$

```
[In] int(cos(c + d*x)^10*(a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] (a^8*(tan(c + d*x)*5i - 5*tan(c + d*x)^2 + 2))/(15*d*(5*tan(c + d*x) - tan(
c + d*x)^2*10i - 10*tan(c + d*x)^3 + tan(c + d*x)^4*5i + tan(c + d*x)^5 + 1
i))
```

3.87 $\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^8\,dx=-\frac{ia^{14}}{3d(a-ia\tan(c+dx))^6}+\frac{ia^{13}}{5d(a-ia\tan(c+dx))^5}$$

```
[Out] -1/3*I*a^14/d/(a-I*a*tan(d*x+c))^6+1/5*I*a^13/d/(a-I*a*tan(d*x+c))^5
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^8\,dx=\frac{ia^{13}}{5d(a-ia\tan(c+dx))^5}-\frac{ia^{14}}{3d(a-ia\tan(c+dx))^6}$$

```
[In] Int[Cos[c + d*x]^12*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((-1/3*I)*a^14)/(d*(a - I*a*Tan[c + d*x])^6) + ((I/5)*a^13)/(d*(a - I*a*Tan
[c + d*x])^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
```

```
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(ia^{13})\,\text{Subst}\left(\int \frac{a+x}{(a-x)^7}\,dx, x, ia\tan(c+dx)\right)}{d} \\
&= -\frac{(ia^{13})\,\text{Subst}\left(\int \left(\frac{2a}{(a-x)^7} - \frac{1}{(a-x)^6}\right)\,dx, x, ia\tan(c+dx)\right)}{d} \\
&= -\frac{ia^{14}}{3d(a-ia\tan(c+dx))^6} + \frac{ia^{13}}{5d(a-ia\tan(c+dx))^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.62

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{a^8(-2i+3\tan(c+dx))}{15d(i+\tan(c+dx))^6}$$

```
[In] Integrate[Cos[c + d*x]^12*(a + I*a*Tan[c + d*x])^8,x]
[Out] -1/15*(a^8*(-2*I + 3*Tan[c + d*x]))/(d*(I + Tan[c + d*x])^6)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 638 vs. 2(47) = 94.

Time = 2.81 (sec) , antiderivative size = 639, normalized size of antiderivative = 11.62

Expression too large to display

```
[In] int(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^8,x)
[Out] 1/d*(a^8*(-1/12*sin(d*x+c)^7*cos(d*x+c)^5-7/120*sin(d*x+c)^5*cos(d*x+c)^5-7
/192*sin(d*x+c)^3*cos(d*x+c)^5-7/384*cos(d*x+c)^5*sin(d*x+c)+7/1536*(cos(d*
x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+7/1024*d*x+7/1024*c)-8*I*a^8*(-1/12*sin(d
*x+c)^6*cos(d*x+c)^6-1/20*sin(d*x+c)^4*cos(d*x+c)^6-1/40*cos(d*x+c)^6*sin(d
*x+c)^2-1/120*cos(d*x+c)^6)-28*a^8*(-1/12*sin(d*x+c)^5*cos(d*x+c)^7-1/24*si
n(d*x+c)^3*cos(d*x+c)^7-1/64*sin(d*x+c)*cos(d*x+c)^7+1/384*(cos(d*x+c)^5+5/
4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/1024*d*x+5/1024*c)+56*I*a^8*(-
1/12*cos(d*x+c)^8*sin(d*x+c)^4-1/30*cos(d*x+c)^8*sin(d*x+c)^2-1/120*cos(d*x
+c)^8)+70*a^8*(-1/12*sin(d*x+c)^3*cos(d*x+c)^9-1/40*sin(d*x+c)*cos(d*x+c)^9
+1/320*(cos(d*x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*
```

```
sin(d*x+c)+7/1024*d*x+7/1024*c)-2/3*I*a^8*cos(d*x+c)^12-28*a^8*(-1/12*sin(d
*x+c)*cos(d*x+c)^11+1/120*(cos(d*x+c)^9+9/8*cos(d*x+c)^7+21/16*cos(d*x+c)^5
+105/64*cos(d*x+c)^3+315/128*cos(d*x+c))*sin(d*x+c)+21/1024*d*x+21/1024*c)-
56*I*a^8*(-1/12*cos(d*x+c)^10*sin(d*x+c)^2-1/60*cos(d*x+c)^10)+a^8*(1/12*(c
os(d*x+c)^11+11/10*cos(d*x+c)^9+99/80*cos(d*x+c)^7+231/160*cos(d*x+c)^5+231
/128*cos(d*x+c)^3+693/256*cos(d*x+c))*sin(d*x+c)+231/1024*d*x+231/1024*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.38

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$=\frac{-5i\,a^8e^{(12i\,dx+12i\,c)}-24i\,a^8e^{(10i\,dx+10i\,c)}-45i\,a^8e^{(8i\,dx+8i\,c)}-40i\,a^8e^{(6i\,dx+6i\,c)}-15i\,a^8e^{(4i\,dx+4i\,c)}}{960\,d}$$

```
[In] integrate(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/960*(-5*I*a^8*e^(12*I*d*x + 12*I*c) - 24*I*a^8*e^(10*I*d*x + 10*I*c) - 45
*I*a^8*e^(8*I*d*x + 8*I*c) - 40*I*a^8*e^(6*I*d*x + 6*I*c) - 15*I*a^8*e^(4*I
*d*x + 4*I*c))/d
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(42) = 84.

Time = 0.52 (sec) , antiderivative size = 197, normalized size of antiderivative = 3.58

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$=\begin{cases}\frac{-3932160ia^8d^4e^{12ic}e^{12idx}-18874368ia^8d^4e^{10ic}e^{10idx}-35389440ia^8d^4e^{8ic}e^{8idx}-31457280ia^8d^4e^{6ic}e^{6idx}-11796480ia^8d^4e^{4ic}e^{4idx}}{754974720d^5} & \text{for } d^5 \\ x\left(\frac{a^8e^{12ic}}{16}+\frac{a^8e^{10ic}}{4}+\frac{3a^8e^{8ic}}{8}+\frac{a^8e^{6ic}}{4}+\frac{a^8e^{4ic}}{16}\right) & \text{other}\end{cases}$$

```
[In] integrate(cos(d*x+c)**12*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise(((-3932160*I*a**8*d**4*exp(12*I*c)*exp(12*I*d*x) - 18874368*I*a**
8*d**4*exp(10*I*c)*exp(10*I*d*x) - 35389440*I*a**8*d**4*exp(8*I*c)*exp(8*I*
d*x) - 31457280*I*a**8*d**4*exp(6*I*c)*exp(6*I*d*x) - 11796480*I*a**8*d**4*
exp(4*I*c)*exp(4*I*d*x))/(754974720*d**5), Ne(d**5, 0)), (x*(a**8*exp(12*I*
c)/16 + a**8*exp(10*I*c)/4 + 3*a**8*exp(8*I*c)/8 + a**8*exp(6*I*c)/4 + a**8
*exp(4*I*c)/16), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(43) = 86$.

Time = 0.33 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.95

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{3\,a^8\tan\left(dx+c\right)^7 - 20i\,a^8\tan\left(dx+c\right)^6 - 57\,a^8\tan\left(dx+c\right)^5 + 90i\,a^8\tan\left(dx+c\right)^4 + 85\,a^8\tan\left(dx+c\right.}{15\left(\tan\left(dx+c\right)^{12} + 6\,\tan\left(dx+c\right)^{10} + 15\,\tan\left(dx+c\right)^8 + 20\,\tan\left(dx+c\right)^6 + 15\,\tan\right.}$$

```
[In] integrate(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/15*(3*a^8*tan(d*x + c)^7 - 20*I*a^8*tan(d*x + c)^6 - 57*a^8*tan(d*x + c)
^5 + 90*I*a^8*tan(d*x + c)^4 + 85*a^8*tan(d*x + c)^3 - 48*I*a^8*tan(d*x + c
)^2 - 15*a^8*tan(d*x + c) + 2*I*a^8)/((tan(d*x + c)^12 + 6*tan(d*x + c)^10
+ 15*tan(d*x + c)^8 + 20*tan(d*x + c)^6 + 15*tan(d*x + c)^4 + 6*tan(d*x + c
)^2 + 1)*d)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(43) = 86$.

Time = 1.27 (sec) , antiderivative size = 437, normalized size of antiderivative = 7.95

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{5i\,a^8e^{(40i\,dx+26i\,c)} + 94i\,a^8e^{(38i\,dx+24i\,c)} + 836i\,a^8e^{(36i\,dx+22i\,c)} + 4674i\,a^8e^{(34i\,dx+20i\,c)} + 18411i\,a^8e^{(32i\,dx+18i}}{960\left(de^{(28i\,dx+14i\,c)} + 14\,de^{(26}\right.}$$

```
[In] integrate(cos(d*x+c)^12*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/960*(5*I*a^8*e^(40*I*d*x + 26*I*c) + 94*I*a^8*e^(38*I*d*x + 24*I*c) + 83
6*I*a^8*e^(36*I*d*x + 22*I*c) + 4674*I*a^8*e^(34*I*d*x + 20*I*c) + 18411*I*
a^8*e^(32*I*d*x + 18*I*c) + 54264*I*a^8*e^(30*I*d*x + 16*I*c) + 124033*I*a^
8*e^(28*I*d*x + 14*I*c) + 224822*I*a^8*e^(26*I*d*x + 12*I*c) + 327613*I*a^8
*e^(24*I*d*x + 10*I*c) + 386672*I*a^8*e^(22*I*d*x + 8*I*c) + 370513*I*a^8*e
^(20*I*d*x + 6*I*c) + 287534*I*a^8*e^(18*I*d*x + 4*I*c) + 179361*I*a^8*e^(1
6*I*d*x + 2*I*c) + 34011*I*a^8*e^(12*I*d*x - 2*I*c) + 9754*I*a^8*e^(10*I*d*
x - 4*I*c) + 1970*I*a^8*e^(8*I*d*x - 6*I*c) + 250*I*a^8*e^(6*I*d*x - 8*I*c)
 + 15*I*a^8*e^(4*I*d*x - 10*I*c) + 88704*I*a^8*e^(14*I*d*x))/(d*e^(28*I*d*x
 + 14*I*c) + 14*d*e^(26*I*d*x + 12*I*c) + 91*d*e^(24*I*d*x + 10*I*c) + 364*
d*e^(22*I*d*x + 8*I*c) + 1001*d*e^(20*I*d*x + 6*I*c) + 2002*d*e^(18*I*d*x +
 4*I*c) + 3003*d*e^(16*I*d*x + 2*I*c) + 3003*d*e^(12*I*d*x - 2*I*c) + 2002*
```

```
d*e^(10*I*d*x - 4*I*c) + 1001*d*e^(8*I*d*x - 6*I*c) + 364*d*e^(6*I*d*x - 8*
I*c) + 91*d*e^(4*I*d*x - 10*I*c) + 14*d*e^(2*I*d*x - 12*I*c) + 3432*d*e^(14
*I*d*x) + d*e^(-14*I*c))
```

Mupad [B] (verification not implemented)

Time = 3.84 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.49

$$\int \cos^{12}(c+dx)(a+ia\tan(c+dx))^8\,dx =$$

$$-\frac{a^8\,(3\tan(c+d\,x)-2\mathrm{i})}{15\,d\,\left({\tan\left(c+d\,x\right)}^6+{\tan\left(c+d\,x\right)}^5\,6\mathrm{i}-15\,{\tan\left(c+d\,x\right)}^4-{\tan\left(c+d\,x\right)}^3\,20\mathrm{i}+15\,{\tan\left(c+d\,x\right)}^2+\tan\left(\right.\right.}$$

```
[In] int(cos(c + d*x)^12*(a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] -(a^8*(3*tan(c + d*x) - 2i))/(15*d*(tan(c + d*x)*6i + 15*tan(c + d*x)^2 - t
an(c + d*x)^3*20i - 15*tan(c + d*x)^4 + tan(c + d*x)^5*6i + tan(c + d*x)^6
- 1))
```

3.88 $\int \cos^{14}(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \cos^{14}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{ia^{15}}{7d(a-ia\tan(c+dx))^7}$$

```
[Out] -1/7*I*a^15/d/(a-I*a*tan(d*x+c))^7
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 32}

$$\int \cos^{14}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{ia^{15}}{7d(a-ia\tan(c+dx))^7}$$

```
[In] Int[Cos[c + d*x]^14*(a + I*a*Tan[c + d*x])^8,x]
[Out] ((-1/7*I)*a^15)/(d*(a - I*a*Tan[c + d*x])^7)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{(ia^{15})\,\text{Subst}\left(\int \frac{1}{(a-x)^8}\,dx, x, ia\tan(c+dx)\right)}{d}$$
$$= -\frac{ia^{15}}{7d(a-ia\tan(c+dx))^7}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 116 vs. 2(27) = 54.

Time = 2.93 (sec) , antiderivative size = 116, normalized size of antiderivative = 4.30

$$\int \cos^{14}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8(35+56\cos(2(c+dx))+28\cos(4(c+dx))+8\cos(6(c+dx))-14i\sin(2(c+dx))-14i\sin(4(c+dx))}{896d(\cos(dx)+i\sin(dx))^8}$$

```
[In] Integrate[Cos[c + d*x]^14*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*(35 + 56*Cos[2*(c + d*x)] + 28*Cos[4*(c + d*x)] + 8*Cos[6*(c + d*x)] -
 (14*I)*Sin[2*(c + d*x)] - (14*I)*Sin[4*(c + d*x)] - (6*I)*Sin[6*(c + d*x)]
)*((-I)*Cos[8*(c + 2*d*x)] + Sin[8*(c + 2*d*x)]))/(896*d*(Cos[d*x] + I*Sin[
d*x])^8)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 688 vs. 2(23) = 46.

Time = 1.40 (sec) , antiderivative size = 689, normalized size of antiderivative = 25.52

Expression too large to display

```
[In] int(cos(d*x+c)^14*(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 1/d*(a^8*(-1/14*sin(d*x+c)^7*cos(d*x+c)^7-1/24*sin(d*x+c)^5*cos(d*x+c)^7-1/
48*sin(d*x+c)^3*cos(d*x+c)^7-1/128*sin(d*x+c)*cos(d*x+c)^7+1/768*(cos(d*x+c
)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/2048*d*x+5/2048*c)-8*I*a
^8*(-1/14*sin(d*x+c)^6*cos(d*x+c)^8-1/28*cos(d*x+c)^8*sin(d*x+c)^4-1/70*cos
(d*x+c)^8*sin(d*x+c)^2-1/280*cos(d*x+c)^8)-28*a^8*(-1/14*sin(d*x+c)^5*cos(d
*x+c)^9-5/168*sin(d*x+c)^3*cos(d*x+c)^9-1/112*sin(d*x+c)*cos(d*x+c)^9+1/896
*(cos(d*x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*sin(d*
x+c)+5/2048*d*x+5/2048*c)-4/7*I*a^8*cos(d*x+c)^14+70*a^8*(-1/14*sin(d*x+c)^
3*cos(d*x+c)^11-1/56*sin(d*x+c)*cos(d*x+c)^11+1/560*(cos(d*x+c)^9+9/8*cos(d
*x+c)^7+21/16*cos(d*x+c)^5+105/64*cos(d*x+c)^3+315/128*cos(d*x+c))*sin(d*x+
```

```
c)+9/2048*d*x+9/2048*c)-56*I*a^8*(-1/14*sin(d*x+c)^2*cos(d*x+c)^12-1/84*cos
(d*x+c)^12)-28*a^8*(-1/14*sin(d*x+c)*cos(d*x+c)^13+1/168*(cos(d*x+c)^11+11/
10*cos(d*x+c)^9+99/80*cos(d*x+c)^7+231/160*cos(d*x+c)^5+231/128*cos(d*x+c)^
3+693/256*cos(d*x+c))*sin(d*x+c)+33/2048*d*x+33/2048*c)+56*I*a^8*(-1/14*sin
(d*x+c)^4*cos(d*x+c)^10-1/42*cos(d*x+c)^10*sin(d*x+c)^2-1/210*cos(d*x+c)^10
)+a^8*(1/14*(cos(d*x+c)^13+13/12*cos(d*x+c)^11+143/120*cos(d*x+c)^9+429/320
*cos(d*x+c)^7+1001/640*cos(d*x+c)^5+1001/512*cos(d*x+c)^3+3003/1024*cos(d*x
+c))*sin(d*x+c)+429/2048*d*x+429/2048*c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(21) = 42.

Time = 0.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.85

$$\int \cos^{14}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{-i\,a^8e^{(14i\,dx+14i\,c)} - 7i\,a^8e^{(12i\,dx+12i\,c)} - 21i\,a^8e^{(10i\,dx+10i\,c)} - 35i\,a^8e^{(8i\,dx+8i\,c)} - 35i\,a^8e^{(6i\,dx+6i\,c)} - 21i\,a^8e^{(4i\ldots}}{896\,d}$$

```
[In] integrate(cos(d*x+c)^14*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/896*(-I*a^8*e^(14*I*d*x + 14*I*c) - 7*I*a^8*e^(12*I*d*x + 12*I*c) - 21*I*
a^8*e^(10*I*d*x + 10*I*c) - 35*I*a^8*e^(8*I*d*x + 8*I*c) - 35*I*a^8*e^(6*I*
d*x + 6*I*c) - 21*I*a^8*e^(4*I*d*x + 4*I*c) - 7*I*a^8*e^(2*I*d*x + 2*I*c))/
d
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(22) = 44.

Time = 0.63 (sec) , antiderivative size = 279, normalized size of antiderivative = 10.33

$$\int \cos^{14}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \begin{cases} \frac{-4398046511104ia^8d^6e^{14ic}e^{14idx}-30786325577728ia^8d^6e^{12ic}e^{12idx}-92358976733184ia^8d^6e^{10ic}e^{10idx}-153931627888640ia^8d^6e^{8ic}e^{8idx}-15\ldots}{3940649673949184d^7} \\ x\left(\frac{a^8e^{14ic}}{64}+\frac{3a^8e^{12ic}}{32}+\frac{15a^8e^{10ic}}{64}+\frac{5a^8e^{8ic}}{16}+\frac{15a^8e^{6ic}}{64}+\frac{3a^8e^{4ic}}{32}+\frac{a^8e^{2ic}}{64}\right) \end{cases}$$

```
[In] integrate(cos(d*x+c)**14*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise(((-4398046511104*I*a**8*d**6*exp(14*I*c)*exp(14*I*d*x) - 30786325
577728*I*a**8*d**6*exp(12*I*c)*exp(12*I*d*x) - 92358976733184*I*a**8*d**6*e
xp(10*I*c)*exp(10*I*d*x) - 153931627888640*I*a**8*d**6*exp(8*I*c)*exp(8*I*d
```

```
*x) - 153931627888640*I*a**8*d**6*exp(6*I*c)*exp(6*I*d*x) - 92358976733184*
I*a**8*d**6*exp(4*I*c)*exp(4*I*d*x) - 30786325577728*I*a**8*d**6*exp(2*I*c)
*exp(2*I*d*x))/(3940649673949184*d**7), Ne(d**7, 0)), (x*(a**8*exp(14*I*c)/
64 + 3*a**8*exp(12*I*c)/32 + 15*a**8*exp(10*I*c)/64 + 5*a**8*exp(8*I*c)/16
+ 15*a**8*exp(6*I*c)/64 + 3*a**8*exp(4*I*c)/32 + a**8*exp(2*I*c)/64), True)
)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(21) = 42.

Time = 0.34 (sec) , antiderivative size = 171, normalized size of antiderivative = 6.33

$$\int \cos^{14}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{a^8\tan(dx+c)^7 - 7i\,a^8\tan(dx+c)^6 - 21\,a^8\tan(dx+c)^5 + 35i\,a^8\tan(dx+c)^4 + 35\,a^8\tan(dx+c)^3 -}{7\left(\tan(dx+c)^{14} + 7\,\tan(dx+c)^{12} + 21\,\tan(dx+c)^{10} + 35\,\tan(dx+c)^8 + 35\,\tan(dx+c)^6 + 2\right.}$$

```
[In] integrate(cos(d*x+c)^14*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/7*(a^8*tan(d*x + c)^7 - 7*I*a^8*tan(d*x + c)^6 - 21*a^8*tan(d*x + c)^5 +
 35*I*a^8*tan(d*x + c)^4 + 35*a^8*tan(d*x + c)^3 - 21*I*a^8*tan(d*x + c)^2
- 7*a^8*tan(d*x + c) + I*a^8)/((tan(d*x + c)^14 + 7*tan(d*x + c)^12 + 21*ta
n(d*x + c)^10 + 35*tan(d*x + c)^8 + 35*tan(d*x + c)^6 + 21*tan(d*x + c)^4 +
 7*tan(d*x + c)^2 + 1)*d)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 465 vs. 2(21) = 42.

Time = 1.31 (sec) , antiderivative size = 465, normalized size of antiderivative = 17.22

$$\int \cos^{14}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{i\,a^8e^{(42i\,dx+28i\,c)} + 21i\,a^8e^{(40i\,dx+26i\,c)} + 210i\,a^8e^{(38i\,dx+24i\,c)} + 1330i\,a^8e^{(36i\,dx+22i\,c)} + 5985i\,a^8e^{(34i\,dx+20i\,c)} +}{896\,(de^{(}}$$

```
[In] integrate(cos(d*x+c)^14*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/896*(I*a^8*e^(42*I*d*x + 28*I*c) + 21*I*a^8*e^(40*I*d*x + 26*I*c) + 210*
I*a^8*e^(38*I*d*x + 24*I*c) + 1330*I*a^8*e^(36*I*d*x + 22*I*c) + 5985*I*a^8
*e^(34*I*d*x + 20*I*c) + 20349*I*a^8*e^(32*I*d*x + 18*I*c) + 54264*I*a^8*e^
(30*I*d*x + 16*I*c) + 116279*I*a^8*e^(28*I*d*x + 14*I*c) + 203476*I*a^8*e^(
26*I*d*x + 12*I*c) + 293839*I*a^8*e^(24*I*d*x + 10*I*c) + 352352*I*a^8*e^(2
```

```
2*I*d*x + 8*I*c) + 351715*I*a^8*e^(20*I*d*x + 6*I*c) + 291928*I*a^8*e^(18*I
*d*x + 4*I*c) + 200487*I*a^8*e^(16*I*d*x + 2*I*c) + 51261*I*a^8*e^(12*I*d*x
 - 2*I*c) + 18347*I*a^8*e^(10*I*d*x - 4*I*c) + 4984*I*a^8*e^(8*I*d*x - 6*I*
c) + 966*I*a^8*e^(6*I*d*x - 8*I*c) + 119*I*a^8*e^(4*I*d*x - 10*I*c) + 7*I*a
^8*e^(2*I*d*x - 12*I*c) + 112848*I*a^8*e^(14*I*d*x))/(d*e^(28*I*d*x + 14*I*
c) + 14*d*e^(26*I*d*x + 12*I*c) + 91*d*e^(24*I*d*x + 10*I*c) + 364*d*e^(22*
I*d*x + 8*I*c) + 1001*d*e^(20*I*d*x + 6*I*c) + 2002*d*e^(18*I*d*x + 4*I*c)
+ 3003*d*e^(16*I*d*x + 2*I*c) + 3003*d*e^(12*I*d*x - 2*I*c) + 2002*d*e^(10*
I*d*x - 4*I*c) + 1001*d*e^(8*I*d*x - 6*I*c) + 364*d*e^(6*I*d*x - 8*I*c) + 9
1*d*e^(4*I*d*x - 10*I*c) + 14*d*e^(2*I*d*x - 12*I*c) + 3432*d*e^(14*I*d*x)
+ d*e^(-14*I*c))
```

Mupad [B] (verification not implemented)

Time = 3.90 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.89

$$\int \cos^{14}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{a^8\cos\left(c+d\,x\right)^8\left(\tan(c+d\,x)-7\mathrm{i}\right)}{7\,d} + \frac{64\,a^8\cos\left(c+d\,x\right)^{14}\left(\tan(c+d\,x)-\mathrm{i}\right)}{7\,d} + \frac{8\,a^8\cos\left(c+d\,x\right)^{10}\left(3\tan(c+d\,x)-7\mathrm{i}\right)}{7\,d} - \frac{16\,a^8\cos\left(c+d\,x\right)^{12}\left(5\tan(c+d\,x)-7\mathrm{i}\right)}{7\,d}$$

```
[In] int(cos(c + d*x)^14*(a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] (64*a^8*cos(c + d*x)^14*(tan(c + d*x) - 1i))/(7*d) - (a^8*cos(c + d*x)^8*(t
an(c + d*x) - 7i))/(7*d) + (8*a^8*cos(c + d*x)^10*(3*tan(c + d*x) - 7i))/(7
*d) - (16*a^8*cos(c + d*x)^12*(5*tan(c + d*x) - 7i))/(7*d)
```

3.89 $\int \cos^{16}(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 24, antiderivative size = 225

$$\begin{aligned}\int \cos^{16}(c+dx)(a+ia\tan(c+dx))^8\,dx &= \frac{a^8x}{256} - \frac{ia^{16}}{16d(a-ia\tan(c+dx))^8} \\ &- \frac{ia^{15}}{28d(a-ia\tan(c+dx))^7} \\ &- \frac{ia^{14}}{48d(a-ia\tan(c+dx))^6} \\ &- \frac{ia^{13}}{80d(a-ia\tan(c+dx))^5} \\ &- \frac{ia^{12}}{128d(a-ia\tan(c+dx))^4} \\ &- \frac{ia^{11}}{192d(a-ia\tan(c+dx))^3} \\ &- \frac{ia^{10}}{256d(a-ia\tan(c+dx))^2} \\ &- \frac{ia^9}{256d(a-ia\tan(c+dx))}\end{aligned}$$

```
[Out] 1/256*a^8*x-1/16*I*a^16/d/(a-I*a*tan(d*x+c))^8-1/28*I*a^15/d/(a-I*a*tan(d*x
+c))^7-1/48*I*a^14/d/(a-I*a*tan(d*x+c))^6-1/80*I*a^13/d/(a-I*a*tan(d*x+c))^
5-1/128*I*a^12/d/(a-I*a*tan(d*x+c))^4-1/192*I*a^11/d/(a-I*a*tan(d*x+c))^3-1
/256*I*a^10/d/(a-I*a*tan(d*x+c))^2-1/256*I*a^9/d/(a-I*a*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3568, 46, 212}

$$\int \cos^{16}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$=-\frac{ia^{16}}{16d(a-ia\tan(c+dx))^8}-\frac{ia^{15}}{28d(a-ia\tan(c+dx))^7}-\frac{ia^{14}}{48d(a-ia\tan(c+dx))^6}$$
$$-\frac{ia^{13}}{80d(a-ia\tan(c+dx))^5}-\frac{ia^{12}}{128d(a-ia\tan(c+dx))^4}-\frac{ia^{11}}{192d(a-ia\tan(c+dx))^3}$$
$$-\frac{ia^{10}}{256d(a-ia\tan(c+dx))^2}-\frac{ia^9}{256d(a-ia\tan(c+dx))}+\frac{a^8x}{256}$$

```
[In] Int[Cos[c + d*x]^16*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*x)/256 - ((I/16)*a^16)/(d*(a - I*a*Tan[c + d*x])^8) - ((I/28)*a^15)/(d
*(a - I*a*Tan[c + d*x])^7) - ((I/48)*a^14)/(d*(a - I*a*Tan[c + d*x])^6) - (
(I/80)*a^13)/(d*(a - I*a*Tan[c + d*x])^5) - ((I/128)*a^12)/(d*(a - I*a*Tan[
c + d*x])^4) - ((I/192)*a^11)/(d*(a - I*a*Tan[c + d*x])^3) - ((I/256)*a^10)
/(d*(a - I*a*Tan[c + d*x])^2) - ((I/256)*a^9)/(d*(a - I*a*Tan[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{(ia^{17})\,\text{Subst}\left(\int \frac{1}{(a-x)^9(a+x)}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^{17})\,\text{Subst}\left(\int \left(\frac{1}{2a(a-x)^9} + \frac{1}{4a^2(a-x)^8} + \frac{1}{8a^3(a-x)^7} + \frac{1}{16a^4(a-x)^6} + \frac{1}{32a^5(a-x)^5} + \frac{1}{64a^6(a-x)^4} + \frac{1}{128a^7(a-x)^3}\right.\right.}{d}$$

$$= -\frac{ia^{16}}{16d(a-ia\tan(c+dx))^8} - \frac{ia^{15}}{28d(a-ia\tan(c+dx))^7} - \frac{ia^{14}}{48d(a-ia\tan(c+dx))^6} - \frac{ia^{13}}{80d(a-ia\tan(c+dx))^5} - \frac{ia^{12}}{128d(a-ia\tan(c+dx))^4} - \frac{ia^{11}}{192d(a-ia\tan(c+dx))^3} - \frac{ia^{10}}{256d(a-ia\tan(c+dx))^2} - \frac{ia^9}{256d(a-ia\tan(c+dx))} - \frac{(ia^9)\,\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, ia\tan(c+dx)\right)}{256d}$$

$$= \frac{a^8x}{256} - \frac{ia^{16}}{16d(a-ia\tan(c+dx))^8} - \frac{ia^{15}}{28d(a-ia\tan(c+dx))^7} - \frac{ia^{14}}{48d(a-ia\tan(c+dx))^6} - \frac{ia^{13}}{80d(a-ia\tan(c+dx))^5} - \frac{ia^{12}}{128d(a-ia\tan(c+dx))^4} - \frac{ia^{11}}{192d(a-ia\tan(c+dx))^3} - \frac{ia^{10}}{256d(a-ia\tan(c+dx))^2} - \frac{ia^9}{256d(a-ia\tan(c+dx))}$$

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.68

$$\int \cos^{16}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{ia^8\sec^8(c+dx)(7350 + 12544\cos(2(c+dx)) + 7840\cos(4(c+dx)) + 3840\cos(6(c+dx)) + 1194\cos(8(}{}$$

```
[In] Integrate[Cos[c + d*x]^16*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((-1/215040*I)*a^8*Sec[c + d*x]^8*(7350 + 12544*Cos[2*(c + d*x)] + 7840*Cos
[4*(c + d*x)] + 3840*Cos[6*(c + d*x)] + 1194*Cos[8*(c + d*x)] - (3136*I)*Si
n[2*(c + d*x)] - (3920*I)*Sin[4*(c + d*x)] - (2880*I)*Sin[6*(c + d*x)] - (1
089*I)*Sin[8*(c + d*x)] + 840*ArcTan[Tan[c + d*x]]*(I*Cos[8*(c + d*x)] + Si
n[8*(c + d*x)])))/(d*(I + Tan[c + d*x])^8)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 738 vs. $2(191) = 382$.

Time = 2.59 (sec) , antiderivative size = 739, normalized size of antiderivative = 3.28

Expression too large to display

```
[In] int(cos(d*x+c)^16*(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 1/d*(a^8*(-1/16*sin(d*x+c)^7*cos(d*x+c)^9-1/32*sin(d*x+c)^5*cos(d*x+c)^9-5/
384*sin(d*x+c)^3*cos(d*x+c)^9-1/256*sin(d*x+c)*cos(d*x+c)^9+1/2048*(cos(d*x
+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+35/3
2768*d*x+35/32768*c)+56*I*a^8*(-1/16*sin(d*x+c)^4*cos(d*x+c)^12-1/56*sin(d*
x+c)^2*cos(d*x+c)^12-1/336*cos(d*x+c)^12)-28*a^8*(-1/16*sin(d*x+c)^5*cos(d*
x+c)^11-5/224*sin(d*x+c)^3*cos(d*x+c)^11-5/896*sin(d*x+c)*cos(d*x+c)^11+1/1
792*(cos(d*x+c)^9+9/8*cos(d*x+c)^7+21/16*cos(d*x+c)^5+105/64*cos(d*x+c)^3+3
15/128*cos(d*x+c))*sin(d*x+c)+45/32768*d*x+45/32768*c)-8*I*a^8*(-1/16*sin(d
*x+c)^6*cos(d*x+c)^10-3/112*sin(d*x+c)^4*cos(d*x+c)^10-1/112*cos(d*x+c)^10*
sin(d*x+c)^2-1/560*cos(d*x+c)^10)+70*a^8*(-1/16*sin(d*x+c)^3*cos(d*x+c)^13-
3/224*sin(d*x+c)*cos(d*x+c)^13+1/896*(cos(d*x+c)^11+11/10*cos(d*x+c)^9+99/8
0*cos(d*x+c)^7+231/160*cos(d*x+c)^5+231/128*cos(d*x+c)^3+693/256*cos(d*x+c)
)*sin(d*x+c)+99/32768*d*x+99/32768*c)-1/2*I*a^8*cos(d*x+c)^16-28*a^8*(-1/16
*sin(d*x+c)*cos(d*x+c)^15+1/224*(cos(d*x+c)^13+13/12*cos(d*x+c)^11+143/120*
cos(d*x+c)^9+429/320*cos(d*x+c)^7+1001/640*cos(d*x+c)^5+1001/512*cos(d*x+c)
^3+3003/1024*cos(d*x+c))*sin(d*x+c)+429/32768*d*x+429/32768*c)-56*I*a^8*(-1
/16*cos(d*x+c)^14*sin(d*x+c)^2-1/112*cos(d*x+c)^14)+a^8*(1/16*(cos(d*x+c)^1
5+15/14*cos(d*x+c)^13+65/56*cos(d*x+c)^11+143/112*cos(d*x+c)^9+1287/896*cos
(d*x+c)^7+429/256*cos(d*x+c)^5+2145/1024*cos(d*x+c)^3+6435/2048*cos(d*x+c))
*sin(d*x+c)+6435/32768*d*x+6435/32768*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.56

$$\int \cos^{16}(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{1680\,a^8dx - 105i\,a^8e^{(16i\,dx+16i\,c)} - 960i\,a^8e^{(14i\,dx+14i\,c)} - 3920i\,a^8e^{(12i\,dx+12i\,c)} - 9408i\,a^8e^{(10i\,dx+10i\,c)} - 1470}{430080\,d}$$

```
[In] integrate(cos(d*x+c)^16*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/430080*(1680*a^8*d*x - 105*I*a^8*e^(16*I*d*x + 16*I*c) - 960*I*a^8*e^(14*
I*d*x + 14*I*c) - 3920*I*a^8*e^(12*I*d*x + 12*I*c) - 9408*I*a^8*e^(10*I*d*x
 + 10*I*c) - 14700*I*a^8*e^(8*I*d*x + 8*I*c) - 15680*I*a^8*e^(6*I*d*x + 6*I
*c) - 11760*I*a^8*e^(4*I*d*x + 4*I*c) - 6720*I*a^8*e^(2*I*d*x + 2*I*c))/d
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.44

$$\int \cos^{16}(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{a^8x}{256} + \begin{cases} \frac{-354658470655426560ia^8d^7e^{16ic}e^{16idx} - 324259173170675712 0ia^8d^7e^{14ic}e^{14idx} - 13240582904469258240ia^8d^7e^{12ic}e^{12idx} - 31777398970726\cdots}{\cdots} \\ x\left(\frac{a^8e^{16ic}}{256} + \frac{a^8e^{14ic}}{32} + \frac{7a^8e^{12ic}}{64} + \frac{7a^8e^{10ic}}{32} + \frac{35a^8e^{8ic}}{128} + \frac{7a^8e^{6ic}}{32} + \frac{7a^8e^{4ic}}{64} + \frac{a^8e^{2ic}}{32}\right) \end{cases}$$

```
[In] integrate(cos(d*x+c)**16*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] a**8*x/256 + Piecewise(((-354658470655426560*I*a**8*d**7*exp(16*I*c)*exp(16
*I*d*x) - 3242591731706757120*I*a**8*d**7*exp(14*I*c)*exp(14*I*d*x) - 13240
582904469258240*I*a**8*d**7*exp(12*I*c)*exp(12*I*d*x) - 3177739897072621977
6*I*a**8*d**7*exp(10*I*c)*exp(10*I*d*x) - 49652185891759718400*I*a**8*d**7*
exp(8*I*c)*exp(8*I*d*x) - 52962331617877032960*I*a**8*d**7*exp(6*I*c)*exp(6
*I*d*x) - 39721748713407774720*I*a**8*d**7*exp(4*I*c)*exp(4*I*d*x) - 226981
42121947299840*I*a**8*d**7*exp(2*I*c)*exp(2*I*d*x))/(145268109580462718976 0
*d**8), Ne(d**8, 0)), (x*(a**8*exp(16*I*c)/256 + a**8*exp(14*I*c)/32 + 7*a*
*8*exp(12*I*c)/64 + 7*a**8*exp(10*I*c)/32 + 35*a**8*exp(8*I*c)/128 + 7*a**8
*exp(6*I*c)/32 + 7*a**8*exp(4*I*c)/64 + a**8*exp(2*I*c)/32), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.09

$$\int \cos^{16}(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{105\,(dx+c)a^8 + \frac{105\,a^8\tan(dx+c)^{15} + 805\,a^8\tan(dx+c)^{13} + 2681\,a^8\tan(dx+c)^{11} + 5053\,a^8\tan(dx+c)^9 + 2883\,a^8\tan(dx+c)^7 + 21504i\,a^8\tan(\cdots}{\tan(dx+c)^{16} + 8\,\tan(dx+c)^{14} + 28\,\tan(dx+c)^{12} + 56\,\tan(dx+\cdots}}{26880\,d}$$

```
[In] integrate(cos(d*x+c)^16*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/26880*(105*(d*x + c)*a^8 + (105*a^8*tan(d*x + c)^15 + 805*a^8*tan(d*x + c
)^13 + 2681*a^8*tan(d*x + c)^11 + 5053*a^8*tan(d*x + c)^9 + 2883*a^8*tan(d*
x + c)^7 + 21504*I*a^8*tan(d*x + c)^6 + 70791*a^8*tan(d*x + c)^5 - 114688*I
*a^8*tan(d*x + c)^4 - 117285*a^8*tan(d*x + c)^3 + 74752*I*a^8*tan(d*x + c)^
2 + 26775*a^8*tan(d*x + c) - 4096*I*a^8)/(tan(d*x + c)^16 + 8*tan(d*x + c)^
14 + 28*tan(d*x + c)^12 + 56*tan(d*x + c)^10 + 70*tan(d*x + c)^8 + 56*tan(d
*x + c)^6 + 28*tan(d*x + c)^4 + 8*tan(d*x + c)^2 + 1))/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1457 vs. 2(175) = 350.

Time = 1.53 (sec) , antiderivative size = 1457, normalized size of antiderivative = 6.48

$$\int \cos^{16}(c+dx)(a+ia\tan(c+dx))^8\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^16*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/13762560*(53760*a^8*d*x*e^(28*I*d*x + 14*I*c) + 752640*a^8*d*x*e^(26*I*d*
x + 12*I*c) + 4892160*a^8*d*x*e^(24*I*d*x + 10*I*c) + 19568640*a^8*d*x*e^(2
2*I*d*x + 8*I*c) + 53813760*a^8*d*x*e^(20*I*d*x + 6*I*c) + 107627520*a^8*d*
x*e^(18*I*d*x + 4*I*c) + 161441280*a^8*d*x*e^(16*I*d*x + 2*I*c) + 161441280
*a^8*d*x*e^(12*I*d*x - 2*I*c) + 107627520*a^8*d*x*e^(10*I*d*x - 4*I*c) + 53
813760*a^8*d*x*e^(8*I*d*x - 6*I*c) + 19568640*a^8*d*x*e^(6*I*d*x - 8*I*c) +
 4892160*a^8*d*x*e^(4*I*d*x - 10*I*c) + 752640*a^8*d*x*e^(2*I*d*x - 12*I*c)
 + 184504320*a^8*d*x*e^(14*I*d*x) + 53760*a^8*d*x*e^(-14*I*c) - 25935*I*a^8
*e^(28*I*d*x + 14*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 363090*I*a^8*e^(26*I*
d*x + 12*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 2360085*I*a^8*e^(24*I*d*x + 10
*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 9440340*I*a^8*e^(22*I*d*x + 8*I*c)*log
(e^(2*I*d*x + 2*I*c) + 1) - 25960935*I*a^8*e^(20*I*d*x + 6*I*c)*log(e^(2*I*
d*x + 2*I*c) + 1) - 51921870*I*a^8*e^(18*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*
I*c) + 1) - 77882805*I*a^8*e^(16*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1
) - 77882805*I*a^8*e^(12*I*d*x - 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 5192
1870*I*a^8*e^(10*I*d*x - 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 25960935*I*a
^8*e^(8*I*d*x - 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 9440340*I*a^8*e^(6*I*
d*x - 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 2360085*I*a^8*e^(4*I*d*x - 10*I
*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 363090*I*a^8*e^(2*I*d*x - 12*I*c)*log(e^
(2*I*d*x + 2*I*c) + 1) - 89008920*I*a^8*e^(14*I*d*x)*log(e^(2*I*d*x + 2*I*c
) + 1) - 25935*I*a^8*e^(-14*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) + 25935*I*a^8
*e^(28*I*d*x + 14*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 363090*I*a^8*e^(26*I
*d*x + 12*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 2360085*I*a^8*e^(24*I*d*x +
10*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 9440340*I*a^8*e^(22*I*d*x + 8*I*c)*
log(e^(2*I*d*x) + e^(-2*I*c)) + 25960935*I*a^8*e^(20*I*d*x + 6*I*c)*log(e^(
2*I*d*x) + e^(-2*I*c)) + 51921870*I*a^8*e^(18*I*d*x + 4*I*c)*log(e^(2*I*d*x
) + e^(-2*I*c)) + 77882805*I*a^8*e^(16*I*d*x + 2*I*c)*log(e^(2*I*d*x) + e^(
-2*I*c)) + 77882805*I*a^8*e^(12*I*d*x - 2*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)
) + 51921870*I*a^8*e^(10*I*d*x - 4*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 259
60935*I*a^8*e^(8*I*d*x - 6*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 9440340*I*a
^8*e^(6*I*d*x - 8*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 2360085*I*a^8*e^(4*I
*d*x - 10*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 363090*I*a^8*e^(2*I*d*x - 12
*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 89008920*I*a^8*e^(14*I*d*x)*log(e^(2*
I*d*x) + e^(-2*I*c)) + 25935*I*a^8*e^(-14*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)
```

```
) - 3360*I*a^8*e^(44*I*d*x + 30*I*c) - 77760*I*a^8*e^(42*I*d*x + 28*I*c) -
861280*I*a^8*e^(40*I*d*x + 26*I*c) - 6075776*I*a^8*e^(38*I*d*x + 24*I*c) -
30645664*I*a^8*e^(36*I*d*x + 22*I*c) - 117621056*I*a^8*e^(34*I*d*x + 20*I*c
) - 356948704*I*a^8*e^(32*I*d*x + 18*I*c) - 878640896*I*a^8*e^(30*I*d*x + 1
6*I*c) - 1785698272*I*a^8*e^(28*I*d*x + 14*I*c) - 3034111808*I*a^8*e^(26*I*
d*x + 12*I*c) - 4346890912*I*a^8*e^(24*I*d*x + 10*I*c) - 5277021568*I*a^8*e
^(22*I*d*x + 8*I*c) - 5435017952*I*a^8*e^(20*I*d*x + 6*I*c) - 4735681216*I*
a^8*e^(18*I*d*x + 4*I*c) - 3464933024*I*a^8*e^(16*I*d*x + 2*I*c) - 10369936
64*I*a^8*e^(12*I*d*x - 2*I*c) - 404782336*I*a^8*e^(10*I*d*x - 4*I*c) - 1200
14720*I*a^8*e^(8*I*d*x - 6*I*c) - 25338880*I*a^8*e^(6*I*d*x - 8*I*c) - 3386
880*I*a^8*e^(4*I*d*x - 10*I*c) - 215040*I*a^8*e^(2*I*d*x - 12*I*c) - 210182
8096*I*a^8*e^(14*I*d*x))/(d*e^(28*I*d*x + 14*I*c) + 14*d*e^(26*I*d*x + 12*I
*c) + 91*d*e^(24*I*d*x + 10*I*c) + 364*d*e^(22*I*d*x + 8*I*c) + 1001*d*e^(2
0*I*d*x + 6*I*c) + 2002*d*e^(18*I*d*x + 4*I*c) + 3003*d*e^(16*I*d*x + 2*I*c
) + 3003*d*e^(12*I*d*x - 2*I*c) + 2002*d*e^(10*I*d*x - 4*I*c) + 1001*d*e^(8
*I*d*x - 6*I*c) + 364*d*e^(6*I*d*x - 8*I*c) + 91*d*e^(4*I*d*x - 10*I*c) + 1
4*d*e^(2*I*d*x - 12*I*c) + 3432*d*e^(14*I*d*x) + d*e^(-14*I*c))
```

Mupad [B] (verification not implemented)

Time = 5.67 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.87

$$\int \cos^{16}(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{a^8\,x}{256} - \frac{-\frac{a^8\tan(c+d\,x)^7}{256} - \frac{a^8\tan(c+d\,x)^6\,1\mathrm{i}}{32} + \frac{85\,a^8\tan(c+d\,x)^5}{768} + \frac{a^8\tan(c+d\,x)^4\,11\mathrm{i}}{48} - \frac{1193\,a^8\tan(c+d\,x)^3}{3840} - \frac{a^8\tan\cdots}{\cdots}}{d\,\left(\tan\left(c+d\,x\right)^8 + \tan\left(c+d\,x\right)^7\,8\mathrm{i} - 28\tan\left(c+d\,x\right)^6 - \tan\left(c+d\,x\right)^5\,56\mathrm{i} + 70\tan\left(c+d\,x\right)^4 + \tan\left(c+\cdots\right.\right.}$$

```
[In] int(cos(c + d*x)^16*(a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] (a^8*x)/256 - ((5993*a^8*tan(c + d*x))/26880 + (a^8*16i)/105 - (a^8*tan(c +
 d*x)^2*143i)/480 - (1193*a^8*tan(c + d*x)^3)/3840 + (a^8*tan(c + d*x)^4*11
i)/48 + (85*a^8*tan(c + d*x)^5)/768 - (a^8*tan(c + d*x)^6*1i)/32 - (a^8*tan
(c + d*x)^7)/256)/(d*(tan(c + d*x)^3*56i - 28*tan(c + d*x)^2 - tan(c + d*x)
*8i + 70*tan(c + d*x)^4 - tan(c + d*x)^5*56i - 28*tan(c + d*x)^6 + tan(c +
d*x)^7*8i + tan(c + d*x)^8 + 1))
```

3.90 $\int \cos^{18}(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 24, antiderivative size = 279

$$
\begin{aligned}
&\int \cos^{18}(c+dx)(a+ia\tan(c+dx))^8\,dx \\
&= \frac{5a^8x}{512} - \frac{ia^{17}}{36d(a-ia\tan(c+dx))^9} - \frac{ia^{16}}{32d(a-ia\tan(c+dx))^8} \\
&\quad - \frac{3ia^{15}}{112d(a-ia\tan(c+dx))^7} - \frac{ia^{14}}{48d(a-ia\tan(c+dx))^6} - \frac{ia^{13}}{64d(a-ia\tan(c+dx))^5} \\
&\quad - \frac{3ia^{12}}{256d(a-ia\tan(c+dx))^4} - \frac{7ia^{11}}{768d(a-ia\tan(c+dx))^3} - \frac{ia^{10}}{128d(a-ia\tan(c+dx))^2} \\
&\quad - \frac{9ia^9}{1024d(a-ia\tan(c+dx))} + \frac{ia^9}{1024d(a+ia\tan(c+dx))}
\end{aligned}
$$

```
[Out] 5/512*a^8*x-1/36*I*a^17/d/(a-I*a*tan(d*x+c))^9-1/32*I*a^16/d/(a-I*a*tan(d*x
+c))^8-3/112*I*a^15/d/(a-I*a*tan(d*x+c))^7-1/48*I*a^14/d/(a-I*a*tan(d*x+c))
^6-1/64*I*a^13/d/(a-I*a*tan(d*x+c))^5-3/256*I*a^12/d/(a-I*a*tan(d*x+c))^4-7
/768*I*a^11/d/(a-I*a*tan(d*x+c))^3-1/128*I*a^10/d/(a-I*a*tan(d*x+c))^2-9/10
24*I*a^9/d/(a-I*a*tan(d*x+c))+1/1024*I*a^9/d/(a+I*a*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used

$= \{3568, 46, 212\}$

$$\int \cos^{18}(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$= -\frac{ia^{17}}{36d(a-ia\tan(c+dx))^9} - \frac{ia^{16}}{32d(a-ia\tan(c+dx))^8} - \frac{3ia^{15}}{112d(a-ia\tan(c+dx))^7} - \frac{ia^{14}}{48d(a-ia\tan(c+dx))^6} - \frac{ia^{13}}{64d(a-ia\tan(c+dx))^5} - \frac{3ia^{12}}{256d(a-ia\tan(c+dx))^4} - \frac{7ia^{11}}{768d(a-ia\tan(c+dx))^3} - \frac{ia^{10}}{128d(a-ia\tan(c+dx))^2} - \frac{9ia^9}{1024d(a-ia\tan(c+dx))} + \frac{ia^9}{1024d(a+ia\tan(c+dx))} + \frac{5a^8x}{512}$$

```
[In] Int[Cos[c + d*x]^18*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (5*a^8*x)/512 - ((I/36)*a^17)/(d*(a - I*a*Tan[c + d*x])^9) - ((I/32)*a^16)/
(d*(a - I*a*Tan[c + d*x])^8) - (((3*I)/112)*a^15)/(d*(a - I*a*Tan[c + d*x])
^7) - ((I/48)*a^14)/(d*(a - I*a*Tan[c + d*x])^6) - ((I/64)*a^13)/(d*(a - I*
a*Tan[c + d*x])^5) - (((3*I)/256)*a^12)/(d*(a - I*a*Tan[c + d*x])^4) - (((7
*I)/768)*a^11)/(d*(a - I*a*Tan[c + d*x])^3) - ((I/128)*a^10)/(d*(a - I*a*Ta
n[c + d*x])^2) - (((9*I)/1024)*a^9)/(d*(a - I*a*Tan[c + d*x])) + ((I/1024)*
a^9)/(d*(a + I*a*Tan[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{(ia^{19})\,\text{Subst}\left(\int \frac{1}{(a-x)^{10}(a+x)^2}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^{19})\,\text{Subst}\left(\int \left(\frac{1}{4a^2(a-x)^{10}} + \frac{1}{4a^3(a-x)^9} + \frac{3}{16a^4(a-x)^8} + \frac{1}{8a^5(a-x)^7} + \frac{5}{64a^6(a-x)^6} + \frac{3}{64a^7(a-x)^5} + \frac{7}{256a^8(a-}\right.\right.}{d}$$

$$= -\frac{ia^{17}}{36d(a-ia\tan(c+dx))^9} - \frac{ia^{16}}{32d(a-ia\tan(c+dx))^8} - \frac{3ia^{15}}{112d(a-ia\tan(c+dx))^7} - \frac{ia^{14}}{48d(a-ia\tan(c+dx))^6} - \frac{ia^{13}}{64d(a-ia\tan(c+dx))^5} - \frac{3ia^{12}}{256d(a-ia\tan(c+dx))^4} - \frac{7ia^{11}}{768d(a-ia\tan(c+dx))^3} - \frac{ia^{10}}{128d(a-ia\tan(c+dx))^2} - \frac{9ia^9}{1024d(a-ia\tan(c+dx))} + \frac{ia^9}{1024d(a+ia\tan(c+dx))} - \frac{(5ia^9)\,\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, ia\tan(c+dx)\right)}{512d}$$

$$= \frac{5a^8x}{512} - \frac{ia^{17}}{36d(a-ia\tan(c+dx))^9} - \frac{ia^{16}}{32d(a-ia\tan(c+dx))^8} - \frac{3ia^{15}}{112d(a-ia\tan(c+dx))^7} - \frac{ia^{14}}{48d(a-ia\tan(c+dx))^6} - \frac{ia^{13}}{64d(a-ia\tan(c+dx))^5} - \frac{3ia^{12}}{256d(a-ia\tan(c+dx))^4} - \frac{7ia^{11}}{768d(a-ia\tan(c+dx))^3} - \frac{ia^{10}}{128d(a-ia\tan(c+dx))^2} - \frac{9ia^9}{1024d(a-ia\tan(c+dx))} + \frac{ia^9}{1024d(a+ia\tan(c+dx))}$$

Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.67

$$\int \cos^{18}(c+dx)(a+ia\tan(c+dx))^8\,dx =$$

$$-\frac{ia^8\sec^{10}(c+dx)(7938 + 14112\cos(2(c+dx)) + 10080\cos(4(c+dx)) + 6480\cos(6(c+dx)) + 2462\,\text{co}}{}$$

```
[In] Integrate[Cos[c + d*x]^18*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((-1/258048*I)*a^8*Sec[c + d*x]^10*(7938 + 14112*Cos[2*(c + d*x)] + 10080*C
os[4*(c + d*x)] + 6480*Cos[6*(c + d*x)] + 2462*Cos[8*(c + d*x)] - 112*Cos[1
```

```
0*(c + d*x)] - (3528*I)*Sin[2*(c + d*x)] - (5040*I)*Sin[4*(c + d*x)] - (486
0*I)*Sin[6*(c + d*x)] - (2147*I)*Sin[8*(c + d*x)] + 2520*ArcTan[Tan[c + d*x
]]*(I*Cos[8*(c + d*x)] + Sin[8*(c + d*x)]) + (140*I)*Sin[10*(c + d*x)]))/(d
*(-I + Tan[c + d*x])*(I + Tan[c + d*x])^9)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 788 vs. 2(237) = 474.

Time = 2.28 (sec) , antiderivative size = 789, normalized size of antiderivative = 2.83

Expression too large to display

```
[In] int(cos(d*x+c)^18*(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 1/d*(a^8*(-1/18*sin(d*x+c)^7*cos(d*x+c)^11-7/288*sin(d*x+c)^5*cos(d*x+c)^11
-5/576*sin(d*x+c)^3*cos(d*x+c)^11-5/2304*sin(d*x+c)*cos(d*x+c)^11+1/4608*(c
os(d*x+c)^9+9/8*cos(d*x+c)^7+21/16*cos(d*x+c)^5+105/64*cos(d*x+c)^3+315/128
*cos(d*x+c))*sin(d*x+c)+35/65536*d*x+35/65536*c)-4/9*I*a^8*cos(d*x+c)^18-28
*a^8*(-1/18*sin(d*x+c)^5*cos(d*x+c)^13-5/288*sin(d*x+c)^3*cos(d*x+c)^13-5/1
344*sin(d*x+c)*cos(d*x+c)^13+5/16128*(cos(d*x+c)^11+11/10*cos(d*x+c)^9+99/8
0*cos(d*x+c)^7+231/160*cos(d*x+c)^5+231/128*cos(d*x+c)^3+693/256*cos(d*x+c)
)*sin(d*x+c)+55/65536*d*x+55/65536*c)-8*I*a^8*(-1/18*cos(d*x+c)^12*sin(d*x+
c)^6-1/48*sin(d*x+c)^4*cos(d*x+c)^12-1/168*sin(d*x+c)^2*cos(d*x+c)^12-1/100
8*cos(d*x+c)^12)+70*a^8*(-1/18*sin(d*x+c)^3*cos(d*x+c)^15-1/96*sin(d*x+c)*c
os(d*x+c)^15+1/1344*(cos(d*x+c)^13+13/12*cos(d*x+c)^11+143/120*cos(d*x+c)^9
+429/320*cos(d*x+c)^7+1001/640*cos(d*x+c)^5+1001/512*cos(d*x+c)^3+3003/1024
*cos(d*x+c))*sin(d*x+c)+143/65536*d*x+143/65536*c)-56*I*a^8*(-1/18*cos(d*x+
c)^16*sin(d*x+c)^2-1/144*cos(d*x+c)^16)-28*a^8*(-1/18*sin(d*x+c)*cos(d*x+c)
^17+1/288*(cos(d*x+c)^15+15/14*cos(d*x+c)^13+65/56*cos(d*x+c)^11+143/112*co
s(d*x+c)^9+1287/896*cos(d*x+c)^7+429/256*cos(d*x+c)^5+2145/1024*cos(d*x+c)^
3+6435/2048*cos(d*x+c))*sin(d*x+c)+715/65536*d*x+715/65536*c)+56*I*a^8*(-1/
18*cos(d*x+c)^14*sin(d*x+c)^4-1/72*cos(d*x+c)^14*sin(d*x+c)^2-1/504*cos(d*x
+c)^14)+a^8*(1/18*(cos(d*x+c)^17+17/16*cos(d*x+c)^15+255/224*cos(d*x+c)^13+
1105/896*cos(d*x+c)^11+2431/1792*cos(d*x+c)^9+21879/14336*cos(d*x+c)^7+7293
/4096*cos(d*x+c)^5+36465/16384*cos(d*x+c)^3+109395/32768*cos(d*x+c))*sin(d*
x+c)+12155/65536*d*x+12155/65536*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.58

$$\int \cos^{18}(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{\left(5040\,a^8dxe^{(2i\,dx+2i\,c)} - 28i\,a^8e^{(20i\,dx+20i\,c)} - 315i\,a^8e^{(18i\,dx+18i\,c)} - 1620i\,a^8e^{(16i\,dx+16i\,c)} - 5040i\,a^8e^{(14i\,dx+14i\,c)}\right.}{}$$

```
[In] integrate(cos(d*x+c)^18*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/516096*(5040*a^8*d*x*e^(2*I*d*x + 2*I*c) - 28*I*a^8*e^(20*I*d*x + 20*I*c)
 - 315*I*a^8*e^(18*I*d*x + 18*I*c) - 1620*I*a^8*e^(16*I*d*x + 16*I*c) - 504
0*I*a^8*e^(14*I*d*x + 14*I*c) - 10584*I*a^8*e^(12*I*d*x + 12*I*c) - 15876*I
*a^8*e^(10*I*d*x + 10*I*c) - 17640*I*a^8*e^(8*I*d*x + 8*I*c) - 15120*I*a^8*
e^(6*I*d*x + 6*I*c) - 11340*I*a^8*e^(4*I*d*x + 4*I*c) + 252*I*a^8)*e^(-2*I*
d*x - 2*I*c)/d
```

Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.48

$$\int \cos^{18}(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{5a^8x}{512} + \begin{cases} \frac{(-277298568799925181577403826176ia^8d^9e^{20ic}e^{18idx} - 311960889899915829274579304448 0ia^8d^9e^{18ic}e^{16idx} - 16043702909138528362692\cdots}{} \\ x\left(-\frac{5a^8}{512} + \frac{(a^8e^{20ic}+10a^8e^{18ic}+45a^8e^{16ic}+120a^8e^{14ic}+210a^8e^{12ic}+252a^8e^{10ic}+210a^8e^{8ic}+120a^8e^{6ic}+45a^8e^{4ic}+10a^8e^{2ic}+a^8)e^{-2ic}}{1024}\right) \end{cases}$$

```
[In] integrate(cos(d*x+c)**18*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] 5*a**8*x/512 + Piecewise(((-277298568799925181577403826176*I*a**8*d**9*exp(
20*I*c)*exp(18*I*d*x) - 3119608898999158292745793044480*I*a**8*d**9*exp(18*
I*c)*exp(16*I*d*x) - 16043702909138528362692649943040*I*a**8*d**9*exp(16*I*
c)*exp(14*I*d*x) - 49913742383986532683932688711680*I*a**8*d**9*exp(14*I*c)
*exp(12*I*d*x) - 104818859006371718636258646294528*I*a**8*d**9*exp(12*I*c)*
exp(10*I*d*x) - 157228288509557577954387969441792*I*a**8*d**9*exp(10*I*c)*e
xp(8*I*d*x) - 174698098343952864393764410490880*I*a**8*d**9*exp(8*I*c)*exp(
6*I*d*x) - 149741227151959598051798066135040*I*a**8*d**9*exp(6*I*c)*exp(4*I
*d*x) - 112305920363969698538848549601280*I*a**8*d**9*exp(4*I*c)*exp(2*I*d*
x) + 24956871191993266341966344355840*I*a**8*d**9*exp(-2*I*d*x))*exp(-2*I*c)
/(5111167220120220946834707324076032*d**10), Ne(d**10*exp(2*I*c), 0)), (x*(
-5*a**8/512 + (a**8*exp(20*I*c) + 10*a**8*exp(18*I*c) + 45*a**8*exp(16*I*c)
 + 120*a**8*exp(14*I*c) + 210*a**8*exp(12*I*c) + 252*a**8*exp(10*I*c) + 210
*a**8*exp(8*I*c) + 120*a**8*exp(6*I*c) + 45*a**8*exp(4*I*c) + 10*a**8*exp(2
*I*c) + a**8)*exp(-2*I*c)/1024), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.96

$$\int \cos^{18}(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{315\,(dx+c)a^8 + \frac{315\,a^8\tan(dx+c)^{17}+2730\,a^8\tan(dx+c)^{15}+10458\,a^8\tan(dx+c)^{13}+23202\,a^8\tan(dx+c)^{11}+32768\,a^8\tan(dx+c)^{9}+27486\,a^8}{\tan(dx+c)^{18}+9\,\tan(dx+c)^{16}+36\,\tan(dx+c)^{14}+84\,\tan(}}{}$$

```
[In] integrate(cos(d*x+c)^18*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/32256*(315*(d*x + c)*a^8 + (315*a^8*tan(d*x + c)^17 + 2730*a^8*tan(d*x +
c)^15 + 10458*a^8*tan(d*x + c)^13 + 23202*a^8*tan(d*x + c)^11 + 32768*a^8*t
an(d*x + c)^9 + 27486*a^8*tan(d*x + c)^7 + 21504*I*a^8*tan(d*x + c)^6 + 863
10*a^8*tan(d*x + c)^5 - 119808*I*a^8*tan(d*x + c)^4 - 121002*a^8*tan(d*x +
c)^3 + 82944*I*a^8*tan(d*x + c)^2 + 31941*a^8*tan(d*x + c) - 5120*I*a^8)/(t
an(d*x + c)^18 + 9*tan(d*x + c)^16 + 36*tan(d*x + c)^14 + 84*tan(d*x + c)^1
2 + 126*tan(d*x + c)^10 + 126*tan(d*x + c)^8 + 84*tan(d*x + c)^6 + 36*tan(d
*x + c)^4 + 9*tan(d*x + c)^2 + 1))/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1514 vs. 2(217) = 434.

Time = 1.63 (sec) , antiderivative size = 1514, normalized size of antiderivative = 5.43

$$\int \cos^{18}(c+dx)(a+ia\tan(c+dx))^8\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^18*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/16515072*(161280*a^8*d*x*e^(30*I*d*x + 16*I*c) + 2257920*a^8*d*x*e^(28*I*
d*x + 14*I*c) + 14676480*a^8*d*x*e^(26*I*d*x + 12*I*c) + 58705920*a^8*d*x*e
^(24*I*d*x + 10*I*c) + 161441280*a^8*d*x*e^(22*I*d*x + 8*I*c) + 322882560*a
^8*d*x*e^(20*I*d*x + 6*I*c) + 484323840*a^8*d*x*e^(18*I*d*x + 4*I*c) + 5535
12960*a^8*d*x*e^(16*I*d*x + 2*I*c) + 322882560*a^8*d*x*e^(12*I*d*x - 2*I*c)
 + 161441280*a^8*d*x*e^(10*I*d*x - 4*I*c) + 58705920*a^8*d*x*e^(8*I*d*x - 6
*I*c) + 14676480*a^8*d*x*e^(6*I*d*x - 8*I*c) + 2257920*a^8*d*x*e^(4*I*d*x -
 10*I*c) + 161280*a^8*d*x*e^(2*I*d*x - 12*I*c) + 484323840*a^8*d*x*e^(14*I*
d*x) - 75789*I*a^8*e^(30*I*d*x + 16*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 106
1046*I*a^8*e^(28*I*d*x + 14*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 6896799*I*a
^8*e^(26*I*d*x + 12*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 27587196*I*a^8*e^(2
4*I*d*x + 10*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 75864789*I*a^8*e^(22*I*d*x
```

+ 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 151729578*I*a^8*e^(20*I*d*x + 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 227594367*I*a^8*e^(18*I*d*x + 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 260107848*I*a^8*e^(16*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 151729578*I*a^8*e^(12*I*d*x - 2*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 75864789*I*a^8*e^(10*I*d*x - 4*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 27587196*I*a^8*e^(8*I*d*x - 6*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 6896799*I*a^8*e^(6*I*d*x - 8*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 1061046*I*a^8*e^(4*I*d*x - 10*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 75789*I*a^8*e^(2*I*d*x - 12*I*c)*log(e^(2*I*d*x + 2*I*c) + 1) - 227594367*I*a^8*e^(14*I*d*x)*log(e^(2*I*d*x + 2*I*c) + 1) + 75789*I*a^8*e^(30*I*d*x + 16*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 1061046*I*a^8*e^(28*I*d*x + 14*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 6896799*I*a^8*e^(26*I*d*x + 12*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 27587196*I*a^8*e^(24*I*d*x + 10*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 75864789*I*a^8*e^(22*I*d*x + 8*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 151729578*I*a^8*e^(20*I*d*x + 6*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 227594367*I*a^8*e^(18*I*d*x + 4*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 260107848*I*a^8*e^(16*I*d*x + 2*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 151729578*I*a^8*e^(12*I*d*x - 2*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 75864789*I*a^8*e^(10*I*d*x - 4*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 27587196*I*a^8*e^(8*I*d*x - 6*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 6896799*I*a^8*e^(6*I*d*x - 8*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 1061046*I*a^8*e^(4*I*d*x - 10*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 75789*I*a^8*e^(2*I*d*x - 12*I*c)*log(e^(2*I*d*x) + e^(-2*I*c)) + 227594367*I*a^8*e^(14*I*d*x)*log(e^(2*I*d*x) + e^(-2*I*c)) - 896*I*a^8*e^(48*I*d*x + 34*I*c) - 22624*I*a^8*e^(46*I*d*x + 32*I*c) - 274496*I*a^8*e^(44*I*d*x + 30*I*c) - 2130464*I*a^8*e^(42*I*d*x + 28*I*c) - 11880064*I*a^8*e^(40*I*d*x + 26*I*c) - 50679776*I*a^8*e^(38*I*d*x + 24*I*c) - 171966144*I*a^8*e^(36*I*d*x + 22*I*c) - 476470176*I*a^8*e^(34*I*d*x + 20*I*c) - 1098297984*I*a^8*e^(32*I*d*x + 18*I*c) - 2135476640*I*a^8*e^(30*I*d*x + 16*I*c) - 3538601920*I*a^8*e^(28*I*d*x + 14*I*c) - 5032909280*I*a^8*e^(26*I*d*x + 12*I*c) - 6165461120*I*a^8*e^(24*I*d*x + 10*I*c) - 6498731680*I*a^8*e^(22*I*d*x + 8*I*c) - 5857001024*I*a^8*e^(20*I*d*x + 6*I*c) - 4459555296*I*a^8*e^(18*I*d*x + 4*I*c) - 2817258624*I*a^8*e^(16*I*d*x + 2*I*c) - 573963264*I*a^8*e^(12*I*d*x - 2*I*c) - 168384384*I*a^8*e^(10*I*d*x - 4*I*c) - 32288256*I*a^8*e^(8*I*d*x - 6*I*c) - 2628864*I*a^8*e^(6*I*d*x - 8*I*c) + 370944*I*a^8*e^(4*I*d*x - 10*I*c) + 112896*I*a^8*e^(2*I*d*x - 12*I*c) - 1439738496*I*a^8*e^(14*I*d*x) + 8064*I*a^8*e^(-14*I*c))/(d*e^(30*I*d*x + 16*I*c) + 14*d*e^(28*I*d*x + 14*I*c) + 91*d*e^(26*I*d*x + 12*I*c) + 364*d*e^(24*I*d*x + 10*I*c) + 1001*d*e^(22*I*d*x + 8*I*c) + 2002*d*e^(20*I*d*x + 6*I*c) + 3003*d*e^(18*I*d*x + 4*I*c) + 3432*d*e^(16*I*d*x + 2*I*c) + 2002*d*e^(12*I*d*x - 2*I*c) + 1001*d*e^(10*I*d*x - 4*I*c) + 364*d*e^(8*I*d*x - 6*I*c) + 91*d*e^(6*I*d*x - 8*I*c) + 14*d*e^(4*I*d*x - 10*I*c) + d*e^(2*I*d*x - 12*I*c) + 3003*d*e^(14*I*d*x))

Mupad [B] (verification not implemented)

Time = 6.04 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.83

$$\int \cos^{18}(c+dx)(a+ia\tan(c+dx))^8\,dx=\frac{5\,a^8\,x}{512}$$
$$+\frac{\frac{5\,a^8\,\tan(c+d\,x)^9}{512}+\frac{a^8\,\tan(c+d\,x)^8\,5\mathrm{i}}{64}-\frac{205\,a^8\,\tan(c+d\,x)^7}{768}-\frac{a^8\,\tan(c+d\,x)^6\,95\mathrm{i}}{192}+\frac{a^8\,\tan(c+d\,x)^5}{2}+\frac{a^8\,\tan(c+d\,x)^4\,11}{64}}{d\,\left(\tan\left(c+d\,x\right)^{10}+\tan\left(c+d\,x\right)^9\,8\mathrm{i}-27\,\tan\left(c+d\,x\right)^8-\tan\left(c+d\,x\right)^7\,48\mathrm{i}+42\,\tan\left(c+d\,x\right)^6+42\,\tan}$$

```
[In] int(cos(c + d*x)^18*(a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] (5*a^8*x)/512 + ((a^8*tan(c + d*x)^2*163i)/448 - (a^8*10i)/63 - (9019*a^8*t
an(c + d*x))/32256 + (393*a^8*tan(c + d*x)^3)/1792 + (a^8*tan(c + d*x)^4*11
i)/64 + (a^8*tan(c + d*x)^5)/2 - (a^8*tan(c + d*x)^6*95i)/192 - (205*a^8*ta
n(c + d*x)^7)/768 + (a^8*tan(c + d*x)^8*5i)/64 + (5*a^8*tan(c + d*x)^9)/512
)/(d*(tan(c + d*x)^3*48i - 27*tan(c + d*x)^2 - tan(c + d*x)*8i + 42*tan(c +
 d*x)^4 + 42*tan(c + d*x)^6 - tan(c + d*x)^7*48i - 27*tan(c + d*x)^8 + tan(
c + d*x)^9*8i + tan(c + d*x)^10 + 1))
```

3.91 $\int \cos(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 22, antiderivative size = 235

$$\int \cos(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{3003a^8\operatorname{arctanh}(\sin(c+dx))}{16d} - \frac{3003ia^8\sec(c+dx)}{16d} - \frac{13ia^3\sec(c+dx)(a+ia\tan(c+dx))^5}{6d} - \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d} - \frac{429ia^2\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^3}{40d} - \frac{143i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^4}{30d} - \frac{1001i\sec(c+dx)\left(a^4+ia^4\tan(c+dx)\right)^2}{40d} - \frac{1001i\sec(c+dx)\left(a^8+ia^8\tan(c+dx)\right)}{16d}$$

```
[Out] -3003/16*a^8*arctanh(sin(d*x+c))/d-3003/16*I*a^8*sec(d*x+c)/d-13/6*I*a^3*se
c(d*x+c)*(a+I*a*tan(d*x+c))^5/d-2*I*a*cos(d*x+c)*(a+I*a*tan(d*x+c))^7/d-429
/40*I*a^2*sec(d*x+c)*(a^2+I*a^2*tan(d*x+c))^3/d-143/30*I*sec(d*x+c)*(a^2+I*
a^2*tan(d*x+c))^4/d-1001/40*I*sec(d*x+c)*(a^4+I*a^4*tan(d*x+c))^2/d-1001/16
*I*sec(d*x+c)*(a^8+I*a^8*tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3577, 3579, 3567, 3855}

$$\int \cos(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{3003a^8\text{arctanh}(\sin(c+dx))}{16d} - \frac{3003ia^8\sec(c+dx)}{16d} - \frac{1001i\sec(c+dx)\left(a^8+ia^8\tan(c+dx)\right)}{16d} - \frac{1001i\sec(c+dx)\left(a^4+ia^4\tan(c+dx)\right)^2}{40d} - \frac{13ia^3\sec(c+dx)(a+ia\tan(c+dx))^5}{6d} - \frac{429ia^2\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^3}{40d} - \frac{143i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^4}{30d} - \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d}$$

```
[In] Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (-3003*a^8*ArcTanh[Sin[c + d*x]])/(16*d) - (((3003*I)/16)*a^8*Sec[c + d*x])
/d - (((13*I)/6)*a^3*Sec[c + d*x]*(a + I*a*Tan[c + d*x])^5)/d - ((2*I)*a*Co
s[c + d*x]*(a + I*a*Tan[c + d*x])^7)/d - (((429*I)/40)*a^2*Sec[c + d*x]*(a^
2 + I*a^2*Tan[c + d*x])^3)/d - (((143*I)/30)*Sec[c + d*x]*(a^2 + I*a^2*Tan[
c + d*x])^4)/d - (((1001*I)/40)*Sec[c + d*x]*(a^4 + I*a^4*Tan[c + d*x])^2)/
d - (((1001*I)/16)*Sec[c + d*x]*(a^8 + I*a^8*Tan[c + d*x]))/d
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
```

```
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d} \\
&\quad -\left(13a^2\right)\int \sec(c+dx)(a+ia\tan(c+dx))^6\,dx \\
&= -\frac{13ia^3\sec(c+dx)(a+ia\tan(c+dx))^5}{6d}-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d} \\
&\quad -\frac{1}{6}\left(143a^3\right)\int \sec(c+dx)(a+ia\tan(c+dx))^5\,dx \\
&= -\frac{13ia^3\sec(c+dx)(a+ia\tan(c+dx))^5}{6d}-\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d} \\
&\quad -\frac{143i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^4}{30d} \\
&\quad -\frac{1}{10}\left(429a^4\right)\int \sec(c+dx)(a+ia\tan(c+dx))^4\,dx \\
&= -\frac{429ia^5\sec(c+dx)(a+ia\tan(c+dx))^3}{40d}-\frac{13ia^3\sec(c+dx)(a+ia\tan(c+dx))^5}{6d} \\
&\quad -\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d}-\frac{143i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^4}{30d} \\
&\quad -\frac{1}{40}\left(3003a^5\right)\int \sec(c+dx)(a+ia\tan(c+dx))^3\,dx
\end{aligned}$$

$$= -\frac{429ia^5\sec(c+dx)(a+ia\tan(c+dx))^3}{40d} - \frac{13ia^3\sec(c+dx)(a+ia\tan(c+dx))^5}{6d} - \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d} - \frac{143i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^4}{30d} - \frac{1001i\sec(c+dx)\left(a^4+ia^4\tan(c+dx)\right)^2}{40d} - \frac{1}{8}\left(1001a^6\right)\int\sec(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$= -\frac{429ia^5\sec(c+dx)(a+ia\tan(c+dx))^3}{40d} - \frac{13ia^3\sec(c+dx)(a+ia\tan(c+dx))^5}{6d} - \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d} - \frac{143i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^4}{30d} - \frac{1001i\sec(c+dx)\left(a^4+ia^4\tan(c+dx)\right)^2}{40d} - \frac{1001i\sec(c+dx)\left(a^8+ia^8\tan(c+dx)\right)}{16d} - \frac{1}{16}\left(3003a^7\right)\int\sec(c+dx)(a+ia\tan(c+dx))\,dx$$

$$= -\frac{3003ia^8\sec(c+dx)}{16d} - \frac{429ia^5\sec(c+dx)(a+ia\tan(c+dx))^3}{40d} - \frac{13ia^3\sec(c+dx)(a+ia\tan(c+dx))^5}{6d} - \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d} - \frac{143i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^4}{30d} - \frac{1001i\sec(c+dx)\left(a^4+ia^4\tan(c+dx)\right)^2}{40d} - \frac{1001i\sec(c+dx)\left(a^8+ia^8\tan(c+dx)\right)}{16d} - \frac{1}{16}\left(3003a^8\right)\int\sec(c+dx)\,dx$$

$$= -\frac{3003a^8\text{arctanh}(\sin(c+dx))}{16d} - \frac{3003ia^8\sec(c+dx)}{16d} - \frac{429ia^5\sec(c+dx)(a+ia\tan(c+dx))^3}{40d} - \frac{13ia^3\sec(c+dx)(a+ia\tan(c+dx))^5}{6d} - \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^7}{d} - \frac{143i\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^4}{30d} - \frac{1001i\sec(c+dx)\left(a^4+ia^4\tan(c+dx)\right)^2}{40d} - \frac{1001i\sec(c+dx)\left(a^8+ia^8\tan(c+dx)\right)}{16d}$$

Mathematica [A] (verified)

Time = 2.76 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.87

$$\int \cos(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$=\frac{a^8\cos^2(c+dx)(\cos(8c)-i\sin(8c))\left(-658944i\cos(c+dx)+720720\cos^6(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\right.\right.\right.}{}$$

```
[In] Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*Cos[c + d*x]^2*(Cos[8*c] - I*Sin[8*c])*((-658944*I)*Cos[c + d*x] + 720
720*Cos[c + d*x]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]]) + 5*((-73216*I)*Cos[3*(c + d*x)] - (19968*I)*
Cos[5*(c + d*x)] - (1536*I)*Cos[7*(c + d*x)] + 12870*Sin[c + d*x] + 22165*S
in[3*(c + d*x)] + 10959*Sin[5*(c + d*x)] + 1536*Sin[7*(c + d*x)]))*(-I + Ta
n[c + d*x])^8)/(3840*d*(Cos[d*x] + I*Sin[d*x])^8)
```

Maple [A] (verified)

Time = 90.46 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.64

method	result
risch	$-\frac{128ia^8e^{i(dx+c)}}{d}-\frac{ia^8\left(62475\,e^{11i(dx+c)}+246505\,e^{9i(dx+c)}+416094\,e^{7i(dx+c)}+364194\,e^{5i(dx+c)}+163095\,e^{3i(dx+c)}+\right.}{120d\left(e^{2i(dx+c)}+1\right)^6}$
derivativedivides	$\frac{a^8\left(\frac{\sin^9(dx+c)}{6\cos(dx+c)^6}-\frac{\sin^9(dx+c)}{8\cos(dx+c)^4}+\frac{5\left(\sin^9(dx+c)\right)}{16\cos(dx+c)^2}+\frac{5\left(\sin^7(dx+c)\right)}{16}+\frac{7\left(\sin^5(dx+c)\right)}{16}+\frac{35\left(\sin^3(dx+c)\right)}{48}+\frac{35\sin(dx+c)}{16}-\frac{35\ln(\sec(\right.}{}$
default	$\frac{a^8\left(\frac{\sin^9(dx+c)}{6\cos(dx+c)^6}-\frac{\sin^9(dx+c)}{8\cos(dx+c)^4}+\frac{5\left(\sin^9(dx+c)\right)}{16\cos(dx+c)^2}+\frac{5\left(\sin^7(dx+c)\right)}{16}+\frac{7\left(\sin^5(dx+c)\right)}{16}+\frac{35\left(\sin^3(dx+c)\right)}{48}+\frac{35\sin(dx+c)}{16}-\frac{35\ln(\sec(\right.}{}$

```
[In] int(cos(d*x+c)*(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] -128*I/d*a^8*exp(I*(d*x+c))-1/120*I*a^8/d/(exp(2*I*(d*x+c))+1)^6*(62475*exp
(11*I*(d*x+c))+246505*exp(9*I*(d*x+c))+416094*exp(7*I*(d*x+c))+364194*exp(5
*I*(d*x+c))+163095*exp(3*I*(d*x+c))+29685*exp(I*(d*x+c)))-3003/16/d*a^8*ln(
exp(I*(d*x+c))+I)+3003/16/d*a^8*ln(exp(I*(d*x+c))-I)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.61

$$\int \cos(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{-30720i\,a^8e^{(13i\,dx+13i\,c)} - 309270i\,a^8e^{(11i\,dx+11i\,c)} - 953810i\,a^8e^{(9i\,dx+9i\,c)} - 1446588i\,a^8e^{(7i\,dx+7i\,c)} - 1189188}{}$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/240*(-30720*I*a^8*e^(13*I*d*x + 13*I*c) - 309270*I*a^8*e^(11*I*d*x + 11*I
*c) - 953810*I*a^8*e^(9*I*d*x + 9*I*c) - 1446588*I*a^8*e^(7*I*d*x + 7*I*c)
- 1189188*I*a^8*e^(5*I*d*x + 5*I*c) - 510510*I*a^8*e^(3*I*d*x + 3*I*c) - 90
090*I*a^8*e^(I*d*x + I*c) - 45045*(a^8*e^(12*I*d*x + 12*I*c) + 6*a^8*e^(10*
I*d*x + 10*I*c) + 15*a^8*e^(8*I*d*x + 8*I*c) + 20*a^8*e^(6*I*d*x + 6*I*c) +
 15*a^8*e^(4*I*d*x + 4*I*c) + 6*a^8*e^(2*I*d*x + 2*I*c) + a^8)*log(e^(I*d*x
 + I*c) + I) + 45045*(a^8*e^(12*I*d*x + 12*I*c) + 6*a^8*e^(10*I*d*x + 10*I*
c) + 15*a^8*e^(8*I*d*x + 8*I*c) + 20*a^8*e^(6*I*d*x + 6*I*c) + 15*a^8*e^(4*
I*d*x + 4*I*c) + 6*a^8*e^(2*I*d*x + 2*I*c) + a^8)*log(e^(I*d*x + I*c) - I))
/(d*e^(12*I*d*x + 12*I*c) + 6*d*e^(10*I*d*x + 10*I*c) + 15*d*e^(8*I*d*x + 8
*I*c) + 20*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*I*d*x + 4*I*c) + 6*d*e^(2*I*d*
x + 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.36

$$\int \cos(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{3003a^8\left(\frac{\log\left(e^{idx}-ie^{-ic}\right)}{16} - \frac{\log\left(e^{idx}+ie^{-ic}\right)}{16}\right)}{d}$$
$$+ \frac{-62475ia^8e^{11ic}e^{11idx} - 246505ia^8e^{9ic}e^{9idx} - 416094ia^8e^{7ic}e^{7idx} - 364194ia^8e^{5ic}e^{5idx} - 163095ia^8e^{3ic}e^{3idx}}{120de^{12ic}e^{12idx} + 720de^{10ic}e^{10idx} + 1800de^{8ic}e^{8idx} + 2400de^{6ic}e^{6idx} + 1800de^{4ic}e^{4idx} + 720de^{2ic}e^{2ic}}$$
$$+ \begin{cases} -\frac{128ia^8e^{ic}e^{idx}}{d} & \text{for } d \neq 0 \\ 128a^8xe^{ic} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] 3003*a**8*(log(exp(I*d*x) - I*exp(-I*c))/16 - log(exp(I*d*x) + I*exp(-I*c))
/16)/d + (-62475*I*a**8*exp(11*I*c)*exp(11*I*d*x) - 246505*I*a**8*exp(9*I*c
)*exp(9*I*d*x) - 416094*I*a**8*exp(7*I*c)*exp(7*I*d*x) - 364194*I*a**8*exp(
5*I*c)*exp(5*I*d*x) - 163095*I*a**8*exp(3*I*c)*exp(3*I*d*x) - 29685*I*a**8*
exp(I*c)*exp(I*d*x))/(120*d*exp(12*I*c)*exp(12*I*d*x) + 720*d*exp(10*I*c)*e
```

```
xp(10*I*d*x) + 1800*d*exp(8*I*c)*exp(8*I*d*x) + 2400*d*exp(6*I*c)*exp(6*I*d
*x) + 1800*d*exp(4*I*c)*exp(4*I*d*x) + 720*d*exp(2*I*c)*exp(2*I*d*x) + 120*
d) + Piecewise((-128*I*a**8*exp(I*c)*exp(I*d*x)/d, Ne(d, 0)), (128*a**8*x*e
xp(I*c), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 396 vs. 2(195) = 390.

Time = 0.27 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.69

$$\int \cos(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{5\,a^8\left(\frac{2\left(87\,\sin(dx+c)^5-136\,\sin(dx+c)^3+57\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}+105\,\log\left(\sin\left(dx+c\right)+1\right)-105\,\log\left(\sin\left(dx+c\right)-1\right)-\right.}{}$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/480*(5*a^8*(2*(87*sin(d*x + c)^5 - 136*sin(d*x + c)^3 + 57*sin(d*x + c))
/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) + 105*log(sin(d
*x + c) + 1) - 105*log(sin(d*x + c) - 1) - 96*sin(d*x + c)) + 840*a^8*(2*(9
*sin(d*x + c)^3 - 7*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) +
 15*log(sin(d*x + c) + 1) - 15*log(sin(d*x + c) - 1) - 16*sin(d*x + c)) + 8
400*a^8*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x + c) + 1) - 3*
log(sin(d*x + c) - 1) - 4*sin(d*x + c)) + 26880*I*a^8*(1/cos(d*x + c) + cos
(d*x + c)) + 8960*I*a^8*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x
+ c)) + 768*I*a^8*((15*cos(d*x + c)^4 - 5*cos(d*x + c)^2 + 1)/cos(d*x + c)^
5 + 5*cos(d*x + c)) + 6720*a^8*(log(sin(d*x + c) + 1) - log(sin(d*x + c) -
1) - 2*sin(d*x + c)) + 3840*I*a^8*cos(d*x + c) - 480*a^8*sin(d*x + c))/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 924 vs. 2(195) = 390.

Time = 1.44 (sec) , antiderivative size = 924, normalized size of antiderivative = 3.93

$$\int \cos(c+dx)(a+ia\tan(c+dx))^8\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/61440*(11512215*a^8*e^(12*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 69
073290*a^8*e^(10*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 172683225*a^8
```

```
*e^(8*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 230244300*a^8*e^(6*I*d*x
+ 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 172683225*a^8*e^(4*I*d*x + 4*I*c)*log
(I*e^(I*d*x + I*c) + 1) + 69073290*a^8*e^(2*I*d*x + 2*I*c)*log(I*e^(I*d*x +
 I*c) + 1) - 19305*a^8*e^(12*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) - 1) - 1
15830*a^8*e^(10*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) - 1) - 289575*a^8*e^(
8*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) - 1) - 386100*a^8*e^(6*I*d*x + 6*I*c
)*log(I*e^(I*d*x + I*c) - 1) - 289575*a^8*e^(4*I*d*x + 4*I*c)*log(I*e^(I*d*
x + I*c) - 1) - 115830*a^8*e^(2*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) -
 11512215*a^8*e^(12*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 69073290*
a^8*e^(10*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 172683225*a^8*e^(8*
I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 230244300*a^8*e^(6*I*d*x + 6*I
*c)*log(-I*e^(I*d*x + I*c) + 1) - 172683225*a^8*e^(4*I*d*x + 4*I*c)*log(-I*
e^(I*d*x + I*c) + 1) - 69073290*a^8*e^(2*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I
*c) + 1) + 19305*a^8*e^(12*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) - 1) + 11
5830*a^8*e^(10*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) - 1) + 289575*a^8*e^(
8*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) + 386100*a^8*e^(6*I*d*x + 6*I*
c)*log(-I*e^(I*d*x + I*c) - 1) + 289575*a^8*e^(4*I*d*x + 4*I*c)*log(-I*e^(I
*d*x + I*c) - 1) + 115830*a^8*e^(2*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) -
1) - 7864320*I*a^8*e^(13*I*d*x + 13*I*c) - 79173120*I*a^8*e^(11*I*d*x + 11*
I*c) - 244175360*I*a^8*e^(9*I*d*x + 9*I*c) - 370326528*I*a^8*e^(7*I*d*x + 7
*I*c) - 304432128*I*a^8*e^(5*I*d*x + 5*I*c) - 130690560*I*a^8*e^(3*I*d*x +
3*I*c) - 23063040*I*a^8*e^(I*d*x + I*c) + 11512215*a^8*log(I*e^(I*d*x + I*c
) + 1) - 19305*a^8*log(I*e^(I*d*x + I*c) - 1) - 11512215*a^8*log(-I*e^(I*d*
x + I*c) + 1) + 19305*a^8*log(-I*e^(I*d*x + I*c) - 1))/(d*e^(12*I*d*x + 12*
I*c) + 6*d*e^(10*I*d*x + 10*I*c) + 15*d*e^(8*I*d*x + 8*I*c) + 20*d*e^(6*I*d
*x + 6*I*c) + 15*d*e^(4*I*d*x + 4*I*c) + 6*d*e^(2*I*d*x + 2*I*c) + d)
```

Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.70

$$\int \cos(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{\frac{3019\,a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{8}+\frac{a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}2891\mathrm{i}}{8}-\frac{52795\,a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{24}-\frac{a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}45115\mathrm{i}}{24}+\frac{22415\,a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}}{4}+\frac{a^8\,\mathrm{t}}{}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}1\mathrm{i}-6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}6\mathrm{i}+15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}+\tan\left(\frac{c}{2}\right.\right.}$$
$$-\frac{3003\,a^8\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}$$

```
[In] int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] ((a^8*tan(c/2 + (d*x)/2)^3*160729i)/120 - (127113*a^8*tan(c/2 + (d*x)/2)^2)
/40 + (167237*a^8*tan(c/2 + (d*x)/2)^4)/24 - (a^8*tan(c/2 + (d*x)/2)^5*1297
7i)/4 - (97811*a^8*tan(c/2 + (d*x)/2)^6)/12 + (a^8*tan(c/2 + (d*x)/2)^7*437
```

57i)/12 + (22415*a^8*tan(c/2 + (d*x)/2)^8)/4 - (a^8*tan(c/2 + (d*x)/2)^9*45
115i)/24 - (52795*a^8*tan(c/2 + (d*x)/2)^10)/24 + (a^8*tan(c/2 + (d*x)/2)^1
1*2891i)/8 + (3019*a^8*tan(c/2 + (d*x)/2)^12)/8 + (8848*a^8)/15 - (a^8*tan(
c/2 + (d*x)/2)*25499i)/120)/(d*(tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2*6
i - 6*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4*15i + 15*tan(c/2 + (d*x)/
2)^5 - tan(c/2 + (d*x)/2)^6*20i - 20*tan(c/2 + (d*x)/2)^7 + tan(c/2 + (d*x)
/2)^8*15i + 15*tan(c/2 + (d*x)/2)^9 - tan(c/2 + (d*x)/2)^10*6i - 6*tan(c/2
+ (d*x)/2)^11 + tan(c/2 + (d*x)/2)^12*1i + tan(c/2 + (d*x)/2)^13 + 1i)) - (
3003*a^8*atanh(tan(c/2 + (d*x)/2)))/(8*d)

3.92 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 24, antiderivative size = 205

$$\begin{aligned}\int \cos^3(c+dx)(a+ia\tan(c+dx))^8\,dx=&\frac{1155a^8\text{arctanh}(\sin(c+dx))}{8d}+\frac{1155ia^8\sec(c+dx)}{8d}\\&+\frac{22ia^3\cos(c+dx)(a+ia\tan(c+dx))^5}{3d}\\&-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d}\\&+\frac{33ia^2\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^3}{4d}\\&+\frac{77i\sec(c+dx)\left(a^4+ia^4\tan(c+dx)\right)^2}{4d}\\&+\frac{385i\sec(c+dx)\left(a^8+ia^8\tan(c+dx)\right)}{8d}\end{aligned}$$

```
[Out] 1155/8*a^8*arctanh(sin(d*x+c))/d+1155/8*I*a^8*sec(d*x+c)/d+22/3*I*a^3*cos(d
*x+c)*(a+I*a*tan(d*x+c))^5/d-2/3*I*a*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^7/d+33
/4*I*a^2*sec(d*x+c)*(a^2+I*a^2*tan(d*x+c))^3/d+77/4*I*sec(d*x+c)*(a^4+I*a^4
*tan(d*x+c))^2/d+385/8*I*sec(d*x+c)*(a^8+I*a^8*tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3577, 3579, 3567, 3855}

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{1155a^8\text{arctanh}(\sin(c+dx))}{8d}+\frac{1155ia^8\sec(c+dx)}{8d}+\frac{385i\sec(c+dx)\left(a^8+ia^8\tan(c+dx)\right)}{8d}+\frac{77i\sec(c+dx)\left(a^4+ia^4\tan(c+dx)\right)^2}{4d}+\frac{22ia^3\cos(c+dx)(a+ia\tan(c+dx))^5}{3d}+\frac{33ia^2\sec(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^3}{4d}-\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d}$$

```
[In] Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (1155*a^8*ArcTanh[Sin[c + d*x]])/(8*d) + (((1155*I)/8)*a^8*Sec[c + d*x])/d
+ (((22*I)/3)*a^3*Cos[c + d*x]*(a + I*a*Tan[c + d*x])^5)/d - (((2*I)/3)*a*C
os[c + d*x]^3*(a + I*a*Tan[c + d*x])^7)/d + (((33*I)/4)*a^2*Sec[c + d*x]*(a
^2 + I*a^2*Tan[c + d*x])^3)/d + (((77*I)/4)*Sec[c + d*x]*(a^4 + I*a^4*Tan[c
 + d*x])^2)/d + (((385*I)/8)*Sec[c + d*x]*(a^8 + I*a^8*Tan[c + d*x]))/d
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d} - \frac{1}{3}(11a^2)\int \cos(c+dx)(a+ia\tan(c+dx))^6\,dx \\
&= \frac{22ia^3\cos(c+dx)(a+ia\tan(c+dx))^5}{3d} - \frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d} + (33a^4)\int \sec(c+dx)(a+ia\tan(c+dx))^4\,dx \\
&= \frac{33ia^5\sec(c+dx)(a+ia\tan(c+dx))^3}{4d} + \frac{22ia^3\cos(c+dx)(a+ia\tan(c+dx))^5}{3d} - \frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d} + \frac{1}{4}(231a^5)\int \sec(c+dx)(a+ia\tan(c+dx))^3\,dx \\
&= \frac{33ia^5\sec(c+dx)(a+ia\tan(c+dx))^3}{4d} + \frac{22ia^3\cos(c+dx)(a+ia\tan(c+dx))^5}{3d} - \frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d} + \frac{77i\sec(c+dx)\left(a^4+ia^4\tan(c+dx)\right)^2}{4d} + \frac{1}{4}(385a^6)\int \sec(c+dx)(a+ia\tan(c+dx))^2\,dx \\
&= \frac{33ia^5\sec(c+dx)(a+ia\tan(c+dx))^3}{4d} + \frac{22ia^3\cos(c+dx)(a+ia\tan(c+dx))^5}{3d} - \frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d} + \frac{77i\sec(c+dx)\left(a^4+ia^4\tan(c+dx)\right)^2}{4d} + \frac{385i\sec(c+dx)\left(a^8+ia^8\tan(c+dx)\right)}{8d} + \frac{1}{8}(1155a^7)\int \sec(c+dx)(a+ia\tan(c+dx))\,dx
\end{aligned}$$

$$= \frac{1155ia^8 \sec(c+dx)}{8d} + \frac{33ia^5 \sec(c+dx)(a+ia\tan(c+dx))^3}{4d} + \frac{22ia^3 \cos(c+dx)(a+ia\tan(c+dx))^5}{3d} - \frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d} + \frac{77i\sec(c+dx)\left(a^4+ia^4\tan(c+dx)\right)^2}{4d} + \frac{385i\sec(c+dx)\left(a^8+ia^8\tan(c+dx)\right)}{8d} + \frac{1}{8}\left(1155a^8\right)\int \sec(c+dx)\,dx$$

$$= \frac{1155a^8\operatorname{arctanh}(\sin(c+dx))}{8d} + \frac{1155ia^8 \sec(c+dx)}{8d} + \frac{33ia^5 \sec(c+dx)(a+ia\tan(c+dx))^3}{4d} + \frac{22ia^3 \cos(c+dx)(a+ia\tan(c+dx))^5}{3d} - \frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^7}{3d} + \frac{77i\sec(c+dx)\left(a^4+ia^4\tan(c+dx)\right)^2}{4d} + \frac{385i\sec(c+dx)\left(a^8+ia^8\tan(c+dx)\right)}{8d}$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1540 vs. $2(205) = 410$.

Time = 8.30 (sec) , antiderivative size = 1540, normalized size of antiderivative = 7.51

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$= -\frac{1155\cos(8c)\cos^8(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(a+ia\tan(c+dx))^8}{8d(\cos(dx)+i\sin(dx))^8}$$
$$+\frac{1155\cos(8c)\cos^8(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(a+ia\tan(c+dx))^8}{8d(\cos(dx)+i\sin(dx))^8}$$
$$+\frac{\cos(3dx)\cos^8(c+dx)\left(-\frac{32}{3}i\cos(5c)-\frac{32}{3}\sin(5c)\right)(a+ia\tan(c+dx))^8}{d(\cos(dx)+i\sin(dx))^8}$$
$$+\frac{\cos(dx)\cos^8(c+dx)(160i\cos(7c)+160\sin(7c))(a+ia\tan(c+dx))^8}{d(\cos(dx)+i\sin(dx))^8}$$
$$+\frac{1155i\cos^8(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sin(8c)(a+ia\tan(c+dx))^8}{8d(\cos(dx)+i\sin(dx))^8}$$
$$-\frac{1155i\cos^8(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sin(8c)(a+ia\tan(c+dx))^8}{8d(\cos(dx)+i\sin(dx))^8}$$
$$+\frac{\cos^8(c+dx)\sec(c)\left(\frac{236}{3}i\cos(8c)+\frac{236}{3}\sin(8c)\right)(a+ia\tan(c+dx))^8}{d(\cos(dx)+i\sin(dx))^8}$$
$$+\frac{\cos^8(c+dx)(-160\cos(7c)+160i\sin(7c))\sin(dx)(a+ia\tan(c+dx))^8}{d(\cos(dx)+i\sin(dx))^8}$$
$$+\frac{\cos^8(c+dx)\left(\frac{32}{3}\cos(5c)-\frac{32}{3}i\sin(5c)\right)\sin(3dx)(a+ia\tan(c+dx))^8}{d(\cos(dx)+i\sin(dx))^8}$$
$$+\frac{\cos^8(c+dx)\left(\frac{1}{16}\cos(8c)-\frac{1}{16}i\sin(8c)\right)(a+ia\tan(c+dx))^8}{d(\cos(dx)+i\sin(dx))^8\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^4}$$
$$-\frac{i\cos^8(c+dx)\left(\frac{4}{3}\cos(8c)-\frac{4}{3}i\sin(8c)\right)\sin\left(\frac{dx}{2}\right)(a+ia\tan(c+dx))^8}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)(\cos(dx)+i\sin(dx))^8\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^3}$$
$$+\frac{\cos^8(c+dx)\left((-375-32i)\cos\left(\frac{c}{2}\right)+(375-32i)\sin\left(\frac{c}{2}\right)\right)\left(\frac{1}{48}\cos(8c)-\frac{1}{48}i\sin(8c)\right)(a+ia\tan(c+dx))^8}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)(\cos(dx)+i\sin(dx))^8\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^2}$$
$$+\frac{i\cos^8(c+dx)\left(\frac{236}{3}\cos(8c)-\frac{236}{3}i\sin(8c)\right)\sin\left(\frac{dx}{2}\right)(a+ia\tan(c+dx))^8}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)(\cos(dx)+i\sin(dx))^8\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}$$
$$+\frac{\cos^8(c+dx)\left(-\frac{1}{16}\cos(8c)+\frac{1}{16}i\sin(8c)\right)(a+ia\tan(c+dx))^8}{d(\cos(dx)+i\sin(dx))^8\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^4}$$
$$+\frac{i\cos^8(c+dx)\left(\frac{4}{3}\cos(8c)-\frac{4}{3}i\sin(8c)\right)\sin\left(\frac{dx}{2}\right)(a+ia\tan(c+dx))^8}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)(\cos(dx)+i\sin(dx))^8\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^3}$$
$$+\frac{\cos^8(c+dx)\left((375-32i)\cos\left(\frac{c}{2}\right)+(375+32i)\sin\left(\frac{c}{2}\right)\right)\left(\frac{1}{48}\cos(8c)-\frac{1}{48}i\sin(8c)\right)(a+ia\tan(c+dx))^8}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)(\cos(dx)+i\sin(dx))^8\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^2}$$
$$-\frac{i\cos^8(c+dx)\left(\frac{236}{3}\cos(8c)-\frac{236}{3}i\sin(8c)\right)\sin\left(\frac{dx}{2}\right)(a+ia\tan(c+dx))^8}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)(\cos(dx)+i\sin(dx))^8\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}$$

```
[In] Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^8,x]

[Out] (-1155*Cos[8*c]*Cos[c + d*x]^8*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]
*(a + I*a*Tan[c + d*x])^8)/(8*d*(Cos[d*x] + I*Sin[d*x])^8) + (1155*Cos[8*c]
*Cos[c + d*x]^8*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*(a + I*a*Tan[c
 + d*x])^8)/(8*d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[3*d*x]*Cos[c + d*x]^8*((
(-32*I)/3)*Cos[5*c] - (32*Sin[5*c])/3)*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*
x] + I*Sin[d*x])^8) + (Cos[d*x]*Cos[c + d*x]^8*((160*I)*Cos[7*c] + 160*Sin[
7*c])*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) + (((1155*I)/
8)*Cos[c + d*x]^8*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sin[8*c]*(a
+ I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) - (((1155*I)/8)*Cos[c
+ d*x]^8*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sin[8*c]*(a + I*a*Tan
[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[c + d*x]^8*Sec[c]*(((236
*I)/3)*Cos[8*c] + (236*Sin[8*c])/3)*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x]
+ I*Sin[d*x])^8) + (Cos[c + d*x]^8*(-160*Cos[7*c] + (160*I)*Sin[7*c])*Sin[d
*x]*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[c + d*x]
^8*((32*Cos[5*c])/3 - ((32*I)/3)*Sin[5*c])*Sin[3*d*x]*(a + I*a*Tan[c + d*x]
)^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[c + d*x]^8*(Cos[8*c]/16 - (I/16)*
Sin[8*c])*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8*(Cos[c/2 +
 (d*x)/2] - Sin[c/2 + (d*x)/2])^4) - (I*Cos[c + d*x]^8*((4*Cos[8*c])/3 - ((
4*I)/3)*Sin[8*c])*Sin[(d*x)/2]*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[c/2] - Sin
[c/2])*(Cos[d*x] + I*Sin[d*x])^8*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^
3) + (Cos[c + d*x]^8*((-375 - 32*I)*Cos[c/2] + (375 - 32*I)*Sin[c/2])*(Cos[
8*c]/48 - (I/48)*Sin[8*c])*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[c/2] - Sin[c/2
])*(Cos[d*x] + I*Sin[d*x])^8*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) +
 (I*Cos[c + d*x]^8*((236*Cos[8*c])/3 - ((236*I)/3)*Sin[8*c])*Sin[(d*x)/2]*(
a + I*a*Tan[c + d*x])^8)/(d*(Cos[c/2] - Sin[c/2])*(Cos[d*x] + I*Sin[d*x])^8
*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) + (Cos[c + d*x]^8*(-1/16*Cos[8*
c] + (I/16)*Sin[8*c])*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^
8*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^4) + (I*Cos[c + d*x]^8*((4*Cos[
8*c])/3 - ((4*I)/3)*Sin[8*c])*Sin[(d*x)/2]*(a + I*a*Tan[c + d*x])^8)/(d*(Co
s[c/2] + Sin[c/2])*(Cos[d*x] + I*Sin[d*x])^8*(Cos[c/2 + (d*x)/2] + Sin[c/2
+ (d*x)/2])^3) + (Cos[c + d*x]^8*((375 - 32*I)*Cos[c/2] + (375 + 32*I)*Sin[
c/2])*(Cos[8*c]/48 - (I/48)*Sin[8*c])*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[c/2
] + Sin[c/2])*(Cos[d*x] + I*Sin[d*x])^8*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*
x)/2])^2) - (I*Cos[c + d*x]^8*((236*Cos[8*c])/3 - ((236*I)/3)*Sin[8*c])*Sin
[(d*x)/2]*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[c/2] + Sin[c/2])*(Cos[d*x] + I*
Sin[d*x])^8*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [A] (verified)

Time = 253.24 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{32 i a^{8} e^{3 i (dx+c)}}{3d}+\frac{160 i a^{8} e^{i (dx+c)}}{d}+\frac{i a^{8} \left(2295 e^{7 i (dx+c)}+5855 e^{5 i (dx+c)}+5153 e^{3 i (dx+c)}+1545 e^{i (dx+c)}\right)}{12 d \left(e^{2 i (dx+c)}+1\right)^{4}}+\frac{1155 a^{8}}{}$
derivativedivides	$a^{8}\left(\frac{\sin^{9}(dx+c)}{4\cos(dx+c)^{4}}-\frac{5\left(\sin^{9}(dx+c)\right)}{8\cos(dx+c)^{2}}-\frac{5\left(\sin^{7}(dx+c)\right)}{8}-\frac{7\left(\sin^{5}(dx+c)\right)}{8}-\frac{35\left(\sin^{3}(dx+c)\right)}{24}-\frac{35\sin(dx+c)}{8}+\frac{35\ln(\sec(dx+c)+\tan(dx+c)}{8}\right.$
default	$a^{8}\left(\frac{\sin^{9}(dx+c)}{4\cos(dx+c)^{4}}-\frac{5\left(\sin^{9}(dx+c)\right)}{8\cos(dx+c)^{2}}-\frac{5\left(\sin^{7}(dx+c)\right)}{8}-\frac{7\left(\sin^{5}(dx+c)\right)}{8}-\frac{35\left(\sin^{3}(dx+c)\right)}{24}-\frac{35\sin(dx+c)}{8}+\frac{35\ln(\sec(dx+c)+\tan(dx+c)}{8}\right.$

```
[In] int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] -32/3*I/d*a^8*exp(3*I*(d*x+c))+160*I/d*a^8*exp(I*(d*x+c))+1/12*I*a^8/d/(exp
(2*I*(d*x+c))+1)^4*(2295*exp(7*I*(d*x+c))+5855*exp(5*I*(d*x+c))+5153*exp(3*
I*(d*x+c))+1545*exp(I*(d*x+c)))+1155/8/d*a^8*ln(exp(I*(d*x+c))+I)-1155/8/d*
a^8*ln(exp(I*(d*x+c))-I)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.39

$$\int \cos^{3}(c+dx)(a+ia\tan(c+dx))^{8}\,dx = \frac{-256 i\,a^{8} e^{(11 i\,dx+11 i\,c)}+2816 i\,a^{8} e^{(9 i\,dx+9 i\,c)}+18414 i\,a^{8} e^{(7 i\,dx+7 i\,c)}+33726 i\,a^{8} e^{(5 i\,dx+5 i\,c)}+25410 i\,a^{8} e^{(3 i\,dx+3 i\,c}}{}$$

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/24*(-256*I*a^8*e^(11*I*d*x + 11*I*c) + 2816*I*a^8*e^(9*I*d*x + 9*I*c) + 1
8414*I*a^8*e^(7*I*d*x + 7*I*c) + 33726*I*a^8*e^(5*I*d*x + 5*I*c) + 25410*I*
a^8*e^(3*I*d*x + 3*I*c) + 6930*I*a^8*e^(I*d*x + I*c) + 3465*(a^8*e^(8*I*d*x
 + 8*I*c) + 4*a^8*e^(6*I*d*x + 6*I*c) + 6*a^8*e^(4*I*d*x + 4*I*c) + 4*a^8*e
^(2*I*d*x + 2*I*c) + a^8)*log(e^(I*d*x + I*c) + I) - 3465*(a^8*e^(8*I*d*x +
 8*I*c) + 4*a^8*e^(6*I*d*x + 6*I*c) + 6*a^8*e^(4*I*d*x + 4*I*c) + 4*a^8*e^(
2*I*d*x + 2*I*c) + a^8)*log(e^(I*d*x + I*c) - I))/(d*e^(8*I*d*x + 8*I*c) +
4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c)
 + d)
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.35

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{1155a^8\left(-\frac{\log\left(e^{idx}-ie^{-ic}\right)}{8}+\frac{\log\left(e^{idx}+ie^{-ic}\right)}{8}\right)}{d} + \frac{2295ia^8e^{7ic}e^{7idx}+5855ia^8e^{5ic}e^{5idx}+5153ia^8e^{3ic}e^{3idx}+1545ia^8e^{ic}e^{idx}}{12de^{8ic}e^{8idx}+48de^{6ic}e^{6idx}+72de^{4ic}e^{4idx}+48de^{2ic}e^{2idx}+12d} + \begin{cases} \frac{-32ia^8de^{3ic}e^{3idx}+480ia^8de^{ic}e^{idx}}{3d^2} & \text{for } d^2\neq 0 \\ x\left(32a^8e^{3ic}-160a^8e^{ic}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] 1155*a**8*(-log(exp(I*d*x) - I*exp(-I*c))/8 + log(exp(I*d*x) + I*exp(-I*c))
/8)/d + (2295*I*a**8*exp(7*I*c)*exp(7*I*d*x) + 5855*I*a**8*exp(5*I*c)*exp(5
*I*d*x) + 5153*I*a**8*exp(3*I*c)*exp(3*I*d*x) + 1545*I*a**8*exp(I*c)*exp(I*
d*x))/(12*d*exp(8*I*c)*exp(8*I*d*x) + 48*d*exp(6*I*c)*exp(6*I*d*x) + 72*d*e
xp(4*I*c)*exp(4*I*d*x) + 48*d*exp(2*I*c)*exp(2*I*d*x) + 12*d) + Piecewise((
(-32*I*a**8*d*exp(3*I*c)*exp(3*I*d*x) + 480*I*a**8*d*exp(I*c)*exp(I*d*x))/(
3*d**2), Ne(d**2, 0)), (x*(32*a**8*exp(3*I*c) - 160*a**8*exp(I*c)), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(169) = 338.

Time = 0.29 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.72

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{128i\,a^8\cos\left(dx+c\right)^3+448\,a^8\sin\left(dx+c\right)^3+896i\left(\cos\left(dx+c\right)^3-\frac{3}{\cos(dx+c)}-6\,\cos\left(dx+c\right)\right)a^8+128i}{}$$

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/48*(128*I*a^8*cos(d*x + c)^3 + 448*a^8*sin(d*x + c)^3 + 896*I*(cos(d*x +
 c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a^8 + 128*I*(cos(d*x + c)^3 - (9*c
os(d*x + c)^2 - 1)/cos(d*x + c)^3 - 9*cos(d*x + c))*a^8 + 896*I*(cos(d*x +
c)^3 - 3*cos(d*x + c))*a^8 + (16*sin(d*x + c)^3 - 6*(13*sin(d*x + c)^3 - 11
*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 105*log(sin(d*x +
c) + 1) + 105*log(sin(d*x + c) - 1) + 144*sin(d*x + c))*a^8 + 112*(4*sin(d*
```

```
x + c)^3 - 6*sin(d*x + c)/(sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) +
 15*log(sin(d*x + c) - 1) + 24*sin(d*x + c))*a^8 + 560*(2*sin(d*x + c)^3 -
3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin(d*x + c))*a^8 + 1
6*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^8)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2835 vs. 2(169) = 338.

Time = 1.35 (sec) , antiderivative size = 2835, normalized size of antiderivative = 13.83

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^8\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/98304*(763587*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1069
0218*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 69486417*a^8*e^
(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 277945668*a^8*e^(22*I*d*x
+ 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 764350587*a^8*e^(20*I*d*x + 6*I*c)*lo
g(I*e^(I*d*x + I*c) + 1) + 1528701174*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d
*x + I*c) + 1) + 2293051761*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c)
+ 1) + 2293051761*a^8*e^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 152
8701174*a^8*e^(10*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 764350587*a^8
*e^(8*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 277945668*a^8*e^(6*I*d*x
- 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 69486417*a^8*e^(4*I*d*x - 10*I*c)*log
(I*e^(I*d*x + I*c) + 1) + 10690218*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x
+ I*c) + 1) + 2620630584*a^8*e^(14*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 7635
87*a^8*e^(-14*I*c)*log(I*e^(I*d*x + I*c) + 1) + 14956128*a^8*e^(28*I*d*x +
14*I*c)*log(I*e^(I*d*x + I*c) - 1) + 209385792*a^8*e^(26*I*d*x + 12*I*c)*lo
g(I*e^(I*d*x + I*c) - 1) + 1361007648*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*
d*x + I*c) - 1) + 5444030592*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c)
 - 1) + 14971084128*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 2
9942168256*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 4491325238
4*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 44913252384*a^8*e^(
12*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 29942168256*a^8*e^(10*I*d*x
- 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 14971084128*a^8*e^(8*I*d*x - 6*I*c)*l
og(I*e^(I*d*x + I*c) - 1) + 5444030592*a^8*e^(6*I*d*x - 8*I*c)*log(I*e^(I*d
*x + I*c) - 1) + 1361007648*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(I*d*x + I*c)
- 1) + 209385792*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x + I*c) - 1) + 5132
9431296*a^8*e^(14*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 14956128*a^8*e^(-14*I
*c)*log(I*e^(I*d*x + I*c) - 1) - 763587*a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^
(I*d*x + I*c) + 1) - 10690218*a^8*e^(26*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I
*c) + 1) - 69486417*a^8*e^(24*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) + 1) -
 277945668*a^8*e^(22*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 764350587
```

*a^8*e^(20*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1528701174*a^8*e^(1
8*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 2293051761*a^8*e^(16*I*d*x +
2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 2293051761*a^8*e^(12*I*d*x - 2*I*c)*l
og(-I*e^(I*d*x + I*c) + 1) - 1528701174*a^8*e^(10*I*d*x - 4*I*c)*log(-I*e^(
I*d*x + I*c) + 1) - 764350587*a^8*e^(8*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c
) + 1) - 277945668*a^8*e^(6*I*d*x - 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 69
486417*a^8*e^(4*I*d*x - 10*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 10690218*a^8*
e^(2*I*d*x - 12*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 2620630584*a^8*e^(14*I*d
*x)*log(-I*e^(I*d*x + I*c) + 1) - 763587*a^8*e^(-14*I*c)*log(-I*e^(I*d*x +
I*c) + 1) - 14956128*a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^(I*d*x + I*c) - 1)
- 209385792*a^8*e^(26*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 1361007
648*a^8*e^(24*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 5444030592*a^8*
e^(22*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 14971084128*a^8*e^(20*I*
d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 29942168256*a^8*e^(18*I*d*x + 4*
I*c)*log(-I*e^(I*d*x + I*c) - 1) - 44913252384*a^8*e^(16*I*d*x + 2*I*c)*log
(-I*e^(I*d*x + I*c) - 1) - 44913252384*a^8*e^(12*I*d*x - 2*I*c)*log(-I*e^(I
*d*x + I*c) - 1) - 29942168256*a^8*e^(10*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I
*c) - 1) - 14971084128*a^8*e^(8*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) - 1)
- 5444030592*a^8*e^(6*I*d*x - 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 13610076
48*a^8*e^(4*I*d*x - 10*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 209385792*a^8*e^(
2*I*d*x - 12*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 51329431296*a^8*e^(14*I*d*x
)*log(-I*e^(I*d*x + I*c) - 1) - 14956128*a^8*e^(-14*I*c)*log(-I*e^(I*d*x +
I*c) - 1) - 99*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 1386
*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 9009*a^8*e^(24*I*d
*x + 10*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 36036*a^8*e^(22*I*d*x + 8*I*c)*l
og(I*e^(I*d*x) + e^(-I*c)) - 99099*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x)
+ e^(-I*c)) - 198198*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c))
- 297297*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 297297*a^8*
e^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 198198*a^8*e^(10*I*d*x -
4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 99099*a^8*e^(8*I*d*x - 6*I*c)*log(I*e
^(I*d*x) + e^(-I*c)) - 36036*a^8*e^(6*I*d*x - 8*I*c)*log(I*e^(I*d*x) + e^(-
I*c)) - 9009*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 1386*a^
8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 339768*a^8*e^(14*I*d*x
)*log(I*e^(I*d*x) + e^(-I*c)) - 99*a^8*e^(-14*I*c)*log(I*e^(I*d*x) + e^(-I*
c)) + 99*a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 1386*a^8*
e^(26*I*d*x + 12*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 9009*a^8*e^(24*I*d*x +
10*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 36036*a^8*e^(22*I*d*x + 8*I*c)*log(
-I*e^(I*d*x) + e^(-I*c)) + 99099*a^8*e^(20*I*d*x + 6*I*c)*log(-I*e^(I*d*x)
+ e^(-I*c)) + 198198*a^8*e^(18*I*d*x + 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c))
+ 297297*a^8*e^(16*I*d*x + 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 297297*a^8
*e^(12*I*d*x - 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 198198*a^8*e^(10*I*d*x
- 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 99099*a^8*e^(8*I*d*x - 6*I*c)*log(
-I*e^(I*d*x) + e^(-I*c)) + 36036*a^8*e^(6*I*d*x - 8*I*c)*log(-I*e^(I*d*x) +
e^(-I*c)) + 9009*a^8*e^(4*I*d*x - 10*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 1
386*a^8*e^(2*I*d*x - 12*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 339768*a^8*e^(1

```
4*I*d*x)*log(-I*e^(I*d*x) + e^(-I*c)) + 99*a^8*e^(-14*I*c)*log(-I*e^(I*d*x)
 + e^(-I*c)) - 1048576*I*a^8*e^(31*I*d*x + 17*I*c) + 1048576*I*a^8*e^(29*I*
d*x + 15*I*c) + 143581184*I*a^8*e^(27*I*d*x + 13*I*c) + 1285595136*I*a^8*e^
(25*I*d*x + 11*I*c) + 6043484160*I*a^8*e^(23*I*d*x + 9*I*c) + 18494373888*I
*a^8*e^(21*I*d*x + 7*I*c) + 40069865472*I*a^8*e^(19*I*d*x + 5*I*c) + 640797
81888*I*a^8*e^(17*I*d*x + 3*I*c) + 77250527232*I*a^8*e^(15*I*d*x + I*c) + 7
0758072320*I*a^8*e^(13*I*d*x - I*c) + 49095122944*I*a^8*e^(11*I*d*x - 3*I*c
) + 25432580096*I*a^8*e^(9*I*d*x - 5*I*c) + 9546645504*I*a^8*e^(7*I*d*x - 7
*I*c) + 2456272896*I*a^8*e^(5*I*d*x - 9*I*c) + 387932160*I*a^8*e^(3*I*d*x -
 11*I*c) + 28385280*I*a^8*e^(I*d*x - 13*I*c))/(d*e^(28*I*d*x + 14*I*c) + 14
*d*e^(26*I*d*x + 12*I*c) + 91*d*e^(24*I*d*x + 10*I*c) + 364*d*e^(22*I*d*x +
 8*I*c) + 1001*d*e^(20*I*d*x + 6*I*c) + 2002*d*e^(18*I*d*x + 4*I*c) + 3003*
d*e^(16*I*d*x + 2*I*c) + 3003*d*e^(12*I*d*x - 2*I*c) + 2002*d*e^(10*I*d*x -
 4*I*c) + 1001*d*e^(8*I*d*x - 6*I*c) + 364*d*e^(6*I*d*x - 8*I*c) + 91*d*e^(
4*I*d*x - 10*I*c) + 14*d*e^(2*I*d*x - 12*I*c) + 3432*d*e^(14*I*d*x) + d*e^(
-14*I*c))
```

Mupad [B] (verification not implemented)

Time = 8.75 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.67

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{\frac{1147\,a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{4}+\frac{a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}3505\mathrm{i}}{4}-\frac{5639\,a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}}{3}-a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}3585\mathrm{i}+\frac{25993\,a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{6}+\cdots}{d\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}3\mathrm{i}+7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}13\mathrm{i}-18\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}-\tan\left(\cdots\right.\right.}$$
$$+\frac{1155\,a^8\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}$$

```
[In] int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] ((27565*a^8*tan(c/2 + (d*x)/2)^2)/12 - (a^8*tan(c/2 + (d*x)/2)^3*12041i)/3
- 4575*a^8*tan(c/2 + (d*x)/2)^4 + (a^8*tan(c/2 + (d*x)/2)^5*33847i)/6 + (25
993*a^8*tan(c/2 + (d*x)/2)^6)/6 - a^8*tan(c/2 + (d*x)/2)^7*3585i - (5639*a^
8*tan(c/2 + (d*x)/2)^8)/3 + (a^8*tan(c/2 + (d*x)/2)^9*3505i)/4 + (1147*a^8*
tan(c/2 + (d*x)/2)^10)/4 - (1360*a^8)/3 + (a^8*tan(c/2 + (d*x)/2)*4293i)/4)
/(d*(3*tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2*7i - 13*tan(c/2 + (d*x)/2)
^3 + tan(c/2 + (d*x)/2)^4*18i + 22*tan(c/2 + (d*x)/2)^5 - tan(c/2 + (d*x)/2
)^6*22i - 18*tan(c/2 + (d*x)/2)^7 + tan(c/2 + (d*x)/2)^8*13i + 7*tan(c/2 +
(d*x)/2)^9 - tan(c/2 + (d*x)/2)^10*3i - tan(c/2 + (d*x)/2)^11 + 1i)) + (115
5*a^8*atanh(tan(c/2 + (d*x)/2)))/(4*d)
```

3.93 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 24, antiderivative size = 173

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{63a^8\text{arctanh}(\sin(c+dx))}{2d} - \frac{63ia^8\sec(c+dx)}{2d} + \frac{6ia^3\cos^3(c+dx)(a+ia\tan(c+dx))^5}{5d} - \frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^7}{5d} - \frac{42ia^2\cos(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^3}{5d} - \frac{21i\sec(c+dx)\left(a^8+ia^8\tan(c+dx)\right)}{2d}$$

```
[Out] -63/2*a^8*arctanh(sin(d*x+c))/d-63/2*I*a^8*sec(d*x+c)/d+6/5*I*a^3*cos(d*x+c
)^3*(a+I*a*tan(d*x+c))^5/d-2/5*I*a*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^7/d-42/5
*I*a^2*cos(d*x+c)*(a^2+I*a^2*tan(d*x+c))^3/d-21/2*I*sec(d*x+c)*(a^8+I*a^8*t
an(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used

$= \{3577, 3579, 3567, 3855\}$

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{63a^8\operatorname{arctanh}(\sin(c+dx))}{2d} - \frac{63ia^8\sec(c+dx)}{2d} - \frac{21i\sec(c+dx)\left(a^8+ia^8\tan(c+dx)\right)}{2d} + \frac{6ia^3\cos^3(c+dx)(a+ia\tan(c+dx))^5}{5d} - \frac{42ia^2\cos(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^3}{5d} - \frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^7}{5d}$$

```
[In] Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (-63*a^8*ArcTanh[Sin[c + d*x]])/(2*d) - (((63*I)/2)*a^8*Sec[c + d*x])/d + (
((6*I)/5)*a^3*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^5)/d - (((2*I)/5)*a*Cos
[c + d*x]^5*(a + I*a*Tan[c + d*x])^7)/d - (((42*I)/5)*a^2*Cos[c + d*x]*(a^2
 + I*a^2*Tan[c + d*x])^3)/d - (((21*I)/2)*Sec[c + d*x]*(a^8 + I*a^8*Tan[c +
 d*x]))/d
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
```

```
2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^7}{5d} \\
&\quad -\frac{1}{5}\left(9a^2\right)\int \cos^3(c+dx)(a+ia\tan(c+dx))^6\,dx \\
&= \frac{6ia^3\cos^3(c+dx)(a+ia\tan(c+dx))^5}{5d}-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^7}{5d} \\
&\quad +\frac{1}{5}\left(21a^4\right)\int \cos(c+dx)(a+ia\tan(c+dx))^4\,dx \\
&= -\frac{42ia^5\cos(c+dx)(a+ia\tan(c+dx))^3}{5d}+\frac{6ia^3\cos^3(c+dx)(a+ia\tan(c+dx))^5}{5d} \\
&\quad -\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^7}{5d}-\left(21a^6\right)\int \sec(c+dx)(a+ia\tan(c+dx))^2\,dx \\
&= -\frac{42ia^5\cos(c+dx)(a+ia\tan(c+dx))^3}{5d}+\frac{6ia^3\cos^3(c+dx)(a+ia\tan(c+dx))^5}{5d} \\
&\quad -\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^7}{5d}-\frac{21i\sec(c+dx)\left(a^8+ia^8\tan(c+dx)\right)}{2d} \\
&\quad -\frac{1}{2}\left(63a^7\right)\int \sec(c+dx)(a+ia\tan(c+dx))\,dx \\
&= -\frac{63ia^8\sec(c+dx)}{2d}-\frac{42ia^5\cos(c+dx)(a+ia\tan(c+dx))^3}{5d} \\
&\quad +\frac{6ia^3\cos^3(c+dx)(a+ia\tan(c+dx))^5}{5d}-\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^7}{5d} \\
&\quad -\frac{21i\sec(c+dx)\left(a^8+ia^8\tan(c+dx)\right)}{2d}-\frac{1}{2}\left(63a^8\right)\int \sec(c+dx)\,dx \\
&= -\frac{63a^8\text{arctanh}(\sin(c+dx))}{2d}-\frac{63ia^8\sec(c+dx)}{2d} \\
&\quad -\frac{42ia^5\cos(c+dx)(a+ia\tan(c+dx))^3}{5d}+\frac{6ia^3\cos^3(c+dx)(a+ia\tan(c+dx))^5}{5d} \\
&\quad -\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^7}{5d}-\frac{21i\sec(c+dx)\left(a^8+ia^8\tan(c+dx)\right)}{2d}
\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1162 vs. 2(173) = 346.

Time = 8.06 (sec) , antiderivative size = 1162, normalized size of antiderivative = 6.72

$$\begin{aligned}
&\int \cos^5(c+dx)(a+ia\tan(c+dx))^8\,dx \\
&= \frac{63\cos(8c)\cos^8(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(a+ia\tan(c+dx))^8}{2d(\cos(dx)+i\sin(dx))^8} \\
&- \frac{63\cos(8c)\cos^8(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(a+ia\tan(c+dx))^8}{2d(\cos(dx)+i\sin(dx))^8} \\
&+ \frac{\cos(5dx)\cos^8(c+dx)\left(-\frac{8}{5}i\cos(3c)-\frac{8}{5}\sin(3c)\right)(a+ia\tan(c+dx))^8}{d(\cos(dx)+i\sin(dx))^8} \\
&+ \frac{\cos(3dx)\cos^8(c+dx)(8i\cos(5c)+8\sin(5c))(a+ia\tan(c+dx))^8}{d(\cos(dx)+i\sin(dx))^8} \\
&+ \frac{\cos(dx)\cos^8(c+dx)(-48i\cos(7c)-48\sin(7c))(a+ia\tan(c+dx))^8}{d(\cos(dx)+i\sin(dx))^8} \\
&+ \frac{\cos^8(c+dx)\sec(c)(-8i\cos(8c)-8\sin(8c))(a+ia\tan(c+dx))^8}{d(\cos(dx)+i\sin(dx))^8} \\
&- \frac{63i\cos^8(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sin(8c)(a+ia\tan(c+dx))^8}{2d(\cos(dx)+i\sin(dx))^8} \\
&+ \frac{63i\cos^8(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sin(8c)(a+ia\tan(c+dx))^8}{2d(\cos(dx)+i\sin(dx))^8} \\
&+ \frac{\cos^8(c+dx)(48\cos(7c)-48i\sin(7c))\sin(dx)(a+ia\tan(c+dx))^8}{d(\cos(dx)+i\sin(dx))^8} \\
&+ \frac{\cos^8(c+dx)(-8\cos(5c)+8i\sin(5c))\sin(3dx)(a+ia\tan(c+dx))^8}{d(\cos(dx)+i\sin(dx))^8} \\
&+ \frac{\cos^8(c+dx)\left(\frac{8}{5}\cos(3c)-\frac{8}{5}i\sin(3c)\right)\sin(5dx)(a+ia\tan(c+dx))^8}{d(\cos(dx)+i\sin(dx))^8} \\
&+ \frac{\cos^8(c+dx)\left(\frac{1}{4}\cos(8c)-\frac{1}{4}i\sin(8c)\right)(a+ia\tan(c+dx))^8}{d(\cos(dx)+i\sin(dx))^8\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^2} \\
&- \frac{i\cos^8(c+dx)(8\cos(8c)-8i\sin(8c))\sin\left(\frac{dx}{2}\right)(a+ia\tan(c+dx))^8}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)(\cos(dx)+i\sin(dx))^8\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)} \\
&+ \frac{\cos^8(c+dx)\left(-\frac{1}{4}\cos(8c)+\frac{1}{4}i\sin(8c)\right)(a+ia\tan(c+dx))^8}{d(\cos(dx)+i\sin(dx))^8\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^2} \\
&+ \frac{i\cos^8(c+dx)(8\cos(8c)-8i\sin(8c))\sin\left(\frac{dx}{2}\right)(a+ia\tan(c+dx))^8}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)(\cos(dx)+i\sin(dx))^8\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}
\end{aligned}$$

```
[In] Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (63*Cos[8*c]*Cos[c + d*x]^8*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*(a
 + I*a*Tan[c + d*x])^8)/(2*d*(Cos[d*x] + I*Sin[d*x])^8) - (63*Cos[8*c]*Cos[
c + d*x]^8*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*(a + I*a*Tan[c + d*
x])^8)/(2*d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[5*d*x]*Cos[c + d*x]^8*(((-8*I
)/5)*Cos[3*c] - (8*Sin[3*c])/5)*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*
Sin[d*x])^8) + (Cos[3*d*x]*Cos[c + d*x]^8*((8*I)*Cos[5*c] + 8*Sin[5*c])*(a
+ I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[d*x]*Cos[c + d*
x]^8*((-48*I)*Cos[7*c] - 48*Sin[7*c])*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x
] + I*Sin[d*x])^8) + (Cos[c + d*x]^8*Sec[c]*((-8*I)*Cos[8*c] - 8*Sin[8*c])*
(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) - (((63*I)/2)*Cos[c
 + d*x]^8*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sin[8*c]*(a + I*a*Ta
n[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) + (((63*I)/2)*Cos[c + d*x]^8*L
og[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sin[8*c]*(a + I*a*Tan[c + d*x])
^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[c + d*x]^8*(48*Cos[7*c] - (48*I)*S
in[7*c])*Sin[d*x]*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) +
 (Cos[c + d*x]^8*(-8*Cos[5*c] + (8*I)*Sin[5*c])*Sin[3*d*x]*(a + I*a*Tan[c +
 d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8) + (Cos[c + d*x]^8*((8*Cos[3*c])/5 -
 ((8*I)/5)*Sin[3*c])*Sin[5*d*x]*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*
Sin[d*x])^8) + (Cos[c + d*x]^8*(Cos[8*c]/4 - (I/4)*Sin[8*c])*(a + I*a*Tan[c
 + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d
*x)/2])^2) - (I*Cos[c + d*x]^8*(8*Cos[8*c] - (8*I)*Sin[8*c])*Sin[(d*x)/2]*(
a + I*a*Tan[c + d*x])^8)/(d*(Cos[c/2] - Sin[c/2])*(Cos[d*x] + I*Sin[d*x])^8
*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) + (Cos[c + d*x]^8*(-1/4*Cos[8*c
] + (I/4)*Sin[8*c])*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[d*x] + I*Sin[d*x])^8*
(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + (I*Cos[c + d*x]^8*(8*Cos[8*c
] - (8*I)*Sin[8*c])*Sin[(d*x)/2]*(a + I*a*Tan[c + d*x])^8)/(d*(Cos[c/2] + S
in[c/2])*(Cos[d*x] + I*Sin[d*x])^8*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]
))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(152) = 304$.

Time = 2.38 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.86

$$\frac{a^8(\sin^9(dx+c))}{2d\cos(dx+c)^2}+\frac{a^8(\sin^7(dx+c))}{2d}+\frac{203a^8(\sin^5(dx+c))}{10d}+\frac{21a^8(\sin^3(dx+c))}{2d}+\frac{283a^8\sin(dx+c)}{10d}-\frac{63a^8\ln}{}$$

```
[In] int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 1/2/d*a^8*sin(d*x+c)^9/cos(d*x+c)^2+1/2*a^8*sin(d*x+c)^7/d+203/10*a^8*sin(d
*x+c)^5/d+21/2*a^8*sin(d*x+c)^3/d+283/10*a^8*sin(d*x+c)/d-63/2/d*a^8*ln(sec
(d*x+c)+tan(d*x+c))-416/15*I/d*a^8*cos(d*x+c)*sin(d*x+c)^2+56/5*I/d*a^8*cos
(d*x+c)^3*sin(d*x+c)^2+112/15*I/d*a^8*cos(d*x+c)^3-8*I/d*a^8*sin(d*x+c)^8/c
os(d*x+c)-8/5*I/d*a^8*cos(d*x+c)^5-104/5*I/d*a^8*cos(d*x+c)*sin(d*x+c)^4-83
```

```
2/15*I/d*a^8*cos(d*x+c)+29/5/d*a^8*cos(d*x+c)^4*sin(d*x+c)-8/5/d*a^8*cos(d*
x+c)^2*sin(d*x+c)-8*I/d*a^8*cos(d*x+c)*sin(d*x+c)^6
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.10

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{-16i\,a^8e^{(9i\,dx+9i\,c)}+48i\,a^8e^{(7i\,dx+7i\,c)}-336i\,a^8e^{(5i\,dx+5i\,c)}-1050i\,a^8e^{(3i\,dx+3i\,c)}-630i\,a^8e^{(i\,dx+i\,c)}-315\left(a^8e^{\cdots}\right.}{10\left(de^{(4i\,dx+4i\,c)}\cdots\right.}$$

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/10*(-16*I*a^8*e^(9*I*d*x + 9*I*c) + 48*I*a^8*e^(7*I*d*x + 7*I*c) - 336*I*
a^8*e^(5*I*d*x + 5*I*c) - 1050*I*a^8*e^(3*I*d*x + 3*I*c) - 630*I*a^8*e^(I*d
*x + I*c) - 315*(a^8*e^(4*I*d*x + 4*I*c) + 2*a^8*e^(2*I*d*x + 2*I*c) + a^8)
*log(e^(I*d*x + I*c) + I) + 315*(a^8*e^(4*I*d*x + 4*I*c) + 2*a^8*e^(2*I*d*x
 + 2*I*c) + a^8)*log(e^(I*d*x + I*c) - I))/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(
2*I*d*x + 2*I*c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.36

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{63a^8\left(\frac{\log\left(e^{idx}-ie^{-ic}\right)}{2}-\frac{\log\left(e^{idx}+ie^{-ic}\right)}{2}\right)}{d} + \frac{-17ia^8e^{3ic}e^{3idx}-15ia^8e^{ic}e^{idx}}{de^{4ic}e^{4idx}+2de^{2ic}e^{2idx}+d} + \begin{cases} \frac{-8ia^8d^2e^{5ic}e^{5idx}+40ia^8d^2e^{3ic}e^{3idx}-240ia^8d^2e^{ic}e^{idx}}{5d^3} & \text{for } d^3 \neq 0 \\ x(8a^8e^{5ic}-24a^8e^{3ic}+48a^8e^{ic}) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] 63*a**8*(log(exp(I*d*x) - I*exp(-I*c))/2 - log(exp(I*d*x) + I*exp(-I*c))/2)
/d + (-17*I*a**8*exp(3*I*c)*exp(3*I*d*x) - 15*I*a**8*exp(I*c)*exp(I*d*x))/(
d*exp(4*I*c)*exp(4*I*d*x) + 2*d*exp(2*I*c)*exp(2*I*d*x) + d) + Piecewise(((
-8*I*a**8*d**2*exp(5*I*c)*exp(5*I*d*x) + 40*I*a**8*d**2*exp(3*I*c)*exp(3*I*
d*x) - 240*I*a**8*d**2*exp(I*c)*exp(I*d*x))/(5*d**3), Ne(d**3, 0)), (x*(8*a
**8*exp(5*I*c) - 24*a**8*exp(3*I*c) + 48*a**8*exp(I*c)), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(143) = 286.

Time = 0.28 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.88

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^8\,dx =$$

$$-\frac{96i\,a^8\cos{(dx+c)}^5 - 840\,a^8\sin{(dx+c)}^5 + 224i\,\left(3\,\cos{(dx+c)}^5 - 5\,\cos{(dx+c)}^3\right)a^8 + 224i\,\left(3\,\cos{(dx+c)}\right.}{}$$

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/60*(96*I*a^8*cos(d*x + c)^5 - 840*a^8*sin(d*x + c)^5 + 224*I*(3*cos(d*x
+ c)^5 - 5*cos(d*x + c)^3)*a^8 + 224*I*(3*cos(d*x + c)^5 - 10*cos(d*x + c)^
3 + 15*cos(d*x + c))*a^8 + 96*I*(cos(d*x + c)^5 - 5*cos(d*x + c)^3 + 5/cos(
d*x + c) + 15*cos(d*x + c))*a^8 - (12*sin(d*x + c)^5 + 40*sin(d*x + c)^3 -
30*sin(d*x + c)/(sin(d*x + c)^2 - 1) - 105*log(sin(d*x + c) + 1) + 105*log(
sin(d*x + c) - 1) + 180*sin(d*x + c))*a^8 - 56*(6*sin(d*x + c)^5 + 10*sin(d
*x + c)^3 - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1) + 30*sin(d*
x + c))*a^8 - 112*(3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*a^8 - 4*(3*sin(d*x
+ c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^8)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2849 vs. 2(143) = 286.

Time = 1.42 (sec) , antiderivative size = 2849, normalized size of antiderivative = 16.47

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^8\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/655360*(42021645*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*c) + 1) + 5
88303030*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 3823969695*
a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 15295878780*a^8*e^(2
2*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 42063666645*a^8*e^(20*I*d*x +
 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 84127333290*a^8*e^(18*I*d*x + 4*I*c)*l
og(I*e^(I*d*x + I*c) + 1) + 126190999935*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(
I*d*x + I*c) + 1) + 126190999935*a^8*e^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x +
I*c) + 1) + 84127333290*a^8*e^(10*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) + 1)
 + 42063666645*a^8*e^(8*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1529587
8780*a^8*e^(6*I*d*x - 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 3823969695*a^8*e^
```

(4*I*d*x - 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 588303030*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 144218285640*a^8*e^(14*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 42021645*a^8*e^(-14*I*c)*log(I*e^(I*d*x + I*c) + 1) + 21376575*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*c) - 1) + 299272050*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1945268325*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) - 1) + 7781073300*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) - 1) + 21397951575*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 42795903150*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 64193854725*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 64193854725*a^8*e^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 42795903150*a^8*e^(10*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 21397951575*a^8*e^(8*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 7781073300*a^8*e^(6*I*d*x - 8*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1945268325*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(I*d*x + I*c) - 1) + 299272050*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x + I*c) - 1) + 73364405400*a^8*e^(14*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 21376575*a^8*e^(-14*I*c)*log(I*e^(I*d*x + I*c) - 1) - 42021645*a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 588303030*a^8*e^(26*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 3823969695*a^8*e^(24*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 15295878780*a^8*e^(22*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 42063666645*a^8*e^(20*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 84127333290*a^8*e^(18*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 126190999935*a^8*e^(16*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 126190999935*a^8*e^(12*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 84127333290*a^8*e^(10*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 42063666645*a^8*e^(8*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 15295878780*a^8*e^(6*I*d*x - 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 3823969695*a^8*e^(4*I*d*x - 10*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 588303030*a^8*e^(2*I*d*x - 12*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 144218285640*a^8*e^(14*I*d*x)*log(-I*e^(I*d*x + I*c) + 1) - 42021645*a^8*e^(-14*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 21376575*a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 299272050*a^8*e^(26*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 1945268325*a^8*e^(24*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 7781073300*a^8*e^(22*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 21397951575*a^8*e^(20*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 42795903150*a^8*e^(18*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 64193854725*a^8*e^(16*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 64193854725*a^8*e^(12*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 42795903150*a^8*e^(10*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 21397951575*a^8*e^(8*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 7781073300*a^8*e^(6*I*d*x - 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 1945268325*a^8*e^(4*I*d*x - 10*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 299272050*a^8*e^(2*I*d*x - 12*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 73364405400*a^8*e^(14*I*d*x)*log(-I*e^(I*d*x + I*c) - 1) - 21376575*a^8*e^(-14*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 1230*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 17220*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 111930*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 447720*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x) +

```
e^(-I*c)) - 1231230*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) -
 2462460*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 3693690*a^8
*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 3693690*a^8*e^(12*I*d*x
 - 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 2462460*a^8*e^(10*I*d*x - 4*I*c)*lo
g(I*e^(I*d*x) + e^(-I*c)) - 1231230*a^8*e^(8*I*d*x - 6*I*c)*log(I*e^(I*d*x)
 + e^(-I*c)) - 447720*a^8*e^(6*I*d*x - 8*I*c)*log(I*e^(I*d*x) + e^(-I*c)) -
 111930*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 17220*a^8*e^
(2*I*d*x - 12*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 4221360*a^8*e^(14*I*d*x)*l
og(I*e^(I*d*x) + e^(-I*c)) - 1230*a^8*e^(-14*I*c)*log(I*e^(I*d*x) + e^(-I*c
)) + 1230*a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 17220*a^
8*e^(26*I*d*x + 12*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 111930*a^8*e^(24*I*d
*x + 10*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 447720*a^8*e^(22*I*d*x + 8*I*c)
*log(-I*e^(I*d*x) + e^(-I*c)) + 1231230*a^8*e^(20*I*d*x + 6*I*c)*log(-I*e^(
I*d*x) + e^(-I*c)) + 2462460*a^8*e^(18*I*d*x + 4*I*c)*log(-I*e^(I*d*x) + e^
(-I*c)) + 3693690*a^8*e^(16*I*d*x + 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 3
693690*a^8*e^(12*I*d*x - 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 2462460*a^8*
e^(10*I*d*x - 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 1231230*a^8*e^(8*I*d*x
- 6*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 447720*a^8*e^(6*I*d*x - 8*I*c)*log(
-I*e^(I*d*x) + e^(-I*c)) + 111930*a^8*e^(4*I*d*x - 10*I*c)*log(-I*e^(I*d*x)
 + e^(-I*c)) + 17220*a^8*e^(2*I*d*x - 12*I*c)*log(-I*e^(I*d*x) + e^(-I*c))
+ 4221360*a^8*e^(14*I*d*x)*log(-I*e^(I*d*x) + e^(-I*c)) + 1230*a^8*e^(-14*I
*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 1048576*I*a^8*e^(33*I*d*x + 19*I*c) - 94
37184*I*a^8*e^(31*I*d*x + 17*I*c) - 53477376*I*a^8*e^(29*I*d*x + 15*I*c) -
356122624*I*a^8*e^(27*I*d*x + 13*I*c) - 2147352576*I*a^8*e^(25*I*d*x + 11*I
*c) - 9154854912*I*a^8*e^(23*I*d*x + 9*I*c) - 27241218048*I*a^8*e^(21*I*d*x
 + 7*I*c) - 58509361152*I*a^8*e^(19*I*d*x + 5*I*c) - 93311336448*I*a^8*e^(1
7*I*d*x + 3*I*c) - 112396337152*I*a^8*e^(15*I*d*x + I*c) - 102926647296*I*a
^8*e^(13*I*d*x - I*c) - 71411564544*I*a^8*e^(11*I*d*x - 3*I*c) - 3699284377
6*I*a^8*e^(9*I*d*x - 5*I*c) - 13886029824*I*a^8*e^(7*I*d*x - 7*I*c) - 35727
60576*I*a^8*e^(5*I*d*x - 9*I*c) - 564264960*I*a^8*e^(3*I*d*x - 11*I*c) - 41
287680*I*a^8*e^(I*d*x - 13*I*c))/(d*e^(28*I*d*x + 14*I*c) + 14*d*e^(26*I*d*
x + 12*I*c) + 91*d*e^(24*I*d*x + 10*I*c) + 364*d*e^(22*I*d*x + 8*I*c) + 100
1*d*e^(20*I*d*x + 6*I*c) + 2002*d*e^(18*I*d*x + 4*I*c) + 3003*d*e^(16*I*d*x
 + 2*I*c) + 3003*d*e^(12*I*d*x - 2*I*c) + 2002*d*e^(10*I*d*x - 4*I*c) + 100
1*d*e^(8*I*d*x - 6*I*c) + 364*d*e^(6*I*d*x - 8*I*c) + 91*d*e^(4*I*d*x - 10*
I*c) + 14*d*e^(2*I*d*x - 12*I*c) + 3432*d*e^(14*I*d*x) + d*e^(-14*I*c))
```

Mupad [B] (verification not implemented)

Time = 8.58 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.62

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{63\,a^8\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

$$+\frac{65\,a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7 309\mathrm{i}-761\,a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5 1109\mathrm{i}+\frac{7351\,a^8\tan\left(\frac{c}{2}-\right.}{5}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8 5\mathrm{i}-12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 20\mathrm{i}+26\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\tan\right.}$$

```
[In] int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] (a^8*tan(c/2 + (d*x)/2)^3*1223i - (4407*a^8*tan(c/2 + (d*x)/2)^2)/5 + (7351
*a^8*tan(c/2 + (d*x)/2)^4)/5 - a^8*tan(c/2 + (d*x)/2)^5*1109i - 761*a^8*tan
(c/2 + (d*x)/2)^6 + a^8*tan(c/2 + (d*x)/2)^7*309i + 65*a^8*tan(c/2 + (d*x)/
2)^8 + (496*a^8)/5 - a^8*tan(c/2 + (d*x)/2)*431i)/(d*(5*tan(c/2 + (d*x)/2)
- tan(c/2 + (d*x)/2)^2*12i - 20*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4
*26i + 26*tan(c/2 + (d*x)/2)^5 - tan(c/2 + (d*x)/2)^6*20i - 12*tan(c/2 + (d
*x)/2)^7 + tan(c/2 + (d*x)/2)^8*5i + tan(c/2 + (d*x)/2)^9 + 1i)) - (63*a^8*
atanh(tan(c/2 + (d*x)/2)))/d
```

3.94 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 24, antiderivative size = 152

$$\begin{aligned}\int \cos^7(c+dx)(a+ia\tan(c+dx))^8\,dx &= \frac{a^8\text{arctanh}(\sin(c+dx))}{d} \\ &+ \frac{2ia^3\cos^5(c+dx)(a+ia\tan(c+dx))^5}{5d} \\ &- \frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{7d} \\ &- \frac{2ia^2\cos^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^3}{3d} \\ &+ \frac{2i\cos(c+dx)\left(a^8+ia^8\tan(c+dx)\right)}{d}\end{aligned}$$

```
[Out] a^8*arctanh(sin(d*x+c))/d+2/5*I*a^3*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^5/d-2/7
*I*a*cos(d*x+c)^7*(a+I*a*tan(d*x+c))^7/d-2/3*I*a^2*cos(d*x+c)^3*(a^2+I*a^2*
tan(d*x+c))^3/d+2*I*cos(d*x+c)*(a^8+I*a^8*tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used

$= \{3577, 3855\}$

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{a^8\operatorname{arctanh}(\sin(c+dx))}{d} + \frac{2i\cos(c+dx)\left(a^8+ia^8\tan(c+dx)\right)}{d} + \frac{2ia^3\cos^5(c+dx)(a+ia\tan(c+dx))^5}{5d} - \frac{2ia^2\cos^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^3}{3d} - \frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{7d}$$

```
[In] Int[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*ArcTanh[Sin[c + d*x]])/d + (((2*I)/5)*a^3*Cos[c + d*x]^5*(a + I*a*Tan[
c + d*x])^5)/d - (((2*I)/7)*a*Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^7)/d -
(((2*I)/3)*a^2*Cos[c + d*x]^3*(a^2 + I*a^2*Tan[c + d*x])^3)/d + ((2*I)*Cos[
c + d*x]*(a^8 + I*a^8*Tan[c + d*x]))/d
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = -\frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{7d} - a^2\int \cos^5(c+dx)(a+ia\tan(c+dx))^6\,dx$$
$$= \frac{2ia^3\cos^5(c+dx)(a+ia\tan(c+dx))^5}{5d} - \frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{7d} + a^4\int \cos^3(c+dx)(a+ia\tan(c+dx))^4\,dx$$

$$= -\frac{2ia^5\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d} + \frac{2ia^3\cos^5(c+dx)(a+ia\tan(c+dx))^5}{5d} - \frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{7d} - a^6\int\cos(c+dx)(a+ia\tan(c+dx))^2\,dx$$

$$= -\frac{2ia^5\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d} + \frac{2ia^3\cos^5(c+dx)(a+ia\tan(c+dx))^5}{5d} - \frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{7d} + \frac{2i\cos(c+dx)\left(a^8+ia^8\tan(c+dx)\right)}{d} + a^8\int\sec(c+dx)\,dx$$

$$= \frac{a^8\text{arctanh}(\sin(c+dx))}{d} - \frac{2ia^5\cos^3(c+dx)(a+ia\tan(c+dx))^3}{3d} + \frac{2ia^3\cos^5(c+dx)(a+ia\tan(c+dx))^5}{5d} - \frac{2ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{7d} + \frac{2i\cos(c+dx)\left(a^8+ia^8\tan(c+dx)\right)}{d}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 305 vs. $2(152) = 304$.

Time = 3.17 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.01

$$\int\cos^7(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8\left(-70i\cos\left(\frac{1}{2}(c+dx)\right)+42i\cos\left(\frac{3}{2}(c+dx)\right)+210i\cos\left(\frac{5}{2}(c+dx)\right)-30i\cos\left(\frac{7}{2}(c+dx)\right)-105\cos\left(\frac{7}{2}(\right.\right.}{}$$

```
[In] Integrate[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*((-70*I)*Cos[(c + d*x)/2] + (42*I)*Cos[(3*(c + d*x))/2] + (210*I)*Cos[
(5*(c + d*x))/2] - (30*I)*Cos[(7*(c + d*x))/2] - 105*Cos[(7*(c + d*x))/2]*L
og[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 105*Cos[(7*(c + d*x))/2]*Log[Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]] - 70*Sin[(c + d*x)/2] - 42*Sin[(3*(c + d*x
))/2] + 210*Sin[(5*(c + d*x))/2] + 30*Sin[(7*(c + d*x))/2] + (105*I)*Log[Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[(7*(c + d*x))/2] - (105*I)*Log[Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[(7*(c + d*x))/2])*(Cos[(7*c + 23*d*x)/2
] + I*Sin[(7*c + 23*d*x)/2]))/(105*d*(Cos[d*x] + I*Sin[d*x])^8)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 384 vs. 2(138) = 276.

Time = 1.56 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.53

$$-\frac{29a^8(\sin^7(dx+c))}{7d}-\frac{a^8(\sin^5(dx+c))}{5d}-\frac{a^8(\sin^3(dx+c))}{3d}+\frac{139a^8\sin(dx+c)}{105d}+\frac{a^8\ln(\sec(dx+c)+\tan(dx$$

```
[In] int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] -29/7*a^8*sin(d*x+c)^7/d-1/5*a^8*sin(d*x+c)^5/d-1/3*a^8*sin(d*x+c)^3/d+139/
105*a^8*sin(d*x+c)/d+1/d*a^8*ln(sec(d*x+c)+tan(d*x+c))+128/35*I/d*a^8*cos(d
*x+c)-8*I/d*a^8*sin(d*x+c)^4*cos(d*x+c)^3-10/d*a^8*sin(d*x+c)^3*cos(d*x+c)^
4-232/35/d*a^8*cos(d*x+c)^4*sin(d*x+c)+122/105/d*a^8*cos(d*x+c)^2*sin(d*x+c
)-64/15*I/d*a^8*cos(d*x+c)^3+48/35*I/d*a^8*cos(d*x+c)*sin(d*x+c)^4+29/7/d*a
^8*cos(d*x+c)^6*sin(d*x+c)-32/5*I/d*a^8*cos(d*x+c)^3*sin(d*x+c)^2+8/7*I/d*a
^8*cos(d*x+c)*sin(d*x+c)^6+64/35*I/d*a^8*cos(d*x+c)*sin(d*x+c)^2-8/7*I/d*a^
8*cos(d*x+c)^7+8*I/d*a^8*cos(d*x+c)^5*sin(d*x+c)^2+16/5*I/d*a^8*cos(d*x+c)^
5
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.63

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$=\frac{-30i\,a^8e^{(7i\,dx+7i\,c)}+42i\,a^8e^{(5i\,dx+5i\,c)}-70i\,a^8e^{(3i\,dx+3i\,c)}+210i\,a^8e^{(i\,dx+i\,c)}+105\,a^8\log\left(e^{(i\,dx+i\,c)}+i\right)-105}{105\,d}$$

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/105*(-30*I*a^8*e^(7*I*d*x + 7*I*c) + 42*I*a^8*e^(5*I*d*x + 5*I*c) - 70*I*
a^8*e^(3*I*d*x + 3*I*c) + 210*I*a^8*e^(I*d*x + I*c) + 105*a^8*log(e^(I*d*x
+ I*c) + I) - 105*a^8*log(e^(I*d*x + I*c) - I))/d
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.23

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{a^8\left(-\log\left(e^{idx}-ie^{-ic}\right)+\log\left(e^{idx}+ie^{-ic}\right)\right)}{d} + \begin{cases} \frac{-30ia^8d^3e^{7ic}e^{7idx}+42ia^8d^3e^{5ic}e^{5idx}-70ia^8d^3e^{3ic}e^{3idx}+210ia^8d^3e^{ic}e^{idx}}{105d^4} & \text{for } d^4\neq 0 \\ x(2a^8e^{7ic}-2a^8e^{5ic}+2a^8e^{3ic}-2a^8e^{ic}) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)**7*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] a**8*(-log(exp(I*d*x) - I*exp(-I*c)) + log(exp(I*d*x) + I*exp(-I*c)))/d + P
iecewise(((-30*I*a**8*d**3*exp(7*I*c)*exp(7*I*d*x) + 42*I*a**8*d**3*exp(5*I
*c)*exp(5*I*d*x) - 70*I*a**8*d**3*exp(3*I*c)*exp(3*I*d*x) + 210*I*a**8*d**3
*exp(I*c)*exp(I*d*x))/(105*d**4), Ne(d**4, 0)), (x*(2*a**8*exp(7*I*c) - 2*a
**8*exp(5*I*c) + 2*a**8*exp(3*I*c) - 2*a**8*exp(I*c)), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 309 vs. 2(130) = 260.

Time = 0.27 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.03

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{240i\,a^8\cos\left(dx+c\right)^7+840\,a^8\sin\left(dx+c\right)^7+112i\,\left(15\,\cos\left(dx+c\right)^7-42\,\cos\left(dx+c\right)^5+35\,\cos\left(dx+c\right)\right.}{}$$

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/210*(240*I*a^8*cos(d*x + c)^7 + 840*a^8*sin(d*x + c)^7 + 112*I*(15*cos(d
*x + c)^7 - 42*cos(d*x + c)^5 + 35*cos(d*x + c)^3)*a^8 + 336*I*(5*cos(d*x +
 c)^7 - 7*cos(d*x + c)^5)*a^8 + 48*I*(5*cos(d*x + c)^7 - 21*cos(d*x + c)^5
+ 35*cos(d*x + c)^3 - 35*cos(d*x + c))*a^8 + (30*sin(d*x + c)^7 + 42*sin(d*
x + c)^5 + 70*sin(d*x + c)^3 - 105*log(sin(d*x + c) + 1) + 105*log(sin(d*x
+ c) - 1) + 210*sin(d*x + c))*a^8 + 56*(15*sin(d*x + c)^7 - 42*sin(d*x + c)
^5 + 35*sin(d*x + c)^3)*a^8 + 420*(5*sin(d*x + c)^7 - 7*sin(d*x + c)^5)*a^8
 + 6*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin(d*x + c)^3 - 35*sin(d*x
 + c))*a^8)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2863 vs. 2(130) = 260.

Time = 1.54 (sec) , antiderivative size = 2863, normalized size of antiderivative = 18.84

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^8\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/55050240*(1635552135*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*c) + 1)
 + 22897729890*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 14883
5244285*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 595340977140
*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1637187687135*a^8*e^
(20*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 3274375374270*a^8*e^(18*I*d
*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 4911563061405*a^8*e^(16*I*d*x + 2*
I*c)*log(I*e^(I*d*x + I*c) + 1) + 4911563061405*a^8*e^(12*I*d*x - 2*I*c)*lo
g(I*e^(I*d*x + I*c) + 1) + 3274375374270*a^8*e^(10*I*d*x - 4*I*c)*log(I*e^(
I*d*x + I*c) + 1) + 1637187687135*a^8*e^(8*I*d*x - 6*I*c)*log(I*e^(I*d*x +
I*c) + 1) + 595340977140*a^8*e^(6*I*d*x - 8*I*c)*log(I*e^(I*d*x + I*c) + 1)
 + 148835244285*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 22897
729890*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x + I*c) + 1) + 5613214927320*
a^8*e^(14*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 1635552135*a^8*e^(-14*I*c)*lo
g(I*e^(I*d*x + I*c) + 1) + 1690450650*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*
d*x + I*c) - 1) + 23666309100*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*
c) - 1) + 153831009150*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) - 1)
 + 615324036600*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) - 1) + 16921
41100650*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 338428220130
0*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 5076423301950*a^8*e
^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 5076423301950*a^8*e^(12*I*
d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 3384282201300*a^8*e^(10*I*d*x - 4
*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1692141100650*a^8*e^(8*I*d*x - 6*I*c)*lo
g(I*e^(I*d*x + I*c) - 1) + 615324036600*a^8*e^(6*I*d*x - 8*I*c)*log(I*e^(I*
d*x + I*c) - 1) + 153831009150*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(I*d*x + I*
c) - 1) + 23666309100*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x + I*c) - 1) +
 5801626630800*a^8*e^(14*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 1690450650*a^8
*e^(-14*I*c)*log(I*e^(I*d*x + I*c) - 1) - 1635552135*a^8*e^(28*I*d*x + 14*I
*c)*log(-I*e^(I*d*x + I*c) + 1) - 22897729890*a^8*e^(26*I*d*x + 12*I*c)*log
(-I*e^(I*d*x + I*c) + 1) - 148835244285*a^8*e^(24*I*d*x + 10*I*c)*log(-I*e^
(I*d*x + I*c) + 1) - 595340977140*a^8*e^(22*I*d*x + 8*I*c)*log(-I*e^(I*d*x
+ I*c) + 1) - 1637187687135*a^8*e^(20*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c)
 + 1) - 3274375374270*a^8*e^(18*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1)
- 4911563061405*a^8*e^(16*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 4911
563061405*a^8*e^(12*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 3274375374
```

270*a^8*e^(10*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1637187687135*a^8*e^(8*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 595340977140*a^8*e^(6*I*d*x - 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 148835244285*a^8*e^(4*I*d*x - 10*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 22897729890*a^8*e^(2*I*d*x - 12*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 5613214927320*a^8*e^(14*I*d*x)*log(-I*e^(I*d*x + I*c) + 1) - 1635552135*a^8*e^(-14*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1690450650*a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 23666309100*a^8*e^(26*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 153831009150*a^8*e^(24*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 615324036600*a^8*e^(22*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 1692141100650*a^8*e^(20*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 3384282201300*a^8*e^(18*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 5076423301950*a^8*e^(16*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 5076423301950*a^8*e^(12*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 3384282201300*a^8*e^(10*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 1692141100650*a^8*e^(8*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 615324036600*a^8*e^(6*I*d*x - 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 153831009150*a^8*e^(4*I*d*x - 10*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 23666309100*a^8*e^(2*I*d*x - 12*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 5801626630800*a^8*e^(14*I*d*x)*log(-I*e^(I*d*x + I*c) - 1) - 1690450650*a^8*e^(-14*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 151725*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 2124150*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 13806975*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 55227900*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 151876725*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 303753450*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 455630175*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 455630175*a^8*e^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 303753450*a^8*e^(10*I*d*x - 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 151876725*a^8*e^(8*I*d*x - 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 55227900*a^8*e^(6*I*d*x - 8*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 13806975*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 2124150*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 520720200*a^8*e^(14*I*d*x)*log(I*e^(I*d*x) + e^(-I*c)) - 151725*a^8*e^(-14*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 151725*a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 2124150*a^8*e^(26*I*d*x + 12*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 13806975*a^8*e^(24*I*d*x + 10*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 55227900*a^8*e^(22*I*d*x + 8*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 151876725*a^8*e^(20*I*d*x + 6*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 303753450*a^8*e^(18*I*d*x + 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 455630175*a^8*e^(16*I*d*x + 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 455630175*a^8*e^(12*I*d*x - 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 303753450*a^8*e^(10*I*d*x - 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 151876725*a^8*e^(8*I*d*x - 6*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 55227900*a^8*e^(6*I*d*x - 8*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 13806975*a^8*e^(4*I*d*x - 10*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 2124150*a^8*e^(2*I*d*x - 12*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 520720200*a^8*e^(14*I*d*x)*log(-I*e^(I*d*x) + e^(-I*c)) + 151725*a^8*e^(-14*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 15728640*I*a^8*e^(

```
35*I*d*x + 21*I*c) - 198180864*I*a^8*e^(33*I*d*x + 19*I*c) - 1159725056*I*a
^8*e^(31*I*d*x + 17*I*c) - 4125097984*I*a^8*e^(29*I*d*x + 15*I*c) - 9527361
536*I*a^8*e^(27*I*d*x + 13*I*c) - 12786335744*I*a^8*e^(25*I*d*x + 11*I*c) +
 190840832*I*a^8*e^(23*I*d*x + 9*I*c) + 48882515968*I*a^8*e^(21*I*d*x + 7*I
*c) + 138550444032*I*a^8*e^(19*I*d*x + 5*I*c) + 239314403328*I*a^8*e^(17*I*
d*x + 3*I*c) + 295994130432*I*a^8*e^(15*I*d*x + I*c) + 273474912256*I*a^8*e
^(13*I*d*x - I*c) + 190268309504*I*a^8*e^(11*I*d*x - 3*I*c) + 98635350016*I
*a^8*e^(9*I*d*x - 5*I*c) + 37029412864*I*a^8*e^(7*I*d*x - 7*I*c) + 95273615
36*I*a^8*e^(5*I*d*x - 9*I*c) + 1504706560*I*a^8*e^(3*I*d*x - 11*I*c) + 1101
00480*I*a^8*e^(I*d*x - 13*I*c))/(d*e^(28*I*d*x + 14*I*c) + 14*d*e^(26*I*d*x
 + 12*I*c) + 91*d*e^(24*I*d*x + 10*I*c) + 364*d*e^(22*I*d*x + 8*I*c) + 1001
*d*e^(20*I*d*x + 6*I*c) + 2002*d*e^(18*I*d*x + 4*I*c) + 3003*d*e^(16*I*d*x
+ 2*I*c) + 3003*d*e^(12*I*d*x - 2*I*c) + 2002*d*e^(10*I*d*x - 4*I*c) + 1001
*d*e^(8*I*d*x - 6*I*c) + 364*d*e^(6*I*d*x - 8*I*c) + 91*d*e^(4*I*d*x - 10*I
*c) + 14*d*e^(2*I*d*x - 12*I*c) + 3432*d*e^(14*I*d*x) + d*e^(-14*I*c))
```

Mupad [B] (verification not implemented)

Time = 8.25 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.36

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{2\,a^8\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} + \frac{a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5 16\mathrm{i} - \frac{80\,a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3} - \frac{a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 224\mathrm{i}}{3} + \frac{224\,a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{5} + \frac{a^8\tan\left(\frac{c}{2}\right.}{}}{d\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7 - \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 7\mathrm{i} + 21\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5 + \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 35\mathrm{i} - 35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 - \tan\right.}$$

```
[In] int(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] (2*a^8*atanh(tan(c/2 + (d*x)/2)))/d + ((224*a^8*tan(c/2 + (d*x)/2)^2)/5 - (
a^8*tan(c/2 + (d*x)/2)^3*224i)/3 - (80*a^8*tan(c/2 + (d*x)/2)^4)/3 + a^8*ta
n(c/2 + (d*x)/2)^5*16i - (304*a^8)/105 + (a^8*tan(c/2 + (d*x)/2)*304i)/15)/
(d*(7*tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2*21i - 35*tan(c/2 + (d*x)/2)
^3 + tan(c/2 + (d*x)/2)^4*35i + 21*tan(c/2 + (d*x)/2)^5 - tan(c/2 + (d*x)/2
)^6*7i - tan(c/2 + (d*x)/2)^7 + 1i))
```

3.95 $\int \cos^9(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 24, antiderivative size = 66

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{63d} - \frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^8}{9d}$$

```
[Out] -1/63*I*a*cos(d*x+c)^7*(a+I*a*tan(d*x+c))^7/d-1/9*I*cos(d*x+c)^9*(a+I*a*tan
(d*x+c))^8/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3578, 3569}

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^8}{9d} - \frac{ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{63d}$$

```
[In] Int[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((-1/63*I)*a*Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^7)/d - ((I/9)*Cos[c + d*
x]^9*(a + I*a*Tan[c + d*x])^8)/d
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
```

```
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\text{integral} = -\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^8}{9d} + \frac{1}{9}a\int \cos^7(c+dx)(a+ia\tan(c+dx))^7\,dx$$
$$= -\frac{ia\cos^7(c+dx)(a+ia\tan(c+dx))^7}{63d} - \frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^8}{9d}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 146 vs. $2(66) = 132$.

Time = 0.81 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.21

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8\sec(c+dx)(-i\cos(5(c+dx))+\sin(5(c+dx)))\left(9\cos(c+dx)+16\cos(3(c+dx))+7\cos(5(c+dx))+\right.}{}$$

```
[In] Integrate[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*Sec[c + d*x]*((-I)*Cos[5*(c + d*x)] + Sin[5*(c + d*x)])*(9*Cos[c + d*x
] + 16*Cos[3*(c + d*x)] + 7*Cos[5*(c + d*x)] + 192*Sqrt[Cos[c + d*x]^2]*Cos
[5*(c + d*x)] + (9*I)*Sin[c + d*x] + (16*I)*Sin[3*(c + d*x)] + (7*I)*Sin[5*
(c + d*x)] - (192*I)*Sqrt[Cos[c + d*x]^2]*Sin[5*(c + d*x)]))/(252*d)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 446 vs. 2(58) = 116.

Time = 1.59 (sec) , antiderivative size = 447, normalized size of antiderivative = 6.77

$$\frac{a^8(\sin^9(dx+c))}{9} - \frac{8ia^8(\cos^9(dx+c))}{9} - 28a^8\left(-\frac{(\cos^4(dx+c))(\sin^5(dx+c))}{9} - \frac{5(\sin^3(dx+c))(\cos^4(dx+c))}{63} - \frac{\sin(dx+c)(\cos^4(dx+c))}{21} + \right.$$

```
[In] int(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 1/d*(1/9*a^8*sin(d*x+c)^9-8/9*I*a^8*cos(d*x+c)^9-28*a^8*(-1/9*cos(d*x+c)^4*
sin(d*x+c)^5-5/63*sin(d*x+c)^3*cos(d*x+c)^4-1/21*sin(d*x+c)*cos(d*x+c)^4+1/
63*(2+cos(d*x+c)^2)*sin(d*x+c))-8*I*a^8*(-1/9*cos(d*x+c)^3*sin(d*x+c)^6-2/2
1*cos(d*x+c)^3*sin(d*x+c)^4-8/105*cos(d*x+c)^3*sin(d*x+c)^2-16/315*cos(d*x+
c)^3)+70*a^8*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(d*x+c)^6+1
/105*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-56*I*a^8*(-1/9*cos(d*x
+c)^7*sin(d*x+c)^2-2/63*cos(d*x+c)^7)-28*a^8*(-1/9*cos(d*x+c)^8*sin(d*x+c)+
1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))+56*I
*a^8*(-1/9*sin(d*x+c)^4*cos(d*x+c)^5-4/63*cos(d*x+c)^5*sin(d*x+c)^2-8/315*c
os(d*x+c)^5)+1/9*a^8*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c)^6+48/35*cos(d*x+c)
^4+64/35*cos(d*x+c)^2)*sin(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.52

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{-7i\,a^8e^{(9i\,dx+9i\,c)} - 9i\,a^8e^{(7i\,dx+7i\,c)}}{126\,d}$$

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/126*(-7*I*a^8*e^(9*I*d*x + 9*I*c) - 9*I*a^8*e^(7*I*d*x + 7*I*c))/d
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.21

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^8\,dx = \begin{cases} \frac{-14ia^8de^{9ic}e^{9idx}-18ia^8de^{7ic}e^{7idx}}{252d^2} & \text{for } d^2\neq 0 \\ x\left(\frac{a^8e^{9ic}}{2}+\frac{a^8e^{7ic}}{2}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)**9*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise(((-14*I*a**8*d*exp(9*I*c)*exp(9*I*d*x) - 18*I*a**8*d*exp(7*I*c)*e
xp(7*I*d*x))/(252*d**2), Ne(d**2, 0)), (x*(a**8*exp(9*I*c)/2 + a**8*exp(7*I
*c)/2), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(54) = 108.

Time = 0.26 (sec) , antiderivative size = 302, normalized size of antiderivative = 4.58

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^8\,dx =$$

$$-\frac{280i\,a^8\cos(dx+c)^9-35\,a^8\sin(dx+c)^9+56i\left(35\cos(dx+c)^9-90\cos(dx+c)^7+63\cos(dx+c)^5\right)\cdots}{}$$

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/315*(280*I*a^8*cos(d*x + c)^9 - 35*a^8*sin(d*x + c)^9 + 56*I*(35*cos(d*x
 + c)^9 - 90*cos(d*x + c)^7 + 63*cos(d*x + c)^5)*a^8 + 8*I*(35*cos(d*x + c)
^9 - 135*cos(d*x + c)^7 + 189*cos(d*x + c)^5 - 105*cos(d*x + c)^3)*a^8 + 28
0*I*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a^8 - 70*(35*sin(d*x + c)^9 - 90*
sin(d*x + c)^7 + 63*sin(d*x + c)^5)*a^8 - 28*(35*sin(d*x + c)^9 - 135*sin(d
*x + c)^7 + 189*sin(d*x + c)^5 - 105*sin(d*x + c)^3)*a^8 - (35*sin(d*x + c)
^9 - 180*sin(d*x + c)^7 + 378*sin(d*x + c)^5 - 420*sin(d*x + c)^3 + 315*sin
(d*x + c))*a^8 - 140*(7*sin(d*x + c)^9 - 9*sin(d*x + c)^7)*a^8)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2451 vs. 2(54) = 108.

Time = 1.62 (sec) , antiderivative size = 2451, normalized size of antiderivative = 37.14

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^8\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/66060288*(1419343317*a^8*e^(24*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) + 1)
 + 17032119804*a^8*e^(22*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 93676
658922*a^8*e^(20*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 312255529740*a
^8*e^(18*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 702574941915*a^8*e^(16
*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1124119907064*a^8*e^(14*I*d*x
+ 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1124119907064*a^8*e^(10*I*d*x - 2*I*c
)*log(I*e^(I*d*x + I*c) + 1) + 702574941915*a^8*e^(8*I*d*x - 4*I*c)*log(I*e
^(I*d*x + I*c) + 1) + 312255529740*a^8*e^(6*I*d*x - 6*I*c)*log(I*e^(I*d*x +
 I*c) + 1) + 93676658922*a^8*e^(4*I*d*x - 8*I*c)*log(I*e^(I*d*x + I*c) + 1)
 + 17032119804*a^8*e^(2*I*d*x - 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 131147
3224908*a^8*e^(12*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 1419343317*a^8*e^(-12
*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1419097050*a^8*e^(24*I*d*x + 12*I*c)*log
(I*e^(I*d*x + I*c) - 1) + 17029164600*a^8*e^(22*I*d*x + 10*I*c)*log(I*e^(I*
d*x + I*c) - 1) + 93660405300*a^8*e^(20*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c
) - 1) + 312201351000*a^8*e^(18*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) - 1) +
 702453039750*a^8*e^(16*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1123924
863600*a^8*e^(14*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1123924863600*
a^8*e^(10*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 702453039750*a^8*e^(8
*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 312201351000*a^8*e^(6*I*d*x -
6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 93660405300*a^8*e^(4*I*d*x - 8*I*c)*log
(I*e^(I*d*x + I*c) - 1) + 17029164600*a^8*e^(2*I*d*x - 10*I*c)*log(I*e^(I*d
*x + I*c) - 1) + 1311245674200*a^8*e^(12*I*d*x)*log(I*e^(I*d*x + I*c) - 1)
+ 1419097050*a^8*e^(-12*I*c)*log(I*e^(I*d*x + I*c) - 1) - 1419343317*a^8*e^
(24*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 17032119804*a^8*e^(22*I*d
*x + 10*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 93676658922*a^8*e^(20*I*d*x + 8*
I*c)*log(-I*e^(I*d*x + I*c) + 1) - 312255529740*a^8*e^(18*I*d*x + 6*I*c)*lo
g(-I*e^(I*d*x + I*c) + 1) - 702574941915*a^8*e^(16*I*d*x + 4*I*c)*log(-I*e^
(I*d*x + I*c) + 1) - 1124119907064*a^8*e^(14*I*d*x + 2*I*c)*log(-I*e^(I*d*x
 + I*c) + 1) - 1124119907064*a^8*e^(10*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c
) + 1) - 702574941915*a^8*e^(8*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) -
 312255529740*a^8*e^(6*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 9367665
8922*a^8*e^(4*I*d*x - 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 17032119804*a^8*
e^(2*I*d*x - 10*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1311473224908*a^8*e^(12*
I*d*x)*log(-I*e^(I*d*x + I*c) + 1) - 1419343317*a^8*e^(-12*I*c)*log(-I*e^(I
```

*d*x + I*c) + 1) - 1419097050*a^8*e^(24*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 17029164600*a^8*e^(22*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 93660405300*a^8*e^(20*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 312201351000*a^8*e^(18*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 702453039750*a^8*e^(16*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 1123924863600*a^8*e^(14*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 1123924863600*a^8*e^(10*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 702453039750*a^8*e^(8*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 312201351000*a^8*e^(6*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 93660405300*a^8*e^(4*I*d*x - 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 17029164600*a^8*e^(2*I*d*x - 10*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 1311245674200*a^8*e^(12*I*d*x)*log(-I*e^(I*d*x + I*c) - 1) - 1419097050*a^8*e^(-12*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 246267*a^8*e^(24*I*d*x + 12*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 2955204*a^8*e^(22*I*d*x + 10*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 16253622*a^8*e^(20*I*d*x + 8*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 54178740*a^8*e^(18*I*d*x + 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 121902165*a^8*e^(16*I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 195043464*a^8*e^(14*I*d*x + 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 195043464*a^8*e^(10*I*d*x - 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 121902165*a^8*e^(8*I*d*x - 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 54178740*a^8*e^(6*I*d*x - 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 16253622*a^8*e^(4*I*d*x - 8*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 2955204*a^8*e^(2*I*d*x - 10*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 227550708*a^8*e^(12*I*d*x)*log(I*e^(I*d*x) + e^(-I*c)) - 246267*a^8*e^(-12*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 246267*a^8*e^(24*I*d*x + 12*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 2955204*a^8*e^(22*I*d*x + 10*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 16253622*a^8*e^(20*I*d*x + 8*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 54178740*a^8*e^(18*I*d*x + 6*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 121902165*a^8*e^(16*I*d*x + 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 195043464*a^8*e^(14*I*d*x + 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 195043464*a^8*e^(10*I*d*x - 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 121902165*a^8*e^(8*I*d*x - 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 54178740*a^8*e^(6*I*d*x - 6*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 16253622*a^8*e^(4*I*d*x - 8*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 2955204*a^8*e^(2*I*d*x - 10*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 227550708*a^8*e^(12*I*d*x)*log(-I*e^(I*d*x) + e^(-I*c)) + 246267*a^8*e^(-12*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 3670016*I*a^8*e^(33*I*d*x + 21*I*c) - 48758784*I*a^8*e^(31*I*d*x + 19*I*c) - 298844160*I*a^8*e^(29*I*d*x + 17*I*c) - 1118830592*I*a^8*e^(27*I*d*x + 15*I*c) - 2854748160*I*a^8*e^(25*I*d*x + 13*I*c) - 5242355712*I*a^8*e^(23*I*d*x + 11*I*c) - 7128219648*I*a^8*e^(21*I*d*x + 9*I*c) - 7266631680*I*a^8*e^(19*I*d*x + 7*I*c) - 5553782784*I*a^8*e^(17*I*d*x + 5*I*c) - 3143106560*I*a^8*e^(15*I*d*x + 3*I*c) - 1280311296*I*a^8*e^(13*I*d*x + I*c) - 355467264*I*a^8*e^(11*I*d*x - I*c) - 60293120*I*a^8*e^(9*I*d*x - 3*I*c) - 4718592*I*a^8*e^(7*I*d*x - 5*I*c))/(d*e^(24*I*d*x + 12*I*c) + 12*d*e^(22*I*d*x + 10*I*c) + 66*d*e^(20*I*d*x + 8*I*c) + 220*d*e^(18*I*d*x + 6*I*c) + 495*d*e^(16*I*d*x + 4*I*c) + 792*d*e^(14*I*d*x + 2*I*c) + 792*d*e^(10*I*d*x - 2*I*c) + 495*d*e^(8*I*d*x - 4*I*c) + 220*d*e^(6*I*d*x - 6*I*c) + 66*d*e^(4*I*d*x - 8*I*c) + 12*d*e^(2*I*d*x - 10*I*c)

```
+ 924*d*e^(12*I*d*x) + d*e^(-12*I*c))
```

Mupad [B] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.56

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{2\,a^8\,\left(\frac{\mathrm{e}^{c\,7\mathrm{i}+d\,x\,7\mathrm{i}}\,9\mathrm{i}}{4}+\frac{\mathrm{e}^{c\,9\mathrm{i}+d\,x\,9\mathrm{i}}\,7\mathrm{i}}{4}\right)}{63\,d}$$

```
[In] int(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^8,x)
[Out] -(2*a^8*((exp(c*7i + d*x*7i)*9i)/4 + (exp(c*9i + d*x*9i)*7i)/4))/(63*d)
```

3.96 $\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 24, antiderivative size = 136

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{2ia^3\cos^5(c+dx)(a+ia\tan(c+dx))^5}{1155d} - \frac{2ia^2\cos^7(c+dx)(a+ia\tan(c+dx))^6}{231d} - \frac{ia\cos^9(c+dx)(a+ia\tan(c+dx))^7}{33d} - \frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^8}{11d}$$

```
[Out] -2/1155*I*a^3*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^5/d-2/231*I*a^2*cos(d*x+c)^7*
(a+I*a*tan(d*x+c))^6/d-1/33*I*a*cos(d*x+c)^9*(a+I*a*tan(d*x+c))^7/d-1/11*I*
cos(d*x+c)^11*(a+I*a*tan(d*x+c))^8/d
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3578, 3569}

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{2ia^3\cos^5(c+dx)(a+ia\tan(c+dx))^5}{1155d} - \frac{2ia^2\cos^7(c+dx)(a+ia\tan(c+dx))^6}{231d} - \frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^8}{11d} - \frac{ia\cos^9(c+dx)(a+ia\tan(c+dx))^7}{33d}$$

```
[In] Int[Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (((-2*I)/1155)*a^3*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^5)/d - (((2*I)/231
)*a^2*Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^6)/d - ((I/33)*a*Cos[c + d*x]^9
*(a + I*a*Tan[c + d*x])^7)/d - ((I/11)*Cos[c + d*x]^11*(a + I*a*Tan[c + d*x
])^8)/d
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^8}{11d} + \frac{1}{11}(3a)\int\cos^9(c+dx)(a+ia\tan(c+dx))^7\,dx \\
&= -\frac{ia\cos^9(c+dx)(a+ia\tan(c+dx))^7}{33d} - \frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^8}{11d} + \frac{1}{33}\left(2a^2\right)\int\cos^7(c+dx)(a+ia\tan(c+dx))^6\,dx \\
&= -\frac{2ia^2\cos^7(c+dx)(a+ia\tan(c+dx))^6}{231d} - \frac{ia\cos^9(c+dx)(a+ia\tan(c+dx))^7}{33d} - \frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^8}{11d} + \frac{1}{231}\left(2a^3\right)\int\cos^5(c+dx)(a+ia\tan(c+dx))^5\,dx \\
&= -\frac{2ia^3\cos^5(c+dx)(a+ia\tan(c+dx))^5}{1155d} - \frac{2ia^2\cos^7(c+dx)(a+ia\tan(c+dx))^6}{231d} - \frac{ia\cos^9(c+dx)(a+ia\tan(c+dx))^7}{33d} - \frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^8}{11d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.11

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$=\frac{a^8\sec(c+dx)(-i\cos(6(c+dx))+\sin(6(c+dx)))\left(726+1111\cos(2(c+dx))+490\cos(4(c+dx))+105\right.}{}$$

```
[In] Integrate[Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*Sec[c + d*x]*((-I)*Cos[6*(c + d*x)] + Sin[6*(c + d*x)])*(726 + 1111*Co
s[2*(c + d*x)] + 490*Cos[4*(c + d*x)] + 105*Cos[6*(c + d*x)] + 11008*Sqrt[C
os[c + d*x]^2]*Cos[6*(c + d*x)] + (649*I)*Sin[2*(c + d*x)] + (490*I)*Sin[4*
(c + d*x)] + (105*I)*Sin[6*(c + d*x)] - (11008*I)*Sqrt[Cos[c + d*x]^2]*Sin[
6*(c + d*x)]))/(18480*d)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 566 vs. 2(120) = 240.

Time = 1.88 (sec) , antiderivative size = 567, normalized size of antiderivative = 4.17

$$\frac{a^8\left(-\frac{(\sin^7(dx+c))(\cos^4(dx+c))}{11}-\frac{7(\cos^4(dx+c))(\sin^5(dx+c))}{99}-\frac{5(\sin^3(dx+c))(\cos^4(dx+c))}{99}-\frac{\sin(dx+c)(\cos^4(dx+c))}{33}+\frac{(2+\cos^2(dx+c)}{99}\right.}{}$$

```
[In] int(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 1/d*(a^8*(-1/11*sin(d*x+c)^7*cos(d*x+c)^4-7/99*cos(d*x+c)^4*sin(d*x+c)^5-5/
99*sin(d*x+c)^3*cos(d*x+c)^4-1/33*sin(d*x+c)*cos(d*x+c)^4+1/99*(2+cos(d*x+c
)^2)*sin(d*x+c))-56*I*a^8*(-1/11*cos(d*x+c)^9*sin(d*x+c)^2-2/99*cos(d*x+c)^
9)-28*a^8*(-1/11*sin(d*x+c)^5*cos(d*x+c)^6-5/99*sin(d*x+c)^3*cos(d*x+c)^6-5
/231*sin(d*x+c)*cos(d*x+c)^6+1/231*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(
d*x+c))-8/11*I*a^8*cos(d*x+c)^11+70*a^8*(-1/11*sin(d*x+c)^3*cos(d*x+c)^8-1/
33*cos(d*x+c)^8*sin(d*x+c)+1/231*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*co
s(d*x+c)^2)*sin(d*x+c))-8*I*a^8*(-1/11*cos(d*x+c)^5*sin(d*x+c)^6-2/33*sin(d
*x+c)^4*cos(d*x+c)^5-8/231*cos(d*x+c)^5*sin(d*x+c)^2-16/1155*cos(d*x+c)^5)-
28*a^8*(-1/11*sin(d*x+c)*cos(d*x+c)^10+1/99*(128/35+cos(d*x+c)^8+8/7*cos(d*
x+c)^6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*x+c))+56*I*a^8*(-1/11*c
os(d*x+c)^7*sin(d*x+c)^4-4/99*cos(d*x+c)^7*sin(d*x+c)^2-8/693*cos(d*x+c)^7)
+1/11*a^8*(256/63+cos(d*x+c)^10+10/9*cos(d*x+c)^8+80/63*cos(d*x+c)^6+32/21*
cos(d*x+c)^4+128/63*cos(d*x+c)^2)*sin(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.46

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{-105i\,a^8e^{(11i\,dx+11i\,c)}-385i\,a^8e^{(9i\,dx+9i\,c)}-495i\,a^8e^{(7i\,dx+7i\,c)}-231i\,a^8e^{(5i\,dx+5i\,c)}}{9240\,d}$$

```
[In] integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/9240*(-105*I*a^8*e^(11*I*d*x + 11*I*c) - 385*I*a^8*e^(9*I*d*x + 9*I*c) -
495*I*a^8*e^(7*I*d*x + 7*I*c) - 231*I*a^8*e^(5*I*d*x + 5*I*c))/d
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.19

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^8\,dx = \begin{cases} \frac{-53760ia^8d^3e^{11ic}e^{11idx}-197120ia^8d^3e^{9ic}e^{9idx}-253440ia^8d^3e^{7ic}e^{7idx}-118272ia^8d^3e^{5ic}e^{5idx}}{4730880d^4} & \text{for } d^4\neq 0 \\ x\left(\frac{a^8e^{11ic}}{8}+\frac{3a^8e^{9ic}}{8}+\frac{3a^8e^{7ic}}{8}+\frac{a^8e^{5ic}}{8}\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)**11*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise(((-53760*I*a**8*d**3*exp(11*I*c)*exp(11*I*d*x) - 197120*I*a**8*d*
*3*exp(9*I*c)*exp(9*I*d*x) - 253440*I*a**8*d**3*exp(7*I*c)*exp(7*I*d*x) - 1
18272*I*a**8*d**3*exp(5*I*c)*exp(5*I*d*x))/(4730880*d**4), Ne(d**4, 0)), (x
*(a**8*exp(11*I*c)/8 + 3*a**8*exp(9*I*c)/8 + 3*a**8*exp(7*I*c)/8 + a**8*exp
(5*I*c)/8), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(112) = 224.

Time = 0.43 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.61

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^8\,dx =$$

$$-\frac{2520i\,a^8\cos(dx+c)^{11}+24i\left(105\cos(dx+c)^{11}-385\cos(dx+c)^9+495\cos(dx+c)^7-231\cos(dx\right.}{}$$

```
[In] integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/3465*(2520*I*a^8*cos(d*x + c)^11 + 24*I*(105*cos(d*x + c)^11 - 385*cos(d
*x + c)^9 + 495*cos(d*x + c)^7 - 231*cos(d*x + c)^5)*a^8 + 280*I*(63*cos(d*
x + c)^11 - 154*cos(d*x + c)^9 + 99*cos(d*x + c)^7)*a^8 + 1960*I*(9*cos(d*x
 + c)^11 - 11*cos(d*x + c)^9)*a^8 + 28*(315*sin(d*x + c)^11 - 1540*sin(d*x
+ c)^9 + 2970*sin(d*x + c)^7 - 2772*sin(d*x + c)^5 + 1155*sin(d*x + c)^3)*a
^8 + 210*(105*sin(d*x + c)^11 - 385*sin(d*x + c)^9 + 495*sin(d*x + c)^7 - 2
31*sin(d*x + c)^5)*a^8 + 140*(63*sin(d*x + c)^11 - 154*sin(d*x + c)^9 + 99*
sin(d*x + c)^7)*a^8 + 5*(63*sin(d*x + c)^11 - 385*sin(d*x + c)^9 + 990*sin(
d*x + c)^7 - 1386*sin(d*x + c)^5 + 1155*sin(d*x + c)^3 - 693*sin(d*x + c))*
a^8 + 35*(9*sin(d*x + c)^11 - 11*sin(d*x + c)^9)*a^8)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2863 vs. $2(112) = 224$.

Time = 1.73 (sec) , antiderivative size = 2863, normalized size of antiderivative = 21.05

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^8\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/4844421120*(82027951005*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*c) +
 1) + 1148391314070*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) + 1) +
7464543541455*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 298581
74165820*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 821099789560
05*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 164219957912010*a^
8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 246329936868015*a^8*e^(
16*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 246329936868015*a^8*e^(12*I*
d*x - 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 164219957912010*a^8*e^(10*I*d*x -
 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 82109978956005*a^8*e^(8*I*d*x - 6*I*c)
*log(I*e^(I*d*x + I*c) + 1) + 29858174165820*a^8*e^(6*I*d*x - 8*I*c)*log(I*
e^(I*d*x + I*c) + 1) + 7464543541455*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(I*d*
x + I*c) + 1) + 1148391314070*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x + I*c
) + 1) + 281519927849160*a^8*e^(14*I*d*x)*log(I*e^(I*d*x + I*c) + 1) + 8202
7951005*a^8*e^(-14*I*c)*log(I*e^(I*d*x + I*c) + 1) + 82004266575*a^8*e^(28*
I*d*x + 14*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1148059732050*a^8*e^(26*I*d*x
+ 12*I*c)*log(I*e^(I*d*x + I*c) - 1) + 7462388258325*a^8*e^(24*I*d*x + 10*I
*c)*log(I*e^(I*d*x + I*c) - 1) + 29849553033300*a^8*e^(22*I*d*x + 8*I*c)*lo
g(I*e^(I*d*x + I*c) - 1) + 82086270841575*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^
(I*d*x + I*c) - 1) + 164172541683150*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*
x + I*c) - 1) + 246258812524725*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x + I
*c) - 1) + 246258812524725*a^8*e^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) -
```

1) + 164172541683150*a^8*e^(10*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 82086270841575*a^8*e^(8*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 29849553033300*a^8*e^(6*I*d*x - 8*I*c)*log(I*e^(I*d*x + I*c) - 1) + 7462388258325*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1148059732050*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x + I*c) - 1) + 281438642885400*a^8*e^(14*I*d*x)*log(I*e^(I*d*x + I*c) - 1) + 82004266575*a^8*e^(-14*I*c)*log(I*e^(I*d*x + I*c) - 1) - 82027951005*a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1148391314070*a^8*e^(26*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 7464543541455*a^8*e^(24*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 29858174165820*a^8*e^(22*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 82109978956005*a^8*e^(20*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 164219957912010*a^8*e^(18*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 246329936868015*a^8*e^(16*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 246329936868015*a^8*e^(12*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 164219957912010*a^8*e^(10*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 82109978956005*a^8*e^(8*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 29858174165820*a^8*e^(6*I*d*x - 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 7464543541455*a^8*e^(4*I*d*x - 10*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1148391314070*a^8*e^(2*I*d*x - 12*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 281519927849160*a^8*e^(14*I*d*x)*log(-I*e^(I*d*x + I*c) + 1) - 82027951005*a^8*e^(-14*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 82004266575*a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 1148059732050*a^8*e^(26*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 7462388258325*a^8*e^(24*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 29849553033300*a^8*e^(22*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 82086270841575*a^8*e^(20*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 164172541683150*a^8*e^(18*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 246258812524725*a^8*e^(16*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 246258812524725*a^8*e^(12*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 164172541683150*a^8*e^(10*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 82086270841575*a^8*e^(8*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 29849553033300*a^8*e^(6*I*d*x - 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 7462388258325*a^8*e^(4*I*d*x - 10*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 1148059732050*a^8*e^(2*I*d*x - 12*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 281438642885400*a^8*e^(14*I*d*x)*log(-I*e^(I*d*x + I*c) - 1) - 82004266575*a^8*e^(-14*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 23684430*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 331582020*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 2155283130*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 8621132520*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 23708114430*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 47416228860*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 71124343290*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 71124343290*a^8*e^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 47416228860*a^8*e^(10*I*d*x - 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 23708114430*a^8*e^(8*I*d*x - 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 8621132520*a^8*e^(6*I*d*x - 8*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 2155283130*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 331582020*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x) + e

```
^(-I*c)) - 81284963760*a^8*e^(14*I*d*x)*log(I*e^(I*d*x) + e^(-I*c)) - 23684
430*a^8*e^(-14*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 23684430*a^8*e^(28*I*d*x
+ 14*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 331582020*a^8*e^(26*I*d*x + 12*I*c
)*log(-I*e^(I*d*x) + e^(-I*c)) + 2155283130*a^8*e^(24*I*d*x + 10*I*c)*log(-
I*e^(I*d*x) + e^(-I*c)) + 8621132520*a^8*e^(22*I*d*x + 8*I*c)*log(-I*e^(I*d
*x) + e^(-I*c)) + 23708114430*a^8*e^(20*I*d*x + 6*I*c)*log(-I*e^(I*d*x) + e
^(-I*c)) + 47416228860*a^8*e^(18*I*d*x + 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)
) + 71124343290*a^8*e^(16*I*d*x + 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 711
24343290*a^8*e^(12*I*d*x - 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 4741622886
0*a^8*e^(10*I*d*x - 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 23708114430*a^8*e
^(8*I*d*x - 6*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 8621132520*a^8*e^(6*I*d*x
 - 8*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 2155283130*a^8*e^(4*I*d*x - 10*I*c
)*log(-I*e^(I*d*x) + e^(-I*c)) + 331582020*a^8*e^(2*I*d*x - 12*I*c)*log(-I*
e^(I*d*x) + e^(-I*c)) + 81284963760*a^8*e^(14*I*d*x)*log(-I*e^(I*d*x) + e^(
-I*c)) + 23684430*a^8*e^(-14*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 55050240*I
*a^8*e^(39*I*d*x + 25*I*c) - 972554240*I*a^8*e^(37*I*d*x + 23*I*c) - 809500
6720*I*a^8*e^(35*I*d*x + 21*I*c) - 42161143808*I*a^8*e^(33*I*d*x + 19*I*c)
- 153891110912*I*a^8*e^(31*I*d*x + 17*I*c) - 417750581248*I*a^8*e^(29*I*d*x
 + 15*I*c) - 873287647232*I*a^8*e^(27*I*d*x + 13*I*c) - 1435886419968*I*a^8
*e^(25*I*d*x + 11*I*c) - 1879877615616*I*a^8*e^(23*I*d*x + 9*I*c) - 1970745
114624*I*a^8*e^(21*I*d*x + 7*I*c) - 1654208331776*I*a^8*e^(19*I*d*x + 5*I*c
) - 1105350098944*I*a^8*e^(17*I*d*x + 3*I*c) - 580728651776*I*a^8*e^(15*I*d
*x + I*c) - 234836983808*I*a^8*e^(13*I*d*x - I*c) - 70581747712*I*a^8*e^(11
*I*d*x - 3*I*c) - 14856224768*I*a^8*e^(9*I*d*x - 5*I*c) - 1955069952*I*a^8*
e^(7*I*d*x - 7*I*c) - 121110528*I*a^8*e^(5*I*d*x - 9*I*c))/(d*e^(28*I*d*x +
 14*I*c) + 14*d*e^(26*I*d*x + 12*I*c) + 91*d*e^(24*I*d*x + 10*I*c) + 364*d*
e^(22*I*d*x + 8*I*c) + 1001*d*e^(20*I*d*x + 6*I*c) + 2002*d*e^(18*I*d*x + 4
*I*c) + 3003*d*e^(16*I*d*x + 2*I*c) + 3003*d*e^(12*I*d*x - 2*I*c) + 2002*d*
e^(10*I*d*x - 4*I*c) + 1001*d*e^(8*I*d*x - 6*I*c) + 364*d*e^(6*I*d*x - 8*I*
c) + 91*d*e^(4*I*d*x - 10*I*c) + 14*d*e^(2*I*d*x - 12*I*c) + 3432*d*e^(14*I
*d*x) + d*e^(-14*I*c))
```

Mupad [B] (verification not implemented)

Time = 4.81 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.48

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{a^8\left(\frac{e^{c\,5i+d\,x\,5i}\,1i}{40}+\frac{e^{c\,7i+d\,x\,7i}\,3i}{56}+\frac{e^{c\,9i+d\,x\,9i}\,1i}{24}+\frac{e^{c\,11i+d\,x\,11i}\,1i}{88}\right)}{d}$$

```
[In] int(cos(c + d*x)^11*(a + a*tan(c + d*x)*1i)^8,x)
[Out] -(a^8*((exp(c*5i + d*x*5i)*1i)/40 + (exp(c*7i + d*x*7i)*3i)/56 + (exp(c*9i
+ d*x*9i)*1i)/24 + (exp(c*11i + d*x*11i)*1i)/88))/d
```

3.97 $\int \cos^{13}(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 24, antiderivative size = 211

$$\begin{aligned}\int \cos^{13}(c+dx)(a+ia\tan(c+dx))^8\,dx =& -\frac{20ia^3\cos^7(c+dx)(a+ia\tan(c+dx))^5}{3003d}\\&-\frac{20ia^2\cos^9(c+dx)(a+ia\tan(c+dx))^6}{1287d}\\&-\frac{5ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^7}{143d}\\&-\frac{i\cos^{13}(c+dx)(a+ia\tan(c+dx))^8}{13d}\\&-\frac{8ia^2\cos^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^3}{9009d}\\&-\frac{8i\cos^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^4}{3003d}\end{aligned}$$

```
[Out] -20/3003*I*a^3*cos(d*x+c)^7*(a+I*a*tan(d*x+c))^5/d-20/1287*I*a^2*cos(d*x+c)
^9*(a+I*a*tan(d*x+c))^6/d-5/143*I*a*cos(d*x+c)^11*(a+I*a*tan(d*x+c))^7/d-1/
13*I*cos(d*x+c)^13*(a+I*a*tan(d*x+c))^8/d-8/9009*I*a^2*cos(d*x+c)^3*(a^2+I*
a^2*tan(d*x+c))^3/d-8/3003*I*cos(d*x+c)^5*(a^2+I*a^2*tan(d*x+c))^4/d
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3578, 3569}

$$\int \cos^{13}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{20ia^3\cos^7(c+dx)(a+ia\tan(c+dx))^5}{3003d} - \frac{20ia^2\cos^9(c+dx)(a+ia\tan(c+dx))^6}{1287d} - \frac{8i\cos^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^4}{3003d} - \frac{8ia^2\cos^3(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^3}{9009d} - \frac{i\cos^{13}(c+dx)(a+ia\tan(c+dx))^8}{13d} - \frac{5ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^7}{143d}$$

```
[In] Int[Cos[c + d*x]^13*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (((-20*I)/3003)*a^3*Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^5)/d - (((20*I)/1
287)*a^2*Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^6)/d - (((5*I)/143)*a*Cos[c
+ d*x]^11*(a + I*a*Tan[c + d*x])^7)/d - ((I/13)*Cos[c + d*x]^13*(a + I*a*Ta
n[c + d*x])^8)/d - (((8*I)/9009)*a^2*Cos[c + d*x]^3*(a^2 + I*a^2*Tan[c + d*
x])^3)/d - (((8*I)/3003)*Cos[c + d*x]^5*(a^2 + I*a^2*Tan[c + d*x])^4)/d
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{i\cos^{13}(c+dx)(a+ia\tan(c+dx))^8}{13d} \\
&\quad + \frac{1}{13}(5a)\int\cos^{11}(c+dx)(a+ia\tan(c+dx))^7\,dx \\
&= -\frac{5ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^7}{143d} - \frac{i\cos^{13}(c+dx)(a+ia\tan(c+dx))^8}{13d} \\
&\quad + \frac{1}{143}\left(20a^2\right)\int\cos^9(c+dx)(a+ia\tan(c+dx))^6\,dx \\
&= -\frac{20ia^2\cos^9(c+dx)(a+ia\tan(c+dx))^6}{1287d} - \frac{5ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^7}{143d} \\
&\quad - \frac{i\cos^{13}(c+dx)(a+ia\tan(c+dx))^8}{13d} + \frac{1}{429}\left(20a^3\right)\int\cos^7(c+dx)(a+ia\tan(c+dx))^5\,dx \\
&= -\frac{20ia^3\cos^7(c+dx)(a+ia\tan(c+dx))^5}{3003d} - \frac{20ia^2\cos^9(c+dx)(a+ia\tan(c+dx))^6}{1287d} \\
&\quad - \frac{5ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^7}{143d} - \frac{i\cos^{13}(c+dx)(a+ia\tan(c+dx))^8}{13d} \\
&\quad + \frac{(40a^4)\int\cos^5(c+dx)(a+ia\tan(c+dx))^4\,dx}{3003} \\
&= -\frac{20ia^3\cos^7(c+dx)(a+ia\tan(c+dx))^5}{3003d} - \frac{20ia^2\cos^9(c+dx)(a+ia\tan(c+dx))^6}{1287d} \\
&\quad - \frac{5ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^7}{143d} - \frac{i\cos^{13}(c+dx)(a+ia\tan(c+dx))^8}{13d} \\
&\quad - \frac{8i\cos^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^4}{3003d} + \frac{(8a^5)\int\cos^3(c+dx)(a+ia\tan(c+dx))^3\,dx}{3003} \\
&= -\frac{8ia^5\cos^3(c+dx)(a+ia\tan(c+dx))^3}{9009d} - \frac{20ia^3\cos^7(c+dx)(a+ia\tan(c+dx))^5}{3003d} \\
&\quad - \frac{20ia^2\cos^9(c+dx)(a+ia\tan(c+dx))^6}{1287d} - \frac{5ia\cos^{11}(c+dx)(a+ia\tan(c+dx))^7}{143d} \\
&\quad - \frac{i\cos^{13}(c+dx)(a+ia\tan(c+dx))^8}{13d} - \frac{8i\cos^5(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^4}{3003d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.78 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.80

$$\int \cos^{13}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$= \frac{a^8\sec(c+dx)(-i\cos(7(c+dx))+\sin(7(c+dx)))\left(44759\cos(c+dx)+26117\cos(3(c+dx))+7791\cos(5\right.}{}$$

```
[In] Integrate[Cos[c + d*x]^13*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*Sec[c + d*x]*((-I)*Cos[7*(c + d*x)] + Sin[7*(c + d*x)])*(44759*Cos[c +
 d*x] + 26117*Cos[3*(c + d*x)] + 7791*Cos[5*(c + d*x)] + 693*Cos[7*(c + d*x
)] + 275456*Sqrt[Cos[c + d*x]^2]*Cos[7*(c + d*x)] + (1001*I)*Sin[c + d*x] +
 (2093*I)*Sin[3*(c + d*x)] + (1785*I)*Sin[5*(c + d*x)] + (693*I)*Sin[7*(c +
 d*x)] - (275456*I)*Sqrt[Cos[c + d*x]^2]*Sin[7*(c + d*x)]))/(576576*d)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 616 vs. 2(187) = 374.

Time = 1.81 (sec) , antiderivative size = 617, normalized size of antiderivative = 2.92

Expression too large to display

```
[In] int(cos(d*x+c)^13*(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 1/d*(a^8*(-1/13*sin(d*x+c)^7*cos(d*x+c)^6-7/143*sin(d*x+c)^5*cos(d*x+c)^6-3
5/1287*sin(d*x+c)^3*cos(d*x+c)^6-5/429*sin(d*x+c)*cos(d*x+c)^6+1/429*(8/3+c
os(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-8/13*I*a^8*cos(d*x+c)^13-28*a^8*(
-1/13*sin(d*x+c)^5*cos(d*x+c)^8-5/143*sin(d*x+c)^3*cos(d*x+c)^8-5/429*cos(d
*x+c)^8*sin(d*x+c)+5/3003*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c
)^2)*sin(d*x+c))-8*I*a^8*(-1/13*cos(d*x+c)^7*sin(d*x+c)^6-6/143*cos(d*x+c)^
7*sin(d*x+c)^4-8/429*cos(d*x+c)^7*sin(d*x+c)^2-16/3003*cos(d*x+c)^7)+70*a^8
*(-1/13*sin(d*x+c)^3*cos(d*x+c)^10-3/143*sin(d*x+c)*cos(d*x+c)^10+1/429*(12
8/35+cos(d*x+c)^8+8/7*cos(d*x+c)^6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*s
in(d*x+c))+56*I*a^8*(-1/13*cos(d*x+c)^9*sin(d*x+c)^4-4/143*cos(d*x+c)^9*sin
(d*x+c)^2-8/1287*cos(d*x+c)^9)-28*a^8*(-1/13*sin(d*x+c)*cos(d*x+c)^12+1/143
*(256/63+cos(d*x+c)^10+10/9*cos(d*x+c)^8+80/63*cos(d*x+c)^6+32/21*cos(d*x+c
)^4+128/63*cos(d*x+c)^2)*sin(d*x+c))-56*I*a^8*(-1/13*cos(d*x+c)^11*sin(d*x+
c)^2-2/143*cos(d*x+c)^11)+1/13*a^8*(1024/231+cos(d*x+c)^12+12/11*cos(d*x+c)
^10+40/33*cos(d*x+c)^8+320/231*cos(d*x+c)^6+128/77*cos(d*x+c)^4+512/231*cos
(d*x+c)^2)*sin(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.43

$$\int \cos^{13}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$=\frac{-693i\,a^8e^{(13i\,dx+13i\,c)}-4095i\,a^8e^{(11i\,dx+11i\,c)}-10010i\,a^8e^{(9i\,dx+9i\,c)}-12870i\,a^8e^{(7i\,dx+7i\,c)}-9009i\,a^8e^{(5i\,dx+}}{288288\,d}$$

```
[In] integrate(cos(d*x+c)^13*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/288288*(-693*I*a^8*e^(13*I*d*x + 13*I*c) - 4095*I*a^8*e^(11*I*d*x + 11*I*
c) - 10010*I*a^8*e^(9*I*d*x + 9*I*c) - 12870*I*a^8*e^(7*I*d*x + 7*I*c) - 90
09*I*a^8*e^(5*I*d*x + 5*I*c) - 3003*I*a^8*e^(3*I*d*x + 3*I*c))/d
```

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.14

$$\int \cos^{13}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$=\begin{cases}\frac{-17439916032ia^8d^5e^{13ic}e^{13idx}-103054049280ia^8d^5e^{11ic}e^{11idx}-251909898240ia^8d^5e^{9ic}e^{9idx}-323884154880ia^8d^5e^{7ic}e^{7idx}-226718908416i}{7255005069312d^6} \\ x\left(\frac{a^8e^{13ic}}{32}+\frac{5a^8e^{11ic}}{32}+\frac{5a^8e^{9ic}}{16}+\frac{5a^8e^{7ic}}{16}+\frac{5a^8e^{5ic}}{32}+\frac{a^8e^{3ic}}{32}\right)\end{cases}$$

```
[In] integrate(cos(d*x+c)**13*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise(((-17439916032*I*a**8*d**5*exp(13*I*c)*exp(13*I*d*x) - 1030540492
80*I*a**8*d**5*exp(11*I*c)*exp(11*I*d*x) - 251909898240*I*a**8*d**5*exp(9*I
*c)*exp(9*I*d*x) - 323884154880*I*a**8*d**5*exp(7*I*c)*exp(7*I*d*x) - 22671
8908416*I*a**8*d**5*exp(5*I*c)*exp(5*I*d*x) - 75572969472*I*a**8*d**5*exp(3
*I*c)*exp(3*I*d*x))/(7255005069312*d**6), Ne(d**6, 0)), (x*(a**8*exp(13*I*c
)/32 + 5*a**8*exp(11*I*c)/32 + 5*a**8*exp(9*I*c)/16 + 5*a**8*exp(7*I*c)/16
+ 5*a**8*exp(5*I*c)/32 + a**8*exp(3*I*c)/32), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 405 vs. 2(175) = 350.

Time = 0.32 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.92

$$\int \cos^{13}(c+dx)(a+ia\tan(c+dx))^8\,dx =$$
$$-\frac{5544i\,a^8\cos(dx+c)^{13}+24i\left(231\cos(dx+c)^{13}-819\cos(dx+c)^{11}+1001\cos(dx+c)^9-429\cos(dx\right.}{}$$

```
[In] integrate(cos(d*x+c)^13*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/9009*(5544*I*a^8*cos(d*x + c)^13 + 24*I*(231*cos(d*x + c)^13 - 819*cos(d
*x + c)^11 + 1001*cos(d*x + c)^9 - 429*cos(d*x + c)^7)*a^8 + 392*I*(99*cos(
d*x + c)^13 - 234*cos(d*x + c)^11 + 143*cos(d*x + c)^9)*a^8 + 3528*I*(11*co
s(d*x + c)^13 - 13*cos(d*x + c)^11)*a^8 - 42*(1155*sin(d*x + c)^13 - 5460*s
in(d*x + c)^11 + 10010*sin(d*x + c)^9 - 8580*sin(d*x + c)^7 + 3003*sin(d*x
+ c)^5)*a^8 - 28*(693*sin(d*x + c)^13 - 4095*sin(d*x + c)^11 + 10010*sin(d*
x + c)^9 - 12870*sin(d*x + c)^7 + 9009*sin(d*x + c)^5 - 3003*sin(d*x + c)^3
)*a^8 - 84*(231*sin(d*x + c)^13 - 819*sin(d*x + c)^11 + 1001*sin(d*x + c)^9
 - 429*sin(d*x + c)^7)*a^8 - 3*(231*sin(d*x + c)^13 - 1638*sin(d*x + c)^11
+ 5005*sin(d*x + c)^9 - 8580*sin(d*x + c)^7 + 9009*sin(d*x + c)^5 - 6006*si
n(d*x + c)^3 + 3003*sin(d*x + c))*a^8 - 7*(99*sin(d*x + c)^13 - 234*sin(d*x
 + c)^11 + 143*sin(d*x + c)^9)*a^8)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2891 vs. 2(175) = 350.

Time = 1.84 (sec) , antiderivative size = 2891, normalized size of antiderivative = 13.70

$$\int \cos^{13}(c+dx)(a+ia\tan(c+dx))^8\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^13*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/151145938944*(1945052766657*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*
c) + 1) + 27230738733198*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) +
1) + 176999801765787*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) + 1) +
 707999207063148*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1946
997819423657*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 38939956
38847314*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 584099345827
0971*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 5840993458270971
```

*a^8*e^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 3893995638847314*a^8
*e^(10*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1946997819423657*a^8*e^(
8*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 707999207063148*a^8*e^(6*I*d*
x - 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 176999801765787*a^8*e^(4*I*d*x - 10
*I*c)*log(I*e^(I*d*x + I*c) + 1) + 27230738733198*a^8*e^(2*I*d*x - 12*I*c)*
log(I*e^(I*d*x + I*c) + 1) + 6675421095166824*a^8*e^(14*I*d*x)*log(I*e^(I*d
*x + I*c) + 1) + 1945052766657*a^8*e^(-14*I*c)*log(I*e^(I*d*x + I*c) + 1) +
1944080407269*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*c) - 1) + 27217
125701766*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1769113170
61479*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) - 1) + 70764526824591
6*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1946024487676269*a^
8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 3892048975352538*a^8*e^
(18*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 5838073463028807*a^8*e^(16*
I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 5838073463028807*a^8*e^(12*I*d*
x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 3892048975352538*a^8*e^(10*I*d*x -
4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 1946024487676269*a^8*e^(8*I*d*x - 6*I*c
)*log(I*e^(I*d*x + I*c) - 1) + 707645268245916*a^8*e^(6*I*d*x - 8*I*c)*log(
I*e^(I*d*x + I*c) - 1) + 176911317061479*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(
I*d*x + I*c) - 1) + 27217125701766*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x
+ I*c) - 1) + 6672083957747208*a^8*e^(14*I*d*x)*log(I*e^(I*d*x + I*c) - 1)
+ 1944080407269*a^8*e^(-14*I*c)*log(I*e^(I*d*x + I*c) - 1) - 1945052766657*
a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 27230738733198*a^8*
e^(26*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 176999801765787*a^8*e^(
24*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 707999207063148*a^8*e^(22*
I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1946997819423657*a^8*e^(20*I*d
*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 3893995638847314*a^8*e^(18*I*d*x
+ 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 5840993458270971*a^8*e^(16*I*d*x + 2
*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 5840993458270971*a^8*e^(12*I*d*x - 2*I*
c)*log(-I*e^(I*d*x + I*c) + 1) - 3893995638847314*a^8*e^(10*I*d*x - 4*I*c)*
log(-I*e^(I*d*x + I*c) + 1) - 1946997819423657*a^8*e^(8*I*d*x - 6*I*c)*log(
-I*e^(I*d*x + I*c) + 1) - 707999207063148*a^8*e^(6*I*d*x - 8*I*c)*log(-I*e^
(I*d*x + I*c) + 1) - 176999801765787*a^8*e^(4*I*d*x - 10*I*c)*log(-I*e^(I*d
*x + I*c) + 1) - 27230738733198*a^8*e^(2*I*d*x - 12*I*c)*log(-I*e^(I*d*x +
I*c) + 1) - 6675421095166824*a^8*e^(14*I*d*x)*log(-I*e^(I*d*x + I*c) + 1) -
1945052766657*a^8*e^(-14*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 1944080407269*
a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 27217125701766*a^8*
e^(26*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 176911317061479*a^8*e^(
24*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 707645268245916*a^8*e^(22*
I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 1946024487676269*a^8*e^(20*I*d
*x + 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 3892048975352538*a^8*e^(18*I*d*x
+ 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 5838073463028807*a^8*e^(16*I*d*x + 2
*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 5838073463028807*a^8*e^(12*I*d*x - 2*I*
c)*log(-I*e^(I*d*x + I*c) - 1) - 3892048975352538*a^8*e^(10*I*d*x - 4*I*c)*
log(-I*e^(I*d*x + I*c) - 1) - 1946024487676269*a^8*e^(8*I*d*x - 6*I*c)*log(
-I*e^(I*d*x + I*c) - 1) - 707645268245916*a^8*e^(6*I*d*x - 8*I*c)*log(-I*e^

(I*d*x + I*c) - 1) - 176911317061479*a^8*e^(4*I*d*x - 10*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 27217125701766*a^8*e^(2*I*d*x - 12*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 6672083957747208*a^8*e^(14*I*d*x)*log(-I*e^(I*d*x + I*c) - 1) - 1944080407269*a^8*e^(-14*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 972359388*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 13613031432*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 88484704308*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 353938817232*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 973331747388*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 1946663494776*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 2919995242164*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 2919995242164*a^8*e^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 1946663494776*a^8*e^(10*I*d*x - 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 973331747388*a^8*e^(8*I*d*x - 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 353938817232*a^8*e^(6*I*d*x - 8*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 88484704308*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 13613031432*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 3337137419616*a^8*e^(14*I*d*x)*log(I*e^(I*d*x) + e^(-I*c)) - 972359388*a^8*e^(-14*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 972359388*a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 13613031432*a^8*e^(26*I*d*x + 12*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 88484704308*a^8*e^(24*I*d*x + 10*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 353938817232*a^8*e^(22*I*d*x + 8*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 973331747388*a^8*e^(20*I*d*x + 6*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 1946663494776*a^8*e^(18*I*d*x + 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 2919995242164*a^8*e^(16*I*d*x + 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 2919995242164*a^8*e^(12*I*d*x - 2*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 1946663494776*a^8*e^(10*I*d*x - 4*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 973331747388*a^8*e^(8*I*d*x - 6*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 353938817232*a^8*e^(6*I*d*x - 8*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 88484704308*a^8*e^(4*I*d*x - 10*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 13613031432*a^8*e^(2*I*d*x - 12*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 3337137419616*a^8*e^(14*I*d*x)*log(-I*e^(I*d*x) + e^(-I*c)) + 972359388*a^8*e^(-14*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) - 363331584*I*a^8*e^(41*I*d*x + 27*I*c) - 7233601536*I*a^8*e^(39*I*d*x + 25*I*c) - 68368728064*I*a^8*e^(37*I*d*x + 23*I*c) - 407847305216*I*a^8*e^(35*I*d*x + 21*I*c) - 1721956827136*I*a^8*e^(33*I*d*x + 19*I*c) - 5468544040960*I*a^8*e^(31*I*d*x + 17*I*c) - 13550653276160*I*a^8*e^(29*I*d*x + 15*I*c) - 26817907916800*I*a^8*e^(27*I*d*x + 13*I*c) - 43029359493120*I*a^8*e^(25*I*d*x + 11*I*c) - 56481348059136*I*a^8*e^(23*I*d*x + 9*I*c) - 60915861946368*I*a^8*e^(21*I*d*x + 7*I*c) - 53989539315712*I*a^8*e^(19*I*d*x + 5*I*c) - 39164164702208*I*a^8*e^(17*I*d*x + 3*I*c) - 23049212526592*I*a^8*e^(15*I*d*x + I*c) - 10844177956864*I*a^8*e^(13*I*d*x - I*c) - 3984947412992*I*a^8*e^(11*I*d*x - 3*I*c) - 1102630617088*I*a^8*e^(9*I*d*x - 5*I*c) - 216147689472*I*a^8*e^(7*I*d*x - 7*I*c) - 26765426688*I*a^8*e^(5*I*d*x - 9*I*c) - 1574436864*I*a^8*e^(3*I*d*x - 11*I*c))/(d*e^(28*I*d*x + 14*I*c) + 14*d*e^(26*I*d*x + 12*I*c) + 91*d*e^(24*I*d*x + 10*I*c) + 364*d*e^(22*I*d*x + 8*I*c) + 1001*d*e^(20*I*d*x + 6*I*c) + 2002*d*e^(18*I*d*x + 4*I*c) + 3003*d*e^(16*I*d*x + 2*I*c) + 3003*d*e^(12*I*d*x

```
 - 2*I*c) + 2002*d*e^(10*I*d*x - 4*I*c) + 1001*d*e^(8*I*d*x - 6*I*c) + 364*
d*e^(6*I*d*x - 8*I*c) + 91*d*e^(4*I*d*x - 10*I*c) + 14*d*e^(2*I*d*x - 12*I*
c) + 3432*d*e^(14*I*d*x) + d*e^(-14*I*c))
```

Mupad [B] (verification not implemented)

Time = 4.84 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.44

$$\int \cos^{13}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$=-\frac{a^8\left(\frac{e^{c\,3i+d\,x\,3i}\,1i}{96}+\frac{e^{c\,5i+d\,x\,5i}\,1i}{32}+\frac{e^{c\,7i+d\,x\,7i}\,5i}{112}+\frac{e^{c\,9i+d\,x\,9i}\,5i}{144}+\frac{e^{c\,11i+d\,x\,11i}\,5i}{352}+\frac{e^{c\,13i+d\,x\,13i}\,1i}{416}\right)}{d}$$

```
[In] int(cos(c + d*x)^13*(a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] -(a^8*((exp(c*3i + d*x*3i)*1i)/96 + (exp(c*5i + d*x*5i)*1i)/32 + (exp(c*7i
+ d*x*7i)*5i)/112 + (exp(c*9i + d*x*9i)*5i)/144 + (exp(c*11i + d*x*11i)*5i)
/352 + (exp(c*13i + d*x*13i)*1i)/416))/d
```

3.98 $\int \cos^{15}(c+dx)(a+ia\tan(c+dx))^8\,dx$

Optimal result

Integrand size = 24, antiderivative size = 212

$$\int \cos^{15}(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{7a^8\sin(c+dx)}{1287d} - \frac{7a^8\sin^3(c+dx)}{1287d} + \frac{7a^8\sin^5(c+dx)}{2145d} - \frac{a^8\sin^7(c+dx)}{1287d} - \frac{2ia^3\cos^{13}(c+dx)(a+ia\tan(c+dx))^5}{195d} - \frac{2ia\cos^{15}(c+dx)(a+ia\tan(c+dx))^7}{15d} - \frac{2ia^2\cos^{11}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^3}{715d} - \frac{2i\cos^9(c+dx)\left(a^8+ia^8\tan(c+dx)\right)}{1287d}$$

```
[Out] 7/1287*a^8*sin(d*x+c)/d-7/1287*a^8*sin(d*x+c)^3/d+7/2145*a^8*sin(d*x+c)^5/d
-1/1287*a^8*sin(d*x+c)^7/d-2/195*I*a^3*cos(d*x+c)^13*(a+I*a*tan(d*x+c))^5/d
-2/15*I*a*cos(d*x+c)^15*(a+I*a*tan(d*x+c))^7/d-2/715*I*a^2*cos(d*x+c)^11*(a
^2+I*a^2*tan(d*x+c))^3/d-2/1287*I*cos(d*x+c)^9*(a^8+I*a^8*tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3577, 2713}

$$\int \cos^{15}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{a^8\sin^7(c+dx)}{1287d}+\frac{7a^8\sin^5(c+dx)}{2145d}-\frac{7a^8\sin^3(c+dx)}{1287d}+\frac{7a^8\sin(c+dx)}{1287d}-\frac{2i\cos^9(c+dx)\left(a^8+ia^8\tan(c+dx)\right)}{1287d}-\frac{2ia^3\cos^{13}(c+dx)(a+ia\tan(c+dx))^5}{195d}-\frac{2ia^2\cos^{11}(c+dx)\left(a^2+ia^2\tan(c+dx)\right)^3}{715d}-\frac{2ia\cos^{15}(c+dx)(a+ia\tan(c+dx))^7}{15d}$$

```
[In] Int[Cos[c + d*x]^15*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (7*a^8*Sin[c + d*x])/(1287*d) - (7*a^8*Sin[c + d*x]^3)/(1287*d) + (7*a^8*Si
n[c + d*x]^5)/(2145*d) - (a^8*Sin[c + d*x]^7)/(1287*d) - (((2*I)/195)*a^3*C
os[c + d*x]^13*(a + I*a*Tan[c + d*x])^5)/d - (((2*I)/15)*a*Cos[c + d*x]^15*
(a + I*a*Tan[c + d*x])^7)/d - (((2*I)/715)*a^2*Cos[c + d*x]^11*(a^2 + I*a^2
*Tan[c + d*x])^3)/d - (((2*I)/1287)*Cos[c + d*x]^9*(a^8 + I*a^8*Tan[c + d*x
]))/d
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2ia\cos^{15}(c+dx)(a+ia\tan(c+dx))^7}{15d} \\
&\quad+\frac{1}{15}a^2\int\cos^{13}(c+dx)(a+ia\tan(c+dx))^6\,dx \\
&= -\frac{2ia^3\cos^{13}(c+dx)(a+ia\tan(c+dx))^5}{195d}-\frac{2ia\cos^{15}(c+dx)(a+ia\tan(c+dx))^7}{15d} \\
&\quad+\frac{1}{65}a^4\int\cos^{11}(c+dx)(a+ia\tan(c+dx))^4\,dx \\
&= -\frac{2ia^5\cos^{11}(c+dx)(a+ia\tan(c+dx))^3}{715d}-\frac{2ia^3\cos^{13}(c+dx)(a+ia\tan(c+dx))^5}{195d} \\
&\quad-\frac{2ia\cos^{15}(c+dx)(a+ia\tan(c+dx))^7}{15d}+\frac{1}{143}a^6\int\cos^9(c+dx)(a+ia\tan(c+dx))^2\,dx \\
&= -\frac{2ia^5\cos^{11}(c+dx)(a+ia\tan(c+dx))^3}{715d}-\frac{2ia^3\cos^{13}(c+dx)(a+ia\tan(c+dx))^5}{195d} \\
&\quad-\frac{2ia\cos^{15}(c+dx)(a+ia\tan(c+dx))^7}{15d} \\
&\quad-\frac{2i\cos^9(c+dx)\left(a^8+ia^8\tan(c+dx)\right)}{1287d}+\frac{(7a^8)\int\cos^7(c+dx)\,dx}{1287} \\
&= -\frac{2ia^5\cos^{11}(c+dx)(a+ia\tan(c+dx))^3}{715d}-\frac{2ia^3\cos^{13}(c+dx)(a+ia\tan(c+dx))^5}{195d} \\
&\quad-\frac{2ia\cos^{15}(c+dx)(a+ia\tan(c+dx))^7}{15d}-\frac{2i\cos^9(c+dx)\left(a^8+ia^8\tan(c+dx)\right)}{1287d} \\
&\quad-\frac{(7a^8)\operatorname{Subst}\left(\int\left(1-3x^2+3x^4-x^6\right)\,dx,x,-\sin(c+dx)\right)}{1287d} \\
&= \frac{7a^8\sin(c+dx)}{1287d}-\frac{7a^8\sin^3(c+dx)}{1287d}+\frac{7a^8\sin^5(c+dx)}{2145d} \\
&\quad-\frac{a^8\sin^7(c+dx)}{1287d}-\frac{2ia^5\cos^{11}(c+dx)(a+ia\tan(c+dx))^3}{715d} \\
&\quad-\frac{2ia^3\cos^{13}(c+dx)(a+ia\tan(c+dx))^5}{195d} \\
&\quad-\frac{2ia\cos^{15}(c+dx)(a+ia\tan(c+dx))^7}{15d}-\frac{2i\cos^9(c+dx)\left(a^8+ia^8\tan(c+dx)\right)}{1287d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.11 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.82

$$\int \cos^{15}(c+dx)(a+ia\tan(c+dx))^8\,dx$$
$$=\frac{a^8\sec(c+dx)(-i\cos(8(c+dx))+\sin(8(c+dx)))\left(28600+48256\cos(2(c+dx))+28896\cos(4(c+dx)\right.}{}$$

```
[In] Integrate[Cos[c + d*x]^15*(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (a^8*Sec[c + d*x]*((-I)*Cos[8*(c + d*x)] + Sin[8*(c + d*x)])*(28600 + 48256
*Cos[2*(c + d*x)] + 28896*Cos[4*(c + d*x)] + 12672*Cos[6*(c + d*x)] + 3432*
Cos[8*(c + d*x)] + 317440*Sqrt[Cos[c + d*x]^2]*Cos[8*(c + d*x)] - (10946*I)
*Sin[2*(c + d*x)] - (13146*I)*Sin[4*(c + d*x)] - (8778*I)*Sin[6*(c + d*x)]
- (3003*I)*Sin[8*(c + d*x)] - (317440*I)*Sqrt[Cos[c + d*x]^2]*Sin[8*(c + d*
x)]))/(823680*d)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 666 vs. 2(188) = 376.

Time = 1.80 (sec) , antiderivative size = 667, normalized size of antiderivative = 3.15

Expression too large to display

```
[In] int(cos(d*x+c)^15*(a+I*a*tan(d*x+c))^8,x)
```

```
[Out] 1/d*(a^8*(-1/15*sin(d*x+c)^7*cos(d*x+c)^8-7/195*sin(d*x+c)^5*cos(d*x+c)^8-7
/429*sin(d*x+c)^3*cos(d*x+c)^8-7/1287*cos(d*x+c)^8*sin(d*x+c)+1/1287*(16/5+
cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))-8/15*I*a^8*cos(
d*x+c)^15-28*a^8*(-1/15*sin(d*x+c)^5*cos(d*x+c)^10-1/39*sin(d*x+c)^3*cos(d*
x+c)^10-1/143*sin(d*x+c)*cos(d*x+c)^10+1/1287*(128/35+cos(d*x+c)^8+8/7*cos(
d*x+c)^6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*x+c))-8*I*a^8*(-1/15*
cos(d*x+c)^9*sin(d*x+c)^6-2/65*cos(d*x+c)^9*sin(d*x+c)^4-8/715*cos(d*x+c)^9
*sin(d*x+c)^2-16/6435*cos(d*x+c)^9)+70*a^8*(-1/15*sin(d*x+c)^3*cos(d*x+c)^1
2-1/65*sin(d*x+c)*cos(d*x+c)^12+1/715*(256/63+cos(d*x+c)^10+10/9*cos(d*x+c)
^8+80/63*cos(d*x+c)^6+32/21*cos(d*x+c)^4+128/63*cos(d*x+c)^2)*sin(d*x+c))+5
6*I*a^8*(-1/15*cos(d*x+c)^11*sin(d*x+c)^4-4/195*cos(d*x+c)^11*sin(d*x+c)^2-
8/2145*cos(d*x+c)^11)-28*a^8*(-1/15*sin(d*x+c)*cos(d*x+c)^14+1/195*(1024/23
1+cos(d*x+c)^12+12/11*cos(d*x+c)^10+40/33*cos(d*x+c)^8+320/231*cos(d*x+c)^6
+128/77*cos(d*x+c)^4+512/231*cos(d*x+c)^2)*sin(d*x+c))-56*I*a^8*(-1/15*cos(
d*x+c)^13*sin(d*x+c)^2-2/195*cos(d*x+c)^13)+1/15*a^8*(2048/429+cos(d*x+c)^1
4+14/13*cos(d*x+c)^12+168/143*cos(d*x+c)^10+560/429*cos(d*x+c)^8+640/429*co
s(d*x+c)^6+256/143*cos(d*x+c)^4+1024/429*cos(d*x+c)^2)*sin(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.56

$$\int \cos^{15}(c+dx)(a+ia\tan(c+dx))^8\,dx = \frac{-429i\,a^8e^{(15i\,dx+15i\,c)} - 3465i\,a^8e^{(13i\,dx+13i\,c)} - 12285i\,a^8e^{(11i\,dx+11i\,c)} - 25025i\,a^8e^{(9i\,dx+9i\,c)} - 32175i\,a^8e^{(7i\,dx}}{823680\,d}$$

```
[In] integrate(cos(d*x+c)^15*(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/823680*(-429*I*a^8*e^(15*I*d*x + 15*I*c) - 3465*I*a^8*e^(13*I*d*x + 13*I*
c) - 12285*I*a^8*e^(11*I*d*x + 11*I*c) - 25025*I*a^8*e^(9*I*d*x + 9*I*c) -
32175*I*a^8*e^(7*I*d*x + 7*I*c) - 27027*I*a^8*e^(5*I*d*x + 5*I*c) - 15015*I
*a^8*e^(3*I*d*x + 3*I*c) - 6435*I*a^8*e^(I*d*x + I*c))/d
```

Sympy [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.48

$$\int \cos^{15}(c+dx)(a+ia\tan(c+dx))^8\,dx = \begin{cases} \frac{-108677488507984281 60ia^8d^7e^{15ic}e^{15idx} - 8777797148721807360 0ia^8d^7e^{13ic}e^{13idx} - 311212808000136806400ia^8d^7e^{11ic}e^{11idx} - 633952016296}{} \\ x\left(\frac{a^8e^{15ic}}{128} + \frac{7a^8e^{13ic}}{128} + \frac{21a^8e^{11ic}}{128} + \frac{35a^8e^{9ic}}{128} + \frac{35a^8e^{7ic}}{128} + \frac{21a^8e^{5ic}}{128} + \frac{7a^8e^{3ic}}{128} + \frac{a^8e^{ic}}{128}\right) \end{cases}$$

```
[In] integrate(cos(d*x+c)**15*(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise(((-10867748850798428160*I*a**8*d**7*exp(15*I*c)*exp(15*I*d*x) - 8
7777971487218073600*I*a**8*d**7*exp(13*I*c)*exp(13*I*d*x) - 311212808000136
806400*I*a**8*d**7*exp(11*I*c)*exp(11*I*d*x) - 633952016296574976000*I*a**8
*d**7*exp(9*I*c)*exp(9*I*d*x) - 815081163809882112000*I*a**8*d**7*exp(7*I*c
)*exp(7*I*d*x) - 684668177600300974080*I*a**8*d**7*exp(5*I*c)*exp(5*I*d*x)
- 380371209777944985600*I*a**8*d**7*exp(3*I*c)*exp(3*I*d*x) - 1630162327619
76422400*I*a**8*d**7*exp(I*c)*exp(I*d*x))/(20866077793532982067200*d**8), N
e(d**8, 0)), (x*(a**8*exp(15*I*c)/128 + 7*a**8*exp(13*I*c)/128 + 21*a**8*ex
p(11*I*c)/128 + 35*a**8*exp(9*I*c)/128 + 35*a**8*exp(7*I*c)/128 + 21*a**8*e
xp(5*I*c)/128 + 7*a**8*exp(3*I*c)/128 + a**8*exp(I*c)/128), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 453 vs. $2(180) = 360$.

Time = 0.50 (sec) , antiderivative size = 453, normalized size of antiderivative = 2.14

$$\int \cos^{15}(c+dx)(a+ia\tan(c+dx))^8\,dx = -\frac{3432i\,a^8\cos{(dx+c)}^{15} + 8i\left(429\cos{(dx+c)}^{15} - 1485\cos{(dx+c)}^{13} + 1755\cos{(dx+c)}^{11} - 715\cos{(}\right.}{}$$

```
[In] integrate(cos(d*x+c)^15*(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/6435*(3432*I*a^8*cos(d*x + c)^15 + 8*I*(429*cos(d*x + c)^15 - 1485*cos(d
*x + c)^13 + 1755*cos(d*x + c)^11 - 715*cos(d*x + c)^9)*a^8 + 168*I*(143*co
s(d*x + c)^15 - 330*cos(d*x + c)^13 + 195*cos(d*x + c)^11)*a^8 + 1848*I*(13
*cos(d*x + c)^15 - 15*cos(d*x + c)^13)*a^8 + 4*(3003*sin(d*x + c)^15 - 1386
0*sin(d*x + c)^13 + 24570*sin(d*x + c)^11 - 20020*sin(d*x + c)^9 + 6435*sin
(d*x + c)^7)*a^8 + 10*(3003*sin(d*x + c)^15 - 17325*sin(d*x + c)^13 + 40950
*sin(d*x + c)^11 - 50050*sin(d*x + c)^9 + 32175*sin(d*x + c)^7 - 9009*sin(d
*x + c)^5)*a^8 + 4*(3003*sin(d*x + c)^15 - 20790*sin(d*x + c)^13 + 61425*si
n(d*x + c)^11 - 100100*sin(d*x + c)^9 + 96525*sin(d*x + c)^7 - 54054*sin(d*
x + c)^5 + 15015*sin(d*x + c)^3)*a^8 + (429*sin(d*x + c)^15 - 1485*sin(d*x
+ c)^13 + 1755*sin(d*x + c)^11 - 715*sin(d*x + c)^9)*a^8 + (429*sin(d*x + c
)^15 - 3465*sin(d*x + c)^13 + 12285*sin(d*x + c)^11 - 25025*sin(d*x + c)^9
+ 32175*sin(d*x + c)^7 - 27027*sin(d*x + c)^5 + 15015*sin(d*x + c)^3 - 6435
*sin(d*x + c))*a^8)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2919 vs. $2(180) = 360$.

Time = 1.89 (sec) , antiderivative size = 2919, normalized size of antiderivative = 13.77

$$\int \cos^{15}(c+dx)(a+ia\tan(c+dx))^8\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^15*(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/863691079680*(5682101344920*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*
c) + 1) + 79549418828880*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) +
1) + 517071222387720*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) + 1) +
 2068284889550880*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 568
7783446264920*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1137556
```

6892529840*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1706335033
8794760*a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1706335033879
4760*a^8*e^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) + 1) + 1137556689252984
0*a^8*e^(10*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) + 1) + 5687783446264920*a^
8*e^(8*I*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) + 1) + 2068284889550880*a^8*e^(
6*I*d*x - 8*I*c)*log(I*e^(I*d*x + I*c) + 1) + 517071222387720*a^8*e^(4*I*d*
x - 10*I*c)*log(I*e^(I*d*x + I*c) + 1) + 79549418828880*a^8*e^(2*I*d*x - 12
*I*c)*log(I*e^(I*d*x + I*c) + 1) + 19500971815765440*a^8*e^(14*I*d*x)*log(I
*e^(I*d*x + I*c) + 1) + 5682101344920*a^8*e^(-14*I*c)*log(I*e^(I*d*x + I*c)
 + 1) + 5674116082635*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x + I*c) - 1)
+ 79437625156890*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x + I*c) - 1) + 516
344563519785*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x + I*c) - 1) + 2065378
254079140*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x + I*c) - 1) + 56797901987
17635*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 113595803974352
70*a^8*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 17039370596152905*
a^8*e^(16*I*d*x + 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 17039370596152905*a^8
*e^(12*I*d*x - 2*I*c)*log(I*e^(I*d*x + I*c) - 1) + 11359580397435270*a^8*e^
(10*I*d*x - 4*I*c)*log(I*e^(I*d*x + I*c) - 1) + 5679790198717635*a^8*e^(8*I
*d*x - 6*I*c)*log(I*e^(I*d*x + I*c) - 1) + 2065378254079140*a^8*e^(6*I*d*x
- 8*I*c)*log(I*e^(I*d*x + I*c) - 1) + 516344563519785*a^8*e^(4*I*d*x - 10*I
*c)*log(I*e^(I*d*x + I*c) - 1) + 79437625156890*a^8*e^(2*I*d*x - 12*I*c)*lo
g(I*e^(I*d*x + I*c) - 1) + 19473566395603320*a^8*e^(14*I*d*x)*log(I*e^(I*d*
x + I*c) - 1) + 5674116082635*a^8*e^(-14*I*c)*log(I*e^(I*d*x + I*c) - 1) -
5682101344920*a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 79549
418828880*a^8*e^(26*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 517071222
387720*a^8*e^(24*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 206828488955
0880*a^8*e^(22*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 568778344626492
0*a^8*e^(20*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 11375566892529840*
a^8*e^(18*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 17063350338794760*a^
8*e^(16*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 17063350338794760*a^8*
e^(12*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 11375566892529840*a^8*e^
(10*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 5687783446264920*a^8*e^(8*
I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 2068284889550880*a^8*e^(6*I*d*
x - 8*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 517071222387720*a^8*e^(4*I*d*x - 1
0*I*c)*log(-I*e^(I*d*x + I*c) + 1) - 79549418828880*a^8*e^(2*I*d*x - 12*I*c
)*log(-I*e^(I*d*x + I*c) + 1) - 19500971815765440*a^8*e^(14*I*d*x)*log(-I*e
^(I*d*x + I*c) + 1) - 5682101344920*a^8*e^(-14*I*c)*log(-I*e^(I*d*x + I*c)
+ 1) - 5674116082635*a^8*e^(28*I*d*x + 14*I*c)*log(-I*e^(I*d*x + I*c) - 1)
- 79437625156890*a^8*e^(26*I*d*x + 12*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 51
6344563519785*a^8*e^(24*I*d*x + 10*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 20653
78254079140*a^8*e^(22*I*d*x + 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 56797901
98717635*a^8*e^(20*I*d*x + 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 11359580397
435270*a^8*e^(18*I*d*x + 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 1703937059615
2905*a^8*e^(16*I*d*x + 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 170393705961529
05*a^8*e^(12*I*d*x - 2*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 11359580397435270

*a^8*e^(10*I*d*x - 4*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 5679790198717635*a^
8*e^(8*I*d*x - 6*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 2065378254079140*a^8*e^
(6*I*d*x - 8*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 516344563519785*a^8*e^(4*I*
d*x - 10*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 79437625156890*a^8*e^(2*I*d*x -
 12*I*c)*log(-I*e^(I*d*x + I*c) - 1) - 19473566395603320*a^8*e^(14*I*d*x)*l
og(-I*e^(I*d*x + I*c) - 1) - 5674116082635*a^8*e^(-14*I*c)*log(-I*e^(I*d*x
+ I*c) - 1) - 7985262285*a^8*e^(28*I*d*x + 14*I*c)*log(I*e^(I*d*x) + e^(-I*
c)) - 111793671990*a^8*e^(26*I*d*x + 12*I*c)*log(I*e^(I*d*x) + e^(-I*c)) -
726658867935*a^8*e^(24*I*d*x + 10*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 290663
5471740*a^8*e^(22*I*d*x + 8*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 799324754728
5*a^8*e^(20*I*d*x + 6*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 15986495094570*a^8
*e^(18*I*d*x + 4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 23979742641855*a^8*e^(1
6*I*d*x + 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 23979742641855*a^8*e^(12*I*d
*x - 2*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 15986495094570*a^8*e^(10*I*d*x -
4*I*c)*log(I*e^(I*d*x) + e^(-I*c)) - 7993247547285*a^8*e^(8*I*d*x - 6*I*c)*
log(I*e^(I*d*x) + e^(-I*c)) - 2906635471740*a^8*e^(6*I*d*x - 8*I*c)*log(I*e
^(I*d*x) + e^(-I*c)) - 726658867935*a^8*e^(4*I*d*x - 10*I*c)*log(I*e^(I*d*x
) + e^(-I*c)) - 111793671990*a^8*e^(2*I*d*x - 12*I*c)*log(I*e^(I*d*x) + e^(
-I*c)) - 27405420162120*a^8*e^(14*I*d*x)*log(I*e^(I*d*x) + e^(-I*c)) - 7985
262285*a^8*e^(-14*I*c)*log(I*e^(I*d*x) + e^(-I*c)) + 7985262285*a^8*e^(28*I
*d*x + 14*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 111793671990*a^8*e^(26*I*d*x
+ 12*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 726658867935*a^8*e^(24*I*d*x + 10*
I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 2906635471740*a^8*e^(22*I*d*x + 8*I*c)*
log(-I*e^(I*d*x) + e^(-I*c)) + 7993247547285*a^8*e^(20*I*d*x + 6*I*c)*log(-
I*e^(I*d*x) + e^(-I*c)) + 15986495094570*a^8*e^(18*I*d*x + 4*I*c)*log(-I*e^
(I*d*x) + e^(-I*c)) + 23979742641855*a^8*e^(16*I*d*x + 2*I*c)*log(-I*e^(I*d
*x) + e^(-I*c)) + 23979742641855*a^8*e^(12*I*d*x - 2*I*c)*log(-I*e^(I*d*x)
+ e^(-I*c)) + 15986495094570*a^8*e^(10*I*d*x - 4*I*c)*log(-I*e^(I*d*x) + e^
(-I*c)) + 7993247547285*a^8*e^(8*I*d*x - 6*I*c)*log(-I*e^(I*d*x) + e^(-I*c)
) + 2906635471740*a^8*e^(6*I*d*x - 8*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 72
6658867935*a^8*e^(4*I*d*x - 10*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 11179367
1990*a^8*e^(2*I*d*x - 12*I*c)*log(-I*e^(I*d*x) + e^(-I*c)) + 27405420162120
*a^8*e^(14*I*d*x)*log(-I*e^(I*d*x) + e^(-I*c)) + 7985262285*a^8*e^(-14*I*c)
*log(-I*e^(I*d*x) + e^(-I*c)) - 449839104*I*a^8*e^(43*I*d*x + 29*I*c) - 993
1063296*I*a^8*e^(41*I*d*x + 27*I*c) - 104683536384*I*a^8*e^(39*I*d*x + 25*I
*c) - 700958375936*I*a^8*e^(37*I*d*x + 23*I*c) - 3346162253824*I*a^8*e^(35*
I*d*x + 21*I*c) - 12115053117440*I*a^8*e^(33*I*d*x + 19*I*c) - 345536410419
20*I*a^8*e^(31*I*d*x + 17*I*c) - 79597529989120*I*a^8*e^(29*I*d*x + 15*I*c)
 - 150652615393280*I*a^8*e^(27*I*d*x + 13*I*c) - 237078702981120*I*a^8*e^(2
5*I*d*x + 11*I*c) - 312733543170048*I*a^8*e^(23*I*d*x + 9*I*c) - 3475582872
45312*I*a^8*e^(21*I*d*x + 7*I*c) - 326158241497088*I*a^8*e^(19*I*d*x + 5*I*
c) - 258238371069952*I*a^8*e^(17*I*d*x + 3*I*c) - 171721273376768*I*a^8*e^(
15*I*d*x + I*c) - 95003913224192*I*a^8*e^(13*I*d*x - I*c) - 43034893877248*
I*a^8*e^(11*I*d*x - 3*I*c) - 15562783588352*I*a^8*e^(9*I*d*x - 5*I*c) - 431
9355076608*I*a^8*e^(7*I*d*x - 7*I*c) - 862791401472*I*a^8*e^(5*I*d*x - 9*I*

```
c) - 110210580480*I*a^8*e^(3*I*d*x - 11*I*c) - 6747586560*I*a^8*e^(I*d*x -
13*I*c))/(d*e^(28*I*d*x + 14*I*c) + 14*d*e^(26*I*d*x + 12*I*c) + 91*d*e^(24
*I*d*x + 10*I*c) + 364*d*e^(22*I*d*x + 8*I*c) + 1001*d*e^(20*I*d*x + 6*I*c)
 + 2002*d*e^(18*I*d*x + 4*I*c) + 3003*d*e^(16*I*d*x + 2*I*c) + 3003*d*e^(12
*I*d*x - 2*I*c) + 2002*d*e^(10*I*d*x - 4*I*c) + 1001*d*e^(8*I*d*x - 6*I*c)
+ 364*d*e^(6*I*d*x - 8*I*c) + 91*d*e^(4*I*d*x - 10*I*c) + 14*d*e^(2*I*d*x -
 12*I*c) + 3432*d*e^(14*I*d*x) + d*e^(-14*I*c))
```

Mupad [B] (verification not implemented)

Time = 6.41 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.05

$$\int \cos^{15}(c+dx)(a+ia\tan(c+dx))^8\,dx$$

$$= \frac{2\,a^8\left(2\sin\left(\frac{c}{4}+\frac{d\,x}{4}\right)^2-1\right)\left(-\frac{44779\sin(c+d\,x)^2}{32}+\frac{\sin(c+d\,x)\,32175\mathrm{i}}{128}-\frac{26075\sin(2\,c+2\,d\,x)^2}{16}-\frac{\sin(2\,c+2\,d\,x)\,3575\mathrm{i}}{8}+\frac{114583\sin}{\ldots}\right.}{\ldots}$$

```
[In] int(cos(c + d*x)^15*(a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] (2*a^8*(2*sin(c/4 + (d*x)/4)^2 - 1)*((sin(c + d*x)*32175i)/128 - (sin(2*c +
 2*d*x)*3575i)/8 + (sin(3*c + 3*d*x)*84227i)/128 - sin(4*c + 4*d*x)*754i +
(sin(5*c + 5*d*x)*111527i)/128 - (sin(6*c + 6*d*x)*7187i)/8 + (sin(7*c + 7*
d*x)*121427i)/128 - (26075*sin(2*c + 2*d*x)^2)/16 + (114583*sin(c/2 + (d*x)
/2)^2)/64 - (57925*sin(3*c + 3*d*x)^2)/32 + (116585*sin((3*c)/2 + (3*d*x)/2
)^2)/64 + (119315*sin((5*c)/2 + (5*d*x)/2)^2)/64 + (122285*sin((7*c)/2 + (7
*d*x)/2)^2)/64 - (44779*sin(c + d*x)^2)/32 - 952))/(6435*d*(sin((15*c)/2 +
(15*d*x)/2) - sin((15*c)/4 + (15*d*x)/4)^2*2i + 1i))
```

3.99 $\int \frac{\sec^{10}(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 107

$$\int \frac{\sec^{10}(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{8i(a-ia\tan(c+dx))^5}{5a^6d} - \frac{2i(a-ia\tan(c+dx))^6}{a^7d} + \frac{6i(a-ia\tan(c+dx))^7}{7a^8d} - \frac{i(a-ia\tan(c+dx))^8}{8a^9d}$$

```
[Out] 8/5*I*(a-I*a*tan(d*x+c))^5/a^6/d-2*I*(a-I*a*tan(d*x+c))^6/a^7/d+6/7*I*(a-I*
a*tan(d*x+c))^7/a^8/d-1/8*I*(a-I*a*tan(d*x+c))^8/a^9/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\int \frac{\sec^{10}(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i(a-ia\tan(c+dx))^8}{8a^9d} + \frac{6i(a-ia\tan(c+dx))^7}{7a^8d} - \frac{2i(a-ia\tan(c+dx))^6}{a^7d} + \frac{8i(a-ia\tan(c+dx))^5}{5a^6d}$$

```
[In] Int[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (((8*I)/5)*(a - I*a*Tan[c + d*x])^5)/(a^6*d) - ((2*I)*(a - I*a*Tan[c + d*x]
)^6)/(a^7*d) + (((6*I)/7)*(a - I*a*Tan[c + d*x])^7)/(a^8*d) - ((I/8)*(a - I
*a*Tan[c + d*x])^8)/(a^9*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)^4(a+x)^3\,dx, x, ia\tan(c+dx)\right)}{a^9d}\\ &= -\frac{i\text{Subst}\left(\int \left(8a^3(a-x)^4-12a^2(a-x)^5+6a(a-x)^6-(a-x)^7\right)\,dx, x, ia\tan(c+dx)\right)}{a^9d}\\ &= \frac{8i(a-ia\tan(c+dx))^5}{5a^6d}-\frac{2i(a-ia\tan(c+dx))^6}{a^7d}\\ &\quad+\frac{6i(a-ia\tan(c+dx))^7}{7a^8d}-\frac{i(a-ia\tan(c+dx))^8}{8a^9d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.52

$$\int \frac{\sec^{10}(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{(i+\tan(c+dx))^5\left(93+185i\tan(c+dx)-135\tan^2(c+dx)-35i\tan^3(c+dx)\right)}{280ad}$$

```
[In] Integrate[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x]),x]
[Out] ((I + Tan[c + d*x])^5*(93 + (185*I)*Tan[c + d*x] - 135*Tan[c + d*x]^2 - (35
*I)*Tan[c + d*x]^3))/(280*a*d)
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.54

method	result
risch	$\frac{32i\left(56\,\mathrm{e}^{6i(dx+c)}+28\,\mathrm{e}^{4i(dx+c)}+8\,\mathrm{e}^{2i(dx+c)}+1\right)}{35da\left(\mathrm{e}^{2i(dx+c)}+1\right)^{8}}$
derivativedivides	$-\frac{-\tan(dx+c)+\frac{i\left(\tan^{8}(dx+c)\right)}{8}-\frac{\left(\tan^{7}(dx+c)\right)}{7}+\frac{i\left(\tan^{6}(dx+c)\right)}{2}-\frac{3\left(\tan^{5}(dx+c)\right)}{5}+\frac{3i\left(\tan^{4}(dx+c)\right)}{4}-\left(\tan^{3}(dx+c)\right)+\frac{i\left(\tan^{2}\right.}{}}{ad}$
default	$-\frac{-\tan(dx+c)+\frac{i\left(\tan^{8}(dx+c)\right)}{8}-\frac{\left(\tan^{7}(dx+c)\right)}{7}+\frac{i\left(\tan^{6}(dx+c)\right)}{2}-\frac{3\left(\tan^{5}(dx+c)\right)}{5}+\frac{3i\left(\tan^{4}(dx+c)\right)}{4}-\left(\tan^{3}(dx+c)\right)+\frac{i\left(\tan^{2}\right.}{}}{ad}$

[In] int(sec(d*x+c)^10/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 32/35*I*(56*exp(6*I*(d*x+c))+28*exp(4*I*(d*x+c))+8*exp(2*I*(d*x+c))+1)/d/a/(exp(2*I*(d*x+c))+1)^8

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.36

$$\int \frac{\sec^{10}(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{32\left(-56i\,e^{(6i\,dx+6i\,c)}-28i\,e^{(4i\,dx+4i\,c)}-8i\,e^{(2i\,dx+2i\,c)}-\right.}{35\left(ade^{(16i\,dx+16i\,c)}+8\,ade^{(14i\,dx+14i\,c)}+28\,ade^{(12i\,dx+12i\,c)}+56\,ade^{(10i\,dx+10i\,c)}+70\,ade^{(8i\,dx+8i\,c)}+56\,ade\right.}$$

[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c)),x, algorithm="fricas")

[Out] -32/35*(-56*I*e^(6*I*d*x + 6*I*c) - 28*I*e^(4*I*d*x + 4*I*c) - 8*I*e^(2*I*d*x + 2*I*c) - I)/(a*d*e^(16*I*d*x + 16*I*c) + 8*a*d*e^(14*I*d*x + 14*I*c) + 28*a*d*e^(12*I*d*x + 12*I*c) + 56*a*d*e^(10*I*d*x + 10*I*c) + 70*a*d*e^(8*I*d*x + 8*I*c) + 56*a*d*e^(6*I*d*x + 6*I*c) + 28*a*d*e^(4*I*d*x + 4*I*c) + 8*a*d*e^(2*I*d*x + 2*I*c) + a*d)

Sympy [F]

$$\int \frac{\sec^{10}(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i\int \frac{\sec^{10}(c+dx)}{\tan(c+dx)-i}\,dx}{a}$$

[In] integrate(sec(d*x+c)**10/(a+I*a*tan(d*x+c)),x)

[Out] -I*Integral(sec(c + d*x)**10/(tan(c + d*x) - I), x)/a

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.81

$$\int \frac{\sec^{10}(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{35i\,\tan\left(dx+c\right)^{8}-40\,\tan\left(dx+c\right)^{7}+140i\,\tan\left(dx+c\right)^{6}-168\,\tan\left(dx+c\right)^{5}+210i\,\tan\left(dx+c\right)^{4}-28}{280\,ad}$$

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
[Out] -1/280*(35*I*tan(d*x + c)^8 - 40*tan(d*x + c)^7 + 140*I*tan(d*x + c)^6 - 16
8*tan(d*x + c)^5 + 210*I*tan(d*x + c)^4 - 280*tan(d*x + c)^3 + 140*I*tan(d*
x + c)^2 - 280*tan(d*x + c))/(a*d)
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.81

$$\int \frac{\sec^{10}(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{35i\,\tan\left(dx+c\right)^{8}-40\,\tan\left(dx+c\right)^{7}+140i\,\tan\left(dx+c\right)^{6}-168\,\tan\left(dx+c\right)^{5}+210i\,\tan\left(dx+c\right)^{4}-28}{280\,ad}$$

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] -1/280*(35*I*tan(d*x + c)^8 - 40*tan(d*x + c)^7 + 140*I*tan(d*x + c)^6 - 16
8*tan(d*x + c)^5 + 210*I*tan(d*x + c)^4 - 280*tan(d*x + c)^3 + 140*I*tan(d*
x + c)^2 - 280*tan(d*x + c))/(a*d)
```

Mupad [B] (verification not implemented)

Time = 4.35 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.86

$$\int \frac{\sec^{10}(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\cos\left(c+d\,x\right)^{8}\,35\mathrm{i}+128\,\sin\left(c+d\,x\right)\,\cos\left(c+d\,x\right)^{7}+64\,\sin\left(c+d\,x\right)\,\cos\left(c+d\,x\right)^{5}+48\,\sin\left(c+d\,x\right)\,\cos\left(c\right.}{280\,a\,d\cos\left(c+d\,x\right)^{8}}$$

```
[In] int(1/(cos(c + d*x)^10*(a + a*tan(c + d*x)*1i)),x)
[Out] (40*cos(c + d*x)*sin(c + d*x) + 48*cos(c + d*x)^3*sin(c + d*x) + 64*cos(c +
 d*x)^5*sin(c + d*x) + 128*cos(c + d*x)^7*sin(c + d*x) + cos(c + d*x)^8*35i
 - 35i)/(280*a*d*cos(c + d*x)^8)
```

3.100 $\int \frac{\sec^8(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 80

$$\int \frac{\sec^8(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{i(a-ia\tan(c+dx))^4}{a^5d} - \frac{4i(a-ia\tan(c+dx))^5}{5a^6d} + \frac{i(a-ia\tan(c+dx))^6}{6a^7d}$$

```
[Out] I*(a-I*a*tan(d*x+c))^4/a^5/d-4/5*I*(a-I*a*tan(d*x+c))^5/a^6/d+1/6*I*(a-I*a*
tan(d*x+c))^6/a^7/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 45}

$$\int \frac{\sec^8(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{i(a-ia\tan(c+dx))^6}{6a^7d} - \frac{4i(a-ia\tan(c+dx))^5}{5a^6d} + \frac{i(a-ia\tan(c+dx))^4}{a^5d}$$

```
[In] Int[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (I*(a - I*a*Tan[c + d*x])^4)/(a^5*d) - (((4*I)/5)*(a - I*a*Tan[c + d*x])^5)
/(a^6*d) + ((I/6)*(a - I*a*Tan[c + d*x])^6)/(a^7*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)^3(a+x)^2\,dx, x, ia\tan(c+dx)\right)}{a^7d}\\
&= -\frac{i\text{Subst}\left(\int \left(4a^2(a-x)^3-4a(a-x)^4+(a-x)^5\right)\,dx, x, ia\tan(c+dx)\right)}{a^7d}\\
&= \frac{i(a-ia\tan(c+dx))^4}{a^5d}-\frac{4i(a-ia\tan(c+dx))^5}{5a^6d}+\frac{i(a-ia\tan(c+dx))^6}{6a^7d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

$$
\int \frac{\sec^8(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i(i+\tan(c+dx))^4\left(-11-14i\tan(c+dx)+5\tan^2(c+dx)\right)}{30ad}
$$

```
[In] Integrate[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x]),x]
[Out] ((-1/30*I)*(I + Tan[c + d*x])^4*(-11 - (14*I)*Tan[c + d*x] + 5*Tan[c + d*x]
^2))/(a*d)
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

method	result	size
risch	$\frac{16i\left(15\,\mathrm{e}^{4i(dx+c)}+6\,\mathrm{e}^{2i(dx+c)}+1\right)}{15da\left(\mathrm{e}^{2i(dx+c)}+1\right)^6}$	47
derivativedivides	$-\frac{-\tan(dx+c)+\frac{i\left(\tan^6(dx+c)\right)}{6}-\frac{\left(\tan^5(dx+c)\right)}{5}+\frac{i\left(\tan^4(dx+c)\right)}{2}-\frac{2\left(\tan^3(dx+c)\right)}{3}+\frac{i\left(\tan^2(dx+c)\right)}{2}}{ad}$	71
default	$-\frac{-\tan(dx+c)+\frac{i\left(\tan^6(dx+c)\right)}{6}-\frac{\left(\tan^5(dx+c)\right)}{5}+\frac{i\left(\tan^4(dx+c)\right)}{2}-\frac{2\left(\tan^3(dx+c)\right)}{3}+\frac{i\left(\tan^2(dx+c)\right)}{2}}{ad}$	71

```
[In] int(sec(d*x+c)^8/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 16/15*I*(15*exp(4*I*(d*x+c))+6*exp(2*I*(d*x+c))+1)/d/a/(exp(2*I*(d*x+c))+1)
^6
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.36

$$\int \frac{\sec^8(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{16\left(-15i\,e^{(4i\,dx+4i\,c)}-6i\,e^{(2i\,dx+2i\,c)}-i\right)}{15\left(ade^{(12i\,dx+12i\,c)}+6\,ade^{(10i\,dx+10i\,c)}+15\,ade^{(8i\,dx+8i\,c)}+20\,ade^{(6i\,dx+6i\,c)}+15\,ade^{(4i\,dx+4i\,c)}+6\,ade^{(2i\,dx}\right.}$$

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
[Out] -16/15*(-15*I*e^(4*I*d*x + 4*I*c) - 6*I*e^(2*I*d*x + 2*I*c) - I)/(a*d*e^(12
*I*d*x + 12*I*c) + 6*a*d*e^(10*I*d*x + 10*I*c) + 15*a*d*e^(8*I*d*x + 8*I*c)
 + 20*a*d*e^(6*I*d*x + 6*I*c) + 15*a*d*e^(4*I*d*x + 4*I*c) + 6*a*d*e^(2*I*d
*x + 2*I*c) + a*d)
```

Sympy [F]

$$\int \frac{\sec^8(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i\int \frac{\sec^8(c+dx)}{\tan(c+dx)-i}\,dx}{a}$$

```
[In] integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c)),x)
[Out] -I*Integral(sec(c + d*x)**8/(tan(c + d*x) - I), x)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.84

$$\int \frac{\sec^8(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{-5i\tan(dx+c)^6+6\tan(dx+c)^5-15i\tan(dx+c)^4+20\tan(dx+c)^3-15i\tan(dx+c)^2+30\tan}{30\,ad}$$

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
[Out] 1/30*(-5*I*tan(d*x + c)^6 + 6*tan(d*x + c)^5 - 15*I*tan(d*x + c)^4 + 20*tan
(d*x + c)^3 - 15*I*tan(d*x + c)^2 + 30*tan(d*x + c))/(a*d)
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.84

$$\int \frac{\sec^8(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{5i\,\tan\left(dx+c\right)^6 - 6\,\tan\left(dx+c\right)^5 + 15i\,\tan\left(dx+c\right)^4 - 20\,\tan\left(dx+c\right)^3 + 15i\,\tan\left(dx+c\right)^2 - 30\,\tan(}{30\,ad}$$

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/30*(5*I*tan(d*x + c)^6 - 6*tan(d*x + c)^5 + 15*I*tan(d*x + c)^4 - 20*tan
(d*x + c)^3 + 15*I*tan(d*x + c)^2 - 30*tan(d*x + c))/(a*d)
```

Mupad [B] (verification not implemented)

Time = 3.94 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.42

$$\int \frac{\sec^8(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\sin\left(c+d\,x\right)\,\left(30\cos\left(c+d\,x\right)^5 - \cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)\,15\mathrm{i} + 20\cos\left(c+d\,x\right)^3\sin\left(c+d\,x\right)^2 - \cos\left(c+\right.\right.}{30\,a\,d\cos\left(c+d\,x\right)^6}$$

```
[In] int(1/(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)),x)
```

```
[Out] (sin(c + d*x)*(6*cos(c + d*x)*sin(c + d*x)^4 - cos(c + d*x)^4*sin(c + d*x)*
15i + 30*cos(c + d*x)^5 - sin(c + d*x)^5*5i - cos(c + d*x)^2*sin(c + d*x)^3
*15i + 20*cos(c + d*x)^3*sin(c + d*x)^2))/(30*a*d*cos(c + d*x)^6)
```

3.101 $\int \frac{\sec^6(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \frac{\sec^6(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{2i(a-ia\tan(c+dx))^3}{3a^4d} - \frac{i(a-ia\tan(c+dx))^4}{4a^5d}$$

```
[Out] 2/3*I*(a-I*a*tan(d*x+c))^3/a^4/d-1/4*I*(a-I*a*tan(d*x+c))^4/a^5/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\int \frac{\sec^6(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{2i(a-ia\tan(c+dx))^3}{3a^4d} - \frac{i(a-ia\tan(c+dx))^4}{4a^5d}$$

```
[In] Int[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (((2*I)/3)*(a - I*a*Tan[c + d*x])^3)/(a^4*d) - ((I/4)*(a - I*a*Tan[c + d*x]
)^4)/(a^5*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)^2(a+x)\,dx, x, ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{i\text{Subst}\left(\int \left(2a(a-x)^2-(a-x)^3\right)\,dx, x, ia\tan(c+dx)\right)}{a^5d}\\ &= \frac{2i(a-ia\tan(c+dx))^3}{3a^4d}-\frac{i(a-ia\tan(c+dx))^4}{4a^5d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{\sec^6(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\tan(c+dx)\left(12-6i\tan(c+dx)+4\tan^2(c+dx)-3i\tan^3(c+dx)\right)}{12ad}$$

```
[In] Integrate[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x]),x]
[Out] (Tan[c + d*x]*(12 - (6*I)*Tan[c + d*x] + 4*Tan[c + d*x]^2 - (3*I)*Tan[c + d
*x]^3))/(12*a*d)
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

method	result	size
risch	$\frac{4i\left(4\,e^{2i(dx+c)}+1\right)}{3da\left(e^{2i(dx+c)}+1\right)^4}$	36
derivativedivides	$-\frac{-\tan(dx+c)+\frac{i\left(\tan^4(dx+c)\right)}{4}-\frac{\left(\tan^3(dx+c)\right)}{3}+\frac{i\left(\tan^2(dx+c)\right)}{2}}{ad}$	50
default	$-\frac{-\tan(dx+c)+\frac{i\left(\tan^4(dx+c)\right)}{4}-\frac{\left(\tan^3(dx+c)\right)}{3}+\frac{i\left(\tan^2(dx+c)\right)}{2}}{ad}$	50

```
[In] int(sec(d*x+c)^6/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 4/3*I*(4*exp(2*I*(d*x+c))+1)/d/a/(exp(2*I*(d*x+c))+1)^4
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.31

$$\int \frac{\sec^6(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{4\left(-4i\,e^{(2i\,dx+2i\,c)}-i\right)}{3\left(ade^{(8i\,dx+8i\,c)}+4\,ade^{(6i\,dx+6i\,c)}+6\,ade^{(4i\,dx+4i\,c)}+4\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
[Out] -4/3*(-4*I*e^(2*I*d*x + 2*I*c) - I)/(a*d*e^(8*I*d*x + 8*I*c) + 4*a*d*e^(6*I
*d*x + 6*I*c) + 6*a*d*e^(4*I*d*x + 4*I*c) + 4*a*d*e^(2*I*d*x + 2*I*c) + a*d
)
```

Sympy [F]

$$\int \frac{\sec^6(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i\int \frac{\sec^6(c+dx)}{\tan(c+dx)-i}\,dx}{a}$$

```
[In] integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c)),x)
[Out] -I*Integral(sec(c + d*x)**6/(tan(c + d*x) - I), x)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{\sec^6(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{-3i\,\tan\left(dx+c\right)^4+4\,\tan\left(dx+c\right)^3-6i\,\tan\left(dx+c\right)^2+12\,\tan\left(dx+c\right)}{12\,ad}$$

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
[Out] 1/12*(-3*I*tan(d*x + c)^4 + 4*tan(d*x + c)^3 - 6*I*tan(d*x + c)^2 + 12*tan(
d*x + c))/(a*d)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{\sec^6(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{3i\,\tan\left(dx+c\right)^4 - 4\,\tan\left(dx+c\right)^3 + 6i\,\tan\left(dx+c\right)^2 - 12\,\tan\left(dx+c\right)}{12\,ad}$$

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(3*I*tan(d*x + c)^4 - 4*tan(d*x + c)^3 + 6*I*tan(d*x + c)^2 - 12*tan(
d*x + c))/(a*d)
```

Mupad [B] (verification not implemented)

Time = 4.52 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.40

$$\int \frac{\sec^6(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\sin\left(c+d\,x\right)\,\left(12\cos\left(c+d\,x\right)^3 - \cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)\,6\mathrm{i} + 4\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)^2 - \sin\left(c+d\,x\right.\right.}{12\,a\,d\cos\left(c+d\,x\right)^4}$$

```
[In] int(1/(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)),x)
```

```
[Out] (sin(c + d*x)*(4*cos(c + d*x)*sin(c + d*x)^2 - cos(c + d*x)^2*sin(c + d*x)*
6i + 12*cos(c + d*x)^3 - sin(c + d*x)^3*3i))/(12*a*d*cos(c + d*x)^4)
```

3.102 $\int \frac{\sec^4(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 34

$$\int \frac{\sec^4(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\tan(c+dx)}{ad} - \frac{i\tan^2(c+dx)}{2ad}$$

```
[Out] tan(d*x+c)/a/d-1/2*I*tan(d*x+c)^2/a/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {3568}

$$\int \frac{\sec^4(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\tan(c+dx)}{ad} - \frac{i\tan^2(c+dx)}{2ad}$$

```
[In] Int[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x]),x]
[Out] Tan[c + d*x]/(a*d) - ((I/2)*Tan[c + d*x]^2)/(a*d)
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)\,dx, x, ia\tan(c+dx)\right)}{a^3d} \\ &= \frac{\tan(c+dx)}{ad} - \frac{i\tan^2(c+dx)}{2ad}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\sec^4(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\tan(c+dx)}{ad} - \frac{i\tan^2(c+dx)}{2ad}$$

```
[In] Integrate[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x]),x]
[Out] Tan[c + d*x]/(a*d) - ((I/2)*Tan[c + d*x]^2)/(a*d)
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

method	result	size
risch	$\frac{2i}{da\left(e^{2i(dx+c)}+1\right)^2}$	23
derivativedivides	$-\frac{i\left(\frac{\left(\tan^2(dx+c)\right)}{2}+i\tan(dx+c)\right)}{ad}$	30
default	$-\frac{i\left(\frac{\left(\tan^2(dx+c)\right)}{2}+i\tan(dx+c)\right)}{ad}$	30

```
[In] int(sec(d*x+c)^4/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 2*I/d/a/(exp(2*I*(d*x+c))+1)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{\sec^4(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{2i}{ade^{(4i\,dx+4i\,c)} + 2\,ade^{(2i\,dx+2i\,c)} + ad}$$

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
[Out] 2*I/(a*d*e^(4*I*d*x + 4*I*c) + 2*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i\int \frac{\sec^4(c+dx)}{\tan(c+dx)-i}\,dx}{a}$$

```
[In] integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c)),x)
[Out] -I*Integral(sec(c + d*x)**4/(tan(c + d*x) - I), x)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{\sec^4(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i\tan(dx+c)^2 - 2\tan(dx+c)}{2\,ad}$$

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
[Out] -1/2*(I*tan(d*x + c)^2 - 2*tan(d*x + c))/(a*d)
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{\sec^4(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i\tan(dx+c)^2 - 2\tan(dx+c)}{2\,ad}$$

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] -1/2*(I*tan(d*x + c)^2 - 2*tan(d*x + c))/(a*d)
```

Mupad [B] (verification not implemented)

Time = 3.90 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int \frac{\sec^4(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{\tan(c+d\,x)\,\left(-2+\tan(c+d\,x)\,1\mathrm{i}\right)}{2\,a\,d}$$

```
[In] int(1/(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)),x)
[Out] -(tan(c + d*x)*(tan(c + d*x)*1i - 2))/(2*a*d)
```

3.103 $\int \frac{\sec^2(c+dx)}{a+ia\tan(c+dx)} dx$

Optimal result

Integrand size = 24, antiderivative size = 23

$$\int \frac{\sec^2(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{x}{a}+\frac{i\log(\cos(c+dx))}{ad}$$

```
[Out] x/a+I*ln(cos(d*x+c))/a/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 31}

$$\int \frac{\sec^2(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{x}{a}+\frac{i\log(\cos(c+dx))}{ad}$$

```
[In] Int[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x]),x]
[Out] x/a + (I*Log[Cos[c + d*x]])/(a*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
```

```
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, ia\tan(c+dx)\right)}{ad}\\ &= \frac{x}{a} + \frac{i\log(\cos(c+dx))}{ad}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i\log(i-\tan(c+dx))}{ad}$$

```
[In] Integrate[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x]),x]
[Out] ((-I)*Log[I - Tan[c + d*x]])/(a*d)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$-\frac{i\ln(a+ia\tan(dx+c))}{ad}$	23
default	$-\frac{i\ln(a+ia\tan(dx+c))}{ad}$	23
risch	$\frac{2x}{a}+\frac{2c}{ad}+\frac{i\ln\left(e^{2i(dx+c)}+1\right)}{ad}$	38

```
[In] int(sec(d*x+c)^2/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] -I/a/d*ln(a+I*a*tan(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{\sec^2(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{2\,dx + i\,\log\left(e^{(2i\,dx+2i\,c)}+1\right)}{ad}$$

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
[Out] (2*d*x + I*log(e^(2*I*d*x + 2*I*c) + 1))/(a*d)
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i\int \frac{\sec^2(c+dx)}{\tan(c+dx)-i}\,dx}{a}$$

```
[In] integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c)),x)
[Out] -I*Integral(sec(c + d*x)**2/(tan(c + d*x) - I), x)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\sec^2(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i\,\log\left(i\,a\tan\left(dx+c\right)+a\right)}{ad}$$

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
[Out] -I*log(I*a*tan(d*x + c) + a)/(a*d)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(21) = 42.

Time = 0.36 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.48

$$\int \frac{\sec^2(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{-\frac{i\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a}+\frac{2i\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}{a}-\frac{i\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a}}{d}$$

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] -(-I*log(tan(1/2*d*x + 1/2*c) + 1)/a + 2*I*log(tan(1/2*d*x + 1/2*c) - I)/a
- I*log(tan(1/2*d*x + 1/2*c) - 1)/a)/d
```

Mupad [B] (verification not implemented)

Time = 4.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\sec^2(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{a\,d}$$

```
[In] int(1/(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)),x)
[Out] -(log(tan(c + d*x) - 1i)*1i)/(a*d)
```

3.104 $\int \frac{1}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 33

$$\int \frac{1}{a+ia\tan(c+dx)}\,dx = \frac{x}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}$$

```
[Out] 1/2*x/a+1/2*I/d/(a+I*a*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3560, 8}

$$\int \frac{1}{a+ia\tan(c+dx)}\,dx = \frac{x}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^(-1),x]
[Out] x/(2*a) + (I/2)/(d*(a + I*a*Tan[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{i}{2d(a+ia\tan(c+dx))}+\frac{\int 1\,dx}{2a}\\ &= \frac{x}{2a}+\frac{i}{2d(a+ia\tan(c+dx))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{1}{a+ia\tan(c+dx)}\,dx=\frac{\frac{\arctan(\tan(c+dx))}{a}+\frac{1}{-ia+a\tan(c+dx)}}{2d}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(-1),x]
[Out] (ArcTan[Tan[c + d*x]]/a + ((-I)*a + a*Tan[c + d*x])^(-1))/(2*d)
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{x}{2a}+\frac{i e^{-2i(dx+c)}}{4ad}$	26
derivativedivides	$\frac{\arctan(\tan(dx+c))}{2da}+\frac{1}{2da(\tan(dx+c)-i)}$	36
default	$\frac{\arctan(\tan(dx+c))}{2da}+\frac{1}{2da(\tan(dx+c)-i)}$	36
norman	$\frac{\frac{x}{2a}+\frac{i}{2ad}+\frac{x\left(\tan^2(dx+c)\right)}{2a}+\frac{\tan(dx+c)}{2ad}}{1+\tan^2(dx+c)}$	58

```
[In] int(1/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/2*x/a+1/4*I/a/d*exp(-2*I*(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{1}{a + ia\tan(c + dx)}\,dx = \frac{\left(2\,dxe^{(2i\,dx+2i\,c)} + i\right)e^{(-2i\,dx-2i\,c)}}{4\,ad}$$

```
[In] integrate(1/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*d*x*e^(2*I*d*x + 2*I*c) + I)*e^(-2*I*d*x - 2*I*c)/(a*d)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.82

$$\int \frac{1}{a + ia\tan(c + dx)}\,dx = \begin{cases} \frac{ie^{-2ic}e^{-2idx}}{4ad} & \text{for } ade^{2ic} \neq 0 \\ x\left(\frac{(e^{2ic}+1)e^{-2ic}}{2a} - \frac{1}{2a}\right) & \text{otherwise} \end{cases} + \frac{x}{2a}$$

```
[In] integrate(1/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise((I*exp(-2*I*c)*exp(-2*I*d*x)/(4*a*d), Ne(a*d*exp(2*I*c), 0)), (x*
((exp(2*I*c) + 1)*exp(-2*I*c)/(2*a) - 1/(2*a)), True)) + x/(2*a)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a + ia\tan(c + dx)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(25) = 50$.

Time = 0.36 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.76

$$\int \frac{1}{a+ia\tan(c+dx)}\,dx = -\frac{-\frac{i\log(\tan(dx+c)+i)}{a}+\frac{i\log(\tan(dx+c)-i)}{a}+\frac{-i\tan(dx+c)-3}{a(\tan(dx+c)-i)}}{4\,d}$$

```
[In] integrate(1/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(-I*log(tan(d*x + c) + I)/a + I*log(tan(d*x + c) - I)/a + (-I*tan(d*x
+ c) - 3)/(a*(tan(d*x + c) - I)))/d
```

Mupad [B] (verification not implemented)

Time = 4.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{1}{a+ia\tan(c+dx)}\,dx = \frac{x}{2\,a}+\frac{1\mathrm{i}}{2\,a\,d\,\left(1+\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}$$

```
[In] int(1/(a + a*tan(c + d*x)*1i),x)
```

```
[Out] x/(2*a) + 1i/(2*a*d*(tan(c + d*x)*1i + 1))
```

3.105 $\int \frac{\cos^2(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \frac{\cos^2(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{3x}{8a} - \frac{i}{8d(a-ia\tan(c+dx))} + \frac{ia}{8d(a+ia\tan(c+dx))^2} + \frac{i}{4d(a+ia\tan(c+dx))}$$

```
[Out] 3/8*x/a-1/8*I/d/(a-I*a*tan(d*x+c))+1/8*I*a/d/(a+I*a*tan(d*x+c))^2+1/4*I/d/(
a+I*a*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3568, 46, 212}

$$\int \frac{\cos^2(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{ia}{8d(a+ia\tan(c+dx))^2} - \frac{i}{8d(a-ia\tan(c+dx))} + \frac{i}{4d(a+ia\tan(c+dx))} + \frac{3x}{8a}$$

```
[In] Int[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (3*x)/(8*a) - (I/8)/(d*(a - I*a*Tan[c + d*x])) + ((I/8)*a)/(d*(a + I*a*Tan[
c + d*x])^2) + (I/4)/(d*(a + I*a*Tan[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
```

```
NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(ia^3)\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^3}\,dx, x, ia\tan(c+dx)\right)}{d}\\
&= -\frac{(ia^3)\,\text{Subst}\left(\int \left(\frac{1}{8a^3(a-x)^2}+\frac{1}{4a^2(a+x)^3}+\frac{1}{4a^3(a+x)^2}+\frac{3}{8a^3(a^2-x^2)}\right)dx, x, ia\tan(c+dx)\right)}{d}\\
&= -\frac{i}{8d(a-ia\tan(c+dx))}+\frac{ia}{8d(a+ia\tan(c+dx))^2}\\
&\quad+\frac{i}{4d(a+ia\tan(c+dx))}-\frac{(3i)\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, ia\tan(c+dx)\right)}{8d}\\
&= \frac{3x}{8a}-\frac{i}{8d(a-ia\tan(c+dx))}+\frac{ia}{8d(a+ia\tan(c+dx))^2}+\frac{i}{4d(a+ia\tan(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.06

$$\int \frac{\cos^2(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{2-3i\tan(c+dx)+3\tan^2(c+dx)+3\arctan(\tan(c+dx))(-i+\tan(c+dx))^2(i+\tan(c+dx))}{8ad(-i+\tan(c+dx))^2(i+\tan(c+dx))}$$

```
[In] Integrate[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (2 - (3*I)*Tan[c + d*x] + 3*Tan[c + d*x]^2 + 3*ArcTan[Tan[c + d*x]]*(-I + T
an[c + d*x])^2*(I + Tan[c + d*x]))/(8*a*d*(-I + Tan[c + d*x])^2*(I + Tan[c
+ d*x]))
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.74

method	result	size
risch	$\frac{3x}{8a}+\frac{i\mathrm{e}^{-4i(dx+c)}}{32ad}+\frac{i\cos(2dx+2c)}{8ad}+\frac{\sin(2dx+2c)}{4ad}$	61
derivativedivides	$\frac{-\frac{3i\ln(\tan(dx+c)-i)}{16}-\frac{i}{8(\tan(dx+c)-i)^2}+\frac{1}{4\tan(dx+c)-4i}+\frac{3i\ln(\tan(dx+c)+i)}{16}+\frac{1}{8\tan(dx+c)+8i}}{da}$	75
default	$\frac{-\frac{3i\ln(\tan(dx+c)-i)}{16}-\frac{i}{8(\tan(dx+c)-i)^2}+\frac{1}{4\tan(dx+c)-4i}+\frac{3i\ln(\tan(dx+c)+i)}{16}+\frac{1}{8\tan(dx+c)+8i}}{da}$	75

```
[In] int(cos(d*x+c)^2/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*x/a+1/32*I/a/d*exp(-4*I*(d*x+c))+1/8*I/a/d*cos(2*d*x+2*c)+1/4/a/d*sin(2
*d*x+2*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.66

$$\int \frac{\cos^2(c+dx)}{a+ia\tan(c+dx)}\,dx=\frac{\left(12\,dxe^{(4i\,dx+4i\,c)}-2i\,e^{(6i\,dx+6i\,c)}+6i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-4i\,dx-4i\,c)}}{32\,ad}$$

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/32*(12*d*x*e^(4*I*d*x + 4*I*c) - 2*I*e^(6*I*d*x + 6*I*c) + 6*I*e^(2*I*d*x
 + 2*I*c) + I)*e^(-4*I*d*x - 4*I*c)/(a*d)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.84

$$\int \frac{\cos^2(c+dx)}{a+ia\tan(c+dx)}\,dx = \begin{cases}\frac{\left(-512ia^2d^2e^{8ic}e^{2idx}+1536ia^2d^2e^{4ic}e^{-2idx}+256ia^2d^2e^{2ic}e^{-4idx}\right)e^{-6ic}}{8192a^3d^3} & \text{for } a^3d^3e^{6ic}\neq 0\\ x\left(\frac{\left(e^{6ic}+3e^{4ic}+3e^{2ic}+1\right)e^{-4ic}}{8a}-\frac{3}{8a}\right) & \text{otherwise}\end{cases}+\frac{3x}{8a}$$

```
[In] integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise(((-512*I*a**2*d**2*exp(8*I*c)*exp(2*I*d*x) + 1536*I*a**2*d**2*exp
(4*I*c)*exp(-2*I*d*x) + 256*I*a**2*d**2*exp(2*I*c)*exp(-4*I*d*x))*exp(-6*I*
c)/(8192*a**3*d**3), Ne(a**3*d**3*exp(6*I*c), 0)), (x*((exp(6*I*c) + 3*exp(
4*I*c) + 3*exp(2*I*c) + 1)*exp(-4*I*c)/(8*a) - 3/(8*a)), True)) + 3*x/(8*a)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.16

$$\int \frac{\cos^2(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{-\frac{6i\log(\tan(dx+c)+i)}{a}+\frac{6i\log(\tan(dx+c)-i)}{a}+\frac{2(3\tan(dx+c)+5i)}{a(-i\tan(dx+c)+1)}+\frac{-9i\tan(dx+c)^2-26\tan(dx+c)+21i}{a(\tan(dx+c)-i)^2}}{32\,d}$$

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] -1/32*(-6*I*log(tan(d*x + c) + I)/a + 6*I*log(tan(d*x + c) - I)/a + 2*(3*ta
n(d*x + c) + 5*I)/(a*(-I*tan(d*x + c) + 1)) + (-9*I*tan(d*x + c)^2 - 26*tan
(d*x + c) + 21*I)/(a*(tan(d*x + c) - I)^2))/d
```

Mupad [B] (verification not implemented)

Time = 4.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.73

$$\int \frac{\cos^2(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{3\,x}{8\,a}-\frac{\frac{3\tan(c+d\,x)^2}{8}-\frac{\tan(c+d\,x)\,3{}\mathrm{i}}{8}+\frac{1}{4}}{a\,d\,{\left(1+\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^2\,\left(\tan\left(c+d\,x\right)+1{}\mathrm{i}\right)}$$

```
[In] int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i),x)
[Out] (3*x)/(8*a) - ((3*tan(c + d*x)^2)/8 - (tan(c + d*x)*3i)/8 + 1/4)/(a*d*(tan(
c + d*x)*1i + 1)^2*(tan(c + d*x) + 1i))
```

3.106 $\int \frac{\cos^4(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 134

$$\int \frac{\cos^4(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{5x}{16a} - \frac{ia}{32d(a-ia\tan(c+dx))^2} - \frac{i}{8d(a-ia\tan(c+dx))} + \frac{ia^2}{24d(a+ia\tan(c+dx))^3} + \frac{3ia}{32d(a+ia\tan(c+dx))^2} + \frac{3i}{16d(a+ia\tan(c+dx))}$$

```
[Out] 5/16*x/a-1/32*I*a/d/(a-I*a*tan(d*x+c))^2-1/8*I/d/(a-I*a*tan(d*x+c))+1/24*I*
a^2/d/(a+I*a*tan(d*x+c))^3+3/32*I*a/d/(a+I*a*tan(d*x+c))^2+3/16*I/d/(a+I*a*
tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3568, 46, 212}

$$\int \frac{\cos^4(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{ia^2}{24d(a+ia\tan(c+dx))^3} - \frac{ia}{32d(a-ia\tan(c+dx))^2} + \frac{3ia}{32d(a+ia\tan(c+dx))^2} - \frac{i}{8d(a-ia\tan(c+dx))} + \frac{3i}{16d(a+ia\tan(c+dx))} + \frac{5x}{16a}$$

```
[In] Int[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (5*x)/(16*a) - ((I/32)*a)/(d*(a - I*a*Tan[c + d*x])^2) - (I/8)/(d*(a - I*a*
Tan[c + d*x])) + ((I/24)*a^2)/(d*(a + I*a*Tan[c + d*x])^3) + (((3*I)/32)*a)
/(d*(a + I*a*Tan[c + d*x])^2) + ((3*I)/16)/(d*(a + I*a*Tan[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{(ia^5)\,\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^4}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^5)\,\text{Subst}\left(\int \left(\frac{1}{16a^4(a-x)^3} + \frac{1}{8a^5(a-x)^2} + \frac{1}{8a^3(a+x)^4} + \frac{3}{16a^4(a+x)^3} + \frac{3}{16a^5(a+x)^2} + \frac{5}{16a^5(a^2-x^2)}\right) dx, x, i}{d}$$

$$= -\frac{ia}{32d(a-ia\tan(c+dx))^2} - \frac{i}{8d(a-ia\tan(c+dx))} + \frac{ia^2}{24d(a+ia\tan(c+dx))^3} + \frac{3ia}{32d(a+ia\tan(c+dx))^2} + \frac{3i}{16d(a+ia\tan(c+dx))} - \frac{(5i)\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, ia\tan(c+dx)\right)}{16d}$$

$$= \frac{5x}{16a} - \frac{ia}{32d(a-ia\tan(c+dx))^2} - \frac{i}{8d(a-ia\tan(c+dx))} + \frac{ia^2}{24d(a+ia\tan(c+dx))^3} + \frac{3ia}{32d(a+ia\tan(c+dx))^2} + \frac{3i}{16d(a+ia\tan(c+dx))}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.98

$$\int \frac{\cos^4(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{\sec^5(c+dx)(-80\cos(c+dx)+15\cos(3(c+dx))+\cos(5(c+dx))+120i\arctan(\tan(c+dx))(\cos(c+}{384ad(-i+\tan(c+dx))^3(i+\tan(}$$

```
[In] Integrate[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] -1/384*(Sec[c + d*x]^5*(-80*Cos[c + d*x] + 15*Cos[3*(c + d*x)] + Cos[5*(c +
 d*x)] + (120*I)*ArcTan[Tan[c + d*x]]*(Cos[c + d*x] + I*Sin[c + d*x]) + (40
*I)*Sin[c + d*x] + (45*I)*Sin[3*(c + d*x)] + (5*I)*Sin[5*(c + d*x)]))/(a*d*
(-I + Tan[c + d*x])^3*(I + Tan[c + d*x])^2)
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.72

method	result
risch	$\frac{5x}{16a}+\frac{i\mathrm{e}^{-6i(dx+c)}}{192ad}+\frac{i\cos(4dx+4c)}{32ad}+\frac{3\sin(4dx+4c)}{64ad}+\frac{5i\cos(2dx+2c)}{64ad}+\frac{15\sin(2dx+2c)}{64ad}$
derivativedivides	$\frac{-\frac{5i\ln(\tan(dx+c)-i)}{32}-\frac{3i}{32(\tan(dx+c)-i)^2}-\frac{1}{24(\tan(dx+c)-i)^3}+\frac{3}{16(\tan(dx+c)-i)}+\frac{i}{32(\tan(dx+c)+i)^2}+\frac{5i\ln(\tan(dx+c)+i)}{32}+\frac{}{8\tan(}}{da}$
default	$\frac{-\frac{5i\ln(\tan(dx+c)-i)}{32}-\frac{3i}{32(\tan(dx+c)-i)^2}-\frac{1}{24(\tan(dx+c)-i)^3}+\frac{3}{16(\tan(dx+c)-i)}+\frac{i}{32(\tan(dx+c)+i)^2}+\frac{5i\ln(\tan(dx+c)+i)}{32}+\frac{}{8\tan(}}{da}$

```
[In] int(cos(d*x+c)^4/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 5/16*x/a+1/192*I/a/d*exp(-6*I*(d*x+c))+1/32*I/a/d*cos(4*d*x+4*c)+3/64/a/d*s
in(4*d*x+4*c)+5/64*I/a/d*cos(2*d*x+2*c)+15/64/a/d*sin(2*d*x+2*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.57

$$\int \frac{\cos^4(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\left(120\,dxe^{(6i\,dx+6i\,c)}-3i\,e^{(10i\,dx+10i\,c)}-30i\,e^{(8i\,dx+8i\,c)}+60i\,e^{(4i\,dx+4i\,c)}+15i\,e^{(2i\,dx+2i\,c)}+2i\right)e^{(-6i\,dx-6i\,c)}}{384\,ad}$$

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/384*(120*d*x*e^(6*I*d*x + 6*I*c) - 3*I*e^(10*I*d*x + 10*I*c) - 30*I*e^(8*
I*d*x + 8*I*c) + 60*I*e^(4*I*d*x + 4*I*c) + 15*I*e^(2*I*d*x + 2*I*c) + 2*I)
*e^(-6*I*d*x - 6*I*c)/(a*d)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.63

$$\int \frac{\cos^4(c+dx)}{a+ia\tan(c+dx)}\,dx$$
$$= \begin{cases} \frac{(-50331648ia^4d^4e^{16ic}e^{4idx}-503316480ia^4d^4e^{14ic}e^{2idx}+1006632960ia^4d^4e^{10ic}e^{-2idx}+251658240ia^4d^4e^{8ic}e^{-4idx}+33554432ia^4d^4e^{6ic}e^{-6i}}{6442450944a^5d^5} \\ x\left(\frac{(e^{10ic}+5e^{8ic}+10e^{6ic}+10e^{4ic}+5e^{2ic}+1)e^{-6ic}}{32a}-\frac{5}{16a}\right) \end{cases}$$
$$+\frac{5x}{16a}$$

```
[In] integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise(((-50331648*I*a**4*d**4*exp(16*I*c)*exp(4*I*d*x) - 503316480*I*a*
*4*d**4*exp(14*I*c)*exp(2*I*d*x) + 1006632960*I*a**4*d**4*exp(10*I*c)*exp(-
2*I*d*x) + 251658240*I*a**4*d**4*exp(8*I*c)*exp(-4*I*d*x) + 33554432*I*a**4
*d**4*exp(6*I*c)*exp(-6*I*d*x))*exp(-12*I*c)/(6442450944*a**5*d**5), Ne(a**
5*d**5*exp(12*I*c), 0)), (x*((exp(10*I*c) + 5*exp(8*I*c) + 10*exp(6*I*c) +
10*exp(4*I*c) + 5*exp(2*I*c) + 1)*exp(-6*I*c)/(32*a) - 5/(16*a)), True)) +
5*x/(16*a)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^4(c+dx)}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.87

$$\int \frac{\cos^4(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{-\frac{30i\log(\tan(dx+c)+i)}{a}+\frac{30i\log(\tan(dx+c)-i)}{a}+\frac{3\left(-15i\tan(dx+c)^2+38\tan(dx+c)+25i\right)}{a(-i\tan(dx+c)+1)^2}-\frac{55i\tan(dx+c)^3+201\tan(dx+c)^2-255i}{a(\tan(dx+c)-i)^3}}{192\,d}$$

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/192*(-30*I*log(tan(d*x + c) + I)/a + 30*I*log(tan(d*x + c) - I)/a + 3*(-
15*I*tan(d*x + c)^2 + 38*tan(d*x + c) + 25*I)/(a*(-I*tan(d*x + c) + 1)^2) -
 (55*I*tan(d*x + c)^3 + 201*tan(d*x + c)^2 - 255*I*tan(d*x + c) - 117)/(a*(
tan(d*x + c) - I)^3))/d
```

Mupad [B] (verification not implemented)

Time = 4.99 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.92

$$\int \frac{\cos^4(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{5\,x}{16\,a} + \frac{\frac{25\tan(c+d\,x)}{48\,a}+\frac{1\mathrm{i}}{6\,a}+\frac{\tan(c+d\,x)^2\,25\mathrm{i}}{48\,a}+\frac{5\tan(c+d\,x)^3}{16\,a}+\frac{\tan(c+d\,x)^4\,5\mathrm{i}}{16\,a}}{d\,\left(\tan\left(c+d\,x\right)^5\,1\mathrm{i}+\tan\left(c+d\,x\right)^4+\tan\left(c+d\,x\right)^3\,2\mathrm{i}+2\tan\left(c+d\,x\right)^2+\tan\left(c+d\,x\right)\,1\mathrm{i}+1\right)}$$

```
[In] int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i),x)
```

```
[Out] (5*x)/(16*a) + ((25*tan(c + d*x))/(48*a) + 1i/(6*a) + (tan(c + d*x)^2*25i)/
(48*a) + (5*tan(c + d*x)^3)/(16*a) + (tan(c + d*x)^4*5i)/(16*a))/(d*(tan(c
+ d*x)*1i + 2*tan(c + d*x)^2 + tan(c + d*x)^3*2i + tan(c + d*x)^4 + tan(c +
 d*x)^5*1i + 1))
```

3.107 $\int \frac{\sec^7(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 84

$$\int \frac{\sec^7(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{3\text{arctanh}(\sin(c+dx))}{8ad} - \frac{i\sec^5(c+dx)}{5ad} + \frac{3\sec(c+dx)\tan(c+dx)}{8ad} + \frac{\sec^3(c+dx)\tan(c+dx)}{4ad}$$

```
[Out] 3/8*arctanh(sin(d*x+c))/a/d-1/5*I*sec(d*x+c)^5/a/d+3/8*sec(d*x+c)*tan(d*x+c
)/a/d+1/4*sec(d*x+c)^3*tan(d*x+c)/a/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3582, 3853, 3855}

$$\int \frac{\sec^7(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{3\text{arctanh}(\sin(c+dx))}{8ad} - \frac{i\sec^5(c+dx)}{5ad} + \frac{\tan(c+dx)\sec^3(c+dx)}{4ad} + \frac{3\tan(c+dx)\sec(c+dx)}{8ad}$$

```
[In] Int[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(8*a*d) - ((I/5)*Sec[c + d*x]^5)/(a*d) + (3*Sec[c
 + d*x]*Tan[c + d*x])/(8*a*d) + (Sec[c + d*x]^3*Tan[c + d*x])/(4*a*d)
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
```

```
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{i\sec^5(c+dx)}{5ad} + \frac{\int \sec^5(c+dx)\,dx}{a} \\
&= -\frac{i\sec^5(c+dx)}{5ad} + \frac{\sec^3(c+dx)\tan(c+dx)}{4ad} + \frac{3\int \sec^3(c+dx)\,dx}{4a} \\
&= -\frac{i\sec^5(c+dx)}{5ad} + \frac{3\sec(c+dx)\tan(c+dx)}{8ad} + \frac{\sec^3(c+dx)\tan(c+dx)}{4ad} + \frac{3\int \sec(c+dx)\,dx}{8a} \\
&= \frac{3\text{arctanh}(\sin(c+dx))}{8ad} - \frac{i\sec^5(c+dx)}{5ad} \\
&\quad + \frac{3\sec(c+dx)\tan(c+dx)}{8ad} + \frac{\sec^3(c+dx)\tan(c+dx)}{4ad}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.71

$$\int \frac{\sec^7(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{240\text{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)+\sec^5(c+dx)(-64i+70\sin(2(c+dx))+15\sin(4(c+dx)))}{320ad}$$

```
[In] Integrate[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (240*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]] + Sec[c + d*x]^5*(-64*I + 70*Sin
[2*(c + d*x)] + 15*Sin[4*(c + d*x)]))/(320*a*d)
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.45

method	result
risch	$-\frac{i\left(15\,\mathrm{e}^{9i(dx+c)}+70\,\mathrm{e}^{7i(dx+c)}+128\,\mathrm{e}^{5i(dx+c)}-70\,\mathrm{e}^{3i(dx+c)}-15\,\mathrm{e}^{i(dx+c)}\right)}{20da\left(\mathrm{e}^{2i(dx+c)}+1\right)^{5}}+\frac{3\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{8ad}-\frac{3\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{8ad}$
derivativedivides	$\frac{\frac{i}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{5}}+\frac{2\left(\frac{7}{16}+\frac{5i}{16}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(\frac{5}{16}+\frac{3i}{16}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2\left(\frac{1}{4}+\frac{3i}{8}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2\left(\frac{1}{8}+\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}-}{ad}$
default	$\frac{\frac{i}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{5}}+\frac{2\left(\frac{7}{16}+\frac{5i}{16}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(\frac{5}{16}+\frac{3i}{16}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2\left(\frac{1}{4}+\frac{3i}{8}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2\left(\frac{1}{8}+\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}-}{ad}$

```
[In] int(sec(d*x+c)^7/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/20*I/d/a/(exp(2*I*(d*x+c))+1)^5*(15*exp(9*I*(d*x+c))+70*exp(7*I*(d*x+c))
+128*exp(5*I*(d*x+c))-70*exp(3*I*(d*x+c))-15*exp(I*(d*x+c)))+3/8/a/d*ln(exp
(I*(d*x+c))+I)-3/8/a/d*ln(exp(I*(d*x+c))-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(74) = 148.

Time = 0.25 (sec) , antiderivative size = 266, normalized size of antiderivative = 3.17

$$\int \frac{\sec^7(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{15\left(e^{(10i\,dx+10i\,c)}+5\,e^{(8i\,dx+8i\,c)}+10\,e^{(6i\,dx+6i\,c)}+10\,e^{(4i\,dx+4i\,c)}+5\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-15\,(}{40\,(ade^{(10i}}$$

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/40*(15*(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*I*d*x + 6
*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x +
I*c) + I) - 15*(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*I*d
*x + 6*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*
d*x + I*c) - I) - 30*I*e^(9*I*d*x + 9*I*c) - 140*I*e^(7*I*d*x + 7*I*c) - 25
6*I*e^(5*I*d*x + 5*I*c) + 140*I*e^(3*I*d*x + 3*I*c) + 30*I*e^(I*d*x + I*c))
/(a*d*e^(10*I*d*x + 10*I*c) + 5*a*d*e^(8*I*d*x + 8*I*c) + 10*a*d*e^(6*I*d*x
 + 6*I*c) + 10*a*d*e^(4*I*d*x + 4*I*c) + 5*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

Sympy [F]

$$\int \frac{\sec^7(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i\int \frac{\sec^7(c+dx)}{\tan(c+dx)-i}\,dx}{a}$$

[In] integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c)),x)

[Out] -I*Integral(sec(c + d*x)**7/(tan(c + d*x) - I), x)/a

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 288 vs. 2(74) = 148.

Time = 0.27 (sec) , antiderivative size = 288, normalized size of antiderivative = 3.43

$$\int \frac{\sec^7(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{3\left(\frac{16\left(\frac{25i\sin(dx+c)}{\cos(dx+c)+1}-\frac{10i\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{80\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10i\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{40\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{25i\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+8\right)}{-120ia+\frac{600ia\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{1200ia\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{1200ia\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{600ia\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{120ia\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{\mathrm{lo}}{}\right.}{8d}$$

[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c)),x, algorithm="maxima")

[Out] -3/8*(16*(25*I*sin(d*x + c)/(cos(d*x + c) + 1) - 10*I*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 80*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*I*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 40*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 25*I*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 8)/(-120*I*a + 600*I*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1200*I*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1200*I*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 600*I*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 120*I*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) - log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a + log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a)/d

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.64

$$\int \frac{\sec^7(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\frac{15\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a}-\frac{15\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a}+\frac{2\left(25\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+40i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8-10\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+80i\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^5a}}{40d}$$

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/40*(15*log(tan(1/2*d*x + 1/2*c) + 1)/a - 15*log(tan(1/2*d*x + 1/2*c) - 1)
/a + 2*(25*tan(1/2*d*x + 1/2*c)^9 + 40*I*tan(1/2*d*x + 1/2*c)^8 - 10*tan(1/
2*d*x + 1/2*c)^7 + 80*I*tan(1/2*d*x + 1/2*c)^4 + 10*tan(1/2*d*x + 1/2*c)^3
- 25*tan(1/2*d*x + 1/2*c) + 8*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^5*a))/d
```

Mupad [B] (verification not implemented)

Time = 8.36 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.30

$$\int \frac{\sec^7(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,a\,d} + \frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2\,a}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2\,a}+\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4\,a}-\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,a}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,4\mathrm{i}}{a}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,2\mathrm{i}}{a}+\frac{2\mathrm{i}}{5\,a}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

```
[In] int(1/(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)),x)
```

```
[Out] (3*atanh(tan(c/2 + (d*x)/2)))/(4*a*d) + (tan(c/2 + (d*x)/2)^3/(2*a) + (tan(
c/2 + (d*x)/2)^4*4i)/a - tan(c/2 + (d*x)/2)^7/(2*a) + (tan(c/2 + (d*x)/2)^8
*2i)/a + (5*tan(c/2 + (d*x)/2)^9)/(4*a) + 2i/(5*a) - (5*tan(c/2 + (d*x)/2))
/(4*a))/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 +
 (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.108 $\int \frac{\sec^5(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 60

$$\int \frac{\sec^5(c+dx)}{a+ia\tan(c+dx)}\,dx=\frac{\operatorname{arctanh}(\sin(c+dx))}{2ad}-\frac{i\sec^3(c+dx)}{3ad}+\frac{\sec(c+dx)\tan(c+dx)}{2ad}$$

```
[Out] 1/2*arctanh(sin(d*x+c))/a/d-1/3*I*sec(d*x+c)^3/a/d+1/2*sec(d*x+c)*tan(d*x+c
)/a/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3582, 3853, 3855}

$$\int \frac{\sec^5(c+dx)}{a+ia\tan(c+dx)}\,dx=\frac{\operatorname{arctanh}(\sin(c+dx))}{2ad}-\frac{i\sec^3(c+dx)}{3ad}+\frac{\tan(c+dx)\sec(c+dx)}{2ad}$$

```
[In] Int[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(2*a*d) - ((I/3)*Sec[c + d*x]^3)/(a*d) + (Sec[c + d*x
]*Tan[c + d*x])/(2*a*d)
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{i\sec^3(c+dx)}{3ad} + \frac{\int \sec^3(c+dx)\,dx}{a} \\
&= -\frac{i\sec^3(c+dx)}{3ad} + \frac{\sec(c+dx)\tan(c+dx)}{2ad} + \frac{\int \sec(c+dx)\,dx}{2a} \\
&= \frac{\operatorname{arctanh}(\sin(c+dx))}{2ad} - \frac{i\sec^3(c+dx)}{3ad} + \frac{\sec(c+dx)\tan(c+dx)}{2ad}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \frac{\sec^5(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{12\operatorname{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)+\sec^3(c+dx)(-4i+3\sin(2(c+dx)))}{12ad}$$

```
[In] Integrate[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x]),x]
[Out] (12*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]] + Sec[c + d*x]^3*(-4*I + 3*Sin[2*
(c + d*x)]))/(12*a*d)
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.67

method	result
risch	$-\frac{i\left(3\,\mathrm{e}^{5i(dx+c)}+8\,\mathrm{e}^{3i(dx+c)}-3\,\mathrm{e}^{i(dx+c)}\right)}{3da\left(\mathrm{e}^{2i(dx+c)}+1\right)^{3}}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2ad}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2ad}$
derivativedivides	$\frac{\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2\left(\frac{1}{4}+\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(\frac{1}{4}+\frac{i}{4}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}-\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{2\left(\frac{1}{4}-\frac{i}{4}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{2(}{(\tan(}}{ad}$
default	$\frac{\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2\left(\frac{1}{4}+\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(\frac{1}{4}+\frac{i}{4}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}-\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{2\left(\frac{1}{4}-\frac{i}{4}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{2(}{(\tan(}}{ad}$

```
[In] int(sec(d*x+c)^5/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*I/d/a/(exp(2*I*(d*x+c))+1)^3*(3*exp(5*I*(d*x+c))+8*exp(3*I*(d*x+c))-3*
exp(I*(d*x+c)))-1/2/a/d*ln(exp(I*(d*x+c))-I)+1/2/a/d*ln(exp(I*(d*x+c))+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(52) = 104$.

Time = 0.24 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.90

$$\int \frac{\sec^5(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{3\left(e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-3\left(e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e^{(2i\,dx+2i}\right.}{6\left(ade^{(6i\,dx+6i\,c)}+3\,ade^{(4i\,dx+4i\,c)}+3\,ade^{(2i\,dx+2}\right.}$$

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I*c)
 + 1)*log(e^(I*d*x + I*c) + I) - 3*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*
I*c) + 3*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) - I) - 6*I*e^(5*I*d*x
 + 5*I*c) - 16*I*e^(3*I*d*x + 3*I*c) + 6*I*e^(I*d*x + I*c))/(a*d*e^(6*I*d*x
 + 6*I*c) + 3*a*d*e^(4*I*d*x + 4*I*c) + 3*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i\int \frac{\sec^5(c+dx)}{\tan(c+dx)-i}\,dx}{a}$$

```
[In] integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -I*Integral(sec(c + d*x)**5/(tan(c + d*x) - I), x)/a
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(52) = 104.

Time = 0.36 (sec) , antiderivative size = 186, normalized size of antiderivative = 3.10

$$\int \frac{\sec^5(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\frac{4\left(\frac{3i\sin(dx+c)}{\cos(dx+c)+1}+\frac{6\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{3i\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+2\right)}{6ia-\frac{18ia\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{18ia\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{6ia\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}}{2d}$$

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(4*(3*I*sin(d*x + c)/(cos(d*x + c) + 1) + 6*sin(d*x + c)^4/(cos(d*x + c
) + 1)^4 - 3*I*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2)/(6*I*a - 18*I*a*sin
(d*x + c)^2/(cos(d*x + c) + 1)^2 + 18*I*a*sin(d*x + c)^4/(cos(d*x + c) + 1)
^4 - 6*I*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + log(sin(d*x + c)/(cos(d*x
 + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a)/d
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.65

$$\int \frac{\sec^5(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\frac{3\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a}-\frac{3\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a}+\frac{2\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+6i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2i\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3a}}{6d}$$

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*log(tan(1/2*d*x + 1/2*c) + 1)/a - 3*log(tan(1/2*d*x + 1/2*c) - 1)/a
+ 2*(3*tan(1/2*d*x + 1/2*c)^5 + 6*I*tan(1/2*d*x + 1/2*c)^4 - 3*tan(1/2*d*x
+ 1/2*c) + 2*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a))/d
```

Mupad [B] (verification not implemented)

Time = 5.91 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.93

$$\int \frac{\sec^5(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a\,d} + \frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{a}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\,2i}{a}+\frac{2i}{3\,a}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}$$

```
[In] int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)),x)
```

```
[Out] atanh(tan(c/2 + (d*x)/2))/(a*d) + ((tan(c/2 + (d*x)/2)^4*2i)/a + tan(c/2 +
(d*x)/2)^5/a + 2i/(3*a) - tan(c/2 + (d*x)/2)/a)/(d*(3*tan(c/2 + (d*x)/2)^2
- 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.109 $\int \frac{\sec^3(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 31

$$\int \frac{\sec^3(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{ad} - \frac{i\sec(c+dx)}{ad}$$

```
[Out] arctanh(sin(d*x+c))/a/d-I*sec(d*x+c)/a/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3582, 3855}

$$\int \frac{\sec^3(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{ad} - \frac{i\sec(c+dx)}{ad}$$

```
[In] Int[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(a*d) - (I*Sec[c + d*x])/(a*d)
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\sec(c+dx)}{ad} + \frac{\int \sec(c+dx)\,dx}{a}\\ &= \frac{\operatorname{arctanh}(\sin(c+dx))}{ad} - \frac{i\sec(c+dx)}{ad}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{\sec^3(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{2\operatorname{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)-i\sec(c+dx)}{ad}$$

```
[In] Integrate[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x]),x]
[Out] (2*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]] - I*Sec[c + d*x])/(a*d)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(30) = 60.

Time = 1.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.26

method	result	size
derivativedivides	$\frac{-\frac{i}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{2i}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2}-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{ad}$	70
default	$\frac{-\frac{i}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{2i}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2}-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{ad}$	70
risch	$-\frac{2i e^{i(dx+c)}}{da\left(e^{2i(dx+c)}+1\right)}-\frac{\ln\left(e^{i(dx+c)}-i\right)}{ad}+\frac{\ln\left(e^{i(dx+c)}+i\right)}{ad}$	74

```
[In] int(sec(d*x+c)^3/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 2/d/a*(-1/2*I/(tan(1/2*d*x+1/2*c)+1)+1/2*ln(tan(1/2*d*x+1/2*c)+1)+1/2*I/(ta
n(1/2*d*x+1/2*c)-1)-1/2*ln(tan(1/2*d*x+1/2*c)-1))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(29) = 58.

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.58

$$\int \frac{\sec^3(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\left(e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-\left(e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-2i\,e^{(i\,dx+i\,c)}}{ade^{(2i\,dx+2i\,c)}+ad}$$

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) + I) - (e^(2*I*d*x + 2*I*c)
+ 1)*log(e^(I*d*x + I*c) - I) - 2*I*e^(I*d*x + I*c))/(a*d*e^(2*I*d*x + 2*I*
c) + a*d)
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{i\int \frac{\sec^3(c+dx)}{\tan(c+dx)-i}\,dx}{a}$$

```
[In] integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -I*Integral(sec(c + d*x)**3/(tan(c + d*x) - I), x)/a
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(29) = 58.

Time = 0.37 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.68

$$\int \frac{\sec^3(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{2}{-i\,a+\frac{i\,a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}}{d}$$

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] (log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x + c
) + 1) - 1)/a - 2/(-I*a + I*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2))/d
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.87

$$\int \frac{\sec^3(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\frac{\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a} - \frac{\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a} + \frac{2i}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a}}{d}$$

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] (log(tan(1/2*d*x + 1/2*c) + 1)/a - log(tan(1/2*d*x + 1/2*c) - 1)/a + 2*I/((
tan(1/2*d*x + 1/2*c)^2 - 1)*a))/d
```

Mupad [B] (verification not implemented)

Time = 4.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.39

$$\int \frac{\sec^3(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d} + \frac{2\mathrm{i}}{a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

```
[In] int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)),x)
```

```
[Out] (2*atanh(tan(c/2 + (d*x)/2)))/(a*d) + 2i/(a*d*(tan(c/2 + (d*x)/2)^2 - 1))
```

3.110 $\int \frac{\sec(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 28

$$\int \frac{\sec(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{i\sec(c+dx)}{d(a+ia\tan(c+dx))}$$

```
[Out] I*sec(d*x+c)/d/(a+I*a*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3569}

$$\int \frac{\sec(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{i\sec(c+dx)}{d(a+ia\tan(c+dx))}$$

```
[In] Int[Sec[c + d*x]/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (I*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x]))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\text{integral} = \frac{i\sec(c+dx)}{d(a+ia\tan(c+dx))}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{\sec(c+dx)}{a+ia\tan(c+dx)}\,dx=\frac{\sec(c+dx)}{ad(-i+\tan(c+dx))}$$

```
[In] Integrate[Sec[c + d*x]/(a + I*a*Tan[c + d*x]),x]
[Out] Sec[c + d*x]/(a*d*(-I + Tan[c + d*x]))
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

method	result	size
risch	$\frac{i\mathrm{e}^{-i(dx+c)}}{ad}$	19
derivativedivides	$\frac{2}{da\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	23
default	$\frac{2}{da\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	23

```
[In] int(sec(d*x+c)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] I/a/d*exp(-I*(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int \frac{\sec(c+dx)}{a+ia\tan(c+dx)}\,dx=\frac{i\,e^{(-i\,dx-i\,c)}}{ad}$$

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
[Out] I*e^(-I*d*x - I*c)/(a*d)
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \frac{\sec(c+dx)}{a+ia\tan(c+dx)}\,dx = \begin{cases} \frac{\sec(c+dx)}{ad\tan(c+dx)-iad} & \text{for } d \neq 0 \\ \frac{x\sec(c)}{ia\tan(c)+a} & \text{otherwise} \end{cases}$$

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c)),x)
[Out] Piecewise((sec(c + d*x)/(a*d*tan(c + d*x) - I*a*d), Ne(d, 0)), (x*sec(c)/(I
*a*tan(c) + a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{\sec(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{2}{\left(-i\,a+\frac{a\sin(dx+c)}{\cos(dx+c)+1}\right)d}$$

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
[Out] 2/((-I*a + a*sin(d*x + c)/(cos(d*x + c) + 1))*d)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int \frac{\sec(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{2}{ad\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}$$

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] 2/(a*d*(tan(1/2*d*x + 1/2*c) - I))
```

Mupad [B] (verification not implemented)

Time = 4.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{\sec(c+dx)}{a+ia\tan(c+dx)}\,dx=\frac{2\mathrm{i}}{a\,d\,\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}\right)}$$

```
[In] int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)),x)
[Out] 2i/(a*d*(tan(c/2 + (d*x)/2)*1i + 1))
```

3.111 $\int \frac{\cos(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 47

$$\int \frac{\cos(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{2\sin(c+dx)}{3ad}+\frac{i\cos(c+dx)}{3d(a+ia\tan(c+dx))}$$

```
[Out] 2/3*sin(d*x+c)/a/d+1/3*I*cos(d*x+c)/d/(a+I*a*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3583, 2717}

$$\int \frac{\cos(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{2\sin(c+dx)}{3ad}+\frac{i\cos(c+dx)}{3d(a+ia\tan(c+dx))}$$

```
[In] Int[Cos[c + d*x]/(a + I*a*Tan[c + d*x]),x]
[Out] (2*Sin[c + d*x])/(3*a*d) + ((I/3)*Cos[c + d*x])/(d*(a + I*a*Tan[c + d*x]))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
```

```
&& EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{i\cos(c+dx)}{3d(a+ia\tan(c+dx))} + \frac{2\int\cos(c+dx)\,dx}{3a}\\ &= \frac{2\sin(c+dx)}{3ad} + \frac{i\cos(c+dx)}{3d(a+ia\tan(c+dx))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06

$$\int \frac{\cos(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{\sec(c+dx)(-3+\cos(2(c+dx))+2i\sin(2(c+dx)))}{6ad(-i+\tan(c+dx))}$$

```
[In] Integrate[Cos[c + d*x]/(a + I*a*Tan[c + d*x]),x]
[Out] -1/6*(Sec[c + d*x]*(-3 + Cos[2*(c + d*x)] + (2*I)*Sin[2*(c + d*x)]))/(a*d*(
-I + Tan[c + d*x]))
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

method	result
risch	$\frac{i e^{-3i(dx+c)}}{12ad}+\frac{i\cos(dx+c)}{4ad}+\frac{3\sin(dx+c)}{4ad}$
derivativedivides	$\frac{-\frac{2}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{3}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{2}{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4i}}{ad}$
default	$\frac{-\frac{2}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{3}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{2}{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4i}}{ad}$
norman	$\frac{\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3ad}+\frac{2\tan(dx+c)}{3ad}-\frac{2i\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{2i\left(\tan^2(dx+c)\right)}{3ad}+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan^2(dx+c)\right)}{3ad}-\frac{2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\tan(dx+c)}{3ad}+}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)(1+\tan^2(dx+c))}$

```
[In] int(cos(d*x+c)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/12*I/a/d*exp(-3*I*(d*x+c))+1/4*I/a/d*cos(d*x+c)+3/4*sin(d*x+c)/a/d
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{\cos(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\left(-3i\,e^{(4i\,dx+4i\,c)}+6i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-3i\,dx-3i\,c)}}{12\,ad}$$

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(-3*I*e^(4*I*d*x + 4*I*c) + 6*I*e^(2*I*d*x + 2*I*c) + I)*e^(-3*I*d*x -
 3*I*c)/(a*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(36) = 72.

Time = 0.17 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.68

$$\int \frac{\cos(c+dx)}{a+ia\tan(c+dx)}\,dx = \begin{cases} \frac{\left(-24ia^2d^2e^{5ic}e^{idx}+48ia^2d^2e^{3ic}e^{-idx}+8ia^2d^2e^{ic}e^{-3idx}\right)e^{-4ic}}{96a^3d^3} & \text{for } a^3d^3e^{4ic}\neq 0 \\ \frac{x\left(e^{4ic}+2e^{2ic}+1\right)e^{-3ic}}{4a} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise((((-24*I*a**2*d**2*exp(5*I*c)*exp(I*d*x) + 48*I*a**2*d**2*exp(3*I*
c)*exp(-I*d*x) + 8*I*a**2*d**2*exp(I*c)*exp(-3*I*d*x))*exp(-4*I*c)/(96*a**3
*d**3), Ne(a**3*d**3*exp(4*I*c), 0)), (x*(exp(4*I*c) + 2*exp(2*I*c) + 1)*ex
p(-3*I*c)/(4*a), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.43

$$\int \frac{\cos(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\frac{3}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)} + \frac{9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-7}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^3}}{6\,d}$$

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3/(a*(tan(1/2*d*x + 1/2*c) + I)) + (9*tan(1/2*d*x + 1/2*c)^2 - 12*I*ta
n(1/2*d*x + 1/2*c) - 7)/(a*(tan(1/2*d*x + 1/2*c) - I)^3))/d
```

Mupad [B] (verification not implemented)

Time = 3.97 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.66

$$\int \frac{\cos(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\left(-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,3\mathrm{i}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)\,2\mathrm{i}}{3\,a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)\,{\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}\right)}^3}$$

```
[In] int(cos(c + d*x)/(a + a*tan(c + d*x)*1i),x)
```

```
[Out] ((tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2*3i - 3*tan(c/2 + (d*x)/2)^3 + 1
i)*2i)/(3*a*d*(tan(c/2 + (d*x)/2) + 1i)*(tan(c/2 + (d*x)/2)*1i + 1)^3)
```

3.112 $\int \frac{\cos^3(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{\cos^3(c+dx)}{a+ia\tan(c+dx)}\,dx=\frac{4\sin(c+dx)}{5ad}-\frac{4\sin^3(c+dx)}{15ad}+\frac{i\cos^3(c+dx)}{5d(a+ia\tan(c+dx))}$$

```
[Out] 4/5*sin(d*x+c)/a/d-4/15*sin(d*x+c)^3/a/d+1/5*I*cos(d*x+c)^3/d/(a+I*a*tan(d*
x+c))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3583, 2713}

$$\int \frac{\cos^3(c+dx)}{a+ia\tan(c+dx)}\,dx=-\frac{4\sin^3(c+dx)}{15ad}+\frac{4\sin(c+dx)}{5ad}+\frac{i\cos^3(c+dx)}{5d(a+ia\tan(c+dx))}$$

```
[In] Int[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (4*Sin[c + d*x])/(5*a*d) - (4*Sin[c + d*x]^3)/(15*a*d) + ((I/5)*Cos[c + d*x
]^3)/(d*(a + I*a*Tan[c + d*x]))
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{i\cos^3(c+dx)}{5d(a+ia\tan(c+dx))}+\frac{4\int\cos^3(c+dx)\,dx}{5a}\\
&= \frac{i\cos^3(c+dx)}{5d(a+ia\tan(c+dx))}-\frac{4\text{Subst}\left(\int(1-x^2)\,dx,x,-\sin(c+dx)\right)}{5ad}\\
&= \frac{4\sin(c+dx)}{5ad}-\frac{4\sin^3(c+dx)}{15ad}+\frac{i\cos^3(c+dx)}{5d(a+ia\tan(c+dx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.07

$$
\int\frac{\cos^3(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{\sec(c+dx)(-45+20\cos(2(c+dx))+\cos(4(c+dx))+40i\sin(2(c+dx))+4i\sin(4(c+dx)))}{120ad(-i+\tan(c+dx))}
$$

```
[In] Integrate[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] -1/120*(Sec[c + d*x]*(-45 + 20*Cos[2*(c + d*x)] + Cos[4*(c + d*x)] + (40*I)
*Sin[2*(c + d*x)] + (4*I)*Sin[4*(c + d*x)]))/(a*d*(-I + Tan[c + d*x]))
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.25

method	result
risch	$\frac{i e^{-5i(dx+c)}}{80ad}+\frac{i\cos(dx+c)}{8ad}+\frac{5\sin(dx+c)}{8ad}+\frac{i\cos(3dx+3c)}{16ad}+\frac{5\sin(3dx+3c)}{48ad}$
derivativedivides	$\frac{-\frac{i}{4(\tan(\frac{dx}{2}+\frac{c}{2})+i)^2}-\frac{1}{6(\tan(\frac{dx}{2}+\frac{c}{2})+i)^3}+\frac{5}{8(\tan(\frac{dx}{2}+\frac{c}{2})+i)}-\frac{i}{(-i+\tan(\frac{dx}{2}+\frac{c}{2}))^4}+\frac{3i}{2(-i+\tan(\frac{dx}{2}+\frac{c}{2}))^2}+\frac{2}{5(-i+\tan(\frac{dx}{2}+}}{ad}$
default	$\frac{-\frac{i}{4(\tan(\frac{dx}{2}+\frac{c}{2})+i)^2}-\frac{1}{6(\tan(\frac{dx}{2}+\frac{c}{2})+i)^3}+\frac{5}{8(\tan(\frac{dx}{2}+\frac{c}{2})+i)}-\frac{i}{(-i+\tan(\frac{dx}{2}+\frac{c}{2}))^4}+\frac{3i}{2(-i+\tan(\frac{dx}{2}+\frac{c}{2}))^2}+\frac{2}{5(-i+\tan(\frac{dx}{2}+}}{ad}$

```
[In] int(cos(d*x+c)^3/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/80*I/a/d*exp(-5*I*(d*x+c))+1/8*I/a/d*cos(d*x+c)+5/8*sin(d*x+c)/a/d+1/16*I
/a/d*cos(3*d*x+3*c)+5/48/a/d*sin(3*d*x+3*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94

$$\int \frac{\cos^3(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\left(-5i\,e^{(8i\,dx+8i\,c)}-60i\,e^{(6i\,dx+6i\,c)}+90i\,e^{(4i\,dx+4i\,c)}+20i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-5i\,dx-5i\,c)}}{240\,ad}$$

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(-5*I*e^(8*I*d*x + 8*I*c) - 60*I*e^(6*I*d*x + 6*I*c) + 90*I*e^(4*I*d*
x + 4*I*c) + 20*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(-5*I*d*x - 5*I*c)/(a*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(53) = 106.

Time = 0.24 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.93

$$\int \frac{\cos^3(c+dx)}{a+ia\tan(c+dx)}\,dx = \begin{cases} \frac{\left(-30720ia^4d^4e^{12ic}e^{3idx}-368640ia^4d^4e^{10ic}e^{idx}+552960ia^4d^4e^{8ic}e^{-idx}+122880ia^4d^4e^{6ic}e^{-3idx}+18432ia^4d^4e^{4ic}e^{-5idx}\right)e^{-9ic}}{1474560a^5d^5} & \text{for } a^5d^5 \\ \frac{x\left(e^{8ic}+4e^{6ic}+6e^{4ic}+4e^{2ic}+1\right)e^{-5ic}}{16a} & \text{otherwi} \end{cases}$$

```
[In] integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise(((-30720*I*a**4*d**4*exp(12*I*c)*exp(3*I*d*x) - 368640*I*a**4*d**
4*exp(10*I*c)*exp(I*d*x) + 552960*I*a**4*d**4*exp(8*I*c)*exp(-I*d*x) + 1228
80*I*a**4*d**4*exp(6*I*c)*exp(-3*I*d*x) + 18432*I*a**4*d**4*exp(4*I*c)*exp(
-5*I*d*x))*exp(-9*I*c)/(1474560*a**5*d**5), Ne(a**5*d**5*exp(9*I*c), 0)), (
x*(exp(8*I*c) + 4*exp(6*I*c) + 6*exp(4*I*c) + 4*exp(2*I*c) + 1)*exp(-5*I*c)
/(16*a), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(57) = 114$.

Time = 0.40 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.78

$$\int \frac{\cos^3(c+dx)}{a+ia\tan(c+dx)}\,dx$$
$$= \frac{\frac{5\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+24i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-13\right)}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^3}+\frac{165\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-480i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-650\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+400i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+113}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^5}}{120\,d}$$

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120*(5*(15*tan(1/2*d*x + 1/2*c)^2 + 24*I*tan(1/2*d*x + 1/2*c) - 13)/(a*(t
an(1/2*d*x + 1/2*c) + I)^3) + (165*tan(1/2*d*x + 1/2*c)^4 - 480*I*tan(1/2*d
*x + 1/2*c)^3 - 650*tan(1/2*d*x + 1/2*c)^2 + 400*I*tan(1/2*d*x + 1/2*c) + 1
13)/(a*(tan(1/2*d*x + 1/2*c) - I)^5))/d
```

Mupad [B] (verification not implemented)

Time = 5.73 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.00

$$\int \frac{\cos^3(c+dx)}{a+ia\tan(c+dx)}\,dx =$$
$$-\frac{\left(-15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 15\mathrm{i}-5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 25\mathrm{i}-13\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\tan\left(\cdots\right.\right.}{15\,a\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\mathrm{i}\right)^3\left(1+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,1\mathrm{i}\right)^5}$$

```
[In] int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i),x)
```

```
[Out] -((9*tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2*21i - 13*tan(c/2 + (d*x)/2)^
3 + tan(c/2 + (d*x)/2)^4*25i - 5*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*x)/2)^
6*15i - 15*tan(c/2 + (d*x)/2)^7 + 3i)*2i)/(15*a*d*(tan(c/2 + (d*x)/2) + 1i)
^3*(tan(c/2 + (d*x)/2)*1i + 1)^5)
```

3.113 $\int \frac{\cos^5(c+dx)}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 85

$$\int \frac{\cos^5(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{6\sin(c+dx)}{7ad} - \frac{4\sin^3(c+dx)}{7ad} + \frac{6\sin^5(c+dx)}{35ad} + \frac{i\cos^5(c+dx)}{7d(a+ia\tan(c+dx))}$$

```
[Out] 6/7*sin(d*x+c)/a/d-4/7*sin(d*x+c)^3/a/d+6/35*sin(d*x+c)^5/a/d+1/7*I*cos(d*x
+c)^5/d/(a+I*a*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3583, 2713}

$$\int \frac{\cos^5(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{6\sin^5(c+dx)}{35ad} - \frac{4\sin^3(c+dx)}{7ad} + \frac{6\sin(c+dx)}{7ad} + \frac{i\cos^5(c+dx)}{7d(a+ia\tan(c+dx))}$$

```
[In] Int[Cos[c + d*x]^5/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (6*Sin[c + d*x])/(7*a*d) - (4*Sin[c + d*x]^3)/(7*a*d) + (6*Sin[c + d*x]^5)/
(35*a*d) + ((I/7)*Cos[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x]))
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{i\cos^5(c+dx)}{7d(a+ia\tan(c+dx))}+\frac{6\int\cos^5(c+dx)\,dx}{7a}\\ &= \frac{i\cos^5(c+dx)}{7d(a+ia\tan(c+dx))}-\frac{6\text{Subst}\left(\int\left(1-2x^2+x^4\right)\,dx,x,-\sin(c+dx)\right)}{7ad}\\ &= \frac{6\sin(c+dx)}{7ad}-\frac{4\sin^3(c+dx)}{7ad}+\frac{6\sin^5(c+dx)}{35ad}+\frac{i\cos^5(c+dx)}{7d(a+ia\tan(c+dx))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.11

$$\int\frac{\cos^5(c+dx)}{a+ia\tan(c+dx)}\,dx = -\frac{\sec(c+dx)(-350+175\cos(2(c+dx))+14\cos(4(c+dx))+\cos(6(c+dx))+350i\sin(2(c+dx))+56}{1120ad(-i+\tan(c+dx))}$$

```
[In] Integrate[Cos[c + d*x]^5/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] -1/1120*(Sec[c + d*x]*(-350 + 175*Cos[2*(c + d*x)] + 14*Cos[4*(c + d*x)] +
Cos[6*(c + d*x)] + (350*I)*Sin[2*(c + d*x)] + (56*I)*Sin[4*(c + d*x)] + (6*
I)*Sin[6*(c + d*x)]))/(a*d*(-I + Tan[c + d*x]))
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.40

method	result
risch	$\frac{i\mathrm{e}^{-7i(dx+c)}}{448ad}+\frac{5i\cos(dx+c)}{64ad}+\frac{35\sin(dx+c)}{64ad}+\frac{i\cos(5dx+5c)}{64ad}+\frac{7\sin(5dx+5c)}{320ad}+\frac{3i\cos(3dx+3c)}{64ad}+\frac{7\sin(3dx+$
derivativedivides	$-\frac{2}{7\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{7}}+\frac{i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{6}}+\frac{15i}{8\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{11i}{4\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{21}{10\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}}-\frac{}{4(-i+}$
default	$-\frac{2}{7\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{7}}+\frac{i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{6}}+\frac{15i}{8\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{11i}{4\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{21}{10\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}}-\frac{}{4(-i+}$

```
[In] int(cos(d*x+c)^5/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/448*I/a/d*exp(-7*I*(d*x+c))+5/64*I/a/d*cos(d*x+c)+35/64*sin(d*x+c)/a/d+1/
64*I/a/d*cos(5*d*x+5*c)+7/320/a/d*sin(5*d*x+5*c)+3/64*I/a/d*cos(3*d*x+3*c)+
7/64/a/d*sin(3*d*x+3*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00

$$\int \frac{\cos^5(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\left(-7i\,e^{(12i\,dx+12i\,c)}-70i\,e^{(10i\,dx+10i\,c)}-525i\,e^{(8i\,dx+8i\,c)}+700i\,e^{(6i\,dx+6i\,c)}+175i\,e^{(4i\,dx+4i\,c)}+42i\,e^{(2i\,dx+2i\,c)}\right.}{2240\,ad}$$

```
[In] integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2240*(-7*I*e^(12*I*d*x + 12*I*c) - 70*I*e^(10*I*d*x + 10*I*c) - 525*I*e^(
8*I*d*x + 8*I*c) + 700*I*e^(6*I*d*x + 6*I*c) + 175*I*e^(4*I*d*x + 4*I*c) +
42*I*e^(2*I*d*x + 2*I*c) + 5*I)*e^(-7*I*d*x - 7*I*c)/(a*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(68) = 136.

Time = 0.34 (sec) , antiderivative size = 264, normalized size of antiderivative = 3.11

$$\int \frac{\cos^5(c+dx)}{a+ia\tan(c+dx)}\,dx = \begin{cases} \frac{\left(-150323855360ia^6d^6e^{21ic}e^{5idx}-1503238553600ia^6d^6e^{19ic}e^{3idx}-11274289152000ia^6d^6e^{17ic}e^{idx}+15032385536000ia^6d^6e^{15ic}e^{-idx}+375809\right.}{48103633715200a^7d^7} \\ \frac{x\left(e^{12ic}+6e^{10ic}+15e^{8ic}+20e^{6ic}+15e^{4ic}+6e^{2ic}+1\right)e^{-7ic}}{64a} \end{cases}$$

```
[In] integrate(cos(d*x+c)**5/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Piecewise(((-150323855360*I*a**6*d**6*exp(21*I*c)*exp(5*I*d*x) - 1503238553
600*I*a**6*d**6*exp(19*I*c)*exp(3*I*d*x) - 11274289152000*I*a**6*d**6*exp(1
7*I*c)*exp(I*d*x) + 15032385536000*I*a**6*d**6*exp(15*I*c)*exp(-I*d*x) + 37
58096384000*I*a**6*d**6*exp(13*I*c)*exp(-3*I*d*x) + 901943132160*I*a**6*d**
6*exp(11*I*c)*exp(-5*I*d*x) + 107374182400*I*a**6*d**6*exp(9*I*c)*exp(-7*I*
d*x))*exp(-16*I*c)/(48103633715200*a**7*d**7), Ne(a**7*d**7*exp(16*I*c), 0)
), (x*(exp(12*I*c) + 6*exp(10*I*c) + 15*exp(8*I*c) + 20*exp(6*I*c) + 15*exp
(4*I*c) + 6*exp(2*I*c) + 1)*exp(-7*I*c)/(64*a), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^5(c+dx)}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(73) = 146$.

Time = 0.43 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.01

$$\int \frac{\cos^5(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\frac{7\left(55\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+180i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-250\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-160i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+43\right)}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^5} + \frac{735\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-3360i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-}{}}{560\,d}$$

```
[In] integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/560*(7*(55*tan(1/2*d*x + 1/2*c)^4 + 180*I*tan(1/2*d*x + 1/2*c)^3 - 250*ta
n(1/2*d*x + 1/2*c)^2 - 160*I*tan(1/2*d*x + 1/2*c) + 43)/(a*(tan(1/2*d*x + 1
/2*c) + I)^5) + (735*tan(1/2*d*x + 1/2*c)^6 - 3360*I*tan(1/2*d*x + 1/2*c)^5
 - 7315*tan(1/2*d*x + 1/2*c)^4 + 8820*I*tan(1/2*d*x + 1/2*c)^3 + 6321*tan(1
/2*d*x + 1/2*c)^2 - 2492*I*tan(1/2*d*x + 1/2*c) - 461)/(a*(tan(1/2*d*x + 1/
2*c) - I)^7))/d
```

Mupad [B] (verification not implemented)

Time = 8.21 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.21

$$\int \frac{\cos^5(c+dx)}{a+ia\tan(c+dx)}\,dx = \frac{\left(-35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}35\mathrm{i}-35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}105\mathrm{i}-126\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}+\mathrm{t}\right.}{35\,a\,d\left(\tan\left(\frac{c}{2}+\right.\right.}$$

```
[In] int(cos(c + d*x)^5/(a + a*tan(c + d*x)*1i),x)
```

```
[Out] ((25*tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2*55i - 15*tan(c/2 + (d*x)/2)^
3 + tan(c/2 + (d*x)/2)^4*130i + 26*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*x)/2
)^6*182i - 126*tan(c/2 + (d*x)/2)^7 + tan(c/2 + (d*x)/2)^8*105i - 35*tan(c/
2 + (d*x)/2)^9 + tan(c/2 + (d*x)/2)^10*35i - 35*tan(c/2 + (d*x)/2)^11 + 5i)
*2i)/(35*a*d*(tan(c/2 + (d*x)/2) + 1i)^5*(tan(c/2 + (d*x)/2)*1i + 1)^7)
```

3.114 $\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{4i(a-ia\tan(c+dx))^5}{5a^7d} - \frac{2i(a-ia\tan(c+dx))^6}{3a^8d} + \frac{i(a-ia\tan(c+dx))^7}{7a^9d}$$

```
[Out] 4/5*I*(a-I*a*tan(d*x+c))^5/a^7/d-2/3*I*(a-I*a*tan(d*x+c))^6/a^8/d+1/7*I*(a-
I*a*tan(d*x+c))^7/a^9/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 45}

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{i(a-ia\tan(c+dx))^7}{7a^9d} - \frac{2i(a-ia\tan(c+dx))^6}{3a^8d} + \frac{4i(a-ia\tan(c+dx))^5}{5a^7d}$$

```
[In] Int[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (((4*I)/5)*(a - I*a*Tan[c + d*x])^5)/(a^7*d) - (((2*I)/3)*(a - I*a*Tan[c +
d*x])^6)/(a^8*d) + ((I/7)*(a - I*a*Tan[c + d*x])^7)/(a^9*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)^4(a+x)^2\,dx, x, ia\tan(c+dx)\right)}{a^9d}\\ &= -\frac{i\text{Subst}\left(\int \left(4a^2(a-x)^4-4a(a-x)^5+(a-x)^6\right)\,dx, x, ia\tan(c+dx)\right)}{a^9d}\\ &= \frac{4i(a-ia\tan(c+dx))^5}{5a^7d}-\frac{2i(a-ia\tan(c+dx))^6}{3a^8d}+\frac{i(a-ia\tan(c+dx))^7}{7a^9d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.54

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{(i+\tan(c+dx))^5\left(-29-40i\tan(c+dx)+15\tan^2(c+dx)\right)}{105a^2d}$$

```
[In] Integrate[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^2,x]
[Out] -1/105*((I + Tan[c + d*x])^5*(-29 - (40*I)*Tan[c + d*x] + 15*Tan[c + d*x]^2
))/(a^2*d)
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.57

method	result	size
risch	$\frac{128i\left(21\,e^{4i(dx+c)}+7\,e^{2i(dx+c)}+1\right)}{105d\,a^2\left(e^{2i(dx+c)}+1\right)^7}$	47
derivativedivides	$\frac{\tan(dx+c)-\frac{\left(\tan^7(dx+c)\right)}{7}-\frac{i\left(\tan^6(dx+c)\right)}{3}-\frac{\left(\tan^5(dx+c)\right)}{5}-i\left(\tan^4(dx+c)\right)+\frac{\left(\tan^3(dx+c)\right)}{3}-i\left(\tan^2(dx+c)\right)}{a^2d}$	78
default	$\frac{\tan(dx+c)-\frac{\left(\tan^7(dx+c)\right)}{7}-\frac{i\left(\tan^6(dx+c)\right)}{3}-\frac{\left(\tan^5(dx+c)\right)}{5}-i\left(\tan^4(dx+c)\right)+\frac{\left(\tan^3(dx+c)\right)}{3}-i\left(\tan^2(dx+c)\right)}{a^2d}$	78

```
[In] int(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 128/105*I*(21*exp(4*I*(d*x+c))+7*exp(2*I*(d*x+c))+1)/d/a^2/(exp(2*I*(d*x+c)
)+1)^7
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(64) = 128.

Time = 0.24 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.68

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx =$$
$$-\frac{128\left(-21i\,e^{(4i\,dx+4i\,c)}-7i\,e^{(2i\,dx+2i\,c)}-i\right)}{105\left(a^2de^{(14i\,dx+14i\,c)}+7\,a^2de^{(12i\,dx+12i\,c)}+21\,a^2de^{(10i\,dx+10i\,c)}+35\,a^2de^{(8i\,dx+8i\,c)}+35\,a^2de^{(6i\,dx+6i\,c)}+21\right.}$$

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -128/105*(-21*I*e^(4*I*d*x + 4*I*c) - 7*I*e^(2*I*d*x + 2*I*c) - I)/(a^2*d*e
^(14*I*d*x + 14*I*c) + 7*a^2*d*e^(12*I*d*x + 12*I*c) + 21*a^2*d*e^(10*I*d*x
 + 10*I*c) + 35*a^2*d*e^(8*I*d*x + 8*I*c) + 35*a^2*d*e^(6*I*d*x + 6*I*c) +
21*a^2*d*e^(4*I*d*x + 4*I*c) + 7*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F]

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\sec^{10}(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

```
[In] integrate(sec(d*x+c)**10/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -Integral(sec(c + d*x)**10/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)/a**
2
```

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.94

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx =$$
$$-\frac{15\tan(dx+c)^7+35i\tan(dx+c)^6+21\tan(dx+c)^5+105i\tan(dx+c)^4-35\tan(dx+c)^3+105i}{105\,a^2d}$$

[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")

[Out] -1/105*(15*tan(d*x + c)^7 + 35*I*tan(d*x + c)^6 + 21*tan(d*x + c)^5 + 105*I*tan(d*x + c)^4 - 35*tan(d*x + c)^3 + 105*I*tan(d*x + c)^2 - 105*tan(d*x + c))/(a^2*d)

Giac [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.94

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{15\,\tan\left(dx+c\right)^{7}+35i\,\tan\left(dx+c\right)^{6}+21\,\tan\left(dx+c\right)^{5}+105i\,\tan\left(dx+c\right)^{4}-35\,\tan\left(dx+c\right)^{3}+105i}{105\,a^{2}d}$$

[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")

[Out] -1/105*(15*tan(d*x + c)^7 + 35*I*tan(d*x + c)^6 + 21*tan(d*x + c)^5 + 105*I*tan(d*x + c)^4 - 35*tan(d*x + c)^3 + 105*I*tan(d*x + c)^2 - 105*tan(d*x + c))/(a^2*d)

Mupad [B] (verification not implemented)

Time = 3.91 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.13

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\cos\left(c+d\,x\right)^{7}\,35\mathrm{i}+64\,\sin\left(c+d\,x\right)\,\cos\left(c+d\,x\right)^{6}+32\,\sin\left(c+d\,x\right)\,\cos\left(c+d\,x\right)^{4}+24\,\sin\left(c+d\,x\right)\,\cos\left(c\right.}{105\,a^{2}\,d\cos\left(c+d\,x\right)^{7}}$$

[In] int(1/(cos(c + d*x)^10*(a + a*tan(c + d*x)*1i)^2),x)

[Out] (24*cos(c + d*x)^2*sin(c + d*x) - 15*sin(c + d*x) - cos(c + d*x)*35i + 32*cos(c + d*x)^4*sin(c + d*x) + 64*cos(c + d*x)^6*sin(c + d*x) + cos(c + d*x)^7*35i)/(105*a^2*d*cos(c + d*x)^7)

3.115 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{i(a-ia\tan(c+dx))^4}{2a^6d} - \frac{i(a-ia\tan(c+dx))^5}{5a^7d}$$

```
[Out] 1/2*I*(a-I*a*tan(d*x+c))^4/a^6/d-1/5*I*(a-I*a*tan(d*x+c))^5/a^7/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{i(a-ia\tan(c+dx))^4}{2a^6d} - \frac{i(a-ia\tan(c+dx))^5}{5a^7d}$$

```
[In] Int[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((I/2)*(a - I*a*Tan[c + d*x])^4)/(a^6*d) - ((I/5)*(a - I*a*Tan[c + d*x])^5)
/(a^7*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)^3(a+x)\,dx, x, ia\tan(c+dx)\right)}{a^7d}\\ &= -\frac{i\text{Subst}\left(\int \left(2a(a-x)^3-(a-x)^4\right)\,dx, x, ia\tan(c+dx)\right)}{a^7d}\\ &= \frac{i(a-ia\tan(c+dx))^4}{2a^6d}-\frac{i(a-ia\tan(c+dx))^5}{5a^7d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\tan(c+dx)\left(-10+10i\tan(c+dx)+5i\tan^3(c+dx)+2\tan^4(c+dx)\right)}{10a^2d}$$

```
[In] Integrate[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^2,x]
[Out] -1/10*(Tan[c + d*x]*(-10 + (10*I)*Tan[c + d*x] + (5*I)*Tan[c + d*x]^3 + 2*T
an[c + d*x]^4))/(a^2*d)
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

method	result	size
risch	$\frac{8i\left(5\,{\mathrm e}^{2i(dx+c)}+1\right)}{5d\,a^{2}\left({\mathrm e}^{2i(dx+c)}+1\right)^{5}}$	36
derivativedivides	$\frac{\tan(dx+c)-\frac{\left(\tan^5(dx+c)\right)}{5}-\frac{i\left(\tan^4(dx+c)\right)}{2}-i\left(\tan^2(dx+c)\right)}{a^2d}$	47
default	$\frac{\tan(dx+c)-\frac{\left(\tan^5(dx+c)\right)}{5}-\frac{i\left(\tan^4(dx+c)\right)}{2}-i\left(\tan^2(dx+c)\right)}{a^2d}$	47

```
[In] int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 8/5*I*(5*exp(2*I*(d*x+c))+1)/d/a^2/(exp(2*I*(d*x+c))+1)^5
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(43) = 86$.

Time = 0.23 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.76

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{8\left(-5i\,e^{(2i\,dx+2i\,c)}-i\right)}{5\left(a^2de^{(10i\,dx+10i\,c)}+5\,a^2de^{(8i\,dx+8i\,c)}+10\,a^2de^{(6i\,dx+6i\,c)}+10\,a^2de^{(4i\,dx+4i\,c)}+5\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -8/5*(-5*I*e^(2*I*d*x + 2*I*c) - I)/(a^2*d*e^(10*I*d*x + 10*I*c) + 5*a^2*d*
e^(8*I*d*x + 8*I*c) + 10*a^2*d*e^(6*I*d*x + 6*I*c) + 10*a^2*d*e^(4*I*d*x +
4*I*c) + 5*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F]

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\sec^8(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

```
[In] integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -Integral(sec(c + d*x)**8/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.73 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{2\,\tan(dx+c)^5+5i\,\tan(dx+c)^4+10i\,\tan(dx+c)^2-10\,\tan(dx+c)}{10\,a^2d}$$

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/10*(2*tan(d*x + c)^5 + 5*I*tan(d*x + c)^4 + 10*I*tan(d*x + c)^2 - 10*tan
(d*x + c))/(a^2*d)
```

Giac [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{2\,\tan\left(dx+c\right)^5+5i\,\tan\left(dx+c\right)^4+10i\,\tan\left(dx+c\right)^2-10\,\tan\left(dx+c\right)}{10\,a^2d}$$

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
[Out] -1/10*(2*tan(d*x + c)^5 + 5*I*tan(d*x + c)^4 + 10*I*tan(d*x + c)^2 - 10*tan
(d*x + c))/(a^2*d)
```

Mupad [B] (verification not implemented)

Time = 3.80 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.40

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\sin\left(c+d\,x\right)\,\left(-10\cos\left(c+d\,x\right)^4+\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)\,10\mathrm{i}+\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)^3\,5\mathrm{i}+2\sin}{10\,a^2\,d\cos\left(c+d\,x\right)^5}$$

```
[In] int(1/(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^2),x)
[Out] -(sin(c + d*x)*(cos(c + d*x)*sin(c + d*x)^3*5i + cos(c + d*x)^3*sin(c + d*x
)*10i - 10*cos(c + d*x)^4 + 2*sin(c + d*x)^4))/(10*a^2*d*cos(c + d*x)^5)
```

3.116 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{i(a-ia\tan(c+dx))^3}{3a^5d}$$

```
[Out] 1/3*I*(a-I*a*tan(d*x+c))^3/a^5/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 32}

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{i(a-ia\tan(c+dx))^3}{3a^5d}$$

```
[In] Int[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((I/3)*(a - I*a*Tan[c + d*x])^3)/(a^5*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
```

```
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)^2\,dx, x, ia\tan(c+dx)\right)}{a^5 d} \\ &= \frac{i(a-ia\tan(c+dx))^3}{3a^5 d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.85

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\tan(c+dx)}{a^2d} - \frac{i\tan^2(c+dx)}{a^2d} - \frac{\tan^3(c+dx)}{3a^2d}$$

```
[In] Integrate[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^2,x]
[Out] Tan[c + d*x]/(a^2*d) - (I*Tan[c + d*x]^2)/(a^2*d) - Tan[c + d*x]^3/(3*a^2*d
)
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$-\frac{(\tan(dx+c)+i)^3}{3a^2d}$	20
default	$-\frac{(\tan(dx+c)+i)^3}{3a^2d}$	20
risch	$\frac{8i}{3d\,a^2\left(e^{2i(dx+c)}+1\right)^3}$	23

```
[In] int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] -1/3/a^2/d*(tan(d*x+c)+I)^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(21) = 42.

Time = 0.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.00

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{8i}{3\left(a^2de^{(6i\,dx+6i\,c)}+3\,a^2de^{(4i\,dx+4i\,c)}+3\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 8/3*I/(a^2*d*e^(6*I*d*x + 6*I*c) + 3*a^2*d*e^(4*I*d*x + 4*I*c) + 3*a^2*d*e^
(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F]

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\sec^6(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

```
[In] integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -Integral(sec(c + d*x)**6/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\tan(dx+c)^3+3i\,\tan(dx+c)^2-3\,\tan(dx+c)}{3\,a^2d}$$

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(tan(d*x + c)^3 + 3*I*tan(d*x + c)^2 - 3*tan(d*x + c))/(a^2*d)
```

Giac [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\tan\left(dx+c\right)^3+3i\,\tan\left(dx+c\right)^2-3\,\tan\left(dx+c\right)}{3\,a^2d}$$

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
[Out] -1/3*(tan(d*x + c)^3 + 3*I*tan(d*x + c)^2 - 3*tan(d*x + c))/(a^2*d)
```

Mupad [B] (verification not implemented)

Time = 4.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\tan(c+d\,x)\,\left(\tan(c+d\,x)^2+\tan(c+d\,x)\,3\mathrm{i}-3\right)}{3\,a^2\,d}$$

```
[In] int(1/(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^2),x)
[Out] -(tan(c + d*x)*(tan(c + d*x)*3i + tan(c + d*x)^2 - 3))/(3*a^2*d)
```

3.117 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 38

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^2} dx = \frac{2x}{a^2} + \frac{2i\log(\cos(c+dx))}{a^2d} - \frac{\tan(c+dx)}{a^2d}$$

```
[Out] 2*x/a^2+2*I*ln(cos(d*x+c))/a^2/d-tan(d*x+c)/a^2/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^2} dx = -\frac{\tan(c+dx)}{a^2d} + \frac{2i\log(\cos(c+dx))}{a^2d} + \frac{2x}{a^2}$$

```
[In] Int[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (2*x)/a^2 + ((2*I)*Log[Cos[c + d*x]])/(a^2*d) - Tan[c + d*x]/(a^2*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
```

```
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int \frac{a-x}{a+x}\,dx, x, ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{i\text{Subst}\left(\int \left(-1+\frac{2a}{a+x}\right)\,dx, x, ia\tan(c+dx)\right)}{a^3d}\\ &= \frac{2x}{a^2}+\frac{2i\log(\cos(c+dx))}{a^2d}-\frac{\tan(c+dx)}{a^2d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{2i\log(i-\tan(c+dx))}{a^2d}-\frac{\tan(c+dx)}{a^2d}$$

```
[In] Integrate[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^2,x]
[Out] ((-2*I)*Log[I - Tan[c + d*x]])/(a^2*d) - Tan[c + d*x]/(a^2*d)
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{-\tan(dx+c)-2i\ln(\tan(dx+c)-i)}{d\,a^2}$	30
default	$\frac{-\tan(dx+c)-2i\ln(\tan(dx+c)-i)}{d\,a^2}$	30
risch	$\frac{4x}{a^2}+\frac{4c}{a^2d}-\frac{2i}{d\,a^2\left(e^{2i(dx+c)}+1\right)}+\frac{2i\ln\left(e^{2i(dx+c)}+1\right)}{a^2d}$	60

```
[In] int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d/a^2*(-tan(d*x+c)-2*I*ln(tan(d*x+c)-I))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.84

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{2\left(2\,dxe^{(2i\,dx+2i\,c)}+2\,dx-\left(-i\,e^{(2i\,dx+2i\,c)}-i\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)-i\right)}{a^2de^{(2i\,dx+2i\,c)}+a^2d}$$

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
[Out] 2*(2*d*x*e^(2*I*d*x + 2*I*c) + 2*d*x - (-I*e^(2*I*d*x + 2*I*c) - I)*log(e^(
2*I*d*x + 2*I*c) + 1) - I)/(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\sec^4(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

```
[In] integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c))**2,x)
[Out] -Integral(sec(c + d*x)**4/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)/a**2
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{-\frac{2i\,\log(i\,\tan(dx+c)+1)}{a^2}-\frac{\tan(dx+c)}{a^2}}{d}$$

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
[Out] (-2*I*log(I*tan(d*x + c) + 1)/a^2 - tan(d*x + c)/a^2)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(36) = 72$.

Time = 0.47 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.63

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{2\left(\frac{i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^2} - \frac{2i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)}{a^2} + \frac{i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^2} + \frac{-i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^2}\right)}{d}$$

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*(I*log(tan(1/2*d*x + 1/2*c) + 1)/a^2 - 2*I*log(tan(1/2*d*x + 1/2*c) - I)/
a^2 + I*log(tan(1/2*d*x + 1/2*c) - 1)/a^2 + (-I*tan(1/2*d*x + 1/2*c)^2 + ta
n(1/2*d*x + 1/2*c) + I)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a^2))/d
```

Mupad [B] (verification not implemented)

Time = 3.75 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\tan(c+d\,x)+\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,2\mathrm{i}}{a^2\,d}$$

```
[In] int(1/(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^2),x)
```

```
[Out] -(log(tan(c + d*x) - 1i)*2i + tan(c + d*x))/(a^2*d)
```

3.118 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 26

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{i}{d\,(a^2+ia^2\tan(c+dx))}$$

```
[Out] I/d/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 32}

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{i}{d\,(a^2+ia^2\tan(c+dx))}$$

```
[In] Int[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] I/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
```

```
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int \frac{1}{(a+x)^2}\,dx, x, ia\tan(c+dx)\right)}{ad}\\ &= \frac{i}{d\,(a^2+ia^2\tan(c+dx))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{1}{a^2d(-i+\tan(c+dx))}$$

```
[In] Integrate[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^2,x]
[Out] 1/(a^2*d*(-I + Tan[c + d*x]))
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{1}{a^2d(\tan(dx+c)-i)}$	19
default	$\frac{1}{a^2d(\tan(dx+c)-i)}$	19
risch	$\frac{i\mathrm{e}^{-2i(dx+c)}}{2a^2d}$	19

```
[In] int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/a^2/d/(tan(d*x+c)-I)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{i\,e^{(-2i\,dx-2i\,c)}}{2\,a^2d}$$

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
[Out] 1/2*I*e^(-2*I*d*x - 2*I*c)/(a^2*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(17) = 34.

Time = 0.51 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.50

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \begin{cases} -\frac{i\sec^2(c+dx)}{2a^2d\tan^2(c+dx)-4ia^2d\tan(c+dx)-2a^2d} & \text{for } d\neq 0 \\ \frac{x\sec^2(c)}{(ia\tan(c)+a)^2} & \text{otherwise} \end{cases}$$

```
[In] integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c))**2,x)
[Out] Piecewise((-I*sec(c + d*x)**2/(2*a**2*d*tan(c + d*x)**2 - 4*I*a**2*d*tan(c
+ d*x) - 2*a**2*d), Ne(d, 0)), (x*sec(c)**2/(I*a*tan(c) + a)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{i}{(i\,a\tan(dx+c)+a)ad}$$

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
[Out] I/((I*a*tan(d*x + c) + a)*a*d)
```

Giac [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^2}$$

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
[Out] -2*tan(1/2*d*x + 1/2*c)/(a^2*d*(tan(1/2*d*x + 1/2*c) - I)^2)
```

Mupad [B] (verification not implemented)

Time = 3.89 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{1\mathrm{i}}{a^2\,d\,\left(1+\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}$$

```
[In] int(1/(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^2),x)
[Out] 1i/(a^2*d*(tan(c + d*x)*1i + 1))
```

3.119 $\int \frac{1}{(a+ia\tan(c+dx))^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 61

$$\int \frac{1}{(a+ia\tan(c+dx))^2} dx = \frac{x}{4a^2} + \frac{i}{4d(a+ia\tan(c+dx))^2} + \frac{i}{4d(a^2+ia^2\tan(c+dx))}$$

```
[Out] 1/4*x/a^2+1/4*I/d/(a+I*a*tan(d*x+c))^2+1/4*I/d/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3560, 8}

$$\int \frac{1}{(a+ia\tan(c+dx))^2} dx = \frac{i}{4d(a^2+ia^2\tan(c+dx))} + \frac{x}{4a^2} + \frac{i}{4d(a+ia\tan(c+dx))^2}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^(-2),x]
```

```
[Out] x/(4*a^2) + (I/4)/(d*(a + I*a*Tan[c + d*x])^2) + (I/4)/(d*(a^2 + I*a^2*Tan[
c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{i}{4d(a+ia\tan(c+dx))^2}+\frac{\int \frac{1}{a+ia\tan(c+dx)}\,dx}{2a}\\
&= \frac{i}{4d(a+ia\tan(c+dx))^2}+\frac{i}{4d\,(a^2+ia^2\tan(c+dx))}+\frac{\int 1\,dx}{4a^2}\\
&= \frac{x}{4a^2}+\frac{i}{4d(a+ia\tan(c+dx))^2}+\frac{i}{4d\,(a^2+ia^2\tan(c+dx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.85

$$
\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx=\frac{-2i+\tan(c+dx)+\arctan(\tan(c+dx))(-i+\tan(c+dx))^2}{4a^2d(-i+\tan(c+dx))^2}
$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(-2),x]
```

```
[Out] (-2*I + Tan[c + d*x] + ArcTan[Tan[c + d*x]]*(-I + Tan[c + d*x])^2)/(4*a^2*d
*(-I + Tan[c + d*x])^2)
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

method	result	size
risch	$\frac{x}{4a^2}+\frac{i\mathrm{e}^{-2i(dx+c)}}{4a^2d}+\frac{i\mathrm{e}^{-4i(dx+c)}}{16a^2d}$	44
derivativedivides	$\frac{\arctan(\tan(dx+c))}{4d\,a^2}-\frac{i}{4d\,a^2(\tan(dx+c)-i)^2}+\frac{1}{4a^2d(\tan(dx+c)-i)}$	56
default	$\frac{\arctan(\tan(dx+c))}{4d\,a^2}-\frac{i}{4d\,a^2(\tan(dx+c)-i)^2}+\frac{1}{4a^2d(\tan(dx+c)-i)}$	56
norman	$\frac{\frac{x}{4a}+\frac{\tan^3(dx+c)}{4ad}+\frac{x\left(\tan^2(dx+c)\right)}{2a}+\frac{x\left(\tan^4(dx+c)\right)}{4a}+\frac{i}{2ad}+\frac{3\tan(dx+c)}{4ad}}{a\left(1+\tan^2(dx+c)\right)^2}$	91

```
[In] int(1/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x/a^2+1/4*I/a^2/d*exp(-2*I*(d*x+c))+1/16*I/a^2/d*exp(-4*I*(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.70

$$\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx = \frac{\left(4\,dxe^{(4i\,dx+4i\,c)} + 4i\,e^{(2i\,dx+2i\,c)} + i\right)e^{(-4i\,dx-4i\,c)}}{16\,a^2d}$$

```
[In] integrate(1/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*d*x*e^(4*I*d*x + 4*I*c) + 4*I*e^(2*I*d*x + 2*I*c) + I)*e^(-4*I*d*x
 - 4*I*c)/(a^2*d)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.92

$$\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx = \begin{cases} \frac{(16ia^2de^{4ic}e^{-2idx}+4ia^2de^{2ic}e^{-4idx})e^{-6ic}}{64a^4d^2} & \text{for } a^4d^2e^{6ic} \neq 0 \\ x\left(\frac{(e^{4ic}+2e^{2ic}+1)e^{-4ic}}{4a^2} - \frac{1}{4a^2}\right) & \text{otherwise} \end{cases} + \frac{x}{4a^2}$$

```
[In] integrate(1/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise((((16*I*a**2*d*exp(4*I*c)*exp(-2*I*d*x) + 4*I*a**2*d*exp(2*I*c)*ex
p(-4*I*d*x))*exp(-6*I*c)/(64*a**4*d**2), Ne(a**4*d**2*exp(6*I*c), 0)), (x*(
(exp(4*I*c) + 2*exp(2*I*c) + 1)*exp(-4*I*c)/(4*a**2) - 1/(4*a**2)), True))
+ x/(4*a**2)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.11

$$\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx = -\frac{-\frac{2i\log(\tan(dx+c)+i)}{a^2}+\frac{2i\log(\tan(dx+c)-i)}{a^2}+\frac{-3i\tan(dx+c)^2-10\tan(dx+c)+11i}{a^2(\tan(dx+c)-i)^2}}{16\,d}$$

```
[In] integrate(1/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
[Out] -1/16*(-2*I*log(tan(d*x + c) + I)/a^2 + 2*I*log(tan(d*x + c) - I)/a^2 + (-3
*I*tan(d*x + c)^2 - 10*tan(d*x + c) + 11*I)/(a^2*(tan(d*x + c) - I)^2))/d
```

Mupad [B] (verification not implemented)

Time = 4.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.64

$$\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx = \frac{x}{4\,a^2} - \frac{\frac{\tan(c+d\,x)}{4}-\frac{1}{2}\mathrm{i}}{a^2\,d\,{\left(1+\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}^2}$$

```
[In] int(1/(a + a*tan(c + d*x)*1i)^2,x)
[Out] x/(4*a^2) - (tan(c + d*x)/4 - 1i/2)/(a^2*d*(tan(c + d*x)*1i + 1)^2)
```

3.120 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 114

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{x}{4a^2}+\frac{ia}{12d(a+ia\tan(c+dx))^3}+\frac{i}{8d(a+ia\tan(c+dx))^2} - \frac{i}{16d\,(a^2-ia^2\tan(c+dx))}+\frac{3i}{16d\,(a^2+ia^2\tan(c+dx))}$$

```
[Out] 1/4*x/a^2+1/12*I*a/d/(a+I*a*tan(d*x+c))^3+1/8*I/d/(a+I*a*tan(d*x+c))^2-1/16
*I/d/(a^2-I*a^2*tan(d*x+c))+3/16*I/d/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3568, 46, 212}

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{i}{16d\,(a^2-ia^2\tan(c+dx))}+\frac{3i}{16d\,(a^2+ia^2\tan(c+dx))} + \frac{x}{4a^2}+\frac{ia}{12d(a+ia\tan(c+dx))^3}+\frac{i}{8d(a+ia\tan(c+dx))^2}$$

```
[In] Int[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] x/(4*a^2) + ((I/12)*a)/(d*(a + I*a*Tan[c + d*x])^3) + (I/8)/(d*(a + I*a*Tan
[c + d*x])^2) - (I/16)/(d*(a^2 - I*a^2*Tan[c + d*x])) + ((3*I)/16)/(d*(a^2
+ I*a^2*Tan[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(ia^3)\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^4}\,dx, x, ia\tan(c+dx)\right)}{d} \\
&= -\frac{(ia^3)\,\text{Subst}\left(\int \left(\frac{1}{16a^4(a-x)^2} + \frac{1}{4a^2(a+x)^4} + \frac{1}{4a^3(a+x)^3} + \frac{3}{16a^4(a+x)^2} + \frac{1}{4a^4(a^2-x^2)}\right) dx, x, ia\tan(c+dx)\right)}{d} \\
&= \frac{ia}{12d(a+ia\tan(c+dx))^3} + \frac{i}{8d(a+ia\tan(c+dx))^2} - \frac{i}{16d\,(a^2-ia^2\tan(c+dx))} \\
&\quad + \frac{3i}{16d\,(a^2+ia^2\tan(c+dx))} - \frac{i\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, ia\tan(c+dx)\right)}{4ad} \\
&= \frac{x}{4a^2} + \frac{ia}{12d(a+ia\tan(c+dx))^3} + \frac{i}{8d(a+ia\tan(c+dx))^2} \\
&\quad - \frac{i}{16d\,(a^2-ia^2\tan(c+dx))} + \frac{3i}{16d\,(a^2+ia^2\tan(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.85

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{4i+\tan(c+dx)+6i\tan^2(c+dx)-3\tan^3(c+dx)-3\arctan(\tan(c+dx))(-i+\tan(c+dx))^3(i+\tan(c+dx))}{12a^2d(-i+\tan(c+dx))^3(i+\tan(c+dx))}$$

```
[In] Integrate[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -1/12*(4*I + Tan[c + d*x] + (6*I)*Tan[c + d*x]^2 - 3*Tan[c + d*x]^3 - 3*Arc
Tan[Tan[c + d*x]]*(-I + Tan[c + d*x])^3*(I + Tan[c + d*x]))/(a^2*d*(-I + Ta
n[c + d*x])^3*(I + Tan[c + d*x]))
```

Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{x}{4a^2}+\frac{i e^{-4i(dx+c)}}{16a^2d}+\frac{i e^{-6i(dx+c)}}{96a^2d}+\frac{5i\cos(2dx+2c)}{32a^2d}+\frac{7\sin(2dx+2c)}{32a^2d}$	79
derivativedivides	$\frac{\frac{i\ln(\tan(dx+c)+i)}{8}+\frac{1}{16\tan(dx+c)+16i}-\frac{i\ln(\tan(dx+c)-i)}{8}-\frac{i}{8(\tan(dx+c)-i)^2}-\frac{1}{12(\tan(dx+c)-i)^3}+\frac{3}{16(\tan(dx+c)-i)}}{d\,a^2}$	88
default	$\frac{\frac{i\ln(\tan(dx+c)+i)}{8}+\frac{1}{16\tan(dx+c)+16i}-\frac{i\ln(\tan(dx+c)-i)}{8}-\frac{i}{8(\tan(dx+c)-i)^2}-\frac{1}{12(\tan(dx+c)-i)^3}+\frac{3}{16(\tan(dx+c)-i)}}{d\,a^2}$	88

```
[In] int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x/a^2+1/16*I/a^2/d*exp(-4*I*(d*x+c))+1/96*I/a^2/d*exp(-6*I*(d*x+c))+5/3
2*I/a^2/d*cos(2*d*x+2*c)+7/32/a^2/d*sin(2*d*x+2*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.57

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\left(24\,dxe^{(6i\,dx+6i\,c)}-3i\,e^{(8i\,dx+8i\,c)}+18i\,e^{(4i\,dx+4i\,c)}+6i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-6i\,dx-6i\,c)}}{96\,a^2d}$$

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/96*(24*d*x*e^(6*I*d*x + 6*I*c) - 3*I*e^(8*I*d*x + 8*I*c) + 18*I*e^(4*I*d*
x + 4*I*c) + 6*I*e^(2*I*d*x + 2*I*c) + I)*e^(-6*I*d*x - 6*I*c)/(a^2*d)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.66

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \begin{cases} \frac{(-24576ia^6d^3e^{14ic}e^{2idx}+147456ia^6d^3e^{10ic}e^{-2idx}+49152ia^6d^3e^{8ic}e^{-4idx}+8192ia^6d^3e^{6ic}e^{-6idx})e^{-12ic}}{786432a^8d^4} & \text{for } a^8d^4e^{12ic}\neq 0 \\ x\left(\frac{(e^{8ic}+4e^{6ic}+6e^{4ic}+4e^{2ic}+1)e^{-6ic}}{16a^2}-\frac{1}{4a^2}\right) & \text{otherwise} \end{cases} + \frac{x}{4a^2}$$

```
[In] integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise(((-24576*I*a**6*d**3*exp(14*I*c)*exp(2*I*d*x) + 147456*I*a**6*d**
3*exp(10*I*c)*exp(-2*I*d*x) + 49152*I*a**6*d**3*exp(8*I*c)*exp(-4*I*d*x) +
8192*I*a**6*d**3*exp(6*I*c)*exp(-6*I*d*x))*exp(-12*I*c)/(786432*a**8*d**4),
 Ne(a**8*d**4*exp(12*I*c), 0)), (x*((exp(8*I*c) + 4*exp(6*I*c) + 6*exp(4*I*
c) + 4*exp(2*I*c) + 1)*exp(-6*I*c)/(16*a**2) - 1/(4*a**2)), True)) + x/(4*a
**2)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [A] (verification not implemented)

none

Time = 0.53 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{-\frac{6i\log(\tan(dx+c)+i)}{a^2}+\frac{6i\log(\tan(dx+c)-i)}{a^2}+\frac{3(2i\tan(dx+c)-3)}{a^2(\tan(dx+c)+i)}+\frac{-11i\tan(dx+c)^3-42\tan(dx+c)^2+57i\tan(dx+c)+30}{a^2(\tan(dx+c)-i)^3}}{48\,d}$$

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/48*(-6*I*log(tan(d*x + c) + I)/a^2 + 6*I*log(tan(d*x + c) - I)/a^2 + 3*(
2*I*tan(d*x + c) - 3)/(a^2*(tan(d*x + c) + I)) + (-11*I*tan(d*x + c)^3 - 42
*tan(d*x + c)^2 + 57*I*tan(d*x + c) + 30)/(a^2*(tan(d*x + c) - I)^3))/d
```

Mupad [B] (verification not implemented)

Time = 3.94 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.62

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{x}{4\,a^2} - \frac{\frac{\tan(c+d\,x)^3\,1\mathrm{i}}{4} + \frac{\tan(c+d\,x)^2}{2} - \frac{\tan(c+d\,x)\,1\mathrm{i}}{12} + \frac{1}{3}}{a^2\,d\,(1+\tan\left(c+d\,x\right)\,1\mathrm{i})^3\,\left(\tan\left(c+d\,x\right)+1\mathrm{i}\right)}$$

```
[In] int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] x/(4*a^2) - (tan(c + d*x)^2/2 - (tan(c + d*x)*1i)/12 + (tan(c + d*x)^3*1i)/
4 + 1/3)/(a^2*d*(tan(c + d*x)*1i + 1)^3*(tan(c + d*x) + 1i))
```

3.121 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 165

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^2} dx = \frac{15x}{64a^2} - \frac{i}{64d(a-ia\tan(c+dx))^2} + \frac{ia^2}{32d(a+ia\tan(c+dx))^4} + \frac{ia}{16d(a+ia\tan(c+dx))^3} + \frac{3i}{32d(a+ia\tan(c+dx))^2} - \frac{5i}{64d(a^2-ia^2\tan(c+dx))} + \frac{5i}{32d(a^2+ia^2\tan(c+dx))}$$

```
[Out] 15/64*x/a^2-1/64*I/d/(a-I*a*tan(d*x+c))^2+1/32*I*a^2/d/(a+I*a*tan(d*x+c))^4
+1/16*I*a/d/(a+I*a*tan(d*x+c))^3+3/32*I/d/(a+I*a*tan(d*x+c))^2-5/64*I/d/(a^
2-I*a^2*tan(d*x+c))+5/32*I/d/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3568, 46, 212}

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^2} dx = \frac{ia^2}{32d(a+ia\tan(c+dx))^4} - \frac{5i}{64d(a^2-ia^2\tan(c+dx))} + \frac{5i}{32d(a^2+ia^2\tan(c+dx))} + \frac{15x}{64a^2} + \frac{ia}{16d(a+ia\tan(c+dx))^3} - \frac{i}{64d(a-ia\tan(c+dx))^2} + \frac{3i}{32d(a+ia\tan(c+dx))^2}$$

```
[In] Int[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (15*x)/(64*a^2) - (I/64)/(d*(a - I*a*Tan[c + d*x])^2) + ((I/32)*a^2)/(d*(a
+ I*a*Tan[c + d*x])^4) + ((I/16)*a)/(d*(a + I*a*Tan[c + d*x])^3) + ((3*I)/3
2)/(d*(a + I*a*Tan[c + d*x])^2) - ((5*I)/64)/(d*(a^2 - I*a^2*Tan[c + d*x]))
 + ((5*I)/32)/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(ia^5)\,\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^5}\,dx, x, ia\tan(c+dx)\right)}{d} \\
&= -\frac{(ia^5)\,\text{Subst}\left(\int \left(\frac{1}{32a^5(a-x)^3}+\frac{5}{64a^6(a-x)^2}+\frac{1}{8a^3(a+x)^5}+\frac{3}{16a^4(a+x)^4}+\frac{3}{16a^5(a+x)^3}+\frac{5}{32a^6(a+x)^2}+\frac{15}{64a^6(a^2-x^2)}\right.\right.}{d} \\
&= -\frac{i}{64d(a-ia\tan(c+dx))^2}+\frac{ia^2}{32d(a+ia\tan(c+dx))^4}+\frac{ia}{16d(a+ia\tan(c+dx))^3} \\
&\quad+\frac{3i}{32d(a+ia\tan(c+dx))^2}-\frac{5i}{64d\,(a^2-ia^2\tan(c+dx))} \\
&\quad+\frac{5i}{32d\,(a^2+ia^2\tan(c+dx))}-\frac{(15i)\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, ia\tan(c+dx)\right)}{64ad} \\
&= \frac{15x}{64a^2}-\frac{i}{64d(a-ia\tan(c+dx))^2}+\frac{ia^2}{32d(a+ia\tan(c+dx))^4}+\frac{ia}{16d(a+ia\tan(c+dx))^3} \\
&\quad+\frac{3i}{32d(a+ia\tan(c+dx))^2}-\frac{5i}{64d\,(a^2-ia^2\tan(c+dx))}+\frac{5i}{32d\,(a^2+ia^2\tan(c+dx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.86

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{i\sec^6(c+dx)(-80-65\cos(2(c+dx))+16\cos(4(c+dx))+\cos(6(c+dx))+120i\arctan(\tan(c+dx))(}{512a^2d(-i+\tan(c+dx))^4(}$$

```
[In] Integrate[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((I/512)*Sec[c + d*x]^6*(-80 - 65*Cos[2*(c + d*x)] + 16*Cos[4*(c + d*x)] +
Cos[6*(c + d*x)] + (120*I)*ArcTan[Tan[c + d*x]]*(Cos[2*(c + d*x)] + I*Sin[2
*(c + d*x)]) - (5*I)*Sin[2*(c + d*x)] + (32*I)*Sin[4*(c + d*x)] + (3*I)*Sin
[6*(c + d*x)]))/(a^2*d*(-I + Tan[c + d*x])^4*(I + Tan[c + d*x])^2)
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.69

method	result
risch	$\frac{15x}{64a^2}+\frac{i\mathrm{e}^{-6i(dx+c)}}{64a^2d}+\frac{i\mathrm{e}^{-8i(dx+c)}}{512a^2d}+\frac{7i\cos(4dx+4c)}{128a^2d}+\frac{\sin(4dx+4c)}{16a^2d}+\frac{7i\cos(2dx+2c)}{64a^2d}+\frac{13\sin(2dx+2c)}{64a^2d}$
derivativedivides	$\frac{-\frac{15i\ln(\tan(dx+c)-i)}{128}+\frac{i}{32(\tan(dx+c)-i)^4}-\frac{3i}{32(\tan(dx+c)-i)^2}-\frac{1}{16(\tan(dx+c)-i)^3}+\frac{5}{32(\tan(dx+c)-i)}+\frac{i}{64(\tan(dx+c)+i)^2}+\frac{15i}{}}{d\,a^2}$
default	$\frac{-\frac{15i\ln(\tan(dx+c)-i)}{128}+\frac{i}{32(\tan(dx+c)-i)^4}-\frac{3i}{32(\tan(dx+c)-i)^2}-\frac{1}{16(\tan(dx+c)-i)^3}+\frac{5}{32(\tan(dx+c)-i)}+\frac{i}{64(\tan(dx+c)+i)^2}+\frac{15i}{}}{d\,a^2}$

```
[In] int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 15/64*x/a^2+1/64*I/a^2/d*exp(-6*I*(d*x+c))+1/512*I/a^2/d*exp(-8*I*(d*x+c))+
7/128*I/a^2/d*cos(4*d*x+4*c)+1/16/a^2/d*sin(4*d*x+4*c)+7/64*I/a^2/d*cos(2*d
*x+2*c)+13/64/a^2/d*sin(2*d*x+2*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.53

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\left(120\,dxe^{(8i\,dx+8i\,c)}-2i\,e^{(12i\,dx+12i\,c)}-24i\,e^{(10i\,dx+10i\,c)}+80i\,e^{(6i\,dx+6i\,c)}+30i\,e^{(4i\,dx+4i\,c)}+8i\,e^{(2i\,dx+2i\,c)}+i\right)}{512\,a^2d}$$

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/512*(120*d*x*e^(8*I*d*x + 8*I*c) - 2*I*e^(12*I*d*x + 12*I*c) - 24*I*e^(10
*I*d*x + 10*I*c) + 80*I*e^(6*I*d*x + 6*I*c) + 30*I*e^(4*I*d*x + 4*I*c) + 8*
I*e^(2*I*d*x + 2*I*c) + I)*e^(-8*I*d*x - 8*I*c)/(a^2*d)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.56

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \begin{cases} \frac{(-17179869184ia^{10}d^5e^{24ic}e^{4idx}-206158430208ia^{10}d^5e^{22ic}e^{2idx}+687194767360ia^{10}d^5e^{18ic}e^{-2idx}+257698037760ia^{10}d^5e^{16ic}e^{-4idx}+6871947}{4398046511104a^{12}d^6} \\ x\left(\frac{(e^{12ic}+6e^{10ic}+15e^{8ic}+20e^{6ic}+15e^{4ic}+6e^{2ic}+1)e^{-8ic}}{64a^2}-\frac{15}{64a^2}\right) \end{cases} + \frac{15x}{64a^2}$$

```
[In] integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise(((-17179869184*I*a**10*d**5*exp(24*I*c)*exp(4*I*d*x) - 2061584302
08*I*a**10*d**5*exp(22*I*c)*exp(2*I*d*x) + 687194767360*I*a**10*d**5*exp(18
*I*c)*exp(-2*I*d*x) + 257698037760*I*a**10*d**5*exp(16*I*c)*exp(-4*I*d*x) +
 68719476736*I*a**10*d**5*exp(14*I*c)*exp(-6*I*d*x) + 8589934592*I*a**10*d*
*5*exp(12*I*c)*exp(-8*I*d*x))*exp(-20*I*c)/(4398046511104*a**12*d**6), Ne(a
**12*d**6*exp(20*I*c), 0)), (x*((exp(12*I*c) + 6*exp(10*I*c) + 15*exp(8*I*c
) + 20*exp(6*I*c) + 15*exp(4*I*c) + 6*exp(2*I*c) + 1)*exp(-8*I*c)/(64*a**2)
 - 15/(64*a**2)), True)) + 15*x/(64*a**2)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [A] (verification not implemented)

none

Time = 0.51 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.75

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx =$$

$$-\frac{-\frac{60i\,\log(\tan(dx+c)+i)}{a^2}+\frac{60i\,\log(\tan(dx+c)-i)}{a^2}+\frac{2\left(45i\,\tan(dx+c)^2-110\,\tan(dx+c)-69i\right)}{a^2(\tan(dx+c)+i)^2}+\frac{-125i\,\tan(dx+c)^4-580\,\tan(dx+c)^3+}{a^2(\tan(}}{512\,d}$$

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/512*(-60*I*log(tan(d*x + c) + I)/a^2 + 60*I*log(tan(d*x + c) - I)/a^2 +
2*(45*I*tan(d*x + c)^2 - 110*tan(d*x + c) - 69*I)/(a^2*(tan(d*x + c) + I)^2
) + (-125*I*tan(d*x + c)^4 - 580*tan(d*x + c)^3 + 1038*I*tan(d*x + c)^2 + 8
68*tan(d*x + c) - 301*I)/(a^2*(tan(d*x + c) - I)^4))/d
```

Mupad [B] (verification not implemented)

Time = 5.25 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.90

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{15\,x}{64\,a^2}$$

$$+\frac{\frac{1}{4\,a^2}-\frac{\tan(c+d\,x)\,17\mathrm{i}}{64\,a^2}+\frac{25\tan(c+d\,x)^2}{32\,a^2}+\frac{\tan(c+d\,x)^3\,5\mathrm{i}}{32\,a^2}+\frac{15\tan(c+d\,x)^4}{32\,a^2}+\frac{\tan(c+d\,x)^5\,15\mathrm{i}}{64\,a^2}}{d\,\left(\tan\left(c+d\,x\right)^6\,1\mathrm{i}+2\tan\left(c+d\,x\right)^5+\tan\left(c+d\,x\right)^4\,1\mathrm{i}+4\tan\left(c+d\,x\right)^3-\tan\left(c+d\,x\right)^2\,1\mathrm{i}+2\tan\left(c\right.\right.}$$

```
[In] int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] (15*x)/(64*a^2) + (1/(4*a^2) - (tan(c + d*x)*17i)/(64*a^2) + (25*tan(c + d*
x)^2)/(32*a^2) + (tan(c + d*x)^3*5i)/(32*a^2) + (15*tan(c + d*x)^4)/(32*a^2
) + (tan(c + d*x)^5*15i)/(64*a^2))/(d*(2*tan(c + d*x) - tan(c + d*x)^2*1i +
 4*tan(c + d*x)^3 + tan(c + d*x)^4*1i + 2*tan(c + d*x)^5 + tan(c + d*x)^6*1
i - 1i))
```

3.122 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 124

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{7\text{arctanh}(\sin(c+dx))}{16a^2d} + \frac{7\sec(c+dx)\tan(c+dx)}{16a^2d} + \frac{7\sec^3(c+dx)\tan(c+dx)}{24a^2d} + \frac{7\sec^5(c+dx)\tan(c+dx)}{30a^2d} - \frac{2i\sec^7(c+dx)}{5d\,(a^2+ia^2\tan(c+dx))}$$

```
[Out] 7/16*arctanh(sin(d*x+c))/a^2/d+7/16*sec(d*x+c)*tan(d*x+c)/a^2/d+7/24*sec(d*
x+c)^3*tan(d*x+c)/a^2/d+7/30*sec(d*x+c)^5*tan(d*x+c)/a^2/d-2/5*I*sec(d*x+c)
^7/d/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3581, 3853, 3855}

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{7\text{arctanh}(\sin(c+dx))}{16a^2d} - \frac{2i\sec^7(c+dx)}{5d\,(a^2+ia^2\tan(c+dx))} + \frac{7\tan(c+dx)\sec^5(c+dx)}{30a^2d} + \frac{7\tan(c+dx)\sec^3(c+dx)}{24a^2d} + \frac{7\tan(c+dx)\sec(c+dx)}{16a^2d}$$

```
[In] Int[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (7*ArcTanh[Sin[c + d*x]])/(16*a^2*d) + (7*Sec[c + d*x]*Tan[c + d*x])/(16*a^
2*d) + (7*Sec[c + d*x]^3*Tan[c + d*x])/(24*a^2*d) + (7*Sec[c + d*x]^5*Tan[c
 + d*x])/(30*a^2*d) - (((2*I)/5)*Sec[c + d*x]^7)/(d*(a^2 + I*a^2*Tan[c + d*
x]))
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2i\sec^7(c+dx)}{5d\,(a^2+ia^2\tan(c+dx))}+\frac{7\int \sec^7(c+dx)\,dx}{5a^2}\\
&= \frac{7\sec^5(c+dx)\tan(c+dx)}{30a^2d}-\frac{2i\sec^7(c+dx)}{5d\,(a^2+ia^2\tan(c+dx))}+\frac{7\int \sec^5(c+dx)\,dx}{6a^2}\\
&= \frac{7\sec^3(c+dx)\tan(c+dx)}{24a^2d}+\frac{7\sec^5(c+dx)\tan(c+dx)}{30a^2d}\\
&\quad -\frac{2i\sec^7(c+dx)}{5d\,(a^2+ia^2\tan(c+dx))}+\frac{7\int \sec^3(c+dx)\,dx}{8a^2}\\
&= \frac{7\sec(c+dx)\tan(c+dx)}{16a^2d}+\frac{7\sec^3(c+dx)\tan(c+dx)}{24a^2d}\\
&\quad +\frac{7\sec^5(c+dx)\tan(c+dx)}{30a^2d}-\frac{2i\sec^7(c+dx)}{5d\,(a^2+ia^2\tan(c+dx))}+\frac{7\int \sec(c+dx)\,dx}{16a^2}\\
&= \frac{7\text{arctanh}(\sin(c+dx))}{16a^2d}+\frac{7\sec(c+dx)\tan(c+dx)}{16a^2d}+\frac{7\sec^3(c+dx)\tan(c+dx)}{24a^2d}\\
&\quad +\frac{7\sec^5(c+dx)\tan(c+dx)}{30a^2d}-\frac{2i\sec^7(c+dx)}{5d\,(a^2+ia^2\tan(c+dx))}
\end{aligned}
$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 294 vs. 2(124) = 248.

Time = 2.42 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.37

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\sec^6(c+dx)\left(3072i\cos(c+dx)+5\left(210\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+21\cos(6(c+dx))\log\right.\right.}{}$$

```
[In] Integrate[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -1/7680*(Sec[c + d*x]^6*((3072*I)*Cos[c + d*x] + 5*(210*Log[Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]] + 21*Cos[6*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2]] + 315*Cos[2*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
- Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 126*Cos[4*(c + d*x)]*(Log[Cos
[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
]) - 210*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 21*Cos[6*(c + d*x)]*Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 60*Sin[c + d*x] - 238*Sin[3*(c + d*
x)] - 42*Sin[5*(c + d*x)])))/(a^2*d)
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.07

method	result
risch	$-\frac{i\left(105\,e^{11i(dx+c)}+595\,e^{9i(dx+c)}+1386\,e^{7i(dx+c)}+1686\,e^{5i(dx+c)}-595\,e^{3i(dx+c)}-105\,e^{i(dx+c)}\right)}{120d\,a^2\left(e^{2i(dx+c)}+1\right)^6}-\frac{7\ln\left(e^{i(dx+c)}-i\right)}{16a^2d}+$
derivativedivides	$\frac{2\left(-\frac{1}{4}+\frac{i}{2}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}+\frac{2\left(\frac{9}{32}+\frac{5i}{8}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{2\left(\frac{9}{32}+\frac{3i}{8}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2\left(-\frac{1}{12}+\frac{3i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}+\frac{2\left(-\frac{1}{4}+\frac{i}{5}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^6}-\frac{7\ln}{}$
default	$\frac{2\left(-\frac{1}{4}+\frac{i}{2}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}+\frac{2\left(\frac{9}{32}+\frac{5i}{8}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{2\left(\frac{9}{32}+\frac{3i}{8}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2\left(-\frac{1}{12}+\frac{3i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}+\frac{2\left(-\frac{1}{4}+\frac{i}{5}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^6}-\frac{7\ln}{}$

```
[In] int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/120*I/d/a^2/(exp(2*I*(d*x+c))+1)^6*(105*exp(11*I*(d*x+c))+595*exp(9*I*(d
*x+c))+1386*exp(7*I*(d*x+c))+1686*exp(5*I*(d*x+c))-595*exp(3*I*(d*x+c))-105
*exp(I*(d*x+c)))-7/16/a^2/d*ln(exp(I*(d*x+c))-I)+7/16/a^2/d*ln(exp(I*(d*x+c
))+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(110) = 220.

Time = 0.24 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.63

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{105\left(e^{(12i\,dx+12i\,c)}+6\,e^{(10i\,dx+10i\,c)}+15\,e^{(8i\,dx+8i\,c)}+20\,e^{(6i\,dx+6i\,c)}+15\,e^{(4i\,dx+4i\,c)}+6\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e\right.}{}$$

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/240*(105*(e^(12*I*d*x + 12*I*c) + 6*e^(10*I*d*x + 10*I*c) + 15*e^(8*I*d*x
 + 8*I*c) + 20*e^(6*I*d*x + 6*I*c) + 15*e^(4*I*d*x + 4*I*c) + 6*e^(2*I*d*x
+ 2*I*c) + 1)*log(e^(I*d*x + I*c) + I) - 105*(e^(12*I*d*x + 12*I*c) + 6*e^(
10*I*d*x + 10*I*c) + 15*e^(8*I*d*x + 8*I*c) + 20*e^(6*I*d*x + 6*I*c) + 15*e
^(4*I*d*x + 4*I*c) + 6*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) - I) -
210*I*e^(11*I*d*x + 11*I*c) - 1190*I*e^(9*I*d*x + 9*I*c) - 2772*I*e^(7*I*d*
x + 7*I*c) - 3372*I*e^(5*I*d*x + 5*I*c) + 1190*I*e^(3*I*d*x + 3*I*c) + 210*
I*e^(I*d*x + I*c))/(a^2*d*e^(12*I*d*x + 12*I*c) + 6*a^2*d*e^(10*I*d*x + 10*
I*c) + 15*a^2*d*e^(8*I*d*x + 8*I*c) + 20*a^2*d*e^(6*I*d*x + 6*I*c) + 15*a^2
*d*e^(4*I*d*x + 4*I*c) + 6*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F]

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\sec^9(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

```
[In] integrate(sec(d*x+c)**9/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -Integral(sec(c + d*x)**9/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)/a**2
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 421 vs. 2(110) = 220.

Time = 0.26 (sec) , antiderivative size = 421, normalized size of antiderivative = 3.40

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{\frac{2\left(\frac{135\sin(dx+c)}{\cos(dx+c)+1}+\frac{96i\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{445\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{960i\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{330\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{960i\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{330\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{480i\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{445\sin(}{(\cos(dx+}\right.}{a^2-\frac{6a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{20a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{6a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a^2\sin(dx}{(\cos(dx+c}}}{240\,d}$$

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/240*(2*(135*sin(d*x + c)/(cos(d*x + c) + 1) + 96*I*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 - 445*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 960*I*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 - 330*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 960*I*
sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 330*sin(d*x + c)^7/(cos(d*x + c) + 1)
^7 - 480*I*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 445*sin(d*x + c)^9/(cos(d*
x + c) + 1)^9 + 480*I*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 135*sin(d*x +
 c)^11/(cos(d*x + c) + 1)^11 - 96*I)/(a^2 - 6*a^2*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 15*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 20*a^2*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6 + 15*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 6*
a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + a^2*sin(d*x + c)^12/(cos(d*x +
c) + 1)^12) + 105*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 - 105*log(si
n(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

Giac [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.64

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{\frac{105 \log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^2} - \frac{105 \log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^2} + \frac{2\left(135\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+480i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-445\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-480i\,\tan\right.}{}}{}$$

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/240*(105*log(tan(1/2*d*x + 1/2*c) + 1)/a^2 - 105*log(tan(1/2*d*x + 1/2*c)
 - 1)/a^2 + 2*(135*tan(1/2*d*x + 1/2*c)^11 + 480*I*tan(1/2*d*x + 1/2*c)^10
- 445*tan(1/2*d*x + 1/2*c)^9 - 480*I*tan(1/2*d*x + 1/2*c)^8 - 330*tan(1/2*d
*x + 1/2*c)^7 + 960*I*tan(1/2*d*x + 1/2*c)^6 - 330*tan(1/2*d*x + 1/2*c)^5 -
 960*I*tan(1/2*d*x + 1/2*c)^4 - 445*tan(1/2*d*x + 1/2*c)^3 + 96*I*tan(1/2*d
*x + 1/2*c)^2 + 135*tan(1/2*d*x + 1/2*c) - 96*I)/((tan(1/2*d*x + 1/2*c)^2 -
 1)^6*a^2))/d
```

Mupad [B] (verification not implemented)

Time = 7.19 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.54

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{7\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{8\,a^2\,d} - \frac{-\frac{9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{8} - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}4\mathrm{i} + \frac{89\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{24} + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}4\mathrm{i} + \frac{11\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{4} - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}8\mathrm{i}}{a^2\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 - \right.}$$

```
[In] int(1/(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^2),x)
```

```
[Out] (7*atanh(tan(c/2 + (d*x)/2)))/(8*a^2*d) - ((89*tan(c/2 + (d*x)/2)^3)/24 - (
tan(c/2 + (d*x)/2)^2*4i)/5 - (9*tan(c/2 + (d*x)/2))/8 + tan(c/2 + (d*x)/2)^
4*8i + (11*tan(c/2 + (d*x)/2)^5)/4 - tan(c/2 + (d*x)/2)^6*8i + (11*tan(c/2
+ (d*x)/2)^7)/4 + tan(c/2 + (d*x)/2)^8*4i + (89*tan(c/2 + (d*x)/2)^9)/24 -
tan(c/2 + (d*x)/2)^10*4i - (9*tan(c/2 + (d*x)/2)^11)/8 + 4i/5)/(a^2*d*(tan(
c/2 + (d*x)/2)^2 - 1)^6)
```

3.123 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 100

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{5\text{arctanh}(\sin(c+dx))}{8a^2d}+\frac{5\sec(c+dx)\tan(c+dx)}{8a^2d}+\frac{5\sec^3(c+dx)\tan(c+dx)}{12a^2d}-\frac{2i\sec^5(c+dx)}{3d\,(a^2+ia^2\tan(c+dx))}$$

```
[Out] 5/8*arctanh(sin(d*x+c))/a^2/d+5/8*sec(d*x+c)*tan(d*x+c)/a^2/d+5/12*sec(d*x+
c)^3*tan(d*x+c)/a^2/d-2/3*I*sec(d*x+c)^5/d/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3581, 3853, 3855}

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{5\text{arctanh}(\sin(c+dx))}{8a^2d}-\frac{2i\sec^5(c+dx)}{3d\,(a^2+ia^2\tan(c+dx))}+\frac{5\tan(c+dx)\sec^3(c+dx)}{12a^2d}+\frac{5\tan(c+dx)\sec(c+dx)}{8a^2d}$$

```
[In] Int[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(8*a^2*d) + (5*Sec[c + d*x]*Tan[c + d*x])/(8*a^2*
d) + (5*Sec[c + d*x]^3*Tan[c + d*x])/(12*a^2*d) - (((2*I)/3)*Sec[c + d*x]^5
)/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2i\sec^5(c+dx)}{3d\,(a^2+ia^2\tan(c+dx))}+\frac{5\int\sec^5(c+dx)\,dx}{3a^2}\\
&= \frac{5\sec^3(c+dx)\tan(c+dx)}{12a^2d}-\frac{2i\sec^5(c+dx)}{3d\,(a^2+ia^2\tan(c+dx))}+\frac{5\int\sec^3(c+dx)\,dx}{4a^2}\\
&= \frac{5\sec(c+dx)\tan(c+dx)}{8a^2d}+\frac{5\sec^3(c+dx)\tan(c+dx)}{12a^2d}\\
&\quad-\frac{2i\sec^5(c+dx)}{3d\,(a^2+ia^2\tan(c+dx))}+\frac{5\int\sec(c+dx)\,dx}{8a^2}\\
&= \frac{5\text{arctanh}(\sin(c+dx))}{8a^2d}+\frac{5\sec(c+dx)\tan(c+dx)}{8a^2d}\\
&\quad+\frac{5\sec^3(c+dx)\tan(c+dx)}{12a^2d}-\frac{2i\sec^5(c+dx)}{3d\,(a^2+ia^2\tan(c+dx))}
\end{aligned}
$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 215 vs. $2(100) = 200$.

Time = 1.46 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.15

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\sec^4(c+dx)\left(128i\cos(c+dx)+45\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+60\cos(2(c+dx))\left(\log\left(\cos\right.\right.\right.}{}$$

```
[In] Integrate[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -1/192*(Sec[c + d*x]^4*((128*I)*Cos[c + d*x] + 45*Log[Cos[(c + d*x)/2] - Si
n[(c + d*x)/2]] + 60*Cos[2*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 15*Cos[4*(c + d*x)]*(Log
[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]]) - 45*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 18*Sin[c + d*x] - 30
*Sin[3*(c + d*x)]))/(a^2*d)
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.11

method	result
risch	$-\frac{i\left(15\,e^{7i(dx+c)}+55\,e^{5i(dx+c)}+73\,e^{3i(dx+c)}-15\,e^{i(dx+c)}\right)}{12d\,a^2\left(e^{2i(dx+c)}+1\right)^4}+\frac{5\ln\left(e^{i(dx+c)}+i\right)}{8a^2d}-\frac{5\ln\left(e^{i(dx+c)}-i\right)}{8a^2d}$
derivativedivides	$\frac{\frac{2\left(\frac{3}{16}+\frac{i}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2\left(\frac{1}{16}+\frac{i}{2}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{2\left(-\frac{1}{4}+\frac{i}{3}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}+\frac{2\left(\frac{3}{16}-\frac{i}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{2(}{(\tan(}}{a^2d}$
default	$\frac{\frac{2\left(\frac{3}{16}+\frac{i}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2\left(\frac{1}{16}+\frac{i}{2}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{2\left(-\frac{1}{4}+\frac{i}{3}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}+\frac{2\left(\frac{3}{16}-\frac{i}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{2(}{(\tan(}}{a^2d}$

```
[In] int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*I/d/a^2/(exp(2*I*(d*x+c))+1)^4*(15*exp(7*I*(d*x+c))+55*exp(5*I*(d*x+c
))+73*exp(3*I*(d*x+c))-15*exp(I*(d*x+c)))+5/8/a^2/d*ln(exp(I*(d*x+c))+I)-5/
8/a^2/d*ln(exp(I*(d*x+c))-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(88) = 176.

Time = 0.24 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.30

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{15\left(e^{(8i\,dx+8i\,c)}+4\,e^{(6i\,dx+6i\,c)}+6\,e^{(4i\,dx+4i\,c)}+4\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-15\left(e^{(8i\,dx+8i\,c)}+4\,e^{(6i\,c}\right.}{24\left(a^2de^{(8i\,dx+8i\,c)}+4\,a^2de^{(6i\,d}\right.}$$

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(15*(e^(8*I*d*x + 8*I*c) + 4*e^(6*I*d*x + 6*I*c) + 6*e^(4*I*d*x + 4*I*
c) + 4*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) + I) - 15*(e^(8*I*d*x +
 8*I*c) + 4*e^(6*I*d*x + 6*I*c) + 6*e^(4*I*d*x + 4*I*c) + 4*e^(2*I*d*x + 2*
I*c) + 1)*log(e^(I*d*x + I*c) - I) - 30*I*e^(7*I*d*x + 7*I*c) - 110*I*e^(5*
I*d*x + 5*I*c) - 146*I*e^(3*I*d*x + 3*I*c) + 30*I*e^(I*d*x + I*c))/(a^2*d*e
^(8*I*d*x + 8*I*c) + 4*a^2*d*e^(6*I*d*x + 6*I*c) + 6*a^2*d*e^(4*I*d*x + 4*I
*c) + 4*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F]

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\sec^7(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

```
[In] integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -Integral(sec(c + d*x)**7/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)/a**2
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 295 vs. 2(88) = 176.

Time = 0.25 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.95

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\frac{2\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{16i\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{33\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{48i\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{33\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{48i\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{9\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-16i\right)}{a^2-\frac{4a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}+\frac{15\log\left(\frac{\sin(dx+}{\cos(dx+c}\right.}{a^2}}{24\,d}$$

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/24*(2*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 16*I*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 - 33*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 48*I*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 - 33*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 48*I*sin(d*x
 + c)^6/(cos(d*x + c) + 1)^6 + 9*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 16*I
)/(a^2 - 4*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^2*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 - 4*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^2*sin(d
*x + c)^8/(cos(d*x + c) + 1)^8) + 15*log(sin(d*x + c)/(cos(d*x + c) + 1) +
1)/a^2 - 15*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

Giac [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.51

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\frac{15\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^2} - \frac{15\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^2} + \frac{2\left(9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+48i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-33\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-48i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\right.}}{24\,d}$$

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(15*log(tan(1/2*d*x + 1/2*c) + 1)/a^2 - 15*log(tan(1/2*d*x + 1/2*c) -
1)/a^2 + 2*(9*tan(1/2*d*x + 1/2*c)^7 + 48*I*tan(1/2*d*x + 1/2*c)^6 - 33*tan
(1/2*d*x + 1/2*c)^5 - 48*I*tan(1/2*d*x + 1/2*c)^4 - 33*tan(1/2*d*x + 1/2*c)
^3 + 16*I*tan(1/2*d*x + 1/2*c)^2 + 9*tan(1/2*d*x + 1/2*c) - 16*I)/((tan(1/2
*d*x + 1/2*c)^2 - 1)^4*a^2))/d
```

Mupad [B] (verification not implemented)

Time = 6.67 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.36

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,a^2\,d} + \frac{\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 4\mathrm{i}-\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 4\mathrm{i}-\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 4\mathrm{i}}{3}+\frac{3\tan\left(\frac{c}{\right.}{4}}{a^2\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)^4}$$

```
[In] int(1/(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^2),x)
```

```
[Out] (5*atanh(tan(c/2 + (d*x)/2)))/(4*a^2*d) + ((3*tan(c/2 + (d*x)/2))/4 + (tan(
c/2 + (d*x)/2)^2*4i)/3 - (11*tan(c/2 + (d*x)/2)^3)/4 - tan(c/2 + (d*x)/2)^4
*4i - (11*tan(c/2 + (d*x)/2)^5)/4 + tan(c/2 + (d*x)/2)^6*4i + (3*tan(c/2 +
(d*x)/2)^7)/4 - 4i/3)/(a^2*d*(tan(c/2 + (d*x)/2)^2 - 1)^4)
```

3.124 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 74

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{3\text{arctanh}(\sin(c+dx))}{2a^2d} + \frac{3\sec(c+dx)\tan(c+dx)}{2a^2d} - \frac{2i\sec^3(c+dx)}{d\,(a^2+ia^2\tan(c+dx))}$$

```
[Out] 3/2*arctanh(sin(d*x+c))/a^2/d+3/2*sec(d*x+c)*tan(d*x+c)/a^2/d-2*I*sec(d*x+c
)^3/d/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3581, 3853, 3855}

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{3\text{arctanh}(\sin(c+dx))}{2a^2d} - \frac{2i\sec^3(c+dx)}{d\,(a^2+ia^2\tan(c+dx))} + \frac{3\tan(c+dx)\sec(c+dx)}{2a^2d}$$

```
[In] Int[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(2*a^2*d) + (3*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*
d) - ((2*I)*Sec[c + d*x]^3)/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
```

```
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2i\sec^3(c+dx)}{d\,(a^2+ia^2\tan(c+dx))}+\frac{3\int\sec^3(c+dx)\,dx}{a^2}\\
&= \frac{3\sec(c+dx)\tan(c+dx)}{2a^2d}-\frac{2i\sec^3(c+dx)}{d\,(a^2+ia^2\tan(c+dx))}+\frac{3\int\sec(c+dx)\,dx}{2a^2}\\
&= \frac{3\text{arctanh}(\sin(c+dx))}{2a^2d}+\frac{3\sec(c+dx)\tan(c+dx)}{2a^2d}-\frac{2i\sec^3(c+dx)}{d\,(a^2+ia^2\tan(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.97

$$\int\frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx =$$
$$-\frac{\sec^2(c+dx)\left(8i\cos(c+dx)+3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+3\cos(2(c+dx))\left(\log\left(\cos\left(\frac{1}{2}(c\right.\right.\right.}{}$$

```
[In] Integrate[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -1/4*(Sec[c + d*x]^2*((8*I)*Cos[c + d*x] + 3*Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]] + 3*Cos[2*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] -
 Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 3*Log[Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]] + 2*Sin[c + d*x]))/(a^2*d)
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.20

method	result
risch	$-\frac{i\left(3\,\mathrm{e}^{3i(dx+c)}+5\,\mathrm{e}^{i(dx+c)}\right)}{d\,a^2\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}+\frac{3\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2a^2d}-\frac{3\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2a^2d}$
derivativedivides	$\frac{\frac{2\left(-\frac{1}{4}+i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}+\frac{2\left(-\frac{1}{4}-i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}}{a^2d}$
default	$\frac{\frac{2\left(-\frac{1}{4}+i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}+\frac{2\left(-\frac{1}{4}-i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}}{a^2d}$

```
[In] int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -I/d/a^2/(exp(2*I*(d*x+c))+1)^2*(3*exp(3*I*(d*x+c))+5*exp(I*(d*x+c)))+3/2/a
^2/d*ln(exp(I*(d*x+c))+I)-3/2/a^2/d*ln(exp(I*(d*x+c))-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(66) = 132.

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.81

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$
$$=\frac{3\left(e^{(4i\,dx+4i\,c)}+2\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-3\left(e^{(4i\,dx+4i\,c)}+2\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-}{2\left(a^2de^{(4i\,dx+4i\,c)}+2\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(3*(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c
) + I) - 3*(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x +
 I*c) - I) - 6*I*e^(3*I*d*x + 3*I*c) - 10*I*e^(I*d*x + I*c))/(a^2*d*e^(4*I*
d*x + 4*I*c) + 2*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\sec^5(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

```
[In] integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c))**2,x)
[Out] -Integral(sec(c + d*x)**5/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)/a**2
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(66) = 132$.

Time = 0.27 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.26

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{4i\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+4i\right)}{a^2-\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}}{2d}$$

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
[Out] -1/2*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 4*I*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 4*I)/(a^2 - 2*a^2*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - 3*l
og(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 3*log(sin(d*x + c)/(cos(d*x +
 c) + 1) - 1)/a^2)/d
```

Giac [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.28

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\frac{3\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^2}-\frac{3\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^2}-\frac{2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-4i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+4i\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^2}}{2d}$$

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
[Out] 1/2*(3*log(tan(1/2*d*x + 1/2*c) + 1)/a^2 - 3*log(tan(1/2*d*x + 1/2*c) - 1)/
a^2 - 2*(tan(1/2*d*x + 1/2*c)^3 - 4*I*tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x
+ 1/2*c) + 4*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^2))/d
```

Mupad [B] (verification not implemented)

Time = 4.92 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.41

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d} - \frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{a^2}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,4\mathrm{i}}{a^2}+\frac{4\mathrm{i}}{a^2}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

```
[In] int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^2),x)
```

```
[Out] (3*atanh(tan(c/2 + (d*x)/2)))/(a^2*d) - (tan(c/2 + (d*x)/2)^3/a^2 - (tan(c/
2 + (d*x)/2)^2*4i)/a^2 + 4i/a^2 + tan(c/2 + (d*x)/2)/a^2)/(d*(tan(c/2 + (d*
x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1))
```

3.125 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 48

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^2} dx = -\frac{\operatorname{arctanh}(\sin(c+dx))}{a^2d} + \frac{2i\sec(c+dx)}{d(a^2+ia^2\tan(c+dx))}$$

```
[Out] -arctanh(sin(d*x+c))/a^2/d+2*I*sec(d*x+c)/d/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3581, 3855}

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^2} dx = -\frac{\operatorname{arctanh}(\sin(c+dx))}{a^2d} + \frac{2i\sec(c+dx)}{d(a^2+ia^2\tan(c+dx))}$$

```
[In] Int[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -(ArcTanh[Sin[c + d*x]]/(a^2*d)) + ((2*I)*Sec[c + d*x])/(d*(a^2 + I*a^2*Tan
[c + d*x]))
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
```

```
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2i\sec(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)}-\frac{\int\sec(c+dx)\,dx}{a^2}\\ &=-\frac{\operatorname{arctanh}(\sin(c+dx))}{a^2d}+\frac{2i\sec(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)}\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 184 vs. $2(48) = 96$.

Time = 0.44 (sec) , antiderivative size = 184, normalized size of antiderivative = 3.83

$$\int\frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\sec^2(c+dx)\left(\cos\left(\frac{1}{2}(c+dx)\right)\left(2i+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\right.\right.\right.}{}$$

```
[In] Integrate[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -((Sec[c + d*x]^2*(Cos[(c + d*x)/2]*(2*I + Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (2 + I*Log[Cos[(c +
d*x)/2] - Sin[(c + d*x)/2]] - I*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])*S
in[(c + d*x)/2])*(Cos[(3*(c + d*x))/2] + I*Sin[(3*(c + d*x))/2]))/(a^2*d*(-
I + Tan[c + d*x])^2))
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12

method	result	size
derivativedivides	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\frac{4}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^2d}$	54
default	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\frac{4}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^2d}$	54
risch	$\frac{2ie^{-i(dx+c)}}{a^2d}+\frac{\ln\left(e^{i(dx+c)}-i\right)}{a^2d}-\frac{\ln\left(e^{i(dx+c)}+i\right)}{a^2d}$	61

```
[In] int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 2/d/a^2*(1/2*ln(tan(1/2*d*x+1/2*c)-1)+2/(-I+tan(1/2*d*x+1/2*c))-1/2*ln(tan(
1/2*d*x+1/2*c)+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.33

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\left(e^{(i\,dx+i\,c)}\log\left(e^{(i\,dx+i\,c)}+i\right)-e^{(i\,dx+i\,c)}\log\left(e^{(i\,dx+i\,c)}-i\right)-2i\right)e^{(-i\,dx-i\,c)}}{a^2d}$$

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
[Out] -(e^(I*d*x + I*c)*log(e^(I*d*x + I*c) + I) - e^(I*d*x + I*c)*log(e^(I*d*x +
 I*c) - I) - 2*I)*e^(-I*d*x - I*c)/(a^2*d)
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\sec^3(c+dx)}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

```
[In] integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c))**2,x)
[Out] -Integral(sec(c + d*x)**3/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)/a**2
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(44) = 88.

Time = 0.56 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.44

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{-2i\,\arctan\left(\cos\left(dx+c\right),\sin\left(dx+c\right)+1\right)-2i\,\arctan\left(\cos\left(dx+c\right),-\sin\left(dx+c\right)+1\right)-4i\,\cos\left(dx-\right.}{}$$

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(-2*I*arctan2(cos(d*x + c), sin(d*x + c) + 1) - 2*I*arctan2(cos(d*x +
c), -sin(d*x + c) + 1) - 4*I*cos(d*x + c) + log(cos(d*x + c)^2 + sin(d*x +
c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*
x + c) + 1) - 4*sin(d*x + c))/(a^2*d)
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\frac{\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^2}-\frac{\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^2}-\frac{4}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}}{d}$$

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(log(tan(1/2*d*x + 1/2*c) + 1)/a^2 - log(tan(1/2*d*x + 1/2*c) - 1)/a^2 - 4
/(a^2*(tan(1/2*d*x + 1/2*c) - I)))/d
```

Mupad [B] (verification not implemented)

Time = 4.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}+\frac{4\mathrm{i}}{a^2\,d\,\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}\right)}$$

```
[In] int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^2),x)
```

```
[Out] 4i/(a^2*d*(tan(c/2 + (d*x)/2)*1i + 1)) - (2*atanh(tan(c/2 + (d*x)/2)))/(a^2
*d)
```

3.126 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 65

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{i\sec(c+dx)}{3d(a+ia\tan(c+dx))^2} + \frac{i\sec(c+dx)}{3d\,(a^2+ia^2\tan(c+dx))}$$

```
[Out] 1/3*I*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^2+1/3*I*sec(d*x+c)/d/(a^2+I*a^2*tan(d
*x+c))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3583, 3569}

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{i\sec(c+dx)}{3d\,(a^2+ia^2\tan(c+dx))} + \frac{i\sec(c+dx)}{3d(a+ia\tan(c+dx))^2}$$

```
[In] Int[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((I/3)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^2) + ((I/3)*Sec[c + d*x])/(d
*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{i\sec(c+dx)}{3d(a+ia\tan(c+dx))^2}+\frac{\int\frac{\sec(c+dx)}{a+ia\tan(c+dx)}\,dx}{3a}\\ &= \frac{i\sec(c+dx)}{3d(a+ia\tan(c+dx))^2}+\frac{i\sec(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.58

$$\int\frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}\,dx=\frac{\sec(c+dx)(-2i+\tan(c+dx))}{3a^2d(-i+\tan(c+dx))^2}$$

```
[In] Integrate[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^2,x]
[Out] (Sec[c + d*x]*(-2*I + Tan[c + d*x]))/(3*a^2*d*(-I + Tan[c + d*x])^2)
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.58

method	result	size
risch	$\frac{i e^{-i(dx+c)}}{2a^2d}+\frac{i e^{-3i(dx+c)}}{6a^2d}$	38
derivativedivides	$\frac{\frac{2i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{4}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}}{a^2d}$	57
default	$\frac{\frac{2i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{4}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}}{a^2d}$	57

```
[In] int(sec(d*x+c)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/2*I/a^2/d*exp(-I*(d*x+c))+1/6*I/a^2/d*exp(-3*I*(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.46

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\left(3i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-3i\,dx-3i\,c)}}{6\,a^2d}$$

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*I*e^(2*I*d*x + 2*I*c) + I)*e^(-3*I*d*x - 3*I*c)/(a^2*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(51) = 102.

Time = 0.54 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.72

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \begin{cases} \frac{\tan(c+dx)\sec(c+dx)}{3a^2d\tan^2(c+dx)-6ia^2d\tan(c+dx)-3a^2d} - \frac{2i\sec(c+dx)}{3a^2d\tan^2(c+dx)-6ia^2d\tan(c+dx)-3a^2d} & \text{for } d\neq 0 \\ \frac{x\sec(c)}{(ia\tan(c)+a)^2} & \text{otherwise} \end{cases}$$

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise((tan(c + d*x)*sec(c + d*x)/(3*a**2*d*tan(c + d*x)**2 - 6*I*a**2*d
*tan(c + d*x) - 3*a**2*d) - 2*I*sec(c + d*x)/(3*a**2*d*tan(c + d*x)**2 - 6*
I*a**2*d*tan(c + d*x) - 3*a**2*d), Ne(d, 0)), (x*sec(c)/(I*a*tan(c) + a)**2
, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{i\,\cos(3\,dx+3\,c)+3i\,\cos(dx+c)+\sin(3\,dx+3\,c)+3\,\sin(dx+c)}{6\,a^2d}$$

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(I*cos(3*d*x + 3*c) + 3*I*cos(d*x + c) + sin(3*d*x + 3*c) + 3*sin(d*x +
 c))/(a^2*d)
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\right)}{3\,a^2d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^3}$$

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
[Out] 2/3*(3*tan(1/2*d*x + 1/2*c)^2 - 3*I*tan(1/2*d*x + 1/2*c) - 2)/(a^2*d*(tan(1
/2*d*x + 1/2*c) - I)^3)
```

Mupad [B] (verification not implemented)

Time = 3.91 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.22

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$
$$= -\frac{2\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,3\mathrm{i}+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\mathrm{i}\right)}{3\,a^2\,d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,1\mathrm{i}-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,3\mathrm{i}+1\right)}$$

```
[In] int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^2),x)
[Out] -(2*(3*tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2*3i - 2i))/(3*a^2*d*(tan(c/
2 + (d*x)/2)*3i - 3*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*1i + 1))
```

3.127 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 71

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2} dx = \frac{3\sin(c+dx)}{5a^2d} - \frac{\sin^3(c+dx)}{5a^2d} + \frac{2i\cos^3(c+dx)}{5d(a^2+ia^2\tan(c+dx))}$$

```
[Out] 3/5*sin(d*x+c)/a^2/d-1/5*sin(d*x+c)^3/a^2/d+2/5*I*cos(d*x+c)^3/d/(a^2+I*a^2
*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3581, 2713}

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2} dx = -\frac{\sin^3(c+dx)}{5a^2d} + \frac{3\sin(c+dx)}{5a^2d} + \frac{2i\cos^3(c+dx)}{5d(a^2+ia^2\tan(c+dx))}$$

```
[In] Int[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (3*Sin[c + d*x])/(5*a^2*d) - Sin[c + d*x]^3/(5*a^2*d) + (((2*I)/5)*Cos[c +
d*x]^3)/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2i\cos^3(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{3\int\cos^3(c+dx)\,dx}{5a^2}\\
&= \frac{2i\cos^3(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}-\frac{3\text{Subst}\left(\int\left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{5a^2d}\\
&= \frac{3\sin(c+dx)}{5a^2d}-\frac{\sin^3(c+dx)}{5a^2d}+\frac{2i\cos^3(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96

$$\begin{aligned}
&\int\frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2}\,dx\\
&=\frac{\sec(c+dx)(-12i+4i\cos(2(c+dx))-3\sec(c+dx)\sin(3(c+dx))+5\tan(c+dx))}{20a^2d(-i+\tan(c+dx))^2}
\end{aligned}$$

```
[In] Integrate[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^2,x]
[Out] (Sec[c + d*x]*(-12*I + (4*I)*Cos[2*(c + d*x)] - 3*Sec[c + d*x]*Sin[3*(c + d
*x)] + 5*Tan[c + d*x]))/(20*a^2*d*(-I + Tan[c + d*x])^2)
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94

method	result
risch	$\frac{i\mathrm{e}^{-3i(dx+c)}}{8a^2d}+\frac{i\mathrm{e}^{-5i(dx+c)}}{40a^2d}+\frac{i\cos(dx+c)}{4a^2d}+\frac{\sin(dx+c)}{2a^2d}$
derivativedivides	$\frac{-\frac{2i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{5i}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{4}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}-\frac{3}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{7}{4\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{2}{8\tan\left(\frac{dx}{2}\right.}}{a^2d}$
default	$\frac{-\frac{2i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{5i}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{4}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}-\frac{3}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{7}{4\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{2}{8\tan\left(\frac{dx}{2}\right.}}{a^2d}$

```
[In] int(cos(d*x+c)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*I/a^2/d*exp(-3*I*(d*x+c))+1/40*I/a^2/d*exp(-5*I*(d*x+c))+1/4*I/a^2/d*co
s(d*x+c)+1/2*sin(d*x+c)/a^2/d
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.73

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\left(-5i\,e^{(6i\,dx+6i\,c)}+15i\,e^{(4i\,dx+4i\,c)}+5i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-5i\,dx-5i\,c)}}{40\,a^2d}$$

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/40*(-5*I*e^(6*I*d*x + 6*I*c) + 15*I*e^(4*I*d*x + 4*I*c) + 5*I*e^(2*I*d*x
 + 2*I*c) + I)*e^(-5*I*d*x - 5*I*c)/(a^2*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(60) = 120.

Time = 0.24 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.30

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \begin{cases} \frac{(-2560ia^6d^3e^{10ic}e^{idx}+7680ia^6d^3e^{8ic}e^{-idx}+2560ia^6d^3e^{6ic}e^{-3idx}+512ia^6d^3e^{4ic}e^{-5idx})e^{-9ic}}{20480a^8d^4} & \text{for } a^8d^4e^{9ic}\neq 0 \\ \frac{x(e^{6ic}+3e^{4ic}+3e^{2ic}+1)e^{-5ic}}{8a^2} & \text{otherwise} \end{cases}$$

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise(((-2560*I*a**6*d**3*exp(10*I*c)*exp(I*d*x) + 7680*I*a**6*d**3*exp
(8*I*c)*exp(-I*d*x) + 2560*I*a**6*d**3*exp(6*I*c)*exp(-3*I*d*x) + 512*I*a**
6*d**3*exp(4*I*c)*exp(-5*I*d*x))*exp(-9*I*c)/(20480*a**8*d**4), Ne(a**8*d**
4*exp(9*I*c), 0)), (x*(exp(6*I*c) + 3*exp(4*I*c) + 3*exp(2*I*c) + 1)*exp(-5
*I*c)/(8*a**2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.31

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\frac{5}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)} + \frac{35\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - 90i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 120\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + 70i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) + 21}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^5}}{20\,d}$$

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/20*(5/(a^2*(tan(1/2*d*x + 1/2*c) + I)) + (35*tan(1/2*d*x + 1/2*c)^4 - 90*
I*tan(1/2*d*x + 1/2*c)^3 - 120*tan(1/2*d*x + 1/2*c)^2 + 70*I*tan(1/2*d*x +
1/2*c) + 21)/(a^2*(tan(1/2*d*x + 1/2*c) - I)^5))/d
```

Mupad [B] (verification not implemented)

Time = 4.85 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.27

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{2\left(-5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 10\mathrm{i} + 10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 + 3\tan\left(\frac{c}{2}+\frac{dx}{2}\right) - 2\mathrm{i}\right)}{5\,a^2\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\mathrm{i}\right)^5\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\mathrm{i}\right)}$$

```
[In] int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] -(2*(3*tan(c/2 + (d*x)/2) + 10*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4*
10i - 5*tan(c/2 + (d*x)/2)^5 - 2i))/(5*a^2*d*(tan(c/2 + (d*x)/2) - 1i)^5*(t
an(c/2 + (d*x)/2) + 1i))
```

3.128 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 89

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2} \, dx = \frac{5\sin(c+dx)}{7a^2d} - \frac{10\sin^3(c+dx)}{21a^2d} + \frac{\sin^5(c+dx)}{7a^2d} + \frac{2i\cos^5(c+dx)}{7d\,(a^2+ia^2\tan(c+dx))}$$

```
[Out] 5/7*sin(d*x+c)/a^2/d-10/21*sin(d*x+c)^3/a^2/d+1/7*sin(d*x+c)^5/a^2/d+2/7*I*
cos(d*x+c)^5/d/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3581, 2713}

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2} \, dx = \frac{\sin^5(c+dx)}{7a^2d} - \frac{10\sin^3(c+dx)}{21a^2d} + \frac{5\sin(c+dx)}{7a^2d} + \frac{2i\cos^5(c+dx)}{7d\,(a^2+ia^2\tan(c+dx))}$$

```
[In] Int[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (5*Sin[c + d*x])/(7*a^2*d) - (10*Sin[c + d*x]^3)/(21*a^2*d) + Sin[c + d*x]^
5/(7*a^2*d) + (((2*I)/7)*Cos[c + d*x]^5)/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2i\cos^5(c+dx)}{7d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{5\int\cos^5(c+dx)\,dx}{7a^2}\\ &= \frac{2i\cos^5(c+dx)}{7d\left(a^2+ia^2\tan(c+dx)\right)}-\frac{5\text{Subst}\left(\int\left(1-2x^2+x^4\right)\,dx,x,-\sin(c+dx)\right)}{7a^2d}\\ &= \frac{5\sin(c+dx)}{7a^2d}-\frac{10\sin^3(c+dx)}{21a^2d}+\frac{\sin^5(c+dx)}{7a^2d}+\frac{2i\cos^5(c+dx)}{7d\left(a^2+ia^2\tan(c+dx)\right)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07

$$\int\frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$
$$=\frac{i\sec^2(c+dx)(-140\cos(c+dx)+42\cos(3(c+dx))+2\cos(5(c+dx))-70i\sin(c+dx)+63i\sin(3(c+}{336a^2d(-i+\tan(c+dx))^2}$$

```
[In] Integrate[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((I/336)*Sec[c + d*x]^2*(-140*Cos[c + d*x] + 42*Cos[3*(c + d*x)] + 2*Cos[5*
(c + d*x)] - (70*I)*Sin[c + d*x] + (63*I)*Sin[3*(c + d*x)] + (5*I)*Sin[5*(c
 + d*x)]))/(a^2*d*(-I + Tan[c + d*x])^2)
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.15

method	result
risch	$\frac{i e^{-5i(dx+c)}}{32a^2d} + \frac{i e^{-7i(dx+c)}}{224a^2d} + \frac{5i\cos(dx+c)}{32a^2d} + \frac{15\sin(dx+c)}{32a^2d} + \frac{3i\cos(3dx+3c)}{32a^2d} + \frac{11\sin(3dx+3c)}{96a^2d}$
derivativedivides	$\frac{-\frac{i}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^2}-\frac{1}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^3}+\frac{3}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}+\frac{2i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{5i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{23i}{8\left(-i+\tan\left(\frac{dx}{2}+\right.\right.}}{a^2d}$
default	$\frac{-\frac{i}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^2}-\frac{1}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^3}+\frac{3}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}+\frac{2i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{5i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{23i}{8\left(-i+\tan\left(\frac{dx}{2}+\right.\right.}}{a^2d}$

```
[In] int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/32*I/a^2/d*exp(-5*I*(d*x+c))+1/224*I/a^2/d*exp(-7*I*(d*x+c))+5/32*I/a^2/d
*cos(d*x+c)+15/32*sin(d*x+c)/a^2/d+3/32*I/a^2/d*cos(3*d*x+3*c)+11/96/a^2/d*
sin(3*d*x+3*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.83

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\left(-7i\,e^{(10i\,dx+10i\,c)}-105i\,e^{(8i\,dx+8i\,c)}+210i\,e^{(6i\,dx+6i\,c)}+70i\,e^{(4i\,dx+4i\,c)}+21i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-7i\,dx-7i\,c)}}{672\,a^2d}$$

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/672*(-7*I*e^(10*I*d*x + 10*I*c) - 105*I*e^(8*I*d*x + 8*I*c) + 210*I*e^(6*
I*d*x + 6*I*c) + 70*I*e^(4*I*d*x + 4*I*c) + 21*I*e^(2*I*d*x + 2*I*c) + 3*I)
*e^(-7*I*d*x - 7*I*c)/(a^2*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(76) = 152.

Time = 0.32 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.60

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \begin{cases} \frac{(-176160768ia^{10}d^5e^{19ic}e^{3idx}-2642411520ia^{10}d^5e^{17ic}e^{idx}+5284823040ia^{10}d^5e^{15ic}e^{-idx}+1761607680ia^{10}d^5e^{13ic}e^{-3idx}+528482304ia^{10}d^5e^{1}}{16911433728a^{12}d^6} \\ \frac{x(e^{10ic}+5e^{8ic}+10e^{6ic}+10e^{4ic}+5e^{2ic}+1)e^{-7ic}}{32a^2} \end{cases}$$

```
[In] integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise(((-176160768*I*a**10*d**5*exp(19*I*c)*exp(3*I*d*x) - 2642411520*I
*a**10*d**5*exp(17*I*c)*exp(I*d*x) + 5284823040*I*a**10*d**5*exp(15*I*c)*ex
p(-I*d*x) + 1761607680*I*a**10*d**5*exp(13*I*c)*exp(-3*I*d*x) + 528482304*I
*a**10*d**5*exp(11*I*c)*exp(-5*I*d*x) + 75497472*I*a**10*d**5*exp(9*I*c)*ex
p(-7*I*d*x))*exp(-16*I*c)/(16911433728*a**12*d**6), Ne(a**12*d**6*exp(16*I*
c), 0)), (x*(exp(10*I*c) + 5*exp(8*I*c) + 10*exp(6*I*c) + 10*exp(4*I*c) + 5
*exp(2*I*c) + 1)*exp(-7*I*c)/(32*a**2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.63

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\frac{7\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+15i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-8\right)}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^3}+\frac{273\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-1155i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-2450\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+2870i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^7}}{168\,d}$$

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/168*(7*(9*tan(1/2*d*x + 1/2*c)^2 + 15*I*tan(1/2*d*x + 1/2*c) - 8)/(a^2*(t
an(1/2*d*x + 1/2*c) + I)^3) + (273*tan(1/2*d*x + 1/2*c)^6 - 1155*I*tan(1/2*
d*x + 1/2*c)^5 - 2450*tan(1/2*d*x + 1/2*c)^4 + 2870*I*tan(1/2*d*x + 1/2*c)^
3 + 2037*tan(1/2*d*x + 1/2*c)^2 - 791*I*tan(1/2*d*x + 1/2*c) - 152)/(a^2*(t
an(1/2*d*x + 1/2*c) - I)^7))/d
```

Mupad [B] (verification not implemented)

Time = 7.59 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.81

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\left(-21\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 42\mathrm{i}+28\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 56\mathrm{i}+42\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\tan\left(\frac{c}{2}\right.\right.}{21\,a^2\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\mathrm{i}\right)^3\left(1+\tan\left(\frac{c}{2}\right.\right.}$$

```
[In] int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] ((3*tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2*24i + 76*tan(c/2 + (d*x)/2)^3
 + tan(c/2 + (d*x)/2)^4*28i + 42*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*x)/2)^
6*56i + 28*tan(c/2 + (d*x)/2)^7 + tan(c/2 + (d*x)/2)^8*42i - 21*tan(c/2 + (
d*x)/2)^9 - 6i)*2i)/(21*a^2*d*(tan(c/2 + (d*x)/2) + 1i)^3*(tan(c/2 + (d*x)/
2)*1i + 1)^7)
```

3.129 $\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 107

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{7\sin(c+dx)}{9a^2d}-\frac{7\sin^3(c+dx)}{9a^2d}+\frac{7\sin^5(c+dx)}{15a^2d}-\frac{\sin^7(c+dx)}{9a^2d}+\frac{2i\cos^7(c+dx)}{9d\,(a^2+ia^2\tan(c+dx))}$$

```
[Out] 7/9*sin(d*x+c)/a^2/d-7/9*sin(d*x+c)^3/a^2/d+7/15*sin(d*x+c)^5/a^2/d-1/9*sin
(d*x+c)^7/a^2/d+2/9*I*cos(d*x+c)^7/d/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3581, 2713}

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\sin^7(c+dx)}{9a^2d}+\frac{7\sin^5(c+dx)}{15a^2d}-\frac{7\sin^3(c+dx)}{9a^2d}+\frac{7\sin(c+dx)}{9a^2d}+\frac{2i\cos^7(c+dx)}{9d\,(a^2+ia^2\tan(c+dx))}$$

```
[In] Int[Cos[c + d*x]^5/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (7*Sin[c + d*x])/(9*a^2*d) - (7*Sin[c + d*x]^3)/(9*a^2*d) + (7*Sin[c + d*x]
^5)/(15*a^2*d) - Sin[c + d*x]^7/(9*a^2*d) + (((2*I)/9)*Cos[c + d*x]^7)/(d*(
a^2 + I*a^2*Tan[c + d*x]))
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2i\cos^7(c+dx)}{9d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{7\int\cos^7(c+dx)\,dx}{9a^2}\\ &= \frac{2i\cos^7(c+dx)}{9d\left(a^2+ia^2\tan(c+dx)\right)}-\frac{7\text{Subst}\left(\int\left(1-3x^2+3x^4-x^6\right)\,dx,x,-\sin(c+dx)\right)}{9a^2d}\\ &= \frac{7\sin(c+dx)}{9a^2d}-\frac{7\sin^3(c+dx)}{9a^2d}+\frac{7\sin^5(c+dx)}{15a^2d}-\frac{\sin^7(c+dx)}{9a^2d}+\frac{2i\cos^7(c+dx)}{9d\left(a^2+ia^2\tan(c+dx)\right)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.09

$$\int\frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{i\sec^2(c+dx)(-1050\cos(c+dx)+378\cos(3(c+dx))+30\cos(5(c+dx))+2\cos(7(c+dx))-525i\sin(c}{2880a^2d(-i+\tan(c+dx))^2}$$

```
[In] Integrate[Cos[c + d*x]^5/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((I/2880)*Sec[c + d*x]^2*(-1050*Cos[c + d*x] + 378*Cos[3*(c + d*x)] + 30*Co
s[5*(c + d*x)] + 2*Cos[7*(c + d*x)] - (525*I)*Sin[c + d*x] + (567*I)*Sin[3*
(c + d*x)] + (75*I)*Sin[5*(c + d*x)] + (7*I)*Sin[7*(c + d*x)]))/(a^2*d*(-I
+ Tan[c + d*x])^2)
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.28

method	result
risch	$\frac{i\mathrm{e}^{-7i(dx+c)}}{128a^2d}+\frac{i\mathrm{e}^{-9i(dx+c)}}{1152a^2d}+\frac{7i\cos(dx+c)}{64a^2d}+\frac{7\sin(dx+c)}{16a^2d}+\frac{i\cos(5dx+5c)}{32a^2d}+\frac{11\sin(5dx+5c)}{320a^2d}+\frac{7i\cos(3dx+3c}{96a^2d}$
derivativedivides	$\frac{\frac{i}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^4}-\frac{9i}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^2}+\frac{1}{20\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^5}-\frac{13}{48\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^3}+\frac{29}{64\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}-\frac{2i}{\left(-i+\tan\left(\frac{dx}{2}\right.\right.}}{}$
default	$\frac{\frac{i}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^4}-\frac{9i}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^2}+\frac{1}{20\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^5}-\frac{13}{48\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^3}+\frac{29}{64\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}-\frac{2i}{\left(-i+\tan\left(\frac{dx}{2}\right.\right.}}{}$

```
[In] int(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/128*I/a^2/d*exp(-7*I*(d*x+c))+1/1152*I/a^2/d*exp(-9*I*(d*x+c))+7/64*I/a^2
/d*cos(d*x+c)+7/16*sin(d*x+c)/a^2/d+1/32*I/a^2/d*cos(5*d*x+5*c)+11/320/a^2/
d*sin(5*d*x+5*c)+7/96*I/a^2/d*cos(3*d*x+3*c)+7/64/a^2/d*sin(3*d*x+3*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.90

$$\int\frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$
$$=\frac{\left(-9i\,e^{(14i\,dx+14i\,c)}-105i\,e^{(12i\,dx+12i\,c)}-945i\,e^{(10i\,dx+10i\,c)}+1575i\,e^{(8i\,dx+8i\,c)}+525i\,e^{(6i\,dx+6i\,c)}+189i\,e^{(4i\,dx+}\right.}{5760\,a^2d}$$

```
[In] integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/5760*(-9*I*e^(14*I*d*x + 14*I*c) - 105*I*e^(12*I*d*x + 12*I*c) - 945*I*e^
(10*I*d*x + 10*I*c) + 1575*I*e^(8*I*d*x + 8*I*c) + 525*I*e^(6*I*d*x + 6*I*c
) + 189*I*e^(4*I*d*x + 4*I*c) + 45*I*e^(2*I*d*x + 2*I*c) + 5*I)*e^(-9*I*d*x
 - 9*I*c)/(a^2*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(94) = 188.

Time = 0.40 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.79

$$\int\frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$
$$=\begin{cases}\frac{\left(-227994731135631360ia^{14}d^7e^{30ic}e^{5idx}-265993852991569920 0ia^{14}d^7e^{28ic}e^{3idx}-23939446769241292800ia^{14}d^7e^{26ic}e^{idx}+39899077948735\right.}{}\\ \frac{x\left(e^{14ic}+7e^{12ic}+21e^{10ic}+35e^{8ic}+35e^{6ic}+21e^{4ic}+7e^{2ic}+1\right)e^{-9ic}}{128a^2}\end{cases}$$

```
[In] integrate(cos(d*x+c)**5/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Piecewise(((-227994731135631360*I*a**14*d**7*exp(30*I*c)*exp(5*I*d*x) - 265
9938529915699200*I*a**14*d**7*exp(28*I*c)*exp(3*I*d*x) - 239394467692412928
00*I*a**14*d**7*exp(26*I*c)*exp(I*d*x) + 39899077948735488000*I*a**14*d**7*
exp(24*I*c)*exp(-I*d*x) + 13299692649578496000*I*a**14*d**7*exp(22*I*c)*exp
(-3*I*d*x) + 4787889353848258560*I*a**14*d**7*exp(20*I*c)*exp(-5*I*d*x) + 1
139973655678156800*I*a**14*d**7*exp(18*I*c)*exp(-7*I*d*x) + 126663739519795
200*I*a**14*d**7*exp(16*I*c)*exp(-9*I*d*x))*exp(-25*I*c)/(14591662792680407
0400*a**16*d**8), Ne(a**16*d**8*exp(25*I*c), 0)), (x*(exp(14*I*c) + 7*exp(1
2*I*c) + 21*exp(10*I*c) + 35*exp(8*I*c) + 35*exp(6*I*c) + 21*exp(4*I*c) + 7
*exp(2*I*c) + 1)*exp(-9*I*c)/(128*a**2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(93) = 186.

Time = 0.49 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.84

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx = \frac{\frac{3\left(435\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+1470i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2060\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1330i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+353\right)}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^5}+\frac{4455\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8-26460i\tan\left(\frac{1}{2}dx\right.}{}}{}$$

```
[In] integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2880*(3*(435*tan(1/2*d*x + 1/2*c)^4 + 1470*I*tan(1/2*d*x + 1/2*c)^3 - 206
0*tan(1/2*d*x + 1/2*c)^2 - 1330*I*tan(1/2*d*x + 1/2*c) + 353)/(a^2*(tan(1/2
*d*x + 1/2*c) + I)^5) + (4455*tan(1/2*d*x + 1/2*c)^8 - 26460*I*tan(1/2*d*x
+ 1/2*c)^7 - 78120*tan(1/2*d*x + 1/2*c)^6 + 137340*I*tan(1/2*d*x + 1/2*c)^5
 + 157374*tan(1/2*d*x + 1/2*c)^4 - 118356*I*tan(1/2*d*x + 1/2*c)^3 - 57744*
tan(1/2*d*x + 1/2*c)^2 + 16596*I*tan(1/2*d*x + 1/2*c) + 2339)/(a^2*(tan(1/2
*d*x + 1/2*c) - I)^9))/d
```

Mupad [B] (verification not implemented)

Time = 6.54 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.02

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{191\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{16}-\frac{1289\sin\left(\frac{3c}{2}+\frac{3dx}{2}\right)}{64}+\frac{649\sin\left(\frac{5c}{2}+\frac{5dx}{2}\right)}{64}-\frac{41\sin\left(\frac{7c}{2}+\frac{7dx}{2}\right)}{32}+\frac{41\sin\left(\frac{9c}{2}+\frac{9dx}{2}\right)}{32}-7\sin\left(\right.\right.}{45\,a^2\,d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\right.}$$

```
[In] int(cos(c + d*x)^5/(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] (cos(c/2 + (d*x)/2)*((cos((3*c)/2 + (3*d*x)/2)*525i)/32 - (cos((5*c)/2 + (5
*d*x)/2)*205i)/32 + (cos((7*c)/2 + (7*d*x)/2)*1i)/2 - (cos((9*c)/2 + (9*d*x
)/2)*1i)/2 + (cos((11*c)/2 + (11*d*x)/2)*1i)/32 - (cos((13*c)/2 + (13*d*x)/
2)*1i)/32 + (191*sin(c/2 + (d*x)/2))/16 - (1289*sin((3*c)/2 + (3*d*x)/2))/6
4 + (649*sin((5*c)/2 + (5*d*x)/2))/64 - (41*sin((7*c)/2 + (7*d*x)/2))/32 +
(41*sin((9*c)/2 + (9*d*x)/2))/32 - (7*sin((11*c)/2 + (11*d*x)/2))/64 + (7*s
in((13*c)/2 + (13*d*x)/2))/64)*2i)/(45*a^2*d*(cos(c/2 + (d*x)/2) + sin(c/2
+ (d*x)/2)*1i)^9*(cos(c/2 + (d*x)/2)*1i + sin(c/2 + (d*x)/2))^5)
```

3.130 $\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 109

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{8i(a-ia\tan(c+dx))^7}{7a^{10}d} - \frac{3i(a-ia\tan(c+dx))^8}{2a^{11}d} + \frac{2i(a-ia\tan(c+dx))^9}{3a^{12}d} - \frac{i(a-ia\tan(c+dx))^{10}}{10a^{13}d}$$

```
[Out] 8/7*I*(a-I*a*tan(d*x+c))^7/a^10/d-3/2*I*(a-I*a*tan(d*x+c))^8/a^11/d+2/3*I*(
a-I*a*tan(d*x+c))^9/a^12/d-1/10*I*(a-I*a*tan(d*x+c))^10/a^13/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{i(a-ia\tan(c+dx))^{10}}{10a^{13}d} + \frac{2i(a-ia\tan(c+dx))^9}{3a^{12}d} - \frac{3i(a-ia\tan(c+dx))^8}{2a^{11}d} + \frac{8i(a-ia\tan(c+dx))^7}{7a^{10}d}$$

```
[In] Int[Sec[c + d*x]^14/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (((8*I)/7)*(a - I*a*Tan[c + d*x])^7)/(a^10*d) - (((3*I)/2)*(a - I*a*Tan[c +
 d*x])^8)/(a^11*d) + (((2*I)/3)*(a - I*a*Tan[c + d*x])^9)/(a^12*d) - ((I/10
)*(a - I*a*Tan[c + d*x])^10)/(a^13*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)^6(a+x)^3\,dx, x, ia\tan(c+dx)\right)}{a^{13}d}\\ &= -\frac{i\text{Subst}\left(\int \left(8a^3(a-x)^6-12a^2(a-x)^7+6a(a-x)^8-(a-x)^9\right)\,dx, x, ia\tan(c+dx)\right)}{a^{13}d}\\ &= \frac{8i(a-ia\tan(c+dx))^7}{7a^{10}d}-\frac{3i(a-ia\tan(c+dx))^8}{2a^{11}d}\\ &\quad+\frac{2i(a-ia\tan(c+dx))^9}{3a^{12}d}-\frac{i(a-ia\tan(c+dx))^{10}}{10a^{13}d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.51

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{(i+\tan(c+dx))^7\left(-44-98i\tan(c+dx)+77\tan^2(c+dx)+21i\tan^3(c+dx)\right)}{210a^3d}$$

```
[In] Integrate[Sec[c + d*x]^14/(a + I*a*Tan[c + d*x])^3,x]
[Out] ((I + Tan[c + d*x])^7*(-44 - (98*I)*Tan[c + d*x] + 77*Tan[c + d*x]^2 + (21*
I)*Tan[c + d*x]^3))/(210*a^3*d)
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.53

method	result
risch	$\frac{128i\left(120\,\mathrm{e}^{6i(dx+c)}+45\,\mathrm{e}^{4i(dx+c)}+10\,\mathrm{e}^{2i(dx+c)}+1\right)}{105d\,a^{3}\left(\mathrm{e}^{2i(dx+c)}+1\right)^{10}}$
derivativedivides	$-\frac{i\left(i\tan(dx+c)-\frac{\left(\tan^{10}(dx+c)\right)}{10}-\frac{i\left(\tan^{9}(dx+c)\right)}{3}-\frac{8i\left(\tan^{7}(dx+c)\right)}{7}+\tan^{6}(dx+c)-\frac{6i\left(\tan^{5}(dx+c)\right)}{5}+2\left(\tan^{4}(dx+c)\right)+\frac{3\left(\tan\right.}{}\right.}{a^{3}d}$
default	$-\frac{i\left(i\tan(dx+c)-\frac{\left(\tan^{10}(dx+c)\right)}{10}-\frac{i\left(\tan^{9}(dx+c)\right)}{3}-\frac{8i\left(\tan^{7}(dx+c)\right)}{7}+\tan^{6}(dx+c)-\frac{6i\left(\tan^{5}(dx+c)\right)}{5}+2\left(\tan^{4}(dx+c)\right)+\frac{3\left(\tan\right.}{}\right.}{a^{3}d}$

```
[In] int(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 128/105*I*(120*exp(6*I*(d*x+c))+45*exp(4*I*(d*x+c))+10*exp(2*I*(d*x+c))+1)/
d/a^3/(exp(2*I*(d*x+c))+1)^10
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(85) = 170$.

Time = 0.25 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.78

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx =$$

$$-\frac{128\left(-120i\,e^{(6i\,dx+6i\,c)}-45i\right.}{105\left(a^3de^{(20i\,dx+20i\,c)}+10\,a^3de^{(18i\,dx+18i\,c)}+45\,a^3de^{(16i\,dx+16i\,c)}+120\,a^3de^{(14i\,dx+14i\,c)}+210\,a^3de^{(12i\,dx+12i\,c}\right.}$$

```
[In] integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -128/105*(-120*I*e^(6*I*d*x + 6*I*c) - 45*I*e^(4*I*d*x + 4*I*c) - 10*I*e^(2
*I*d*x + 2*I*c) - I)/(a^3*d*e^(20*I*d*x + 20*I*c) + 10*a^3*d*e^(18*I*d*x +
18*I*c) + 45*a^3*d*e^(16*I*d*x + 16*I*c) + 120*a^3*d*e^(14*I*d*x + 14*I*c)
+ 210*a^3*d*e^(12*I*d*x + 12*I*c) + 252*a^3*d*e^(10*I*d*x + 10*I*c) + 210*a
^3*d*e^(8*I*d*x + 8*I*c) + 120*a^3*d*e^(6*I*d*x + 6*I*c) + 45*a^3*d*e^(4*I*
d*x + 4*I*c) + 10*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

Sympy [F]

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{\sec^{14}(c+dx)}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

```
[In] integrate(sec(d*x+c)**14/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] I*Integral(sec(c + d*x)**14/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*tan(
c + d*x) + I), x)/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.80

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{-21i\,\tan(dx+c)^{10}+70\,\tan(dx+c)^9+240\,\tan(dx+c)^7+210i\,\tan(dx+c)^6+252\,\tan(dx+c)^5+}{210\,a^3d}$$

[In] integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")

[Out] -1/210*(-21*I*tan(d*x + c)^10 + 70*tan(d*x + c)^9 + 240*tan(d*x + c)^7 + 210*I*tan(d*x + c)^6 + 252*tan(d*x + c)^5 + 420*I*tan(d*x + c)^4 + 315*I*tan(d*x + c)^2 - 210*tan(d*x + c))/(a^3*d)

Giac [A] (verification not implemented)

none

Time = 0.68 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.80

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{-21i\,\tan(dx+c)^{10}+70\,\tan(dx+c)^9+240\,\tan(dx+c)^7+210i\,\tan(dx+c)^6+252\,\tan(dx+c)^5+}{210\,a^3d}$$

[In] integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")

[Out] -1/210*(-21*I*tan(d*x + c)^10 + 70*tan(d*x + c)^9 + 240*tan(d*x + c)^7 + 210*I*tan(d*x + c)^6 + 252*tan(d*x + c)^5 + 420*I*tan(d*x + c)^4 + 315*I*tan(d*x + c)^2 - 210*tan(d*x + c))/(a^3*d)

Mupad [B] (verification not implemented)

Time = 4.54 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.09

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\cos(c+d\,x)^{10}\,84\mathrm{i}+128\,\sin(c+d\,x)\,\cos(c+d\,x)^9+64\,\sin(c+d\,x)\,\cos(c+d\,x)^7+48\,\sin(c+d\,x)\,\cos}{210\,a^3\,d\,\cos(c}$$

[In] int(1/(cos(c + d*x)^14*(a + a*tan(c + d*x)*1i)^3),x)

[Out] (40*cos(c + d*x)^3*sin(c + d*x) - 70*cos(c + d*x)*sin(c + d*x) + 48*cos(c + d*x)^5*sin(c + d*x) + 64*cos(c + d*x)^7*sin(c + d*x) + 128*cos(c + d*x)^9*sin(c + d*x) - cos(c + d*x)^2*105i + cos(c + d*x)^10*84i + 21i)/(210*a^3*d*cos(c + d*x)^10)

3.131 $\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^3} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^3} \, dx = \frac{2i(a-ia\tan(c+dx))^6}{3a^9d} - \frac{4i(a-ia\tan(c+dx))^7}{7a^{10}d} + \frac{i(a-ia\tan(c+dx))^8}{8a^{11}d}$$

```
[Out] 2/3*I*(a-I*a*tan(d*x+c))^6/a^9/d-4/7*I*(a-I*a*tan(d*x+c))^7/a^10/d+1/8*I*(a
-I*a*tan(d*x+c))^8/a^11/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^3} \, dx = \frac{i(a-ia\tan(c+dx))^8}{8a^{11}d} - \frac{4i(a-ia\tan(c+dx))^7}{7a^{10}d} + \frac{2i(a-ia\tan(c+dx))^6}{3a^9d}$$

```
[In] Int[Sec[c + d*x]^12/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (((2*I)/3)*(a - I*a*Tan[c + d*x])^6)/(a^9*d) - (((4*I)/7)*(a - I*a*Tan[c +
d*x])^7)/(a^10*d) + ((I/8)*(a - I*a*Tan[c + d*x])^8)/(a^11*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)^5(a+x)^2\,dx, x, ia\tan(c+dx)\right)}{a^{11}d}\\
&= -\frac{i\text{Subst}\left(\int \left(4a^2(a-x)^5-4a(a-x)^6+(a-x)^7\right)\,dx, x, ia\tan(c+dx)\right)}{a^{11}d}\\
&= \frac{2i(a-ia\tan(c+dx))^6}{3a^9d}-\frac{4i(a-ia\tan(c+dx))^7}{7a^{10}d}+\frac{i(a-ia\tan(c+dx))^8}{8a^{11}d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.56

$$
\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{(i+\tan(c+dx))^6\left(-37i+54\tan(c+dx)+21i\tan^2(c+dx)\right)}{168a^3d}
$$

```
[In] Integrate[Sec[c + d*x]^12/(a + I*a*Tan[c + d*x])^3,x]
[Out] ((I + Tan[c + d*x])^6*(-37*I + 54*Tan[c + d*x] + (21*I)*Tan[c + d*x]^2))/(1
68*a^3*d)
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.57

method	result
risch	$\frac{32i\left(28\,e^{4i(dx+c)}+8\,e^{2i(dx+c)}+1\right)}{21d\,a^3\left(e^{2i(dx+c)}+1\right)^8}$
derivativedivides	$\frac{i\left(\frac{\left(\tan^8(dx+c)\right)}{8}-\frac{\left(\tan^6(dx+c)\right)}{6}+\frac{3i\left(\tan^7(dx+c)\right)}{7}-\frac{5\left(\tan^4(dx+c)\right)}{4}+i\left(\tan^5(dx+c)\right)-\frac{3\left(\tan^2(dx+c)\right)}{2}+\frac{i\left(\tan^3(dx+c)\right)}{3}-i\tan\right.}{a^3d}$
default	$\frac{i\left(\frac{\left(\tan^8(dx+c)\right)}{8}-\frac{\left(\tan^6(dx+c)\right)}{6}+\frac{3i\left(\tan^7(dx+c)\right)}{7}-\frac{5\left(\tan^4(dx+c)\right)}{4}+i\left(\tan^5(dx+c)\right)-\frac{3\left(\tan^2(dx+c)\right)}{2}+\frac{i\left(\tan^3(dx+c)\right)}{3}-i\tan\right.}{a^3d}$

```
[In] int(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 32/21*I*(28*exp(4*I*(d*x+c))+8*exp(2*I*(d*x+c))+1)/d/a^3/(exp(2*I*(d*x+c))+
1)^8
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(64) = 128.

Time = 0.24 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.87

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{32\left(-28i\,e^{(4i\,dx+4i\,c)}-8i\,e^{(2i\,dx+2i\,c)}-i\right)}{21\left(a^3de^{(16i\,dx+16i\,c)}+8\,a^3de^{(14i\,dx+14i\,c)}+28\,a^3de^{(12i\,dx+12i\,c)}+56\,a^3de^{(10i\,dx+10i\,c)}+70\,a^3de^{(8i\,dx+8i\,c)}+56\right.}$$

```
[In] integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -32/21*(-28*I*e^(4*I*d*x + 4*I*c) - 8*I*e^(2*I*d*x + 2*I*c) - I)/(a^3*d*e^(
16*I*d*x + 16*I*c) + 8*a^3*d*e^(14*I*d*x + 14*I*c) + 28*a^3*d*e^(12*I*d*x +
 12*I*c) + 56*a^3*d*e^(10*I*d*x + 10*I*c) + 70*a^3*d*e^(8*I*d*x + 8*I*c) +
56*a^3*d*e^(6*I*d*x + 6*I*c) + 28*a^3*d*e^(4*I*d*x + 4*I*c) + 8*a^3*d*e^(2*
I*d*x + 2*I*c) + a^3*d)
```

Sympy [F]

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{\sec^{12}(c+dx)}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

```
[In] integrate(sec(d*x+c)**12/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] I*Integral(sec(c + d*x)**12/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*tan(
c + d*x) + I), x)/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.06

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{-21i\,\tan(dx+c)^8+72\,\tan(dx+c)^7+28i\,\tan(dx+c)^6+168\,\tan(dx+c)^5+210i\,\tan(dx+c)^4+5}{168\,a^3d}$$

```
[In] integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/168*(-21*I*tan(d*x + c)^8 + 72*tan(d*x + c)^7 + 28*I*tan(d*x + c)^6 + 16
8*tan(d*x + c)^5 + 210*I*tan(d*x + c)^4 + 56*tan(d*x + c)^3 + 252*I*tan(d*x
 + c)^2 - 168*tan(d*x + c))/(a^3*d)
```

Giac [A] (verification not implemented)

none

Time = 0.77 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.06

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{-21i\,\tan\left(dx+c\right)^8+72\,\tan\left(dx+c\right)^7+28i\,\tan\left(dx+c\right)^6+168\,\tan\left(dx+c\right)^5+210i\,\tan\left(dx+c\right)^4+}{168\,a^3d}$$

```
[In] integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/168*(-21*I*tan(d*x + c)^8 + 72*tan(d*x + c)^7 + 28*I*tan(d*x + c)^6 + 16
8*tan(d*x + c)^5 + 210*I*tan(d*x + c)^4 + 56*tan(d*x + c)^3 + 252*I*tan(d*x
 + c)^2 - 168*tan(d*x + c))/(a^3*d)
```

Mupad [B] (verification not implemented)

Time = 4.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.26

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\cos\left(c+d\,x\right)^8\,91\mathrm{i}+128\,\sin\left(c+d\,x\right)\,\cos\left(c+d\,x\right)^7+64\,\sin\left(c+d\,x\right)\,\cos\left(c+d\,x\right)^5+48\,\sin\left(c+d\,x\right)\,\cos}{168\,a^3\,d\cos\left(c+d\,x\right)^8}$$

```
[In] int(1/(cos(c + d*x)^12*(a + a*tan(c + d*x)*1i)^3),x)
```

```
[Out] (48*cos(c + d*x)^3*sin(c + d*x) - 72*cos(c + d*x)*sin(c + d*x) + 64*cos(c +
 d*x)^5*sin(c + d*x) + 128*cos(c + d*x)^7*sin(c + d*x) - cos(c + d*x)^2*112
i + cos(c + d*x)^8*91i + 21i)/(168*a^3*d*cos(c + d*x)^8)
```

3.132 $\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{2i(a-ia\tan(c+dx))^5}{5a^8d} - \frac{i(a-ia\tan(c+dx))^6}{6a^9d}$$

```
[Out] 2/5*I*(a-I*a*tan(d*x+c))^5/a^8/d-1/6*I*(a-I*a*tan(d*x+c))^6/a^9/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{2i(a-ia\tan(c+dx))^5}{5a^8d} - \frac{i(a-ia\tan(c+dx))^6}{6a^9d}$$

```
[In] Int[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (((2*I)/5)*(a - I*a*Tan[c + d*x])^5)/(a^8*d) - ((I/6)*(a - I*a*Tan[c + d*x]
)^6)/(a^9*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)^4(a+x)\,dx, x, ia\tan(c+dx)\right)}{a^9d}\\ &= -\frac{i\text{Subst}\left(\int \left(2a(a-x)^4-(a-x)^5\right)\,dx, x, ia\tan(c+dx)\right)}{a^9d}\\ &= \frac{2i(a-ia\tan(c+dx))^5}{5a^8d}-\frac{i(a-ia\tan(c+dx))^6}{6a^9d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.62

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{(7+5i\tan(c+dx))(i+\tan(c+dx))^5}{30a^3d}$$

```
[In] Integrate[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^3,x]
[Out] ((7 + (5*I)*Tan[c + d*x])*(I + Tan[c + d*x])^5)/(30*a^3*d)
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

method	result	size
risch	$\frac{32i\left(6\,\mathrm{e}^{2i(dx+c)}+1\right)}{15d\,a^3\left(\mathrm{e}^{2i(dx+c)}+1\right)^6}$	36
derivativedivides	$-\frac{i\left(i\tan(dx+c)-\frac{\left(\tan^6(dx+c)\right)}{6}-\frac{3i\left(\tan^5(dx+c)\right)}{5}+\frac{\left(\tan^4(dx+c)\right)}{2}-\frac{2i\left(\tan^3(dx+c)\right)}{3}+\frac{3\left(\tan^2(dx+c)\right)}{2}\right)}{a^3d}$	72
default	$-\frac{i\left(i\tan(dx+c)-\frac{\left(\tan^6(dx+c)\right)}{6}-\frac{3i\left(\tan^5(dx+c)\right)}{5}+\frac{\left(\tan^4(dx+c)\right)}{2}-\frac{2i\left(\tan^3(dx+c)\right)}{3}+\frac{3\left(\tan^2(dx+c)\right)}{2}\right)}{a^3d}$	72

```
[In] int(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 32/15*I*(6*exp(2*I*(d*x+c))+1)/d/a^3/(exp(2*I*(d*x+c))+1)^6
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(43) = 86.

Time = 0.24 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.04

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{32\left(-6i\,e^{(2i\,dx+2i\,c)}-i\right)}{15\left(a^3de^{(12i\,dx+12i\,c)}+6\,a^3de^{(10i\,dx+10i\,c)}+15\,a^3de^{(8i\,dx+8i\,c)}+20\,a^3de^{(6i\,dx+6i\,c)}+15\,a^3de^{(4i\,dx+4i\,c)}+6\,a^3de\right.}$$

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -32/15*(-6*I*e^(2*I*d*x + 2*I*c) - I)/(a^3*d*e^(12*I*d*x + 12*I*c) + 6*a^3*
d*e^(10*I*d*x + 10*I*c) + 15*a^3*d*e^(8*I*d*x + 8*I*c) + 20*a^3*d*e^(6*I*d*
x + 6*I*c) + 15*a^3*d*e^(4*I*d*x + 4*I*c) + 6*a^3*d*e^(2*I*d*x + 2*I*c) + a
^3*d)
```

Sympy [F]

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{\sec^{10}(c+dx)}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

```
[In] integrate(sec(d*x+c)**10/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] I*Integral(sec(c + d*x)**10/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*tan(
c + d*x) + I), x)/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.22

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{5i\,\tan(dx+c)^6-18\,\tan(dx+c)^5-15i\,\tan(dx+c)^4-20\,\tan(dx+c)^3-45i\,\tan(dx+c)^2+30\,\tan(d}{30\,a^3d}$$

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/30*(5*I*tan(d*x + c)^6 - 18*tan(d*x + c)^5 - 15*I*tan(d*x + c)^4 - 20*tan
(d*x + c)^3 - 45*I*tan(d*x + c)^2 + 30*tan(d*x + c))/(a^3*d)
```

Giac [A] (verification not implemented)

none

Time = 0.64 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.22

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{-5i\,\tan{(dx+c)}^6+18\,\tan{(dx+c)}^5+15i\,\tan{(dx+c)}^4+20\,\tan{(dx+c)}^3+45i\,\tan{(dx+c)}^2-30\,\mathrm{t}}{30\,a^3d}$$

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/30*(-5*I*tan(d*x + c)^6 + 18*tan(d*x + c)^5 + 15*I*tan(d*x + c)^4 + 20*t
an(d*x + c)^3 + 45*I*tan(d*x + c)^2 - 30*tan(d*x + c))/(a^3*d)
```

Mupad [B] (verification not implemented)

Time = 3.79 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.07

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\sin{(c+d\,x)}\,\left(-30\cos{(c+d\,x)}^5+\cos{(c+d\,x)}^4\,\sin{(c+d\,x)}\,45\mathrm{i}+20\cos{(c+d\,x)}^3\sin{(c+d\,x)}^2+\mathrm{cos}\right.}{30\,a^3\,d\cos{(c+d\,x)}^6}$$

```
[In] int(1/(cos(c + d*x)^10*(a + a*tan(c + d*x)*1i)^3),x)
```

```
[Out] -(sin(c + d*x)*(18*cos(c + d*x)*sin(c + d*x)^4 + cos(c + d*x)^4*sin(c + d*x
)*45i - 30*cos(c + d*x)^5 - sin(c + d*x)^5*5i + cos(c + d*x)^2*sin(c + d*x)
^3*15i + 20*cos(c + d*x)^3*sin(c + d*x)^2))/(30*a^3*d*cos(c + d*x)^6)
```

3.133 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i(a-ia\tan(c+dx))^4}{4a^7d}$$

```
[Out] 1/4*I*(a-I*a*tan(d*x+c))^4/a^7/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 32}

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i(a-ia\tan(c+dx))^4}{4a^7d}$$

```
[In] Int[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((I/4)*(a - I*a*Tan[c + d*x])^4)/(a^7*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
```

```
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)^3\,dx, x, ia\tan(c+dx)\right)}{a^7 d}\\ &= \frac{i(a-ia\tan(c+dx))^4}{4a^7 d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.85

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\tan(c+dx)\left(4-6i\tan(c+dx)-4\tan^2(c+dx)+i\tan^3(c+dx)\right)}{4a^3 d}$$

```
[In] Integrate[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^3,x]
[Out] (Tan[c + d*x]*(4 - (6*I)*Tan[c + d*x] - 4*Tan[c + d*x]^2 + I*Tan[c + d*x]^3
))/(4*a^3*d)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{i(\tan(dx+c)+i)^4}{4a^3 d}$	21
default	$\frac{i(\tan(dx+c)+i)^4}{4a^3 d}$	21
risch	$\frac{4i}{d\,a^3\left(e^{2i(dx+c)}+1\right)^4}$	23

```
[In] int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 1/4*I/a^3/d*(tan(d*x+c)+I)^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(21) = 42$.

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.56

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{4i}{a^3de^{(8i\,dx+8i\,c)}+4\,a^3de^{(6i\,dx+6i\,c)}+6\,a^3de^{(4i\,dx+4i\,c)}+4\,a^3de^{(2i\,dx+2i\,c)}+a^3d}$$

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 4*I/(a^3*d*e^(8*I*d*x + 8*I*c) + 4*a^3*d*e^(6*I*d*x + 6*I*c) + 6*a^3*d*e^(4
*I*d*x + 4*I*c) + 4*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

Sympy [F]

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{\sec^8(c+dx)}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

```
[In] integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] I*Integral(sec(c + d*x)**8/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*tan(c
 + d*x) + I), x)/a**3
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(21) = 42$.

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.74

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{-i\tan(dx+c)^4+4\tan(dx+c)^3+6i\tan(dx+c)^2-4\tan(dx+c)}{4\,a^3d}$$

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(-I*tan(d*x + c)^4 + 4*tan(d*x + c)^3 + 6*I*tan(d*x + c)^2 - 4*tan(d*x
 + c))/(a^3*d)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(21) = 42.

Time = 0.60 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.74

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{-i\,\tan\left(dx+c\right)^4+4\,\tan\left(dx+c\right)^3+6i\,\tan\left(dx+c\right)^2-4\,\tan\left(dx+c\right)}{4\,a^3d}$$

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/4*(-I*tan(d*x + c)^4 + 4*tan(d*x + c)^3 + 6*I*tan(d*x + c)^2 - 4*tan(d*x
 + c))/(a^3*d)
```

Mupad [B] (verification not implemented)

Time = 4.45 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.85

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\sin\left(c+d\,x\right)\,\left(-4\cos\left(c+d\,x\right)^3+\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)\,6\mathrm{i}+4\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)^2-\sin\left(c\right.\right.}{4\,a^3\,d\cos\left(c+d\,x\right)^4}$$

```
[In] int(1/(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^3),x)
```

```
[Out] -(sin(c + d*x)*(4*cos(c + d*x)*sin(c + d*x)^2 + cos(c + d*x)^2*sin(c + d*x)
*6i - 4*cos(c + d*x)^3 - sin(c + d*x)^3*1i))/(4*a^3*d*cos(c + d*x)^4)
```

3.134 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 58

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^3}\,dx=\frac{4x}{a^3}+\frac{4i\log(\cos(c+dx))}{a^3d}-\frac{3\tan(c+dx)}{a^3d}+\frac{i\tan^2(c+dx)}{2a^3d}$$

```
[Out] 4*x/a^3+4*I*ln(cos(d*x+c))/a^3/d-3*tan(d*x+c)/a^3/d+1/2*I*tan(d*x+c)^2/a^3/
d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^3}\,dx=\frac{i\tan^2(c+dx)}{2a^3d}-\frac{3\tan(c+dx)}{a^3d}+\frac{4i\log(\cos(c+dx))}{a^3d}+\frac{4x}{a^3}$$

```
[In] Int[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (4*x)/a^3 + ((4*I)*Log[Cos[c + d*x]])/(a^3*d) - (3*Tan[c + d*x])/(a^3*d) +
((I/2)*Tan[c + d*x]^2)/(a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int \frac{(a-x)^2}{a+x}\,dx, x, ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{i\text{Subst}\left(\int \left(-3a+x+\frac{4a^2}{a+x}\right)\,dx, x, ia\tan(c+dx)\right)}{a^5d}\\ &= \frac{4x}{a^3}+\frac{4i\log(\cos(c+dx))}{a^3d}-\frac{3\tan(c+dx)}{a^3d}+\frac{i\tan^2(c+dx)}{2a^3d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{-8i\log(i-\tan(c+dx))-6\tan(c+dx)+i\tan^2(c+dx)}{2a^3d}$$

```
[In] Integrate[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^3,x]
[Out] ((-8*I)*Log[I - Tan[c + d*x]] - 6*Tan[c + d*x] + I*Tan[c + d*x]^2)/(2*a^3*d
)
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$-\frac{3\tan(dx+c)}{a^3d}+\frac{i(\tan^2(dx+c))}{2a^3d}+\frac{4\arctan(\tan(dx+c))}{a^3d}-\frac{2i\ln(1+\tan^2(dx+c))}{a^3d}$	68
default	$-\frac{3\tan(dx+c)}{a^3d}+\frac{i(\tan^2(dx+c))}{2a^3d}+\frac{4\arctan(\tan(dx+c))}{a^3d}-\frac{2i\ln(1+\tan^2(dx+c))}{a^3d}$	68
risch	$\frac{8x}{a^3}+\frac{8c}{a^3d}-\frac{2i(2\,e^{2i(dx+c)}+3)}{d\,a^3(e^{2i(dx+c)}+1)^2}+\frac{4i\ln(e^{2i(dx+c)}+1)}{a^3d}$	73

```
[In] int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] -3*tan(d*x+c)/a^3/d+1/2*I*tan(d*x+c)^2/a^3/d+4/a^3/d*arctan(tan(d*x+c))-2*I
/a^3/d*ln(1+tan(d*x+c)^2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(52) = 104$.

Time = 0.24 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.95

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{2\left(4\,dxe^{(4i\,dx+4i\,c)}+4\,dx+2\,(4\,dx-i)e^{(2i\,dx+2i\,c)}-2\left(-i\,e^{(4i\,dx+4i\,c)}-2i\,e^{(2i\,dx+2i\,c)}-i\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)}{a^3de^{(4i\,dx+4i\,c)}+2\,a^3de^{(2i\,dx+2i\,c)}+a^3d}$$

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 2*(4*d*x*e^(4*I*d*x + 4*I*c) + 4*d*x + 2*(4*d*x - I)*e^(2*I*d*x + 2*I*c) -
2*(-I*e^(4*I*d*x + 4*I*c) - 2*I*e^(2*I*d*x + 2*I*c) - I)*log(e^(2*I*d*x + 2
*I*c) + 1) - 3*I)/(a^3*d*e^(4*I*d*x + 4*I*c) + 2*a^3*d*e^(2*I*d*x + 2*I*c)
+ a^3*d)
```

Sympy [F]

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\int\frac{\sec^6(c+dx)}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

```
[In] integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] I*Integral(sec(c + d*x)**6/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*tan(c
 + d*x) + I), x)/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.78

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\frac{i\tan(dx+c)^2-6\tan(dx+c)}{a^3}-\frac{8i\log(i\tan(dx+c)+1)}{a^3}}{2\,d}$$

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((I*tan(d*x + c)^2 - 6*tan(d*x + c))/a^3 - 8*I*log(I*tan(d*x + c) + 1)/
a^3)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(52) = 104$.

Time = 0.59 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.21

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{2\left(\frac{2i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^3} - \frac{4i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)}{a^3} + \frac{2i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^3} + \frac{-3i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+7i\,\text{ta}}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\right.}\right.}{d}$$

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 2*(2*I*log(tan(1/2*d*x + 1/2*c) + 1)/a^3 - 4*I*log(tan(1/2*d*x + 1/2*c) - I
)/a^3 + 2*I*log(tan(1/2*d*x + 1/2*c) - 1)/a^3 + (-3*I*tan(1/2*d*x + 1/2*c)^
4 + 3*tan(1/2*d*x + 1/2*c)^3 + 7*I*tan(1/2*d*x + 1/2*c)^2 - 3*tan(1/2*d*x +
 1/2*c) - 3*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3))/d
```

Mupad [B] (verification not implemented)

Time = 3.67 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,8\mathrm{i}+6\tan(c+d\,x)-\tan(c+d\,x)^2\,1\mathrm{i}}{2\,a^3\,d}$$

```
[In] int(1/(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^3),x)
```

```
[Out] -(log(tan(c + d*x) - 1i)*8i + 6*tan(c + d*x) - tan(c + d*x)^2*1i)/(2*a^3*d)
```

3.135 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 50

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^3} dx = -\frac{x}{a^3} - \frac{i\log(\cos(c+dx))}{a^3d} + \frac{2i}{d(a^3+ia^3\tan(c+dx))}$$

```
[Out] -x/a^3-I*ln(cos(d*x+c))/a^3/d+2*I/d/(a^3+I*a^3*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^3} dx = \frac{2i}{d(a^3+ia^3\tan(c+dx))} - \frac{i\log(\cos(c+dx))}{a^3d} - \frac{x}{a^3}$$

```
[In] Int[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] -(x/a^3) - (I*Log[Cos[c + d*x]])/(a^3*d) + (2*I)/(d*(a^3 + I*a^3*Tan[c + d*
x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int \frac{a-x}{(a+x)^2}\,dx, x, ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{i\text{Subst}\left(\int \left(\frac{1}{-a-x}+\frac{2a}{(a+x)^2}\right)\,dx, x, ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{x}{a^3}-\frac{i\log(\cos(c+dx))}{a^3d}+\frac{2i}{d\left(a^3+ia^3\tan(c+dx)\right)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{i\left(-\log(i-\tan(c+dx))-\frac{2a}{a+ia\tan(c+dx)}\right)}{a^3d}$$

```
[In] Integrate[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^3,x]
[Out] ((-I)*(-Log[I - Tan[c + d*x]] - (2*a)/(a + I*a*Tan[c + d*x])))/(a^3*d)
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.12

method	result	size
derivativedivides	$\frac{2}{a^3d(\tan(dx+c)-i)}+\frac{i\ln\left(1+\tan^2(dx+c)\right)}{2a^3d}-\frac{\arctan(\tan(dx+c))}{a^3d}$	56
default	$\frac{2}{a^3d(\tan(dx+c)-i)}+\frac{i\ln\left(1+\tan^2(dx+c)\right)}{2a^3d}-\frac{\arctan(\tan(dx+c))}{a^3d}$	56
risch	$\frac{i\mathrm{e}^{-2i(dx+c)}}{a^3d}-\frac{2x}{a^3}-\frac{2c}{a^3d}-\frac{i\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{a^3d}$	56

```
[In] int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 2/a^3/d/(tan(d*x+c)-I)+1/2*I/a^3/d*ln(1+tan(d*x+c)^2)-1/a^3/d*arctan(tan(d*
x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.10

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\left(2\,dxe^{(2i\,dx+2i\,c)}+i\,e^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-i\right)e^{(-2i\,dx-2i\,c)}}{a^3d}$$

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
[Out] -(2*d*x*e^(2*I*d*x + 2*I*c) + I*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c)
 + 1) - I)*e^(-2*I*d*x - 2*I*c)/(a^3*d)
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{\sec^4(c+dx)}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

```
[In] integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c))**3,x)
[Out] I*Integral(sec(c + d*x)**4/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*tan(c
 + d*x) + I), x)/a**3
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.32

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\frac{4\,(-i\,\tan(dx+c)-1)}{2i\,a^3\tan(dx+c)^2+4\,a^3\tan(dx+c)-2i\,a^3}-\frac{i\,\log(i\,\tan(dx+c)+1)}{a^3}}{d}$$

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
[Out] -(4*(-I*tan(d*x + c) - 1)/(2*I*a^3*tan(d*x + c)^2 + 4*a^3*tan(d*x + c) - 2*
I*a^3) - I*log(I*tan(d*x + c) + 1)/a^3)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(44) = 88$.

Time = 0.65 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.00

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\frac{i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^3}-\frac{2i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}{a^3}+\frac{i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^3}+\frac{3i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+10\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3i}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^2}}{d}$$

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(I*log(tan(1/2*d*x + 1/2*c) + 1)/a^3 - 2*I*log(tan(1/2*d*x + 1/2*c) - I)/a
^3 + I*log(tan(1/2*d*x + 1/2*c) - 1)/a^3 + (3*I*tan(1/2*d*x + 1/2*c)^2 + 10
*tan(1/2*d*x + 1/2*c) - 3*I)/(a^3*(tan(1/2*d*x + 1/2*c) - I)^2))/d
```

Mupad [B] (verification not implemented)

Time = 3.93 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,1\mathrm{i}}{a^3\,d}+\frac{2\mathrm{i}}{a^3\,d\,\left(1+\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}$$

```
[In] int(1/(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^3),x)
```

```
[Out] (log(tan(c + d*x) - 1i)*1i)/(a^3*d) + 2i/(a^3*d*(tan(c + d*x)*1i + 1))
```

3.136 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i}{2ad(a+ia\tan(c+dx))^2}$$

```
[Out] 1/2*I/a/d/(a+I*a*tan(d*x+c))^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 32}

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i}{2ad(a+ia\tan(c+dx))^2}$$

```
[In] Int[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (I/2)/(a*d*(a + I*a*Tan[c + d*x])^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
```

```
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{i\text{Subst}\left(\int \frac{1}{(a+x)^3} dx, x, ia\tan(c+dx)\right)}{ad}$$
$$= \frac{i}{2ad(a+ia\tan(c+dx))^2}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^3} dx = -\frac{i}{2a^3d(-i+\tan(c+dx))^2}$$

```
[In] Integrate[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^3,x]
[Out] (-1/2*I)/(a^3*d*(-I + Tan[c + d*x])^2)
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{i}{2ad(a+ia\tan(dx+c))^2}$	24
default	$\frac{i}{2ad(a+ia\tan(dx+c))^2}$	24
risch	$\frac{i\mathrm{e}^{-2i(dx+c)}}{4a^3d}+\frac{i\mathrm{e}^{-4i(dx+c)}}{8a^3d}$	38

```
[In] int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 1/2*I/a/d/(a+I*a*tan(d*x+c))^2
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\left(2i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-4i\,dx-4i\,c)}}{8\,a^3d}$$

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
[Out] 1/8*(2*I*e^(2*I*d*x + 2*I*c) + I)*e^(-4*I*d*x - 4*I*c)/(a^3*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(19) = 38.

Time = 0.82 (sec) , antiderivative size = 153, normalized size of antiderivative = 5.67

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \begin{cases} -\frac{i\tan(c+dx)\sec^2(c+dx)}{8a^3d\tan^3(c+dx)-24ia^3d\tan^2(c+dx)-24a^3d\tan(c+dx)+8ia^3d} - \frac{3\sec^2(c+dx)}{8a^3d\tan^3(c+dx)-24ia^3d\tan^2(c+dx)-24a^3d\tan(c+dx)+8ia^3d} & \text{fo} \\ \frac{x\sec^2(c)}{(ia\tan(c)+a)^3} & \text{ot} \end{cases}$$

```
[In] integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c))**3,x)
[Out] Piecewise((-I*tan(c + d*x)*sec(c + d*x)**2/(8*a**3*d*tan(c + d*x)**3 - 24*I
*a**3*d*tan(c + d*x)**2 - 24*a**3*d*tan(c + d*x) + 8*I*a**3*d) - 3*sec(c +
d*x)**2/(8*a**3*d*tan(c + d*x)**3 - 24*I*a**3*d*tan(c + d*x)**2 - 24*a**3*d
*tan(c + d*x) + 8*I*a**3*d), Ne(d, 0)), (x*sec(c)**2/(I*a*tan(c) + a)**3, T
rue))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i}{2\left(i\,a\tan\left(dx+c\right)+a\right)^2ad}$$

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
[Out] 1/2*I/((I*a*tan(d*x + c) + a)^2*a*d)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(21) = 42$.

Time = 0.55 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.11

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^3d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^4}$$

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
[Out] -2*(tan(1/2*d*x + 1/2*c)^3 - I*tan(1/2*d*x + 1/2*c)^2 - tan(1/2*d*x + 1/2*c
))/(a^3*d*(tan(1/2*d*x + 1/2*c) - I)^4)
```

Mupad [B] (verification not implemented)

Time = 4.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{1\mathrm{i}}{2\,a^3\,d\,(\tan(c+d\,x)-\mathrm{i})^2}$$

```
[In] int(1/(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^3),x)
[Out] -1i/(2*a^3*d*(tan(c + d*x) - 1i)^2)
```

3.137 $\int \frac{1}{(a+ia\tan(c+dx))^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 88

$$\int \frac{1}{(a+ia\tan(c+dx))^3} dx = \frac{x}{8a^3} + \frac{i}{6d(a+ia\tan(c+dx))^3} + \frac{i}{8ad(a+ia\tan(c+dx))^2} + \frac{i}{8d(a^3+ia^3\tan(c+dx))}$$

```
[Out] 1/8*x/a^3+1/6*I/d/(a+I*a*tan(d*x+c))^3+1/8*I/a/d/(a+I*a*tan(d*x+c))^2+1/8*I
/d/(a^3+I*a^3*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3560, 8}

$$\int \frac{1}{(a+ia\tan(c+dx))^3} dx = \frac{i}{8d(a^3+ia^3\tan(c+dx))} + \frac{x}{8a^3} + \frac{i}{8ad(a+ia\tan(c+dx))^2} + \frac{i}{6d(a+ia\tan(c+dx))^3}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^(-3),x]
```

```
[Out] x/(8*a^3) + (I/6)/(d*(a + I*a*Tan[c + d*x])^3) + (I/8)/(a*d*(a + I*a*Tan[c
+ d*x])^2) + (I/8)/(d*(a^3 + I*a^3*Tan[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{i}{6d(a+ia\tan(c+dx))^3}+\frac{\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx}{2a}\\
&= \frac{i}{6d(a+ia\tan(c+dx))^3}+\frac{i}{8ad(a+ia\tan(c+dx))^2}+\frac{\int \frac{1}{a+ia\tan(c+dx)}\,dx}{4a^2}\\
&= \frac{i}{6d(a+ia\tan(c+dx))^3}+\frac{i}{8ad(a+ia\tan(c+dx))^2}+\frac{i}{8d\left(a^3+ia^3\tan(c+dx)\right)}+\frac{\int 1\,dx}{8a^3}\\
&= \frac{x}{8a^3}+\frac{i}{6d(a+ia\tan(c+dx))^3}+\frac{i}{8ad(a+ia\tan(c+dx))^2}+\frac{i}{8d\left(a^3+ia^3\tan(c+dx)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.74

$$\int \frac{1}{(a+ia\tan(c+dx))^3}\,dx = \frac{-10-9i\tan(c+dx)+3\tan^2(c+dx)+3\arctan(\tan(c+dx))(-i+\tan(c+dx))^3}{24a^3d(-i+\tan(c+dx))^3}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(-3),x]
[Out] (-10 - (9*I)*Tan[c + d*x] + 3*Tan[c + d*x]^2 + 3*ArcTan[Tan[c + d*x]]*(-I +
 Tan[c + d*x])^3)/(24*a^3*d*(-I + Tan[c + d*x])^3)
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.70

method	result	si
risch	$\frac{x}{8a^3}+\frac{3ie^{-2i(dx+c)}}{16a^3d}+\frac{3ie^{-4i(dx+c)}}{32a^3d}+\frac{ie^{-6i(dx+c)}}{48a^3d}$	6
derivativedivides	$\frac{\arctan(\tan(dx+c))}{8a^3d}-\frac{i}{8d\,a^3(\tan(dx+c)-i)^2}-\frac{1}{6d\,a^3(\tan(dx+c)-i)^3}+\frac{1}{8a^3d(\tan(dx+c)-i)}$	7
default	$\frac{\arctan(\tan(dx+c))}{8a^3d}-\frac{i}{8d\,a^3(\tan(dx+c)-i)^2}-\frac{1}{6d\,a^3(\tan(dx+c)-i)^3}+\frac{1}{8a^3d(\tan(dx+c)-i)}$	7
norman	$\frac{\frac{x}{8a}+\frac{\tan^3(dx+c)}{3ad}+\frac{\tan^5(dx+c)}{8ad}+\frac{3x\left(\tan^2(dx+c)\right)}{8a}+\frac{3x\left(\tan^4(dx+c)\right)}{8a}+\frac{x\left(\tan^6(dx+c)\right)}{8a}+\frac{5i}{12ad}+\frac{7\tan(dx+c)}{8ad}-\frac{i\left(\tan^2(dx+c)\right)}{4ad}}{a^2(1+\tan^2(dx+c))^3}$	1

```
[In] int(1/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 1/8*x/a^3+3/16*I/a^3/d*exp(-2*I*(d*x+c))+3/32*I/a^3/d*exp(-4*I*(d*x+c))+1/4
8*I/a^3/d*exp(-6*I*(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.61

$$\int \frac{1}{(a+ia\tan(c+dx))^3}\,dx = \frac{\left(12\,dxe^{(6i\,dx+6i\,c)}+18i\,e^{(4i\,dx+4i\,c)}+9i\,e^{(2i\,dx+2i\,c)}+2i\right)e^{(-6i\,dx-6i\,c)}}{96\,a^3d}$$

```
[In] integrate(1/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
[Out] 1/96*(12*d*x*e^(6*I*d*x + 6*I*c) + 18*I*e^(4*I*d*x + 4*I*c) + 9*I*e^(2*I*d*
x + 2*I*c) + 2*I)*e^(-6*I*d*x - 6*I*c)/(a^3*d)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.76

$$\int \frac{1}{(a+ia\tan(c+dx))^3}\,dx = \begin{cases} \frac{(4608ia^6d^2e^{10ic}e^{-2idx}+2304ia^6d^2e^{8ic}e^{-4idx}+512ia^6d^2e^{6ic}e^{-6idx})e^{-12ic}}{24576a^9d^3} & \text{for } a^9d^3e^{12ic}\neq 0 \\ x\left(\frac{(e^{6ic}+3e^{4ic}+3e^{2ic}+1)e^{-6ic}}{8a^3}-\frac{1}{8a^3}\right) & \text{otherwise} \end{cases} + \frac{x}{8a^3}$$

```
[In] integrate(1/(a+I*a*tan(d*x+c))**3,x)
[Out] Piecewise(((4608*I*a**6*d**2*exp(10*I*c)*exp(-2*I*d*x) + 2304*I*a**6*d**2*e
xp(8*I*c)*exp(-4*I*d*x) + 512*I*a**6*d**2*exp(6*I*c)*exp(-6*I*d*x))*exp(-12
*I*c)/(24576*a**9*d**3), Ne(a**9*d**3*exp(12*I*c), 0)), (x*((exp(6*I*c) + 3
*exp(4*I*c) + 3*exp(2*I*c) + 1)*exp(-6*I*c)/(8*a**3) - 1/(8*a**3)), True))
+ x/(8*a**3)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a+ia\tan(c+dx))^3}\,dx = -\frac{-\frac{6i\log(\tan(dx+c)+i)}{a^3} + \frac{6i\log(\tan(dx+c)-i)}{a^3} + \frac{-11i\tan(dx+c)^3-45\tan(dx+c)^2+69i\tan(dx+c)+51}{a^3(\tan(dx+c)-i)^3}}{96\,d}$$

```
[In] integrate(1/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/96*(-6*I*log(tan(d*x + c) + I)/a^3 + 6*I*log(tan(d*x + c) - I)/a^3 + (-1
1*I*tan(d*x + c)^3 - 45*tan(d*x + c)^2 + 69*I*tan(d*x + c) + 51)/(a^3*(tan(
d*x + c) - I)^3))/d
```

Mupad [B] (verification not implemented)

Time = 4.00 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.57

$$\int \frac{1}{(a+ia\tan(c+dx))^3}\,dx = \frac{x}{8\,a^3} - \frac{\frac{\tan(c+d\,x)^2\,1\mathrm{i}}{8} + \frac{3\tan(c+d\,x)}{8} - \frac{5}{12}\mathrm{i}}{a^3\,d\,(1+\tan(c+d\,x)\,1\mathrm{i})^3}$$

```
[In] int(1/(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] x/(8*a^3) - ((3*tan(c + d*x))/8 + (tan(c + d*x)^2*1i)/8 - 5i/12)/(a^3*d*(ta
n(c + d*x)*1i + 1)^3)
```

3.138 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 141

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{5x}{32a^3}+\frac{ia}{16d(a+ia\tan(c+dx))^4}+\frac{i}{12d(a+ia\tan(c+dx))^3}+\frac{3i}{32ad(a+ia\tan(c+dx))^2}-\frac{i}{32d\,(a^3-ia^3\tan(c+dx))}+\frac{i}{8d\,(a^3+ia^3\tan(c+dx))}$$

```
[Out] 5/32*x/a^3+1/16*I*a/d/(a+I*a*tan(d*x+c))^4+1/12*I/d/(a+I*a*tan(d*x+c))^3+3/
32*I/a/d/(a+I*a*tan(d*x+c))^2-1/32*I/d/(a^3-I*a^3*tan(d*x+c))+1/8*I/d/(a^3+
I*a^3*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3568, 46, 212}

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{i}{32d\,(a^3-ia^3\tan(c+dx))}+\frac{i}{8d\,(a^3+ia^3\tan(c+dx))}+\frac{5x}{32a^3}+\frac{ia}{16d(a+ia\tan(c+dx))^4}+\frac{i}{12d(a+ia\tan(c+dx))^3}+\frac{3i}{32ad(a+ia\tan(c+dx))^2}$$

```
[In] Int[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (5*x)/(32*a^3) + ((I/16)*a)/(d*(a + I*a*Tan[c + d*x])^4) + (I/12)/(d*(a + I
*a*Tan[c + d*x])^3) + ((3*I)/32)/(a*d*(a + I*a*Tan[c + d*x])^2) - (I/32)/(d
*(a^3 - I*a^3*Tan[c + d*x])) + (I/8)/(d*(a^3 + I*a^3*Tan[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{(ia^3)\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^5}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^3)\,\text{Subst}\left(\int \left(\frac{1}{32a^5(a-x)^2} + \frac{1}{4a^2(a+x)^5} + \frac{1}{4a^3(a+x)^4} + \frac{3}{16a^4(a+x)^3} + \frac{1}{8a^5(a+x)^2} + \frac{5}{32a^5(a^2-x^2)}\right) dx, x, ia}{d}$$

$$= \frac{ia}{16d(a+ia\tan(c+dx))^4} + \frac{i}{12d(a+ia\tan(c+dx))^3} + \frac{3i}{32ad(a+ia\tan(c+dx))^2} - \frac{i}{32d\,(a^3-ia^3\tan(c+dx))} + \frac{i}{8d\,(a^3+ia^3\tan(c+dx))} - \frac{(5i)\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, ia\tan(c+dx)\right)}{32a^2d}$$

$$= \frac{5x}{32a^3} + \frac{ia}{16d(a+ia\tan(c+dx))^4} + \frac{i}{12d(a+ia\tan(c+dx))^3} + \frac{3i}{32ad(a+ia\tan(c+dx))^2} - \frac{i}{32d\,(a^3-ia^3\tan(c+dx))} + \frac{i}{8d\,(a^3+ia^3\tan(c+dx))}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.97

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx =$$

$$-\frac{ia^3\left(\frac{5i\arctan(\tan(c+dx))}{32a^6}+\frac{1}{32a^5(a-ia\tan(c+dx))}-\frac{1}{16a^2(a+ia\tan(c+dx))^4}-\frac{1}{12a^3(a+ia\tan(c+dx))^3}-\frac{3}{32a^4(a+ia\tan(c+dx))^2}-\right.}{d}$$

```
[In] Integrate[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((-I)*a^3*((((5*I)/32)*ArcTan[Tan[c + d*x]])/a^6 + 1/(32*a^5*(a - I*a*Tan[c
 + d*x])) - 1/(16*a^2*(a + I*a*Tan[c + d*x])^4) - 1/(12*a^3*(a + I*a*Tan[c
+ d*x])^3) - 3/(32*a^4*(a + I*a*Tan[c + d*x])^2) - 1/(8*a^5*(a + I*a*Tan[c
+ d*x]))))/d
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.69

method	result
risch	$\frac{5x}{32a^3}+\frac{5i\,e^{-4i(dx+c)}}{64a^3d}+\frac{5i\,e^{-6i(dx+c)}}{192a^3d}+\frac{i\,e^{-8i(dx+c)}}{256a^3d}+\frac{9i\cos(2dx+2c)}{64a^3d}+\frac{11\sin(2dx+2c)}{64a^3d}$
derivativedivides	$\frac{-\frac{5i\ln(\tan(dx+c)-i)}{64}+\frac{i}{16(\tan(dx+c)-i)^4}-\frac{3i}{32(\tan(dx+c)-i)^2}-\frac{1}{12(\tan(dx+c)-i)^3}+\frac{1}{8\tan(dx+c)-8i}+\frac{5i\ln(\tan(dx+c)+i)}{64}+\frac{}{32\tan(}}{d\,a^3}$
default	$\frac{-\frac{5i\ln(\tan(dx+c)-i)}{64}+\frac{i}{16(\tan(dx+c)-i)^4}-\frac{3i}{32(\tan(dx+c)-i)^2}-\frac{1}{12(\tan(dx+c)-i)^3}+\frac{1}{8\tan(dx+c)-8i}+\frac{5i\ln(\tan(dx+c)+i)}{64}+\frac{}{32\tan(}}{d\,a^3}$

```
[In] int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 5/32*x/a^3+5/64*I/a^3/d*exp(-4*I*(d*x+c))+5/192*I/a^3/d*exp(-6*I*(d*x+c))+1
/256*I/a^3/d*exp(-8*I*(d*x+c))+9/64*I/a^3/d*cos(2*d*x+2*c)+11/64/a^3/d*sin(
2*d*x+2*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.54

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$

$$=\frac{\left(120\,dxe^{(8i\,dx+8i\,c)}-12i\,e^{(10i\,dx+10i\,c)}+120i\,e^{(6i\,dx+6i\,c)}+60i\,e^{(4i\,dx+4i\,c)}+20i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-8i\,dx-8i\,c)}}{768\,a^3d}$$

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/768*(120*d*x*e^(8*I*d*x + 8*I*c) - 12*I*e^(10*I*d*x + 10*I*c) + 120*I*e^(
6*I*d*x + 6*I*c) + 60*I*e^(4*I*d*x + 4*I*c) + 20*I*e^(2*I*d*x + 2*I*c) + 3*
I)*e^(-8*I*d*x - 8*I*c)/(a^3*d)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.59

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \begin{cases} \frac{(-100663296ia^{12}d^4e^{22ic}e^{2idx}+1006632960ia^{12}d^4e^{18ic}e^{-2idx}+503316480ia^{12}d^4e^{16ic}e^{-4idx}+167772160ia^{12}d^4e^{14ic}e^{-6idx}+25165824ia^{12}d^4 \cdots}{6442450944a^{15}d^5} \\ x\left(\frac{(e^{10ic}+5e^{8ic}+10e^{6ic}+10e^{4ic}+5e^{2ic}+1)e^{-8ic}}{32a^3}-\frac{5}{32a^3}\right) \end{cases}$$
$$+ \frac{5x}{32a^3}$$

```
[In] integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise(((-100663296*I*a**12*d**4*exp(22*I*c)*exp(2*I*d*x) + 1006632960*I
*a**12*d**4*exp(18*I*c)*exp(-2*I*d*x) + 503316480*I*a**12*d**4*exp(16*I*c)*
exp(-4*I*d*x) + 167772160*I*a**12*d**4*exp(14*I*c)*exp(-6*I*d*x) + 25165824
*I*a**12*d**4*exp(12*I*c)*exp(-8*I*d*x))*exp(-20*I*c)/(6442450944*a**15*d**
5), Ne(a**15*d**5*exp(20*I*c), 0)), (x*((exp(10*I*c) + 5*exp(8*I*c) + 10*ex
p(6*I*c) + 10*exp(4*I*c) + 5*exp(2*I*c) + 1)*exp(-8*I*c)/(32*a**3) - 5/(32*
a**3)), True)) + 5*x/(32*a**3)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [A] (verification not implemented)

none

Time = 0.66 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.82

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{-\frac{60i\log(\tan(dx+c)+i)}{a^3}+\frac{60i\log(\tan(dx+c)-i)}{a^3}-\frac{12\,(5\tan(dx+c)+7i)}{a^3(i\tan(dx+c)-1)}+\frac{-125i\tan(dx+c)^4-596\tan(dx+c)^3+1110i\tan(dx+c)^2+996}{a^3(\tan(dx+c)-i)^4}}{768\,d}$$

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/768*(-60*I*log(tan(d*x + c) + I)/a^3 + 60*I*log(tan(d*x + c) - I)/a^3 -
12*(5*tan(d*x + c) + 7*I)/(a^3*(I*tan(d*x + c) - 1)) + (-125*I*tan(d*x + c)
^4 - 596*tan(d*x + c)^3 + 1110*I*tan(d*x + c)^2 + 996*tan(d*x + c) - 405*I)
/(a^3*(tan(d*x + c) - I)^4))/d
```

Mupad [B] (verification not implemented)

Time = 5.22 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.88

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{5\,x}{32\,a^3} + \frac{\frac{1}{3\,a^3}+\frac{35\tan(c+d\,x)^2}{96\,a^3}-\frac{5\tan(c+d\,x)^4}{32\,a^3}+\frac{\tan(c+d\,x)\,5\mathrm{i}}{32\,a^3}+\frac{\tan(c+d\,x)^3\,15\mathrm{i}}{32\,a^3}}{d\,\left(-\tan\left(c+d\,x\right)^5+\tan\left(c+d\,x\right)^4\,3\mathrm{i}+2\tan\left(c+d\,x\right)^3+\tan\left(c+d\,x\right)^2\,2\mathrm{i}+3\tan\left(c+d\,x\right)-\mathrm{i}\right)}$$

```
[In] int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] (5*x)/(32*a^3) + ((tan(c + d*x)*5i)/(32*a^3) + 1/(3*a^3) + (35*tan(c + d*x)
^2)/(96*a^3) + (tan(c + d*x)^3*15i)/(32*a^3) - (5*tan(c + d*x)^4)/(32*a^3))
/(d*(3*tan(c + d*x) + tan(c + d*x)^2*2i + 2*tan(c + d*x)^3 + tan(c + d*x)^4
*3i - tan(c + d*x)^5 - 1i))
```

3.139 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 195

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx &= \frac{21x}{128a^3}-\frac{i}{128ad(a-ia\tan(c+dx))^2}\\ &\quad+\frac{ia^2}{40d(a+ia\tan(c+dx))^5}+\frac{3ia}{64d(a+ia\tan(c+dx))^4}\\ &\quad+\frac{i}{16d(a+ia\tan(c+dx))^3}+\frac{5i}{64ad(a+ia\tan(c+dx))^2}\\ &\quad-\frac{3i}{64d\,(a^3-ia^3\tan(c+dx))}+\frac{15i}{128d\,(a^3+ia^3\tan(c+dx))}\end{aligned}$$

```
[Out] 21/128*x/a^3-1/128*I/a/d/(a-I*a*tan(d*x+c))^2+1/40*I*a^2/d/(a+I*a*tan(d*x+c
))^5+3/64*I*a/d/(a+I*a*tan(d*x+c))^4+1/16*I/d/(a+I*a*tan(d*x+c))^3+5/64*I/a
/d/(a+I*a*tan(d*x+c))^2-3/64*I/d/(a^3-I*a^3*tan(d*x+c))+15/128*I/d/(a^3+I*a
^3*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used

$= \{3568, 46, 212\}$

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{3i}{64d\,(a^3-ia^3\tan(c+dx))} + \frac{15i}{128d\,(a^3+ia^3\tan(c+dx))} + \frac{21x}{128a^3} + \frac{ia^2}{40d(a+ia\tan(c+dx))^5} + \frac{3ia}{64d(a+ia\tan(c+dx))^4} + \frac{i}{16d(a+ia\tan(c+dx))^3} - \frac{i}{128ad(a-ia\tan(c+dx))^2} + \frac{5i}{64ad(a+ia\tan(c+dx))^2}$$

```
[In] Int[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (21*x)/(128*a^3) - (I/128)/(a*d*(a - I*a*Tan[c + d*x])^2) + ((I/40)*a^2)/(d
*(a + I*a*Tan[c + d*x])^5) + (((3*I)/64)*a)/(d*(a + I*a*Tan[c + d*x])^4) +
(I/16)/(d*(a + I*a*Tan[c + d*x])^3) + ((5*I)/64)/(a*d*(a + I*a*Tan[c + d*x]
)^2) - ((3*I)/64)/(d*(a^3 - I*a^3*Tan[c + d*x])) + ((15*I)/128)/(d*(a^3 + I
*a^3*Tan[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{(ia^5)\,\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^6}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^5)\,\text{Subst}\left(\int \left(\frac{1}{64a^6(a-x)^3} + \frac{3}{64a^7(a-x)^2} + \frac{1}{8a^3(a+x)^6} + \frac{3}{16a^4(a+x)^5} + \frac{3}{16a^5(a+x)^4} + \frac{5}{32a^6(a+x)^3} + \frac{15}{128a^7(a+x)^2}\right.\right.}{d}$$

$$= -\frac{i}{128ad(a-ia\tan(c+dx))^2}+\frac{ia^2}{40d(a+ia\tan(c+dx))^5}+\frac{3ia}{64d(a+ia\tan(c+dx))^4}+\frac{i}{16d(a+ia\tan(c+dx))^3}+\frac{5i}{64ad(a+ia\tan(c+dx))^2}-\frac{3i}{64d(a^3-ia^3\tan(c+dx))}+\frac{15i}{128d(a^3+ia^3\tan(c+dx))}-\frac{(21i)\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx,x,ia\tan(c+dx)\right)}{128a^2d}$$

$$= \frac{21x}{128a^3}-\frac{i}{128ad(a-ia\tan(c+dx))^2}+\frac{ia^2}{40d(a+ia\tan(c+dx))^5}+\frac{3ia}{64d(a+ia\tan(c+dx))^4}+\frac{i}{16d(a+ia\tan(c+dx))^3}+\frac{5i}{64ad(a+ia\tan(c+dx))^2}-\frac{3i}{64d(a^3-ia^3\tan(c+dx))}+\frac{15i}{128d(a^3+ia^3\tan(c+dx))}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.82

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\sec^7(c+dx)(-1050\cos(c+dx)-469\cos(3(c+dx))+105\cos(5(c+dx))+6\cos(7(c+dx))-350i\sin(}{5120a^3d}$$

```
[In] Integrate[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^7*(-1050*Cos[c + d*x] - 469*Cos[3*(c + d*x)] + 105*Cos[5*(c +
 d*x)] + 6*Cos[7*(c + d*x)] - (350*I)*Sin[c + d*x] + (840*I)*ArcTan[Tan[c +
 d*x]]*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x)]) - (189*I)*Sin[3*(c + d*x)] +
 (175*I)*Sin[5*(c + d*x)] + (14*I)*Sin[7*(c + d*x)]))/(5120*a^3*d*(-I + Tan
[c + d*x])^5*(I + Tan[c + d*x])^2)
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.66

method	result
derivativedivides	$\frac{-\frac{21i\ln(\tan(dx+c)-i)}{256}+\frac{3i}{64(\tan(dx+c)-i)^4}-\frac{5i}{64(\tan(dx+c)-i)^2}+\frac{1}{40(\tan(dx+c)-i)^5}-\frac{1}{16(\tan(dx+c)-i)^3}+\frac{15}{128(\tan(dx+c)-i)}+\frac{}{128(}}{d\,a^3}$
default	$\frac{-\frac{21i\ln(\tan(dx+c)-i)}{256}+\frac{3i}{64(\tan(dx+c)-i)^4}-\frac{5i}{64(\tan(dx+c)-i)^2}+\frac{1}{40(\tan(dx+c)-i)^5}-\frac{1}{16(\tan(dx+c)-i)^3}+\frac{15}{128(\tan(dx+c)-i)}+\frac{}{128(}}{d\,a^3}$
risch	$\frac{21x}{128a^3}+\frac{7i e^{-6i(dx+c)}}{256a^3d}+\frac{7i e^{-8i(dx+c)}}{1024a^3d}+\frac{i e^{-10i(dx+c)}}{1280a^3d}+\frac{17i\cos(4dx+4c)}{256a^3d}+\frac{9\sin(4dx+4c)}{128a^3d}+\frac{7i\cos(2dx+2c)}{64a^3d}+$

```
[In] int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(-21/256*I*ln(tan(d*x+c)-I)+3/64*I/(tan(d*x+c)-I)^4-5/64*I/(tan(d*x
+c)-I)^2+1/40/(tan(d*x+c)-I)^5-1/16/(tan(d*x+c)-I)^3+15/128/(tan(d*x+c)-I)+
1/128*I/(tan(d*x+c)+I)^2+21/256*I*ln(tan(d*x+c)+I)+3/64/(tan(d*x+c)+I))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.50

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\left(840\,dxe^{(10i\,dx+10i\,c)}-10i\,e^{(14i\,dx+14i\,c)}-140i\,e^{(12i\,dx+12i\,c)}+700i\,e^{(8i\,dx+8i\,c)}+350i\,e^{(6i\,dx+6i\,c)}+140i\,e^{(4i\,dx+4}\right.}{5120\,a^3d}$$

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/5120*(840*d*x*e^(10*I*d*x + 10*I*c) - 10*I*e^(14*I*d*x + 14*I*c) - 140*I*
e^(12*I*d*x + 12*I*c) + 700*I*e^(8*I*d*x + 8*I*c) + 350*I*e^(6*I*d*x + 6*I*
c) + 140*I*e^(4*I*d*x + 4*I*c) + 35*I*e^(2*I*d*x + 2*I*c) + 4*I)*e^(-10*I*d
*x - 10*I*c)/(a^3*d)
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.50

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \begin{cases} \frac{\left(-1125899906842624 0ia^{18}d^6e^{34ic}e^{4idx}-15762598695796736 0ia^{18}d^6e^{32ic}e^{2idx}+78812993478983680 0ia^{18}d^6e^{28ic}e^{-2idx}+394064967394918400\right.}{57646075230} \\ x\left(\frac{(e^{14ic}+7e^{12ic}+21e^{10ic}+35e^{8ic}+35e^{6ic}+21e^{4ic}+7e^{2ic}+1)e^{-10ic}}{128a^3}-\frac{21}{128a^3}\right) \end{cases} + \frac{21x}{128a^3}$$

```
[In] integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise(((-11258999068426240*I*a**18*d**6*exp(34*I*c)*exp(4*I*d*x) - 1576
25986957967360*I*a**18*d**6*exp(32*I*c)*exp(2*I*d*x) + 788129934789836800*I
*a**18*d**6*exp(28*I*c)*exp(-2*I*d*x) + 394064967394918400*I*a**18*d**6*exp
(26*I*c)*exp(-4*I*d*x) + 157625986957967360*I*a**18*d**6*exp(24*I*c)*exp(-6
*I*d*x) + 39406496739491840*I*a**18*d**6*exp(22*I*c)*exp(-8*I*d*x) + 450359
9627370496*I*a**18*d**6*exp(20*I*c)*exp(-10*I*d*x))*exp(-30*I*c)/(576460752
3034234880*a**21*d**7), Ne(a**21*d**7*exp(30*I*c), 0)), (x*((exp(14*I*c) +
7*exp(12*I*c) + 21*exp(10*I*c) + 35*exp(8*I*c) + 35*exp(6*I*c) + 21*exp(4*I
*c) + 7*exp(2*I*c) + 1)*exp(-10*I*c)/(128*a**3) - 21/(128*a**3)), True)) +
21*x/(128*a**3)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [A] (verification not implemented)

none

Time = 0.68 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.70

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{-\frac{420i\log(\tan(dx+c)+i)}{a^3} + \frac{420i\log(\tan(dx+c)-i)}{a^3} + \frac{10\left(-63i\tan(dx+c)^2+150\tan(dx+c)+91i\right)}{a^3(i\tan(dx+c)-1)^2} - \frac{959i\tan(dx+c)^5+5395\tan(dx+c)^4 \cdots}{\cdots}}{5120\,d}$$

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/5120*(-420*I*log(tan(d*x + c) + I)/a^3 + 420*I*log(tan(d*x + c) - I)/a^3
 + 10*(-63*I*tan(d*x + c)^2 + 150*tan(d*x + c) + 91*I)/(a^3*(I*tan(d*x + c)
 - 1)^2) - (959*I*tan(d*x + c)^5 + 5395*tan(d*x + c)^4 - 12390*I*tan(d*x +
c)^3 - 14710*tan(d*x + c)^2 + 9275*I*tan(d*x + c) + 2647)/(a^3*(tan(d*x + c
) - I)^5))/d
```

Mupad [B] (verification not implemented)

Time = 5.63 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.89

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{21\,x}{128\,a^3}$$

$$+\frac{\frac{7\tan(c+d\,x)}{640\,a^3}+\frac{11\mathrm{i}}{40\,a^3}+\frac{\tan(c+d\,x)^2\,469\mathrm{i}}{640\,a^3}-\frac{21\tan(c+d\,x)^3}{32\,a^3}+\frac{\tan(c+d\,x)^4\,7\mathrm{i}}{32\,a^3}-\frac{63\tan(c+d\,x)^5}{128\,a^3}-\frac{\tan(c\cdots}{1\cdots}}{d\,\left(-\tan\left(c+d\,x\right)^7\,1\mathrm{i}-3\tan\left(c+d\,x\right)^6+\tan\left(c+d\,x\right)^5\,1\mathrm{i}-5\tan\left(c+d\,x\right)^4+\tan\left(c+d\,x\right)^3\,5\mathrm{i}-\tan\left(c\cdots\right.\right.}$$

```
[In] int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] (21*x)/(128*a^3) + ((7*tan(c + d*x))/(640*a^3) + 11i/(40*a^3) + (tan(c + d*
x)^2*469i)/(640*a^3) - (21*tan(c + d*x)^3)/(32*a^3) + (tan(c + d*x)^4*7i)/(
32*a^3) - (63*tan(c + d*x)^5)/(128*a^3) - (tan(c + d*x)^6*21i)/(128*a^3))/(
d*(tan(c + d*x)*3i - tan(c + d*x)^2 + tan(c + d*x)^3*5i - 5*tan(c + d*x)^4
+ tan(c + d*x)^5*1i - 3*tan(c + d*x)^6 - tan(c + d*x)^7*1i + 1))
```

3.140 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 119

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{7\text{arctanh}(\sin(c+dx))}{8a^3d} - \frac{7i\sec^5(c+dx)}{15a^3d} + \frac{7\sec(c+dx)\tan(c+dx)}{8a^3d} + \frac{7\sec^3(c+dx)\tan(c+dx)}{12a^3d} - \frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^2}$$

```
[Out] 7/8*arctanh(sin(d*x+c))/a^3/d-7/15*I*sec(d*x+c)^5/a^3/d+7/8*sec(d*x+c)*tan(
d*x+c)/a^3/d+7/12*sec(d*x+c)^3*tan(d*x+c)/a^3/d-2/3*I*sec(d*x+c)^7/a/d/(a+I
*a*tan(d*x+c))^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3581, 3582, 3853, 3855}

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{7\text{arctanh}(\sin(c+dx))}{8a^3d} - \frac{7i\sec^5(c+dx)}{15a^3d} + \frac{7\tan(c+dx)\sec^3(c+dx)}{12a^3d} + \frac{7\tan(c+dx)\sec(c+dx)}{8a^3d} - \frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^2}$$

```
[In] Int[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (7*ArcTanh[Sin[c + d*x]])/(8*a^3*d) - (((7*I)/15)*Sec[c + d*x]^5)/(a^3*d) +
 (7*Sec[c + d*x]*Tan[c + d*x])/(8*a^3*d) + (7*Sec[c + d*x]^3*Tan[c + d*x])/
(12*a^3*d) - (((2*I)/3)*Sec[c + d*x]^7)/(a*d*(a + I*a*Tan[c + d*x])^2)
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^2}+\frac{7\int\frac{\sec^7(c+dx)}{a+ia\tan(c+dx)}\,dx}{3a^2}\\
&= -\frac{7i\sec^5(c+dx)}{15a^3d}-\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^2}+\frac{7\int\sec^5(c+dx)\,dx}{3a^3}\\
&= -\frac{7i\sec^5(c+dx)}{15a^3d}+\frac{7\sec^3(c+dx)\tan(c+dx)}{12a^3d}\\
&\quad-\frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^2}+\frac{7\int\sec^3(c+dx)\,dx}{4a^3}
\end{aligned}$$

$$= -\frac{7i\sec^5(c+dx)}{15a^3d} + \frac{7\sec(c+dx)\tan(c+dx)}{8a^3d} + \frac{7\sec^3(c+dx)\tan(c+dx)}{12a^3d} - \frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^2} + \frac{7\int\sec(c+dx)\,dx}{8a^3}$$

$$= \frac{7\text{arctanh}(\sin(c+dx))}{8a^3d} - \frac{7i\sec^5(c+dx)}{15a^3d} + \frac{7\sec(c+dx)\tan(c+dx)}{8a^3d} + \frac{7\sec^3(c+dx)\tan(c+dx)}{12a^3d} - \frac{2i\sec^7(c+dx)}{3ad(a+ia\tan(c+dx))^2}$$

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.95

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{\sec^8(c+dx)(\cos(3(c+dx))+i\sin(3(c+dx)))\left(448+1680i\text{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)\cos^5(c+d\ldots\right.}{960a^3d(-i+\tan(c+dx))^3}$$

```
[In] Integrate[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^8*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x)])*(448 + (1680*I)*Arc
Tanh[Sin[c] + Cos[c]*Tan[(d*x)/2]]*Cos[c + d*x]^5 + 640*Cos[2*(c + d*x)] -
(150*I)*Sin[2*(c + d*x)] + (105*I)*Sin[4*(c + d*x)]))/(960*a^3*d*(-I + Tan[
c + d*x])^3)
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{i\left(105\,\mathrm{e}^{9i(dx+c)}+490\,\mathrm{e}^{7i(dx+c)}+896\,\mathrm{e}^{5i(dx+c)}+790\,\mathrm{e}^{3i(dx+c)}-105\,\mathrm{e}^{i(dx+c)}\right)}{60d\,a^3\left(\mathrm{e}^{2i(dx+c)}+1\right)^5}+\frac{7\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{8a^3d}-\frac{7\ln\left(\mathrm{e}^{i(dx+c)}-i\ldots\right.}{8a^3d}$
derivativedivides	$\frac{-\frac{i}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}+\frac{2\left(\frac{1}{16}+\frac{13i}{16}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2\left(-\frac{3}{8}-\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}+\frac{2\left(-\frac{5}{16}+\frac{11i}{16}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{2\left(-\frac{3}{4}+\frac{7i}{24}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\ldots}{8}}{a^3\ldots}$
default	$\frac{-\frac{i}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}+\frac{2\left(\frac{1}{16}+\frac{13i}{16}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2\left(-\frac{3}{8}-\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}+\frac{2\left(-\frac{5}{16}+\frac{11i}{16}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{2\left(-\frac{3}{4}+\frac{7i}{24}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\ldots}{8}}{a^3\ldots}$

```
[In] int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/60*I/d/a^3/(exp(2*I*(d*x+c))+1)^5*(105*exp(9*I*(d*x+c))+490*exp(7*I*(d*x
+c))+896*exp(5*I*(d*x+c))+790*exp(3*I*(d*x+c))-105*exp(I*(d*x+c)))+7/8/a^3/
d*ln(exp(I*(d*x+c))+I)-7/8/a^3/d*ln(exp(I*(d*x+c))-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(103) = 206$.

Time = 0.25 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.34

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{105\left(e^{(10i\,dx+10i\,c)}+5\,e^{(8i\,dx+8i\,c)}+10\,e^{(6i\,dx+6i\,c)}+10\,e^{(4i\,dx+4i\,c)}+5\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-105}{120\,(a^3de^{(10}}$$

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/120*(105*(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*I*d*x +
 6*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x
+ I*c) + I) - 105*(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*
I*d*x + 6*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*log(e^
(I*d*x + I*c) - I) - 210*I*e^(9*I*d*x + 9*I*c) - 980*I*e^(7*I*d*x + 7*I*c)
- 1792*I*e^(5*I*d*x + 5*I*c) - 1580*I*e^(3*I*d*x + 3*I*c) + 210*I*e^(I*d*x
+ I*c))/(a^3*d*e^(10*I*d*x + 10*I*c) + 5*a^3*d*e^(8*I*d*x + 8*I*c) + 10*a^3
*d*e^(6*I*d*x + 6*I*c) + 10*a^3*d*e^(4*I*d*x + 4*I*c) + 5*a^3*d*e^(2*I*d*x
+ 2*I*c) + a^3*d)
```

Sympy [F]

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{\sec^9(c+dx)}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

```
[In] integrate(sec(d*x+c)**9/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] I*Integral(sec(c + d*x)**9/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*tan(c
 + d*x) + I), x)/a**3
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(103) = 206$.

Time = 0.23 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.87

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\frac{16\left(-\frac{15i\sin(dx+c)}{\cos(dx+c)+1}+\frac{320\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{390i\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{400\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{960\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{390i\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{360\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{15i\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-136\right)}{-120i\,a^3+\frac{600i\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{1200i\,a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{1200i\,a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{600i\,a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{120i\,a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}}{8\,d}$$

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(16*(-15*I*sin(d*x + c)/(cos(d*x + c) + 1) + 320*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 + 390*I*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 400*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 + 960*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 390*I*
sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 360*sin(d*x + c)^8/(cos(d*x + c) + 1)
^8 + 15*I*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 136)/(-120*I*a^3 + 600*I*a^
3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1200*I*a^3*sin(d*x + c)^4/(cos(d*x
+ c) + 1)^4 + 1200*I*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 600*I*a^3*si
n(d*x + c)^8/(cos(d*x + c) + 1)^8 + 120*I*a^3*sin(d*x + c)^10/(cos(d*x + c)
 + 1)^10) + 7*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 - 7*log(sin(d*x
+ c)/(cos(d*x + c) + 1) - 1)/a^3)/d
```

Giac [A] (verification not implemented)

none

Time = 0.73 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.38

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{105\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^3} - \frac{105\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^3} + \frac{2\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+360i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-390\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-960i\,\tan(\cdots\right.}{\cdots}}{120\,d}$$

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/120*(105*log(tan(1/2*d*x + 1/2*c) + 1)/a^3 - 105*log(tan(1/2*d*x + 1/2*c)
 - 1)/a^3 + 2*(15*tan(1/2*d*x + 1/2*c)^9 + 360*I*tan(1/2*d*x + 1/2*c)^8 - 3
90*tan(1/2*d*x + 1/2*c)^7 - 960*I*tan(1/2*d*x + 1/2*c)^6 + 400*I*tan(1/2*d*
x + 1/2*c)^4 + 390*tan(1/2*d*x + 1/2*c)^3 - 320*I*tan(1/2*d*x + 1/2*c)^2 -
15*tan(1/2*d*x + 1/2*c) + 136*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^5*a^3))/d
```

Mupad [B] (verification not implemented)

Time = 6.98 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.26

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{7\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,a^3\,d}$$
$$+ \frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4} + \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8 6\mathrm{i} - \frac{13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2} - \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 16\mathrm{i} + \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 20\mathrm{i}}{3} + \frac{13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2} - \frac{\tan(\cdots}{\cdots}}{a^3\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)^5}$$

```
[In] int(1/(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^3),x)
```

```
[Out] (7*atanh(tan(c/2 + (d*x)/2)))/(4*a^3*d) + ((13*tan(c/2 + (d*x)/2)^3)/2 - (t
an(c/2 + (d*x)/2)^2*16i)/3 - tan(c/2 + (d*x)/2)/4 + (tan(c/2 + (d*x)/2)^4*2
0i)/3 - tan(c/2 + (d*x)/2)^6*16i - (13*tan(c/2 + (d*x)/2)^7)/2 + tan(c/2 +
(d*x)/2)^8*6i + tan(c/2 + (d*x)/2)^9/4 + 34i/15)/(a^3*d*(tan(c/2 + (d*x)/2)
^2 - 1)^5)
```

3.141 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 93

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{5\text{arctanh}(\sin(c+dx))}{2a^3d} - \frac{5i\sec^3(c+dx)}{3a^3d} + \frac{5\sec(c+dx)\tan(c+dx)}{2a^3d} - \frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^2}$$

```
[Out] 5/2*arctanh(sin(d*x+c))/a^3/d-5/3*I*sec(d*x+c)^3/a^3/d+5/2*sec(d*x+c)*tan(d
*x+c)/a^3/d-2*I*sec(d*x+c)^5/a/d/(a+I*a*tan(d*x+c))^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3581, 3582, 3853, 3855}

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{5\text{arctanh}(\sin(c+dx))}{2a^3d} - \frac{5i\sec^3(c+dx)}{3a^3d} + \frac{5\tan(c+dx)\sec(c+dx)}{2a^3d} - \frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^2}$$

```
[In] Int[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - (((5*I)/3)*Sec[c + d*x]^3)/(a^3*d) +
(5*Sec[c + d*x]*Tan[c + d*x])/(2*a^3*d) - ((2*I)*Sec[c + d*x]^5)/(a*d*(a +
I*a*Tan[c + d*x])^2)
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^2}+\frac{5\int\frac{\sec^5(c+dx)}{a+ia\tan(c+dx)}\,dx}{a^2}\\
&= -\frac{5i\sec^3(c+dx)}{3a^3d}-\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^2}+\frac{5\int\sec^3(c+dx)\,dx}{a^3}\\
&= -\frac{5i\sec^3(c+dx)}{3a^3d}+\frac{5\sec(c+dx)\tan(c+dx)}{2a^3d}-\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^2}+\frac{5\int\sec(c+dx)\,dx}{2a^3}\\
&= \frac{5\text{arctanh}(\sin(c+dx))}{2a^3d}-\frac{5i\sec^3(c+dx)}{3a^3d}\\
&\quad+\frac{5\sec(c+dx)\tan(c+dx)}{2a^3d}-\frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.68

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{60\text{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)-i\sec^3(c+dx)(20+24\cos(2(c+dx))-9i\sin(2(c+dx)))}{12a^3d}$$

```
[In] Integrate[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (60*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]] - I*Sec[c + d*x]^3*(20 + 24*Cos[2
*(c + d*x)] - (9*I)*Sin[2*(c + d*x)]))/(12*a^3*d)
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.08

method	result
risch	$-\frac{i\left(15\,\mathrm{e}^{5i(dx+c)}+40\,\mathrm{e}^{3i(dx+c)}+33\,\mathrm{e}^{i(dx+c)}\right)}{3d\,a^3\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}+\frac{5\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2a^3d}-\frac{5\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2a^3d}$
derivativedivides	$\frac{\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{2\left(\frac{3}{4}-\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{2\left(-\frac{3}{4}-\frac{7i}{4}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}-\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}+\frac{2\left(-\frac{3}{4}-\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-}{a^3d}$
default	$\frac{\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{2\left(\frac{3}{4}-\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{2\left(-\frac{3}{4}-\frac{7i}{4}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}-\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}+\frac{2\left(-\frac{3}{4}-\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-}{a^3d}$

```
[In] int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*I/d/a^3/(exp(2*I*(d*x+c))+1)^3*(15*exp(5*I*(d*x+c))+40*exp(3*I*(d*x+c)
)+33*exp(I*(d*x+c)))+5/2/a^3/d*ln(exp(I*(d*x+c))+I)-5/2/a^3/d*ln(exp(I*(d*x
+c))-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(81) = 162.

Time = 0.24 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.96

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{15\left(e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-15\left(e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e^{(2i\,c}\right.}{6\left(a^3de^{(6i\,dx+6i\,c)}+3\,a^3de^{(4i\,dx+4i\,c)}+3\,a^3de^{(\right.}}$$

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(15*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I*c
) + 1)*log(e^(I*d*x + I*c) + I) - 15*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x +
4*I*c) + 3*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) - I) - 30*I*e^(5*I*
d*x + 5*I*c) - 80*I*e^(3*I*d*x + 3*I*c) - 66*I*e^(I*d*x + I*c))/(a^3*d*e^(6
*I*d*x + 6*I*c) + 3*a^3*d*e^(4*I*d*x + 4*I*c) + 3*a^3*d*e^(2*I*d*x + 2*I*c)
 + a^3*d)
```

Sympy [F]

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{\sec^7(c+dx)}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

```
[In] integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] I*Integral(sec(c + d*x)**7/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*tan(c
 + d*x) + I), x)/a**3
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(81) = 162$.

Time = 0.32 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.31

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\frac{4\left(-\frac{9i\sin(dx+c)}{\cos(dx+c)+1}-\frac{48\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{18\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{9i\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+22\right)}{6ia^3-\frac{18ia^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{18ia^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{6ia^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{5\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}-\frac{5\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}}{2d}$$

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(4*(-9*I*sin(d*x + c)/(cos(d*x + c) + 1) - 48*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 18*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 9*I*sin(d*x + c)^5/(c
os(d*x + c) + 1)^5 + 22)/(6*I*a^3 - 18*I*a^3*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 18*I*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 6*I*a^3*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6) + 5*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 -
 5*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3)/d
```

Giac [A] (verification not implemented)

none

Time = 0.62 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.20

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{15\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^3} - \frac{15\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^3} - \frac{2\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 18i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 + 48i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 9\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3a^3}}{6d}$$

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(15*log(tan(1/2*d*x + 1/2*c) + 1)/a^3 - 15*log(tan(1/2*d*x + 1/2*c) - 1
)/a^3 - 2*(9*tan(1/2*d*x + 1/2*c)^5 - 18*I*tan(1/2*d*x + 1/2*c)^4 + 48*I*ta
n(1/2*d*x + 1/2*c)^2 - 9*tan(1/2*d*x + 1/2*c) - 22*I)/((tan(1/2*d*x + 1/2*c
)^2 - 1)^3*a^3))/d
```

Mupad [B] (verification not implemented)

Time = 6.07 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.45

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d} + \frac{\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^3} - \frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{a^3} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,16\mathrm{i}}{a^3} + \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,6\mathrm{i}}{a^3} + \frac{22\mathrm{i}}{3\,a^3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 - 3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + 3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 - 1\right)}$$

```
[In] int(1/(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^3),x)
```

```
[Out] (5*atanh(tan(c/2 + (d*x)/2)))/(a^3*d) + ((tan(c/2 + (d*x)/2)^4*6i)/a^3 - (t
an(c/2 + (d*x)/2)^2*16i)/a^3 - (3*tan(c/2 + (d*x)/2)^5)/a^3 + 22i/(3*a^3) +
 (3*tan(c/2 + (d*x)/2))/a^3)/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)
/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.142 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^3} dx = -\frac{3\text{arctanh}(\sin(c+dx))}{a^3d} + \frac{3i\sec(c+dx)}{a^3d} + \frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^2}$$

```
[Out] -3*arctanh(sin(d*x+c))/a^3/d+3*I*sec(d*x+c)/a^3/d+2*I*sec(d*x+c)^3/a/d/(a+I
*a*tan(d*x+c))^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3581, 3582, 3855}

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^3} dx = -\frac{3\text{arctanh}(\sin(c+dx))}{a^3d} + \frac{3i\sec(c+dx)}{a^3d} + \frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^2}$$

```
[In] Int[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (-3*ArcTanh[Sin[c + d*x]])/(a^3*d) + ((3*I)*Sec[c + d*x])/(a^3*d) + ((2*I)*
Sec[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*x])^2)
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
```

```
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^2} - \frac{3\int \frac{\sec^3(c+dx)}{a+ia\tan(c+dx)}\,dx}{a^2} \\
&= \frac{3i\sec(c+dx)}{a^3d} + \frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^2} - \frac{3\int \sec(c+dx)\,dx}{a^3} \\
&= -\frac{3\text{arctanh}(\sin(c+dx))}{a^3d} + \frac{3i\sec(c+dx)}{a^3d} + \frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.66

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\sec^3(c+dx)(i\cos(dx)-\sin(dx))^3\left(6\text{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)(\cos(3c)+i\sin(3c))+(\cos(2c-d\right.}{a^3d(-i+\tan(c+dx))^3}$$

```
[In] Integrate[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^3*(I*Cos[d*x] - Sin[d*x])^3*(6*ArcTanh[Sin[c] + Cos[c]*Tan[(d
*x)/2]]*(Cos[3*c] + I*Sin[3*c]) + (Cos[2*c - d*x] + I*Sin[2*c - d*x])*(-5*I
 + Tan[c + d*x])))/(a^3*d*(-I + Tan[c + d*x])^3)
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.32

method	result	size
derivativedivides	$\frac{\frac{8}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{i}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\frac{2i}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2}-3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^3 d}$	86
default	$\frac{\frac{8}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{i}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\frac{2i}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2}-3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^3 d}$	86
risch	$\frac{4i e^{-i(dx+c)}}{a^3 d}+\frac{2i e^{i(dx+c)}}{d a^3\left(e^{2i(dx+c)}+1\right)}-\frac{3\ln\left(e^{i(dx+c)}+i\right)}{a^3 d}+\frac{3\ln\left(e^{i(dx+c)}-i\right)}{a^3 d}$	93

```
[In] int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^3*(4/(-I+tan(1/2*d*x+1/2*c))-1/2*I/(tan(1/2*d*x+1/2*c)-1)+3/2*ln(tan(
1/2*d*x+1/2*c)-1)+1/2*I/(tan(1/2*d*x+1/2*c)+1)-3/2*ln(tan(1/2*d*x+1/2*c)+1)
)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.72

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{3\left(e^{(3i\,dx+3i\,c)}+e^{(i\,dx+i\,c)}\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-3\left(e^{(3i\,dx+3i\,c)}+e^{(i\,dx+i\,c)}\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-6i\,e^{(2i\,dx+2i\,c)}}{a^3de^{(3i\,dx+3i\,c)}+a^3de^{(i\,dx+i\,c)}}$$

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -(3*(e^(3*I*d*x + 3*I*c) + e^(I*d*x + I*c))*log(e^(I*d*x + I*c) + I) - 3*(e
^(3*I*d*x + 3*I*c) + e^(I*d*x + I*c))*log(e^(I*d*x + I*c) - I) - 6*I*e^(2*I
*d*x + 2*I*c) - 4*I)/(a^3*d*e^(3*I*d*x + 3*I*c) + a^3*d*e^(I*d*x + I*c))
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{\sec^5(c+dx)}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

```
[In] integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] I*Integral(sec(c + d*x)**5/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*tan(c
 + d*x) + I), x)/a**3
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 319 vs. $2(59) = 118$.

Time = 0.41 (sec) , antiderivative size = 319, normalized size of antiderivative = 4.91

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{6\left(\cos\left(3\,dx+3\,c\right)+\cos\left(dx+c\right)+i\,\sin\left(3\,dx+3\,c\right)+i\,\sin\left(dx+c\right)\right)\arctan\left(\cos\left(dx+c\right),\sin\left(dx+c\right)+\right.}{}$$

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] (6*(cos(3*d*x + 3*c) + cos(d*x + c) + I*sin(3*d*x + 3*c) + I*sin(d*x + c))*
arctan2(cos(d*x + c), sin(d*x + c) + 1) + 6*(cos(3*d*x + 3*c) + cos(d*x + c
) + I*sin(3*d*x + 3*c) + I*sin(d*x + c))*arctan2(cos(d*x + c), -sin(d*x + c
) + 1) + 3*(I*cos(3*d*x + 3*c) + I*cos(d*x + c) - sin(3*d*x + 3*c) - sin(d*
x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) + 3*(-I*c
os(3*d*x + 3*c) - I*cos(d*x + c) + sin(3*d*x + 3*c) + sin(d*x + c))*log(cos
(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1) + 12*cos(2*d*x + 2*c) +
12*I*sin(2*d*x + 2*c) + 8)/((-2*I*a^3*cos(3*d*x + 3*c) - 2*I*a^3*cos(d*x +
c) + 2*a^3*sin(3*d*x + 3*c) + 2*a^3*sin(d*x + c))*d)
```

Giac [A] (verification not implemented)

none

Time = 0.60 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.69

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\frac{3\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^3}-\frac{3\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^3}-\frac{2\left(4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-5\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)a^3}}{d}$$

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(3*log(tan(1/2*d*x + 1/2*c) + 1)/a^3 - 3*log(tan(1/2*d*x + 1/2*c) - 1)/a^3
 - 2*(4*tan(1/2*d*x + 1/2*c)^2 - I*tan(1/2*d*x + 1/2*c) - 5)/((tan(1/2*d*x
+ 1/2*c)^3 - I*tan(1/2*d*x + 1/2*c)^2 - tan(1/2*d*x + 1/2*c) + I)*a^3))/d
```

Mupad [B] (verification not implemented)

Time = 4.42 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.62

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= -\frac{6\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d} - \frac{\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^3}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,8\mathrm{i}}{a^3}-\frac{10\mathrm{i}}{a^3}}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,1\mathrm{i}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}+1\right)}$$

```
[In] int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^3),x)
```

```
[Out] - (6*atanh(tan(c/2 + (d*x)/2)))/(a^3*d) - ((tan(c/2 + (d*x)/2)^2*8i)/a^3 -
10i/a^3 + (2*tan(c/2 + (d*x)/2))/a^3)/(d*(tan(c/2 + (d*x)/2)*1i - tan(c/2 +
 (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*1i + 1))
```

3.143 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 32

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx=\frac{i\sec^3(c+dx)}{3d(a+ia\tan(c+dx))^3}$$

```
[Out] 1/3*I*sec(d*x+c)^3/d/(a+I*a*tan(d*x+c))^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {3569}

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx=\frac{i\sec^3(c+dx)}{3d(a+ia\tan(c+dx))^3}$$

```
[In] Int[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^3,x]
[Out] ((I/3)*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^3)
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\text{integral}=\frac{i\sec^3(c+dx)}{3d(a+ia\tan(c+dx))^3}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\sec^3(c+dx)}{3d(a+ia\tan(c+dx))^3}$$

```
[In] Integrate[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^3,x]
[Out] ((I/3)*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^3)
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.59

method	result	size
risch	$\frac{i\mathrm{e}^{-3i(dx+c)}}{3a^3d}$	19
derivativedivides	$\frac{-\frac{8}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}}{a^3d}$	57
default	$\frac{-\frac{8}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}}{a^3d}$	57

```
[In] int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 1/3*I/a^3/d*exp(-3*I*(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.53

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\,e^{(-3i\,dx-3i\,c)}}{3\,a^3d}$$

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
[Out] 1/3*I*e^(-3*I*d*x - 3*I*c)/(a^3*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(26) = 52.

Time = 0.80 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.50

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \begin{cases} -\frac{\sec^3(c+dx)}{3a^3d\tan^3(c+dx)-9ia^3d\tan^2(c+dx)-9a^3d\tan(c+dx)+3ia^3d} & \text{for } d\neq 0 \\ \frac{x\sec^3(c)}{(ia\tan(c)+a)^3} & \text{otherwise} \end{cases}$$

```
[In] integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise((-sec(c + d*x)**3/(3*a**3*d*tan(c + d*x)**3 - 9*I*a**3*d*tan(c +
d*x)**2 - 9*a**3*d*tan(c + d*x) + 3*I*a**3*d), Ne(d, 0)), (x*sec(c)**3/(I*a
*tan(c) + a)**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\cos(3\,dx+3\,c)+\sin(3\,dx+3\,c)}{3\,a^3d}$$

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*(I*cos(3*d*x + 3*c) + sin(3*d*x + 3*c))/(a^3*d)
```

Giac [A] (verification not implemented)

none

Time = 0.56 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{2\left(3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}{3\,a^3d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^3}$$

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 2/3*(3*tan(1/2*d*x + 1/2*c)^2 - 1)/(a^3*d*(tan(1/2*d*x + 1/2*c) - I)^3)
```

Mupad [B] (verification not implemented)

Time = 3.92 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.12

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{2\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 3\mathrm{i}-\mathrm{i}\right)}{3\,a^3\,d\,\left(-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 1\mathrm{i}-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,3\mathrm{i}+1\right)}$$

```
[In] int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^3),x)
```

```
[Out] -(2*(tan(c/2 + (d*x)/2)^2*3i - 1i))/(3*a^3*d*(tan(c/2 + (d*x)/2)*3i - 3*tan
(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*1i + 1))
```

3.144 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 98

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^3} dx = \frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3} + \frac{2i\sec(c+dx)}{15ad(a+ia\tan(c+dx))^2} + \frac{2i\sec(c+dx)}{15d(a^3+ia^3\tan(c+dx))}$$

```
[Out] 1/5*I*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^3+2/15*I*sec(d*x+c)/a/d/(a+I*a*tan(d*
x+c))^2+2/15*I*sec(d*x+c)/d/(a^3+I*a^3*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3583, 3569}

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^3} dx = \frac{2i\sec(c+dx)}{15d(a^3+ia^3\tan(c+dx))} + \frac{2i\sec(c+dx)}{15ad(a+ia\tan(c+dx))^2} + \frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}$$

```
[In] Int[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((I/5)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^3) + (((2*I)/15)*Sec[c + d*x
])/(a*d*(a + I*a*Tan[c + d*x])^2) + (((2*I)/15)*Sec[c + d*x])/(d*(a^3 + I*a
^3*Tan[c + d*x]))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
```

```
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}+\frac{2\int\frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{5a}\\ &= \frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}+\frac{2i\sec(c+dx)}{15ad(a+ia\tan(c+dx))^2}+\frac{2\int\frac{\sec(c+dx)}{a+ia\tan(c+dx)}\,dx}{15a^2}\\ &= \frac{i\sec(c+dx)}{5d(a+ia\tan(c+dx))^3}+\frac{2i\sec(c+dx)}{15ad(a+ia\tan(c+dx))^2}+\frac{2i\sec(c+dx)}{15d\left(a^3+ia^3\tan(c+dx)\right)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.55

$$\int\frac{\sec(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\sec^3(c+dx)(5+9\cos(2(c+dx))+6i\sin(2(c+dx)))}{30a^3d(-i+\tan(c+dx))^3}$$

```
[In] Integrate[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^3,x]
[Out] -1/30*(Sec[c + d*x]^3*(5 + 9*Cos[2*(c + d*x)] + (6*I)*Sin[2*(c + d*x)]))/(a
^3*d*(-I + Tan[c + d*x])^3)
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.57

method	result	size
risch	$\frac{i e^{-i(dx+c)}}{4a^3d}+\frac{i e^{-3i(dx+c)}}{6a^3d}+\frac{i e^{-5i(dx+c)}}{20a^3d}$	56
derivativedivides	$\frac{\frac{8}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}-\frac{16}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}}{a^3d}$	90
default	$\frac{\frac{8}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}-\frac{16}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}}{a^3d}$	90

```
[In] int(sec(d*x+c)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 1/4*I/a^3/d*exp(-I*(d*x+c))+1/6*I/a^3/d*exp(-3*I*(d*x+c))+1/20*I/a^3/d*exp(
-5*I*(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.42

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^3}\,dx=\frac{\left(15i\,e^{(4i\,dx+4i\,c)}+10i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-5i\,dx-5i\,c)}}{60\,a^3d}$$

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
[Out] 1/60*(15*I*e^(4*I*d*x + 4*I*c) + 10*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(-5*I*d*
x - 5*I*c)/(a^3*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(82) = 164.

Time = 0.85 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.23

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$=\begin{cases}\frac{2\tan^2(c+dx)\sec(c+dx)}{15a^3d\tan^3(c+dx)-45ia^3d\tan^2(c+dx)-45a^3d\tan(c+dx)+15ia^3d}-\frac{6i\tan(c+dx)\sec(c+dx)}{15a^3d\tan^3(c+dx)-45ia^3d\tan^2(c+dx)-45a^3d\tan(c+dx)+15ia^3d}- \\ \frac{x\sec(c)}{(ia\tan(c)+a)^3}\end{cases}$$

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))**3,x)
[Out] Piecewise((2*tan(c + d*x)**2*sec(c + d*x)/(15*a**3*d*tan(c + d*x)**3 - 45*I
*a**3*d*tan(c + d*x)**2 - 45*a**3*d*tan(c + d*x) + 15*I*a**3*d) - 6*I*tan(c
 + d*x)*sec(c + d*x)/(15*a**3*d*tan(c + d*x)**3 - 45*I*a**3*d*tan(c + d*x)*
*2 - 45*a**3*d*tan(c + d*x) + 15*I*a**3*d) - 7*sec(c + d*x)/(15*a**3*d*tan(
c + d*x)**3 - 45*I*a**3*d*tan(c + d*x)**2 - 45*a**3*d*tan(c + d*x) + 15*I*a
**3*d), Ne(d, 0)), (x*sec(c)/(I*a*tan(c) + a)**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.70

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{3i\cos(5dx+5c)+10i\cos(3dx+3c)+15i\cos(dx+c)+3\sin(5dx+5c)+10\sin(3dx+3c)+15\,\mathrm{si}}{60a^3d}$$

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(3*I*cos(5*d*x + 5*c) + 10*I*cos(3*d*x + 3*c) + 15*I*cos(d*x + c) + 3*
sin(5*d*x + 5*c) + 10*sin(3*d*x + 3*c) + 15*sin(d*x + c))/(a^3*d)
```

Giac [A] (verification not implemented)

none

Time = 0.55 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.74

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{2\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-30i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-40\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+20i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+7\right)}{15a^3d\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^5}$$

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 2/15*(15*tan(1/2*d*x + 1/2*c)^4 - 30*I*tan(1/2*d*x + 1/2*c)^3 - 40*tan(1/2*
d*x + 1/2*c)^2 + 20*I*tan(1/2*d*x + 1/2*c) + 7)/(a^3*d*(tan(1/2*d*x + 1/2*c
) - I)^5)
```

Mupad [B] (verification not implemented)

Time = 4.74 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.36

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{2\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 15\mathrm{i}+30\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 40\mathrm{i}-20\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+7\mathrm{i}\right)}{15a^3d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 1\mathrm{i}+5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 10\mathrm{i}-10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)5\mathrm{i}+1\right)}$$

```
[In] int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^3),x)
```

```
[Out] (2*(30*tan(c/2 + (d*x)/2)^3 - tan(c/2 + (d*x)/2)^2*40i - 20*tan(c/2 + (d*x)
/2) + tan(c/2 + (d*x)/2)^4*15i + 7i))/(15*a^3*d*(tan(c/2 + (d*x)/2)*5i - 10
*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*10i + 5*tan(c/2 + (d*x)/2)^4 +
 tan(c/2 + (d*x)/2)^5*1i + 1))
```

3.145 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 101

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{12\sin(c+dx)}{35a^3d} - \frac{4\sin^3(c+dx)}{35a^3d} + \frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3} + \frac{8i\cos^3(c+dx)}{35d\,(a^3+ia^3\tan(c+dx))}$$

```
[Out] 12/35*sin(d*x+c)/a^3/d-4/35*sin(d*x+c)^3/a^3/d+1/7*I*cos(d*x+c)/d/(a+I*a*ta
n(d*x+c))^3+8/35*I*cos(d*x+c)^3/d/(a^3+I*a^3*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3583, 3581, 2713}

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{4\sin^3(c+dx)}{35a^3d} + \frac{12\sin(c+dx)}{35a^3d} + \frac{8i\cos^3(c+dx)}{35d\,(a^3+ia^3\tan(c+dx))} + \frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}$$

```
[In] Int[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (12*Sin[c + d*x])/(35*a^3*d) - (4*Sin[c + d*x]^3)/(35*a^3*d) + ((I/7)*Cos[c
 + d*x])/(d*(a + I*a*Tan[c + d*x])^3) + (((8*I)/35)*Cos[c + d*x]^3)/(d*(a^3
 + I*a^3*Tan[c + d*x]))
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}+\frac{4\int\frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{7a}\\
&= \frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}+\frac{8i\cos^3(c+dx)}{35d\,(a^3+ia^3\tan(c+dx))}+\frac{12\int\cos^3(c+dx)\,dx}{35a^3}\\
&= \frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}+\frac{8i\cos^3(c+dx)}{35d\,(a^3+ia^3\tan(c+dx))}\\
&\quad-\frac{12\text{Subst}\left(\int\left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{35a^3d}\\
&= \frac{12\sin(c+dx)}{35a^3d}-\frac{4\sin^3(c+dx)}{35a^3d}+\frac{i\cos(c+dx)}{7d(a+ia\tan(c+dx))^3}+\frac{8i\cos^3(c+dx)}{35d\,(a^3+ia^3\tan(c+dx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.75

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\sec^3(c+dx)(35+84\cos(2(c+dx))-15\cos(4(c+dx))+56i\sin(2(c+dx))-20i\sin(4(c+dx)))}{280a^3d(-i+\tan(c+dx))^3}$$

```
[In] Integrate[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] -1/280*(Sec[c + d*x]^3*(35 + 84*Cos[2*(c + d*x)] - 15*Cos[4*(c + d*x)] + (5
6*I)*Sin[2*(c + d*x)] - (20*I)*Sin[4*(c + d*x)]))/(a^3*d*(-I + Tan[c + d*x]
)^3)
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.84

method	result
risch	$\frac{i\mathrm{e}^{-3i(dx+c)}}{8a^3d}+\frac{i\mathrm{e}^{-5i(dx+c)}}{20a^3d}+\frac{i\mathrm{e}^{-7i(dx+c)}}{112a^3d}+\frac{3i\cos(dx+c)}{16a^3d}+\frac{5\sin(dx+c)}{16a^3d}$
derivativedivides	$\frac{\frac{2}{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+16i}+\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{9i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{17i}{4\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{8}{7\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}+\frac{38}{5(-i+\tan(\cdots}}{a^3d}$
default	$\frac{\frac{2}{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+16i}+\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{9i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{17i}{4\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{8}{7\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}+\frac{38}{5(-i+\tan(\cdots}}{a^3d}$

```
[In] int(cos(d*x+c)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*I/a^3/d*exp(-3*I*(d*x+c))+1/20*I/a^3/d*exp(-5*I*(d*x+c))+1/112*I/a^3/d*
exp(-7*I*(d*x+c))+3/16*I/a^3/d*cos(d*x+c)+5/16*sin(d*x+c)/a^3/d
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.62

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\left(-35i\,e^{(8i\,dx+8i\,c)}+140i\,e^{(6i\,dx+6i\,c)}+70i\,e^{(4i\,dx+4i\,c)}+28i\,e^{(2i\,dx+2i\,c)}+5i\right)e^{(-7i\,dx-7i\,c)}}{560\,a^3d}$$

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/560*(-35*I*e^(8*I*d*x + 8*I*c) + 140*I*e^(6*I*d*x + 6*I*c) + 70*I*e^(4*I*
d*x + 4*I*c) + 28*I*e^(2*I*d*x + 2*I*c) + 5*I)*e^(-7*I*d*x - 7*I*c)/(a^3*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(87) = 174.

Time = 0.30 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.95

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \begin{cases} \frac{(-71680ia^{12}d^4e^{17ic}e^{idx}+286720ia^{12}d^4e^{15ic}e^{-idx}+143360ia^{12}d^4e^{13ic}e^{-3idx}+57344ia^{12}d^4e^{11ic}e^{-5idx}+10240ia^{12}d^4e^{9ic}e^{-7idx})e^{-16ic}}{1146880a^{15}d^5} & \text{for} \\ \frac{x(e^{8ic}+4e^{6ic}+6e^{4ic}+4e^{2ic}+1)e^{-7ic}}{16a^3} & \text{otl} \end{cases}$$

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise((((-71680*I*a**12*d**4*exp(17*I*c)*exp(I*d*x) + 286720*I*a**12*d**
4*exp(15*I*c)*exp(-I*d*x) + 143360*I*a**12*d**4*exp(13*I*c)*exp(-3*I*d*x) +
 57344*I*a**12*d**4*exp(11*I*c)*exp(-5*I*d*x) + 10240*I*a**12*d**4*exp(9*I*
c)*exp(-7*I*d*x))*exp(-16*I*c)/(1146880*a**15*d**5), Ne(a**15*d**5*exp(16*I
*c), 0)), (x*(exp(8*I*c) + 4*exp(6*I*c) + 6*exp(4*I*c) + 4*exp(2*I*c) + 1)*
exp(-7*I*c)/(16*a**3), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [A] (verification not implemented)

none

Time = 0.64 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.18

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{35}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)} + \frac{525\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-1960i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-4025\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+4480i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+3143\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^7}}{280\,d}$$

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/280*(35/(a^3*(tan(1/2*d*x + 1/2*c) + I)) + (525*tan(1/2*d*x + 1/2*c)^6 -
1960*I*tan(1/2*d*x + 1/2*c)^5 - 4025*tan(1/2*d*x + 1/2*c)^4 + 4480*I*tan(1/
2*d*x + 1/2*c)^3 + 3143*tan(1/2*d*x + 1/2*c)^2 - 1176*I*tan(1/2*d*x + 1/2*c
) - 243)/(a^3*(tan(1/2*d*x + 1/2*c) - I)^7))/d
```

Mupad [B] (verification not implemented)

Time = 6.59 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.33

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = -\frac{\left(35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 105\mathrm{i}-175\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 105\mathrm{i}-7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\mathrm{ta}\right.}{35\,a^3\,d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\mathrm{i}\right)\,\left(1+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,1\mathrm{i}\right)^7}$$

```
[In] int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] -((43*tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2*77i - 7*tan(c/2 + (d*x)/2)^
3 + tan(c/2 + (d*x)/2)^4*105i - 175*tan(c/2 + (d*x)/2)^5 - tan(c/2 + (d*x)/
2)^6*105i + 35*tan(c/2 + (d*x)/2)^7 - 13i)*2i)/(35*a^3*d*(tan(c/2 + (d*x)/2
) + 1i)*(tan(c/2 + (d*x)/2)*1i + 1)^7)
```

3.146 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 121

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{10\sin(c+dx)}{21a^3d} - \frac{20\sin^3(c+dx)}{63a^3d} + \frac{2\sin^5(c+dx)}{21a^3d} + \frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3} + \frac{4i\cos^5(c+dx)}{21d\left(a^3+ia^3\tan(c+dx)\right)}$$

```
[Out] 10/21*sin(d*x+c)/a^3/d-20/63*sin(d*x+c)^3/a^3/d+2/21*sin(d*x+c)^5/a^3/d+1/9
*I*cos(d*x+c)^3/d/(a+I*a*tan(d*x+c))^3+4/21*I*cos(d*x+c)^5/d/(a^3+I*a^3*tan
(d*x+c))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3583, 3581, 2713}

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{2\sin^5(c+dx)}{21a^3d} - \frac{20\sin^3(c+dx)}{63a^3d} + \frac{10\sin(c+dx)}{21a^3d} + \frac{4i\cos^5(c+dx)}{21d\left(a^3+ia^3\tan(c+dx)\right)} + \frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}$$

```
[In] Int[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (10*Sin[c + d*x])/(21*a^3*d) - (20*Sin[c + d*x]^3)/(63*a^3*d) + (2*Sin[c +
d*x]^5)/(21*a^3*d) + ((I/9)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^3) +
(((4*I)/21)*Cos[c + d*x]^5)/(d*(a^3 + I*a^3*Tan[c + d*x]))
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}+\frac{2\int\frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{3a}\\
&= \frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}+\frac{4i\cos^5(c+dx)}{21d\,(a^3+ia^3\tan(c+dx))}+\frac{10\int\cos^5(c+dx)\,dx}{21a^3}\\
&= \frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}+\frac{4i\cos^5(c+dx)}{21d\,(a^3+ia^3\tan(c+dx))}\\
&\quad -\frac{10\mathrm{Subst}\left(\int\left(1-2x^2+x^4\right)\,dx,x,-\sin(c+dx)\right)}{21a^3d}\\
&= \frac{10\sin(c+dx)}{21a^3d}-\frac{20\sin^3(c+dx)}{63a^3d}+\frac{2\sin^5(c+dx)}{21a^3d}\\
&\quad +\frac{i\cos^3(c+dx)}{9d(a+ia\tan(c+dx))^3}+\frac{4i\cos^5(c+dx)}{21d\,(a^3+ia^3\tan(c+dx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.81

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\sec^3(c+dx)(-210-567\cos(2(c+dx))+162\cos(4(c+dx))+7\cos(6(c+dx))-378i\sin(2(c+dx))+2\cdots}{2016a^3d(-i+\tan(c+dx))^3}$$

```
[In] Integrate[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^3*(-210 - 567*Cos[2*(c + d*x)] + 162*Cos[4*(c + d*x)] + 7*Cos
[6*(c + d*x)] - (378*I)*Sin[2*(c + d*x)] + (216*I)*Sin[4*(c + d*x)] + (14*I
)*Sin[6*(c + d*x)]))/(2016*a^3*d*(-I + Tan[c + d*x])^3)
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.99

method	result
risch	$\frac{3i e^{-5i(dx+c)}}{64a^3d}+\frac{3i e^{-7i(dx+c)}}{224a^3d}+\frac{i e^{-9i(dx+c)}}{576a^3d}+\frac{9i\cos(dx+c)}{64a^3d}+\frac{21\sin(dx+c)}{64a^3d}+\frac{19i\cos(3dx+3c)}{192a^3d}+\frac{7\sin(3dx+3c)}{64a^3d}\cdots$
derivativedivides	$\frac{46i}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}+\frac{9i}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{59i}{4\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{8}{9\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9}-\cdots$
default	$\frac{46i}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}+\frac{9i}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{59i}{4\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{8}{9\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9}-\cdots$

```
[In] int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/64*I/a^3/d*exp(-5*I*(d*x+c))+3/224*I/a^3/d*exp(-7*I*(d*x+c))+1/576*I/a^3/
d*exp(-9*I*(d*x+c))+9/64*I/a^3/d*cos(d*x+c)+21/64*sin(d*x+c)/a^3/d+19/192*I
/a^3/d*cos(3*d*x+3*c)+7/64/a^3/d*sin(3*d*x+3*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.70

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\left(-21i\,e^{(12i\,dx+12i\,c)}-378i\,e^{(10i\,dx+10i\,c)}+945i\,e^{(8i\,dx+8i\,c)}+420i\,e^{(6i\,dx+6i\,c)}+189i\,e^{(4i\,dx+4i\,c)}+54i\,e^{(2i\,dx+2i\,c)}\cdots\right.}{4032\,a^3d}$$

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

[Out] 1/4032*(-21*I*e^(12*I*d*x + 12*I*c) - 378*I*e^(10*I*d*x + 10*I*c) + 945*I*e ^(8*I*d*x + 8*I*c) + 420*I*e^(6*I*d*x + 6*I*c) + 189*I*e^(4*I*d*x + 4*I*c) + 54*I*e^(2*I*d*x + 2*I*c) + 7*I)*e^(-9*I*d*x - 9*I*c)/(a^3*d)

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(105) = 210$.

Time = 0.36 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.19

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \begin{cases} \frac{(-811748818944ia^{18}d^6e^{28ic}e^{3idx}-14611478740992ia^{18}d^6e^{26ic}e^{idx}+36528696852480ia^{18}d^6e^{24ic}e^{-idx}+16234976378880ia^{18}d^6e^{22ic}e^{-3idx}+}{155855773237248a^{21}d^7} \\ \frac{x(e^{12ic}+6e^{10ic}+15e^{8ic}+20e^{6ic}+15e^{4ic}+6e^{2ic}+1)e^{-9ic}}{64a^3} \end{cases}$$

[In] integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c))**3,x)

[Out] Piecewise(((-811748818944*I*a**18*d**6*exp(28*I*c)*exp(3*I*d*x) - 146114787 40992*I*a**18*d**6*exp(26*I*c)*exp(I*d*x) + 36528696852480*I*a**18*d**6*exp (24*I*c)*exp(-I*d*x) + 16234976378880*I*a**18*d**6*exp(22*I*c)*exp(-3*I*d*x) + 7305739370496*I*a**18*d**6*exp(20*I*c)*exp(-5*I*d*x) + 2087354105856*I* a**18*d**6*exp(18*I*c)*exp(-7*I*d*x) + 270582939648*I*a**18*d**6*exp(16*I*c)*exp(-9*I*d*x))*exp(-25*I*c)/(155855773237248*a**21*d**7), Ne(a**21*d**7*e xp(25*I*c), 0)), (x*(exp(12*I*c) + 6*exp(10*I*c) + 15*exp(8*I*c) + 20*exp(6 *I*c) + 15*exp(4*I*c) + 6*exp(2*I*c) + 1)*exp(-9*I*c)/(64*a**3), True))

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ e exponent.

Giac [A] (verification not implemented)

none

Time = 0.76 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.41

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\frac{21\left(21\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+36i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-19\right)}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^3}+\frac{3591\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8-19656i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-56196\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+95760i\tan\left(\frac{1}{2}dx+\right.}{a}}{2016\,d}$$

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2016*(21*(21*tan(1/2*d*x + 1/2*c)^2 + 36*I*tan(1/2*d*x + 1/2*c) - 19)/(a^
3*(tan(1/2*d*x + 1/2*c) + I)^3) + (3591*tan(1/2*d*x + 1/2*c)^8 - 19656*I*ta
n(1/2*d*x + 1/2*c)^7 - 56196*tan(1/2*d*x + 1/2*c)^6 + 95760*I*tan(1/2*d*x +
 1/2*c)^5 + 107730*tan(1/2*d*x + 1/2*c)^4 - 79464*I*tan(1/2*d*x + 1/2*c)^3
- 38484*tan(1/2*d*x + 1/2*c)^2 + 10944*I*tan(1/2*d*x + 1/2*c) + 1615)/(a^3*
(tan(1/2*d*x + 1/2*c) - I)^9))/d
```

Mupad [B] (verification not implemented)

Time = 7.19 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.55

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \frac{\left(63\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}189\mathrm{i}-273\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}63\mathrm{i}-378\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}+\tan\right.}{63\,a^3\,d\left(\tan\left(\frac{c}{2}+\right.\right.}$$

```
[In] int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] ((51*tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2*39i + 235*tan(c/2 + (d*x)/2)
^3 + tan(c/2 + (d*x)/2)^4*450i - 306*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*x)
/2)^6*294i - 378*tan(c/2 + (d*x)/2)^7 - tan(c/2 + (d*x)/2)^8*63i - 273*tan(
c/2 + (d*x)/2)^9 - tan(c/2 + (d*x)/2)^10*189i + 63*tan(c/2 + (d*x)/2)^11 -
19i)*2i)/(63*a^3*d*(tan(c/2 + (d*x)/2) + 1i)^3*(tan(c/2 + (d*x)/2)*1i + 1)^
9)
```

3.147 $\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^3} dx$

Optimal result

Integrand size = 24, antiderivative size = 139

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^3} dx = \frac{56\sin(c+dx)}{99a^3d} - \frac{56\sin^3(c+dx)}{99a^3d} + \frac{56\sin^5(c+dx)}{165a^3d} - \frac{8\sin^7(c+dx)}{99a^3d} + \frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+dx))^3} + \frac{16i\cos^7(c+dx)}{99d\,(a^3+ia^3\tan(c+dx))}$$

```
[Out] 56/99*sin(d*x+c)/a^3/d-56/99*sin(d*x+c)^3/a^3/d+56/165*sin(d*x+c)^5/a^3/d-8
/99*sin(d*x+c)^7/a^3/d+1/11*I*cos(d*x+c)^5/d/(a+I*a*tan(d*x+c))^3+16/99*I*c
os(d*x+c)^7/d/(a^3+I*a^3*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3583, 3581, 2713}

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^3} dx = -\frac{8\sin^7(c+dx)}{99a^3d} + \frac{56\sin^5(c+dx)}{165a^3d} - \frac{56\sin^3(c+dx)}{99a^3d} + \frac{56\sin(c+dx)}{99a^3d} + \frac{16i\cos^7(c+dx)}{99d\,(a^3+ia^3\tan(c+dx))} + \frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+dx))^3}$$

```
[In] Int[Cos[c + d*x]^5/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (56*Sin[c + d*x])/(99*a^3*d) - (56*Sin[c + d*x]^3)/(99*a^3*d) + (56*Sin[c +
 d*x]^5)/(165*a^3*d) - (8*Sin[c + d*x]^7)/(99*a^3*d) + ((I/11)*Cos[c + d*x]
^5)/(d*(a + I*a*Tan[c + d*x])^3) + (((16*I)/99)*Cos[c + d*x]^7)/(d*(a^3 + I
*a^3*Tan[c + d*x]))
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+dx))^3}+\frac{8\int\frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{11a}\\
&= \frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+dx))^3}+\frac{16i\cos^7(c+dx)}{99d\,(a^3+ia^3\tan(c+dx))}+\frac{56\int\cos^7(c+dx)\,dx}{99a^3}\\
&= \frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+dx))^3}+\frac{16i\cos^7(c+dx)}{99d\,(a^3+ia^3\tan(c+dx))}\\
&\quad-\frac{56\text{Subst}\left(\int\left(1-3x^2+3x^4-x^6\right)\,dx,x,-\sin(c+dx)\right)}{99a^3d}\\
&= \frac{56\sin(c+dx)}{99a^3d}-\frac{56\sin^3(c+dx)}{99a^3d}+\frac{56\sin^5(c+dx)}{165a^3d}-\frac{8\sin^7(c+dx)}{99a^3d}\\
&\quad+\frac{i\cos^5(c+dx)}{11d(a+ia\tan(c+dx))^3}+\frac{16i\cos^7(c+dx)}{99d\,(a^3+ia^3\tan(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.86

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{\sec^3(c+dx)(-5775-16632\cos(2(c+dx))+5940\cos(4(c+dx))+440\cos(6(c+dx))+27\cos(8(c+dx$$
$$63360a^3d(-i+\tan($$

```
[In] Integrate[Cos[c + d*x]^5/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^3*(-5775 - 16632*Cos[2*(c + d*x)] + 5940*Cos[4*(c + d*x)] + 4
40*Cos[6*(c + d*x)] + 27*Cos[8*(c + d*x)] - (11088*I)*Sin[2*(c + d*x)] + (7
920*I)*Sin[4*(c + d*x)] + (880*I)*Sin[6*(c + d*x)] + (72*I)*Sin[8*(c + d*x)
]))/(63360*a^3*d*(-I + Tan[c + d*x])^3)
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.12

method	result
risch	$\frac{i\mathrm{e}^{-7i(dx+c)}}{64a^3d}+\frac{i\mathrm{e}^{-9i(dx+c)}}{288a^3d}+\frac{i\mathrm{e}^{-11i(dx+c)}}{2816a^3d}+\frac{7i\cos(dx+c)}{64a^3d}+\frac{21\sin(dx+c)}{64a^3d}+\frac{11i\cos(5dx+5c)}{256a^3d}+\frac{57\sin(5dx+5}{1280a^3d}$
derivativedivides	$\frac{i}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^4}+\frac{217i}{6\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+\frac{1}{40\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^5}-\frac{7}{48\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^3}+\frac{37}{128\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}-\frac{23}{\left(-i+\tan\left(\right.\right.}$
default	$\frac{i}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^4}+\frac{217i}{6\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+\frac{1}{40\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^5}-\frac{7}{48\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^3}+\frac{37}{128\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}-\frac{23}{\left(-i+\tan\left(\right.\right.}$

```
[In] int(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/64*I/a^3/d*exp(-7*I*(d*x+c))+1/288*I/a^3/d*exp(-9*I*(d*x+c))+1/2816*I/a^3
/d*exp(-11*I*(d*x+c))+7/64*I/a^3/d*cos(d*x+c)+21/64*sin(d*x+c)/a^3/d+11/256
*I/a^3/d*cos(5*d*x+5*c)+57/1280/a^3/d*sin(5*d*x+5*c)+31/384*I/a^3/d*cos(3*d
*x+3*c)+13/128/a^3/d*sin(3*d*x+3*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.77

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{\left(-99i\,e^{(16i\,dx+16i\,c)}-1320i\,e^{(14i\,dx+14i\,c)}-13860i\,e^{(12i\,dx+12i\,c)}+27720i\,e^{(10i\,dx+10i\,c)}+11550i\,e^{(8i\,dx+8i\,c)}+5}{126720\,a^3d}$$

```
[In] integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/126720*(-99*I*e^(16*I*d*x + 16*I*c) - 1320*I*e^(14*I*d*x + 14*I*c) - 1386
0*I*e^(12*I*d*x + 12*I*c) + 27720*I*e^(10*I*d*x + 10*I*c) + 11550*I*e^(8*I*
d*x + 8*I*c) + 5544*I*e^(6*I*d*x + 6*I*c) + 1980*I*e^(4*I*d*x + 4*I*c) + 44
0*I*e^(2*I*d*x + 2*I*c) + 45*I)*e^(-11*I*d*x - 11*I*c)/(a^3*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(122) = 244.

Time = 0.45 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.40

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \begin{cases} \frac{(-626985510622986240ia^{24}d^8e^{41ic}e^{5idx}-8359806808306483200ia^{24}d^8e^{39ic}e^{3idx}-87777971487218073600ia^{24}d^8e^{37ic}e^{idx}+1755559429744361 \cdots}{} \\ \frac{x(e^{16ic}+8e^{14ic}+28e^{12ic}+56e^{10ic}+70e^{8ic}+56e^{6ic}+28e^{4ic}+8e^{2ic}+1)e^{-11ic}}{256a^3} \end{cases}$$

```
[In] integrate(cos(d*x+c)**5/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Piecewise(((-626985510622986240*I*a**24*d**8*exp(41*I*c)*exp(5*I*d*x) - 835
9806808306483200*I*a**24*d**8*exp(39*I*c)*exp(3*I*d*x) - 877779714872180736
00*I*a**24*d**8*exp(37*I*c)*exp(I*d*x) + 175555942974436147200*I*a**24*d**8
*exp(35*I*c)*exp(-I*d*x) + 73148309572681728000*I*a**24*d**8*exp(33*I*c)*ex
p(-3*I*d*x) + 35111188594887229440*I*a**24*d**8*exp(31*I*c)*exp(-5*I*d*x) +
 12539710212459724800*I*a**24*d**8*exp(29*I*c)*exp(-7*I*d*x) + 278660226943
5494400*I*a**24*d**8*exp(27*I*c)*exp(-9*I*d*x) + 284993413919539200*I*a**24
*d**8*exp(25*I*c)*exp(-11*I*d*x))*exp(-36*I*c)/(802541453597422387200*a**27
*d**9), Ne(a**27*d**9*exp(36*I*c), 0)), (x*(exp(16*I*c) + 8*exp(14*I*c) + 2
8*exp(12*I*c) + 56*exp(10*I*c) + 70*exp(8*I*c) + 56*exp(6*I*c) + 28*exp(4*I
*c) + 8*exp(2*I*c) + 1)*exp(-11*I*c)/(256*a**3), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [A] (verification not implemented)

none

Time = 0.70 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.60

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{33\left(555\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+1920i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2710\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1760i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+463\right)}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)^5}+\frac{108405\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-784080i\,\mathrm{ta}}{}}{}$$

```
[In] integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/63360*(33*(555*tan(1/2*d*x + 1/2*c)^4 + 1920*I*tan(1/2*d*x + 1/2*c)^3 - 2
710*tan(1/2*d*x + 1/2*c)^2 - 1760*I*tan(1/2*d*x + 1/2*c) + 463)/(a^3*(tan(1
/2*d*x + 1/2*c) + I)^5) + (108405*tan(1/2*d*x + 1/2*c)^10 - 784080*I*tan(1/
2*d*x + 1/2*c)^9 - 2901195*tan(1/2*d*x + 1/2*c)^8 + 6652800*I*tan(1/2*d*x +
 1/2*c)^7 + 10407474*tan(1/2*d*x + 1/2*c)^6 - 11435424*I*tan(1/2*d*x + 1/2*
c)^5 - 8949270*tan(1/2*d*x + 1/2*c)^4 + 4899840*I*tan(1/2*d*x + 1/2*c)^3 +
1816265*tan(1/2*d*x + 1/2*c)^2 - 411664*I*tan(1/2*d*x + 1/2*c) - 47279)/(a^
3*(tan(1/2*d*x + 1/2*c) - I)^11))/d
```

Mupad [B] (verification not implemented)

Time = 6.77 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.98

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{\left(\frac{\cos(7\,c+7\,d\,x)}{64}+\frac{\cos(9\,c+9\,d\,x)}{288}+\frac{\cos(11\,c+11\,d\,x)}{2816}-\frac{\sin(7\,c+7\,d\,x)\,1\mathrm{i}}{64}-\frac{\sin(9\,c+9\,d\,x)\,1\mathrm{i}}{288}-\frac{\sin(11\,c+11\,d\,x)\,1\mathrm{i}}{2816}+\frac{\sqrt{224}\,\cos(5\,c+5\,d\,x-}{128\dots}\right.}{a^3\,d}$$

```
[In] int(cos(c + d*x)^5/(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] ((cos(7*c + 7*d*x)/64 + cos(9*c + 9*d*x)/288 + cos(11*c + 11*d*x)/2816 - (s
in(7*c + 7*d*x)*1i)/64 - (sin(9*c + 9*d*x)*1i)/288 - (sin(11*c + 11*d*x)*1i
)/2816 + (224^(1/2)*cos(5*c + atanh(57/55)*1i + 5*d*x)*1i)/1280 + (560^(1/2
)*cos(3*c + atanh(39/31)*1i + 3*d*x)*1i)/384 + (2^(1/2)*cos(c + atanh(3)*1i
 + d*x)*7i)/32)*1i)/(a^3*d)
```

3.148 $\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{4i(a-ia\tan(c+dx))^7}{7a^{11}d} - \frac{i(a-ia\tan(c+dx))^8}{2a^{12}d} + \frac{i(a-ia\tan(c+dx))^9}{9a^{13}d}$$

```
[Out] 4/7*I*(a-I*a*tan(d*x+c))^7/a^11/d-1/2*I*(a-I*a*tan(d*x+c))^8/a^12/d+1/9*I*(
a-I*a*tan(d*x+c))^9/a^13/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{i(a-ia\tan(c+dx))^9}{9a^{13}d} - \frac{i(a-ia\tan(c+dx))^8}{2a^{12}d} + \frac{4i(a-ia\tan(c+dx))^7}{7a^{11}d}$$

```
[In] Int[Sec[c + d*x]^14/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (((4*I)/7)*(a - I*a*Tan[c + d*x])^7)/(a^11*d) - ((I/2)*(a - I*a*Tan[c + d*x
])^8)/(a^12*d) + ((I/9)*(a - I*a*Tan[c + d*x])^9)/(a^13*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)^6(a+x)^2\,dx, x, ia\tan(c+dx)\right)}{a^{13}d}\\ &= -\frac{i\text{Subst}\left(\int \left(4a^2(a-x)^6-4a(a-x)^7+(a-x)^8\right)\,dx, x, ia\tan(c+dx)\right)}{a^{13}d}\\ &= \frac{4i(a-ia\tan(c+dx))^7}{7a^{11}d}-\frac{i(a-ia\tan(c+dx))^8}{2a^{12}d}+\frac{i(a-ia\tan(c+dx))^9}{9a^{13}d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.54

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{(i+\tan(c+dx))^7\left(-23-35i\tan(c+dx)+14\tan^2(c+dx)\right)}{126a^4d}$$

```
[In] Integrate[Sec[c + d*x]^14/(a + I*a*Tan[c + d*x])^4,x]
[Out] ((I + Tan[c + d*x])^7*(-23 - (35*I)*Tan[c + d*x] + 14*Tan[c + d*x]^2))/(126
*a^4*d)
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.57

method	result
risch	$\frac{128i\left(36\,\mathrm{e}^{4i(dx+c)}+9\,\mathrm{e}^{2i(dx+c)}+1\right)}{63d\,a^4\left(\mathrm{e}^{2i(dx+c)}+1\right)^9}$
derivativedivides	$-\frac{-\tan(dx+c)-\frac{\left(\tan^9(dx+c)\right)}{9}-\frac{i\left(\tan^8(dx+c)\right)}{2}+\frac{4\left(\tan^7(dx+c)\right)}{7}-\frac{2i\left(\tan^6(dx+c)\right)}{3}+2\left(\tan^5(dx+c)\right)+i\left(\tan^4(dx+c)\right)+\frac{4\left(\mathrm{ta}\right.}{}}{a^4d}$
default	$-\frac{-\tan(dx+c)-\frac{\left(\tan^9(dx+c)\right)}{9}-\frac{i\left(\tan^8(dx+c)\right)}{2}+\frac{4\left(\tan^7(dx+c)\right)}{7}-\frac{2i\left(\tan^6(dx+c)\right)}{3}+2\left(\tan^5(dx+c)\right)+i\left(\tan^4(dx+c)\right)+\frac{4\left(\mathrm{ta}\right.}{}}{a^4d}$

```
[In] int(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 128/63*I*(36*exp(4*I*(d*x+c))+9*exp(2*I*(d*x+c))+1)/d/a^4/(exp(2*I*(d*x+c))
+1)^9
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(64) = 128.

Time = 0.27 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.05

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{128\left(-36i\,e^{(4i\,dx+4i\,c)} - 9i\,e^{(2i\,dx+}\right.}{63\left(a^4de^{(18i\,dx+18i\,c)} + 9\,a^4de^{(16i\,dx+16i\,c)} + 36\,a^4de^{(14i\,dx+14i\,c)} + 84\,a^4de^{(12i\,dx+12i\,c)} + 126\,a^4de^{(10i\,dx+10i\,c)} + \right.}$$

```
[In] integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -128/63*(-36*I*e^(4*I*d*x + 4*I*c) - 9*I*e^(2*I*d*x + 2*I*c) - I)/(a^4*d*e^
(18*I*d*x + 18*I*c) + 9*a^4*d*e^(16*I*d*x + 16*I*c) + 36*a^4*d*e^(14*I*d*x
+ 14*I*c) + 84*a^4*d*e^(12*I*d*x + 12*I*c) + 126*a^4*d*e^(10*I*d*x + 10*I*c
) + 126*a^4*d*e^(8*I*d*x + 8*I*c) + 84*a^4*d*e^(6*I*d*x + 6*I*c) + 36*a^4*d
*e^(4*I*d*x + 4*I*c) + 9*a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)
```

Sympy [F]

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\int \frac{\sec^{14}(c+dx)}{\tan^4(c+dx)-4i\tan^3(c+dx)-6\tan^2(c+dx)+4i\tan(c+dx)+1}\,dx}{a^4}$$

```
[In] integrate(sec(d*x+c)**14/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**14/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6*tan(c
+ d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{14\,\tan(dx+c)^9 + 63i\,\tan(dx+c)^8 - 72\,\tan(dx+c)^7 + 84i\,\tan(dx+c)^6 - 252\,\tan(dx+c)^5 - 126i\,\tan}{126\,a^4d}$$

```
[In] integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/126*(14*tan(d*x + c)^9 + 63*I*tan(d*x + c)^8 - 72*tan(d*x + c)^7 + 84*I*t
an(d*x + c)^6 - 252*tan(d*x + c)^5 - 126*I*tan(d*x + c)^4 - 168*tan(d*x + c
)^3 - 252*I*tan(d*x + c)^2 + 126*tan(d*x + c))/(a^4*d)
```

Giac [A] (verification not implemented)

none

Time = 0.87 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{14\,\tan{(dx+c)}^9 + 63i\,\tan{(dx+c)}^8 - 72\,\tan{(dx+c)}^7 + 84i\,\tan{(dx+c)}^6 - 252\,\tan{(dx+c)}^5 - 126i\,\mathrm{t}}{126\,a^4 d}$$

```
[In] integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/126*(14*tan(d*x + c)^9 + 63*I*tan(d*x + c)^8 - 72*tan(d*x + c)^7 + 84*I*t
an(d*x + c)^6 - 252*tan(d*x + c)^5 - 126*I*tan(d*x + c)^4 - 168*tan(d*x + c
)^3 - 252*I*tan(d*x + c)^2 + 126*tan(d*x + c))/(a^4*d)
```

Mupad [B] (verification not implemented)

Time = 4.06 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.46

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\cos{(c+d\,x)}^9\,105\mathrm{i} + 128\,\sin{(c+d\,x)}\,\cos{(c+d\,x)}^8 + 64\,\sin{(c+d\,x)}\,\cos{(c+d\,x)}^6 + 48\,\sin{(c+d\,x)}\,\mathrm{cos}}{126\,a^4\,d\cos{(}}$$

```
[In] int(1/(cos(c + d*x)^14*(a + a*tan(c + d*x)*1i)^4),x)
```

```
[Out] (cos(c + d*x)*63i + 14*sin(c + d*x) - 128*cos(c + d*x)^2*sin(c + d*x) + 48*
cos(c + d*x)^4*sin(c + d*x) + 64*cos(c + d*x)^6*sin(c + d*x) + 128*cos(c +
d*x)^8*sin(c + d*x) - cos(c + d*x)^3*168i + cos(c + d*x)^9*105i)/(126*a^4*d
*cos(c + d*x)^9)
```

3.149 $\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{i(a-ia\tan(c+dx))^6}{3a^{10}d} - \frac{i(a-ia\tan(c+dx))^7}{7a^{11}d}$$

```
[Out] 1/3*I*(a-I*a*tan(d*x+c))^6/a^10/d-1/7*I*(a-I*a*tan(d*x+c))^7/a^11/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{i(a-ia\tan(c+dx))^6}{3a^{10}d} - \frac{i(a-ia\tan(c+dx))^7}{7a^{11}d}$$

```
[In] Int[Sec[c + d*x]^12/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((I/3)*(a - I*a*Tan[c + d*x])^6)/(a^10*d) - ((I/7)*(a - I*a*Tan[c + d*x])^7
)/(a^11*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)^5(a+x)\,dx, x, ia\tan(c+dx)\right)}{a^{11}d}\\ &= -\frac{i\text{Subst}\left(\int \left(2a(a-x)^5-(a-x)^6\right)\,dx, x, ia\tan(c+dx)\right)}{a^{11}d}\\ &= \frac{i(a-ia\tan(c+dx))^6}{3a^{10}d}-\frac{i(a-ia\tan(c+dx))^7}{7a^{11}d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.62

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{(i+\tan(c+dx))^6(-4i+3\tan(c+dx))}{21a^4d}$$

```
[In] Integrate[Sec[c + d*x]^12/(a + I*a*Tan[c + d*x])^4,x]
[Out] ((I + Tan[c + d*x])^6*(-4*I + 3*Tan[c + d*x]))/(21*a^4*d)
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

method	result	size
risch	$\frac{64i\left(7\,\mathrm{e}^{2i(dx+c)}+1\right)}{21d\,a^4\left(\mathrm{e}^{2i(dx+c)}+1\right)^7}$	36
derivativedivides	$-\frac{-\tan(dx+c)-\frac{\left(\tan^7(dx+c)\right)}{7}-\frac{2i\left(\tan^6(dx+c)\right)}{3}+\tan^5(dx+c)+\frac{5\left(\tan^3(dx+c)\right)}{3}+2i\left(\tan^2(dx+c)\right)}{a^4d}$	68
default	$-\frac{-\tan(dx+c)-\frac{\left(\tan^7(dx+c)\right)}{7}-\frac{2i\left(\tan^6(dx+c)\right)}{3}+\tan^5(dx+c)+\frac{5\left(\tan^3(dx+c)\right)}{3}+2i\left(\tan^2(dx+c)\right)}{a^4d}$	68

```
[In] int(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
[Out] 64/21*I*(7*exp(2*I*(d*x+c))+1)/d/a^4/(exp(2*I*(d*x+c))+1)^7
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(43) = 86.

Time = 0.24 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.31

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{64\left(-7i\,e^{(2i\,dx+2i\,c)}-i\right)}{21\left(a^4de^{(14i\,dx+14i\,c)}+7\,a^4de^{(12i\,dx+12i\,c)}+21\,a^4de^{(10i\,dx+10i\,c)}+35\,a^4de^{(8i\,dx+8i\,c)}+35\,a^4de^{(6i\,dx+6i\,c)}+21\,a^4\right.}$$

```
[In] integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -64/21*(-7*I*e^(2*I*d*x + 2*I*c) - I)/(a^4*d*e^(14*I*d*x + 14*I*c) + 7*a^4*
d*e^(12*I*d*x + 12*I*c) + 21*a^4*d*e^(10*I*d*x + 10*I*c) + 35*a^4*d*e^(8*I*
d*x + 8*I*c) + 35*a^4*d*e^(6*I*d*x + 6*I*c) + 21*a^4*d*e^(4*I*d*x + 4*I*c)
+ 7*a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)
```

Sympy [F]

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\int \frac{\sec^{12}(c+dx)}{\tan^4(c+dx)-4i\tan^3(c+dx)-6\tan^2(c+dx)+4i\tan(c+dx)+1}\,dx}{a^4}$$

```
[In] integrate(sec(d*x+c)**12/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**12/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6*tan(c
+ d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.22

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{3\tan(dx+c)^7+14i\tan(dx+c)^6-21\tan(dx+c)^5-35\tan(dx+c)^3-42i\tan(dx+c)^2+21\tan(dx}{21\,a^4d}$$

```
[In] integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/21*(3*tan(d*x + c)^7 + 14*I*tan(d*x + c)^6 - 21*tan(d*x + c)^5 - 35*tan(d
*x + c)^3 - 42*I*tan(d*x + c)^2 + 21*tan(d*x + c))/(a^4*d)
```

Giac [A] (verification not implemented)

none

Time = 0.80 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.22

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{3\tan(dx+c)^7 + 14i\tan(dx+c)^6 - 21\tan(dx+c)^5 - 35\tan(dx+c)^3 - 42i\tan(dx+c)^2 + 21\tan(dx+c)}{21a^4d}$$

```
[In] integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/21*(3*tan(d*x + c)^7 + 14*I*tan(d*x + c)^6 - 21*tan(d*x + c)^5 - 35*tan(d
*x + c)^3 - 42*I*tan(d*x + c)^2 + 21*tan(d*x + c))/(a^4*d)
```

Mupad [B] (verification not implemented)

Time = 4.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.05

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\sin(c+d\,x)\,\left(21\cos(c+d\,x)^6 - \cos(c+d\,x)^5\sin(c+d\,x)\,42\mathrm{i} - 35\cos(c+d\,x)^4\sin(c+d\,x)^2 - 21\cos(\ldots\right)}{21\,a^4\,d\cos(c+d\,x)^7}$$

```
[In] int(1/(cos(c + d*x)^12*(a + a*tan(c + d*x)*1i)^4),x)
```

```
[Out] (sin(c + d*x)*(cos(c + d*x)*sin(c + d*x)^5*14i - cos(c + d*x)^5*sin(c + d*x
)*42i + 21*cos(c + d*x)^6 + 3*sin(c + d*x)^6 - 21*cos(c + d*x)^2*sin(c + d*
x)^4 - 35*cos(c + d*x)^4*sin(c + d*x)^2))/(21*a^4*d*cos(c + d*x)^7)
```

3.150 $\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{i(a-ia\tan(c+dx))^5}{5a^9d}$$

```
[Out] 1/5*I*(a-I*a*tan(d*x+c))^5/a^9/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 32}

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{i(a-ia\tan(c+dx))^5}{5a^9d}$$

```
[In] Int[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((I/5)*(a - I*a*Tan[c + d*x])^5)/(a^9*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
```

```
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)^4\,dx, x, ia\tan(c+dx)\right)}{a^9 d}\\ &= \frac{i(a-ia\tan(c+dx))^5}{5a^9 d}\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 58 vs. $2(27) = 54$.

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\tan(c+dx)\left(5-10i\tan(c+dx)-10\tan^2(c+dx)+5i\tan^3(c+dx)+\tan^4(c+dx)\right)}{5a^4 d}$$

```
[In] Integrate[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^4,x]
[Out] (Tan[c + d*x]*(5 - (10*I)*Tan[c + d*x] - 10*Tan[c + d*x]^2 + (5*I)*Tan[c +
d*x]^3 + Tan[c + d*x]^4))/(5*a^4*d)
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$\frac{(\tan(dx+c)+i)^5}{5a^4 d}$	20
default	$\frac{(\tan(dx+c)+i)^5}{5a^4 d}$	20
risch	$\frac{32i}{5d\,a^4\left(e^{2i(dx+c)}+1\right)^5}$	23

```
[In] int(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
[Out] 1/5/a^4/d*(tan(d*x+c)+I)^5
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(21) = 42.

Time = 0.23 (sec) , antiderivative size = 84, normalized size of antiderivative = 3.11

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{32i}{5\left(a^4de^{(10i\,dx+10i\,c)}+5\,a^4de^{(8i\,dx+8i\,c)}+10\,a^4de^{(6i\,dx+6i\,c)}+10\,a^4de^{(4i\,dx+4i\,c)}+5\,a^4de^{(2i\,dx+2i\,c)}+a^4d\right)}$$

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 32/5*I/(a^4*d*e^(10*I*d*x + 10*I*c) + 5*a^4*d*e^(8*I*d*x + 8*I*c) + 10*a^4*
d*e^(6*I*d*x + 6*I*c) + 10*a^4*d*e^(4*I*d*x + 4*I*c) + 5*a^4*d*e^(2*I*d*x +
 2*I*c) + a^4*d)
```

Sympy [F]

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\int \frac{\sec^{10}(c+dx)}{\tan^4(c+dx)-4i\tan^3(c+dx)-6\tan^2(c+dx)+4i\tan(c+dx)+1}\,dx}{a^4}$$

```
[In] integrate(sec(d*x+c)**10/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**10/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6*tan(c
+ d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(21) = 42.

Time = 0.35 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.04

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{\tan(dx+c)^5+5i\,\tan(dx+c)^4-10\,\tan(dx+c)^3-10i\,\tan(dx+c)^2+5\,\tan(dx+c)}{5\,a^4d}$$

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/5*(tan(d*x + c)^5 + 5*I*tan(d*x + c)^4 - 10*tan(d*x + c)^3 - 10*I*tan(d*x
 + c)^2 + 5*tan(d*x + c))/(a^4*d)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(21) = 42$.

Time = 0.75 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.04

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\tan(dx+c)^5 + 5i\tan(dx+c)^4 - 10\tan(dx+c)^3 - 10i\tan(dx+c)^2 + 5\tan(dx+c)}{5a^4d}$$

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/5*(tan(d*x + c)^5 + 5*I*tan(d*x + c)^4 - 10*tan(d*x + c)^3 - 10*I*tan(d*x
 + c)^2 + 5*tan(d*x + c))/(a^4*d)
```

Mupad [B] (verification not implemented)

Time = 4.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.44

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\sin(c+dx)\left(5\cos(c+dx)^4 - \cos(c+dx)^3\sin(c+dx)\,10\mathrm{i} - 10\cos(c+dx)^2\sin(c+dx)^2 + \cos(c+\right.}{5a^4d\cos(c+dx)^5}$$

```
[In] int(1/(cos(c + d*x)^10*(a + a*tan(c + d*x)*1i)^4),x)
```

```
[Out] (sin(c + d*x)*(cos(c + d*x)*sin(c + d*x)^3*5i - cos(c + d*x)^3*sin(c + d*x)
*10i + 5*cos(c + d*x)^4 + sin(c + d*x)^4 - 10*cos(c + d*x)^2*sin(c + d*x)^2
))/(5*a^4*d*cos(c + d*x)^5)
```

3.151 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 90

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{8x}{a^4}+\frac{8i\log(\cos(c+dx))}{a^4d}-\frac{4\tan(c+dx)}{a^4d} - \frac{i(a-ia\tan(c+dx))^2}{a^6d}-\frac{i(a-ia\tan(c+dx))^3}{3a^7d}$$

```
[Out] 8*x/a^4+8*I*ln(cos(d*x+c))/a^4/d-4*tan(d*x+c)/a^4/d-I*(a-I*a*tan(d*x+c))^2/
a^6/d-1/3*I*(a-I*a*tan(d*x+c))^3/a^7/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{i(a-ia\tan(c+dx))^3}{3a^7d}-\frac{i(a-ia\tan(c+dx))^2}{a^6d} - \frac{4\tan(c+dx)}{a^4d}+\frac{8i\log(\cos(c+dx))}{a^4d}+\frac{8x}{a^4}$$

```
[In] Int[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (8*x)/a^4 + ((8*I)*Log[Cos[c + d*x]])/(a^4*d) - (4*Tan[c + d*x])/(a^4*d) -
(I*(a - I*a*Tan[c + d*x])^2)/(a^6*d) - ((I/3)*(a - I*a*Tan[c + d*x])^3)/(a^
7*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int \frac{(a-x)^3}{a+x}\,dx, x, ia\tan(c+dx)\right)}{a^7d}\\ &= -\frac{i\text{Subst}\left(\int \left(-4a^2-2a(a-x)-(a-x)^2+\frac{8a^3}{a+x}\right)\,dx, x, ia\tan(c+dx)\right)}{a^7d}\\ &= \frac{8x}{a^4}+\frac{8i\log(\cos(c+dx))}{a^4d}-\frac{4\tan(c+dx)}{a^4d}-\frac{i(a-ia\tan(c+dx))^2}{a^6d}-\frac{i(a-ia\tan(c+dx))^3}{3a^7d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{-24i\log(i-\tan(c+dx))-21\tan(c+dx)+6i\tan^2(c+dx)+\tan^3(c+dx)}{3a^4d}$$

```
[In] Integrate[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^4,x]
[Out] ((-24*I)*Log[I - Tan[c + d*x]] - 21*Tan[c + d*x] + (6*I)*Tan[c + d*x]^2 + T
an[c + d*x]^3)/(3*a^4*d)
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{7\tan(dx+c)}{a^4d}+\frac{\tan^3(dx+c)}{3a^4d}+\frac{2i(\tan^2(dx+c))}{a^4d}+\frac{8\arctan(\tan(dx+c))}{a^4d}-\frac{4i\ln(1+\tan^2(dx+c))}{a^4d}$	84
default	$-\frac{7\tan(dx+c)}{a^4d}+\frac{\tan^3(dx+c)}{3a^4d}+\frac{2i(\tan^2(dx+c))}{a^4d}+\frac{8\arctan(\tan(dx+c))}{a^4d}-\frac{4i\ln(1+\tan^2(dx+c))}{a^4d}$	84
risch	$\frac{16x}{a^4}+\frac{16c}{a^4d}-\frac{4i(6\,e^{4i(dx+c)}+15\,e^{2i(dx+c)}+11)}{3d\,a^4(e^{2i(dx+c)}+1)^3}+\frac{8i\ln(e^{2i(dx+c)}+1)}{a^4d}$	84

```
[In] int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -7*tan(d*x+c)/a^4/d+1/3/a^4/d*tan(d*x+c)^3+2*I/a^4/d*tan(d*x+c)^2+8/a^4/d*a
rctan(tan(d*x+c))-4*I/a^4/d*ln(1+tan(d*x+c)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.73

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{4\left(12\,dxe^{(6i\,dx+6i\,c)}+12\,dx+6\,(6\,dx-i)e^{(4i\,dx+4i\,c)}+3\,(12\,dx-5i)e^{(2i\,dx+2i\,c)}-6\left(-i\,e^{(6i\,dx+6i\,c)}-3i\,e^{(4i\,dx-}\right.\right.}{3\left(a^4de^{(6i\,dx+6i\,c)}+3\,a^4de^{(4i\,dx+4i\,c)}+3\,a^4de^{(2i\,dx+2i\,c)}+a^4\right.}$$

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 4/3*(12*d*x*e^(6*I*d*x + 6*I*c) + 12*d*x + 6*(6*d*x - I)*e^(4*I*d*x + 4*I*c
) + 3*(12*d*x - 5*I)*e^(2*I*d*x + 2*I*c) - 6*(-I*e^(6*I*d*x + 6*I*c) - 3*I*
e^(4*I*d*x + 4*I*c) - 3*I*e^(2*I*d*x + 2*I*c) - I)*log(e^(2*I*d*x + 2*I*c)
+ 1) - 11*I)/(a^4*d*e^(6*I*d*x + 6*I*c) + 3*a^4*d*e^(4*I*d*x + 4*I*c) + 3*a
^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)
```

Sympy [F]

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\int \frac{\sec^8{(c+dx)}}{\tan^4{(c+dx)}-4i\tan^3{(c+dx)}-6\tan^2{(c+dx)}+4i\tan{(c+dx)}+1}\,dx}{a^4}$$

```
[In] integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**8/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6*tan(c +
 d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.59

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\frac{\tan(dx+c)^3+6i\tan(dx+c)^2-21\tan(dx+c)}{a^4} - \frac{24i\log(i\tan(dx+c)+1)}{a^4}}{3\,d}$$

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*((tan(d*x + c)^3 + 6*I*tan(d*x + c)^2 - 21*tan(d*x + c))/a^4 - 24*I*log
(I*tan(d*x + c) + 1)/a^4)/d
```

Giac [A] (verification not implemented)

none

Time = 0.73 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.71

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{2\left(-\frac{12i\log(\tan(\frac{1}{2}dx+\frac{1}{2}c)+1)}{a^4} + \frac{24i\log(\tan(\frac{1}{2}dx+\frac{1}{2}c)-i)}{a^4} - \frac{12i\log(\tan(\frac{1}{2}dx+\frac{1}{2}c)-1)}{a^4} + \frac{22i\tan(\frac{1}{2}dx+\frac{1}{2}c)^6-21\tan(\frac{1}{2}dx+\frac{1}{2}\right.}{3\,d}$$

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -2/3*(-12*I*log(tan(1/2*d*x + 1/2*c) + 1)/a^4 + 24*I*log(tan(1/2*d*x + 1/2*
c) - I)/a^4 - 12*I*log(tan(1/2*d*x + 1/2*c) - 1)/a^4 + (22*I*tan(1/2*d*x +
1/2*c)^6 - 21*tan(1/2*d*x + 1/2*c)^5 - 78*I*tan(1/2*d*x + 1/2*c)^4 + 46*tan
(1/2*d*x + 1/2*c)^3 + 78*I*tan(1/2*d*x + 1/2*c)^2 - 21*tan(1/2*d*x + 1/2*c)
 - 22*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a^4))/d
```

Mupad [B] (verification not implemented)

Time = 3.76 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.67

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{\frac{7\tan(c+dx)}{a^4} - \frac{\tan(c+dx)^3}{3a^4} + \frac{\ln(\tan(c+dx)-\mathrm{i})\,8\mathrm{i}}{a^4} - \frac{\tan(c+dx)^2\,2\mathrm{i}}{a^4}}{d}$$

```
[In] int(1/(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^4),x)
```

```
[Out] -((log(tan(c + d*x) - 1i)*8i)/a^4 + (7*tan(c + d*x))/a^4 - (tan(c + d*x)^2*
2i)/a^4 - tan(c + d*x)^3/(3*a^4))/d
```

3.152 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 63

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{4x}{a^4} - \frac{4i\log(\cos(c+dx))}{a^4d} + \frac{\tan(c+dx)}{a^4d} + \frac{4i}{d\left(a^4+ia^4\tan(c+dx)\right)}$$

```
[Out] -4*x/a^4-4*I*ln(cos(d*x+c))/a^4/d+tan(d*x+c)/a^4/d+4*I/d/(a^4+I*a^4*tan(d*x
+c))
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 45}

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\tan(c+dx)}{a^4d} + \frac{4i}{d\left(a^4+ia^4\tan(c+dx)\right)} - \frac{4i\log(\cos(c+dx))}{a^4d} - \frac{4x}{a^4}$$

```
[In] Int[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (-4*x)/a^4 - ((4*I)*Log[Cos[c + d*x]])/(a^4*d) + Tan[c + d*x]/(a^4*d) + (4*
I)/(d*(a^4 + I*a^4*Tan[c + d*x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int \frac{(a-x)^2}{(a+x)^2}\,dx, x, ia\tan(c+dx)\right)}{a^5 d}\\ &= -\frac{i\text{Subst}\left(\int \left(1+\frac{4a^2}{(a+x)^2}-\frac{4a}{a+x}\right)\,dx, x, ia\tan(c+dx)\right)}{a^5 d}\\ &= -\frac{4x}{a^4}-\frac{4i\log(\cos(c+dx))}{a^4 d}+\frac{\tan(c+dx)}{a^4 d}+\frac{4i}{d\,(a^4+ia^4\tan(c+dx))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{i\left(-4\log(i-\tan(c+dx))+i\tan(c+dx)+\frac{4i}{-i+\tan(c+dx)}\right)}{a^4 d}$$

```
[In] Integrate[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^4,x]
[Out] ((-I)*(-4*Log[I - Tan[c + d*x]] + I*Tan[c + d*x] + (4*I)/(-I + Tan[c + d*x]
)))/(a^4*d)
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\frac{\tan(dx+c)}{a^4 d}-\frac{4\arctan(\tan(dx+c))}{a^4 d}+\frac{2i\ln(1+\tan^2(dx+c))}{a^4 d}+\frac{4}{a^4 d(\tan(dx+c)-i)}$	69
default	$\frac{\tan(dx+c)}{a^4 d}-\frac{4\arctan(\tan(dx+c))}{a^4 d}+\frac{2i\ln(1+\tan^2(dx+c))}{a^4 d}+\frac{4}{a^4 d(\tan(dx+c)-i)}$	69
risch	$\frac{2i\mathrm{e}^{-2i(dx+c)}}{a^4 d}-\frac{8x}{a^4}-\frac{8c}{a^4 d}+\frac{2i}{d\,a^4\left(\mathrm{e}^{2i(dx+c)}+1\right)}-\frac{4i\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{a^4 d}$	78

```
[In] int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
[Out] tan(d*x+c)/a^4/d-4/a^4/d*arctan(tan(d*x+c))+2*I/a^4/d*ln(1+tan(d*x+c)^2)+4/
a^4/d/(tan(d*x+c)-I)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.62

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{2\left(4\,dxe^{(4i\,dx+4i\,c)}+2\,(2\,dx-i)e^{(2i\,dx+2i\,c)}+2\left(i\,e^{(4i\,dx+4i\,c)}+i\,e^{(2i\,dx+2i\,c)}\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)-i\right)}{a^4de^{(4i\,dx+4i\,c)}+a^4de^{(2i\,dx+2i\,c)}}$$

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
[Out] -2*(4*d*x*e^(4*I*d*x + 4*I*c) + 2*(2*d*x - I)*e^(2*I*d*x + 2*I*c) + 2*(I*e^
(4*I*d*x + 4*I*c) + I*e^(2*I*d*x + 2*I*c))*log(e^(2*I*d*x + 2*I*c) + 1) - I
)/(a^4*d*e^(4*I*d*x + 4*I*c) + a^4*d*e^(2*I*d*x + 2*I*c))
```

Sympy [F]

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\int \frac{\sec^6(c+dx)}{\tan^4(c+dx)-4i\tan^3(c+dx)-6\tan^2(c+dx)+4i\tan(c+dx)+1}\,dx}{a^4}$$

```
[In] integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c))**4,x)
[Out] Integral(sec(c + d*x)**6/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6*tan(c +
 d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.51

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\frac{4\left(\tan(dx+c)^2-2i\,\tan(dx+c)-1\right)}{a^4\tan(dx+c)^3-3i\,a^4\tan(dx+c)^2-3\,a^4\tan(dx+c)+i\,a^4}+\frac{4i\,\log(i\,\tan(dx+c)+1)}{a^4}+\frac{\tan(dx+c)}{a^4}}{d}$$

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
[Out] (4*(tan(d*x + c)^2 - 2*I*tan(d*x + c) - 1)/(a^4*tan(d*x + c)^3 - 3*I*a^4*ta
n(d*x + c)^2 - 3*a^4*tan(d*x + c) + I*a^4) + 4*I*log(I*tan(d*x + c) + 1)/a^
4 + tan(d*x + c)/a^4)/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(57) = 114.

Time = 0.72 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.32

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{2\left(-\frac{2i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^4}+\frac{4i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}{a^4}-\frac{2i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^4}+\frac{2i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2i}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a^4}-\right.}{d}$$

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 2*(-2*I*log(tan(1/2*d*x + 1/2*c) + 1)/a^4 + 4*I*log(tan(1/2*d*x + 1/2*c) -
I)/a^4 - 2*I*log(tan(1/2*d*x + 1/2*c) - 1)/a^4 + (2*I*tan(1/2*d*x + 1/2*c)^
2 - tan(1/2*d*x + 1/2*c) - 2*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a^4) - 2*(3*I
*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) - 3*I)/(a^4*(tan(1/2*d*x +
 1/2*c) - I)^2))/d
```

Mupad [B] (verification not implemented)

Time = 4.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,4\mathrm{i}}{a^4\,d}+\frac{\tan(c+d\,x)}{a^4\,d} + \frac{4\mathrm{i}}{a^4\,d\,\left(1+\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}$$

```
[In] int(1/(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^4),x)
```

```
[Out] (log(tan(c + d*x) - 1i)*4i)/(a^4*d) + tan(c + d*x)/(a^4*d) + 4i/(a^4*d*(tan
(c + d*x)*1i + 1))
```

3.153 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^4} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 29

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^4} \, dx = \frac{\tan(c+dx)}{d\,(a^2+ia^2\tan(c+dx))^2}$$

```
[Out] tan(d*x+c)/d/(a^2+I*a^2*tan(d*x+c))^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 34}

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^4} \, dx = \frac{\tan(c+dx)}{d\,(a^2+ia^2\tan(c+dx))^2}$$

```
[In] Int[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] Tan[c + d*x]/(d*(a^2 + I*a^2*Tan[c + d*x])^2)
```

Rule 34

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)), x_Symbol] :> Simp[d*x*((
a + b*x)^(m + 1)/(b*(m + 2))), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a*d -
 b*c*(m + 2), 0]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
```

```
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int \frac{a-x}{(a+x)^3}\,dx, x, ia\tan(c+dx)\right)}{a^3d}\\ &= \frac{\tan(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.24

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{i(i+\tan(c+dx))^2}{4a^4d(-i+\tan(c+dx))^2}$$

```
[In] Integrate[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^4,x]
[Out] ((I/4)*(I + Tan[c + d*x])^2)/(a^4*d*(-I + Tan[c + d*x])^2)
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

method	result	size
risch	$\frac{i e^{-4i(dx+c)}}{4a^4d}$	19
derivativedivides	$\frac{-\frac{i}{(\tan(dx+c)-i)^2}-\frac{1}{\tan(dx+c)-i}}{a^4d}$	36
default	$\frac{-\frac{i}{(\tan(dx+c)-i)^2}-\frac{1}{\tan(dx+c)-i}}{a^4d}$	36

```
[In] int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
[Out] 1/4*I/a^4/d*exp(-4*I*(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.59

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{i\,e^{(-4i\,dx-4i\,c)}}{4\,a^4d}$$

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
[Out] 1/4*I*e^(-4*I*d*x - 4*I*c)/(a^4*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(24) = 48.

Time = 1.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.28

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \begin{cases} \frac{i\sec^4(c+dx)}{4a^4d\tan^4(c+dx)-16ia^4d\tan^3(c+dx)-24a^4d\tan^2(c+dx)+16ia^4d\tan(c+dx)+4a^4d} & \text{for } d\neq 0 \\ \frac{x\sec^4(c)}{(ia\tan(c)+a)^4} & \text{otherwise} \end{cases}$$

```
[In] integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c))**4,x)
[Out] Piecewise((I*sec(c + d*x)**4/(4*a**4*d*tan(c + d*x)**4 - 16*I*a**4*d*tan(c
+ d*x)**3 - 24*a**4*d*tan(c + d*x)**2 + 16*I*a**4*d*tan(c + d*x) + 4*a**4*d
), Ne(d, 0)), (x*sec(c)**4/(I*a*tan(c) + a)**4, True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(27) = 54.

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.28

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{\tan(dx+c)^2 - i\,\tan(dx+c)}{(a^4\tan(dx+c)^3 - 3i\,a^4\tan(dx+c)^2 - 3\,a^4\tan(dx+c) + i\,a^4)d}$$

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
[Out] -(tan(d*x + c)^2 - I*tan(d*x + c))/((a^4*tan(d*x + c)^3 - 3*I*a^4*tan(d*x +
 c)^2 - 3*a^4*tan(d*x + c) + I*a^4)*d)
```

Giac [A] (verification not implemented)

none

Time = 0.62 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.52

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^4d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^4}$$

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
[Out] -2*(tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c))/(a^4*d*(tan(1/2*d*x + 1/
2*c) - I)^4)
```

Mupad [B] (verification not implemented)

Time = 3.91 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{\tan(c+d\,x)}{a^4\,d\,\left(\tan\left(c+d\,x\right)-\mathrm{i}\right)^2}$$

```
[In] int(1/(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^4),x)
[Out] -tan(c + d*x)/(a^4*d*(tan(c + d*x) - 1i)^2)
```

3.154 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{i}{3ad(a+ia\tan(c+dx))^3}$$

```
[Out] 1/3*I/a/d/(a+I*a*tan(d*x+c))^3
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 32}

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{i}{3ad(a+ia\tan(c+dx))^3}$$

```
[In] Int[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (I/3)/(a*d*(a + I*a*Tan[c + d*x])^3)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
```

```
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{i\text{Subst}\left(\int \frac{1}{(a+x)^4}\,dx, x, ia\tan(c+dx)\right)}{ad}$$
$$= \frac{i}{3ad(a+ia\tan(c+dx))^3}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{1}{3a^4d(-i+\tan(c+dx))^3}$$

```
[In] Integrate[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^4,x]
[Out] -1/3*1/(a^4*d*(-I + Tan[c + d*x])^3)
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{i}{3ad(a+ia\tan(dx+c))^3}$	24
default	$\frac{i}{3ad(a+ia\tan(dx+c))^3}$	24
risch	$\frac{i\mathrm{e}^{-2i(dx+c)}}{8a^4d}+\frac{i\mathrm{e}^{-4i(dx+c)}}{8a^4d}+\frac{i\mathrm{e}^{-6i(dx+c)}}{24a^4d}$	56

```
[In] int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
[Out] 1/3*I/a/d/(a+I*a*tan(d*x+c))^3
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\left(3i\,e^{(4i\,dx+4i\,c)}+3i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-6i\,dx-6i\,c)}}{24\,a^4d}$$

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/24*(3*I*e^(4*I*d*x + 4*I*c) + 3*I*e^(2*I*d*x + 2*I*c) + I)*e^(-6*I*d*x -
6*I*c)/(a^4*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(19) = 38.

Time = 1.20 (sec) , antiderivative size = 272, normalized size of antiderivative = 10.07

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \begin{cases} -\frac{i\tan^2(c+dx)\sec^2(c+dx)}{24a^4d\tan^4(c+dx)-96ia^4d\tan^3(c+dx)-144a^4d\tan^2(c+dx)+96ia^4d\tan(c+dx)+24a^4d} - \frac{4\tan(c+dx)\sec}{24a^4d\tan^4(c+dx)-96ia^4d\tan^3(c+dx)-144a^4} \\ \frac{x\sec^2(c)}{(ia\tan(c)+a)^4} \end{cases}$$

```
[In] integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise((-I*tan(c + d*x)**2*sec(c + d*x)**2/(24*a**4*d*tan(c + d*x)**4 -
96*I*a**4*d*tan(c + d*x)**3 - 144*a**4*d*tan(c + d*x)**2 + 96*I*a**4*d*tan(
c + d*x) + 24*a**4*d) - 4*tan(c + d*x)*sec(c + d*x)**2/(24*a**4*d*tan(c + d
*x)**4 - 96*I*a**4*d*tan(c + d*x)**3 - 144*a**4*d*tan(c + d*x)**2 + 96*I*a*
*4*d*tan(c + d*x) + 24*a**4*d) + 7*I*sec(c + d*x)**2/(24*a**4*d*tan(c + d*x
)**4 - 96*I*a**4*d*tan(c + d*x)**3 - 144*a**4*d*tan(c + d*x)**2 + 96*I*a**4
*d*tan(c + d*x) + 24*a**4*d), Ne(d, 0)), (x*sec(c)**2/(I*a*tan(c) + a)**4,
True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{i}{3\left(i\,a\tan\left(dx+c\right)+a\right)^3ad}$$

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*I/((I*a*tan(d*x + c) + a)^3*a*d)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(21) = 42.

Time = 0.63 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.15

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{2\left(3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-10\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\tan\left(\frac{1}{2}\,c\right.\right.}{3\,a^4d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^6}$$

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -2/3*(3*tan(1/2*d*x + 1/2*c)^5 - 6*I*tan(1/2*d*x + 1/2*c)^4 - 10*tan(1/2*d*
x + 1/2*c)^3 + 6*I*tan(1/2*d*x + 1/2*c)^2 + 3*tan(1/2*d*x + 1/2*c))/(a^4*d*
(tan(1/2*d*x + 1/2*c) - I)^6)
```

Mupad [B] (verification not implemented)

Time = 3.74 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{1}{3\,a^4\,d\left(\tan\left(c+d\,x\right)-\mathrm{i}\right)^3}$$

```
[In] int(1/(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^4),x)
```

```
[Out] -1/(3*a^4*d*(tan(c + d*x) - 1i)^3)
```

3.155 $\int \frac{1}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 116

$$\int \frac{1}{(a+ia\tan(c+dx))^4}\,dx=\frac{x}{16a^4}+\frac{i}{8d(a+ia\tan(c+dx))^4}+\frac{i}{12ad(a+ia\tan(c+dx))^3}+\frac{i}{16d\,(a^2+ia^2\tan(c+dx))^2}+\frac{i}{16d\,(a^4+ia^4\tan(c+dx))}$$

```
[Out] 1/16*x/a^4+1/8*I/d/(a+I*a*tan(d*x+c))^4+1/12*I/a/d/(a+I*a*tan(d*x+c))^3+1/1
6*I/d/(a^2+I*a^2*tan(d*x+c))^2+1/16*I/d/(a^4+I*a^4*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3560, 8}

$$\int \frac{1}{(a+ia\tan(c+dx))^4}\,dx=\frac{i}{16d\,(a^4+ia^4\tan(c+dx))}+\frac{x}{16a^4}+\frac{i}{16d\,(a^2+ia^2\tan(c+dx))^2}+\frac{i}{12ad(a+ia\tan(c+dx))^3}+\frac{i}{8d(a+ia\tan(c+dx))^4}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^(-4),x]
```

```
[Out] x/(16*a^4) + (I/8)/(d*(a + I*a*Tan[c + d*x])^4) + (I/12)/(a*d*(a + I*a*Tan[
c + d*x])^3) + (I/16)/(d*(a^2 + I*a^2*Tan[c + d*x])^2) + (I/16)/(d*(a^4 + I
*a^4*Tan[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{i}{8d(a+ia\tan(c+dx))^4}+\frac{\int \frac{1}{(a+ia\tan(c+dx))^3}\,dx}{2a}\\
&= \frac{i}{8d(a+ia\tan(c+dx))^4}+\frac{i}{12ad(a+ia\tan(c+dx))^3}+\frac{\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx}{4a^2}\\
&= \frac{i}{8d(a+ia\tan(c+dx))^4}+\frac{i}{12ad(a+ia\tan(c+dx))^3}\\
&\quad+\frac{i}{16d\left(a^2+ia^2\tan(c+dx)\right)^2}+\frac{\int \frac{1}{a+ia\tan(c+dx)}\,dx}{8a^3}\\
&= \frac{i}{8d(a+ia\tan(c+dx))^4}+\frac{i}{12ad(a+ia\tan(c+dx))^3}\\
&\quad+\frac{i}{16d\left(a^2+ia^2\tan(c+dx)\right)^2}+\frac{i}{16d\left(a^4+ia^4\tan(c+dx)\right)}+\frac{\int 1\,dx}{16a^4}\\
&= \frac{x}{16a^4}+\frac{i}{8d(a+ia\tan(c+dx))^4}+\frac{i}{12ad(a+ia\tan(c+dx))^3}\\
&\quad+\frac{i}{16d\left(a^2+ia^2\tan(c+dx)\right)^2}+\frac{i}{16d\left(a^4+ia^4\tan(c+dx)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97

$$\int \frac{1}{(a+ia\tan(c+dx))^4}\,dx = -\frac{ia\left(\frac{i\arctan(\tan(c+dx))}{16a^5}-\frac{1}{8a(a+ia\tan(c+dx))^4}-\frac{1}{12a^2(a+ia\tan(c+dx))^3}-\frac{1}{16a^3(a+ia\tan(c+dx))^2}-\frac{1}{16a^4(a+ia\tan(c+dx))}\right)}{d}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(-4),x]
```

```
[Out] ((-I)*a*(((I/16)*ArcTan[Tan[c + d*x]])/a^5 - 1/(8*a*(a + I*a*Tan[c + d*x])^
4) - 1/(12*a^2*(a + I*a*Tan[c + d*x])^3) - 1/(16*a^3*(a + I*a*Tan[c + d*x])
^2) - 1/(16*a^4*(a + I*a*Tan[c + d*x]))))/d
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.69

method	result
risch	$\frac{x}{16a^4}+\frac{i\mathrm{e}^{-2i(dx+c)}}{8a^4d}+\frac{3i\mathrm{e}^{-4i(dx+c)}}{32a^4d}+\frac{i\mathrm{e}^{-6i(dx+c)}}{24a^4d}+\frac{i\mathrm{e}^{-8i(dx+c)}}{128a^4d}$
derivativedivides	$\frac{i}{8d\,a^4(\tan(dx+c)-i)^4}+\frac{\arctan(\tan(dx+c))}{16a^4d}-\frac{i}{16d\,a^4(\tan(dx+c)-i)^2}-\frac{1}{12d\,a^4(\tan(dx+c)-i)^3}+\frac{1}{16a^4d(\tan(dx+c$
default	$\frac{i}{8d\,a^4(\tan(dx+c)-i)^4}+\frac{\arctan(\tan(dx+c))}{16a^4d}-\frac{i}{16d\,a^4(\tan(dx+c)-i)^2}-\frac{1}{12d\,a^4(\tan(dx+c)-i)^3}+\frac{1}{16a^4d(\tan(dx+c$
norman	$\frac{\frac{x}{16a}+\frac{5\left(\tan^3(dx+c)\right)}{48ad}+\frac{11\left(\tan^5(dx+c)\right)}{48ad}+\frac{\tan^7(dx+c)}{16ad}+\frac{x\left(\tan^2(dx+c)\right)}{4a}+\frac{3x\left(\tan^4(dx+c)\right)}{8a}+\frac{x\left(\tan^6(dx+c)\right)}{4a}+\frac{x\left(\tan^8(dx+c)\right)}{16a}+\frac{i}{3a}}{a^3(1+\tan^2(dx+c))^4}$

```
[In] int(1/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*x/a^4+1/8*I/a^4/d*exp(-2*I*(d*x+c))+3/32*I/a^4/d*exp(-4*I*(d*x+c))+1/2
4*I/a^4/d*exp(-6*I*(d*x+c))+1/128*I/a^4/d*exp(-8*I*(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.56

$$\int \frac{1}{(a+ia\tan(c+dx))^4}\,dx = \frac{\left(24\,dxe^{(8i\,dx+8i\,c)}+48i\,e^{(6i\,dx+6i\,c)}+36i\,e^{(4i\,dx+4i\,c)}+16i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-8i\,dx-8i\,c)}}{384\,a^4d}$$

```
[In] integrate(1/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/384*(24*d*x*e^(8*I*d*x + 8*I*c) + 48*I*e^(6*I*d*x + 6*I*c) + 36*I*e^(4*I*
d*x + 4*I*c) + 16*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(-8*I*d*x - 8*I*c)/(a^4*d)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.63

$$\int \frac{1}{(a+ia\tan(c+dx))^4}\,dx = \begin{cases} \frac{(98304ia^{12}d^3e^{18ic}e^{-2idx}+73728ia^{12}d^3e^{16ic}e^{-4idx}+32768ia^{12}d^3e^{14ic}e^{-6idx}+6144ia^{12}d^3e^{12ic}e^{-8idx})e^{-20ic}}{786432a^{16}d^4} & \text{for } a^{16}d^4e^{20ic}\neq 0 \\ x\left(\frac{(e^{8ic}+4e^{6ic}+6e^{4ic}+4e^{2ic}+1)e^{-8ic}}{16a^4}-\frac{1}{16a^4}\right) & \text{otherwise} \end{cases} + \frac{x}{16a^4}$$

```
[In] integrate(1/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise(((98304*I*a**12*d**3*exp(18*I*c)*exp(-2*I*d*x) + 73728*I*a**12*d*
*3*exp(16*I*c)*exp(-4*I*d*x) + 32768*I*a**12*d**3*exp(14*I*c)*exp(-6*I*d*x)
 + 6144*I*a**12*d**3*exp(12*I*c)*exp(-8*I*d*x))*exp(-20*I*c)/(786432*a**16*
d**4), Ne(a**16*d**4*exp(20*I*c), 0)), (x*((exp(8*I*c) + 4*exp(6*I*c) + 6*e
xp(4*I*c) + 4*exp(2*I*c) + 1)*exp(-8*I*c)/(16*a**4) - 1/(16*a**4)), True))
+ x/(16*a**4)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.76

$$\int \frac{1}{(a+ia\tan(c+dx))^4}\,dx = -\frac{-\frac{12i\log(\tan(dx+c)+i)}{a^4}+\frac{12i\log(\tan(dx+c)-i)}{a^4}+\frac{-25i\tan(dx+c)^4-124\tan(dx+c)^3+246i\tan(dx+c)^2+252\tan(dx+c)-153i}{a^4(\tan(dx+c)-i)^4}}{384\,d}$$

```
[In] integrate(1/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/384*(-12*I*log(tan(d*x + c) + I)/a^4 + 12*I*log(tan(d*x + c) - I)/a^4 +
(-25*I*tan(d*x + c)^4 - 124*tan(d*x + c)^3 + 246*I*tan(d*x + c)^2 + 252*tan
(d*x + c) - 153*I)/(a^4*(tan(d*x + c) - I)^4))/d
```

Mupad [B] (verification not implemented)

Time = 4.52 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.52

$$\int \frac{1}{(a+ia\tan(c+dx))^4}\,dx = \frac{x}{16\,a^4} - \frac{-\frac{\tan(c+d\,x)^3}{16} + \frac{\tan(c+d\,x)^2\,1\mathrm{i}}{4} + \frac{19\tan(c+d\,x)}{48} - \frac{1}{3}\mathrm{i}}{a^4\,d\,(1+\tan\left(c+d\,x\right)\,1\mathrm{i})^4}$$

```
[In] int(1/(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] x/(16*a^4) - ((19*tan(c + d*x))/48 + (tan(c + d*x)^2*1i)/4 - tan(c + d*x)^3
/16 - 1i/3)/(a^4*d*(tan(c + d*x)*1i + 1)^4)
```

3.156 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 169

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{3x}{32a^4}+\frac{ia}{20d(a+ia\tan(c+dx))^5}+\frac{i}{16d(a+ia\tan(c+dx))^4}+\frac{i}{16ad(a+ia\tan(c+dx))^3}+\frac{i}{16d\left(a^2+ia^2\tan(c+dx)\right)^2}-\frac{i}{64d\left(a^4-ia^4\tan(c+dx)\right)}+\frac{5i}{64d\left(a^4+ia^4\tan(c+dx)\right)}$$

```
[Out] 3/32*x/a^4+1/20*I*a/d/(a+I*a*tan(d*x+c))^5+1/16*I/d/(a+I*a*tan(d*x+c))^4+1/
16*I/a/d/(a+I*a*tan(d*x+c))^3+1/16*I/d/(a^2+I*a^2*tan(d*x+c))^2-1/64*I/d/(a
^4-I*a^4*tan(d*x+c))+5/64*I/d/(a^4+I*a^4*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3568, 46, 212}

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{i}{64d\left(a^4-ia^4\tan(c+dx)\right)}+\frac{5i}{64d\left(a^4+ia^4\tan(c+dx)\right)}+\frac{3x}{32a^4}+\frac{i}{16d\left(a^2+ia^2\tan(c+dx)\right)^2}+\frac{ia}{20d(a+ia\tan(c+dx))^5}+\frac{i}{16d(a+ia\tan(c+dx))^4}+\frac{i}{16ad(a+ia\tan(c+dx))^3}$$

```
[In] Int[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (3*x)/(32*a^4) + ((I/20)*a)/(d*(a + I*a*Tan[c + d*x])^5) + (I/16)/(d*(a + I
*a*Tan[c + d*x])^4) + (I/16)/(a*d*(a + I*a*Tan[c + d*x])^3) + (I/16)/(d*(a^
2 + I*a^2*Tan[c + d*x])^2) - (I/64)/(d*(a^4 - I*a^4*Tan[c + d*x])) + ((5*I)
/64)/(d*(a^4 + I*a^4*Tan[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(ia^3)\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^6}\,dx, x, ia\tan(c+dx)\right)}{d} \\
&= -\frac{(ia^3)\,\text{Subst}\left(\int \left(\frac{1}{64a^6(a-x)^2} + \frac{1}{4a^2(a+x)^6} + \frac{1}{4a^3(a+x)^5} + \frac{3}{16a^4(a+x)^4} + \frac{1}{8a^5(a+x)^3} + \frac{5}{64a^6(a+x)^2} + \frac{3}{32a^6(a^2-x^2)}\right.\right.}{d} \\
&= \frac{ia}{20d(a+ia\tan(c+dx))^5} + \frac{i}{16d(a+ia\tan(c+dx))^4} + \frac{i}{16ad(a+ia\tan(c+dx))^3} \\
&\quad + \frac{i}{16d\left(a^2+ia^2\tan(c+dx)\right)^2} - \frac{i}{64d\left(a^4-ia^4\tan(c+dx)\right)} \\
&\quad + \frac{5i}{64d\left(a^4+ia^4\tan(c+dx)\right)} - \frac{(3i)\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, ia\tan(c+dx)\right)}{32a^3d} \\
&= \frac{3x}{32a^4} + \frac{ia}{20d(a+ia\tan(c+dx))^5} + \frac{i}{16d(a+ia\tan(c+dx))^4} + \frac{i}{16ad(a+ia\tan(c+dx))^3} \\
&\quad + \frac{i}{16d\left(a^2+ia^2\tan(c+dx)\right)^2} - \frac{i}{64d\left(a^4-ia^4\tan(c+dx)\right)} + \frac{5i}{64d\left(a^4+ia^4\tan(c+dx)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.84

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{\sec^6(c+dx)(50i+100i\cos(2(c+dx))+46i\cos(4(c+dx))-4i\cos(6(c+dx))-50\sin(2(c+dx))+60}{640a^4d(-i+\tan(c+dx))^5(}$$

```
[In] Integrate[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^6*(50*I + (100*I)*Cos[2*(c + d*x)] + (46*I)*Cos[4*(c + d*x)]
- (4*I)*Cos[6*(c + d*x)] - 50*Sin[2*(c + d*x)] + 60*ArcTan[Tan[c + d*x]]*(C
os[4*(c + d*x)] + I*Sin[4*(c + d*x)]) - 31*Sin[4*(c + d*x)] + 6*Sin[6*(c +
d*x)]))/(640*a^4*d*(-I + Tan[c + d*x])^5*(I + Tan[c + d*x]))
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.68

method	result
derivativedivides	$\frac{\frac{3i\ln(\tan(dx+c)+i)}{64}+\frac{1}{64\tan(dx+c)+64i}-\frac{3i\ln(\tan(dx+c)-i)}{64}+\frac{i}{16(\tan(dx+c)-i)^4}-\frac{i}{16(\tan(dx+c)-i)^2}+\frac{1}{20(\tan(dx+c)-i)^5}-\frac{}{16(\text{ta}}}{d\,a^4}$
default	$\frac{\frac{3i\ln(\tan(dx+c)+i)}{64}+\frac{1}{64\tan(dx+c)+64i}-\frac{3i\ln(\tan(dx+c)-i)}{64}+\frac{i}{16(\tan(dx+c)-i)^4}-\frac{i}{16(\tan(dx+c)-i)^2}+\frac{1}{20(\tan(dx+c)-i)^5}-\frac{}{16(\text{ta}}}{d\,a^4}$
risch	$\frac{3x}{32a^4}+\frac{5i\mathrm{e}^{-4i(dx+c)}}{64a^4d}+\frac{5i\mathrm{e}^{-6i(dx+c)}}{128a^4d}+\frac{3i\mathrm{e}^{-8i(dx+c)}}{256a^4d}+\frac{i\mathrm{e}^{-10i(dx+c)}}{640a^4d}+\frac{7i\cos(2dx+2c)}{64a^4d}+\frac{\sin(2dx+2c)}{8a^4d}$

```
[In] int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^4*(3/64*I*ln(tan(d*x+c)+I)+1/64/(tan(d*x+c)+I)-3/64*I*ln(tan(d*x+c)-I
)+1/16*I/(tan(d*x+c)-I)^4-1/16*I/(tan(d*x+c)-I)^2+1/20/(tan(d*x+c)-I)^5-1/1
6/(tan(d*x+c)-I)^3+5/64/(tan(d*x+c)-I))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.51

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{(120\,dxe^{(10i\,dx+10i\,c)}-10i\,e^{(12i\,dx+12i\,c)}+150i\,e^{(8i\,dx+8i\,c)}+100i\,e^{(6i\,dx+6i\,c)}+50i\,e^{(4i\,dx+4i\,c)}+15i\,e^{(2i\,dx+2i\,c)}}{1280\,a^4d}$$

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/1280*(120*d*x*e^(10*I*d*x + 10*I*c) - 10*I*e^(12*I*d*x + 12*I*c) + 150*I*
e^(8*I*d*x + 8*I*c) + 100*I*e^(6*I*d*x + 6*I*c) + 50*I*e^(4*I*d*x + 4*I*c)
+ 15*I*e^(2*I*d*x + 2*I*c) + 2*I)*e^(-10*I*d*x - 10*I*c)/(a^4*d)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.53

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \begin{cases} \frac{(-171798691840ia^{20}d^5e^{32ic}e^{2idx}+2576980377600ia^{20}d^5e^{28ic}e^{-2idx}+1717986918400ia^{20}d^5e^{26ic}e^{-4idx}+858993459200ia^{20}d^5e^{24ic}e^{-6idx}+257\cdots}{21990232555520a^{24}d^6} \\ x\left(\frac{(e^{12ic}+6e^{10ic}+15e^{8ic}+20e^{6ic}+15e^{4ic}+6e^{2ic}+1)e^{-10ic}}{64a^4}-\frac{3}{32a^4}\right) \end{cases} + \frac{3x}{32a^4}$$

```
[In] integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise(((-171798691840*I*a**20*d**5*exp(32*I*c)*exp(2*I*d*x) + 257698037
7600*I*a**20*d**5*exp(28*I*c)*exp(-2*I*d*x) + 1717986918400*I*a**20*d**5*ex
p(26*I*c)*exp(-4*I*d*x) + 858993459200*I*a**20*d**5*exp(24*I*c)*exp(-6*I*d*
x) + 257698037760*I*a**20*d**5*exp(22*I*c)*exp(-8*I*d*x) + 34359738368*I*a*
*20*d**5*exp(20*I*c)*exp(-10*I*d*x))*exp(-30*I*c)/(21990232555520*a**24*d**
6), Ne(a**24*d**6*exp(30*I*c), 0)), (x*((exp(12*I*c) + 6*exp(10*I*c) + 15*e
xp(8*I*c) + 20*exp(6*I*c) + 15*exp(4*I*c) + 6*exp(2*I*c) + 1)*exp(-10*I*c)/
(64*a**4) - 3/(32*a**4)), True)) + 3*x/(32*a**4)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [A] (verification not implemented)

none

Time = 0.80 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.73

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx =$$
$$-\frac{-\frac{60i\,\log(\tan(dx+c)+i)}{a^4}+\frac{60i\,\log(\tan(dx+c)-i)}{a^4}+\frac{20\,(3i\,\tan(dx+c)-4)}{a^4(\tan(dx+c)+i)}+\frac{-137i\,\tan(dx+c)^5-785\,\tan(dx+c)^4+1850i\,\tan(dx+c)^3+2}{a^4(\tan(dx+c)-i)^5}}{1280\,d}$$

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/1280*(-60*I*log(tan(d*x + c) + I)/a^4 + 60*I*log(tan(d*x + c) - I)/a^4 +
 20*(3*I*tan(d*x + c) - 4)/(a^4*(tan(d*x + c) + I)) + (-137*I*tan(d*x + c)^
5 - 785*tan(d*x + c)^4 + 1850*I*tan(d*x + c)^3 + 2290*tan(d*x + c)^2 - 1565
*I*tan(d*x + c) - 541)/(a^4*(tan(d*x + c) - I)^5))/d
```

Mupad [B] (verification not implemented)

Time = 4.60 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.53

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{3\,x}{32\,a^4}-\frac{-\frac{3\tan(c+d\,x)^5}{32}+\frac{\tan(c+d\,x)^4\,3\mathrm{i}}{8}+\frac{\tan(c+d\,x)^3}{2}-\frac{\tan(c+d\,x)^2\,1\mathrm{i}}{8}+\frac{47\tan(c+d\,x)}{160}-\frac{3}{10}\mathrm{i}}{a^4\,d\,(\tan(c+d\,x)-\mathrm{i})^5\,(\tan(c+d\,x)+1\mathrm{i})}$$

```
[In] int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] (3*x)/(32*a^4) - ((47*tan(c + d*x))/160 - (tan(c + d*x)^2*1i)/8 + tan(c + d
*x)^3/2 + (tan(c + d*x)^4*3i)/8 - (3*tan(c + d*x)^5)/32 - 3i/10)/(a^4*d*(ta
n(c + d*x) - 1i)^5*(tan(c + d*x) + 1i))
```

3.157 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 224

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx &= \frac{7x}{64a^4}+\frac{ia^2}{48d(a+ia\tan(c+dx))^6}+\frac{3ia}{80d(a+ia\tan(c+dx))^5}\\&\quad+\frac{3i}{64d(a+ia\tan(c+dx))^4}+\frac{5i}{96ad(a+ia\tan(c+dx))^3}\\&\quad-\frac{i}{256d\,(a^2-ia^2\tan(c+dx))^2}+\frac{15i}{256d\,(a^2+ia^2\tan(c+dx))^2}\\&\quad-\frac{7i}{256d\,(a^4-ia^4\tan(c+dx))}+\frac{21i}{256d\,(a^4+ia^4\tan(c+dx))}\end{aligned}$$

```
[Out] 7/64*x/a^4+1/48*I*a^2/d/(a+I*a*tan(d*x+c))^6+3/80*I*a/d/(a+I*a*tan(d*x+c))^
5+3/64*I/d/(a+I*a*tan(d*x+c))^4+5/96*I/a/d/(a+I*a*tan(d*x+c))^3-1/256*I/d/(
a^2-I*a^2*tan(d*x+c))^2+15/256*I/d/(a^2+I*a^2*tan(d*x+c))^2-7/256*I/d/(a^4-
I*a^4*tan(d*x+c))+21/256*I/d/(a^4+I*a^4*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used

= {3568, 46, 212}

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{7i}{256d\,(a^4-ia^4\tan(c+dx))}+\frac{21i}{256d\,(a^4+ia^4\tan(c+dx))}+\frac{7x}{64a^4}+\frac{ia^2}{48d(a+ia\tan(c+dx))^6}-\frac{i}{256d\,(a^2-ia^2\tan(c+dx))^2}+\frac{15i}{256d\,(a^2+ia^2\tan(c+dx))^2}+\frac{3ia}{80d(a+ia\tan(c+dx))^5}+\frac{3i}{64d(a+ia\tan(c+dx))^4}+\frac{5i}{96ad(a+ia\tan(c+dx))^3}$$

```
[In] Int[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (7*x)/(64*a^4) + ((I/48)*a^2)/(d*(a + I*a*Tan[c + d*x])^6) + (((3*I)/80)*a)
/(d*(a + I*a*Tan[c + d*x])^5) + ((3*I)/64)/(d*(a + I*a*Tan[c + d*x])^4) + (
(5*I)/96)/(a*d*(a + I*a*Tan[c + d*x])^3) - (I/256)/(d*(a^2 - I*a^2*Tan[c +
d*x])^2) + ((15*I)/256)/(d*(a^2 + I*a^2*Tan[c + d*x])^2) - ((7*I)/256)/(d*(
a^4 - I*a^4*Tan[c + d*x])) + ((21*I)/256)/(d*(a^4 + I*a^4*Tan[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{(ia^5)\,\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^7}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^5)\,\text{Subst}\Big(\int \Big(\frac{1}{128a^7(a-x)^3} + \frac{7}{256a^8(a-x)^2} + \frac{1}{8a^3(a+x)^7} + \frac{3}{16a^4(a+x)^6} + \frac{3}{16a^5(a+x)^5} + \frac{5}{32a^6(a+x)^4} + \frac{15}{128a^7(a \cdots}{d}$$

$$= \frac{ia^2}{48d(a+ia\tan(c+dx))^6} + \frac{3ia}{80d(a+ia\tan(c+dx))^5} + \frac{3i}{64d(a+ia\tan(c+dx))^4} + \frac{5i}{96ad(a+ia\tan(c+dx))^3} - \frac{i}{256d\,(a^2-ia^2\tan(c+dx))^2} + \frac{15i}{256d\,(a^2+ia^2\tan(c+dx))^2} - \frac{7i}{256d\,(a^4-ia^4\tan(c+dx))} + \frac{21i}{256d\,(a^4+ia^4\tan(c+dx))} - \frac{(7i)\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, ia\tan(c+dx)\right)}{64a^3d}$$

$$= \frac{7x}{64a^4} + \frac{ia^2}{48d(a+ia\tan(c+dx))^6} + \frac{3ia}{80d(a+ia\tan(c+dx))^5} + \frac{3i}{64d(a+ia\tan(c+dx))^4} + \frac{5i}{96ad(a+ia\tan(c+dx))^3} - \frac{i}{256d\,(a^2-ia^2\tan(c+dx))^2} + \frac{15i}{256d\,(a^2+ia^2\tan(c+dx))^2} - \frac{7i}{256d\,(a^4-ia^4\tan(c+dx))} + \frac{21i}{256d\,(a^4+ia^4\tan(c+dx))}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.73

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\sec^8(c+dx)(525i + 1120i\cos(2(c+dx)) + 504i\cos(4(c+dx)) - 96i\cos(6(c+dx)) - 5i\cos(8(c+dx)) - \cdots}{7680 \cdots}$$

```
[In] Integrate[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^8*(525*I + (1120*I)*Cos[2*(c + d*x)] + (504*I)*Cos[4*(c + d*x
)] - (96*I)*Cos[6*(c + d*x)] - (5*I)*Cos[8*(c + d*x)] - 560*Sin[2*(c + d*x)
] + 840*ArcTan[Tan[c + d*x]]*(Cos[4*(c + d*x)] + I*Sin[4*(c + d*x)]) - 294*
Sin[4*(c + d*x)] + 144*Sin[6*(c + d*x)] + 10*Sin[8*(c + d*x)]))/(7680*a^4*d
*(-I + Tan[c + d*x])^6*(I + Tan[c + d*x])^2)
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.64

method	result
derivativedivides	$\frac{-\frac{7i\ln(\tan(dx+c)-i)}{128}+\frac{3i}{64(\tan(dx+c)-i)^4}-\frac{i}{48(\tan(dx+c)-i)^6}-\frac{15i}{256(\tan(dx+c)-i)^2}+\frac{3}{80(\tan(dx+c)-i)^5}-\frac{5}{96(\tan(dx+c)-i)^3}+\frac{25}{}}{d\,a^4}$
default	$\frac{-\frac{7i\ln(\tan(dx+c)-i)}{128}+\frac{3i}{64(\tan(dx+c)-i)^4}-\frac{i}{48(\tan(dx+c)-i)^6}-\frac{15i}{256(\tan(dx+c)-i)^2}+\frac{3}{80(\tan(dx+c)-i)^5}-\frac{5}{96(\tan(dx+c)-i)^3}+\frac{25}{}}{d\,a^4}$
risch	$\frac{7x}{64a^4}+\frac{7ie^{-6i(dx+c)}}{192a^4d}+\frac{7ie^{-8i(dx+c)}}{512a^4d}+\frac{ie^{-10i(dx+c)}}{320a^4d}+\frac{ie^{-12i(dx+c)}}{3072a^4d}+\frac{69i\cos(4dx+4c)}{1024a^4d}+\frac{71\sin(4dx+4c)}{1024a^4d}+$

```
[In] int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^4*(-7/128*I*ln(tan(d*x+c)-I)+3/64*I/(tan(d*x+c)-I)^4-1/48*I/(tan(d*x+
c)-I)^6-15/256*I/(tan(d*x+c)-I)^2+3/80/(tan(d*x+c)-I)^5-5/96/(tan(d*x+c)-I)
^3+21/256/(tan(d*x+c)-I)+1/256*I/(tan(d*x+c)+I)^2+7/128*I*ln(tan(d*x+c)+I)+
7/256/(tan(d*x+c)+I))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.49

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\left(1680\,dxe^{(12i\,dx+12i\,c)}-15i\,e^{(16i\,dx+16i\,c)}-240i\,e^{(14i\,dx+14i\,c)}+1680i\,e^{(10i\,dx+10i\,c)}+1050i\,e^{(8i\,dx+8i\,c)}+560i\,e\right.}{15360\,a^4d}$$

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/15360*(1680*d*x*e^(12*I*d*x + 12*I*c) - 15*I*e^(16*I*d*x + 16*I*c) - 240*
I*e^(14*I*d*x + 14*I*c) + 1680*I*e^(10*I*d*x + 10*I*c) + 1050*I*e^(8*I*d*x
+ 8*I*c) + 560*I*e^(6*I*d*x + 6*I*c) + 210*I*e^(4*I*d*x + 4*I*c) + 48*I*e^(
2*I*d*x + 2*I*c) + 5*I)*e^(-12*I*d*x - 12*I*c)/(a^4*d)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.46

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \begin{cases} \frac{(-202661983231672320ia^{28}d^7e^{46ic}e^{4idx}-3242591731706757120ia^{28}d^7e^{44ic}e^{2idx}+22698142121947299840ia^{28}d^7e^{40ic}e^{-2idx}+141863388262}{} \\ x\left(\frac{(e^{16ic}+8e^{14ic}+28e^{12ic}+56e^{10ic}+70e^{8ic}+56e^{6ic}+28e^{4ic}+8e^{2ic}+1)e^{-12ic}}{256a^4}-\frac{7}{64a^4}\right) \end{cases} + \frac{7x}{64a^4}$$

```
[In] integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise(((-202661983231672320*I*a**28*d**7*exp(46*I*c)*exp(4*I*d*x) - 324
2591731706757120*I*a**28*d**7*exp(44*I*c)*exp(2*I*d*x) + 226981421219472998
40*I*a**28*d**7*exp(40*I*c)*exp(-2*I*d*x) + 14186338826217062400*I*a**28*d*
*7*exp(38*I*c)*exp(-4*I*d*x) + 7566047373982433280*I*a**28*d**7*exp(36*I*c)
*exp(-6*I*d*x) + 2837267765243412480*I*a**28*d**7*exp(34*I*c)*exp(-8*I*d*x)
 + 648518346341351424*I*a**28*d**7*exp(32*I*c)*exp(-10*I*d*x) + 67553994410
557440*I*a**28*d**7*exp(30*I*c)*exp(-12*I*d*x))*exp(-42*I*c)/(2075258708292
32455680*a**32*d**8), Ne(a**32*d**8*exp(42*I*c), 0)), (x*((exp(16*I*c) + 8*
exp(14*I*c) + 28*exp(12*I*c) + 56*exp(10*I*c) + 70*exp(8*I*c) + 56*exp(6*I*
c) + 28*exp(4*I*c) + 8*exp(2*I*c) + 1)*exp(-12*I*c)/(256*a**4) - 7/(64*a**4
)), True)) + 7*x/(64*a**4)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [A] (verification not implemented)

none

Time = 0.81 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.64

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{-\frac{420i\log(\tan(dx+c)+i)}{a^4}+\frac{420i\log(\tan(dx+c)-i)}{a^4}+\frac{30\left(21i\tan(dx+c)^2-49\tan(dx+c)-29i\right)}{a^4(\tan(dx+c)+i)^2}+\frac{-1029i\tan(dx+c)^6-6804\tan(dx+c)^5}{}}{7680\,d}$$

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/7680*(-420*I*log(tan(d*x + c) + I)/a^4 + 420*I*log(tan(d*x + c) - I)/a^4
 + 30*(21*I*tan(d*x + c)^2 - 49*tan(d*x + c) - 29*I)/(a^4*(tan(d*x + c) + I
)^2) + (-1029*I*tan(d*x + c)^6 - 6804*tan(d*x + c)^5 + 19035*I*tan(d*x + c)
^4 + 29080*tan(d*x + c)^3 - 25995*I*tan(d*x + c)^2 - 13332*tan(d*x + c) + 3
317*I)/(a^4*(tan(d*x + c) - I)^6))/d
```

Mupad [B] (verification not implemented)

Time = 6.07 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.88

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{7\,x}{64\,a^4} + \frac{\frac{\tan(c+d\,x)\,169\mathrm{i}}{960\,a^4}+\frac{4}{15\,a^4}+\frac{119\,\tan(c+d\,x)^2}{240\,a^4}+\frac{\tan(c+d\,x)^3\,889\mathrm{i}}{960\,a^4}-\frac{7\,\tan(c+d\,x)^4}{24\,a^4}+\frac{\tan(c+d\,x)^5\,91\mathrm{i}}{192\,a^4}}{d\,\left(-\tan\left(c+d\,x\right)^8\,1\mathrm{i}-4\,\tan\left(c+d\,x\right)^7+\tan\left(c+d\,x\right)^6\,4\mathrm{i}-4\,\tan\left(c+d\,x\right)^5+\tan\left(c+d\,x\right)^4\,10\mathrm{i}+4\,\tan\right.}$$

```
[In] int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] (7*x)/(64*a^4) + ((tan(c + d*x)*169i)/(960*a^4) + 4/(15*a^4) + (119*tan(c +
 d*x)^2)/(240*a^4) + (tan(c + d*x)^3*889i)/(960*a^4) - (7*tan(c + d*x)^4)/(
24*a^4) + (tan(c + d*x)^5*91i)/(192*a^4) - (7*tan(c + d*x)^6)/(16*a^4) - (t
an(c + d*x)^7*7i)/(64*a^4))/(d*(4*tan(c + d*x) + tan(c + d*x)^2*4i + 4*tan(
c + d*x)^3 + tan(c + d*x)^4*10i - 4*tan(c + d*x)^5 + tan(c + d*x)^6*4i - 4*
tan(c + d*x)^7 - tan(c + d*x)^8*1i - 1i))
```

3.158 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 133

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{35\text{arctanh}(\sin(c+dx))}{8a^4d} + \frac{35\sec(c+dx)\tan(c+dx)}{8a^4d} + \frac{35\sec^3(c+dx)\tan(c+dx)}{12a^4d} - \frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3} - \frac{14i\sec^5(c+dx)}{3d\,(a^4+ia^4\tan(c+dx))}$$

```
[Out] 35/8*arctanh(sin(d*x+c))/a^4/d+35/8*sec(d*x+c)*tan(d*x+c)/a^4/d+35/12*sec(d
*x+c)^3*tan(d*x+c)/a^4/d-2*I*sec(d*x+c)^7/a/d/(a+I*a*tan(d*x+c))^3-14/3*I*s
ec(d*x+c)^5/d/(a^4+I*a^4*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3581, 3853, 3855}

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{35\text{arctanh}(\sin(c+dx))}{8a^4d} - \frac{14i\sec^5(c+dx)}{3d\,(a^4+ia^4\tan(c+dx))} + \frac{35\tan(c+dx)\sec^3(c+dx)}{12a^4d} + \frac{35\tan(c+dx)\sec(c+dx)}{8a^4d} - \frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}$$

```
[In] Int[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (35*ArcTanh[Sin[c + d*x]])/(8*a^4*d) + (35*Sec[c + d*x]*Tan[c + d*x])/(8*a^
4*d) + (35*Sec[c + d*x]^3*Tan[c + d*x])/(12*a^4*d) - ((2*I)*Sec[c + d*x]^7)
/(a*d*(a + I*a*Tan[c + d*x])^3) - (((14*I)/3)*Sec[c + d*x]^5)/(d*(a^4 + I*a
^4*Tan[c + d*x]))
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}+\frac{7\int\frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{a^2}\\
&= -\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}-\frac{14i\sec^5(c+dx)}{3d\,(a^4+ia^4\tan(c+dx))}+\frac{35\int\sec^5(c+dx)\,dx}{3a^4}\\
&= \frac{35\sec^3(c+dx)\tan(c+dx)}{12a^4d}-\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}\\
&\quad-\frac{14i\sec^5(c+dx)}{3d\,(a^4+ia^4\tan(c+dx))}+\frac{35\int\sec^3(c+dx)\,dx}{4a^4}\\
&= \frac{35\sec(c+dx)\tan(c+dx)}{8a^4d}+\frac{35\sec^3(c+dx)\tan(c+dx)}{12a^4d}\\
&\quad-\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}-\frac{14i\sec^5(c+dx)}{3d\,(a^4+ia^4\tan(c+dx))}+\frac{35\int\sec(c+dx)\,dx}{8a^4}\\
&= \frac{35\operatorname{arctanh}(\sin(c+dx))}{8a^4d}+\frac{35\sec(c+dx)\tan(c+dx)}{8a^4d}+\frac{35\sec^3(c+dx)\tan(c+dx)}{12a^4d}\\
&\quad-\frac{2i\sec^7(c+dx)}{ad(a+ia\tan(c+dx))^3}-\frac{14i\sec^5(c+dx)}{3d\,(a^4+ia^4\tan(c+dx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.54 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.78

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^4}\,dx =$$
$$-\frac{\sec^4(c+dx)\left(896i\cos(c+dx)+3\left(128i\cos(3(c+dx))+105\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+3\right.\right.}{}$$

```
[In] Integrate[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] -1/192*(Sec[c + d*x]^4*((896*I)*Cos[c + d*x] + 3*((128*I)*Cos[3*(c + d*x)]
+ 105*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 35*Cos[4*(c + d*x)]*Log[Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]] + 140*Cos[2*(c + d*x)]*(Log[Cos[(c + d*x
)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 105*
Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 35*Cos[4*(c + d*x)]*Log[Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]] + 42*Sin[c + d*x] + 58*Sin[3*(c + d*x)])))/(a^
4*d)
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.83

method	result
risch	$-\frac{i\left(105\,\mathrm{e}^{7i(dx+c)}+385\,\mathrm{e}^{5i(dx+c)}+511\,\mathrm{e}^{3i(dx+c)}+279\,\mathrm{e}^{i(dx+c)}\right)}{12d\,a^4\left(\mathrm{e}^{2i(dx+c)}+1\right)^4}+\frac{35\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{8a^4d}-\frac{35\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{8a^4d}$
derivativedivides	$\frac{\frac{2\left(\frac{1}{4}-\frac{2i}{3}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}+\frac{2\left(-\frac{25}{16}-i\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{2\left(-\frac{27}{16}+3i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{35\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}+\frac{2\left(\frac{25}{16}-i\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+}{a^4d}$
default	$\frac{\frac{2\left(\frac{1}{4}-\frac{2i}{3}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}+\frac{2\left(-\frac{25}{16}-i\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{2\left(-\frac{27}{16}+3i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{35\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}+\frac{2\left(\frac{25}{16}-i\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+}{a^4d}$

```
[In] int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*I/d/a^4/(exp(2*I*(d*x+c))+1)^4*(105*exp(7*I*(d*x+c))+385*exp(5*I*(d*x
+c))+511*exp(3*I*(d*x+c))+279*exp(I*(d*x+c)))+35/8/a^4/d*ln(exp(I*(d*x+c))+
I)-35/8/a^4/d*ln(exp(I*(d*x+c))-I)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.73

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{105\left(e^{(8i\,dx+8i\,c)}+4\,e^{(6i\,dx+6i\,c)}+6\,e^{(4i\,dx+4i\,c)}+4\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-105\left(e^{(8i\,dx+8i\,c)}+4\,e^{(\cdots}\right.}{24\left(a^4de^{(8i\,dx+8i\,c)}+4\,a^4de^{(\cdots}\right.}$$

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/24*(105*(e^(8*I*d*x + 8*I*c) + 4*e^(6*I*d*x + 6*I*c) + 6*e^(4*I*d*x + 4*I
*c) + 4*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) + I) - 105*(e^(8*I*d*x
 + 8*I*c) + 4*e^(6*I*d*x + 6*I*c) + 6*e^(4*I*d*x + 4*I*c) + 4*e^(2*I*d*x +
2*I*c) + 1)*log(e^(I*d*x + I*c) - I) - 210*I*e^(7*I*d*x + 7*I*c) - 770*I*e^
(5*I*d*x + 5*I*c) - 1022*I*e^(3*I*d*x + 3*I*c) - 558*I*e^(I*d*x + I*c))/(a^
4*d*e^(8*I*d*x + 8*I*c) + 4*a^4*d*e^(6*I*d*x + 6*I*c) + 6*a^4*d*e^(4*I*d*x
+ 4*I*c) + 4*a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)
```

Sympy [F]

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\int \frac{\sec^9(c+dx)}{\tan^4(c+dx)-4i\tan^3(c+dx)-6\tan^2(c+dx)+4i\tan(c+dx)+1}\,dx}{a^4}$$

```
[In] integrate(sec(d*x+c)**9/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**9/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6*tan(c +
 d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 295 vs. 2(117) = 234.

Time = 0.23 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.22

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{\frac{2\left(\frac{81\sin(dx+c)}{\cos(dx+c)+1}-\frac{544i\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{105\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{480i\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{105\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{96i\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{81\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+160i\right)}{a^4-\frac{4a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a^4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^4\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{105\log\left(\frac{\sin\cdots}{\cos\cdots}\right.}{\cdots}}{24\,d}$$

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/24*(2*(81*sin(d*x + c)/(cos(d*x + c) + 1) - 544*I*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 - 105*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 480*I*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 - 105*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 96*I*s
in(d*x + c)^6/(cos(d*x + c) + 1)^6 + 81*sin(d*x + c)^7/(cos(d*x + c) + 1)^7
 + 160*I)/(a^4 - 4*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^4*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4 - 4*a^4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a
^4*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 105*log(sin(d*x + c)/(cos(d*x + c
) + 1) + 1)/a^4 + 105*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4)/d
```

Giac [A] (verification not implemented)

none

Time = 0.79 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.14

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\frac{105\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^4} - \frac{105\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^4} - \frac{2\left(81\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-96i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-105\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+480i\,\tan\left(\frac{1}{2}\,d\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\right.\right.}}{24\,d}$$

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(105*log(tan(1/2*d*x + 1/2*c) + 1)/a^4 - 105*log(tan(1/2*d*x + 1/2*c)
- 1)/a^4 - 2*(81*tan(1/2*d*x + 1/2*c)^7 - 96*I*tan(1/2*d*x + 1/2*c)^6 - 105
*tan(1/2*d*x + 1/2*c)^5 + 480*I*tan(1/2*d*x + 1/2*c)^4 - 105*tan(1/2*d*x +
1/2*c)^3 - 544*I*tan(1/2*d*x + 1/2*c)^2 + 81*tan(1/2*d*x + 1/2*c) + 160*I)/
((tan(1/2*d*x + 1/2*c)^2 - 1)^4*a^4))/d
```

Mupad [B] (verification not implemented)

Time = 7.75 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.48

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{35\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,a^4\,d} + \frac{\frac{35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4\,a^4} + \frac{35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4\,a^4} - \frac{27\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4\,a^4} - \frac{27\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,a^4} + \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,136\mathrm{i}}{3\,a^4} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,40\mathrm{i}}{a^4} + \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

```
[In] int(1/(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^4),x)
```

```
[Out] (35*atanh(tan(c/2 + (d*x)/2)))/(4*a^4*d) + ((tan(c/2 + (d*x)/2)^2*136i)/(3*
a^4) + (35*tan(c/2 + (d*x)/2)^3)/(4*a^4) - (tan(c/2 + (d*x)/2)^4*40i)/a^4 +
```

```
 (35*tan(c/2 + (d*x)/2)^5)/(4*a^4) + (tan(c/2 + (d*x)/2)^6*8i)/a^4 - (27*ta
n(c/2 + (d*x)/2)^7)/(4*a^4) - 40i/(3*a^4) - (27*tan(c/2 + (d*x)/2))/(4*a^4)
)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2
)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

3.159 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 107

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{15\text{arctanh}(\sin(c+dx))}{2a^4d} - \frac{15\sec(c+dx)\tan(c+dx)}{2a^4d} + \frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3} + \frac{10i\sec^3(c+dx)}{d\,(a^4+ia^4\tan(c+dx))}$$

```
[Out] -15/2*arctanh(sin(d*x+c))/a^4/d-15/2*sec(d*x+c)*tan(d*x+c)/a^4/d+2*I*sec(d*
x+c)^5/a/d/(a+I*a*tan(d*x+c))^3+10*I*sec(d*x+c)^3/d/(a^4+I*a^4*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3581, 3853, 3855}

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{15\text{arctanh}(\sin(c+dx))}{2a^4d} + \frac{10i\sec^3(c+dx)}{d\,(a^4+ia^4\tan(c+dx))} - \frac{15\tan(c+dx)\sec(c+dx)}{2a^4d} + \frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3}$$

```
[In] Int[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (-15*ArcTanh[Sin[c + d*x]])/(2*a^4*d) - (15*Sec[c + d*x]*Tan[c + d*x])/(2*a
^4*d) + ((2*I)*Sec[c + d*x]^5)/(a*d*(a + I*a*Tan[c + d*x])^3) + ((10*I)*Sec
[c + d*x]^3)/(d*(a^4 + I*a^4*Tan[c + d*x]))
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3} - \frac{5\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{a^2} \\
&= \frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3} + \frac{10i\sec^3(c+dx)}{d\left(a^4+ia^4\tan(c+dx)\right)} - \frac{15\int \sec^3(c+dx)\,dx}{a^4} \\
&= -\frac{15\sec(c+dx)\tan(c+dx)}{2a^4d} + \frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3} \\
&\quad + \frac{10i\sec^3(c+dx)}{d\left(a^4+ia^4\tan(c+dx)\right)} - \frac{15\int \sec(c+dx)\,dx}{2a^4} \\
&= -\frac{15\text{arctanh}(\sin(c+dx))}{2a^4d} - \frac{15\sec(c+dx)\tan(c+dx)}{2a^4d} \\
&\quad + \frac{2i\sec^5(c+dx)}{ad(a+ia\tan(c+dx))^3} + \frac{10i\sec^3(c+dx)}{d\left(a^4+ia^4\tan(c+dx)\right)}
\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 988 vs. $2(107) = 214$.

Time = 6.64 (sec) , antiderivative size = 988, normalized size of antiderivative = 9.23

$$
\begin{aligned}
&\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^4}\,dx \\
&= \frac{15\cos(4c)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^4(c+dx)(\cos(dx)+i\sin(dx))^4}{2d(a+ia\tan(c+dx))^4} \\
&\quad - \frac{15\cos(4c)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^4(c+dx)(\cos(dx)+i\sin(dx))^4}{2d(a+ia\tan(c+dx))^4} \\
&\quad + \frac{\cos(dx)\sec^4(c+dx)(8i\cos(3c)-8\sin(3c))(\cos(dx)+i\sin(dx))^4}{d(a+ia\tan(c+dx))^4} \\
&\quad + \frac{\sec(c)\sec^4(c+dx)(4i\cos(4c)-4\sin(4c))(\cos(dx)+i\sin(dx))^4}{d(a+ia\tan(c+dx))^4} \\
&\quad + \frac{15i\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^4(c+dx)\sin(4c)(\cos(dx)+i\sin(dx))^4}{2d(a+ia\tan(c+dx))^4} \\
&\quad - \frac{15i\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^4(c+dx)\sin(4c)(\cos(dx)+i\sin(dx))^4}{2d(a+ia\tan(c+dx))^4} \\
&\quad + \frac{\sec^4(c+dx)(8\cos(3c)+8i\sin(3c))(\cos(dx)+i\sin(dx))^4\sin(dx)}{d(a+ia\tan(c+dx))^4} \\
&\quad + \frac{\sec^4(c+dx)\left(\frac{1}{4}\cos(4c)+\frac{1}{4}i\sin(4c)\right)(\cos(dx)+i\sin(dx))^4}{d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^2(a+ia\tan(c+dx))^4} \\
&\quad + \frac{\sec^4(c+dx)\left(-\frac{1}{4}\cos(4c)-\frac{1}{4}i\sin(4c)\right)(\cos(dx)+i\sin(dx))^4}{d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^2(a+ia\tan(c+dx))^4} \\
&\quad + \frac{4\sec^4(c+dx)(\cos(dx)+i\sin(dx))^4\left(\frac{1}{2}\cos\left(4c-\frac{dx}{2}\right)-\frac{1}{2}\cos\left(4c+\frac{dx}{2}\right)+\frac{1}{2}i\sin\left(4c-\frac{dx}{2}\right)-\frac{1}{2}i\sin\left(4c+\right.\right.}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(a+ia\tan(c+dx))^4} \\
&\quad + \frac{4\sec^4(c+dx)(\cos(dx)+i\sin(dx))^4\left(-\frac{1}{2}\cos\left(4c-\frac{dx}{2}\right)+\frac{1}{2}\cos\left(4c+\frac{dx}{2}\right)-\frac{1}{2}i\sin\left(4c-\frac{dx}{2}\right)+\frac{1}{2}i\sin\left(4c+\right.\right.}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(a+ia\tan(c+dx))^4}
\end{aligned}
$$

```
[In] Integrate[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (15*Cos[4*c]*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^4*(C
os[d*x] + I*Sin[d*x])^4)/(2*d*(a + I*a*Tan[c + d*x])^4) - (15*Cos[4*c]*Log[
Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^4*(Cos[d*x] + I*Sin[d
*x])^4)/(2*d*(a + I*a*Tan[c + d*x])^4) + (Cos[d*x]*Sec[c + d*x]^4*((8*I)*Co
s[3*c] - 8*Sin[3*c])*(Cos[d*x] + I*Sin[d*x])^4)/(d*(a + I*a*Tan[c + d*x])^4
) + (Sec[c]*Sec[c + d*x]^4*((4*I)*Cos[4*c] - 4*Sin[4*c])*(Cos[d*x] + I*Sin[
d*x])^4)/(d*(a + I*a*Tan[c + d*x])^4) + (((15*I)/2)*Log[Cos[c/2 + (d*x)/2]
- Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^4*Sin[4*c]*(Cos[d*x] + I*Sin[d*x])^4)/(d
```

```
*(a + I*a*Tan[c + d*x])^4) - (((15*I)/2)*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 +
 (d*x)/2]]*Sec[c + d*x]^4*Sin[4*c]*(Cos[d*x] + I*Sin[d*x])^4)/(d*(a + I*a*T
an[c + d*x])^4) + (Sec[c + d*x]^4*(8*Cos[3*c] + (8*I)*Sin[3*c])*(Cos[d*x] +
 I*Sin[d*x])^4*Sin[d*x])/(d*(a + I*a*Tan[c + d*x])^4) + (Sec[c + d*x]^4*(Co
s[4*c]/4 + (I/4)*Sin[4*c])*(Cos[d*x] + I*Sin[d*x])^4)/(d*(Cos[c/2 + (d*x)/2
] - Sin[c/2 + (d*x)/2])^2*(a + I*a*Tan[c + d*x])^4) + (Sec[c + d*x]^4*(-1/4
*Cos[4*c] - (I/4)*Sin[4*c])*(Cos[d*x] + I*Sin[d*x])^4)/(d*(Cos[c/2 + (d*x)/
2] + Sin[c/2 + (d*x)/2])^2*(a + I*a*Tan[c + d*x])^4) + (4*Sec[c + d*x]^4*(C
os[d*x] + I*Sin[d*x])^4*(Cos[4*c - (d*x)/2]/2 - Cos[4*c + (d*x)/2]/2 + (I/2
)*Sin[4*c - (d*x)/2] - (I/2)*Sin[4*c + (d*x)/2]))/(d*(Cos[c/2] + Sin[c/2])*
(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])*(a + I*a*Tan[c + d*x])^4) + (4*Se
c[c + d*x]^4*(Cos[d*x] + I*Sin[d*x])^4*(-1/2*Cos[4*c - (d*x)/2] + Cos[4*c +
 (d*x)/2]/2 - (I/2)*Sin[4*c - (d*x)/2] + (I/2)*Sin[4*c + (d*x)/2]))/(d*(Cos
[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])*(a + I*a*Tan[c
+ d*x])^4)
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00

method	result
risch	$\frac{8i\mathrm{e}^{-i(dx+c)}}{a^4 d}+\frac{i\left(7\,\mathrm{e}^{3i(dx+c)}+9\,\mathrm{e}^{i(dx+c)}\right)}{d\,a^4\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}-\frac{15\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2a^4 d}+\frac{15\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2a^4 d}$
derivativedivides	$\frac{\frac{16}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2\left(\frac{1}{4}-2i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{15\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}+\frac{2\left(\frac{1}{4}+2i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-15\,\mathrm{l}}{a^4 d}$
default	$\frac{\frac{16}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2\left(\frac{1}{4}-2i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{15\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}+\frac{2\left(\frac{1}{4}+2i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-15\,\mathrm{l}}{a^4 d}$

```
[In] int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 8*I/a^4/d*exp(-I*(d*x+c))+I/d/a^4/(exp(2*I*(d*x+c))+1)^2*(7*exp(3*I*(d*x+c)
)+9*exp(I*(d*x+c)))-15/2/a^4/d*ln(exp(I*(d*x+c))+I)+15/2/a^4/d*ln(exp(I*(d*
x+c))-I)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.50

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^4}\,dx =$$
$$-\frac{15\left(e^{(5i\,dx+5i\,c)}+2\,e^{(3i\,dx+3i\,c)}+e^{(i\,dx+i\,c)}\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-15\left(e^{(5i\,dx+5i\,c)}+2\,e^{(3i\,dx+3i\,c)}+e^{(i\,dx+i\,c)}\right)\mathrm{l}}{2\left(a^4de^{(5i\,dx+5i\,c)}+2\,a^4de^{(3i\,dx+3i\,c)}+a^4de^{(i\,dx+i\,c)}\right.}$$

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/2*(15*(e^(5*I*d*x + 5*I*c) + 2*e^(3*I*d*x + 3*I*c) + e^(I*d*x + I*c))*lo
g(e^(I*d*x + I*c) + I) - 15*(e^(5*I*d*x + 5*I*c) + 2*e^(3*I*d*x + 3*I*c) +
e^(I*d*x + I*c))*log(e^(I*d*x + I*c) - I) - 30*I*e^(4*I*d*x + 4*I*c) - 50*I
*e^(2*I*d*x + 2*I*c) - 16*I)/(a^4*d*e^(5*I*d*x + 5*I*c) + 2*a^4*d*e^(3*I*d*
x + 3*I*c) + a^4*d*e^(I*d*x + I*c))
```

Sympy [F]

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\int \frac{\sec^7(c+dx)}{\tan^4(c+dx)-4i\tan^3(c+dx)-6\tan^2(c+dx)+4i\tan(c+dx)+1}\,dx}{a^4}$$

```
[In] integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**7/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6*tan(c +
 d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(95) = 190$.

Time = 0.32 (sec) , antiderivative size = 457, normalized size of antiderivative = 4.27

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{30\left(\cos\left(5\,dx+5\,c\right)+2\,\cos\left(3\,dx+3\,c\right)+\cos\left(dx+c\right)+i\,\sin\left(5\,dx+5\,c\right)+2i\,\sin\left(3\,dx+3\,c\right)+i\,\sin\left(dx+\right.\right.}{}$$

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] (30*(cos(5*d*x + 5*c) + 2*cos(3*d*x + 3*c) + cos(d*x + c) + I*sin(5*d*x + 5
*c) + 2*I*sin(3*d*x + 3*c) + I*sin(d*x + c))*arctan2(cos(d*x + c), sin(d*x
+ c) + 1) + 30*(cos(5*d*x + 5*c) + 2*cos(3*d*x + 3*c) + cos(d*x + c) + I*si
n(5*d*x + 5*c) + 2*I*sin(3*d*x + 3*c) + I*sin(d*x + c))*arctan2(cos(d*x + c
), -sin(d*x + c) + 1) + 15*(I*cos(5*d*x + 5*c) + 2*I*cos(3*d*x + 3*c) + I*c
os(d*x + c) - sin(5*d*x + 5*c) - 2*sin(3*d*x + 3*c) - sin(d*x + c))*log(cos
(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) + 15*(-I*cos(5*d*x + 5*c
) - 2*I*cos(3*d*x + 3*c) - I*cos(d*x + c) + sin(5*d*x + 5*c) + 2*sin(3*d*x
+ 3*c) + sin(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c)
 + 1) + 60*cos(4*d*x + 4*c) + 100*cos(2*d*x + 2*c) + 60*I*sin(4*d*x + 4*c)
+ 100*I*sin(2*d*x + 2*c) + 32)/((-4*I*a^4*cos(5*d*x + 5*c) - 8*I*a^4*cos(3*
d*x + 3*c) - 4*I*a^4*cos(d*x + c) + 4*a^4*sin(5*d*x + 5*c) + 8*a^4*sin(3*d*
x + 3*c) + 4*a^4*sin(d*x + c))*d)
```

Giac [A] (verification not implemented)

none

Time = 0.77 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.06

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^4}\,dx =$$
$$-\frac{\frac{15\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^4}-\frac{15\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^4}-\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-8i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8i\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2a^4}-\frac{32}{a^4\left(\tan\left(\frac{1}{2}\,dx\right.\right.}}{2\,d}$$

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/2*(15*log(tan(1/2*d*x + 1/2*c) + 1)/a^4 - 15*log(tan(1/2*d*x + 1/2*c) -
1)/a^4 - 2*(tan(1/2*d*x + 1/2*c)^3 - 8*I*tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d
*x + 1/2*c) + 8*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^4) - 32/(a^4*(tan(1/2*
d*x + 1/2*c) - I)))/d
```

Mupad [B] (verification not implemented)

Time = 6.26 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.51

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{15\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^4\,d}$$
$$+\frac{\frac{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{a^4}-\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^4}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 39\mathrm{i}}{a^4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 17\mathrm{i}}{a^4}+\frac{24\mathrm{i}}{a^4}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,1\mathrm{i}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,2\mathrm{i}-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}+1\right)}$$

```
[In] int(1/(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^4),x)
```

```
[Out] ((9*tan(c/2 + (d*x)/2)^3)/a^4 - (tan(c/2 + (d*x)/2)^2*39i)/a^4 + (tan(c/2 +
 (d*x)/2)^4*17i)/a^4 + 24i/a^4 - (7*tan(c/2 + (d*x)/2))/a^4)/(d*(tan(c/2 +
(d*x)/2)*1i - 2*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*2i + tan(c/2 +
(d*x)/2)^4 + tan(c/2 + (d*x)/2)^5*1i + 1)) - (15*atanh(tan(c/2 + (d*x)/2)))
/(a^4*d)
```

3.160 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{a^4d} + \frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^3} - \frac{2i\sec(c+dx)}{d\,(a^4+ia^4\tan(c+dx))}$$

```
[Out] arctanh(sin(d*x+c))/a^4/d+2/3*I*sec(d*x+c)^3/a/d/(a+I*a*tan(d*x+c))^3-2*I*s
ec(d*x+c)/d/(a^4+I*a^4*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3581, 3855}

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{a^4d} - \frac{2i\sec(c+dx)}{d\,(a^4+ia^4\tan(c+dx))} + \frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^3}$$

```
[In] Int[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(a^4*d) + (((2*I)/3)*Sec[c + d*x]^3)/(a*d*(a + I*a*Ta
n[c + d*x])^3) - ((2*I)*Sec[c + d*x])/(d*(a^4 + I*a^4*Tan[c + d*x]))
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
```

```
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^3} - \frac{\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{a^2}\\ &= \frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^3} - \frac{2i\sec(c+dx)}{d\,(a^4+ia^4\tan(c+dx))} + \frac{\int \sec(c+dx)\,dx}{a^4}\\ &= \frac{\operatorname{arctanh}(\sin(c+dx))}{a^4d} + \frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^3} - \frac{2i\sec(c+dx)}{d\,(a^4+ia^4\tan(c+dx))}\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 247 vs. 2(82) = 164.

Time = 0.61 (sec) , antiderivative size = 247, normalized size of antiderivative = 3.01

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\sec^4(c+dx)(\cos(dx)+i\sin(dx))^4\left(-3\cos(4c)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+3\cos(4c)\log\left(\cos\right.\right.}{}$$

```
[In] Integrate[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^4*(Cos[d*x] + I*Sin[d*x])^4*(-3*Cos[4*c]*Log[Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2]] + 3*Cos[4*c]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
- 2*Cos[3*d*x]*Sin[c] + 6*Cos[d*x]*Sin[3*c] - (3*I)*Log[Cos[(c + d*x)/2] -
Sin[(c + d*x)/2]]*Sin[4*c] + (3*I)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
*Sin[4*c] + Cos[3*c]*((-6*I)*Cos[d*x] - 6*Sin[d*x]) - (6*I)*Sin[3*c]*Sin[d*
x] + (2*I)*Sin[c]*Sin[3*d*x] + 2*Cos[c]*(I*Cos[3*d*x] + Sin[3*d*x])))/(3*a^
4*d*(-I + Tan[c + d*x])^4)
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{8i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{16}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^{4}d}$	71
default	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{8i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{16}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^{4}d}$	71
risch	$-\frac{2i\mathrm{e}^{-i(dx+c)}}{a^{4}d}+\frac{2i\mathrm{e}^{-3i(dx+c)}}{3a^{4}d}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{a^{4}d}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{a^{4}d}$	79

```
[In] int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^4*(1/2*ln(tan(1/2*d*x+1/2*c)+1)+4*I/(-I+tan(1/2*d*x+1/2*c))^2-8/3/(-I
+tan(1/2*d*x+1/2*c))^3-1/2*ln(tan(1/2*d*x+1/2*c)-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.93

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\left(3\,e^{(3i\,dx+3i\,c)}\log\left(e^{(i\,dx+i\,c)}+i\right)-3\,e^{(3i\,dx+3i\,c)}\log\left(e^{(i\,dx+i\,c)}-i\right)-6i\,e^{(2i\,dx+2i\,c)}+2i\right)e^{(-3i\,dx-3i\,c)}}{3\,a^4d}$$

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*e^(3*I*d*x + 3*I*c)*log(e^(I*d*x + I*c) + I) - 3*e^(3*I*d*x + 3*I*c)
*log(e^(I*d*x + I*c) - I) - 6*I*e^(2*I*d*x + 2*I*c) + 2*I)*e^(-3*I*d*x - 3*
I*c)/(a^4*d)
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\int \frac{\sec^5(c+dx)}{\tan^4(c+dx)-4i\tan^3(c+dx)-6\tan^2(c+dx)+4i\tan(c+dx)+1}\,dx}{a^4}$$

```
[In] integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**5/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6*tan(c +
 d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.72

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{-6i\arctan\left(\cos\left(dx+c\right),\sin\left(dx+c\right)+1\right) - 6i\arctan\left(\cos\left(dx+c\right),-\sin\left(dx+c\right)+1\right) + 4i\cos\left(3\,dx+\right.}{}$$

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/6*(-6*I*arctan2(cos(d*x + c), sin(d*x + c) + 1) - 6*I*arctan2(cos(d*x + c
), -sin(d*x + c) + 1) + 4*I*cos(3*d*x + 3*c) - 12*I*cos(d*x + c) + 3*log(co
s(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - 3*log(cos(d*x + c)^2
+ sin(d*x + c)^2 - 2*sin(d*x + c) + 1) + 4*sin(3*d*x + 3*c) - 12*sin(d*x +
c))/(a^4*d)
```

Giac [A] (verification not implemented)

none

Time = 0.65 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.87

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\frac{3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^4} - \frac{3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^4} + \frac{8\,\left(3i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^4\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^3}}{3\,d}$$

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*log(tan(1/2*d*x + 1/2*c) + 1)/a^4 - 3*log(tan(1/2*d*x + 1/2*c) - 1)/
a^4 + 8*(3*I*tan(1/2*d*x + 1/2*c) + 1)/(a^4*(tan(1/2*d*x + 1/2*c) - I)^3))/
d
```

Mupad [B] (verification not implemented)

Time = 4.39 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.07

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^4\,d} - \frac{-\frac{8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^4}+\frac{8\mathrm{i}}{3\,a^4}}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,1\mathrm{i}-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,3\mathrm{i}+1\right)}$$

```
[In] int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^4),x)
```

```
[Out] (2*atanh(tan(c/2 + (d*x)/2)))/(a^4*d) - (8i/(3*a^4) - (8*tan(c/2 + (d*x)/2)
)/a^4)/(d*(tan(c/2 + (d*x)/2)*3i - 3*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)
/2)^3*1i + 1))
```

3.161 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 68

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{i\sec^3(c+dx)}{5d(a+ia\tan(c+dx))^4} + \frac{i\sec^3(c+dx)}{15ad(a+ia\tan(c+dx))^3}$$

```
[Out] 1/5*I*sec(d*x+c)^3/d/(a+I*a*tan(d*x+c))^4+1/15*I*sec(d*x+c)^3/a/d/(a+I*a*ta
n(d*x+c))^3
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3583, 3569}

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{i\sec^3(c+dx)}{15ad(a+ia\tan(c+dx))^3} + \frac{i\sec^3(c+dx)}{5d(a+ia\tan(c+dx))^4}$$

```
[In] Int[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((I/5)*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^4) + ((I/15)*Sec[c + d*x]^
3)/(a*d*(a + I*a*Tan[c + d*x])^3)
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{i\sec^3(c+dx)}{5d(a+ia\tan(c+dx))^4}+\frac{\int\frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx}{5a}\\ &= \frac{i\sec^3(c+dx)}{5d(a+ia\tan(c+dx))^4}+\frac{i\sec^3(c+dx)}{15ad(a+ia\tan(c+dx))^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.59

$$\int\frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx=-\frac{\sec^3(c+dx)(-4i+\tan(c+dx))}{15a^4d(-i+\tan(c+dx))^4}$$

```
[In] Integrate[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^4,x]
[Out] -1/15*(Sec[c + d*x]^3*(-4*I + Tan[c + d*x]))/(a^4*d*(-I + Tan[c + d*x])^4)
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.56

method	result	size
risch	$\frac{i e^{-3i(dx+c)}}{6a^4d}+\frac{i e^{-5i(dx+c)}}{10a^4d}$	38
derivativedivides	$\frac{-\frac{28}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{6i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{8i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{16}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}}{a^4d}$	90
default	$\frac{-\frac{28}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{6i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{8i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{16}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}}{a^4d}$	90

```
[In] int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
[Out] 1/6*I/a^4/d*exp(-3*I*(d*x+c))+1/10*I/a^4/d*exp(-5*I*(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.44

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\left(5i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-5i\,dx-5i\,c)}}{30\,a^4d}$$

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
[Out] 1/30*(5*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(-5*I*d*x - 5*I*c)/(a^4*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(54) = 108.

Time = 1.15 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.68

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \begin{cases} -\frac{\tan(c+dx)\sec^3(c+dx)}{15a^4d\tan^4(c+dx)-60ia^4d\tan^3(c+dx)-90a^4d\tan^2(c+dx)+60ia^4d\tan(c+dx)+15a^4d}+\frac{4i\sec^3(c+d}{15a^4d\tan^4(c+dx)-60ia^4d\tan^3(c+dx)-90a^4d\,\mathrm{t}} \\ \frac{x\sec^3(c)}{(ia\tan(c)+a)^4} \end{cases}$$

```
[In] integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c))**4,x)
[Out] Piecewise((-tan(c + d*x)*sec(c + d*x)**3/(15*a**4*d*tan(c + d*x)**4 - 60*I*
a**4*d*tan(c + d*x)**3 - 90*a**4*d*tan(c + d*x)**2 + 60*I*a**4*d*tan(c + d*
x) + 15*a**4*d) + 4*I*sec(c + d*x)**3/(15*a**4*d*tan(c + d*x)**4 - 60*I*a**
4*d*tan(c + d*x)**3 - 90*a**4*d*tan(c + d*x)**2 + 60*I*a**4*d*tan(c + d*x)
+ 15*a**4*d), Ne(d, 0)), (x*sec(c)**3/(I*a*tan(c) + a)**4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.78

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{3i\,\cos(5\,dx+5\,c)+5i\,\cos(3\,dx+3\,c)+3\,\sin(5\,dx+5\,c)+5\,\sin(3\,dx+3\,c)}{30\,a^4d}$$

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
[Out] 1/30*(3*I*cos(5*d*x + 5*c) + 5*I*cos(3*d*x + 3*c) + 3*sin(5*d*x + 5*c) + 5*
sin(3*d*x + 3*c))/(a^4*d)
```

Giac [A] (verification not implemented)

none

Time = 0.64 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.07

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{2\left(15\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 - 15i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 25\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + 5i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + 4\right)}{15\,a^4d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^5}$$

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 2/15*(15*tan(1/2*d*x + 1/2*c)^4 - 15*I*tan(1/2*d*x + 1/2*c)^3 - 25*tan(1/2*
d*x + 1/2*c)^2 + 5*I*tan(1/2*d*x + 1/2*c) + 4)/(a^4*d*(tan(1/2*d*x + 1/2*c)
 - I)^5)
```

Mupad [B] (verification not implemented)

Time = 4.41 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.96

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{2\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 15\mathrm{i} + 15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 - \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 25\mathrm{i} - 5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right) + 4\mathrm{i}\right)}{15\,a^4\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5 1\mathrm{i} + 5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 - \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 10\mathrm{i} - 10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 + \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,5\mathrm{i} + 1\right.}$$

```
[In] int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^4),x)
```

```
[Out] (2*(15*tan(c/2 + (d*x)/2)^3 - tan(c/2 + (d*x)/2)^2*25i - 5*tan(c/2 + (d*x)/
2) + tan(c/2 + (d*x)/2)^4*15i + 4i))/(15*a^4*d*(tan(c/2 + (d*x)/2)*5i - 10*
tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*10i + 5*tan(c/2 + (d*x)/2)^4 +
tan(c/2 + (d*x)/2)^5*1i + 1))
```

3.162 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 132

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4} + \frac{3i\sec(c+dx)}{35ad(a+ia\tan(c+dx))^3} + \frac{2i\sec(c+dx)}{35d\,(a^2+ia^2\tan(c+dx))^2} + \frac{2i\sec(c+dx)}{35d\,(a^4+ia^4\tan(c+dx))}$$

```
[Out] 1/7*I*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^4+3/35*I*sec(d*x+c)/a/d/(a+I*a*tan(d*
x+c))^3+2/35*I*sec(d*x+c)/d/(a^2+I*a^2*tan(d*x+c))^2+2/35*I*sec(d*x+c)/d/(a
^4+I*a^4*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3583, 3569}

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{2i\sec(c+dx)}{35d\,(a^4+ia^4\tan(c+dx))} + \frac{2i\sec(c+dx)}{35d\,(a^2+ia^2\tan(c+dx))^2} + \frac{3i\sec(c+dx)}{35ad(a+ia\tan(c+dx))^3} + \frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4}$$

```
[In] Int[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((I/7)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^4) + (((3*I)/35)*Sec[c + d*x
])/(a*d*(a + I*a*Tan[c + d*x])^3) + (((2*I)/35)*Sec[c + d*x])/(d*(a^2 + I*a
^2*Tan[c + d*x])^2) + (((2*I)/35)*Sec[c + d*x])/(d*(a^4 + I*a^4*Tan[c + d*x
]))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4}+\frac{3\int\frac{\sec(c+dx)}{(a+ia\tan(c+dx))^3}\,dx}{7a}\\
&= \frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4}+\frac{3i\sec(c+dx)}{35ad(a+ia\tan(c+dx))^3}+\frac{6\int\frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{35a^2}\\
&= \frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4}+\frac{3i\sec(c+dx)}{35ad(a+ia\tan(c+dx))^3}\\
&\quad+\frac{2i\sec(c+dx)}{35d\left(a^2+ia^2\tan(c+dx)\right)^2}+\frac{2\int\frac{\sec(c+dx)}{a+ia\tan(c+dx)}\,dx}{35a^3}\\
&= \frac{i\sec(c+dx)}{7d(a+ia\tan(c+dx))^4}+\frac{3i\sec(c+dx)}{35ad(a+ia\tan(c+dx))^3}\\
&\quad+\frac{2i\sec(c+dx)}{35d\left(a^2+ia^2\tan(c+dx)\right)^2}+\frac{2i\sec(c+dx)}{35d\left(a^4+ia^4\tan(c+dx)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.55

$$\int\frac{\sec(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{i\sec^4(c+dx)(28\cos(c+dx)+20\cos(3(c+dx))+7i\sin(c+dx)+15i\sin(3(c+dx)))}{140a^4d(-i+\tan(c+dx))^4}$$

```
[In] Integrate[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^4,x]
[Out] ((I/140)*Sec[c + d*x]^4*(28*Cos[c + d*x] + 20*Cos[3*(c + d*x)] + (7*I)*Sin[
c + d*x] + (15*I)*Sin[3*(c + d*x)]))/(a^4*d*(-I + Tan[c + d*x])^4)
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.56

method	result
risch	$\frac{i e^{-i(dx+c)}}{8a^4d}+\frac{i e^{-3i(dx+c)}}{8a^4d}+\frac{3i e^{-5i(dx+c)}}{40a^4d}+\frac{i e^{-7i(dx+c)}}{56a^4d}$
derivativedivides	$\frac{-\frac{16i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{8i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+\frac{6i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{12}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{72}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{a^4d}$
default	$\frac{-\frac{16i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{8i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+\frac{6i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{12}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{72}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{a^4d}$

```
[In] int(sec(d*x+c)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*I/a^4/d*exp(-I*(d*x+c))+1/8*I/a^4/d*exp(-3*I*(d*x+c))+3/40*I/a^4/d*exp(
-5*I*(d*x+c))+1/56*I/a^4/d*exp(-7*I*(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.39

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\left(35i\,e^{(6i\,dx+6i\,c)}+35i\,e^{(4i\,dx+4i\,c)}+21i\,e^{(2i\,dx+2i\,c)}+5i\right)e^{(-7i\,dx-7i\,c)}}{280\,a^4d}$$

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/280*(35*I*e^(6*I*d*x + 6*I*c) + 35*I*e^(4*I*d*x + 4*I*c) + 21*I*e^(2*I*d*
x + 2*I*c) + 5*I)*e^(-7*I*d*x - 7*I*c)/(a^4*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 354 vs. 2(112) = 224.

Time = 1.18 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.68

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \begin{cases} \frac{2\tan^3(c+dx)\sec(c+dx)}{35a^4d\tan^4(c+dx)-140ia^4d\tan^3(c+dx)-210a^4d\tan^2(c+dx)+140ia^4d\tan(c+dx)+35a^4d} - \frac{8i\tan^2(c+dx)}{35a^4d\tan^4(c+dx)-140ia^4d\tan^3(c+dx)-210a} \\ \frac{x\sec(c)}{(ia\tan(c)+a)^4} \end{cases}$$

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise((2*tan(c + d*x)**3*sec(c + d*x)/(35*a**4*d*tan(c + d*x)**4 - 140*
I*a**4*d*tan(c + d*x)**3 - 210*a**4*d*tan(c + d*x)**2 + 140*I*a**4*d*tan(c
+ d*x) + 35*a**4*d) - 8*I*tan(c + d*x)**2*sec(c + d*x)/(35*a**4*d*tan(c + d
*x)**4 - 140*I*a**4*d*tan(c + d*x)**3 - 210*a**4*d*tan(c + d*x)**2 + 140*I*
a**4*d*tan(c + d*x) + 35*a**4*d) - 13*tan(c + d*x)*sec(c + d*x)/(35*a**4*d*
tan(c + d*x)**4 - 140*I*a**4*d*tan(c + d*x)**3 - 210*a**4*d*tan(c + d*x)**2
 + 140*I*a**4*d*tan(c + d*x) + 35*a**4*d) + 12*I*sec(c + d*x)/(35*a**4*d*ta
n(c + d*x)**4 - 140*I*a**4*d*tan(c + d*x)**3 - 210*a**4*d*tan(c + d*x)**2 +
 140*I*a**4*d*tan(c + d*x) + 35*a**4*d), Ne(d, 0)), (x*sec(c)/(I*a*tan(c) +
 a)**4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.69

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{5i\cos(7\,dx+7\,c)+21i\cos(5\,dx+5\,c)+35i\cos(3\,dx+3\,c)+35i\cos(dx+c)+5\sin(7\,dx+7\,c)+21}{280\,a^4d}$$

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/280*(5*I*cos(7*d*x + 7*c) + 21*I*cos(5*d*x + 5*c) + 35*I*cos(3*d*x + 3*c)
 + 35*I*cos(d*x + c) + 5*sin(7*d*x + 7*c) + 21*sin(5*d*x + 5*c) + 35*sin(3*
d*x + 3*c) + 35*sin(d*x + c))/(a^4*d)
```

Giac [A] (verification not implemented)

none

Time = 0.62 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.75

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{2\left(35\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-105i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-210\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+210i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+147}{35\,a^4d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^7}$$

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 2/35*(35*tan(1/2*d*x + 1/2*c)^6 - 105*I*tan(1/2*d*x + 1/2*c)^5 - 210*tan(1/
2*d*x + 1/2*c)^4 + 210*I*tan(1/2*d*x + 1/2*c)^3 + 147*tan(1/2*d*x + 1/2*c)^
2 - 49*I*tan(1/2*d*x + 1/2*c) - 12)/(a^4*d*(tan(1/2*d*x + 1/2*c) - I)^7)
```

Mupad [B] (verification not implemented)

Time = 4.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.48

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,1\mathrm{i}}{8}+\frac{\mathrm{e}^{-c\,3\mathrm{i}-d\,x\,3\mathrm{i}}\,1\mathrm{i}}{8}+\frac{\mathrm{e}^{-c\,5\mathrm{i}-d\,x\,5\mathrm{i}}\,3\mathrm{i}}{40}+\frac{\mathrm{e}^{-c\,7\mathrm{i}-d\,x\,7\mathrm{i}}\,1\mathrm{i}}{56}}{a^4\,d}$$

```
[In] int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^4),x)
[Out] ((exp(- c*1i - d*x*1i)*1i)/8 + (exp(- c*3i - d*x*3i)*1i)/8 + (exp(- c*5i -
d*x*5i)*3i)/40 + (exp(- c*7i - d*x*7i)*1i)/56)/(a^4*d)
```

3.163 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 134

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{4\sin(c+dx)}{21a^4d} - \frac{4\sin^3(c+dx)}{63a^4d} + \frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4} + \frac{5i\cos(c+dx)}{63ad(a+ia\tan(c+dx))^3} + \frac{8i\cos^3(c+dx)}{63d\,(a^4+ia^4\tan(c+dx))}$$

```
[Out] 4/21*sin(d*x+c)/a^4/d-4/63*sin(d*x+c)^3/a^4/d+1/9*I*cos(d*x+c)/d/(a+I*a*tan
(d*x+c))^4+5/63*I*cos(d*x+c)/a/d/(a+I*a*tan(d*x+c))^3+8/63*I*cos(d*x+c)^3/d
/(a^4+I*a^4*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3583, 3581, 2713}

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{4\sin^3(c+dx)}{63a^4d} + \frac{4\sin(c+dx)}{21a^4d} + \frac{8i\cos^3(c+dx)}{63d\,(a^4+ia^4\tan(c+dx))} + \frac{5i\cos(c+dx)}{63ad(a+ia\tan(c+dx))^3} + \frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}$$

```
[In] Int[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (4*Sin[c + d*x])/(21*a^4*d) - (4*Sin[c + d*x]^3)/(63*a^4*d) + ((I/9)*Cos[c
+ d*x])/(d*(a + I*a*Tan[c + d*x])^4) + (((5*I)/63)*Cos[c + d*x])/(a*d*(a +
I*a*Tan[c + d*x])^3) + (((8*I)/63)*Cos[c + d*x]^3)/(d*(a^4 + I*a^4*Tan[c +
d*x]))
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}+\frac{5\int\frac{\cos(c+dx)}{(a+ia\tan(c+dx))^3}\,dx}{9a}\\
&= \frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}+\frac{5i\cos(c+dx)}{63ad(a+ia\tan(c+dx))^3}+\frac{20\int\frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{63a^2}\\
&= \frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}+\frac{5i\cos(c+dx)}{63ad(a+ia\tan(c+dx))^3}\\
&\quad+\frac{8i\cos^3(c+dx)}{63d\,(a^4+ia^4\tan(c+dx))}+\frac{4\int\cos^3(c+dx)\,dx}{21a^4}\\
&= \frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}+\frac{5i\cos(c+dx)}{63ad(a+ia\tan(c+dx))^3}\\
&\quad+\frac{8i\cos^3(c+dx)}{63d\,(a^4+ia^4\tan(c+dx))}-\frac{4\text{Subst}\left(\int\left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{21a^4d}\\
&= \frac{4\sin(c+dx)}{21a^4d}-\frac{4\sin^3(c+dx)}{63a^4d}+\frac{i\cos(c+dx)}{9d(a+ia\tan(c+dx))^4}\\
&\quad+\frac{5i\cos(c+dx)}{63ad(a+ia\tan(c+dx))^3}+\frac{8i\cos^3(c+dx)}{63d\,(a^4+ia^4\tan(c+dx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.71

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{i\sec^4(c+dx)(-168\cos(c+dx)-180\cos(3(c+dx))+28\cos(5(c+dx))-42i\sin(c+dx)-135i\sin(}{1008a^4d(-i+\tan(c+dx))^4}$$

```
[In] Integrate[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((-1/1008*I)*Sec[c + d*x]^4*(-168*Cos[c + d*x] - 180*Cos[3*(c + d*x)] + 28*
Cos[5*(c + d*x)] - (42*I)*Sin[c + d*x] - (135*I)*Sin[3*(c + d*x)] + (35*I)*
Sin[5*(c + d*x)]))/(a^4*d*(-I + Tan[c + d*x])^4)
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.77

method	result
risch	$\frac{5i\mathrm{e}^{-3i(dx+c)}}{48a^4d}+\frac{i\mathrm{e}^{-5i(dx+c)}}{16a^4d}+\frac{5i\mathrm{e}^{-7i(dx+c)}}{224a^4d}+\frac{i\mathrm{e}^{-9i(dx+c)}}{288a^4d}+\frac{i\cos(dx+c)}{8a^4d}+\frac{3\sin(dx+c)}{16a^4d}$
derivativedivides	$\frac{\frac{86i}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{8i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}-\frac{49i}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{49i}{8\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{16}{9\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9}-}{a^4d}$
default	$\frac{\frac{86i}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{8i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}-\frac{49i}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{49i}{8\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{16}{9\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9}-}{a^4d}$

```
[In] int(cos(d*x+c)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 5/48*I/a^4/d*exp(-3*I*(d*x+c))+1/16*I/a^4/d*exp(-5*I*(d*x+c))+5/224*I/a^4/d
*exp(-7*I*(d*x+c))+1/288*I/a^4/d*exp(-9*I*(d*x+c))+1/8*I/a^4/d*cos(d*x+c)+3
/16*sin(d*x+c)/a^4/d
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.55

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\left(-63i\,e^{(10i\,dx+10i\,c)}+315i\,e^{(8i\,dx+8i\,c)}+210i\,e^{(6i\,dx+6i\,c)}+126i\,e^{(4i\,dx+4i\,c)}+45i\,e^{(2i\,dx+2i\,c)}+7i\right)e^{(-9i\,dx-9i\,c)}}{2016\,a^4d}$$

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/2016*(-63*I*e^(10*I*d*x + 10*I*c) + 315*I*e^(8*I*d*x + 8*I*c) + 210*I*e^(
6*I*d*x + 6*I*c) + 126*I*e^(4*I*d*x + 4*I*c) + 45*I*e^(2*I*d*x + 2*I*c) + 7
*I)*e^(-9*I*d*x - 9*I*c)/(a^4*d)
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.72

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$
$$= \begin{cases} \frac{(-1585446912ia^{20}d^5e^{26ic}e^{idx}+7927234560ia^{20}d^5e^{24ic}e^{-idx}+5284823040ia^{20}d^5e^{22ic}e^{-3idx}+3170893824ia^{20}d^5e^{20ic}e^{-5idx}+1132462080ia^{20}d}{50734301184a^{24}d^6} \\ \frac{x(e^{10ic}+5e^{8ic}+10e^{6ic}+10e^{4ic}+5e^{2ic}+1)e^{-9ic}}{32a^4} \end{cases}$$

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise(((-1585446912*I*a**20*d**5*exp(26*I*c)*exp(I*d*x) + 7927234560*I*
a**20*d**5*exp(24*I*c)*exp(-I*d*x) + 5284823040*I*a**20*d**5*exp(22*I*c)*ex
p(-3*I*d*x) + 3170893824*I*a**20*d**5*exp(20*I*c)*exp(-5*I*d*x) + 113246208
0*I*a**20*d**5*exp(18*I*c)*exp(-7*I*d*x) + 176160768*I*a**20*d**5*exp(16*I*
c)*exp(-9*I*d*x))*exp(-25*I*c)/(50734301184*a**24*d**6), Ne(a**24*d**6*exp(
25*I*c), 0)), (x*(exp(10*I*c) + 5*exp(8*I*c) + 10*exp(6*I*c) + 10*exp(4*I*c
) + 5*exp(2*I*c) + 1)*exp(-9*I*c)/(32*a**4), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [A] (verification not implemented)

none

Time = 0.81 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.08

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\frac{63}{a^4(\tan(\frac{1}{2}dx+\frac{1}{2}c)+i)} + \frac{1953\tan(\frac{1}{2}dx+\frac{1}{2}c)^8-9450i\tan(\frac{1}{2}dx+\frac{1}{2}c)^7-25998\tan(\frac{1}{2}dx+\frac{1}{2}c)^6+42210i\tan(\frac{1}{2}dx+\frac{1}{2}c)^5+46368\tan(\frac{1}{2}dx+\frac{1}{2}}{a^4(\tan(\frac{1}{2}dx+\frac{1}{2}c)-i)^9}}{1008\,d}$$

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/1008*(63/(a^4*(tan(1/2*d*x + 1/2*c) + I)) + (1953*tan(1/2*d*x + 1/2*c)^8
- 9450*I*tan(1/2*d*x + 1/2*c)^7 - 25998*tan(1/2*d*x + 1/2*c)^6 + 42210*I*ta
n(1/2*d*x + 1/2*c)^5 + 46368*tan(1/2*d*x + 1/2*c)^4 - 33054*I*tan(1/2*d*x +
 1/2*c)^3 - 15858*tan(1/2*d*x + 1/2*c)^2 + 4374*I*tan(1/2*d*x + 1/2*c) + 70
3)/(a^4*(tan(1/2*d*x + 1/2*c) - I)^9))/d
```

Mupad [B] (verification not implemented)

Time = 8.14 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.20

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\left(63\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 252\mathrm{i}-588\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 672\mathrm{i}+378\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\mathrm{ta}\right.}{63\,a^4\,d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\mathrm{i}\right)\,\left(1+\mathrm{ta}\right.}$$

```
[In] int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] ((372*tan(c/2 + (d*x)/2)^3 - tan(c/2 + (d*x)/2)^2*288i - 97*tan(c/2 + (d*x)
/2) + tan(c/2 + (d*x)/2)^4*168i + 378*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*x
)/2)^6*672i - 588*tan(c/2 + (d*x)/2)^7 - tan(c/2 + (d*x)/2)^8*252i + 63*tan
(c/2 + (d*x)/2)^9 + 20i)*2i)/(63*a^4*d*(tan(c/2 + (d*x)/2) + 1i)*(tan(c/2 +
 (d*x)/2)*1i + 1)^9)
```

3.164 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 156

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{10\sin(c+dx)}{33a^4d} - \frac{20\sin^3(c+dx)}{99a^4d} + \frac{2\sin^5(c+dx)}{33a^4d} + \frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4} + \frac{7i\cos^3(c+dx)}{99ad(a+ia\tan(c+dx))^3} + \frac{4i\cos^5(c+dx)}{33d\,(a^4+ia^4\tan(c+dx))}$$

```
[Out] 10/33*sin(d*x+c)/a^4/d-20/99*sin(d*x+c)^3/a^4/d+2/33*sin(d*x+c)^5/a^4/d+1/1
1*I*cos(d*x+c)^3/d/(a+I*a*tan(d*x+c))^4+7/99*I*cos(d*x+c)^3/a/d/(a+I*a*tan(
d*x+c))^3+4/33*I*cos(d*x+c)^5/d/(a^4+I*a^4*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3583, 3581, 2713}

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{2\sin^5(c+dx)}{33a^4d} - \frac{20\sin^3(c+dx)}{99a^4d} + \frac{10\sin(c+dx)}{33a^4d} + \frac{4i\cos^5(c+dx)}{33d\,(a^4+ia^4\tan(c+dx))} + \frac{7i\cos^3(c+dx)}{99ad(a+ia\tan(c+dx))^3} + \frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}$$

```
[In] Int[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (10*Sin[c + d*x])/(33*a^4*d) - (20*Sin[c + d*x]^3)/(99*a^4*d) + (2*Sin[c +
d*x]^5)/(33*a^4*d) + ((I/11)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^4) +
 (((7*I)/99)*Cos[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*x])^3) + (((4*I)/33)*C
os[c + d*x]^5)/(d*(a^4 + I*a^4*Tan[c + d*x]))
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}+\frac{7\int\frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx}{11a}\\
&= \frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}+\frac{7i\cos^3(c+dx)}{99ad(a+ia\tan(c+dx))^3}+\frac{14\int\frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{33a^2}\\
&= \frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}+\frac{7i\cos^3(c+dx)}{99ad(a+ia\tan(c+dx))^3}\\
&\quad+\frac{4i\cos^5(c+dx)}{33d\left(a^4+ia^4\tan(c+dx)\right)}+\frac{10\int\cos^5(c+dx)\,dx}{33a^4}\\
&= \frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4}+\frac{7i\cos^3(c+dx)}{99ad(a+ia\tan(c+dx))^3}\\
&\quad+\frac{4i\cos^5(c+dx)}{33d\left(a^4+ia^4\tan(c+dx)\right)}-\frac{10\text{Subst}\left(\int\left(1-2x^2+x^4\right)\,dx,x,-\sin(c+dx)\right)}{33a^4d}
\end{aligned}
$$

$$= \frac{10\sin(c+dx)}{33a^4d} - \frac{20\sin^3(c+dx)}{99a^4d} + \frac{2\sin^5(c+dx)}{33a^4d} + \frac{i\cos^3(c+dx)}{11d(a+ia\tan(c+dx))^4} + \frac{7i\cos^3(c+dx)}{99ad(a+ia\tan(c+dx))^3} + \frac{4i\cos^5(c+dx)}{33d\left(a^4+ia^4\tan(c+dx)\right)}$$

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.75

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{i\sec^4(c+dx)(-924\cos(c+dx)-1188\cos(3(c+dx))+308\cos(5(c+dx))+12\cos(7(c+dx))-231i\,s}{6336a^4d(-i+\tan(c+dx))^4}$$

```
[In] Integrate[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((-1/6336*I)*Sec[c + d*x]^4*(-924*Cos[c + d*x] - 1188*Cos[3*(c + d*x)] + 30
8*Cos[5*(c + d*x)] + 12*Cos[7*(c + d*x)] - (231*I)*Sin[c + d*x] - (891*I)*S
in[3*(c + d*x)] + (385*I)*Sin[5*(c + d*x)] + (21*I)*Sin[7*(c + d*x)]))/(a^4
*d*(-I + Tan[c + d*x])^4)
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.88

method	result
risch	$\frac{7i\mathrm{e}^{-5i(dx+c)}}{128a^4d}+\frac{3i\mathrm{e}^{-7i(dx+c)}}{128a^4d}+\frac{7i\mathrm{e}^{-9i(dx+c)}}{1152a^4d}+\frac{i\mathrm{e}^{-11i(dx+c)}}{1408a^4d}+\frac{7i\cos(dx+c)}{64a^4d}+\frac{7\sin(dx+c)}{32a^4d}+\frac{17i\cos(3dx+3c)}{192a^4d}$
derivativedivides	$\frac{\frac{8i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{10}}-\frac{67i}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{44i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}+\frac{385i}{6\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+\frac{201i}{32\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-}{}$
default	$\frac{\frac{8i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{10}}-\frac{67i}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{44i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}+\frac{385i}{6\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+\frac{201i}{32\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-}{}$

```
[In] int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 7/128*I/a^4/d*exp(-5*I*(d*x+c))+3/128*I/a^4/d*exp(-7*I*(d*x+c))+7/1152*I/a^
4/d*exp(-9*I*(d*x+c))+1/1408*I/a^4/d*exp(-11*I*(d*x+c))+7/64*I/a^4/d*cos(d*
x+c)+7/32*sin(d*x+c)/a^4/d+17/192*I/a^4/d*cos(3*d*x+3*c)+3/32/a^4/d*sin(3*d
*x+3*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.62

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\left(-33i\,e^{(14i\,dx+14i\,c)} - 693i\,e^{(12i\,dx+12i\,c)} + 2079i\,e^{(10i\,dx+10i\,c)} + 1155i\,e^{(8i\,dx+8i\,c)} + 693i\,e^{(6i\,dx+6i\,c)} + 297i\,e^{(4i}}{12672\,a^4d}$$

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/12672*(-33*I*e^(14*I*d*x + 14*I*c) - 693*I*e^(12*I*d*x + 12*I*c) + 2079*I
*e^(10*I*d*x + 10*I*c) + 1155*I*e^(8*I*d*x + 8*I*c) + 693*I*e^(6*I*d*x + 6*
I*c) + 297*I*e^(4*I*d*x + 4*I*c) + 77*I*e^(2*I*d*x + 2*I*c) + 9*I)*e^(-11*I
*d*x - 11*I*c)/(a^4*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(136) = 272.

Time = 0.42 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.92

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \begin{cases} \frac{(-16719613616612966 4ia^{28}d^7e^{39ic}e^{3idx} - 35111188594887229 44ia^{28}d^7e^{37ic}e^{idx} + 10533356578466168832ia^{28}d^7e^{35ic}e^{-idx} + 58518647658145}{} \\ \frac{x(e^{14ic}+7e^{12ic}+21e^{10ic}+35e^{8ic}+35e^{6ic}+21e^{4ic}+7e^{2ic}+1)e^{-11ic}}{128a^4} \end{cases}$$

```
[In] integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise(((-167196136166129664*I*a**28*d**7*exp(39*I*c)*exp(3*I*d*x) - 351
1118859488722944*I*a**28*d**7*exp(37*I*c)*exp(I*d*x) + 10533356578466168832
*I*a**28*d**7*exp(35*I*c)*exp(-I*d*x) + 5851864765814538240*I*a**28*d**7*ex
p(33*I*c)*exp(-3*I*d*x) + 3511118859488722944*I*a**28*d**7*exp(31*I*c)*exp(
-5*I*d*x) + 1504765225495166976*I*a**28*d**7*exp(29*I*c)*exp(-7*I*d*x) + 39
0124317720969216*I*a**28*d**7*exp(27*I*c)*exp(-9*I*d*x) + 45598946227126272
*I*a**28*d**7*exp(25*I*c)*exp(-11*I*d*x))*exp(-36*I*c)/(642033162877937909 7
6*a**32*d**8), Ne(a**32*d**8*exp(36*I*c), 0)), (x*(exp(14*I*c) + 7*exp(12*I
*c) + 21*exp(10*I*c) + 35*exp(8*I*c) + 35*exp(6*I*c) + 21*exp(4*I*c) + 7*ex
p(2*I*c) + 1)*exp(-11*I*c)/(128*a**4), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [A] (verification not implemented)

none

Time = 0.85 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.26

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\frac{33\left(12\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+21i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-11\right)}{a^4\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^3}+\frac{5940\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10}-39501i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-141075\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8+313236i\tan\left(\frac{1}{2}c\right)}{}}{}$$

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3168*(33*(12*tan(1/2*d*x + 1/2*c)^2 + 21*I*tan(1/2*d*x + 1/2*c) - 11)/(a^
4*(tan(1/2*d*x + 1/2*c) + I)^3) + (5940*tan(1/2*d*x + 1/2*c)^10 - 39501*I*t
an(1/2*d*x + 1/2*c)^9 - 141075*tan(1/2*d*x + 1/2*c)^8 + 313236*I*tan(1/2*d*
x + 1/2*c)^7 + 479556*tan(1/2*d*x + 1/2*c)^6 - 516054*I*tan(1/2*d*x + 1/2*c
)^5 - 397914*tan(1/2*d*x + 1/2*c)^4 + 214500*I*tan(1/2*d*x + 1/2*c)^3 + 790
24*tan(1/2*d*x + 1/2*c)^2 - 17765*I*tan(1/2*d*x + 1/2*c) - 2155)/(a^4*(tan(
1/2*d*x + 1/2*c) - I)^11))/d
```

Mupad [B] (verification not implemented)

Time = 6.74 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.38

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{269\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{16}-\frac{1307\sin\left(\frac{3c}{2}+\frac{3dx}{2}\right)}{64}+\frac{1307\sin\left(\frac{5c}{2}+\frac{5dx}{2}\right)}{64}-\frac{1099\sin\left(\frac{7c}{2}+\frac{7dx}{2}\right)}{32}+\frac{203\sin\left(\frac{9c}{2}+\frac{9dx}{2}\right)}{32}-\right.}{99\,a^4\,d\left(\cos\left(\frac{c}{2}\right.\right.}$$

```
[In] int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] -(cos(c/2 + (d*x)/2)*((cos((3*c)/2 + (3*d*x)/2)*231i)/16 - (cos((5*c)/2 + (
5*d*x)/2)*231i)/16 + cos((7*c)/2 + (7*d*x)/2)*33i - cos((9*c)/2 + (9*d*x)/2
)*5i + (cos((11*c)/2 + (11*d*x)/2)*3i)/16 - (cos((13*c)/2 + (13*d*x)/2)*3i)
/16 + (269*sin(c/2 + (d*x)/2))/16 - (1307*sin((3*c)/2 + (3*d*x)/2))/64 + (1
307*sin((5*c)/2 + (5*d*x)/2))/64 - (1099*sin((7*c)/2 + (7*d*x)/2))/32 + (20
3*sin((9*c)/2 + (9*d*x)/2))/32 - (21*sin((11*c)/2 + (11*d*x)/2))/64 + (21*s
in((13*c)/2 + (13*d*x)/2))/64)*2i)/(99*a^4*d*(cos(c/2 + (d*x)/2) + sin(c/2
+ (d*x)/2)*1i)^11*(cos(c/2 + (d*x)/2)*1i + sin(c/2 + (d*x)/2))^3)
```

3.165 $\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 174

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{56\sin(c+dx)}{143a^4d} - \frac{56\sin^3(c+dx)}{143a^4d} + \frac{168\sin^5(c+dx)}{715a^4d} - \frac{8\sin^7(c+dx)}{143a^4d} + \frac{i\cos^5(c+dx)}{13d(a+ia\tan(c+dx))^4} + \frac{9i\cos^5(c+dx)}{143ad(a+ia\tan(c+dx))^3} + \frac{16i\cos^7(c+dx)}{143d\,(a^4+ia^4\tan(c+dx))}$$

```
[Out] 56/143*sin(d*x+c)/a^4/d-56/143*sin(d*x+c)^3/a^4/d+168/715*sin(d*x+c)^5/a^4/
d-8/143*sin(d*x+c)^7/a^4/d+1/13*I*cos(d*x+c)^5/d/(a+I*a*tan(d*x+c))^4+9/143
*I*cos(d*x+c)^5/a/d/(a+I*a*tan(d*x+c))^3+16/143*I*cos(d*x+c)^7/d/(a^4+I*a^4
*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3583, 3581, 2713}

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{8\sin^7(c+dx)}{143a^4d} + \frac{168\sin^5(c+dx)}{715a^4d} - \frac{56\sin^3(c+dx)}{143a^4d} + \frac{56\sin(c+dx)}{143a^4d} + \frac{16i\cos^7(c+dx)}{143d\,(a^4+ia^4\tan(c+dx))} + \frac{9i\cos^5(c+dx)}{143ad(a+ia\tan(c+dx))^3} + \frac{i\cos^5(c+dx)}{13d(a+ia\tan(c+dx))^4}$$

```
[In] Int[Cos[c + d*x]^5/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (56*Sin[c + d*x])/(143*a^4*d) - (56*Sin[c + d*x]^3)/(143*a^4*d) + (168*Sin[
c + d*x]^5)/(715*a^4*d) - (8*Sin[c + d*x]^7)/(143*a^4*d) + ((I/13)*Cos[c +
d*x]^5)/(d*(a + I*a*Tan[c + d*x])^4) + (((9*I)/143)*Cos[c + d*x]^5)/(a*d*(a
 + I*a*Tan[c + d*x])^3) + (((16*I)/143)*Cos[c + d*x]^7)/(d*(a^4 + I*a^4*Tan
[c + d*x]))
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\text{integral} = \frac{i\cos^5(c+dx)}{13d(a+ia\tan(c+dx))^4} + \frac{9\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^3}\,dx}{13a}$$

$$= \frac{i\cos^5(c+dx)}{13d(a+ia\tan(c+dx))^4} + \frac{9i\cos^5(c+dx)}{143ad(a+ia\tan(c+dx))^3} + \frac{72\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{143a^2}$$

$$= \frac{i\cos^5(c+dx)}{13d(a+ia\tan(c+dx))^4} + \frac{9i\cos^5(c+dx)}{143ad(a+ia\tan(c+dx))^3} + \frac{16i\cos^7(c+dx)}{143d\,(a^4+ia^4\tan(c+dx))} + \frac{56\int \cos^7(c+dx)\,dx}{143a^4}$$

$$= \frac{i\cos^5(c+dx)}{13d(a+ia\tan(c+dx))^4} + \frac{9i\cos^5(c+dx)}{143ad(a+ia\tan(c+dx))^3} + \frac{16i\cos^7(c+dx)}{143d\,(a^4+ia^4\tan(c+dx))} - \frac{56\text{Subst}\left(\int(1-3x^2+3x^4-x^6)\;dx,x,-\sin(c+dx)\right)}{143a^4d}$$

$$= \frac{56\sin(c+dx)}{143a^4d} - \frac{56\sin^3(c+dx)}{143a^4d} + \frac{168\sin^5(c+dx)}{715a^4d} - \frac{8\sin^7(c+dx)}{143a^4d} + \frac{i\cos^5(c+dx)}{13d(a+ia\tan(c+dx))^4} + \frac{9i\cos^5(c+dx)}{143ad(a+ia\tan(c+dx))^3} + \frac{16i\cos^7(c+dx)}{143d\,(a^4+ia^4\tan(c+dx))}$$

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.80

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = -\frac{i\sec^4(c+dx)(-24024\cos(c+dx)-34320\cos(3(c+dx))+11440\cos(5(c+dx))+780\cos(7(c+dx))\cdots}{1\cdots}$$

```
[In] Integrate[Cos[c + d*x]^5/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((-1/183040*I)*Sec[c + d*x]^4*(-24024*Cos[c + d*x] - 34320*Cos[3*(c + d*x)]
 + 11440*Cos[5*(c + d*x)] + 780*Cos[7*(c + d*x)] + 44*Cos[9*(c + d*x)] - (6
006*I)*Sin[c + d*x] - (25740*I)*Sin[3*(c + d*x)] + (14300*I)*Sin[5*(c + d*x
)] + (1365*I)*Sin[7*(c + d*x)] + (99*I)*Sin[9*(c + d*x)]))/(a^4*d*(-I + Tan
[c + d*x])^4)
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.99

method	result
risch	$\frac{3i\mathrm{e}^{-7i(dx+c)}}{128a^4d}+\frac{i\mathrm{e}^{-9i(dx+c)}}{128a^4d}+\frac{9i\mathrm{e}^{-11i(dx+c)}}{5632a^4d}+\frac{i\mathrm{e}^{-13i(dx+c)}}{6656a^4d}+\frac{3i\cos(dx+c)}{32a^4d}+\frac{15\sin(dx+c)}{64a^4d}+\frac{25i\cos(5dx+5c)}{512a^4d}\cdots$
derivativedivides	$\frac{-\frac{135i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}+\frac{i}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^4}+\frac{1}{80\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^5}-\frac{5}{64\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^3}+\frac{23}{128\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}-\frac{137}{32\left(-i+\tan\left(\cdots\right.\right.}\cdots}{\cdots}$
default	$\frac{-\frac{135i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}+\frac{i}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^4}+\frac{1}{80\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^5}-\frac{5}{64\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^3}+\frac{23}{128\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}-\frac{137}{32\left(-i+\tan\left(\cdots\right.\right.}\cdots}{\cdots}$

```
[In] int(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 3/128*I/a^4/d*exp(-7*I*(d*x+c))+1/128*I/a^4/d*exp(-9*I*(d*x+c))+9/5632*I/a^
4/d*exp(-11*I*(d*x+c))+1/6656*I/a^4/d*exp(-13*I*(d*x+c))+3/32*I/a^4/d*cos(d
*x+c)+15/64*sin(d*x+c)/a^4/d+25/512*I/a^4/d*cos(5*d*x+5*c)+127/2560/a^4/d*s
in(5*d*x+5*c)+39/512*I/a^4/d*cos(3*d*x+3*c)+45/512/a^4/d*sin(3*d*x+3*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.68

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\left(-143i\,e^{(18i\,dx+18i\,c)} - 2145i\,e^{(16i\,dx+16i\,c)} - 25740i\,e^{(14i\,dx+14i\,c)} + 60060i\,e^{(12i\,dx+12i\,c)} + 30030i\,e^{(10i\,dx+10i\,c)}\right.}{366080\,a^4 d}$$

```
[In] integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/366080*(-143*I*e^(18*I*d*x + 18*I*c) - 2145*I*e^(16*I*d*x + 16*I*c) - 257
40*I*e^(14*I*d*x + 14*I*c) + 60060*I*e^(12*I*d*x + 12*I*c) + 30030*I*e^(10*
I*d*x + 10*I*c) + 18018*I*e^(8*I*d*x + 8*I*c) + 8580*I*e^(6*I*d*x + 6*I*c)
+ 2860*I*e^(4*I*d*x + 4*I*c) + 585*I*e^(2*I*d*x + 2*I*c) + 55*I)*e^(-13*I*d
*x - 13*I*c)/(a^4*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 367 vs. 2(153) = 306.

Time = 0.50 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.11

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \begin{cases} \frac{(-1688246017625898163896320ia^{36}d^9e^{54ic}e^{5idx} - 2532369026438847245844480 0ia^{36}d^9e^{52ic}e^{3idx} - 3038842831726616695013376 00ia^{36}d^9e^{50\cdots}}{\cdots} \\ \frac{x\left(e^{18ic}+9e^{16ic}+36e^{14ic}+84e^{12ic}+126e^{10ic}+126e^{8ic}+84e^{6ic}+36e^{4ic}+9e^{2ic}+1\right)e^{-13ic}}{512a^4} \end{cases}$$

```
[In] integrate(cos(d*x+c)**5/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Piecewise(((-1688246017625898163896320*I*a**36*d**9*exp(54*I*c)*exp(5*I*d*x
) - 25323690264388472458444800*I*a**36*d**9*exp(52*I*c)*exp(3*I*d*x) - 3038
84283172661669501337600*I*a**36*d**9*exp(50*I*c)*exp(I*d*x) + 7090633274028
77228836454400*I*a**36*d**9*exp(48*I*c)*exp(-I*d*x) + 354531663701438614418
227200*I*a**36*d**9*exp(46*I*c)*exp(-3*I*d*x) + 212718998220863168650936320
*I*a**36*d**9*exp(44*I*c)*exp(-5*I*d*x) + 101294761057553889833779200*I*a**
```

36*d**9*exp(42*I*c)*exp(-7*I*d*x) + 337649203525179632779264OO*I*a**36*d**9
*exp(40*I*c)*exp(-9*I*d*x) + 690646098119685612503O4OO*I*a**36*d**9*exp(38*
I*c)*exp(-11*I*d*x) + 649325391394576216883200*I*a**36*d**9*exp(36*I*c)*exp
(-13*I*d*x))*exp(-49*I*c)/(4321909805122299299574579200*a**40*d**10), Ne(a*
*40*d**10*exp(49*I*c), 0)), (x*(exp(18*I*c) + 9*exp(16*I*c) + 36*exp(14*I*c
) + 84*exp(12*I*c) + 126*exp(10*I*c) + 126*exp(8*I*c) + 84*exp(6*I*c) + 36*
exp(4*I*c) + 9*exp(2*I*c) + 1)*exp(-13*I*c)/(512*a**4), True))

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

[In] integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.

Giac [A] (verification not implemented)

none

Time = 0.78 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.43

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx = \frac{\frac{143\left(115\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+405i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-575\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-375i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+98\right)}{a^4\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^5}+\frac{166595\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{12}-1409265i\tan\left(\frac{1}{2}\right.}{}}{}$$

[In] integrate(cos(d*x+c)^5/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")

[Out] 1/91520*(143*(115*tan(1/2*d*x + 1/2*c)^4 + 405*I*tan(1/2*d*x + 1/2*c)^3 - 5
75*tan(1/2*d*x + 1/2*c)^2 - 375*I*tan(1/2*d*x + 1/2*c) + 98)/(a^4*(tan(1/2*
d*x + 1/2*c) + I)^5) + (166595*tan(1/2*d*x + 1/2*c)^12 - 1409265*I*tan(1/2*
d*x + 1/2*c)^11 - 6232655*tan(1/2*d*x + 1/2*c)^10 + 17535375*I*tan(1/2*d*x
+ 1/2*c)^9 + 34610004*tan(1/2*d*x + 1/2*c)^8 - 49771722*I*tan(1/2*d*x + 1/2
*c)^7 - 53349582*tan(1/2*d*x + 1/2*c)^6 + 42730974*I*tan(1/2*d*x + 1/2*c)^5
 + 25431835*tan(1/2*d*x + 1/2*c)^4 - 10954229*I*tan(1/2*d*x + 1/2*c)^3 - 32
78067*tan(1/2*d*x + 1/2*c)^2 + 614627*I*tan(1/2*d*x + 1/2*c) + 60094)/(a^4*
(tan(1/2*d*x + 1/2*c) - I)^13))/d

Mupad [B] (verification not implemented)

Time = 7.93 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.51

$$\int \frac{\cos^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx$$

$$= \frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{15049\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{128}-\frac{4513\sin\left(\frac{3c}{2}+\frac{3dx}{2}\right)}{32}+\frac{4513\sin\left(\frac{5c}{2}+\frac{5dx}{2}\right)}{32}-\frac{15461\sin\left(\frac{7c}{2}+\frac{7dx}{2}\right)}{64}+\frac{3941\sin\left(\frac{9c}{2}+\frac{9dx}{2}\right)}{64}\right.}{}$$

```
[In] int(cos(c + d*x)^5/(a + a*tan(c + d*x)*1i)^4,x)
```

```
[Out] (cos(c/2 + (d*x)/2)*((cos((3*c)/2 + (3*d*x)/2)*3003i)/32 - (cos((5*c)/2 + (
5*d*x)/2)*3003i)/32 + (cos((7*c)/2 + (7*d*x)/2)*7293i)/32 - (cos((9*c)/2 +
(9*d*x)/2)*1533i)/32 + (cos((11*c)/2 + (11*d*x)/2)*103i)/32 - (cos((13*c)/2
 + (13*d*x)/2)*103i)/32 + (cos((15*c)/2 + (15*d*x)/2)*11i)/64 - (cos((17*c)
/2 + (17*d*x)/2)*11i)/64 + (15049*sin(c/2 + (d*x)/2))/128 - (4513*sin((3*c)
/2 + (3*d*x)/2))/32 + (4513*sin((5*c)/2 + (5*d*x)/2))/32 - (15461*sin((7*c)
/2 + (7*d*x)/2))/64 + (3941*sin((9*c)/2 + (9*d*x)/2))/64 - (183*sin((11*c)/
2 + (11*d*x)/2))/32 + (183*sin((13*c)/2 + (13*d*x)/2))/32 - (99*sin((15*c)/
2 + (15*d*x)/2))/256 + (99*sin((17*c)/2 + (17*d*x)/2))/256)*2i)/(715*a^4*d*
(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2)*1i)^13*(cos(c/2 + (d*x)/2)*1i + si
n(c/2 + (d*x)/2))^5)
```

3.166 $\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^8} dx$

Optimal result

Integrand size = 24, antiderivative size = 134

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^8} dx = -\frac{192x}{a^8} - \frac{192i\log(\cos(c+dx))}{a^8d} + \frac{129\tan(c+dx)}{a^8d} - \frac{36i\tan^2(c+dx)}{a^8d} - \frac{10\tan^3(c+dx)}{a^8d} + \frac{2i\tan^4(c+dx)}{a^8d} + \frac{\tan^5(c+dx)}{5a^8d} + \frac{64i}{d(a^8+ia^8\tan(c+dx))}$$

```
[Out] -192*x/a^8-192*I*ln(cos(d*x+c))/a^8/d+129*tan(d*x+c)/a^8/d-36*I*tan(d*x+c)^
2/a^8/d-10*tan(d*x+c)^3/a^8/d+2*I*tan(d*x+c)^4/a^8/d+1/5*tan(d*x+c)^5/a^8/d
+64*I/d/(a^8+I*a^8*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^8} dx = \frac{\tan^5(c+dx)}{5a^8d} + \frac{2i\tan^4(c+dx)}{a^8d} - \frac{10\tan^3(c+dx)}{a^8d} - \frac{36i\tan^2(c+dx)}{a^8d} + \frac{129\tan(c+dx)}{a^8d} + \frac{64i}{d(a^8+ia^8\tan(c+dx))} - \frac{192i\log(\cos(c+dx))}{a^8d} - \frac{192x}{a^8}$$

```
[In] Int[Sec[c + d*x]^14/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (-192*x)/a^8 - ((192*I)*Log[Cos[c + d*x]])/(a^8*d) + (129*Tan[c + d*x])/(a^
8*d) - ((36*I)*Tan[c + d*x]^2)/(a^8*d) - (10*Tan[c + d*x]^3)/(a^8*d) + ((2*
I)*Tan[c + d*x]^4)/(a^8*d) + Tan[c + d*x]^5/(5*a^8*d) + (64*I)/(d*(a^8 + I*
a^8*Tan[c + d*x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int \frac{(a-x)^6}{(a+x)^2}\,dx, x, ia\tan(c+dx)\right)}{a^{13}d}\\ &= -\frac{i\text{Subst}\left(\int \left(129a^4-72a^3x+30a^2x^2-8ax^3+x^4+\frac{64a^6}{(a+x)^2}-\frac{192a^5}{a+x}\right)dx, x, ia\tan(c+dx)\right)}{a^{13}d}\\ &= -\frac{192x}{a^8}-\frac{192i\log(\cos(c+dx))}{a^8d}+\frac{129\tan(c+dx)}{a^8d}-\frac{36i\tan^2(c+dx)}{a^8d}\\ &\quad-\frac{10\tan^3(c+dx)}{a^8d}+\frac{2i\tan^4(c+dx)}{a^8d}+\frac{\tan^5(c+dx)}{5a^8d}+\frac{64i}{d\left(a^8+ia^8\tan(c+dx)\right)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.73

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{i\left(-960\log(i-\tan(c+dx))+645i\tan(c+dx)+180\tan^2(c+dx)-50i\tan^3(c+dx)-10\tan^4(c+dx)\right.}{5a^8d}$$

```
[In] Integrate[Sec[c + d*x]^14/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((-1/5*I)*(-960*Log[I - Tan[c + d*x]] + (645*I)*Tan[c + d*x] + 180*Tan[c +
d*x]^2 - (50*I)*Tan[c + d*x]^3 - 10*Tan[c + d*x]^4 + I*Tan[c + d*x]^5 + (32
0*I)/(-I + Tan[c + d*x])))/(a^8*d)
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.93

method	result
risch	$\frac{32 i \,{\mathrm e}^{-2 i (d x +c)}}{a^{8} d}-\frac{384 x}{a^{8}}-\frac{384 c}{a^{8} d}+\frac{16 i \left(50 \,{\mathrm e}^{8 i (d x +c)}+220 \,{\mathrm e}^{6 i (d x +c)}+370 \,{\mathrm e}^{4 i (d x +c)}+285 \,{\mathrm e}^{2 i (d x +c)}+87\right)}{5 d \,a^{8} \left({\mathrm e}^{2 i (d x +c)}+1\right)^{5}}-\frac{192 i \ln \left({\mathrm e}^{2 i (d\right.}}{a^{8} d}$
derivativedivides	$\frac{129 \tan \left(d x +c \right)}{a^{8} d}+\frac{\tan^{5}\left(d x +c \right)}{5 a^{8} d}+\frac{2 i \left(\tan^{4}\left(d x +c \right)\right)}{a^{8} d}-\frac{10 \left(\tan^{3}\left(d x +c \right)\right)}{a^{8} d}-\frac{36 i \left(\tan^{2}\left(d x +c \right)\right)}{a^{8} d}+\frac{64}{a^{8} d \left(\tan \left(d x +c \right)-i\right)}-$
default	$\frac{129 \tan \left(d x +c \right)}{a^{8} d}+\frac{\tan^{5}\left(d x +c \right)}{5 a^{8} d}+\frac{2 i \left(\tan^{4}\left(d x +c \right)\right)}{a^{8} d}-\frac{10 \left(\tan^{3}\left(d x +c \right)\right)}{a^{8} d}-\frac{36 i \left(\tan^{2}\left(d x +c \right)\right)}{a^{8} d}+\frac{64}{a^{8} d \left(\tan \left(d x +c \right)-i\right)}-$

```
[In] int(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 32*I/a^8/d*exp(-2*I*(d*x+c))-384*x/a^8-384/a^8/d*c+16/5*I*(50*exp(8*I*(d*x+
c))+220*exp(6*I*(d*x+c))+370*exp(4*I*(d*x+c))+285*exp(2*I*(d*x+c))+87)/d/a^
8/(exp(2*I*(d*x+c))+1)^5-192*I/a^8/d*ln(exp(2*I*(d*x+c))+1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 273 vs. 2(122) = 244.

Time = 0.30 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.04

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{16\left(120\,dxe^{(12i\,dx+12i\,c)}+60\,(10\,dx-i)e^{(10i\,dx+10i\,c)}+30\,(40\,dx-9i)e^{(8i\,dx+8i\,c)}+10\,(120\,dx-47i)e^{(6i\,dx+}\right.}{5\left(a^8de^{(12i\,dx+}\right.}$$

```
[In] integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -16/5*(120*d*x*e^(12*I*d*x + 12*I*c) + 60*(10*d*x - I)*e^(10*I*d*x + 10*I*c
) + 30*(40*d*x - 9*I)*e^(8*I*d*x + 8*I*c) + 10*(120*d*x - 47*I)*e^(6*I*d*x
+ 6*I*c) + 5*(120*d*x - 77*I)*e^(4*I*d*x + 4*I*c) + (120*d*x - 137*I)*e^(2*
I*d*x + 2*I*c) + 60*(I*e^(12*I*d*x + 12*I*c) + 5*I*e^(10*I*d*x + 10*I*c) +
10*I*e^(8*I*d*x + 8*I*c) + 10*I*e^(6*I*d*x + 6*I*c) + 5*I*e^(4*I*d*x + 4*I*
c) + I*e^(2*I*d*x + 2*I*c))*log(e^(2*I*d*x + 2*I*c) + 1) - 10*I)/(a^8*d*e^(
12*I*d*x + 12*I*c) + 5*a^8*d*e^(10*I*d*x + 10*I*c) + 10*a^8*d*e^(8*I*d*x +
8*I*c) + 10*a^8*d*e^(6*I*d*x + 6*I*c) + 5*a^8*d*e^(4*I*d*x + 4*I*c) + a^8*d
*e^(2*I*d*x + 2*I*c))
```

Sympy [F]

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{\int \frac{\sec^{14}(c+dx)}{\tan^8(c+dx)-8i\tan^7(c+dx)-28\tan^6(c+dx)+56i\tan^5(c+dx)+70\tan^4(c+dx)-56i\tan^3(c+dx)-28\tan^2(c+dx)+8i\tan(c+dx)+1}\,dx}{a^8}$$

```
[In] integrate(sec(d*x+c)**14/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Integral(sec(c + d*x)**14/(tan(c + d*x)**8 - 8*I*tan(c + d*x)**7 - 28*tan(c
 + d*x)**6 + 56*I*tan(c + d*x)**5 + 70*tan(c + d*x)**4 - 56*I*tan(c + d*x)*
*3 - 28*tan(c + d*x)**2 + 8*I*tan(c + d*x) + 1), x)/a**8
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.71

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{\frac{320\left(\tan(dx+c)^6-6i\tan(dx+c)^5-15\tan(dx+c)^4+20i\tan(dx+c)^3+15\tan(dx+c)^2-6i\tan(dx+c)-1\right)}{a^8\tan(dx+c)^7-7i\,a^8\tan(dx+c)^6-21\,a^8\tan(dx+c)^5+35i\,a^8\tan(dx+c)^4+35\,a^8\tan(dx+c)^3-21i\,a^8\tan(dx+c)^2-7\,a^8\tan(dx+c)+i\,a^8}+\frac{\tan(d}{}}{5\,d}$$

```
[In] integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/5*(320*(tan(d*x + c)^6 - 6*I*tan(d*x + c)^5 - 15*tan(d*x + c)^4 + 20*I*ta
n(d*x + c)^3 + 15*tan(d*x + c)^2 - 6*I*tan(d*x + c) - 1)/(a^8*tan(d*x + c)^
7 - 7*I*a^8*tan(d*x + c)^6 - 21*a^8*tan(d*x + c)^5 + 35*I*a^8*tan(d*x + c)^
4 + 35*a^8*tan(d*x + c)^3 - 21*I*a^8*tan(d*x + c)^2 - 7*a^8*tan(d*x + c) +
I*a^8) + (tan(d*x + c)^5 + 10*I*tan(d*x + c)^4 - 50*tan(d*x + c)^3 - 180*I*
tan(d*x + c)^2 + 645*tan(d*x + c))/a^8 + 960*I*log(I*tan(d*x + c) + 1)/a^8)
/d
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(122) = 244.

Time = 1.93 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.87

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx =$$
$$-\frac{2\left(\frac{480i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^8}-\frac{960i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}{a^8}+\frac{480i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^8}+\frac{160\left(9i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+20\tan\left(\frac{1}{2}\,d\right.\right.}{a^8\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^2}\right.}{}$$

```
[In] integrate(sec(d*x+c)^14/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -2/5*(480*I*log(tan(1/2*d*x + 1/2*c) + 1)/a^8 - 960*I*log(tan(1/2*d*x + 1/2
*c) - I)/a^8 + 480*I*log(tan(1/2*d*x + 1/2*c) - 1)/a^8 + 160*(9*I*tan(1/2*d
*x + 1/2*c)^2 + 20*tan(1/2*d*x + 1/2*c) - 9*I)/(a^8*(tan(1/2*d*x + 1/2*c) -
 I)^2) + (-1096*I*tan(1/2*d*x + 1/2*c)^10 + 645*tan(1/2*d*x + 1/2*c)^9 + 58
40*I*tan(1/2*d*x + 1/2*c)^8 - 2780*tan(1/2*d*x + 1/2*c)^7 - 12120*I*tan(1/2
*d*x + 1/2*c)^6 + 4286*tan(1/2*d*x + 1/2*c)^5 + 12120*I*tan(1/2*d*x + 1/2*c
)^4 - 2780*tan(1/2*d*x + 1/2*c)^3 - 5840*I*tan(1/2*d*x + 1/2*c)^2 + 645*tan
(1/2*d*x + 1/2*c) + 1096*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^5*a^8))/d
```

Mupad [B] (verification not implemented)

Time = 4.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.78

$$\int \frac{\sec^{14}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\frac{129\tan(c+d\,x)}{a^8}-\frac{10\tan(c+d\,x)^3}{a^8}+\frac{\tan(c+d\,x)^5}{5\,a^8}+\frac{\ln(\tan(c+d\,x)-\mathrm{i})\,192\mathrm{i}}{a^8}+\frac{64\mathrm{i}}{a^8\,(1+\tan(c+d\,x)\,1\mathrm{i})}-\frac{\tan(c+d\,x)^2\,36\mathrm{i}}{a^8}+\frac{\tan(c+d\,x)^4\,2\mathrm{i}}{a^8}}{d}$$

```
[In] int(1/(cos(c + d*x)^14*(a + a*tan(c + d*x)*1i)^8),x)
```

```
[Out] ((log(tan(c + d*x) - 1i)*192i)/a^8 + (129*tan(c + d*x))/a^8 + 64i/(a^8*(tan
(c + d*x)*1i + 1)) - (tan(c + d*x)^2*36i)/a^8 - (10*tan(c + d*x)^3)/a^8 + (
tan(c + d*x)^4*2i)/a^8 + tan(c + d*x)^5/(5*a^8))/d
```

3.167 $\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 126

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{80x}{a^8}+\frac{80i\log(\cos(c+dx))}{a^8d}-\frac{31\tan(c+dx)}{a^8d}+\frac{4i\tan^2(c+dx)}{a^8d}+\frac{\tan^3(c+dx)}{3a^8d}+\frac{16i}{d\,(a^4+ia^4\tan(c+dx))^2}-\frac{80i}{d\,(a^8+ia^8\tan(c+dx))}$$

```
[Out] 80*x/a^8+80*I*ln(cos(d*x+c))/a^8/d-31*tan(d*x+c)/a^8/d+4*I*tan(d*x+c)^2/a^8
/d+1/3*tan(d*x+c)^3/a^8/d+16*I/d/(a^4+I*a^4*tan(d*x+c))^2-80*I/d/(a^8+I*a^8
*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\tan^3(c+dx)}{3a^8d}+\frac{4i\tan^2(c+dx)}{a^8d}-\frac{31\tan(c+dx)}{a^8d}-\frac{80i}{d\,(a^8+ia^8\tan(c+dx))}+\frac{80i\log(\cos(c+dx))}{a^8d}+\frac{80x}{a^8}+\frac{16i}{d\,(a^4+ia^4\tan(c+dx))^2}$$

```
[In] Int[Sec[c + d*x]^12/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (80*x)/a^8 + ((80*I)*Log[Cos[c + d*x]])/(a^8*d) - (31*Tan[c + d*x])/(a^8*d)
 + ((4*I)*Tan[c + d*x]^2)/(a^8*d) + Tan[c + d*x]^3/(3*a^8*d) + (16*I)/(d*(a
^4 + I*a^4*Tan[c + d*x])^2) - (80*I)/(d*(a^8 + I*a^8*Tan[c + d*x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int \frac{(a-x)^5}{(a+x)^3}\,dx, x, ia\tan(c+dx)\right)}{a^{11}d}\\ &= -\frac{i\text{Subst}\left(\int \left(-31a^2+8ax-x^2+\frac{32a^5}{(a+x)^3}-\frac{80a^4}{(a+x)^2}+\frac{80a^3}{a+x}\right)\,dx, x, ia\tan(c+dx)\right)}{a^{11}d}\\ &= \frac{80x}{a^8}+\frac{80i\log(\cos(c+dx))}{a^8d}-\frac{31\tan(c+dx)}{a^8d}+\frac{4i\tan^2(c+dx)}{a^8d}\\ &\quad+\frac{\tan^3(c+dx)}{3a^8d}+\frac{16i}{d\left(a^4+ia^4\tan(c+dx)\right)^2}-\frac{80i}{d\left(a^8+ia^8\tan(c+dx)\right)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.70

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{i\left(-93i\tan(c+dx)-12\tan^2(c+dx)+i\tan^3(c+dx)+48\left(5\log(i-\tan(c+dx))+\frac{-4-5i\tan(c+dx)}{(-i+\tan(c+dx))^2}\right)\right)}{3a^8d}$$

```
[In] Integrate[Sec[c + d*x]^12/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((-1/3*I)*((-93*I)*Tan[c + d*x] - 12*Tan[c + d*x]^2 + I*Tan[c + d*x]^3 + 48
*(5*Log[I - Tan[c + d*x]] + (-4 - (5*I)*Tan[c + d*x])/(-I + Tan[c + d*x])^2
)))/(a^8*d)
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.95

method	result
risch	$-\frac{32i\mathrm{e}^{-2i(dx+c)}}{a^8d}+\frac{4i\mathrm{e}^{-4i(dx+c)}}{a^8d}+\frac{160x}{a^8}+\frac{160c}{a^8d}-\frac{4i\left(36\,\mathrm{e}^{4i(dx+c)}+81\,\mathrm{e}^{2i(dx+c)}+47\right)}{3d\,a^8\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}+\frac{80i\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{a^8d}$
derivativedivides	$-\frac{31\tan(dx+c)}{a^8d}+\frac{\tan^3(dx+c)}{3a^8d}+\frac{4i\left(\tan^2(dx+c)\right)}{a^8d}+\frac{80\arctan(\tan(dx+c))}{a^8d}-\frac{40i\ln\left(1+\tan^2(dx+c)\right)}{a^8d}-\frac{}{a^8d(\tan}$
default	$-\frac{31\tan(dx+c)}{a^8d}+\frac{\tan^3(dx+c)}{3a^8d}+\frac{4i\left(\tan^2(dx+c)\right)}{a^8d}+\frac{80\arctan(\tan(dx+c))}{a^8d}-\frac{40i\ln\left(1+\tan^2(dx+c)\right)}{a^8d}-\frac{}{a^8d(\tan}$

[In] int(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)

[Out] -32*I/a^8/d*exp(-2*I*(d*x+c))+4*I/a^8/d*exp(-4*I*(d*x+c))+160*x/a^8+160/a^8
/d*c-4/3*I*(36*exp(4*I*(d*x+c))+81*exp(2*I*(d*x+c))+47)/d/a^8/(exp(2*I*(d*x
+c))+1)^3+80*I/a^8/d*ln(exp(2*I*(d*x+c))+1)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.58

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$=\frac{4\left(120\,dxe^{(10i\,dx+10i\,c)}+60\,(6\,dx-i)e^{(8i\,dx+8i\,c)}+30\,(12\,dx-5i)e^{(6i\,dx+6i\,c)}+10\,(12\,dx-11i)e^{(4i\,dx+4i\,c)}-\right.}{3\left(a^8de^{(10i\,dx+10i\,c)}+3\,a^8de^{(8i\,dx+8i\,c}\right.}$$

[In] integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")

[Out] 4/3*(120*d*x*e^(10*I*d*x + 10*I*c) + 60*(6*d*x - I)*e^(8*I*d*x + 8*I*c) + 3
0*(12*d*x - 5*I)*e^(6*I*d*x + 6*I*c) + 10*(12*d*x - 11*I)*e^(4*I*d*x + 4*I*
c) - 60*(-I*e^(10*I*d*x + 10*I*c) - 3*I*e^(8*I*d*x + 8*I*c) - 3*I*e^(6*I*d*
x + 6*I*c) - I*e^(4*I*d*x + 4*I*c))*log(e^(2*I*d*x + 2*I*c) + 1) - 15*I*e^(
2*I*d*x + 2*I*c) + 3*I)/(a^8*d*e^(10*I*d*x + 10*I*c) + 3*a^8*d*e^(8*I*d*x +
 8*I*c) + 3*a^8*d*e^(6*I*d*x + 6*I*c) + a^8*d*e^(4*I*d*x + 4*I*c))

Sympy [F]

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$=\frac{\int \frac{\sec^{12}(c+dx)}{\tan^8(c+dx)-8i\tan^7(c+dx)-28\tan^6(c+dx)+56i\tan^5(c+dx)+70\tan^4(c+dx)-56i\tan^3(c+dx)-28\tan^2(c+dx)+8i\tan(c+dx)+1}\,dx}{a^8}$$

```
[In] integrate(sec(d*x+c)**12/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Integral(sec(c + d*x)**12/(tan(c + d*x)**8 - 8*I*tan(c + d*x)**7 - 28*tan(c
 + d*x)**6 + 56*I*tan(c + d*x)**5 + 70*tan(c + d*x)**4 - 56*I*tan(c + d*x)*
*3 - 28*tan(c + d*x)**2 + 8*I*tan(c + d*x) + 1), x)/a**8
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.68

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx =$$
$$-\frac{\frac{48\left(5\tan(dx+c)^6-29i\tan(dx+c)^5-70\tan(dx+c)^4+90i\tan(dx+c)^3+65\tan(dx+c)^2-25i\tan(dx+c)-4\right)}{a^8\tan(dx+c)^7-7ia^8\tan(dx+c)^6-21a^8\tan(dx+c)^5+35ia^8\tan(dx+c)^4+35a^8\tan(dx+c)^3-21ia^8\tan(dx+c)^2-7a^8\tan(dx+c)+ia^8}-\tan(}{3d}$$

```
[In] integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/3*(48*(5*tan(d*x + c)^6 - 29*I*tan(d*x + c)^5 - 70*tan(d*x + c)^4 + 90*I
*tan(d*x + c)^3 + 65*tan(d*x + c)^2 - 25*I*tan(d*x + c) - 4)/(a^8*tan(d*x +
 c)^7 - 7*I*a^8*tan(d*x + c)^6 - 21*a^8*tan(d*x + c)^5 + 35*I*a^8*tan(d*x +
 c)^4 + 35*a^8*tan(d*x + c)^3 - 21*I*a^8*tan(d*x + c)^2 - 7*a^8*tan(d*x + c
) + I*a^8) - (tan(d*x + c)^3 + 12*I*tan(d*x + c)^2 - 93*tan(d*x + c))/a^8 +
 240*I*log(I*tan(d*x + c) + 1)/a^8)/d
```

Giac [A] (verification not implemented)

none

Time = 1.98 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.78

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx =$$
$$-\frac{2\left(-\frac{120i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^8}+\frac{240i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)}{a^8}-\frac{120i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^8}+\frac{220i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-93\tan\left(\frac{1}{2}dx+\right.}{}\right.}{}$$

```
[In] integrate(sec(d*x+c)^12/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -2/3*(-120*I*log(tan(1/2*d*x + 1/2*c) + 1)/a^8 + 240*I*log(tan(1/2*d*x + 1/
2*c) - I)/a^8 - 120*I*log(tan(1/2*d*x + 1/2*c) - 1)/a^8 + (220*I*tan(1/2*d*
x + 1/2*c)^6 - 93*tan(1/2*d*x + 1/2*c)^5 - 684*I*tan(1/2*d*x + 1/2*c)^4 + 1
90*tan(1/2*d*x + 1/2*c)^3 + 684*I*tan(1/2*d*x + 1/2*c)^2 - 93*tan(1/2*d*x +
 1/2*c) - 220*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a^8) + 4*(-125*I*tan(1/2*d
*x + 1/2*c)^4 - 536*tan(1/2*d*x + 1/2*c)^3 + 846*I*tan(1/2*d*x + 1/2*c)^2 +
 536*tan(1/2*d*x + 1/2*c) - 125*I)/(a^8*(tan(1/2*d*x + 1/2*c) - I)^4))/d
```

Mupad [B] (verification not implemented)

Time = 3.81 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.90

$$\int \frac{\sec^{12}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\tan(c+d\,x)^3}{3\,a^8\,d} - \frac{31\tan(c+d\,x)}{a^8\,d} + \frac{\tan(c+d\,x)^2\,4\mathrm{i}}{a^8\,d} - \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,80\mathrm{i}}{a^8\,d} - \frac{\frac{64}{a^8}+\frac{\tan(c+d\,x)\,80\mathrm{i}}{a^8}}{d\,\left(\tan\left(c+d\,x\right)^2\,1\mathrm{i}+2\tan\left(c+d\,x\right)-\mathrm{i}\right)}$$

```
[In] int(1/(cos(c + d*x)^12*(a + a*tan(c + d*x)*1i)^8),x)
```

```
[Out] (tan(c + d*x)^2*4i)/(a^8*d) - (31*tan(c + d*x))/(a^8*d) - (log(tan(c + d*x)
 - 1i)*80i)/(a^8*d) + tan(c + d*x)^3/(3*a^8*d) - ((tan(c + d*x)*80i)/a^8 +
64/a^8)/(d*(2*tan(c + d*x) + tan(c + d*x)^2*1i - 1i))
```

3.168 $\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 116

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{8x}{a^8} - \frac{8i\log(\cos(c+dx))}{a^8d} + \frac{\tan(c+dx)}{a^8d} + \frac{16i}{3a^5d(a+ia\tan(c+dx))^3} - \frac{16i}{d\left(a^4+ia^4\tan(c+dx)\right)^2} + \frac{24i}{d\left(a^8+ia^8\tan(c+dx)\right)}$$

```
[Out] -8*x/a^8-8*I*ln(cos(d*x+c))/a^8/d+tan(d*x+c)/a^8/d+16/3*I/a^5/d/(a+I*a*tan(
d*x+c))^3-16*I/d/(a^4+I*a^4*tan(d*x+c))^2+24*I/d/(a^8+I*a^8*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\tan(c+dx)}{a^8d} + \frac{24i}{d\left(a^8+ia^8\tan(c+dx)\right)} - \frac{8i\log(\cos(c+dx))}{a^8d} - \frac{8x}{a^8} + \frac{16i}{3a^5d(a+ia\tan(c+dx))^3} - \frac{16i}{d\left(a^4+ia^4\tan(c+dx)\right)^2}$$

```
[In] Int[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (-8*x)/a^8 - ((8*I)*Log[Cos[c + d*x]])/(a^8*d) + Tan[c + d*x]/(a^8*d) + ((1
6*I)/3)/(a^5*d*(a + I*a*Tan[c + d*x])^3) - (16*I)/(d*(a^4 + I*a^4*Tan[c + d
*x])^2) + (24*I)/(d*(a^8 + I*a^8*Tan[c + d*x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{i\text{Subst}\left(\int \frac{(a-x)^4}{(a+x)^4}\,dx, x, ia\tan(c+dx)\right)}{a^9 d}\\
&= -\frac{i\text{Subst}\left(\int \left(1+\frac{16a^4}{(a+x)^4}-\frac{32a^3}{(a+x)^3}+\frac{24a^2}{(a+x)^2}-\frac{8a}{a+x}\right)dx, x, ia\tan(c+dx)\right)}{a^9 d}\\
&= -\frac{8x}{a^8}-\frac{8i\log(\cos(c+dx))}{a^8 d}+\frac{\tan(c+dx)}{a^8 d}+\frac{16i}{3a^5 d(a+ia\tan(c+dx))^3}\\
&\quad -\frac{16i}{d\left(a^4+ia^4\tan(c+dx)\right)^2}+\frac{24i}{d\left(a^8+ia^8\tan(c+dx)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.68

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{i\left(-8a\log(i-\tan(c+dx))+ia\tan(c+dx)+\frac{8a(-5i+12\tan(c+dx)+9i\tan^2(c+dx))}{3(-i+\tan(c+dx))^3}\right)}{a^9 d}$$

```
[In] Integrate[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((-I)*(-8*a*Log[I - Tan[c + d*x]] + I*a*Tan[c + d*x] + (8*a*(-5*I + 12*Tan[
c + d*x] + (9*I)*Tan[c + d*x]^2))/(3*(-I + Tan[c + d*x])^3)))/(a^9*d)
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{\tan(dx+c)}{a^8d}+\frac{16i}{a^8d(\tan(dx+c)-i)^2}-\frac{16}{3a^8d(\tan(dx+c)-i)^3}-\frac{8\arctan(\tan(dx+c))}{a^8d}+\frac{4i\ln(1+\tan^2(dx+c))}{a^8d}+\frac{}{a^8d(}$
default	$\frac{\tan(dx+c)}{a^8d}+\frac{16i}{a^8d(\tan(dx+c)-i)^2}-\frac{16}{3a^8d(\tan(dx+c)-i)^3}-\frac{8\arctan(\tan(dx+c))}{a^8d}+\frac{4i\ln(1+\tan^2(dx+c))}{a^8d}+\frac{}{a^8d(}$
risch	$\frac{6i\mathrm{e}^{-2i(dx+c)}}{a^8d}-\frac{2i\mathrm{e}^{-4i(dx+c)}}{a^8d}+\frac{2i\mathrm{e}^{-6i(dx+c)}}{3a^8d}-\frac{16x}{a^8}-\frac{16c}{a^8d}+\frac{2i}{d\,a^8\left(\mathrm{e}^{2i(dx+c)}+1\right)}-\frac{8i\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{a^8d}$

```
[In] int(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
[Out] tan(d*x+c)/a^8/d+16*I/a^8/d/(tan(d*x+c)-I)^2-16/3/a^8/d/(tan(d*x+c)-I)^3-8/
a^8/d*arctan(tan(d*x+c))+4*I/a^8/d*ln(1+tan(d*x+c)^2)+24/a^8/d/(tan(d*x+c)-
I)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.07

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{2\left(24\,dxe^{(8i\,dx+8i\,c)}+12\,(2\,dx-i)e^{(6i\,dx+6i\,c)}+12\left(i\,e^{(8i\,dx+8i\,c)}+i\,e^{(6i\,dx+6i\,c)}\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)-6i\,e^{(4}\right.}{3\left(a^8de^{(8i\,dx+8i\,c)}+a^8de^{(6i\,dx+6i\,c)}\right)}$$

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
[Out] -2/3*(24*d*x*e^(8*I*d*x + 8*I*c) + 12*(2*d*x - I)*e^(6*I*d*x + 6*I*c) + 12*
(I*e^(8*I*d*x + 8*I*c) + I*e^(6*I*d*x + 6*I*c))*log(e^(2*I*d*x + 2*I*c) + 1
) - 6*I*e^(4*I*d*x + 4*I*c) + 2*I*e^(2*I*d*x + 2*I*c) - I)/(a^8*d*e^(8*I*d*
x + 8*I*c) + a^8*d*e^(6*I*d*x + 6*I*c))
```

Sympy [F]

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\int \frac{\sec^{10}(c+dx)}{\tan^8(c+dx)-8i\tan^7(c+dx)-28\tan^6(c+dx)+56i\tan^5(c+dx)+70\tan^4(c+dx)-56i\tan^3(c+dx)-28\tan^2(c+dx)+8i\tan(c+dx)+1}\,dx}{a^8}$$

```
[In] integrate(sec(d*x+c)**10/(a+I*a*tan(d*x+c))**8,x)
[Out] Integral(sec(c + d*x)**10/(tan(c + d*x)**8 - 8*I*tan(c + d*x)**7 - 28*tan(c
 + d*x)**6 + 56*I*tan(c + d*x)**5 + 70*tan(c + d*x)**4 - 56*I*tan(c + d*x)*
*3 - 28*tan(c + d*x)**2 + 8*I*tan(c + d*x) + 1), x)/a**8
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.65

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\frac{8\left(9\tan(dx+c)^6-48i\tan(dx+c)^5-107\tan(dx+c)^4+128i\tan(dx+c)^3+87\tan(dx+c)^2-32i\tan(dx+c)-5\right)}{a^8\tan(dx+c)^7-7i\,a^8\tan(dx+c)^6-21\,a^8\tan(dx+c)^5+35i\,a^8\tan(dx+c)^4+35\,a^8\tan(dx+c)^3-21i\,a^8\tan(dx+c)^2-7\,a^8\tan(dx+c)+i\,a^8}+\frac{24i\log\ldots}{}}{3\,d}$$

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/3*(8*(9*tan(d*x + c)^6 - 48*I*tan(d*x + c)^5 - 107*tan(d*x + c)^4 + 128*I
*tan(d*x + c)^3 + 87*tan(d*x + c)^2 - 32*I*tan(d*x + c) - 5)/(a^8*tan(d*x +
 c)^7 - 7*I*a^8*tan(d*x + c)^6 - 21*a^8*tan(d*x + c)^5 + 35*I*a^8*tan(d*x +
 c)^4 + 35*a^8*tan(d*x + c)^3 - 21*I*a^8*tan(d*x + c)^2 - 7*a^8*tan(d*x + c
) + I*a^8) + 24*I*log(I*tan(d*x + c) + 1)/a^8 + 3*tan(d*x + c)/a^8)/d
```

Giac [A] (verification not implemented)

none

Time = 1.79 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.72

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{2\left(\frac{60i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^8}-\frac{120i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)}{a^8}+\frac{60i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^8}-\frac{15\left(4i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\ldots\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^8}\ldots\right)}{}$$

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -2/15*(60*I*log(tan(1/2*d*x + 1/2*c) + 1)/a^8 - 120*I*log(tan(1/2*d*x + 1/2
*c) - I)/a^8 + 60*I*log(tan(1/2*d*x + 1/2*c) - 1)/a^8 - 15*(4*I*tan(1/2*d*x
 + 1/2*c)^2 - tan(1/2*d*x + 1/2*c) - 4*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a^8
) + 2*(147*I*tan(1/2*d*x + 1/2*c)^6 + 942*tan(1/2*d*x + 1/2*c)^5 - 2445*I*t
an(1/2*d*x + 1/2*c)^4 - 3460*tan(1/2*d*x + 1/2*c)^3 + 2445*I*tan(1/2*d*x +
1/2*c)^2 + 942*tan(1/2*d*x + 1/2*c) - 147*I)/(a^8*(tan(1/2*d*x + 1/2*c) - I
)^6))/d
```

Mupad [B] (verification not implemented)

Time = 4.53 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\tan(c+d\,x)}{a^8\,d} - \frac{\frac{32\tan(c+d\,x)}{a^8} - \frac{40\mathrm{i}}{3\,a^8} + \frac{\tan(c+d\,x)^2\,24\mathrm{i}}{a^8}}{d\,\left(-\tan\left(c+d\,x\right)^3\,1\mathrm{i} - 3\tan\left(c+d\,x\right)^2 + \tan\left(c+d\,x\right)\,3\mathrm{i} + 1\right)} + \frac{\ln\left(\tan(c+d\,x) - \mathrm{i}\right)\,8\mathrm{i}}{a^8\,d}$$

```
[In] int(1/(cos(c + d*x)^10*(a + a*tan(c + d*x)*1i)^8),x)
```

```
[Out] (log(tan(c + d*x) - 1i)*8i)/(a^8*d) - ((32*tan(c + d*x))/a^8 - 40i/(3*a^8)
+ (tan(c + d*x)^2*24i)/a^8)/(d*(tan(c + d*x)*3i - 3*tan(c + d*x)^2 - tan(c
+ d*x)^3*1i + 1)) + tan(c + d*x)/(a^8*d)
```

3.169 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^8} dx$

Optimal result

Integrand size = 24, antiderivative size = 43

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^8} dx = \frac{i(a-ia\tan(c+dx))^4}{8d\left(a^3+ia^3\tan(c+dx)\right)^4}$$

```
[Out] 1/8*I*(a-I*a*tan(d*x+c))^4/d/(a^3+I*a^3*tan(d*x+c))^4
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 37}

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^8} dx = \frac{i(a-ia\tan(c+dx))^4}{8d\left(a^3+ia^3\tan(c+dx)\right)^4}$$

```
[In] Int[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((I/8)*(a - I*a*Tan[c + d*x])^4)/(d*(a^3 + I*a^3*Tan[c + d*x])^4)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
```

```
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{i\text{Subst}\left(\int \frac{(a-x)^3}{(a+x)^5}\,dx, x, ia\tan(c+dx)\right)}{a^7 d}$$
$$= \frac{i(a-ia\tan(c+dx))^4}{8d\left(a^3+ia^3\tan(c+dx)\right)^4}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{i(i+\tan(c+dx))^4}{8a^8 d(-i+\tan(c+dx))^4}$$

```
[In] Integrate[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^8,x]
[Out] ((I/8)*(I + Tan[c + d*x])^4)/(a^8*d*(-I + Tan[c + d*x])^4)
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.44

method	result	size
risch	$\frac{i e^{-8i(dx+c)}}{8a^8 d}$	19
derivativedivides	$-\frac{-\frac{2i}{(\tan(dx+c)-i)^4}+\frac{3i}{(\tan(dx+c)-i)^2}+\frac{1}{\tan(dx+c)-i}-\frac{4}{(\tan(dx+c)-i)^3}}{a^8 d}$	62
default	$-\frac{-\frac{2i}{(\tan(dx+c)-i)^4}+\frac{3i}{(\tan(dx+c)-i)^2}+\frac{1}{\tan(dx+c)-i}-\frac{4}{(\tan(dx+c)-i)^3}}{a^8 d}$	62

```
[In] int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
[Out] 1/8*I/a^8/d*exp(-8*I*(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.40

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{i\,e^{(-8i\,dx-8i\,c)}}{8\,a^8d}$$

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
[Out] 1/8*I*e^(-8*I*d*x - 8*I*c)/(a^8*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(34) = 68.

Time = 8.75 (sec) , antiderivative size = 160, normalized size of antiderivative = 3.72

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \begin{cases} \frac{i\sec^8(c+dx)}{8a^8d\tan^8(c+dx)-64ia^8d\tan^7(c+dx)-224a^8d\tan^6(c+dx)+448ia^8d\tan^5(c+dx)+560a^8d\tan^4(c+dx)-448ia^8d\tan^3(c+dx)-224a^8d\tan^2(c+dx)+64ia^8d\tan(c+dx)+8a^8d} \\ \frac{x\sec^8(c)}{(ia\tan(c)+a)^8} \end{cases}$$

```
[In] integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c))**8,x)
[Out] Piecewise((I*sec(c + d*x)**8/(8*a**8*d*tan(c + d*x)**8 - 64*I*a**8*d*tan(c
+ d*x)**7 - 224*a**8*d*tan(c + d*x)**6 + 448*I*a**8*d*tan(c + d*x)**5 + 560
*a**8*d*tan(c + d*x)**4 - 448*I*a**8*d*tan(c + d*x)**3 - 224*a**8*d*tan(c +
 d*x)**2 + 64*I*a**8*d*tan(c + d*x) + 8*a**8*d), Ne(d, 0)), (x*sec(c)**8/(I
*a*tan(c) + a)**8, True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(35) = 70.

Time = 0.23 (sec) , antiderivative size = 158, normalized size of antiderivative = 3.67

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{\tan(dx+c)^6-3i\tan(dx+c)^5-4\tan(dx+c)^4+4i\tan(dx+c)^3+3\tan(dx+c)^2 \cdots}{(a^8\tan(dx+c)^7-7i\,a^8\tan(dx+c)^6-21\,a^8\tan(dx+c)^5+35i\,a^8\tan(dx+c)^4+35\,a^8\tan(dx+c)^3 \cdots}$$

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -(tan(d*x + c)^6 - 3*I*tan(d*x + c)^5 - 4*tan(d*x + c)^4 + 4*I*tan(d*x + c)
^3 + 3*tan(d*x + c)^2 - I*tan(d*x + c))/((a^8*tan(d*x + c)^7 - 7*I*a^8*tan(
d*x + c)^6 - 21*a^8*tan(d*x + c)^5 + 35*I*a^8*tan(d*x + c)^4 + 35*a^8*tan(d
*x + c)^3 - 21*I*a^8*tan(d*x + c)^2 - 7*a^8*tan(d*x + c) + I*a^8)*d)
```

Giac [A] (verification not implemented)

none

Time = 1.57 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.63

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^8d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^8}$$

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -2*(tan(1/2*d*x + 1/2*c)^7 - 7*tan(1/2*d*x + 1/2*c)^5 + 7*tan(1/2*d*x + 1/2
*c)^3 - tan(1/2*d*x + 1/2*c))/(a^8*d*(tan(1/2*d*x + 1/2*c) - I)^8)
```

Mupad [B] (verification not implemented)

Time = 3.97 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.70

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{\tan(c+d\,x)\,\left(\tan(c+d\,x)^2\,1\mathrm{i}-\mathrm{i}\right)}{a^8\,d\,\left(\tan\left(c+d\,x\right)^4\,1\mathrm{i}+4\tan\left(c+d\,x\right)^3-\tan\left(c+d\,x\right)^2\,6\mathrm{i}-4\tan\left(c+d\,x\right)+1\mathrm{i}\right)}$$

```
[In] int(1/(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^8),x)
```

```
[Out] -(tan(c + d*x)*(tan(c + d*x)^2*1i - 1i))/(a^8*d*(4*tan(c + d*x)^3 - tan(c +
 d*x)^2*6i - 4*tan(c + d*x) + tan(c + d*x)^4*1i + 1i))
```

3.170 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 81

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{4i}{5a^3d(a+ia\tan(c+dx))^5}+\frac{i}{3a^5d(a+ia\tan(c+dx))^3}-\frac{i}{d\left(a^2+ia^2\tan(c+dx)\right)^4}$$

```
[Out] 4/5*I/a^3/d/(a+I*a*tan(d*x+c))^5+1/3*I/a^5/d/(a+I*a*tan(d*x+c))^3-I/d/(a^2+
I*a^2*tan(d*x+c))^4
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{i}{3a^5d(a+ia\tan(c+dx))^3}+\frac{4i}{5a^3d(a+ia\tan(c+dx))^5}-\frac{i}{d\left(a^2+ia^2\tan(c+dx)\right)^4}$$

```
[In] Int[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((4*I)/5)/(a^3*d*(a + I*a*Tan[c + d*x])^5) + (I/3)/(a^5*d*(a + I*a*Tan[c +
d*x])^3) - I/(d*(a^2 + I*a^2*Tan[c + d*x])^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{i\text{Subst}\left(\int \frac{(a-x)^2}{(a+x)^6}\,dx, x, ia\tan(c+dx)\right)}{a^5d} \\
&= -\frac{i\text{Subst}\left(\int \left(\frac{4a^2}{(a+x)^6}-\frac{4a}{(a+x)^5}+\frac{1}{(a+x)^4}\right)dx, x, ia\tan(c+dx)\right)}{a^5d} \\
&= \frac{4i}{5a^3d(a+ia\tan(c+dx))^5}+\frac{i}{3a^5d(a+ia\tan(c+dx))^3}-\frac{i}{d\,(a^2+ia^2\tan(c+dx))^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.54

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{2-5i\tan(c+dx)-5\tan^2(c+dx)}{15a^8d(-i+\tan(c+dx))^5}$$

```
[In] Integrate[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^8,x]
[Out] (2 - (5*I)*Tan[c + d*x] - 5*Tan[c + d*x]^2)/(15*a^8*d*(-I + Tan[c + d*x])^5
)
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

method	result	size
derivativedivides	$-\frac{\frac{i}{(\tan(dx+c)-i)^4}-\frac{4}{5(\tan(dx+c)-i)^5}+\frac{1}{3(\tan(dx+c)-i)^3}}{a^8d}$	50
default	$-\frac{\frac{i}{(\tan(dx+c)-i)^4}-\frac{4}{5(\tan(dx+c)-i)^5}+\frac{1}{3(\tan(dx+c)-i)^3}}{a^8d}$	50
risch	$\frac{i\mathrm{e}^{-6i(dx+c)}}{24a^8d}+\frac{i\mathrm{e}^{-8i(dx+c)}}{16a^8d}+\frac{i\mathrm{e}^{-10i(dx+c)}}{40a^8d}$	56

```
[In] int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^8/d*(I/(tan(d*x+c)-I)^4-4/5/(tan(d*x+c)-I)^5+1/3/(tan(d*x+c)-I)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.51

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\left(10i\,e^{(4i\,dx+4i\,c)}+15i\,e^{(2i\,dx+2i\,c)}+6i\right)e^{(-10i\,dx-10i\,c)}}{240\,a^8d}$$

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/240*(10*I*e^(4*I*d*x + 4*I*c) + 15*I*e^(2*I*d*x + 2*I*c) + 6*I)*e^(-10*I*
d*x - 10*I*c)/(a^8*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 466 vs. 2(65) = 130.

Time = 8.73 (sec) , antiderivative size = 466, normalized size of antiderivative = 5.75

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \begin{cases} -\frac{i\tan^2(c+dx)\sec^6(c+dx)}{240a^8d\tan^8(c+dx)-1920ia^8d\tan^7(c+dx)-6720a^8d\tan^6(c+dx)+13440ia^8d\tan^5(c+dx)+16800a^8d\tan^4(c+dx)-13440ia^8d\tan^3(c+dx)} \\ \frac{x\sec^6(c)}{(ia\tan(c)+a)^8} \end{cases}$$

```
[In] integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise((-I*tan(c + d*x)**2*sec(c + d*x)**6/(240*a**8*d*tan(c + d*x)**8 -
 1920*I*a**8*d*tan(c + d*x)**7 - 6720*a**8*d*tan(c + d*x)**6 + 13440*I*a**8
*d*tan(c + d*x)**5 + 16800*a**8*d*tan(c + d*x)**4 - 13440*I*a**8*d*tan(c +
d*x)**3 - 6720*a**8*d*tan(c + d*x)**2 + 1920*I*a**8*d*tan(c + d*x) + 240*a*
*8*d) - 8*tan(c + d*x)*sec(c + d*x)**6/(240*a**8*d*tan(c + d*x)**8 - 1920*I
*a**8*d*tan(c + d*x)**7 - 6720*a**8*d*tan(c + d*x)**6 + 13440*I*a**8*d*tan(
c + d*x)**5 + 16800*a**8*d*tan(c + d*x)**4 - 13440*I*a**8*d*tan(c + d*x)**3
 - 6720*a**8*d*tan(c + d*x)**2 + 1920*I*a**8*d*tan(c + d*x) + 240*a**8*d) +
 31*I*sec(c + d*x)**6/(240*a**8*d*tan(c + d*x)**8 - 1920*I*a**8*d*tan(c + d
*x)**7 - 6720*a**8*d*tan(c + d*x)**6 + 13440*I*a**8*d*tan(c + d*x)**5 + 168
00*a**8*d*tan(c + d*x)**4 - 13440*I*a**8*d*tan(c + d*x)**3 - 6720*a**8*d*ta
n(c + d*x)**2 + 1920*I*a**8*d*tan(c + d*x) + 240*a**8*d), Ne(d, 0)), (x*sec
(c)**6/(I*a*tan(c) + a)**8, True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(65) = 130$.

Time = 0.20 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.74

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^8}\,dx =$$

$$-\frac{5\,\tan\left(dx+c\right)^4 - 5i\,\tan\left(dx+c\right)^3 + 3\,\tan\left(dx+c\right)^2 - i\,\tan\left(d\right.}{15\,\left(a^8\tan\left(dx+c\right)^7 - 7i\,a^8\tan\left(dx+c\right)^6 - 21\,a^8\tan\left(dx+c\right)^5 + 35i\,a^8\tan\left(dx+c\right)^4 + 35\,a^8\tan\left(dx+c\right.\right.}$$

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/15*(5*tan(d*x + c)^4 - 5*I*tan(d*x + c)^3 + 3*tan(d*x + c)^2 - I*tan(d*x
 + c) + 2)/((a^8*tan(d*x + c)^7 - 7*I*a^8*tan(d*x + c)^6 - 21*a^8*tan(d*x +
 c)^5 + 35*I*a^8*tan(d*x + c)^4 + 35*a^8*tan(d*x + c)^3 - 21*I*a^8*tan(d*x
+ c)^2 - 7*a^8*tan(d*x + c) + I*a^8)*d)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(65) = 130$.

Time = 1.50 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.69

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^8}\,dx =$$

$$-\frac{2\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 - 30i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8 - 140\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 170i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6 + 282\,\mathrm{t}\right.}{15\,a^8d\left(\mathrm{tan}\right.}$$

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -2/15*(15*tan(1/2*d*x + 1/2*c)^9 - 30*I*tan(1/2*d*x + 1/2*c)^8 - 140*tan(1/
2*d*x + 1/2*c)^7 + 170*I*tan(1/2*d*x + 1/2*c)^6 + 282*tan(1/2*d*x + 1/2*c)^
5 - 170*I*tan(1/2*d*x + 1/2*c)^4 - 140*tan(1/2*d*x + 1/2*c)^3 + 30*I*tan(1/
2*d*x + 1/2*c)^2 + 15*tan(1/2*d*x + 1/2*c))/(a^8*d*(tan(1/2*d*x + 1/2*c) -
I)^10)
```

Mupad [B] (verification not implemented)

Time = 4.23 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$=\frac{-\tan(c+d\,x)^2\,5\mathrm{i}+5\tan(c+d\,x)+2\mathrm{i}}{15\,a^8\,d\,\left(\tan\left(c+d\,x\right)^5\,1\mathrm{i}+5\tan\left(c+d\,x\right)^4-\tan\left(c+d\,x\right)^3\,10\mathrm{i}-10\tan\left(c+d\,x\right)^2+\tan\left(c+d\,x\right)\,5\mathrm{i}+1\right)}$$

```
[In] int(1/(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^8),x)
```

```
[Out] (5*tan(c + d*x) - tan(c + d*x)^2*5i + 2i)/(15*a^8*d*(tan(c + d*x)*5i - 10*t
an(c + d*x)^2 - tan(c + d*x)^3*10i + 5*tan(c + d*x)^4 + tan(c + d*x)^5*1i +
 1))
```

3.171 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 55

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{i}{3a^2d(a+ia\tan(c+dx))^6}-\frac{i}{5a^3d(a+ia\tan(c+dx))^5}$$

```
[Out] 1/3*I/a^2/d/(a+I*a*tan(d*x+c))^6-1/5*I/a^3/d/(a+I*a*tan(d*x+c))^5
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{i}{3a^2d(a+ia\tan(c+dx))^6}-\frac{i}{5a^3d(a+ia\tan(c+dx))^5}$$

```
[In] Int[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (I/3)/(a^2*d*(a + I*a*Tan[c + d*x])^6) - (I/5)/(a^3*d*(a + I*a*Tan[c + d*x]
)^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int \frac{a-x}{(a+x)^7}\,dx, x, ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{i\text{Subst}\left(\int \left(\frac{2a}{(a+x)^7}-\frac{1}{(a+x)^6}\right)\,dx, x, ia\tan(c+dx)\right)}{a^3d}\\ &= \frac{i}{3a^2d(a+ia\tan(c+dx))^6}-\frac{i}{5a^3d(a+ia\tan(c+dx))^5}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.62

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{2i+3\tan(c+dx)}{15a^8d(-i+\tan(c+dx))^6}$$

```
[In] Integrate[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^8,x]
[Out] -1/15*(2*I + 3*Tan[c + d*x])/(a^8*d*(-I + Tan[c + d*x])^6)
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

method	result	size
derivativedivides	$\frac{-\frac{i}{3(\tan(dx+c)-i)^6}-\frac{1}{5(\tan(dx+c)-i)^5}}{a^8d}$	36
default	$\frac{-\frac{i}{3(\tan(dx+c)-i)^6}-\frac{1}{5(\tan(dx+c)-i)^5}}{a^8d}$	36
risch	$\frac{i\mathrm{e}^{-4i(dx+c)}}{64a^8d}+\frac{i\mathrm{e}^{-6i(dx+c)}}{24a^8d}+\frac{3i\mathrm{e}^{-8i(dx+c)}}{64a^8d}+\frac{i\mathrm{e}^{-10i(dx+c)}}{40a^8d}+\frac{i\mathrm{e}^{-12i(dx+c)}}{192a^8d}$	92

```
[In] int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
[Out] 1/a^8/d*(-1/3*I/(tan(d*x+c)-I)^6-1/5/(tan(d*x+c)-I)^5)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\left(15i\,e^{(8i\,dx+8i\,c)}+40i\,e^{(6i\,dx+6i\,c)}+45i\,e^{(4i\,dx+4i\,c)}+24i\,e^{(2i\,dx+2i\,c)}+5i\right)e^{(-12i\,dx-12i\,c)}}{960\,a^8d}$$

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/960*(15*I*e^(8*I*d*x + 8*I*c) + 40*I*e^(6*I*d*x + 6*I*c) + 45*I*e^(4*I*d*
x + 4*I*c) + 24*I*e^(2*I*d*x + 2*I*c) + 5*I)*e^(-12*I*d*x - 12*I*c)/(a^8*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 774 vs. 2(42) = 84.

Time = 8.87 (sec) , antiderivative size = 774, normalized size of antiderivative = 14.07

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \begin{cases} \frac{i\tan^4(c+dx)\sec^4(c+dx)}{960a^8d\tan^8(c+dx)-7680ia^8d\tan^7(c+dx)-26880a^8d\tan^6(c+dx)+53760ia^8d\tan^5(c+dx)+67200a^8d\tan^4(c+dx)-53760ia^8d\tan^3(c+dx)-26} \\ \frac{x\sec^4(c)}{(ia\tan(c)+a)^8} \end{cases}$$

```
[In] integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise((I*tan(c + d*x)**4*sec(c + d*x)**4/(960*a**8*d*tan(c + d*x)**8 -
7680*I*a**8*d*tan(c + d*x)**7 - 26880*a**8*d*tan(c + d*x)**6 + 53760*I*a**8
*d*tan(c + d*x)**5 + 67200*a**8*d*tan(c + d*x)**4 - 53760*I*a**8*d*tan(c +
d*x)**3 - 26880*a**8*d*tan(c + d*x)**2 + 7680*I*a**8*d*tan(c + d*x) + 960*a
**8*d) + 8*tan(c + d*x)**3*sec(c + d*x)**4/(960*a**8*d*tan(c + d*x)**8 - 76
80*I*a**8*d*tan(c + d*x)**7 - 26880*a**8*d*tan(c + d*x)**6 + 53760*I*a**8*d
*tan(c + d*x)**5 + 67200*a**8*d*tan(c + d*x)**4 - 53760*I*a**8*d*tan(c + d*
x)**3 - 26880*a**8*d*tan(c + d*x)**2 + 7680*I*a**8*d*tan(c + d*x) + 960*a**
8*d) - 30*I*tan(c + d*x)**2*sec(c + d*x)**4/(960*a**8*d*tan(c + d*x)**8 - 7
680*I*a**8*d*tan(c + d*x)**7 - 26880*a**8*d*tan(c + d*x)**6 + 53760*I*a**8*
d*tan(c + d*x)**5 + 67200*a**8*d*tan(c + d*x)**4 - 53760*I*a**8*d*tan(c + d
*x)**3 - 26880*a**8*d*tan(c + d*x)**2 + 7680*I*a**8*d*tan(c + d*x) + 960*a*
*8*d) - 72*tan(c + d*x)*sec(c + d*x)**4/(960*a**8*d*tan(c + d*x)**8 - 7680*
I*a**8*d*tan(c + d*x)**7 - 26880*a**8*d*tan(c + d*x)**6 + 53760*I*a**8*d*ta
n(c + d*x)**5 + 67200*a**8*d*tan(c + d*x)**4 - 53760*I*a**8*d*tan(c + d*x)*
```

```
*3 - 26880*a**8*d*tan(c + d*x)**2 + 7680*I*a**8*d*tan(c + d*x) + 960*a**8*d
) + 129*I*sec(c + d*x)**4/(960*a**8*d*tan(c + d*x)**8 - 7680*I*a**8*d*tan(c
 + d*x)**7 - 26880*a**8*d*tan(c + d*x)**6 + 53760*I*a**8*d*tan(c + d*x)**5
+ 67200*a**8*d*tan(c + d*x)**4 - 53760*I*a**8*d*tan(c + d*x)**3 - 26880*a**
8*d*tan(c + d*x)**2 + 7680*I*a**8*d*tan(c + d*x) + 960*a**8*d), Ne(d, 0)),
(x*sec(c)**4/(I*a*tan(c) + a)**8, True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(43) = 86.

Time = 0.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.20

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{3\tan(dx+c)^2 - i\tan(dx+c) + 2}{15\left(a^8\tan(dx+c)^7 - 7i\,a^8\tan(dx+c)^6 - 21\,a^8\tan(dx+c)^5 + 35i\,a^8\tan(dx+c)^4 + 35\,a^8\tan(dx+$$

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -1/15*(3*tan(d*x + c)^2 - I*tan(d*x + c) + 2)/((a^8*tan(d*x + c)^7 - 7*I*a^
8*tan(d*x + c)^6 - 21*a^8*tan(d*x + c)^5 + 35*I*a^8*tan(d*x + c)^4 + 35*a^8
*tan(d*x + c)^3 - 21*I*a^8*tan(d*x + c)^2 - 7*a^8*tan(d*x + c) + I*a^8)*d)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(43) = 86.

Time = 1.31 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.96

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{2\left(15\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11} - 60i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10} - 235\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9} + 480i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8} + 82$$

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -2/15*(15*tan(1/2*d*x + 1/2*c)^11 - 60*I*tan(1/2*d*x + 1/2*c)^10 - 235*tan(
1/2*d*x + 1/2*c)^9 + 480*I*tan(1/2*d*x + 1/2*c)^8 + 822*tan(1/2*d*x + 1/2*c
)^7 - 904*I*tan(1/2*d*x + 1/2*c)^6 - 822*tan(1/2*d*x + 1/2*c)^5 + 480*I*tan
(1/2*d*x + 1/2*c)^4 + 235*tan(1/2*d*x + 1/2*c)^3 - 60*I*tan(1/2*d*x + 1/2*c
)^2 - 15*tan(1/2*d*x + 1/2*c))/(a^8*d*(tan(1/2*d*x + 1/2*c) - I)^12)
```

Mupad [B] (verification not implemented)

Time = 4.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.55

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx =$$

$$-\frac{-2+\tan(c+d\,x)\,3\mathrm{i}}{15\,a^8\,d\,\left(\tan\left(c+d\,x\right)^6\,1\mathrm{i}+6\tan\left(c+d\,x\right)^5-\tan\left(c+d\,x\right)^4\,15\mathrm{i}-20\tan\left(c+d\,x\right)^3+\tan\left(c+d\,x\right)^2\,15\mathrm{i}+\right.}$$

```
[In] int(1/(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^8),x)
```

```
[Out] -(tan(c + d*x)*3i - 2)/(15*a^8*d*(6*tan(c + d*x) + tan(c + d*x)^2*15i - 20*
tan(c + d*x)^3 - tan(c + d*x)^4*15i + 6*tan(c + d*x)^5 + tan(c + d*x)^6*1i
- 1i))
```

3.172 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 27

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{i}{7ad(a+ia\tan(c+dx))^7}$$

```
[Out] 1/7*I/a/d/(a+I*a*tan(d*x+c))^7
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 32}

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{i}{7ad(a+ia\tan(c+dx))^7}$$

```
[In] Int[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (I/7)/(a*d*(a + I*a*Tan[c + d*x])^7)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
```

```
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int \frac{1}{(a+x)^8}\,dx, x, ia\tan(c+dx)\right)}{ad}\\ &= \frac{i}{7ad(a+ia\tan(c+dx))^7}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{1}{7a^8d(-i+\tan(c+dx))^7}$$

```
[In] Integrate[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^8,x]
[Out] -1/7*1/(a^8*d*(-I + Tan[c + d*x])^7)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{i}{7ad(a+ia\tan(dx+c))^7}$
default	$\frac{i}{7ad(a+ia\tan(dx+c))^7}$
risch	$\frac{i\mathrm{e}^{-2i(dx+c)}}{128a^8d}+\frac{3i\mathrm{e}^{-4i(dx+c)}}{128a^8d}+\frac{5i\mathrm{e}^{-6i(dx+c)}}{128a^8d}+\frac{5i\mathrm{e}^{-8i(dx+c)}}{128a^8d}+\frac{3i\mathrm{e}^{-10i(dx+c)}}{128a^8d}+\frac{i\mathrm{e}^{-12i(dx+c)}}{128a^8d}+\frac{i\mathrm{e}^{-14i(dx+c)}}{896a^8d}$

```
[In] int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
[Out] 1/7*I/a/d/(a+I*a*tan(d*x+c))^7
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(21) = 42$.

Time = 0.23 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.15

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\left(7i\,e^{(12i\,dx+12i\,c)}+21i\,e^{(10i\,dx+10i\,c)}+35i\,e^{(8i\,dx+8i\,c)}+35i\,e^{(6i\,dx+6i\,c)}+21i\,e^{(4i\,dx+4i\,c)}+7i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-}}{896\,a^8d}$$

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/896*(7*I*e^(12*I*d*x + 12*I*c) + 21*I*e^(10*I*d*x + 10*I*c) + 35*I*e^(8*I
*d*x + 8*I*c) + 35*I*e^(6*I*d*x + 6*I*c) + 21*I*e^(4*I*d*x + 4*I*c) + 7*I*e
^(2*I*d*x + 2*I*c) + I)*e^(-14*I*d*x - 14*I*c)/(a^8*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1081 vs. $2(19) = 38$.

Time = 8.89 (sec) , antiderivative size = 1081, normalized size of antiderivative = 40.04

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise((-I*tan(c + d*x)**6*sec(c + d*x)**2/(896*a**8*d*tan(c + d*x)**8 -
 7168*I*a**8*d*tan(c + d*x)**7 - 25088*a**8*d*tan(c + d*x)**6 + 50176*I*a**
8*d*tan(c + d*x)**5 + 62720*a**8*d*tan(c + d*x)**4 - 50176*I*a**8*d*tan(c +
 d*x)**3 - 25088*a**8*d*tan(c + d*x)**2 + 7168*I*a**8*d*tan(c + d*x) + 896*
a**8*d) - 8*tan(c + d*x)**5*sec(c + d*x)**2/(896*a**8*d*tan(c + d*x)**8 - 7
168*I*a**8*d*tan(c + d*x)**7 - 25088*a**8*d*tan(c + d*x)**6 + 50176*I*a**8*
d*tan(c + d*x)**5 + 62720*a**8*d*tan(c + d*x)**4 - 50176*I*a**8*d*tan(c + d
*x)**3 - 25088*a**8*d*tan(c + d*x)**2 + 7168*I*a**8*d*tan(c + d*x) + 896*a*
*8*d) + 29*I*tan(c + d*x)**4*sec(c + d*x)**2/(896*a**8*d*tan(c + d*x)**8 -
7168*I*a**8*d*tan(c + d*x)**7 - 25088*a**8*d*tan(c + d*x)**6 + 50176*I*a**8
*d*tan(c + d*x)**5 + 62720*a**8*d*tan(c + d*x)**4 - 50176*I*a**8*d*tan(c +
d*x)**3 - 25088*a**8*d*tan(c + d*x)**2 + 7168*I*a**8*d*tan(c + d*x) + 896*a
**8*d) + 64*tan(c + d*x)**3*sec(c + d*x)**2/(896*a**8*d*tan(c + d*x)**8 - 7
168*I*a**8*d*tan(c + d*x)**7 - 25088*a**8*d*tan(c + d*x)**6 + 50176*I*a**8*
d*tan(c + d*x)**5 + 62720*a**8*d*tan(c + d*x)**4 - 50176*I*a**8*d*tan(c + d
*x)**3 - 25088*a**8*d*tan(c + d*x)**2 + 7168*I*a**8*d*tan(c + d*x) + 896*a*
```

```
*8*d) - 99*I*tan(c + d*x)**2*sec(c + d*x)**2/(896*a**8*d*tan(c + d*x)**8 -
7168*I*a**8*d*tan(c + d*x)**7 - 25088*a**8*d*tan(c + d*x)**6 + 50176*I*a**8
*d*tan(c + d*x)**5 + 62720*a**8*d*tan(c + d*x)**4 - 50176*I*a**8*d*tan(c +
d*x)**3 - 25088*a**8*d*tan(c + d*x)**2 + 7168*I*a**8*d*tan(c + d*x) + 896*a
**8*d) - 120*tan(c + d*x)*sec(c + d*x)**2/(896*a**8*d*tan(c + d*x)**8 - 716
8*I*a**8*d*tan(c + d*x)**7 - 25088*a**8*d*tan(c + d*x)**6 + 50176*I*a**8*d*
tan(c + d*x)**5 + 62720*a**8*d*tan(c + d*x)**4 - 50176*I*a**8*d*tan(c + d*x
)**3 - 25088*a**8*d*tan(c + d*x)**2 + 7168*I*a**8*d*tan(c + d*x) + 896*a**8
*d) + 127*I*sec(c + d*x)**2/(896*a**8*d*tan(c + d*x)**8 - 7168*I*a**8*d*tan
(c + d*x)**7 - 25088*a**8*d*tan(c + d*x)**6 + 50176*I*a**8*d*tan(c + d*x)**
5 + 62720*a**8*d*tan(c + d*x)**4 - 50176*I*a**8*d*tan(c + d*x)**3 - 25088*a
**8*d*tan(c + d*x)**2 + 7168*I*a**8*d*tan(c + d*x) + 896*a**8*d), Ne(d, 0))
, (x*sec(c)**2/(I*a*tan(c) + a)**8, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{i}{7\,(i\,a\tan{(dx+c)}+a)^7ad}$$

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/7*I/((I*a*tan(d*x + c) + a)^7*a*d)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(21) = 42.

Time = 1.21 (sec) , antiderivative size = 189, normalized size of antiderivative = 7.00

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{2\left(7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}-42i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}-182\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+490i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+100\right.}{}$$

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -2/7*(7*tan(1/2*d*x + 1/2*c)^13 - 42*I*tan(1/2*d*x + 1/2*c)^12 - 182*tan(1/
2*d*x + 1/2*c)^11 + 490*I*tan(1/2*d*x + 1/2*c)^10 + 1001*tan(1/2*d*x + 1/2*
c)^9 - 1484*I*tan(1/2*d*x + 1/2*c)^8 - 1716*tan(1/2*d*x + 1/2*c)^7 + 1484*I
*tan(1/2*d*x + 1/2*c)^6 + 1001*tan(1/2*d*x + 1/2*c)^5 - 490*I*tan(1/2*d*x +
 1/2*c)^4 - 182*tan(1/2*d*x + 1/2*c)^3 + 42*I*tan(1/2*d*x + 1/2*c)^2 + 7*ta
n(1/2*d*x + 1/2*c))/(a^8*d*(tan(1/2*d*x + 1/2*c) - I)^14)
```

Mupad [B] (verification not implemented)

Time = 4.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx=-\frac{1}{7\,a^8\,d\,(\tan\left(c+d\,x\right)-\mathrm{i})^7}$$

```
[In] int(1/(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^8),x)
[Out] -1/(7*a^8*d*(tan(c + d*x) - 1i)^7)
```

3.173 $\int \frac{1}{(a+ia\tan(c+dx))^8}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 229

$$\begin{aligned}\int \frac{1}{(a+ia\tan(c+dx))^8}\,dx =& \frac{x}{256a^8}+\frac{i}{16d(a+ia\tan(c+dx))^8}\\ &+\frac{i}{28ad(a+ia\tan(c+dx))^7}+\frac{i}{48a^2d(a+ia\tan(c+dx))^6}\\ &+\frac{i}{80a^3d(a+ia\tan(c+dx))^5}+\frac{i}{128d\left(a^2+ia^2\tan(c+dx)\right)^4}\\ &+\frac{i}{192a^2d\left(a^2+ia^2\tan(c+dx)\right)^3}\\ &+\frac{i}{256d\left(a^4+ia^4\tan(c+dx)\right)^2}+\frac{i}{256d\left(a^8+ia^8\tan(c+dx)\right)}\end{aligned}$$

```
[Out] 1/256*x/a^8+1/16*I/d/(a+I*a*tan(d*x+c))^8+1/28*I/a/d/(a+I*a*tan(d*x+c))^7+1
/48*I/a^2/d/(a+I*a*tan(d*x+c))^6+1/80*I/a^3/d/(a+I*a*tan(d*x+c))^5+1/128*I/
d/(a^2+I*a^2*tan(d*x+c))^4+1/192*I/a^2/d/(a^2+I*a^2*tan(d*x+c))^3+1/256*I/d
/(a^4+I*a^4*tan(d*x+c))^2+1/256*I/d/(a^8+I*a^8*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used

$= \{3560, 8\}$

$$\int \frac{1}{(a+ia\tan(c+dx))^8}\,dx = \frac{i}{256d\,(a^8+ia^8\tan(c+dx))}+\frac{x}{256a^8} + \frac{i}{256d\,(a^4+ia^4\tan(c+dx))^2}+\frac{i}{80a^3d(a+ia\tan(c+dx))^5} + \frac{i}{192a^2d\,(a^2+ia^2\tan(c+dx))^3} + \frac{i}{128d\,(a^2+ia^2\tan(c+dx))^4}+\frac{i}{48a^2d(a+ia\tan(c+dx))^6} + \frac{i}{28ad(a+ia\tan(c+dx))^7}+\frac{i}{16d(a+ia\tan(c+dx))^8}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^(-8),x]
```

```
[Out] x/(256*a^8) + (I/16)/(d*(a + I*a*Tan[c + d*x])^8) + (I/28)/(a*d*(a + I*a*Ta
n[c + d*x])^7) + (I/48)/(a^2*d*(a + I*a*Tan[c + d*x])^6) + (I/80)/(a^3*d*(a
 + I*a*Tan[c + d*x])^5) + (I/128)/(d*(a^2 + I*a^2*Tan[c + d*x])^4) + (I/192
)/(a^2*d*(a^2 + I*a^2*Tan[c + d*x])^3) + (I/256)/(d*(a^4 + I*a^4*Tan[c + d*
x])^2) + (I/256)/(d*(a^8 + I*a^8*Tan[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\text{integral} = \frac{i}{16d(a+ia\tan(c+dx))^8}+\frac{\int \frac{1}{(a+ia\tan(c+dx))^7}\,dx}{2a}$$

$$= \frac{i}{16d(a+ia\tan(c+dx))^8}+\frac{i}{28ad(a+ia\tan(c+dx))^7}+\frac{\int \frac{1}{(a+ia\tan(c+dx))^6}\,dx}{4a^2}$$

$$= \frac{i}{16d(a+ia\tan(c+dx))^8}+\frac{i}{28ad(a+ia\tan(c+dx))^7} + \frac{i}{48a^2d(a+ia\tan(c+dx))^6}+\frac{\int \frac{1}{(a+ia\tan(c+dx))^5}\,dx}{8a^3}$$

$$= \frac{i}{16d(a+ia\tan(c+dx))^8}+\frac{i}{28ad(a+ia\tan(c+dx))^7} + \frac{i}{48a^2d(a+ia\tan(c+dx))^6}+\frac{i}{80a^3d(a+ia\tan(c+dx))^5}+\frac{\int \frac{1}{(a+ia\tan(c+dx))^4}\,dx}{16a^4}$$

$$= \frac{i}{16d(a+ia\tan(c+dx))^8} + \frac{i}{28ad(a+ia\tan(c+dx))^7} + \frac{i}{48a^2d(a+ia\tan(c+dx))^6} + \frac{i}{80a^3d(a+ia\tan(c+dx))^5} + \frac{i}{128d\left(a^2+ia^2\tan(c+dx)\right)^4} + \frac{\int \frac{1}{(a+ia\tan(c+dx))^3}\,dx}{32a^5}$$

$$= \frac{i}{16d(a+ia\tan(c+dx))^8} + \frac{i}{28ad(a+ia\tan(c+dx))^7} + \frac{i}{48a^2d(a+ia\tan(c+dx))^6} + \frac{i}{80a^3d(a+ia\tan(c+dx))^5} + \frac{i}{192a^5d(a+ia\tan(c+dx))^3} + \frac{i}{128d\left(a^2+ia^2\tan(c+dx)\right)^4} + \frac{\int \frac{1}{(a+ia\tan(c+dx))^2}\,dx}{64a^6}$$

$$= \frac{i}{16d(a+ia\tan(c+dx))^8} + \frac{i}{28ad(a+ia\tan(c+dx))^7} + \frac{i}{48a^2d(a+ia\tan(c+dx))^6} + \frac{i}{80a^3d(a+ia\tan(c+dx))^5} + \frac{i}{192a^5d(a+ia\tan(c+dx))^3} + \frac{i}{128d\left(a^2+ia^2\tan(c+dx)\right)^4} + \frac{i}{256d\left(a^4+ia^4\tan(c+dx)\right)^2} + \frac{\int \frac{1}{a+ia\tan(c+dx)}\,dx}{128a^7}$$

$$= \frac{i}{16d(a+ia\tan(c+dx))^8} + \frac{i}{28ad(a+ia\tan(c+dx))^7} + \frac{i}{48a^2d(a+ia\tan(c+dx))^6} + \frac{i}{80a^3d(a+ia\tan(c+dx))^5} + \frac{i}{192a^5d(a+ia\tan(c+dx))^3} + \frac{i}{128d\left(a^2+ia^2\tan(c+dx)\right)^4} + \frac{i}{256d\left(a^4+ia^4\tan(c+dx)\right)^2} + \frac{i}{256d\left(a^8+ia^8\tan(c+dx)\right)} + \frac{\int 1\,dx}{256a^8}$$

$$= \frac{x}{256a^8} + \frac{i}{16d(a+ia\tan(c+dx))^8} + \frac{i}{28ad(a+ia\tan(c+dx))^7} + \frac{i}{48a^2d(a+ia\tan(c+dx))^6} + \frac{i}{80a^3d(a+ia\tan(c+dx))^5} + \frac{i}{192a^5d(a+ia\tan(c+dx))^3} + \frac{i}{128d\left(a^2+ia^2\tan(c+dx)\right)^4} + \frac{i}{256d\left(a^4+ia^4\tan(c+dx)\right)^2} + \frac{i}{256d\left(a^8+ia^8\tan(c+dx)\right)}$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.66

$$\int \frac{1}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{i\sec^8(c+dx)(7350+12544\cos(2(c+dx))+7840\cos(4(c+dx))+3840\cos(6(c+dx))+1194\cos(8(c+$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(-8),x]
```

```
[Out] ((I/215040)*Sec[c + d*x]^8*(7350 + 12544*Cos[2*(c + d*x)] + 7840*Cos[4*(c +
 d*x)] + 3840*Cos[6*(c + d*x)] + 1194*Cos[8*(c + d*x)] + (3136*I)*Sin[2*(c
+ d*x)] + (3920*I)*Sin[4*(c + d*x)] + (2880*I)*Sin[6*(c + d*x)] + (1089*I)*
Sin[8*(c + d*x)] + 840*ArcTan[Tan[c + d*x]]*((-I)*Cos[8*(c + d*x)] + Sin[8*
(c + d*x)])))/(a^8*d*(-I + Tan[c + d*x])^8)
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.66

method	result
risch	$\frac{x}{256a^8}+\frac{ie^{-2i(dx+c)}}{64a^8d}+\frac{7ie^{-4i(dx+c)}}{256a^8d}+\frac{7ie^{-6i(dx+c)}}{192a^8d}+\frac{35ie^{-8i(dx+c)}}{1024a^8d}+\frac{7ie^{-10i(dx+c)}}{320a^8d}+\frac{7ie^{-12i(dx+c)}}{768a^8d}+$
derivativedivides	$\frac{\arctan(\tan(dx+c))}{256a^8d}+\frac{i}{16d\,a^8(\tan(dx+c)-i)^8}+\frac{i}{128d\,a^8(\tan(dx+c)-i)^4}-\frac{i}{48d\,a^8(\tan(dx+c)-i)^6}-$
default	$\frac{\arctan(\tan(dx+c))}{256a^8d}+\frac{i}{16d\,a^8(\tan(dx+c)-i)^8}+\frac{i}{128d\,a^8(\tan(dx+c)-i)^4}-\frac{i}{48d\,a^8(\tan(dx+c)-i)^6}-$
norman	$\frac{x}{256a}+\frac{961\left(\tan^7(dx+c)\right)}{8960ad}+\frac{7x\left(\tan^4(dx+c)\right)}{64a}-\frac{1117\left(\tan^3(dx+c)\right)}{256ad}+\frac{x\left(\tan^2(dx+c)\right)}{32a}+\frac{35x\left(\tan^8(dx+c)\right)}{128a}+\frac{7x\left(\tan^{10}(dx+c)\right)}{32a}+$

```
[In] int(1/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/256*x/a^8+1/64*I/a^8/d*exp(-2*I*(d*x+c))+7/256*I/a^8/d*exp(-4*I*(d*x+c))+
7/192*I/a^8/d*exp(-6*I*(d*x+c))+35/1024*I/a^8/d*exp(-8*I*(d*x+c))+7/320*I/a
^8/d*exp(-10*I*(d*x+c))+7/768*I/a^8/d*exp(-12*I*(d*x+c))+1/448*I/a^8/d*exp(
-14*I*(d*x+c))+1/4096*I/a^8/d*exp(-16*I*(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.48

$$\int \frac{1}{(a+ia\tan(c+dx))^8}\,dx = \frac{\left(1680\,dxe^{(16i\,dx+16i\,c)}+6720i\,e^{(14i\,dx+14i\,c)}+11760i\,e^{(12i\,dx+12i\,c)}+15680i\,e^{(10i\,dx+10i\,c)}+14700i\,e^{(8i\,dx+8i\,c)}+\right.}{430080\,a^8d}$$

[In] integrate(1/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")

[Out] 1/430080*(1680*d*x*e^(16*I*d*x + 16*I*c) + 6720*I*e^(14*I*d*x + 14*I*c) + 11760*I*e^(12*I*d*x + 12*I*c) + 15680*I*e^(10*I*d*x + 10*I*c) + 14700*I*e^(8*I*d*x + 8*I*c) + 9408*I*e^(6*I*d*x + 6*I*c) + 3920*I*e^(4*I*d*x + 4*I*c) + 960*I*e^(2*I*d*x + 2*I*c) + 105*I)*e^(-16*I*d*x - 16*I*c)/(a^8*d)

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.42

$$\int \frac{1}{(a+ia\tan(c+dx))^8}\,dx = \begin{cases} \frac{(2269814212194729984 0ia^{56}d^7e^{70ic}e^{-2idx}+39721748713407774720ia^{56}d^7e^{68ic}e^{-4idx}+52962331617877032960ia^{56}d^7e^{66ic}e^{-6idx}+4965218589\cdots}{} \\ x\left(\frac{(e^{16ic}+8e^{14ic}+28e^{12ic}+56e^{10ic}+70e^{8ic}+56e^{6ic}+28e^{4ic}+8e^{2ic}+1)e^{-16ic}}{256a^8}-\frac{1}{256a^8}\right) \end{cases} + \frac{x}{256a^8}$$

[In] integrate(1/(a+I*a*tan(d*x+c))**8,x)

[Out] Piecewise(((22698142121947299840*I*a**56*d**7*exp(70*I*c)*exp(-2*I*d*x) + 39721748713407774720*I*a**56*d**7*exp(68*I*c)*exp(-4*I*d*x) + 52962331617877032960*I*a**56*d**7*exp(66*I*c)*exp(-6*I*d*x) + 49652185891759718400*I*a**56*d**7*exp(64*I*c)*exp(-8*I*d*x) + 31777398970726219776*I*a**56*d**7*exp(62*I*c)*exp(-10*I*d*x) + 13240582904469258240*I*a**56*d**7*exp(60*I*c)*exp(-12*I*d*x) + 3242591731706757120*I*a**56*d**7*exp(58*I*c)*exp(-14*I*d*x) + 354658470655426560*I*a**56*d**7*exp(56*I*c)*exp(-16*I*d*x))*exp(-72*I*c)/(145268109580462718976 0*a**64*d**8), Ne(a**64*d**8*exp(72*I*c), 0)), (x*((exp(16*I*c) + 8*exp(14*I*c) + 28*exp(12*I*c) + 56*exp(10*I*c) + 70*exp(8*I*c) + 56*exp(6*I*c) + 28*exp(4*I*c) + 8*exp(2*I*c) + 1)*exp(-16*I*c)/(256*a**8) - 1/(256*a**8)), True)) + x/(256*a**8)

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+ia\tan(c+dx))^8}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [A] (verification not implemented)

none

Time = 0.68 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.56

$$\int \frac{1}{(a+ia\tan(c+dx))^8}\,dx =$$
$$-\frac{-\frac{840i\log(\tan(dx+c)+i)}{a^8}+\frac{840i\log(\tan(dx+c)-i)}{a^8}+\frac{-2283i\tan(dx+c)^8-19944\tan(dx+c)^7+77364i\tan(dx+c)^6+175448\tan(dx+c)^5}{a^8(}}{430080\,d}$$

```
[In] integrate(1/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/430080*(-840*I*log(tan(d*x + c) + I)/a^8 + 840*I*log(tan(d*x + c) - I)/a
^8 + (-2283*I*tan(d*x + c)^8 - 19944*tan(d*x + c)^7 + 77364*I*tan(d*x + c)^
6 + 175448*tan(d*x + c)^5 - 258370*I*tan(d*x + c)^4 - 261464*tan(d*x + c)^3
 + 192052*I*tan(d*x + c)^2 + 114152*tan(d*x + c) - 67819*I)/(a^8*(tan(d*x +
 c) - I)^8))/d
```

Mupad [B] (verification not implemented)

Time = 6.01 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+ia\tan(c+dx))^8}\,dx = \frac{x}{256\,a^8}$$
$$-\frac{\frac{\tan(c+d\,x)\,5993\mathrm{i}}{26880\,a^8}+\frac{16}{105\,a^8}-\frac{143\tan(c+d\,x)^2}{480\,a^8}-\frac{\tan(c+d\,x)^3\,1193\mathrm{i}}{3840\,a^8}+\frac{11\tan(c+d\,x)^4}{48\,a^8}+\frac{\tan(c+d\,x)}{768\,a^8}}{d\,\left(\tan(c+d\,x)^8\,1\mathrm{i}+8\tan(c+d\,x)^7-\tan(c+d\,x)^6\,28\mathrm{i}-56\tan(c+d\,x)^5+\tan(c+d\,x)^4\,70\mathrm{i}+56\,\mathrm{t}\right.}$$

```
[In] int(1/(a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] x/(256*a^8) - ((tan(c + d*x)*5993i)/(26880*a^8) + 16/(105*a^8) - (143*tan(c
 + d*x)^2)/(480*a^8) - (tan(c + d*x)^3*1193i)/(3840*a^8) + (11*tan(c + d*x)
^4)/(48*a^8) + (tan(c + d*x)^5*85i)/(768*a^8) - tan(c + d*x)^6/(32*a^8) - (
tan(c + d*x)^7*1i)/(256*a^8))/(d*(56*tan(c + d*x)^3 - tan(c + d*x)^2*28i -
8*tan(c + d*x) + tan(c + d*x)^4*70i - 56*tan(c + d*x)^5 - tan(c + d*x)^6*28
i + 8*tan(c + d*x)^7 + tan(c + d*x)^8*1i + 1i))
```

3.174 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 278

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx &= \frac{5x}{512a^8}+\frac{ia}{36d(a+ia\tan(c+dx))^9}+\frac{i}{32d(a+ia\tan(c+dx))^8}\\ &\quad+\frac{3i}{112ad(a+ia\tan(c+dx))^7}+\frac{i}{48a^2d(a+ia\tan(c+dx))^6}\\ &\quad+\frac{i}{64a^3d(a+ia\tan(c+dx))^5}+\frac{7i}{768a^5d(a+ia\tan(c+dx))^3}\\ &\quad+\frac{3i}{256d\,(a^2+ia^2\tan(c+dx))^4}+\frac{i}{128d\,(a^4+ia^4\tan(c+dx))^2}\\ &\quad-\frac{i}{1024d\,(a^8-ia^8\tan(c+dx))}+\frac{9i}{1024d\,(a^8+ia^8\tan(c+dx))}\end{aligned}$$

```
[Out] 5/512*x/a^8+1/36*I*a/d/(a+I*a*tan(d*x+c))^9+1/32*I/d/(a+I*a*tan(d*x+c))^8+3
/112*I/a/d/(a+I*a*tan(d*x+c))^7+1/48*I/a^2/d/(a+I*a*tan(d*x+c))^6+1/64*I/a^
3/d/(a+I*a*tan(d*x+c))^5+7/768*I/a^5/d/(a+I*a*tan(d*x+c))^3+3/256*I/d/(a^2+
I*a^2*tan(d*x+c))^4+1/128*I/d/(a^4+I*a^4*tan(d*x+c))^2-1/1024*I/d/(a^8-I*a^
8*tan(d*x+c))+9/1024*I/d/(a^8+I*a^8*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used

$= \{3568, 46, 212\}$

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{i}{1024d\,(a^8-ia^8\tan(c+dx))}+\frac{9i}{1024d\,(a^8+ia^8\tan(c+dx))}+\frac{5x}{512a^8}+\frac{7i}{768a^5d(a+ia\tan(c+dx))^3}+\frac{i}{128d\,(a^4+ia^4\tan(c+dx))^2}+\frac{i}{64a^3d(a+ia\tan(c+dx))^5}+\frac{3i}{256d\,(a^2+ia^2\tan(c+dx))^4}+\frac{i}{48a^2d(a+ia\tan(c+dx))^6}+\frac{ia}{36d(a+ia\tan(c+dx))^9}+\frac{i}{32d(a+ia\tan(c+dx))^8}+\frac{3i}{112ad(a+ia\tan(c+dx))^7}$$

```
[In] Int[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (5*x)/(512*a^8) + ((I/36)*a)/(d*(a + I*a*Tan[c + d*x])^9) + (I/32)/(d*(a +
I*a*Tan[c + d*x])^8) + ((3*I)/112)/(a*d*(a + I*a*Tan[c + d*x])^7) + (I/48)/
(a^2*d*(a + I*a*Tan[c + d*x])^6) + (I/64)/(a^3*d*(a + I*a*Tan[c + d*x])^5)
+ ((7*I)/768)/(a^5*d*(a + I*a*Tan[c + d*x])^3) + ((3*I)/256)/(d*(a^2 + I*a^
2*Tan[c + d*x])^4) + (I/128)/(d*(a^4 + I*a^4*Tan[c + d*x])^2) - (I/1024)/(d
*(a^8 - I*a^8*Tan[c + d*x])) + ((9*I)/1024)/(d*(a^8 + I*a^8*Tan[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{(ia^3)\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{10}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^3)\,\text{Subst}\left(\int \left(\frac{1}{1024a^{10}(a-x)^2} + \frac{1}{4a^2(a+x)^{10}} + \frac{1}{4a^3(a+x)^9} + \frac{3}{16a^4(a+x)^8} + \frac{1}{8a^5(a+x)^7} + \frac{5}{64a^6(a+x)^6} + \frac{3}{64a^7(a+\cdots}{d}$$

$$= \frac{ia}{36d(a+ia\tan(c+dx))^9} + \frac{i}{32d(a+ia\tan(c+dx))^8} + \frac{3i}{112ad(a+ia\tan(c+dx))^7} + \frac{i}{48a^2d(a+ia\tan(c+dx))^6} + \frac{i}{64a^3d(a+ia\tan(c+dx))^5} + \frac{7i}{768a^5d(a+ia\tan(c+dx))^3} + \frac{3i}{256d\,(a^2+ia^2\tan(c+dx))^4} + \frac{i}{128d\,(a^4+ia^4\tan(c+dx))^2} - \frac{i}{1024d\,(a^8-ia^8\tan(c+dx))} + \frac{9i}{1024d\,(a^8+ia^8\tan(c+dx))} - \frac{(5i)\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, ia\tan(c+dx)\right)}{512a^7d}$$

$$= \frac{5x}{512a^8} + \frac{ia}{36d(a+ia\tan(c+dx))^9} + \frac{i}{32d(a+ia\tan(c+dx))^8} + \frac{3i}{112ad(a+ia\tan(c+dx))^7} + \frac{i}{48a^2d(a+ia\tan(c+dx))^6} + \frac{i}{64a^3d(a+ia\tan(c+dx))^5} + \frac{7i}{768a^5d(a+ia\tan(c+dx))^3} + \frac{3i}{256d\,(a^2+ia^2\tan(c+dx))^4} + \frac{i}{128d\,(a^4+ia^4\tan(c+dx))^2} - \frac{i}{1024d\,(a^8-ia^8\tan(c+dx))} + \frac{9i}{1024d\,(a^8+ia^8\tan(c+dx))}$$

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.68

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\sec^{10}(c+dx)(2520\arctan(\tan(c+dx))(\cos(8(c+dx))+i\sin(8(c+dx)))+i(7938+14112\cos(2(c+dx)\cdots}{\cdots}$$

```
[In] Integrate[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (Sec[c + d*x]^10*(2520*ArcTan[Tan[c + d*x]]*(Cos[8*(c + d*x)] + I*Sin[8*(c
+ d*x)]) + I*(7938 + 14112*Cos[2*(c + d*x)] + 10080*Cos[4*(c + d*x)] + 6480
```

```
*Cos[6*(c + d*x)] + 2462*Cos[8*(c + d*x)] - 112*Cos[10*(c + d*x)] + (3528*I
)*Sin[2*(c + d*x)] + (5040*I)*Sin[4*(c + d*x)] + (4860*I)*Sin[6*(c + d*x)]
+ (2147*I)*Sin[8*(c + d*x)] - (140*I)*Sin[10*(c + d*x)])))/(258048*a^8*d*(-
I + Tan[c + d*x])^9*(I + Tan[c + d*x]))
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.61

method	result
derivativedivides	$\frac{-\frac{5i\ln(\tan(dx+c)-i)}{1024}+\frac{3i}{256(\tan(dx+c)-i)^4}+\frac{i}{32(\tan(dx+c)-i)^8}-\frac{i}{48(\tan(dx+c)-i)^6}-\frac{i}{128(\tan(dx+c)-i)^2}+\frac{1}{36(\tan(dx+c)-i)^9}-}{da}$
default	$\frac{-\frac{5i\ln(\tan(dx+c)-i)}{1024}+\frac{3i}{256(\tan(dx+c)-i)^4}+\frac{i}{32(\tan(dx+c)-i)^8}-\frac{i}{48(\tan(dx+c)-i)^6}-\frac{i}{128(\tan(dx+c)-i)^2}+\frac{1}{36(\tan(dx+c)-i)^9}-}{da}$
risch	$\frac{5x}{512a^8}+\frac{15ie^{-4i(dx+c)}}{512a^8d}+\frac{35ie^{-6i(dx+c)}}{1024a^8d}+\frac{63ie^{-8i(dx+c)}}{2048a^8d}+\frac{21ie^{-10i(dx+c)}}{1024a^8d}+\frac{5ie^{-12i(dx+c)}}{512a^8d}+\frac{45ie^{-14i(dx+c}}{14336a^8d}$

```
[In] int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^8*(-5/1024*I*ln(tan(d*x+c)-I)+3/256*I/(tan(d*x+c)-I)^4+1/32*I/(tan(d*
x+c)-I)^8-1/48*I/(tan(d*x+c)-I)^6-1/128*I/(tan(d*x+c)-I)^2+1/36/(tan(d*x+c)
-I)^9-3/112/(tan(d*x+c)-I)^7+1/64/(tan(d*x+c)-I)^5-7/768/(tan(d*x+c)-I)^3+9
/1024/(tan(d*x+c)-I)+5/1024*I*ln(tan(d*x+c)+I)+1/1024/(tan(d*x+c)+I))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.47

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\left(5040\,dxe^{(18i\,dx+18i\,c)}-252i\,e^{(20i\,dx+20i\,c)}+11340i\,e^{(16i\,dx+16i\,c)}+15120i\,e^{(14i\,dx+14i\,c)}+17640i\,e^{(12i\,dx+12i\,c)}\right.}{}$$

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/516096*(5040*d*x*e^(18*I*d*x + 18*I*c) - 252*I*e^(20*I*d*x + 20*I*c) + 11
340*I*e^(16*I*d*x + 16*I*c) + 15120*I*e^(14*I*d*x + 14*I*c) + 17640*I*e^(12
*I*d*x + 12*I*c) + 15876*I*e^(10*I*d*x + 10*I*c) + 10584*I*e^(8*I*d*x + 8*I
*c) + 5040*I*e^(6*I*d*x + 6*I*c) + 1620*I*e^(4*I*d*x + 4*I*c) + 315*I*e^(2*
I*d*x + 2*I*c) + 28*I)*e^(-18*I*d*x - 18*I*c)/(a^8*d)
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.42

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \begin{cases} \frac{(-2495687119199326634196634435584ia^{72}d^9e^{92ic}e^{2idx}+1123059203639696985388485496012 80ia^{72}d^9e^{88ic}e^{-2idx}+1497412271519595980517 9\cdots}{} \\ x\left(\frac{(e^{20ic}+10e^{18ic}+45e^{16ic}+120e^{14ic}+210e^{12ic}+252e^{10ic}+210e^{8ic}+120e^{6ic}+45e^{4ic}+10e^{2ic}+1)e^{-18ic}}{1024a^8}-\frac{5}{512a^8}\right) \end{cases}$$
$$+\frac{5x}{512a^8}$$

```
[In] integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise(((-2495687119199326634196634435584*I*a**72*d**9*exp(92*I*c)*exp(2
*I*d*x) + 112305920363969698538848549601280*I*a**72*d**9*exp(88*I*c)*exp(-2
*I*d*x) + 149741227151959598051798066135040*I*a**72*d**9*exp(86*I*c)*exp(-4
*I*d*x) + 174698098343952864393764410490880*I*a**72*d**9*exp(84*I*c)*exp(-6
*I*d*x) + 157228288509557577954387969441792*I*a**72*d**9*exp(82*I*c)*exp(-8
*I*d*x) + 104818859006371718636258646294528*I*a**72*d**9*exp(80*I*c)*exp(-1
0*I*d*x) + 49913742383986532683932688711680*I*a**72*d**9*exp(78*I*c)*exp(-1
2*I*d*x) + 16043702909138528362692649943040*I*a**72*d**9*exp(76*I*c)*exp(-1
4*I*d*x) + 3119608898999158292745793044480*I*a**72*d**9*exp(74*I*c)*exp(-16
*I*d*x) + 277298568799925181577403826176*I*a**72*d**9*exp(72*I*c)*exp(-18*I
*d*x))*exp(-90*I*c)/(511116722012022094683470732407 6032*a**80*d**10), Ne(a*
*80*d**10*exp(90*I*c), 0)), (x*((exp(20*I*c) + 10*exp(18*I*c) + 45*exp(16*I
*c) + 120*exp(14*I*c) + 210*exp(12*I*c) + 252*exp(10*I*c) + 210*exp(8*I*c)
+ 120*exp(6*I*c) + 45*exp(4*I*c) + 10*exp(2*I*c) + 1)*exp(-18*I*c)/(1024*a*
*8) - 5/(512*a**8)), True)) + 5*x/(512*a**8)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [A] (verification not implemented)

none

Time = 1.28 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.59

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{-\frac{2520i\log(\tan(dx+c)+i)}{a^8}+\frac{2520i\log(\tan(dx+c)-i)}{a^8}+\frac{504(5i\tan(dx+c)-6)}{a^8(\tan(dx+c)+i)}+\frac{-7129i\tan(dx+c)^9-68697\tan(dx+c)^8+296964i\tan}{}}{5}$$

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/516096*(-2520*I*log(tan(d*x + c) + I)/a^8 + 2520*I*log(tan(d*x + c) - I)
/a^8 + 504*(5*I*tan(d*x + c) - 6)/(a^8*(tan(d*x + c) + I)) + (-7129*I*tan(d
*x + c)^9 - 68697*tan(d*x + c)^8 + 296964*I*tan(d*x + c)^7 + 758772*tan(d*x
 + c)^6 - 1271214*I*tan(d*x + c)^5 - 1465758*tan(d*x + c)^4 + 1191540*I*tan
(d*x + c)^3 + 693828*tan(d*x + c)^2 - 295425*I*tan(d*x + c) - 89553)/(a^8*(
tan(d*x + c) - I)^9))/d
```

Mupad [B] (verification not implemented)

Time = 6.25 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.85

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{5x}{512a^8} + \frac{\frac{163\tan(c+dx)^2}{448a^8}-\frac{10}{63a^8}-\frac{\tan(c+dx)\,9019\mathrm{i}}{32256a^8}+\frac{\tan(c+dx)^3\,393\mathrm{i}}{1792a^8}+\frac{11\tan(c+dx)^4}{64a^8}+\frac{\tan(c+dx)^5\,1\mathrm{i}}{2a^8}-}{d\left(\tan(c+dx)^{10}\,1\mathrm{i}+8\tan(c+dx)^9-\tan(c+dx)^8\,27\mathrm{i}-48\tan(c+dx)^7+\tan(c+dx)^6\,42\mathrm{i}+\tan\right.}$$

```
[In] int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] (5*x)/(512*a^8) + ((163*tan(c + d*x)^2)/(448*a^8) - 10/(63*a^8) - (tan(c +
d*x)*9019i)/(32256*a^8) + (tan(c + d*x)^3*393i)/(1792*a^8) + (11*tan(c + d*
x)^4)/(64*a^8) + (tan(c + d*x)^5*1i)/(2*a^8) - (95*tan(c + d*x)^6)/(192*a^8
) - (tan(c + d*x)^7*205i)/(768*a^8) + (5*tan(c + d*x)^8)/(64*a^8) + (tan(c
+ d*x)^9*5i)/(512*a^8))/(d*(48*tan(c + d*x)^3 - tan(c + d*x)^2*27i - 8*tan(
c + d*x) + tan(c + d*x)^4*42i + tan(c + d*x)^6*42i - 48*tan(c + d*x)^7 - ta
n(c + d*x)^8*27i + 8*tan(c + d*x)^9 + tan(c + d*x)^10*1i + 1i))
```

3.175 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 333

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx &= \frac{33x}{2048a^8}+\frac{ia^2}{80d(a+ia\tan(c+dx))^{10}}\\
&+\frac{ia}{48d(a+ia\tan(c+dx))^9}+\frac{3i}{128d(a+ia\tan(c+dx))^8}\\
&+\frac{5i}{224ad(a+ia\tan(c+dx))^7}+\frac{5i}{256a^2d(a+ia\tan(c+dx))^6}\\
&+\frac{21i}{1280a^3d(a+ia\tan(c+dx))^5}\\
&+\frac{3i}{256a^5d(a+ia\tan(c+dx))^3}+\frac{7i}{512d\,(a^2+ia^2\tan(c+dx))^4}\\
&-\frac{i}{4096d\,(a^4-ia^4\tan(c+dx))^2}\\
&+\frac{45i}{4096d\,(a^4+ia^4\tan(c+dx))^2}\\
&-\frac{11i}{4096d\,(a^8-ia^8\tan(c+dx))}+\frac{55i}{4096d\,(a^8+ia^8\tan(c+dx))}\end{aligned}$$

```
[Out] 33/2048*x/a^8+1/80*I*a^2/d/(a+I*a*tan(d*x+c))^10+1/48*I*a/d/(a+I*a*tan(d*x+
c))^9+3/128*I/d/(a+I*a*tan(d*x+c))^8+5/224*I/a/d/(a+I*a*tan(d*x+c))^7+5/256
*I/a^2/d/(a+I*a*tan(d*x+c))^6+21/1280*I/a^3/d/(a+I*a*tan(d*x+c))^5+3/256*I/
a^5/d/(a+I*a*tan(d*x+c))^3+7/512*I/d/(a^2+I*a^2*tan(d*x+c))^4-1/4096*I/d/(a
^4-I*a^4*tan(d*x+c))^2+45/4096*I/d/(a^4+I*a^4*tan(d*x+c))^2-11/4096*I/d/(a^
8-I*a^8*tan(d*x+c))+55/4096*I/d/(a^8+I*a^8*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3568, 46, 212}

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{11i}{4096d\,(a^8-ia^8\tan(c+dx))}+\frac{55i}{4096d\,(a^8+ia^8\tan(c+dx))}+\frac{33x}{2048a^8}+\frac{3i}{256a^5d(a+ia\tan(c+dx))^3}-\frac{i}{4096d\,(a^4-ia^4\tan(c+dx))^2}+\frac{45i}{4096d\,(a^4+ia^4\tan(c+dx))^2}+\frac{21i}{1280a^3d(a+ia\tan(c+dx))^5}+\frac{ia^2}{80d(a+ia\tan(c+dx))^{10}}+\frac{7i}{512d\,(a^2+ia^2\tan(c+dx))^4}+\frac{5i}{256a^2d(a+ia\tan(c+dx))^6}+\frac{ia}{48d(a+ia\tan(c+dx))^9}+\frac{3i}{128d(a+ia\tan(c+dx))^8}+\frac{5i}{224ad(a+ia\tan(c+dx))^7}$$

```
[In] Int[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (33*x)/(2048*a^8) + ((I/80)*a^2)/(d*(a + I*a*Tan[c + d*x])^10) + ((I/48)*a)
/(d*(a + I*a*Tan[c + d*x])^9) + ((3*I)/128)/(d*(a + I*a*Tan[c + d*x])^8) +
((5*I)/224)/(a*d*(a + I*a*Tan[c + d*x])^7) + ((5*I)/256)/(a^2*d*(a + I*a*Ta
n[c + d*x])^6) + ((21*I)/1280)/(a^3*d*(a + I*a*Tan[c + d*x])^5) + ((3*I)/25
6)/(a^5*d*(a + I*a*Tan[c + d*x])^3) + ((7*I)/512)/(d*(a^2 + I*a^2*Tan[c + d
*x])^4) - (I/4096)/(d*(a^4 - I*a^4*Tan[c + d*x])^2) + ((45*I)/4096)/(d*(a^4
 + I*a^4*Tan[c + d*x])^2) - ((11*I)/4096)/(d*(a^8 - I*a^8*Tan[c + d*x])) +
((55*I)/4096)/(d*(a^8 + I*a^8*Tan[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{(ia^5)\,\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^{11}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{(ia^5)\,\text{Subst}\left(\int \left(\frac{1}{2048a^{11}(a-x)^3} + \frac{11}{4096a^{12}(a-x)^2} + \frac{1}{8a^3(a+x)^{11}} + \frac{3}{16a^4(a+x)^{10}} + \frac{3}{16a^5(a+x)^9} + \frac{5}{32a^6(a+x)^8} + \cdots\right.\right.}{}$$

$$= \frac{ia^2}{80d(a+ia\tan(c+dx))^{10}} + \frac{ia}{48d(a+ia\tan(c+dx))^9} + \frac{3i}{128d(a+ia\tan(c+dx))^8} + \frac{5i}{224ad(a+ia\tan(c+dx))^7} + \frac{5i}{256a^2d(a+ia\tan(c+dx))^6} + \frac{21i}{1280a^3d(a+ia\tan(c+dx))^5} + \frac{3i}{256a^5d(a+ia\tan(c+dx))^3} + \frac{7i}{512d\,(a^2+ia^2\tan(c+dx))^4} - \frac{i}{4096d\,(a^4-ia^4\tan(c+dx))^2} + \frac{45i}{4096d\,(a^4+ia^4\tan(c+dx))^2} - \frac{11i}{4096d\,(a^8-ia^8\tan(c+dx))} + \frac{55i}{4096d\,(a^8+ia^8\tan(c+dx))} - \frac{(33i)\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, ia\tan(c+dx)\right)}{2048a^7d}$$

$$= \frac{33x}{2048a^8} + \frac{ia^2}{80d(a+ia\tan(c+dx))^{10}} + \frac{ia}{48d(a+ia\tan(c+dx))^9} + \frac{3i}{128d(a+ia\tan(c+dx))^8} + \frac{5i}{224ad(a+ia\tan(c+dx))^7} + \frac{5i}{256a^2d(a+ia\tan(c+dx))^6} + \frac{21i}{1280a^3d(a+ia\tan(c+dx))^5} + \frac{3i}{256a^5d(a+ia\tan(c+dx))^3} + \frac{7i}{512d\,(a^2+ia^2\tan(c+dx))^4} - \frac{i}{4096d\,(a^4-ia^4\tan(c+dx))^2} + \frac{45i}{4096d\,(a^4+ia^4\tan(c+dx))^2} - \frac{11i}{4096d\,(a^8-ia^8\tan(c+dx))} + \frac{55i}{4096d\,(a^8+ia^8\tan(c+dx))}$$

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.62

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{\sec^{12}(c+dx)(48510i+88704i\cos(2(c+dx))+69300i\cos(4(c+dx))+52800i\cos(6(c+dx))+21538i\,c}{}$$

```
[In] Integrate[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (Sec[c + d*x]^12*(48510*I + (88704*I)*Cos[2*(c + d*x)] + (69300*I)*Cos[4*(c
 + d*x)] + (52800*I)*Cos[6*(c + d*x)] + (21538*I)*Cos[8*(c + d*x)] - (2240*
I)*Cos[10*(c + d*x)] - (84*I)*Cos[12*(c + d*x)] - 22176*Sin[2*(c + d*x)] -
34650*Sin[4*(c + d*x)] - 39600*Sin[6*(c + d*x)] + 27720*ArcTan[Tan[c + d*x]
]*(Cos[8*(c + d*x)] + I*Sin[8*(c + d*x)]) - 18073*Sin[8*(c + d*x)] + 2800*S
in[10*(c + d*x)] + 126*Sin[12*(c + d*x)]))/(1720320*a^8*d*(-I + Tan[c + d*x
])^10*(I + Tan[c + d*x])^2)
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.59

method	result
derivativedivides	$\frac{i}{4096(\tan(dx+c)+i)^2}+\frac{33i\ln(\tan(dx+c)+i)}{4096}+\frac{11}{4096(\tan(dx+c)+i)}-\frac{33i\ln(\tan(dx+c)-i)}{4096}+\frac{7i}{512(\tan(dx+c)-i)^4}+\frac{3i}{128(\tan(dx+c)-i)^8}$
default	$\frac{i}{4096(\tan(dx+c)+i)^2}+\frac{33i\ln(\tan(dx+c)+i)}{4096}+\frac{11}{4096(\tan(dx+c)+i)}-\frac{33i\ln(\tan(dx+c)-i)}{4096}+\frac{7i}{512(\tan(dx+c)-i)^4}+\frac{3i}{128(\tan(dx+c)-i)^8}$
risch	$\frac{33x}{2048a^8}+\frac{33ie^{-6i(dx+c)}}{1024a^8d}+\frac{231ie^{-8i(dx+c)}}{8192a^8d}+\frac{99ie^{-10i(dx+c)}}{5120a^8d}+\frac{165ie^{-12i(dx+c)}}{16384a^8d}+\frac{55ie^{-14i(dx+c)}}{14336a^8d}+\frac{33ie^{-16i}}{32768}$

```
[In] int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^8*(1/4096*I/(tan(d*x+c)+I)^2+33/4096*I*ln(tan(d*x+c)+I)+11/4096/(tan(
d*x+c)+I)-33/4096*I*ln(tan(d*x+c)-I)+7/512*I/(tan(d*x+c)-I)^4+3/128*I/(tan(
d*x+c)-I)^8-1/80*I/(tan(d*x+c)-I)^10-5/256*I/(tan(d*x+c)-I)^6-45/4096*I/(ta
n(d*x+c)-I)^2+1/48/(tan(d*x+c)-I)^9-5/224/(tan(d*x+c)-I)^7+21/1280/(tan(d*x
+c)-I)^5-3/256/(tan(d*x+c)-I)^3+55/4096/(tan(d*x+c)-I))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.46

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\left(55440\,dxe^{(20i\,dx+20i\,c)} - 210i\,e^{(24i\,dx+24i\,c)} - 5040i\,e^{(22i\,dx+22i\,c)} + 92400i\,e^{(18i\,dx+18i\,c)} + 103950i\,e^{(16i\,dx+16i\,c)}\right.}{}$$

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/3440640*(55440*d*x*e^(20*I*d*x + 20*I*c) - 210*I*e^(24*I*d*x + 24*I*c) -
5040*I*e^(22*I*d*x + 22*I*c) + 92400*I*e^(18*I*d*x + 18*I*c) + 103950*I*e^(
16*I*d*x + 16*I*c) + 110880*I*e^(14*I*d*x + 14*I*c) + 97020*I*e^(12*I*d*x +
 12*I*c) + 66528*I*e^(10*I*d*x + 10*I*c) + 34650*I*e^(8*I*d*x + 8*I*c) + 13
200*I*e^(6*I*d*x + 6*I*c) + 3465*I*e^(4*I*d*x + 4*I*c) + 560*I*e^(2*I*d*x +
 2*I*c) + 42*I)*e^(-20*I*d*x - 20*I*c)/(a^8*d)
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.39

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \begin{cases} \frac{(-1143348752854353237236938680970741190492160\,0ia^{88}d^{11}e^{114ic}e^{4idx} - 2744037006850447769368652834329778857181184\,00ia^{88}d^{11}e^{112ic}e^{2\cdots}}{} \\ x\left(\frac{\left(e^{24ic}+12e^{22ic}+66e^{20ic}+220e^{18ic}+495e^{16ic}+792e^{14ic}+924e^{12ic}+792e^{10ic}+495e^{8ic}+220e^{6ic}+66e^{4ic}+12e^{2ic}+1\right)e^{-20ic}}{4096a^8} - \frac{33}{2048a^8}\right) \end{cases} + \frac{33x}{2048a^8}$$

```
[In] integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise(((-11433487528543532372369386809707411904921600*I*a**88*d**11*exp
(114*I*c)*exp(4*I*d*x) - 274403700685044776936865283432977885718118400*I*a*
*88*d**11*exp(112*I*c)*exp(2*I*d*x) + 5030734512559154243842530196271261238
165504000*I*a**88*d**11*exp(108*I*c)*exp(-2*I*d*x) + 565957632662904852432
2846470805168892936192000*I*a**88*d**11*exp(106*I*c)*exp(-4*I*d*x) + 6036881
415070985092611036235525513485798604800*I*a**88*d**11*exp(104*I*c)*exp(-6*I
*d*x) + 5282271238187111956034656706084824300073779200*I*a**88*d**11*exp(10
2*I*c)*exp(-8*I*d*x) + 3622128849042591055566621741315308091479162880*I*a**
88*d**11*exp(100*I*c)*exp(-10*I*d*x) + 188652544220968284144094882360172296
4312064000*I*a**88*d**11*exp(98*I*c)*exp(-12*I*d*x) + 718676358937022034834
647170895894462595072000*I*a**88*d**11*exp(96*I*c)*exp(-14*I*d*x) + 1886525
```

```
44220968284144094882360172296431206400*I*a**88*d**11*exp(94*I*c)*exp(-16*I*
d*x) + 304893000761116086326318364825886431746457600*I*a**88*d**11*exp(92*I*
c)*exp(-18*I*d*x) + 2286697505708706474473877361941482380984320*I*a**88*d**
11*exp(90*I*c)*exp(-20*I*d*x))*exp(-110*I*c)/(18732625966765723438890003349
0246236650235494400*a**96*d**12), Ne(a**96*d**12*exp(110*I*c), 0)), (x*((ex
p(24*I*c) + 12*exp(22*I*c) + 66*exp(20*I*c) + 220*exp(18*I*c) + 495*exp(16*
I*c) + 792*exp(14*I*c) + 924*exp(12*I*c) + 792*exp(10*I*c) + 495*exp(8*I*c)
 + 220*exp(6*I*c) + 66*exp(4*I*c) + 12*exp(2*I*c) + 1)*exp(-20*I*c)/(4096*a
**8) - 33/(2048*a**8)), True)) + 33*x/(2048*a**8)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [A] (verification not implemented)

none

Time = 1.25 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.55

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{-\frac{27720i\log(\tan(dx+c)+i)}{a^8} + \frac{27720i\log(\tan(dx+c)-i)}{a^8} + \frac{420\left(99i\tan(dx+c)^2-220\tan(dx+c)-123i\right)}{a^8(\tan(dx+c)+i)^2} - \frac{81191i\tan(dx+c)^{10}+85811}{}}{}$$

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/3440640*(-27720*I*log(tan(d*x + c) + I)/a^8 + 27720*I*log(tan(d*x + c) -
 I)/a^8 + 420*(99*I*tan(d*x + c)^2 - 220*tan(d*x + c) - 123*I)/(a^8*(tan(d*
x + c) + I)^2) - (81191*I*tan(d*x + c)^10 + 858110*tan(d*x + c)^9 - 4107195
*I*tan(d*x + c)^8 - 11748840*tan(d*x + c)^7 + 22318590*I*tan(d*x + c)^6 + 2
9583540*tan(d*x + c)^5 - 27983550*I*tan(d*x + c)^4 - 19002600*tan(d*x + c)^
3 + 9206235*I*tan(d*x + c)^2 + 3108990*tan(d*x + c) - 648327*I)/(a^8*(tan(d
*x + c) - I)^10))/d
```

Mupad [B] (verification not implemented)

Time = 6.41 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.88

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{33\,x}{2048\,a^8} - \frac{\frac{\tan(c+d\,x)\,66953\mathrm{i}}{215040\,a^8}+\frac{17}{105\,a^8}-\frac{9097\tan(c+d\,x)^2}{26880\,a^8}+\frac{\tan(c+d\,x)^3\,4279\mathrm{i}}{43008\,a^8}-\frac{99\tan(c+d\,x)^4}{112\,a^8}-\tan}{d\,\left(\tan(c+d\,x)^{12}\,1\mathrm{i}+8\tan(c+d\,x)^{11}-\tan(c+d\,x)^{10}\,26\mathrm{i}-40\tan(c+d\,x)^9+\tan(c+d\,x)^8\,15\mathrm{i}-48\right.}$$

```
[In] int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] (33*x)/(2048*a^8) - ((tan(c + d*x)*66953i)/(215040*a^8) + 17/(105*a^8) - (9
097*tan(c + d*x)^2)/(26880*a^8) + (tan(c + d*x)^3*4279i)/(43008*a^8) - (99*
tan(c + d*x)^4)/(112*a^8) - (tan(c + d*x)^5*42537i)/(35840*a^8) + (341*tan(
c + d*x)^6)/(640*a^8) - (tan(c + d*x)^7*1969i)/(5120*a^8) + (11*tan(c + d*x
)^8)/(16*a^8) + (tan(c + d*x)^9*869i)/(2048*a^8) - (33*tan(c + d*x)^10)/(25
6*a^8) - (tan(c + d*x)^11*33i)/(2048*a^8))/(d*(40*tan(c + d*x)^3 - tan(c +
d*x)^2*26i - 8*tan(c + d*x) + tan(c + d*x)^4*15i + 48*tan(c + d*x)^5 + tan(
c + d*x)^6*84i - 48*tan(c + d*x)^7 + tan(c + d*x)^8*15i - 40*tan(c + d*x)^9
 - tan(c + d*x)^10*26i + 8*tan(c + d*x)^11 + tan(c + d*x)^12*1i + 1i))
```

3.176 $\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 205

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{1155\text{arctanh}(\sin(c+dx))}{8a^8d} + \frac{1155\sec(c+dx)\tan(c+dx)}{8a^8d} + \frac{385\sec^3(c+dx)\tan(c+dx)}{4a^8d} + \frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7} - \frac{22i\sec^9(c+dx)}{3a^3d(a+ia\tan(c+dx))^5} - \frac{66i\sec^7(c+dx)}{a^2d\,(a^2+ia^2\tan(c+dx))^3} - \frac{154i\sec^5(c+dx)}{d\,(a^8+ia^8\tan(c+dx))}$$

```
[Out] 1155/8*arctanh(sin(d*x+c))/a^8/d+1155/8*sec(d*x+c)*tan(d*x+c)/a^8/d+385/4*s
ec(d*x+c)^3*tan(d*x+c)/a^8/d+2/3*I*sec(d*x+c)^11/a/d/(a+I*a*tan(d*x+c))^7-2
2/3*I*sec(d*x+c)^9/a^3/d/(a+I*a*tan(d*x+c))^5-66*I*sec(d*x+c)^7/a^2/d/(a^2+
I*a^2*tan(d*x+c))^3-154*I*sec(d*x+c)^5/d/(a^8+I*a^8*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used

$= \{3581, 3853, 3855\}$

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{1155\text{arctanh}(\sin(c+dx))}{8a^8d} - \frac{154i\sec^5(c+dx)}{d\,(a^8+ia^8\tan(c+dx))} + \frac{385\tan(c+dx)\sec^3(c+dx)}{4a^8d} + \frac{1155\tan(c+dx)\sec(c+dx)}{8a^8d} - \frac{22i\sec^9(c+dx)}{3a^3d(a+ia\tan(c+dx))^5} - \frac{66i\sec^7(c+dx)}{a^2d\,(a^2+ia^2\tan(c+dx))^3} + \frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7}$$

```
[In] Int[Sec[c + d*x]^13/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (1155*ArcTanh[Sin[c + d*x]])/(8*a^8*d) + (1155*Sec[c + d*x]*Tan[c + d*x])/(
8*a^8*d) + (385*Sec[c + d*x]^3*Tan[c + d*x])/(4*a^8*d) + (((2*I)/3)*Sec[c +
 d*x]^11)/(a*d*(a + I*a*Tan[c + d*x])^7) - (((22*I)/3)*Sec[c + d*x]^9)/(a^3
*d*(a + I*a*Tan[c + d*x])^5) - ((66*I)*Sec[c + d*x]^7)/(a^2*d*(a^2 + I*a^2*
Tan[c + d*x])^3) - ((154*I)*Sec[c + d*x]^5)/(d*(a^8 + I*a^8*Tan[c + d*x]))
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7} - \frac{11\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^6}\,dx}{3a^2}$$

$$= \frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7} - \frac{22i\sec^9(c+dx)}{3a^3d(a+ia\tan(c+dx))^5} + \frac{33\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^4}\,dx}{a^4}$$

$$= \frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7} - \frac{22i\sec^9(c+dx)}{3a^3d(a+ia\tan(c+dx))^5} - \frac{66i\sec^7(c+dx)}{a^5d(a+ia\tan(c+dx))^3} + \frac{231\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{a^6}$$

$$= \frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7} - \frac{22i\sec^9(c+dx)}{3a^3d(a+ia\tan(c+dx))^5} - \frac{66i\sec^7(c+dx)}{a^5d(a+ia\tan(c+dx))^3} - \frac{154i\sec^5(c+dx)}{d\,(a^8+ia^8\tan(c+dx))} + \frac{385\int \sec^5(c+dx)\,dx}{a^8}$$

$$= \frac{385\sec^3(c+dx)\tan(c+dx)}{4a^8d} + \frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7} - \frac{22i\sec^9(c+dx)}{3a^3d(a+ia\tan(c+dx))^5} - \frac{66i\sec^7(c+dx)}{a^5d(a+ia\tan(c+dx))^3} - \frac{154i\sec^5(c+dx)}{d\,(a^8+ia^8\tan(c+dx))} + \frac{1155\int \sec^3(c+dx)\,dx}{4a^8}$$

$$= \frac{1155\sec(c+dx)\tan(c+dx)}{8a^8d} + \frac{385\sec^3(c+dx)\tan(c+dx)}{4a^8d} + \frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7} - \frac{22i\sec^9(c+dx)}{3a^3d(a+ia\tan(c+dx))^5} - \frac{66i\sec^7(c+dx)}{a^5d(a+ia\tan(c+dx))^3} - \frac{154i\sec^5(c+dx)}{d\,(a^8+ia^8\tan(c+dx))} + \frac{1155\int \sec(c+dx)\,dx}{8a^8}$$

$$= \frac{1155\text{arctanh}(\sin(c+dx))}{8a^8d} + \frac{1155\sec(c+dx)\tan(c+dx)}{8a^8d} + \frac{385\sec^3(c+dx)\tan(c+dx)}{4a^8d} + \frac{2i\sec^{11}(c+dx)}{3ad(a+ia\tan(c+dx))^7} - \frac{22i\sec^9(c+dx)}{3a^3d(a+ia\tan(c+dx))^5} - \frac{66i\sec^7(c+dx)}{a^5d(a+ia\tan(c+dx))^3} - \frac{154i\sec^5(c+dx)}{d\,(a^8+ia^8\tan(c+dx))}$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1704 vs. $2(205) = 410$.

Time = 7.39 (sec) , antiderivative size = 1704, normalized size of antiderivative = 8.31

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= -\frac{1155\cos(8c)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^8(c+dx)(\cos(dx)+i\sin(dx))^8}{8d(a+ia\tan(c+dx))^8}$$
$$+\frac{1155\cos(8c)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^8(c+dx)(\cos(dx)+i\sin(dx))^8}{8d(a+ia\tan(c+dx))^8}$$
$$+\frac{\cos(3dx)\sec^8(c+dx)\left(\frac{32}{3}i\cos(5c)-\frac{32}{3}\sin(5c)\right)(\cos(dx)+i\sin(dx))^8}{d(a+ia\tan(c+dx))^8}$$
$$+\frac{\cos(dx)\sec^8(c+dx)(-160i\cos(7c)+160\sin(7c))(\cos(dx)+i\sin(dx))^8}{d(a+ia\tan(c+dx))^8}$$
$$-\frac{1155i\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^8(c+dx)\sin(8c)(\cos(dx)+i\sin(dx))^8}{8d(a+ia\tan(c+dx))^8}$$
$$+\frac{1155i\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^8(c+dx)\sin(8c)(\cos(dx)+i\sin(dx))^8}{8d(a+ia\tan(c+dx))^8}$$
$$+\frac{\sec(c)\sec^8(c+dx)\left(-\frac{236}{3}i\cos(8c)+\frac{236}{3}\sin(8c)\right)(\cos(dx)+i\sin(dx))^8}{d(a+ia\tan(c+dx))^8}$$
$$+\frac{\sec^8(c+dx)(-160\cos(7c)-160i\sin(7c))(\cos(dx)+i\sin(dx))^8\sin(dx)}{d(a+ia\tan(c+dx))^8}$$
$$+\frac{\sec^8(c+dx)\left(\frac{32}{3}\cos(5c)+\frac{32}{3}i\sin(5c)\right)(\cos(dx)+i\sin(dx))^8\sin(3dx)}{d(a+ia\tan(c+dx))^8}$$
$$+\frac{\sec^8(c+dx)\left(\frac{1}{16}\cos(8c)+\frac{1}{16}i\sin(8c)\right)(\cos(dx)+i\sin(dx))^8}{d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^4(a+ia\tan(c+dx))^8}$$
$$-\frac{\left(\frac{1}{96}+\frac{i}{96}\right)\sec^8(c+dx)\left(-407i\cos\left(\frac{15c}{2}\right)+343\cos\left(\frac{17c}{2}\right)+407\sin\left(\frac{15c}{2}\right)+343i\sin\left(\frac{17c}{2}\right)\right)(\cos(dx)+i\sin(}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^2(a+ia\tan(c+dx))^8}$$
$$+\frac{\sec^8(c+dx)\left(-\frac{1}{16}\cos(8c)-\frac{1}{16}i\sin(8c)\right)(\cos(dx)+i\sin(dx))^8}{d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^4(a+ia\tan(c+dx))^8}$$
$$+\frac{\left(\frac{1}{96}+\frac{i}{96}\right)\sec^8(c+dx)\left(407\cos\left(\frac{15c}{2}\right)-343i\cos\left(\frac{17c}{2}\right)+407i\sin\left(\frac{15c}{2}\right)+343\sin\left(\frac{17c}{2}\right)\right)(\cos(dx)+i\sin(d}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^2(a+ia\tan(c+dx))^8}$$
$$+\frac{236\sec^8(c+dx)(\cos(dx)+i\sin(dx))^8\left(\frac{1}{2}\cos\left(8c-\frac{dx}{2}\right)-\frac{1}{2}\cos\left(8c+\frac{dx}{2}\right)+\frac{1}{2}i\sin\left(8c-\frac{dx}{2}\right)-\frac{1}{2}i\sin\left(8c-\right.\right.}{3d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(a+ia\tan(c+dx))^8}$$
$$+\frac{4\sec^8(c+dx)(\cos(dx)+i\sin(dx))^8\left(\frac{1}{2}\cos\left(8c-\frac{dx}{2}\right)-\frac{1}{2}\cos\left(8c+\frac{dx}{2}\right)+\frac{1}{2}i\sin\left(8c-\frac{dx}{2}\right)-\frac{1}{2}i\sin\left(8c+\right.\right.}{3d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^3(a+ia\tan(c+dx))^8}$$
$$+\frac{4\sec^8(c+dx)(\cos(dx)+i\sin(dx))^8\left(-\frac{1}{2}\cos\left(8c-\frac{dx}{2}\right)+\frac{1}{2}\cos\left(8c+\frac{dx}{2}\right)-\frac{1}{2}i\sin\left(8c-\frac{dx}{2}\right)+\frac{1}{2}i\sin\left(8c+\right.\right.}{3d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^3(a+ia\tan(c+dx))^8}$$
$$+\frac{236\sec^8(c+dx)(\cos(dx)+i\sin(dx))^8\left(-\frac{1}{2}\cos\left(8c-\frac{dx}{2}\right)+\frac{1}{2}\cos\left(8c+\frac{dx}{2}\right)-\frac{1}{2}i\sin\left(8c-\frac{dx}{2}\right)+\frac{1}{2}i\sin\left(8c\right.\right.}{3d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(a+ia\tan(c+dx))^8}$$

```
[In] Integrate[Sec[c + d*x]^13/(a + I*a*Tan[c + d*x])^8,x]

[Out] (-1155*Cos[8*c]*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^8
*(Cos[d*x] + I*Sin[d*x])^8)/(8*d*(a + I*a*Tan[c + d*x])^8) + (1155*Cos[8*c]
*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^8*(Cos[d*x] + I*
Sin[d*x])^8)/(8*d*(a + I*a*Tan[c + d*x])^8) + (Cos[3*d*x]*Sec[c + d*x]^8*((
(32*I)/3)*Cos[5*c] - (32*Sin[5*c])/3)*(Cos[d*x] + I*Sin[d*x])^8)/(d*(a + I*
a*Tan[c + d*x])^8) + (Cos[d*x]*Sec[c + d*x]^8*((-160*I)*Cos[7*c] + 160*Sin[
7*c])*(Cos[d*x] + I*Sin[d*x])^8)/(d*(a + I*a*Tan[c + d*x])^8) - (((1155*I)/
8)*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^8*Sin[8*c]*(Co
s[d*x] + I*Sin[d*x])^8)/(d*(a + I*a*Tan[c + d*x])^8) + (((1155*I)/8)*Log[Co
s[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^8*Sin[8*c]*(Cos[d*x] +
I*Sin[d*x])^8)/(d*(a + I*a*Tan[c + d*x])^8) + (Sec[c]*Sec[c + d*x]^8*(((-23
6*I)/3)*Cos[8*c] + (236*Sin[8*c])/3)*(Cos[d*x] + I*Sin[d*x])^8)/(d*(a + I*a
*Tan[c + d*x])^8) + (Sec[c + d*x]^8*(-160*Cos[7*c] - (160*I)*Sin[7*c])*(Cos
[d*x] + I*Sin[d*x])^8*Sin[d*x])/(d*(a + I*a*Tan[c + d*x])^8) + (Sec[c + d*x
]^8*((32*Cos[5*c])/3 + ((32*I)/3)*Sin[5*c])*(Cos[d*x] + I*Sin[d*x])^8*Sin[3
*d*x])/(d*(a + I*a*Tan[c + d*x])^8) + (Sec[c + d*x]^8*(Cos[8*c]/16 + (I/16)
*Sin[8*c])*(Cos[d*x] + I*Sin[d*x])^8)/(d*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d
*x)/2])^4*(a + I*a*Tan[c + d*x])^8) - ((1/96 + I/96)*Sec[c + d*x]^8*((-407*
I)*Cos[(15*c)/2] + 343*Cos[(17*c)/2] + 407*Sin[(15*c)/2] + (343*I)*Sin[(17*
c)/2])*(Cos[d*x] + I*Sin[d*x])^8)/(d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)
/2] - Sin[c/2 + (d*x)/2])^2*(a + I*a*Tan[c + d*x])^8) + (Sec[c + d*x]^8*(-1
/16*Cos[8*c] - (I/16)*Sin[8*c])*(Cos[d*x] + I*Sin[d*x])^8)/(d*(Cos[c/2 + (d
*x)/2] + Sin[c/2 + (d*x)/2])^4*(a + I*a*Tan[c + d*x])^8) + ((1/96 + I/96)*S
ec[c + d*x]^8*(407*Cos[(15*c)/2] - (343*I)*Cos[(17*c)/2] + (407*I)*Sin[(15*
c)/2] + 343*Sin[(17*c)/2])*(Cos[d*x] + I*Sin[d*x])^8)/(d*(Cos[c/2] + Sin[c/
2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2*(a + I*a*Tan[c + d*x])^8) +
 (236*Sec[c + d*x]^8*(Cos[d*x] + I*Sin[d*x])^8*(Cos[8*c - (d*x)/2]/2 - Cos[
8*c + (d*x)/2]/2 + (I/2)*Sin[8*c - (d*x)/2] - (I/2)*Sin[8*c + (d*x)/2]))/(3
*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])*(a + I*a
*Tan[c + d*x])^8) + (4*Sec[c + d*x]^8*(Cos[d*x] + I*Sin[d*x])^8*(Cos[8*c -
(d*x)/2]/2 - Cos[8*c + (d*x)/2]/2 + (I/2)*Sin[8*c - (d*x)/2] - (I/2)*Sin[8*
c + (d*x)/2]))/(3*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (
d*x)/2])^3*(a + I*a*Tan[c + d*x])^8) + (4*Sec[c + d*x]^8*(Cos[d*x] + I*Sin[
d*x])^8*(-1/2*Cos[8*c - (d*x)/2] + Cos[8*c + (d*x)/2]/2 - (I/2)*Sin[8*c - (
d*x)/2] + (I/2)*Sin[8*c + (d*x)/2]))/(3*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 +
(d*x)/2] - Sin[c/2 + (d*x)/2])^3*(a + I*a*Tan[c + d*x])^8) + (236*Sec[c + d
*x]^8*(Cos[d*x] + I*Sin[d*x])^8*(-1/2*Cos[8*c - (d*x)/2] + Cos[8*c + (d*x)/
2]/2 - (I/2)*Sin[8*c - (d*x)/2] + (I/2)*Sin[8*c + (d*x)/2]))/(3*d*(Cos[c/2]
 + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])*(a + I*a*Tan[c + d*x
])^8)
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{160 i \mathrm{e}^{-i(dx+c)}}{a^{8} d}+\frac{32 i \mathrm{e}^{-3 i(dx+c)}}{3 a^{8} d}-\frac{i\left(1545 \mathrm{e}^{7 i(dx+c)}+5153 \mathrm{e}^{5 i(dx+c)}+5855 \mathrm{e}^{3 i(dx+c)}+2295 \mathrm{e}^{i(dx+c)}\right)}{12 d\, a^{8}\left(\mathrm{e}^{2 i(dx+c)}+1\right)^{4}}+\frac{1155 \ln\left(\mathrm{e}^{i(}\right.}{8 a^{8}}$
derivativedivides	$\frac{\frac{2\left(\frac{1}{4}-\frac{4 i}{3}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2\left(-\frac{121}{16}-2 i\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(-\frac{123}{16}+38 i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}-\frac{1155 \ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}+\frac{128 i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}}{}$
default	$\frac{\frac{2\left(\frac{1}{4}-\frac{4 i}{3}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2\left(-\frac{121}{16}-2 i\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(-\frac{123}{16}+38 i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}-\frac{1155 \ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}+\frac{128 i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}}{}$

```
[In] int(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] -160*I/a^8/d*exp(-I*(d*x+c))+32/3*I/a^8/d*exp(-3*I*(d*x+c))-1/12*I/d/a^8/(e
xp(2*I*(d*x+c))+1)^4*(1545*exp(7*I*(d*x+c))+5153*exp(5*I*(d*x+c))+5855*exp(
3*I*(d*x+c))+2295*exp(I*(d*x+c)))+1155/8/a^8/d*ln(exp(I*(d*x+c))+I)-1155/8/
a^8/d*ln(exp(I*(d*x+c))-I)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.30

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{8}}\,dx$$
$$=\frac{3465\left(e^{(11 i\,dx+11 i\,c)}+4\,e^{(9 i\,dx+9 i\,c)}+6\,e^{(7 i\,dx+7 i\,c)}+4\,e^{(5 i\,dx+5 i\,c)}+e^{(3 i\,dx+3 i\,c)}\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-3465\left(e^{(11 i\right.}{24\left(\right.}$$

```
[In] integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/24*(3465*(e^(11*I*d*x + 11*I*c) + 4*e^(9*I*d*x + 9*I*c) + 6*e^(7*I*d*x +
7*I*c) + 4*e^(5*I*d*x + 5*I*c) + e^(3*I*d*x + 3*I*c))*log(e^(I*d*x + I*c) +
 I) - 3465*(e^(11*I*d*x + 11*I*c) + 4*e^(9*I*d*x + 9*I*c) + 6*e^(7*I*d*x +
7*I*c) + 4*e^(5*I*d*x + 5*I*c) + e^(3*I*d*x + 3*I*c))*log(e^(I*d*x + I*c) -
 I) - 6930*I*e^(10*I*d*x + 10*I*c) - 25410*I*e^(8*I*d*x + 8*I*c) - 33726*I*
e^(6*I*d*x + 6*I*c) - 18414*I*e^(4*I*d*x + 4*I*c) - 2816*I*e^(2*I*d*x + 2*I
*c) + 256*I)/(a^8*d*e^(11*I*d*x + 11*I*c) + 4*a^8*d*e^(9*I*d*x + 9*I*c) + 6
*a^8*d*e^(7*I*d*x + 7*I*c) + 4*a^8*d*e^(5*I*d*x + 5*I*c) + a^8*d*e^(3*I*d*x
 + 3*I*c))
```

Sympy [F]

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$=\frac{\int \frac{\sec^{13}(c+dx)}{\tan^8(c+dx)-8i\tan^7(c+dx)-28\tan^6(c+dx)+56i\tan^5(c+dx)+70\tan^4(c+dx)-56i\tan^3(c+dx)-28\tan^2(c+dx)+8i\tan(c+dx)+1}\,dx}{a^8}$$

```
[In] integrate(sec(d*x+c)**13/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Integral(sec(c + d*x)**13/(tan(c + d*x)**8 - 8*I*tan(c + d*x)**7 - 28*tan(c
 + d*x)**6 + 56*I*tan(c + d*x)**5 + 70*tan(c + d*x)**4 - 56*I*tan(c + d*x)*
*3 - 28*tan(c + d*x)**2 + 8*I*tan(c + d*x) + 1), x)/a**8
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 786 vs. 2(179) = 358.

Time = 0.39 (sec) , antiderivative size = 786, normalized size of antiderivative = 3.83

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] -(6930*(cos(11*d*x + 11*c) + 4*cos(9*d*x + 9*c) + 6*cos(7*d*x + 7*c) + 4*co
s(5*d*x + 5*c) + cos(3*d*x + 3*c) + I*sin(11*d*x + 11*c) + 4*I*sin(9*d*x +
9*c) + 6*I*sin(7*d*x + 7*c) + 4*I*sin(5*d*x + 5*c) + I*sin(3*d*x + 3*c))*ar
ctan2(cos(d*x + c), sin(d*x + c) + 1) + 6930*(cos(11*d*x + 11*c) + 4*cos(9*
d*x + 9*c) + 6*cos(7*d*x + 7*c) + 4*cos(5*d*x + 5*c) + cos(3*d*x + 3*c) + I
*sin(11*d*x + 11*c) + 4*I*sin(9*d*x + 9*c) + 6*I*sin(7*d*x + 7*c) + 4*I*sin
(5*d*x + 5*c) + I*sin(3*d*x + 3*c))*arctan2(cos(d*x + c), -sin(d*x + c) + 1
) + 3465*(I*cos(11*d*x + 11*c) + 4*I*cos(9*d*x + 9*c) + 6*I*cos(7*d*x + 7*c
) + 4*I*cos(5*d*x + 5*c) + I*cos(3*d*x + 3*c) - sin(11*d*x + 11*c) - 4*sin(
9*d*x + 9*c) - 6*sin(7*d*x + 7*c) - 4*sin(5*d*x + 5*c) - sin(3*d*x + 3*c))*
log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) + 3465*(-I*cos(11
*d*x + 11*c) - 4*I*cos(9*d*x + 9*c) - 6*I*cos(7*d*x + 7*c) - 4*I*cos(5*d*x
+ 5*c) - I*cos(3*d*x + 3*c) + sin(11*d*x + 11*c) + 4*sin(9*d*x + 9*c) + 6*s
in(7*d*x + 7*c) + 4*sin(5*d*x + 5*c) + sin(3*d*x + 3*c))*log(cos(d*x + c)^2
 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1) + 13860*cos(10*d*x + 10*c) + 50820*
cos(8*d*x + 8*c) + 67452*cos(6*d*x + 6*c) + 36828*cos(4*d*x + 4*c) + 5632*c
os(2*d*x + 2*c) + 13860*I*sin(10*d*x + 10*c) + 50820*I*sin(8*d*x + 8*c) + 6
7452*I*sin(6*d*x + 6*c) + 36828*I*sin(4*d*x + 4*c) + 5632*I*sin(2*d*x + 2*c
) - 512)/((-48*I*a^8*cos(11*d*x + 11*c) - 192*I*a^8*cos(9*d*x + 9*c) - 288*
I*a^8*cos(7*d*x + 7*c) - 192*I*a^8*cos(5*d*x + 5*c) - 48*I*a^8*cos(3*d*x +
3*c) + 48*a^8*sin(11*d*x + 11*c) + 192*a^8*sin(9*d*x + 9*c) + 288*a^8*sin(7
*d*x + 7*c) + 192*a^8*sin(5*d*x + 5*c) + 48*a^8*sin(3*d*x + 3*c))*d)
```

Giac [A] (verification not implemented)

none

Time = 1.88 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.95

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{\frac{3465\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^8} - \frac{3465\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^8} - \frac{1024\left(6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-7\right)}{a^8\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^3} - \frac{2\left(369\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-\cdots\right.}{\cdots}}{24\cdots}$$

```
[In] integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/24*(3465*log(tan(1/2*d*x + 1/2*c) + 1)/a^8 - 3465*log(tan(1/2*d*x + 1/2*c
) - 1)/a^8 - 1024*(6*tan(1/2*d*x + 1/2*c)^2 - 15*I*tan(1/2*d*x + 1/2*c) - 7
)/(a^8*(tan(1/2*d*x + 1/2*c) - I)^3) - 2*(369*tan(1/2*d*x + 1/2*c)^7 - 1728
*I*tan(1/2*d*x + 1/2*c)^6 - 393*tan(1/2*d*x + 1/2*c)^5 + 5568*I*tan(1/2*d*x
 + 1/2*c)^4 - 393*tan(1/2*d*x + 1/2*c)^3 - 5696*I*tan(1/2*d*x + 1/2*c)^2 +
369*tan(1/2*d*x + 1/2*c) + 1856*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^4*a^8))/d
```

Mupad [B] (verification not implemented)

Time = 8.50 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.68

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{\frac{33847\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{6\,a^8} - \frac{12041\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3\,a^8} - \frac{3585\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{a^8} + \frac{3505\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4\,a^8} + \frac{4293\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,a^8} + \cdots}{d\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}1\mathrm{i} - 3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10} + \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9 7\mathrm{i} + 13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8 - \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7 18\mathrm{i} - 22\,\mathrm{t}\cdots\right.}$$
$$+ \frac{1155\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,a^8\,d}$$

```
[In] int(1/(cos(c + d*x)^13*(a + a*tan(c + d*x)*1i)^8),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^2*27565i)/(12*a^8) - (12041*tan(c/2 + (d*x)/2)^3)/(3*a
^8) - (tan(c/2 + (d*x)/2)^4*4575i)/a^8 + (33847*tan(c/2 + (d*x)/2)^5)/(6*a^
8) + (tan(c/2 + (d*x)/2)^6*25993i)/(6*a^8) - (3585*tan(c/2 + (d*x)/2)^7)/a^
8 - (tan(c/2 + (d*x)/2)^8*5639i)/(3*a^8) + (3505*tan(c/2 + (d*x)/2)^9)/(4*a
^8) + (tan(c/2 + (d*x)/2)^10*1147i)/(4*a^8) - 1360i/(3*a^8) + (4293*tan(c/2
 + (d*x)/2))/(4*a^8))/(d*(tan(c/2 + (d*x)/2)*3i - 7*tan(c/2 + (d*x)/2)^2 -
tan(c/2 + (d*x)/2)^3*13i + 18*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^5*2
2i - 22*tan(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^7*18i + 13*tan(c/2 + (d*x
)/2)^8 + tan(c/2 + (d*x)/2)^9*7i - 3*tan(c/2 + (d*x)/2)^10 - tan(c/2 + (d*x
)/2)^11*1i + 1)) + (1155*atanh(tan(c/2 + (d*x)/2)))/(4*a^8*d)
```

3.177 $\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 183

$$\begin{aligned}\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = &-\frac{63\text{arctanh}(\sin(c+dx))}{2a^8d}-\frac{63\sec(c+dx)\tan(c+dx)}{2a^8d}\\&+\frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7}-\frac{6i\sec^7(c+dx)}{5a^3d(a+ia\tan(c+dx))^5}\\&+\frac{42i\sec^5(c+dx)}{5a^2d\left(a^2+ia^2\tan(c+dx)\right)^3}+\frac{42i\sec^3(c+dx)}{d\left(a^8+ia^8\tan(c+dx)\right)}\end{aligned}$$

```
[Out] -63/2*arctanh(sin(d*x+c))/a^8/d-63/2*sec(d*x+c)*tan(d*x+c)/a^8/d+2/5*I*sec(
d*x+c)^9/a/d/(a+I*a*tan(d*x+c))^7-6/5*I*sec(d*x+c)^7/a^3/d/(a+I*a*tan(d*x+c
))^5+42/5*I*sec(d*x+c)^5/a^2/d/(a^2+I*a^2*tan(d*x+c))^3+42*I*sec(d*x+c)^3/d
/(a^8+I*a^8*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3581, 3853, 3855}

$$\begin{aligned}\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = &-\frac{63\text{arctanh}(\sin(c+dx))}{2a^8d}+\frac{42i\sec^3(c+dx)}{d\left(a^8+ia^8\tan(c+dx)\right)}\\&-\frac{63\tan(c+dx)\sec(c+dx)}{2a^8d}-\frac{6i\sec^7(c+dx)}{5a^3d(a+ia\tan(c+dx))^5}\\&+\frac{42i\sec^5(c+dx)}{5a^2d\left(a^2+ia^2\tan(c+dx)\right)^3}+\frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7}\end{aligned}$$

```
[In] Int[Sec[c + d*x]^11/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (-63*ArcTanh[Sin[c + d*x]])/(2*a^8*d) - (63*Sec[c + d*x]*Tan[c + d*x])/(2*a
^8*d) + (((2*I)/5)*Sec[c + d*x]^9)/(a*d*(a + I*a*Tan[c + d*x])^7) - (((6*I)
/5)*Sec[c + d*x]^7)/(a^3*d*(a + I*a*Tan[c + d*x])^5) + (((42*I)/5)*Sec[c +
d*x]^5)/(a^2*d*(a^2 + I*a^2*Tan[c + d*x])^3) + ((42*I)*Sec[c + d*x]^3)/(d*(
a^8 + I*a^8*Tan[c + d*x]))
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7}-\frac{9\int\frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^6}\,dx}{5a^2}\\
&= \frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7}-\frac{6i\sec^7(c+dx)}{5a^3d(a+ia\tan(c+dx))^5}+\frac{21\int\frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^4}\,dx}{5a^4}\\
&= \frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7}-\frac{6i\sec^7(c+dx)}{5a^3d(a+ia\tan(c+dx))^5}\\
&\quad+\frac{42i\sec^5(c+dx)}{5a^5d(a+ia\tan(c+dx))^3}-\frac{21\int\frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{a^6}\\
&= \frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7}-\frac{6i\sec^7(c+dx)}{5a^3d(a+ia\tan(c+dx))^5}\\
&\quad+\frac{42i\sec^5(c+dx)}{5a^5d(a+ia\tan(c+dx))^3}+\frac{42i\sec^3(c+dx)}{d\left(a^8+ia^8\tan(c+dx)\right)}-\frac{63\int\sec^3(c+dx)\,dx}{a^8}
\end{aligned}$$

$$= -\frac{63\sec(c+dx)\tan(c+dx)}{2a^8d} + \frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7} - \frac{6i\sec^7(c+dx)}{5a^3d(a+ia\tan(c+dx))^5} + \frac{42i\sec^5(c+dx)}{5a^5d(a+ia\tan(c+dx))^3} + \frac{42i\sec^3(c+dx)}{d(a^8+ia^8\tan(c+dx))} - \frac{63\int\sec(c+dx)\,dx}{2a^8}$$

$$= -\frac{63\text{arctanh}(\sin(c+dx))}{2a^8d} - \frac{63\sec(c+dx)\tan(c+dx)}{2a^8d} + \frac{2i\sec^9(c+dx)}{5ad(a+ia\tan(c+dx))^7} - \frac{6i\sec^7(c+dx)}{5a^3d(a+ia\tan(c+dx))^5} + \frac{42i\sec^5(c+dx)}{5a^5d(a+ia\tan(c+dx))^3} + \frac{42i\sec^3(c+dx)}{d(a^8+ia^8\tan(c+dx))}$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1244 vs. $2(183) = 366$.

Time = 7.08 (sec) , antiderivative size = 1244, normalized size of antiderivative = 6.80

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{63\cos(8c)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^8(c+dx)(\cos(dx)+i\sin(dx))^8}{2d(a+ia\tan(c+dx))^8}$$
$$- \frac{63\cos(8c)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^8(c+dx)(\cos(dx)+i\sin(dx))^8}{2d(a+ia\tan(c+dx))^8}$$
$$+ \frac{\cos(5dx)\sec^8(c+dx)\left(\frac{8}{5}i\cos(3c)-\frac{8}{5}\sin(3c)\right)(\cos(dx)+i\sin(dx))^8}{d(a+ia\tan(c+dx))^8}$$
$$+ \frac{\cos(3dx)\sec^8(c+dx)(-8i\cos(5c)+8\sin(5c))(\cos(dx)+i\sin(dx))^8}{d(a+ia\tan(c+dx))^8}$$
$$+ \frac{\cos(dx)\sec^8(c+dx)(48i\cos(7c)-48\sin(7c))(\cos(dx)+i\sin(dx))^8}{d(a+ia\tan(c+dx))^8}$$
$$+ \frac{\sec(c)\sec^8(c+dx)(8i\cos(8c)-8\sin(8c))(\cos(dx)+i\sin(dx))^8}{d(a+ia\tan(c+dx))^8}$$
$$+ \frac{63i\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^8(c+dx)\sin(8c)(\cos(dx)+i\sin(dx))^8}{2d(a+ia\tan(c+dx))^8}$$
$$- \frac{63i\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^8(c+dx)\sin(8c)(\cos(dx)+i\sin(dx))^8}{2d(a+ia\tan(c+dx))^8}$$
$$+ \frac{\sec^8(c+dx)(48\cos(7c)+48i\sin(7c))(\cos(dx)+i\sin(dx))^8\sin(dx)}{d(a+ia\tan(c+dx))^8}$$
$$+ \frac{\sec^8(c+dx)(-8\cos(5c)-8i\sin(5c))(\cos(dx)+i\sin(dx))^8\sin(3dx)}{d(a+ia\tan(c+dx))^8}$$
$$+ \frac{\sec^8(c+dx)\left(\frac{8}{5}\cos(3c)+\frac{8}{5}i\sin(3c)\right)(\cos(dx)+i\sin(dx))^8\sin(5dx)}{d(a+ia\tan(c+dx))^8}$$
$$+ \frac{\sec^8(c+dx)\left(\frac{1}{4}\cos(8c)+\frac{1}{4}i\sin(8c)\right)(\cos(dx)+i\sin(dx))^8}{d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^2(a+ia\tan(c+dx))^8}$$
$$+ \frac{\sec^8(c+dx)\left(-\frac{1}{4}\cos(8c)-\frac{1}{4}i\sin(8c)\right)(\cos(dx)+i\sin(dx))^8}{d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^2(a+ia\tan(c+dx))^8}$$
$$+ \frac{8\sec^8(c+dx)(\cos(dx)+i\sin(dx))^8\left(\frac{1}{2}\cos\left(8c-\frac{dx}{2}\right)-\frac{1}{2}\cos\left(8c+\frac{dx}{2}\right)+\frac{1}{2}i\sin\left(8c-\frac{dx}{2}\right)-\frac{1}{2}i\sin\left(8c+\right.\right.}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(a+ia\tan(c+dx))^8}$$
$$+ \frac{8\sec^8(c+dx)(\cos(dx)+i\sin(dx))^8\left(-\frac{1}{2}\cos\left(8c-\frac{dx}{2}\right)+\frac{1}{2}\cos\left(8c+\frac{dx}{2}\right)-\frac{1}{2}i\sin\left(8c-\frac{dx}{2}\right)+\frac{1}{2}i\sin\left(8c+\right.\right.}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(a+ia\tan(c+dx))^8}$$

```
[In] Integrate[Sec[c + d*x]^11/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (63*Cos[8*c]*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^8*(C
os[d*x] + I*Sin[d*x])^8)/(2*d*(a + I*a*Tan[c + d*x])^8) - (63*Cos[8*c]*Log[
Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^8*(Cos[d*x] + I*Sin[d
```

```
*x])^8)/(2*d*(a + I*a*Tan[c + d*x])^8) + (Cos[5*d*x]*Sec[c + d*x]^8*(((8*I)
/5)*Cos[3*c] - (8*Sin[3*c])/5)*(Cos[d*x] + I*Sin[d*x])^8)/(d*(a + I*a*Tan[c
 + d*x])^8) + (Cos[3*d*x]*Sec[c + d*x]^8*((-8*I)*Cos[5*c] + 8*Sin[5*c])*(Co
s[d*x] + I*Sin[d*x])^8)/(d*(a + I*a*Tan[c + d*x])^8) + (Cos[d*x]*Sec[c + d*
x]^8*((48*I)*Cos[7*c] - 48*Sin[7*c])*(Cos[d*x] + I*Sin[d*x])^8)/(d*(a + I*a
*Tan[c + d*x])^8) + (Sec[c]*Sec[c + d*x]^8*((8*I)*Cos[8*c] - 8*Sin[8*c])*(C
os[d*x] + I*Sin[d*x])^8)/(d*(a + I*a*Tan[c + d*x])^8) + (((63*I)/2)*Log[Cos
[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^8*Sin[8*c]*(Cos[d*x] + I
*Sin[d*x])^8)/(d*(a + I*a*Tan[c + d*x])^8) - (((63*I)/2)*Log[Cos[c/2 + (d*x
)/2] + Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^8*Sin[8*c]*(Cos[d*x] + I*Sin[d*x])^
8)/(d*(a + I*a*Tan[c + d*x])^8) + (Sec[c + d*x]^8*(48*Cos[7*c] + (48*I)*Sin
[7*c])*(Cos[d*x] + I*Sin[d*x])^8*Sin[d*x])/(d*(a + I*a*Tan[c + d*x])^8) + (
Sec[c + d*x]^8*(-8*Cos[5*c] - (8*I)*Sin[5*c])*(Cos[d*x] + I*Sin[d*x])^8*Sin
[3*d*x])/(d*(a + I*a*Tan[c + d*x])^8) + (Sec[c + d*x]^8*((8*Cos[3*c])/5 + (
(8*I)/5)*Sin[3*c])*(Cos[d*x] + I*Sin[d*x])^8*Sin[5*d*x])/(d*(a + I*a*Tan[c
+ d*x])^8) + (Sec[c + d*x]^8*(Cos[8*c]/4 + (I/4)*Sin[8*c])*(Cos[d*x] + I*Si
n[d*x])^8)/(d*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2*(a + I*a*Tan[c +
d*x])^8) + (Sec[c + d*x]^8*(-1/4*Cos[8*c] - (I/4)*Sin[8*c])*(Cos[d*x] + I*S
in[d*x])^8)/(d*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2*(a + I*a*Tan[c +
 d*x])^8) + (8*Sec[c + d*x]^8*(Cos[d*x] + I*Sin[d*x])^8*(Cos[8*c - (d*x)/2]
/2 - Cos[8*c + (d*x)/2]/2 + (I/2)*Sin[8*c - (d*x)/2] - (I/2)*Sin[8*c + (d*x
)/2]))/(d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])*(
a + I*a*Tan[c + d*x])^8) + (8*Sec[c + d*x]^8*(Cos[d*x] + I*Sin[d*x])^8*(-1/
2*Cos[8*c - (d*x)/2] + Cos[8*c + (d*x)/2]/2 - (I/2)*Sin[8*c - (d*x)/2] + (I
/2)*Sin[8*c + (d*x)/2]))/(d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin
[c/2 + (d*x)/2])*(a + I*a*Tan[c + d*x])^8)
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.78

method	result
risch	$\frac{48i\,\mathrm{e}^{-i(dx+c)}}{a^8 d}-\frac{8i\,\mathrm{e}^{-3i(dx+c)}}{a^8 d}+\frac{8i\,\mathrm{e}^{-5i(dx+c)}}{5a^8 d}+\frac{i\left(15\,\mathrm{e}^{3i(dx+c)}+17\,\mathrm{e}^{i(dx+c)}\right)}{d\,a^8\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}-\frac{63\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2a^8 d}+\frac{63\ln\left(\mathrm{e}^{i(dx-}\right.}{2a^8 d}$
derivativedivides	$\frac{\frac{2\left(\frac{1}{4}+4i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{63\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}+\frac{2\left(\frac{1}{4}-4i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{63\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}}{a^8 d}$
default	$\frac{\frac{2\left(\frac{1}{4}+4i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{63\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}+\frac{2\left(\frac{1}{4}-4i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{63\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}}{a^8 d}$

```
[In] int(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 48*I/a^8/d*exp(-I*(d*x+c))-8*I/a^8/d*exp(-3*I*(d*x+c))+8/5*I/a^8/d*exp(-5*I
*(d*x+c))+I/d/a^8/(exp(2*I*(d*x+c))+1)^2*(15*exp(3*I*(d*x+c))+17*exp(I*(d*x
+c)))-63/2/a^8/d*ln(exp(I*(d*x+c))+I)+63/2/a^8/d*ln(exp(I*(d*x+c))-I)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.99

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{315\left(e^{(9i\,dx+9i\,c)}+2\,e^{(7i\,dx+7i\,c)}+e^{(5i\,dx+5i\,c)}\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-315\left(e^{(9i\,dx+9i\,c)}+2\,e^{(7i\,dx+7i\,c)}+e^{(5i\,dx+5i\,c)}\right.}{10\left(a^8de^{(9i\,dx+9i\,c)}+2\,a^8de^{(7i\,d}\right.}$$

```
[In] integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] -1/10*(315*(e^(9*I*d*x + 9*I*c) + 2*e^(7*I*d*x + 7*I*c) + e^(5*I*d*x + 5*I*
c))*log(e^(I*d*x + I*c) + I) - 315*(e^(9*I*d*x + 9*I*c) + 2*e^(7*I*d*x + 7*
I*c) + e^(5*I*d*x + 5*I*c))*log(e^(I*d*x + I*c) - I) - 630*I*e^(8*I*d*x + 8
*I*c) - 1050*I*e^(6*I*d*x + 6*I*c) - 336*I*e^(4*I*d*x + 4*I*c) + 48*I*e^(2*
I*d*x + 2*I*c) - 16*I)/(a^8*d*e^(9*I*d*x + 9*I*c) + 2*a^8*d*e^(7*I*d*x + 7*
I*c) + a^8*d*e^(5*I*d*x + 5*I*c))
```

Sympy [F]

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\int \frac{\sec^{11}(c+dx)}{\tan^8(c+dx)-8i\tan^7(c+dx)-28\tan^6(c+dx)+56i\tan^5(c+dx)+70\tan^4(c+dx)-56i\tan^3(c+dx)-28\tan^2(c+dx)+8i\tan(c+dx)+1}\,dx}{a^8}$$

```
[In] integrate(sec(d*x+c)**11/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Integral(sec(c + d*x)**11/(tan(c + d*x)**8 - 8*I*tan(c + d*x)**7 - 28*tan(c
 + d*x)**6 + 56*I*tan(c + d*x)**5 + 70*tan(c + d*x)**4 - 56*I*tan(c + d*x)*
*3 - 28*tan(c + d*x)**2 + 8*I*tan(c + d*x) + 1), x)/a**8
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 531 vs. 2(157) = 314.

Time = 0.34 (sec) , antiderivative size = 531, normalized size of antiderivative = 2.90

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{630\left(\cos(9\,dx+9\,c)+2\,\cos(7\,dx+7\,c)+\cos(5\,dx+5\,c)+i\,\sin(9\,dx+9\,c)+2i\,\sin(7\,dx+7\,c)+i\,\sin(\right.}{}$$

```
[In] integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] (630*(cos(9*d*x + 9*c) + 2*cos(7*d*x + 7*c) + cos(5*d*x + 5*c) + I*sin(9*d*
x + 9*c) + 2*I*sin(7*d*x + 7*c) + I*sin(5*d*x + 5*c))*arctan2(cos(d*x + c),
 sin(d*x + c) + 1) + 630*(cos(9*d*x + 9*c) + 2*cos(7*d*x + 7*c) + cos(5*d*x
 + 5*c) + I*sin(9*d*x + 9*c) + 2*I*sin(7*d*x + 7*c) + I*sin(5*d*x + 5*c))*a
rctan2(cos(d*x + c), -sin(d*x + c) + 1) + 315*(I*cos(9*d*x + 9*c) + 2*I*cos
(7*d*x + 7*c) + I*cos(5*d*x + 5*c) - sin(9*d*x + 9*c) - 2*sin(7*d*x + 7*c)
- sin(5*d*x + 5*c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) +
1) + 315*(-I*cos(9*d*x + 9*c) - 2*I*cos(7*d*x + 7*c) - I*cos(5*d*x + 5*c) +
 sin(9*d*x + 9*c) + 2*sin(7*d*x + 7*c) + sin(5*d*x + 5*c))*log(cos(d*x + c)
^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1) + 1260*cos(8*d*x + 8*c) + 2100*co
s(6*d*x + 6*c) + 672*cos(4*d*x + 4*c) - 96*cos(2*d*x + 2*c) + 1260*I*sin(8*
d*x + 8*c) + 2100*I*sin(6*d*x + 6*c) + 672*I*sin(4*d*x + 4*c) - 96*I*sin(2*
d*x + 2*c) + 32)/((-20*I*a^8*cos(9*d*x + 9*c) - 40*I*a^8*cos(7*d*x + 7*c) -
 20*I*a^8*cos(5*d*x + 5*c) + 20*a^8*sin(9*d*x + 9*c) + 40*a^8*sin(7*d*x + 7
*c) + 20*a^8*sin(5*d*x + 5*c))*d)
```

Giac [A] (verification not implemented)

none

Time = 1.81 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.90

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{\frac{315\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^8} - \frac{315\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^8} - \frac{10\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-16i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+16i\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^8} - \frac{64\left(10\cdots\right.}{}}{10\,d}$$

```
[In] integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] -1/10*(315*log(tan(1/2*d*x + 1/2*c) + 1)/a^8 - 315*log(tan(1/2*d*x + 1/2*c)
 - 1)/a^8 - 10*(tan(1/2*d*x + 1/2*c)^3 - 16*I*tan(1/2*d*x + 1/2*c)^2 + tan(
1/2*d*x + 1/2*c) + 16*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^8) - 64*(10*tan(
1/2*d*x + 1/2*c)^4 - 45*I*tan(1/2*d*x + 1/2*c)^3 - 85*tan(1/2*d*x + 1/2*c)^
2 + 55*I*tan(1/2*d*x + 1/2*c) + 13)/(a^8*(tan(1/2*d*x + 1/2*c) - I)^5))/d
```

Mupad [B] (verification not implemented)

Time = 8.67 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.55

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{63\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^8\,d} + \frac{\frac{1223\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{a^8} - \frac{1109\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{a^8} + \frac{309\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{a^8} - \frac{431\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^8} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 4407\mathrm{i}}{5\,a^8} +}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9 1\mathrm{i} + 5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7 12\mathrm{i} - 20\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 26\mathrm{i} + 26\,\mathrm{t}\right.}$$

```
[In] int(1/(cos(c + d*x)^11*(a + a*tan(c + d*x)*1i)^8),x)
```

```
[Out] ((1223*tan(c/2 + (d*x)/2)^3)/a^8 - (tan(c/2 + (d*x)/2)^2*4407i)/(5*a^8) + (
tan(c/2 + (d*x)/2)^4*7351i)/(5*a^8) - (1109*tan(c/2 + (d*x)/2)^5)/a^8 - (ta
n(c/2 + (d*x)/2)^6*761i)/a^8 + (309*tan(c/2 + (d*x)/2)^7)/a^8 + (tan(c/2 +
(d*x)/2)^8*65i)/a^8 + 496i/(5*a^8) - (431*tan(c/2 + (d*x)/2))/a^8)/(d*(tan(
c/2 + (d*x)/2)*5i - 12*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*20i + 26
*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^5*26i - 20*tan(c/2 + (d*x)/2)^6
- tan(c/2 + (d*x)/2)^7*12i + 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^9*
1i + 1)) - (63*atanh(tan(c/2 + (d*x)/2)))/(a^8*d)
```

3.178 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 156

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{a^8d} + \frac{2i\sec^7(c+dx)}{7ad(a+ia\tan(c+dx))^7} - \frac{2i\sec^5(c+dx)}{5a^3d(a+ia\tan(c+dx))^5} + \frac{2i\sec^3(c+dx)}{3a^2d\,(a^2+ia^2\tan(c+dx))^3} - \frac{2i\sec(c+dx)}{d\,(a^8+ia^8\tan(c+dx))}$$

```
[Out] arctanh(sin(d*x+c))/a^8/d+2/7*I*sec(d*x+c)^7/a/d/(a+I*a*tan(d*x+c))^7-2/5*I
*sec(d*x+c)^5/a^3/d/(a+I*a*tan(d*x+c))^5+2/3*I*sec(d*x+c)^3/a^2/d/(a^2+I*a^
2*tan(d*x+c))^3-2*I*sec(d*x+c)/d/(a^8+I*a^8*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3581, 3855}

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{a^8d} - \frac{2i\sec(c+dx)}{d\,(a^8+ia^8\tan(c+dx))} - \frac{2i\sec^5(c+dx)}{5a^3d(a+ia\tan(c+dx))^5} + \frac{2i\sec^3(c+dx)}{3a^2d\,(a^2+ia^2\tan(c+dx))^3} + \frac{2i\sec^7(c+dx)}{7ad(a+ia\tan(c+dx))^7}$$

```
[In] Int[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(a^8*d) + (((2*I)/7)*Sec[c + d*x]^7)/(a*d*(a + I*a*Ta
n[c + d*x])^7) - (((2*I)/5)*Sec[c + d*x]^5)/(a^3*d*(a + I*a*Tan[c + d*x])^5
) + (((2*I)/3)*Sec[c + d*x]^3)/(a^2*d*(a^2 + I*a^2*Tan[c + d*x])^3) - ((2*I
)*Sec[c + d*x])/(d*(a^8 + I*a^8*Tan[c + d*x]))
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2i\sec^7(c+dx)}{7ad(a+ia\tan(c+dx))^7} - \frac{\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^6}\,dx}{a^2} \\
&= \frac{2i\sec^7(c+dx)}{7ad(a+ia\tan(c+dx))^7} - \frac{2i\sec^5(c+dx)}{5a^3d(a+ia\tan(c+dx))^5} + \frac{\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^4}\,dx}{a^4} \\
&= \frac{2i\sec^7(c+dx)}{7ad(a+ia\tan(c+dx))^7} - \frac{2i\sec^5(c+dx)}{5a^3d(a+ia\tan(c+dx))^5} \\
&\quad + \frac{2i\sec^3(c+dx)}{3a^5d(a+ia\tan(c+dx))^3} - \frac{\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{a^6} \\
&= \frac{2i\sec^7(c+dx)}{7ad(a+ia\tan(c+dx))^7} - \frac{2i\sec^5(c+dx)}{5a^3d(a+ia\tan(c+dx))^5} \\
&\quad + \frac{2i\sec^3(c+dx)}{3a^5d(a+ia\tan(c+dx))^3} - \frac{2i\sec(c+dx)}{d(a^8+ia^8\tan(c+dx))} + \frac{\int \sec(c+dx)\,dx}{a^8} \\
&= \frac{\operatorname{arctanh}(\sin(c+dx))}{a^8d} + \frac{2i\sec^7(c+dx)}{7ad(a+ia\tan(c+dx))^7} - \frac{2i\sec^5(c+dx)}{5a^3d(a+ia\tan(c+dx))^5} \\
&\quad + \frac{2i\sec^3(c+dx)}{3a^5d(a+ia\tan(c+dx))^3} - \frac{2i\sec(c+dx)}{d(a^8+ia^8\tan(c+dx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.95

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{\sec^8(c+dx)\left(70i\cos\left(\frac{1}{2}(c+dx)\right)-42i\cos\left(\frac{3}{2}(c+dx)\right)-210i\cos\left(\frac{5}{2}(c+dx)\right)+30i\cos\left(\frac{7}{2}(c+dx)\right)-10\right.}{}$$

```
[In] Integrate[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (Sec[c + d*x]^8*((70*I)*Cos[(c + d*x)/2] - (42*I)*Cos[(3*(c + d*x))/2] - (2
10*I)*Cos[(5*(c + d*x))/2] + (30*I)*Cos[(7*(c + d*x))/2] - 105*Cos[(7*(c +
d*x))/2]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 105*Cos[(7*(c + d*x))/2
]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 70*Sin[(c + d*x)/2] - 42*Sin[(
3*(c + d*x))/2] + 210*Sin[(5*(c + d*x))/2] + 30*Sin[(7*(c + d*x))/2] - (105
*I)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[(7*(c + d*x))/2] + (105*I)
*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[(7*(c + d*x))/2])*(Cos[(9*(c
+ d*x))/2] + I*Sin[(9*(c + d*x))/2]))/(105*a^8*d*(-I + Tan[c + d*x])^8)
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{2i\mathrm{e}^{-i(dx+c)}}{a^8d}+\frac{2i\mathrm{e}^{-3i(dx+c)}}{3a^8d}-\frac{2i\mathrm{e}^{-5i(dx+c)}}{5a^8d}+\frac{2i\mathrm{e}^{-7i(dx+c)}}{7a^8d}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{a^8d}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{a^8d}$
derivativedivides	$\frac{-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{128i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+\frac{16i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{128i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{}{7\left(-i+\tan\right.}}{a^8d}$
default	$\frac{-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{128i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+\frac{16i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{128i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{}{7\left(-i+\tan\right.}}{a^8d}$

```
[In] int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] -2*I/a^8/d*exp(-I*(d*x+c))+2/3*I/a^8/d*exp(-3*I*(d*x+c))-2/5*I/a^8/d*exp(-5
*I*(d*x+c))+2/7*I/a^8/d*exp(-7*I*(d*x+c))-1/a^8/d*ln(exp(I*(d*x+c))-I)+1/a^
8/d*ln(exp(I*(d*x+c))+I)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.63

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\left(105\,e^{(7i\,dx+7i\,c)}\log\left(e^{(i\,dx+i\,c)}+i\right)-105\,e^{(7i\,dx+7i\,c)}\log\left(e^{(i\,dx+i\,c)}-i\right)-210i\,e^{(6i\,dx+6i\,c)}+70i\,e^{(4i\,dx+4i\,c)}-42}{105\,a^8d}$$

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/105*(105*e^(7*I*d*x + 7*I*c)*log(e^(I*d*x + I*c) + I) - 105*e^(7*I*d*x +
7*I*c)*log(e^(I*d*x + I*c) - I) - 210*I*e^(6*I*d*x + 6*I*c) + 70*I*e^(4*I*d
*x + 4*I*c) - 42*I*e^(2*I*d*x + 2*I*c) + 30*I)*e^(-7*I*d*x - 7*I*c)/(a^8*d)
```

Sympy [F]

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\int \frac{\sec^9(c+dx)}{\tan^8(c+dx)-8i\tan^7(c+dx)-28\tan^6(c+dx)+56i\tan^5(c+dx)+70\tan^4(c+dx)-56i\tan^3(c+dx)-28\tan^2(c+dx)+8i\tan(c+dx)+1}\,dx}{a^8}$$

```
[In] integrate(sec(d*x+c)**9/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Integral(sec(c + d*x)**9/(tan(c + d*x)**8 - 8*I*tan(c + d*x)**7 - 28*tan(c
+ d*x)**6 + 56*I*tan(c + d*x)**5 + 70*tan(c + d*x)**4 - 56*I*tan(c + d*x)**
3 - 28*tan(c + d*x)**2 + 8*I*tan(c + d*x) + 1), x)/a**8
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.19

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{-210i\arctan\left(\cos\left(dx+c\right),\sin\left(dx+c\right)+1\right)-210i\arctan\left(\cos\left(dx+c\right),-\sin\left(dx+c\right)+1\right)+60i\cos\left(7\right.}{}$$

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/210*(-210*I*arctan2(cos(d*x + c), sin(d*x + c) + 1) - 210*I*arctan2(cos(d
*x + c), -sin(d*x + c) + 1) + 60*I*cos(7*d*x + 7*c) - 84*I*cos(5*d*x + 5*c)
```

```
 + 140*I*cos(3*d*x + 3*c) - 420*I*cos(d*x + c) + 105*log(cos(d*x + c)^2 + s
in(d*x + c)^2 + 2*sin(d*x + c) + 1) - 105*log(cos(d*x + c)^2 + sin(d*x + c)
^2 - 2*sin(d*x + c) + 1) + 60*sin(7*d*x + 7*c) - 84*sin(5*d*x + 5*c) + 140*
sin(3*d*x + 3*c) - 420*sin(d*x + c))/(a^8*d)
```

Giac [A] (verification not implemented)

none

Time = 1.64 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.79

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\frac{105\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^8} - \frac{105\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^8} - \frac{16\left(-105i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-175\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+490i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+294\,\mathrm{t}\right.}{a^8\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^7}}{105\,d}$$

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/105*(105*log(tan(1/2*d*x + 1/2*c) + 1)/a^8 - 105*log(tan(1/2*d*x + 1/2*c)
 - 1)/a^8 - 16*(-105*I*tan(1/2*d*x + 1/2*c)^5 - 175*tan(1/2*d*x + 1/2*c)^4
+ 490*I*tan(1/2*d*x + 1/2*c)^3 + 294*tan(1/2*d*x + 1/2*c)^2 - 133*I*tan(1/2
*d*x + 1/2*c) - 19)/(a^8*(tan(1/2*d*x + 1/2*c) - I)^7))/d
```

Mupad [B] (verification not implemented)

Time = 8.00 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.33

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^8\,d} + \frac{\frac{16\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{a^8} - \frac{224\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3a^8} + \frac{304\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{15a^8} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 224\mathrm{i}}{5a^8} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^8}}{d\left(-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7 1\mathrm{i} - 7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 21\mathrm{i} + 35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 35\mathrm{i} - \right.}$$

```
[In] int(1/(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^8),x)
```

```
[Out] (2*atanh(tan(c/2 + (d*x)/2)))/(a^8*d) + ((tan(c/2 + (d*x)/2)^2*224i)/(5*a^8
) - (224*tan(c/2 + (d*x)/2)^3)/(3*a^8) - (tan(c/2 + (d*x)/2)^4*80i)/(3*a^8)
 + (16*tan(c/2 + (d*x)/2)^5)/a^8 - 304i/(105*a^8) + (304*tan(c/2 + (d*x)/2)
)/(15*a^8))/(d*(tan(c/2 + (d*x)/2)*7i - 21*tan(c/2 + (d*x)/2)^2 - tan(c/2 +
 (d*x)/2)^3*35i + 35*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^5*21i - 7*ta
n(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^7*1i + 1))
```

3.179 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 68

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{i\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^8} + \frac{i\sec^7(c+dx)}{63ad(a+ia\tan(c+dx))^7}$$

```
[Out] 1/9*I*sec(d*x+c)^7/d/(a+I*a*tan(d*x+c))^8+1/63*I*sec(d*x+c)^7/a/d/(a+I*a*ta
n(d*x+c))^7
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3583, 3569}

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{i\sec^7(c+dx)}{63ad(a+ia\tan(c+dx))^7} + \frac{i\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^8}$$

```
[In] Int[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((I/9)*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^8) + ((I/63)*Sec[c + d*x]^
7)/(a*d*(a + I*a*Tan[c + d*x])^7)
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{i\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^8} + \frac{\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^7}\,dx}{9a} \\
&= \frac{i\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^8} + \frac{i\sec^7(c+dx)}{63ad(a+ia\tan(c+dx))^7}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.59

$$
\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{\sec^7(c+dx)(-8i+\tan(c+dx))}{63a^8d(-i+\tan(c+dx))^8}
$$

```
[In] Integrate[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^8,x]
[Out] -1/63*(Sec[c + d*x]^7*(-8*I + Tan[c + d*x]))/(a^8*d*(-I + Tan[c + d*x])^8)
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.56

method	result
risch	$\frac{i e^{-7i(dx+c)}}{14a^8d} + \frac{i e^{-9i(dx+c)}}{18a^8d}$
derivativedivides	$\frac{-\frac{1856}{7\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7} - \frac{152i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4} + \frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)} - \frac{172}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3} + \frac{992i}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6} + \frac{14i}{\left(-i+\tan\left(\frac{d}{2}\right.\right.}}{a^8d}$
default	$\frac{-\frac{1856}{7\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7} - \frac{152i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4} + \frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)} - \frac{172}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3} + \frac{992i}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6} + \frac{14i}{\left(-i+\tan\left(\frac{d}{2}\right.\right.}}{a^8d}$

```
[In] int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
[Out] 1/14*I/a^8/d*exp(-7*I*(d*x+c))+1/18*I/a^8/d*exp(-9*I*(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.44

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\left(9i\,e^{(2i\,dx+2i\,c)}+7i\right)e^{(-9i\,dx-9i\,c)}}{126\,a^8d}$$

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
[Out] 1/126*(9*I*e^(2*I*d*x + 2*I*c) + 7*I)*e^(-9*I*d*x - 9*I*c)/(a^8*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(54) = 108.

Time = 8.97 (sec) , antiderivative size = 311, normalized size of antiderivative = 4.57

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \begin{cases} -\frac{\tan(c+dx)\sec^7(c+dx)}{63a^8d\tan^8(c+dx)-504ia^8d\tan^7(c+dx)-1764a^8d\tan^6(c+dx)+3528ia^8d\tan^5(c+dx)+4410a^8d\tan^4(c+dx)-3528ia^8d\tan^3(c+dx)-1764a} \\ \frac{x\sec^7(c)}{(ia\tan(c)+a)^8} \end{cases}$$

```
[In] integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c))**8,x)
[Out] Piecewise((-tan(c + d*x)*sec(c + d*x)**7/(63*a**8*d*tan(c + d*x)**8 - 504*I
*a**8*d*tan(c + d*x)**7 - 1764*a**8*d*tan(c + d*x)**6 + 3528*I*a**8*d*tan(c
 + d*x)**5 + 4410*a**8*d*tan(c + d*x)**4 - 3528*I*a**8*d*tan(c + d*x)**3 -
1764*a**8*d*tan(c + d*x)**2 + 504*I*a**8*d*tan(c + d*x) + 63*a**8*d) + 8*I*
sec(c + d*x)**7/(63*a**8*d*tan(c + d*x)**8 - 504*I*a**8*d*tan(c + d*x)**7 -
 1764*a**8*d*tan(c + d*x)**6 + 3528*I*a**8*d*tan(c + d*x)**5 + 4410*a**8*d*
tan(c + d*x)**4 - 3528*I*a**8*d*tan(c + d*x)**3 - 1764*a**8*d*tan(c + d*x)*
*2 + 504*I*a**8*d*tan(c + d*x) + 63*a**8*d), Ne(d, 0)), (x*sec(c)**7/(I*a*t
an(c) + a)**8, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.78

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{7i\,\cos\left(9\,dx+9\,c\right)+9i\,\cos\left(7\,dx+7\,c\right)+7\,\sin\left(9\,dx+9\,c\right)+9\,\sin\left(7\,dx+7\,c\right)}{126\,a^8d}$$

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/126*(7*I*cos(9*d*x + 9*c) + 9*I*cos(7*d*x + 7*c) + 7*sin(9*d*x + 9*c) + 9
*sin(7*d*x + 7*c))/(a^8*d)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(56) = 112.

Time = 1.49 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.84

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{2\left(63\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-63i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-483\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+315i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+693\,\mathrm{ta}\right.}{63\,a^8d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}$$

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 2/63*(63*tan(1/2*d*x + 1/2*c)^8 - 63*I*tan(1/2*d*x + 1/2*c)^7 - 483*tan(1/2
*d*x + 1/2*c)^6 + 315*I*tan(1/2*d*x + 1/2*c)^5 + 693*tan(1/2*d*x + 1/2*c)^4
 - 189*I*tan(1/2*d*x + 1/2*c)^3 - 225*tan(1/2*d*x + 1/2*c)^2 + 9*I*tan(1/2*
d*x + 1/2*c) + 8)/(a^8*d*(tan(1/2*d*x + 1/2*c) - I)^9)
```

Mupad [B] (verification not implemented)

Time = 4.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.54

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{2\left(\frac{\mathrm{e}^{-c\,7\mathrm{i}-d\,x\,7\mathrm{i}}\,9\mathrm{i}}{4}+\frac{\mathrm{e}^{-c\,9\mathrm{i}-d\,x\,9\mathrm{i}}\,7\mathrm{i}}{4}\right)}{63\,a^8\,d}$$

```
[In] int(1/(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^8),x)
```

```
[Out] (2*((exp(- c*7i - d*x*7i)*9i)/4 + (exp(- c*9i - d*x*9i)*7i)/4))/(63*a^8*d)
```

3.180 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^8} dx$

Optimal result

Integrand size = 24, antiderivative size = 138

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^8} dx = \frac{i\sec^5(c+dx)}{11d(a+ia\tan(c+dx))^8} + \frac{i\sec^5(c+dx)}{33ad(a+ia\tan(c+dx))^7} + \frac{2i\sec^5(c+dx)}{231a^2d(a+ia\tan(c+dx))^6} + \frac{2i\sec^5(c+dx)}{1155a^3d(a+ia\tan(c+dx))^5}$$

```
[Out] 1/11*I*sec(d*x+c)^5/d/(a+I*a*tan(d*x+c))^8+1/33*I*sec(d*x+c)^5/a/d/(a+I*a*t
an(d*x+c))^7+2/231*I*sec(d*x+c)^5/a^2/d/(a+I*a*tan(d*x+c))^6+2/1155*I*sec(d
*x+c)^5/a^3/d/(a+I*a*tan(d*x+c))^5
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3583, 3569}

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^8} dx = \frac{2i\sec^5(c+dx)}{1155a^3d(a+ia\tan(c+dx))^5} + \frac{2i\sec^5(c+dx)}{231a^2d(a+ia\tan(c+dx))^6} + \frac{i\sec^5(c+dx)}{33ad(a+ia\tan(c+dx))^7} + \frac{i\sec^5(c+dx)}{11d(a+ia\tan(c+dx))^8}$$

```
[In] Int[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((I/11)*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^8) + ((I/33)*Sec[c + d*x]
^5)/(a*d*(a + I*a*Tan[c + d*x])^7) + (((2*I)/231)*Sec[c + d*x]^5)/(a^2*d*(a
```

```
 + I*a*Tan[c + d*x])^6) + (((2*I)/1155)*Sec[c + d*x]^5)/(a^3*d*(a + I*a*Tan
[c + d*x])^5)
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{i\sec^5(c+dx)}{11d(a+ia\tan(c+dx))^8}+\frac{3\int\frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^7}\,dx}{11a}\\
&= \frac{i\sec^5(c+dx)}{11d(a+ia\tan(c+dx))^8}+\frac{i\sec^5(c+dx)}{33ad(a+ia\tan(c+dx))^7}+\frac{2\int\frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^6}\,dx}{33a^2}\\
&= \frac{i\sec^5(c+dx)}{11d(a+ia\tan(c+dx))^8}+\frac{i\sec^5(c+dx)}{33ad(a+ia\tan(c+dx))^7}\\
&\quad+\frac{2i\sec^5(c+dx)}{231a^2d(a+ia\tan(c+dx))^6}+\frac{2\int\frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^5}\,dx}{231a^3}\\
&= \frac{i\sec^5(c+dx)}{11d(a+ia\tan(c+dx))^8}+\frac{i\sec^5(c+dx)}{33ad(a+ia\tan(c+dx))^7}\\
&\quad+\frac{2i\sec^5(c+dx)}{231a^2d(a+ia\tan(c+dx))^6}+\frac{2i\sec^5(c+dx)}{1155a^3d(a+ia\tan(c+dx))^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.53

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{i\sec^8(c+dx)(440\cos(c+dx)+168\cos(3(c+dx))+55i\sin(c+dx)+63i\sin(3(c+dx)))}{4620a^8d(-i+\tan(c+dx))^8}$$

```
[In] Integrate[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((I/4620)*Sec[c + d*x]^8*(440*Cos[c + d*x] + 168*Cos[3*(c + d*x)] + (55*I)*
Sin[c + d*x] + (63*I)*Sin[3*(c + d*x)]))/(a^8*d*(-I + Tan[c + d*x])^8)
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.54

method	result
risch	$\frac{ie^{-5i(dx+c)}}{40a^8d}+\frac{3ie^{-7i(dx+c)}}{56a^8d}+\frac{ie^{-9i(dx+c)}}{24a^8d}+\frac{ie^{-11i(dx+c)}}{88a^8d}$
derivativedivides	$\frac{\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{576i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}-\frac{4752}{7\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}+\frac{1024}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9}-\frac{176i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{1864}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{a^8d}$
default	$\frac{\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{576i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}-\frac{4752}{7\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}+\frac{1024}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9}-\frac{176i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{1864}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{a^8d}$

```
[In] int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/40*I/a^8/d*exp(-5*I*(d*x+c))+3/56*I/a^8/d*exp(-7*I*(d*x+c))+1/24*I/a^8/d*
exp(-9*I*(d*x+c))+1/88*I/a^8/d*exp(-11*I*(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.38

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\left(231i\,e^{(6i\,dx+6i\,c)}+495i\,e^{(4i\,dx+4i\,c)}+385i\,e^{(2i\,dx+2i\,c)}+105i\right)e^{(-11i\,dx-11i\,c)}}{9240\,a^8d}$$

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/9240*(231*I*e^(6*I*d*x + 6*I*c) + 495*I*e^(4*I*d*x + 4*I*c) + 385*I*e^(2*
I*d*x + 2*I*c) + 105*I)*e^(-11*I*d*x - 11*I*c)/(a^8*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 620 vs. 2(119) = 238.

Time = 8.79 (sec) , antiderivative size = 620, normalized size of antiderivative = 4.49

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \begin{cases} \frac{2\tan^3(c+dx)\sec^5(c+dx)}{1155a^8d\tan^8(c+dx)-9240ia^8d\tan^7(c+dx)-32340a^8d\tan^6(c+dx)+64680ia^8d\tan^5(c+dx)+80850a^8d\tan^4(c+dx)-64680ia^8d\tan^3(c+dx)} \\ \frac{x\sec^5(c)}{(ia\tan(c)+a)^8} \end{cases}$$

```
[In] integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise((2*tan(c + d*x)**3*sec(c + d*x)**5/(1155*a**8*d*tan(c + d*x)**8 -
 9240*I*a**8*d*tan(c + d*x)**7 - 32340*a**8*d*tan(c + d*x)**6 + 64680*I*a**
8*d*tan(c + d*x)**5 + 80850*a**8*d*tan(c + d*x)**4 - 64680*I*a**8*d*tan(c +
 d*x)**3 - 32340*a**8*d*tan(c + d*x)**2 + 9240*I*a**8*d*tan(c + d*x) + 1155
*a**8*d) - 16*I*tan(c + d*x)**2*sec(c + d*x)**5/(1155*a**8*d*tan(c + d*x)**
8 - 9240*I*a**8*d*tan(c + d*x)**7 - 32340*a**8*d*tan(c + d*x)**6 + 64680*I*
a**8*d*tan(c + d*x)**5 + 80850*a**8*d*tan(c + d*x)**4 - 64680*I*a**8*d*tan(
c + d*x)**3 - 32340*a**8*d*tan(c + d*x)**2 + 9240*I*a**8*d*tan(c + d*x) + 1
155*a**8*d) - 61*tan(c + d*x)*sec(c + d*x)**5/(1155*a**8*d*tan(c + d*x)**8
- 9240*I*a**8*d*tan(c + d*x)**7 - 32340*a**8*d*tan(c + d*x)**6 + 64680*I*a*
*8*d*tan(c + d*x)**5 + 80850*a**8*d*tan(c + d*x)**4 - 64680*I*a**8*d*tan(c
+ d*x)**3 - 32340*a**8*d*tan(c + d*x)**2 + 9240*I*a**8*d*tan(c + d*x) + 115
5*a**8*d) + 152*I*sec(c + d*x)**5/(1155*a**8*d*tan(c + d*x)**8 - 9240*I*a**
8*d*tan(c + d*x)**7 - 32340*a**8*d*tan(c + d*x)**6 + 64680*I*a**8*d*tan(c +
 d*x)**5 + 80850*a**8*d*tan(c + d*x)**4 - 64680*I*a**8*d*tan(c + d*x)**3 -
32340*a**8*d*tan(c + d*x)**2 + 9240*I*a**8*d*tan(c + d*x) + 1155*a**8*d), N
e(d, 0)), (x*sec(c)**5/(I*a*tan(c) + a)**8, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.70

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{105i\cos(11\,dx+11\,c)+385i\cos(9\,dx+9\,c)+495i\cos(7\,dx+7\,c)+231i\cos(5\,dx+5\,c)+105\sin(1}{9240\,a^8d}$$

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/9240*(105*I*cos(11*d*x + 11*c) + 385*I*cos(9*d*x + 9*c) + 495*I*cos(7*d*x
 + 7*c) + 231*I*cos(5*d*x + 5*c) + 105*sin(11*d*x + 11*c) + 385*sin(9*d*x +
 9*c) + 495*sin(7*d*x + 7*c) + 231*sin(5*d*x + 5*c))/(a^8*d)
```

Giac [A] (verification not implemented)

none

Time = 1.42 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.09

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{2\left(1155\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-3465i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-13860\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+23100i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}\right.}{}$$

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 2/1155*(1155*tan(1/2*d*x + 1/2*c)^10 - 3465*I*tan(1/2*d*x + 1/2*c)^9 - 1386
0*tan(1/2*d*x + 1/2*c)^8 + 23100*I*tan(1/2*d*x + 1/2*c)^7 + 37422*tan(1/2*d
*x + 1/2*c)^6 - 32802*I*tan(1/2*d*x + 1/2*c)^5 - 27060*tan(1/2*d*x + 1/2*c)
^4 + 11220*I*tan(1/2*d*x + 1/2*c)^3 + 4895*tan(1/2*d*x + 1/2*c)^2 - 517*I*t
an(1/2*d*x + 1/2*c) - 152)/(a^8*d*(tan(1/2*d*x + 1/2*c) - I)^11)
```

Mupad [B] (verification not implemented)

Time = 4.38 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.46

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\frac{e^{-c\,5i-d\,x\,5i}\,1i}{40}+\frac{e^{-c\,7i-d\,x\,7i}\,3i}{56}+\frac{e^{-c\,9i-d\,x\,9i}\,1i}{24}+\frac{e^{-c\,11i-d\,x\,11i}\,1i}{88}}{a^8\,d}$$

```
[In] int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^8),x)
```

```
[Out] ((exp(- c*5i - d*x*5i)*1i)/40 + (exp(- c*7i - d*x*7i)*3i)/56 + (exp(- c*9i
- d*x*9i)*1i)/24 + (exp(- c*11i - d*x*11i)*1i)/88)/(a^8*d)
```

3.181 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^8} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 213

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^8} \, dx = \frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8} + \frac{5i\sec^3(c+dx)}{143ad(a+ia\tan(c+dx))^7} + \frac{20i\sec^3(c+dx)}{1287a^2d(a+ia\tan(c+dx))^6} + \frac{20i\sec^3(c+dx)}{3003a^3d(a+ia\tan(c+dx))^5} + \frac{8i\sec^3(c+dx)}{3003d\left(a^2+ia^2\tan(c+dx)\right)^4} + \frac{8i\sec^3(c+dx)}{9009a^2d\left(a^2+ia^2\tan(c+dx)\right)^3}$$

```
[Out] 1/13*I*sec(d*x+c)^3/d/(a+I*a*tan(d*x+c))^8+5/143*I*sec(d*x+c)^3/a/d/(a+I*a*
tan(d*x+c))^7+20/1287*I*sec(d*x+c)^3/a^2/d/(a+I*a*tan(d*x+c))^6+20/3003*I*s
ec(d*x+c)^3/a^3/d/(a+I*a*tan(d*x+c))^5+8/3003*I*sec(d*x+c)^3/d/(a^2+I*a^2*t
an(d*x+c))^4+8/9009*I*sec(d*x+c)^3/a^2/d/(a^2+I*a^2*tan(d*x+c))^3
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used

$= \{3583, 3569\}$

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{20i\sec^3(c+dx)}{3003a^3d(a+ia\tan(c+dx))^5} + \frac{8i\sec^3(c+dx)}{9009a^2d\left(a^2+ia^2\tan(c+dx)\right)^3} + \frac{8i\sec^3(c+dx)}{3003d\left(a^2+ia^2\tan(c+dx)\right)^4} + \frac{20i\sec^3(c+dx)}{1287a^2d(a+ia\tan(c+dx))^6} + \frac{5i\sec^3(c+dx)}{143ad(a+ia\tan(c+dx))^7} + \frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8}$$

```
[In] Int[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((I/13)*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^8) + (((5*I)/143)*Sec[c +
 d*x]^3)/(a*d*(a + I*a*Tan[c + d*x])^7) + (((20*I)/1287)*Sec[c + d*x]^3)/(a
^2*d*(a + I*a*Tan[c + d*x])^6) + (((20*I)/3003)*Sec[c + d*x]^3)/(a^3*d*(a +
 I*a*Tan[c + d*x])^5) + (((8*I)/3003)*Sec[c + d*x]^3)/(d*(a^2 + I*a^2*Tan[c
 + d*x])^4) + (((8*I)/9009)*Sec[c + d*x]^3)/(a^2*d*(a^2 + I*a^2*Tan[c + d*x
])^3)
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\text{integral} = \frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8} + \frac{5\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^7}\,dx}{13a}$$

$$= \frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8} + \frac{5i\sec^3(c+dx)}{143ad(a+ia\tan(c+dx))^7} + \frac{20\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^6}\,dx}{143a^2}$$

$$= \frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8} + \frac{5i\sec^3(c+dx)}{143ad(a+ia\tan(c+dx))^7} + \frac{20i\sec^3(c+dx)}{1287a^2d(a+ia\tan(c+dx))^6} + \frac{20\int\frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^5}\,dx}{429a^3}$$

$$= \frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8} + \frac{5i\sec^3(c+dx)}{143ad(a+ia\tan(c+dx))^7} + \frac{20i\sec^3(c+dx)}{1287a^2d(a+ia\tan(c+dx))^6} + \frac{20i\sec^3(c+dx)}{3003a^3d(a+ia\tan(c+dx))^5} + \frac{40\int\frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx}{3003a^4}$$

$$= \frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8} + \frac{5i\sec^3(c+dx)}{143ad(a+ia\tan(c+dx))^7} + \frac{20i\sec^3(c+dx)}{1287a^2d(a+ia\tan(c+dx))^6} + \frac{20i\sec^3(c+dx)}{3003a^3d(a+ia\tan(c+dx))^5} + \frac{8i\sec^3(c+dx)}{3003d\left(a^2+ia^2\tan(c+dx)\right)^4} + \frac{8\int\frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx}{3003a^5}$$

$$= \frac{i\sec^3(c+dx)}{13d(a+ia\tan(c+dx))^8} + \frac{5i\sec^3(c+dx)}{143ad(a+ia\tan(c+dx))^7} + \frac{20i\sec^3(c+dx)}{1287a^2d(a+ia\tan(c+dx))^6} + \frac{20i\sec^3(c+dx)}{3003a^3d(a+ia\tan(c+dx))^5} + \frac{8i\sec^3(c+dx)}{9009a^5d(a+ia\tan(c+dx))^3} + \frac{8i\sec^3(c+dx)}{3003d\left(a^2+ia^2\tan(c+dx)\right)^4}$$

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.45

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{i\sec^8(c+dx)(11440\cos(c+dx)+6552\cos(3(c+dx))+1848\cos(5(c+dx))+1430i\sin(c+dx)+2457}{144144a^8d(-i+\tan(c+dx))^8}$$

```
[In] Integrate[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((I/144144)*Sec[c + d*x]^8*(11440*Cos[c + d*x] + 6552*Cos[3*(c + d*x)] + 18
48*Cos[5*(c + d*x)] + (1430*I)*Sin[c + d*x] + (2457*I)*Sin[3*(c + d*x)] + (
1155*I)*Sin[5*(c + d*x)]))/(a^8*d*(-I + Tan[c + d*x])^8)
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.52

method	result
risch	$\frac{i e^{-3i(dx+c)}}{96a^8d}+\frac{i e^{-5i(dx+c)}}{32a^8d}+\frac{5i e^{-7i(dx+c)}}{112a^8d}+\frac{5i e^{-9i(dx+c)}}{144a^8d}+\frac{5i e^{-11i(dx+c)}}{352a^8d}+\frac{i e^{-13i(dx+c)}}{416a^8d}$
derivativedivides	$-\frac{188}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{256}{13\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{13}}+\frac{480}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}+\frac{2672i}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{9056}{7\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}+\frac{}{9(-i+\text{t}}$
default	$-\frac{188}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{256}{13\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{13}}+\frac{480}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}+\frac{2672i}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{9056}{7\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}+\frac{}{9(-i+\text{t}}$

```
[In] int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/96*I/a^8/d*exp(-3*I*(d*x+c))+1/32*I/a^8/d*exp(-5*I*(d*x+c))+5/112*I/a^8/d
*exp(-7*I*(d*x+c))+5/144*I/a^8/d*exp(-9*I*(d*x+c))+5/352*I/a^8/d*exp(-11*I*
(d*x+c))+1/416*I/a^8/d*exp(-13*I*(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.35

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\left(3003i\,e^{(10i\,dx+10i\,c)}+9009i\,e^{(8i\,dx+8i\,c)}+12870i\,e^{(6i\,dx+6i\,c)}+10010i\,e^{(4i\,dx+4i\,c)}+4095i\,e^{(2i\,dx+2i\,c)}+693i\right)e^{(-}}{288288\,a^8d}$$

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/288288*(3003*I*e^(10*I*d*x + 10*I*c) + 9009*I*e^(8*I*d*x + 8*I*c) + 12870
*I*e^(6*I*d*x + 6*I*c) + 10010*I*e^(4*I*d*x + 4*I*c) + 4095*I*e^(2*I*d*x +
2*I*c) + 693*I)*e^(-13*I*d*x - 13*I*c)/(a^8*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 928 vs. 2(189) = 378.

Time = 8.74 (sec) , antiderivative size = 928, normalized size of antiderivative = 4.36

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise((-8*tan(c + d*x)**5*sec(c + d*x)**3/(9009*a**8*d*tan(c + d*x)**8
- 72072*I*a**8*d*tan(c + d*x)**7 - 252252*a**8*d*tan(c + d*x)**6 + 504504*I
*a**8*d*tan(c + d*x)**5 + 630630*a**8*d*tan(c + d*x)**4 - 504504*I*a**8*d*t
an(c + d*x)**3 - 252252*a**8*d*tan(c + d*x)**2 + 72072*I*a**8*d*tan(c + d*x
) + 9009*a**8*d) + 64*I*tan(c + d*x)**4*sec(c + d*x)**3/(9009*a**8*d*tan(c
+ d*x)**8 - 72072*I*a**8*d*tan(c + d*x)**7 - 252252*a**8*d*tan(c + d*x)**6
+ 504504*I*a**8*d*tan(c + d*x)**5 + 630630*a**8*d*tan(c + d*x)**4 - 504504*
I*a**8*d*tan(c + d*x)**3 - 252252*a**8*d*tan(c + d*x)**2 + 72072*I*a**8*d*t
an(c + d*x) + 9009*a**8*d) + 236*tan(c + d*x)**3*sec(c + d*x)**3/(9009*a**8
*d*tan(c + d*x)**8 - 72072*I*a**8*d*tan(c + d*x)**7 - 252252*a**8*d*tan(c +
 d*x)**6 + 504504*I*a**8*d*tan(c + d*x)**5 + 630630*a**8*d*tan(c + d*x)**4
- 504504*I*a**8*d*tan(c + d*x)**3 - 252252*a**8*d*tan(c + d*x)**2 + 72072*I
*a**8*d*tan(c + d*x) + 9009*a**8*d) - 544*I*tan(c + d*x)**2*sec(c + d*x)**3
/(9009*a**8*d*tan(c + d*x)**8 - 72072*I*a**8*d*tan(c + d*x)**7 - 252252*a**
8*d*tan(c + d*x)**6 + 504504*I*a**8*d*tan(c + d*x)**5 + 630630*a**8*d*tan(c
 + d*x)**4 - 504504*I*a**8*d*tan(c + d*x)**3 - 252252*a**8*d*tan(c + d*x)**
2 + 72072*I*a**8*d*tan(c + d*x) + 9009*a**8*d) - 911*tan(c + d*x)*sec(c + d
*x)**3/(9009*a**8*d*tan(c + d*x)**8 - 72072*I*a**8*d*tan(c + d*x)**7 - 2522
52*a**8*d*tan(c + d*x)**6 + 504504*I*a**8*d*tan(c + d*x)**5 + 630630*a**8*d
*tan(c + d*x)**4 - 504504*I*a**8*d*tan(c + d*x)**3 - 252252*a**8*d*tan(c +
d*x)**2 + 72072*I*a**8*d*tan(c + d*x) + 9009*a**8*d) + 1240*I*sec(c + d*x)*
*3/(9009*a**8*d*tan(c + d*x)**8 - 72072*I*a**8*d*tan(c + d*x)**7 - 252252*a
**8*d*tan(c + d*x)**6 + 504504*I*a**8*d*tan(c + d*x)**5 + 630630*a**8*d*tan
(c + d*x)**4 - 504504*I*a**8*d*tan(c + d*x)**3 - 252252*a**8*d*tan(c + d*x)
**2 + 72072*I*a**8*d*tan(c + d*x) + 9009*a**8*d), Ne(d, 0)), (x*sec(c)**3/(
I*a*tan(c) + a)**8, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.66

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{693i\cos(13\,dx+13\,c)+4095i\cos(11\,dx+11\,c)+10010i\cos(9\,dx+9\,c)+12870i\cos(7\,dx+7\,c)+90}{}$$

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/288288*(693*I*cos(13*d*x + 13*c) + 4095*I*cos(11*d*x + 11*c) + 10010*I*co
s(9*d*x + 9*c) + 12870*I*cos(7*d*x + 7*c) + 9009*I*cos(5*d*x + 5*c) + 3003*
I*cos(3*d*x + 3*c) + 693*sin(13*d*x + 13*c) + 4095*sin(11*d*x + 11*c) + 100
10*sin(9*d*x + 9*c) + 12870*sin(7*d*x + 7*c) + 9009*sin(5*d*x + 5*c) + 3003
*sin(3*d*x + 3*c))/(a^8*d)
```

Giac [A] (verification not implemented)

none

Time = 1.27 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.83

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{2\left(9009\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}-45045i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-183183\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+435435i\tan\left(\frac{1}{2}\,dx+\right.}{}$$

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 2/9009*(9009*tan(1/2*d*x + 1/2*c)^12 - 45045*I*tan(1/2*d*x + 1/2*c)^11 - 18
3183*tan(1/2*d*x + 1/2*c)^10 + 435435*I*tan(1/2*d*x + 1/2*c)^9 + 810810*tan
(1/2*d*x + 1/2*c)^8 - 1051050*I*tan(1/2*d*x + 1/2*c)^7 - 1076790*tan(1/2*d*
x + 1/2*c)^6 + 785070*I*tan(1/2*d*x + 1/2*c)^5 + 451165*tan(1/2*d*x + 1/2*c
)^4 - 171457*I*tan(1/2*d*x + 1/2*c)^3 - 51675*tan(1/2*d*x + 1/2*c)^2 + 7111
*I*tan(1/2*d*x + 1/2*c) + 1240)/(a^8*d*(tan(1/2*d*x + 1/2*c) - I)^13)
```

Mupad [B] (verification not implemented)

Time = 5.17 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.75

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\frac{\cos(3\,c+3\,d\,x)^3\,5\mathrm{i}}{36}+\frac{5\,\sin(3\,c+3\,d\,x)\cos(3\,c+3\,d\,x)^2}{36}-\frac{\cos(3\,c+3\,d\,x)\,3\mathrm{i}}{32}+\frac{\cos(5\,c+5\,d\,x)\,1\mathrm{i}}{32}+\frac{\cos(7\,c+7\,d\,x)\,5\mathrm{i}}{112}+\frac{\cos(11\,c+11\,d\,x)\,5\mathrm{i}}{352}+\cos}{a^8\,d}$$

```
[In] int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^8),x)
```

```
[Out] ((cos(5*c + 5*d*x)*1i)/32 - (cos(3*c + 3*d*x)*3i)/32 + (cos(7*c + 7*d*x)*5i
)/112 + (cos(11*c + 11*d*x)*5i)/352 + (cos(13*c + 13*d*x)*1i)/416 - (7*sin(
3*c + 3*d*x))/288 + sin(5*c + 5*d*x)/32 + (5*sin(7*c + 7*d*x))/112 + (5*sin
(11*c + 11*d*x))/352 + sin(13*c + 13*d*x)/416 + (cos(3*c + 3*d*x)^3*5i)/36
+ (5*cos(3*c + 3*d*x)^2*sin(3*c + 3*d*x))/36)/(a^8*d)
```

3.182 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 269

$$
\begin{aligned}
\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx =& \frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8}+\frac{7i\sec(c+dx)}{195ad(a+ia\tan(c+dx))^7}\\
&+\frac{14i\sec(c+dx)}{715a^2d(a+ia\tan(c+dx))^6}\\
&+\frac{14i\sec(c+dx)}{1287a^3d(a+ia\tan(c+dx))^5}\\
&+\frac{8i\sec(c+dx)}{1287d\,(a^2+ia^2\tan(c+dx))^4}\\
&+\frac{8i\sec(c+dx)}{2145a^2d\,(a^2+ia^2\tan(c+dx))^3}\\
&+\frac{16i\sec(c+dx)}{6435d\,(a^4+ia^4\tan(c+dx))^2}\\
&+\frac{16i\sec(c+dx)}{6435d\,(a^8+ia^8\tan(c+dx))}
\end{aligned}
$$

```
[Out] 1/15*I*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^8+7/195*I*sec(d*x+c)/a/d/(a+I*a*tan(
d*x+c))^7+14/715*I*sec(d*x+c)/a^2/d/(a+I*a*tan(d*x+c))^6+14/1287*I*sec(d*x+
c)/a^3/d/(a+I*a*tan(d*x+c))^5+8/1287*I*sec(d*x+c)/d/(a^2+I*a^2*tan(d*x+c))^
4+8/2145*I*sec(d*x+c)/a^2/d/(a^2+I*a^2*tan(d*x+c))^3+16/6435*I*sec(d*x+c)/d
/(a^4+I*a^4*tan(d*x+c))^2+16/6435*I*sec(d*x+c)/d/(a^8+I*a^8*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3583, 3569}

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{16i\sec(c+dx)}{6435d\left(a^8+ia^8\tan(c+dx)\right)}+\frac{16i\sec(c+dx)}{6435d\left(a^4+ia^4\tan(c+dx)\right)^2} + \frac{14i\sec(c+dx)}{1287a^3d(a+ia\tan(c+dx))^5} + \frac{8i\sec(c+dx)}{2145a^2d\left(a^2+ia^2\tan(c+dx)\right)^3} + \frac{8i\sec(c+dx)}{1287d\left(a^2+ia^2\tan(c+dx)\right)^4} + \frac{14i\sec(c+dx)}{715a^2d(a+ia\tan(c+dx))^6} + \frac{7i\sec(c+dx)}{195ad(a+ia\tan(c+dx))^7}+\frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8}$$

```
[In] Int[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((I/15)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^8) + (((7*I)/195)*Sec[c + d
*x])/(a*d*(a + I*a*Tan[c + d*x])^7) + (((14*I)/715)*Sec[c + d*x])/(a^2*d*(a
 + I*a*Tan[c + d*x])^6) + (((14*I)/1287)*Sec[c + d*x])/(a^3*d*(a + I*a*Tan[
c + d*x])^5) + (((8*I)/1287)*Sec[c + d*x])/(d*(a^2 + I*a^2*Tan[c + d*x])^4)
 + (((8*I)/2145)*Sec[c + d*x])/(a^2*d*(a^2 + I*a^2*Tan[c + d*x])^3) + (((16
*I)/6435)*Sec[c + d*x])/(d*(a^4 + I*a^4*Tan[c + d*x])^2) + (((16*I)/6435)*S
ec[c + d*x])/(d*(a^8 + I*a^8*Tan[c + d*x]))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\text{integral} = \frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8} + \frac{7\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^7}\,dx}{15a}$$

$$= \frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8} + \frac{7i\sec(c+dx)}{195ad(a+ia\tan(c+dx))^7} + \frac{14\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^6}\,dx}{65a^2}$$

$$= \frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8} + \frac{7i\sec(c+dx)}{195ad(a+ia\tan(c+dx))^7} + \frac{14i\sec(c+dx)}{715a^2d(a+ia\tan(c+dx))^6} + \frac{14\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^5}\,dx}{143a^3}$$

$$= \frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8} + \frac{7i\sec(c+dx)}{195ad(a+ia\tan(c+dx))^7} + \frac{14i\sec(c+dx)}{715a^2d(a+ia\tan(c+dx))^6} + \frac{14i\sec(c+dx)}{1287a^3d(a+ia\tan(c+dx))^5} + \frac{56\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^4}\,dx}{1287a^4}$$

$$= \frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8} + \frac{7i\sec(c+dx)}{195ad(a+ia\tan(c+dx))^7} + \frac{14i\sec(c+dx)}{715a^2d(a+ia\tan(c+dx))^6} + \frac{14i\sec(c+dx)}{1287a^3d(a+ia\tan(c+dx))^5} + \frac{8i\sec(c+dx)}{1287d\left(a^2+ia^2\tan(c+dx)\right)^4} + \frac{8\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^3}\,dx}{429a^5}$$

$$= \frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8} + \frac{7i\sec(c+dx)}{195ad(a+ia\tan(c+dx))^7} + \frac{14i\sec(c+dx)}{715a^2d(a+ia\tan(c+dx))^6} + \frac{14i\sec(c+dx)}{1287a^3d(a+ia\tan(c+dx))^5} + \frac{8i\sec(c+dx)}{2145a^5d(a+ia\tan(c+dx))^3} + \frac{8i\sec(c+dx)}{1287d\left(a^2+ia^2\tan(c+dx)\right)^4} + \frac{16\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{2145a^6}$$

$$= \frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8} + \frac{7i\sec(c+dx)}{195ad(a+ia\tan(c+dx))^7} + \frac{14i\sec(c+dx)}{715a^2d(a+ia\tan(c+dx))^6} + \frac{14i\sec(c+dx)}{1287a^3d(a+ia\tan(c+dx))^5} + \frac{8i\sec(c+dx)}{2145a^5d(a+ia\tan(c+dx))^3} + \frac{8i\sec(c+dx)}{1287d\left(a^2+ia^2\tan(c+dx)\right)^4} + \frac{16i\sec(c+dx)}{6435d\left(a^4+ia^4\tan(c+dx)\right)^2} + \frac{16\int \frac{\sec(c+dx)}{a+ia\tan(c+dx)}\,dx}{6435a^7}$$

$$= \frac{i\sec(c+dx)}{15d(a+ia\tan(c+dx))^8} + \frac{7i\sec(c+dx)}{195ad(a+ia\tan(c+dx))^7} + \frac{14i\sec(c+dx)}{715a^2d(a+ia\tan(c+dx))^6} + \frac{14i\sec(c+dx)}{1287a^3d(a+ia\tan(c+dx))^5} + \frac{8i\sec(c+dx)}{2145a^5d(a+ia\tan(c+dx))^3} + \frac{8i\sec(c+dx)}{1287d\left(a^2+ia^2\tan(c+dx)\right)^4} + \frac{16i\sec(c+dx)}{6435d\left(a^4+ia^4\tan(c+dx)\right)^2} + \frac{16i\sec(c+dx)}{6435d\left(a^8+ia^8\tan(c+dx)\right)}$$

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.43

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{i\sec^8(c+dx)(28600\cos(c+dx)+19656\cos(3(c+dx))+9240\cos(5(c+dx))+3432\cos(7(c+dx))+35}{411840a^8d(-i+\tan(c+a}$$

```
[In] Integrate[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((I/411840)*Sec[c + d*x]^8*(28600*Cos[c + d*x] + 19656*Cos[3*(c + d*x)] + 9
240*Cos[5*(c + d*x)] + 3432*Cos[7*(c + d*x)] + (3575*I)*Sin[c + d*x] + (737
1*I)*Sin[3*(c + d*x)] + (5775*I)*Sin[5*(c + d*x)] + (3003*I)*Sin[7*(c + d*x
)]))/(a^8*d*(-I + Tan[c + d*x])^8)
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.54

method	result
risch	$\frac{i e^{-i(dx+c)}}{128a^8d} + \frac{7i e^{-3i(dx+c)}}{384a^8d} + \frac{21i e^{-5i(dx+c)}}{640a^8d} + \frac{5i e^{-7i(dx+c)}}{128a^8d} + \frac{35i e^{-9i(dx+c)}}{1152a^8d} + \frac{21i e^{-11i(dx+c)}}{1408a^8d} + \frac{7i e^{-13i(dx+c)}}{1664a^8d}$
derivativedivides	$\frac{29792}{9\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9} + \frac{15008i}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{10}} - \frac{196}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3} - \frac{224i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4} - \frac{3584i}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{12}} - \frac{29}{\left(-i+\tan\right.}$
default	$\frac{29792}{9\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9} + \frac{15008i}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{10}} - \frac{196}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3} - \frac{224i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4} - \frac{3584i}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{12}} - \frac{29}{\left(-i+\tan\right.}$

```
[In] int(sec(d*x+c)/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/128*I/a^8/d*exp(-I*(d*x+c))+7/384*I/a^8/d*exp(-3*I*(d*x+c))+21/640*I/a^8/
d*exp(-5*I*(d*x+c))+5/128*I/a^8/d*exp(-7*I*(d*x+c))+35/1152*I/a^8/d*exp(-9*
I*(d*x+c))+21/1408*I/a^8/d*exp(-11*I*(d*x+c))+7/1664*I/a^8/d*exp(-13*I*(d*x
+c))+1/1920*I/a^8/d*exp(-15*I*(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.36

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{(6435i\,e^{(14i\,dx+14i\,c)} + 15015i\,e^{(12i\,dx+12i\,c)} + 27027i\,e^{(10i\,dx+10i\,c)} + 32175i\,e^{(8i\,dx+8i\,c)} + 25025i\,e^{(6i\,dx+6i\,c)} + \cdots}{823680\,a^8d}$$

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/823680*(6435*I*e^(14*I*d*x + 14*I*c) + 15015*I*e^(12*I*d*x + 12*I*c) + 27
027*I*e^(10*I*d*x + 10*I*c) + 32175*I*e^(8*I*d*x + 8*I*c) + 25025*I*e^(6*I*
d*x + 6*I*c) + 12285*I*e^(4*I*d*x + 4*I*c) + 3465*I*e^(2*I*d*x + 2*I*c) + 4
29*I)*e^(-15*I*d*x - 15*I*c)/(a^8*d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1221 vs. 2(238) = 476.

Time = 8.86 (sec) , antiderivative size = 1221, normalized size of antiderivative = 4.54

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise((16*tan(c + d*x)**7*sec(c + d*x)/(6435*a**8*d*tan(c + d*x)**8 - 5
1480*I*a**8*d*tan(c + d*x)**7 - 180180*a**8*d*tan(c + d*x)**6 + 360360*I*a*
*8*d*tan(c + d*x)**5 + 450450*a**8*d*tan(c + d*x)**4 - 360360*I*a**8*d*tan(
c + d*x)**3 - 180180*a**8*d*tan(c + d*x)**2 + 51480*I*a**8*d*tan(c + d*x) +
 6435*a**8*d) - 128*I*tan(c + d*x)**6*sec(c + d*x)/(6435*a**8*d*tan(c + d*x
)**8 - 51480*I*a**8*d*tan(c + d*x)**7 - 180180*a**8*d*tan(c + d*x)**6 + 360
360*I*a**8*d*tan(c + d*x)**5 + 450450*a**8*d*tan(c + d*x)**4 - 360360*I*a**
8*d*tan(c + d*x)**3 - 180180*a**8*d*tan(c + d*x)**2 + 51480*I*a**8*d*tan(c
+ d*x) + 6435*a**8*d) - 456*tan(c + d*x)**5*sec(c + d*x)/(6435*a**8*d*tan(c
 + d*x)**8 - 51480*I*a**8*d*tan(c + d*x)**7 - 180180*a**8*d*tan(c + d*x)**6
 + 360360*I*a**8*d*tan(c + d*x)**5 + 450450*a**8*d*tan(c + d*x)**4 - 360360
*I*a**8*d*tan(c + d*x)**3 - 180180*a**8*d*tan(c + d*x)**2 + 51480*I*a**8*d*
tan(c + d*x) + 6435*a**8*d) + 960*I*tan(c + d*x)**4*sec(c + d*x)/(6435*a**8
*d*tan(c + d*x)**8 - 51480*I*a**8*d*tan(c + d*x)**7 - 180180*a**8*d*tan(c +
 d*x)**6 + 360360*I*a**8*d*tan(c + d*x)**5 + 450450*a**8*d*tan(c + d*x)**4
- 360360*I*a**8*d*tan(c + d*x)**3 - 180180*a**8*d*tan(c + d*x)**2 + 51480*I
```

```
*a**8*d*tan(c + d*x) + 6435*a**8*d) + 1350*tan(c + d*x)**3*sec(c + d*x)/(64
35*a**8*d*tan(c + d*x)**8 - 51480*I*a**8*d*tan(c + d*x)**7 - 180180*a**8*d*
tan(c + d*x)**6 + 360360*I*a**8*d*tan(c + d*x)**5 + 450450*a**8*d*tan(c + d
*x)**4 - 360360*I*a**8*d*tan(c + d*x)**3 - 180180*a**8*d*tan(c + d*x)**2 +
51480*I*a**8*d*tan(c + d*x) + 6435*a**8*d) - 1392*I*tan(c + d*x)**2*sec(c +
 d*x)/(6435*a**8*d*tan(c + d*x)**8 - 51480*I*a**8*d*tan(c + d*x)**7 - 18018
0*a**8*d*tan(c + d*x)**6 + 360360*I*a**8*d*tan(c + d*x)**5 + 450450*a**8*d*
tan(c + d*x)**4 - 360360*I*a**8*d*tan(c + d*x)**3 - 180180*a**8*d*tan(c + d
*x)**2 + 51480*I*a**8*d*tan(c + d*x) + 6435*a**8*d) - 1181*tan(c + d*x)*sec
(c + d*x)/(6435*a**8*d*tan(c + d*x)**8 - 51480*I*a**8*d*tan(c + d*x)**7 - 1
80180*a**8*d*tan(c + d*x)**6 + 360360*I*a**8*d*tan(c + d*x)**5 + 450450*a**
8*d*tan(c + d*x)**4 - 360360*I*a**8*d*tan(c + d*x)**3 - 180180*a**8*d*tan(c
 + d*x)**2 + 51480*I*a**8*d*tan(c + d*x) + 6435*a**8*d) + 952*I*sec(c + d*x
)/(6435*a**8*d*tan(c + d*x)**8 - 51480*I*a**8*d*tan(c + d*x)**7 - 180180*a*
*8*d*tan(c + d*x)**6 + 360360*I*a**8*d*tan(c + d*x)**5 + 450450*a**8*d*tan(
c + d*x)**4 - 360360*I*a**8*d*tan(c + d*x)**3 - 180180*a**8*d*tan(c + d*x)*
*2 + 51480*I*a**8*d*tan(c + d*x) + 6435*a**8*d), Ne(d, 0)), (x*sec(c)/(I*a*
tan(c) + a)**8, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.67

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{429i\,\cos(15\,dx+15\,c)+3465i\,\cos(13\,dx+13\,c)+12285i\,\cos(11\,dx+11\,c)+25025i\,\cos(9\,dx+9\,c)+3}{}$$

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] 1/823680*(429*I*cos(15*d*x + 15*c) + 3465*I*cos(13*d*x + 13*c) + 12285*I*co
s(11*d*x + 11*c) + 25025*I*cos(9*d*x + 9*c) + 32175*I*cos(7*d*x + 7*c) + 27
027*I*cos(5*d*x + 5*c) + 15015*I*cos(3*d*x + 3*c) + 6435*I*cos(d*x + c) + 4
29*sin(15*d*x + 15*c) + 3465*sin(13*d*x + 13*c) + 12285*sin(11*d*x + 11*c)
+ 25025*sin(9*d*x + 9*c) + 32175*sin(7*d*x + 7*c) + 27027*sin(5*d*x + 5*c)
+ 15015*sin(3*d*x + 3*c) + 6435*sin(d*x + c))/(a^8*d)
```

Giac [A] (verification not implemented)

none

Time = 1.17 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.75

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{2\left(6435\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{14} - 45045i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13} - 210210\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12} + 630630i\,\tan\left(\frac{1}{2}\,dx+\right.\right.}{}$$

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 2/6435*(6435*tan(1/2*d*x + 1/2*c)^14 - 45045*I*tan(1/2*d*x + 1/2*c)^13 - 21
0210*tan(1/2*d*x + 1/2*c)^12 + 630630*I*tan(1/2*d*x + 1/2*c)^11 + 1414413*t
an(1/2*d*x + 1/2*c)^10 - 2357355*I*tan(1/2*d*x + 1/2*c)^9 - 3063060*tan(1/2
*d*x + 1/2*c)^8 + 3063060*I*tan(1/2*d*x + 1/2*c)^7 + 2407405*tan(1/2*d*x +
1/2*c)^6 - 1444443*I*tan(1/2*d*x + 1/2*c)^5 - 668850*tan(1/2*d*x + 1/2*c)^4
 + 222950*I*tan(1/2*d*x + 1/2*c)^3 + 54915*tan(1/2*d*x + 1/2*c)^2 - 7845*I*
tan(1/2*d*x + 1/2*c) - 952)/(a^8*d*(tan(1/2*d*x + 1/2*c) - I)^15)
```

Mupad [B] (verification not implemented)

Time = 6.15 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.83

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{2\left(2\sin\left(\frac{c}{4}+\frac{d\,x}{4}\right)^2-1\right)\left(-\frac{\sin(c+d\,x)^2\,44779\mathrm{i}}{32}+\frac{32175\,\sin(c+d\,x)}{128}-\frac{\sin(2\,c+2\,d\,x)^2\,26075\mathrm{i}}{16}-\frac{3575\,\sin(2\,c+2\,d\,x)}{8}+\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{6}\right.}{}$$

```
[In] int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^8),x)
```

```
[Out] (2*(2*sin(c/4 + (d*x)/4)^2 - 1)*((32175*sin(c + d*x))/128 - (3575*sin(2*c +
 2*d*x))/8 + (84227*sin(3*c + 3*d*x))/128 - 754*sin(4*c + 4*d*x) + (111527*
sin(5*c + 5*d*x))/128 - (7187*sin(6*c + 6*d*x))/8 + (121427*sin(7*c + 7*d*x
))/128 - (sin(2*c + 2*d*x)^2*26075i)/16 + (sin(c/2 + (d*x)/2)^2*114583i)/64
 - (sin(3*c + 3*d*x)^2*57925i)/32 + (sin((3*c)/2 + (3*d*x)/2)^2*116585i)/64
 + (sin((5*c)/2 + (5*d*x)/2)^2*119315i)/64 + (sin((7*c)/2 + (7*d*x)/2)^2*12
2285i)/64 - (sin(c + d*x)^2*44779i)/32 - 952i))/(6435*a^8*d*(sin((15*c)/2 +
 (15*d*x)/2)*1i - 2*sin((15*c)/4 + (15*d*x)/4)^2 + 1))
```

3.183 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 271

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{192\sin(c+dx)}{12155a^8d} - \frac{64\sin^3(c+dx)}{12155a^8d} + \frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8} + \frac{3i\cos(c+dx)}{85ad(a+ia\tan(c+dx))^7} + \frac{24i\cos(c+dx)}{1105a^2d(a+ia\tan(c+dx))^6} + \frac{168i\cos(c+dx)}{12155a^3d(a+ia\tan(c+dx))^5} + \frac{112i\cos(c+dx)}{12155d\left(a^2+ia^2\tan(c+dx)\right)^4} + \frac{16i\cos(c+dx)}{2431a^2d\left(a^2+ia^2\tan(c+dx)\right)^3} + \frac{128i\cos^3(c+dx)}{12155d\left(a^8+ia^8\tan(c+dx)\right)}$$

```
[Out] 192/12155*sin(d*x+c)/a^8/d-64/12155*sin(d*x+c)^3/a^8/d+1/17*I*cos(d*x+c)/d/
(a+I*a*tan(d*x+c))^8+3/85*I*cos(d*x+c)/a/d/(a+I*a*tan(d*x+c))^7+24/1105*I*c
os(d*x+c)/a^2/d/(a+I*a*tan(d*x+c))^6+168/12155*I*cos(d*x+c)/a^3/d/(a+I*a*ta
n(d*x+c))^5+112/12155*I*cos(d*x+c)/d/(a^2+I*a^2*tan(d*x+c))^4+16/2431*I*cos
(d*x+c)/a^2/d/(a^2+I*a^2*tan(d*x+c))^3+128/12155*I*cos(d*x+c)^3/d/(a^8+I*a^
8*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3583, 3581, 2713}

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{64\sin^3(c+dx)}{12155a^8d}+\frac{192\sin(c+dx)}{12155a^8d}+\frac{128i\cos^3(c+dx)}{12155d\,(a^8+ia^8\tan(c+dx))}+\frac{168i\cos(c+dx)}{12155a^3d(a+ia\tan(c+dx))^5}+\frac{16i\cos(c+dx)}{2431a^2d\,(a^2+ia^2\tan(c+dx))^3}+\frac{112i\cos(c+dx)}{12155d\,(a^2+ia^2\tan(c+dx))^4}+\frac{24i\cos(c+dx)}{1105a^2d(a+ia\tan(c+dx))^6}+\frac{3i\cos(c+dx)}{85ad(a+ia\tan(c+dx))^7}+\frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8}$$

```
[In] Int[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (192*Sin[c + d*x])/(12155*a^8*d) - (64*Sin[c + d*x]^3)/(12155*a^8*d) + ((I/
17)*Cos[c + d*x])/(d*(a + I*a*Tan[c + d*x])^8) + (((3*I)/85)*Cos[c + d*x])/
(a*d*(a + I*a*Tan[c + d*x])^7) + (((24*I)/1105)*Cos[c + d*x])/(a^2*d*(a + I
*a*Tan[c + d*x])^6) + (((168*I)/12155)*Cos[c + d*x])/(a^3*d*(a + I*a*Tan[c
+ d*x])^5) + (((112*I)/12155)*Cos[c + d*x])/(d*(a^2 + I*a^2*Tan[c + d*x])^4
) + (((16*I)/2431)*Cos[c + d*x])/(a^2*d*(a^2 + I*a^2*Tan[c + d*x])^3) + (((
128*I)/12155)*Cos[c + d*x]^3)/(d*(a^8 + I*a^8*Tan[c + d*x]))
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
```

```
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8}+\frac{9\int\frac{\cos(c+dx)}{(a+ia\tan(c+dx))^7}\,dx}{17a}\\
&= \frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8}+\frac{3i\cos(c+dx)}{85ad(a+ia\tan(c+dx))^7}+\frac{24\int\frac{\cos(c+dx)}{(a+ia\tan(c+dx))^6}\,dx}{85a^2}\\
&= \frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8}+\frac{3i\cos(c+dx)}{85ad(a+ia\tan(c+dx))^7}\\
&\quad+\frac{24i\cos(c+dx)}{1105a^2d(a+ia\tan(c+dx))^6}+\frac{168\int\frac{\cos(c+dx)}{(a+ia\tan(c+dx))^5}\,dx}{1105a^3}\\
&= \frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8}+\frac{3i\cos(c+dx)}{85ad(a+ia\tan(c+dx))^7}\\
&\quad+\frac{24i\cos(c+dx)}{1105a^2d(a+ia\tan(c+dx))^6}\\
&\quad+\frac{168i\cos(c+dx)}{12155a^3d(a+ia\tan(c+dx))^5}+\frac{1008\int\frac{\cos(c+dx)}{(a+ia\tan(c+dx))^4}\,dx}{12155a^4}\\
&= \frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8}+\frac{3i\cos(c+dx)}{85ad(a+ia\tan(c+dx))^7}\\
&\quad+\frac{24i\cos(c+dx)}{1105a^2d(a+ia\tan(c+dx))^6}+\frac{168i\cos(c+dx)}{12155a^3d(a+ia\tan(c+dx))^5}\\
&\quad+\frac{112i\cos(c+dx)}{12155d\left(a^2+ia^2\tan(c+dx)\right)^4}+\frac{112\int\frac{\cos(c+dx)}{(a+ia\tan(c+dx))^3}\,dx}{2431a^5}
\end{aligned}
$$

$$= \frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8} + \frac{3i\cos(c+dx)}{85ad(a+ia\tan(c+dx))^7} + \frac{24i\cos(c+dx)}{1105a^2d(a+ia\tan(c+dx))^6} + \frac{168i\cos(c+dx)}{12155a^3d(a+ia\tan(c+dx))^5} + \frac{16i\cos(c+dx)}{2431a^5d(a+ia\tan(c+dx))^3} + \frac{112i\cos(c+dx)}{12155d\left(a^2+ia^2\tan(c+dx)\right)^4} + \frac{64\int\frac{\cos(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{2431a^6}$$

$$= \frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8} + \frac{3i\cos(c+dx)}{85ad(a+ia\tan(c+dx))^7} + \frac{24i\cos(c+dx)}{1105a^2d(a+ia\tan(c+dx))^6} + \frac{168i\cos(c+dx)}{12155a^3d(a+ia\tan(c+dx))^5} + \frac{16i\cos(c+dx)}{2431a^5d(a+ia\tan(c+dx))^3} + \frac{112i\cos(c+dx)}{12155d\left(a^2+ia^2\tan(c+dx)\right)^4} + \frac{128i\cos^3(c+dx)}{12155d\left(a^8+ia^8\tan(c+dx)\right)} + \frac{192\int\cos^3(c+dx)\,dx}{12155a^8}$$

$$= \frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8} + \frac{3i\cos(c+dx)}{85ad(a+ia\tan(c+dx))^7} + \frac{24i\cos(c+dx)}{1105a^2d(a+ia\tan(c+dx))^6} + \frac{168i\cos(c+dx)}{12155a^3d(a+ia\tan(c+dx))^5} + \frac{16i\cos(c+dx)}{2431a^5d(a+ia\tan(c+dx))^3} + \frac{112i\cos(c+dx)}{12155d\left(a^2+ia^2\tan(c+dx)\right)^4} + \frac{128i\cos^3(c+dx)}{12155d\left(a^8+ia^8\tan(c+dx)\right)} - \frac{192\text{Subst}\left(\int\left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{12155a^8d}$$

$$= \frac{192\sin(c+dx)}{12155a^8d} - \frac{64\sin^3(c+dx)}{12155a^8d} + \frac{i\cos(c+dx)}{17d(a+ia\tan(c+dx))^8} + \frac{3i\cos(c+dx)}{85ad(a+ia\tan(c+dx))^7} + \frac{24i\cos(c+dx)}{1105a^2d(a+ia\tan(c+dx))^6} + \frac{168i\cos(c+dx)}{12155a^3d(a+ia\tan(c+dx))^5} + \frac{16i\cos(c+dx)}{2431a^5d(a+ia\tan(c+dx))^3} + \frac{112i\cos(c+dx)}{12155d\left(a^2+ia^2\tan(c+dx)\right)^4} + \frac{128i\cos^3(c+dx)}{12155d\left(a^8+ia^8\tan(c+dx)\right)}$$

Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.51

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx =$$
$$-\frac{i\sec^8(c+dx)(-194480\cos(c+dx)-148512\cos(3(c+dx))-89760\cos(5(c+dx))-58344\cos(7(c+d}{}$$

```
[In] Integrate[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((-1/3111680*I)*Sec[c + d*x]^8*(-194480*Cos[c + d*x] - 148512*Cos[3*(c + d*
x)] - 89760*Cos[5*(c + d*x)] - 58344*Cos[7*(c + d*x)] + 5720*Cos[9*(c + d*x
)] - (24310*I)*Sin[c + d*x] - (55692*I)*Sin[3*(c + d*x)] - (56100*I)*Sin[5*
(c + d*x)] - (51051*I)*Sin[7*(c + d*x)] + (6435*I)*Sin[9*(c + d*x)]))/(a^8*
d*(-I + Tan[c + d*x])^8)
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.65

method	result
risch	$\frac{3i e^{-3i(dx+c)}}{128a^8d}+\frac{21i e^{-5i(dx+c)}}{640a^8d}+\frac{9i e^{-7i(dx+c)}}{256a^8d}+\frac{7i e^{-9i(dx+c)}}{256a^8d}+\frac{21i e^{-11i(dx+c)}}{1408a^8d}+\frac{9i e^{-13i(dx+c)}}{1664a^8d}+\frac{3i e^{-15i(dx+}}{2560a^8d}$
derivativedivides	$\frac{2}{512\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+512i}+\frac{38218i}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{10}}-\frac{7937i}{32\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{1793i}{128\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{128i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{16}}+\frac{}{\left(-i+\right.}$
default	$\frac{2}{512\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+512i}+\frac{38218i}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{10}}-\frac{7937i}{32\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{1793i}{128\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{128i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{16}}+\frac{}{\left(-i+\right.}$

```
[In] int(cos(d*x+c)/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 3/128*I/a^8/d*exp(-3*I*(d*x+c))+21/640*I/a^8/d*exp(-5*I*(d*x+c))+9/256*I/a^
8/d*exp(-7*I*(d*x+c))+7/256*I/a^8/d*exp(-9*I*(d*x+c))+21/1408*I/a^8/d*exp(-
11*I*(d*x+c))+9/1664*I/a^8/d*exp(-13*I*(d*x+c))+3/2560*I/a^8/d*exp(-15*I*(d
*x+c))+1/8704*I/a^8/d*exp(-17*I*(d*x+c))+1/64*I/a^8/d*cos(d*x+c)+5/256*sin(
d*x+c)/a^8/d
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.44

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\left(-12155i\,e^{(18i\,dx+18i\,c)}+109395i\,e^{(16i\,dx+16i\,c)}+145860i\,e^{(14i\,dx+14i\,c)}+204204i\,e^{(12i\,dx+12i\,c)}+218790i\,e^{(10i\,\ldots}\right.}{6223\ldots}$$

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/6223360*(-12155*I*e^(18*I*d*x + 18*I*c) + 109395*I*e^(16*I*d*x + 16*I*c)
+ 145860*I*e^(14*I*d*x + 14*I*c) + 204204*I*e^(12*I*d*x + 12*I*c) + 218790*
I*e^(10*I*d*x + 10*I*c) + 170170*I*e^(8*I*d*x + 8*I*c) + 92820*I*e^(6*I*d*x
 + 6*I*c) + 33660*I*e^(4*I*d*x + 4*I*c) + 7293*I*e^(2*I*d*x + 2*I*c) + 715*
I)*e^(-17*I*d*x - 17*I*c)/(a^8*d)
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.35

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \begin{cases} \frac{\left(-1435009114982013439311872 00ia^{72}d^9e^{82ic}e^{idx}+129150820348381209538068480 0ia^{72}d^9e^{80ic}e^{-idx}+1722010937978416127174246400ia^{72}\ldots\right.}{\ldots} \\ \frac{x\left(e^{18ic}+9e^{16ic}+36e^{14ic}+84e^{12ic}+126e^{10ic}+126e^{8ic}+84e^{6ic}+36e^{4ic}+9e^{2ic}+1\right)e^{-17ic}}{512a^8} \end{cases}$$

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise(((-143500911498201343931187200*I*a**72*d**9*exp(82*I*c)*exp(I*d*x
) + 1291508203483812095380684800*I*a**72*d**9*exp(80*I*c)*exp(-I*d*x) + 172
2010937978416127174246400*I*a**72*d**9*exp(78*I*c)*exp(-3*I*d*x) + 24108153
13169782578043944960*I*a**72*d**9*exp(76*I*c)*exp(-5*I*d*x) + 2583016406967
624190761369600*I*a**72*d**9*exp(74*I*c)*exp(-7*I*d*x) + 200901276097481881
5036620800*I*a**72*d**9*exp(72*I*c)*exp(-9*I*d*x) + 109582514234990117183 81
56800*I*a**72*d**9*exp(70*I*c)*exp(-11*I*d*x) + 397387139533480644732518400
*I*a**72*d**9*exp(68*I*c)*exp(-13*I*d*x) + 86100546898920806358712320*I*a**
72*d**9*exp(66*I*c)*exp(-15*I*d*x) + 8441230088129490819481600*I*a**72*d**9
*exp(64*I*c)*exp(-17*I*d*x))*exp(-81*I*c)/(7347246668707908809276784 6400*a*
*80*d**10), Ne(a**80*d**10*exp(81*I*c), 0)), (x*(exp(18*I*c) + 9*exp(16*I*c
) + 36*exp(14*I*c) + 84*exp(12*I*c) + 126*exp(10*I*c) + 126*exp(8*I*c) + 84
*exp(6*I*c) + 36*exp(4*I*c) + 9*exp(2*I*c) + 1)*exp(-17*I*c)/(512*a**8), Tr
ue))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [A] (verification not implemented)

none

Time = 1.26 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.92

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\frac{12155}{a^8\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)} + \frac{6211205\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{16} - 55791450i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15} - 303072770\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{14} + 1091397450i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}}{}}{}$$

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
[Out] 1/3111680*(12155/(a^8*(tan(1/2*d*x + 1/2*c) + I)) + (6211205*tan(1/2*d*x +
1/2*c)^16 - 55791450*I*tan(1/2*d*x + 1/2*c)^15 - 303072770*tan(1/2*d*x + 1/
2*c)^14 + 1091397450*I*tan(1/2*d*x + 1/2*c)^13 + 2909561798*tan(1/2*d*x + 1
/2*c)^12 - 5901218466*I*tan(1/2*d*x + 1/2*c)^11 - 9405145178*tan(1/2*d*x +
1/2*c)^10 + 11877161010*I*tan(1/2*d*x + 1/2*c)^9 + 12017308160*tan(1/2*d*x
+ 1/2*c)^8 - 9710430158*I*tan(1/2*d*x + 1/2*c)^7 - 6263238566*tan(1/2*d*x +
 1/2*c)^6 + 3172666718*I*tan(1/2*d*x + 1/2*c)^5 + 1247921210*tan(1/2*d*x +
1/2*c)^4 - 365303990*I*tan(1/2*d*x + 1/2*c)^3 - 77883902*tan(1/2*d*x + 1/2*
c)^2 + 10498214*I*tan(1/2*d*x + 1/2*c) + 982907)/(a^8*(tan(1/2*d*x + 1/2*c)
 - I)^17))/d
```

Mupad [B] (verification not implemented)

Time = 7.47 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.97

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{152329\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128} - \frac{41121\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{32} + \frac{41121\,\sin\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{32} - \frac{96165\,\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{64} + \frac{96165\,\sin\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)}{64}\right.}{}$$

```
[In] int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^8,x)

[Out] (cos(c/2 + (d*x)/2)*((cos((3*c)/2 + (3*d*x)/2)*12155i)/16 - (cos((5*c)/2 +
(5*d*x)/2)*12155i)/16 + (cos((7*c)/2 + (7*d*x)/2)*21437i)/16 - (cos((9*c)/2
 + (9*d*x)/2)*21437i)/16 + (cos((11*c)/2 + (11*d*x)/2)*27047i)/16 - (cos((1
3*c)/2 + (13*d*x)/2)*27047i)/16 + (cos((15*c)/2 + (15*d*x)/2)*61387i)/32 -
(cos((17*c)/2 + (17*d*x)/2)*715i)/32 + (152329*sin(c/2 + (d*x)/2))/128 - (4
1121*sin((3*c)/2 + (3*d*x)/2))/32 + (41121*sin((5*c)/2 + (5*d*x)/2))/32 - (
96165*sin((7*c)/2 + (7*d*x)/2))/64 + (96165*sin((9*c)/2 + (9*d*x)/2))/64 -
(55095*sin((11*c)/2 + (11*d*x)/2))/32 + (55095*sin((13*c)/2 + (13*d*x)/2))/
32 - (491811*sin((15*c)/2 + (15*d*x)/2))/256 + (6435*sin((17*c)/2 + (17*d*x
)/2))/256)*2i)/(12155*a^8*d*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2)*1i)^17
*(cos(c/2 + (d*x)/2)*1i + sin(c/2 + (d*x)/2)))
```

3.184 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 301

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx &= \frac{160\sin(c+dx)}{4199a^8d}-\frac{320\sin^3(c+dx)}{12597a^8d}\\&\quad+\frac{32\sin^5(c+dx)}{4199a^8d}+\frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8}\\&\quad+\frac{11i\cos^3(c+dx)}{323ad(a+ia\tan(c+dx))^7}+\frac{22i\cos^3(c+dx)}{969a^2d(a+ia\tan(c+dx))^6}\\&\quad+\frac{66i\cos^3(c+dx)}{4199a^3d(a+ia\tan(c+dx))^5}\\&\quad+\frac{48i\cos^3(c+dx)}{4199d\,(a^2+ia^2\tan(c+dx))^4}\\&\quad+\frac{112i\cos^3(c+dx)}{12597a^2d\,(a^2+ia^2\tan(c+dx))^3}\\&\quad+\frac{64i\cos^5(c+dx)}{4199d\,(a^8+ia^8\tan(c+dx))}\end{aligned}$$

```
[Out] 160/4199*sin(d*x+c)/a^8/d-320/12597*sin(d*x+c)^3/a^8/d+32/4199*sin(d*x+c)^5
/a^8/d+1/19*I*cos(d*x+c)^3/d/(a+I*a*tan(d*x+c))^8+11/323*I*cos(d*x+c)^3/a/d
/(a+I*a*tan(d*x+c))^7+22/969*I*cos(d*x+c)^3/a^2/d/(a+I*a*tan(d*x+c))^6+66/4
199*I*cos(d*x+c)^3/a^3/d/(a+I*a*tan(d*x+c))^5+48/4199*I*cos(d*x+c)^3/d/(a^2
+I*a^2*tan(d*x+c))^4+112/12597*I*cos(d*x+c)^3/a^2/d/(a^2+I*a^2*tan(d*x+c))^
3+64/4199*I*cos(d*x+c)^5/d/(a^8+I*a^8*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3583, 3581, 2713}

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{32\sin^5(c+dx)}{4199a^8d}-\frac{320\sin^3(c+dx)}{12597a^8d}+\frac{160\sin(c+dx)}{4199a^8d}+\frac{64i\cos^5(c+dx)}{4199d\,(a^8+ia^8\tan(c+dx))}+\frac{66i\cos^3(c+dx)}{4199a^3d(a+ia\tan(c+dx))^5}+\frac{112i\cos^3(c+dx)}{12597a^2d\,(a^2+ia^2\tan(c+dx))^3}+\frac{48i\cos^3(c+dx)}{4199d\,(a^2+ia^2\tan(c+dx))^4}+\frac{22i\cos^3(c+dx)}{969a^2d(a+ia\tan(c+dx))^6}+\frac{11i\cos^3(c+dx)}{323ad(a+ia\tan(c+dx))^7}+\frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8}$$

```
[In] Int[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] (160*Sin[c + d*x])/(4199*a^8*d) - (320*Sin[c + d*x]^3)/(12597*a^8*d) + (32*
Sin[c + d*x]^5)/(4199*a^8*d) + ((I/19)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[c +
d*x])^8) + (((11*I)/323)*Cos[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*x])^7) + (
((22*I)/969)*Cos[c + d*x]^3)/(a^2*d*(a + I*a*Tan[c + d*x])^6) + (((66*I)/41
99)*Cos[c + d*x]^3)/(a^3*d*(a + I*a*Tan[c + d*x])^5) + (((48*I)/4199)*Cos[c
 + d*x]^3)/(d*(a^2 + I*a^2*Tan[c + d*x])^4) + (((112*I)/12597)*Cos[c + d*x]
^3)/(a^2*d*(a^2 + I*a^2*Tan[c + d*x])^3) + (((64*I)/4199)*Cos[c + d*x]^5)/(
d*(a^8 + I*a^8*Tan[c + d*x]))
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
```

```
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8}+\frac{11\int\frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^7}\,dx}{19a}\\
&= \frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8}+\frac{11i\cos^3(c+dx)}{323ad(a+ia\tan(c+dx))^7}+\frac{110\int\frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^6}\,dx}{323a^2}\\
&= \frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8}+\frac{11i\cos^3(c+dx)}{323ad(a+ia\tan(c+dx))^7}\\
&\quad+\frac{22i\cos^3(c+dx)}{969a^2d(a+ia\tan(c+dx))^6}+\frac{66\int\frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^5}\,dx}{323a^3}\\
&= \frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8}+\frac{11i\cos^3(c+dx)}{323ad(a+ia\tan(c+dx))^7}\\
&\quad+\frac{22i\cos^3(c+dx)}{969a^2d(a+ia\tan(c+dx))^6}\\
&\quad+\frac{66i\cos^3(c+dx)}{4199a^3d(a+ia\tan(c+dx))^5}+\frac{528\int\frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^4}\,dx}{4199a^4}\\
&= \frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8}+\frac{11i\cos^3(c+dx)}{323ad(a+ia\tan(c+dx))^7}+\frac{22i\cos^3(c+dx)}{969a^2d(a+ia\tan(c+dx))^6}\\
&\quad+\frac{66i\cos^3(c+dx)}{4199a^3d(a+ia\tan(c+dx))^5}+\frac{48i\cos^3(c+dx)}{4199d\left(a^2+ia^2\tan(c+dx)\right)^4}+\frac{336\int\frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^3}\,dx}{4199a^5}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8}+\frac{11i\cos^3(c+dx)}{323ad(a+ia\tan(c+dx))^7} \\
&+\frac{22i\cos^3(c+dx)}{969a^2d(a+ia\tan(c+dx))^6}+\frac{66i\cos^3(c+dx)}{4199a^3d(a+ia\tan(c+dx))^5} \\
&+\frac{112i\cos^3(c+dx)}{12597a^5d(a+ia\tan(c+dx))^3} \\
&+\frac{48i\cos^3(c+dx)}{4199d\left(a^2+ia^2\tan(c+dx)\right)^4}+\frac{224\int\frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^2}\,dx}{4199a^6}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8}+\frac{11i\cos^3(c+dx)}{323ad(a+ia\tan(c+dx))^7} \\
&+\frac{22i\cos^3(c+dx)}{969a^2d(a+ia\tan(c+dx))^6}+\frac{66i\cos^3(c+dx)}{4199a^3d(a+ia\tan(c+dx))^5} \\
&+\frac{112i\cos^3(c+dx)}{12597a^5d(a+ia\tan(c+dx))^3}+\frac{48i\cos^3(c+dx)}{4199d\left(a^2+ia^2\tan(c+dx)\right)^4} \\
&+\frac{64i\cos^5(c+dx)}{4199d\left(a^8+ia^8\tan(c+dx)\right)}+\frac{160\int\cos^5(c+dx)\,dx}{4199a^8}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8}+\frac{11i\cos^3(c+dx)}{323ad(a+ia\tan(c+dx))^7} \\
&+\frac{22i\cos^3(c+dx)}{969a^2d(a+ia\tan(c+dx))^6}+\frac{66i\cos^3(c+dx)}{4199a^3d(a+ia\tan(c+dx))^5} \\
&+\frac{112i\cos^3(c+dx)}{12597a^5d(a+ia\tan(c+dx))^3}+\frac{48i\cos^3(c+dx)}{4199d\left(a^2+ia^2\tan(c+dx)\right)^4} \\
&+\frac{64i\cos^5(c+dx)}{4199d\left(a^8+ia^8\tan(c+dx)\right)}-\frac{160\text{Subst}\left(\int\left(1-2x^2+x^4\right)\,dx,x,-\sin(c+dx)\right)}{4199a^8d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{160\sin(c+dx)}{4199a^8d}-\frac{320\sin^3(c+dx)}{12597a^8d}+\frac{32\sin^5(c+dx)}{4199a^8d}+\frac{i\cos^3(c+dx)}{19d(a+ia\tan(c+dx))^8} \\
&+\frac{11i\cos^3(c+dx)}{323ad(a+ia\tan(c+dx))^7}+\frac{22i\cos^3(c+dx)}{969a^2d(a+ia\tan(c+dx))^6} \\
&+\frac{66i\cos^3(c+dx)}{4199a^3d(a+ia\tan(c+dx))^5}+\frac{112i\cos^3(c+dx)}{12597a^5d(a+ia\tan(c+dx))^3} \\
&+\frac{48i\cos^3(c+dx)}{4199d\left(a^2+ia^2\tan(c+dx)\right)^4}+\frac{64i\cos^5(c+dx)}{4199d\left(a^8+ia^8\tan(c+dx)\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.53

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx =$$
$$-\frac{i\sec^8(c+dx)(-739024\cos(c+dx)-604656\cos(3(c+dx))-426360\cos(5(c+dx))-369512\cos(7(c+\cdots}{}$$

```
[In] Integrate[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^8,x]
```

```
[Out] ((-1/12899328*I)*Sec[c + d*x]^8*(-739024*Cos[c + d*x] - 604656*Cos[3*(c + d
*x)] - 426360*Cos[5*(c + d*x)] - 369512*Cos[7*(c + d*x)] + 65208*Cos[9*(c +
 d*x)] + 1768*Cos[11*(c + d*x)] - (92378*I)*Sin[c + d*x] - (226746*I)*Sin[3
*(c + d*x)] - (266475*I)*Sin[5*(c + d*x)] - (323323*I)*Sin[7*(c + d*x)] + (
73359*I)*Sin[9*(c + d*x)] + (2431*I)*Sin[11*(c + d*x)]))/(a^8*d*(-I + Tan[c
 + d*x])^8)
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.70

method	result
risch	$\frac{33i\mathrm{e}^{-5i(dx+c)}}{1024a^8d}+\frac{33i\mathrm{e}^{-7i(dx+c)}}{1024a^8d}+\frac{77i\mathrm{e}^{-9i(dx+c)}}{3072a^8d}+\frac{15i\mathrm{e}^{-11i(dx+c)}}{1024a^8d}+\frac{165i\mathrm{e}^{-13i(dx+c)}}{26624a^8d}+\frac{11i\mathrm{e}^{-15i(dx+c)}}{6144a^8d}+\frac{11i\mathrm{e}\cdots}{34\cdots}$
derivativedivides	$-\frac{1984i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{16}}-\frac{1}{768\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^3}+\frac{3}{256\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}-\frac{32525i}{4\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}-\frac{i}{512\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^2}-\frac{\cdots}{8(-i+\mathrm{ta}\cdots}$
default	$-\frac{1984i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{16}}-\frac{1}{768\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^3}+\frac{3}{256\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}-\frac{32525i}{4\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}-\frac{i}{512\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^2}-\frac{\cdots}{8(-i+\mathrm{ta}\cdots}$

```
[In] int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 33/1024*I/a^8/d*exp(-5*I*(d*x+c))+33/1024*I/a^8/d*exp(-7*I*(d*x+c))+77/3072
*I/a^8/d*exp(-9*I*(d*x+c))+15/1024*I/a^8/d*exp(-11*I*(d*x+c))+165/26624*I/a
^8/d*exp(-13*I*(d*x+c))+11/6144*I/a^8/d*exp(-15*I*(d*x+c))+11/34816*I/a^8/d
*exp(-17*I*(d*x+c))+1/38912*I/a^8/d*exp(-19*I*(d*x+c))+11/512*I/a^8/d*cos(d
*x+c)+33/1024*sin(d*x+c)/a^8/d+41/1536*I/a^8/d*cos(3*d*x+3*c)+83/3072/a^8/d
*sin(3*d*x+3*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.47

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \frac{\left(-4199i\,e^{(22i\,dx+22i\,c)} - 138567i\,e^{(20i\,dx+20i\,c)} + 692835i\,e^{(18i\,dx+18i\,c)} + 692835i\,e^{(16i\,dx+16i\,c)} + 831402i\,e^{(14i\,d}\right.}{}$$

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x, algorithm="fricas")
```

```
[Out] 1/25798656*(-4199*I*e^(22*I*d*x + 22*I*c) - 138567*I*e^(20*I*d*x + 20*I*c)
+ 692835*I*e^(18*I*d*x + 18*I*c) + 692835*I*e^(16*I*d*x + 16*I*c) + 831402*
I*e^(14*I*d*x + 14*I*c) + 831402*I*e^(12*I*d*x + 12*I*c) + 646646*I*e^(10*I
*d*x + 10*I*c) + 377910*I*e^(8*I*d*x + 8*I*c) + 159885*I*e^(6*I*d*x + 6*I*c
) + 46189*I*e^(4*I*d*x + 4*I*c) + 8151*I*e^(2*I*d*x + 2*I*c) + 663*I)*e^(-1
9*I*d*x - 19*I*c)/(a^8*d)
```

Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.45

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx$$
$$= \begin{cases} \frac{(-627910689858846946911347157688181273395 2ia^{88}d^{11}e^{103ic}e^{3idx} - 20721052765341949248074456203709982022041 6ia^{88}d^{11}e^{101ic}e^{idx} + 10}{} \\ \frac{x\left(e^{22ic}+11e^{20ic}+55e^{18ic}+165e^{16ic}+330e^{14ic}+462e^{12ic}+462e^{10ic}+330e^{8ic}+165e^{6ic}+55e^{4ic}+11e^{2ic}+1\right)e^{-19ic}}{2048a^8} \end{cases}$$

```
[In] integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c))**8,x)
```

```
[Out] Piecewise(((-6279106898588469469113471576881812733952*I*a**88*d**11*exp(103
*I*c)*exp(3*I*d*x) - 207210527653419492480744562037099820220416*I*a**88*d**
11*exp(101*I*c)*exp(I*d*x) + 1036052638267097462403722810185499101102080*I*
a**88*d**11*exp(99*I*c)*exp(-I*d*x) + 1036052638267097462403722810185499101
102080*I*a**88*d**11*exp(97*I*c)*exp(-3*I*d*x) + 12432631659205169548844673
72222598921322496*I*a**88*d**11*exp(95*I*c)*exp(-5*I*d*x) + 124326316592051
6954884467372222598921322496*I*a**88*d**11*exp(93*I*c)*exp(-7*I*d*x) + 9669
82462382624298243474622839799161028608*I*a**88*d**11*exp(91*I*c)*exp(-9*I*d
*x) + 565119620872962252220212441919363146055680*I*a**88*d**11*exp(89*I*c)*
exp(-11*I*d*x) + 239089070369330183631628340812038254100480*I*a**88*d**11*e
xp(87*I*c)*exp(-13*I*d*x) + 69070175884473164160248187345699940073472*I*a**
88*d**11*exp(85*I*c)*exp(-15*I*d*x) + 121888545678482054400437977668882247 1
8848*I*a**88*d**11*exp(83*I*c)*exp(-17*I*d*x) + 9914379313560741267021270 91
```

```
086602010624*I*a**88*d**11*exp(81*I*c)*exp(-19*I*d*x))*exp(-100*I*c)/(38578
8327849275564182331693683618574374010 88*a**96*d**12), Ne(a**96*d**12*exp(10
0*I*c), 0)), (x*(exp(22*I*c) + 11*exp(20*I*c) + 55*exp(18*I*c) + 165*exp(16
*I*c) + 330*exp(14*I*c) + 462*exp(12*I*c) + 462*exp(10*I*c) + 330*exp(8*I*c
) + 165*exp(6*I*c) + 55*exp(4*I*c) + 11*exp(2*I*c) + 1)*exp(-19*I*c)/(2048*
a**8), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [A] (verification not implemented)

none

Time = 1.30 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.00

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = \frac{\frac{4199\left(18\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+33i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-17\right)}{a^8\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^3} + \frac{12823746\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{18}-140368371i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{17}-879644311\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{16}}{}}{}$$

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^8,x, algorithm="giac")
```

```
[Out] 1/6449664*(4199*(18*tan(1/2*d*x + 1/2*c)^2 + 33*I*tan(1/2*d*x + 1/2*c) - 17
)/(a^8*(tan(1/2*d*x + 1/2*c) + I)^3) + (12823746*tan(1/2*d*x + 1/2*c)^18 -
140368371*I*tan(1/2*d*x + 1/2*c)^17 - 879644311*tan(1/2*d*x + 1/2*c)^16 + 3
693272440*I*tan(1/2*d*x + 1/2*c)^15 + 11467502592*tan(1/2*d*x + 1/2*c)^14 -
 27403194676*I*tan(1/2*d*x + 1/2*c)^13 - 51919375300*tan(1/2*d*x + 1/2*c)^1
2 + 79183835016*I*tan(1/2*d*x + 1/2*c)^11 + 98304418212*tan(1/2*d*x + 1/2*c
)^10 - 99750226290*I*tan(1/2*d*x + 1/2*c)^9 - 82860874122*tan(1/2*d*x + 1/2
*c)^8 + 56110430792*I*tan(1/2*d*x + 1/2*c)^7 + 30766700912*tan(1/2*d*x + 1/
2*c)^6 - 13462452660*I*tan(1/2*d*x + 1/2*c)^5 - 4616712644*tan(1/2*d*x + 1/
2*c)^4 + 1197851960*I*tan(1/2*d*x + 1/2*c)^3 + 226248618*tan(1/2*d*x + 1/2*
c)^2 - 27911475*I*tan(1/2*d*x + 1/2*c) - 2143959)/(a^8*(tan(1/2*d*x + 1/2*c
) - I)^19))/d
```

Mupad [B] (verification not implemented)

Time = 10.46 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.02

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^8}\,dx = -\frac{2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{46189\cos\left(\frac{5c}{2}+\frac{5dx}{2}\right)}{64}-\frac{46189\cos\left(\frac{3c}{2}+\frac{3dx}{2}\right)}{64}-\frac{20995\cos\left(\frac{7c}{2}+\frac{7dx}{2}\right)}{16}+\frac{20995\cos\left(\frac{9c}{2}+\frac{9dx}{2}\right)}{16}-\frac{221255\cos\left(\cdots\right)}{\cdots}\right.}{\cdots}$$

```
[In] int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^8,x)
```

```
[Out] -(2*cos(c/2 + (d*x)/2)*((46189*cos((5*c)/2 + (5*d*x)/2))/64 - (46189*cos((3
*c)/2 + (3*d*x)/2))/64 - (20995*cos((7*c)/2 + (7*d*x)/2))/16 + (20995*cos((
9*c)/2 + (9*d*x)/2))/16 - (221255*cos((11*c)/2 + (11*d*x)/2))/128 + (221255
*cos((13*c)/2 + (13*d*x)/2))/128 - (66861*cos((15*c)/2 + (15*d*x)/2))/32 +
(2093*cos((17*c)/2 + (17*d*x)/2))/32 - (221*cos((19*c)/2 + (19*d*x)/2))/128
 + (221*cos((21*c)/2 + (21*d*x)/2))/128 + (sin(c/2 + (d*x)/2)*309861i)/256
- (sin((3*c)/2 + (3*d*x)/2)*665911i)/512 + (sin((5*c)/2 + (5*d*x)/2)*665911
i)/512 - (sin((7*c)/2 + (7*d*x)/2)*194821i)/128 + (sin((9*c)/2 + (9*d*x)/2)
*194821i)/128 - (sin((11*c)/2 + (11*d*x)/2)*1825043i)/1024 + (sin((13*c)/2
+ (13*d*x)/2)*1825043i)/1024 - (sin((15*c)/2 + (15*d*x)/2)*1074183i)/512 +
(sin((17*c)/2 + (17*d*x)/2)*37895i)/512 - (sin((19*c)/2 + (19*d*x)/2)*2431i
)/1024 + (sin((21*c)/2 + (21*d*x)/2)*2431i)/1024))/(12597*a^8*d*(cos(c/2 +
(d*x)/2) + sin(c/2 + (d*x)/2)*1i)^19*(cos(c/2 + (d*x)/2)*1i + sin(c/2 + (d*
x)/2))^3)
```

3.185 $\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 123

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx = -\frac{6ae^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}+\frac{6ae^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{5d}+\frac{2ae(e\sec(c+dx))^{5/2}\sin(c+dx)}{5d}$$

```
[Out] 2/7*I*a*(e*sec(d*x+c))^(7/2)/d+2/5*a*e*(e*sec(d*x+c))^(5/2)*sin(d*x+c)/d-6/
5*a*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+6/5*a*e^3*sin(d*
x+c)*(e*sec(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3567, 3853, 3856, 2719}

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx = -\frac{6ae^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{6ae^3\sin(c+dx)\sqrt{e\sec(c+dx)}}{5d}+\frac{2ia(e\sec(c+dx))^{7/2}}{7d}+\frac{2ae\sin(c+dx)(e\sec(c+dx))^{5/2}}{5d}$$

```
[In] Int[(e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (-6*a*e^4*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c +
 d*x]]) + (((2*I)/7)*a*(e*Sec[c + d*x])^(7/2))/d + (6*a*e^3*Sqrt[e*Sec[c +
d*x]]*Sin[c + d*x])/(5*d) + (2*a*e*(e*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(5*
d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2ia(e\sec(c+dx))^{7/2}}{7d} + a\int (e\sec(c+dx))^{7/2}\,dx \\
&= \frac{2ia(e\sec(c+dx))^{7/2}}{7d} + \frac{2ae(e\sec(c+dx))^{5/2}\sin(c+dx)}{5d} + \frac{1}{5}\left(3ae^2\right)\int (e\sec(c+dx))^{3/2}\,dx \\
&= \frac{2ia(e\sec(c+dx))^{7/2}}{7d} + \frac{6ae^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{5d} \\
&\quad + \frac{2ae(e\sec(c+dx))^{5/2}\sin(c+dx)}{5d} - \frac{1}{5}\left(3ae^4\right)\int \frac{1}{\sqrt{e\sec(c+dx)}}\,dx \\
&= \frac{2ia(e\sec(c+dx))^{7/2}}{7d} + \frac{6ae^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{5d} \\
&\quad + \frac{2ae(e\sec(c+dx))^{5/2}\sin(c+dx)}{5d} - \frac{\left(3ae^4\right)\int \sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}
\end{aligned}
$$

$$= -\frac{6ae^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2ia(e\sec(c+dx))^{7/2}}{7d} + \frac{6ae^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2ae(e\sec(c+dx))^{5/2}\sin(c+dx)}{5d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.22 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.27

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx = \frac{aee^{-idx}(e\sec(c+dx))^{5/2}(\cos(dx)-i\sin(dx))(\cos(c+3dx)+i\sin(c+3dx))\left(-36i-2\right.}{}$$

```
[In] Integrate[(e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (a*e*(e*Sec[c + d*x])^(5/2)*(Cos[d*x] - I*Sin[d*x])*(Cos[c + 3*d*x] + I*Sin
[c + 3*d*x])*(-36*I - (28*I)*Cos[2*(c + d*x)] + ((7*I)*(1 + E^((2*I)*(c + d
*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((2*I
)*(c + d*x)) + 7*Sec[c + d*x]*Sin[3*(c + d*x)] + 27*Tan[c + d*x]))/(70*d*E^
(I*d*x))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 433 vs. 2(130) = 260.

Time = 27.13 (sec) , antiderivative size = 434, normalized size of antiderivative = 3.53

method	result
default	$\frac{2a\sqrt{e\sec(dx+c)}\,e^3\left(3iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}\,(\cos^2(dx+c))-3iE(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\right.}{}$
parts	$\frac{2a\sqrt{e\sec(dx+c)}\,e^3\left(3iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}\,(\cos^2(dx+c))-3iE(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\right.}{}$

```
[In] int((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*a/d*(e*sec(d*x+c))^(1/2)*e^3/(cos(d*x+c)+1)*(3*I*EllipticF(I*(csc(d*x+c
)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)
*cos(d*x+c)^2-3*I*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2+6*I*cos(d*x+c)*Ellipt
icF(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(
d*x+c)+1))^(1/2)-6*I*cos(d*x+c)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos
```

```
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+3*I*EllipticF(I*(csc
(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)-3*I*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+3*sin(d*x+c)+tan(d*x+c)+sec(d*x+c)*tan(d*
x+c))+2/7*I*a*(e*sec(d*x+c))^(7/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.69

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx =$$

$$-\frac{2\left(\sqrt{2}\left(21i\,ae^3e^{(7i\,dx+7i\,c)}+77i\,ae^3e^{(5i\,dx+5i\,c)}+23i\,ae^3e^{(3i\,dx+3i\,c)}+7i\,ae^3e^{(i\,dx+i\,c)}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{35\,(de^{(6i\,dx}}$$

```
[In] integrate((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -2/35*(sqrt(2)*(21*I*a*e^3*e^(7*I*d*x + 7*I*c) + 77*I*a*e^3*e^(5*I*d*x + 5*
I*c) + 23*I*a*e^3*e^(3*I*d*x + 3*I*c) + 7*I*a*e^3*e^(I*d*x + I*c))*sqrt(e/(
e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 21*sqrt(2)*(I*a*e^3*e^(
6*I*d*x + 6*I*c) + 3*I*a*e^3*e^(4*I*d*x + 4*I*c) + 3*I*a*e^3*e^(2*I*d*x + 2
*I*c) + I*a*e^3)*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
e^(I*d*x + I*c))))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e
^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx = \text{Timed out}$$

```
[In] integrate((e*sec(d*x+c))**(7/2)*(a+I*a*tan(d*x+c)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx = \int (e\sec(dx+c))^{\frac{7}{2}}\,(i\,a\tan(dx+c)+a)\,dx$$

```
[In] integrate((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*sec(d*x + c))^(7/2)*(I*a*tan(d*x + c) + a), x)
```

Giac [F]

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx = \int (e\sec(dx+c))^{\frac{7}{2}}\,(i\,a\tan(dx+c)+a)\,dx$$

```
[In] integrate((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(7/2)*(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}\,(a+a\tan(c+d\,x)\ 1\mathrm{i})\ dx$$

```
[In] int((e/cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i),x)
[Out] int((e/cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i), x)
```

3.186 $\int (e \sec(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 94

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx = \frac{2ae^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3d} + \frac{2ia(e\sec(c+dx))^{5/2}}{5d} + \frac{2ae(e\sec(c+dx))^{3/2}\sin(c+dx)}{3d}$$

```
[Out] 2/5*I*a*(e*sec(d*x+c))^(5/2)/d+2/3*a*e*(e*sec(d*x+c))^(3/2)*sin(d*x+c)/d+2/
3*a*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d
*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3567, 3853, 3856, 2720}

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx = \frac{2ae^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3d} + \frac{2ia(e\sec(c+dx))^{5/2}}{5d} + \frac{2ae\sin(c+dx)(e\sec(c+dx))^{3/2}}{3d}$$

```
[In] Int[(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (2*a*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])
/(3*d) + (((2*I)/5)*a*(e*Sec[c + d*x])^(5/2))/d + (2*a*e*(e*Sec[c + d*x])^(
3/2)*Sin[c + d*x])/(3*d)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2ia(e\sec(c+dx))^{5/2}}{5d} + a\int (e\sec(c+dx))^{5/2}\,dx \\
&= \frac{2ia(e\sec(c+dx))^{5/2}}{5d} + \frac{2ae(e\sec(c+dx))^{3/2}\sin(c+dx)}{3d} + \frac{1}{3}\left(ae^2\right)\int \sqrt{e\sec(c+dx)}\,dx \\
&= \frac{2ia(e\sec(c+dx))^{5/2}}{5d} + \frac{2ae(e\sec(c+dx))^{3/2}\sin(c+dx)}{3d} \\
&\quad + \frac{1}{3}\left(ae^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx \\
&= \frac{2ae^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3d} \\
&\quad + \frac{2ia(e\sec(c+dx))^{5/2}}{5d} + \frac{2ae(e\sec(c+dx))^{3/2}\sin(c+dx)}{3d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.61

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx = \frac{a(e\sec(c+dx))^{5/2}\left(6i+10\cos^{\frac{5}{2}}(c+dx)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+5\sin(2(c+dx))\right)}{15d}$$

```
[In] Integrate[(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (a*(e*Sec[c + d*x])^(5/2)*(6*I + 10*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/
2, 2] + 5*Sin[2*(c + d*x)]))/(15*d)
```

Maple [A] (verified)

Time = 24.87 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.73

method	result
default	$-\frac{2a\sqrt{e\sec(dx+c)}\,e^2\left(i\cos(dx+c)F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}+iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{c}{co}}\right.}{3d}$
parts	$-\frac{2a\sqrt{e\sec(dx+c)}\,e^2\left(i\cos(dx+c)F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}+iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{c}{co}}\right.}{3d}$

```
[In] int((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*a/d*(e*sec(d*x+c))^(1/2)*e^2*(I*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+I*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(
cos(d*x+c)+1))^(1/2)-tan(d*x+c))+2/5*I*a*(e*sec(d*x+c))^(5/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.64

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx = -\frac{2\left(\sqrt{2}\left(5i\,ae^2e^{(4i\,dx+4i\,c)}-12i\,ae^2e^{(2i\,dx+2i\,c)}-5i\,ae^2\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+5\sqrt{2}\left(i\,ae^2e^{(4i\,dx+4i\,c)}+2i\right.}{15\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

```
[In] integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -2/15*(sqrt(2)*(5*I*a*e^2*e^(4*I*d*x + 4*I*c) - 12*I*a*e^2*e^(2*I*d*x + 2*I
*c) - 5*I*a*e^2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)
+ 5*sqrt(2)*(I*a*e^2*e^(4*I*d*x + 4*I*c) + 2*I*a*e^2*e^(2*I*d*x + 2*I*c) +
I*a*e^2)*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))/(d*e^(4*I*d*x
 + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx = ia\left(\int\left(-i(e\sec{(c+dx)})^{\frac{5}{2}}\right)dx + \int (e\sec{(c+dx)})^{\frac{5}{2}}\tan{(c+dx)}\,dx\right)$$

```
[In] integrate((e*sec(d*x+c))**(5/2)*(a+I*a*tan(d*x+c)),x)
[Out] I*a*(Integral(-I*(e*sec(c + d*x))**(5/2), x) + Integral((e*sec(c + d*x))**(
5/2)*tan(c + d*x), x))
```

Maxima [F]

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx = \int (e\sec{(dx+c)})^{\frac{5}{2}}(i\,a\tan{(dx+c)}+a)\,dx$$

```
[In] integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*sec(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a), x)
```

Giac [F]

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx = \int (e\sec{(dx+c)})^{\frac{5}{2}}(i\,a\tan{(dx+c)}+a)\,dx$$

```
[In] integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx = \int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^{5/2}\,\left(a+a\tan(c+d\,x)\,\,1\mathrm{i}\right)\,dx$$

```
[In] int((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i),x)
[Out] int((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i), x)
```

3.187 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 90

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx = -\frac{2ae^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2ia(e\sec(c+dx))^{3/2}}{3d} + \frac{2ae\sqrt{e\sec(c+dx)}\sin(c+dx)}{d}$$

```
[Out] 2/3*I*a*(e*sec(d*x+c))^(3/2)/d-2*a*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(e*sec
(d*x+c))^(1/2)+2*a*e*sin(d*x+c)*(e*sec(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3567, 3853, 3856, 2719}

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx = -\frac{2ae^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2ia(e\sec(c+dx))^{3/2}}{3d} + \frac{2ae\sin(c+dx)\sqrt{e\sec(c+dx)}}{d}$$

```
[In] Int[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (-2*a*e^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d
*x]]) + (((2*I)/3)*a*(e*Sec[c + d*x])^(3/2))/d + (2*a*e*Sqrt[e*Sec[c + d*x]
]*Sin[c + d*x])/d
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2ia(e\sec(c+dx))^{3/2}}{3d} + a\int (e\sec(c+dx))^{3/2}\,dx \\
&= \frac{2ia(e\sec(c+dx))^{3/2}}{3d} + \frac{2ae\sqrt{e\sec(c+dx)}\sin(c+dx)}{d} - \left(ae^2\right)\int \frac{1}{\sqrt{e\sec(c+dx)}}\,dx \\
&= \frac{2ia(e\sec(c+dx))^{3/2}}{3d} + \frac{2ae\sqrt{e\sec(c+dx)}\sin(c+dx)}{d} - \frac{\left(ae^2\right)\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} \\
&= -\frac{2ae^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2ia(e\sec(c+dx))^{3/2}}{3d} + \frac{2ae\sqrt{e\sec(c+dx)}\sin(c+dx)}{d}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.01 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.13

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx = \frac{2aee^{-2idx}\sqrt{e\sec(c+dx)}(\cos(c+3dx)+i\sin(c+3dx))\left(-2i+i\sqrt{1+e^{2i(c+dx)}}\,\text{Hyperge}\right.}{3d}$$

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (2*a*e*Sqrt[e*Sec[c + d*x]]*(Cos[c + 3*d*x] + I*Sin[c + 3*d*x])*(-2*I + I*S
qrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c
+ d*x))] + Tan[c + d*x]))/(3*d*E^((2*I)*d*x))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 410 vs. 2(105) = 210.

Time = 3.33 (sec) , antiderivative size = 411, normalized size of antiderivative = 4.57

method	result
default	$\frac{2a\left(iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}(\cos^2(dx+c))-iE(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{}{\cos(d}}\right.}{}$
parts	$\frac{2a\left(iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}(\cos^2(dx+c))-iE(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{}{\cos(d}}\right.}{}$

```
[In] int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a/d*(I*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2-I*EllipticE(I*(csc(d*x+c)-cot(
d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d
*x+c)^2+2*I*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-2*I*EllipticE(I*(csc(d*x+c)-c
ot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*co
s(d*x+c)+I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*
x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)-I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)+sin(d*x+c))*(e*s
ec(d*x+c))^(1/2)*e/(cos(d*x+c)+1)+2/3*I*a*(e*sec(d*x+c))^(3/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.29

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx =$$
$$-\frac{2\left(\sqrt{2}(3i\,aee^{(3i\,dx+3i\,c)}+i\,aee^{(i\,dx+i\,c)})\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+3\sqrt{2}(i\,aee^{(2i\,dx+2i\,c)}+i\,ae)\sqrt{e}\text{weierstras}\right.}{3\left(de^{(2i\,dx+2i\,c)}+d\right)}$$

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(2)*(3*I*a*e*e^(3*I*d*x + 3*I*c) + I*a*e*e^(I*d*x + I*c))*sqrt(e/
(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 3*sqrt(2)*(I*a*e*e^(2*
I*d*x + 2*I*c) + I*a*e)*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, e^(I*d*x + I*c))))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx = ia\left(\int \left(-i(e\sec(c+dx))^{\frac{3}{2}}\right)\,dx + \int (e\sec(c+dx))^{\frac{3}{2}}\tan(c+dx)\,dx\right)$$

```
[In] integrate((e*sec(d*x+c))**(3/2)*(a+I*a*tan(d*x+c)),x)
```

```
[Out] I*a*(Integral(-I*(e*sec(c + d*x))**(3/2), x) + Integral((e*sec(c + d*x))**(
3/2)*tan(c + d*x), x))
```

Maxima [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx = \int (e\sec(dx+c))^{\frac{3}{2}}(i\,a\tan(dx+c)+a)\,dx$$

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a), x)
```

Giac [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx = \int (e\sec(dx+c))^{\frac{3}{2}}\,(i\,a\tan(dx+c)+a)\,dx$$

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}\,(a+a\tan(c+d\,x)\;1\mathrm{i})\;dx$$

```
[In] int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i),x)
[Out] int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i), x)
```

3.188 $\int \sqrt{e \sec(c+dx)}(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 60

$$\int \sqrt{e \sec(c+dx)}(a+ia\tan(c+dx))\,dx = \frac{2ia\sqrt{e\sec(c+dx)}}{d} + \frac{2a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{d}$$

```
[Out] 2*I*a*(e*sec(d*x+c))^(1/2)/d+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))
^(1/2)/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3567, 3856, 2720}

$$\int \sqrt{e \sec(c+dx)}(a+ia\tan(c+dx))\,dx = \frac{2a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{d} + \frac{2ia\sqrt{e\sec(c+dx)}}{d}$$

```
[In] Int[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((2*I)*a*Sqrt[e*Sec[c + d*x]])/d + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/d
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2ia\sqrt{e\sec(c+dx)}}{d} + a\int\sqrt{e\sec(c+dx)}\,dx \\ &= \frac{2ia\sqrt{e\sec(c+dx)}}{d} + \left(a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx \\ &= \frac{2ia\sqrt{e\sec(c+dx)}}{d} + \frac{2a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

$$\int\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))\,dx = \frac{2a\left(i+\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\right)\sqrt{e\sec(c+dx)}}{d}$$

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x]),x]
[Out] (2*a*(I + Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])*Sqrt[e*Sec[c + d*x]
])/d
```

Maple [A] (verified)

Time = 7.39 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.60

method	result
parts	$-\frac{2ia(\cos(dx+c)+1)\sqrt{e\sec(dx+c)}\,\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,F(i(\csc(dx+c)-\cot(dx+c)),i)}{d}+\frac{2ia\sqrt{e\sec(dx+c)}}{d}$
default	$-\frac{2ia\left(F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\cos(dx+c)+F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos}{\cos(}}\right.}{d}$

```
[In] int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] -2*I*a/d*(cos(d*x+c)+1)*(e*sec(d*x+c))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)+2*I*a*(
e*sec(d*x+c))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98

$$\int\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))\,dx = -\frac{2\left(-i\sqrt{2}a\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+i\sqrt{2}a\sqrt{e}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)}{d}$$

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
[Out] -2*(-I*sqrt(2)*a*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)
+ I*sqrt(2)*a*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))/d
```

Sympy [F]

$$\int\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))\,dx = ia\left(\int\left(-i\sqrt{e\sec(c+dx)}\right)\,dx+\int\sqrt{e\sec(c+dx)}\tan(c+dx)\,dx\right)$$

```
[In] integrate((e*sec(d*x+c))**(1/2)*(a+I*a*tan(d*x+c)),x)
[Out] I*a*(Integral(-I*sqrt(e*sec(c + d*x)), x) + Integral(sqrt(e*sec(c + d*x))*t
an(c + d*x), x))
```

Maxima [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))\,dx = \int \sqrt{e\sec\left(dx+c\right)}(i\,a\tan\left(dx+c\right)+a)\,dx$$

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
[Out] integrate(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{-1,[2,0]%%%}+%%%{%%{[-2,0]:[1,0,%%%{1,[1]%%%}]%%},[1,0]%%%}+%
%%{%%%
```

Mupad [B] (verification not implemented)

Time = 4.52 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.67

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))\,dx = \frac{2\,a\,\left(\sqrt{\cos\left(c+d\,x\right)}\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+1{}\mathrm{i}\right)\,\sqrt{\frac{e}{\cos\left(c+d\,x\right)}}}{d}$$

```
[In] int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i),x)
[Out] (2*a*(cos(c + d*x)^(1/2)*ellipticF(c/2 + (d*x)/2, 2) + 1i)*(e/cos(c + d*x))
^(1/2))/d
```

3.189 $\int \frac{a+ia\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 60

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx=-\frac{2ia}{d\sqrt{e\sec(c+dx)}}+\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

```
[Out] -2*I*a/d/(e*sec(d*x+c))^(1/2)+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(e*sec(d*x+
c))^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3567, 3856, 2719}

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx=\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{2ia}{d\sqrt{e\sec(c+dx)}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] ((-2*I)*a)/(d*Sqrt[e*Sec[c + d*x]]) + (2*a*EllipticE[(c + d*x)/2, 2])/(d*Sq
rt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2ia}{d\sqrt{e\sec(c+dx)}} + a\int \frac{1}{\sqrt{e\sec(c+dx)}}\,dx \\
&= -\frac{2ia}{d\sqrt{e\sec(c+dx)}} + \frac{a\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} \\
&= -\frac{2ia}{d\sqrt{e\sec(c+dx)}} + \frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.70 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.22

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx = -\frac{4iae^{2i(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)}{3d\sqrt{1+e^{2i(c+dx)}}\sqrt{e\sec(c+dx)}}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])/Sqrt[e*Sec[c + d*x]],x]
[Out] (((-4*I)/3)*a*E^((2*I)*(c + d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I
)*(c + d*x))])/(d*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[e*Sec[c + d*x]])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 300 vs. 2(79) = 158.

Time = 6.64 (sec) , antiderivative size = 301, normalized size of antiderivative = 5.02

method	result
risch	$-\frac{2ia\sqrt{2}}{d\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}-\frac{i\left(-\frac{2\left(e\,\mathrm{e}^{2i(dx+c)}+e\right)}{e\sqrt{\mathrm{e}^{i(dx+c)}\left(e\,\mathrm{e}^{2i(dx+c)}+e\right)}}+\frac{i\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(\mathrm{e}^{i(dx+c)}-i\right)}\,\sqrt{i\mathrm{e}^{i(dx+c)}}\,\left(-2iE\left(\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)},\frac{\sqrt{2}}{2}\right.\right.}{\sqrt{e\,\mathrm{e}^{3i(dx+c)}+e\,\mathrm{e}^{i(dx+c)}}}}{d\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,\left(\mathrm{e}^{2i(dx+c)}+1\right)}$
parts	$\frac{2a\left(i\cos(dx+c)E(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}-i\cos(dx+c)F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+}}\right.}{}$
default	Expression too large to display

```
[In] int((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*I/d*a*2^(1/2)/(e*exp(I*(d*x+c))/(exp(I*(d*x+c))^2+1))^(1/2)-I/d*(-2*(e*e
xp(I*(d*x+c))^2+e)/e/(exp(I*(d*x+c))*(e*exp(I*(d*x+c))^2+e))^(1/2)+I*(-I*(e
xp(I*(d*x+c))+I))^(1/2)*2^(1/2)*(I*(exp(I*(d*x+c))-I))^(1/2)*(I*exp(I*(d*x+
c)))^(1/2)/(e*exp(I*(d*x+c))^3+e*exp(I*(d*x+c)))^(1/2)*(-2*I*EllipticE((-I*
(exp(I*(d*x+c))+I))^(1/2),1/2*2^(1/2))+I*EllipticF((-I*(exp(I*(d*x+c))+I))^
(1/2),1/2*2^(1/2))))*a*2^(1/2)/(e*exp(I*(d*x+c))/(exp(I*(d*x+c))^2+1))^(1/2
)*(e*exp(I*(d*x+c))*(exp(I*(d*x+c))^2+1))^(1/2)/(exp(I*(d*x+c))^2+1)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.45

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx = \frac{2i\,\sqrt{2}a\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)}{d\sqrt{e}}$$

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] 2*I*sqrt(2)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x +
I*c)))/(d*sqrt(e))
```

Sympy [F]

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx = ia\left(\int\left(-\frac{i}{\sqrt{e\sec(c+dx)}}\right)dx+\int\frac{\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx\right)$$

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))**(1/2),x)
[Out] I*a*(Integral(-I/sqrt(e*sec(c + d*x)), x) + Integral(tan(c + d*x)/sqrt(e*se
c(c + d*x)), x))
```

Maxima [F]

$$\int \frac{a + ia\tan(c + dx)}{\sqrt{e\sec(c + dx)}}\,dx = \int \frac{i\,a\tan\left(dx + c\right) + a}{\sqrt{e\sec\left(dx + c\right)}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)/sqrt(e*sec(d*x + c)), x)
```

Giac [F]

$$\int \frac{a + ia\tan(c + dx)}{\sqrt{e\sec(c + dx)}}\,dx = \int \frac{i\,a\tan\left(dx + c\right) + a}{\sqrt{e\sec\left(dx + c\right)}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)/sqrt(e*sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + ia\tan(c + dx)}{\sqrt{e\sec(c + dx)}}\,dx = \int \frac{a + a\tan(c + d\,x)\,1\mathrm{i}}{\sqrt{\frac{e}{\cos(c+d\,x)}}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)/(e/cos(c + d*x))^(1/2),x)
[Out] int((a + a*tan(c + d*x)*1i)/(e/cos(c + d*x))^(1/2), x)
```

3.190 $\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 96

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{2ia}{3d(e\sec(c+dx))^{3/2}} + \frac{2a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2} + \frac{2a\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}$$

```
[Out] -2/3*I*a/d/(e*sec(d*x+c))^(3/2)+2/3*a*sin(d*x+c)/d/e/(e*sec(d*x+c))^(1/2)+2
/3*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/d/e^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3567, 3854, 3856, 2720}

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{3/2}}\,dx = \frac{2a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2} - \frac{2ia}{3d(e\sec(c+dx))^{3/2}} + \frac{2a\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])/(e*Sec[c + d*x])^(3/2),x]
```

```
[Out] (((-2*I)/3)*a)/(d*(e*Sec[c + d*x])^(3/2)) + (2*a*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d*e^2) + (2*a*Sin[c + d*x])/(3*
d*e*Sqrt[e*Sec[c + d*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2ia}{3d(e\sec(c+dx))^{3/2}}+a\int \frac{1}{(e\sec(c+dx))^{3/2}}\,dx\\
&= -\frac{2ia}{3d(e\sec(c+dx))^{3/2}}+\frac{2a\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}+\frac{a\int \sqrt{e\sec(c+dx)}\,dx}{3e^2}\\
&= -\frac{2ia}{3d(e\sec(c+dx))^{3/2}}+\frac{2a\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}\\
&\quad+\frac{\left(a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3e^2}\\
&= -\frac{2ia}{3d(e\sec(c+dx))^{3/2}}+\frac{2a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}\\
&\quad+\frac{2a\sin(c+dx)}{3de\sqrt{e\sec(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.65

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{3/2}}\,dx = \frac{2a\left(-i\cos(c+dx)+\frac{\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{\sqrt{\cos(c+dx)}}+\sin(c+dx)\right)}{3de\sqrt{e\sec(c+dx)}}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])/(e*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*a*((-I)*Cos[c + d*x] + EllipticF[(c + d*x)/2, 2]/Sqrt[Cos[c + d*x]] + Si
n[c + d*x]))/(3*d*e*Sqrt[e*Sec[c + d*x]])
```

Maple [A] (verified)

Time = 6.75 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.58

method	result
default	$\frac{2a\left(iF(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+i\sec(dx+c)F(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{c}{co}}\right.}{3ed\sqrt{e\sec(dx+c)}}$
parts	$-\frac{2a\left(iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}+i\sec(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{}{\cos(}}\right.}{3d\sqrt{e\sec(dx+c)}\,e}$
risch	$-\frac{i e^{i(dx+c)}a\sqrt{2}}{3de\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}+\frac{2\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{i\left(e^{i(dx+c)}-i\right)}\sqrt{i e^{i(dx+c)}}\,F\left(\sqrt{-i\left(e^{i(dx+c)}+i\right)},\frac{\sqrt{2}}{2}\right)a\sqrt{e\,e^{i(dx+c)}\left(e^{2i(dx+c)}+1\right)}}{3d\sqrt{e\,e^{3i(dx+c)}+e\,e^{i(dx+c)}}\,e\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}$

```
[In] int((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*a/e/d/(e*sec(d*x+c))^(1/2)*(I*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(
1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*sec(d*x+c)*Elli
pticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)-I*cos(d*x+c)+sin(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.79

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{2}\left(-i\,ae^{(2i\,dx+2i\,c)}-i\,a\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}-2i\,\sqrt{2}a\sqrt{e}\text{weierstrassPInver}}{3\,de^2}$$

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-I*a*e^(2*I*d*x + 2*I*c) - I*a)*sqrt(e/(e^(2*I*d*x + 2*I*c) +
 1))*e^(1/2*I*d*x + 1/2*I*c) - 2*I*sqrt(2)*a*sqrt(e)*weierstrassPInverse(-4
, 0, e^(I*d*x + I*c)))/(d*e^2)
```

Sympy [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{3/2}}\,dx = ia\left(\int\left(-\frac{i}{(e\sec(c+dx))^{\frac{3}{2}}}\right)dx+\int\frac{\tan(c+dx)}{(e\sec(c+dx))^{\frac{3}{2}}}\,dx\right)$$

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))**(3/2),x)
[Out] I*a*(Integral(-I/(e*sec(c + d*x))**(3/2), x) + Integral(tan(c + d*x)/(e*sec
(c + d*x))**(3/2), x))
```

Maxima [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{i\,a\tan(dx+c)+a}{(e\sec(dx+c))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)/(e*sec(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{i\,a\tan(dx+c)+a}{(e\sec(dx+c))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)/(e*sec(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{3/2}}\,dx=\int \frac{a+a\tan(c+d\,x)\ 1\mathrm{i}}{{\left(\frac{e}{\cos(c+d\,x)}\right)}^{3/2}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)/(e/cos(c + d*x))^(3/2),x)
[Out] int((a + a*tan(c + d*x)*1i)/(e/cos(c + d*x))^(3/2), x)
```

3.191 $\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 96

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{2ia}{5d(e\sec(c+dx))^{5/2}} + \frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2a\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}$$

```
[Out] -2/5*I*a/d/(e*sec(d*x+c))^(5/2)+2/5*a*sin(d*x+c)/d/e/(e*sec(d*x+c))^(3/2)+6
/5*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/d/e^2/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3567, 3854, 3856, 2719}

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{5/2}}\,dx = \frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{2ia}{5d(e\sec(c+dx))^{5/2}} + \frac{2a\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])/(e*Sec[c + d*x])^(5/2),x]
```

```
[Out] (((-2*I)/5)*a)/(d*(e*Sec[c + d*x])^(5/2)) + (6*a*EllipticE[(c + d*x)/2, 2])
/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (2*a*Sin[c + d*x])/(5*
d*e*(e*Sec[c + d*x])^(3/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2ia}{5d(e\sec(c+dx))^{5/2}}+a\int\frac{1}{(e\sec(c+dx))^{5/2}}\,dx\\
&= -\frac{2ia}{5d(e\sec(c+dx))^{5/2}}+\frac{2a\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}+\frac{(3a)\int\frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{5e^2}\\
&= -\frac{2ia}{5d(e\sec(c+dx))^{5/2}}+\frac{2a\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}+\frac{(3a)\int\sqrt{\cos(c+dx)}\,dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\\
&= -\frac{2ia}{5d(e\sec(c+dx))^{5/2}}+\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2a\sin(c+dx)}{5de(e\sec(c+dx))^{3/2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.13 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.03

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{a\left(2+2\cos(2(c+dx))-2\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)-3i\sin(2(c+dx))\right)(-i}{5de^2\sqrt{e\sec(c+dx)}}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])/(e*Sec[c + d*x])^(5/2),x]
```

```
[Out] -1/5*(a*(2 + 2*Cos[2*(c + d*x)] - 2*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeom
etric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] - (3*I)*Sin[2*(c + d*x)])*(-I
 + Tan[c + d*x]))/(d*e^2*Sqrt[e*Sec[c + d*x]])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 319 vs. 2(107) = 214.

Time = 6.19 (sec) , antiderivative size = 320, normalized size of antiderivative = 3.33

method	result
risch	$-\frac{i\left(e^{2i(dx+c)}+7\right)a\sqrt{2}}{10d\,e^2\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}-\frac{3i\left(-\frac{2\left(e\,e^{2i(dx+c)}+e\right)}{e\sqrt{e^{i(dx+c)}\left(e\,e^{2i(dx+c)}+e\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(e^{i(dx+c)}-i\right)}\,\sqrt{ie^{i(dx+c)}}\left(-2iE\left(\sqrt{-i\left(e^{i(dx+c)}+\cdots\right.}\right.\right.}{\sqrt{e\,e^{3i(dx+c)}+e\,e^{i(dx+c)}}}\right.}{5d\,e^2\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}$
default	$\frac{2a\left(-3i\cos(dx+c)F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}+3i\cos(dx+c)E(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+\cdots}{\cos(dx+c\cdots}}\right.}{}$
parts	$\frac{2a\left(-3i\cos(dx+c)F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}+3i\cos(dx+c)E(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+\cdots}{\cos(dx+c\cdots}}\right.}{}$

```
[In] int((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*I*(exp(I*(d*x+c))^2+7)/d*a*2^(1/2)/e^2/(e*exp(I*(d*x+c))/(exp(I*(d*x+
c))^2+1))^(1/2)-3/5*I/d*(-2*(e*exp(I*(d*x+c))^2+e)/e/(exp(I*(d*x+c))*(e*exp
(I*(d*x+c))^2+e))^(1/2)+I*(-I*(exp(I*(d*x+c))+I))^(1/2)*2^(1/2)*(I*(exp(I*(
d*x+c))-I))^(1/2)*(I*exp(I*(d*x+c)))^(1/2)/(e*exp(I*(d*x+c))^3+e*exp(I*(d*x
+c)))^(1/2)*(-2*I*EllipticE((-I*(exp(I*(d*x+c))+I))^(1/2),1/2*2^(1/2))+I*El
lipticF((-I*(exp(I*(d*x+c))+I))^(1/2),1/2*2^(1/2))))*a*2^(1/2)/e^2/(exp(I*(
d*x+c))^2+1)/(e*exp(I*(d*x+c))/(exp(I*(d*x+c))^2+1))^(1/2)*(e*exp(I*(d*x+c)
)*(exp(I*(d*x+c))^2+1))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.14

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{5/2}}\,dx = \frac{\left(12i\sqrt{2}a\sqrt{e}e^{(i\,dx+i\,c)}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right.\right.}{10}$$

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/10*(12*I*sqrt(2)*a*sqrt(e)*e^(I*d*x + I*c)*weierstrassZeta(-4, 0, weierst
rassPInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(-I*a*e^(4*I*d*x + 4*I*c) +
 4*I*a*e^(2*I*d*x + 2*I*c) + 5*I*a)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/
2*I*d*x + 1/2*I*c))*e^(-I*d*x - I*c)/(d*e^3)
```

Sympy [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{5/2}}\,dx = ia\left(\int \left(-\frac{i}{(e\sec{(c+dx)})^{\frac{5}{2}}}\right)dx + \int \frac{\tan{(c+dx)}}{(e\sec{(c+dx)})^{\frac{5}{2}}}\,dx\right)$$

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))**(5/2),x)
```

```
[Out] I*a*(Integral(-I/(e*sec(c + d*x))**(5/2), x) + Integral(tan(c + d*x)/(e*sec
(c + d*x))**(5/2), x))
```

Maxima [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{i\,a\tan{(dx+c)}+a}{(e\sec{(dx+c)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)/(e*sec(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{i\,a\tan\left(dx+c\right)+a}{\left(e\sec\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)/(e*sec(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{a+a\tan(c+d\,x)\ 1\mathrm{i}}{{\left(\frac{e}{\cos(c+d\,x)}\right)}^{5/2}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)/(e/cos(c + d*x))^(5/2),x)
[Out] int((a + a*tan(c + d*x)*1i)/(e/cos(c + d*x))^(5/2), x)
```

3.192 $\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 125

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{7/2}}\,dx = -\frac{2ia}{7d(e\sec(c+dx))^{7/2}} + \frac{10a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{21de^4} + \frac{2a\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}} + \frac{10a\sin(c+dx)}{21de^3\sqrt{e\sec(c+dx)}}$$

```
[Out] -2/7*I*a/d/(e*sec(d*x+c))^(7/2)+2/7*a*sin(d*x+c)/d/e/(e*sec(d*x+c))^(5/2)+1
0/21*a*sin(d*x+c)/d/e^3/(e*sec(d*x+c))^(1/2)+10/21*a*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^
(1/2)*(e*sec(d*x+c))^(1/2)/d/e^4
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3567, 3854, 3856, 2720}

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{7/2}}\,dx = \frac{10a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{21de^4} + \frac{10a\sin(c+dx)}{21de^3\sqrt{e\sec(c+dx)}} - \frac{2ia}{7d(e\sec(c+dx))^{7/2}} + \frac{2a\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])/(e*Sec[c + d*x])^(7/2),x]
```

```
[Out] (((-2*I)/7)*a)/(d*(e*Sec[c + d*x])^(7/2)) + (10*a*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(21*d*e^4) + (2*a*Sin[c + d*x])/(
```

```
7*d*e*(e*Sec[c + d*x])^(5/2)) + (10*a*Sin[c + d*x])/(21*d*e^3*Sqrt[e*Sec[c
+ d*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2ia}{7d(e\sec(c+dx))^{7/2}} + a\int \frac{1}{(e\sec(c+dx))^{7/2}}\,dx \\
&= -\frac{2ia}{7d(e\sec(c+dx))^{7/2}} + \frac{2a\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}} + \frac{(5a)\int \frac{1}{(e\sec(c+dx))^{3/2}}\,dx}{7e^2} \\
&= -\frac{2ia}{7d(e\sec(c+dx))^{7/2}} + \frac{2a\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}} \\
&\quad + \frac{10a\sin(c+dx)}{21de^3\sqrt{e\sec(c+dx)}} + \frac{(5a)\int \sqrt{e\sec(c+dx)}\,dx}{21e^4} \\
&= -\frac{2ia}{7d(e\sec(c+dx))^{7/2}} + \frac{2a\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}} + \frac{10a\sin(c+dx)}{21de^3\sqrt{e\sec(c+dx)}} \\
&\quad + \frac{\left(5a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21e^4}
\end{aligned}
$$

$$= -\frac{2ia}{7d(e\sec(c+dx))^{7/2}} + \frac{10a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{21de^4} + \frac{2a\sin(c+dx)}{7de(e\sec(c+dx))^{5/2}} + \frac{10a\sin(c+dx)}{21de^3\sqrt{e\sec(c+dx)}}$$

Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.97

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{7/2}}\,dx = \frac{a\sqrt{e\sec(c+dx)}(\cos(c+dx)+i\sin(c+dx))\left(-14i\cos(c+dx)+2i\cos(3(c+\right.}{}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])/(e*Sec[c + d*x])^(7/2),x]
```

```
[Out] (a*Sqrt[e*Sec[c + d*x]]*(Cos[c + d*x] + I*Sin[c + d*x])*((-14*I)*Cos[c + d*
x] + (2*I)*Cos[3*(c + d*x)] + 20*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*(Cos[c + d*x] - I*Sin[c + d*x]) + 5*Sin[c + d*x] + 5*Sin[3*(c + d*x)]))/
(42*d*e^4)
```

Maple [A] (verified)

Time = 5.82 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.43

method	result
default	$-\frac{2a\left(5iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}+5i\sec(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{}{c}}\right.}{21d\sqrt{e\sec(dx+c)}\,e^3}$
parts	$-\frac{2a\left(5iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}+5i\sec(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{}{c}}\right.}{21d\sqrt{e\sec(dx+c)}\,e^3}$

```
[In] int((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21*a/d/(e*sec(d*x+c))^(1/2)/e^3*(5*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*E
llipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)+5*I*sec(d*x+
c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)
*(1/(cos(d*x+c)+1))^(1/2)-3*cos(d*x+c)^2*sin(d*x+c)-5*sin(d*x+c))-2/7*I*a/d
/(e*sec(d*x+c))^(7/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.94

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{7/2}}\,dx = \frac{\left(-40i\sqrt{2}a\sqrt{e}e^{(2i\,dx+2i\,c)}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(-3i\,ae^{(6i\,dx+6i}\right.\right.}{84\,d}$$

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/84*(-40*I*sqrt(2)*a*sqrt(e)*e^(2*I*d*x + 2*I*c)*weierstrassPInverse(-4, 0
, e^(I*d*x + I*c)) + sqrt(2)*(-3*I*a*e^(6*I*d*x + 6*I*c) - 19*I*a*e^(4*I*d*
x + 4*I*c) - 9*I*a*e^(2*I*d*x + 2*I*c) + 7*I*a)*sqrt(e/(e^(2*I*d*x + 2*I*c)
 + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-2*I*d*x - 2*I*c)/(d*e^4)
```

Sympy [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{7/2}}\,dx = ia\left(\int\left(-\frac{i}{(e\sec(c+dx))^{\frac{7}{2}}}\right)dx+\int\frac{\tan(c+dx)}{(e\sec(c+dx))^{\frac{7}{2}}}\,dx\right)$$

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))**(7/2),x)
```

```
[Out] I*a*(Integral(-I/(e*sec(c + d*x))**(7/2), x) + Integral(tan(c + d*x)/(e*sec
(c + d*x))**(7/2), x))
```

Maxima [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{7/2}}\,dx = \int \frac{i\,a\tan(dx+c)+a}{(e\sec(dx+c))^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)/(e*sec(d*x + c))^(7/2), x)
```

Giac [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{7/2}}\,dx = \int \frac{i\,a\tan\left(dx+c\right)+a}{\left(e\sec\left(dx+c\right)\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))/(e*sec(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)/(e*sec(d*x + c))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{7/2}}\,dx = \int \frac{a+a\tan(c+d\,x)\ 1\mathrm{i}}{{\left(\frac{e}{\cos(c+d\,x)}\right)}^{7/2}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)/(e/cos(c + d*x))^(7/2),x)
[Out] int((a + a*tan(c + d*x)*1i)/(e/cos(c + d*x))^(7/2), x)
```

3.193 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 28, antiderivative size = 138

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2\,dx = -\frac{14a^2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{14ia^2(e\sec(c+dx))^{3/2}}{15d}+\frac{14a^2e\sqrt{e\sec(c+dx)}\sin(c+dx)}{5d}+\frac{2i(e\sec(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)}{5d}$$

```
[Out] 14/15*I*a^2*(e*sec(d*x+c))^(3/2)/d-14/5*a^2*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/
2)/(e*sec(d*x+c))^(1/2)+14/5*a^2*e*sin(d*x+c)*(e*sec(d*x+c))^(1/2)/d+2/5*I*
(e*sec(d*x+c))^(3/2)*(a^2+I*a^2*tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3579, 3567, 3853, 3856, 2719}

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2\,dx = -\frac{14a^2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{14ia^2(e\sec(c+dx))^{3/2}}{15d}+\frac{14a^2e\sin(c+dx)\sqrt{e\sec(c+dx)}}{5d}+\frac{2i\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}}{5d}$$

```
[In] Int[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (-14*a^2*e^2*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) + (((14*I)/15)*a^2*(e*Sec[c + d*x])^(3/2))/d + (14*a^2*e*Sqrt[e*
Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (((2*I)/5)*(e*Sec[c + d*x])^(3/2)*(a^2
+ I*a^2*Tan[c + d*x]))/d
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\text{integral} = \frac{2i(e\sec(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)}{5d} + \frac{1}{5}(7a)\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx$$

$$
\begin{aligned}
&= \frac{14ia^2(e\sec(c+dx))^{3/2}}{15d} + \frac{2i(e\sec(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)}{5d} \\
&\quad + \frac{1}{5}\left(7a^2\right)\int (e\sec(c+dx))^{3/2}\,dx \\
&= \frac{14ia^2(e\sec(c+dx))^{3/2}}{15d} + \frac{14a^2e\sqrt{e\sec(c+dx)}\sin(c+dx)}{5d} \\
&\quad + \frac{2i(e\sec(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)}{5d} - \frac{1}{5}\left(7a^2e^2\right)\int \frac{1}{\sqrt{e\sec(c+dx)}}\,dx \\
&= \frac{14ia^2(e\sec(c+dx))^{3/2}}{15d} + \frac{14a^2e\sqrt{e\sec(c+dx)}\sin(c+dx)}{5d} \\
&\quad + \frac{2i(e\sec(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)}{5d} - \frac{\left(7a^2e^2\right)\int \sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} \\
&= -\frac{14a^2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{14ia^2(e\sec(c+dx))^{3/2}}{15d} \\
&\quad + \frac{14a^2e\sqrt{e\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2i(e\sec(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)}{5d}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.30 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.93

$$
\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2\,dx = \frac{(e\sec(c+dx))^{3/2}\left(-\frac{14i\sqrt{2}\left(3\sqrt{1+e^{2i(c+dx)}}-e^{2idx}\left(-1+e^{2ic}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)\right)}{\left(-1+e^{2ic}\right)\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}} + \cdots\right.}{}
$$

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((e*Sec[c + d*x])^(3/2)*(((-14*I)*Sqrt[2]*(3*Sqrt[1 + E^((2*I)*(c + d*x))]
- E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*
I)*(c + d*x))]))/((-1 + E^((2*I)*c))*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
+ d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]) + (Csc[c]*Sec[c + d*x]^(5/2)*(Cos[
2*c] - I*Sin[2*c])*(36*Cos[d*x] + 27*Cos[2*c + d*x] + 21*Cos[2*c + 3*d*x] -
 (20*I)*Sin[d*x] + (20*I)*Sin[2*c + d*x]))/2)*(a + I*a*Tan[c + d*x])^2)/(15
*d*Sec[c + d*x]^(7/2)*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 429 vs. 2(143) = 286.

Time = 14.59 (sec) , antiderivative size = 430, normalized size of antiderivative = 3.12

method	result
default	$\frac{2e\, a^2 \sqrt{e \sec(dx+c)} \left(21iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}\left(\cos^2(dx+c)\right)-21iE(i(\csc(dx+c)-\cot(dx+c)),\right.}{}$
parts	Expression too large to display

```
[In] int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*e*a^2/d*(e*sec(d*x+c))^(1/2)/(cos(d*x+c)+1)*(21*I*EllipticF(I*(csc(d*x
+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/
2)*cos(d*x+c)^2-21*I*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2+42*I*cos(d*x+c)*El
lipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
cos(d*x+c)+1))^(1/2)-42*I*cos(d*x+c)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+21*I*EllipticF(
I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+
c)+1))^(1/2)-21*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(csc(d*x+c)
-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)+10*I+21*sin(d*x+c)+10*I*sec(d*x+c)
-3*tan(d*x+c)-3*sec(d*x+c)*tan(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.20

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2\,dx =$$

$$-\frac{2\left(\sqrt{2}\left(21i\,a^2ee^{(5i\,dx+5i\,c)}+16i\,a^2ee^{(3i\,dx+3i\,c)}+7i\,a^2ee^{(i\,dx+i\,c)}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+21\sqrt{2}\left(i\,a^2ee^{(4i\,dx}\right.\right.}{15\left(de^{(4i\,dx+4i\,c)}+2\,de\right.}$$

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/15*(sqrt(2)*(21*I*a^2*e*e^(5*I*d*x + 5*I*c) + 16*I*a^2*e*e^(3*I*d*x + 3*
I*c) + 7*I*a^2*e*e^(I*d*x + I*c))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*
I*d*x + 1/2*I*c) + 21*sqrt(2)*(I*a^2*e*e^(4*I*d*x + 4*I*c) + 2*I*a^2*e*e^(2
*I*d*x + 2*I*c) + I*a^2*e)*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, e^(I*d*x + I*c))))/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*
c) + d)
```

Sympy [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2\,dx = -a^2\left(\int\left(-(e\sec(c+dx))^{\frac{3}{2}}\right)\,dx + \int (e\sec(c+dx))^{\frac{3}{2}}\tan^2(c+dx)\,dx + \int\left(-2i(e\sec(c+dx))^{\frac{3}{2}}\tan(c+dx)\right)\,dx\right)$$

```
[In] integrate((e*sec(d*x+c))**(3/2)*(a+I*a*tan(d*x+c))**2,x)
[Out] -a**2*(Integral(-(e*sec(c + d*x))**(3/2), x) + Integral((e*sec(c + d*x))**(
3/2)*tan(c + d*x)**2, x) + Integral(-2*I*(e*sec(c + d*x))**(3/2)*tan(c + d*
x), x))
```

Maxima [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2\,dx = \int (e\sec(dx+c))^{\frac{3}{2}}(i\,a\tan(dx+c)+a)^2\,dx$$

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
[Out] integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^2, x)
```

Giac [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2\,dx = \int (e\sec(dx+c))^{\frac{3}{2}}(i\,a\tan(dx+c)+a)^2\,dx$$

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}(a+a\tan(c+d\,x)\,1\mathrm{i})^2\,dx$$

```
[In] int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^2,x)
[Out] int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^2, x)
```

3.194 $\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 28, antiderivative size = 106

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2\,dx = \frac{10ia^2\sqrt{e\sec(c+dx)}}{3d} + \frac{10a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3d} + \frac{2i\sqrt{e\sec(c+dx)}(a^2+ia^2\tan(c+dx))}{3d}$$

```
[Out] 10/3*I*a^2*(e*sec(d*x+c))^(1/2)/d+10/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*s
ec(d*x+c))^(1/2)/d+2/3*I*(e*sec(d*x+c))^(1/2)*(a^2+I*a^2*tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3579, 3567, 3856, 2720}

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2\,dx = \frac{10ia^2\sqrt{e\sec(c+dx)}}{3d} + \frac{2i(a^2+ia^2\tan(c+dx))\sqrt{e\sec(c+dx)}}{3d} + \frac{10a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3d}$$

```
[In] Int[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (((10*I)/3)*a^2*Sqrt[e*Sec[c + d*x]])/d + (10*a^2*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d) + (((2*I)/3)*Sqrt[e*Sec[c +
 d*x]]*(a^2 + I*a^2*Tan[c + d*x]))/d
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2i\sqrt{e\sec(c+dx)}\left(a^2+ia^2\tan(c+dx)\right)}{3d} \\
&\quad+\frac{1}{3}(5a)\int\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))\,dx \\
&= \frac{10ia^2\sqrt{e\sec(c+dx)}}{3d}+\frac{2i\sqrt{e\sec(c+dx)}\left(a^2+ia^2\tan(c+dx)\right)}{3d} \\
&\quad+\frac{1}{3}\left(5a^2\right)\int\sqrt{e\sec(c+dx)}\,dx \\
&= \frac{10ia^2\sqrt{e\sec(c+dx)}}{3d}+\frac{2i\sqrt{e\sec(c+dx)}\left(a^2+ia^2\tan(c+dx)\right)}{3d} \\
&\quad+\frac{1}{3}\left(5a^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx
\end{aligned}$$

$$= \frac{10ia^2\sqrt{e\sec(c+dx)}}{3d} + \frac{10a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3d} + \frac{2i\sqrt{e\sec(c+dx)}(a^2+ia^2\tan(c+dx))}{3d}$$

Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.63

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2\,dx$$
$$= \frac{2a^2(e\sec(c+dx))^{3/2}\left(6i\cos(c+dx)+5\cos^{\frac{3}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-\sin(c+dx)\right)}{3de}$$

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (2*a^2*(e*Sec[c + d*x])^(3/2)*((6*I)*Cos[c + d*x] + 5*Cos[c + d*x]^(3/2)*El
lipticF[(c + d*x)/2, 2] - Sin[c + d*x]))/(3*d*e)
```

Maple [A] (verified)

Time = 10.73 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.36

method	result
default	$-\frac{2a^2\sqrt{e\sec(dx+c)}\left(-5i\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}F(i(-\csc(dx+c)+\cot(dx+c)),i)\cos(dx+c)-5iF(i(-\csc(dx+c)+\cot(dx+c)\right.}{3d}$
parts	$-\frac{2ia^2(\cos(dx+c)+1)\sqrt{e\sec(dx+c)}\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}F(i(\csc(dx+c)-\cot(dx+c)),i)}{d}+\frac{4ia^2\sqrt{e\sec(dx+c)}}{d}-\frac{2a^2\sqrt{e}}{}$

```
[In] int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*a^2/d*(e*sec(d*x+c))^(1/2)*(-5*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/
(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*cos(d*x+c)-5*
I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-
csc(d*x+c)+cot(d*x+c)),I)-6*I+tan(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.02

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2\,dx =$$
$$-\frac{2\left(\sqrt{2}\left(-7i\,a^2e^{(2i\,dx+2i\,c)}-5i\,a^2\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+5\sqrt{2}\left(i\,a^2e^{(2i\,dx+2i\,c)}+i\,a^2\right)\sqrt{e}\text{weierstrassPIn}\right.}{3\left(de^{(2i\,dx+2i\,c)}+d\right)}$$

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(2)*(-7*I*a^2*e^(2*I*d*x + 2*I*c) - 5*I*a^2)*sqrt(e/(e^(2*I*d*x +
 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 5*sqrt(2)*(I*a^2*e^(2*I*d*x + 2*I*c
) + I*a^2)*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))/(d*e^(2*I*d
*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2\,dx = -a^2\left(\int\left(-\sqrt{e\sec(c+dx)}\right)dx + \int\sqrt{e\sec(c+dx)}\tan^2(c+dx)\,dx + \int\left(-2i\sqrt{e\sec(c+dx)}\tan(c+dx)\right)dx\right)$$

```
[In] integrate((e*sec(d*x+c))**(1/2)*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -a**2*(Integral(-sqrt(e*sec(c + d*x)), x) + Integral(sqrt(e*sec(c + d*x))*t
an(c + d*x)**2, x) + Integral(-2*I*sqrt(e*sec(c + d*x))*tan(c + d*x), x))
```

Maxima [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2\,dx = \int\sqrt{e\sec(dx+c)}(i\,a\tan(dx+c)+a)^2\,dx$$

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{-1,[2,0]%%%}+%%%{%%{[-2,0]:[1,0,%%%{1,[1]%%%}]%%},[1,0]%%%}+%
%%{%%%
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2\,dx = \int \sqrt{\frac{e}{\cos(c+d\,x)}}\,(a+a\tan(c+d\,x)\;1\mathrm{i})^2\,dx$$

```
[In] int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^2, x)
```

3.195 $\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 107

$$\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}\,dx = \frac{6a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{6a^2\sqrt{e\sec(c+dx)}\sin(c+dx)}{de} - \frac{4i(a^2+ia^2\tan(c+dx))}{d\sqrt{e\sec(c+dx)}}$$

```
[Out] 6*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)-6*a^2*sin(d*x+c)*(
e*sec(d*x+c))^(1/2)/d/e-4*I*(a^2+I*a^2*tan(d*x+c))/d/(e*sec(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3577, 3853, 3856, 2719}

$$\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}\,dx = -\frac{6a^2\sin(c+dx)\sqrt{e\sec(c+dx)}}{de} - \frac{4i(a^2+ia^2\tan(c+dx))}{d\sqrt{e\sec(c+dx)}} + \frac{6a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^2/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] (6*a^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]
]) - (6*a^2*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/(d*e) - ((4*I)*(a^2 + I*a^2*
Tan[c + d*x]))/(d*Sqrt[e*Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{4i(a^2+ia^2\tan(c+dx))}{d\sqrt{e\sec(c+dx)}}-\frac{(3a^2)\int(e\sec(c+dx))^{3/2}\,dx}{e^2}\\
&= -\frac{6a^2\sqrt{e\sec(c+dx)}\sin(c+dx)}{de}-\frac{4i(a^2+ia^2\tan(c+dx))}{d\sqrt{e\sec(c+dx)}}+(3a^2)\int\frac{1}{\sqrt{e\sec(c+dx)}}\,dx\\
&= -\frac{6a^2\sqrt{e\sec(c+dx)}\sin(c+dx)}{de}-\frac{4i(a^2+ia^2\tan(c+dx))}{d\sqrt{e\sec(c+dx)}}+\frac{(3a^2)\int\sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\\
&= \frac{6a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{6a^2\sqrt{e\sec(c+dx)}\sin(c+dx)}{de}-\frac{4i(a^2+ia^2\tan(c+dx))}{d\sqrt{e\sec(c+dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.18 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.23

$$\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}\,dx = -\frac{2i\sqrt{2}a^2e^{2i(c+dx)}\left(-\sqrt{1+e^{2i(c+dx)}}+\left(1+e^{2i(c+dx)}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)\right)}{d\sqrt{\frac{ee^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(1+e^{2i(c+dx)}\right)^{3/2}}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^2/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] ((-2*I)*Sqrt[2]*a^2*E^((2*I)*(c + d*x))*(-Sqrt[1 + E^((2*I)*(c + d*x))] + (
1 + E^((2*I)*(c + d*x)))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*
x))]))/(d*Sqrt[(e*E^(I*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(1 + E^((2*I)
*(c + d*x)))^(3/2))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 806 vs. 2(123) = 246.

Time = 10.34 (sec) , antiderivative size = 807, normalized size of antiderivative = 7.54

method	result
parts	$\frac{2a^2\left(i\cos(dx+c)E(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}-i\cos(dx+c)F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right.}{}$
default	Expression too large to display

```
[In] int((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a^2/d/(cos(d*x+c)+1)/(e*sec(d*x+c))^(1/2)*(I*EllipticE(I*(csc(d*x+c)-cot(
d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d
*x+c)-I*cos(d*x+c)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1)
)^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+2*I*(1/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)-2*I*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(
cos(d*x+c)+1))^(1/2)+I*sec(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)-I*sec(d*x+c)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos
(d*x+c)+1))^(1/2)+sin(d*x+c))-4*I*a^2/(e*sec(d*x+c))^(1/2)/d+2*a^2/d/(cos(d
*x+c)+1)/(e*sec(d*x+c))^(1/2)*(2*I*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(
1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-2*I*El
```

```
lipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*cos(d*x+c)+4*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)-4*I*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)
+1))^(1/2)+2*I*sec(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)-2*I*sec(d*x+c)*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c
)+1))^(1/2)+sin(d*x+c)-tan(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.64

$$\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}\,dx = -\frac{2\left(-i\sqrt{2}a^2\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{3}{2}i\,dx+\frac{3}{2}i\,c\right)} - 3i\sqrt{2}a^2\sqrt{e}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i}\right.\right.\right.}{de}$$

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(-I*sqrt(2)*a^2*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c
) - 3*I*sqrt(2)*a^2*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, e^(I*d*x + I*c))))/(d*e)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}\,dx = -a^2\left(\int\left(-\frac{1}{\sqrt{e\sec(c+dx)}}\right)dx + \int\frac{\tan^2(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx + \int\left(-\frac{2i\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\right)dx\right)$$

```
[In] integrate((a+I*a*tan(d*x+c))**2/(e*sec(d*x+c))**(1/2),x)
```

```
[Out] -a**2*(Integral(-1/sqrt(e*sec(c + d*x)), x) + Integral(tan(c + d*x)**2/sqrt
(e*sec(c + d*x)), x) + Integral(-2*I*tan(c + d*x)/sqrt(e*sec(c + d*x)), x))
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^2}{\sqrt{e\sec(dx+c)}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^2/sqrt(e*sec(d*x + c)), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^2}{\sqrt{e\sec(dx+c)}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^2/sqrt(e*sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}\,dx = \int \frac{(a+a\tan(c+d\,x)\;1\mathrm{i})^2}{\sqrt{\frac{e}{\cos(c+d\,x)}}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(1/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(1/2), x)
```

3.196 $\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 85

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{2a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{3d(e\sec(c+dx))^{3/2}}$$

```
[Out] -2/3*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*
d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/d/e^2-4/3*I*(a^2+
I*a^2*tan(d*x+c))/d/(e*sec(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3577, 3856, 2720}

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{2a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{3d(e\sec(c+dx))^{3/2}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(3*d*e^2) - (((4*I)/3)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(3/2
))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{4i(a^2+ia^2\tan(c+dx))}{3d(e\sec(c+dx))^{3/2}}-\frac{a^2\int\sqrt{e\sec(c+dx)}\,dx}{3e^2}\\
&= -\frac{4i(a^2+ia^2\tan(c+dx))}{3d(e\sec(c+dx))^{3/2}}-\frac{\left(a^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3e^2}\\
&= -\frac{2a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2}-\frac{4i(a^2+ia^2\tan(c+dx))}{3d(e\sec(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.53 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int\frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{2a^2\sec^2(c+dx)\left(2i\cos(c+dx)+\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(\cos(c+dx)-i\sin(c+dx))\right)(\cos\ldots}{3d(e\sec(c+dx))^{3/2}(\cos(dx)+i\sin(dx))^2}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*a^2*Sec[c + d*x]^2*((2*I)*Cos[c + d*x] + Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2]*(Cos[c + d*x] - I*Sin[c + d*x]))*(Cos[c + 3*d*x] + I*Sin[c +
 3*d*x]))/(3*d*(e*Sec[c + d*x])^(3/2)*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [A] (verified)

Time = 9.08 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.84

method	result
default	$-\frac{2a^2\left(iF(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+i\sec(dx+c)F(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\right.}{3ed\sqrt{e\sec(dx+c)}}$
risch	$-\frac{2ie^{i(dx+c)}a^2\sqrt{2}}{3de\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}-\frac{2\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{i\left(e^{i(dx+c)}-i\right)}\sqrt{ie^{i(dx+c)}}F\left(\sqrt{-i\left(e^{i(dx+c)}+i\right)},\frac{\sqrt{2}}{2}\right)a^2\sqrt{e\,e^{i(dx+c)}\left(e^{2i(dx+c)}+1\right)}}{3d\sqrt{e\,e^{3i(dx+c)}+e\,e^{i(dx+c)}}\,e\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}$
parts	$-\frac{2a^2\left(iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}+i\sec(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\right.}{3d\sqrt{e\sec(dx+c)}\,e}$

```
[In] int((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*a^2/e/d/(e*sec(d*x+c))^(1/2)*(I*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I
)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*sec(d*x+c)*E
llipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)+2*I*cos(d*x+c)-2*sin(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{2\left(-i\sqrt{2}a^2\sqrt{e}\,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(i\,a^2e^{(2i\,dx+2i\,c)}+i\,a^2\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}\right)}{3\,de^2}$$

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(-I*sqrt(2)*a^2*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)) +
sqrt(2)*(I*a^2*e^(2*I*d*x + 2*I*c) + I*a^2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1
))*e^(1/2*I*d*x + 1/2*I*c))/(d*e^2)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{3/2}}\,dx = -a^2\left(\int \left(-\frac{1}{(e\sec{(c+dx)})^{\frac{3}{2}}}\right)dx + \int \frac{\tan^2{(c+dx)}}{(e\sec{(c+dx)})^{\frac{3}{2}}}\,dx + \int \left(-\frac{2i\tan{(c+dx)}}{(e\sec{(c+dx)})^{\frac{3}{2}}}\right)dx\right)$$

```
[In] integrate((a+I*a*tan(d*x+c))**2/(e*sec(d*x+c))**(3/2),x)
[Out] -a**2*(Integral(-1/(e*sec(c + d*x))**(3/2), x) + Integral(tan(c + d*x)**2/(
e*sec(c + d*x))**(3/2), x) + Integral(-2*I*tan(c + d*x)/(e*sec(c + d*x))**(
3/2), x))
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{(i\,a\tan{(dx+c)}+a)^2}{(e\sec{(dx+c)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^2/(e*sec(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{(i\,a\tan{(dx+c)}+a)^2}{(e\sec{(dx+c)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^2/(e*sec(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{(a+a\tan(c+d\,x)\;1\mathrm{i})^2}{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(3/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(3/2), x)
```

3.197 $\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 85

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}\,dx=\frac{2a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i(a^2+ia^2\tan(c+dx))}{5d(e\sec(c+dx))^{5/2}}$$

```
[Out] 2/5*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/d/e^2/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)-4/5*I*(a^2+I
*a^2*tan(d*x+c))/d/(e*sec(d*x+c))^(5/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3577, 3856, 2719}

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}\,dx=\frac{2a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i(a^2+ia^2\tan(c+dx))}{5d(e\sec(c+dx))^{5/2}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*EllipticE[(c + d*x)/2, 2])/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c
+ d*x]]) - (((4*I)/5)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(5/2)
)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{4i(a^2+ia^2\tan(c+dx))}{5d(e\sec(c+dx))^{5/2}}+\frac{a^2\int\frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{5e^2}\\ &= -\frac{4i(a^2+ia^2\tan(c+dx))}{5d(e\sec(c+dx))^{5/2}}+\frac{a^2\int\sqrt{\cos(c+dx)}\,dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\\ &= \frac{2a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4i(a^2+ia^2\tan(c+dx))}{5d(e\sec(c+dx))^{5/2}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.99 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int\frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{i\sqrt{2}a^2\left(\frac{ee^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{3/2}\left(1+e^{2i(c+dx)}\right)^{3/2}\left(3\sqrt{1+e^{2i(c+dx)}}+2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)\right)}{15de^4}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((-1/15*I)*Sqrt[2]*a^2*((e*E^(I*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))^(3/2
)*(1 + E^((2*I)*(c + d*x)))^(3/2)*(3*Sqrt[1 + E^((2*I)*(c + d*x))] + 2*Hype
rgeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/(d*e^4)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(99) = 198.

Time = 13.22 (sec) , antiderivative size = 324, normalized size of antiderivative = 3.81

method	result
risch	$-\frac{i\left(\mathrm{e}^{2 i (d x+c)}+2\right) a^{2} \sqrt{2}}{5 d\, e^{2} \sqrt{\frac{e\, \mathrm{e}^{i (d x+c)}}{\mathrm{e}^{2 i (d x+c)}+1}}}-\frac{i\left(-\frac{2\left(e\, \mathrm{e}^{2 i (d x+c)}+e\right)}{e \sqrt{\mathrm{e}^{i (d x+c)}\left(e\, \mathrm{e}^{2 i (d x+c)}+e\right)}}+\frac{i \sqrt{-i\left(\mathrm{e}^{i (d x+c)}+i\right)}\, \sqrt{2}\, \sqrt{i\left(\mathrm{e}^{i (d x+c)}-i\right)}\, \sqrt{i \mathrm{e}^{i (d x+c)}}\, \left(-2 i E\left(\sqrt{-i\left(\mathrm{e}^{i (d x+c)}+\right.}\right.\right.}{\sqrt{e\, \mathrm{e}^{3 i (d x+c)}+e\, \mathrm{e}^{i (d x+c)}}}}{5 d\, e^{2}\left(\mathrm{e}^{2 i (d x+c)}+1\right) \sqrt{\frac{e\, \mathrm{e}^{i (d x+c)}}{\mathrm{e}^{2 i (d x+c)}+}}}$
default	$\frac{2 i a^{2}\left(\sqrt{\frac{\cos (d x+c)}{\cos (d x+c)+1}}\, F(i(-\csc (d x+c)+\cot (d x+c)), i) \sqrt{\frac{1}{\cos (d x+c)+1}}\, \cos (d x+c)-\sqrt{\frac{\cos (d x+c)}{\cos (d x+c)+1}}\, E(i(-\csc (d x+c)+\cot (d x+c)), i) \sqrt{\frac{}{c}}\right.}{}$
parts	Expression too large to display

```
[In] int((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*I*(exp(I*(d*x+c))^2+2)/d*a^2*2^(1/2)/e^2/(e*exp(I*(d*x+c))/(exp(I*(d*x
+c))^2+1))^(1/2)-1/5*I/d*(-2*(e*exp(I*(d*x+c))^2+e)/e/(exp(I*(d*x+c))*(e*ex
p(I*(d*x+c))^2+e))^(1/2)+I*(-I*(exp(I*(d*x+c))+I))^(1/2)*2^(1/2)*(I*(exp(I*
(d*x+c))-I))^(1/2)*(I*exp(I*(d*x+c)))^(1/2)/(e*exp(I*(d*x+c))^3+e*exp(I*(d*
x+c)))^(1/2)*(-2*I*EllipticE((-I*(exp(I*(d*x+c))+I))^(1/2),1/2*2^(1/2))+I*E
llipticF((-I*(exp(I*(d*x+c))+I))^(1/2),1/2*2^(1/2))))*a^2*2^(1/2)/e^2/(exp(
I*(d*x+c))^2+1)/(e*exp(I*(d*x+c))/(exp(I*(d*x+c))^2+1))^(1/2)*(e*exp(I*(d*x
+c))*(exp(I*(d*x+c))^2+1))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.11

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}\,dx = \frac{2i\sqrt{2}a^2\sqrt{e}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\sqrt{}}{5\,de^3}$$

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*I*sqrt(2)*a^2*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, e^(I*d*x + I*c))) + sqrt(2)*(-I*a^2*e^(3*I*d*x + 3*I*c) - I*a^2*e^(I*d*
x + I*c))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*e^3
)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}\,dx = -a^2\left(\int\left(-\frac{1}{(e\sec{(c+dx)})^{\frac{5}{2}}}\right)dx + \int\frac{\tan^2{(c+dx)}}{(e\sec{(c+dx)})^{\frac{5}{2}}}\,dx + \int\left(-\frac{2i\tan{(c+dx)}}{(e\sec{(c+dx)})^{\frac{5}{2}}}\right)dx\right)$$

```
[In] integrate((a+I*a*tan(d*x+c))**2/(e*sec(d*x+c))**(5/2),x)
[Out] -a**2*(Integral(-1/(e*sec(c + d*x))**(5/2), x) + Integral(tan(c + d*x)**2/(
e*sec(c + d*x))**(5/2), x) + Integral(-2*I*tan(c + d*x)/(e*sec(c + d*x))**(
5/2), x))
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{(i\,a\tan{(dx+c)}+a)^2}{(e\sec{(dx+c)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^2/(e*sec(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{(i\,a\tan{(dx+c)}+a)^2}{(e\sec{(dx+c)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^2/(e*sec(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{(a+a\tan(c+d\,x)\;1\mathrm{i})^2}{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(5/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(5/2), x)
```

3.198 $\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 116

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}\,dx = \frac{2a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{7de^4} + \frac{2a^2\sin(c+dx)}{7de^3\sqrt{e\sec(c+dx)}} - \frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}$$

```
[Out] 2/7*a^2*sin(d*x+c)/d/e^3/(e*sec(d*x+c))^(1/2)+2/7*a^2*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)
^(1/2)*(e*sec(d*x+c))^(1/2)/d/e^4-4/7*I*(a^2+I*a^2*tan(d*x+c))/d/(e*sec(d*x
+c))^(7/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3577, 3854, 3856, 2720}

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}\,dx = \frac{2a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{7de^4} + \frac{2a^2\sin(c+dx)}{7de^3\sqrt{e\sec(c+dx)}} - \frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(7/2),x]
```

```
[Out] (2*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(
7*d*e^4) + (2*a^2*Sin[c + d*x])/(7*d*e^3*Sqrt[e*Sec[c + d*x]]) - (((4*I)/7)
*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(7/2))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}+\frac{(3a^2)\int\frac{1}{(e\sec(c+dx))^{3/2}}\,dx}{7e^2}\\
&= \frac{2a^2\sin(c+dx)}{7de^3\sqrt{e\sec(c+dx)}}-\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}+\frac{a^2\int\sqrt{e\sec(c+dx)}\,dx}{7e^4}\\
&= \frac{2a^2\sin(c+dx)}{7de^3\sqrt{e\sec(c+dx)}}-\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}\\
&\quad+\frac{\left(a^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{7e^4}\\
&= \frac{2a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{7de^4}\\
&\quad+\frac{2a^2\sin(c+dx)}{7de^3\sqrt{e\sec(c+dx)}}-\frac{4i(a^2+ia^2\tan(c+dx))}{7d(e\sec(c+dx))^{7/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.15

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}\,dx = \frac{a^2\sqrt{e\sec(c+dx)}\left(-2i-2i\cos(2(c+dx))+2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+\right.\right.}{7d}$$

[In] Integrate[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(7/2),x]

[Out] (a^2*Sqrt[e*Sec[c + d*x]]*(-2*I - (2*I)*Cos[2*(c + d*x)] + 2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)]) - Sin[2*(c + d*x)])*(Cos[2*(c + 2*d*x)] + I*Sin[2*(c + 2*d*x)]))/(7*d*e^4*(Cos[d*x] + I*Sin[d*x])^2)

Maple [A] (verified)

Time = 11.33 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.48

method	result
default	$\frac{2a^2\left(-2i(\cos^3(dx+c))+iF(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+2(\cos^2(dx+c))\sin(dx+c)+i\sec(dx+c)F(i(\right.}{7e^3d\sqrt{e\sec(dx+c)}}$
risch	$-\frac{ie^{i(dx+c)}\left(e^{2i(dx+c)}+3\right)a^2\sqrt{2}}{14de^3\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}+\frac{2\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{i\left(e^{i(dx+c)}-i\right)}\sqrt{ie^{i(dx+c)}}\,F\left(\sqrt{-i\left(e^{i(dx+c)}+i\right)},\frac{\sqrt{2}}{2}\right)a^2\sqrt{e\,e^{i(dx+c)}\left(e^{2i(dx\right.}}}{7d\sqrt{e\,e^{3i(dx+c)}+e\,e^{i(dx+c)}}\,e^3\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}$
parts	$-\frac{2a^2\left(5iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}+5i\sec(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\right.}}{21d\sqrt{e\sec(dx+c)}\,e^3}$

[In] int((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)

[Out] 2/7*a^2/e^3/d/(e*sec(d*x+c))^(1/2)*(-2*I*cos(d*x+c)^3+I*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+2*cos(d*x+c)^2*sin(d*x+c)+I*sec(d*x+c)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+sin(d*x+c))

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.83

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}\,dx = \frac{-4i\sqrt{2}a^2\sqrt{e}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(-i\,a^2e^{(4i\,dx+4i\,c)}-4i\,a^2\right.}{14\,de^4}$$

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/14*(-4*I*sqrt(2)*a^2*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c))
+ sqrt(2)*(-I*a^2*e^(4*I*d*x + 4*I*c) - 4*I*a^2*e^(2*I*d*x + 2*I*c) - 3*I*a
^2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*e^4)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}\,dx = -a^2\left(\int\left(-\frac{1}{(e\sec(c+dx))^{\frac{7}{2}}}\right)dx + \int\frac{\tan^2(c+dx)}{(e\sec(c+dx))^{\frac{7}{2}}}\,dx + \int\left(-\frac{2i\tan(c+dx)}{(e\sec(c+dx))^{\frac{7}{2}}}\right)dx\right)$$

```
[In] integrate((a+I*a*tan(d*x+c))**2/(e*sec(d*x+c))**(7/2),x)
```

```
[Out] -a**2*(Integral(-1/(e*sec(c + d*x))**(7/2), x) + Integral(tan(c + d*x)**2/(
e*sec(c + d*x))**(7/2), x) + Integral(-2*I*tan(c + d*x)/(e*sec(c + d*x))**(
7/2), x))
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^2}{(e\sec(dx+c))^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^2/(e*sec(d*x + c))^(7/2), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}\,dx = \int \frac{(i\,a\tan\left(dx+c\right)+a)^2}{(e\sec\left(dx+c\right))^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^2/(e*sec(d*x + c))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}\,dx = \int \frac{\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^2}{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(7/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(7/2), x)
```

3.199 $\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{9/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 116

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{9/2}}\,dx = \frac{2a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2a^2\sin(c+dx)}{9de^3(e\sec(c+dx))^{3/2}} - \frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}$$

```
[Out] 2/9*a^2*sin(d*x+c)/d/e^3/(e*sec(d*x+c))^(3/2)+2/3*a^2*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/e^4/cos(
d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)-4/9*I*(a^2+I*a^2*tan(d*x+c))/d/(e*sec(d*x
+c))^(9/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3577, 3854, 3856, 2719}

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{9/2}}\,dx = \frac{2a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2a^2\sin(c+dx)}{9de^3(e\sec(c+dx))^{3/2}} - \frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(9/2),x]
```

```
[Out] (2*a^2*EllipticE[(c + d*x)/2, 2])/(3*d*e^4*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c
+ d*x]]) + (2*a^2*Sin[c + d*x])/(9*d*e^3*(e*Sec[c + d*x])^(3/2)) - (((4*I)/
9)*(a^2 + I*a^2*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(9/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}+\frac{(5a^2)\int\frac{1}{(e\sec(c+dx))^{5/2}}\,dx}{9e^2}\\
&= \frac{2a^2\sin(c+dx)}{9de^3(e\sec(c+dx))^{3/2}}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}+\frac{a^2\int\frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{3e^4}\\
&= \frac{2a^2\sin(c+dx)}{9de^3(e\sec(c+dx))^{3/2}}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}+\frac{a^2\int\sqrt{\cos(c+dx)}\,dx}{3e^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\\
&= \frac{2a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3de^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2a^2\sin(c+dx)}{9de^3(e\sec(c+dx))^{3/2}}-\frac{4i(a^2+ia^2\tan(c+dx))}{9d(e\sec(c+dx))^{9/2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.56 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.15

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{9/2}}\,dx = \frac{ia^2\left(9-4e^{2i(c+dx)}-e^{4i(c+dx)}-\frac{8e^{2i(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}\right)}{18\sqrt{2}de^4\sqrt{\frac{ee^{i(c+dx)}}{1+e^{2i(c+dx)}}}}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(9/2),x]
```

```
[Out] ((I/18)*a^2*(9 - 4*E^((2*I)*(c + d*x)) - E^((4*I)*(c + d*x)) - (8*E^((2*I)*
(c + d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/Sqrt[1 +
 E^((2*I)*(c + d*x))]))/(Sqrt[2]*d*e^4*Sqrt[(e*E^(I*(c + d*x)))/(1 + E^((2*
I)*(c + d*x)))])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 336 vs. 2(126) = 252.

Time = 19.86 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.91

method	result
risch	$-\frac{i\left(\mathrm{e}^{4i(dx+c)}+4\,\mathrm{e}^{2i(dx+c)}+15\right)a^2\sqrt{2}}{36d\,e^4\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}-\frac{i\left(-\frac{2\left(e\,\mathrm{e}^{2i(dx+c)}+e\right)}{e\sqrt{\mathrm{e}^{i(dx+c)}\left(e\,\mathrm{e}^{2i(dx+c)}+e\right)}}+\frac{i\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(\mathrm{e}^{i(dx+c)}-i\right)}\sqrt{i\mathrm{e}^{i(dx+c)}}\left(-2iE\left(\sqrt{-\cdots}\right.\right.}{\sqrt{e\,\mathrm{e}^{3i(dx+c)}+e\,\mathrm{e}^{\cdots}}}\right.}{3d\,e^4\left(\mathrm{e}^{2i(dx+c)}+1\right)\sqrt{\cdots}}$
default	$-\frac{2ia^2\left(2\left(\cos^5(dx+c)\right)+3\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,E(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\cos(dx+c)-3\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,F(i(-\csc(dx+c)\cdots\right.}{\cdots}$
parts	Expression too large to display

```
[In] int((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/36*I*(exp(I*(d*x+c))^4+4*exp(I*(d*x+c))^2+15)/d*a^2*2^(1/2)/e^4/(e*exp(I
*(d*x+c))/(exp(I*(d*x+c))^2+1))^(1/2)-1/3*I/d*(-2*(e*exp(I*(d*x+c))^2+e)/e/
(exp(I*(d*x+c))*(e*exp(I*(d*x+c))^2+e))^(1/2)+I*(-I*(exp(I*(d*x+c))+I))^(1/
2)*2^(1/2)*(I*(exp(I*(d*x+c))-I))^(1/2)*(I*exp(I*(d*x+c)))^(1/2)/(e*exp(I*(
d*x+c))^3+e*exp(I*(d*x+c)))^(1/2)*(-2*I*EllipticE((-I*(exp(I*(d*x+c))+I))^(
1/2),1/2*2^(1/2))+I*EllipticF((-I*(exp(I*(d*x+c))+I))^(1/2),1/2*2^(1/2))))*
a^2*2^(1/2)/e^4/(exp(I*(d*x+c))^2+1)/(e*exp(I*(d*x+c))/(exp(I*(d*x+c))^2+1)
)^(1/2)*(e*exp(I*(d*x+c))*(exp(I*(d*x+c))^2+1))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.13

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{9/2}}\,dx = \frac{\left(24i\sqrt{2}a^2\sqrt{e}e^{(i\,dx+i\,c)}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx-}\right.\right.\right.}{}$$

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(9/2),x, algorithm="fricas")
```

```
[Out] 1/36*(24*I*sqrt(2)*a^2*sqrt(e)*e^(I*d*x + I*c)*weierstrassZeta(-4, 0, weier
strassPInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(-I*a^2*e^(6*I*d*x + 6*I*
c) - 5*I*a^2*e^(4*I*d*x + 4*I*c) + 5*I*a^2*e^(2*I*d*x + 2*I*c) + 9*I*a^2)*s
qrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-I*d*x - I*c)/
(d*e^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+I*a*tan(d*x+c))**2/(e*sec(d*x+c))**(9/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{9/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^2}{(e\sec(dx+c))^{\frac{9}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^2/(e*sec(d*x + c))^(9/2), x)
```

Giac [F]

$$\int \frac{(a + ia\tan(c + dx))^2}{(e\sec(c + dx))^{9/2}}\, dx = \int \frac{(i\,a\tan\left(dx + c\right) + a)^2}{(e\sec\left(dx + c\right))^{\frac{9}{2}}}\, dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(9/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^2/(e*sec(d*x + c))^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + ia\tan(c + dx))^2}{(e\sec(c + dx))^{9/2}}\, dx = \int \frac{(a + a\tan(c + d\,x)\ 1i)^2}{\left(\frac{e}{\cos(c+d\,x)}\right)^{9/2}}\, dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(9/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(9/2), x)
```

3.200 $\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 147

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}\,dx=\frac{10a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{33de^6}+\frac{2a^2\sin(c+dx)}{11de^3(e\sec(c+dx))^{5/2}}+\frac{10a^2\sin(c+dx)}{33de^5\sqrt{e\sec(c+dx)}}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}$$

```
[Out] 2/11*a^2*sin(d*x+c)/d/e^3/(e*sec(d*x+c))^(5/2)+10/33*a^2*sin(d*x+c)/d/e^5/(
e*sec(d*x+c))^(1/2)+10/33*a^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1
/2)/d/e^6-4/11*I*(a^2+I*a^2*tan(d*x+c))/d/(e*sec(d*x+c))^(11/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3577, 3854, 3856, 2720}

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}\,dx=\frac{10a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{33de^6}+\frac{10a^2\sin(c+dx)}{33de^5\sqrt{e\sec(c+dx)}}+\frac{2a^2\sin(c+dx)}{11de^3(e\sec(c+dx))^{5/2}}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(11/2),x]
```

```
[Out] (10*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(33*d*e^6) + (2*a^2*Sin[c + d*x])/(11*d*e^3*(e*Sec[c + d*x])^(5/2)) + (10*a
```

```
^2*Sin[c + d*x])/(33*d*e^5*Sqrt[e*Sec[c + d*x]]) - (((4*I)/11)*(a^2 + I*a^2
*Tan[c + d*x]))/(d*(e*Sec[c + d*x])^(11/2))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}+\frac{(7a^2)\int\frac{1}{(e\sec(c+dx))^{7/2}}\,dx}{11e^2}\\
&= \frac{2a^2\sin(c+dx)}{11de^3(e\sec(c+dx))^{5/2}}-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}+\frac{(5a^2)\int\frac{1}{(e\sec(c+dx))^{3/2}}\,dx}{11e^4}\\
&= \frac{2a^2\sin(c+dx)}{11de^3(e\sec(c+dx))^{5/2}}+\frac{10a^2\sin(c+dx)}{33de^5\sqrt{e\sec(c+dx)}}\\
&\quad-\frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}+\frac{(5a^2)\int\sqrt{e\sec(c+dx)}\,dx}{33e^6}
\end{aligned}$$

$$= \frac{2a^2\sin(c+dx)}{11de^3(e\sec(c+dx))^{5/2}} + \frac{10a^2\sin(c+dx)}{33de^5\sqrt{e\sec(c+dx)}} - \frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}} + \frac{\left(5a^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{33e^6}$$

$$= \frac{10a^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{33de^6} + \frac{2a^2\sin(c+dx)}{11de^3(e\sec(c+dx))^{5/2}} + \frac{10a^2\sin(c+dx)}{33de^5\sqrt{e\sec(c+dx)}} - \frac{4i(a^2+ia^2\tan(c+dx))}{11d(e\sec(c+dx))^{11/2}}$$

Mathematica [A] (verified)

Time = 2.26 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.05

$$\int\frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}\,dx = \frac{a^2\sqrt{e\sec(c+dx)}\Big(-28i-24i\cos(2(c+dx))+4i\cos(4(c+dx))+40\sqrt{\cos(c}}{}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^2/(e*Sec[c + d*x])^(11/2),x]
```

```
[Out] (a^2*Sqrt[e*Sec[c + d*x]]*(-28*I - (24*I)*Cos[2*(c + d*x)] + (4*I)*Cos[4*(c
 + d*x)] + 40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)
] - I*Sin[2*(c + d*x)]) - 6*Sin[2*(c + d*x)] + 7*Sin[4*(c + d*x)])*(Cos[2*(
c + 2*d*x)] + I*Sin[2*(c + 2*d*x)]))/(132*d*e^6*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [A] (verified)

Time = 20.16 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.29

method	result
default	$\frac{2a^2\left(-6i(\cos^5(dx+c))+6\sin(dx+c)(\cos^4(dx+c))+5iF(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+5i\sec(dx+c)F\right.}{33e^5d\sqrt{e\sec(dx+c)}}$
parts	$-\frac{2a^2\left(-7\sin(dx+c)(\cos^4(dx+c))+15iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}+15i\sec(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}F\right.}{77d\sqrt{e\sec(dx+c)}\,e^5}$

```
[In] int((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/33*a^2/e^5/d/(e*sec(d*x+c))^(1/2)*(-6*I*cos(d*x+c)^5+6*sin(d*x+c)*cos(d*x
+c)^4+5*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*
x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)+5*I*sec(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)+3*
cos(d*x+c)^2*sin(d*x+c)+5*sin(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.97

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}\,dx = \frac{\left(-80i\,\sqrt{2}a^2\sqrt{e}e^{(2i\,dx+2i\,c)}\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(-3i\,a^2e^{(8i}\right.\right.}{}$$

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(11/2),x, algorithm="fricas")
```

```
[Out] 1/264*(-80*I*sqrt(2)*a^2*sqrt(e)*e^(2*I*d*x + 2*I*c)*weierstrassPInverse(-4
, 0, e^(I*d*x + I*c)) + sqrt(2)*(-3*I*a^2*e^(8*I*d*x + 8*I*c) - 18*I*a^2*e^
(6*I*d*x + 6*I*c) - 56*I*a^2*e^(4*I*d*x + 4*I*c) - 30*I*a^2*e^(2*I*d*x + 2*
I*c) + 11*I*a^2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))
*e^(-2*I*d*x - 2*I*c)/(d*e^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+I*a*tan(d*x+c))**2/(e*sec(d*x+c))**(11/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^2}{(e\sec(dx+c))^{\frac{11}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^2/(e*sec(d*x + c))^(11/2), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^2}{(e\sec(dx+c))^{\frac{11}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^2/(e*sec(d*x+c))^(11/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^2/(e*sec(d*x + c))^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}\,dx = \int \frac{(a+a\tan(c+d\,x)\;1\mathrm{i})^2}{\left(\frac{e}{\cos(c+d\,x)}\right)^{11/2}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(11/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^2/(e/cos(c + d*x))^(11/2), x)
```

3.201 $\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 28, antiderivative size = 202

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^3\,dx = -\frac{2a^3e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{10ia^3(e\sec(c+dx))^{7/2}}{21d} + \frac{2a^3e^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{d} + \frac{2a^3e(e\sec(c+dx))^{5/2}\sin(c+dx)}{3d} + \frac{2ia(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}{11d} + \frac{10i(e\sec(c+dx))^{7/2}\left(a^3+ia^3\tan(c+dx)\right)}{33d}$$

```
[Out] 10/21*I*a^3*(e*sec(d*x+c))^(7/2)/d+2/3*a^3*e*(e*sec(d*x+c))^(5/2)*sin(d*x+c
)/d-2*a^3*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+2*a^3*e^3*
sin(d*x+c)*(e*sec(d*x+c))^(1/2)/d+2/11*I*a*(e*sec(d*x+c))^(7/2)*(a+I*a*tan(
d*x+c))^2/d+10/33*I*(e*sec(d*x+c))^(7/2)*(a^3+I*a^3*tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used

= {3579, 3567, 3853, 3856, 2719}

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^3\,dx = -\frac{2a^3e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2a^3e^3\sin(c+dx)\sqrt{e\sec(c+dx)}}{d} + \frac{10ia^3(e\sec(c+dx))^{7/2}}{21d} + \frac{2a^3e\sin(c+dx)(e\sec(c+dx))^{5/2}}{3d} + \frac{10i(a^3+ia^3\tan(c+dx))\,(e\sec(c+dx))^{7/2}}{33d} + \frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{7/2}}{11d}$$

```
[In] Int[(e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (-2*a^3*e^4*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c +
 d*x]]) + (((10*I)/21)*a^3*(e*Sec[c + d*x])^(7/2))/d + (2*a^3*e^3*Sqrt[e*Se
c[c + d*x]]*Sin[c + d*x])/d + (2*a^3*e*(e*Sec[c + d*x])^(5/2)*Sin[c + d*x])
/(3*d) + (((2*I)/11)*a*(e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^2)/d +
 (((10*I)/33)*(e*Sec[c + d*x])^(7/2)*(a^3 + I*a^3*Tan[c + d*x]))/d
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2ia(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}{11d} \\
&\quad+\frac{1}{11}(15a)\int(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2\,dx \\
&= \frac{2ia(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}{11d} \\
&\quad+\frac{10i(e\sec(c+dx))^{7/2}\left(a^3+ia^3\tan(c+dx)\right)}{33d} \\
&\quad+\frac{1}{3}\left(5a^2\right)\int(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx \\
&= \frac{10ia^3(e\sec(c+dx))^{7/2}}{21d}+\frac{2ia(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}{11d} \\
&\quad+\frac{10i(e\sec(c+dx))^{7/2}\left(a^3+ia^3\tan(c+dx)\right)}{33d}+\frac{1}{3}\left(5a^3\right)\int(e\sec(c+dx))^{7/2}\,dx \\
&= \frac{10ia^3(e\sec(c+dx))^{7/2}}{21d}+\frac{2a^3e(e\sec(c+dx))^{5/2}\sin(c+dx)}{3d} \\
&\quad+\frac{2ia(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}{11d} \\
&\quad+\frac{10i(e\sec(c+dx))^{7/2}\left(a^3+ia^3\tan(c+dx)\right)}{33d}+\left(a^3e^2\right)\int(e\sec(c+dx))^{3/2}\,dx \\
&= \frac{10ia^3(e\sec(c+dx))^{7/2}}{21d}+\frac{2a^3e^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{d} \\
&\quad+\frac{2a^3e(e\sec(c+dx))^{5/2}\sin(c+dx)}{3d}+\frac{2ia(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}{11d} \\
&\quad+\frac{10i(e\sec(c+dx))^{7/2}\left(a^3+ia^3\tan(c+dx)\right)}{33d}-\left(a^3e^4\right)\int\frac{1}{\sqrt{e\sec(c+dx)}}\,dx \\
&= \frac{10ia^3(e\sec(c+dx))^{7/2}}{21d}+\frac{2a^3e^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{d} \\
&\quad+\frac{2a^3e(e\sec(c+dx))^{5/2}\sin(c+dx)}{3d}+\frac{2ia(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}{11d} \\
&\quad+\frac{10i(e\sec(c+dx))^{7/2}\left(a^3+ia^3\tan(c+dx)\right)}{33d}-\frac{\left(a^3e^4\right)\int\sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2a^3e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{10ia^3(e\sec(c+dx))^{7/2}}{21d} \\
&+ \frac{2a^3e^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{d} + \frac{2a^3e(e\sec(c+dx))^{5/2}\sin(c+dx)}{3d} \\
&+ \frac{2ia(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}{11d} \\
&+ \frac{10i(e\sec(c+dx))^{7/2}\left(a^3+ia^3\tan(c+dx)\right)}{33d}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.84 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.80

$$
\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^3\,dx =
$$

$$
-\frac{a^3e^3\sec^4(c+dx)\sqrt{e\sec(c+dx)}\left(-908\cos(c+dx)-858\cos(3(c+dx))-154\cos(5(c+dx))+\frac{77}{2}e^{-5i(c+
$$

```
[In] Integrate[(e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] -1/1848*(a^3*e^3*Sec[c + d*x]^4*Sqrt[e*Sec[c + d*x]]*(-908*Cos[c + d*x] - 8
58*Cos[3*(c + d*x)] - 154*Cos[5*(c + d*x)] + (77*(1 + E^((2*I)*(c + d*x)))^
(11/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/(2*E^((5*I)*
(c + d*x))) - (38*I)*Sin[c + d*x] - (451*I)*Sin[3*(c + d*x)] - (77*I)*Sin[5
*(c + d*x)])*(-I + Tan[c + d*x]))/d
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 487 vs. 2(201) = 402.

Time = 11.29 (sec) , antiderivative size = 488, normalized size of antiderivative = 2.42

$$
-\frac{2ie^3a^3\sqrt{e\sec(dx+c)}\left(231(\cos^2(dx+c))\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}E(i(\csc(dx+c)-\cot(dx+c)),i)-
$$

```
[In] int((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] -2/231*I*e^3*a^3/d*(e*sec(d*x+c))^(1/2)/(cos(d*x+c)+1)*(231*cos(d*x+c)^2*(1
/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(csc(d
*x+c)-cot(d*x+c)),I)-231*cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)+462*cos(d*x+c)
*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(cs
```

```
c(d*x+c)-cot(d*x+c)),I)-462*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(
d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+231*(1/(cos(d
*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(csc(d*x+c)-c
ot(d*x+c)),I)-231*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))
^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-231*I*sin(d*x+c)*tan(d*x+c)^2-77*I
*tan(d*x+c)^3-308*I*tan(d*x+c)^3*sec(d*x+c)+231*I*tan(d*x+c)*sec(d*x+c)^3-1
32*sec(d*x+c)^2-132*sec(d*x+c)^3+21*sec(d*x+c)^4+21*sec(d*x+c)^5)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.56

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^3\,dx = -\frac{2\left(\sqrt{2}(231i\,a^3e^3e^{(11i\,dx+11i\,c)}+1309i\,a^3e^3e^{(9i\,dx+9i\,c)}+946i\,a^3e^3e^{(7i\,dx+7i\,c)}+870i\,a^3e^3e^{(5i\,dx+5i\,c)}+407i\,a^3e^3e^{\cdots}\right.}{\cdots}$$

```
[In] integrate((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -2/231*(sqrt(2)*(231*I*a^3*e^3*e^(11*I*d*x + 11*I*c) + 1309*I*a^3*e^3*e^(9*
I*d*x + 9*I*c) + 946*I*a^3*e^3*e^(7*I*d*x + 7*I*c) + 870*I*a^3*e^3*e^(5*I*d
*x + 5*I*c) + 407*I*a^3*e^3*e^(3*I*d*x + 3*I*c) + 77*I*a^3*e^3*e^(I*d*x + I
*c))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 231*sqrt(2
)*(I*a^3*e^3*e^(10*I*d*x + 10*I*c) + 5*I*a^3*e^3*e^(8*I*d*x + 8*I*c) + 10*I
*a^3*e^3*e^(6*I*d*x + 6*I*c) + 10*I*a^3*e^3*e^(4*I*d*x + 4*I*c) + 5*I*a^3*e
^3*e^(2*I*d*x + 2*I*c) + I*a^3*e^3)*sqrt(e)*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, e^(I*d*x + I*c))))/(d*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*I
*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) + 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e
^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^3\,dx = \text{Timed out}$$

```
[In] integrate((e*sec(d*x+c))**(7/2)*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^3\,dx = \int (e\sec(dx+c))^{\frac{7}{2}}(i\,a\tan(dx+c)+a)^3\,dx$$

```
[In] integrate((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
[Out] integrate((e*sec(d*x + c))^(7/2)*(I*a*tan(d*x + c) + a)^3, x)
```

Giac [F]

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^3\,dx = \int (e\sec(dx+c))^{\frac{7}{2}}(i\,a\tan(dx+c)+a)^3\,dx$$

```
[In] integrate((e*sec(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(7/2)*(I*a*tan(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^3\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}(a+a\tan(c+d\,x)\ 1\mathrm{i})^3\,dx$$

```
[In] int((e/cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)^3,x)
[Out] int((e/cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)^3, x)
```

3.202 $\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 28, antiderivative size = 175

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^3\,dx = \frac{26a^3e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{21d} + \frac{26ia^3(e\sec(c+dx))^{5/2}}{35d} + \frac{26a^3e(e\sec(c+dx))^{3/2}\sin(c+dx)}{21d} + \frac{2ia(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}{9d} + \frac{26i(e\sec(c+dx))^{5/2}\left(a^3+ia^3\tan(c+dx)\right)}{63d}$$

```
[Out] 26/35*I*a^3*(e*sec(d*x+c))^(5/2)/d+26/21*a^3*e*(e*sec(d*x+c))^(3/2)*sin(d*x
+c)/d+26/21*a^3*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/d+2/9*
I*a*(e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^2/d+26/63*I*(e*sec(d*x+c))^(5/2
)*(a^3+I*a^3*tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used

$= \{3579, 3567, 3853, 3856, 2720\}$

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^3\,dx = \frac{26a^3e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{21d} + \frac{26ia^3(e\sec(c+dx))^{5/2}}{35d} + \frac{26a^3e\sin(c+dx)(e\sec(c+dx))^{3/2}}{21d} + \frac{26i(a^3+ia^3\tan(c+dx))\,(e\sec(c+dx))^{5/2}}{63d} + \frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{5/2}}{9d}$$

```
[In] Int[(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (26*a^3*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x
]])/(21*d) + (((26*I)/35)*a^3*(e*Sec[c + d*x])^(5/2))/d + (26*a^3*e*(e*Sec[
c + d*x])^(3/2)*Sin[c + d*x])/(21*d) + (((2*I)/9)*a*(e*Sec[c + d*x])^(5/2)*
(a + I*a*Tan[c + d*x])^2)/d + (((26*I)/63)*(e*Sec[c + d*x])^(5/2)*(a^3 + I*
a^3*Tan[c + d*x]))/d
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
```

```
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2ia(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}{9d} \\
&\quad + \frac{1}{9}(13a)\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2\,dx \\
&= \frac{2ia(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}{9d} \\
&\quad + \frac{26i(e\sec(c+dx))^{5/2}\left(a^3+ia^3\tan(c+dx)\right)}{63d} \\
&\quad + \frac{1}{7}\left(13a^2\right)\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx \\
&= \frac{26ia^3(e\sec(c+dx))^{5/2}}{35d} + \frac{2ia(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}{9d} \\
&\quad + \frac{26i(e\sec(c+dx))^{5/2}\left(a^3+ia^3\tan(c+dx)\right)}{63d} + \frac{1}{7}\left(13a^3\right)\int (e\sec(c+dx))^{5/2}\,dx \\
&= \frac{26ia^3(e\sec(c+dx))^{5/2}}{35d} + \frac{26a^3e(e\sec(c+dx))^{3/2}\sin(c+dx)}{21d} \\
&\quad + \frac{2ia(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}{9d} \\
&\quad + \frac{26i(e\sec(c+dx))^{5/2}\left(a^3+ia^3\tan(c+dx)\right)}{63d} + \frac{1}{21}\left(13a^3e^2\right)\int \sqrt{e\sec(c+dx)}\,dx \\
&= \frac{26ia^3(e\sec(c+dx))^{5/2}}{35d} + \frac{26a^3e(e\sec(c+dx))^{3/2}\sin(c+dx)}{21d} \\
&\quad + \frac{2ia(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}{9d} \\
&\quad + \frac{26i(e\sec(c+dx))^{5/2}\left(a^3+ia^3\tan(c+dx)\right)}{63d} \\
&\quad + \frac{1}{21}\left(13a^3e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx
\end{aligned}
$$

$$= \frac{26a^3e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{21d} + \frac{26ia^3(e\sec(c+dx))^{5/2}}{35d} + \frac{26a^3e(e\sec(c+dx))^{3/2}\sin(c+dx)}{21d} + \frac{2ia(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}{9d} + \frac{26i(e\sec(c+dx))^{5/2}\left(a^3+ia^3\tan(c+dx)\right)}{63d}$$

Mathematica [A] (verified)

Time = 2.74 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.51

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^3\,dx = \frac{a^3\sec^2(c+dx)(e\sec(c+dx))^{5/2}\left(728i+1008i\cos(2(c+dx))+1560\cos^{\frac{9}{2}}(c+dx)\,\text{El}\right.}{1260d}$$

```
[In] Integrate[(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*Sec[c + d*x]^2*(e*Sec[c + d*x])^(5/2)*(728*I + (1008*I)*Cos[2*(c + d*x
)] + 1560*Cos[c + d*x]^(9/2)*EllipticF[(c + d*x)/2, 2] - 150*Sin[2*(c + d*x
)] + 195*Sin[4*(c + d*x)]))/(1260*d)
```

Maple [A] (verified)

Time = 7.50 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.05

$$-\frac{2ie^2a^3\sqrt{e\sec(dx+c)}\left(195F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\cos(dx+c)+1\right.}{}$$

```
[In] int((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] -2/315*I*e^2*a^3/d*(e*sec(d*x+c))^(1/2)*(195*EllipticF(I*(csc(d*x+c)-cot(d*
x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x
+c)+195*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)+195*I*tan(d*x+c)-252*sec(d*x+c)^2-135*I*sec(
d*x+c)^2*tan(d*x+c)+35*sec(d*x+c)^4)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.47

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^3\,dx =$$
$$-\frac{2\left(\sqrt{2}(195i\,a^3e^2e^{(8i\,dx+8i\,c)}-1158i\,a^3e^2e^{(6i\,dx+6i\,c)}-1456i\,a^3e^2e^{(4i\,dx+4i\,c)}-858i\,a^3e^2e^{(2i\,dx+2i\,c)}-195i\,a^3e^2\right)}{315\,(de}$$

```
[In] integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -2/315*(sqrt(2)*(195*I*a^3*e^2*e^(8*I*d*x + 8*I*c) - 1158*I*a^3*e^2*e^(6*I*
d*x + 6*I*c) - 1456*I*a^3*e^2*e^(4*I*d*x + 4*I*c) - 858*I*a^3*e^2*e^(2*I*d*
x + 2*I*c) - 195*I*a^3*e^2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x
+ 1/2*I*c) + 195*sqrt(2)*(I*a^3*e^2*e^(8*I*d*x + 8*I*c) + 4*I*a^3*e^2*e^(6*
I*d*x + 6*I*c) + 6*I*a^3*e^2*e^(4*I*d*x + 4*I*c) + 4*I*a^3*e^2*e^(2*I*d*x +
 2*I*c) + I*a^3*e^2)*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))/(
d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) +
 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^3\,dx =$$
$$-ia^3\left(\int i(e\sec{(c+dx)})^{\frac{5}{2}}\,dx+\int\left(-3(e\sec{(c+dx)})^{\frac{5}{2}}\tan{(c+dx)}\right)dx\right.$$
$$\left.+\int(e\sec{(c+dx)})^{\frac{5}{2}}\tan^3{(c+dx)}\,dx+\int\left(-3i(e\sec{(c+dx)})^{\frac{5}{2}}\tan^2{(c+dx)}\right)dx\right)$$

```
[In] integrate((e*sec(d*x+c))**(5/2)*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] -I*a**3*(Integral(I*(e*sec(c + d*x))**(5/2), x) + Integral(-3*(e*sec(c + d*
x))**(5/2)*tan(c + d*x), x) + Integral((e*sec(c + d*x))**(5/2)*tan(c + d*x)
**3, x) + Integral(-3*I*(e*sec(c + d*x))**(5/2)*tan(c + d*x)**2, x))
```

Maxima [F]

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^3\,dx = \int (e\sec(dx+c))^{\frac{5}{2}}(i\,a\tan(dx+c)+a)^3\,dx$$

```
[In] integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
[Out] integrate((e*sec(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a)^3, x)
```

Giac [F]

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^3\,dx = \int (e\sec(dx+c))^{\frac{5}{2}}(i\,a\tan(dx+c)+a)^3\,dx$$

```
[In] integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^3\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}(a+a\tan(c+d\,x)\;1\mathrm{i})^3\,dx$$

```
[In] int((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^3,x)
[Out] int((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^3, x)
```

3.203 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 28, antiderivative size = 175

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3\,dx = -\frac{22a^3e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{22ia^3(e\sec(c+dx))^{3/2}}{15d}+\frac{22a^3e\sqrt{e\sec(c+dx)}\sin(c+dx)}{5d}+\frac{2ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}{7d}+\frac{22i(e\sec(c+dx))^{3/2}\left(a^3+ia^3\tan(c+dx)\right)}{35d}$$

```
[Out] 22/15*I*a^3*(e*sec(d*x+c))^(3/2)/d-22/5*a^3*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/
2)/(e*sec(d*x+c))^(1/2)+22/5*a^3*e*sin(d*x+c)*(e*sec(d*x+c))^(1/2)/d+2/7*I*
a*(e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^2/d+22/35*I*(e*sec(d*x+c))^(3/2)*
(a^3+I*a^3*tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used

= {3579, 3567, 3853, 3856, 2719}

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3\,dx = -\frac{22a^3e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{22ia^3(e\sec(c+dx))^{3/2}}{15d} + \frac{22a^3e\sin(c+dx)\sqrt{e\sec(c+dx)}}{5d} + \frac{22i(a^3+ia^3\tan(c+dx))\,(e\sec(c+dx))^{3/2}}{35d} + \frac{2ia(a+ia\tan(c+dx))^2(e\sec(c+dx))^{3/2}}{7d}$$

```
[In] Int[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (-22*a^3*e^2*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) + (((22*I)/15)*a^3*(e*Sec[c + d*x])^(3/2))/d + (22*a^3*e*Sqrt[e*
Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (((2*I)/7)*a*(e*Sec[c + d*x])^(3/2)*(a
+ I*a*Tan[c + d*x])^2)/d + (((22*I)/35)*(e*Sec[c + d*x])^(3/2)*(a^3 + I*a^3
*Tan[c + d*x]))/d
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}{7d} \\
&\quad+\frac{1}{7}(11a)\int(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2\,dx \\
&= \frac{2ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}{7d} \\
&\quad+\frac{22i(e\sec(c+dx))^{3/2}\left(a^3+ia^3\tan(c+dx)\right)}{35d} \\
&\quad+\frac{1}{5}\left(11a^2\right)\int(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx \\
&= \frac{22ia^3(e\sec(c+dx))^{3/2}}{15d}+\frac{2ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}{7d} \\
&\quad+\frac{22i(e\sec(c+dx))^{3/2}\left(a^3+ia^3\tan(c+dx)\right)}{35d}+\frac{1}{5}\left(11a^3\right)\int(e\sec(c+dx))^{3/2}\,dx \\
&= \frac{22ia^3(e\sec(c+dx))^{3/2}}{15d}+\frac{22a^3e\sqrt{e\sec(c+dx)}\sin(c+dx)}{5d} \\
&\quad+\frac{2ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}{7d} \\
&\quad+\frac{22i(e\sec(c+dx))^{3/2}\left(a^3+ia^3\tan(c+dx)\right)}{35d}-\frac{1}{5}\left(11a^3e^2\right)\int\frac{1}{\sqrt{e\sec(c+dx)}}\,dx \\
&= \frac{22ia^3(e\sec(c+dx))^{3/2}}{15d}+\frac{22a^3e\sqrt{e\sec(c+dx)}\sin(c+dx)}{5d} \\
&\quad+\frac{2ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}{7d} \\
&\quad+\frac{22i(e\sec(c+dx))^{3/2}\left(a^3+ia^3\tan(c+dx)\right)}{35d}-\frac{\left(11a^3e^2\right)\int\sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} \\
&= -\frac{22a^3e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{22ia^3(e\sec(c+dx))^{3/2}}{15d} \\
&\quad+\frac{22a^3e\sqrt{e\sec(c+dx)}\sin(c+dx)}{5d}+\frac{2ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}{7d} \\
&\quad+\frac{22i(e\sec(c+dx))^{3/2}\left(a^3+ia^3\tan(c+dx)\right)}{35d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.37 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.74

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3\,dx = \frac{a^3(e\sec(c+dx))^{3/2}(1+i\tan(c+dx))\left(-116i-308i\cos(2(c+dx))+77ie^{-2i(c+dx)}\left(\right.\right.}{}$$

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*(e*Sec[c + d*x])^(3/2)*(1 + I*Tan[c + d*x])*(-116*I - (308*I)*Cos[2*(c
 + d*x)] + ((77*I)*(1 + E^((2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3
/4, 7/4, -E^((2*I)*(c + d*x))])/E^((2*I)*(c + d*x)) + 77*Sec[c + d*x]*Sin[3
*(c + d*x)] + 17*Tan[c + d*x]))/(210*d)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 451 vs. 2(174) = 348.

Time = 16.21 (sec) , antiderivative size = 452, normalized size of antiderivative = 2.58

method	result
default	$-\frac{2e\,a^3\sqrt{e\sec(dx+c)}\left(231i(\cos^2(dx+c))F(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}-231i(\cos^2(dx+c))E(i(-\mathrm{cs}\right.}{}$
parts	Expression too large to display

```
[In] int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/105*e*a^3/d*(e*sec(d*x+c))^(1/2)/(cos(d*x+c)+1)*(231*I*cos(d*x+c)^2*Elli
pticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(c
os(d*x+c)+1))^(1/2)-231*I*cos(d*x+c)^2*EllipticE(I*(-csc(d*x+c)+cot(d*x+c))
,I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+462*I*cos(d*
x+c)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*(1/(cos(d*x+c)+1))^(1/2)-462*I*cos(d*x+c)*EllipticE(I*(-csc(d*x+c)+cot(
d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+231*I
*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
(1/(cos(d*x+c)+1))^(1/2)-231*I*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)-140*I-231*sin(d*x+c)
-140*I*sec(d*x+c)+63*tan(d*x+c)+15*I*sec(d*x+c)^2+63*sec(d*x+c)*tan(d*x+c)+
15*I*sec(d*x+c)^3)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.19

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3\,dx =$$

$$-\frac{2\left(\sqrt{2}(231i\,a^3ee^{(7i\,dx+7i\,c)}+287i\,a^3ee^{(5i\,dx+5i\,c)}+253i\,a^3ee^{(3i\,dx+3i\,c)}+77i\,a^3ee^{(i\,dx+i\,c)}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\right.}}{105\,(de^{(6i}}$$

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -2/105*(sqrt(2)*(231*I*a^3*e*e^(7*I*d*x + 7*I*c) + 287*I*a^3*e*e^(5*I*d*x +
 5*I*c) + 253*I*a^3*e*e^(3*I*d*x + 3*I*c) + 77*I*a^3*e*e^(I*d*x + I*c))*sqr
t(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 231*sqrt(2)*(I*a^3
*e*e^(6*I*d*x + 6*I*c) + 3*I*a^3*e*e^(4*I*d*x + 4*I*c) + 3*I*a^3*e*e^(2*I*d
*x + 2*I*c) + I*a^3*e)*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, e^(I*d*x + I*c))))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) +
 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3\,dx =$$
$$-ia^3\left(\int i(e\sec(c+dx))^{\frac{3}{2}}\,dx+\int\left(-3(e\sec(c+dx))^{\frac{3}{2}}\tan(c+dx)\right)\,dx\right.$$
$$\left.+\int (e\sec(c+dx))^{\frac{3}{2}}\tan^3(c+dx)\,dx+\int\left(-3i(e\sec(c+dx))^{\frac{3}{2}}\tan^2(c+dx)\right)\,dx\right)$$

```
[In] integrate((e*sec(d*x+c))**(3/2)*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] -I*a**3*(Integral(I*(e*sec(c + d*x))**(3/2), x) + Integral(-3*(e*sec(c + d*
x))**(3/2)*tan(c + d*x), x) + Integral((e*sec(c + d*x))**(3/2)*tan(c + d*x)
**3, x) + Integral(-3*I*(e*sec(c + d*x))**(3/2)*tan(c + d*x)**2, x))
```

Maxima [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3\,dx = \int (e\sec(dx+c))^{\frac{3}{2}}(i\,a\tan(dx+c)+a)^3\,dx$$

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
[Out] integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^3, x)
```

Giac [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3\,dx = \int (e\sec(dx+c))^{\frac{3}{2}}(i\,a\tan(dx+c)+a)^3\,dx$$

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}(a+a\tan(c+d\,x)\ 1\mathrm{i})^3\,dx$$

```
[In] int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^3,x)
[Out] int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^3, x)
```

3.204 $\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 28, antiderivative size = 139

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3\,dx = \frac{6ia^3\sqrt{e\sec(c+dx)}}{d}+\frac{6a^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{d}+\frac{2ia\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}{5d}+\frac{6i\sqrt{e\sec(c+dx)}(a^3+ia^3\tan(c+dx))}{5d}$$

```
[Out] 6*I*a^3*(e*sec(d*x+c))^(1/2)/d+6*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x
+c))^(1/2)/d+2/5*I*a*(e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^2/d+6/5*I*(e*s
ec(d*x+c))^(1/2)*(a^3+I*a^3*tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3579, 3567, 3856, 2720}

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3\,dx = \frac{6ia^3\sqrt{e\sec(c+dx)}}{d}+\frac{6i(a^3+ia^3\tan(c+dx))\sqrt{e\sec(c+dx)}}{5d}+\frac{6a^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{d}+\frac{2ia(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}{5d}$$

```
[In] Int[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((6*I)*a^3*Sqrt[e*Sec[c + d*x]])/d + (6*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/d + (((2*I)/5)*a*Sqrt[e*Sec[c + d*x]]*(
a + I*a*Tan[c + d*x])^2)/d + (((6*I)/5)*Sqrt[e*Sec[c + d*x]]*(a^3 + I*a^3*T
an[c + d*x]))/d
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2ia\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}{5d} \\
&\quad + \frac{1}{5}(9a)\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2\,dx \\
&= \frac{2ia\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}{5d} + \frac{6i\sqrt{e\sec(c+dx)}(a^3+ia^3\tan(c+dx))}{5d} \\
&\quad + (3a^2)\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))\,dx
\end{aligned}
$$

$$
\begin{aligned}
&=\frac{6ia^3\sqrt{e\sec(c+dx)}}{d}+\frac{2ia\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}{5d}\\
&\quad+\frac{6i\sqrt{e\sec(c+dx)}(a^3+ia^3\tan(c+dx))}{5d}+(3a^3)\int\sqrt{e\sec(c+dx)}\,dx\\
&=\frac{6ia^3\sqrt{e\sec(c+dx)}}{d}+\frac{2ia\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}{5d}\\
&\quad+\frac{6i\sqrt{e\sec(c+dx)}(a^3+ia^3\tan(c+dx))}{5d}\\
&\quad+\left(3a^3\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx\\
&=\frac{6ia^3\sqrt{e\sec(c+dx)}}{d}+\frac{6a^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{d}\\
&\quad+\frac{2ia\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}{5d}+\frac{6i\sqrt{e\sec(c+dx)}(a^3+ia^3\tan(c+dx))}{5d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.37 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.57

$$
\int\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3\,dx
$$

$$
=\frac{a^3\sec^2(c+dx)\sqrt{e\sec(c+dx)}\left(18i+20i\cos(2(c+dx))+30\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)-5\sin(\right.}{5d}
$$

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*Sec[c + d*x]^2*Sqrt[e*Sec[c + d*x]]*(18*I + (20*I)*Cos[2*(c + d*x)] +
30*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] - 5*Sin[2*(c + d*x)]))/(5*d
)
```

Maple [A] (verified)

Time = 14.26 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.13

method	result
default	$\frac{2a^3\sqrt{e\sec(dx+c)}\left(15i\cos(dx+c)F(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}+15iF(i(-\csc(dx+c)+\cot(dx+c)),i)\right.}{5d}$
parts	$-\frac{2ia^3(\cos(dx+c)+1)\sqrt{e\sec(dx+c)}\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}F(i(\csc(dx+c)-\cot(dx+c)),i)}{d}-\frac{ia^3\sqrt{e\sec(dx+c)}\left(5\cos(dx+c)\right.}{}$

```
[In] int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*a^3/d*(e*sec(d*x+c))^(1/2)*(15*I*cos(d*x+c)*EllipticF(I*(-csc(d*x+c)+co
t(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+15*
I*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(cos(d*x+c)+1))^(1/2)+20*I-5*tan(d*x+c)-I*sec(d*x+c)^2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.06

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3\,dx = -\frac{2\left(\sqrt{2}\left(-25i\,a^3e^{(4i\,dx+4i\,c)}-36i\,a^3e^{(2i\,dx+2i\,c)}-15i\,a^3\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+15\sqrt{2}\left(i\,a^3e^{(4i\,dx+4i\,c)}+\right.}{5\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -2/5*(sqrt(2)*(-25*I*a^3*e^(4*I*d*x + 4*I*c) - 36*I*a^3*e^(2*I*d*x + 2*I*c)
 - 15*I*a^3)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 15
*sqrt(2)*(I*a^3*e^(4*I*d*x + 4*I*c) + 2*I*a^3*e^(2*I*d*x + 2*I*c) + I*a^3)*
sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))/(d*e^(4*I*d*x + 4*I*c)
 + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3\,dx = -ia^3\left(\int i\sqrt{e\sec(c+dx)}\,dx + \int\left(-3\sqrt{e\sec(c+dx)}\tan(c+dx)\right)dx + \int\sqrt{e\sec(c+dx)}\tan^3(c+dx)\,dx + \int\left(-3i\sqrt{e\sec(c+dx)}\tan^2(c+dx)\right)dx\right)$$

```
[In] integrate((e*sec(d*x+c))**(1/2)*(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] -I*a**3*(Integral(I*sqrt(e*sec(c + d*x)), x) + Integral(-3*sqrt(e*sec(c + d
*x))*tan(c + d*x), x) + Integral(sqrt(e*sec(c + d*x))*tan(c + d*x)**3, x) +
 Integral(-3*I*sqrt(e*sec(c + d*x))*tan(c + d*x)**2, x))
```

Maxima [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3\,dx = \int \sqrt{e\sec\left(dx+c\right)}\left(i\,a\tan\left(dx+c\right)+a\right)^3\,dx$$

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
[Out] integrate(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^3, x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{-1,[2,0]%%%}+%%%{%%{[-2,0]:[1,0,%%%{1,[1]%%%}]%%},[1,0]%%%}+%
%%{%%%
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3\,dx = \int \sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^3\,dx$$

```
[In] int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^3,x)
[Out] int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^3, x)
```

3.205 $\int \frac{(a+ia\tan(c+dx))^3}{\sqrt{e\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 124

$$\int \frac{(a+ia\tan(c+dx))^3}{\sqrt{e\sec(c+dx)}}\,dx=-\frac{26ia^3}{3d\sqrt{e\sec(c+dx)}}+\frac{14a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{6a^3\tan(c+dx)}{d\sqrt{e\sec(c+dx)}}-\frac{2ia^3\tan^2(c+dx)}{3d\sqrt{e\sec(c+dx)}}$$

```
[Out] -26/3*I*a^3/d/(e*sec(d*x+c))^(1/2)+14*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(e*
sec(d*x+c))^(1/2)-6*a^3*tan(d*x+c)/d/(e*sec(d*x+c))^(1/2)-2/3*I*a^3*tan(d*x
+c)^2/d/(e*sec(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3579, 3577, 3853, 3856, 2719}

$$\int \frac{(a+ia\tan(c+dx))^3}{\sqrt{e\sec(c+dx)}}\,dx=-\frac{14a^3\sin(c+dx)\sqrt{e\sec(c+dx)}}{de}-\frac{28i(a^3+ia^3\tan(c+dx))}{3d\sqrt{e\sec(c+dx)}}+\frac{14a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2ia(a+ia\tan(c+dx))^2}{3d\sqrt{e\sec(c+dx)}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^3/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] (14*a^3*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x
]]) - (14*a^3*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/(d*e) + (((2*I)/3)*a*(a +
```

```
I*a*Tan[c + d*x])^2)/(d*Sqrt[e*Sec[c + d*x]]) - (((28*I)/3)*(a^3 + I*a^3*Ta
n[c + d*x]))/(d*Sqrt[e*Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\text{integral} = \frac{2ia(a+ia\tan(c+dx))^2}{3d\sqrt{e\sec(c+dx)}} + \frac{1}{3}(7a)\int \frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}\,dx$$

$$= \frac{2ia(a+ia\tan(c+dx))^2}{3d\sqrt{e\sec(c+dx)}} - \frac{28i(a^3+ia^3\tan(c+dx))}{3d\sqrt{e\sec(c+dx)}} - \frac{(7a^3)\int(e\sec(c+dx))^{3/2}\,dx}{e^2}$$

$$= -\frac{14a^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{de} + \frac{2ia(a+ia\tan(c+dx))^2}{3d\sqrt{e\sec(c+dx)}} - \frac{28i(a^3+ia^3\tan(c+dx))}{3d\sqrt{e\sec(c+dx)}} + (7a^3)\int\frac{1}{\sqrt{e\sec(c+dx)}}\,dx$$

$$= -\frac{14a^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{de} + \frac{2ia(a+ia\tan(c+dx))^2}{3d\sqrt{e\sec(c+dx)}} - \frac{28i(a^3+ia^3\tan(c+dx))}{3d\sqrt{e\sec(c+dx)}} + \frac{(7a^3)\int\sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$= \frac{14a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{14a^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{de} + \frac{2ia(a+ia\tan(c+dx))^2}{3d\sqrt{e\sec(c+dx)}} - \frac{28i(a^3+ia^3\tan(c+dx))}{3d\sqrt{e\sec(c+dx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.67 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.81

$$\int\frac{(a+ia\tan(c+dx))^3}{\sqrt{e\sec(c+dx)}}\,dx$$

$$= \frac{2a^3\sqrt{e\sec(c+dx)}(\cos(c)+i\sin(c))(-i\cos(dx)+\sin(dx))\left(-8+7\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{2}\right.\right.}{3de}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^3/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] (2*a^3*Sqrt[e*Sec[c + d*x]]*(Cos[c] + I*Sin[c])*((-I)*Cos[d*x] + Sin[d*x])*
(-8 + 7*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^(
(2*I)*(c + d*x))] - I*Tan[c + d*x]))/(3*d*e)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1113 vs. 2(134) = 268.

Time = 12.23 (sec) , antiderivative size = 1114, normalized size of antiderivative = 8.98

method	result	size
parts	Expression too large to display	1114
default	Expression too large to display	1306

```
[In] int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a^3/d/(cos(d*x+c)+1)/(e*sec(d*x+c))^(1/2)*(I*EllipticE(I*(csc(d*x+c)-cot(
d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d
*x+c)-I*cos(d*x+c)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1)
)^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+2*I*(1/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)-2*I*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(
cos(d*x+c)+1))^(1/2)+I*sec(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)-I*sec(d*x+c)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos
(d*x+c)+1))^(1/2)+sin(d*x+c))+1/6*I*a^3/d/(cos(d*x+c)+1)/(e*sec(d*x+c))^(1/
2)/(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*(-12*cos(d*x+c)*(-cos(d*x+c)/(cos(d
*x+c)+1)^2)^(1/2)+3*ln((2*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2
*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)-cos(d*x+c)+1)/(cos(d*x+c)+1))-3*ln(2*
(2*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*(-cos(d*x+c)/(cos(d*x+
c)+1)^2)^(1/2)-cos(d*x+c)+1)/(cos(d*x+c)+1))-12*(-cos(d*x+c)/(cos(d*x+c)+1)
^2)^(1/2)-4*sec(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)-4*sec(d*x+c)^2*
(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2))-6*I*a^3/(e*sec(d*x+c))^(1/2)/d+6*a^3/
d/(cos(d*x+c)+1)/(e*sec(d*x+c))^(1/2)*(2*I*EllipticE(I*(csc(d*x+c)-cot(d*x+
c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c
)-2*I*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+4*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)-4*I*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(co
s(d*x+c)+1))^(1/2)+2*I*sec(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)-2*I*sec(d*x+c)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(c
os(d*x+c)+1))^(1/2)+sin(d*x+c)-tan(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.99

$$\int \frac{(a+ia\tan(c+dx))^3}{\sqrt{e\sec(c+dx)}}\,dx = -\frac{2\left(\sqrt{2}\left(-9i\,a^3e^{(3i\,dx+3i\,c)} - 7i\,a^3e^{(i\,dx+i\,c)}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)} + 21\,\sqrt{2}\left(-i\,a^3e^{(2i\,dx+2i\,c)} - i\,a^3\right)\sqrt{e}\mathrm{we}}{3\left(dee^{(2i\,dx+2i\,c)} + de\right)}$$

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] -2/3*(sqrt(2)*(-9*I*a^3*e^(3*I*d*x + 3*I*c) - 7*I*a^3*e^(I*d*x + I*c))*sqrt
(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 21*sqrt(2)*(-I*a^3*
e^(2*I*d*x + 2*I*c) - I*a^3)*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, e^(I*d*x + I*c))))/(d*e*e^(2*I*d*x + 2*I*c) + d*e)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{\sqrt{e\sec(c+dx)}}\,dx = -ia^3\left(\int \frac{i}{\sqrt{e\sec{(c+dx)}}}\,dx + \int \left(-\frac{3\tan{(c+dx)}}{\sqrt{e\sec{(c+dx)}}}\right)dx + \int \frac{\tan^3{(c+dx)}}{\sqrt{e\sec{(c+dx)}}}\,dx + \int \left(-\frac{3i\tan^2{(c+dx)}}{\sqrt{e\sec{(c+dx)}}}\right)dx\right)$$

```
[In] integrate((a+I*a*tan(d*x+c))**3/(e*sec(d*x+c))**(1/2),x)
[Out] -I*a**3*(Integral(I/sqrt(e*sec(c + d*x)), x) + Integral(-3*tan(c + d*x)/sqr
t(e*sec(c + d*x)), x) + Integral(tan(c + d*x)**3/sqrt(e*sec(c + d*x)), x) +
 Integral(-3*I*tan(c + d*x)**2/sqrt(e*sec(c + d*x)), x))
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{\sqrt{e\sec(c+dx)}}\,dx = \int \frac{(i\,a\tan{(dx+c)}+a)^3}{\sqrt{e\sec{(dx+c)}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^3/sqrt(e*sec(d*x + c)), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{\sqrt{e\sec(c+dx)}}\,dx=\int \frac{(i\,a\tan\left(dx+c\right)+a)^3}{\sqrt{e\sec\left(dx+c\right)}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^3/sqrt(e*sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^3}{\sqrt{e\sec(c+dx)}}\,dx=\int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^3}{\sqrt{\frac{e}{\cos(c+d\,x)}}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(1/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(1/2), x)
```

3.206 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 111

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{10ia^3\sqrt{e\sec(c+dx)}}{3de^2} - \frac{10a^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2} - \frac{4ia(a+ia\tan(c+dx))^2}{3d(e\sec(c+dx))^{3/2}}$$

```
[Out] -10/3*I*a^3*(e*sec(d*x+c))^(1/2)/d/e^2-10/3*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)
*(e*sec(d*x+c))^(1/2)/d/e^2-4/3*I*a*(a+I*a*tan(d*x+c))^2/d/(e*sec(d*x+c))^(
3/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3577, 3567, 3856, 2720}

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{10ia^3\sqrt{e\sec(c+dx)}}{3de^2} - \frac{10a^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2} - \frac{4ia(a+ia\tan(c+dx))^2}{3d(e\sec(c+dx))^{3/2}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(3/2),x]
```

```
[Out] (((-10*I)/3)*a^3*Sqrt[e*Sec[c + d*x]])/(d*e^2) - (10*a^3*Sqrt[Cos[c + d*x]]
*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*d*e^2) - (((4*I)/3)*a*(
a + I*a*Tan[c + d*x])^2)/(d*(e*Sec[c + d*x])^(3/2))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{4ia(a+ia\tan(c+dx))^2}{3d(e\sec(c+dx))^{3/2}} - \frac{(5a^2)\int\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))\,dx}{3e^2} \\
&= -\frac{10ia^3\sqrt{e\sec(c+dx)}}{3de^2} - \frac{4ia(a+ia\tan(c+dx))^2}{3d(e\sec(c+dx))^{3/2}} - \frac{(5a^3)\int\sqrt{e\sec(c+dx)}\,dx}{3e^2} \\
&= -\frac{10ia^3\sqrt{e\sec(c+dx)}}{3de^2} - \frac{4ia(a+ia\tan(c+dx))^2}{3d(e\sec(c+dx))^{3/2}} \\
&\quad - \frac{\left(5a^3\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3e^2} \\
&= -\frac{10ia^3\sqrt{e\sec(c+dx)}}{3de^2} - \frac{10a^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3de^2} \\
&\quad - \frac{4ia(a+ia\tan(c+dx))^2}{3d(e\sec(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.97 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.11

$$\int \frac{(a + ia\tan(c + dx))^3}{(e\sec(c + dx))^{3/2}}\,dx =$$
$$-\frac{2a^3\sec^2(c+dx)\left(7i\cos(c+dx)+5\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(\cos(c+dx)-i\sin(c+dx))+\right.}{3d(e\sec(c+dx))^{3/2}(\cos(dx)+i\sin(dx))^3}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(3/2),x]
[Out] (-2*a^3*Sec[c + d*x]^2*((7*I)*Cos[c + d*x] + 5*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*(Cos[c + d*x] - I*Sin[c + d*x]) + 3*Sin[c + d*x])*(Cos[c +
 4*d*x] + I*Sin[c + 4*d*x]))/(3*d*(e*Sec[c + d*x])^(3/2)*(Cos[d*x] + I*Sin[
d*x])^3)
```

Maple [A] (verified)

Time = 12.28 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.49

method	result
default	$-\frac{2a^3\left(5iF(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+5i\sec(dx+c)F(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+}}\right.}{3ed\sqrt{e\sec(dx+c)}}$
parts	Expression too large to display

```
[In] int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/3*a^3/e/d/(e*sec(d*x+c))^(1/2)*(5*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*El
lipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)+5*I*sec(d*x+
c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I
)*(1/(cos(d*x+c)+1))^(1/2)+4*I*cos(d*x+c)-4*sin(d*x+c)+3*I*sec(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.74

$$\int \frac{(a + ia\tan(c + dx))^3}{(e\sec(c + dx))^{3/2}}\,dx =$$
$$-\frac{2\left(-5i\sqrt{2}a^3\sqrt{e}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(2i\,a^3e^{(2i\,dx+2i\,c)}+5i\,a^3\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\right.}\right.}{3\,de^2}$$

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(-5*I*sqrt(2)*a^3*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c))
+ sqrt(2)*(2*I*a^3*e^(2*I*d*x + 2*I*c) + 5*I*a^3)*sqrt(e/(e^(2*I*d*x + 2*I*
c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*e^2)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{3/2}}\,dx = -ia^3\left(\int \frac{i}{(e\sec(c+dx))^{\frac{3}{2}}}\,dx + \int \left(-\frac{3\tan(c+dx)}{(e\sec(c+dx))^{\frac{3}{2}}}\right)dx + \int \frac{\tan^3(c+dx)}{(e\sec(c+dx))^{\frac{3}{2}}}\,dx + \int \left(-\frac{3i\tan^2(c+dx)}{(e\sec(c+dx))^{\frac{3}{2}}}\right)dx\right)$$

```
[In] integrate((a+I*a*tan(d*x+c))**3/(e*sec(d*x+c))**(3/2),x)
[Out] -I*a**3*(Integral(I/(e*sec(c + d*x))**(3/2), x) + Integral(-3*tan(c + d*x)/
(e*sec(c + d*x))**(3/2), x) + Integral(tan(c + d*x)**3/(e*sec(c + d*x))**(3
/2), x) + Integral(-3*I*tan(c + d*x)**2/(e*sec(c + d*x))**(3/2), x))
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^3}{(e\sec(dx+c))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^3}{(e\sec(dx+c))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^3}{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(3/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(3/2), x)
```

3.207 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 111

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{5/2}}\,dx = \frac{6ia^3}{5de^2\sqrt{e\sec(c+dx)}} - \frac{6a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{4ia(a+ia\tan(c+dx))^2}{5d(e\sec(c+dx))^{5/2}}$$

```
[Out] 6/5*I*a^3/d/e^2/(e*sec(d*x+c))^(1/2)-6/5*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/e^2/cos(d*x+c)^(1
/2)/(e*sec(d*x+c))^(1/2)-4/5*I*a*(a+I*a*tan(d*x+c))^2/d/(e*sec(d*x+c))^(5/2
)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3577, 3567, 3856, 2719}

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{5/2}}\,dx = \frac{6ia^3}{5de^2\sqrt{e\sec(c+dx)}} - \frac{6a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{4ia(a+ia\tan(c+dx))^2}{5d(e\sec(c+dx))^{5/2}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(5/2),x]
```

```
[Out] (((6*I)/5)*a^3)/(d*e^2*Sqrt[e*Sec[c + d*x]]) - (6*a^3*EllipticE[(c + d*x)/2
, 2])/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) - (((4*I)/5)*a*(a +
 I*a*Tan[c + d*x])^2)/(d*(e*Sec[c + d*x])^(5/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{4ia(a+ia\tan(c+dx))^2}{5d(e\sec(c+dx))^{5/2}}-\frac{(3a^2)\int \frac{a+ia\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx}{5e^2}\\
&= \frac{6ia^3}{5de^2\sqrt{e\sec(c+dx)}}-\frac{4ia(a+ia\tan(c+dx))^2}{5d(e\sec(c+dx))^{5/2}}-\frac{(3a^3)\int \frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{5e^2}\\
&= \frac{6ia^3}{5de^2\sqrt{e\sec(c+dx)}}-\frac{4ia(a+ia\tan(c+dx))^2}{5d(e\sec(c+dx))^{5/2}}-\frac{(3a^3)\int \sqrt{\cos(c+dx)}\,dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\\
&= \frac{6ia^3}{5de^2\sqrt{e\sec(c+dx)}}-\frac{6a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4ia(a+ia\tan(c+dx))^2}{5d(e\sec(c+dx))^{5/2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.40 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.97

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{4ia^3e^{2i(c+dx)}\left(1+e^{2i(c+dx)}-\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)\right)}{5de^2\left(1+e^{2i(c+dx)}\right)\sqrt{e\sec(c+dx)}}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(5/2),x]
```

```
[Out] (((-4*I)/5)*a^3*E^((2*I)*(c + d*x))*(1 + E^((2*I)*(c + d*x)) - Sqrt[1 + E^(
(2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/
(d*e^2*(1 + E^((2*I)*(c + d*x)))*Sqrt[e*Sec[c + d*x]])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(120) = 240.

Time = 14.92 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.92

method	result
risch	$-\frac{2i\left(e^{2i(dx+c)}-3\right)a^3\sqrt{2}}{5d\,e^2\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}+\frac{3i\left(-\frac{2\left(e\,e^{2i(dx+c)}+e\right)}{e\sqrt{e^{i(dx+c)}\left(e\,e^{2i(dx+c)}+e\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(dx+c)}-i\right)}\sqrt{ie^{i(dx+c)}}\left(-2iE\left(\sqrt{-i\left(e^{i(dx+c)}\cdots\right.}\right.\right.}{\sqrt{e\,e^{3i(dx+c)}+e\,e^{i(dx+c)}}}\cdots\right)}{5d\,e^2\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}$
default	Expression too large to display
parts	Expression too large to display

```
[In] int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*I*(exp(I*(d*x+c))^2-3)/d*a^3*2^(1/2)/e^2/(e*exp(I*(d*x+c))/(exp(I*(d*x
+c))^2+1))^(1/2)+3/5*I/d*(-2*(e*exp(I*(d*x+c))^2+e)/e/(exp(I*(d*x+c))*(e*ex
p(I*(d*x+c))^2+e))^(1/2)+I*(-I*(exp(I*(d*x+c))+I))^(1/2)*2^(1/2)*(I*(exp(I*
(d*x+c))-I))^(1/2)*(I*exp(I*(d*x+c)))^(1/2)/(e*exp(I*(d*x+c))^3+e*exp(I*(d*
x+c)))^(1/2)*(-2*I*EllipticE((-I*(exp(I*(d*x+c))+I))^(1/2),1/2*2^(1/2))+I*E
llipticF((-I*(exp(I*(d*x+c))+I))^(1/2),1/2*2^(1/2))))*a^3*2^(1/2)/e^2/(exp(
I*(d*x+c))^2+1)/(e*exp(I*(d*x+c))/(exp(I*(d*x+c))^2+1))^(1/2)*(e*exp(I*(d*x
+c))*(exp(I*(d*x+c))^2+1))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.85

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{2\left(3i\sqrt{2}a^3\sqrt{e}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\sqrt{2}\left(i\,a^3e^{(3i\,dx+3i\,c)}+i\,a^3e^{(i\,d}\right.\right.}{5\,de^3}$$

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/5*(3*I*sqrt(2)*a^3*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, e^(I*d*x + I*c))) + sqrt(2)*(I*a^3*e^(3*I*d*x + 3*I*c) + I*a^3*e^(I*d*
x + I*c))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*e^3
)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{5/2}}\,dx = -ia^3\left(\int \frac{i}{(e\sec{(c+dx)})^{\frac{5}{2}}}\,dx + \int \left(-\frac{3\tan{(c+dx)}}{(e\sec{(c+dx)})^{\frac{5}{2}}}\right)dx + \int \frac{\tan^3{(c+dx)}}{(e\sec{(c+dx)})^{\frac{5}{2}}}\,dx + \int \left(-\frac{3i\tan^2{(c+dx)}}{(e\sec{(c+dx)})^{\frac{5}{2}}}\right)dx\right)$$

```
[In] integrate((a+I*a*tan(d*x+c))**3/(e*sec(d*x+c))**(5/2),x)
```

```
[Out] -I*a**3*(Integral(I/(e*sec(c + d*x))**(5/2), x) + Integral(-3*tan(c + d*x)/
(e*sec(c + d*x))**(5/2), x) + Integral(tan(c + d*x)**3/(e*sec(c + d*x))**(5
/2), x) + Integral(-3*I*tan(c + d*x)**2/(e*sec(c + d*x))**(5/2), x))
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{(i\,a\tan{(dx+c)}+a)^3}{(e\sec{(dx+c)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{(i\,a\tan\left(dx+c\right)+a)^3}{(e\sec\left(dx+c\right))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^3}{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(5/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(5/2), x)
```

3.208 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 124

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{7/2}}\,dx = -\frac{2a^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{21de^4} - \frac{2i(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}} - \frac{4i(a^3+ia^3\tan(c+dx))}{21de^2(e\sec(c+dx))^{3/2}}$$

```
[Out] -2/21*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/d/e^4-2/7*I*(a+I
*a*tan(d*x+c))^3/d/(e*sec(d*x+c))^(7/2)-4/21*I*(a^3+I*a^3*tan(d*x+c))/d/e^2
/(e*sec(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3578, 3577, 3856, 2720}

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{7/2}}\,dx = -\frac{2a^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{21de^4} - \frac{4i(a^3+ia^3\tan(c+dx))}{21de^2(e\sec(c+dx))^{3/2}} - \frac{2i(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(7/2),x]
```

```
[Out] (-2*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(21*d*e^4) - (((2*I)/7)*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(7/2)
) - (((4*I)/21)*(a^3 + I*a^3*Tan[c + d*x]))/(d*e^2*(e*Sec[c + d*x])^(3/2))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2i(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}+\frac{a\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{3/2}}\,dx}{7e^2}\\
&= -\frac{2i(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}-\frac{4i(a^3+ia^3\tan(c+dx))}{21de^2(e\sec(c+dx))^{3/2}}-\frac{a^3\int \sqrt{e\sec(c+dx)}\,dx}{21e^4}\\
&= -\frac{2i(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}-\frac{4i(a^3+ia^3\tan(c+dx))}{21de^2(e\sec(c+dx))^{3/2}}\\
&\quad -\frac{\left(a^3\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21e^4}
\end{aligned}$$

$$= -\frac{2a^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{21de^4} - \frac{2i(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}} - \frac{4i(a^3+ia^3\tan(c+dx))}{21de^2(e\sec(c+dx))^{3/2}}$$

Mathematica [A] (verified)

Time = 1.87 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.07

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{7/2}}\,dx = -\frac{a^3\sqrt{e\sec(c+dx)}\Big(5i+5i\cos(2(c+dx))+2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(\cos(2(c+dx))-i\sin}{21de^4(\cos(dx)+i\sin(dx))^3}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(7/2),x]
```

```
[Out] -1/21*(a^3*Sqrt[e*Sec[c + d*x]]*(5*I + (5*I)*Cos[2*(c + d*x)] + 2*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)])
 - Sin[2*(c + d*x)])*(Cos[2*c + 5*d*x] + I*Sin[2*c + 5*d*x]))/(d*e^4*(Cos[d
*x] + I*Sin[d*x])^3)
```

Maple [A] (verified)

Time = 13.45 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.46

method	result
default	$-\frac{2a^3\left(12i(\cos^3(dx+c))+iF(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}-12(\cos^2(dx+c))\sin(dx+c)+i\sec(dx+c)F}{21e^3d\sqrt{e\sec(dx+c)}}$
risch	$-\frac{ie^{i(dx+c)}(3e^{2i(dx+c)}+2)a^3\sqrt{2}}{21de^3\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}-\frac{2\sqrt{-i(e^{i(dx+c)}+i)}\sqrt{i(e^{i(dx+c)}-i)}\sqrt{ie^{i(dx+c)}}F\left(\sqrt{-i(e^{i(dx+c)}+i)},\frac{\sqrt{2}}{2}\right)a^3\sqrt{e\,e^{i(dx+c)}}(e^{2i(}}{21d\sqrt{e\,e^{3i(dx+c)}+e\,e^{i(dx+c)}}\,e^3(e^{2i(dx+c)}+1)\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}$
parts	$-\frac{2a^3\left(5iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}+5i\sec(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{}}{21d\sqrt{e\sec(dx+c)}\,e^3}$

```
[In] int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21*a^3/e^3/d/(e*sec(d*x+c))^(1/2)*(12*I*cos(d*x+c)^3+I*EllipticF(I*(-csc
(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)-12*cos(d*x+c)^2*sin(d*x+c)+I*sec(d*x+c)*EllipticF(I*(-csc(d*x+c)+cot
(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-7*I*
cos(d*x+c)+sin(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.77

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{7/2}}\,dx = \frac{2i\sqrt{2}a^3\sqrt{e}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(-3i\,a^3e^{(4i\,dx+4i\,c)}-5i\,a^3\right.}{21\,de^4}$$

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(7/2),x, algorithm="fricas")
[Out] 1/21*(2*I*sqrt(2)*a^3*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)) +
 sqrt(2)*(-3*I*a^3*e^(4*I*d*x + 4*I*c) - 5*I*a^3*e^(2*I*d*x + 2*I*c) - 2*I*
a^3)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*e^4)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{7/2}}\,dx = -ia^3\left(\int \frac{i}{(e\sec{(c+dx)})^{\frac{7}{2}}}\,dx + \int \left(-\frac{3\tan{(c+dx)}}{(e\sec{(c+dx)})^{\frac{7}{2}}}\right)dx + \int \frac{\tan^3{(c+dx)}}{(e\sec{(c+dx)})^{\frac{7}{2}}}\,dx + \int \left(-\frac{3i\tan^2{(c+dx)}}{(e\sec{(c+dx)})^{\frac{7}{2}}}\right)dx\right)$$

```
[In] integrate((a+I*a*tan(d*x+c))**3/(e*sec(d*x+c))**(7/2),x)
[Out] -I*a**3*(Integral(I/(e*sec(c + d*x))**(7/2), x) + Integral(-3*tan(c + d*x)/
(e*sec(c + d*x))**(7/2), x) + Integral(tan(c + d*x)**3/(e*sec(c + d*x))**(7
/2), x) + Integral(-3*I*tan(c + d*x)**2/(e*sec(c + d*x))**(7/2), x))
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{7/2}}\,dx = \int \frac{(i\,a\tan{(dx+c)}+a)^3}{(e\sec{(dx+c)})^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(7/2), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{7/2}}\,dx = \int \frac{\left(i\,a\tan\left(dx+c\right)+a\right)^3}{\left(e\sec\left(dx+c\right)\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{7/2}}\,dx = \int \frac{\left(a+a\tan(c+d\,x)\;1\mathrm{i}\right)^3}{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(7/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(7/2), x)
```

3.209 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{9/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 124

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{9/2}}\,dx = \frac{2a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15de^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{2i(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}} - \frac{4i(a^3+ia^3\tan(c+dx))}{15de^2(e\sec(c+dx))^{5/2}}$$

```
[Out] 2/15*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))/d/e^4/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)-2/9*I*(a+I*
a*tan(d*x+c))^3/d/(e*sec(d*x+c))^(9/2)-4/15*I*(a^3+I*a^3*tan(d*x+c))/d/e^2/
(e*sec(d*x+c))^(5/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3578, 3577, 3856, 2719}

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{9/2}}\,dx = \frac{2a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15de^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{4i(a^3+ia^3\tan(c+dx))}{15de^2(e\sec(c+dx))^{5/2}} - \frac{2i(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(9/2),x]
```

```
[Out] (2*a^3*EllipticE[(c + d*x)/2, 2])/(15*d*e^4*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c
 + d*x]]) - (((2*I)/9)*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(9/2))
 - (((4*I)/15)*(a^3 + I*a^3*Tan[c + d*x]))/(d*e^2*(e*Sec[c + d*x])^(5/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2i(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}+\frac{a\int\frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}\,dx}{3e^2}\\
&= -\frac{2i(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}-\frac{4i(a^3+ia^3\tan(c+dx))}{15de^2(e\sec(c+dx))^{5/2}}+\frac{a^3\int\frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{15e^4}\\
&= -\frac{2i(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}-\frac{4i(a^3+ia^3\tan(c+dx))}{15de^2(e\sec(c+dx))^{5/2}}+\frac{a^3\int\sqrt{\cos(c+dx)}\,dx}{15e^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\\
&= \frac{2a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15de^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{2i(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}-\frac{4i(a^3+ia^3\tan(c+dx))}{15de^2(e\sec(c+dx))^{5/2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.95 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.95

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{9/2}}\,dx =$$
$$-\frac{a^3e^{-2i(c+dx)}\left(11+16e^{2i(c+dx)}+5e^{4i(c+dx)}+4\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)\right)(-i+}{90de^2(e\sec(c+dx))^{5/2}}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(9/2),x]
```

```
[Out] -1/90*(a^3*(11 + 16*E^((2*I)*(c + d*x)) + 5*E^((4*I)*(c + d*x)) + 4*Sqrt[1
+ E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x)
)])*(-I + Tan[c + d*x])^3)/(d*e^2*E^((2*I)*(c + d*x))*(e*Sec[c + d*x])^(5/2
))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 338 vs. 2(132) = 264.

Time = 14.16 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.73

method	result
risch	$-\frac{i\left(5\,e^{4i(dx+c)}+11\,e^{2i(dx+c)}+12\right)a^3\sqrt{2}}{90d\,e^4\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}-\frac{i\left(-\frac{2\left(e\,e^{2i(dx+c)}+e\right)}{e\sqrt{e^{i(dx+c)}\left(e\,e^{2i(dx+c)}+e\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(dx+c)}-i\right)}\sqrt{ie^{i(dx+c)}}\left(-2iE\left(\right.\right.}{\sqrt{e\,e^{3i(dx+c)}+}}\right.}{15d\,e^4\left(e^{2i(dx+c)}+1\right.}$
default	$\frac{2ia^3\left(-20i\sin(dx+c)\left(\cos^4(dx+c)\right)-20i\left(\cos^3(dx+c)\right)\sin(dx+c)-20\left(\cos^5(dx+c)\right)+3\cos(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}E(i(\csc(\right.}{}$
parts	Expression too large to display

```
[In] int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/90*I*(5*exp(I*(d*x+c))^4+11*exp(I*(d*x+c))^2+12)/d*a^3*2^(1/2)/e^4/(e*ex
p(I*(d*x+c))/(exp(I*(d*x+c))^2+1))^(1/2)-1/15*I/d*(-2*(e*exp(I*(d*x+c))^2+e
)/e/(exp(I*(d*x+c))*(e*exp(I*(d*x+c))^2+e))^(1/2)+I*(-I*(exp(I*(d*x+c))+I))
^(1/2)*2^(1/2)*(I*(exp(I*(d*x+c))-I))^(1/2)*(I*exp(I*(d*x+c)))^(1/2)/(e*exp
(I*(d*x+c))^3+e*exp(I*(d*x+c)))^(1/2)*(-2*I*EllipticE((-I*(exp(I*(d*x+c))+I
))^(1/2),1/2*2^(1/2))+I*EllipticF((-I*(exp(I*(d*x+c))+I))^(1/2),1/2*2^(1/2)
)))*a^3*2^(1/2)/e^4/(exp(I*(d*x+c))^2+1)/(e*exp(I*(d*x+c))/(exp(I*(d*x+c))^
2+1))^(1/2)*(e*exp(I*(d*x+c))*(exp(I*(d*x+c))^2+1))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.87

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{9/2}}\,dx = \frac{12i\sqrt{2}a^3\sqrt{e}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\sqrt{}}{90}$$

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(9/2),x, algorithm="fricas")
```

```
[Out] 1/90*(12*I*sqrt(2)*a^3*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, e^(I*d*x + I*c))) + sqrt(2)*(-5*I*a^3*e^(5*I*d*x + 5*I*c) - 16*I*a^3*
e^(3*I*d*x + 3*I*c) - 11*I*a^3*e^(I*d*x + I*c))*sqrt(e/(e^(2*I*d*x + 2*I*c)
 + 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*e^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+I*a*tan(d*x+c))**3/(e*sec(d*x+c))**(9/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{9/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^3}{(e\sec(dx+c))^{\frac{9}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(9/2), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{9/2}}\,dx = \int \frac{(i\,a\tan\left(dx+c\right)+a)^3}{\left(e\sec\left(dx+c\right)\right)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(9/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{9/2}}\,dx = \int \frac{\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^3}{\left(\frac{e}{\cos(c+d\,x)}\right)^{9/2}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(9/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(9/2), x)
```

3.210 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{11/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 155

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{11/2}}\,dx = \frac{10a^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{77de^6} + \frac{10a^3\sin(c+dx)}{77de^5\sqrt{e\sec(c+dx)}} - \frac{2i(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}} - \frac{20i(a^3+ia^3\tan(c+dx))}{77de^2(e\sec(c+dx))^{7/2}}$$

```
[Out] 10/77*a^3*sin(d*x+c)/d/e^5/(e*sec(d*x+c))^(1/2)+10/77*a^3*(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*
x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/d/e^6-2/11*I*(a+I*a*tan(d*x+c))^3/d/(e*sec(
d*x+c))^(11/2)-20/77*I*(a^3+I*a^3*tan(d*x+c))/d/e^2/(e*sec(d*x+c))^(7/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3578, 3577, 3854, 3856, 2720}

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{11/2}}\,dx = \frac{10a^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{77de^6} + \frac{10a^3\sin(c+dx)}{77de^5\sqrt{e\sec(c+dx)}} - \frac{20i(a^3+ia^3\tan(c+dx))}{77de^2(e\sec(c+dx))^{7/2}} - \frac{2i(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(11/2),x]
```

```
[Out] (10*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(77*d*e^6) + (10*a^3*Sin[c + d*x])/(77*d*e^5*Sqrt[e*Sec[c + d*x]]) - (((2*I
```

)/11)*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(11/2)) - (((20*I)/77)*
(a^3 + I*a^3*Tan[c + d*x]))/(d*e^2*(e*Sec[c + d*x])^(7/2))

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2i(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}+\frac{(5a)\int\frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}\,dx}{11e^2}\\
&= -\frac{2i(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}-\frac{20i(a^3+ia^3\tan(c+dx))}{77de^2(e\sec(c+dx))^{7/2}}+\frac{(15a^3)\int\frac{1}{(e\sec(c+dx))^{3/2}}\,dx}{77e^4}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{10a^3\sin(c+dx)}{77de^5\sqrt{e\sec(c+dx)}} - \frac{2i(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}} \\
&\quad - \frac{20i(a^3+ia^3\tan(c+dx))}{77de^2(e\sec(c+dx))^{7/2}} + \frac{(5a^3)\int\sqrt{e\sec(c+dx)}\,dx}{77e^6} \\
&= \frac{10a^3\sin(c+dx)}{77de^5\sqrt{e\sec(c+dx)}} - \frac{2i(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}} - \frac{20i(a^3+ia^3\tan(c+dx))}{77de^2(e\sec(c+dx))^{7/2}} \\
&\quad + \frac{\left(5a^3\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{77e^6} \\
&= \frac{10a^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{77de^6} \\
&\quad + \frac{10a^3\sin(c+dx)}{77de^5\sqrt{e\sec(c+dx)}} - \frac{2i(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}} - \frac{20i(a^3+ia^3\tan(c+dx))}{77de^2(e\sec(c+dx))^{7/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.95

$$
\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{11/2}}\,dx = \frac{a^3\sqrt{e\sec(c+dx)}\Big(-46i\cos(c+dx)-22i\cos(3(c+dx))-15\sin(c+dx)+2}{}
$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(11/2),x]
```

```
[Out] (a^3*Sqrt[e*Sec[c + d*x]]*((-46*I)*Cos[c + d*x] - (22*I)*Cos[3*(c + d*x)] -
 15*Sin[c + d*x] + 20*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[3*(
c + d*x)] - I*Sin[3*(c + d*x)]) - 15*Sin[3*(c + d*x)])*(Cos[3*(c + 2*d*x)]
+ I*Sin[3*(c + 2*d*x)]))/(154*d*e^6*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [A] (verified)

Time = 20.10 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.30

method	result
default	$-\frac{2a^3\left(28i(\cos^5(dx+c))-28\sin(dx+c)(\cos^4(dx+c))+5iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}-11i(\cos^3(dx+\right.}{77e^5d\sqrt{e\sec(d}}$
risch	$-\frac{ie^{i(dx+c)}\left(7\,e^{4i(dx+c)}+24\,e^{2i(dx+c)}+37\right)a^3\sqrt{2}}{308d\,e^5\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}+\frac{10\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{i\left(e^{i(dx+c)}-i\right)}\sqrt{ie^{i(dx+c)}}\,F\left(\sqrt{-i\left(e^{i(dx+c)}+i\right)},\frac{\sqrt{2}}{2}\right)a^3}{77d\sqrt{e\,e^{3i(dx+c)}+e\,e^{i(dx+c)}}\,e^5\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{e\,e^{i(dx+}}{e^{2i(dx+c)}}}}$
parts	$-\frac{2a^3\left(-7\sin(dx+c)(\cos^4(dx+c))+15iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}+15i\sec(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}F\right.}{77d\sqrt{e\sec(dx+c)}\,e^5}$

```
[In] int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/77*a^3/e^5/d/(e*sec(d*x+c))^(1/2)*(28*I*cos(d*x+c)^5-28*sin(d*x+c)*cos(d
*x+c)^4+5*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d
*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)-11*I*cos(d*x+c)^3+5*I*sec(d*x+c)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d
*x+c)+1))^(1/2)-3*cos(d*x+c)^2*sin(d*x+c)-5*sin(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.71

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{11/2}}\,dx = \frac{-40i\sqrt{2}a^3\sqrt{e}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(-7i\,a^3e^{(6i\,dx+6i\,c)}-3\right.}{308\,de^6}$$

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(11/2),x, algorithm="fricas")
```

```
[Out] 1/308*(-40*I*sqrt(2)*a^3*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)
) + sqrt(2)*(-7*I*a^3*e^(6*I*d*x + 6*I*c) - 31*I*a^3*e^(4*I*d*x + 4*I*c) -
61*I*a^3*e^(2*I*d*x + 2*I*c) - 37*I*a^3)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*
e^(1/2*I*d*x + 1/2*I*c))/(d*e^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{11/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+I*a*tan(d*x+c))**3/(e*sec(d*x+c))**(11/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{11/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^3}{(e\sec(dx+c))^{\frac{11}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(11/2), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{11/2}}\,dx = \int \frac{\left(i\,a\tan\left(dx+c\right)+a\right)^3}{\left(e\sec\left(dx+c\right)\right)^{\frac{11}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(11/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{11/2}}\,dx = \int \frac{\left(a+a\tan(c+d\,x)\;1\mathrm{i}\right)^3}{\left(\frac{e}{\cos(c+d\,x)}\right)^{11/2}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(11/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(11/2), x)
```

3.211 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{13/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 155

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{13/2}}\,dx = \frac{14a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39de^6\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{14a^3\sin(c+dx)}{117de^5(e\sec(c+dx))^{3/2}} - \frac{2i(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}} - \frac{28i(a^3+ia^3\tan(c+dx))}{117de^2(e\sec(c+dx))^{9/2}}$$

```
[Out] 14/117*a^3*sin(d*x+c)/d/e^5/(e*sec(d*x+c))^(3/2)+14/39*a^3*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/e^6
/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)-2/13*I*(a+I*a*tan(d*x+c))^3/d/(e*sec
(d*x+c))^(13/2)-28/117*I*(a^3+I*a^3*tan(d*x+c))/d/e^2/(e*sec(d*x+c))^(9/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3578, 3577, 3854, 3856, 2719}

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{13/2}}\,dx = \frac{14a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39de^6\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{14a^3\sin(c+dx)}{117de^5(e\sec(c+dx))^{3/2}} - \frac{28i(a^3+ia^3\tan(c+dx))}{117de^2(e\sec(c+dx))^{9/2}} - \frac{2i(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(13/2),x]
```

```
[Out] (14*a^3*EllipticE[(c + d*x)/2, 2])/(39*d*e^6*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) + (14*a^3*Sin[c + d*x])/(117*d*e^5*(e*Sec[c + d*x])^(3/2)) - (((
```

```
2*I)/13)*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(13/2)) - (((28*I)/1
17)*(a^3 + I*a^3*Tan[c + d*x]))/(d*e^2*(e*Sec[c + d*x])^(9/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2i(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}+\frac{(7a)\int\frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{9/2}}\,dx}{13e^2}\\ &= -\frac{2i(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}-\frac{28i(a^3+ia^3\tan(c+dx))}{117de^2(e\sec(c+dx))^{9/2}}+\frac{(35a^3)\int\frac{1}{(e\sec(c+dx))^{5/2}}\,dx}{117e^4}\end{aligned}$$

$$= \frac{14a^3\sin(c+dx)}{117de^5(e\sec(c+dx))^{3/2}} - \frac{2i(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}} - \frac{28i(a^3+ia^3\tan(c+dx))}{117de^2(e\sec(c+dx))^{9/2}} + \frac{(7a^3)\int\frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{39e^6}$$

$$= \frac{14a^3\sin(c+dx)}{117de^5(e\sec(c+dx))^{3/2}} - \frac{2i(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}} - \frac{28i(a^3+ia^3\tan(c+dx))}{117de^2(e\sec(c+dx))^{9/2}} + \frac{(7a^3)\int\sqrt{\cos(c+dx)}\,dx}{39e^6\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$= \frac{14a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39de^6\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{14a^3\sin(c+dx)}{117de^5(e\sec(c+dx))^{3/2}} - \frac{2i(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}} - \frac{28i(a^3+ia^3\tan(c+dx))}{117de^2(e\sec(c+dx))^{9/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.31 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.94

$$\int\frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{13/2}}\,dx = \frac{a^3\sqrt{e\sec(c+dx)}(-i\cos(3(c+dx))+\sin(3(c+dx)))\left(62+8\cos(2(c+dx))\right.}{}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(13/2),x]
```

```
[Out] (a^3*Sqrt[e*Sec[c + d*x]]*((-I)*Cos[3*(c + d*x)] + Sin[3*(c + d*x)])*(62 +
8*Cos[2*(c + d*x)] - 54*Cos[4*(c + d*x)] + (56*Sqrt[1 + E^((2*I)*(c + d*x))
]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((2*I)*(c + d*x
)) + (42*I)*Sin[2*(c + d*x)] + (63*I)*Sin[4*(c + d*x)]))/(468*d*e^7)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 351 vs. 2(159) = 318.

Time = 25.77 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.27

method	result
risch	$-\frac{i(9\,\mathrm{e}^{6i(dx+c)}+41\,\mathrm{e}^{4i(dx+c)}+83\,\mathrm{e}^{2i(dx+c)}+219)a^3\sqrt{2}}{936d\,e^6\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}} - \frac{7i\left(-\frac{2\left(e\,\mathrm{e}^{2i(dx+c)}+e\right)}{e\sqrt{\mathrm{e}^{i(dx+c)}\left(e\,\mathrm{e}^{2i(dx+c)}+e\right)}}+\frac{i\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(\mathrm{e}^{i(dx+c)}-i\right)}}{}\right.}{39}$
default	$-\frac{2ia^3\left(36(\cos^7(dx+c))+36(\cos^6(dx+c))+7i(\cos^2(dx+c))\sin(dx+c)-13(\cos^5(dx+c))+5i\sin(dx+c)(\cos^4(dx+c))+21\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)}}\right.}{}$
parts	Expression too large to display

```
[In] int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(13/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/936*I*(9*exp(I*(d*x+c))^6+41*exp(I*(d*x+c))^4+83*exp(I*(d*x+c))^2+219)/d
*a^3*2^(1/2)/e^6/(e*exp(I*(d*x+c))/(exp(I*(d*x+c))^2+1))^(1/2)-7/39*I/d*(-2
*(e*exp(I*(d*x+c))^2+e)/e/(exp(I*(d*x+c))*(e*exp(I*(d*x+c))^2+e))^(1/2)+I*(
-I*(exp(I*(d*x+c))+I))^(1/2)*2^(1/2)*(I*(exp(I*(d*x+c))-I))^(1/2)*(I*exp(I*
(d*x+c)))^(1/2)/(e*exp(I*(d*x+c))^3+e*exp(I*(d*x+c)))^(1/2)*(-2*I*EllipticE
((-I*(exp(I*(d*x+c))+I))^(1/2),1/2*2^(1/2))+I*EllipticF((-I*(exp(I*(d*x+c))
+I))^(1/2),1/2*2^(1/2))))*a^3*2^(1/2)/e^6/(exp(I*(d*x+c))^2+1)/(e*exp(I*(d*
x+c))/(exp(I*(d*x+c))^2+1))^(1/2)*(e*exp(I*(d*x+c))*(exp(I*(d*x+c))^2+1))^(
1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.94

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{13/2}}\,dx = \frac{\left(336i\sqrt{2}a^3\sqrt{e}e^{(i\,dx+i\,c)}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,d}\right.\right.\right.}{}$$

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(13/2),x, algorithm="fricas")
```

```
[Out] 1/936*(336*I*sqrt(2)*a^3*sqrt(e)*e^(I*d*x + I*c)*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(-9*I*a^3*e^(8*I*d*x +
8*I*c) - 50*I*a^3*e^(6*I*d*x + 6*I*c) - 124*I*a^3*e^(4*I*d*x + 4*I*c) + 34*
I*a^3*e^(2*I*d*x + 2*I*c) + 117*I*a^3)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^
(1/2*I*d*x + 1/2*I*c))*e^(-I*d*x - I*c)/(d*e^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{13/2}}\,dx=\text{Timed out}$$

```
[In] integrate((a+I*a*tan(d*x+c))**3/(e*sec(d*x+c))**(13/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{13/2}}\,dx=\int \frac{(i\,a\tan\left(dx+c\right)+a)^3}{\left(e\sec\left(dx+c\right)\right)^{\frac{13}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(13/2),x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(13/2), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{13/2}}\,dx=\int \frac{(i\,a\tan\left(dx+c\right)+a)^3}{\left(e\sec\left(dx+c\right)\right)^{\frac{13}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(13/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(13/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{13/2}}\,dx=\int \frac{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^3}{{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{13/2}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(13/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(13/2), x)
```

3.212 $\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{15/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 186

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{15/2}}\,dx = \frac{2a^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{11de^8} + \frac{6a^3\sin(c+dx)}{55de^5(e\sec(c+dx))^{5/2}} + \frac{2a^3\sin(c+dx)}{11de^7\sqrt{e\sec(c+dx)}} - \frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}} - \frac{12i(a^3+ia^3\tan(c+dx))}{55de^2(e\sec(c+dx))^{11/2}}$$

```
[Out] 6/55*a^3*sin(d*x+c)/d/e^5/(e*sec(d*x+c))^(5/2)+2/11*a^3*sin(d*x+c)/d/e^7/(e
*sec(d*x+c))^(1/2)+2/11*a^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2
)/d/e^8-2/15*I*(a+I*a*tan(d*x+c))^3/d/(e*sec(d*x+c))^(15/2)-12/55*I*(a^3+I*
a^3*tan(d*x+c))/d/e^2/(e*sec(d*x+c))^(11/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3578, 3577, 3854, 3856, 2720}

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{15/2}}\,dx = \frac{2a^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{11de^8} + \frac{2a^3\sin(c+dx)}{11de^7\sqrt{e\sec(c+dx)}} + \frac{6a^3\sin(c+dx)}{55de^5(e\sec(c+dx))^{5/2}} - \frac{12i(a^3+ia^3\tan(c+dx))}{55de^2(e\sec(c+dx))^{11/2}} - \frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(15/2),x]
```

```
[Out] (2*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(
11*d*e^8) + (6*a^3*Sin[c + d*x])/(55*d*e^5*(e*Sec[c + d*x])^(5/2)) + (2*a^3
*Sin[c + d*x])/(11*d*e^7*Sqrt[e*Sec[c + d*x]]) - (((2*I)/15)*(a + I*a*Tan[c
 + d*x])^3)/(d*(e*Sec[c + d*x])^(15/2)) - (((12*I)/55)*(a^3 + I*a^3*Tan[c +
 d*x]))/(d*e^2*(e*Sec[c + d*x])^(11/2))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}+\frac{(3a)\int\frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}\,dx}{5e^2}\\
&= -\frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}-\frac{12i(a^3+ia^3\tan(c+dx))}{55de^2(e\sec(c+dx))^{11/2}}+\frac{(21a^3)\int\frac{1}{(e\sec(c+dx))^{7/2}}\,dx}{55e^4}\\
&= \frac{6a^3\sin(c+dx)}{55de^5(e\sec(c+dx))^{5/2}}-\frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}\\
&\quad-\frac{12i(a^3+ia^3\tan(c+dx))}{55de^2(e\sec(c+dx))^{11/2}}+\frac{(3a^3)\int\frac{1}{(e\sec(c+dx))^{3/2}}\,dx}{11e^6}\\
&= \frac{6a^3\sin(c+dx)}{55de^5(e\sec(c+dx))^{5/2}}+\frac{2a^3\sin(c+dx)}{11de^7\sqrt{e\sec(c+dx)}}-\frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}\\
&\quad-\frac{12i(a^3+ia^3\tan(c+dx))}{55de^2(e\sec(c+dx))^{11/2}}+\frac{a^3\int\sqrt{e\sec(c+dx)}\,dx}{11e^8}\\
&= \frac{6a^3\sin(c+dx)}{55de^5(e\sec(c+dx))^{5/2}}+\frac{2a^3\sin(c+dx)}{11de^7\sqrt{e\sec(c+dx)}}-\frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}\\
&\quad-\frac{12i(a^3+ia^3\tan(c+dx))}{55de^2(e\sec(c+dx))^{11/2}}+\frac{\left(a^3\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{11e^8}\\
&= \frac{2a^3\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{11de^8}+\frac{6a^3\sin(c+dx)}{55de^5(e\sec(c+dx))^{5/2}}\\
&\quad+\frac{2a^3\sin(c+dx)}{11de^7\sqrt{e\sec(c+dx)}}-\frac{2i(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}-\frac{12i(a^3+ia^3\tan(c+dx))}{55de^2(e\sec(c+dx))^{11/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.59 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.91

$$\int\frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{15/2}}\,dx=\frac{a^3\sqrt{e\sec(c+dx)}\left(-332i\cos(c+dx)-154i\cos(3(c+dx))+22i\cos(5(c+d\right.}{}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^3/(e*Sec[c + d*x])^(15/2),x]
```

```
[Out] (a^3*Sqrt[e*Sec[c + d*x]]*((-332*I)*Cos[c + d*x] - (154*I)*Cos[3*(c + d*x)]
 + (22*I)*Cos[5*(c + d*x)] - 114*Sin[c + d*x] + 240*Sqrt[Cos[c + d*x]]*Elli
pticF[(c + d*x)/2, 2]*(Cos[3*(c + d*x)] - I*Sin[3*(c + d*x)]) - 81*Sin[3*(c
 + d*x)] + 33*Sin[5*(c + d*x)])*(Cos[3*(c + 2*d*x)] + I*Sin[3*(c + 2*d*x)])
)/(1320*d*e^8*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [A] (verified)

Time = 24.65 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.17

method	result
default	$\frac{2a^3\left(-44i\left(\cos^7(dx+c)\right)+44\sin(dx+c)\left(\cos^6(dx+c)\right)+15i\left(\cos^5(dx+c)\right)+7\sin(dx+c)\left(\cos^4(dx+c)\right)+15iF(i(-\csc(dx+c)+\cot(dx+c)),$
parts	$-\frac{2a^3\left(-77\sin(dx+c)\left(\cos^6(dx+c)\right)-91\sin(dx+c)\left(\cos^4(dx+c)\right)+195iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}+19}{1155d\sqrt{e\sec(dx-}}$

```
[In] int((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(15/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/165*a^3/e^7/d/(e*sec(d*x+c))^(1/2)*(-44*I*cos(d*x+c)^7+44*sin(d*x+c)*cos(
d*x+c)^6+15*I*cos(d*x+c)^5+7*sin(d*x+c)*cos(d*x+c)^4+15*I*EllipticF(I*(-csc
(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)+15*I*sec(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)+9*cos(d*x+c)^2*sin(d*x+c)+15
*sin(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.84

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{15/2}}\,dx = \frac{\left(-480i\sqrt{2}a^3\sqrt{e}e^{(2i\,dx+2i\,c)}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(-11i\,a^3e\right.\right.}{}$$

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(15/2),x, algorithm="fricas")
```

```
[Out] 1/2640*(-480*I*sqrt(2)*a^3*sqrt(e)*e^(2*I*d*x + 2*I*c)*weierstrassPInverse(
-4, 0, e^(I*d*x + I*c)) + sqrt(2)*(-11*I*a^3*e^(10*I*d*x + 10*I*c) - 73*I*a
^3*e^(8*I*d*x + 8*I*c) - 218*I*a^3*e^(6*I*d*x + 6*I*c) - 446*I*a^3*e^(4*I*d
*x + 4*I*c) - 235*I*a^3*e^(2*I*d*x + 2*I*c) + 55*I*a^3)*sqrt(e/(e^(2*I*d*x
+ 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-2*I*d*x - 2*I*c)/(d*e^8)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{15/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+I*a*tan(d*x+c))**3/(e*sec(d*x+c))**(15/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{15/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^3}{(e\sec(dx+c))^{\frac{15}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(15/2),x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(15/2), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{15/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^3}{(e\sec(dx+c))^{\frac{15}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^3/(e*sec(d*x+c))^(15/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^3/(e*sec(d*x + c))^(15/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^3}{(e\sec(c+dx))^{15/2}}\,dx = \int \frac{(a+a\tan(c+d\,x)\;1\mathrm{i})^3}{\left(\frac{e}{\cos(c+d\,x)}\right)^{15/2}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(15/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^3/(e/cos(c + d*x))^(15/2), x)
```

3.213 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^4\,dx$

Optimal result

Integrand size = 28, antiderivative size = 215

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^4\,dx = -\frac{22a^4e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{22ia^4(e\sec(c+dx))^{3/2}}{9d} + \frac{22a^4e\sqrt{e\sec(c+dx)}\sin(c+dx)}{3d}+\frac{2ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}{9d} + \frac{10i(e\sec(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)^2}{21d} + \frac{22i(e\sec(c+dx))^{3/2}\left(a^4+ia^4\tan(c+dx)\right)}{21d}$$

```
[Out] 22/9*I*a^4*(e*sec(d*x+c))^(3/2)/d-22/3*a^4*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2
)/(e*sec(d*x+c))^(1/2)+22/3*a^4*e*sin(d*x+c)*(e*sec(d*x+c))^(1/2)/d+2/9*I*a
*(e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^3/d+10/21*I*(e*sec(d*x+c))^(3/2)*(
a^2+I*a^2*tan(d*x+c))^2/d+22/21*I*(e*sec(d*x+c))^(3/2)*(a^4+I*a^4*tan(d*x+c
))/d
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3579, 3567, 3853, 3856, 2719}

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^4\,dx = -\frac{22a^4e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{22ia^4(e\sec(c+dx))^{3/2}}{9d}+\frac{22a^4e\sin(c+dx)\sqrt{e\sec(c+dx)}}{3d}+\frac{22i(a^4+ia^4\tan(c+dx))\,(e\sec(c+dx))^{3/2}}{21d}+\frac{10i(a^2+ia^2\tan(c+dx))^2\,(e\sec(c+dx))^{3/2}}{21d}+\frac{2ia(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}{9d}$$

```
[In] Int[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (-22*a^4*e^2*EllipticE[(c + d*x)/2, 2])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) + (((22*I)/9)*a^4*(e*Sec[c + d*x])^(3/2))/d + (22*a^4*e*Sqrt[e*S
ec[c + d*x]]*Sin[c + d*x])/(3*d) + (((2*I)/9)*a*(e*Sec[c + d*x])^(3/2)*(a +
 I*a*Tan[c + d*x])^3)/d + (((10*I)/21)*(e*Sec[c + d*x])^(3/2)*(a^2 + I*a^2*
Tan[c + d*x])^2)/d + (((22*I)/21)*(e*Sec[c + d*x])^(3/2)*(a^4 + I*a^4*Tan[c
 + d*x]))/d
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}{9d} \\
&\quad + \frac{1}{3}(5a)\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3\,dx \\
&= \frac{2ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}{9d} \\
&\quad + \frac{10i(e\sec(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)^2}{21d} \\
&\quad + \frac{1}{21}\left(55a^2\right)\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2\,dx \\
&= \frac{2ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}{9d} \\
&\quad + \frac{10i(e\sec(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)^2}{21d} \\
&\quad + \frac{22i(e\sec(c+dx))^{3/2}\left(a^4+ia^4\tan(c+dx)\right)}{21d} \\
&\quad + \frac{1}{3}\left(11a^3\right)\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx \\
&= \frac{22ia^4(e\sec(c+dx))^{3/2}}{9d} + \frac{2ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}{9d} \\
&\quad + \frac{10i(e\sec(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)^2}{21d} \\
&\quad + \frac{22i(e\sec(c+dx))^{3/2}\left(a^4+ia^4\tan(c+dx)\right)}{21d} + \frac{1}{3}\left(11a^4\right)\int (e\sec(c+dx))^{3/2}\,dx
\end{aligned}
$$

$$= \frac{22ia^4(e\sec(c+dx))^{3/2}}{9d} + \frac{22a^4e\sqrt{e\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}{9d} + \frac{10i(e\sec(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)^2}{21d} + \frac{22i(e\sec(c+dx))^{3/2}\left(a^4+ia^4\tan(c+dx)\right)}{21d} - \frac{1}{3}\left(11a^4e^2\right)\int \frac{1}{\sqrt{e\sec(c+dx)}}\,dx$$

$$= \frac{22ia^4(e\sec(c+dx))^{3/2}}{9d} + \frac{22a^4e\sqrt{e\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}{9d} + \frac{10i(e\sec(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)^2}{21d} + \frac{22i(e\sec(c+dx))^{3/2}\left(a^4+ia^4\tan(c+dx)\right)}{21d} - \frac{\left(11a^4e^2\right)\int \sqrt{\cos(c+dx)}\,dx}{3\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$= -\frac{22a^4e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{22ia^4(e\sec(c+dx))^{3/2}}{9d} + \frac{22a^4e\sqrt{e\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}{9d} + \frac{10i(e\sec(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)^2}{21d} + \frac{22i(e\sec(c+dx))^{3/2}\left(a^4+ia^4\tan(c+dx)\right)}{21d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.21 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.52

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^4\,dx = \frac{(e\sec(c+dx))^{3/2}\left(\frac{22i\sqrt{2}e^{-i(3c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\left(-3\sqrt{1+e^{2i(c+dx)}}+e^{2idx}\left(-1+e^{2ic}\right)\text{Hype}\right.}{-1+e^{2ic}}}{}$$

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((e*Sec[c + d*x])^(3/2)*(((22*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)
*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*(-3*Sqrt[1 + E^((2*I)*(c + d*x)
```

```
)] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^(
(2*I)*(c + d*x))]))/(E^(I*(3*c + d*x))*(-1 + E^((2*I)*c))) + (Csc[c]*Sec[c
+ d*x]^(9/2)*(Cos[4*c] - I*Sin[4*c])*(1260*Cos[d*x] + 1050*Cos[2*c + d*x] +
 742*Cos[2*c + 3*d*x] + 413*Cos[4*c + 3*d*x] + 231*Cos[4*c + 5*d*x] - (720*
I)*Sin[d*x] + (720*I)*Sin[2*c + d*x] - (336*I)*Sin[2*c + 3*d*x] + (336*I)*S
in[4*c + 3*d*x]))/56)*(a + I*a*Tan[c + d*x])^4)/(9*d*Sec[c + d*x]^(11/2)*(C
os[d*x] + I*Sin[d*x])^4)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(208) = 416$.

Time = 24.52 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.23

method	result
default	$-\frac{2ie\,a^4\sqrt{e\sec(dx+c)}\left(231\left(\cos^2(dx+c)\right)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,E(i(\csc(dx+c)-\cot(dx+c)),i)-231\left(\cos^2(dx+c)\right)\sqrt{\frac{1}{\cos(dx+c)+1}}$
parts	Expression too large to display

```
[In] int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2/63*I*e*a^4/d*(e*sec(d*x+c))^(1/2)/(cos(d*x+c)+1)*(231*cos(d*x+c)^2*(1/(c
os(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(csc(d*x+
c)-cot(d*x+c)),I)-231*cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)+462*cos(d*x+c)*(1
/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(csc(d
*x+c)-cot(d*x+c)),I)-462*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x
+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+231*(1/(cos(d*x+
c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(
d*x+c)),I)-231*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-168+231*I*sin(d*x+c)-168*sec(d*x+c)-9
1*I*tan(d*x+c)+36*sec(d*x+c)^2-91*I*tan(d*x+c)*sec(d*x+c)+36*sec(d*x+c)^3+7
*I*tan(d*x+c)*sec(d*x+c)^2+7*I*tan(d*x+c)*sec(d*x+c)^3)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.16

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^4\,dx =$$

$$-\frac{2\left(\sqrt{2}\left(231i\,a^4ee^{(9i\,dx+9i\,c)}+406i\,a^4ee^{(7i\,dx+7i\,c)}+540i\,a^4ee^{(5i\,dx+5i\,c)}+330i\,a^4ee^{(3i\,dx+3i\,c)}+77i\,a^4ee^{(i\,dx+i\,c)}\right)\right.}{}$$

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -2/63*(sqrt(2)*(231*I*a^4*e*e^(9*I*d*x + 9*I*c) + 406*I*a^4*e*e^(7*I*d*x +
7*I*c) + 540*I*a^4*e*e^(5*I*d*x + 5*I*c) + 330*I*a^4*e*e^(3*I*d*x + 3*I*c)
+ 77*I*a^4*e*e^(I*d*x + I*c))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*
x + 1/2*I*c) + 231*sqrt(2)*(I*a^4*e*e^(8*I*d*x + 8*I*c) + 4*I*a^4*e*e^(6*I*
d*x + 6*I*c) + 6*I*a^4*e*e^(4*I*d*x + 4*I*c) + 4*I*a^4*e*e^(2*I*d*x + 2*I*c
) + I*a^4*e)*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I
*d*x + I*c))))/(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*
I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^4\,dx = a^4\left(\int (e\sec{(c+dx)})^{\frac{3}{2}}\,dx + \int \left(-6(e\sec{(c+dx)})^{\frac{3}{2}}\tan^2{(c+dx)}\right)dx + \int (e\sec{(c+dx)})^{\frac{3}{2}}\tan^4{(c+dx)}\,dx + \int 4i(e\sec{(c+dx)})^{\frac{3}{2}}\tan{(c+dx)}\,dx + \int \left(-4i(e\sec{(c+dx)})^{\frac{3}{2}}\tan^3{(c+dx)}\right)dx\right)$$

```
[In] integrate((e*sec(d*x+c))**(3/2)*(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] a**4*(Integral((e*sec(c + d*x))**(3/2), x) + Integral(-6*(e*sec(c + d*x))**
(3/2)*tan(c + d*x)**2, x) + Integral((e*sec(c + d*x))**(3/2)*tan(c + d*x)**
4, x) + Integral(4*I*(e*sec(c + d*x))**(3/2)*tan(c + d*x), x) + Integral(-4
*I*(e*sec(c + d*x))**(3/2)*tan(c + d*x)**3, x))
```

Maxima [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^4\,dx = \int (e\sec{(dx+c)})^{\frac{3}{2}}\,(i\,a\tan{(dx+c)}+a)^4\,dx$$

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^4, x)
```

Giac [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^4\,dx = \int (e\sec(dx+c))^{\frac{3}{2}}(i\,a\tan(dx+c)+a)^4\,dx$$

[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")

[Out] integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^4, x)

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^4\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}(a+a\tan(c+d\,x)\,1\mathrm{i})^4\,dx$$

[In] int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^4,x)

[Out] int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^4, x)

3.214 $\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^4\,dx$

Optimal result

Integrand size = 28, antiderivative size = 183

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^4\,dx = \frac{78ia^4\sqrt{e\sec(c+dx)}}{7d}+\frac{78a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{7d}+\frac{2ia\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}{7d}+\frac{26i\sqrt{e\sec(c+dx)}\left(a^2+ia^2\tan(c+dx)\right)^2}{35d}+\frac{78i\sqrt{e\sec(c+dx)}\left(a^4+ia^4\tan(c+dx)\right)}{35d}$$

```
[Out] 78/7*I*a^4*(e*sec(d*x+c))^(1/2)/d+78/7*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*s
ec(d*x+c))^(1/2)/d+2/7*I*a*(e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^3/d+26/3
5*I*(e*sec(d*x+c))^(1/2)*(a^2+I*a^2*tan(d*x+c))^2/d+78/35*I*(e*sec(d*x+c))^
(1/2)*(a^4+I*a^4*tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used

$= \{3579, 3567, 3856, 2720\}$

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^4\,dx$$
$$= \frac{78ia^4\sqrt{e\sec(c+dx)}}{7d} + \frac{78i(a^4+ia^4\tan(c+dx))\sqrt{e\sec(c+dx)}}{35d} + \frac{78a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{7d} + \frac{26i(a^2+ia^2\tan(c+dx))^2\sqrt{e\sec(c+dx)}}{35d} + \frac{2ia(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}{7d}$$

```
[In] Int[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (((78*I)/7)*a^4*Sqrt[e*Sec[c + d*x]])/d + (78*a^4*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(7*d) + (((2*I)/7)*a*Sqrt[e*Sec[c
 + d*x]]*(a + I*a*Tan[c + d*x])^3)/d + (((26*I)/35)*Sqrt[e*Sec[c + d*x]]*(a
^2 + I*a^2*Tan[c + d*x])^2)/d + (((78*I)/35)*Sqrt[e*Sec[c + d*x]]*(a^4 + I*
a^4*Tan[c + d*x]))/d
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\text{integral} = \frac{2ia\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}{7d} + \frac{1}{7}(13a)\int\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3\,dx$$

$$= \frac{2ia\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}{7d} + \frac{26i\sqrt{e\sec(c+dx)}\left(a^2+ia^2\tan(c+dx)\right)^2}{35d} + \frac{1}{35}\left(117a^2\right)\int\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2\,dx$$

$$= \frac{2ia\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}{7d} + \frac{26i\sqrt{e\sec(c+dx)}\left(a^2+ia^2\tan(c+dx)\right)^2}{35d} + \frac{78i\sqrt{e\sec(c+dx)}\left(a^4+ia^4\tan(c+dx)\right)}{35d} + \frac{1}{7}\left(39a^3\right)\int\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))\,dx$$

$$= \frac{78ia^4\sqrt{e\sec(c+dx)}}{7d} + \frac{2ia\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}{7d} + \frac{26i\sqrt{e\sec(c+dx)}\left(a^2+ia^2\tan(c+dx)\right)^2}{35d} + \frac{78i\sqrt{e\sec(c+dx)}\left(a^4+ia^4\tan(c+dx)\right)}{35d} + \frac{1}{7}\left(39a^4\right)\int\sqrt{e\sec(c+dx)}\,dx$$

$$= \frac{78ia^4\sqrt{e\sec(c+dx)}}{7d} + \frac{2ia\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}{7d} + \frac{26i\sqrt{e\sec(c+dx)}\left(a^2+ia^2\tan(c+dx)\right)^2}{35d} + \frac{78i\sqrt{e\sec(c+dx)}\left(a^4+ia^4\tan(c+dx)\right)}{35d} + \frac{1}{7}\left(39a^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx$$

$$= \frac{78ia^4\sqrt{e\sec(c+dx)}}{7d} + \frac{78a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{7d} + \frac{2ia\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}{7d} + \frac{26i\sqrt{e\sec(c+dx)}\left(a^2+ia^2\tan(c+dx)\right)^2}{35d} + \frac{78i\sqrt{e\sec(c+dx)}\left(a^4+ia^4\tan(c+dx)\right)}{35d}$$

Mathematica [A] (verified)

Time = 2.93 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.55

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^4\,dx$$
$$= \frac{a^4\sec^4(c+dx)\sqrt{e\sec(c+dx)}\left(728i+1008i\cos(2(c+dx))+280i\cos(4(c+dx))+1560\cos^{\frac{9}{2}}(c+dx)\,\text{Elli}\right.}{140d}$$

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (a^4*Sec[c + d*x]^4*Sqrt[e*Sec[c + d*x]]*(728*I + (1008*I)*Cos[2*(c + d*x)]
 + (280*I)*Cos[4*(c + d*x)] + 1560*Cos[c + d*x]^(9/2)*EllipticF[(c + d*x)/2
, 2] - 150*Sin[2*(c + d*x)] - 85*Sin[4*(c + d*x)]))/(140*d)
```

Maple [A] (verified)

Time = 19.38 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.94

method	result
default	$\frac{2ia^4\sqrt{e\sec(dx+c)}\left(195\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,F(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\cos(dx+c)+195F(i(-\csc(dx+c)+\cot(dx+c)),i\right.}{35d}$
parts	$-\frac{2ia^4(\cos(dx+c)+1)F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{e\sec(dx+c)}}{d}+\frac{2ia^4\sqrt{e\sec(dx+c)}\left(-4F(i(\csc(d\right.}{}$

```
[In] int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/35*I*a^4/d*(e*sec(d*x+c))^(1/2)*(195*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*El
lipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+1
95*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)+280+85*I*tan(d*x+c)-28*sec(d*x+c)^2-5*I*sec(d*x+
c)^2*tan(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.03

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^4\,dx = -\frac{2\left(\sqrt{2}\left(-365i\,a^4e^{(6i\,dx+6i\,c)}-793i\,a^4e^{(4i\,dx+4i\,c)}-663i\,a^4e^{(2i\,dx+2i\,c)}-195i\,a^4\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+\ldots}{35\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\ldots}}$$

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -2/35*(sqrt(2)*(-365*I*a^4*e^(6*I*d*x + 6*I*c) - 793*I*a^4*e^(4*I*d*x + 4*I
*c) - 663*I*a^4*e^(2*I*d*x + 2*I*c) - 195*I*a^4)*sqrt(e/(e^(2*I*d*x + 2*I*c
) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 195*sqrt(2)*(I*a^4*e^(6*I*d*x + 6*I*c) +
3*I*a^4*e^(4*I*d*x + 4*I*c) + 3*I*a^4*e^(2*I*d*x + 2*I*c) + I*a^4)*sqrt(e)*
weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e
^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^4\,dx = a^4\left(\int \sqrt{e\sec{(c+dx)}}\,dx + \int \left(-6\sqrt{e\sec{(c+dx)}}\tan^2{(c+dx)}\right)\,dx + \int \sqrt{e\sec{(c+dx)}}\tan^4{(c+dx)}\,dx + \int 4i\sqrt{e\sec{(c+dx)}}\tan{(c+dx)}\,dx + \int \left(-4i\sqrt{e\sec{(c+dx)}}\tan^3{(c+dx)}\right)\,dx\right)$$

```
[In] integrate((e*sec(d*x+c))**(1/2)*(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] a**4*(Integral(sqrt(e*sec(c + d*x)), x) + Integral(-6*sqrt(e*sec(c + d*x))*
tan(c + d*x)**2, x) + Integral(sqrt(e*sec(c + d*x))*tan(c + d*x)**4, x) + I
ntegral(4*I*sqrt(e*sec(c + d*x))*tan(c + d*x), x) + Integral(-4*I*sqrt(e*se
c(c + d*x))*tan(c + d*x)**3, x))
```

Maxima [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^4\,dx = \int \sqrt{e\sec\left(dx+c\right)}\left(i\,a\tan\left(dx+c\right)+a\right)^4\,dx$$

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
[Out] integrate(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^4, x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^4\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{-1,[2,0]%%%}+%%%{%%{[-2,0]:[1,0,%%%{1,[1]%%%}]%%},[1,0]%%%}+%
%%{%%%
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^4\,dx = \int \sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^4\,dx$$

```
[In] int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^4,x)
[Out] int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^4, x)
```

3.215 $\int \frac{(a+ia\tan(c+dx))^4}{\sqrt{e\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 178

$$\int \frac{(a+ia\tan(c+dx))^4}{\sqrt{e\sec(c+dx)}}\,dx = \frac{154a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{154ia^4(e\sec(c+dx))^{3/2}}{15de^2} - \frac{154a^4\sqrt{e\sec(c+dx)}\sin(c+dx)}{5de} - \frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}} - \frac{22i(e\sec(c+dx))^{3/2}\left(a^4+ia^4\tan(c+dx)\right)}{5de^2}$$

```
[Out] -154/15*I*a^4*(e*sec(d*x+c))^(3/2)/d/e^2+154/5*a^4*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^
(1/2)/(e*sec(d*x+c))^(1/2)-154/5*a^4*sin(d*x+c)*(e*sec(d*x+c))^(1/2)/d/e-4*
I*a*(a+I*a*tan(d*x+c))^3/d/(e*sec(d*x+c))^(1/2)-22/5*I*(e*sec(d*x+c))^(3/2)
*(a^4+I*a^4*tan(d*x+c))/d/e^2
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3577, 3579, 3567, 3853, 3856, 2719}

$$\int \frac{(a+ia\tan(c+dx))^4}{\sqrt{e\sec(c+dx)}}\,dx = -\frac{154ia^4(e\sec(c+dx))^{3/2}}{15de^2} - \frac{22i(a^4+ia^4\tan(c+dx))\,(e\sec(c+dx))^{3/2}}{5de^2} - \frac{154a^4\sin(c+dx)\sqrt{e\sec(c+dx)}}{5de} + \frac{154a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^4/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] (154*a^4*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c +
d*x]]) - (((154*I)/15)*a^4*(e*Sec[c + d*x])^(3/2))/(d*e^2) - (154*a^4*Sqrt[
e*Sec[c + d*x]]*Sin[c + d*x])/(5*d*e) - ((4*I)*a*(a + I*a*Tan[c + d*x])^3)/
(d*Sqrt[e*Sec[c + d*x]]) - (((22*I)/5)*(e*Sec[c + d*x])^(3/2)*(a^4 + I*a^4*
Tan[c + d*x]))/(d*e^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}} - \frac{(11a^2)\int(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2\,dx}{e^2}\\
&= -\frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}} - \frac{22i(e\sec(c+dx))^{3/2}\left(a^4+ia^4\tan(c+dx)\right)}{5de^2}\\
&\quad - \frac{(77a^3)\int(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx}{5e^2}\\
&= -\frac{154ia^4(e\sec(c+dx))^{3/2}}{15de^2} - \frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}}\\
&\quad - \frac{22i(e\sec(c+dx))^{3/2}\left(a^4+ia^4\tan(c+dx)\right)}{5de^2} - \frac{(77a^4)\int(e\sec(c+dx))^{3/2}\,dx}{5e^2}\\
&= -\frac{154ia^4(e\sec(c+dx))^{3/2}}{15de^2} - \frac{154a^4\sqrt{e\sec(c+dx)}\sin(c+dx)}{5de} - \frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}}\\
&\quad - \frac{22i(e\sec(c+dx))^{3/2}\left(a^4+ia^4\tan(c+dx)\right)}{5de^2} + \frac{1}{5}(77a^4)\int\frac{1}{\sqrt{e\sec(c+dx)}}\,dx\\
&= -\frac{154ia^4(e\sec(c+dx))^{3/2}}{15de^2} - \frac{154a^4\sqrt{e\sec(c+dx)}\sin(c+dx)}{5de} - \frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}}\\
&\quad - \frac{22i(e\sec(c+dx))^{3/2}\left(a^4+ia^4\tan(c+dx)\right)}{5de^2} + \frac{(77a^4)\int\sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}
\end{aligned}
$$

$$= \frac{154a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{154ia^4(e\sec(c+dx))^{3/2}}{15de^2} - \frac{154a^4\sqrt{e\sec(c+dx)}\sin(c+dx)}{5de} - \frac{4ia(a+ia\tan(c+dx))^3}{d\sqrt{e\sec(c+dx)}} - \frac{22i(e\sec(c+dx))^{3/2}\left(a^4+ia^4\tan(c+dx)\right)}{5de^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.69

$$\int \frac{(a+ia\tan(c+dx))^4}{\sqrt{e\sec(c+dx)}}\,dx = -\frac{2ia^4e^{i(c+dx)}\left(-77-176e^{2i(c+dx)}-111e^{4i(c+dx)}+77\left(1+e^{2i(c+dx)}\right)^{5/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+\ldots}\right.\right.}{15de\left(1+e^{2i(c+dx)}\right)^2}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^4/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] (((-2*I)/15)*a^4*E^(I*(c + d*x))*(-77 - 176*E^((2*I)*(c + d*x)) - 111*E^((4
*I)*(c + d*x)) + 77*(1 + E^((2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2,
3/4, 7/4, -E^((2*I)*(c + d*x))])*Sqrt[e*Sec[c + d*x]])/(d*e*(1 + E^((2*I)*(
c + d*x)))^2)
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1377 vs. 2(179) = 358.

Time = 19.09 (sec) , antiderivative size = 1378, normalized size of antiderivative = 7.74

method	result	size
default	Expression too large to display	1378
parts	Expression too large to display	1543

```
[In] int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*I*a^4/d/(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)/(cos(d*x+c)+1)^3/(e*sec(d
*x+c))^(1/2)*(231*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3
/2)*cos(d*x+c)^3-231*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)
^(3/2)*cos(d*x+c)^3+924*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)
```

```
/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1)
^2)^(3/2)*cos(d*x+c)^2-924*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)/(cos(d*x+c)
+1)^2)^(3/2)*cos(d*x+c)^2+1386*cos(d*x+c)*EllipticF(I*(-csc(d*x+c)+cot(d*x+
c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(-cos(d*x
+c)/(cos(d*x+c)+1)^2)^(3/2)-1386*cos(d*x+c)*EllipticE(I*(-csc(d*x+c)+cot(d*
x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*(-cos(d
*x+c)/(cos(d*x+c)+1)^2)^(3/2)+99*I*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2
)^(3/2)+924*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(
d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)-92
4*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)
*(1/(cos(d*x+c)+1))^(1/2)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)-120*cos(d*x+
c)^3*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)-120*I*cos(d*x+c)^2*sin(d*x+c)*(-c
os(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)+231*sec(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(-
cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)-231*sec(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(
-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)-360*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+
c)+1)^2)^(3/2)-9*I*tan(d*x+c)*sec(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/
2)-380*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)-129*I*cos(d*x+c)*sin
(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)-180*(-cos(d*x+c)/(cos(d*x+c)+1
)^2)^(3/2)-3*I*sec(d*x+c)^2*tan(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2)
-30*cos(d*x+c)*ln(2*(2*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*(-
cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)-cos(d*x+c)+1)/(cos(d*x+c)+1))+30*cos(d*x
+c)*ln((2*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*(-cos(d*x+c)/(c
os(d*x+c)+1)^2)^(1/2)-cos(d*x+c)+1)/(cos(d*x+c)+1))-60*sec(d*x+c)*(-cos(d*x
+c)/(cos(d*x+c)+1)^2)^(3/2)+102*I*tan(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)
^(3/2)-20*sec(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(3/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.92

$$\int \frac{(a+ia\tan(c+dx))^4}{\sqrt{e\sec(c+dx)}}\,dx = -\frac{2\left(\sqrt{2}\left(-111i\,a^4e^{(5i\,dx+5i\,c)}-176i\,a^4e^{(3i\,dx+3i\,c)}-77i\,a^4e^{(i\,dx+i\,c)}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+231\sqrt{2}(-i\cdots\right.}{15\left(dee^{(4i\,dx+4i\,c)}+2\cdots\right.}$$

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*(sqrt(2)*(-111*I*a^4*e^(5*I*d*x + 5*I*c) - 176*I*a^4*e^(3*I*d*x + 3*I
*c) - 77*I*a^4*e^(I*d*x + I*c))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*
```

```
d*x + 1/2*I*c) + 231*sqrt(2)*(-I*a^4*e^(4*I*d*x + 4*I*c) - 2*I*a^4*e^(2*I*d
*x + 2*I*c) - I*a^4)*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, e^(I*d*x + I*c))))/(d*e*e^(4*I*d*x + 4*I*c) + 2*d*e*e^(2*I*d*x + 2*I*c)
 + d*e)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{\sqrt{e\sec(c+dx)}}\,dx = a^4\left(\int \frac{1}{\sqrt{e\sec{(c+dx)}}}\,dx + \int\left(-\frac{6\tan^2{(c+dx)}}{\sqrt{e\sec{(c+dx)}}}\right)dx + \int \frac{\tan^4{(c+dx)}}{\sqrt{e\sec{(c+dx)}}}\,dx + \int \frac{4i\tan{(c+dx)}}{\sqrt{e\sec{(c+dx)}}}\,dx + \int\left(-\frac{4i\tan^3{(c+dx)}}{\sqrt{e\sec{(c+dx)}}}\right)dx\right)$$

```
[In] integrate((a+I*a*tan(d*x+c))**4/(e*sec(d*x+c))**(1/2),x)
```

```
[Out] a**4*(Integral(1/sqrt(e*sec(c + d*x)), x) + Integral(-6*tan(c + d*x)**2/sqr
t(e*sec(c + d*x)), x) + Integral(tan(c + d*x)**4/sqrt(e*sec(c + d*x)), x) +
 Integral(4*I*tan(c + d*x)/sqrt(e*sec(c + d*x)), x) + Integral(-4*I*tan(c +
 d*x)**3/sqrt(e*sec(c + d*x)), x))
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{\sqrt{e\sec(c+dx)}}\,dx = \int \frac{(i\,a\tan{(dx+c)}+a)^4}{\sqrt{e\sec{(dx+c)}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^4/sqrt(e*sec(d*x + c)), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{\sqrt{e\sec(c+dx)}}\,dx = \int \frac{(i\,a\tan{(dx+c)}+a)^4}{\sqrt{e\sec{(dx+c)}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^4/sqrt(e*sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^4}{\sqrt{e\sec(c+dx)}}\,dx=\int \frac{{\left(a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^4}{\sqrt{\frac{e}{\cos\left(c+d\,x\right)}}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(1/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(1/2), x)
```

3.216 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 146

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{10ia^4\sqrt{e\sec(c+dx)}}{de^2} - \frac{10a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{de^2} - \frac{4ia(a+ia\tan(c+dx))^3}{3d(e\sec(c+dx))^{3/2}} - \frac{2i\sqrt{e\sec(c+dx)}(a^4+ia^4\tan(c+dx))}{de^2}$$

```
[Out] -10*I*a^4*(e*sec(d*x+c))^(1/2)/d/e^2-10*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*
sec(d*x+c))^(1/2)/d/e^2-4/3*I*a*(a+I*a*tan(d*x+c))^3/d/(e*sec(d*x+c))^(3/2)
-2*I*(e*sec(d*x+c))^(1/2)*(a^4+I*a^4*tan(d*x+c))/d/e^2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3577, 3579, 3567, 3856, 2720}

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{10ia^4\sqrt{e\sec(c+dx)}}{de^2} - \frac{2i(a^4+ia^4\tan(c+dx))\sqrt{e\sec(c+dx)}}{de^2} - \frac{10a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{de^2} - \frac{4ia(a+ia\tan(c+dx))^3}{3d(e\sec(c+dx))^{3/2}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((-10*I)*a^4*Sqrt[e*Sec[c + d*x]])/(d*e^2) - (10*a^4*Sqrt[Cos[c + d*x]]*Ell
ipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(d*e^2) - (((4*I)/3)*a*(a + I*
a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(3/2)) - ((2*I)*Sqrt[e*Sec[c + d*x]]
*(a^4 + I*a^4*Tan[c + d*x]))/(d*e^2)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{4ia(a+ia\tan(c+dx))^3}{3d(e\sec(c+dx))^{3/2}} - \frac{(3a^2)\int\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2\,dx}{e^2}\\
&= -\frac{4ia(a+ia\tan(c+dx))^3}{3d(e\sec(c+dx))^{3/2}} - \frac{2i\sqrt{e\sec(c+dx)}(a^4+ia^4\tan(c+dx))}{de^2}\\
&\quad - \frac{(5a^3)\int\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))\,dx}{e^2}\\
&= -\frac{10ia^4\sqrt{e\sec(c+dx)}}{de^2} - \frac{4ia(a+ia\tan(c+dx))^3}{3d(e\sec(c+dx))^{3/2}}\\
&\quad - \frac{2i\sqrt{e\sec(c+dx)}(a^4+ia^4\tan(c+dx))}{de^2} - \frac{(5a^4)\int\sqrt{e\sec(c+dx)}\,dx}{e^2}\\
&= -\frac{10ia^4\sqrt{e\sec(c+dx)}}{de^2} - \frac{4ia(a+ia\tan(c+dx))^3}{3d(e\sec(c+dx))^{3/2}}\\
&\quad - \frac{2i\sqrt{e\sec(c+dx)}(a^4+ia^4\tan(c+dx))}{de^2}\\
&\quad - \frac{\left(5a^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{e^2}\\
&= -\frac{10ia^4\sqrt{e\sec(c+dx)}}{de^2} - \frac{10a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{de^2}\\
&\quad - \frac{4ia(a+ia\tan(c+dx))^3}{3d(e\sec(c+dx))^{3/2}} - \frac{2i\sqrt{e\sec(c+dx)}(a^4+ia^4\tan(c+dx))}{de^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.97 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.89

$$
\int\frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{3/2}}\,dx = \frac{a^4\sec^3(c+dx)\left(21+19\cos(2(c+dx))-30i\cos^{\frac{3}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx\right.\right.}{3d(e\sec(c+dx)}
$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(3/2),x]
```

```
[Out] (a^4*Sec[c + d*x]^3*(21 + 19*Cos[2*(c + d*x)] - (30*I)*Cos[c + d*x]^(3/2)*E
llipticF[(c + d*x)/2, 2]*(Cos[c + d*x] - I*Sin[c + d*x]) - (11*I)*Sin[2*(c
+ d*x)])*((-I)*Cos[c + 5*d*x] + Sin[c + 5*d*x]))/(3*d*(e*Sec[c + d*x])^(3/2
)*(Cos[d*x] + I*Sin[d*x])^4)
```

Maple [A] (verified)

Time = 17.23 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.23

method	result
default	$-\frac{2a^4\left(15iF(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}+15i\sec(dx+c)F(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c}}\right.}{3ed\sqrt{e\sec(dx+c)}}$
parts	Expression too large to display

```
[In] int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*a^4/e/d/(e*sec(d*x+c))^(1/2)*(15*I*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)
),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+15*I*sec(d*
x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I
*(-csc(d*x+c)+cot(d*x+c)),I)+8*I*cos(d*x+c)-8*sin(d*x+c)+12*I*sec(d*x+c)-se
c(d*x+c)*tan(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.88

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{2\left(\sqrt{2}\left(4i\,a^4e^{(4i\,dx+4i\,c)}+21i\,a^4e^{(2i\,dx+2i\,c)}+15i\,a^4\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+15\sqrt{2}\left(-i\,a^4e^{(2i\,dx+2i\,c)}-i\,a^4\right.\right.}{3\left(de^2e^{(2i\,dx+2i\,c)}+de^2\right)}$$

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(2)*(4*I*a^4*e^(4*I*d*x + 4*I*c) + 21*I*a^4*e^(2*I*d*x + 2*I*c) +
 15*I*a^4)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 15*s
qrt(2)*(-I*a^4*e^(2*I*d*x + 2*I*c) - I*a^4)*sqrt(e)*weierstrassPInverse(-4,
 0, e^(I*d*x + I*c)))/(d*e^2*e^(2*I*d*x + 2*I*c) + d*e^2)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{3/2}}\,dx = a^4\left(\int \frac{1}{(e\sec(c+dx))^{\frac{3}{2}}}\,dx + \int \left(-\frac{6\tan^2(c+dx)}{(e\sec(c+dx))^{\frac{3}{2}}}\right)dx + \int \frac{\tan^4(c+dx)}{(e\sec(c+dx))^{\frac{3}{2}}}\,dx + \int \frac{4i\tan(c+dx)}{(e\sec(c+dx))^{\frac{3}{2}}}\,dx + \int \left(-\frac{4i\tan^3(c+dx)}{(e\sec(c+dx))^{\frac{3}{2}}}\right)dx\right)$$

```
[In] integrate((a+I*a*tan(d*x+c))**4/(e*sec(d*x+c))**(3/2),x)
```

```
[Out] a**4*(Integral((e*sec(c + d*x))**(-3/2), x) + Integral(-6*tan(c + d*x)**2/(
e*sec(c + d*x))**(3/2), x) + Integral(tan(c + d*x)**4/(e*sec(c + d*x))**(3/
2), x) + Integral(4*I*tan(c + d*x)/(e*sec(c + d*x))**(3/2), x) + Integral(-
4*I*tan(c + d*x)**3/(e*sec(c + d*x))**(3/2), x))
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^4}{(e\sec(dx+c))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^4}{(e\sec(dx+c))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^4}{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(3/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(3/2), x)
```

3.217 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 156

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{42a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{42a^4\sqrt{e\sec(c+dx)}\sin(c+dx)}{5de^3} - \frac{4ia(a+ia\tan(c+dx))^3}{5d(e\sec(c+dx))^{5/2}} + \frac{28i(a^4+ia^4\tan(c+dx))}{5de^2\sqrt{e\sec(c+dx)}}$$

```
[Out] -42/5*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d/e^2/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+42/5*a^4*s
in(d*x+c)*(e*sec(d*x+c))^(1/2)/d/e^3-4/5*I*a*(a+I*a*tan(d*x+c))^3/d/(e*sec(
d*x+c))^(5/2)+28/5*I*(a^4+I*a^4*tan(d*x+c))/d/e^2/(e*sec(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3577, 3853, 3856, 2719}

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{5/2}}\,dx = \frac{42a^4\sin(c+dx)\sqrt{e\sec(c+dx)}}{5de^3} + \frac{28i(a^4+ia^4\tan(c+dx))}{5de^2\sqrt{e\sec(c+dx)}} - \frac{42a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{4ia(a+ia\tan(c+dx))^3}{5d(e\sec(c+dx))^{5/2}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-42*a^4*EllipticE[(c + d*x)/2, 2])/(5*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) + (42*a^4*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/(5*d*e^3) - (((4*I)
/5)*a*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(5/2)) + (((28*I)/5)*(a
^4 + I*a^4*Tan[c + d*x]))/(d*e^2*Sqrt[e*Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{4ia(a+ia\tan(c+dx))^3}{5d(e\sec(c+dx))^{5/2}}-\frac{(7a^2)\int\frac{(a+ia\tan(c+dx))^2}{\sqrt{e\sec(c+dx)}}\,dx}{5e^2}\\
&=-\frac{4ia(a+ia\tan(c+dx))^3}{5d(e\sec(c+dx))^{5/2}}+\frac{28i(a^4+ia^4\tan(c+dx))}{5de^2\sqrt{e\sec(c+dx)}}+\frac{(21a^4)\int(e\sec(c+dx))^{3/2}\,dx}{5e^4}\\
&=\frac{42a^4\sqrt{e\sec(c+dx)}\sin(c+dx)}{5de^3}-\frac{4ia(a+ia\tan(c+dx))^3}{5d(e\sec(c+dx))^{5/2}}\\
&\quad+\frac{28i(a^4+ia^4\tan(c+dx))}{5de^2\sqrt{e\sec(c+dx)}}-\frac{(21a^4)\int\frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{5e^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{42a^4\sqrt{e\sec(c+dx)}\sin(c+dx)}{5de^3} - \frac{4ia(a+ia\tan(c+dx))^3}{5d(e\sec(c+dx))^{5/2}} \\
&\quad + \frac{28i(a^4+ia^4\tan(c+dx))}{5de^2\sqrt{e\sec(c+dx)}} - \frac{(21a^4)\int\sqrt{\cos(c+dx)}\,dx}{5e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} \\
&= -\frac{42a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{42a^4\sqrt{e\sec(c+dx)}\sin(c+dx)}{5de^3} \\
&\quad - \frac{4ia(a+ia\tan(c+dx))^3}{5d(e\sec(c+dx))^{5/2}} + \frac{28i(a^4+ia^4\tan(c+dx))}{5de^2\sqrt{e\sec(c+dx)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.01 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.71

$$
\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{4ia^4e^{2i(c+dx)}\left(7+2e^{2i(c+dx)}-7\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)\right)}{5de^2\left(1+e^{2i(c+dx)}\right)\sqrt{e\sec(c+dx)}}
$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(5/2),x]
```

```
[Out] (((-4*I)/5)*a^4*E^((2*I)*(c + d*x))*(7 + 2*E^((2*I)*(c + d*x)) - 7*Sqrt[1 +
 E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))
]))/(d*e^2*(1 + E^((2*I)*(c + d*x)))*Sqrt[e*Sec[c + d*x]])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1720 vs. 2(160) = 320.

Time = 22.49 (sec) , antiderivative size = 1721, normalized size of antiderivative = 11.03

method	result	size
parts	Expression too large to display	1721
default	Expression too large to display	1949

```
[In] int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*a^4/d/(cos(d*x+c)+1)/(e*sec(d*x+c))^(1/2)/e^2*(3*I*cos(d*x+c)*Elliptic
F(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)-3*I*cos(d*x+c)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+6*I*(cos(d*x+c)/(cos(d
```

*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(
1/2)-6*I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Ellipti
cE(I*(csc(d*x+c)-cot(d*x+c)),I)+3*I*sec(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)-3*I*s
ec(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Ellipt
icE(I*(csc(d*x+c)-cot(d*x+c)),I)-cos(d*x+c)^2*sin(d*x+c)-sin(d*x+c)*cos(d*x
+c)-3*sin(d*x+c))+2/5*a^4/d/(cos(d*x+c)+1)/(e*sec(d*x+c))^(1/2)/e^2*(-12*I*
cos(d*x+c)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*
(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+12*I*cos(d*x+c)*EllipticF(I*(csc(d*x+c)-c
ot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-24
I(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(
csc(d*x+c)-cot(d*x+c)),I)+24*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(
I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)+cos(d*x+c)^2*sin(d*x+
c)-12*I*sec(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)+12*I*sec(d*x+c)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^
(1/2)+sin(d*x+c)*cos(d*x+c)-7*sin(d*x+c)+5*tan(d*x+c))+2/5*I*a^4/d/(e*sec(d
x+c))^(1/2)/e^2(5*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*ln((2*cos(d*x+c)*(
-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)-
cos(d*x+c)+1)/(cos(d*x+c)+1))-5*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*ln(2*(
2*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*(-cos(d*x+c)/(cos(d*x+c
)+1)^2)^(1/2)-cos(d*x+c)+1)/(cos(d*x+c)+1))-4*cos(d*x+c)^2+5*sec(d*x+c)*(-c
os(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*ln((2*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)
+1)^2)^(1/2)+2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)-cos(d*x+c)+1)/(cos(d*x+
c)+1))-5*sec(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)*ln(2*(2*cos(d*x+c)
*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2)+2*(-cos(d*x+c)/(cos(d*x+c)+1)^2)^(1/2
)-cos(d*x+c)+1)/(cos(d*x+c)+1))+20)-8/5*I*a^4/d/(e*sec(d*x+c))^(5/2)+12/5*a
^4/d/(cos(d*x+c)+1)/(e*sec(d*x+c))^(1/2)/e^2*(-2*I*EllipticE(I*(csc(d*x+c)-
cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*c
os(d*x+c)+2*I*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/
2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-4*I*EllipticE(I*(csc(d*x+c)
-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+
4*I*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)+cos(d*x+c)^2*sin(d*x+c)-2*I*sec(d*x+c)*(1/(cos(d
x+c)+1))^(1/2)(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(csc(d*x+c)-c
ot(d*x+c)),I)+2*I*sec(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*
(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)+sin(d*x+c)*cos(d*x+c)-2
*sin(d*x+c))

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.60

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{5/2}}\,dx =$$
$$-\frac{2\left(21i\sqrt{2}a^4\sqrt{e}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\sqrt{2}\left(2i\,a^4e^{(3i\,dx+3i\,c)}+7i\,a^4\right.\right.}{5\,de^3}$$

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/5*(21*I*sqrt(2)*a^4*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, e^(I*d*x + I*c))) + sqrt(2)*(2*I*a^4*e^(3*I*d*x + 3*I*c) + 7*I*a^4*e^
(I*d*x + I*c))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/(
d*e^3)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{5/2}}\,dx = a^4\left(\int \frac{1}{(e\sec{(c+dx)})^{\frac{5}{2}}}\,dx + \int \left(-\frac{6\tan^2{(c+dx)}}{(e\sec{(c+dx)})^{\frac{5}{2}}}\right)dx + \int \frac{\tan^4{(c+dx)}}{(e\sec{(c+dx)})^{\frac{5}{2}}}\,dx + \int \frac{4i\tan{(c+dx)}}{(e\sec{(c+dx)})^{\frac{5}{2}}}\,dx + \int \left(-\frac{4i\tan^3{(c+dx)}}{(e\sec{(c+dx)})^{\frac{5}{2}}}\right)dx\right)$$

```
[In] integrate((a+I*a*tan(d*x+c))**4/(e*sec(d*x+c))**(5/2),x)
```

```
[Out] a**4*(Integral((e*sec(c + d*x))**(-5/2), x) + Integral(-6*tan(c + d*x)**2/(
e*sec(c + d*x))**(5/2), x) + Integral(tan(c + d*x)**4/(e*sec(c + d*x))**(5/
2), x) + Integral(4*I*tan(c + d*x)/(e*sec(c + d*x))**(5/2), x) + Integral(-
4*I*tan(c + d*x)**3/(e*sec(c + d*x))**(5/2), x))
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{(i\,a\tan{(dx+c)}+a)^4}{(e\sec{(dx+c)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{(i\,a\tan\left(dx+c\right)+a)^4}{(e\sec\left(dx+c\right))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^4}{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(5/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(5/2), x)
```

3.218 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 125

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{7/2}}\,dx = \frac{10a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{21de^4} - \frac{4ia(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}} + \frac{20i(a^4+ia^4\tan(c+dx))}{21de^2(e\sec(c+dx))^{3/2}}$$

```
[Out] 10/21*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/d/e^4-4/7*I*a*(a
+I*a*tan(d*x+c))^3/d/(e*sec(d*x+c))^(7/2)+20/21*I*(a^4+I*a^4*tan(d*x+c))/d/
e^2/(e*sec(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3577, 3856, 2720}

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{7/2}}\,dx = \frac{10a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{21de^4} + \frac{20i(a^4+ia^4\tan(c+dx))}{21de^2(e\sec(c+dx))^{3/2}} - \frac{4ia(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(7/2),x]
```

```
[Out] (10*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(21*d*e^4) - (((4*I)/7)*a*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(7/
```

```
2)) + (((20*I)/21)*(a^4 + I*a^4*Tan[c + d*x]))/(d*e^2*(e*Sec[c + d*x])^(3/2
))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{4ia(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}} - \frac{(5a^2)\int \frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{3/2}}\,dx}{7e^2} \\
&= -\frac{4ia(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}} + \frac{20i(a^4+ia^4\tan(c+dx))}{21de^2(e\sec(c+dx))^{3/2}} + \frac{(5a^4)\int \sqrt{e\sec(c+dx)}\,dx}{21e^4} \\
&= -\frac{4ia(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}} + \frac{20i(a^4+ia^4\tan(c+dx))}{21de^2(e\sec(c+dx))^{3/2}} \\
&\quad + \frac{\left(5a^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21e^4} \\
&= \frac{10a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{21de^4} \\
&\quad - \frac{4ia(a+ia\tan(c+dx))^3}{7d(e\sec(c+dx))^{7/2}} + \frac{20i(a^4+ia^4\tan(c+dx))}{21de^2(e\sec(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.71 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.06

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{7/2}}\,dx = \frac{2a^4\sqrt{e\sec(c+dx)}\Big(2i+2i\cos(2(c+dx))+5\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\tfrac{1}{2}(c+\right.}{21}$$

[In] Integrate[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(7/2),x]

[Out] (2*a^4*Sqrt[e*Sec[c + d*x]]*(2*I + (2*I)*Cos[2*(c + d*x)] + 5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)]) + 8*Sin[2*(c + d*x)])*(Cos[2*(c + 3*d*x)] + I*Sin[2*(c + 3*d*x)]))/(21*d*e^4*(Cos[d*x] + I*Sin[d*x])^4)

Maple [A] (verified)

Time = 14.52 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.46

method	result
default	$\frac{2a^4\left(5iF(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}-24i(\cos^3(dx+c))+5i\sec(dx+c)F(i(-\csc(dx+c)+\cot(dx+c))\right.}{21e^3d\sqrt{e\sec(dx+c)}}$
risch	$-\frac{2ie^{i(dx+c)}\left(3e^{2i(dx+c)}-5\right)a^4\sqrt{2}}{21de^3\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}+\frac{10\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{i\left(e^{i(dx+c)}-i\right)}\sqrt{ie^{i(dx+c)}}\,F\left(\sqrt{-i\left(e^{i(dx+c)}+i\right)},\frac{\sqrt{2}}{2}\right)a^4\sqrt{e\,e^{i(dx+c)}\left(e\right.}}{21d\sqrt{e\,e^{3i(dx+c)}+e\,e^{i(dx+c)}}\,e^3\left(e^{2i(dx+c)}+1\right)\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}$
parts	$-\frac{2a^4\left(5iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}+5i\sec(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\right.}}{21d\sqrt{e\sec(dx+c)}\,e^3}$

[In] int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)

[Out] 2/21*a^4/e^3/d/(e*sec(d*x+c))^(1/2)*(5*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)-24*I*cos(d*x+c)^3+5*I*sec(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)+24*cos(d*x+c)^2*sin(d*x+c)+28*I*cos(d*x+c)-16*sin(d*x+c))

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.77

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{7/2}}\,dx = -\frac{2\left(5i\sqrt{2}a^4\sqrt{e}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(3i\,a^4e^{(4i\,dx+4i\,c)}-2i\,a^4e^{(2i\,dx+2i\,c)}-5i\,a^4\right)\sqrt{\frac{e}{e^{(2i\,dx+}}}}{21\,de^4}$$

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -2/21*(5*I*sqrt(2)*a^4*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c))
+ sqrt(2)*(3*I*a^4*e^(4*I*d*x + 4*I*c) - 2*I*a^4*e^(2*I*d*x + 2*I*c) - 5*I*
a^4)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*e^4)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{7/2}}\,dx = a^4\left(\int \frac{1}{(e\sec(c+dx))^{\frac{7}{2}}}\,dx+\int \left(-\frac{6\tan^2(c+dx)}{(e\sec(c+dx))^{\frac{7}{2}}}\right)dx + \int \frac{\tan^4(c+dx)}{(e\sec(c+dx))^{\frac{7}{2}}}\,dx+\int \frac{4i\tan(c+dx)}{(e\sec(c+dx))^{\frac{7}{2}}}\,dx+\int \left(-\frac{4i\tan^3(c+dx)}{(e\sec(c+dx))^{\frac{7}{2}}}\right)dx\right)$$

```
[In] integrate((a+I*a*tan(d*x+c))**4/(e*sec(d*x+c))**(7/2),x)
```

```
[Out] a**4*(Integral((e*sec(c + d*x))**(-7/2), x) + Integral(-6*tan(c + d*x)**2/(
e*sec(c + d*x))**(7/2), x) + Integral(tan(c + d*x)**4/(e*sec(c + d*x))**(7/
2), x) + Integral(4*I*tan(c + d*x)/(e*sec(c + d*x))**(7/2), x) + Integral(-
4*I*tan(c + d*x)**3/(e*sec(c + d*x))**(7/2), x))
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{7/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^4}{(e\sec(dx+c))^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(7/2), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{7/2}}\,dx=\int \frac{(i\,a\tan\left(dx+c\right)+a)^4}{(e\sec\left(dx+c\right))^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{7/2}}\,dx=\int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^4}{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(7/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(7/2), x)
```

3.219 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{9/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 125

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{9/2}}\,dx = -\frac{2a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15de^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{4ia(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}} + \frac{4i(a^4+ia^4\tan(c+dx))}{15de^2(e\sec(c+dx))^{5/2}}$$

```
[Out] -2/15*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d/e^4/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)-4/9*I*a*(a
+I*a*tan(d*x+c))^3/d/(e*sec(d*x+c))^(9/2)+4/15*I*(a^4+I*a^4*tan(d*x+c))/d/e
^2/(e*sec(d*x+c))^(5/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3577, 3856, 2719}

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{9/2}}\,dx = -\frac{2a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15de^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{4i(a^4+ia^4\tan(c+dx))}{15de^2(e\sec(c+dx))^{5/2}} - \frac{4ia(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(9/2),x]
```

```
[Out] (-2*a^4*EllipticE[(c + d*x)/2, 2])/(15*d*e^4*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) - (((4*I)/9)*a*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(9/
```

```
2)) + (((4*I)/15)*(a^4 + I*a^4*Tan[c + d*x]))/(d*e^2*(e*Sec[c + d*x])^(5/2)
)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{4ia(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}-\frac{a^2\int\frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{5/2}}\,dx}{3e^2}\\
&= -\frac{4ia(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}+\frac{4i(a^4+ia^4\tan(c+dx))}{15de^2(e\sec(c+dx))^{5/2}}-\frac{a^4\int\frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{15e^4}\\
&= -\frac{4ia(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}+\frac{4i(a^4+ia^4\tan(c+dx))}{15de^2(e\sec(c+dx))^{5/2}}-\frac{a^4\int\sqrt{\cos(c+dx)}\,dx}{15e^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\\
&= -\frac{2a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15de^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{4ia(a+ia\tan(c+dx))^3}{9d(e\sec(c+dx))^{9/2}}+\frac{4i(a^4+ia^4\tan(c+dx))}{15de^2(e\sec(c+dx))^{5/2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.18 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.86

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{9/2}}\,dx =$$
$$-\frac{ia^4e^{i(c+dx)}\left(2+7e^{2i(c+dx)}+5e^{4i(c+dx)}-2\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)\right)\sqrt{e\sec(c-}{45de^5}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(9/2),x]
```

```
[Out] ((-1/45*I)*a^4*E^(I*(c + d*x))*(2 + 7*E^((2*I)*(c + d*x)) + 5*E^((4*I)*(c +
 d*x)) - 2*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(c + d*x))])*Sqrt[e*Sec[c + d*x]])/(d*e^5)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 338 vs. 2(133) = 266.

Time = 25.45 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.71

method	result
risch	$-\frac{i\left(5\,e^{4i(dx+c)}+2\,e^{2i(dx+c)}-6\right)a^4\sqrt{2}}{45d\,e^4\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}+\frac{i\left(-\frac{2\left(e\,e^{2i(dx+c)}+e\right)}{e\sqrt{e^{i(dx+c)}\left(e\,e^{2i(dx+c)}+e\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(e^{i(dx+c)}-i\right)}\,\sqrt{ie^{i(dx+c)}}\,\left(-2iE\left(\sqrt{\cdot}\right.\right.}{\sqrt{e\,e^{3i(dx+c)}+e\,e}}\right.}{15d\,e^4\left(e^{2i(dx+c)}+1\right)\sqrt{\cdot}}$
default	$-\frac{2ia^4\left(40\left(\cos^5(dx+c)\right)+3\cos(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,E(i(\csc(dx+c)-\cot(dx+c)),i)-3F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\cdot}\right.}{}$
parts	Expression too large to display

```
[In] int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/45*I*(5*exp(I*(d*x+c))^4+2*exp(I*(d*x+c))^2-6)/d*a^4*2^(1/2)/e^4/(e*exp(
I*(d*x+c))/(exp(I*(d*x+c))^2+1))^(1/2)+1/15*I/d*(-2*(e*exp(I*(d*x+c))^2+e)/
e/(exp(I*(d*x+c))*(e*exp(I*(d*x+c))^2+e))^(1/2)+I*(-I*(exp(I*(d*x+c))+I))^(
1/2)*2^(1/2)*(I*(exp(I*(d*x+c))-I))^(1/2)*(I*exp(I*(d*x+c)))^(1/2)/(e*exp(I
*(d*x+c))^3+e*exp(I*(d*x+c)))^(1/2)*(-2*I*EllipticE((-I*(exp(I*(d*x+c))+I))
^(1/2),1/2*2^(1/2))+I*EllipticF((-I*(exp(I*(d*x+c))+I))^(1/2),1/2*2^(1/2)))
)*a^4*2^(1/2)/e^4/(exp(I*(d*x+c))^2+1)/(e*exp(I*(d*x+c))/(exp(I*(d*x+c))^2+
1))^(1/2)*(e*exp(I*(d*x+c))*(exp(I*(d*x+c))^2+1))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.86

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{9/2}}\,dx = \frac{-6i\,\sqrt{2}a^4\sqrt{e}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\cdots}{45\cdots}$$

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(9/2),x, algorithm="fricas")
```

```
[Out] 1/45*(-6*I*sqrt(2)*a^4*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, e^(I*d*x + I*c))) + sqrt(2)*(-5*I*a^4*e^(5*I*d*x + 5*I*c) - 7*I*a^4*e
^(3*I*d*x + 3*I*c) - 2*I*a^4*e^(I*d*x + I*c))*sqrt(e/(e^(2*I*d*x + 2*I*c) +
 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*e^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+I*a*tan(d*x+c))**4/(e*sec(d*x+c))**(9/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{9/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^4}{(e\sec(dx+c))^{\frac{9}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(9/2), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{9/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^4}{(e\sec(dx+c))^{\frac{9}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(9/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{9/2}}\,dx = \int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^4}{\left(\frac{e}{\cos(c+d\,x)}\right)^{9/2}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(9/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(9/2), x)
```

3.220 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{11/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 156

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{11/2}}\,dx = -\frac{2a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{77de^6} - \frac{2a^4\sin(c+dx)}{77de^5\sqrt{e\sec(c+dx)}} - \frac{4ia(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}} + \frac{4i(a^4+ia^4\tan(c+dx))}{77de^2(e\sec(c+dx))^{7/2}}$$

```
[Out] -2/77*a^4*sin(d*x+c)/d/e^5/(e*sec(d*x+c))^(1/2)-2/77*a^4*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x
+c)^(1/2)*(e*sec(d*x+c))^(1/2)/d/e^6-4/11*I*a*(a+I*a*tan(d*x+c))^3/d/(e*sec
(d*x+c))^(11/2)+4/77*I*(a^4+I*a^4*tan(d*x+c))/d/e^2/(e*sec(d*x+c))^(7/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3577, 3854, 3856, 2720}

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{11/2}}\,dx = -\frac{2a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{77de^6} - \frac{2a^4\sin(c+dx)}{77de^5\sqrt{e\sec(c+dx)}} + \frac{4i(a^4+ia^4\tan(c+dx))}{77de^2(e\sec(c+dx))^{7/2}} - \frac{4ia(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(11/2),x]
```

```
[Out] (-2*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(77*d*e^6) - (2*a^4*Sin[c + d*x])/(77*d*e^5*Sqrt[e*Sec[c + d*x]]) - (((4*I)
/11)*a*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(11/2)) + (((4*I)/77)*
(a^4 + I*a^4*Tan[c + d*x]))/(d*e^2*(e*Sec[c + d*x])^(7/2))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{4ia(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}-\frac{a^2\int\frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{7/2}}\,dx}{11e^2}\\
&= -\frac{4ia(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}+\frac{4i(a^4+ia^4\tan(c+dx))}{77de^2(e\sec(c+dx))^{7/2}}-\frac{(3a^4)\int\frac{1}{(e\sec(c+dx))^{3/2}}\,dx}{77e^4}\\
&= -\frac{2a^4\sin(c+dx)}{77de^5\sqrt{e\sec(c+dx)}}-\frac{4ia(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}}\\
&\quad+\frac{4i(a^4+ia^4\tan(c+dx))}{77de^2(e\sec(c+dx))^{7/2}}-\frac{a^4\int\sqrt{e\sec(c+dx)}\,dx}{77e^6}
\end{aligned}$$

$$= -\frac{2a^4\sin(c+dx)}{77de^5\sqrt{e\sec(c+dx)}} - \frac{4ia(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}} + \frac{4i(a^4+ia^4\tan(c+dx))}{77de^2(e\sec(c+dx))^{7/2}} - \frac{\left(a^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{77e^6}$$

$$= -\frac{2a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{77de^6} - \frac{2a^4\sin(c+dx)}{77de^5\sqrt{e\sec(c+dx)}} - \frac{4ia(a+ia\tan(c+dx))^3}{11d(e\sec(c+dx))^{11/2}} + \frac{4i(a^4+ia^4\tan(c+dx))}{77de^2(e\sec(c+dx))^{7/2}}$$

Mathematica [A] (verified)

Time = 2.50 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.95

$$\int\frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{11/2}}\,dx = -\frac{a^4\sqrt{e\sec(c+dx)}\left(37i\cos(c+dx)+11i\cos(3(c+dx))-3\sin(c+dx)+4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+\right.\right.}{154de^6(\cos(dx)+i}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(11/2),x]
```

```
[Out] -1/154*(a^4*Sqrt[e*Sec[c + d*x]]*((37*I)*Cos[c + d*x] + (11*I)*Cos[3*(c + d
*x)] - 3*Sin[c + d*x] + 4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos
[3*(c + d*x)] - I*Sin[3*(c + d*x)]) - 3*Sin[3*(c + d*x)])*(Cos[3*c + 7*d*x]
 + I*Sin[3*c + 7*d*x]))/(d*e^6*(Cos[d*x] + I*Sin[d*x])^4)
```

Maple [A] (verified)

Time = 22.54 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.29

method	result
default	$\frac{2a^4\left(-56i(\cos^5(dx+c))+56\sin(dx+c)(\cos^4(dx+c))+iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}+44i(\cos^3(dx+c)\right.}{77e^5d\sqrt{e\sec(dx+}}$
risch	$-\frac{ie^{i(dx+c)}(7\,e^{4i(dx+c)}+13\,e^{2i(dx+c)}+4)a^4\sqrt{2}}{154d\,e^5\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}} - \frac{2\sqrt{-i(e^{i(dx+c)}+i)}\sqrt{i(e^{i(dx+c)}-i)}\sqrt{ie^{i(dx+c)}}F\left(\sqrt{-i(e^{i(dx+c)}+i)},\frac{\sqrt{2}}{2}\right)a^4\sqrt{e}}{77d\sqrt{e\,e^{3i(dx+c)}+e\,e^{i(dx+c)}}\,e^5\,(e^{2i(dx+c)}+1)\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+}}}$
parts	$-\frac{2a^4\left(-7\sin(dx+c)(\cos^4(dx+c))+15iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}+15i\sec(dx+c)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}F\right.}{77d\sqrt{e\sec(dx+c)}\,e^5}$

```
[In] int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/77*a^4/e^5/d/(e*sec(d*x+c))^(1/2)*(-56*I*cos(d*x+c)^5+56*sin(d*x+c)*cos(d
*x+c)^4+I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x
```

```
+c)),I)*(1/(cos(d*x+c)+1))^(1/2)+44*I*cos(d*x+c)^3+I*sec(d*x+c)*(cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c
)+1))^(1/2)-16*cos(d*x+c)^2*sin(d*x+c)-sin(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.71

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{11/2}}\,dx = \frac{4i\,\sqrt{2}a^4\sqrt{e}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(-7i\,a^4e^{(6i\,dx+6i\,c)}-20i\,a\right.}{154\,de^6}$$

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(11/2),x, algorithm="fricas")
```

```
[Out] 1/154*(4*I*sqrt(2)*a^4*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c))
+ sqrt(2)*(-7*I*a^4*e^(6*I*d*x + 6*I*c) - 20*I*a^4*e^(4*I*d*x + 4*I*c) - 17
*I*a^4*e^(2*I*d*x + 2*I*c) - 4*I*a^4)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(
1/2*I*d*x + 1/2*I*c))/(d*e^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{11/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+I*a*tan(d*x+c))**4/(e*sec(d*x+c))**(11/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{11/2}}\,dx = \int \frac{(i\,a\tan{(dx+c)}+a)^4}{(e\sec{(dx+c)})^{\frac{11}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(11/2), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{11/2}}\,dx = \int \frac{(i\,a\tan\left(dx+c\right)+a)^4}{(e\sec\left(dx+c\right))^{\frac{11}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(11/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{11/2}}\,dx = \int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^4}{\left(\frac{e}{\cos(c+d\,x)}\right)^{11/2}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(11/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(11/2), x)
```

3.221 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{13/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 156

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{13/2}}\,dx = \frac{2a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39de^6\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2a^4\sin(c+dx)}{117de^5(e\sec(c+dx))^{3/2}} - \frac{4ia(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}} - \frac{4i(a^4+ia^4\tan(c+dx))}{117de^2(e\sec(c+dx))^{9/2}}$$

```
[Out] 2/117*a^4*sin(d*x+c)/d/e^5/(e*sec(d*x+c))^(3/2)+2/39*a^4*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/e^6/c
os(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)-4/13*I*a*(a+I*a*tan(d*x+c))^3/d/(e*sec
(d*x+c))^(13/2)-4/117*I*(a^4+I*a^4*tan(d*x+c))/d/e^2/(e*sec(d*x+c))^(9/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3577, 3854, 3856, 2719}

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{13/2}}\,dx = \frac{2a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39de^6\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2a^4\sin(c+dx)}{117de^5(e\sec(c+dx))^{3/2}} - \frac{4i(a^4+ia^4\tan(c+dx))}{117de^2(e\sec(c+dx))^{9/2}} - \frac{4ia(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(13/2),x]
```

```
[Out] (2*a^4*EllipticE[(c + d*x)/2, 2])/(39*d*e^6*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c
 + d*x]]) + (2*a^4*Sin[c + d*x])/(117*d*e^5*(e*Sec[c + d*x])^(3/2)) - (((4*
```

```
I)/13)*a*(a + I*a*Tan[c + d*x])^3)/(d*(e*Sec[c + d*x])^(13/2)) - (((4*I)/11
7)*(a^4 + I*a^4*Tan[c + d*x]))/(d*e^2*(e*Sec[c + d*x])^(9/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{4ia(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}+\frac{a^2\int\frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{9/2}}\,dx}{13e^2}\\
&= -\frac{4ia(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}-\frac{4i(a^4+ia^4\tan(c+dx))}{117de^2(e\sec(c+dx))^{9/2}}+\frac{(5a^4)\int\frac{1}{(e\sec(c+dx))^{5/2}}\,dx}{117e^4}\\
&= \frac{2a^4\sin(c+dx)}{117de^5(e\sec(c+dx))^{3/2}}-\frac{4ia(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}}\\
&\quad -\frac{4i(a^4+ia^4\tan(c+dx))}{117de^2(e\sec(c+dx))^{9/2}}+\frac{a^4\int\frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{39e^6}
\end{aligned}$$

$$= \frac{2a^4 \sin(c+dx)}{117de^5(e\sec(c+dx))^{3/2}} - \frac{4ia(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}} - \frac{4i(a^4+ia^4\tan(c+dx))}{117de^2(e\sec(c+dx))^{9/2}} + \frac{a^4 \int \sqrt{\cos(c+dx)}\,dx}{39e^6\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$= \frac{2a^4 E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39de^6\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2a^4\sin(c+dx)}{117de^5(e\sec(c+dx))^{3/2}} - \frac{4ia(a+ia\tan(c+dx))^3}{13d(e\sec(c+dx))^{13/2}} - \frac{4i(a^4+ia^4\tan(c+dx))}{117de^2(e\sec(c+dx))^{9/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.21 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.78

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{13/2}}\,dx = -\frac{ia^4e^{i(c+dx)}\left(31+59e^{2i(c+dx)}+37e^{4i(c+dx)}+9e^{6i(c+dx)}+8\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+\cdots}\right.\right.}{468de^7}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(13/2),x]
```

```
[Out] ((-1/468*I)*a^4*E^(I*(c + d*x))*(31 + 59*E^((2*I)*(c + d*x)) + 37*E^((4*I)*
(c + d*x)) + 9*E^((6*I)*(c + d*x)) + 8*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyperg
eometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sqrt[e*Sec[c + d*x]])/(d*
e^7)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 351 vs. 2(160) = 320.

Time = 26.29 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.26

method	result
risch	$-\frac{i(9\,e^{6i(dx+c)}+28\,e^{4i(dx+c)}+31\,e^{2i(dx+c)}+24)a^4\sqrt{2}}{468d\,e^6\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}} - \frac{i\left(-\frac{2\left(e\,e^{2i(dx+c)}+e\right)}{e\sqrt{e^{i(dx+c)}\left(e\,e^{2i(dx+c)}+e\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(dx+c)}-i\right)}\sqrt{ie^{i(\cdots}}}{\cdots}\right.}{39d\,e^{\cdots}}$
default	$-\frac{2ia^4\left(72(\cos^7(dx+c))+72(\cos^6(dx+c))+i(\cos^2(dx+c))\sin(dx+c)-52(\cos^5(dx+c))-16i\sin(dx+c)(\cos^4(dx+c))+3\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\cdots\right.}{\cdots}$
parts	Expression too large to display

```
[In] int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(13/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/468*I*(9*exp(I*(d*x+c))^6+28*exp(I*(d*x+c))^4+31*exp(I*(d*x+c))^2+24)/d*
a^4*2^(1/2)/e^6/(e*exp(I*(d*x+c))/(exp(I*(d*x+c))^2+1))^(1/2)-1/39*I/d*(-2*
(e*exp(I*(d*x+c))^2+e)/e/(exp(I*(d*x+c))*(e*exp(I*(d*x+c))^2+e))^(1/2)+I*(-
I*(exp(I*(d*x+c))+I))^(1/2)*2^(1/2)*(I*(exp(I*(d*x+c))-I))^(1/2)*(I*exp(I*(
d*x+c)))^(1/2)/(e*exp(I*(d*x+c))^3+e*exp(I*(d*x+c)))^(1/2)*(-2*I*EllipticE(
(-I*(exp(I*(d*x+c))+I))^(1/2),1/2*2^(1/2))+I*EllipticF((-I*(exp(I*(d*x+c))+
I))^(1/2),1/2*2^(1/2))))*a^4*2^(1/2)/e^6/(exp(I*(d*x+c))^2+1)/(e*exp(I*(d*x
+c))/(exp(I*(d*x+c))^2+1))^(1/2)*(e*exp(I*(d*x+c))*(exp(I*(d*x+c))^2+1))^(1
/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.78

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{13/2}}\,dx = \frac{24i\sqrt{2}a^4\sqrt{e}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\sqrt{}}{}$$

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(13/2),x, algorithm="fricas")
```

```
[Out] 1/468*(24*I*sqrt(2)*a^4*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(-9*I*a^4*e^(7*I*d*x + 7*I*c) - 37*I*a^4
*e^(5*I*d*x + 5*I*c) - 59*I*a^4*e^(3*I*d*x + 3*I*c) - 31*I*a^4*e^(I*d*x + I
*c))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*e^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{13/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+I*a*tan(d*x+c))**4/(e*sec(d*x+c))**(13/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{13/2}}\,dx = \int \frac{(i\,a\tan\left(dx+c\right)+a)^4}{(e\sec\left(dx+c\right))^{\frac{13}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(13/2),x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(13/2), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{13/2}}\,dx = \int \frac{(i\,a\tan\left(dx+c\right)+a)^4}{(e\sec\left(dx+c\right))^{\frac{13}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(13/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(13/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{13/2}}\,dx = \int \frac{{\left(a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^4}{{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{13/2}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(13/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(13/2), x)
```

3.222 $\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{15/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 187

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{15/2}}\,dx = \frac{2a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{33de^8} + \frac{2a^4\sin(c+dx)}{55de^5(e\sec(c+dx))^{5/2}} + \frac{2a^4\sin(c+dx)}{33de^7\sqrt{e\sec(c+dx)}} - \frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}} - \frac{4i(a^4+ia^4\tan(c+dx))}{55de^2(e\sec(c+dx))^{11/2}}$$

```
[Out] 2/55*a^4*sin(d*x+c)/d/e^5/(e*sec(d*x+c))^(5/2)+2/33*a^4*sin(d*x+c)/d/e^7/(e
*sec(d*x+c))^(1/2)+2/33*a^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2
)/d/e^8-4/15*I*a*(a+I*a*tan(d*x+c))^3/d/(e*sec(d*x+c))^(15/2)-4/55*I*(a^4+I
*a^4*tan(d*x+c))/d/e^2/(e*sec(d*x+c))^(11/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3577, 3854, 3856, 2720}

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{15/2}}\,dx = \frac{2a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{33de^8} + \frac{2a^4\sin(c+dx)}{33de^7\sqrt{e\sec(c+dx)}} + \frac{2a^4\sin(c+dx)}{55de^5(e\sec(c+dx))^{5/2}} - \frac{4i(a^4+ia^4\tan(c+dx))}{55de^2(e\sec(c+dx))^{11/2}} - \frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(15/2),x]
```

```
[Out] (2*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(
33*d*e^8) + (2*a^4*Sin[c + d*x])/(55*d*e^5*(e*Sec[c + d*x])^(5/2)) + (2*a^4
*Sin[c + d*x])/(33*d*e^7*Sqrt[e*Sec[c + d*x]]) - (((4*I)/15)*a*(a + I*a*Tan
[c + d*x])^3)/(d*(e*Sec[c + d*x])^(15/2)) - (((4*I)/55)*(a^4 + I*a^4*Tan[c
+ d*x]))/(d*e^2*(e*Sec[c + d*x])^(11/2))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}+\frac{a^2\int\frac{(a+ia\tan(c+dx))^2}{(e\sec(c+dx))^{11/2}}\,dx}{5e^2}\\
&= -\frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}}-\frac{4i(a^4+ia^4\tan(c+dx))}{55de^2(e\sec(c+dx))^{11/2}}+\frac{(7a^4)\int\frac{1}{(e\sec(c+dx))^{7/2}}\,dx}{55e^4}
\end{aligned}$$

$$= \frac{2a^4\sin(c+dx)}{55de^5(e\sec(c+dx))^{5/2}} - \frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}} - \frac{4i(a^4+ia^4\tan(c+dx))}{55de^2(e\sec(c+dx))^{11/2}} + \frac{a^4\int\frac{1}{(e\sec(c+dx))^{3/2}}\,dx}{11e^6}$$

$$= \frac{2a^4\sin(c+dx)}{55de^5(e\sec(c+dx))^{5/2}} + \frac{2a^4\sin(c+dx)}{33de^7\sqrt{e\sec(c+dx)}} - \frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}} - \frac{4i(a^4+ia^4\tan(c+dx))}{55de^2(e\sec(c+dx))^{11/2}} + \frac{a^4\int\sqrt{e\sec(c+dx)}\,dx}{33e^8}$$

$$= \frac{2a^4\sin(c+dx)}{55de^5(e\sec(c+dx))^{5/2}} + \frac{2a^4\sin(c+dx)}{33de^7\sqrt{e\sec(c+dx)}} - \frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}} - \frac{4i(a^4+ia^4\tan(c+dx))}{55de^2(e\sec(c+dx))^{11/2}} + \frac{\left(a^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{33e^8}$$

$$= \frac{2a^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{33de^8} + \frac{2a^4\sin(c+dx)}{55de^5(e\sec(c+dx))^{5/2}} + \frac{2a^4\sin(c+dx)}{33de^7\sqrt{e\sec(c+dx)}} - \frac{4ia(a+ia\tan(c+dx))^3}{15d(e\sec(c+dx))^{15/2}} - \frac{4i(a^4+ia^4\tan(c+dx))}{55de^2(e\sec(c+dx))^{11/2}}$$

Mathematica [A] (verified)

Time = 3.19 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.83

$$\int\frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{15/2}}\,dx = -\frac{ia^4\sqrt{e\sec(c+dx)}\left(64+112\cos(2(c+dx))+48\cos(4(c+dx))-54i\sin(2(c+dx))-37i\sin(4(c+dx))\right.}{660de^8(c}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^4/(e*Sec[c + d*x])^(15/2),x]
```

```
[Out] ((-1/660*I)*a^4*Sqrt[e*Sec[c + d*x]]*(64 + 112*Cos[2*(c + d*x)] + 48*Cos[4*
(c + d*x)] - (54*I)*Sin[2*(c + d*x)] - (37*I)*Sin[4*(c + d*x)] + 40*Sqrt[Co
s[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(I*Cos[4*(c + d*x)] + Sin[4*(c + d*x)
]))*(Cos[4*(c + 2*d*x)] + I*Sin[4*(c + 2*d*x)]))/(d*e^8*(Cos[d*x] + I*Sin[d
*x])^4)
```

Maple [A] (verified)

Time = 33.74 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.16

method	result
default	$\frac{2a^4\left(-88i\left(\cos^7(dx+c)\right)+88\sin(dx+c)\left(\cos^6(dx+c)\right)+60i\left(\cos^5(dx+c)\right)-16\sin(dx+c)\left(\cos^4(dx+c)\right)+5iF(i(-\csc(dx+c)+\cot(dx+c)),}{165}$
risch	$-\frac{i e^{i(dx+c)}\left(11\,e^{6i(dx+c)}+47\,e^{4i(dx+c)}+81\,e^{2i(dx+c)}+85\right)a^4\sqrt{2}}{1320d\,e^7\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}+\frac{2\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{i\left(e^{i(dx+c)}-i\right)}\sqrt{ie^{i(dx+c)}}\,F\left(\sqrt{-i\left(e^{i(dx+c)}-\right.}\right.}{33d\sqrt{e\,e^{3i(dx+c)}+e\,e^{i(dx+c)}}\,e^7\left(e^{2i(dx+c)}+1\right.}$
parts	$-\frac{2a^4\left(-77\sin(dx+c)\left(\cos^6(dx+c)\right)-91\sin(dx+c)\left(\cos^4(dx+c)\right)+195iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}+19}{1155d\sqrt{e\sec(dx-}}$

```
[In] int((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(15/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/165*a^4/e^7/d/(e*sec(d*x+c))^(1/2)*(-88*I*cos(d*x+c)^7+88*sin(d*x+c)*cos(
d*x+c)^6+60*I*cos(d*x+c)^5-16*sin(d*x+c)*cos(d*x+c)^4+5*I*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))
^(1/2)+5*I*sec(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d
*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)+3*cos(d*x+c)^2*sin(d*x+c)+5*s
in(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.66

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{15/2}}\,dx = \frac{-80i\,\sqrt{2}a^4\sqrt{e}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(-11i\,a^4e^{(8i\,dx+8i\,c)}-5\right.}{}$$

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(15/2),x, algorithm="fricas")
```

```
[Out] 1/1320*(-80*I*sqrt(2)*a^4*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c
)) + sqrt(2)*(-11*I*a^4*e^(8*I*d*x + 8*I*c) - 58*I*a^4*e^(6*I*d*x + 6*I*c)
- 128*I*a^4*e^(4*I*d*x + 4*I*c) - 166*I*a^4*e^(2*I*d*x + 2*I*c) - 85*I*a^4)
*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*e^8)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{15/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+I*a*tan(d*x+c))**4/(e*sec(d*x+c))**(15/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{15/2}}\,dx = \int \frac{(i\,a\tan{(dx+c)}+a)^4}{(e\sec{(dx+c)})^{\frac{15}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(15/2),x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(15/2), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{15/2}}\,dx = \int \frac{(i\,a\tan{(dx+c)}+a)^4}{(e\sec{(dx+c)})^{\frac{15}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^4/(e*sec(d*x+c))^(15/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^4/(e*sec(d*x + c))^(15/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^4}{(e\sec(c+dx))^{15/2}}\,dx = \int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^4}{\left(\frac{e}{\cos(c+d\,x)}\right)^{15/2}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(15/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^4/(e/cos(c + d*x))^(15/2), x)
```

3.223 $\int \frac{(e\sec(c+dx))^{11/2}}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 136

$$\int \frac{(e\sec(c+dx))^{11/2}}{a+ia\tan(c+dx)}\,dx = -\frac{6e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad} + \frac{6e^5\sqrt{e\sec(c+dx)}\sin(c+dx)}{5ad} + \frac{2e^3(e\sec(c+dx))^{5/2}\sin(c+dx)}{5ad}$$

```
[Out] -2/7*I*e^2*(e*sec(d*x+c))^(7/2)/a/d+2/5*e^3*(e*sec(d*x+c))^(5/2)*sin(d*x+c)
/a/d-6/5*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2))/a/d/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+6/5*e^5*s
in(d*x+c)*(e*sec(d*x+c))^(1/2)/a/d
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3582, 3853, 3856, 2719}

$$\int \frac{(e\sec(c+dx))^{11/2}}{a+ia\tan(c+dx)}\,dx = -\frac{6e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{6e^5\sin(c+dx)\sqrt{e\sec(c+dx)}}{5ad} + \frac{2e^3\sin(c+dx)(e\sec(c+dx))^{5/2}}{5ad} - \frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad}$$

```
[In] Int[(e*Sec[c + d*x])^(11/2)/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (-6*e^6*EllipticE[(c + d*x)/2, 2])/(5*a*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c +
 d*x]]) - (((2*I)/7)*e^2*(e*Sec[c + d*x])^(7/2))/(a*d) + (6*e^5*Sqrt[e*Sec[
```

```
c + d*x]]*Sin[c + d*x])/(5*a*d) + (2*e^3*(e*Sec[c + d*x])^(5/2)*Sin[c + d*x
])/(5*a*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad} + \frac{e^2\int (e\sec(c+dx))^{7/2}\,dx}{a} \\
&= -\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad} + \frac{2e^3(e\sec(c+dx))^{5/2}\sin(c+dx)}{5ad} + \frac{(3e^4)\int (e\sec(c+dx))^{3/2}\,dx}{5a} \\
&= -\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad} + \frac{6e^5\sqrt{e\sec(c+dx)}\sin(c+dx)}{5ad} \\
&\quad + \frac{2e^3(e\sec(c+dx))^{5/2}\sin(c+dx)}{5ad} - \frac{(3e^6)\int \frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{5a} \\
&= -\frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad} + \frac{6e^5\sqrt{e\sec(c+dx)}\sin(c+dx)}{5ad} \\
&\quad + \frac{2e^3(e\sec(c+dx))^{5/2}\sin(c+dx)}{5ad} - \frac{(3e^6)\int \sqrt{\cos(c+dx)}\,dx}{5a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}
\end{aligned}
$$

$$= -\frac{6e^6 E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{2ie^2(e\sec(c+dx))^{7/2}}{7ad} + \frac{6e^5\sqrt{e\sec(c+dx)}\sin(c+dx)}{5ad} + \frac{2e^3(e\sec(c+dx))^{5/2}\sin(c+dx)}{5ad}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.23 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.94

$$\int \frac{(e\sec(c+dx))^{11/2}}{a+ia\tan(c+dx)}\,dx = \frac{e^4(e\sec(c+dx))^{3/2}\left(76+28\cos(2(c+dx))-7e^{-2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{5/2}\text{Hyperg}\right.}{}$$

```
[In] Integrate[(e*Sec[c + d*x])^(11/2)/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (e^4*(e*Sec[c + d*x])^(3/2)*(76 + 28*Cos[2*(c + d*x)] - (7*(1 + E^((2*I)*(c
 + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^(
(2*I)*(c + d*x)) + (7*I)*Sec[c + d*x]*Sin[3*(c + d*x)] - (13*I)*Tan[c + d*x
])*(-I + Tan[c + d*x]))/(70*a*d)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 442 vs. 2(143) = 286.

Time = 8.74 (sec) , antiderivative size = 443, normalized size of antiderivative = 3.26

method	result
default	$\frac{2e^5\sqrt{e\sec(dx+c)}\left(21iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}(\cos^2(dx+c))-21iE(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{}\right.}{}$

```
[In] int((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/35*e^5/a/d*(e*sec(d*x+c))^(1/2)/(cos(d*x+c)+1)*(21*I*EllipticF(I*(csc(d*x
+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/
2)*cos(d*x+c)^2-21*I*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2+42*I*cos(d*x+c)*El
lipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
cos(d*x+c)+1))^(1/2)-42*I*cos(d*x+c)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+21*I*EllipticF(
I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+
c)+1))^(1/2)-21*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(csc(d*x+c)
-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)+21*sin(d*x+c)+7*tan(d*x+c)-5*I*sec
(d*x+c)^2+7*sec(d*x+c)*tan(d*x+c)-5*I*sec(d*x+c)^3)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.51

$$\int \frac{(e\sec(c+dx))^{11/2}}{a+ia\tan(c+dx)}\,dx = -\frac{2\left(\sqrt{2}\left(21i\,e^5e^{(7i\,dx+7i\,c)}+77i\,e^5e^{(5i\,dx+5i\,c)}+103i\,e^5e^{(3i\,dx+3i\,c)}+7i\,e^5e^{(i\,dx+i\,c)}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+2\right.}{35\left(ade^{(6i\,dx+6i\,c)}\right.}$$

```
[In] integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -2/35*(sqrt(2)*(21*I*e^5*e^(7*I*d*x + 7*I*c) + 77*I*e^5*e^(5*I*d*x + 5*I*c)
 + 103*I*e^5*e^(3*I*d*x + 3*I*c) + 7*I*e^5*e^(I*d*x + I*c))*sqrt(e/(e^(2*I*
d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 21*sqrt(2)*(I*e^5*e^(6*I*d*x +
 6*I*c) + 3*I*e^5*e^(4*I*d*x + 4*I*c) + 3*I*e^5*e^(2*I*d*x + 2*I*c) + I*e^5
)*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x + I*c)
)))/(a*d*e^(6*I*d*x + 6*I*c) + 3*a*d*e^(4*I*d*x + 4*I*c) + 3*a*d*e^(2*I*d*x
 + 2*I*c) + a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{11/2}}{a+ia\tan(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((e*sec(d*x+c))**(11/2)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{11/2}}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{11/2}}{a+ia\tan(c+dx)}\,dx = \int \frac{(e\sec{(dx+c)})^{\frac{11}{2}}}{i\,a\tan{(dx+c)}+a}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(11/2)/(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{11/2}}{a+ia\tan(c+dx)}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{11/2}}{a+a\tan{(c+d\,x)}\,1\mathrm{i}}\,dx$$

```
[In] int((e/cos(c + d*x))^(11/2)/(a + a*tan(c + d*x)*1i),x)
[Out] int((e/cos(c + d*x))^(11/2)/(a + a*tan(c + d*x)*1i), x)
```

3.224 $\int \frac{(e\sec(c+dx))^{9/2}}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 105

$$\int \frac{(e\sec(c+dx))^{9/2}}{a+ia\tan(c+dx)}\,dx = \frac{2e^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3ad} - \frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad} + \frac{2e^3(e\sec(c+dx))^{3/2}\sin(c+dx)}{3ad}$$

```
[Out] -2/5*I*e^2*(e*sec(d*x+c))^(5/2)/a/d+2/3*e^3*(e*sec(d*x+c))^(3/2)*sin(d*x+c)
/a/d+2/3*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(
1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/a/d
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3582, 3853, 3856, 2720}

$$\int \frac{(e\sec(c+dx))^{9/2}}{a+ia\tan(c+dx)}\,dx = \frac{2e^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3ad} + \frac{2e^3\sin(c+dx)(e\sec(c+dx))^{3/2}}{3ad} - \frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad}$$

```
[In] Int[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (2*e^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(
3*a*d) - (((2*I)/5)*e^2*(e*Sec[c + d*x])^(5/2))/(a*d) + (2*e^3*(e*Sec[c + d
*x])^(3/2)*Sin[c + d*x])/(3*a*d)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad}+\frac{e^2\int(e\sec(c+dx))^{5/2}\,dx}{a}\\
&= -\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad}+\frac{2e^3(e\sec(c+dx))^{3/2}\sin(c+dx)}{3ad}+\frac{e^4\int\sqrt{e\sec(c+dx)}\,dx}{3a}\\
&= -\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad}+\frac{2e^3(e\sec(c+dx))^{3/2}\sin(c+dx)}{3ad}\\
&\quad+\frac{\left(e^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3a}\\
&= \frac{2e^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3ad}\\
&\quad-\frac{2ie^2(e\sec(c+dx))^{5/2}}{5ad}+\frac{2e^3(e\sec(c+dx))^{3/2}\sin(c+dx)}{3ad}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.58 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.59

$$\int \frac{(e\sec(c+dx))^{9/2}}{a+ia\tan(c+dx)}\,dx = \frac{e^2(e\sec(c+dx))^{5/2}\left(-6i+10\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+5\sin(2(c\right.}{15ad}$$

[In] Integrate[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x]),x]

[Out] (e^2*(e*Sec[c + d*x])^(5/2)*(-6*I + 10*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 5*Sin[2*(c + d*x)]))/(15*a*d)

Maple [A] (verified)

Time = 7.63 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.50

method	result
default	$\frac{2e^4\sqrt{e\sec(dx+c)}\left(5i\cos(dx+c)F(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}+5iF(i(-\csc(dx+c)+\cot(dx+c)),i)\right.}{15ad}$

[In] int((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 2/15*e^4/a/d*(e*sec(d*x+c))^(1/2)*(5*I*cos(d*x+c)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+5*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)+5*tan(d*x+c)-3*I*sec(d*x+c)^2)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.45

$$\int \frac{(e\sec(c+dx))^{9/2}}{a+ia\tan(c+dx)}\,dx =$$

$$-\frac{2\left(\sqrt{2}\left(5i\,e^4e^{(4i\,dx+4i\,c)}+12i\,e^4e^{(2i\,dx+2i\,c)}-5i\,e^4\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+5\sqrt{2}\left(i\,e^4e^{(4i\,dx+4i\,c)}+2i\,e^4e^{(2i\right.\right.}{15\left(ade^{(4i\,dx+4i\,c)}+2\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")

[Out] -2/15*(sqrt(2)*(5*I*e^4*e^(4*I*d*x + 4*I*c) + 12*I*e^4*e^(2*I*d*x + 2*I*c) - 5*I*e^4)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 5*sqrt(2)*(I*e^4*e^(4*I*d*x + 4*I*c) + 2*I*e^4*e^(2*I*d*x + 2*I*c) + I*e^4)*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))/(a*d*e^(4*I*d*x + 4*I*c) + 2*a*d*e^(2*I*d*x + 2*I*c) + a*d)

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{9/2}}{a+ia\tan(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((e*sec(d*x+c))**(9/2)/(a+I*a*tan(d*x+c)),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{9/2}}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{9/2}}{a+ia\tan(c+dx)}\,dx = \int \frac{(e\sec(dx+c))^{\frac{9}{2}}}{i\,a\tan(dx+c)+a}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(9/2)/(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{9/2}}{a+ia\tan(c+dx)}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{9/2}}{a+a\tan(c+d\,x)\,1\mathrm{i}}\,dx$$

```
[In] int((e/cos(c + d*x))^(9/2)/(a + a*tan(c + d*x)*1i),x)
[Out] int((e/cos(c + d*x))^(9/2)/(a + a*tan(c + d*x)*1i), x)
```

3.225 $\int \frac{(e\sec(c+dx))^{7/2}}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 101

$$\int \frac{(e\sec(c+dx))^{7/2}}{a+ia\tan(c+dx)}\,dx = -\frac{2e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad} + \frac{2e^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{ad}$$

```
[Out] -2/3*I*e^2*(e*sec(d*x+c))^(3/2)/a/d-2*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a/d/cos(d*x+c)^(1/2)/(
e*sec(d*x+c))^(1/2)+2*e^3*sin(d*x+c)*(e*sec(d*x+c))^(1/2)/a/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3582, 3853, 3856, 2719}

$$\int \frac{(e\sec(c+dx))^{7/2}}{a+ia\tan(c+dx)}\,dx = -\frac{2e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2e^3\sin(c+dx)\sqrt{e\sec(c+dx)}}{ad} - \frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}$$

```
[In] Int[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (-2*e^4*EllipticE[(c + d*x)/2, 2])/(a*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d
*x]]) - (((2*I)/3)*e^2*(e*Sec[c + d*x])^(3/2))/(a*d) + (2*e^3*Sqrt[e*Sec[c
+ d*x]]*Sin[c + d*x])/(a*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}+\frac{e^2\int(e\sec(c+dx))^{3/2}\,dx}{a}\\
&= -\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}+\frac{2e^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{ad}-\frac{e^4\int\frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{a}\\
&= -\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}+\frac{2e^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{ad}-\frac{e^4\int\sqrt{\cos(c+dx)}\,dx}{a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\\
&= -\frac{2e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}-\frac{2ie^2(e\sec(c+dx))^{3/2}}{3ad}+\frac{2e^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{ad}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.75 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.01

$$\int \frac{(e\sec(c+dx))^{7/2}}{a+ia\tan(c+dx)}\,dx = \frac{2ie^3\sqrt{e\sec(c+dx)}(\cos(c)+i\sin(c))(\cos(dx)+i\sin(dx))\left(-4+\sqrt{1+e^{2i(c+dx)}}\right.}{3ad}$$

```
[In] Integrate[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (((2*I)/3)*e^3*Sqrt[e*Sec[c + d*x]]*(Cos[c] + I*Sin[c])*(Cos[d*x] + I*Sin[d
*x])*(-4 + Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(c + d*x))] + I*Tan[c + d*x]))/(a*d)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 409 vs. 2(116) = 232.

Time = 6.54 (sec) , antiderivative size = 410, normalized size of antiderivative = 4.06

method	result
default	$\frac{2e^3\sqrt{e\sec(dx+c)}\left(3iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}\left(\cos^2(dx+c)\right)-3iE(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{}\right.}{}$

```
[In] int((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*e^3/a/d*(e*sec(d*x+c))^(1/2)/(cos(d*x+c)+1)*(3*I*EllipticF(I*(csc(d*x+c
)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)
*cos(d*x+c)^2-3*I*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2+6*I*cos(d*x+c)*Ellipt
icF(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(
d*x+c)+1))^(1/2)-6*I*cos(d*x+c)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+3*I*EllipticF(I*(csc
(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))
^(1/2)-3*I*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)-I+3*sin(d*x+c)-I*sec(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.22

$$\int \frac{(e \sec(c+dx))^{7/2}}{a+ia\tan(c+dx)}\,dx = -\frac{2\left(\sqrt{2}\left(3i\,e^3e^{(3i\,dx+3i\,c)}+5i\,e^3e^{(i\,dx+i\,c)}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+3\sqrt{2}\left(i\,e^3e^{(2i\,dx+2i\,c)}+i\,e^3\right)\sqrt{e}\text{weierstrassZ}\right.}{3\left(ade^{(2i\,dx+2i\,c)}+ad\right)}$$

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(2)*(3*I*e^3*e^(3*I*d*x + 3*I*c) + 5*I*e^3*e^(I*d*x + I*c))*sqrt(
e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 3*sqrt(2)*(I*e^3*e^(
2*I*d*x + 2*I*c) + I*e^3)*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, e^(I*d*x + I*c))))/(a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e \sec(c+dx))^{7/2}}{a+ia\tan(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e \sec(c+dx))^{7/2}}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{7/2}}{a+ia\tan(c+dx)}\,dx = \int \frac{(e\sec(dx+c))^{\frac{7}{2}}}{i\,a\tan(dx+c)+a}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(7/2)/(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/2}}{a+ia\tan(c+dx)}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}}{a+a\tan(c+d\,x)\ 1\mathrm{i}}\,dx$$

```
[In] int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i),x)
[Out] int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i), x)
```

3.226 $\int \frac{(e\sec(c+dx))^{5/2}}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 70

$$\int \frac{(e\sec(c+dx))^{5/2}}{a+ia\tan(c+dx)}\,dx = -\frac{2ie^2\sqrt{e\sec(c+dx)}}{ad} + \frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{ad}$$

```
[Out] -2*I*e^2*(e*sec(d*x+c))^(1/2)/a/d+2*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(
d*x+c))^(1/2)/a/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3582, 3856, 2720}

$$\int \frac{(e\sec(c+dx))^{5/2}}{a+ia\tan(c+dx)}\,dx = \frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{ad} - \frac{2ie^2\sqrt{e\sec(c+dx)}}{ad}$$

```
[In] Int[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((-2*I)*e^2*Sqrt[e*Sec[c + d*x]])/(a*d) + (2*e^2*Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(a*d)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2ie^2\sqrt{e\sec(c+dx)}}{ad}+\frac{e^2\int\sqrt{e\sec(c+dx)}\,dx}{a}\\
&= -\frac{2ie^2\sqrt{e\sec(c+dx)}}{ad}+\frac{\left(e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{a}\\
&= -\frac{2ie^2\sqrt{e\sec(c+dx)}}{ad}+\frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{ad}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.36 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.70

$$
\int\frac{(e\sec(c+dx))^{5/2}}{a+ia\tan(c+dx)}\,dx=\frac{2e^2\left(-i+\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\right)\sqrt{e\sec(c+dx)}}{ad}
$$

```
[In] Integrate[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x]),x]
[Out] (2*e^2*(-I + Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])*Sqrt[e*Sec[c + d
*x]])/(a*d)
```

Maple [A] (verified)

Time = 6.63 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.96

method	result
default	$-\frac{2ie^2\left(F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\cos(dx+c)+F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(}{\cos(d}}$ ad

[In] int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)

[Out] -2*I*e^2/a/d*(EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+1)*(e*sec(d*x+c))^(1/2)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

$$\int \frac{(e\sec(c+dx))^{5/2}}{a+ia\tan(c+dx)}\,dx = -\frac{2\left(i\sqrt{2}e^2\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+i\sqrt{2}e^{\frac{5}{2}}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)}{ad}$$

[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")

[Out] -2*(I*sqrt(2)*e^2*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + I*sqrt(2)*e^(5/2)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))/(a*d)

Sympy [F]

$$\int \frac{(e\sec(c+dx))^{5/2}}{a+ia\tan(c+dx)}\,dx = -\frac{i\int\frac{(e\sec{(c+dx)})^{\frac{5}{2}}}{\tan{(c+dx)}-i}\,dx}{a}$$

[In] integrate((e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c)),x)

[Out] -I*Integral((e*sec(c + d*x))**(5/2)/(tan(c + d*x) - I), x)/a

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{5/2}}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{5/2}}{a+ia\tan(c+dx)}\,dx = \int \frac{(e\sec(dx+c))^{\frac{5}{2}}}{i\,a\tan(dx+c)+a}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(5/2)/(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{5/2}}{a+ia\tan(c+dx)}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}}{a+a\tan(c+d\,x)\ 1\mathrm{i}}\,dx$$

```
[In] int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i),x)
[Out] int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i), x)
```

3.227 $\int \frac{(e\sec(c+dx))^{3/2}}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 70

$$\int \frac{(e\sec(c+dx))^{3/2}}{a+ia\tan(c+dx)}\,dx = \frac{2ie^2}{ad\sqrt{e\sec(c+dx)}} + \frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

```
[Out] 2*I*e^2/a/d/(e*sec(d*x+c))^(1/2)+2*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a/d/cos(d*x+c)^(1/2)/(e*s
ec(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3582, 3856, 2719}

$$\int \frac{(e\sec(c+dx))^{3/2}}{a+ia\tan(c+dx)}\,dx = \frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2ie^2}{ad\sqrt{e\sec(c+dx)}}$$

```
[In] Int[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((2*I)*e^2)/(a*d*Sqrt[e*Sec[c + d*x]]) + (2*e^2*EllipticE[(c + d*x)/2, 2])/
(a*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2ie^2}{ad\sqrt{e\sec(c+dx)}}+\frac{e^2\int\frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{a}\\ &= \frac{2ie^2}{ad\sqrt{e\sec(c+dx)}}+\frac{e^2\int\sqrt{\cos(c+dx)}\,dx}{a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\\ &= \frac{2ie^2}{ad\sqrt{e\sec(c+dx)}}+\frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.37 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.06

$$\int\frac{(e\sec(c+dx))^{3/2}}{a+ia\tan(c+dx)}\,dx=\frac{2iee^{-i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)\sqrt{e\sec(c+dx)}}{ad}$$

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x]),x]
[Out] ((2*I)*e*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E
^((2*I)*(c + d*x))]*Sqrt[e*Sec[c + d*x]])/(a*d*E^(I*(c + d*x)))
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 779 vs. $2(89) = 178$.

Time = 4.51 (sec) , antiderivative size = 780, normalized size of antiderivative = 11.14

method	result	size
default	Expression too large to display	780

```
[In] int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*e/a/d*(-e*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^2*(1-cos(d*x+c
))^2-1))^(1/2)*(4*I*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(-csc(d*x+c)^2*
(1-cos(d*x+c))^2+1)^(1/2)*((csc(d*x+c)^2*(1-cos(d*x+c))^2+1)*(csc(d*x+c)^2*
(1-cos(d*x+c))^2-1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)-4*I*(csc(
d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)
*((csc(d*x+c)^2*(1-cos(d*x+c))^2+1)*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1))^(1/2
)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)-I*ln(2*csc(d*x+c)^2*(1-cos(d*x+c))
^2+2*(csc(d*x+c)^4*(1-cos(d*x+c))^4-1)^(1/2))*(csc(d*x+c)^4*(1-cos(d*x+c))^
4-1)^(1/2)*((csc(d*x+c)^2*(1-cos(d*x+c))^2+1)*(csc(d*x+c)^2*(1-cos(d*x+c))^
2-1))^(1/2)+I*(csc(d*x+c)^4*(1-cos(d*x+c))^4-1)^(1/2)*((csc(d*x+c)^2*(1-cos
(d*x+c))^2+1)*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1))^(1/2)*ln(csc(d*x+c)^2*(1-c
os(d*x+c))^2+(csc(d*x+c)^4*(1-cos(d*x+c))^4-1)^(1/2))+4*I*csc(d*x+c)^2*(csc
(d*x+c)^4*(1-cos(d*x+c))^4-1)^(1/2)*(1-cos(d*x+c))^2+4*csc(d*x+c)^3*(csc(d*
x+c)^4*(1-cos(d*x+c))^4-1)^(1/2)*(1-cos(d*x+c))^3-4*I*(csc(d*x+c)^4*(1-cos(
d*x+c))^4-1)^(1/2)-4*(csc(d*x+c)^4*(1-cos(d*x+c))^4-1)^(1/2)*(csc(d*x+c)-co
t(d*x+c)))/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^4*(1-cos(d*x+c))^4
-1)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.37

$$\int \frac{(e\sec(c+dx))^{3/2}}{a+ia\tan(c+dx)}\,dx = -\frac{2\left(-i\sqrt{2}e^{\frac{3}{2}}e^{(i\,dx+i\,c)}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\sqrt{2}\left(-i\,ee^{(2i\,dx+2i\,c)}-i\,e\right.\right.}{ad}$$

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -2*(-I*sqrt(2)*e^(3/2)*e^(I*d*x + I*c)*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(-I*e*e^(2*I*d*x + 2*I*c) - I*e)*
sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-I*d*x - I*c)
/(a*d)
```

Sympy [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{a+ia\tan(c+dx)}\,dx = -\frac{i\int \frac{(e\sec(c+dx))^{\frac{3}{2}}}{\tan(c+dx)-i}\,dx}{a}$$

```
[In] integrate((e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c)),x)
[Out] -I*Integral((e*sec(c + d*x))**(3/2)/(tan(c + d*x) - I), x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{3/2}}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{a+ia\tan(c+dx)}\,dx = \int \frac{(e\sec(dx+c))^{\frac{3}{2}}}{i\,a\tan(dx+c)+a}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(3/2)/(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{3/2}}{a+ia\tan(c+dx)}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}{a+a\tan(c+d\,x)\;1\mathrm{i}}\,dx$$

```
[In] int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i),x)
[Out] int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i), x)
```

3.228 $\int \frac{\sqrt{e \sec(c+dx)}}{a+ia \tan(c+dx)} dx$

Optimal result

Integrand size = 28, antiderivative size = 80

$$\int \frac{\sqrt{e \sec(c+dx)}}{a+ia \tan(c+dx)} dx = \frac{2\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{e \sec(c+dx)}}{3ad} + \frac{2i\sqrt{e \sec(c+dx)}}{3d(a+ia \tan(c+dx))}$$

```
[Out] 2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/a/d+2/3*I*(e*sec(d*x+c
))^(1/2)/d/(a+I*a*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3583, 3856, 2720}

$$\int \frac{\sqrt{e \sec(c+dx)}}{a+ia \tan(c+dx)} dx = \frac{2\sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx), 2\right) \sqrt{e \sec(c+dx)}}{3ad} + \frac{2i\sqrt{e \sec(c+dx)}}{3d(a+ia \tan(c+dx))}$$

```
[In] Int[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*a*
d) + (((2*I)/3)*Sqrt[e*Sec[c + d*x]])/(d*(a + I*a*Tan[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2i\sqrt{e\sec(c+dx)}}{3d(a+ia\tan(c+dx))} + \frac{\int \sqrt{e\sec(c+dx)}\,dx}{3a} \\
&= \frac{2i\sqrt{e\sec(c+dx)}}{3d(a+ia\tan(c+dx))} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3a} \\
&= \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3ad} + \frac{2i\sqrt{e\sec(c+dx)}}{3d(a+ia\tan(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{e\sec(c+dx)}}{a+ia\tan(c+dx)}\,dx = \frac{2(e\sec(c+dx))^{3/2}\left(\cos(c+dx)+\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(-i\cos(c+dx)+\sin(c+dx))\right)}{3ade(-i+\tan(c+dx))}$$

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (2*(e*Sec[c + d*x])^(3/2)*(Cos[c + d*x] + Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*((-I)*Cos[c + d*x] + Sin[c + d*x])))/(3*a*d*e*(-I + Tan[c + d*x
]))
```

Maple [A] (verified)

Time = 6.35 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.00

method	result
default	$\frac{2\sqrt{e\sec(dx+c)}\left(i\cos(dx+c)F(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}+iF(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{}{\cos(dx}}\right.}{3ad}$

[In] int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 2/3/a/d*(e*sec(d*x+c))^(1/2)*(I*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*cos(d*x+c)+I*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+I*cos(d*x+c)^2+sin(d*x+c)*cos(d*x+c))

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{e\sec(c+dx)}}{a+ia\tan(c+dx)}\,dx = \frac{\left(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(i\,e^{(2i\,dx+2i\,c)}+i\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}-2i\,\sqrt{2}\sqrt{e}e^{(2i\,dx+2i\,c)}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)e}{3\,ad}$$

[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")

[Out] 1/3*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(I*e^(2*I*d*x + 2*I*c) + I)*e^(1/2*I*d*x + 1/2*I*c) - 2*I*sqrt(2)*sqrt(e)*e^(2*I*d*x + 2*I*c)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))*e^(-2*I*d*x - 2*I*c)/(a*d)

Sympy [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{a+ia\tan(c+dx)}\,dx = -\frac{i\int\frac{\sqrt{e\sec(c+dx)}}{\tan(c+dx)-i}\,dx}{a}$$

[In] integrate((e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c)),x)

[Out] -I*Integral(sqrt(e*sec(c + d*x))/(tan(c + d*x) - I), x)/a

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e\sec(c+dx)}}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{a+ia\tan(c+dx)}\,dx = \int \frac{\sqrt{e\sec\left(dx+c\right)}}{i\,a\tan\left(dx+c\right)+a}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] integrate(sqrt(e*sec(d*x + c))/(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\sec(c+dx)}}{a+ia\tan(c+dx)}\,dx = \int \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}}{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

```
[In] int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i),x)
[Out] int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i), x)
```

3.229 $\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 80

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))}\,dx = \frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2i}{5d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))}$$

```
[Out] 6/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))/a/d/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+2/5*I/d/(e*sec(d*x
+c))^(1/2)/(a+I*a*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3583, 3856, 2719}

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))}\,dx = \frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2i}{5d(a+ia\tan(c+dx))\sqrt{e\sec(c+dx)}}$$

```
[In] Int[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])),x]
```

```
[Out] (6*EllipticE[(c + d*x)/2, 2])/(5*a*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]
]) + ((2*I)/5)/(d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2i}{5d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))}+\frac{3\int\frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{5a}\\
&= \frac{2i}{5d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))}+\frac{3\int\sqrt{\cos(c+dx)}\,dx}{5a\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\\
&= \frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{2i}{5d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.57 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.36

$$\int\frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))}\,dx$$
$$=\frac{\left(4+4\cos(2(c+dx))-2e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)+3i\sin(2(c+dx\right.}{5ad\sqrt{e\sec(c+dx)}}$$

```
[In] Integrate[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])),x]
```

```
[Out] ((4 + 4*Cos[2*(c + d*x)] - 2*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x
))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + (3*I)*Sin[2*(c
 + d*x)])*(I + Tan[c + d*x]))/(5*a*d*Sqrt[e*Sec[c + d*x]])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 439 vs. 2(94) = 188.

Time = 7.54 (sec) , antiderivative size = 440, normalized size of antiderivative = 5.50

method	result
default	$-\frac{2i\left(i(\cos^2(dx+c))\sin(dx+c)+3\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,E(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\cos(dx+c)-3\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,F(i(-\mathrm{cs}\right.}{}$

```
[In] int(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*I/a/d/(cos(d*x+c)+1)/(e*sec(d*x+c))^(1/2)*(I*cos(d*x+c)^2*sin(d*x+c)+3
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*
(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Ell
ipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+I*
sin(d*x+c)*cos(d*x+c)-cos(d*x+c)^3+6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Elli
pticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)-6*EllipticF(I*
(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)+3*I*sin(d*x+c)-cos(d*x+c)^2+3*sec(d*x+c)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2
)-3*sec(d*x+c)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(
1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.34

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))}\,dx = \frac{\left(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(7i\,e^{(4i\,dx+4i\,c)}+8i\,e^{(2i\,dx+2i\,c)}+i\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+12i\,\sqrt{2}\sqrt{e}e^{(3i\,dx+3i\,c)}\mathrm{weierstrassZeta}\left(-4,0\right.\right.}{10\,ade}$$

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/10*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(7*I*e^(4*I*d*x + 4*I*c) +
8*I*e^(2*I*d*x + 2*I*c) + I)*e^(1/2*I*d*x + 1/2*I*c) + 12*I*sqrt(2)*sqrt(e)
*e^(3*I*d*x + 3*I*c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I
*d*x + I*c))))*e^(-3*I*d*x - 3*I*c)/(a*d*e)
```

Sympy [F]

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))}\,dx = -\frac{i\int \frac{1}{\sqrt{e\sec{(c+dx)}}\tan{(c+dx)}-i\sqrt{e\sec{(c+dx)}}}\,dx}{a}$$

```
[In] integrate(1/(e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c)),x)
[Out] -I*Integral(1/(sqrt(e*sec(c + d*x))*tan(c + d*x) - I*sqrt(e*sec(c + d*x))),
 x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))}\,dx = \int \frac{1}{\sqrt{e\sec{(dx+c)}}(i\,a\tan{(dx+c)}+a)}\,dx$$

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] integrate(1/(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))}\,dx = \int \frac{1}{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\left(a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}\,dx$$

```
[In] int(1/((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)),x)
[Out] int(1/((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)), x)
```

3.230 $\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 114

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))}\,dx = \frac{10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{21ade^2} + \frac{10\sin(c+dx)}{21ade\sqrt{e\sec(c+dx)}} + \frac{2i}{7d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))}$$

```
[Out] 10/21*sin(d*x+c)/a/d/e/(e*sec(d*x+c))^(1/2)+10/21*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)*(e*sec(d*x+c))^(1/2)/a/d/e^2+2/7*I/d/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x
+c))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3583, 3854, 3856, 2720}

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))}\,dx = \frac{10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{21ade^2} + \frac{10\sin(c+dx)}{21ade\sqrt{e\sec(c+dx)}} + \frac{2i}{7d(a+ia\tan(c+dx))(e\sec(c+dx))^{3/2}}$$

```
[In] Int[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])),x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(21*
a*d*e^2) + (10*Sin[c + d*x])/(21*a*d*e*Sqrt[e*Sec[c + d*x]]) + ((2*I)/7)/(d
*(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2i}{7d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))}+\frac{5\int\frac{1}{(e\sec(c+dx))^{3/2}}\,dx}{7a}\\
&= \frac{10\sin(c+dx)}{21ade\sqrt{e\sec(c+dx)}}+\frac{2i}{7d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))}+\frac{5\int\sqrt{e\sec(c+dx)}\,dx}{21ae^2}\\
&= \frac{10\sin(c+dx)}{21ade\sqrt{e\sec(c+dx)}}+\frac{2i}{7d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))}\\
&\quad+\frac{\left(5\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21ae^2}\\
&= \frac{10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{21ade^2}\\
&\quad+\frac{10\sin(c+dx)}{21ade\sqrt{e\sec(c+dx)}}+\frac{2i}{7d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.10

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))}\,dx = -\frac{\sec^3(c+dx)\left(-14\cos(c+dx)+2\cos(3(c+dx))+20i\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(\cos(c+dx)\right.}{42ad(e\sec(c+dx))^{3/2}(-i+\tan(c+dx))}$$

[In] Integrate[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])),x]

[Out] -1/42*(Sec[c + d*x]^3*(-14*Cos[c + d*x] + 2*Cos[3*(c + d*x)] + (20*I)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[c + d*x] + I*Sin[c + d*x]) + (5*I)*Sin[c + d*x] + (5*I)*Sin[3*(c + d*x)]))/(a*d*(e*Sec[c + d*x])^(3/2)*(-I + Tan[c + d*x]))

Maple [A] (verified)

Time = 8.86 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.53

method	result
default	$\frac{\frac{10iF(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}}{21}+\frac{2i\left(\cos^3(dx+c)\right)}{7}+\frac{10i\sec(dx+c)F(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}}{21}}{ad\sqrt{e\sec(dx+c)}\,e}$

[In] int(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 2/21/a/d/(e*sec(d*x+c))^(1/2)/e*(5*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)+3*I*cos(d*x+c)^3+5*I*sec(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)+3*cos(d*x+c)^2*sin(d*x+c)+5*sin(d*x+c))

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.01

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))}\,dx = \frac{\left(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-7i\,e^{(6i\,dx+6i\,c)}+9i\,e^{(4i\,dx+4i\,c)}+19i\,e^{(2i\,dx+2}\right.\right.}{}$$

[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")

```
[Out] 1/84*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-7*I*e^(6*I*d*x + 6*I*c) +
 9*I*e^(4*I*d*x + 4*I*c) + 19*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(1/2*I*d*x + 1
/2*I*c) - 40*I*sqrt(2)*sqrt(e)*e^(4*I*d*x + 4*I*c)*weierstrassPInverse(-4,
0, e^(I*d*x + I*c)))*e^(-4*I*d*x - 4*I*c)/(a*d*e^2)
```

Sympy [F]

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))}\,dx = -\frac{i\int \frac{1}{(e\sec{(c+dx)})^{\frac{3}{2}}\tan{(c+dx)}-i(e\sec{(c+dx)})^{\frac{3}{2}}}\,dx}{a}$$

```
[In] integrate(1/(e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -I*Integral(1/((e*sec(c + d*x))**(3/2)*tan(c + d*x) - I*(e*sec(c + d*x))**(
3/2)), x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))}\,dx = \int \frac{1}{(e\sec{(dx+c)})^{\frac{3}{2}}\,(i\,a\tan{(dx+c)}+a)}\,dx$$

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))}\,dx = \int \frac{1}{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}\,dx$$

```
[In] int(1/((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)),x)
[Out] int(1/((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)), x)
```

3.231 $\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 114

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))}\,dx = \frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15ade^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{14\sin(c+dx)}{45ade(e\sec(c+dx))^{3/2}} + \frac{2i}{9d(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))}$$

```
[Out] 14/45*sin(d*x+c)/a/d/e/(e*sec(d*x+c))^(3/2)+14/15*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a/d/e^2/cos(d*
x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+2/9*I/d/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x
+c))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3583, 3854, 3856, 2719}

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))}\,dx = \frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15ade^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{14\sin(c+dx)}{45ade(e\sec(c+dx))^{3/2}} + \frac{2i}{9d(a+ia\tan(c+dx))(e\sec(c+dx))^{5/2}}$$

```
[In] Int[1/((e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])),x]
```

```
[Out] (14*EllipticE[(c + d*x)/2, 2])/(15*a*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c
+ d*x]]) + (14*Sin[c + d*x])/(45*a*d*e*(e*Sec[c + d*x])^(3/2)) + ((2*I)/9)/
(d*(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2i}{9d(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))}+\frac{7\int\frac{1}{(e\sec(c+dx))^{5/2}}\,dx}{9a}\\
&= \frac{14\sin(c+dx)}{45ade(e\sec(c+dx))^{3/2}}+\frac{2i}{9d(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))}+\frac{7\int\frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{15ae^2}\\
&= \frac{14\sin(c+dx)}{45ade(e\sec(c+dx))^{3/2}}+\frac{2i}{9d(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))}\\
&\quad+\frac{7\int\sqrt{\cos(c+dx)}\,dx}{15ae^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\\
&= \frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15ade^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{14\sin(c+dx)}{45ade(e\sec(c+dx))^{3/2}}\\
&\quad+\frac{2i}{9d(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.04 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.18

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))}\,dx = \frac{\left(106+104\cos(2(c+dx))-2\cos(4(c+dx))-56e^{2i(c+dx)}\sqrt{\cdots}\right.}{\cdots}$$

```
[In] Integrate[1/((e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])),x]
```

```
[Out] ((106 + 104*Cos[2*(c + d*x)] - 2*Cos[4*(c + d*x)] - 56*E^((2*I)*(c + d*x))*
Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c
 + d*x))] + (70*I)*Sin[2*(c + d*x)] - (7*I)*Sin[4*(c + d*x)])*(I + Tan[c +
d*x]))/(180*a*d*e^2*Sqrt[e*Sec[c + d*x]])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 476 vs. 2(124) = 248.

Time = 10.56 (sec) , antiderivative size = 477, normalized size of antiderivative = 4.18

method	result
default	$-\frac{2i\left(5i\sin(dx+c)\left(\cos^4(dx+c)\right)+5i\left(\cos^3(dx+c)\right)\sin(dx+c)-5\left(\cos^5(dx+c)\right)+7i\left(\cos^2(dx+c)\right)\sin(dx+c)-5\left(\cos^4(dx+c)\right)-21\sqrt{\frac{\cos\cdots}{\cos(\cdots}}\right.}{\cdots}$

```
[In] int(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45*I/a/d/(cos(d*x+c)+1)/(e*sec(d*x+c))^(1/2)/e^2*(5*I*sin(d*x+c)*cos(d*x
+c)^4+5*I*cos(d*x+c)^3*sin(d*x+c)-5*cos(d*x+c)^5+7*I*cos(d*x+c)^2*sin(d*x+c
)-5*cos(d*x+c)^4-21*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x
+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+21*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))
^(1/2)*cos(d*x+c)+7*I*cos(d*x+c)*sin(d*x+c)-42*EllipticF(I*(-csc(d*x+c)+cot
(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+42*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1
/(cos(d*x+c)+1))^(1/2)+21*I*sin(d*x+c)-21*sec(d*x+c)*EllipticF(I*(-csc(d*x+
c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)+21*sec(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+c)+
cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.13

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))}\,dx = \frac{\left(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-9i\,e^{(8i\,dx+8i\,c)}+174i\,e^{(6i\,dx+6i\,c)}+212i\,e^{(4i\,$$

```
[In] integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/360*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-9*I*e^(8*I*d*x + 8*I*c)
+ 174*I*e^(6*I*d*x + 6*I*c) + 212*I*e^(4*I*d*x + 4*I*c) + 34*I*e^(2*I*d*x +
 2*I*c) + 5*I)*e^(1/2*I*d*x + 1/2*I*c) + 336*I*sqrt(2)*sqrt(e)*e^(5*I*d*x +
 5*I*c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))
)*e^(-5*I*d*x - 5*I*c)/(a*d*e^3)
```

Sympy [F]

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))}\,dx = -\frac{i\int \frac{1}{(e\sec(c+dx))^{\frac{5}{2}}\tan(c+dx)-i(e\sec(c+dx))^{\frac{5}{2}}}\,dx}{a}$$

```
[In] integrate(1/(e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c)),x)
```

```
[Out] -I*Integral(1/((e*sec(c + d*x))**(5/2)*tan(c + d*x) - I*(e*sec(c + d*x))**(
5/2)), x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))}\,dx = \int \frac{1}{(e\sec(dx+c))^{\frac{5}{2}}\,(i\,a\tan(dx+c)+a)}\,dx$$

```
[In] integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] integrate(1/((e*sec(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))}\,dx = \int \frac{1}{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}\,(a+a\tan(c+d\,x)\,1\mathrm{i})}\,dx$$

```
[In] int(1/((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)),x)
[Out] int(1/((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)), x)
```

3.232 $\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 145

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))}\,dx = \frac{30\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{77ade^4} + \frac{18\sin(c+dx)}{77ade(e\sec(c+dx))^{5/2}} + \frac{30\sin(c+dx)}{77ade^3\sqrt{e\sec(c+dx)}} + \frac{2i}{11d(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))}$$

```
[Out] 18/77*sin(d*x+c)/a/d/e/(e*sec(d*x+c))^(5/2)+30/77*sin(d*x+c)/a/d/e^3/(e*sec
(d*x+c))^(1/2)+30/77*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/a/d/e
^4+2/11*I/d/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3583, 3854, 3856, 2720}

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))}\,dx = \frac{30\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{77ade^4} + \frac{30\sin(c+dx)}{77ade^3\sqrt{e\sec(c+dx)}} + \frac{18\sin(c+dx)}{77ade(e\sec(c+dx))^{5/2}} + \frac{2i}{11d(a+ia\tan(c+dx))(e\sec(c+dx))^{7/2}}$$

```
[In] Int[1/((e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])),x]
```

```
[Out] (30*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(77*
a*d*e^4) + (18*Sin[c + d*x])/(77*a*d*e*(e*Sec[c + d*x])^(5/2)) + (30*Sin[c
+ d*x])/(77*a*d*e^3*Sqrt[e*Sec[c + d*x]]) + ((2*I)/11)/(d*(e*Sec[c + d*x])^
(7/2)*(a + I*a*Tan[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2i}{11d(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))}+\frac{9\int\frac{1}{(e\sec(c+dx))^{7/2}}\,dx}{11a}\\
&= \frac{18\sin(c+dx)}{77ade(e\sec(c+dx))^{5/2}}+\frac{2i}{11d(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))}+\frac{45\int\frac{1}{(e\sec(c+dx))^{3/2}}\,dx}{77ae^2}\\
&= \frac{18\sin(c+dx)}{77ade(e\sec(c+dx))^{5/2}}+\frac{30\sin(c+dx)}{77ade^3\sqrt{e\sec(c+dx)}}\\
&\quad+\frac{2i}{11d(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))}+\frac{15\int\sqrt{e\sec(c+dx)}\,dx}{77ae^4}\\
&= \frac{18\sin(c+dx)}{77ade(e\sec(c+dx))^{5/2}}+\frac{30\sin(c+dx)}{77ade^3\sqrt{e\sec(c+dx)}}\\
&\quad+\frac{2i}{11d(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))}\\
&\quad+\frac{\left(15\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{77ae^4}
\end{aligned}
$$

$$= \frac{30\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{77ade^4} + \frac{18\sin(c+dx)}{77ade(e\sec(c+dx))^{5/2}} + \frac{30\sin(c+dx)}{77ade^3\sqrt{e\sec(c+dx)}} + \frac{2i}{11d(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))}$$

Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.98

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))}\,dx = -\frac{(e\sec(c+dx))^{3/2}\left(-148\cos(c+dx)+34\cos(3(c+dx))+2\cos(5(c+dx))+240i\sqrt{\cos(c+dx)}\,\text{EllipticF}\right.}{616ade^5(-i+\text{ta}}$$

```
[In] Integrate[1/((e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])),x]
```

```
[Out] -1/616*((e*Sec[c + d*x])^(3/2)*(-148*Cos[c + d*x] + 34*Cos[3*(c + d*x)] + 2
*Cos[5*(c + d*x)] + (240*I)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(C
os[c + d*x] + I*Sin[c + d*x]) + (78*I)*Sin[c + d*x] + (87*I)*Sin[3*(c + d*x
)] + (9*I)*Sin[5*(c + d*x)]))/(a*d*e^5*(-I + Tan[c + d*x]))
```

Maple [A] (verified)

Time = 8.92 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.31

method	result
default	$\frac{\frac{2i\left(\cos^5(dx+c)\right)}{11}+\frac{2\sin(dx+c)\left(\cos^4(dx+c)\right)}{11}+\frac{30iF(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}}{77}+\frac{18\left(\cos^2(dx+c)\right)\sin(dx+c)}{77}+\frac{30i}{}}{ad\sqrt{e\sec(dx+c)}\,e^3}$

```
[In] int(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/77/a/d/(e*sec(d*x+c))^(1/2)/e^3*(7*I*cos(d*x+c)^5+7*sin(d*x+c)*cos(d*x+c)
^4+15*I*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+9*cos(d*x+c)^2*sin(d*x+c)+15*I*sec(d*x+c)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc
(d*x+c)+cot(d*x+c)),I)+15*sin(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.94

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))}\,dx = \frac{\left(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-11i\,e^{(10i\,dx+10i\,c)} - 121i\,e^{(8i\,dx+8i\,c)} + 70i\,e\right.\right.}{}$$

[In] integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x, algorithm="fricas")

[Out] 1/1232*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-11*I*e^(10*I*d*x + 10*I*c) - 121*I*e^(8*I*d*x + 8*I*c) + 70*I*e^(6*I*d*x + 6*I*c) + 226*I*e^(4*I*d*x + 4*I*c) + 53*I*e^(2*I*d*x + 2*I*c) + 7*I)*e^(1/2*I*d*x + 1/2*I*c) - 480*I*sqrt(2)*sqrt(e)*e^(6*I*d*x + 6*I*c)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))*e^(-6*I*d*x - 6*I*c)/(a*d*e^4)

Sympy [F]

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))}\,dx = -\frac{i\int \frac{1}{(e\sec{(c+dx)})^{\frac{7}{2}}\tan{(c+dx)}-i(e\sec{(c+dx)})^{\frac{7}{2}}}\,dx}{a}$$

[In] integrate(1/(e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c)),x)

[Out] -I*Integral(1/((e*sec(c + d*x))**(7/2)*tan(c + d*x) - I*(e*sec(c + d*x))**(7/2)), x)/a

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))}\,dx = \text{Exception raised: RuntimeError}$$

[In] integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

Giac [F]

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))}\,dx = \int \frac{1}{\left(e\sec\left(dx+c\right)\right)^{\frac{7}{2}}\left(i\,a\tan\left(dx+c\right)+a\right)}\,dx$$

```
[In] integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] integrate(1/((e*sec(d*x + c))^(7/2)*(I*a*tan(d*x + c) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))}\,dx = \int \frac{1}{{\left(\frac{e}{\cos(c+d\,x)}\right)}^{7/2}\,\left(a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}\,dx$$

```
[In] int(1/((e/cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)),x)
[Out] int(1/((e/cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)), x)
```

3.233 $\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 183

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{22e^8E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{22e^7\sqrt{e\sec(c+dx)}\sin(c+dx)}{15a^2d} + \frac{22e^5(e\sec(c+dx))^{5/2}\sin(c+dx)}{45a^2d} + \frac{22e^3(e\sec(c+dx))^{9/2}\sin(c+dx)}{63a^2d} - \frac{4ie^2(e\sec(c+dx))^{11/2}}{7d\,(a^2+ia^2\tan(c+dx))}$$

```
[Out] 22/45*e^5*(e*sec(d*x+c))^(5/2)*sin(d*x+c)/a^2/d+22/63*e^3*(e*sec(d*x+c))^(9
/2)*sin(d*x+c)/a^2/d-22/15*e^8*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/cos(d*x+c)^(1/2)/(e*sec(d*x
+c))^(1/2)+22/15*e^7*sin(d*x+c)*(e*sec(d*x+c))^(1/2)/a^2/d-4/7*I*e^2*(e*sec
(d*x+c))^(11/2)/d/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3581, 3853, 3856, 2719}

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{22e^8E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{22e^7\sin(c+dx)\sqrt{e\sec(c+dx)}}{15a^2d} + \frac{22e^5\sin(c+dx)(e\sec(c+dx))^{5/2}}{45a^2d} + \frac{22e^3\sin(c+dx)(e\sec(c+dx))^{9/2}}{63a^2d} - \frac{4ie^2(e\sec(c+dx))^{11/2}}{7d\,(a^2+ia^2\tan(c+dx))}$$

```
[In] Int[(e*Sec[c + d*x])^(15/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (-22*e^8*EllipticE[(c + d*x)/2, 2])/(15*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec
[c + d*x]]) + (22*e^7*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/(15*a^2*d) + (22*e
^5*(e*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(45*a^2*d) + (22*e^3*(e*Sec[c + d*x
])^(9/2)*Sin[c + d*x])/(63*a^2*d) - (((4*I)/7)*e^2*(e*Sec[c + d*x])^(11/2))
/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{4ie^2(e\sec(c+dx))^{11/2}}{7d\,(a^2+ia^2\tan(c+dx))}+\frac{(11e^2)\int(e\sec(c+dx))^{11/2}\,dx}{7a^2} \\
&= \frac{22e^3(e\sec(c+dx))^{9/2}\sin(c+dx)}{63a^2d}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{7d\,(a^2+ia^2\tan(c+dx))}+\frac{(11e^4)\int(e\sec(c+dx))^{7/2}\,dx}{9a^2} \\
&= \frac{22e^5(e\sec(c+dx))^{5/2}\sin(c+dx)}{45a^2d}+\frac{22e^3(e\sec(c+dx))^{9/2}\sin(c+dx)}{63a^2d} \\
&\quad -\frac{4ie^2(e\sec(c+dx))^{11/2}}{7d\,(a^2+ia^2\tan(c+dx))}+\frac{(11e^6)\int(e\sec(c+dx))^{3/2}\,dx}{15a^2}
\end{aligned}
$$

$$= \frac{22e^7\sqrt{e\sec(c+dx)}\sin(c+dx)}{15a^2d} + \frac{22e^5(e\sec(c+dx))^{5/2}\sin(c+dx)}{45a^2d} + \frac{22e^3(e\sec(c+dx))^{9/2}\sin(c+dx)}{63a^2d} - \frac{4ie^2(e\sec(c+dx))^{11/2}}{7d\,(a^2+ia^2\tan(c+dx))} - \frac{(11e^8)\int\frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{15a^2}$$

$$= \frac{22e^7\sqrt{e\sec(c+dx)}\sin(c+dx)}{15a^2d} + \frac{22e^5(e\sec(c+dx))^{5/2}\sin(c+dx)}{45a^2d} + \frac{22e^3(e\sec(c+dx))^{9/2}\sin(c+dx)}{63a^2d} - \frac{4ie^2(e\sec(c+dx))^{11/2}}{7d\,(a^2+ia^2\tan(c+dx))} - \frac{(11e^8)\int\sqrt{\cos(c+dx)}\,dx}{15a^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$= -\frac{22e^8E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{15a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{22e^7\sqrt{e\sec(c+dx)}\sin(c+dx)}{15a^2d} + \frac{22e^5(e\sec(c+dx))^{5/2}\sin(c+dx)}{45a^2d} + \frac{22e^3(e\sec(c+dx))^{9/2}\sin(c+dx)}{63a^2d} - \frac{4ie^2(e\sec(c+dx))^{11/2}}{7d\,(a^2+ia^2\tan(c+dx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.88 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.65

$$\int\frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{(e\sec(c+dx))^{15/2}(\cos(dx)+i\sin(dx))^2\left(\frac{22i\sqrt{2}e^{3ic-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\,(-\right.}{}$$

```
[In] Integrate[(e*Sec[c + d*x])^(15/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((e*Sec[c + d*x])^(15/2)*(Cos[d*x] + I*Sin[d*x])^2*(((22*I)*Sqrt[2]*E^((3*I
)*c - I*d*x)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2
*I)*(c + d*x))]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^(
(2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/(-1 + E^
((2*I)*c)) + (Csc[c]*Sec[c + d*x]^(9/2)*(Cos[2*c] + I*Sin[2*c])*(1260*Cos[d
*x] + 1050*Cos[2*c + d*x] + 1078*Cos[2*c + 3*d*x] + 77*Cos[4*c + 3*d*x] + 2
31*Cos[4*c + 5*d*x] + (720*I)*Sin[d*x] - (720*I)*Sin[2*c + d*x]))/56))/(45*
d*Sec[c + d*x]^(11/2)*(a + I*a*Tan[c + d*x])^2)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 472 vs. 2(185) = 370.

Time = 9.20 (sec) , antiderivative size = 473, normalized size of antiderivative = 2.58

method	result
default	$-\frac{2ie^7\sqrt{e\sec(dx+c)}\left(-231(\cos^2(dx+c))E(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}+231(\cos^2(dx+c))F(i(-\csc(\right.}{}$

```
[In] int((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/315*I*e^7/a^2/d*(e*sec(d*x+c))^(1/2)/(cos(d*x+c)+1)*(-231*cos(d*x+c)^2*E
llipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(cos(d*x+c)+1))^(1/2)+231*cos(d*x+c)^2*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)
),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)-462*(cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(
d*x+c)+1))^(1/2)*cos(d*x+c)+462*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF
(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-231*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(c
os(d*x+c)+1))^(1/2)+231*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x
+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+231*I*sin(d*x+c)+77*I*tan(d
*x+c)+77*I*tan(d*x+c)*sec(d*x+c)-35*I*tan(d*x+c)*sec(d*x+c)^2-35*I*tan(d*x+
c)*sec(d*x+c)^3+90*sec(d*x+c)^2+90*sec(d*x+c)^3)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.40

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^2}\,dx =$$
$$-\frac{2\left(\sqrt{2}(231i\,e^7e^{(9i\,dx+9i\,c)}+1078i\,e^7e^{(7i\,dx+7i\,c)}+1980i\,e^7e^{(5i\,dx+5i\,c)}+1770i\,e^7e^{(3i\,dx+3i\,c)}+77i\,e^7e^{(i\,dx+i\,c)}\right)\sqrt{}}{315\,(a^2}$$

```
[In] integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/315*(sqrt(2)*(231*I*e^7*e^(9*I*d*x + 9*I*c) + 1078*I*e^7*e^(7*I*d*x + 7*
I*c) + 1980*I*e^7*e^(5*I*d*x + 5*I*c) + 1770*I*e^7*e^(3*I*d*x + 3*I*c) + 77
*I*e^7*e^(I*d*x + I*c))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/
2*I*c) + 231*sqrt(2)*(I*e^7*e^(8*I*d*x + 8*I*c) + 4*I*e^7*e^(6*I*d*x + 6*I*
c) + 6*I*e^7*e^(4*I*d*x + 4*I*c) + 4*I*e^7*e^(2*I*d*x + 2*I*c) + I*e^7)*sqr
t(e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x + I*c))))/(
a^2*d*e^(8*I*d*x + 8*I*c) + 4*a^2*d*e^(6*I*d*x + 6*I*c) + 6*a^2*d*e^(4*I*d*
x + 4*I*c) + 4*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*sec(d*x+c))**(15/2)/(a+I*a*tan(d*x+c))**2,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\sec(dx+c))^{\frac{15}{2}}}{(ia\tan(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(15/2)/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{15/2}}{(a+a\tan(c+d\,x)\,1\mathrm{i})^2}\,dx$$

```
[In] int((e/cos(c + d*x))^(15/2)/(a + a*tan(c + d*x)*1i)^2,x)
[Out] int((e/cos(c + d*x))^(15/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

3.234 $\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 152

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{6e^6\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{7a^2d} + \frac{6e^5(e\sec(c+dx))^{3/2}\sin(c+dx)}{7a^2d} + \frac{18e^3(e\sec(c+dx))^{7/2}\sin(c+dx)}{35a^2d} - \frac{4ie^2(e\sec(c+dx))^{9/2}}{5d\,(a^2+ia^2\tan(c+dx))}$$

```
[Out] 6/7*e^5*(e*sec(d*x+c))^(3/2)*sin(d*x+c)/a^2/d+18/35*e^3*(e*sec(d*x+c))^(7/2
)*sin(d*x+c)/a^2/d+6/7*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)
/a^2/d-4/5*I*e^2*(e*sec(d*x+c))^(9/2)/d/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3581, 3853, 3856, 2720}

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{6e^6\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{7a^2d} + \frac{6e^5\sin(c+dx)(e\sec(c+dx))^{3/2}}{7a^2d} + \frac{18e^3\sin(c+dx)(e\sec(c+dx))^{7/2}}{35a^2d} - \frac{4ie^2(e\sec(c+dx))^{9/2}}{5d\,(a^2+ia^2\tan(c+dx))}$$

```
[In] Int[(e*Sec[c + d*x])^(13/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (6*e^6*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(
7*a^2*d) + (6*e^5*(e*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(7*a^2*d) + (18*e^3*
(e*Sec[c + d*x])^(7/2)*Sin[c + d*x])/(35*a^2*d) - (((4*I)/5)*e^2*(e*Sec[c +
 d*x])^(9/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{4ie^2(e\sec(c+dx))^{9/2}}{5d\,(a^2+ia^2\tan(c+dx))}+\frac{(9e^2)\int(e\sec(c+dx))^{9/2}\,dx}{5a^2}\\
&=\frac{18e^3(e\sec(c+dx))^{7/2}\sin(c+dx)}{35a^2d}-\frac{4ie^2(e\sec(c+dx))^{9/2}}{5d\,(a^2+ia^2\tan(c+dx))}+\frac{(9e^4)\int(e\sec(c+dx))^{5/2}\,dx}{7a^2}\\
&=\frac{6e^5(e\sec(c+dx))^{3/2}\sin(c+dx)}{7a^2d}+\frac{18e^3(e\sec(c+dx))^{7/2}\sin(c+dx)}{35a^2d}\\
&\quad-\frac{4ie^2(e\sec(c+dx))^{9/2}}{5d\,(a^2+ia^2\tan(c+dx))}+\frac{(3e^6)\int\sqrt{e\sec(c+dx)}\,dx}{7a^2}
\end{aligned}$$

$$= \frac{6e^5(e\sec(c+dx))^{3/2}\sin(c+dx)}{7a^2d} + \frac{18e^3(e\sec(c+dx))^{7/2}\sin(c+dx)}{35a^2d} - \frac{4ie^2(e\sec(c+dx))^{9/2}}{5d\,(a^2+ia^2\tan(c+dx))} + \frac{\left(3e^6\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{7a^2}$$

$$= \frac{6e^6\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{7a^2d} + \frac{6e^5(e\sec(c+dx))^{3/2}\sin(c+dx)}{7a^2d} + \frac{18e^3(e\sec(c+dx))^{7/2}\sin(c+dx)}{35a^2d} - \frac{4ie^2(e\sec(c+dx))^{9/2}}{5d\,(a^2+ia^2\tan(c+dx))}$$

Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.56

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{e^6\sec^3(c+dx)\sqrt{e\sec(c+dx)}\left(-56i\cos(c+dx)+60\cos^{\frac{7}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}\right.\right.}{70a^2d}$$

```
[In] Integrate[(e*Sec[c + d*x])^(13/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (e^6*Sec[c + d*x]^3*Sqrt[e*Sec[c + d*x]]*((-56*I)*Cos[c + d*x] + 60*Cos[c +
 d*x]^(7/2)*EllipticF[(c + d*x)/2, 2] - 5*Sin[c + d*x] + 15*Sin[3*(c + d*x)
]))/(70*a^2*d)
```

Maple [A] (verified)

Time = 8.13 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.14

method	result
default	$-\frac{2ie^6\sqrt{e\sec(dx+c)}\left(15F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\cos(dx+c)+15F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{}{c}}\right.}{35a^2d}$

```
[In] int((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/35*I*e^6/a^2/d*(e*sec(d*x+c))^(1/2)*(15*EllipticF(I*(csc(d*x+c)-cot(d*x+
c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c
)+15*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)+15*I*tan(d*x+c)+14*sec(d*x+c)^2-5*I*sec(d*x+c)^
2*tan(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.32

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{2\left(\sqrt{2}(15i\,e^6e^{(6i\,dx+6i\,c)}+51i\,e^6e^{(4i\,dx+4i\,c)}+61i\,e^6e^{(2i\,dx+2i\,c)}-15i\,e^6)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+15\sqrt{2}(i\,e\right.}{35\left(a^2de^{(6i\,dx+6i\,c)}+3\,a^2de^{(4i\,dx+4i\,c)}\right.}$$

```
[In] integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/35*(sqrt(2)*(15*I*e^6*e^(6*I*d*x + 6*I*c) + 51*I*e^6*e^(4*I*d*x + 4*I*c)
 + 61*I*e^6*e^(2*I*d*x + 2*I*c) - 15*I*e^6)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1
))*e^(1/2*I*d*x + 1/2*I*c) + 15*sqrt(2)*(I*e^6*e^(6*I*d*x + 6*I*c) + 3*I*e^
6*e^(4*I*d*x + 4*I*c) + 3*I*e^6*e^(2*I*d*x + 2*I*c) + I*e^6)*sqrt(e)*weiers
trassPInverse(-4, 0, e^(I*d*x + I*c)))/(a^2*d*e^(6*I*d*x + 6*I*c) + 3*a^2*d
*e^(4*I*d*x + 4*I*c) + 3*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*sec(d*x+c))**(13/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\sec(dx+c))^{\frac{13}{2}}}{(i\,a\tan(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(13/2)/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{13/2}}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^2}\,dx$$

```
[In] int((e/cos(c + d*x))^(13/2)/(a + a*tan(c + d*x)*1i)^2,x)
[Out] int((e/cos(c + d*x))^(13/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

3.235 $\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 152

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{14e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{14e^5\sqrt{e\sec(c+dx)}\sin(c+dx)}{5a^2d} + \frac{14e^3(e\sec(c+dx))^{5/2}\sin(c+dx)}{15a^2d} - \frac{4ie^2(e\sec(c+dx))^{7/2}}{3d\,(a^2+ia^2\tan(c+dx))}$$

```
[Out] 14/15*e^3*(e*sec(d*x+c))^(5/2)*sin(d*x+c)/a^2/d-14/5*e^6*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/c
os(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+14/5*e^5*sin(d*x+c)*(e*sec(d*x+c))^(1/
2)/a^2/d-4/3*I*e^2*(e*sec(d*x+c))^(7/2)/d/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3581, 3853, 3856, 2719}

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{14e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{14e^5\sin(c+dx)\sqrt{e\sec(c+dx)}}{5a^2d} + \frac{14e^3\sin(c+dx)(e\sec(c+dx))^{5/2}}{15a^2d} - \frac{4ie^2(e\sec(c+dx))^{7/2}}{3d\,(a^2+ia^2\tan(c+dx))}$$

```
[In] Int[(e*Sec[c + d*x])^(11/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (-14*e^6*EllipticE[(c + d*x)/2, 2])/(5*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) + (14*e^5*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/(5*a^2*d) + (14*e^3
*(e*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(15*a^2*d) - (((4*I)/3)*e^2*(e*Sec[c
+ d*x])^(7/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{\left(7e^2\right)\int(e\sec(c+dx))^{7/2}\,dx}{3a^2}\\
&=\frac{14e^3(e\sec(c+dx))^{5/2}\sin(c+dx)}{15a^2d}-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{\left(7e^4\right)\int(e\sec(c+dx))^{3/2}\,dx}{5a^2}\\
&=\frac{14e^5\sqrt{e\sec(c+dx)}\sin(c+dx)}{5a^2d}+\frac{14e^3(e\sec(c+dx))^{5/2}\sin(c+dx)}{15a^2d}\\
&\quad-\frac{4ie^2(e\sec(c+dx))^{7/2}}{3d\left(a^2+ia^2\tan(c+dx)\right)}-\frac{\left(7e^6\right)\int\frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{5a^2}
\end{aligned}
$$

$$= \frac{14e^5\sqrt{e\sec(c+dx)}\sin(c+dx)}{5a^2d} + \frac{14e^3(e\sec(c+dx))^{5/2}\sin(c+dx)}{15a^2d} - \frac{4ie^2(e\sec(c+dx))^{7/2}}{3d\,(a^2+ia^2\tan(c+dx))} - \frac{(7e^6)\int\sqrt{\cos(c+dx)}\,dx}{5a^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$= -\frac{14e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{14e^5\sqrt{e\sec(c+dx)}\sin(c+dx)}{5a^2d} + \frac{14e^3(e\sec(c+dx))^{5/2}\sin(c+dx)}{15a^2d} - \frac{4ie^2(e\sec(c+dx))^{7/2}}{3d\,(a^2+ia^2\tan(c+dx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.90 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.81

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{2ie^5e^{i(c+dx)}\left(-47-56e^{2i(c+dx)}-21e^{4i(c+dx)}+7\left(1+e^{2i(c+dx)}\right)^{5/2}\text{Hypergeome}\right.}{15a^2d\left(1+e^{2i(c+dx)}\right)^2}$$

```
[In] Integrate[(e*Sec[c + d*x])^(11/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (((2*I)/15)*e^5*E^(I*(c + d*x))*(-47 - 56*E^((2*I)*(c + d*x)) - 21*E^((4*I)
*(c + d*x)) + 7*(1 + E^((2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4,
 7/4, -E^((2*I)*(c + d*x))])*Sqrt[e*Sec[c + d*x]])/(a^2*d*(1 + E^((2*I)*(c
+ d*x)))^2)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 431 vs. 2(158) = 316.

Time = 7.88 (sec) , antiderivative size = 432, normalized size of antiderivative = 2.84

method	result
default	$\frac{2e^5\sqrt{e\sec(dx+c)}\left(21iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}\left(\cos^2(dx+c)\right)-21iE(i(\csc(dx+c)-\cot(dx+c)),i)\right.}{}$

```
[In] int((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*e^5/a^2/d*(e*sec(d*x+c))^(1/2)/(cos(d*x+c)+1)*(21*I*EllipticF(I*(csc(d
*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(
1/2)*cos(d*x+c)^2-21*I*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2+42*I*cos(d*x+c)*
EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1
/(cos(d*x+c)+1))^(1/2)-42*I*cos(d*x+c)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),
```

```
I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+21*I*Elliptic
F(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*
x+c)+1))^(1/2)-21*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(csc(d*x+
c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)-10*I+21*sin(d*x+c)-10*I*sec(d*x+
c)-3*tan(d*x+c)-3*sec(d*x+c)*tan(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.12

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{2\left(\sqrt{2}\left(21i\,e^5e^{(5i\,dx+5i\,c)}+56i\,e^5e^{(3i\,dx+3i\,c)}+47i\,e^5e^{(i\,dx+i\,c)}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+21\sqrt{2}\left(i\,e^5e^{(4i\,dx+4i\,c)}\right.}{15\left(a^2de^{(4i\,dx+4i\,c)}+2\,a^2de^{(2i\,d}\right.}$$

```
[In] integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/15*(sqrt(2)*(21*I*e^5*e^(5*I*d*x + 5*I*c) + 56*I*e^5*e^(3*I*d*x + 3*I*c)
 + 47*I*e^5*e^(I*d*x + I*c))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x
 + 1/2*I*c) + 21*sqrt(2)*(I*e^5*e^(4*I*d*x + 4*I*c) + 2*I*e^5*e^(2*I*d*x +
2*I*c) + I*e^5)*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e
^(I*d*x + I*c))))/(a^2*d*e^(4*I*d*x + 4*I*c) + 2*a^2*d*e^(2*I*d*x + 2*I*c)
+ a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*sec(d*x+c))**(11/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\sec(dx+c))^{\frac{11}{2}}}{(i\,a\tan(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(11/2)/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{11/2}}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^2}\,dx$$

```
[In] int((e/cos(c + d*x))^(11/2)/(a + a*tan(c + d*x)*1i)^2,x)
[Out] int((e/cos(c + d*x))^(11/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

3.236 $\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 119

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{10e^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3a^2d} + \frac{10e^3(e\sec(c+dx))^{3/2}\sin(c+dx)}{3a^2d} - \frac{4ie^2(e\sec(c+dx))^{5/2}}{d\,(a^2+ia^2\tan(c+dx))}$$

```
[Out] 10/3*e^3*(e*sec(d*x+c))^(3/2)*sin(d*x+c)/a^2/d+10/3*e^4*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+
c)^(1/2)*(e*sec(d*x+c))^(1/2)/a^2/d-4*I*e^2*(e*sec(d*x+c))^(5/2)/d/(a^2+I*a
^2*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3581, 3853, 3856, 2720}

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{10e^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3a^2d} + \frac{10e^3\sin(c+dx)(e\sec(c+dx))^{3/2}}{3a^2d} - \frac{4ie^2(e\sec(c+dx))^{5/2}}{d\,(a^2+ia^2\tan(c+dx))}$$

```
[In] Int[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (10*e^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(3*a^2*d) + (10*e^3*(e*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*a^2*d) - ((4*I)
*e^2*(e*Sec[c + d*x])^(5/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{4ie^2(e\sec(c+dx))^{5/2}}{d\,(a^2+ia^2\tan(c+dx))}+\frac{(5e^2)\int(e\sec(c+dx))^{5/2}\,dx}{a^2}\\
&= \frac{10e^3(e\sec(c+dx))^{3/2}\sin(c+dx)}{3a^2d}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d\,(a^2+ia^2\tan(c+dx))}+\frac{(5e^4)\int\sqrt{e\sec(c+dx)}\,dx}{3a^2}\\
&= \frac{10e^3(e\sec(c+dx))^{3/2}\sin(c+dx)}{3a^2d}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d\,(a^2+ia^2\tan(c+dx))}\\
&\quad+\frac{\left(5e^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3a^2}\\
&= \frac{10e^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3a^2d}\\
&\quad+\frac{10e^3(e\sec(c+dx))^{3/2}\sin(c+dx)}{3a^2d}-\frac{4ie^2(e\sec(c+dx))^{5/2}}{d\,(a^2+ia^2\tan(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.56

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{2e^3(e\sec(c+dx))^{3/2}\left(-6i\cos(c+dx)+5\cos^{\frac{3}{2}}(c+dx)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\right.}{3a^2d}$$

[In] Integrate[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x])^2,x]

[Out] (2*e^3*(e*Sec[c + d*x])^(3/2)*((-6*I)*Cos[c + d*x] + 5*Cos[c + d*x]^(3/2)*E llipticF[(c + d*x)/2, 2] - Sin[c + d*x]))/(3*a^2*d)

Maple [A] (verified)

Time = 6.86 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.25

method	result
default	$\frac{2e^4\sqrt{e\sec(dx+c)}\left(5i\cos(dx+c)F(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}+5iF(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\right.}{3a^2d}$

[In] int((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 2/3*e^4/a^2/d*(e*sec(d*x+c))^(1/2)*(5*I*cos(d*x+c)*EllipticF(I*(-csc(d*x+c) +cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+ 5*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)), I)*(1/(cos(d*x+c)+1))^(1/2)-6*I-tan(d*x+c))

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.97

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{2\left(\sqrt{2}\left(5i\,e^4e^{(2i\,dx+2i\,c)}+7i\,e^4\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+5\sqrt{2}\left(i\,e^4e^{(2i\,dx+2i\,c)}+i\,e^4\right)\sqrt{e}\text{weierstrassPInverse}\right.}{3\left(a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")

[Out] -2/3*(sqrt(2)*(5*I*e^4*e^(2*I*d*x + 2*I*c) + 7*I*e^4)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 5*sqrt(2)*(I*e^4*e^(2*I*d*x + 2*I*c) + I*e^4)*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))/(a^2*d*e^(2* I*d*x + 2*I*c) + a^2*d)

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*sec(d*x+c))**(9/2)/(a+I*a*tan(d*x+c))**2,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\sec(dx+c))^{\frac{9}{2}}}{(i\,a\tan(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(9/2)/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{9/2}}{(a+a\tan(c+d\,x)\;1\mathrm{i})^2}\,dx$$

```
[In] int((e/cos(c + d*x))^(9/2)/(a + a*tan(c + d*x)*1i)^2,x)
[Out] int((e/cos(c + d*x))^(9/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

3.237 $\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 115

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{6e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{6e^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{a^2d} + \frac{4ie^2(e\sec(c+dx))^{3/2}}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

```
[Out] 6*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))/a^2/d/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)-6*e^3*sin(d*x+
c)*(e*sec(d*x+c))^(1/2)/a^2/d+4*I*e^2*(e*sec(d*x+c))^(3/2)/d/(a^2+I*a^2*tan
(d*x+c))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3581, 3853, 3856, 2719}

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{6e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{6e^3\sin(c+dx)\sqrt{e\sec(c+dx)}}{a^2d} + \frac{4ie^2(e\sec(c+dx))^{3/2}}{d\left(a^2+ia^2\tan(c+dx)\right)}$$

```
[In] Int[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (6*e^4*EllipticE[(c + d*x)/2, 2])/(a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c +
d*x]]) - (6*e^3*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/(a^2*d) + ((4*I)*e^2*(e*
Sec[c + d*x])^(3/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{4ie^2(e\sec(c+dx))^{3/2}}{d\,(a^2+ia^2\tan(c+dx))} - \frac{(3e^2)\int (e\sec(c+dx))^{3/2}\,dx}{a^2} \\
&= -\frac{6e^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{a^2d} + \frac{4ie^2(e\sec(c+dx))^{3/2}}{d\,(a^2+ia^2\tan(c+dx))} + \frac{(3e^4)\int \frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{a^2} \\
&= -\frac{6e^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{a^2d} + \frac{4ie^2(e\sec(c+dx))^{3/2}}{d\,(a^2+ia^2\tan(c+dx))} + \frac{(3e^4)\int \sqrt{\cos(c+dx)}\,dx}{a^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} \\
&= \frac{6e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{6e^3\sqrt{e\sec(c+dx)}\sin(c+dx)}{a^2d} + \frac{4ie^2(e\sec(c+dx))^{3/2}}{d\,(a^2+ia^2\tan(c+dx))}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.49 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.70

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{2ie^3e^{-i(c+dx)}\left(-1+3\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)\right)}{a^2d}$$

```
[In] Integrate[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((2*I)*e^3*(-1 + 3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/
2, 3/4, -E^((2*I)*(c + d*x))])*Sqrt[e*Sec[c + d*x]])/(a^2*d*E^(I*(c + d*x))
)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 430 vs. 2(131) = 262.

Time = 8.13 (sec) , antiderivative size = 431, normalized size of antiderivative = 3.75

method	result
default	$-\frac{2\left(3i(\cos^2(dx+c))E(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}-3i(\cos^2(dx+c))F(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{}\right.}{}$

```
[In] int((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/a^2/d*(3*I*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1
/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2-3*I*EllipticF(I*(-csc(d*
x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*cos(d*x+c)^2+6*I*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)
+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-6*I*EllipticF(I*(-c
sc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*cos(d*x+c)+3*I*(1/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+c)+c
ot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-3*I*(1/(cos(d*x+c)+1))^(1/2
)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
-2*I*cos(d*x+c)^2-2*I*cos(d*x+c)-2*sin(d*x+c)*cos(d*x+c)+sin(d*x+c))*(e*sec
(d*x+c))^(1/2)*e^3/(cos(d*x+c)+1)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.87

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{2\left(-3i\sqrt{2}e^{\frac{7}{2}}e^{(i\,dx+i\,c)}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\sqrt{2}\left(-3i\,e^{3}e^{(2i\,dx+2i\,c)}\right.\right.}{a^2d}$$

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2*(-3*I*sqrt(2)*e^(7/2)*e^(I*d*x + I*c)*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(-3*I*e^3*e^(2*I*d*x + 2*I*c) -
 2*I*e^3)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-I*
d*x - I*c)/(a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\sec(dx+c))^{\frac{7}{2}}}{(i\,a\tan(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(7/2)/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^2}\,dx$$

```
[In] int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^2,x)
[Out] int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

3.238 $\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 90

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3a^2d}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{3d\,(a^2+ia^2\tan(c+dx))}$$

```
[Out] -2/3*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*
d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/a^2/d+4/3*I*e^2*(
e*sec(d*x+c))^(1/2)/d/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3581, 3856, 2720}

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3a^2d}+\frac{4ie^2\sqrt{e\sec(c+dx)}}{3d\,(a^2+ia^2\tan(c+dx))}$$

```
[In] Int[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (-2*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(3*a^2*d) + (((4*I)/3)*e^2*Sqrt[e*Sec[c + d*x]])/(d*(a^2 + I*a^2*Tan[c + d*
x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{4ie^2\sqrt{e\sec(c+dx)}}{3d\left(a^2+ia^2\tan(c+dx)\right)} - \frac{e^2\int\sqrt{e\sec(c+dx)}\,dx}{3a^2} \\
&= \frac{4ie^2\sqrt{e\sec(c+dx)}}{3d\left(a^2+ia^2\tan(c+dx)\right)} - \frac{\left(e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3a^2} \\
&= -\frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3a^2d} + \frac{4ie^2\sqrt{e\sec(c+dx)}}{3d\left(a^2+ia^2\tan(c+dx)\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.12

$$
\int\frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{2(e\sec(c+dx))^{5/2}\left(-2i\cos(c+dx)+\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(\cdots\right.}{3a^2d(-i+\tan(c+dx)\cdots}
$$

```
[In] Integrate[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (2*(e*Sec[c + d*x])^(5/2)*((-2*I)*Cos[c + d*x] + Sqrt[Cos[c + d*x]]*Ellipti
cF[(c + d*x)/2, 2]*(Cos[c + d*x] + I*Sin[c + d*x]))*(Cos[c + d*x] + I*Sin[c
 + d*x]))/(3*a^2*d*(-I + Tan[c + d*x])^2)
```

Maple [A] (verified)

Time = 7.51 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.82

method	result
default	$\frac{2e^2\left(i\cos(dx+c)F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}+iF(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}\right.}{3a^2d}$

```
[In] int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/a^2/d*e^2*(I*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+I*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)
+2*I*cos(d*x+c)^2+2*sin(d*x+c)*cos(d*x+c))*(e*sec(d*x+c))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.08

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{2\left(-i\sqrt{2}e^{\frac{5}{2}}e^{(2i\,dx+2i\,c)}\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(-i\,e^2e^{(2i\,dx+2i\,c)}-i\,e^2\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\right.}}{3\,a^2d}$$

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(-I*sqrt(2)*e^(5/2)*e^(2*I*d*x + 2*I*c)*weierstrassPInverse(-4, 0, e^(
I*d*x + I*c)) + sqrt(2)*(-I*e^2*e^(2*I*d*x + 2*I*c) - I*e^2)*sqrt(e/(e^(2*I
*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-2*I*d*x - 2*I*c)/(a^2*d)
```

Sympy [F]

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{(e\sec{(c+dx)})^{\frac{5}{2}}}{\tan^2{(c+dx)}-2i\tan{(c+dx)}-1}\,dx}{a^2}$$

```
[In] integrate((e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -Integral((e*sec(c + d*x))**(5/2)/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1),
 x)/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\sec(dx+c))^{\frac{5}{2}}}{(ia\tan(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(5/2)/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^2}\,dx$$

```
[In] int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

3.239 $\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 90

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{4ie^2}{5d\sqrt{e\sec(c+dx)}\,(a^2+ia^2\tan(c+dx))}$$

```
[Out] 2/5*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/a^2/d/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+4/5*I*e^2/d/
(e*sec(d*x+c))^(1/2)/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3581, 3856, 2719}

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{4ie^2}{5d\,(a^2+ia^2\tan(c+dx))\sqrt{e\sec(c+dx)}}$$

```
[In] Int[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (2*e^2*EllipticE[(c + d*x)/2, 2])/(5*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c
+ d*x]]) + (((4*I)/5)*e^2)/(d*Sqrt[e*Sec[c + d*x]]*(a^2 + I*a^2*Tan[c + d*x
]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{4ie^2}{5d\sqrt{e\sec(c+dx)}\,(a^2+ia^2\tan(c+dx))}+\frac{e^2\int\frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{5a^2}\\
&= \frac{4ie^2}{5d\sqrt{e\sec(c+dx)}\,(a^2+ia^2\tan(c+dx))}+\frac{e^2\int\sqrt{\cos(c+dx)}\,dx}{5a^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\\
&= \frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2}{5d\sqrt{e\sec(c+dx)}\,(a^2+ia^2\tan(c+dx))}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.82 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.13

$$\int\frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx=\frac{iee^{-3i(c+dx)}\left(1+e^{2i(c+dx)}+2e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\right.\right.}{5a^2d}$$

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((I/5)*e*(1 + E^((2*I)*(c + d*x)) + 2*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)
*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])*Sqrt[
e*Sec[c + d*x]])/(a^2*d*E^((3*I)*(c + d*x)))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 450 vs. 2(104) = 208.

Time = 7.07 (sec) , antiderivative size = 451, normalized size of antiderivative = 5.01

method	result
default	$-\frac{2i\sqrt{e\sec(dx+c)}\,\left(2i\left(\cos^3(dx+c)\right)\sin(dx+c)+\left(\cos^2(dx+c)\right)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,F(i(\csc(dx+c)-\cot(dx+c)),i)-(\cos^2(d}{}$

```
[In] int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*I/a^2/d*(e*sec(d*x+c))^(1/2)*(2*I*cos(d*x+c)^3*sin(d*x+c)+cos(d*x+c)^2
*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(cs
c(d*x+c)-cot(d*x+c)),I)-cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)+2*I*cos(d*x+c)^
2*sin(d*x+c)-2*cos(d*x+c)^4+2*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-2*cos(d*x+c
)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(c
sc(d*x+c)-cot(d*x+c)),I)+I*sin(d*x+c)*cos(d*x+c)-2*cos(d*x+c)^3+EllipticF(I
*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)-(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Elli
pticE(I*(csc(d*x+c)-cot(d*x+c)),I))*e/(cos(d*x+c)+1)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.20

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{\left(2i\sqrt{2}e^{\frac{3}{2}}e^{(3i\,dx+3i\,c)}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c}\right.\right.\right.}{}$$

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/5*(2*I*sqrt(2)*e^(3/2)*e^(3*I*d*x + 3*I*c)*weierstrassZeta(-4, 0, weierst
rassPInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(2*I*e*e^(4*I*d*x + 4*I*c)
+ 3*I*e*e^(2*I*d*x + 2*I*c) + I*e)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2
*I*d*x + 1/2*I*c))*e^(-3*I*d*x - 3*I*c)/(a^2*d)
```

Sympy [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{(e\sec(c+dx))^{\frac{3}{2}}}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

```
[In] integrate((e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**2,x)
[Out] -Integral((e*sec(c + d*x))**(3/2)/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1),
 x)/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\sec(dx+c))^{\frac{3}{2}}}{(i\,a\tan(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(3/2)/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^2}\,dx$$

```
[In] int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^2,x)
[Out] int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

3.240 $\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 116

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{7a^2d} + \frac{2e\sin(c+dx)}{7a^2d\sqrt{e\sec(c+dx)}} + \frac{4ie^2}{7d(e\sec(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)}$$

```
[Out] 2/7*e*sin(d*x+c)/a^2/d/(e*sec(d*x+c))^(1/2)+2/7*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)
*(e*sec(d*x+c))^(1/2)/a^2/d+4/7*I*e^2/d/(e*sec(d*x+c))^(3/2)/(a^2+I*a^2*tan
(d*x+c))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3581, 3854, 3856, 2720}

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx = \frac{4ie^2}{7d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{3/2}} + \frac{2e\sin(c+dx)}{7a^2d\sqrt{e\sec(c+dx)}} + \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{7a^2d}$$

```
[In] Int[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(7*a^
2*d) + (2*e*Sin[c + d*x])/(7*a^2*d*Sqrt[e*Sec[c + d*x]]) + (((4*I)/7)*e^2)/
(d*(e*Sec[c + d*x])^(3/2)*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{4ie^2}{7d(e\sec(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)}+\frac{\left(3e^2\right)\int\frac{1}{(e\sec(c+dx))^{3/2}}\,dx}{7a^2}\\
&=\frac{2e\sin(c+dx)}{7a^2d\sqrt{e\sec(c+dx)}}+\frac{4ie^2}{7d(e\sec(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)}+\frac{\int\sqrt{e\sec(c+dx)}\,dx}{7a^2}\\
&=\frac{2e\sin(c+dx)}{7a^2d\sqrt{e\sec(c+dx)}}+\frac{4ie^2}{7d(e\sec(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)}\\
&\quad+\frac{\left(\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{7a^2}
\end{aligned}
$$

$$= \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{7a^2d} + \frac{2e\sin(c+dx)}{7a^2d\sqrt{e\sec(c+dx)}} + \frac{4ie^2}{7d(e\sec(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)}$$

Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\sec^2(c+dx)\sqrt{e\sec(c+dx)}\Big(2i+2i\cos(2(c+dx))+2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(\cos(2(c-}{7a^2d(-i+\tan(c+dx))^2}$$

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] -1/7*(Sec[c + d*x]^2*Sqrt[e*Sec[c + d*x]]*(2*I + (2*I)*Cos[2*(c + d*x)] + 2
*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] + I*Sin[2*(
c + d*x)]) - Sin[2*(c + d*x)]))/(a^2*d*(-I + Tan[c + d*x])^2)
```

Maple [A] (verified)

Time = 5.16 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.53

method	result
default	$-\frac{2\sqrt{e\sec(dx+c)}\left(i\cos(dx+c)F(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}-2i\left(\cos^4(dx+c)\right)+iF(i(\csc(dx+c)-\cot(\right.}{7a^2d}$

```
[In] int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/7/a^2/d*(e*sec(d*x+c))^(1/2)*(I*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-2*I*co
s(d*x+c)^4+I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(
d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)-2*cos(d*x+c)^3*sin(d*x+c)-sin(d*x+c)*co
s(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{\left(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(3i\,e^{(4i\,dx+4i\,c)}+4i\,e^{(2i\,dx+2i\,c)}+i\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}-4i\,\sqrt{2}\sqrt{e}e^{(4i\,dx+4i\,c)}\text{weierstrassPInverse}(-\right.}{14\,a^2d}$$

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/14*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(3*I*e^(4*I*d*x + 4*I*c) +
4*I*e^(2*I*d*x + 2*I*c) + I)*e^(1/2*I*d*x + 1/2*I*c) - 4*I*sqrt(2)*sqrt(e)*
e^(4*I*d*x + 4*I*c)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))*e^(-4*I*d*
x - 4*I*c)/(a^2*d)
```

Sympy [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\sqrt{e\sec{(c+dx)}}}{\tan^2{(c+dx)}-2i\tan{(c+dx)}-1}\,dx}{a^2}$$

```
[In] integrate((e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -Integral(sqrt(e*sec(c + d*x))/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)
/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{\sqrt{e\sec\left(dx+c\right)}}{\left(i\,a\tan\left(dx+c\right)+a\right)^2}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
[Out] integrate(sqrt(e*sec(d*x + c))/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^2}\,dx$$

```
[In] int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^2,x)
[Out] int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

3.241 $\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 116

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}\,dx = \frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2e\sin(c+dx)}{9a^2d(e\sec(c+dx))^{3/2}} + \frac{4ie^2}{9d(e\sec(c+dx))^{5/2}\,(a^2+ia^2\tan(c+dx))}$$

```
[Out] 2/9*e*sin(d*x+c)/a^2/d/(e*sec(d*x+c))^(3/2)+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/cos(d*x+c)
^(1/2)/(e*sec(d*x+c))^(1/2)+4/9*I*e^2/d/(e*sec(d*x+c))^(5/2)/(a^2+I*a^2*tan
(d*x+c))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3581, 3854, 3856, 2719}

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}\,dx = \frac{4ie^2}{9d\,(a^2+ia^2\tan(c+dx))\,(e\sec(c+dx))^{5/2}} + \frac{2e\sin(c+dx)}{9a^2d(e\sec(c+dx))^{3/2}} + \frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

```
[In] Int[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (2*EllipticE[(c + d*x)/2, 2])/(3*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*
x]]) + (2*e*Sin[c + d*x])/(9*a^2*d*(e*Sec[c + d*x])^(3/2)) + (((4*I)/9)*e^2
)/(d*(e*Sec[c + d*x])^(5/2)*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{4ie^2}{9d(e\sec(c+dx))^{5/2}\,(a^2+ia^2\tan(c+dx))}+\frac{(5e^2)\int \frac{1}{(e\sec(c+dx))^{5/2}}\,dx}{9a^2}\\
&= \frac{2e\sin(c+dx)}{9a^2d(e\sec(c+dx))^{3/2}}+\frac{4ie^2}{9d(e\sec(c+dx))^{5/2}\,(a^2+ia^2\tan(c+dx))}+\frac{\int \frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{3a^2}\\
&= \frac{2e\sin(c+dx)}{9a^2d(e\sec(c+dx))^{3/2}}+\frac{4ie^2}{9d(e\sec(c+dx))^{5/2}\,(a^2+ia^2\tan(c+dx))}\\
&\quad+\frac{\int \sqrt{\cos(c+dx)}\,dx}{3a^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}
\end{aligned}
$$

$$= \frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2e\sin(c+dx)}{9a^2d(e\sec(c+dx))^{3/2}} + \frac{4ie^2}{9d(e\sec(c+dx))^{5/2}\left(a^2+ia^2\tan(c+dx)\right)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.39 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}\,dx = \frac{\left(-\frac{8e^{4i(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} + 2(2+8\cos(2(c+dx))+7i\sin(2(c+dx)))\right)(i\cos(2(c+dx)) -}{18a^2d\sqrt{e\sec(c+dx)}}$$

```
[In] Integrate[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (((-8*E^((4*I)*(c + d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d
*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] + 2*(2 + 8*Cos[2*(c + d*x)] + (7*I)*Si
n[2*(c + d*x)]))*(I*Cos[2*(c + d*x)] + Sin[2*(c + d*x)]))/(18*a^2*d*Sqrt[e*
Sec[c + d*x]])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 473 vs. 2(126) = 252.

Time = 8.48 (sec) , antiderivative size = 474, normalized size of antiderivative = 4.09

method	result
default	$-\frac{2i\left(2i\sin(dx+c)\left(\cos^4(dx+c)\right)+2i\left(\cos^3(dx+c)\right)\sin(dx+c)-2\left(\cos^5(dx+c)\right)+i\left(\cos^2(dx+c)\right)\sin(dx+c)-2\left(\cos^4(dx+c)\right)-3\sqrt{\frac{\cos(dx+}{\cos(dx+c}}\right.}{}$

```
[In] int(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/9*I/a^2/d/(cos(d*x+c)+1)/(e*sec(d*x+c))^(1/2)*(2*I*sin(d*x+c)*cos(d*x+c)
^4+2*I*cos(d*x+c)^3*sin(d*x+c)-2*cos(d*x+c)^5+I*cos(d*x+c)^2*sin(d*x+c)-2*c
os(d*x+c)^4-3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+co
t(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+3*(cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*
cos(d*x+c)+I*sin(d*x+c)*cos(d*x+c)-6*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I
)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+6*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c
```

```
)+1))^(1/2)+3*I*sin(d*x+c)-3*sec(d*x+c)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)
),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+3*sec(d*x+c
)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)
*(1/(cos(d*x+c)+1))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.02

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}\,dx = \frac{\left(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(15i\,e^{(6i\,dx+6i\,c)}+19i\,e^{(4i\,dx+4i\,c)}+5i\,e^{(2i\,dx+2i\,c)}+i\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+24i\,\sqrt{2}\sqrt{e}e^{(5i\,dx+5i\,c)}\text{we}\right.}{36\,a^2de}$$

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas"
)
```

```
[Out] 1/36*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(15*I*e^(6*I*d*x + 6*I*c) +
 19*I*e^(4*I*d*x + 4*I*c) + 5*I*e^(2*I*d*x + 2*I*c) + I)*e^(1/2*I*d*x + 1/2
*I*c) + 24*I*sqrt(2)*sqrt(e)*e^(5*I*d*x + 5*I*c)*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, e^(I*d*x + I*c))))*e^(-5*I*d*x - 5*I*c)/(a^2*d*e)
```

Sympy [F]

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{1}{\sqrt{e\sec{(c+dx)}}\tan^2{(c+dx)}-2i\sqrt{e\sec{(c+dx)}}\tan{(c+dx)}-\sqrt{e\sec{(c+dx)}}}\,dx}{a^2}$$

```
[In] integrate(1/(e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -Integral(1/(sqrt(e*sec(c + d*x))*tan(c + d*x)**2 - 2*I*sqrt(e*sec(c + d*x)
)*tan(c + d*x) - sqrt(e*sec(c + d*x))), x)/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{\sqrt{e\sec\left(dx+c\right)}\left(i\,a\tan\left(dx+c\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^2}\,dx$$

```
[In] int(1/((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

```
[Out] int(1/((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

3.242 $\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 150

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = \frac{10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{33a^2de^2} + \frac{2e\sin(c+dx)}{11a^2d(e\sec(c+dx))^{5/2}} + \frac{10\sin(c+dx)}{33a^2de\sqrt{e\sec(c+dx)}} + \frac{4ie^2}{11d(e\sec(c+dx))^{7/2}\left(a^2+ia^2\tan(c+dx)\right)}$$

```
[Out] 2/11*e*sin(d*x+c)/a^2/d/(e*sec(d*x+c))^(5/2)+10/33*sin(d*x+c)/a^2/d/e/(e*se
c(d*x+c))^(1/2)+10/33*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/a^2/
d/e^2+4/11*I*e^2/d/(e*sec(d*x+c))^(7/2)/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3581, 3854, 3856, 2720}

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = \frac{4ie^2}{11d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{7/2}} + \frac{10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{33a^2de^2} + \frac{2e\sin(c+dx)}{11a^2d(e\sec(c+dx))^{5/2}} + \frac{10\sin(c+dx)}{33a^2de\sqrt{e\sec(c+dx)}}$$

```
[In] Int[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(33*
a^2*d*e^2) + (2*e*Sin[c + d*x])/(11*a^2*d*(e*Sec[c + d*x])^(5/2)) + (10*Sin
[c + d*x])/(33*a^2*d*e*Sqrt[e*Sec[c + d*x]]) + (((4*I)/11)*e^2)/(d*(e*Sec[c
 + d*x])^(7/2)*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{4ie^2}{11d(e\sec(c+dx))^{7/2}\left(a^2+ia^2\tan(c+dx)\right)}+\frac{(7e^2)\int\frac{1}{(e\sec(c+dx))^{7/2}}\,dx}{11a^2}\\
&= \frac{2e\sin(c+dx)}{11a^2d(e\sec(c+dx))^{5/2}}+\frac{4ie^2}{11d(e\sec(c+dx))^{7/2}\left(a^2+ia^2\tan(c+dx)\right)}+\frac{5\int\frac{1}{(e\sec(c+dx))^{3/2}}\,dx}{11a^2}\\
&= \frac{2e\sin(c+dx)}{11a^2d(e\sec(c+dx))^{5/2}}+\frac{10\sin(c+dx)}{33a^2de\sqrt{e\sec(c+dx)}}\\
&\quad+\frac{4ie^2}{11d(e\sec(c+dx))^{7/2}\left(a^2+ia^2\tan(c+dx)\right)}+\frac{5\int\sqrt{e\sec(c+dx)}\,dx}{33a^2e^2}
\end{aligned}$$

$$= \frac{2e\sin(c+dx)}{11a^2d(e\sec(c+dx))^{5/2}} + \frac{10\sin(c+dx)}{33a^2de\sqrt{e\sec(c+dx)}} + \frac{4ie^2}{11d(e\sec(c+dx))^{7/2}(a^2+ia^2\tan(c+dx))} + \frac{\left(5\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{33a^2e^2}$$

$$= \frac{10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{33a^2de^2} + \frac{2e\sin(c+dx)}{11a^2d(e\sec(c+dx))^{5/2}} + \frac{10\sin(c+dx)}{33a^2de\sqrt{e\sec(c+dx)}} + \frac{4ie^2}{11d(e\sec(c+dx))^{7/2}(a^2+ia^2\tan(c+dx))}$$

Mathematica [A] (verified)

Time = 1.54 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.89

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = -\frac{\sec^4(c+dx)\left(28i+24i\cos(2(c+dx))-4i\cos(4(c+dx))+40\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(\cos}{132a^2d(e\sec(c+dx))^{3/2}(-i+\tan(c+a}$$

```
[In] Integrate[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] -1/132*(Sec[c + d*x]^4*(28*I + (24*I)*Cos[2*(c + d*x)] - (4*I)*Cos[4*(c + d
*x)] + 40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] +
I*Sin[2*(c + d*x)]) - 6*Sin[2*(c + d*x)] + 7*Sin[4*(c + d*x)]))/(a^2*d*(e*S
ec[c + d*x])^(3/2)*(-I + Tan[c + d*x])^2)
```

Maple [A] (verified)

Time = 8.66 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.27

method	result
default	$\frac{\frac{4i\left(\cos^5(dx+c)\right)}{11}+\frac{4\sin(dx+c)\left(\cos^4(dx+c)\right)}{11}-\frac{10iF\left(i(\csc(dx+c)-\cot(dx+c)),i\right)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}}{33}+\frac{2\left(\cos^2(dx+c)\right)\sin(dx+c)}{11}-\frac{10i\,s}{}}{a^2d\sqrt{e\sec(dx+c)}\,e}$

```
[In] int(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/33/a^2/d/(e*sec(d*x+c))^(1/2)/e*(6*I*cos(d*x+c)^5+6*sin(d*x+c)*cos(d*x+c)
^4-5*I*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)+3*cos(d*x+c)^2*sin(d*x+c)-5*I*sec(d*x+c)*Elli
pticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)+5*sin(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.84

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = \frac{\left(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-11i\,e^{(8i\,dx+8i\,c)}+30i\,e^{(6i\,dx+6i\,c)}+56i\,e^{(4i\,c}\right.\right.}{}$$

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas"
)
```

```
[Out] 1/264*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-11*I*e^(8*I*d*x + 8*I*c)
 + 30*I*e^(6*I*d*x + 6*I*c) + 56*I*e^(4*I*d*x + 4*I*c) + 18*I*e^(2*I*d*x +
2*I*c) + 3*I)*e^(1/2*I*d*x + 1/2*I*c) - 80*I*sqrt(2)*sqrt(e)*e^(6*I*d*x + 6
*I*c)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))*e^(-6*I*d*x - 6*I*c)/(a^
2*d*e^2)
```

Sympy [F]

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{1}{(e\sec(c+dx))^{\frac{3}{2}}\tan^2(c+dx)-2i(e\sec(c+dx))^{\frac{3}{2}}\tan(c+dx)-(e\sec(c+dx))^{\frac{3}{2}}}\,dx}{a^2}$$

```
[In] integrate(1/(e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -Integral(1/((e*sec(c + d*x))**(3/2)*tan(c + d*x)**2 - 2*I*(e*sec(c + d*x))
**(3/2)*tan(c + d*x) - (e*sec(c + d*x))**(3/2)), x)/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{\left(e\sec\left(dx+c\right)\right)^{\frac{3}{2}}\left(i\,a\tan\left(dx+c\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
[Out] integrate(1/((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^2}\,dx$$

```
[In] int(1/((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^2),x)
[Out] int(1/((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

3.243 $\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 150

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = \frac{42E\left(\frac{1}{2}(c+dx)\middle|2\right)}{65a^2de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2e\sin(c+dx)}{13a^2d(e\sec(c+dx))^{7/2}} + \frac{14\sin(c+dx)}{65a^2de(e\sec(c+dx))^{3/2}} + \frac{4ie^2}{13d(e\sec(c+dx))^{9/2}\,(a^2+ia^2\tan(c+dx))}$$

```
[Out] 2/13*e*sin(d*x+c)/a^2/d/(e*sec(d*x+c))^(7/2)+14/65*sin(d*x+c)/a^2/d/e/(e*se
c(d*x+c))^(3/2)+42/65*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/e^2/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^
(1/2)+4/13*I*e^2/d/(e*sec(d*x+c))^(9/2)/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3581, 3854, 3856, 2719}

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = \frac{4ie^2}{13d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{9/2}} + \frac{42E\left(\frac{1}{2}(c+dx)\middle|2\right)}{65a^2de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2e\sin(c+dx)}{13a^2d(e\sec(c+dx))^{7/2}} + \frac{14\sin(c+dx)}{65a^2de(e\sec(c+dx))^{3/2}}$$

```
[In] Int[1/((e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (42*EllipticE[(c + d*x)/2, 2])/(65*a^2*d*e^2*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) + (2*e*Sin[c + d*x])/(13*a^2*d*(e*Sec[c + d*x])^(7/2)) + (14*Sin
```

```
[c + d*x])/(65*a^2*d*e*(e*Sec[c + d*x])^(3/2)) + (((4*I)/13)*e^2)/(d*(e*Sec
[c + d*x])^(9/2)*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{4ie^2}{13d(e\sec(c+dx))^{9/2}\left(a^2+ia^2\tan(c+dx)\right)}+\frac{(9e^2)\int\frac{1}{(e\sec(c+dx))^{9/2}}\,dx}{13a^2}\\
&= \frac{2e\sin(c+dx)}{13a^2d(e\sec(c+dx))^{7/2}}+\frac{4ie^2}{13d(e\sec(c+dx))^{9/2}\left(a^2+ia^2\tan(c+dx)\right)}+\frac{7\int\frac{1}{(e\sec(c+dx))^{5/2}}\,dx}{13a^2}\\
&= \frac{2e\sin(c+dx)}{13a^2d(e\sec(c+dx))^{7/2}}+\frac{14\sin(c+dx)}{65a^2de(e\sec(c+dx))^{3/2}}\\
&\quad+\frac{4ie^2}{13d(e\sec(c+dx))^{9/2}\left(a^2+ia^2\tan(c+dx)\right)}+\frac{21\int\frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{65a^2e^2}
\end{aligned}$$

$$= \frac{2e\sin(c+dx)}{13a^2d(e\sec(c+dx))^{7/2}} + \frac{14\sin(c+dx)}{65a^2de(e\sec(c+dx))^{3/2}} + \frac{4ie^2}{13d(e\sec(c+dx))^{9/2}(a^2+ia^2\tan(c+dx))} + \frac{21\int\sqrt{\cos(c+dx)}dx}{65a^2e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$= \frac{42E\left(\frac{1}{2}(c+dx)\middle|2\right)}{65a^2de^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2e\sin(c+dx)}{13a^2d(e\sec(c+dx))^{7/2}} + \frac{14\sin(c+dx)}{65a^2de(e\sec(c+dx))^{3/2}} + \frac{4ie^2}{13d(e\sec(c+dx))^{9/2}(a^2+ia^2\tan(c+dx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.82 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.99

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = \frac{(\cos(2(c+dx)) - i\sin(2(c+dx)))\left(88i + 416i\cos(2(c+dx)\right.}{}$$

```
[In] Integrate[1/((e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^2),x]
[Out] ((Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)])*(88*I + (416*I)*Cos[2*(c + d*x)] -
 (8*I)*Cos[4*(c + d*x)] - ((224*I)*E^((4*I)*(c + d*x))*Hypergeometric2F1[1/
2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - 356*Sin
[2*(c + d*x)] + 18*Sin[4*(c + d*x)]))/(520*a^2*d*e^2*Sqrt[e*Sec[c + d*x]])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 510 vs. 2(156) = 312.

Time = 11.16 (sec) , antiderivative size = 511, normalized size of antiderivative = 3.41

method	result
default	$-\frac{2i\left(10i(\cos^6(dx+c))\sin(dx+c)+10i(\cos^5(dx+c))\sin(dx+c)-10(\cos^7(dx+c))+5i\sin(dx+c)(\cos^4(dx+c))-10(\cos^6(dx+c))+5i(\cos\right.}{}$

```
[In] int(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] -2/65*I/a^2/d/(cos(d*x+c)+1)/(e*sec(d*x+c))^(1/2)/e^2*(10*I*sin(d*x+c)*cos(
d*x+c)^6+10*I*cos(d*x+c)^5*sin(d*x+c)-10*cos(d*x+c)^7+5*I*sin(d*x+c)*cos(d*
x+c)^4-10*cos(d*x+c)^6+5*I*cos(d*x+c)^3*sin(d*x+c)+7*I*cos(d*x+c)^2*sin(d*x
```

```
+c)-21*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)+21*EllipticF(I*(csc(d*x+c)-cot(d*x
+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+
c)+7*I*cos(d*x+c)*sin(d*x+c)-42*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)+42*EllipticF(I*(csc(
d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)+21*I*sin(d*x+c)-21*sec(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Ellip
ticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)+21*sec(d*x+c)*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/(
cos(d*x+c)+1))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.93

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = \frac{\left(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-13i\,e^{(10i\,dx+10i\,c)}+373i\,e^{(8i\,dx+8i\,c)}+474\right.\right.}{}$$

```
[In] integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas"
)
```

```
[Out] 1/1040*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-13*I*e^(10*I*d*x + 10*I
*c) + 373*I*e^(8*I*d*x + 8*I*c) + 474*I*e^(6*I*d*x + 6*I*c) + 118*I*e^(4*I*
d*x + 4*I*c) + 35*I*e^(2*I*d*x + 2*I*c) + 5*I)*e^(1/2*I*d*x + 1/2*I*c) + 67
2*I*sqrt(2)*sqrt(e)*e^(7*I*d*x + 7*I*c)*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, e^(I*d*x + I*c))))*e^(-7*I*d*x - 7*I*c)/(a^2*d*e^3)
```

Sympy [F]

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{1}{(e\sec{(c+dx)})^{\frac{5}{2}}\tan^2{(c+dx)}-2i(e\sec{(c+dx)})^{\frac{5}{2}}\tan{(c+dx)}-(e\sec{(c+dx)})^{\frac{5}{2}}}\,dx}{a^2}$$

```
[In] integrate(1/(e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -Integral(1/((e*sec(c + d*x))**(5/2)*tan(c + d*x)**2 - 2*I*(e*sec(c + d*x))
**(5/2)*tan(c + d*x) - (e*sec(c + d*x))**(5/2)), x)/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{(e\sec(dx+c))^{\frac{5}{2}}(ia\tan(dx+c)+a)^2}\,dx$$

```
[In] integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*sec(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}(a+a\tan(c+d\,x)\,1\mathrm{i})^2}\,dx$$

```
[In] int(1/((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

```
[Out] int(1/((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

3.244 $\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 181

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{7a^2de^4} + \frac{2e\sin(c+dx)}{15a^2d(e\sec(c+dx))^{9/2}} + \frac{6\sin(c+dx)}{35a^2de(e\sec(c+dx))^{5/2}} + \frac{2\sin(c+dx)}{7a^2de^3\sqrt{e\sec(c+dx)}} + \frac{4ie^2}{15d(e\sec(c+dx))^{11/2}\left(a^2+ia^2\tan(c+dx)\right)}$$

```
[Out] 2/15*e*sin(d*x+c)/a^2/d/(e*sec(d*x+c))^(9/2)+6/35*sin(d*x+c)/a^2/d/e/(e*sec
(d*x+c))^(5/2)+2/7*sin(d*x+c)/a^2/d/e^3/(e*sec(d*x+c))^(1/2)+2/7*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))
*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/a^2/d/e^4+4/15*I*e^2/d/(e*sec(d*x+c)
)^(11/2)/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3581, 3854, 3856, 2720}

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{7a^2de^4} + \frac{2\sin(c+dx)}{7a^2de^3\sqrt{e\sec(c+dx)}} + \frac{4ie^2}{15d\left(a^2+ia^2\tan(c+dx)\right)(e\sec(c+dx))^{11/2}} + \frac{2e\sin(c+dx)}{15a^2d(e\sec(c+dx))^{9/2}} + \frac{6\sin(c+dx)}{35a^2de(e\sec(c+dx))^{5/2}}$$

[In] Int[1/((e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^2),x]

[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(7*a^2*d*e^4) + (2*e*Sin[c + d*x])/(15*a^2*d*(e*Sec[c + d*x])^(9/2)) + (6*Sin[c + d*x])/(35*a^2*d*e*(e*Sec[c + d*x])^(5/2)) + (2*Sin[c + d*x])/(7*a^2*d*e^3*Sqrt[e*Sec[c + d*x]]) + (((4*I)/15)*e^2)/(d*(e*Sec[c + d*x])^(11/2)*(a^2 + I*a^2*Tan[c + d*x]))

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e + f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0] && IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{4ie^2}{15d(e\sec(c+dx))^{11/2}\left(a^2+ia^2\tan(c+dx)\right)}+\frac{\left(11e^2\right)\int\frac{1}{(e\sec(c+dx))^{11/2}}\,dx}{15a^2}\\
&=\frac{2e\sin(c+dx)}{15a^2d(e\sec(c+dx))^{9/2}}+\frac{4ie^2}{15d(e\sec(c+dx))^{11/2}\left(a^2+ia^2\tan(c+dx)\right)}+\frac{3\int\frac{1}{(e\sec(c+dx))^{7/2}}\,dx}{5a^2}\\
&=\frac{2e\sin(c+dx)}{15a^2d(e\sec(c+dx))^{9/2}}+\frac{6\sin(c+dx)}{35a^2de(e\sec(c+dx))^{5/2}}\\
&\quad+\frac{4ie^2}{15d(e\sec(c+dx))^{11/2}\left(a^2+ia^2\tan(c+dx)\right)}+\frac{3\int\frac{1}{(e\sec(c+dx))^{3/2}}\,dx}{7a^2e^2}
\end{aligned}$$

$$= \frac{2e\sin(c+dx)}{15a^2d(e\sec(c+dx))^{9/2}} + \frac{6\sin(c+dx)}{35a^2de(e\sec(c+dx))^{5/2}} + \frac{2\sin(c+dx)}{7a^2de^3\sqrt{e\sec(c+dx)}} + \frac{4ie^2}{15d(e\sec(c+dx))^{11/2}(a^2+ia^2\tan(c+dx))} + \frac{\int\sqrt{e\sec(c+dx)}\,dx}{7a^2e^4}$$

$$= \frac{2e\sin(c+dx)}{15a^2d(e\sec(c+dx))^{9/2}} + \frac{6\sin(c+dx)}{35a^2de(e\sec(c+dx))^{5/2}} + \frac{2\sin(c+dx)}{7a^2de^3\sqrt{e\sec(c+dx)}} + \frac{4ie^2}{15d(e\sec(c+dx))^{11/2}(a^2+ia^2\tan(c+dx))} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{7a^2e^4}$$

$$= \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{7a^2de^4} + \frac{2e\sin(c+dx)}{15a^2d(e\sec(c+dx))^{9/2}} + \frac{6\sin(c+dx)}{35a^2de(e\sec(c+dx))^{5/2}} + \frac{2\sin(c+dx)}{7a^2de^3\sqrt{e\sec(c+dx)}} + \frac{4ie^2}{15d(e\sec(c+dx))^{11/2}(a^2+ia^2\tan(c+dx))}$$

Mathematica [A] (verified)

Time = 1.90 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.83

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx =$$

$$-\frac{(e\sec(c+dx))^{5/2}\left(296i+228i\cos(2(c+dx))-72i\cos(4(c+dx))-4i\cos(6(c+dx))+480\sqrt{\cos(c+dx}\right.}{1680a^{2}}$$

```
[In] Integrate[1/((e*Sec[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] -1/1680*((e*Sec[c + d*x])^(5/2)*(296*I + (228*I)*Cos[2*(c + d*x)] - (72*I)*
Cos[4*(c + d*x)] - (4*I)*Cos[6*(c + d*x)] + 480*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]) - 17*Sin[2*(c + d
*x)] + 128*Sin[4*(c + d*x)] + 11*Sin[6*(c + d*x)]))/(a^2*d*e^6*(-I + Tan[c
+ d*x])^2)
```

Maple [A] (verified)

Time = 9.96 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.14

method	result
default	$\frac{\frac{4i\left(\cos^{7}(dx+c)\right)}{15}+\frac{4\sin(dx+c)\left(\cos^{6}(dx+c)\right)}{15}+\frac{2\sin(dx+c)\left(\cos^{4}(dx+c)\right)}{15}+\frac{2iF(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}}{7}+\frac{6\left(\cos\right.}{}}{a^{2}d\sqrt{e\sec(dx+c)}\,e^{3}}$

[In] int(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 2/105/a^2/d/(e*sec(d*x+c))^(1/2)/e^3*(14*I*cos(d*x+c)^7+14*sin(d*x+c)*cos(d*x+c)^6+7*sin(d*x+c)*cos(d*x+c)^4+15*I*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+9*cos(d*x+c)^2*sin(d*x+c)+15*I*sec(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)+15*sin(d*x+c))

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.82

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx = \frac{\left(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-15i\,e^{(12i\,dx+12i\,c)}-200i\,e^{(10i\,dx+10i\,c)}+245\right.\right.}{}$$

[In] integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")

[Out] 1/3360*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-15*I*e^(12*I*d*x + 12*I*c) - 200*I*e^(10*I*d*x + 10*I*c) + 245*I*e^(8*I*d*x + 8*I*c) + 592*I*e^(6*I*d*x + 6*I*c) + 211*I*e^(4*I*d*x + 4*I*c) + 56*I*e^(2*I*d*x + 2*I*c) + 7*I)*e^(1/2*I*d*x + 1/2*I*c) - 960*I*sqrt(2)*sqrt(e)*e^(8*I*d*x + 8*I*c)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))*e^(-8*I*d*x - 8*I*c)/(a^2*d*e^4)

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

[In] integrate(1/(e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**2,x)

[Out] Timed out

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{(e\sec(dx+c))^{\frac{7}{2}}(i\,a\tan(dx+c)+a)^2}\,dx$$

```
[In] integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*sec(d*x + c))^(7/2)*(I*a*tan(d*x + c) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}(a+a\tan(c+d\,x)\,1\mathrm{i})^2}\,dx$$

```
[In] int(1/((e/cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)^2),x)
```

```
[Out] int(1/((e/cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

3.245 $\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 178

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^3}\,dx = -\frac{22e^8E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{22ie^4(e\sec(c+dx))^{7/2}}{21a^3d} + \frac{22e^7\sqrt{e\sec(c+dx)}\sin(c+dx)}{5a^3d} + \frac{22e^5(e\sec(c+dx))^{5/2}\sin(c+dx)}{15a^3d} - \frac{4ie^2(e\sec(c+dx))^{11/2}}{3ad(a+ia\tan(c+dx))^2}$$

```
[Out] -22/21*I*e^4*(e*sec(d*x+c))^(7/2)/a^3/d+22/15*e^5*(e*sec(d*x+c))^(5/2)*sin(
d*x+c)/a^3/d-22/5*e^8*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2
)+22/5*e^7*sin(d*x+c)*(e*sec(d*x+c))^(1/2)/a^3/d-4/3*I*e^2*(e*sec(d*x+c))^(
11/2)/a/d/(a+I*a*tan(d*x+c))^2
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3581, 3582, 3853, 3856, 2719}

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^3}\,dx = -\frac{22e^8E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{22e^7\sin(c+dx)\sqrt{e\sec(c+dx)}}{5a^3d} + \frac{22e^5\sin(c+dx)(e\sec(c+dx))^{5/2}}{15a^3d} - \frac{22ie^4(e\sec(c+dx))^{7/2}}{21a^3d} - \frac{4ie^2(e\sec(c+dx))^{11/2}}{3ad(a+ia\tan(c+dx))^2}$$

```
[In] Int[(e*Sec[c + d*x])^(15/2)/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (-22*e^8*EllipticE[(c + d*x)/2, 2])/(5*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) - (((22*I)/21)*e^4*(e*Sec[c + d*x])^(7/2))/(a^3*d) + (22*e^7*Sqr
t[e*Sec[c + d*x]]*Sin[c + d*x])/(5*a^3*d) + (22*e^5*(e*Sec[c + d*x])^(5/2)*
Sin[c + d*x])/(15*a^3*d) - (((4*I)/3)*e^2*(e*Sec[c + d*x])^(11/2))/(a*d*(a
+ I*a*Tan[c + d*x])^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{4ie^2(e\sec(c+dx))^{11/2}}{3ad(a+ia\tan(c+dx))^2}+\frac{(11e^2)\int\frac{(e\sec(c+dx))^{11/2}}{a+ia\tan(c+dx)}\,dx}{3a^2}\\
&= -\frac{22ie^4(e\sec(c+dx))^{7/2}}{21a^3d}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{3ad(a+ia\tan(c+dx))^2}+\frac{(11e^4)\int(e\sec(c+dx))^{7/2}\,dx}{3a^3}\\
&= -\frac{22ie^4(e\sec(c+dx))^{7/2}}{21a^3d}+\frac{22e^5(e\sec(c+dx))^{5/2}\sin(c+dx)}{15a^3d}\\
&\quad-\frac{4ie^2(e\sec(c+dx))^{11/2}}{3ad(a+ia\tan(c+dx))^2}+\frac{(11e^6)\int(e\sec(c+dx))^{3/2}\,dx}{5a^3}\\
&= -\frac{22ie^4(e\sec(c+dx))^{7/2}}{21a^3d}+\frac{22e^7\sqrt{e\sec(c+dx)}\sin(c+dx)}{5a^3d}\\
&\quad+\frac{22e^5(e\sec(c+dx))^{5/2}\sin(c+dx)}{15a^3d}\\
&\quad-\frac{4ie^2(e\sec(c+dx))^{11/2}}{3ad(a+ia\tan(c+dx))^2}-\frac{(11e^8)\int\frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{5a^3}\\
&= -\frac{22ie^4(e\sec(c+dx))^{7/2}}{21a^3d}+\frac{22e^7\sqrt{e\sec(c+dx)}\sin(c+dx)}{5a^3d}\\
&\quad+\frac{22e^5(e\sec(c+dx))^{5/2}\sin(c+dx)}{15a^3d}\\
&\quad-\frac{4ie^2(e\sec(c+dx))^{11/2}}{3ad(a+ia\tan(c+dx))^2}-\frac{(11e^8)\int\sqrt{\cos(c+dx)}\,dx}{5a^3\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\\
&= -\frac{22e^8E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\\
&\quad-\frac{22ie^4(e\sec(c+dx))^{7/2}}{21a^3d}+\frac{22e^7\sqrt{e\sec(c+dx)}\sin(c+dx)}{5a^3d}\\
&\quad+\frac{22e^5(e\sec(c+dx))^{5/2}\sin(c+dx)}{15a^3d}-\frac{4ie^2(e\sec(c+dx))^{11/2}}{3ad(a+ia\tan(c+dx))^2}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.33 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.72

$$\int\frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^3}\,dx =$$

$$-\frac{e^6(e\sec(c+dx))^{3/2}\left(-556-868\cos(2(c+dx))+77e^{-2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{5/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\right.\right.}{210a^3d}$$

```
[In] Integrate[(e*Sec[c + d*x])^(15/2)/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] -1/210*(e^6*(e*Sec[c + d*x])^(3/2)*(-556 - 868*Cos[2*(c + d*x)] + (77*(1 +
E^((2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c +
d*x))])/E^((2*I)*(c + d*x)) + (203*I)*Sec[c + d*x]*Sin[3*(c + d*x)] + (143*
I)*Tan[c + d*x])*(-I + Tan[c + d*x]))/(a^3*d)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 453 vs. 2(179) = 358.

Time = 9.28 (sec) , antiderivative size = 454, normalized size of antiderivative = 2.55

method	result
default	$-\frac{2\sqrt{e\sec(dx+c)}\, e^{7}\left(231iE(i(\csc(dx+c)-\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}\,(\cos^{2}(dx+c))-231iF(i(\csc(dx+c)-\cot(dx+c$

```
[In] int((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/105/a^3/d*(e*sec(d*x+c))^(1/2)*e^7/(cos(d*x+c)+1)*(231*I*cos(d*x+c)^2*El
lipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
cos(d*x+c)+1))^(1/2)-231*I*cos(d*x+c)^2*EllipticF(I*(csc(d*x+c)-cot(d*x+c))
,I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+462*I*cos(d*
x+c)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*(1/(cos(d*x+c)+1))^(1/2)-462*I*cos(d*x+c)*EllipticF(I*(csc(d*x+c)-cot(d*
x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+231*I*(
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(csc(d*x+c)-cot(d*x+c)),I)*(1/
(cos(d*x+c)+1))^(1/2)-231*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(
csc(d*x+c)-cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)+140*I-231*sin(d*x+c)+140
*I*sec(d*x+c)+63*tan(d*x+c)-15*I*sec(d*x+c)^2+63*sec(d*x+c)*tan(d*x+c)-15*I
*sec(d*x+c)^3)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.20

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^3}\,dx =$$
$$-\frac{2\left(\sqrt{2}\left(231i\,e^{7}e^{(7i\,dx+7i\,c)}+847i\,e^{7}e^{(5i\,dx+5i\,c)}+1133i\,e^{7}e^{(3i\,dx+3i\,c)}+637i\,e^{7}e^{(i\,dx+i\,c)}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}\right.}}{105\,(a^{3}de^{(6i\,dx+6i}}$$

```
[In] integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -2/105*(sqrt(2)*(231*I*e^7*e^(7*I*d*x + 7*I*c) + 847*I*e^7*e^(5*I*d*x + 5*I
*c) + 1133*I*e^7*e^(3*I*d*x + 3*I*c) + 637*I*e^7*e^(I*d*x + I*c))*sqrt(e/(e
^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 231*sqrt(2)*(I*e^7*e^(6*
I*d*x + 6*I*c) + 3*I*e^7*e^(4*I*d*x + 4*I*c) + 3*I*e^7*e^(2*I*d*x + 2*I*c)
+ I*e^7)*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x
 + I*c))))/(a^3*d*e^(6*I*d*x + 6*I*c) + 3*a^3*d*e^(4*I*d*x + 4*I*c) + 3*a^3
*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((e*sec(d*x+c))**(15/2)/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{(e\sec(dx+c))^{\frac{15}{2}}}{(ia\tan(dx+c)+a)^3}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(15/2)/(I*a*tan(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{15/2}}{(a+a\tan\left(c+d\,x\right)\,1\mathrm{i})^3}\,dx$$

```
[In] int((e/cos(c + d*x))^(15/2)/(a + a*tan(c + d*x)*1i)^3,x)
[Out] int((e/cos(c + d*x))^(15/2)/(a + a*tan(c + d*x)*1i)^3, x)
```

3.246 $\int \frac{(e \sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^3} dx$

Optimal result

Integrand size = 28, antiderivative size = 141

$$\int \frac{(e \sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^3} dx = \frac{6e^6 \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right) \sqrt{e\sec(c+dx)}}{a^3 d} - \frac{18ie^4(e\sec(c+dx))^{5/2}}{5a^3d} + \frac{6e^5(e\sec(c+dx))^{3/2}\sin(c+dx)}{a^3d} - \frac{4ie^2(e\sec(c+dx))^{9/2}}{ad(a+ia\tan(c+dx))^2}$$

```
[Out] -18/5*I*e^4*(e*sec(d*x+c))^(5/2)/a^3/d+6*e^5*(e*sec(d*x+c))^(3/2)*sin(d*x+c
)/a^3/d+6*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/a^3/d-4*I*e^
2*(e*sec(d*x+c))^(9/2)/a/d/(a+I*a*tan(d*x+c))^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3581, 3582, 3853, 3856, 2720}

$$\int \frac{(e \sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^3} dx = \frac{6e^6 \sqrt{\cos(c+dx)} \operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right) \sqrt{e\sec(c+dx)}}{a^3 d} + \frac{6e^5\sin(c+dx)(e\sec(c+dx))^{3/2}}{a^3d} - \frac{18ie^4(e\sec(c+dx))^{5/2}}{5a^3d} - \frac{4ie^2(e\sec(c+dx))^{9/2}}{ad(a+ia\tan(c+dx))^2}$$

```
[In] Int[(e*Sec[c + d*x])^(13/2)/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (6*e^6*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(
a^3*d) - (((18*I)/5)*e^4*(e*Sec[c + d*x])^(5/2))/(a^3*d) + (6*e^5*(e*Sec[c
```

```
+ d*x])^(3/2)*Sin[c + d*x])/(a^3*d) - ((4*I)*e^2*(e*Sec[c + d*x])^(9/2))/(a
*d*(a + I*a*Tan[c + d*x])^2)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{4ie^2(e\sec(c+dx))^{9/2}}{ad(a+ia\tan(c+dx))^2}+\frac{(9e^2)\int\frac{(e\sec(c+dx))^{9/2}}{a+ia\tan(c+dx)}\,dx}{a^2}\\
&= -\frac{18ie^4(e\sec(c+dx))^{5/2}}{5a^3d}-\frac{4ie^2(e\sec(c+dx))^{9/2}}{ad(a+ia\tan(c+dx))^2}+\frac{(9e^4)\int(e\sec(c+dx))^{5/2}\,dx}{a^3}
\end{aligned}$$

$$= -\frac{18ie^4(e\sec(c+dx))^{5/2}}{5a^3d} + \frac{6e^5(e\sec(c+dx))^{3/2}\sin(c+dx)}{a^3d} - \frac{4ie^2(e\sec(c+dx))^{9/2}}{ad(a+ia\tan(c+dx))^2} + \frac{(3e^6)\int\sqrt{e\sec(c+dx)}\,dx}{a^3}$$

$$= -\frac{18ie^4(e\sec(c+dx))^{5/2}}{5a^3d} + \frac{6e^5(e\sec(c+dx))^{3/2}\sin(c+dx)}{a^3d} - \frac{4ie^2(e\sec(c+dx))^{9/2}}{ad(a+ia\tan(c+dx))^2} + \frac{\left(3e^6\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{a^3}$$

$$= \frac{6e^6\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{a^3d} - \frac{18ie^4(e\sec(c+dx))^{5/2}}{5a^3d} + \frac{6e^5(e\sec(c+dx))^{3/2}\sin(c+dx)}{a^3d} - \frac{4ie^2(e\sec(c+dx))^{9/2}}{ad(a+ia\tan(c+dx))^2}$$

Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.52

$$\int\frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{e^4(e\sec(c+dx))^{5/2}\left(-18i-20i\cos(2(c+dx))+30\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(\right.\right.}{5a^3d}$$

```
[In] Integrate[(e*Sec[c + d*x])^(13/2)/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (e^4*(e*Sec[c + d*x])^(5/2)*(-18*I - (20*I)*Cos[2*(c + d*x)] + 30*Cos[c + d
*x]^(5/2)*EllipticF[(c + d*x)/2, 2] - 5*Sin[2*(c + d*x)]))/(5*a^3*d)
```

Maple [A] (verified)

Time = 8.77 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.13

method	result
default	$\frac{2e^6\sqrt{e\sec(dx+c)}\left(15i\cos(dx+c)F(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}+15iF(i(-\csc(dx+c)+\cot(dx+c)),i)\right.}{5a^3d}$

```
[In] int((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*e^6/a^3/d*(e*sec(d*x+c))^(1/2)*(15*I*cos(d*x+c)*EllipticF(I*(-csc(d*x+c
)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)
+15*I*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(cos(d*x+c)+1))^(1/2)-20*I-5*tan(d*x+c)+I*sec(d*x+c)^2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.12

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^3}\,dx = -\frac{2\left(\sqrt{2}\left(15i\,e^6e^{(4i\,dx+4i\,c)}+36i\,e^6e^{(2i\,dx+2i\,c)}+25i\,e^6\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+15\sqrt{2}\left(i\,e^6e^{(4i\,dx+4i\,c)}+2i\,e^6\cdots\right.\right.}{5\left(a^3de^{(4i\,dx+4i\,c)}+2\,a^3de^{(2i\,dx+2i\,c)}+a^3d\right)}$$

```
[In] integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -2/5*(sqrt(2)*(15*I*e^6*e^(4*I*d*x + 4*I*c) + 36*I*e^6*e^(2*I*d*x + 2*I*c)
+ 25*I*e^6)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 15*
sqrt(2)*(I*e^6*e^(4*I*d*x + 4*I*c) + 2*I*e^6*e^(2*I*d*x + 2*I*c) + I*e^6)*s
qrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))/(a^3*d*e^(4*I*d*x + 4*I
*c) + 2*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((e*sec(d*x+c))**(13/2)/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{(e\sec(dx+c))^{\frac{13}{2}}}{(i\,a\tan(dx+c)+a)^3}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(13/2)/(I*a*tan(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{13/2}}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^3}\,dx$$

```
[In] int((e/cos(c + d*x))^(13/2)/(a + a*tan(c + d*x)*1i)^3,x)
[Out] int((e/cos(c + d*x))^(13/2)/(a + a*tan(c + d*x)*1i)^3, x)
```

3.247 $\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 141

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{14e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{14ie^4(e\sec(c+dx))^{3/2}}{3a^3d} - \frac{14e^5\sqrt{e\sec(c+dx)}\sin(c+dx)}{a^3d} + \frac{4ie^2(e\sec(c+dx))^{7/2}}{ad(a+ia\tan(c+dx))^2}$$

```
[Out] 14/3*I*e^4*(e*sec(d*x+c))^(3/2)/a^3/d+14*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d/cos(d*x+c)^(1
/2)/(e*sec(d*x+c))^(1/2)-14*e^5*sin(d*x+c)*(e*sec(d*x+c))^(1/2)/a^3/d+4*I*e
^2*(e*sec(d*x+c))^(7/2)/a/d/(a+I*a*tan(d*x+c))^2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3581, 3582, 3853, 3856, 2719}

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{14e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{14e^5\sin(c+dx)\sqrt{e\sec(c+dx)}}{a^3d} + \frac{14ie^4(e\sec(c+dx))^{3/2}}{3a^3d} + \frac{4ie^2(e\sec(c+dx))^{7/2}}{ad(a+ia\tan(c+dx))^2}$$

```
[In] Int[(e*Sec[c + d*x])^(11/2)/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (14*e^6*EllipticE[(c + d*x)/2, 2])/(a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c +
 d*x]]) + (((14*I)/3)*e^4*(e*Sec[c + d*x])^(3/2))/(a^3*d) - (14*e^5*Sqrt[e*
```

```
Sec[c + d*x]]*Sin[c + d*x])/(a^3*d) + ((4*I)*e^2*(e*Sec[c + d*x])^(7/2))/(a
*d*(a + I*a*Tan[c + d*x])^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{4ie^2(e\sec(c+dx))^{7/2}}{ad(a+ia\tan(c+dx))^2}-\frac{(7e^2)\int\frac{(e\sec(c+dx))^{7/2}}{a+ia\tan(c+dx)}\,dx}{a^2}\\ &= \frac{14ie^4(e\sec(c+dx))^{3/2}}{3a^3d}+\frac{4ie^2(e\sec(c+dx))^{7/2}}{ad(a+ia\tan(c+dx))^2}-\frac{(7e^4)\int(e\sec(c+dx))^{3/2}\,dx}{a^3}\end{aligned}$$

$$= \frac{14ie^4(e\sec(c+dx))^{3/2}}{3a^3d} - \frac{14e^5\sqrt{e\sec(c+dx)}\sin(c+dx)}{a^3d} + \frac{4ie^2(e\sec(c+dx))^{7/2}}{ad(a+ia\tan(c+dx))^2} + \frac{(7e^6)\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{a^3}$$

$$= \frac{14ie^4(e\sec(c+dx))^{3/2}}{3a^3d} - \frac{14e^5\sqrt{e\sec(c+dx)}\sin(c+dx)}{a^3d} + \frac{4ie^2(e\sec(c+dx))^{7/2}}{ad(a+ia\tan(c+dx))^2} + \frac{(7e^6)\int \sqrt{\cos(c+dx)}dx}{a^3\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$= \frac{14e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{14ie^4(e\sec(c+dx))^{3/2}}{3a^3d} - \frac{14e^5\sqrt{e\sec(c+dx)}\sin(c+dx)}{a^3d} + \frac{4ie^2(e\sec(c+dx))^{7/2}}{ad(a+ia\tan(c+dx))^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.85 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.66

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^3}dx = \frac{ie^4(e\sec(c+dx))^{3/2}\left(35+33\cos(2(c+dx))-7\left(1+e^{2i(c+dx)}\right)^{3/2}\text{Hypergeom}\right.}{3a^3d}$$

```
[In] Integrate[(e*Sec[c + d*x])^(11/2)/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((I/3)*e^4*(e*Sec[c + d*x])^(3/2)*(35 + 33*Cos[2*(c + d*x)] - 7*(1 + E^((2*
I)*(c + d*x)))^(3/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]
 + (9*I)*Sin[2*(c + d*x)]))/(a^3*d)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 443 vs. 2(152) = 304.

Time = 10.35 (sec) , antiderivative size = 444, normalized size of antiderivative = 3.15

method	result
default	$\frac{2\sqrt{e\sec(dx+c)}\,e^5\left(21i(\cos^2(dx+c))F(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}-21i(\cos^2(dx+c))E(i(-\csc(dx+\right.}{}$

```
[In] int((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/a^3/d*(e*sec(d*x+c))^(1/2)*e^5/(cos(d*x+c)+1)*(21*I*cos(d*x+c)^2*(1/(co
s(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+
```

```
c)+cot(d*x+c)),I)-21*I*cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)+42*I*cos(d*x+c)
*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-c
sc(d*x+c)+cot(d*x+c)),I)-42*I*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)+21*I*(1/(c
os(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)-21*I*(1/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+c)+c
ot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+12*I*cos(d*x+c)^2+12*I*cos(
d*x+c)+12*sin(d*x+c)*cos(d*x+c)+I-9*sin(d*x+c)+I*sec(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.06

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^3}\,dx = -\frac{2\left(\sqrt{2}\left(-21i\,e^5e^{(4i\,dx+4i\,c)}-35i\,e^5e^{(2i\,dx+2i\,c)}-12i\,e^5\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+21\,\sqrt{2}\left(-i\,e^5e^{(3i\,dx+3i\,c)}-i\,e\right.}{3\left(a^3de^{(3i\,dx+3i\,c)}+a^3de^{(i\,dx+i\,c)}\right)}$$

```
[In] integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(2)*(-21*I*e^5*e^(4*I*d*x + 4*I*c) - 35*I*e^5*e^(2*I*d*x + 2*I*c)
 - 12*I*e^5)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 21
*sqrt(2)*(-I*e^5*e^(3*I*d*x + 3*I*c) - I*e^5*e^(I*d*x + I*c))*sqrt(e)*weier
strassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x + I*c))))/(a^3*d*e^(3
*I*d*x + 3*I*c) + a^3*d*e^(I*d*x + I*c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((e*sec(d*x+c))**(11/2)/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{(e\sec(dx+c))^{\frac{11}{2}}}{(i\,a\tan(dx+c)+a)^3}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(11/2)/(I*a*tan(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{11/2}}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^3}\,dx$$

```
[In] int((e/cos(c + d*x))^(11/2)/(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] int((e/cos(c + d*x))^(11/2)/(a + a*tan(c + d*x)*1i)^3, x)
```

3.248 $\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^3} dx$

Optimal result

Integrand size = 28, antiderivative size = 116

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^3} dx = \frac{10ie^4\sqrt{e\sec(c+dx)}}{3a^3d} - \frac{10e^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3a^3d} + \frac{4ie^2(e\sec(c+dx))^{5/2}}{3ad(a+ia\tan(c+dx))^2}$$

```
[Out] 10/3*I*e^4*(e*sec(d*x+c))^(1/2)/a^3/d-10/3*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*
(e*sec(d*x+c))^(1/2)/a^3/d+4/3*I*e^2*(e*sec(d*x+c))^(5/2)/a/d/(a+I*a*tan(d*
x+c))^2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3581, 3582, 3856, 2720}

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^3} dx = \frac{10ie^4\sqrt{e\sec(c+dx)}}{3a^3d} - \frac{10e^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3a^3d} + \frac{4ie^2(e\sec(c+dx))^{5/2}}{3ad(a+ia\tan(c+dx))^2}$$

```
[In] Int[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (((10*I)/3)*e^4*Sqrt[e*Sec[c + d*x]])/(a^3*d) - (10*e^4*Sqrt[Cos[c + d*x]]*
EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(3*a^3*d) + (((4*I)/3)*e^2*
(e*Sec[c + d*x])^(5/2))/(a*d*(a + I*a*Tan[c + d*x])^2)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{4ie^2(e\sec(c+dx))^{5/2}}{3ad(a+ia\tan(c+dx))^2} - \frac{(5e^2)\int\frac{(e\sec(c+dx))^{5/2}}{a+ia\tan(c+dx)}\,dx}{3a^2} \\
&= \frac{10ie^4\sqrt{e\sec(c+dx)}}{3a^3d} + \frac{4ie^2(e\sec(c+dx))^{5/2}}{3ad(a+ia\tan(c+dx))^2} - \frac{(5e^4)\int\sqrt{e\sec(c+dx)}\,dx}{3a^3} \\
&= \frac{10ie^4\sqrt{e\sec(c+dx)}}{3a^3d} + \frac{4ie^2(e\sec(c+dx))^{5/2}}{3ad(a+ia\tan(c+dx))^2} \\
&\quad - \frac{\left(5e^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3a^3} \\
&= \frac{10ie^4\sqrt{e\sec(c+dx)}}{3a^3d} - \frac{10e^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{3a^3d} \\
&\quad + \frac{4ie^2(e\sec(c+dx))^{5/2}}{3ad(a+ia\tan(c+dx))^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.08

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{2e^4\sec^3(c+dx)\sqrt{e\sec(c+dx)}\left(-7i\cos(c+dx)+5\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\right.\right.}{3a^3d}$$

[In] Integrate[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x])^3,x]

[Out] (2*e^4*Sec[c + d*x]^3*Sqrt[e*Sec[c + d*x]]*((-7*I)*Cos[c + d*x] + 5*Sqrt[Co s[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[c + d*x] + I*Sin[c + d*x]) + 3*S in[c + d*x])*((-I)*Cos[2*(c + d*x)] + Sin[2*(c + d*x)]))/(3*a^3*d*(-I + Tan [c + d*x])^3)

Maple [A] (verified)

Time = 9.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.01

method	result
default	$\frac{2\left(i(-5\cos(dx+c)-5)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}\,F(i(-\csc(dx+c)+\cot(dx+c)),i)+i\left(4\left(\cos^2(dx+c)\right)+3\right)+4\sin(dx+c)\cos(dx+c)\right)\sqrt{}}{3a^3d}$

[In] int((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 2/3/a^3/d*(I*(-5*cos(d*x+c)-5)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d* x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)+I*(4*cos(d*x+c)^2+3) +4*sin(d*x+c)*cos(d*x+c))*(e*sec(d*x+c))^(1/2)*e^4

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.84

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^3}\,dx =$$
$$-\frac{2\left(-5i\sqrt{2}e^{\frac{9}{2}}e^{(2i\,dx+2i\,c)}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(-5i\,e^4e^{(2i\,dx+2i\,c)}-2i\,e^4\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{(}\right.}{3\,a^3d}$$

[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")

[Out] -2/3*(-5*I*sqrt(2)*e^(9/2)*e^(2*I*d*x + 2*I*c)*weierstrassPInverse(-4, 0, e ^(I*d*x + I*c)) + sqrt(2)*(-5*I*e^4*e^(2*I*d*x + 2*I*c) - 2*I*e^4)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-2*I*d*x - 2*I*c)/(a^ 3*d)

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((e*sec(d*x+c))**(9/2)/(a+I*a*tan(d*x+c))**3,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{(e\sec(dx+c))^{\frac{9}{2}}}{(i\,a\tan(dx+c)+a)^3}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(9/2)/(I*a*tan(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{9/2}}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^3}\,dx$$

```
[In] int((e/cos(c + d*x))^(9/2)/(a + a*tan(c + d*x)*1i)^3,x)
[Out] int((e/cos(c + d*x))^(9/2)/(a + a*tan(c + d*x)*1i)^3, x)
```

3.249 $\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 116

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^3}\,dx = -\frac{6ie^4}{5a^3d\sqrt{e\sec(c+dx)}} - \frac{6e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{4ie^2(e\sec(c+dx))^{3/2}}{5ad(a+ia\tan(c+dx))^2}$$

```
[Out] -6/5*I*e^4/a^3/d/(e*sec(d*x+c))^(1/2)-6/5*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d/cos(d*x+c)^(
1/2)/(e*sec(d*x+c))^(1/2)+4/5*I*e^2*(e*sec(d*x+c))^(3/2)/a/d/(a+I*a*tan(d*x
+c))^2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3581, 3582, 3856, 2719}

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^3}\,dx = -\frac{6ie^4}{5a^3d\sqrt{e\sec(c+dx)}} - \frac{6e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{4ie^2(e\sec(c+dx))^{3/2}}{5ad(a+ia\tan(c+dx))^2}$$

```
[In] Int[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (((-6*I)/5)*e^4)/(a^3*d*Sqrt[e*Sec[c + d*x]]) - (6*e^4*EllipticE[(c + d*x)/
2, 2])/(5*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c + d*x]]) + (((4*I)/5)*e^2*(
e*Sec[c + d*x])^(3/2))/(a*d*(a + I*a*Tan[c + d*x])^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{4ie^2(e\sec(c+dx))^{3/2}}{5ad(a+ia\tan(c+dx))^2}-\frac{(3e^2)\int\frac{(e\sec(c+dx))^{3/2}}{a+ia\tan(c+dx)}\,dx}{5a^2}\\ &=-\frac{6ie^4}{5a^3d\sqrt{e\sec(c+dx)}}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{5ad(a+ia\tan(c+dx))^2}-\frac{(3e^4)\int\frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{5a^3}\\ &=-\frac{6ie^4}{5a^3d\sqrt{e\sec(c+dx)}}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{5ad(a+ia\tan(c+dx))^2}-\frac{(3e^4)\int\sqrt{\cos(c+dx)}\,dx}{5a^3\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\\ &=-\frac{6ie^4}{5a^3d\sqrt{e\sec(c+dx)}}-\frac{6e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{5ad(a+ia\tan(c+dx))^2}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.64 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.01

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{2ee^{-idx}\left(-2+\frac{6e^{2i(c+dx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}\right)(e\sec(c+dx))^{5/2}(\cos(c}{5a^3d(-i+\tan(c+dx))^3}$$

```
[In] Integrate[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (2*e*(-2 + (6*E^((2*I)*(c + d*x))*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*
I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))])*(e*Sec[c + d*x])^(5/2)*(Cos[
c + 2*d*x] + I*Sin[c + 2*d*x]))/(5*a^3*d*E^(I*d*x)*(-I + Tan[c + d*x])^3)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 472 vs. 2(125) = 250.

Time = 8.08 (sec) , antiderivative size = 473, normalized size of antiderivative = 4.08

method	result
default	$-\frac{2i\sqrt{e\sec(dx+c)}\left(4i(\cos^3(dx+c))\sin(dx+c)-3(\cos^2(dx+c))E(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}+3(\cos^2}{}$

```
[In] int((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*I/a^3/d*(e*sec(d*x+c))^(1/2)*(4*I*sin(d*x+c)*cos(d*x+c)^3-3*cos(d*x+c)
^2*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*(1/(cos(d*x+c)+1))^(1/2)+3*cos(d*x+c)^2*EllipticF(I*(-csc(d*x+c)+cot(d*x+
c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+4*I*cos(d
*x+c)^2*sin(d*x+c)-4*cos(d*x+c)^4-6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Ellip
ticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+6*(c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/
(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-3*I*cos(d*x+c)*sin(d*x+c)-4*cos(d*x+c)^3-3
*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*
(1/(cos(d*x+c)+1))^(1/2)+3*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(
d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+5*cos(d*x+c)^2+5*cos(d*x
+c))*e^3/(cos(d*x+c)+1)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.98

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^3}\,dx = -\frac{2\left(3i\sqrt{2}e^{\frac{7}{2}}e^{(3i\,dx+3i\,c)}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\sqrt{2}\left(3i\,e^{3}e^{(4i\,dx+4i\,c)}+\right.\right.}{5\,a^{3}d}$$

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -2/5*(3*I*sqrt(2)*e^(7/2)*e^(3*I*d*x + 3*I*c)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(3*I*e^3*e^(4*I*d*x + 4*I*
c) + 2*I*e^3*e^(2*I*d*x + 2*I*c) - I*e^3)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))
*e^(1/2*I*d*x + 1/2*I*c))*e^(-3*I*d*x - 3*I*c)/(a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{(e\sec(dx+c))^{\frac{7}{2}}}{(i\,a\tan(dx+c)+a)^3}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(7/2)/(I*a*tan(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^3}\,dx$$

```
[In] int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^3,x)
[Out] int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^3, x)
```

3.250 $\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 132

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^3}\,dx = -\frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{21a^3d} + \frac{4ie^2\sqrt{e\sec(c+dx)}}{7ad(a+ia\tan(c+dx))^2} - \frac{2ie^2\sqrt{e\sec(c+dx)}}{21d\left(a^3+ia^3\tan(c+dx)\right)}$$

```
[Out] -2/21*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/a^3/d+4/7*I*e^2*
(e*sec(d*x+c))^(1/2)/a/d/(a+I*a*tan(d*x+c))^2-2/21*I*e^2*(e*sec(d*x+c))^(1/
2)/d/(a^3+I*a^3*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3581, 3583, 3856, 2720}

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^3}\,dx = -\frac{2ie^2\sqrt{e\sec(c+dx)}}{21d\left(a^3+ia^3\tan(c+dx)\right)} - \frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{21a^3d} + \frac{4ie^2\sqrt{e\sec(c+dx)}}{7ad(a+ia\tan(c+dx))^2}$$

```
[In] Int[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (-2*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(21*a^3*d) + (((4*I)/7)*e^2*Sqrt[e*Sec[c + d*x]])/(a*d*(a + I*a*Tan[c + d*x
])^2) - (((2*I)/21)*e^2*Sqrt[e*Sec[c + d*x]])/(d*(a^3 + I*a^3*Tan[c + d*x])
)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{4ie^2\sqrt{e\sec(c+dx)}}{7ad(a+ia\tan(c+dx))^2} - \frac{e^2\int\frac{\sqrt{e\sec(c+dx)}}{a+ia\tan(c+dx)}\,dx}{7a^2} \\
&= \frac{4ie^2\sqrt{e\sec(c+dx)}}{7ad(a+ia\tan(c+dx))^2} - \frac{2ie^2\sqrt{e\sec(c+dx)}}{21d\,(a^3+ia^3\tan(c+dx))} - \frac{e^2\int\sqrt{e\sec(c+dx)}\,dx}{21a^3} \\
&= \frac{4ie^2\sqrt{e\sec(c+dx)}}{7ad(a+ia\tan(c+dx))^2} - \frac{2ie^2\sqrt{e\sec(c+dx)}}{21d\,(a^3+ia^3\tan(c+dx))} \\
&\quad - \frac{\left(e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21a^3}
\end{aligned}
$$

$$= -\frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{21a^3d} + \frac{4ie^2\sqrt{e\sec(c+dx)}}{7ad(a+ia\tan(c+dx))^2} - \frac{2ie^2\sqrt{e\sec(c+dx)}}{21d\,(a^3+ia^3\tan(c+dx))}$$

Mathematica [A] (verified)

Time = 1.57 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.79

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{(e\sec(c+dx))^{5/2}\left(-5i-5i\cos(2(c+dx))+2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+\right.\right.}{21a^3d(-i+\tan(c+}$$

```
[In] Integrate[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((e*Sec[c + d*x])^(5/2)*(-5*I - (5*I)*Cos[2*(c + d*x)] + 2*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]) - Sin[
2*(c + d*x)]))/(21*a^3*d*(-I + Tan[c + d*x])^2)
```

Maple [A] (verified)

Time = 6.61 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.45

method	result
default	$\frac{2e^2\left(12i\left(\cos^4(dx+c)\right)-i\cos(dx+c)F(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}-iF(i(-\csc(dx+c)+\cot(dx+c)),i)\right.}{21a^3d}$

```
[In] int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/21/a^3/d*e^2*(12*I*cos(d*x+c)^4-I*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-
csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-I*(1
/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)+12*cos(d*x+c)^3*sin(d*x+c)-7*I*cos(d*x+c)^2-sin(d*x+c
)*cos(d*x+c))*(e*sec(d*x+c))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.84

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{\left(2i\sqrt{2}e^{\frac{5}{2}}e^{(4i\,dx+4i\,c)}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(2i\,e^{2}e^{(4i\,dx+4i\,c)}\right.\right.}{21\,a^{3}d}$$

[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")

[Out] 1/21*(2*I*sqrt(2)*e^(5/2)*e^(4*I*d*x + 4*I*c)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)) + sqrt(2)*(2*I*e^2*e^(4*I*d*x + 4*I*c) + 5*I*e^2*e^(2*I*d*x + 2*I*c) + 3*I*e^2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-4*I*d*x - 4*I*c)/(a^3*d)

Sympy [F]

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{(e\sec(c+dx))^{\frac{5}{2}}}{\tan^{3}(c+dx)-3i\tan^{2}(c+dx)-3\tan(c+dx)+i}\,dx}{a^{3}}$$

[In] integrate((e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**3,x)

[Out] I*Integral((e*sec(c + d*x))**(5/2)/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*tan(c + d*x) + I), x)/a**3

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

Giac [F]

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{(e\sec(dx+c))^{\frac{5}{2}}}{(i\,a\tan(dx+c)+a)^3}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(5/2)/(I*a*tan(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^3}\,dx$$

```
[In] int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^3,x)
[Out] int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^3, x)
```

3.251 $\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 132

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{4ie^2}{9ad\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2} + \frac{2ie^2}{45d\sqrt{e\sec(c+dx)}\,(a^3+ia^3\tan(c+dx))}$$

```
[Out] 2/15*e^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))/a^3/d/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+4/9*I*e^2/a
/d/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2+2/45*I*e^2/d/(e*sec(d*x+c))^(1
/2)/(a^3+I*a^3*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3581, 3583, 3856, 2719}

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{2ie^2}{45d\,(a^3+ia^3\tan(c+dx))\,\sqrt{e\sec(c+dx)}} + \frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{4ie^2}{9ad(a+ia\tan(c+dx))^2\sqrt{e\sec(c+dx)}}$$

```
[In] Int[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (2*e^2*EllipticE[(c + d*x)/2, 2])/(15*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c
 + d*x]]) + (((4*I)/9)*e^2)/(a*d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x]
)^2) + (((2*I)/45)*e^2)/(d*Sqrt[e*Sec[c + d*x]]*(a^3 + I*a^3*Tan[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{4ie^2}{9ad\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2} + \frac{e^2\int\frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))}\,dx}{9a^2} \\
&= \frac{4ie^2}{9ad\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2} \\
&\quad + \frac{2ie^2}{45d\sqrt{e\sec(c+dx)}\,(a^3+ia^3\tan(c+dx))} + \frac{e^2\int\frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{15a^3} \\
&= \frac{4ie^2}{9ad\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2} \\
&\quad + \frac{2ie^2}{45d\sqrt{e\sec(c+dx)}\,(a^3+ia^3\tan(c+dx))} + \frac{e^2\int\sqrt{\cos(c+dx)}\,dx}{15a^3\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}
\end{aligned}
$$

$$= \frac{2e^2 E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^3 d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{4ie^2}{9ad\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2} + \frac{2ie^2}{45d\sqrt{e\sec(c+dx)}\,(a^3+ia^3\tan(c+dx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.74 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.06

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^3}\,dx = -\frac{e^{-idx}\sec^2(c+dx)(e\sec(c+dx))^{3/2}(\cos(dx)+i\sin(dx))\left(8+8\cos(2(c+dx))+6e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,\mathrm{Hy}\right.}{45a^3d(-i+\tan(c+dx))^3}$$

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] -1/45*(Sec[c + d*x]^2*(e*Sec[c + d*x])^(3/2)*(Cos[d*x] + I*Sin[d*x])*(8 + 8
*Cos[2*(c + d*x)] + 6*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyp
ergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + (3*I)*Sin[2*(c + d*x
)]))/(a^3*d*E^(I*d*x)*(-I + Tan[c + d*x])^3)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 506 vs. 2(140) = 280.

Time = 7.50 (sec) , antiderivative size = 507, normalized size of antiderivative = 3.84

method	result
default	$-\frac{2i\sqrt{e\sec(dx+c)}\left(20i(\cos^5(dx+c))\sin(dx+c)+20i\sin(dx+c)(\cos^4(dx+c))-20(\cos^6(dx+c))+i(\cos^3(dx+c))\sin(dx+c)-20(\cos^5(d\right.}{}$

```
[In] int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/45*I/a^3/d*(e*sec(d*x+c))^(1/2)*(20*I*cos(d*x+c)^5*sin(d*x+c)+20*I*cos(d
*x+c)^4*sin(d*x+c)-20*cos(d*x+c)^6+I*sin(d*x+c)*cos(d*x+c)^3-20*cos(d*x+c)^
5-3*cos(d*x+c)^2*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+3*cos(d*x+c)^2*EllipticE(I*(-csc(d
*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(
1/2)+I*cos(d*x+c)^2*sin(d*x+c)+9*cos(d*x+c)^4-6*(cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*cos
(d*x+c)+6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+c)+cot(d*
x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+3*I*sin(d*x+c)*cos(d*x+c)+9*co
```

```
s(d*x+c)^3-3*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/
2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*El
lipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2))*e/(cos(d*x+
c)+1)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.91

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{\left(12i\sqrt{2}e^{\frac{3}{2}}e^{(5i\,dx+5i\,c)}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right.\right.\right.}{}$$

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/90*(12*I*sqrt(2)*e^(3/2)*e^(5*I*d*x + 5*I*c)*weierstrassZeta(-4, 0, weier
strassPInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(12*I*e*e^(6*I*d*x + 6*I*
c) + 23*I*e*e^(4*I*d*x + 4*I*c) + 16*I*e*e^(2*I*d*x + 2*I*c) + 5*I*e)*sqrt(
e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-5*I*d*x - 5*I*c)/
(a^3*d)
```

Sympy [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{(e\sec(c+dx))^{\frac{3}{2}}}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

```
[In] integrate((e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] I*Integral((e*sec(c + d*x))**(3/2)/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 -
 3*tan(c + d*x) + I), x)/a**3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{(e\sec(dx+c))^{\frac{3}{2}}}{(i\,a\tan(dx+c)+a)^3}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(3/2)/(I*a*tan(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^3}\,dx$$

```
[In] int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^3,x)
[Out] int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^3, x)
```

3.252 $\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 152

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^3}\,dx = \frac{10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{77a^3d} + \frac{10e\sin(c+dx)}{77a^3d\sqrt{e\sec(c+dx)}} + \frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3} + \frac{20ie^2}{77d(e\sec(c+dx))^{3/2}\left(a^3+ia^3\tan(c+dx)\right)}$$

```
[Out] 10/77*e*sin(d*x+c)/a^3/d/(e*sec(d*x+c))^(1/2)+10/77*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(
1/2)*(e*sec(d*x+c))^(1/2)/a^3/d+2/11*I*(e*sec(d*x+c))^(1/2)/d/(a+I*a*tan(d*
x+c))^3+20/77*I*e^2/d/(e*sec(d*x+c))^(3/2)/(a^3+I*a^3*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3583, 3581, 3854, 3856, 2720}

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^3}\,dx = \frac{20ie^2}{77d\left(a^3+ia^3\tan(c+dx)\right)(e\sec(c+dx))^{3/2}} + \frac{10e\sin(c+dx)}{77a^3d\sqrt{e\sec(c+dx)}} + \frac{10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{77a^3d} + \frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3}$$

```
[In] Int[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(77*
a^3*d) + (10*e*Sin[c + d*x])/(77*a^3*d*Sqrt[e*Sec[c + d*x]]) + (((2*I)/11)*
Sqrt[e*Sec[c + d*x]])/(d*(a + I*a*Tan[c + d*x])^3) + (((20*I)/77)*e^2)/(d*(
e*Sec[c + d*x])^(3/2)*(a^3 + I*a^3*Tan[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\text{integral} = \frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3} + \frac{5\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx}{11a}$$

$$
\begin{aligned}
&= \frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3} + \frac{20ie^2}{77d(e\sec(c+dx))^{3/2}\,(a^3+ia^3\tan(c+dx))} \\
&\quad + \frac{(15e^2)\int \frac{1}{(e\sec(c+dx))^{3/2}}\,dx}{77a^3} \\
&= \frac{10e\sin(c+dx)}{77a^3d\sqrt{e\sec(c+dx)}} + \frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3} \\
&\quad + \frac{20ie^2}{77d(e\sec(c+dx))^{3/2}\,(a^3+ia^3\tan(c+dx))} + \frac{5\int \sqrt{e\sec(c+dx)}\,dx}{77a^3} \\
&= \frac{10e\sin(c+dx)}{77a^3d\sqrt{e\sec(c+dx)}} + \frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3} \\
&\quad + \frac{20ie^2}{77d(e\sec(c+dx))^{3/2}\,(a^3+ia^3\tan(c+dx))} \\
&\quad + \frac{\left(5\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{77a^3} \\
&= \frac{10\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{77a^3d} + \frac{10e\sin(c+dx)}{77a^3d\sqrt{e\sec(c+dx)}} \\
&\quad + \frac{2i\sqrt{e\sec(c+dx)}}{11d(a+ia\tan(c+dx))^3} + \frac{20ie^2}{77d(e\sec(c+dx))^{3/2}\,(a^3+ia^3\tan(c+dx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.85

$$
\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\sec^3(c+dx)\sqrt{e\sec(c+dx)}\left(46i\cos(c+dx)+22i\cos(3(c+dx))-15\sin(c+dx)+20\sqrt{\cos(c+dx)}\,\text{El}\right.}{154a^3d(-i+\tan(c+dx))^3}
$$

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((I/154)*Sec[c + d*x]^3*Sqrt[e*Sec[c + d*x]]*((46*I)*Cos[c + d*x] + (22*I)*
Cos[3*(c + d*x)] - 15*Sin[c + d*x] + 20*Sqrt[Cos[c + d*x]]*EllipticF[(c + d
*x)/2, 2]*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x)]) - 15*Sin[3*(c + d*x)]))/(
a^3*d*(-I + Tan[c + d*x])^3)
```

Maple [A] (verified)

Time = 6.30 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.93

method	result
default	$-\frac{2i\left(\left(-5\cos(dx+c)-5\right)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,F(i(-\csc(dx+c)+\cot(dx+c)),i)+i\sin(dx+c)\cos(dx+c)\left(28\left(\cos^4(dx+c)\right)+3\left(\mathrm{co}\right.\right.\right.}{77a^3d}$

```
[In] int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/77*I/a^3/d*((-5*cos(d*x+c)-5)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)+I*sin(d*x+c)*cos(d
*x+c)*(28*cos(d*x+c)^4+3*cos(d*x+c)^2+5)+cos(d*x+c)^4*(-28*cos(d*x+c)^2+11)
)*(e*sec(d*x+c))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^3}\,dx = \frac{\left(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(37i\,e^{(6i\,dx+6i\,c)}+61i\,e^{(4i\,dx+4i\,c)}+31i\,e^{(2i\,dx+2i\,c)}+7i\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}-40i\,\sqrt{2}\sqrt{e}e^{(6i\,dx+6i\,c)}\mathrm{w}\right.}{308\,a^3d}$$

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/308*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(37*I*e^(6*I*d*x + 6*I*c)
+ 61*I*e^(4*I*d*x + 4*I*c) + 31*I*e^(2*I*d*x + 2*I*c) + 7*I)*e^(1/2*I*d*x +
 1/2*I*c) - 40*I*sqrt(2)*sqrt(e)*e^(6*I*d*x + 6*I*c)*weierstrassPInverse(-4
, 0, e^(I*d*x + I*c)))*e^(-6*I*d*x - 6*I*c)/(a^3*d)
```

Sympy [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{\sqrt{e\sec{(c+dx)}}}{\tan^3{(c+dx)}-3i\tan^2{(c+dx)}-3\tan{(c+dx)}+i}\,dx}{a^3}$$

```
[In] integrate((e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] I*Integral(sqrt(e*sec(c + d*x))/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*
tan(c + d*x) + I), x)/a**3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{\sqrt{e\sec(dx+c)}}{(i\,a\tan(dx+c)+a)^3}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
[Out] integrate(sqrt(e*sec(d*x + c))/(I*a*tan(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}}{(a+a\tan(c+d\,x)\,1\mathrm{i})^3}\,dx$$

```
[In] int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^3,x)
[Out] int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^3, x)
```

3.253 $\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 152

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}\,dx = \frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{14e\sin(c+dx)}{117a^3d(e\sec(c+dx))^{3/2}} + \frac{2i}{13d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3} + \frac{28ie^2}{117d(e\sec(c+dx))^{5/2}\left(a^3+ia^3\tan(c+dx)\right)}$$

```
[Out] 14/117*e*sin(d*x+c)/a^3/d/(e*sec(d*x+c))^(3/2)+14/39*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d/cos(d
*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+2/13*I/d/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d
*x+c))^3+28/117*I*e^2/d/(e*sec(d*x+c))^(5/2)/(a^3+I*a^3*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used

= {3583, 3581, 3854, 3856, 2719}

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}\,dx = \frac{28ie^2}{117d\left(a^3+ia^3\tan(c+dx)\right)(e\sec(c+dx))^{5/2}} + \frac{14e\sin(c+dx)}{117a^3d(e\sec(c+dx))^{3/2}} + \frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2i}{13d(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}$$

```
[In] Int[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^3),x]
```

```
[Out] (14*EllipticE[(c + d*x)/2, 2])/(39*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c +
d*x]]) + (14*e*Sin[c + d*x])/(117*a^3*d*(e*Sec[c + d*x])^(3/2)) + ((2*I)/13
)/(d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^3) + (((28*I)/117)*e^2)/(d
*(e*Sec[c + d*x])^(5/2)*(a^3 + I*a^3*Tan[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2i}{13d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}+\frac{7\int\frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}\,dx}{13a}\\
&= \frac{2i}{13d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}\\
&\quad+\frac{28ie^2}{117d(e\sec(c+dx))^{5/2}\,(a^3+ia^3\tan(c+dx))}+\frac{(35e^2)\int\frac{1}{(e\sec(c+dx))^{5/2}}\,dx}{117a^3}\\
&= \frac{14e\sin(c+dx)}{117a^3d(e\sec(c+dx))^{3/2}}+\frac{2i}{13d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}\\
&\quad+\frac{28ie^2}{117d(e\sec(c+dx))^{5/2}\,(a^3+ia^3\tan(c+dx))}+\frac{7\int\frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{39a^3}\\
&= \frac{14e\sin(c+dx)}{117a^3d(e\sec(c+dx))^{3/2}}+\frac{2i}{13d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}\\
&\quad+\frac{28ie^2}{117d(e\sec(c+dx))^{5/2}\,(a^3+ia^3\tan(c+dx))}\\
&\quad+\frac{7\int\sqrt{\cos(c+dx)}\,dx}{39a^3\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\\
&= \frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39a^3d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{14e\sin(c+dx)}{117a^3d(e\sec(c+dx))^{3/2}}\\
&\quad+\frac{2i}{13d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}\\
&\quad+\frac{28ie^2}{117d(e\sec(c+dx))^{5/2}\,(a^3+ia^3\tan(c+dx))}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.18 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{\sqrt{e\sec(c+dx)}(i\cos(3(c+dx))+\sin(3(c+dx)))\left(62+176\cos(2(c+dx))+114\cos(4(c+dx))-56e^{4i}\right.}{468a}$$

[In] Integrate[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^3),x]

[Out] (Sqrt[e*Sec[c + d*x]]*(I*Cos[3*(c + d*x)] + Sin[3*(c + d*x)])*(62 + 176*Cos[2*(c + d*x)] + 114*Cos[4*(c + d*x)] - 56*E^((4*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + (126*I)*Sin[2*(c + d*x)] + (105*I)*Sin[4*(c + d*x)]))/(468*a^3*d*e)

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 527 vs. 2(156) = 312.

Time = 8.70 (sec) , antiderivative size = 528, normalized size of antiderivative = 3.47

method	result
default	$-\frac{2i\left(7i(\cos^2(dx+c))\sin(dx+c)+5i\sin(dx+c)(\cos^4(dx+c))-36(\cos^7(dx+c))+36i(\cos^6(dx+c))\sin(dx+c)-36(\cos^6(dx+c))+5i(\cos\right.}{}$

[In] int(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] -2/117*I/a^3/d/(cos(d*x+c)+1)/(e*sec(d*x+c))^(1/2)*(7*I*cos(d*x+c)^2*sin(d*x+c)+5*I*sin(d*x+c)*cos(d*x+c)^4-36*cos(d*x+c)^7+36*I*cos(d*x+c)^6*sin(d*x+c)-36*cos(d*x+c)^6+5*I*cos(d*x+c)^3*sin(d*x+c)+13*cos(d*x+c)^5+21*I*sin(d*x+c)+13*cos(d*x+c)^4+21*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-21*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+7*I*cos(d*x+c)*sin(d*x+c)+42*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)-42*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+36*I*cos(d*x+c)^5*sin(d*x+c)+21*sec(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)-21*sec(d*x+c)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2))

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{e \sec(c+dx)}(a+ia\tan(c+dx))^3} dx = \frac{\left(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(219i\,e^{(8i\,dx+8i\,c)}+302i\,e^{(6i\,dx+6i\,c)}+124i\,e^{(4i\,dx+4i\,c)}+50i\,e^{(2i\,dx+2i\,c)}+9i\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+3\right.}{936\,a^3de}$$

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas"
)
```

```
[Out] 1/936*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(219*I*e^(8*I*d*x + 8*I*c)
 + 302*I*e^(6*I*d*x + 6*I*c) + 124*I*e^(4*I*d*x + 4*I*c) + 50*I*e^(2*I*d*x
+ 2*I*c) + 9*I)*e^(1/2*I*d*x + 1/2*I*c) + 336*I*sqrt(2)*sqrt(e)*e^(7*I*d*x
+ 7*I*c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x + I*c))
))*e^(-7*I*d*x - 7*I*c)/(a^3*d*e)
```

Sympy [F]

$$\int \frac{1}{\sqrt{e \sec(c+dx)}(a+ia\tan(c+dx))^3} dx = \frac{i\int \frac{1}{\sqrt{e\sec{(c+dx)}}\tan^3{(c+dx)}-3i\sqrt{e\sec{(c+dx)}}\tan^2{(c+dx)}-3\sqrt{e\sec{(c+dx)}}\tan{(c+dx)}+i\sqrt{e\sec{(c+dx)}}}\,dx}{a^3}$$

```
[In] integrate(1/(e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] I*Integral(1/(sqrt(e*sec(c + d*x))*tan(c + d*x)**3 - 3*I*sqrt(e*sec(c + d*x
))*tan(c + d*x)**2 - 3*sqrt(e*sec(c + d*x))*tan(c + d*x) + I*sqrt(e*sec(c +
 d*x))), x)/a**3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{e \sec(c+dx)}(a+ia\tan(c+dx))^3} dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}\,dx = \int \frac{1}{\sqrt{e\sec\left(dx+c\right)}{\left(i\,a\tan\left(dx+c\right)+a\right)}^3}\,dx$$

[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")

[Out] integrate(1/(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^3), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3}\,dx = \int \frac{1}{\sqrt{\frac{e}{\cos(c+d\,x)}}\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^3}\,dx$$

[In] int(1/((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^3),x)

[Out] int(1/((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^3), x)

3.254 $\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 186

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{11a^3de^2} + \frac{6e\sin(c+dx)}{55a^3d(e\sec(c+dx))^{5/2}} + \frac{2\sin(c+dx)}{11a^3de\sqrt{e\sec(c+dx)}} + \frac{2i}{15d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3} + \frac{12ie^2}{55d(e\sec(c+dx))^{7/2}\left(a^3+ia^3\tan(c+dx)\right)}$$

```
[Out] 6/55*e*sin(d*x+c)/a^3/d/(e*sec(d*x+c))^(5/2)+2/11*sin(d*x+c)/a^3/d/e/(e*sec
(d*x+c))^(1/2)+2/11*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/a^3/d/
e^2+2/15*I/d/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3+12/55*I*e^2/d/(e*sec
(d*x+c))^(7/2)/(a^3+I*a^3*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3583, 3581, 3854, 3856, 2720}

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}\,dx = \frac{12ie^2}{55d\left(a^3+ia^3\tan(c+dx)\right)(e\sec(c+dx))^{7/2}} + \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{11a^3de^2} + \frac{6e\sin(c+dx)}{55a^3d(e\sec(c+dx))^{5/2}} + \frac{2\sin(c+dx)}{11a^3de\sqrt{e\sec(c+dx)}} + \frac{2i}{15d(a+ia\tan(c+dx))^3(e\sec(c+dx))^{3/2}}$$

```
[In] Int[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^3),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(11*a
^3*d*e^2) + (6*e*Sin[c + d*x])/(55*a^3*d*(e*Sec[c + d*x])^(5/2)) + (2*Sin[c
 + d*x])/(11*a^3*d*e*Sqrt[e*Sec[c + d*x]]) + ((2*I)/15)/(d*(e*Sec[c + d*x])
^(3/2)*(a + I*a*Tan[c + d*x])^3) + (((12*I)/55)*e^2)/(d*(e*Sec[c + d*x])^(7
/2)*(a^3 + I*a^3*Tan[c + d*x]))
```

<u>Rule 2720</u>

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

<u>Rule 3581</u>

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

<u>Rule 3583</u>

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

<u>Rule 3854</u>

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

<u>Rule 3856</u>

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2i}{15d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}+\frac{3\int\frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx}{5a}\\
&= \frac{2i}{15d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}\\
&\quad+\frac{12ie^2}{55d(e\sec(c+dx))^{7/2}\left(a^3+ia^3\tan(c+dx)\right)}+\frac{\left(21e^2\right)\int\frac{1}{(e\sec(c+dx))^{7/2}}\,dx}{55a^3}\\
&= \frac{6e\sin(c+dx)}{55a^3d(e\sec(c+dx))^{5/2}}+\frac{2i}{15d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}\\
&\quad+\frac{12ie^2}{55d(e\sec(c+dx))^{7/2}\left(a^3+ia^3\tan(c+dx)\right)}+\frac{3\int\frac{1}{(e\sec(c+dx))^{3/2}}\,dx}{11a^3}\\
&= \frac{6e\sin(c+dx)}{55a^3d(e\sec(c+dx))^{5/2}}+\frac{2\sin(c+dx)}{11a^3de\sqrt{e\sec(c+dx)}}\\
&\quad+\frac{2i}{15d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}\\
&\quad+\frac{12ie^2}{55d(e\sec(c+dx))^{7/2}\left(a^3+ia^3\tan(c+dx)\right)}+\frac{\int\sqrt{e\sec(c+dx)}\,dx}{11a^3e^2}\\
&= \frac{6e\sin(c+dx)}{55a^3d(e\sec(c+dx))^{5/2}}+\frac{2\sin(c+dx)}{11a^3de\sqrt{e\sec(c+dx)}}\\
&\quad+\frac{2i}{15d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}\\
&\quad+\frac{12ie^2}{55d(e\sec(c+dx))^{7/2}\left(a^3+ia^3\tan(c+dx)\right)}\\
&\quad+\frac{\left(\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{11a^3e^2}\\
&= \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{11a^3de^2}+\frac{6e\sin(c+dx)}{55a^3d(e\sec(c+dx))^{5/2}}\\
&\quad+\frac{2\sin(c+dx)}{11a^3de\sqrt{e\sec(c+dx)}}+\frac{2i}{15d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}\\
&\quad+\frac{12ie^2}{55d(e\sec(c+dx))^{7/2}\left(a^3+ia^3\tan(c+dx)\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.67 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.81

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}\,dx = \frac{\sec^5(c+dx)\left(-332\cos(c+dx)-154\cos(3(c+dx))+22\text{c}\right.}{}$$

```
[In] Integrate[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^3),x]
```

```
[Out] (Sec[c + d*x]^5*(-332*Cos[c + d*x] - 154*Cos[3*(c + d*x)] + 22*Cos[5*(c + d
*x)] - (114*I)*Sin[c + d*x] + (240*I)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x
)/2, 2]*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x)]) - (81*I)*Sin[3*(c + d*x)] +
 (33*I)*Sin[5*(c + d*x)]))/(1320*a^3*d*(e*Sec[c + d*x])^(3/2)*(-I + Tan[c +
 d*x])^3)
```

Maple [A] (verified)

Time = 9.31 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.17

method	result
default	$\frac{\frac{8i\left(\cos^7(dx+c)\right)}{15}+\frac{8\sin(dx+c)\left(\cos^6(dx+c)\right)}{15}-\frac{2i\left(\cos^5(dx+c)\right)}{11}+\frac{14\sin(dx+c)\left(\cos^4(dx+c)\right)}{165}+\frac{2iF(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\sqrt{}}{11}}{a^3d\sqrt{e\sec(dx+}}$

```
[In] int(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/165/a^3/d/(e*sec(d*x+c))^(1/2)/e*(44*I*cos(d*x+c)^7+44*sin(d*x+c)*cos(d*x
+c)^6-15*I*cos(d*x+c)^5+7*sin(d*x+c)*cos(d*x+c)^4+15*I*EllipticF(I*(-csc(d*
x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1
/2)+9*cos(d*x+c)^2*sin(d*x+c)+15*I*sec(d*x+c)*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)+15*si
n(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.74

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}\,dx = \frac{\left(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-55i\,e^{(10i\,dx+10i\,c)}+235i\,e^{(8i\,dx+8i\,c)}+446\right.\right.}{}$$

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas"
)
```

```
[Out] 1/2640*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-55*I*e^(10*I*d*x + 10*I
*c) + 235*I*e^(8*I*d*x + 8*I*c) + 446*I*e^(6*I*d*x + 6*I*c) + 218*I*e^(4*I*
d*x + 4*I*c) + 73*I*e^(2*I*d*x + 2*I*c) + 11*I)*e^(1/2*I*d*x + 1/2*I*c) - 4
80*I*sqrt(2)*sqrt(e)*e^(8*I*d*x + 8*I*c)*weierstrassPInverse(-4, 0, e^(I*d*
x + I*c)))*e^(-8*I*d*x - 8*I*c)/(a^3*d*e^2)
```

Sympy [F]

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{1}{(e\sec(c+dx))^{\frac{3}{2}}\tan^3(c+dx)-3i(e\sec(c+dx))^{\frac{3}{2}}\tan^2(c+dx)-3(e\sec(c+dx))^{\frac{3}{2}}}}{a^3}$$

```
[In] integrate(1/(e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] I*Integral(1/((e*sec(c + d*x))**(3/2)*tan(c + d*x)**3 - 3*I*(e*sec(c + d*x)
)**(3/2)*tan(c + d*x)**2 - 3*(e*sec(c + d*x))**(3/2)*tan(c + d*x) + I*(e*se
c(c + d*x))**(3/2)), x)/a**3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}\,dx = \int \frac{1}{(e\sec(dx+c))^{\frac{3}{2}}(ia\tan(dx+c)+a)^3}\,dx$$

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^3}\,dx = \int \frac{1}{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^3}\,dx$$

```
[In] int(1/((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^3),x)
[Out] int(1/((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^3), x)
```

3.255 $\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 192

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{154e^8E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^4d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{154e^7\sqrt{e\sec(c+dx)}\sin(c+dx)}{5a^4d} - \frac{154e^5(e\sec(c+dx))^{5/2}\sin(c+dx)}{15a^4d} + \frac{4ie^2(e\sec(c+dx))^{11/2}}{ad(a+ia\tan(c+dx))^3} + \frac{44ie^4(e\sec(c+dx))^{7/2}}{3d\,(a^4+ia^4\tan(c+dx))}$$

```
[Out] -154/15*e^5*(e*sec(d*x+c))^(5/2)*sin(d*x+c)/a^4/d+154/5*e^8*(cos(1/2*d*x+1/
2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^4/
d/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)-154/5*e^7*sin(d*x+c)*(e*sec(d*x+c))
^(1/2)/a^4/d+4*I*e^2*(e*sec(d*x+c))^(11/2)/a/d/(a+I*a*tan(d*x+c))^3+44/3*I*
e^4*(e*sec(d*x+c))^(7/2)/d/(a^4+I*a^4*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3581, 3853, 3856, 2719}

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{154e^8E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^4d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{154e^7\sin(c+dx)\sqrt{e\sec(c+dx)}}{5a^4d} - \frac{154e^5\sin(c+dx)(e\sec(c+dx))^{5/2}}{15a^4d} + \frac{44ie^4(e\sec(c+dx))^{7/2}}{3d\,(a^4+ia^4\tan(c+dx))} + \frac{4ie^2(e\sec(c+dx))^{11/2}}{ad(a+ia\tan(c+dx))^3}$$

```
[In] Int[(e*Sec[c + d*x])^(15/2)/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (154*e^8*EllipticE[(c + d*x)/2, 2])/(5*a^4*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) - (154*e^7*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/(5*a^4*d) - (154*e
^5*(e*Sec[c + d*x])^(5/2)*Sin[c + d*x])/(15*a^4*d) + ((4*I)*e^2*(e*Sec[c +
d*x])^(11/2))/(a*d*(a + I*a*Tan[c + d*x])^3) + (((44*I)/3)*e^4*(e*Sec[c + d
*x])^(7/2))/(d*(a^4 + I*a^4*Tan[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{4ie^2(e\sec(c+dx))^{11/2}}{ad(a+ia\tan(c+dx))^3}-\frac{(11e^2)\int\frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}\,dx}{a^2}\\
&= \frac{4ie^2(e\sec(c+dx))^{11/2}}{ad(a+ia\tan(c+dx))^3}+\frac{44ie^4(e\sec(c+dx))^{7/2}}{3d\,(a^4+ia^4\tan(c+dx))}-\frac{(77e^4)\int(e\sec(c+dx))^{7/2}\,dx}{3a^4}\\
&= -\frac{154e^5(e\sec(c+dx))^{5/2}\sin(c+dx)}{15a^4d}+\frac{4ie^2(e\sec(c+dx))^{11/2}}{ad(a+ia\tan(c+dx))^3}\\
&\quad+\frac{44ie^4(e\sec(c+dx))^{7/2}}{3d\,(a^4+ia^4\tan(c+dx))}-\frac{(77e^6)\int(e\sec(c+dx))^{3/2}\,dx}{5a^4}
\end{aligned}
$$

$$= -\frac{154e^7\sqrt{e\sec(c+dx)}\sin(c+dx)}{5a^4d} - \frac{154e^5(e\sec(c+dx))^{5/2}\sin(c+dx)}{15a^4d} + \frac{4ie^2(e\sec(c+dx))^{11/2}}{ad(a+ia\tan(c+dx))^3} + \frac{44ie^4(e\sec(c+dx))^{7/2}}{3d\left(a^4+ia^4\tan(c+dx)\right)} + \frac{(77e^8)\int\frac{1}{\sqrt{e\sec(c+dx)}}dx}{5a^4}$$

$$= -\frac{154e^7\sqrt{e\sec(c+dx)}\sin(c+dx)}{5a^4d} - \frac{154e^5(e\sec(c+dx))^{5/2}\sin(c+dx)}{15a^4d} + \frac{4ie^2(e\sec(c+dx))^{11/2}}{ad(a+ia\tan(c+dx))^3} + \frac{44ie^4(e\sec(c+dx))^{7/2}}{3d\left(a^4+ia^4\tan(c+dx)\right)} + \frac{(77e^8)\int\sqrt{\cos(c+dx)}dx}{5a^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$= \frac{154e^8E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^4d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{154e^7\sqrt{e\sec(c+dx)}\sin(c+dx)}{5a^4d} - \frac{154e^5(e\sec(c+dx))^{5/2}\sin(c+dx)}{15a^4d} + \frac{4ie^2(e\sec(c+dx))^{11/2}}{ad(a+ia\tan(c+dx))^3} + \frac{44ie^4(e\sec(c+dx))^{7/2}}{3d\left(a^4+ia^4\tan(c+dx)\right)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.17 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.65

$$\int\frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^4}dx = -\frac{ie^5(e\sec(c+dx))^{5/2}\left(-1133\cos(c+dx)+77e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{5/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-e^{2i(c+}\right.\right.}{30a^4d}$$

```
[In] Integrate[(e*Sec[c + d*x])^(15/2)/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((-1/30*I)*e^5*(e*Sec[c + d*x])^(5/2)*(-1133*Cos[c + d*x] + (77*(1 + E^((2*
I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]
)/E^(I*(c + d*x)) - 3*(117*Cos[3*(c + d*x)] + (33*I)*Sin[c + d*x] + (37*I)*
Sin[3*(c + d*x)])))/(a^4*d)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(194) = 388$.

Time = 11.66 (sec) , antiderivative size = 466, normalized size of antiderivative = 2.43

method	result
default	$\frac{2\sqrt{e\sec(dx+c)}\, e^7 \left(-231i(\cos^2(dx+c))E(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}+231i(\cos^2(dx+c))F(i(-\csc(\cdots\right.}{\cdots}$

```
[In] int((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/a^4/d*(e*sec(d*x+c))^(1/2)*e^7/(cos(d*x+c)+1)*(-231*I*cos(d*x+c)^2*Ell
ipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(
cos(d*x+c)+1))^(1/2)+231*I*cos(d*x+c)^2*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)
),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)-462*I*cos(d
*x+c)*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*(1/(cos(d*x+c)+1))^(1/2)+462*I*cos(d*x+c)*EllipticF(I*(-csc(d*x+c)+cot
(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)-231*
I*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*(1/(cos(d*x+c)+1))^(1/2)+231*I*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+120*I*cos(d*x+c)^2+
120*sin(d*x+c)*cos(d*x+c)+120*I*cos(d*x+c)-111*sin(d*x+c)+20*I+3*tan(d*x+c)
+20*I*sec(d*x+c)+3*sec(d*x+c)*tan(d*x+c))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.01

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^4}\,dx =$$
$$-\frac{2\left(\sqrt{2}\left(-231i\,e^7e^{(6i\,dx+6i\,c)}-616i\,e^7e^{(4i\,dx+4i\,c)}-517i\,e^7e^{(2i\,dx+2i\,c)}-120i\,e^7\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+23\cdots\right.}{15\left(a^4de^{(5i\,dx+5i\,c)}\cdots\right.}$$

```
[In] integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -2/15*(sqrt(2)*(-231*I*e^7*e^(6*I*d*x + 6*I*c) - 616*I*e^7*e^(4*I*d*x + 4*I
*c) - 517*I*e^7*e^(2*I*d*x + 2*I*c) - 120*I*e^7)*sqrt(e/(e^(2*I*d*x + 2*I*c
) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 231*sqrt(2)*(-I*e^7*e^(5*I*d*x + 5*I*c) -
 2*I*e^7*e^(3*I*d*x + 3*I*c) - I*e^7*e^(I*d*x + I*c))*sqrt(e)*weierstrassZe
ta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x + I*c))))/(a^4*d*e^(5*I*d*x +
 5*I*c) + 2*a^4*d*e^(3*I*d*x + 3*I*c) + a^4*d*e^(I*d*x + I*c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^4}\,dx = \text{Timed out}$$

```
[In] integrate((e*sec(d*x+c))**(15/2)/(a+I*a*tan(d*x+c))**4,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^4}\,dx = \int \frac{(e\sec(dx+c))^{\frac{15}{2}}}{(i\,a\tan(dx+c)+a)^4}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(15/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(15/2)/(I*a*tan(d*x + c) + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{15/2}}{(a+ia\tan(c+dx))^4}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{15/2}}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^4}\,dx$$

```
[In] int((e/cos(c + d*x))^(15/2)/(a + a*tan(c + d*x)*1i)^4,x)
[Out] int((e/cos(c + d*x))^(15/2)/(a + a*tan(c + d*x)*1i)^4, x)
```

3.256 $\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 157

$$\begin{aligned}\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^4}\,dx = & -\frac{10e^6\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{a^4d} \\ & -\frac{10e^5(e\sec(c+dx))^{3/2}\sin(c+dx)}{a^4d} \\ & +\frac{4ie^2(e\sec(c+dx))^{9/2}}{3ad(a+ia\tan(c+dx))^3}+\frac{12ie^4(e\sec(c+dx))^{5/2}}{d\,(a^4+ia^4\tan(c+dx))}\end{aligned}$$

```
[Out] -10*e^5*(e*sec(d*x+c))^(3/2)*sin(d*x+c)/a^4/d-10*e^6*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^
(1/2)*(e*sec(d*x+c))^(1/2)/a^4/d+4/3*I*e^2*(e*sec(d*x+c))^(9/2)/a/d/(a+I*a*
tan(d*x+c))^3+12*I*e^4*(e*sec(d*x+c))^(5/2)/d/(a^4+I*a^4*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used

$= \{3581, 3853, 3856, 2720\}$

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^4}\,dx = -\frac{10e^6\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{a^4d} - \frac{10e^5\sin(c+dx)(e\sec(c+dx))^{3/2}}{a^4d} + \frac{12ie^4(e\sec(c+dx))^{5/2}}{d\left(a^4+ia^4\tan(c+dx)\right)} + \frac{4ie^2(e\sec(c+dx))^{9/2}}{3ad(a+ia\tan(c+dx))^3}$$

```
[In] Int[(e*Sec[c + d*x])^(13/2)/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (-10*e^6*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])
/(a^4*d) - (10*e^5*(e*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(a^4*d) + (((4*I)/3
)*e^2*(e*Sec[c + d*x])^(9/2))/(a*d*(a + I*a*Tan[c + d*x])^3) + ((12*I)*e^4*
(e*Sec[c + d*x])^(5/2))/(d*(a^4 + I*a^4*Tan[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{4ie^2(e\sec(c+dx))^{9/2}}{3ad(a+ia\tan(c+dx))^3}-\frac{(3e^2)\int\frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^2}\,dx}{a^2}\\
&= \frac{4ie^2(e\sec(c+dx))^{9/2}}{3ad(a+ia\tan(c+dx))^3}+\frac{12ie^4(e\sec(c+dx))^{5/2}}{d\,(a^4+ia^4\tan(c+dx))}-\frac{(15e^4)\int(e\sec(c+dx))^{5/2}\,dx}{a^4}\\
&= -\frac{10e^5(e\sec(c+dx))^{3/2}\sin(c+dx)}{a^4d}+\frac{4ie^2(e\sec(c+dx))^{9/2}}{3ad(a+ia\tan(c+dx))^3}\\
&\quad+\frac{12ie^4(e\sec(c+dx))^{5/2}}{d\,(a^4+ia^4\tan(c+dx))}-\frac{(5e^6)\int\sqrt{e\sec(c+dx)}\,dx}{a^4}\\
&= -\frac{10e^5(e\sec(c+dx))^{3/2}\sin(c+dx)}{a^4d}+\frac{4ie^2(e\sec(c+dx))^{9/2}}{3ad(a+ia\tan(c+dx))^3}\\
&\quad+\frac{12ie^4(e\sec(c+dx))^{5/2}}{d\,(a^4+ia^4\tan(c+dx))}-\frac{\left(5e^6\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{a^4}\\
&= -\frac{10e^6\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{a^4d}\\
&\quad-\frac{10e^5(e\sec(c+dx))^{3/2}\sin(c+dx)}{a^4d}\\
&\quad+\frac{4ie^2(e\sec(c+dx))^{9/2}}{3ad(a+ia\tan(c+dx))^3}+\frac{12ie^4(e\sec(c+dx))^{5/2}}{d\,(a^4+ia^4\tan(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.85

$$\int\frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^4}\,dx=\frac{ie^6\sec^5(c+dx)\sqrt{e\sec(c+dx)}\left(21+19\cos(2(c+dx))+30i\cos^{\frac{3}{2}}(c+dx)\,\text{Ell}\right.}{}$$

```
[In] Integrate[(e*Sec[c + d*x])^(13/2)/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((I/3)*e^6*Sec[c + d*x]^5*Sqrt[e*Sec[c + d*x]]*(21 + 19*Cos[2*(c + d*x)] +
(30*I)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2]*(Cos[c + d*x] + I*Sin[c
 + d*x]) + (11*I)*Sin[2*(c + d*x)])*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x)])
)/(a^4*d*(-I + Tan[c + d*x])^4)
```

Maple [A] (verified)

Time = 10.10 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.10

method	result
default	$\frac{2e^{6}\left(-15i\cos(dx+c)F(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\sqrt{\frac{1}{\cos(dx+c)+1}}-15iF(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{1}{\cos(dx+c)+1}}\right.}{3a^{4}d}$

```
[In] int((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/a^4/d*e^6*(-15*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*
x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-15*I*(cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+
1))^(1/2)+8*sin(d*x+c)*cos(d*x+c)+8*I*cos(d*x+c)^2+tan(d*x+c)+12*I)*(e*sec(
d*x+c))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.94

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^4}\,dx =$$
$$-\frac{2\left(\sqrt{2}\left(-15i\,e^{6}e^{(4i\,dx+4i\,c)}-21i\,e^{6}e^{(2i\,dx+2i\,c)}-4i\,e^{6}\right)\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+15\sqrt{2}\left(-i\,e^{6}e^{(4i\,dx+4i\,c)}-i\,e^{6}\right.\right.}{3\left(a^{4}de^{(4i\,dx+4i\,c)}+a^{4}de^{(2i\,dx+2i\,c)}\right)}$$

```
[In] integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(2)*(-15*I*e^6*e^(4*I*d*x + 4*I*c) - 21*I*e^6*e^(2*I*d*x + 2*I*c)
 - 4*I*e^6)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 15*
sqrt(2)*(-I*e^6*e^(4*I*d*x + 4*I*c) - I*e^6*e^(2*I*d*x + 2*I*c))*sqrt(e)*we
ierstrassPInverse(-4, 0, e^(I*d*x + I*c)))/(a^4*d*e^(4*I*d*x + 4*I*c) + a^4
*d*e^(2*I*d*x + 2*I*c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^4}\,dx = \text{Timed out}$$

```
[In] integrate((e*sec(d*x+c))**(13/2)/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^4}\,dx = \int \frac{(e\sec(dx+c))^{\frac{13}{2}}}{(i\,a\tan(dx+c)+a)^4}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(13/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(13/2)/(I*a*tan(d*x + c) + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{13/2}}{(a+ia\tan(c+dx))^4}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{13/2}}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^4}\,dx$$

```
[In] int((e/cos(c + d*x))^(13/2)/(a + a*tan(c + d*x)*1i)^4,x)
[Out] int((e/cos(c + d*x))^(13/2)/(a + a*tan(c + d*x)*1i)^4, x)
```

3.257 $\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 163

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^4}\,dx = -\frac{42e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^4d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{42e^5\sqrt{e\sec(c+dx)}\sin(c+dx)}{5a^4d} + \frac{4ie^2(e\sec(c+dx))^{7/2}}{5ad(a+ia\tan(c+dx))^3} - \frac{28ie^4(e\sec(c+dx))^{3/2}}{5d\left(a^4+ia^4\tan(c+dx)\right)}$$

```
[Out] -42/5*e^6*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/a^4/d/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+42/5*e^5*s
in(d*x+c)*(e*sec(d*x+c))^(1/2)/a^4/d+4/5*I*e^2*(e*sec(d*x+c))^(7/2)/a/d/(a+
I*a*tan(d*x+c))^3-28/5*I*e^4*(e*sec(d*x+c))^(3/2)/d/(a^4+I*a^4*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3581, 3853, 3856, 2719}

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^4}\,dx = -\frac{42e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^4d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{42e^5\sin(c+dx)\sqrt{e\sec(c+dx)}}{5a^4d} - \frac{28ie^4(e\sec(c+dx))^{3/2}}{5d\left(a^4+ia^4\tan(c+dx)\right)} + \frac{4ie^2(e\sec(c+dx))^{7/2}}{5ad(a+ia\tan(c+dx))^3}$$

```
[In] Int[(e*Sec[c + d*x])^(11/2)/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (-42*e^6*EllipticE[(c + d*x)/2, 2])/(5*a^4*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) + (42*e^5*Sqrt[e*Sec[c + d*x]]*Sin[c + d*x])/(5*a^4*d) + (((4*I)
/5)*e^2*(e*Sec[c + d*x])^(7/2))/(a*d*(a + I*a*Tan[c + d*x])^3) - (((28*I)/5
)*e^4*(e*Sec[c + d*x])^(3/2))/(d*(a^4 + I*a^4*Tan[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{4ie^2(e\sec(c+dx))^{7/2}}{5ad(a+ia\tan(c+dx))^3} - \frac{(7e^2)\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx}{5a^2} \\
&= \frac{4ie^2(e\sec(c+dx))^{7/2}}{5ad(a+ia\tan(c+dx))^3} - \frac{28ie^4(e\sec(c+dx))^{3/2}}{5d\,(a^4+ia^4\tan(c+dx))} + \frac{(21e^4)\int (e\sec(c+dx))^{3/2}\,dx}{5a^4} \\
&= \frac{42e^5\sqrt{e\sec(c+dx)}\sin(c+dx)}{5a^4d} + \frac{4ie^2(e\sec(c+dx))^{7/2}}{5ad(a+ia\tan(c+dx))^3} \\
&\quad - \frac{28ie^4(e\sec(c+dx))^{3/2}}{5d\,(a^4+ia^4\tan(c+dx))} - \frac{(21e^6)\int \frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{5a^4}
\end{aligned}
$$

$$= \frac{42e^5\sqrt{e\sec(c+dx)}\sin(c+dx)}{5a^4d} + \frac{4ie^2(e\sec(c+dx))^{7/2}}{5ad(a+ia\tan(c+dx))^3} - \frac{28ie^4(e\sec(c+dx))^{3/2}}{5d\left(a^4+ia^4\tan(c+dx)\right)} - \frac{(21e^6)\int\sqrt{\cos(c+dx)}\,dx}{5a^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$= -\frac{42e^6E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^4d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{42e^5\sqrt{e\sec(c+dx)}\sin(c+dx)}{5a^4d} + \frac{4ie^2(e\sec(c+dx))^{7/2}}{5ad(a+ia\tan(c+dx))^3} - \frac{28ie^4(e\sec(c+dx))^{3/2}}{5d\left(a^4+ia^4\tan(c+dx)\right)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.59 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.65

$$\int\frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^4}\,dx = -\frac{2ie^5e^{-3i(c+dx)}\left(-2-7e^{2i(c+dx)}+21e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)\right)\sqrt{e\,\text{se}}}{5a^4d}$$

```
[In] Integrate[(e*Sec[c + d*x])^(11/2)/(a + I*a*Tan[c + d*x])^4,x]
[Out] (((-2*I)/5)*e^5*(-2 - 7*E^((2*I)*(c + d*x)) + 21*E^((2*I)*(c + d*x))*Sqrt[1
 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*
x))])*Sqrt[e*Sec[c + d*x]])/(a^4*d*E^((3*I)*(c + d*x)))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 481 vs. 2(167) = 334.

Time = 9.68 (sec) , antiderivative size = 482, normalized size of antiderivative = 2.96

method	result
default	$-\frac{2i\sqrt{e\sec(dx+c)}\left(21\left(\cos^2(dx+c)\right)F(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}-21\left(\cos^2(dx+c)\right)E(i(-\csc(dx+c)\right.}{}$

```
[In] int((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
[Out] -2/5*I/a^4/d*(e*sec(d*x+c))^(1/2)*(21*cos(d*x+c)^2*EllipticF(I*(-csc(d*x+c)
+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)-
21*cos(d*x+c)^2*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)+8*I*sin(d*x+c)*cos(d*x+c)^3+42*(cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(c
```

```
os(d*x+c)+1))^(1/2)*cos(d*x+c)-42*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*Ellipti
cE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-8*cos(
d*x+c)^4+8*I*sin(d*x+c)*cos(d*x+c)^2+21*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)
),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-21*(cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d
*x+c)+1))^(1/2)-8*cos(d*x+c)^3-16*I*sin(d*x+c)*cos(d*x+c)+20*cos(d*x+c)^2+5
*I*sin(d*x+c)+20*cos(d*x+c))*e^5/(cos(d*x+c)+1)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.70

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^4}\,dx =$$
$$-\frac{2\left(21i\sqrt{2}e^{\frac{11}{2}}e^{(3i\,dx+3i\,c)}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\sqrt{2}\left(21i\,e^5e^{(4i\,dx+4i\,c)}\right.\right.}{5\,a^4d}$$

```
[In] integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
[Out] -2/5*(21*I*sqrt(2)*e^(11/2)*e^(3*I*d*x + 3*I*c)*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(21*I*e^5*e^(4*I*d*x + 4
*I*c) + 14*I*e^5*e^(2*I*d*x + 2*I*c) - 2*I*e^5)*sqrt(e/(e^(2*I*d*x + 2*I*c)
 + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-3*I*d*x - 3*I*c)/(a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^4}\,dx = \text{Timed out}$$

```
[In] integrate((e*sec(d*x+c))**(11/2)/(a+I*a*tan(d*x+c))**4,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^4}\,dx = \int \frac{(e\sec(dx+c))^{\frac{11}{2}}}{(i\,a\tan(dx+c)+a)^4}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(11/2)/(I*a*tan(d*x + c) + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^4}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{11/2}}{(a+a\tan(c+d\,x)\;1\mathrm{i})^4}\,dx$$

```
[In] int((e/cos(c + d*x))^(11/2)/(a + a*tan(c + d*x)*1i)^4,x)
[Out] int((e/cos(c + d*x))^(11/2)/(a + a*tan(c + d*x)*1i)^4, x)
```

3.258 $\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 132

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^4}\,dx=\frac{10e^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{21a^4d}+\frac{4ie^2(e\sec(c+dx))^{5/2}}{7ad(a+ia\tan(c+dx))^3}-\frac{20ie^4\sqrt{e\sec(c+dx)}}{21d\left(a^4+ia^4\tan(c+dx)\right)}$$

```
[Out] 10/21*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(e*sec(d*x+c))^(1/2)/a^4/d+4/7*I*e^2*
(e*sec(d*x+c))^(5/2)/a/d/(a+I*a*tan(d*x+c))^3-20/21*I*e^4*(e*sec(d*x+c))^(1
/2)/d/(a^4+I*a^4*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3581, 3856, 2720}

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^4}\,dx=-\frac{20ie^4\sqrt{e\sec(c+dx)}}{21d\left(a^4+ia^4\tan(c+dx)\right)}+\frac{10e^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{21a^4d}+\frac{4ie^2(e\sec(c+dx))^{5/2}}{7ad(a+ia\tan(c+dx))^3}$$

```
[In] Int[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (10*e^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(21*a^4*d) + (((4*I)/7)*e^2*(e*Sec[c + d*x])^(5/2))/(a*d*(a + I*a*Tan[c + d
```

```
*x])^3) - (((20*I)/21)*e^4*Sqrt[e*Sec[c + d*x]])/(d*(a^4 + I*a^4*Tan[c + d*
x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{4ie^2(e\sec(c+dx))^{5/2}}{7ad(a+ia\tan(c+dx))^3}-\frac{(5e^2)\int\frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx}{7a^2}\\
&= \frac{4ie^2(e\sec(c+dx))^{5/2}}{7ad(a+ia\tan(c+dx))^3}-\frac{20ie^4\sqrt{e\sec(c+dx)}}{21d\,(a^4+ia^4\tan(c+dx))}+\frac{(5e^4)\int\sqrt{e\sec(c+dx)}\,dx}{21a^4}\\
&= \frac{4ie^2(e\sec(c+dx))^{5/2}}{7ad(a+ia\tan(c+dx))^3}-\frac{20ie^4\sqrt{e\sec(c+dx)}}{21d\,(a^4+ia^4\tan(c+dx))}\\
&\quad+\frac{\left(5e^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21a^4}\\
&= \frac{10e^4\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{21a^4d}\\
&\quad+\frac{4ie^2(e\sec(c+dx))^{5/2}}{7ad(a+ia\tan(c+dx))^3}-\frac{20ie^4\sqrt{e\sec(c+dx)}}{21d\,(a^4+ia^4\tan(c+dx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.04

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{2e^4\sec^4(c+dx)\sqrt{e\sec(c+dx)}\left(5\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(\cos(\right.}{}$$

[In] Integrate[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x])^4,x]

[Out] (2*e^4*Sec[c + d*x]^4*Sqrt[e*Sec[c + d*x]]*(5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]) - (2*I)*(1 + Cos[2*(c + d*x)] + (4*I)*Sin[2*(c + d*x)]))*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]))/(21*a^4*d*(-I + Tan[c + d*x])^4)

Maple [A] (verified)

Time = 7.63 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.45

method	result
default	$\frac{2e^4\left(24i(\cos^4(dx+c))+5i\cos(dx+c)F(i(-\csc(dx+c)+\cot(dx+c)),i)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}+24(\cos^3(dx+c))\sin(dx+c)+5iF\right.}{21a^4d}$

[In] int((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)

[Out] 2/21/a^4/d*e^4*(24*I*cos(d*x+c)^4+5*I*cos(d*x+c)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+24*cos(d*x+c)^3*sin(d*x+c)+5*I*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)-28*I*cos(d*x+c)^2-16*sin(d*x+c)*cos(d*x+c))*(e*sec(d*x+c))^(1/2)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.84

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^4}\,dx =$$

$$-\frac{2\left(5i\sqrt{2}e^{\frac{9}{2}}e^{(4i\,dx+4i\,c)}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(5i\,e^4e^{(4i\,dx+4i\,c)}+2i\,e^4e^{(2i\,dx+2i\,c)}-3i\,e^4\right)\right.}{21\,a^4d}$$

[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")

[Out] -2/21*(5*I*sqrt(2)*e^(9/2)*e^(4*I*d*x + 4*I*c)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)) + sqrt(2)*(5*I*e^4*e^(4*I*d*x + 4*I*c) + 2*I*e^4*e^(2*I*d*x

```
 + 2*I*c) - 3*I*e^4)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I
*c))*e^(-4*I*d*x - 4*I*c)/(a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^4}\,dx = \text{Timed out}$$

```
[In] integrate((e*sec(d*x+c))**(9/2)/(a+I*a*tan(d*x+c))**4,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^4}\,dx = \int \frac{(e\sec(dx+c))^{\frac{9}{2}}}{(i\,a\tan(dx+c)+a)^4}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(9/2)/(I*a*tan(d*x + c) + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^4}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{9/2}}{(a+a\tan(c+d\,x)\;1\mathrm{i})^4}\,dx$$

```
[In] int((e/cos(c + d*x))^(9/2)/(a + a*tan(c + d*x)*1i)^4,x)
[Out] int((e/cos(c + d*x))^(9/2)/(a + a*tan(c + d*x)*1i)^4, x)
```

3.259 $\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 132

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^4}\,dx = -\frac{2e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^4d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{4ie^2(e\sec(c+dx))^{3/2}}{9ad(a+ia\tan(c+dx))^3} - \frac{4ie^4}{15d\sqrt{e\sec(c+dx)}\,(a^4+ia^4\tan(c+dx))}$$

```
[Out] -2/15*e^4*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/a^4/d/cos(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+4/9*I*e^2*
(e*sec(d*x+c))^(3/2)/a/d/(a+I*a*tan(d*x+c))^3-4/15*I*e^4/d/(e*sec(d*x+c))^(
1/2)/(a^4+I*a^4*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3581, 3856, 2719}

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^4}\,dx = -\frac{4ie^4}{15d\,(a^4+ia^4\tan(c+dx))\,\sqrt{e\sec(c+dx)}} - \frac{2e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^4d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{4ie^2(e\sec(c+dx))^{3/2}}{9ad(a+ia\tan(c+dx))^3}$$

```
[In] Int[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (-2*e^4*EllipticE[(c + d*x)/2, 2])/(15*a^4*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[
c + d*x]]) + (((4*I)/9)*e^2*(e*Sec[c + d*x])^(3/2))/(a*d*(a + I*a*Tan[c + d
```

```
*x])^3) - (((4*I)/15)*e^4)/(d*Sqrt[e*Sec[c + d*x]]*(a^4 + I*a^4*Tan[c + d*x
]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{4ie^2(e\sec(c+dx))^{3/2}}{9ad(a+ia\tan(c+dx))^3}-\frac{e^2\int\frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx}{3a^2}\\
&= \frac{4ie^2(e\sec(c+dx))^{3/2}}{9ad(a+ia\tan(c+dx))^3}-\frac{4ie^4}{15d\sqrt{e\sec(c+dx)}\,(a^4+ia^4\tan(c+dx))}-\frac{e^4\int\frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{15a^4}\\
&= \frac{4ie^2(e\sec(c+dx))^{3/2}}{9ad(a+ia\tan(c+dx))^3}-\frac{4ie^4}{15d\sqrt{e\sec(c+dx)}\,(a^4+ia^4\tan(c+dx))}\\
&\quad-\frac{e^4\int\sqrt{\cos(c+dx)}\,dx}{15a^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}\\
&= -\frac{2e^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15a^4d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}+\frac{4ie^2(e\sec(c+dx))^{3/2}}{9ad(a+ia\tan(c+dx))^3}\\
&\quad-\frac{4ie^4}{15d\sqrt{e\sec(c+dx)}\,(a^4+ia^4\tan(c+dx))}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.72 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.13

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{e^3e^{-idx}\sec^4(c+dx)\sqrt{e\sec(c+dx)}\left(-7-7\cos(2(c+dx))+6e^{2i(c+dx)}\sqrt{1+e\right.}{}$$

```
[In] Integrate[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (e^3*Sec[c + d*x]^4*Sqrt[e*Sec[c + d*x]]*(-7 - 7*Cos[2*(c + d*x)] + 6*E^((2
*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3
/4, -E^((2*I)*(c + d*x))] + (3*I)*Sin[2*(c + d*x)])*((-I)*Cos[c + 2*d*x] +
Sin[c + 2*d*x]))/(45*a^4*d*E^(I*d*x)*(-I + Tan[c + d*x])^4)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 508 vs. 2(140) = 280.

Time = 8.04 (sec) , antiderivative size = 509, normalized size of antiderivative = 3.86

method	result
default	$-\frac{2i\sqrt{e\sec(dx+c)}\left(40i(\cos^5(dx+c))\sin(dx+c)+40i\sin(dx+c)(\cos^4(dx+c))-40(\cos^6(dx+c))-16i(\cos^3(dx+c))\sin(dx+c)-40(\cos\right.}{}$

```
[In] int((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2/45*I/a^4/d*(e*sec(d*x+c))^(1/2)*(40*I*sin(d*x+c)*cos(d*x+c)^5+40*I*cos(d
*x+c)^4*sin(d*x+c)-40*cos(d*x+c)^6-16*I*sin(d*x+c)*cos(d*x+c)^3-40*cos(d*x+
c)^5+3*cos(d*x+c)^2*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)-3*cos(d*x+c)^2*EllipticE(I*(-cs
c(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1)
)^(1/2)-16*I*cos(d*x+c)^2*sin(d*x+c)+36*cos(d*x+c)^4+6*(cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1
/2)*cos(d*x+c)-6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+c)
+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-3*I*cos(d*x+c)*sin(d*x+
c)+36*cos(d*x+c)^3+3*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)
+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-3*(cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2))*e^3
/(cos(d*x+c)+1)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.97

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{\left(-6i\sqrt{2}e^{\frac{7}{2}}e^{(5i\,dx+5i\,c)}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right.\right.\right.}{}$$

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/45*(-6*I*sqrt(2)*e^(7/2)*e^(5*I*d*x + 5*I*c)*weierstrassZeta(-4, 0, weier
strassPInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(-6*I*e^3*e^(6*I*d*x + 6*
I*c) - 4*I*e^3*e^(4*I*d*x + 4*I*c) + 7*I*e^3*e^(2*I*d*x + 2*I*c) + 5*I*e^3)
*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-5*I*d*x - 5
*I*c)/(a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^4}\,dx = \text{Timed out}$$

```
[In] integrate((e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^4}\,dx = \int \frac{(e\sec(dx+c))^{\frac{7}{2}}}{(ia\tan(dx+c)+a)^4}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(7/2)/(I*a*tan(d*x + c) + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^4}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^4}\,dx$$

```
[In] int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^4,x)
[Out] int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^4, x)
```

3.260 $\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 163

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^4}\,dx = -\frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{77a^4d} - \frac{2e^3\sin(c+dx)}{77a^4d\sqrt{e\sec(c+dx)}} + \frac{4ie^2\sqrt{e\sec(c+dx)}}{11ad(a+ia\tan(c+dx))^3} - \frac{4ie^4}{77d(e\sec(c+dx))^{3/2}\,(a^4+ia^4\tan(c+dx))}$$

```
[Out] -2/77*e^3*sin(d*x+c)/a^4/d/(e*sec(d*x+c))^(1/2)-2/77*e^2*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x
+c)^(1/2)*(e*sec(d*x+c))^(1/2)/a^4/d+4/11*I*e^2*(e*sec(d*x+c))^(1/2)/a/d/(a
+I*a*tan(d*x+c))^3-4/77*I*e^4/d/(e*sec(d*x+c))^(3/2)/(a^4+I*a^4*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3581, 3854, 3856, 2720}

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^4}\,dx = -\frac{4ie^4}{77d\left(a^4+ia^4\tan(c+dx)\right)(e\sec(c+dx))^{3/2}} - \frac{2e^3\sin(c+dx)}{77a^4d\sqrt{e\sec(c+dx)}} - \frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{77a^4d} + \frac{4ie^2\sqrt{e\sec(c+dx)}}{11ad(a+ia\tan(c+dx))^3}$$

```
[In] Int[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (-2*e^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/
(77*a^4*d) - (2*e^3*Sin[c + d*x])/(77*a^4*d*Sqrt[e*Sec[c + d*x]]) + (((4*I)
/11)*e^2*Sqrt[e*Sec[c + d*x]])/(a*d*(a + I*a*Tan[c + d*x])^3) - (((4*I)/77)
*e^4)/(d*(e*Sec[c + d*x])^(3/2)*(a^4 + I*a^4*Tan[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{4ie^2\sqrt{e\sec(c+dx)}}{11ad(a+ia\tan(c+dx))^3} - \frac{e^2\int\frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx}{11a^2} \\
&= \frac{4ie^2\sqrt{e\sec(c+dx)}}{11ad(a+ia\tan(c+dx))^3} - \frac{4ie^4}{77d(e\sec(c+dx))^{3/2}\left(a^4+ia^4\tan(c+dx)\right)} \\
&\quad - \frac{\left(3e^4\right)\int\frac{1}{(e\sec(c+dx))^{3/2}}\,dx}{77a^4}
\end{aligned}$$

$$= -\frac{2e^3\sin(c+dx)}{77a^4d\sqrt{e\sec(c+dx)}} + \frac{4ie^2\sqrt{e\sec(c+dx)}}{11ad(a+ia\tan(c+dx))^3} - \frac{4ie^4}{77d(e\sec(c+dx))^{3/2}\,(a^4+ia^4\tan(c+dx))} - \frac{e^2\int\sqrt{e\sec(c+dx)}\,dx}{77a^4}$$

$$= -\frac{2e^3\sin(c+dx)}{77a^4d\sqrt{e\sec(c+dx)}} + \frac{4ie^2\sqrt{e\sec(c+dx)}}{11ad(a+ia\tan(c+dx))^3} - \frac{4ie^4}{77d(e\sec(c+dx))^{3/2}\,(a^4+ia^4\tan(c+dx))} - \frac{\left(e^2\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{77a^4}$$

$$= -\frac{2e^2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{77a^4d} - \frac{2e^3\sin(c+dx)}{77a^4d\sqrt{e\sec(c+dx)}} + \frac{4ie^2\sqrt{e\sec(c+dx)}}{11ad(a+ia\tan(c+dx))^3} - \frac{4ie^4}{77d(e\sec(c+dx))^{3/2}\,(a^4+ia^4\tan(c+dx))}$$

Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.88

$$\int\frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{\sec^2(c+dx)(e\sec(c+dx))^{5/2}(\cos(c+dx)+i\sin(c+dx))\left(37i\cos(c+dx)+\right.}{}$$

```
[In] Integrate[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^2*(e*Sec[c + d*x])^(5/2)*(Cos[c + d*x] + I*Sin[c + d*x])*((37
*I)*Cos[c + d*x] + (11*I)*Cos[3*(c + d*x)] + 3*Sin[c + d*x] - 4*Sqrt[Cos[c
+ d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[3*(c + d*x)] + I*Sin[3*(c + d*x)]) +
 3*Sin[3*(c + d*x)]))/(154*a^4*d*(-I + Tan[c + d*x])^4)
```

Maple [A] (verified)

Time = 6.85 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.88

method	result
default	$-\frac{2i\left((\cos(dx+c)+1)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}\,F(i(-\csc(dx+c)+\cot(dx+c)),i)+i\sin(dx+c)\cos(dx+c)(56(\cos^4(dx+c))-16(\cos^2\right.}{77a^4d}$

```
[In] int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2/77*I/a^4/d*((cos(d*x+c)+1)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x
+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)+I*sin(d*x+c)*cos(d*x+
```

c)*(56*cos(d*x+c)^4-16*cos(d*x+c)^2-1)+cos(d*x+c)^4*(-56*cos(d*x+c)^2+44))*
(e*sec(d*x+c))^(1/2)*e^2

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.77

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{\left(4i\sqrt{2}e^{\frac{5}{2}}e^{(6i\,dx+6i\,c)}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{2}\left(4i\,e^2e^{(6i\,dx+6i\,c)}\right.\right.}{1}$$

[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")

[Out] 1/154*(4*I*sqrt(2)*e^(5/2)*e^(6*I*d*x + 6*I*c)*weierstrassPInverse(-4, 0, e
^(I*d*x + I*c)) + sqrt(2)*(4*I*e^2*e^(6*I*d*x + 6*I*c) + 17*I*e^2*e^(4*I*d*
x + 4*I*c) + 20*I*e^2*e^(2*I*d*x + 2*I*c) + 7*I*e^2)*sqrt(e/(e^(2*I*d*x + 2
*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-6*I*d*x - 6*I*c)/(a^4*d)

Sympy [F]

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{\int \frac{(e\sec{(c+dx)})^{\frac{5}{2}}}{\tan^4{(c+dx)}-4i\tan^3{(c+dx)}-6\tan^2{(c+dx)}+4i\tan{(c+dx)}+1}\,dx}{a^4}$$

[In] integrate((e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**4,x)

[Out] Integral((e*sec(c + d*x))**(5/2)/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6
*tan(c + d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.

Giac [F]

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^4}\,dx = \int \frac{(e\sec(dx+c))^{\frac{5}{2}}}{(i\,a\tan(dx+c)+a)^4}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(5/2)/(I*a*tan(d*x + c) + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^4}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^4}\,dx$$

```
[In] int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^4,x)
[Out] int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^4, x)
```

3.261 $\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 163

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39a^4d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2e^3\sin(c+dx)}{117a^4d(e\sec(c+dx))^{3/2}} + \frac{4ie^2}{13ad\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3} + \frac{4ie^4}{117d(e\sec(c+dx))^{5/2}\left(a^4+ia^4\tan(c+dx)\right)}$$

```
[Out] 2/117*e^3*sin(d*x+c)/a^4/d/(e*sec(d*x+c))^(3/2)+2/39*e^2*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^4/d/c
os(d*x+c)^(1/2)/(e*sec(d*x+c))^(1/2)+4/13*I*e^2/a/d/(e*sec(d*x+c))^(1/2)/(a
+I*a*tan(d*x+c))^3+4/117*I*e^4/d/(e*sec(d*x+c))^(5/2)/(a^4+I*a^4*tan(d*x+c)
)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3581, 3854, 3856, 2719}

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{4ie^4}{117d\left(a^4+ia^4\tan(c+dx)\right)(e\sec(c+dx))^{5/2}} + \frac{2e^3\sin(c+dx)}{117a^4d(e\sec(c+dx))^{3/2}} + \frac{2e^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{39a^4d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{4ie^2}{13ad(a+ia\tan(c+dx))^3\sqrt{e\sec(c+dx)}}$$

```
[In] Int[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (2*e^2*EllipticE[(c + d*x)/2, 2])/(39*a^4*d*Sqrt[Cos[c + d*x]]*Sqrt[e*Sec[c
 + d*x]]) + (2*e^3*Sin[c + d*x])/(117*a^4*d*(e*Sec[c + d*x])^(3/2)) + (((4*
I)/13)*e^2)/(a*d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^3) + (((4*I)/1
17)*e^4)/(d*(e*Sec[c + d*x])^(5/2)*(a^4 + I*a^4*Tan[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\text{integral} = \frac{4ie^2}{13ad\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3} + \frac{e^2\int\frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}\,dx}{13a^2}$$

$$= \frac{4ie^2}{13ad\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3} + \frac{4ie^4}{117d(e\sec(c+dx))^{5/2}\left(a^4+ia^4\tan(c+dx)\right)} + \frac{(5e^4)\int\frac{1}{(e\sec(c+dx))^{5/2}}\,dx}{117a^4}$$

$$= \frac{2e^3 \sin(c+dx)}{117a^4 d(e\sec(c+dx))^{3/2}} + \frac{4ie^2}{13ad\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3} + \frac{4ie^4}{117d(e\sec(c+dx))^{5/2}\,(a^4+ia^4\tan(c+dx))} + \frac{e^2\int \frac{1}{\sqrt{e\sec(c+dx)}}dx}{39a^4}$$

$$= \frac{2e^3 \sin(c+dx)}{117a^4 d(e\sec(c+dx))^{3/2}} + \frac{4ie^2}{13ad\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3} + \frac{4ie^4}{117d(e\sec(c+dx))^{5/2}\,(a^4+ia^4\tan(c+dx))} + \frac{e^2\int \sqrt{\cos(c+dx)}\,dx}{39a^4\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$= \frac{2e^2 E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{39a^4 d\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2e^3 \sin(c+dx)}{117a^4 d(e\sec(c+dx))^{3/2}} + \frac{4ie^2}{13ad\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^3} + \frac{4ie^4}{117d(e\sec(c+dx))^{5/2}\,(a^4+ia^4\tan(c+dx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.18 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.87

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{ie^{-idx}\sec^2(c+dx)(e\sec(c+dx))^{3/2}(\cos(dx)+i\sin(dx))\left(28+40\cos(2(c+\right.}{234a^4 d(-i+\tan(}$$

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] ((I/234)*Sec[c + d*x]^2*(e*Sec[c + d*x])^(3/2)*(Cos[d*x] + I*Sin[d*x])*(28
+ 40*Cos[2*(c + d*x)] + (24*E^((4*I)*(c + d*x))*Hypergeometric2F1[-1/4, 1/2
, 3/4, -E^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] + (22*I)*Sin[2*
(c + d*x)]))/(a^4*d*E^(I*d*x)*(-I + Tan[c + d*x])^4)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 540 vs. 2(167) = 334.

Time = 8.18 (sec) , antiderivative size = 541, normalized size of antiderivative = 3.32

method	result
default	$\frac{2i\sqrt{e\sec(dx+c)}\left(-i(\cos^2(dx+c))\sin(dx+c)+72(\cos^8(dx+c))-72i(\cos^7(dx+c))\sin(dx+c)+72(\cos^7(dx+c))+16i\sin(dx+c)(\cos^4(dx\right.}{}$

[In] int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)

[Out] 2/117*I/a^4/d*(e*sec(d*x+c))^(1/2)*(-I*cos(d*x+c)^2*sin(d*x+c)+72*cos(d*x+c)^8-72*I*cos(d*x+c)^7*sin(d*x+c)+72*cos(d*x+c)^7+16*I*cos(d*x+c)^4*sin(d*x+c)-52*cos(d*x+c)^6-72*I*cos(d*x+c)^6*sin(d*x+c)-52*cos(d*x+c)^5-I*cos(d*x+c)^3*sin(d*x+c)+3*cos(d*x+c)^2*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)-3*cos(d*x+c)^2*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)-3*I*cos(d*x+c)*sin(d*x+c)+6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-6*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+16*I*cos(d*x+c)^5*sin(d*x+c)+3*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-3*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*EllipticE(I*(-csc(d*x+c)+cot(d*x+c)),I)*(1/(cos(d*x+c)+1))^(1/2))*e/(cos(d*x+c)+1)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.81

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{\left(24i\sqrt{2}e^{\frac{3}{2}}e^{(7i\,dx+7i\,c)}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c}\right.\right.\right.}{}$$

[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")

[Out] 1/468*(24*I*sqrt(2)*e^(3/2)*e^(7*I*d*x + 7*I*c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(2)*(24*I*e*e^(8*I*d*x + 8*I*c) + 55*I*e*e^(6*I*d*x + 6*I*c) + 59*I*e*e^(4*I*d*x + 4*I*c) + 37*I*e*e^(2*I*d*x + 2*I*c) + 9*I*e)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-7*I*d*x - 7*I*c)/(a^4*d)

Sympy [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^4}\,dx = \frac{\int \frac{(e\sec(c+dx))^{\frac{3}{2}}}{\tan^4(c+dx)-4i\tan^3(c+dx)-6\tan^2(c+dx)+4i\tan(c+dx)+1}\,dx}{a^4}$$

```
[In] integrate((e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**4,x)
[Out] Integral((e*sec(c + d*x))**(3/2)/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6
*tan(c + d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^4}\,dx = \int \frac{(e\sec(dx+c))^{\frac{3}{2}}}{(i\,a\tan(dx+c)+a)^4}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(3/2)/(I*a*tan(d*x + c) + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^4}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}{(a+a\tan(c+d\,x)\,\,1\mathrm{i})^4}\,dx$$

```
[In] int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^4,x)
[Out] int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^4, x)
```

3.262 $\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^4}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 191

$$\begin{aligned}\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^4}\,dx &= \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{33a^4d} \\ &\quad+\frac{2e\sin(c+dx)}{33a^4d\sqrt{e\sec(c+dx)}}+\frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4} \\ &\quad+\frac{14i\sqrt{e\sec(c+dx)}}{165ad(a+ia\tan(c+dx))^3} \\ &\quad+\frac{4ie^2}{33d(e\sec(c+dx))^{3/2}\left(a^4+ia^4\tan(c+dx)\right)}\end{aligned}$$

```
[Out] 2/33*e*sin(d*x+c)/a^4/d/(e*sec(d*x+c))^(1/2)+2/33*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)*(e*sec(d*x+c))^(1/2)/a^4/d+2/15*I*(e*sec(d*x+c))^(1/2)/d/(a+I*a*tan(d*x+
c))^4+14/165*I*(e*sec(d*x+c))^(1/2)/a/d/(a+I*a*tan(d*x+c))^3+4/33*I*e^2/d/(
e*sec(d*x+c))^(3/2)/(a^4+I*a^4*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used

$= \{3583, 3581, 3854, 3856, 2720\}$

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^4}\,dx = \frac{4ie^2}{33d\,(a^4+ia^4\tan(c+dx))\,(e\sec(c+dx))^{3/2}} + \frac{2e\sin(c+dx)}{33a^4d\sqrt{e\sec(c+dx)}} + \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{33a^4d} + \frac{14i\sqrt{e\sec(c+dx)}}{165ad(a+ia\tan(c+dx))^3} + \frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4}$$

```
[In] Int[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^4,x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[e*Sec[c + d*x]])/(33*a
^4*d) + (2*e*Sin[c + d*x])/(33*a^4*d*Sqrt[e*Sec[c + d*x]]) + (((2*I)/15)*Sq
rt[e*Sec[c + d*x]])/(d*(a + I*a*Tan[c + d*x])^4) + (((14*I)/165)*Sqrt[e*Sec
[c + d*x]])/(a*d*(a + I*a*Tan[c + d*x])^3) + (((4*I)/33)*e^2)/(d*(e*Sec[c +
 d*x])^(3/2)*(a^4 + I*a^4*Tan[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4}+\frac{7\int\frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^3}\,dx}{15a}\\
&= \frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4}+\frac{14i\sqrt{e\sec(c+dx)}}{165ad(a+ia\tan(c+dx))^3}+\frac{7\int\frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx}{33a^2}\\
&= \frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4}+\frac{14i\sqrt{e\sec(c+dx)}}{165ad(a+ia\tan(c+dx))^3}\\
&\quad+\frac{4ie^2}{33d(e\sec(c+dx))^{3/2}\,(a^4+ia^4\tan(c+dx))}+\frac{e^2\int\frac{1}{(e\sec(c+dx))^{3/2}}\,dx}{11a^4}\\
&= \frac{2e\sin(c+dx)}{33a^4d\sqrt{e\sec(c+dx)}}+\frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4}+\frac{14i\sqrt{e\sec(c+dx)}}{165ad(a+ia\tan(c+dx))^3}\\
&\quad+\frac{4ie^2}{33d(e\sec(c+dx))^{3/2}\,(a^4+ia^4\tan(c+dx))}+\frac{\int\sqrt{e\sec(c+dx)}\,dx}{33a^4}\\
&= \frac{2e\sin(c+dx)}{33a^4d\sqrt{e\sec(c+dx)}}+\frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4}\\
&\quad+\frac{14i\sqrt{e\sec(c+dx)}}{165ad(a+ia\tan(c+dx))^3}+\frac{4ie^2}{33d(e\sec(c+dx))^{3/2}\,(a^4+ia^4\tan(c+dx))}\\
&\quad+\frac{\left(\sqrt{\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{33a^4}\\
&= \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\sqrt{e\sec(c+dx)}}{33a^4d}\\
&\quad+\frac{2e\sin(c+dx)}{33a^4d\sqrt{e\sec(c+dx)}}+\frac{2i\sqrt{e\sec(c+dx)}}{15d(a+ia\tan(c+dx))^4}\\
&\quad+\frac{14i\sqrt{e\sec(c+dx)}}{165ad(a+ia\tan(c+dx))^3}+\frac{4ie^2}{33d(e\sec(c+dx))^{3/2}\,(a^4+ia^4\tan(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^4}\,dx = \frac{\sec^4(c+dx)\sqrt{e\sec(c+dx)}\Big(40\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(\cos(4(c+dx))+i\sin(4(c+dx)))}{660a^4d(-i+\tan(c-}$$

[In] Integrate[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^4,x]

[Out] (Sec[c + d*x]^4*Sqrt[e*Sec[c + d*x]]*(40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[4*(c + d*x)] + I*Sin[4*(c + d*x)]) + I*(64 + 112*Cos[2*(c + d*x)] + 48*Cos[4*(c + d*x)] + (54*I)*Sin[2*(c + d*x)] + (37*I)*Sin[4*(c + d*x)])))/(660*a^4*d*(-I + Tan[c + d*x])^4)

Maple [A] (verified)

Time = 6.55 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.80

method	result
default	$\frac{2i\left((5\cos(dx+c)+5)\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\sqrt{\frac{1}{\cos(dx+c)+1}}\,F(i(-\csc(dx+c)+\cot(dx+c)),i)+i\sin(dx+c)\cos(dx+c)\left(-88\left(\cos^6(dx+c)\right)+16(c\right.\right.}{165a^4d}$

[In] int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^4,x,method=_RETURNVERBOSE)

[Out] 2/165*I/a^4/d*((5*cos(d*x+c)+5)*(cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*(1/(cos(d*x+c)+1))^(1/2)*EllipticF(I*(-csc(d*x+c)+cot(d*x+c)),I)+I*sin(d*x+c)*cos(d*x+c)*(-88*cos(d*x+c)^6+16*cos(d*x+c)^4-3*cos(d*x+c)^2-5)+cos(d*x+c)^6*(88*cos(d*x+c)^2-60))*(e*sec(d*x+c))^(1/2)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^4}\,dx = \frac{\left(\sqrt{2}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(85i\,e^{(8i\,dx+8i\,c)}+166i\,e^{(6i\,dx+6i\,c)}+128i\,e^{(4i\,dx+4i\,c)}+58i\,e^{(2i\,dx+2i\,c)}+11i\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}-\right.}{1320\,a^4d}$$

[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="fricas")

```
[Out] 1/1320*(sqrt(2)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(85*I*e^(8*I*d*x + 8*I*c)
 + 166*I*e^(6*I*d*x + 6*I*c) + 128*I*e^(4*I*d*x + 4*I*c) + 58*I*e^(2*I*d*x
+ 2*I*c) + 11*I)*e^(1/2*I*d*x + 1/2*I*c) - 80*I*sqrt(2)*sqrt(e)*e^(8*I*d*x
+ 8*I*c)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))*e^(-8*I*d*x - 8*I*c)/
(a^4*d)
```

Sympy [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^4}\,dx = \frac{\int \frac{\sqrt{e\sec{(c+dx)}}}{\tan^4{(c+dx)}-4i\tan^3{(c+dx)}-6\tan^2{(c+dx)}+4i\tan{(c+dx)}+1}\,dx}{a^4}$$

```
[In] integrate((e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**4,x)
```

```
[Out] Integral(sqrt(e*sec(c + d*x))/(tan(c + d*x)**4 - 4*I*tan(c + d*x)**3 - 6*ta
n(c + d*x)**2 + 4*I*tan(c + d*x) + 1), x)/a**4
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^4}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^4}\,dx = \int \frac{\sqrt{e\sec{(dx+c)}}}{(i\,a\tan{(dx+c)}+a)^4}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*sec(d*x + c))/(I*a*tan(d*x + c) + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^4}\,dx=\int \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}}{{\left(a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^4}\,dx$$

```
[In] int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^4,x)
[Out] int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^4, x)
```

3.263 $\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 69

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))\,dx=\frac{6i2^{5/6}a\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{5}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{5/3}}{5f(1+i\tan(e+fx))^{5/6}}$$

```
[Out] 6/5*I*2^(5/6)*a*hypergeom([-5/6, 5/6],[11/6],1/2-1/2*I*tan(f*x+e))*(d*sec(f
*x+e))^(5/3)/f/(1+I*tan(f*x+e))^(5/6)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3586, 3604, 72, 71}

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))\,dx=\frac{6i2^{5/6}a(d\sec(e+fx))^{5/3}\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{5}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{5f(1+i\tan(e+fx))^{5/6}}$$

```
[In] Int[(d*Sec[e + f*x])^(5/3)*(a + I*a*Tan[e + f*x]),x]
```

```
[Out] (((6*I)/5)*2^(5/6)*a*Hypergeometric2F1[-5/6, 5/6, 11/6, (1 - I*Tan[e + f*x]
)/2]*(d*Sec[e + f*x])^(5/3))/(f*(1 + I*Tan[e + f*x])^(5/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{(d\sec(e+fx))^{5/3}\int(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{11/6}\,dx}{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}$$

$$= \frac{\left(a^2(d\sec(e+fx))^{5/3}\right)\text{Subst}\left(\int\frac{(a+iax)^{5/6}}{\sqrt[6]{a-iax}}\,dx,x,\tan(e+fx)\right)}{f(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}$$

$$= \frac{\left(2^{5/6}a^2(d\sec(e+fx))^{5/3}\right)\text{Subst}\left(\int\frac{\left(\frac{1}{2}+\frac{ix}{2}\right)^{5/6}}{\sqrt[6]{a-iax}}\,dx,x,\tan(e+fx)\right)}{f(a-ia\tan(e+fx))^{5/6}\left(\frac{a+ia\tan(e+fx)}{a}\right)^{5/6}}$$

$$= \frac{6i2^{5/6}a\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{5}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{5/3}}{5f(1+i\tan(e+fx))^{5/6}}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))\,dx = \frac{3ad(d\sec(e+fx))^{2/3}\left(i\sec(e+fx)+\csc(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\sec^2(e\right.\right.}{5f}$$

```
[In] Integrate[(d*Sec[e + f*x])^(5/3)*(a + I*a*Tan[e + f*x]),x]
[Out] (3*a*d*(d*Sec[e + f*x])^(2/3)*(I*Sec[e + f*x] + Csc[e + f*x]*Hypergeometric
2F1[1/2, 5/6, 11/6, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2]))/(5*f)
```

Maple [F]

$$\int (d\sec(fx+e))^{\frac{5}{3}}(a+ia\tan(fx+e))\,dx$$

```
[In] int((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e)),x)
[Out] int((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e)),x)
```

Fricas [F]

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))\,dx = \int (d\sec(fx+e))^{\frac{5}{3}}(i\,a\tan(fx+e)+a)\,dx$$

```
[In] integrate((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e)),x, algorithm="fricas")
[Out] -1/10*(3*2^(2/3)*(5*I*a*d*e^(3*I*f*x + 3*I*e) + I*a*d*e^(I*f*x + I*e))*(d/(
e^(2*I*f*x + 2*I*e) + 1))^(2/3)*e^(2/3*I*f*x + 2/3*I*e) - 10*(f*e^(2*I*f*x
+ 2*I*e) + f)*integral(1/2*I*2^(2/3)*a*d*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3
)*e^(2/3*I*f*x + 2/3*I*e)/f, x))/(f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F]

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))\,dx = ia\left(\int\left(-i(d\sec(e+fx))^{\frac{5}{3}}\right)dx + \int (d\sec(e+fx))^{\frac{5}{3}}\tan(e+fx)\,dx\right)$$

```
[In] integrate((d*sec(f*x+e))**(5/3)*(a+I*a*tan(f*x+e)),x)
[Out] I*a*(Integral(-I*(d*sec(e + f*x))**(5/3), x) + Integral((d*sec(e + f*x))**(
5/3)*tan(e + f*x), x))
```

Maxima [F]

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))\,dx = \int (d\sec(fx+e))^{\frac{5}{3}}\,(i\,a\tan(fx+e)+a)\,dx$$

```
[In] integrate((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e)),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^(5/3)*(I*a*tan(f*x + e) + a), x)
```

Giac [F]

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))\,dx = \int (d\sec(fx+e))^{\frac{5}{3}}\,(i\,a\tan(fx+e)+a)\,dx$$

```
[In] integrate((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e)),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(5/3)*(I*a*tan(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))\,dx = \int \left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}(a+a\tan(e+f\,x)\;1\mathrm{i})\;dx$$

```
[In] int((d/cos(e + f*x))^(5/3)*(a + a*tan(e + f*x)*1i),x)
[Out] int((d/cos(e + f*x))^(5/3)*(a + a*tan(e + f*x)*1i), x)
```

3.264 $\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 67

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))\,dx = \frac{6i\sqrt[6]{2}a\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{6},\frac{7}{6},\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[3]{d\sec(e+fx)}}{f\sqrt[6]{1+i\tan(e+fx)}}$$

```
[Out] 6*I*2^(1/6)*a*hypergeom([-1/6, 1/6],[7/6],1/2-1/2*I*tan(f*x+e))*(d*sec(f*x+
e))^(1/3)/f/(1+I*tan(f*x+e))^(1/6)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3586, 3604, 72, 71}

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))\,dx = \frac{6i\sqrt[6]{2}a\sqrt[3]{d\sec(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{6},\frac{7}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{f\sqrt[6]{1+i\tan(e+fx)}}$$

```
[In] Int[(d*Sec[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x]),x]
```

```
[Out] ((6*I)*2^(1/6)*a*Hypergeometric2F1[-1/6, 1/6, 7/6, (1 - I*Tan[e + f*x])/2]*
(d*Sec[e + f*x])^(1/3))/(f*(1 + I*Tan[e + f*x])^(1/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt[3]{d\sec(e+fx)}\int \sqrt[6]{a-ia\tan(e+fx)}(a+ia\tan(e+fx))^{7/6}\,dx}{\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}} \\
&= \frac{\left(a^2\sqrt[3]{d\sec(e+fx)}\right)\operatorname{Subst}\left(\int \frac{\sqrt[6]{a+iax}}{(a-iax)^{5/6}}\,dx,x,\tan(e+fx)\right)}{f\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}} \\
&= \frac{\left(\sqrt[6]{2}a^2\sqrt[3]{d\sec(e+fx)}\right)\operatorname{Subst}\left(\int \frac{\sqrt[6]{\frac{1}{2}+\frac{ix}{2}}}{(a-iax)^{5/6}}\,dx,x,\tan(e+fx)\right)}{f\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{\frac{a+ia\tan(e+fx)}{a}}} \\
&= \frac{6i\sqrt[6]{2}a\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{6},\frac{7}{6},\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[3]{d\sec(e+fx)}}{f\sqrt[6]{1+i\tan(e+fx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))\,dx = \frac{3a\sqrt[3]{d\sec(e+fx)}\left(i+\cot(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\sec^2(e+fx)\right)\sqrt{-\tan^2(e+fx)}\right)}{f}$$

```
[In] Integrate[(d*Sec[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x]),x]
[Out] (3*a*(d*Sec[e + f*x])^(1/3)*(I + Cot[e + f*x]*Hypergeometric2F1[1/6, 1/2, 7
/6, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2]))/f
```

Maple [F]

$$\int (d\sec(fx+e))^{\frac{1}{3}}\,(a+ia\tan(fx+e))\,dx$$

```
[In] int((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e)),x)
[Out] int((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e)),x)
```

Fricas [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))\,dx = \int (d\sec(fx+e))^{\frac{1}{3}}\,(i\,a\tan(fx+e)+a)\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e)),x, algorithm="fricas")
[Out] (3*I*2^(1/3)*a*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3)*e^(1/3*I*f*x + 1/3*I*e)
+ f*integral(-I*2^(1/3)*a*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3)*e^(-2/3*I*f*x
 - 2/3*I*e)/f, x))/f
```

Sympy [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))\,dx = ia\left(\int \left(-i\sqrt[3]{d\sec(e+fx)}\right)dx + \int \sqrt[3]{d\sec(e+fx)}\tan(e+fx)\,dx\right)$$

```
[In] integrate((d*sec(f*x+e))**(1/3)*(a+I*a*tan(f*x+e)),x)
[Out] I*a*(Integral(-I*(d*sec(e + f*x))**(1/3), x) + Integral((d*sec(e + f*x))**(
1/3)*tan(e + f*x), x))
```

Maxima [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))\,dx = \int (d\sec(fx+e))^{\frac{1}{3}}\,(i\,a\tan(fx+e)+a)\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e)),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^(1/3)*(I*a*tan(f*x + e) + a), x)
```

Giac [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))\,dx = \int (d\sec(fx+e))^{\frac{1}{3}}\,(i\,a\tan(fx+e)+a)\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e)),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(1/3)*(I*a*tan(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))\,dx = \int \left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}\,(a+a\tan(e+f\,x)\ 1\mathrm{i})\,dx$$

```
[In] int((d/cos(e + f*x))^(1/3)*(a + a*tan(e + f*x)*1i),x)
[Out] int((d/cos(e + f*x))^(1/3)*(a + a*tan(e + f*x)*1i), x)
```

3.265 $\int \frac{a+ia\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 67

$$\int \frac{a+ia\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = -\frac{3i2^{5/6}a\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{6},\frac{5}{6},\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[6]{1+i\tan(e+fx)}}{f\sqrt[3]{d\sec(e+fx)}}$$

```
[Out] -3*I*2^(5/6)*a*hypergeom([-1/6, 1/6],[5/6],1/2-1/2*I*tan(f*x+e))*(1+I*tan(f
*x+e))^(1/6)/f/(d*sec(f*x+e))^(1/3)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3586, 3604, 72, 71}

$$\int \frac{a+ia\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = -\frac{3i2^{5/6}a\sqrt[6]{1+i\tan(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{6},\frac{5}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{f\sqrt[3]{d\sec(e+fx)}}$$

```
[In] Int[(a + I*a*Tan[e + f*x])/(d*Sec[e + f*x])^(1/3),x]
```

```
[Out] ((-3*I)*2^(5/6)*a*Hypergeometric2F1[-1/6, 1/6, 5/6, (1 - I*Tan[e + f*x])/2]
*(1 + I*Tan[e + f*x])^(1/6))/(f*(d*Sec[e + f*x])^(1/3))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}\right)\int \frac{(a+ia\tan(e+fx))^{5/6}}{\sqrt[6]{a-ia\tan(e+fx)}}\,dx}{\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{\left(a^2\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}\right)\text{Subst}\left(\int \frac{1}{(a-iax)^{7/6}\sqrt[6]{a+iax}}\,dx, x, \tan(e+fx)\right)}{f\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{\left(a^2\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{\frac{a+ia\tan(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt[6]{\frac{1}{2}+\frac{ix}{2}}(a-iax)^{7/6}}\,dx, x, \tan(e+fx)\right)}{\sqrt[6]{2}f\sqrt[3]{d\sec(e+fx)}}$$

$$= -\frac{3i2^{5/6}a\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{6},\frac{5}{6},\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[6]{1+i\tan(e+fx)}}{f\sqrt[3]{d\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \frac{a+ia\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = -\frac{3a\left(i+\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\sec^2(e+fx)\right)\sqrt{-\tan^2(e+fx)}\right)}{f\sqrt[3]{d\sec(e+fx)}}$$

```
[In] Integrate[(a + I*a*Tan[e + f*x])/(d*Sec[e + f*x])^(1/3),x]
[Out] (-3*a*(I + Cot[e + f*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[e + f*x]^2]*S
qrt[-Tan[e + f*x]^2]))/(f*(d*Sec[e + f*x])^(1/3))
```

Maple [F]

$$\int \frac{a+ia\tan(fx+e)}{(d\sec(fx+e))^{\frac{1}{3}}}dx$$

```
[In] int((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x)
[Out] int((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int \frac{a+ia\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{i\,a\tan(fx+e)+a}{(d\sec(fx+e))^{\frac{1}{3}}}\,dx$$

```
[In] integrate((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x, algorithm="fricas")
[Out] -(3*2^(2/3)*(I*a*e^(2*I*f*x + 2*I*e) + I*a)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(
2/3)*e^(2/3*I*f*x + 2/3*I*e) - (d*f*e^(I*f*x + I*e) - d*f)*integral(-2*2^(2
/3)*(I*a*e^(2*I*f*x + 2*I*e) + I*a*e^(I*f*x + I*e) + I*a)*(d/(e^(2*I*f*x +
2*I*e) + 1))^(2/3)*e^(2/3*I*f*x + 2/3*I*e)/(d*f*e^(3*I*f*x + 3*I*e) - 2*d*f
*e^(2*I*f*x + 2*I*e) + d*f*e^(I*f*x + I*e)), x))/(d*f*e^(I*f*x + I*e) - d*f
)
```

Sympy [F]

$$\int \frac{a+ia\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = ia\left(\int \left(-\frac{i}{\sqrt[3]{d\sec{(e+fx)}}}\right)dx + \int \frac{\tan{(e+fx)}}{\sqrt[3]{d\sec{(e+fx)}}}\,dx\right)$$

```
[In] integrate((a+I*a*tan(f*x+e))/(d*sec(f*x+e))**(1/3),x)
[Out] I*a*(Integral(-I/(d*sec(e + f*x))**(1/3), x) + Integral(tan(e + f*x)/(d*sec
(e + f*x))**(1/3), x))
```

Maxima [F]

$$\int \frac{a+ia\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{i\,a\tan{(fx+e)}+a}{(d\sec{(fx+e)})^{\frac{1}{3}}}\,dx$$

```
[In] integrate((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate((I*a*tan(f*x + e) + a)/(d*sec(f*x + e))^(1/3), x)
```

Giac [F]

$$\int \frac{a+ia\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{i\,a\tan{(fx+e)}+a}{(d\sec{(fx+e)})^{\frac{1}{3}}}\,dx$$

```
[In] integrate((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate((I*a*tan(f*x + e) + a)/(d*sec(f*x + e))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+ia\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{a+a\tan(e+f\,x)\,1\mathrm{i}}{{\left(\frac{d}{\cos(e+f\,x)}\right)}^{1/3}}\,dx$$

```
[In] int((a + a*tan(e + f*x)*1i)/(d/cos(e + f*x))^(1/3),x)
[Out] int((a + a*tan(e + f*x)*1i)/(d/cos(e + f*x))^(1/3), x)
```

3.266 $\int \frac{a+ia\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 69

$$\int \frac{a+ia\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx = -\frac{3i\sqrt[6]{2}a\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{5}{6},\frac{1}{6},\frac{1}{2}(1-i\tan(e+fx))\right)(1+i\tan(e+fx))^{5/6}}{5f(d\sec(e+fx))^{5/3}}$$

```
[Out] -3/5*I*2^(1/6)*a*hypergeom([-5/6, 5/6],[1/6],1/2-1/2*I*tan(f*x+e))*(1+I*tan
(f*x+e))^(5/6)/f/(d*sec(f*x+e))^(5/3)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3586, 3604, 72, 71}

$$\int \frac{a+ia\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx = -\frac{3i\sqrt[6]{2}a(1+i\tan(e+fx))^{5/6}\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{5}{6},\frac{1}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{5f(d\sec(e+fx))^{5/3}}$$

```
[In] Int[(a + I*a*Tan[e + f*x])/(d*Sec[e + f*x])^(5/3),x]
```

```
[Out] (((-3*I)/5)*2^(1/6)*a*Hypergeometric2F1[-5/6, 5/6, 1/6, (1 - I*Tan[e + f*x]
)/2]*(1 + I*Tan[e + f*x])^(5/6))/(f*(d*Sec[e + f*x])^(5/3))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\left((a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\right)\int \frac{\sqrt[6]{a+ia\tan(e+fx)}}{(a-ia\tan(e+fx))^{5/6}}\,dx}{(d\sec(e+fx))^{5/3}}$$

$$= \frac{\left(a^2(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\right)\operatorname{Subst}\left(\int \frac{1}{(a-iax)^{11/6}(a+iax)^{5/6}}\,dx,x,\tan(e+fx)\right)}{f(d\sec(e+fx))^{5/3}}$$

$$= \frac{\left(a^2(a-ia\tan(e+fx))^{5/6}\left(\frac{a+ia\tan(e+fx)}{a}\right)^{5/6}\right)\operatorname{Subst}\left(\int \frac{1}{\left(\frac{1}{2}+\frac{ix}{2}\right)^{5/6}(a-iax)^{11/6}}\,dx,x,\tan(e+fx)\right)}{2^{5/6}f(d\sec(e+fx))^{5/3}}$$

$$= -\frac{3i\sqrt[6]{2}a\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{5}{6},\frac{1}{6},\frac{1}{2}(1-i\tan(e+fx))\right)(1+i\tan(e+fx))^{5/6}}{5f(d\sec(e+fx))^{5/3}}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \frac{a+ia\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx = -\frac{3a\Big(i+\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},\sec^2(e+fx)\right)\sqrt{-\tan^2(e+fx)}\Big)}{5f(d\sec(e+fx))^{5/3}}$$

```
[In] Integrate[(a + I*a*Tan[e + f*x])/(d*Sec[e + f*x])^(5/3),x]
```

```
[Out] (-3*a*(I + Cot[e + f*x]*Hypergeometric2F1[-5/6, 1/2, 1/6, Sec[e + f*x]^2]*S
qrt[-Tan[e + f*x]^2]))/(5*f*(d*Sec[e + f*x])^(5/3))
```

Maple [F]

$$\int \frac{a+ia\tan\left(fx+e\right)}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}}dx$$

```
[In] int((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x)
```

```
[Out] int((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x)
```

Fricas [F]

$$\int \frac{a+ia\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx = \int \frac{i\,a\tan\left(fx+e\right)+a}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}}\,dx$$

```
[In] integrate((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x, algorithm="fricas")
```

```
[Out] 1/10*(10*d^2*f*integral(-2/5*I*2^(1/3)*a*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3
)*e^(-2/3*I*f*x - 2/3*I*e)/(d^2*f), x) - 3*2^(1/3)*(I*a*e^(2*I*f*x + 2*I*e)
 + I*a)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3)*e^(1/3*I*f*x + 1/3*I*e))/(d^2*f
)
```

Sympy [F]

$$\int \frac{a+ia\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx = ia\left(\int \left(-\frac{i}{(d\sec(e+fx))^{\frac{5}{3}}}\right)dx + \int \frac{\tan(e+fx)}{(d\sec(e+fx))^{\frac{5}{3}}}\,dx\right)$$

```
[In] integrate((a+I*a*tan(f*x+e))/(d*sec(f*x+e))**(5/3),x)
[Out] I*a*(Integral(-I/(d*sec(e + f*x))**(5/3), x) + Integral(tan(e + f*x)/(d*sec
(e + f*x))**(5/3), x))
```

Maxima [F]

$$\int \frac{a+ia\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx = \int \frac{i\,a\tan(fx+e)+a}{(d\sec(fx+e))^{\frac{5}{3}}}\,dx$$

```
[In] integrate((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x, algorithm="maxima")
[Out] integrate((I*a*tan(f*x + e) + a)/(d*sec(f*x + e))^(5/3), x)
```

Giac [F]

$$\int \frac{a+ia\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx = \int \frac{i\,a\tan(fx+e)+a}{(d\sec(fx+e))^{\frac{5}{3}}}\,dx$$

```
[In] integrate((a+I*a*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x, algorithm="giac")
[Out] integrate((I*a*tan(f*x + e) + a)/(d*sec(f*x + e))^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+ia\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx = \int \frac{a+a\tan(e+f\,x)\;1\mathrm{i}}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}}\,dx$$

```
[In] int((a + a*tan(e + f*x)*1i)/(d/cos(e + f*x))^(5/3),x)
[Out] int((a + a*tan(e + f*x)*1i)/(d/cos(e + f*x))^(5/3), x)
```

3.267 $\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2\,dx$

Optimal result

Integrand size = 28, antiderivative size = 71

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2\,dx=\frac{12i2^{5/6}a^2\,\text{Hypergeometric2F1}\left(-\frac{11}{6},\frac{5}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{5/3}}{5f(1+i\tan(e+fx))^{5/6}}$$

```
[Out] 12/5*I*2^(5/6)*a^2*hypergeom([-11/6, 5/6],[11/6],1/2-1/2*I*tan(f*x+e))*(d*s
ec(f*x+e))^(5/3)/f/(1+I*tan(f*x+e))^(5/6)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3586, 3604, 72, 71}

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2\,dx=\frac{12i2^{5/6}a^2(d\sec(e+fx))^{5/3}\,\text{Hypergeometric2F1}\left(-\frac{11}{6},\frac{5}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{5f(1+i\tan(e+fx))^{5/6}}$$

```
[In] Int[(d*Sec[e + f*x])^(5/3)*(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (((12*I)/5)*2^(5/6)*a^2*Hypergeometric2F1[-11/6, 5/6, 11/6, (1 - I*Tan[e +
f*x])/2]*(d*Sec[e + f*x])^(5/3))/(f*(1 + I*Tan[e + f*x])^(5/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(d\sec(e+fx))^{5/3}\int(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{17/6}\,dx}{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}} \\
&= \frac{\left(a^2(d\sec(e+fx))^{5/3}\right)\text{Subst}\left(\int\frac{(a+iax)^{11/6}}{\sqrt[6]{a-iax}}\,dx,x,\tan(e+fx)\right)}{f(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}} \\
&= \frac{\left(2\,2^{5/6}a^3(d\sec(e+fx))^{5/3}\right)\text{Subst}\left(\int\frac{\left(\frac{1}{2}+\frac{ix}{2}\right)^{11/6}}{\sqrt[6]{a-iax}}\,dx,x,\tan(e+fx)\right)}{f(a-ia\tan(e+fx))^{5/6}\left(\frac{a+ia\tan(e+fx)}{a}\right)^{5/6}} \\
&= \frac{12i2^{5/6}a^2\,\text{Hypergeometric2F1}\left(-\frac{11}{6},\frac{5}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{5/3}}{5f(1+i\tan(e+fx))^{5/6}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.56

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2\,dx = \frac{3ia^2(d\sec(e+fx))^{5/3}\left(i\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{5}{6},\frac{11}{6},\sec^2(e+fx)\right)\tan(e+fx)+i\right.}{5f\sqrt{-\tan^2(e}}$$

```
[In] Integrate[(d*Sec[e + f*x])^(5/3)*(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (((3*I)/5)*a^2*(d*Sec[e + f*x])^(5/3)*(I*Hypergeometric2F1[-1/2, 5/6, 11/6,
 Sec[e + f*x]^2]*Tan[e + f*x] + I*Hypergeometric2F1[1/2, 5/6, 11/6, Sec[e +
 f*x]^2]*Tan[e + f*x] + 2*Sqrt[-Tan[e + f*x]^2]))/(f*Sqrt[-Tan[e + f*x]^2])
```

Maple [F]

$$\int (d\sec(fx+e))^{\frac{5}{3}}(a+ia\tan(fx+e))^2\,dx$$

```
[In] int((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e))^2,x)
```

```
[Out] int((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e))^2,x)
```

Fricas [F]

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2\,dx = \int (d\sec(fx+e))^{\frac{5}{3}}(i\,a\tan(fx+e)+a)^2\,dx$$

```
[In] integrate((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/80*(3*2^(2/3)*(55*I*a^2*d*e^(5*I*f*x + 5*I*e) + 26*I*a^2*d*e^(3*I*f*x +
3*I*e) + 11*I*a^2*d*e^(I*f*x + I*e))*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*e^
(2/3*I*f*x + 2/3*I*e) - 80*(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2*I*e)
 + f)*integral(11/16*I*2^(2/3)*a^2*d*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*e^
(2/3*I*f*x + 2/3*I*e)/f, x))/(f*e^(4*I*f*x + 4*I*e) + 2*f*e^(2*I*f*x + 2*I*
e) + f)
```

Sympy [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2\,dx=\text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))**(5/3)*(a+I*a*tan(f*x+e))**2,x)
[Out] Timed out
```

Maxima [F]

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2\,dx=\int (d\sec(fx+e))^{\frac{5}{3}}(ia\tan(fx+e)+a)^2\,dx$$

```
[In] integrate((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^(5/3)*(I*a*tan(f*x + e) + a)^2, x)
```

Giac [F]

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2\,dx=\int (d\sec(fx+e))^{\frac{5}{3}}(ia\tan(fx+e)+a)^2\,dx$$

```
[In] integrate((d*sec(f*x+e))^(5/3)*(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(5/3)*(I*a*tan(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2\,dx=\int \left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}(a+a\tan(e+f\,x)\;1\mathrm{i})^2\,dx$$

```
[In] int((d/cos(e + f*x))^(5/3)*(a + a*tan(e + f*x)*1i)^2,x)
[Out] int((d/cos(e + f*x))^(5/3)*(a + a*tan(e + f*x)*1i)^2, x)
```

3.268 $\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2\,dx$

Optimal result

Integrand size = 28, antiderivative size = 69

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2\,dx = \frac{12i\sqrt[6]{2}a^2\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{6},\frac{7}{6},\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[3]{d\sec(e+fx)}}{f\sqrt[6]{1+i\tan(e+fx)}}$$

```
[Out] 12*I*2^(1/6)*a^2*hypergeom([-7/6, 1/6],[7/6],1/2-1/2*I*tan(f*x+e))*(d*sec(f
*x+e))^(1/3)/f/(1+I*tan(f*x+e))^(1/6)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3586, 3604, 72, 71}

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2\,dx = \frac{12i\sqrt[6]{2}a^2\sqrt[3]{d\sec(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{6},\frac{7}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{f\sqrt[6]{1+i\tan(e+fx)}}$$

```
[In] Int[(d*Sec[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] ((12*I)*2^(1/6)*a^2*Hypergeometric2F1[-7/6, 1/6, 7/6, (1 - I*Tan[e + f*x])/
2]*(d*Sec[e + f*x])^(1/3))/(f*(1 + I*Tan[e + f*x])^(1/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt[3]{d\sec(e+fx)}\int \sqrt[6]{a-ia\tan(e+fx)}(a+ia\tan(e+fx))^{13/6}\,dx}{\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}}\\
&= \frac{\left(a^2\sqrt[3]{d\sec(e+fx)}\right)\text{Subst}\left(\int\frac{(a+iax)^{7/6}}{(a-iax)^{5/6}}\,dx,x,\tan(e+fx)\right)}{f\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}}\\
&= \frac{\left(2\sqrt[6]{2}a^3\sqrt[3]{d\sec(e+fx)}\right)\text{Subst}\left(\int\frac{\left(\frac{1}{2}+\frac{ix}{2}\right)^{7/6}}{(a-iax)^{5/6}}\,dx,x,\tan(e+fx)\right)}{f\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{\frac{a+ia\tan(e+fx)}{a}}}\\
&= \frac{12i\sqrt[6]{2}a^2\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{6},\frac{7}{6},\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[3]{d\sec(e+fx)}}{f\sqrt[6]{1+i\tan(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.48

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2\,dx$$
$$=\frac{3a^2\sqrt[3]{d\sec(e+fx)}\Big(2i+\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{6},\frac{7}{6},\sec^2(e+fx)\right)\sqrt{-\tan^2(e+fx)}+\cot$$
$$f$$

```
[In] Integrate[(d*Sec[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (3*a^2*(d*Sec[e + f*x])^(1/3)*(2*I + Cot[e + f*x]*Hypergeometric2F1[-1/2, 1
/6, 7/6, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2] + Cot[e + f*x]*Hypergeometri
c2F1[1/6, 1/2, 7/6, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2]))/f
```

Maple [F]

$$\int (d\sec(fx+e))^{\frac{1}{3}}(a+ia\tan(fx+e))^2\,dx$$

```
[In] int((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e))^2,x)
```

```
[Out] int((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e))^2,x)
```

Fricas [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2\,dx=\int (d\sec(fx+e))^{\frac{1}{3}}(i\,a\tan(fx+e)+a)^2\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/4*(3*2^(1/3)*(-9*I*a^2*e^(2*I*f*x + 2*I*e) - 7*I*a^2)*(d/(e^(2*I*f*x + 2
*I*e) + 1))^(1/3)*e^(1/3*I*f*x + 1/3*I*e) - 4*(f*e^(2*I*f*x + 2*I*e) + f)*i
ntegral(-7/4*I*2^(1/3)*a^2*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3)*e^(-2/3*I*f*
x - 2/3*I*e)/f, x))/(f*e^(2*I*f*x + 2*I*e) + f)
```

Sympy [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2\,dx = -a^2\left(\int \left(-\sqrt[3]{d\sec(e+fx)}\right)dx + \int \sqrt[3]{d\sec(e+fx)}\tan^2(e+fx)\,dx + \int \left(-2i\sqrt[3]{d\sec(e+fx)}\tan(e+fx)\right)dx\right)$$

```
[In] integrate((d*sec(f*x+e))**(1/3)*(a+I*a*tan(f*x+e))**2,x)
[Out] -a**2*(Integral(-(d*sec(e + f*x))**(1/3), x) + Integral((d*sec(e + f*x))**(
1/3)*tan(e + f*x)**2, x) + Integral(-2*I*(d*sec(e + f*x))**(1/3)*tan(e + f*
x), x))
```

Maxima [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2\,dx = \int (d\sec(fx+e))^{\frac{1}{3}}(i\,a\tan(fx+e)+a)^2\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^(1/3)*(I*a*tan(f*x + e) + a)^2, x)
```

Giac [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2\,dx = \int (d\sec(fx+e))^{\frac{1}{3}}(i\,a\tan(fx+e)+a)^2\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/3)*(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(1/3)*(I*a*tan(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2\,dx = \int \left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}(a+a\tan(e+f\,x)\;1\mathrm{i})^2\,dx$$

```
[In] int((d/cos(e + f*x))^(1/3)*(a + a*tan(e + f*x)*1i)^2,x)
[Out] int((d/cos(e + f*x))^(1/3)*(a + a*tan(e + f*x)*1i)^2, x)
```

3.269 $\int \frac{(a+ia\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 83

$$\int \frac{(a+ia\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = -\frac{6i2^{5/6}\,\text{Hypergeometric2F1}\left(-\frac{5}{6},-\frac{1}{6},\frac{5}{6},\frac{1}{2}(1-i\tan(e+fx))\right)\left(a^2+ia^2\tan(e+fx)\right)}{f\sqrt[3]{d\sec(e+fx)}(1+i\tan(e+fx))^{5/6}}$$

```
[Out] -6*I*2^(5/6)*hypergeom([-5/6, -1/6],[5/6],1/2-1/2*I*tan(f*x+e))*(a^2+I*a^2*
tan(f*x+e))/f/(d*sec(f*x+e))^(1/3)/(1+I*tan(f*x+e))^(5/6)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3586, 3604, 72, 71}

$$\int \frac{(a+ia\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = -\frac{6i2^{5/6}(a^2+ia^2\tan(e+fx))\,\text{Hypergeometric2F1}\left(-\frac{5}{6},-\frac{1}{6},\frac{5}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{f(1+i\tan(e+fx))^{5/6}\sqrt[3]{d\sec(e+fx)}}$$

```
[In] Int[(a + I*a*Tan[e + f*x])^2/(d*Sec[e + f*x])^(1/3),x]
```

```
[Out] ((-6*I)*2^(5/6)*Hypergeometric2F1[-5/6, -1/6, 5/6, (1 - I*Tan[e + f*x])/2]*
(a^2 + I*a^2*Tan[e + f*x]))/(f*(d*Sec[e + f*x])^(1/3)*(1 + I*Tan[e + f*x])^
(5/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}\right)\int \frac{(a+ia\tan(e+fx))^{11/6}}{\sqrt[6]{a-ia\tan(e+fx)}}\,dx}{\sqrt[3]{d\sec(e+fx)}}\\
&= \frac{\left(a^2\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}\right)\text{Subst}\left(\int \frac{(a+iax)^{5/6}}{(a-iax)^{7/6}}\,dx,x,\tan(e+fx)\right)}{f\sqrt[3]{d\sec(e+fx)}}\\
&= \frac{\left(2^{5/6}a^2\sqrt[6]{a-ia\tan(e+fx)}(a+ia\tan(e+fx))\right)\text{Subst}\left(\int \frac{\left(\frac{1}{2}+\frac{ix}{2}\right)^{5/6}}{(a-iax)^{7/6}}\,dx,x,\tan(e+fx)\right)}{f\sqrt[3]{d\sec(e+fx)}\left(\frac{a+ia\tan(e+fx)}{a}\right)^{5/6}}\\
&= -\frac{6i2^{5/6}\,\text{Hypergeometric2F1}\left(-\frac{5}{6},-\frac{1}{6},\frac{5}{6},\frac{1}{2}(1-i\tan(e+fx))\right)\left(a^2+ia^2\tan(e+fx)\right)}{f\sqrt[3]{d\sec(e+fx)}(1+i\tan(e+fx))^{5/6}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.24

$$\int \frac{(a+ia\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx$$
$$= \frac{3a^2\left(\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{6},\frac{5}{6},\sec^2(e+fx)\right)\tan(e+fx)+\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\sec^2(e+\right.\right.}{f\sqrt[3]{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}$$

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2/(d*Sec[e + f*x])^(1/3),x]
```

```
[Out] (3*a^2*(Hypergeometric2F1[-1/2, -1/6, 5/6, Sec[e + f*x]^2]*Tan[e + f*x] + H
ypergeometric2F1[-1/6, 1/2, 5/6, Sec[e + f*x]^2]*Tan[e + f*x] - (2*I)*Sqrt[
-Tan[e + f*x]^2]))/(f*(d*Sec[e + f*x])^(1/3)*Sqrt[-Tan[e + f*x]^2])
```

Maple [F]

$$\int \frac{(a+ia\tan(fx+e))^2}{(d\sec(fx+e))^{\frac{1}{3}}}dx$$

```
[In] int((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x)
```

```
[Out] int((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int \frac{(a+ia\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{(i\,a\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{1}{3}}}\,dx$$

```
[In] integrate((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] -1/2*(3*2^(2/3)*(4*I*a^2*e^(2*I*f*x + 2*I*e) + I*a^2*e^(I*f*x + I*e) + 5*I*
a^2)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*e^(2/3*I*f*x + 2/3*I*e) - 2*(d*f*e
^(I*f*x + I*e) - d*f)*integral(-5*2^(2/3)*(I*a^2*e^(2*I*f*x + 2*I*e) + I*a^
2*e^(I*f*x + I*e) + I*a^2)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*e^(2/3*I*f*x
 + 2/3*I*e)/(d*f*e^(3*I*f*x + 3*I*e) - 2*d*f*e^(2*I*f*x + 2*I*e) + d*f*e^(I
*f*x + I*e)), x))/(d*f*e^(I*f*x + I*e) - d*f)
```

Sympy [F]

$$\int \frac{(a+ia\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = -a^2\left(\int\left(-\frac{1}{\sqrt[3]{d\sec(e+fx)}}\right)dx + \int\frac{\tan^2(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx + \int\left(-\frac{2i\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\right)dx\right)$$

```
[In] integrate((a+I*a*tan(f*x+e))**2/(d*sec(f*x+e))**(1/3),x)
[Out] -a**2*(Integral(-1/(d*sec(e + f*x))**(1/3), x) + Integral(tan(e + f*x)**2/(
d*sec(e + f*x))**(1/3), x) + Integral(-2*I*tan(e + f*x)/(d*sec(e + f*x))**(
1/3), x))
```

Maxima [F]

$$\int \frac{(a+ia\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{(i\,a\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{1}{3}}}\,dx$$

```
[In] integrate((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate((I*a*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(1/3), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{(i\,a\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{1}{3}}}\,dx$$

```
[In] integrate((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate((I*a*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{(a+a\tan(e+f\,x)\ 1\mathrm{i})^2}{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}}\,dx$$

```
[In] int((a + a*tan(e + f*x)*1i)^2/(d/cos(e + f*x))^(1/3),x)
[Out] int((a + a*tan(e + f*x)*1i)^2/(d/cos(e + f*x))^(1/3), x)
```

3.270 $\int \frac{(a+ia\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 85

$$\int \frac{(a+ia\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx = -\frac{6i\sqrt[6]{2}\,\text{Hypergeometric2F1}\left(-\frac{5}{6},-\frac{1}{6},\frac{1}{6},\frac{1}{2}(1-i\tan(e+fx))\right)(a^2+ia^2\tan(e+fx))}{5f(d\sec(e+fx))^{5/3}\sqrt[6]{1+i\tan(e+fx)}}$$

```
[Out] -6/5*I*2^(1/6)*hypergeom([-5/6, -1/6],[1/6],1/2-1/2*I*tan(f*x+e))*(a^2+I*a^
2*tan(f*x+e))/f/(d*sec(f*x+e))^(5/3)/(1+I*tan(f*x+e))^(1/6)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3586, 3604, 72, 71}

$$\int \frac{(a+ia\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx = -\frac{6i\sqrt[6]{2}(a^2+ia^2\tan(e+fx))\,\text{Hypergeometric2F1}\left(-\frac{5}{6},-\frac{1}{6},\frac{1}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{5f\sqrt[6]{1+i\tan(e+fx)}(d\sec(e+fx))^{5/3}}$$

```
[In] Int[(a + I*a*Tan[e + f*x])^2/(d*Sec[e + f*x])^(5/3),x]
```

```
[Out] (((-6*I)/5)*2^(1/6)*Hypergeometric2F1[-5/6, -1/6, 1/6, (1 - I*Tan[e + f*x])
/2]*(a^2 + I*a^2*Tan[e + f*x]))/(f*(d*Sec[e + f*x])^(5/3)*(1 + I*Tan[e + f*
x])^(1/6))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left((a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\right)\int\frac{(a+ia\tan(e+fx))^{7/6}}{(a-ia\tan(e+fx))^{5/6}}\,dx}{(d\sec(e+fx))^{5/3}}\\
&= \frac{\left(a^2(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\right)\operatorname{Subst}\left(\int\frac{\sqrt[6]{a+iax}}{(a-iax)^{11/6}}\,dx,x,\tan(e+fx)\right)}{f(d\sec(e+fx))^{5/3}}\\
&= \frac{\left(\sqrt[6]{2}a^2(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))\right)\operatorname{Subst}\left(\int\frac{\sqrt[6]{\frac{1}{2}+\frac{ix}{2}}}{(a-iax)^{11/6}}\,dx,x,\tan(e+fx)\right)}{f(d\sec(e+fx))^{5/3}\sqrt[6]{\frac{a+ia\tan(e+fx)}{a}}}\end{aligned}$$

$$= -\frac{6i\sqrt[6]{2}\,\text{Hypergeometric2F1}\left(-\frac{5}{6},-\frac{1}{6},\frac{1}{6},\frac{1}{2}(1-i\tan(e+fx))\right)\left(a^2+ia^2\tan(e+fx)\right)}{5f(d\sec(e+fx))^{5/3}\sqrt[6]{1+i\tan(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.24

$$\int \frac{(a+ia\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx = \frac{3a^2\Big(\text{Hypergeometric2F1}\left(-\frac{5}{6},-\frac{1}{2},\frac{1}{6},\sec^2(e+fx)\right)\tan(e+fx)+\text{Hypergeom}}{5f(d\sec(e+fx))^{5/3}\sqrt{-}}$$

```
[In] Integrate[(a + I*a*Tan[e + f*x])^2/(d*Sec[e + f*x])^(5/3),x]
```

```
[Out] (3*a^2*(Hypergeometric2F1[-5/6, -1/2, 1/6, Sec[e + f*x]^2]*Tan[e + f*x] + H
ypergeometric2F1[-5/6, 1/2, 1/6, Sec[e + f*x]^2]*Tan[e + f*x] - (2*I)*Sqrt[
-Tan[e + f*x]^2]))/(5*f*(d*Sec[e + f*x])^(5/3)*Sqrt[-Tan[e + f*x]^2])
```

Maple [F]

$$\int \frac{(a+ia\tan(fx+e))^2}{(d\sec(fx+e))^{\frac{5}{3}}}dx$$

```
[In] int((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x)
```

```
[Out] int((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x)
```

Fricas [F]

$$\int \frac{(a+ia\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx = \int \frac{(i\,a\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{5}{3}}}\,dx$$

```
[In] integrate((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x, algorithm="fricas")
```

```
[Out] 1/5*(5*d^2*f*integral(1/5*I*2^(1/3)*a^2*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3)
*e^(-2/3*I*f*x - 2/3*I*e)/(d^2*f), x) - 3*2^(1/3)*(I*a^2*e^(2*I*f*x + 2*I*e
) + I*a^2)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3)*e^(1/3*I*f*x + 1/3*I*e))/(d^
2*f)
```

Sympy [F]

$$\int \frac{(a + ia\tan(e + fx))^2}{(d\sec(e + fx))^{5/3}}\,dx = -a^2\left(\int \left(-\frac{1}{(d\sec(e + fx))^{\frac{5}{3}}}\right)dx + \int \frac{\tan^2(e + fx)}{(d\sec(e + fx))^{\frac{5}{3}}}\,dx + \int \left(-\frac{2i\tan(e + fx)}{(d\sec(e + fx))^{\frac{5}{3}}}\right)dx\right)$$

```
[In] integrate((a+I*a*tan(f*x+e))**2/(d*sec(f*x+e))**(5/3),x)
```

```
[Out] -a**2*(Integral(-1/(d*sec(e + f*x))**(5/3), x) + Integral(tan(e + f*x)**2/(
d*sec(e + f*x))**(5/3), x) + Integral(-2*I*tan(e + f*x)/(d*sec(e + f*x))**(
5/3), x))
```

Maxima [F]

$$\int \frac{(a + ia\tan(e + fx))^2}{(d\sec(e + fx))^{5/3}}\,dx = \int \frac{(i\,a\tan(fx + e) + a)^2}{(d\sec(fx + e))^{\frac{5}{3}}}\,dx$$

```
[In] integrate((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(5/3), x)
```

Giac [F]

$$\int \frac{(a + ia\tan(e + fx))^2}{(d\sec(e + fx))^{5/3}}\,dx = \int \frac{(i\,a\tan(fx + e) + a)^2}{(d\sec(fx + e))^{\frac{5}{3}}}\,dx$$

```
[In] integrate((a+I*a*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + ia\tan(e + fx))^2}{(d\sec(e + fx))^{5/3}}\,dx = \int \frac{(a + a\tan(e + f\,x)\ 1\mathrm{i})^2}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}}\,dx$$

```
[In] int((a + a*tan(e + f*x)*1i)^2/(d/cos(e + f*x))^(5/3),x)
```

```
[Out] int((a + a*tan(e + f*x)*1i)^2/(d/cos(e + f*x))^(5/3), x)
```

3.271 $\int \frac{(d\sec(e+fx))^{5/3}}{a+ia\tan(e+fx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 83

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+ia\tan(e+fx)}\,dx = \frac{3i\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{7}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{5/3}\sqrt[6]{1+i\,\text{ta}}}{5\sqrt[6]{2}f(a+ia\tan(e+fx))}$$

```
[Out] 3/10*I*hypergeom([5/6, 7/6],[11/6],1/2-1/2*I*tan(f*x+e))*(d*sec(f*x+e))^(5/
3)*(1+I*tan(f*x+e))^(1/6)*2^(5/6)/f/(a+I*a*tan(f*x+e))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3586, 3604, 72, 71}

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+ia\tan(e+fx)}\,dx = \frac{3i\sqrt[6]{1+i\tan(e+fx)}(d\sec(e+fx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{7}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(\right.}{5\sqrt[6]{2}f(a+ia\tan(e+fx))}$$

```
[In] Int[(d*Sec[e + f*x])^(5/3)/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] (((3*I)/5)*Hypergeometric2F1[5/6, 7/6, 11/6, (1 - I*Tan[e + f*x])/2]*(d*Sec
[e + f*x])^(5/3)*(1 + I*Tan[e + f*x])^(1/6))/(2^(1/6)*f*(a + I*a*Tan[e + f*
x]))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(d\sec(e+fx))^{5/3}\int \frac{(a-ia\tan(e+fx))^{5/6}}{\sqrt[6]{a+ia\tan(e+fx)}}\,dx}{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}\\
&= \frac{\left(a^2(d\sec(e+fx))^{5/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[6]{a-iax}(a+iax)^{7/6}}\,dx, x, \tan(e+fx)\right)}{f(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}\\
&= \frac{\left(a(d\sec(e+fx))^{5/3}\sqrt[6]{\frac{a+ia\tan(e+fx)}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{1}{2}+\frac{ix}{2}\right)^{7/6}\sqrt[6]{a-iax}}\,dx, x, \tan(e+fx)\right)}{2\sqrt[6]{2}f(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))}\\
&= \frac{3i\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{7}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{5/3}\sqrt[6]{1+i\tan(e+fx)}}{5\sqrt[6]{2}f(a+ia\tan(e+fx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.01

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+ia\tan(e+fx)}\,dx = \frac{6de^{i(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{2}{3},\frac{5}{6},-e^{2i(e+fx)}\right)(d\sec(e+fx))^{2/3}}{a\sqrt[3]{1+e^{2i(e+fx)}}f(-i+\tan(e+fx))}$$

```
[In] Integrate[(d*Sec[e + f*x])^(5/3)/(a + I*a*Tan[e + f*x]),x]
[Out] (6*d*E^(I*(e + f*x))*Hypergeometric2F1[-1/6, 2/3, 5/6, -E^((2*I)*(e + f*x))
]*(d*Sec[e + f*x])^(2/3))/(a*(1 + E^((2*I)*(e + f*x)))^(1/3)*f*(-I + Tan[e
+ f*x]))
```

Maple [F]

$$\int \frac{(d\sec(fx+e))^{\frac{5}{3}}}{a+ia\tan(fx+e)}dx$$

```
[In] int((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x)
[Out] int((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x)
```

Fricas [F]

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+ia\tan(e+fx)}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{3}}}{i\,a\tan(fx+e)+a}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
[Out] (a*f*e^(I*f*x + I*e)*integral(-I*2^(2/3)*d*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2
/3)*e^(2/3*I*f*x + 2/3*I*e)/(a*f), x) - 3*2^(2/3)*(-I*d*e^(2*I*f*x + 2*I*e)
 - I*d)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*e^(2/3*I*f*x + 2/3*I*e))*e^(-I*
f*x - I*e)/(a*f)
```

Sympy [F]

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+ia\tan(e+fx)}\,dx = -\frac{i\int \frac{(d\sec(e+fx))^{\frac{5}{3}}}{\tan(e+fx)-i}\,dx}{a}$$

```
[In] integrate((d*sec(f*x+e))**(5/3)/(a+I*a*tan(f*x+e)),x)
[Out] -I*Integral((d*sec(e + f*x))**(5/3)/(tan(e + f*x) - I), x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+ia\tan(e+fx)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+ia\tan(e+fx)}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{3}}}{i\,a\tan(fx+e)+a}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(5/3)/(I*a*tan(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+ia\tan(e+fx)}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}}{a+a\tan(e+f\,x)\,1\mathrm{i}}\,dx$$

```
[In] int((d/cos(e + f*x))^(5/3)/(a + a*tan(e + f*x)*1i),x)
[Out] int((d/cos(e + f*x))^(5/3)/(a + a*tan(e + f*x)*1i), x)
```

3.272 $\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+ia\tan(e+fx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 81

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+ia\tan(e+fx)}\,dx = \frac{3i\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{11}{6},\frac{7}{6},\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[3]{d\sec(e+fx)}(1+i\tan(e+fx))^{5/6}}{2^{5/6}f(a+ia\tan(e+fx))}$$

```
[Out] 3/2*I*hypergeom([1/6, 11/6],[7/6],1/2-1/2*I*tan(f*x+e))*(d*sec(f*x+e))^(1/3
)*(1+I*tan(f*x+e))^(5/6)*2^(1/6)/f/(a+I*a*tan(f*x+e))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3586, 3604, 72, 71}

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+ia\tan(e+fx)}\,dx = \frac{3i(1+i\tan(e+fx))^{5/6}\sqrt[3]{d\sec(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{11}{6},\frac{7}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{2^{5/6}f(a+ia\tan(e+fx))}$$

```
[In] Int[(d*Sec[e + f*x])^(1/3)/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] ((3*I)*Hypergeometric2F1[1/6, 11/6, 7/6, (1 - I*Tan[e + f*x])/2]*(d*Sec[e +
 f*x])^(1/3)*(1 + I*Tan[e + f*x])^(5/6))/(2^(5/6)*f*(a + I*a*Tan[e + f*x]))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt[3]{d\sec(e+fx)}\int\frac{\sqrt[6]{a-ia\tan(e+fx)}}{(a+ia\tan(e+fx))^{5/6}}\,dx}{\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}}\\
&= \frac{\left(a^2\sqrt[3]{d\sec(e+fx)}\right)\text{Subst}\left(\int\frac{1}{(a-iax)^{5/6}(a+iax)^{11/6}}\,dx,x,\tan(e+fx)\right)}{f\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}}\\
&= \frac{\left(a\sqrt[3]{d\sec(e+fx)}\left(\frac{a+ia\tan(e+fx)}{a}\right)^{5/6}\right)\text{Subst}\left(\int\frac{1}{\left(\frac{1}{2}+\frac{ix}{2}\right)^{11/6}(a-iax)^{5/6}}\,dx,x,\tan(e+fx)\right)}{2\,2^{5/6}f\sqrt[6]{a-ia\tan(e+fx)}(a+ia\tan(e+fx))}\\
&= \frac{3i\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{11}{6},\frac{7}{6},\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[3]{d\sec(e+fx)}(1+i\tan(e+fx))^{5/6}}{2^{5/6}f(a+ia\tan(e+fx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+ia\tan(e+fx)}\,dx = -\frac{3ie^{-2i(e+fx)}\left(-1-e^{2i(e+fx)}+4e^{2i(e+fx)}\sqrt[3]{1+e^{2i(e+fx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{3},\frac{7}{6},-e^{2i(e+fx)}\right)\right)\sqrt[3]{d\sec(e+fx)}}{10af}$$

```
[In] Integrate[(d*Sec[e + f*x])^(1/3)/(a + I*a*Tan[e + f*x]),x]
```

```
[Out] (((-3*I)/10)*(-1 - E^((2*I)*(e + f*x)) + 4*E^((2*I)*(e + f*x))*(1 + E^((2*I
)*(e + f*x)))^(1/3)*Hypergeometric2F1[1/6, 1/3, 7/6, -E^((2*I)*(e + f*x))])
*(d*Sec[e + f*x])^(1/3))/(a*E^((2*I)*(e + f*x))*f)
```

Maple [F]

$$\int \frac{(d\sec(fx+e))^{\frac{1}{3}}}{a+ia\tan(fx+e)}dx$$

```
[In] int((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x)
```

```
[Out] int((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x)
```

Fricas [F]

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+ia\tan(e+fx)}\,dx = \int \frac{(d\sec(fx+e))^{\frac{1}{3}}}{i\,a\tan(fx+e)+a}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/10*(10*a*f*e^(2*I*f*x + 2*I*e)*integral(-2/5*I*2^(1/3)*(d/(e^(2*I*f*x + 2
*I*e) + 1))^(1/3)*e^(-2/3*I*f*x - 2/3*I*e)/(a*f), x) - 3*2^(1/3)*(d/(e^(2*I
*f*x + 2*I*e) + 1))^(1/3)*(-I*e^(2*I*f*x + 2*I*e) - I)*e^(1/3*I*f*x + 1/3*I
*e))*e^(-2*I*f*x - 2*I*e)/(a*f)
```

Sympy [F]

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+ia\tan(e+fx)}\,dx = -\frac{i\int \frac{\sqrt[3]{d\sec(e+fx)}}{\tan(e+fx)-i}\,dx}{a}$$

```
[In] integrate((d*sec(f*x+e))**(1/3)/(a+I*a*tan(f*x+e)),x)
[Out] -I*Integral((d*sec(e + f*x))**(1/3)/(tan(e + f*x) - I), x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+ia\tan(e+fx)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+ia\tan(e+fx)}\,dx = \int \frac{(d\sec(fx+e))^{\frac{1}{3}}}{i\,a\tan(fx+e)+a}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(1/3)/(I*a*tan(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+ia\tan(e+fx)}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}}{a+a\tan(e+f\,x)\;1\mathrm{i}}\,dx$$

```
[In] int((d/cos(e + f*x))^(1/3)/(a + a*tan(e + f*x)*1i),x)
[Out] int((d/cos(e + f*x))^(1/3)/(a + a*tan(e + f*x)*1i), x)
```

3.273 $\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 71

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))}\,dx = -\frac{3i\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{13}{6},\frac{5}{6},\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[6]{1+i\tan(e+fx)}}{2\sqrt[6]{2}af\sqrt[3]{d\sec(e+fx)}}$$

```
[Out] -3/4*I*hypergeom([-1/6, 13/6],[5/6],1/2-1/2*I*tan(f*x+e))*(1+I*tan(f*x+e))^
(1/6)*2^(5/6)/a/f/(d*sec(f*x+e))^(1/3)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3586, 3604, 72, 71}

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))}\,dx = -\frac{3i\sqrt[6]{1+i\tan(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{13}{6},\frac{5}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{2\sqrt[6]{2}af\sqrt[3]{d\sec(e+fx)}}$$

```
[In] Int[1/((d*Sec[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])),x]
```

```
[Out] (((-3*I)/2)*Hypergeometric2F1[-1/6, 13/6, 5/6, (1 - I*Tan[e + f*x])/2]*(1 +
 I*Tan[e + f*x])^(1/6))/(2^(1/6)*a*f*(d*Sec[e + f*x])^(1/3))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

integral

$$= \frac{\left(\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}\right)\int \frac{1}{\sqrt[6]{a-ia\tan(e+fx)}(a+ia\tan(e+fx))^{7/6}}\,dx}{\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{\left(a^2\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}\right)\operatorname{Subst}\left(\int \frac{1}{(a-iax)^{7/6}(a+iax)^{13/6}}\,dx,x,\tan(e+fx)\right)}{f\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{\left(\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{\frac{a+ia\tan(e+fx)}{a}}\right)\operatorname{Subst}\left(\int \frac{1}{\left(\frac{1}{2}+\frac{ix}{2}\right)^{13/6}(a-iax)^{7/6}}\,dx,x,\tan(e+fx)\right)}{4\sqrt[6]{2}f\sqrt[3]{d\sec(e+fx)}}$$

$$= -\frac{3i\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{13}{6},\frac{5}{6},\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[6]{1+i\tan(e+fx)}}{2\sqrt[6]{2}af\sqrt[3]{d\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 1.73 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.58

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))}\,dx = \frac{3\left(-8e^{2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{6},\frac{11}{6},-e^{2i(e+fx)}\right)+5(5+5\cos(2(e+fx))+4i\sin(\right.}{70af\sqrt[3]{d\sec(e+fx)}}$$

```
[In] Integrate[1/((d*Sec[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])),x]
```

```
[Out] (3*(-8*E^((2*I)*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^(2/3)*Hypergeometric2F
1[2/3, 5/6, 11/6, -E^((2*I)*(e + f*x))] + 5*(5 + 5*Cos[2*(e + f*x)] + (4*I)
*Sin[2*(e + f*x)]))*(I + Tan[e + f*x]))/(70*a*f*(d*Sec[e + f*x])^(1/3))
```

Maple [F]

$$\int \frac{1}{(d\sec(fx+e))^{\frac{1}{3}}(a+ia\tan(fx+e))}dx$$

```
[In] int(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x)
```

```
[Out] int(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{1}{3}}(i\,a\tan(fx+e)+a)}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/28*(3*2^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(7*I*e^(5*I*f*x + 5*I*
e) + 9*I*e^(4*I*f*x + 4*I*e) + 6*I*e^(3*I*f*x + 3*I*e) + 10*I*e^(2*I*f*x +
2*I*e) - I*e^(I*f*x + I*e) + I)*e^(2/3*I*f*x + 2/3*I*e) - 28*(a*d*f*e^(4*I*
f*x + 4*I*e) - a*d*f*e^(3*I*f*x + 3*I*e))*integral(-8/7*2^(2/3)*(d/(e^(2*I*
f*x + 2*I*e) + 1))^(2/3)*(I*e^(2*I*f*x + 2*I*e) + I*e^(I*f*x + I*e) + I)*e^
(2/3*I*f*x + 2/3*I*e)/(a*d*f*e^(3*I*f*x + 3*I*e) - 2*a*d*f*e^(2*I*f*x + 2*I
*e) + a*d*f*e^(I*f*x + I*e)), x))/(a*d*f*e^(4*I*f*x + 4*I*e) - a*d*f*e^(3*I
*f*x + 3*I*e))
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))}\,dx = -\frac{i\int \frac{1}{\sqrt[3]{d\sec{(e+fx)}}\tan{(e+fx)}-i\sqrt[3]{d\sec{(e+fx)}}}\,dx}{a}$$

```
[In] integrate(1/(d*sec(f*x+e))**(1/3)/(a+I*a*tan(f*x+e)),x)
[Out] -I*Integral(1/((d*sec(e + f*x))**(1/3)*tan(e + f*x) - I*(d*sec(e + f*x))**(
1/3)), x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))}\,dx = \int \frac{1}{(d\sec{(fx+e)})^{\frac{1}{3}}\,(i\,a\tan{(fx+e)}+a)}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e)),x, algorithm="giac")
[Out] integrate(1/((d*sec(f*x + e))^(1/3)*(I*a*tan(f*x + e) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))}\,dx = \int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}\,\left(a+a\tan\left(e+f\,x\right)\,1\mathrm{i}\right)}\,dx$$

```
[In] int(1/((d/cos(e + f*x))^(1/3)*(a + a*tan(e + f*x)*1i)),x)
[Out] int(1/((d/cos(e + f*x))^(1/3)*(a + a*tan(e + f*x)*1i)), x)
```

3.274 $\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))} dx$

Optimal result

Integrand size = 28, antiderivative size = 71

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))} dx = -\frac{3i\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{17}{6},\frac{1}{6},\frac{1}{2}(1-i\tan(e+fx))\right)(1+i\tan(e+fx))^{5/6}}{10\ 2^{5/6}af(d\sec(e+fx))^{5/3}}$$

```
[Out] -3/20*I*hypergeom([-5/6, 17/6],[1/6],1/2-1/2*I*tan(f*x+e))*(1+I*tan(f*x+e))
^(5/6)*2^(1/6)/a/f/(d*sec(f*x+e))^(5/3)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3586, 3604, 72, 71}

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))} dx = -\frac{3i(1+i\tan(e+fx))^{5/6}\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{17}{6},\frac{1}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{10\ 2^{5/6}af(d\sec(e+fx))^{5/3}}$$

```
[In] Int[1/((d*Sec[e + f*x])^(5/3)*(a + I*a*Tan[e + f*x])),x]
```

```
[Out] (((-3*I)/10)*Hypergeometric2F1[-5/6, 17/6, 1/6, (1 - I*Tan[e + f*x])/2]*(1
+ I*Tan[e + f*x])^(5/6))/(2^(5/6)*a*f*(d*Sec[e + f*x])^(5/3))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

integral

$$= \frac{\left((a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\right)\int \frac{1}{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{11/6}}\,dx}{(d\sec(e+fx))^{5/3}}$$

$$= \frac{\left(a^2(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\right)\mathrm{Subst}\left(\int \frac{1}{(a-iax)^{11/6}(a+iax)^{17/6}}\,dx,x,\tan(e+fx)\right)}{f(d\sec(e+fx))^{5/3}}$$

$$= \frac{\left((a-ia\tan(e+fx))^{5/6}\left(\frac{a+ia\tan(e+fx)}{a}\right)^{5/6}\right)\mathrm{Subst}\left(\int \frac{1}{\left(\frac{1}{2}+\frac{ix}{2}\right)^{17/6}(a-iax)^{11/6}}\,dx,x,\tan(e+fx)\right)}{4\ 2^{5/6}f(d\sec(e+fx))^{5/3}}$$

$$= -\frac{3i\,\mathrm{Hypergeometric2F1}\left(-\frac{5}{6},\frac{17}{6},\frac{1}{6},\frac{1}{2}(1-i\tan(e+fx))\right)(1+i\tan(e+fx))^{5/6}}{10\ 2^{5/6}af(d\sec(e+fx))^{5/3}}$$

Mathematica [A] (verified)

Time = 1.89 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.68

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))}\,dx = -\frac{3\sec^2(e+fx)\left(-26+6\cos(2(e+fx))+\frac{128e^{2i(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{3},\frac{7}{6},-e^{2i(e+fx)}\right)}{\left(1+e^{2i(e+fx)}\right)^{2/3}}+16i\sin(2(e+fx))\right)}{220af(d\sec(e+fx))^{5/3}(-i+\tan(e+fx))}$$

```
[In] Integrate[1/((d*Sec[e + f*x])^(5/3)*(a + I*a*Tan[e + f*x])),x]
```

```
[Out] (-3*Sec[e + f*x]^2*(-26 + 6*Cos[2*(e + f*x)] + (128*E^((2*I)*(e + f*x))*Hyp
ergeometric2F1[1/6, 1/3, 7/6, -E^((2*I)*(e + f*x))])/(1 + E^((2*I)*(e + f*x
)))^(2/3) + (16*I)*Sin[2*(e + f*x)]))/(220*a*f*(d*Sec[e + f*x])^(5/3)*(-I +
 Tan[e + f*x]))
```

Maple [F]

$$\int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}\left(a+ia\tan\left(fx+e\right)\right)}dx$$

```
[In] int(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x)
```

```
[Out] int(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x)
```

Fricas [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))}\,dx = \int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}\left(i\,a\tan\left(fx+e\right)+a\right)}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/440*(440*a*d^2*f*e^(4*I*f*x + 4*I*e)*integral(-16/55*I*2^(1/3)*(d/(e^(2*I
*f*x + 2*I*e) + 1))^(1/3)*e^(-2/3*I*f*x - 2/3*I*e)/(a*d^2*f), x) - 3*2^(1/3
)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3)*(11*I*e^(6*I*f*x + 6*I*e) - 15*I*e^(4
*I*f*x + 4*I*e) - 31*I*e^(2*I*f*x + 2*I*e) - 5*I)*e^(1/3*I*f*x + 1/3*I*e))*
e^(-4*I*f*x - 4*I*e)/(a*d^2*f)
```

Sympy [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))}\,dx = -\frac{i\int \frac{1}{(d\sec(e+fx))^{\frac{5}{3}}\tan(e+fx)-i(d\sec(e+fx))^{\frac{5}{3}}}\,dx}{a}$$

[In] integrate(1/(d*sec(f*x+e))**(5/3)/(a+I*a*tan(f*x+e)),x)

[Out] -I*Integral(1/((d*sec(e + f*x))**(5/3)*tan(e + f*x) - I*(d*sec(e + f*x))**(5/3)), x)/a

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))}\,dx = \text{Exception raised: RuntimeError}$$

[In] integrate(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{5}{3}}(ia\tan(fx+e)+a)}\,dx$$

[In] integrate(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e)),x, algorithm="giac")

[Out] integrate(1/((d*sec(f*x + e))^(5/3)*(I*a*tan(f*x + e) + a)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))}\,dx = \int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}\,(a+a\tan(e+f\,x)\,1\mathrm{i})}\,dx$$

[In] int(1/((d/cos(e + f*x))^(5/3)*(a + a*tan(e + f*x)*1i)),x)

[Out] int(1/((d/cos(e + f*x))^(5/3)*(a + a*tan(e + f*x)*1i)), x)

3.275 $\int \frac{(d\sec(e+fx))^{5/3}}{(a+ia\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 87

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+ia\tan(e+fx))^2}\,dx = \frac{3i\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{13}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{5/3}\sqrt[6]{1+}}{10\sqrt[6]{2}f\,(a^2+ia^2\tan(e+fx))}$$

```
[Out] 3/20*I*hypergeom([5/6, 13/6],[11/6],1/2-1/2*I*tan(f*x+e))*(d*sec(f*x+e))^(5
/3)*(1+I*tan(f*x+e))^(1/6)*2^(5/6)/f/(a^2+I*a^2*tan(f*x+e))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3586, 3604, 72, 71}

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+ia\tan(e+fx))^2}\,dx = \frac{3i\sqrt[6]{1+i\tan(e+fx)}(d\sec(e+fx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{13}{6},\frac{11}{6},\frac{1}{2}(1-i\right.}{10\sqrt[6]{2}f\,(a^2+ia^2\tan(e+fx))}$$

```
[In] Int[(d*Sec[e + f*x])^(5/3)/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (((3*I)/10)*Hypergeometric2F1[5/6, 13/6, 11/6, (1 - I*Tan[e + f*x])/2]*(d*S
ec[e + f*x])^(5/3)*(1 + I*Tan[e + f*x])^(1/6))/(2^(1/6)*f*(a^2 + I*a^2*Tan[
e + f*x]))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(d\sec(e+fx))^{5/3}\int\frac{(a-ia\tan(e+fx))^{5/6}}{(a+ia\tan(e+fx))^{7/6}}\,dx}{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}\\
&= \frac{\left(a^2(d\sec(e+fx))^{5/3}\right)\text{Subst}\left(\int\frac{1}{\sqrt[6]{a-iax}(a+iax)^{13/6}}\,dx,x,\tan(e+fx)\right)}{f(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}}\\
&= \frac{\left((d\sec(e+fx))^{5/3}\sqrt[6]{\frac{a+ia\tan(e+fx)}{a}}\right)\text{Subst}\left(\int\frac{1}{\left(\frac{1}{2}+\frac{ix}{2}\right)^{13/6}\sqrt[6]{a-iax}}\,dx,x,\tan(e+fx)\right)}{4\sqrt[6]{2}f(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))}\\
&= \frac{3i\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{13}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{5/3}\sqrt[6]{1+i\tan(e+fx)}}{10\sqrt[6]{2}f\left(a^2+ia^2\tan(e+fx)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.47

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+ia\tan(e+fx))^2}\,dx = -\frac{3e^{-i(4e+5fx)}\left(1+e^{2i(e+fx)}\right)\left(1+e^{2i(e+fx)}+2e^{2i(e+fx)}\left(1+e^{2i(e+fx)}\right)^{2/3}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{2}{3},\frac{5}{6},-e^{2i(e+}\right.\right.}{28a^2f}$$

[In] Integrate[(d*Sec[e + f*x])^(5/3)/(a + I*a*Tan[e + f*x])^2,x]

[Out] (-3*(1 + E^((2*I)*(e + f*x)))*(1 + E^((2*I)*(e + f*x)) + 2*E^((2*I)*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^(2/3)*Hypergeometric2F1[-1/6, 2/3, 5/6, -E^((2*I)*(e + f*x))])*(d*Sec[e + f*x])^(5/3)*((-I)*Cos[f*x] + Sin[f*x]))/(28*a^2*E^(I*(4*e + 5*f*x))*f)

Maple [F]

$$\int \frac{(d\sec(fx+e))^{\frac{5}{3}}}{(a+ia\tan(fx+e))^2}dx$$

[In] int((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x)

[Out] int((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x)

Fricas [F]

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+ia\tan(e+fx))^2}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{3}}}{(i\,a\tan(fx+e)+a)^2}\,dx$$

[In] integrate((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")

[Out] 1/14*(14*a^2*f*e^(3*I*f*x + 3*I*e)*integral(-1/7*I*2^(2/3)*d*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*e^(2/3*I*f*x + 2/3*I*e)/(a^2*f), x) - 3*2^(2/3)*(-2*I*d*e^(4*I*f*x + 4*I*e) - 3*I*d*e^(2*I*f*x + 2*I*e) - I*d)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*e^(2/3*I*f*x + 2/3*I*e))*e^(-3*I*f*x - 3*I*e)/(a^2*f)

Sympy [F]

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+ia\tan(e+fx))^2}\,dx = -\frac{\int \frac{(d\sec(e+fx))^{\frac{5}{3}}}{\tan^2(e+fx)-2i\tan(e+fx)-1}\,dx}{a^2}$$

```
[In] integrate((d*sec(f*x+e))**(5/3)/(a+I*a*tan(f*x+e))**2,x)
[Out] -Integral((d*sec(e + f*x))**(5/3)/(tan(e + f*x)**2 - 2*I*tan(e + f*x) - 1),
 x)/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+ia\tan(e+fx))^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+ia\tan(e+fx))^2}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{3}}}{(i\,a\tan(fx+e)+a)^2}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(5/3)/(I*a*tan(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+ia\tan(e+fx))^2}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}}{(a+a\tan(e+f\,x)\;1\mathrm{i})^2}\,dx$$

```
[In] int((d/cos(e + f*x))^(5/3)/(a + a*tan(e + f*x)*1i)^2,x)
[Out] int((d/cos(e + f*x))^(5/3)/(a + a*tan(e + f*x)*1i)^2, x)
```

3.276 $\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 87

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx = \frac{3i\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{17}{6},\frac{7}{6},\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[3]{d\sec(e+fx)}(1+i\tan(e+fx))^{5/6}}{2\,2^{5/6}f\,(a^2+ia^2\tan(e+fx))}$$

```
[Out] 3/4*I*hypergeom([1/6, 17/6],[7/6],1/2-1/2*I*tan(f*x+e))*(d*sec(f*x+e))^(1/3
)*(1+I*tan(f*x+e))^(5/6)*2^(1/6)/f/(a^2+I*a^2*tan(f*x+e))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3586, 3604, 72, 71}

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx = \frac{3i(1+i\tan(e+fx))^{5/6}\sqrt[3]{d\sec(e+fx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{17}{6},\frac{7}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{2\,2^{5/6}f\,(a^2+ia^2\tan(e+fx))}$$

```
[In] Int[(d*Sec[e + f*x])^(1/3)/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (((3*I)/2)*Hypergeometric2F1[1/6, 17/6, 7/6, (1 - I*Tan[e + f*x])/2]*(d*Sec
[e + f*x])^(1/3)*(1 + I*Tan[e + f*x])^(5/6))/(2^(5/6)*f*(a^2 + I*a^2*Tan[e
+ f*x]))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt[3]{d\sec(e+fx)}\int \frac{\sqrt[6]{a-ia\tan(e+fx)}}{(a+ia\tan(e+fx))^{11/6}}\,dx}{\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}}$$

$$= \frac{\left(a^2\sqrt[3]{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{(a-iax)^{5/6}(a+iax)^{17/6}}\,dx, x, \tan(e+fx)\right)}{f\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}}$$

$$= \frac{\left(\sqrt[3]{d\sec(e+fx)}\left(\frac{a+ia\tan(e+fx)}{a}\right)^{5/6}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{1}{2}+\frac{ix}{2}\right)^{17/6}(a-iax)^{5/6}}\,dx, x, \tan(e+fx)\right)}{4\ 2^{5/6}f\sqrt[6]{a-ia\tan(e+fx)}(a+ia\tan(e+fx))}$$

$$= \frac{3i\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{17}{6},\frac{7}{6},\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[3]{d\sec(e+fx)}(1+i\tan(e+fx))^{5/6}}{2\ 2^{5/6}f\left(a^2+ia^2\tan(e+fx)\right)}$$

Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx = \frac{3\sec^2(e+fx)\sqrt[3]{d\sec(e+fx)}\left(-2i-2i\cos(2(e+fx))+4ie^{2i(e+fx)}\sqrt[3]{1+e^{2i(e+fx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{6}\right.\right.}{22a^2f(-i+\tan(e+fx))^2}$$

```
[In] Integrate[(d*Sec[e + f*x])^(1/3)/(a + I*a*Tan[e + f*x])^2,x]
```

```
[Out] (3*Sec[e + f*x]^2*(d*Sec[e + f*x])^(1/3)*(-2*I - (2*I)*Cos[2*(e + f*x)] + (
4*I)*E^((2*I)*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^(1/3)*Hypergeometric2F1[
1/6, 1/3, 7/6, -E^((2*I)*(e + f*x))] + Sin[2*(e + f*x)]))/(22*a^2*f*(-I + T
an[e + f*x])^2)
```

Maple [F]

$$\int \frac{(d\sec(fx+e))^{\frac{1}{3}}}{(a+ia\tan(fx+e))^2}dx$$

```
[In] int((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x)
```

```
[Out] int((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x)
```

Fricas [F]

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx = \int \frac{(d\sec(fx+e))^{\frac{1}{3}}}{(i\,a\tan(fx+e)+a)^2}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/44*(44*a^2*f*e^(4*I*f*x + 4*I*e)*integral(-2/11*I*2^(1/3)*(d/(e^(2*I*f*x
+ 2*I*e) + 1))^(1/3)*e^(-2/3*I*f*x - 2/3*I*e)/(a^2*f), x) - 3*2^(1/3)*(d/(e
^(2*I*f*x + 2*I*e) + 1))^(1/3)*(-3*I*e^(4*I*f*x + 4*I*e) - 4*I*e^(2*I*f*x +
 2*I*e) - I)*e^(1/3*I*f*x + 1/3*I*e))*e^(-4*I*f*x - 4*I*e)/(a^2*f)
```

Sympy [F]

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx = -\frac{\int \frac{\sqrt[3]{d\sec{(e+fx)}}}{\tan^2{(e+fx)}-2i\tan{(e+fx)}-1}\,dx}{a^2}$$

```
[In] integrate((d*sec(f*x+e))**(1/3)/(a+I*a*tan(f*x+e))**2,x)
[Out] -Integral((d*sec(e + f*x))**(1/3)/(tan(e + f*x)**2 - 2*I*tan(e + f*x) - 1),
 x)/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx = \int \frac{(d\sec{(fx+e)})^{\frac{1}{3}}}{(i\,a\tan{(fx+e)}+a)^2}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(1/3)/(I*a*tan(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}}{(a+a\tan{(e+f\,x)}\,1\mathrm{i})^2}\,dx$$

```
[In] int((d/cos(e + f*x))^(1/3)/(a + a*tan(e + f*x)*1i)^2,x)
[Out] int((d/cos(e + f*x))^(1/3)/(a + a*tan(e + f*x)*1i)^2, x)
```

3.277 $\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2} dx$

Optimal result

Integrand size = 28, antiderivative size = 71

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2} dx = -\frac{3i\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{19}{6},\frac{5}{6},\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[6]{1+i\tan(e+fx)}}{4\sqrt[6]{2}a^2f\sqrt[3]{d\sec(e+fx)}}$$

```
[Out] -3/8*I*hypergeom([-1/6, 19/6],[5/6],1/2-1/2*I*tan(f*x+e))*(1+I*tan(f*x+e))^
(1/6)*2^(5/6)/a^2/f/(d*sec(f*x+e))^(1/3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3586, 3604, 72, 71}

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2} dx = -\frac{3i\sqrt[6]{1+i\tan(e+fx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{19}{6},\frac{5}{6},\frac{1}{2}(1-i\tan(e+fx))\right)}{4\sqrt[6]{2}a^2f\sqrt[3]{d\sec(e+fx)}}$$

```
[In] Int[1/((d*Sec[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^2),x]
```

```
[Out] (((-3*I)/4)*Hypergeometric2F1[-1/6, 19/6, 5/6, (1 - I*Tan[e + f*x])/2]*(1 +
 I*Tan[e + f*x])^(1/6))/(2^(1/6)*a^2*f*(d*Sec[e + f*x])^(1/3))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

integral

$$=\frac{\left(\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}\right)\int\frac{1}{\sqrt[6]{a-ia\tan(e+fx)}(a+ia\tan(e+fx))^{13/6}}dx}{\sqrt[3]{d\sec(e+fx)}}$$

$$=\frac{\left(a^2\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{a+ia\tan(e+fx)}\right)\operatorname{Subst}\left(\int\frac{1}{(a-iax)^{7/6}(a+iax)^{19/6}}dx,x,\tan(e+fx)\right)}{f\sqrt[3]{d\sec(e+fx)}}$$

$$=\frac{\left(\sqrt[6]{a-ia\tan(e+fx)}\sqrt[6]{\frac{a+ia\tan(e+fx)}{a}}\right)\operatorname{Subst}\left(\int\frac{1}{\left(\frac{1}{2}+\frac{ix}{2}\right)^{19/6}(a-iax)^{7/6}}dx,x,\tan(e+fx)\right)}{8\sqrt[6]{2}af\sqrt[3]{d\sec(e+fx)}}$$

$$=-\frac{3i\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{19}{6},\frac{5}{6},\frac{1}{2}(1-i\tan(e+fx))\right)\sqrt[6]{1+i\tan(e+fx)}}{4\sqrt[6]{2}a^2f\sqrt[3]{d\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 2.16 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.99

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2}\,dx = \frac{(d\sec(e+fx))^{2/3}\left(16e^{3i(e+fx)}\left(1+e^{2i(e+fx)}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{6},\frac{11}{6},-e^{2i(e+fx)}\right)-10(7\cos(e+fx\right.}{260a^2df}$$

```
[In] Integrate[1/((d*Sec[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^2),x]
```

```
[Out] ((d*Sec[e + f*x])^(2/3)*(16*E^((3*I)*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^(
2/3)*Hypergeometric2F1[2/3, 5/6, 11/6, -E^((2*I)*(e + f*x))] - 10*(7*Cos[e
+ f*x] + 5*Cos[3*(e + f*x)] + (18*I)*Cos[e + f*x]^2*Sin[e + f*x])*((-3*I)*
Cos[2*(e + f*x)] - 3*Sin[2*(e + f*x)])))/(260*a^2*d*f)
```

Maple [F]

$$\int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}\left(a+ia\tan\left(fx+e\right)\right)^{2}}dx$$

```
[In] int(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x)
```

```
[Out] int(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2}\,dx = \int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}\left(i\,a\tan\left(fx+e\right)+a\right)^{2}}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas"
)
```

```
[Out] -1/104*(3*2^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(13*I*e^(7*I*f*x + 7*
I*e) + 19*I*e^(6*I*f*x + 6*I*e) + 9*I*e^(5*I*f*x + 5*I*e) + 23*I*e^(4*I*f*x
 + 4*I*e) - 5*I*e^(3*I*f*x + 3*I*e) + 5*I*e^(2*I*f*x + 2*I*e) - I*e^(I*f*x
+ I*e) + I)*e^(2/3*I*f*x + 2/3*I*e) - 104*(a^2*d*f*e^(6*I*f*x + 6*I*e) - a^
2*d*f*e^(5*I*f*x + 5*I*e))*integral(-8/13*2^(2/3)*(d/(e^(2*I*f*x + 2*I*e) +
 1))^(2/3)*(I*e^(2*I*f*x + 2*I*e) + I*e^(I*f*x + I*e) + I)*e^(2/3*I*f*x + 2
/3*I*e)/(a^2*d*f*e^(3*I*f*x + 3*I*e) - 2*a^2*d*f*e^(2*I*f*x + 2*I*e) + a^2*
d*f*e^(I*f*x + I*e)), x))/(a^2*d*f*e^(6*I*f*x + 6*I*e) - a^2*d*f*e^(5*I*f*x
 + 5*I*e))
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2}\,dx = -\frac{\int \frac{1}{\sqrt[3]{d\sec(e+fx)}\tan^2(e+fx)-2i\sqrt[3]{d\sec(e+fx)}\tan(e+fx)-\sqrt[3]{d\sec(e+fx)}}\,dx}{a^2}$$

```
[In] integrate(1/(d*sec(f*x+e))**(1/3)/(a+I*a*tan(f*x+e))**2,x)
```

```
[Out] -Integral(1/((d*sec(e + f*x))**(1/3)*tan(e + f*x)**2 - 2*I*(d*sec(e + f*x))
**(1/3)*tan(e + f*x) - (d*sec(e + f*x))**(1/3)), x)/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{1}{3}}(i\,a\tan(fx+e)+a)^2}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*sec(f*x + e))^(1/3)*(I*a*tan(f*x + e) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+ia\tan(e+fx))^2}\,dx = \int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}\,(a+a\tan\left(e+f\,x\right)\,1\mathrm{i})^2}\,dx$$

```
[In] int(1/((d/cos(e + f*x))^(1/3)*(a + a*tan(e + f*x)*1i)^2),x)
[Out] int(1/((d/cos(e + f*x))^(1/3)*(a + a*tan(e + f*x)*1i)^2), x)
```

3.278 $\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2} dx$

Optimal result

Integrand size = 28, antiderivative size = 71

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2} dx = -\frac{3i \operatorname{Hypergeometric2F1}\left(-\frac{5}{6}, \frac{23}{6}, \frac{1}{6}, \frac{1}{2}(1-i\tan(e+fx))\right)(1+i\tan(e+fx))^{5/6}}{20\ 2^{5/6}a^2f(d\sec(e+fx))^{5/3}}$$

```
[Out] -3/40*I*hypergeom([-5/6, 23/6],[1/6],1/2-1/2*I*tan(f*x+e))*(1+I*tan(f*x+e))
^(5/6)*2^(1/6)/a^2/f/(d*sec(f*x+e))^(5/3)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3586, 3604, 72, 71}

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2} dx = -\frac{3i(1+i\tan(e+fx))^{5/6} \operatorname{Hypergeometric2F1}\left(-\frac{5}{6}, \frac{23}{6}, \frac{1}{6}, \frac{1}{2}(1-i\tan(e+fx))\right)}{20\ 2^{5/6}a^2f(d\sec(e+fx))^{5/3}}$$

```
[In] Int[1/((d*Sec[e + f*x])^(5/3)*(a + I*a*Tan[e + f*x])^2),x]
```

```
[Out] (((-3*I)/20)*Hypergeometric2F1[-5/6, 23/6, 1/6, (1 - I*Tan[e + f*x])/2]*(1
+ I*Tan[e + f*x])^(5/6))/(2^(5/6)*a^2*f*(d*Sec[e + f*x])^(5/3))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

integral

$$= \frac{\left((a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\right)\int \frac{1}{(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{17/6}}\,dx}{(d\sec(e+fx))^{5/3}}$$

$$= \frac{\left(a^2(a-ia\tan(e+fx))^{5/6}(a+ia\tan(e+fx))^{5/6}\right)\mathrm{Subst}\left(\int \frac{1}{(a-iax)^{11/6}(a+iax)^{23/6}}\,dx, x, \tan(e+fx)\right)}{f(d\sec(e+fx))^{5/3}}$$

$$= \frac{\left((a-ia\tan(e+fx))^{5/6}\left(\frac{a+ia\tan(e+fx)}{a}\right)^{5/6}\right)\mathrm{Subst}\left(\int \frac{1}{\left(\frac{1}{2}+\frac{ix}{2}\right)^{23/6}(a-iax)^{11/6}}\,dx, x, \tan(e+fx)\right)}{8\ 2^{5/6}af(d\sec(e+fx))^{5/3}}$$

$$= -\frac{3i\,\mathrm{Hypergeometric2F1}\left(-\frac{5}{6},\frac{23}{6},\frac{1}{6},\frac{1}{2}(1-i\tan(e+fx))\right)(1+i\tan(e+fx))^{5/6}}{20\ 2^{5/6}a^2f(d\sec(e+fx))^{5/3}}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 143 vs. 2(71) = 142.

Time = 1.82 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.01

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2}\,dx = \frac{3i\sec^4(e+fx)\left(-46-40\cos(2(e+fx))+6\cos(4(e+fx\right.}{}$$

```
[In] Integrate[1/((d*Sec[e + f*x])^(5/3)*(a + I*a*Tan[e + f*x])^2),x]
```

```
[Out] (((3*I)/680)*Sec[e + f*x]^4*(-46 - 40*Cos[2*(e + f*x)] + 6*Cos[4*(e + f*x)]
 + 128*E^((2*I)*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^(1/3)*Hypergeometric2F
1[1/6, 1/3, 7/6, -E^((2*I)*(e + f*x))] - (10*I)*Sin[2*(e + f*x)] + (11*I)*S
in[4*(e + f*x)]))/(a^2*f*(d*Sec[e + f*x])^(5/3)*(-I + Tan[e + f*x])^2)
```

Maple [F]

$$\int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}\left(a+ia\tan\left(fx+e\right)\right)^{2}}dx$$

```
[In] int(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x)
```

```
[Out] int(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x)
```

Fricas [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2}\,dx = \int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}\left(i\,a\tan\left(fx+e\right)+a\right)^{2}}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="fricas"
)
```

```
[Out] 1/1360*(1360*a^2*d^2*f*e^(6*I*f*x + 6*I*e)*integral(-16/85*I*2^(1/3)*(d/(e^
(2*I*f*x + 2*I*e) + 1))^(1/3)*e^(-2/3*I*f*x - 2/3*I*e)/(a^2*d^2*f), x) - 3*
2^(1/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(1/3)*(17*I*e^(8*I*f*x + 8*I*e) - 50*
I*e^(6*I*f*x + 6*I*e) - 92*I*e^(4*I*f*x + 4*I*e) - 30*I*e^(2*I*f*x + 2*I*e)
 - 5*I)*e^(1/3*I*f*x + 1/3*I*e))*e^(-6*I*f*x - 6*I*e)/(a^2*d^2*f)
```

Sympy [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2}\,dx = -\frac{\int \frac{1}{(d\sec{(e+fx)})^{\frac{5}{3}}\tan^2{(e+fx)}-2i(d\sec{(e+fx)})^{\frac{5}{3}}\tan{(e+fx)}-(d\sec{(e+fx)})^{\frac{5}{3}}}\,dx}{a^2}$$

```
[In] integrate(1/(d*sec(f*x+e))**(5/3)/(a+I*a*tan(f*x+e))**2,x)
[Out] -Integral(1/((d*sec(e + f*x))**(5/3)*tan(e + f*x)**2 - 2*I*(d*sec(e + f*x))
**(5/3)*tan(e + f*x) - (d*sec(e + f*x))**(5/3)), x)/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="maxima"
)
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2}\,dx = \int \frac{1}{(d\sec{(fx+e)})^{\frac{5}{3}}(i\,a\tan{(fx+e)}+a)^2}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(5/3)/(a+I*a*tan(f*x+e))^2,x, algorithm="giac")
[Out] integrate(1/((d*sec(f*x + e))^(5/3)*(I*a*tan(f*x + e) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+ia\tan(e+fx))^2}\,dx = \int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}\,\left(a+a\tan\left(e+f\,x\right)\,1\mathrm{i}\right)^2}\,dx$$

```
[In] int(1/((d/cos(e + f*x))^(5/3)*(a + a*tan(e + f*x)*1i)^2),x)
[Out] int(1/((d/cos(e + f*x))^(5/3)*(a + a*tan(e + f*x)*1i)^2), x)
```

3.279 $\int \sec^8(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 117

$$\int \sec^8(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{16i(a+ia\tan(c+dx))^{9/2}}{9a^4d} + \frac{24i(a+ia\tan(c+dx))^{11/2}}{11a^5d} - \frac{12i(a+ia\tan(c+dx))^{13/2}}{13a^6d} + \frac{2i(a+ia\tan(c+dx))^{15/2}}{15a^7d}$$

```
[Out] -16/9*I*(a+I*a*tan(d*x+c))^(9/2)/a^4/d+24/11*I*(a+I*a*tan(d*x+c))^(11/2)/a^
5/d-12/13*I*(a+I*a*tan(d*x+c))^(13/2)/a^6/d+2/15*I*(a+I*a*tan(d*x+c))^(15/2
)/a^7/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3568, 45}

$$\int \sec^8(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{2i(a+ia\tan(c+dx))^{15/2}}{15a^7d} - \frac{12i(a+ia\tan(c+dx))^{13/2}}{13a^6d} + \frac{24i(a+ia\tan(c+dx))^{11/2}}{11a^5d} - \frac{16i(a+ia\tan(c+dx))^{9/2}}{9a^4d}$$

```
[In] Int[Sec[c + d*x]^8*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((-16*I)/9)*(a + I*a*Tan[c + d*x])^(9/2))/(a^4*d) + (((24*I)/11)*(a + I*a*
Tan[c + d*x])^(11/2))/(a^5*d) - (((12*I)/13)*(a + I*a*Tan[c + d*x])^(13/2))
/(a^6*d) + (((2*I)/15)*(a + I*a*Tan[c + d*x])^(15/2))/(a^7*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)^3(a+x)^{7/2}\,dx, x, ia\tan(c+dx)\right)}{a^7d} \\
&= -\frac{i\text{Subst}\left(\int \left(8a^3(a+x)^{7/2}-12a^2(a+x)^{9/2}+6a(a+x)^{11/2}-(a+x)^{13/2}\right)\,dx, x, ia\tan(c+dx)\right)}{a^7d} \\
&= -\frac{16i(a+ia\tan(c+dx))^{9/2}}{9a^4d}+\frac{24i(a+ia\tan(c+dx))^{11/2}}{11a^5d} \\
&\quad -\frac{12i(a+ia\tan(c+dx))^{13/2}}{13a^6d}+\frac{2i(a+ia\tan(c+dx))^{15/2}}{15a^7d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.60

$$\int \sec^8(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$
$$=\frac{2(-i+\tan(c+dx))^4\sqrt{a+ia\tan(c+dx)}(-1241i-2367\tan(c+dx)+1683i\tan^2(c+dx)+429\tan^3(c+dx))}{6435d}$$

```
[In] Integrate[Sec[c + d*x]^8*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (2*(-I + Tan[c + d*x])^4*Sqrt[a + I*a*Tan[c + d*x]]*(-1241*I - 2367*Tan[c +
 d*x] + (1683*I)*Tan[c + d*x]^2 + 429*Tan[c + d*x]^3))/(6435*d)
```

Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{15}{2}}}{15}-\frac{6a(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}+\frac{12a^{2}(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}-\frac{8a^{3}(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}\right)}{d\,a^{7}}$	82
default	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{15}{2}}}{15}-\frac{6a(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}+\frac{12a^{2}(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}-\frac{8a^{3}(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}\right)}{d\,a^{7}}$	82

```
[In] int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/d/a^7*(1/15*(a+I*a*tan(d*x+c))^(15/2)-6/13*a*(a+I*a*tan(d*x+c))^(13/2)+
12/11*a^2*(a+I*a*tan(d*x+c))^(11/2)-8/9*a^3*(a+I*a*tan(d*x+c))^(9/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.32

$$\int \sec^{8}(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{256\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(16i\,e^{(15i\,dx+15i\,c)}+120i\,e^{(13i\,dx+13i\,c)}+390i\,e^{(11i\,dx+11i\,c)}+715i\,e^{(9i\,\cdots}\right.}{6435\left(de^{(14i\,dx+14i\,c)}+7\,de^{(12i\,dx+12i\,c)}+21\,de^{(10i\,dx+10i\,c)}+35\,de^{(8i\,dx+8i\,c)}+35\,de^{(6i\,dx+6i\,c)}+21\,de^{(4i\,dx+4\cdots}\right.}$$

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -256/6435*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(16*I*e^(15*I*d*x + 15*
I*c) + 120*I*e^(13*I*d*x + 13*I*c) + 390*I*e^(11*I*d*x + 11*I*c) + 715*I*e^
(9*I*d*x + 9*I*c))/(d*e^(14*I*d*x + 14*I*c) + 7*d*e^(12*I*d*x + 12*I*c) + 2
1*d*e^(10*I*d*x + 10*I*c) + 35*d*e^(8*I*d*x + 8*I*c) + 35*d*e^(6*I*d*x + 6*
I*c) + 21*d*e^(4*I*d*x + 4*I*c) + 7*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^{8}(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\sec^{8}\left(c+dx\right)dx$$

```
[In] integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*sec(c + d*x)**8, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.65

$$\int \sec^8(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$
$$=\frac{2i\left(429\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{15}{2}}-2970\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{13}{2}}a+7020\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{11}{2}}a^2-5720\left(i\right.\right.}{6435\,a^7d}$$

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/6435*I*(429*(I*a*tan(d*x + c) + a)^(15/2) - 2970*(I*a*tan(d*x + c) + a)^(
13/2)*a + 7020*(I*a*tan(d*x + c) + a)^(11/2)*a^2 - 5720*(I*a*tan(d*x + c) +
 a)^(9/2)*a^3)/(a^7*d)
```

Giac [F]

$$\int \sec^8(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx=\int \sqrt{i\,a\tan\left(dx+c\right)+a}\sec\left(dx+c\right)^8\,dx$$

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*sec(d*x + c)^8, x)
```

Mupad [B] (verification not implemented)

Time = 13.14 (sec) , antiderivative size = 474, normalized size of antiderivative = 4.05

$$\int \sec^8(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,4096\mathrm{i}}{6435\,d} - \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,2048\mathrm{i}}{6435\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)} - \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,512\mathrm{i}}{2145\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2} - \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,256\mathrm{i}}{1287\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3} + \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,40960\mathrm{i}}{1287\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4} - \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,52736\mathrm{i}}{715\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^5} + \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,11776\mathrm{i}}{195\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^6} - \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,256\mathrm{i}}{15\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^7}$$

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2)/cos(c + d*x)^8,x)
```

```
[Out] ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*4
0960i)/(1287*d*(exp(c*2i + d*x*2i) + 1)^4) - ((a - (a*(exp(c*2i + d*x*2i)*1
i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*2048i)/(6435*d*(exp(c*2i + d*x*
2i) + 1)) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) +
 1))^(1/2)*512i)/(2145*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (a*(exp(c*2i +
 d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*256i)/(1287*d*(exp(c*
2i + d*x*2i) + 1)^3) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i
+ d*x*2i) + 1))^(1/2)*4096i)/(6435*d) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1
i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*52736i)/(715*d*(exp(c*2i + d*x*2i) +
 1)^5) + ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1)
)^(1/2)*11776i)/(195*d*(exp(c*2i + d*x*2i) + 1)^6) - ((a - (a*(exp(c*2i + d
*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*256i)/(15*d*(exp(c*2i +
 d*x*2i) + 1)^7)
```

3.280 $\int \sec^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \sec^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{8i(a+ia\tan(c+dx))^{7/2}}{7a^3d} + \frac{8i(a+ia\tan(c+dx))^{9/2}}{9a^4d} - \frac{2i(a+ia\tan(c+dx))^{11/2}}{11a^5d}$$

```
[Out] -8/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^3/d+8/9*I*(a+I*a*tan(d*x+c))^(9/2)/a^4/d-
2/11*I*(a+I*a*tan(d*x+c))^(11/2)/a^5/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3568, 45}

$$\int \sec^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{2i(a+ia\tan(c+dx))^{11/2}}{11a^5d} + \frac{8i(a+ia\tan(c+dx))^{9/2}}{9a^4d} - \frac{8i(a+ia\tan(c+dx))^{7/2}}{7a^3d}$$

```
[In] Int[Sec[c + d*x]^6*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((-8*I)/7)*(a + I*a*Tan[c + d*x])^(7/2))/(a^3*d) + (((8*I)/9)*(a + I*a*Tan
[c + d*x])^(9/2))/(a^4*d) - (((2*I)/11)*(a + I*a*Tan[c + d*x])^(11/2))/(a^5
*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)^2(a+x)^{5/2}\,dx,x,ia\tan(c+dx)\right)}{a^5d} \\
&= -\frac{i\text{Subst}\left(\int \left(4a^2(a+x)^{5/2}-4a(a+x)^{7/2}+(a+x)^{9/2}\right)\,dx,x,ia\tan(c+dx)\right)}{a^5d} \\
&= -\frac{8i(a+ia\tan(c+dx))^{7/2}}{7a^3d}+\frac{8i(a+ia\tan(c+dx))^{9/2}}{9a^4d}-\frac{2i(a+ia\tan(c+dx))^{11/2}}{11a^5d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.66

$$
\int \sec^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{2(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}(-151+182i\tan(c+dx)+63\tan^2(c+dx))}{693d}
$$

```
[In] Integrate[Sec[c + d*x]^6*Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] (2*(-I + Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c + d*x]]*(-151 + (182*I)*Tan[c +
 d*x] + 63*Tan[c + d*x]^2))/(693*d)
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}+\frac{4a(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}-\frac{4a^{2}(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}\right)}{d\,a^{5}}$	63
default	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}+\frac{4a(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}-\frac{4a^{2}(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}\right)}{d\,a^{5}}$	63

```
[In] int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*I/d/a^5*(-1/11*(a+I*a*tan(d*x+c))^(11/2)+4/9*a*(a+I*a*tan(d*x+c))^(9/2)-4
/7*a^2*(a+I*a*tan(d*x+c))^(7/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.35

$$\int \sec^{6}(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{64\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(8i\,e^{(11i\,dx+11i\,c)}+44i\,e^{(9i\,dx+9i\,c)}+99i\,e^{(7i\,dx+7i\,c)}\right)}{693\left(de^{(10i\,dx+10i\,c)}+5\,de^{(8i\,dx+8i\,c)}+10\,de^{(6i\,dx+6i\,c)}+10\,de^{(4i\,dx+4i\,c)}+5\,de^{(2i\,dx+2i\,c)}+d\right)}$$

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
[Out] -64/693*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(8*I*e^(11*I*d*x + 11*I*c
) + 44*I*e^(9*I*d*x + 9*I*c) + 99*I*e^(7*I*d*x + 7*I*c))/(d*e^(10*I*d*x + 1
0*I*c) + 5*d*e^(8*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) + 10*d*e^(4*I*d
*x + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^{6}(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\sec^{6}\left(c+dx\right)dx$$

```
[In] integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*sec(c + d*x)**6, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.66

$$\int \sec^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{2i\left(63\,(i\,a\tan\left(dx+c\right)+a)^{\frac{11}{2}}-308\,(i\,a\tan\left(dx+c\right)+a)^{\frac{9}{2}}a+396\,(i\,a\tan\left(dx+c\right)+a)^{\frac{7}{2}}a^2\right)}{693\,a^5d}$$

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] -2/693*I*(63*(I*a*tan(d*x + c) + a)^(11/2) - 308*(I*a*tan(d*x + c) + a)^(9/
2)*a + 396*(I*a*tan(d*x + c) + a)^(7/2)*a^2)/(a^5*d)
```

Giac [F]

$$\int \sec^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{i\,a\tan\left(dx+c\right)+a}\sec\left(dx+c\right)^6\,dx$$

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*sec(d*x + c)^6, x)
```

Mupad [B] (verification not implemented)

Time = 7.76 (sec) , antiderivative size = 352, normalized size of antiderivative = 4.00

$$\begin{aligned}\int \sec^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = &-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,512\mathrm{i}}{693\,d}\\ &-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,256\mathrm{i}}{693\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}\\ &-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{231\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}\\ &+\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,7232\mathrm{i}}{693\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}\\ &-\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,1472\mathrm{i}}{99\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4}\\ &+\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{11\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^5}\end{aligned}$$

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2)/cos(c + d*x)^6,x)
```

```
[Out] ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*7
232i)/(693*d*(exp(c*2i + d*x*2i) + 1)^3) - ((a - (a*(exp(c*2i + d*x*2i)*1i
- 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*256i)/(693*d*(exp(c*2i + d*x*2i)
+ 1)) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))
^(1/2)*64i)/(231*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (a*(exp(c*2i + d*x*2
i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*512i)/(693*d) - ((a - (a*(e
xp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1472i)/(99*d
*(exp(c*2i + d*x*2i) + 1)^4) + ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(e
xp(c*2i + d*x*2i) + 1))^(1/2)*64i)/(11*d*(exp(c*2i + d*x*2i) + 1)^5)
```

3.281 $\int \sec^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 59

$$\int \sec^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{4i(a+ia\tan(c+dx))^{5/2}}{5a^2d}+\frac{2i(a+ia\tan(c+dx))^{7/2}}{7a^3d}$$

```
[Out] -4/5*I*(a+I*a*tan(d*x+c))^(5/2)/a^2/d+2/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^3/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3568, 45}

$$\int \sec^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{2i(a+ia\tan(c+dx))^{7/2}}{7a^3d}-\frac{4i(a+ia\tan(c+dx))^{5/2}}{5a^2d}$$

```
[In] Int[Sec[c + d*x]^4*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((-4*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(a^2*d) + (((2*I)/7)*(a + I*a*Tan
[c + d*x])^(7/2))/(a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)(a+x)^{3/2}\,dx, x, ia\tan(c+dx)\right)}{a^3d} \\ &= -\frac{i\text{Subst}\left(\int \left(2a(a+x)^{3/2}-(a+x)^{5/2}\right)\,dx, x, ia\tan(c+dx)\right)}{a^3d} \\ &= -\frac{4i(a+ia\tan(c+dx))^{5/2}}{5a^2d}+\frac{2i(a+ia\tan(c+dx))^{7/2}}{7a^3d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.81

$$\int \sec^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{2(-i+\tan(c+dx))^2(9i+5\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{35d}$$

```
[In] Integrate[Sec[c + d*x]^4*Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] (2*(-I + Tan[c + d*x])^2*(9*I + 5*Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
/(35*d)
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{2a(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}\right)}{d\,a^3}$	44
default	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{2a(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}\right)}{d\,a^3}$	44

```
[In] int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*I/d/a^3*(1/7*(a+I*a*tan(d*x+c))^(7/2)-2/5*a*(a+I*a*tan(d*x+c))^(5/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.42

$$\int \sec^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{16\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(2i\,e^{(7i\,dx+7i\,c)}+7i\,e^{(5i\,dx+5i\,c)}\right)}{35\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)}$$

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
[Out] -16/35*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(2*I*e^(7*I*d*x + 7*I*c) +
 7*I*e^(5*I*d*x + 5*I*c))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c)
+ 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\sec^4\left(c+dx\right)dx$$

```
[In] integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*sec(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.68

$$\int \sec^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{2i\left(5\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}-14\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}a\right)}{35\,a^3d}$$

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] 2/35*I*(5*(I*a*tan(d*x + c) + a)^(7/2) - 14*(I*a*tan(d*x + c) + a)^(5/2)*a)
/(a^3*d)
```

Giac [F]

$$\int \sec^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{i\,a\tan\left(dx+c\right)+a}\sec\left(dx+c\right)^4\,dx$$

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*sec(d*x + c)^4, x)
```

Mupad [B] (verification not implemented)

Time = 7.37 (sec) , antiderivative size = 230, normalized size of antiderivative = 3.90

$$\int \sec^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{35\,d} - \frac{\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,16\mathrm{i}}{35\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)} + \frac{\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{35\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2} - \frac{\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,16\mathrm{i}}{7\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}$$

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2)/cos(c + d*x)^4,x)
[Out] ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1
28i)/(35*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (a*(exp(c*2i + d*x*2i)*1i -
1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*16i)/(35*d*(exp(c*2i + d*x*2i) + 1)
) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/
2)*32i)/(35*d) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*
2i) + 1))^(1/2)*16i)/(7*d*(exp(c*2i + d*x*2i) + 1)^3)
```

3.282 $\int \sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 29

$$\int \sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{2i(a+ia\tan(c+dx))^{3/2}}{3ad}$$

```
[Out] -2/3*I*(a+I*a*tan(d*x+c))^(3/2)/a/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3568, 32}

$$\int \sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{2i(a+ia\tan(c+dx))^{3/2}}{3ad}$$

```
[In] Int[Sec[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((-2*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(a*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{i\text{Subst}\left(\int \sqrt{a+x}\,dx, x, ia\tan(c+dx)\right)}{ad}$$
$$= -\frac{2i(a+ia\tan(c+dx))^{3/2}}{3ad}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{2i(a+ia\tan(c+dx))^{3/2}}{3ad}$$

```
[In] Integrate[Sec[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] (((-2*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(a*d)
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{2i(a+ia\tan(dx+c))^{\frac{3}{2}}}{3ad}$	24
default	$-\frac{2i(a+ia\tan(dx+c))^{\frac{3}{2}}}{3ad}$	24

```
[In] int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*I*(a+I*a*tan(d*x+c))^(3/2)/a/d
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(21) = 42.

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.59

$$\int \sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{4i\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(3i\,dx+3i\,c)}}{3\left(de^{(2i\,dx+2i\,c)}+d\right)}$$

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
[Out] -4/3*I*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(3*I*d*x + 3*I*c)/(d*e^(
2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\sec^2\left(c+dx\right)dx$$

```
[In] integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*sec(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{2i\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}}{3\,ad}$$

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] -2/3*I*(I*a*tan(d*x + c) + a)^(3/2)/(a*d)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(21) = 42.

Time = 0.52 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.90

$$\int \sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{2i\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2i\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}\right)^{\frac{3}{2}}}{3\,ad}$$

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] -2/3*I*((a*tan(1/2*d*x + 1/2*c)^2 - 2*I*a*tan(1/2*d*x + 1/2*c) - a)/(tan(1/
2*d*x + 1/2*c)^2 - 1))^(3/2)/(a*d)
```

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.83

$$\int \sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$
$$= -\frac{(\cos(2c+2dx)+1+\sin(2c+2dx)\ 1\mathrm{i})\ \sqrt{\frac{a\,(\cos(2c+2dx)+1+\sin(2c+2dx)\ 1\mathrm{i})}{\cos(2c+2dx)+1}}\ 2\mathrm{i}}{3\,d\ (\cos(2c+2dx)+1)}$$

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2)/cos(c + d*x)^2,x)
```

```
[Out] -((cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1)*((a*(cos(2*c + 2*d*x) + sin(
2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2)*2i)/(3*d*(cos(2*c + 2*d
*x) + 1))
```

3.283 $\int \cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 120

$$\int \cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{3i\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}d} + \frac{3ia}{4d\sqrt{a+ia\tan(c+dx)}} - \frac{ia^2}{2d(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] -3/8*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^(1/2)/d*2^(1
/2)+3/4*I*a/d/(a+I*a*tan(d*x+c))^(1/2)-1/2*I*a^2/d/(a+I*a*tan(d*x+c))^(1/2)
/(a-I*a*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3568, 44, 53, 65, 212}

$$\int \cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{ia^2}{2d(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}} - \frac{3i\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}d} + \frac{3ia}{4d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[Cos[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((-3*I)/4)*Sqrt[a]*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/
(Sqrt[2]*d) + (((3*I)/4)*a)/(d*Sqrt[a + I*a*Tan[c + d*x]]) - ((I/2)*a^2)/(d
*(a - I*a*Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{(ia^3)\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{3/2}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^2}{2d(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}} - \frac{(3ia^2)\,\text{Subst}\left(\int\frac{1}{(a-x)(a+x)^{3/2}}\,dx,x,ia\tan(c+dx)\right)}{4d}$$

$$= \frac{3ia}{4d\sqrt{a+ia\tan(c+dx)}} - \frac{ia^2}{2d(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}} - \frac{(3ia)\text{Subst}\left(\int\frac{1}{(a-x)\sqrt{a+x}}\,dx,x,ia\tan(c+dx)\right)}{8d}$$

$$= \frac{3ia}{4d\sqrt{a+ia\tan(c+dx)}} - \frac{ia^2}{2d(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}} - \frac{(3ia)\text{Subst}\left(\int\frac{1}{2a-x^2}\,dx,x,\sqrt{a+ia\tan(c+dx)}\right)}{4d}$$

$$= -\frac{3i\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{4\sqrt{2}d} + \frac{3ia}{4d\sqrt{a+ia\tan(c+dx)}} - \frac{ia^2}{2d(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.42

$$\int\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{ia\,\text{Hypergeometric2F1}\left(-\frac{1}{2},2,\frac{1}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{2d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] ((I/2)*a*Hypergeometric2F1[-1/2, 2, 1/2, (1 + I*Tan[c + d*x])/2])/(d*Sqrt[a
 + I*a*Tan[c + d*x]])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 383 vs. 2(94) = 188.

Time = 38.56 (sec) , antiderivative size = 384, normalized size of antiderivative = 3.20

method	result
default	$-\frac{i\sqrt{a(1+i\tan(dx+c))}\left(3i\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)+3i\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\arctan\left(\sqrt{-\frac{\cos}{\cos}}\right.\right.}{}$

[In] int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/8*I/d*(a*(1+I*tan(d*x+c)))^(1/2)*(3*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)+3*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+3*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+6*I*cos(d*x+c)*sin(d*x+c)+3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-2*cos(d*x+c)^2)

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(87) = 174.

Time = 0.25 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.11

$$\int \cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{\left(3\sqrt{\frac{1}{2}}d\sqrt{-\frac{a}{d^2}}e^{(i\,dx+i\,c)}\log\left(-4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{-\frac{a}{d^2}}-ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}\right)\right.}{}$$

[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")

[Out] -1/8*(3*sqrt(1/2)*d*sqrt(-a/d^2)*e^(I*d*x + I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-a/d^2) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 3*sqrt(1/2)*d*sqrt(-a/d^2)*e^(I*d*x + I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-a/d^2) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-I*e^(4*I*d*x + 4*I*c) + I*e^(2*I*d*x + 2*I*c) + 2*I))*e^(-I*d*x - I*c)/d

Sympy [F]

$$\int \cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\cos^2\left(c+dx\right)dx$$

```
[In] integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*cos(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.02

$$\int \cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{i\left(3\sqrt{2}a^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+\frac{4\left(3\left(i\,a\tan(dx+c)+a\right)a^2-4\,a^3\right)}{\left(i\,a\tan(dx+c)+a\right)^{\frac{3}{2}}-2\sqrt{i\,a\tan(dx+c)+a}a}\right)}{16\,ad}$$

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] 1/16*I*(3*sqrt(2)*a^(3/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a
))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(3*(I*a*tan(d*x + c)
 + a)*a^2 - 4*a^3)/((I*a*tan(d*x + c) + a)^(3/2) - 2*sqrt(I*a*tan(d*x + c)
+ a)*a))/(a*d)
```

Giac [F]

$$\int \cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{i\,a\tan\left(dx+c\right)+a}\cos\left(dx+c\right)^2\,dx$$

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*cos(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \cos\left(c+d\,x\right)^2\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

```
[In] int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.284 $\int \cos^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 193

$$\int \cos^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{35i\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}d} + \frac{35ia^2}{96d(a+ia\tan(c+dx))^{3/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{3/2}} - \frac{7ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}} + \frac{35ia}{64d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] -35/128*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^(1/2)/d*2
^(1/2)+35/64*I*a/d/(a+I*a*tan(d*x+c))^(1/2)+35/96*I*a^2/d/(a+I*a*tan(d*x+c)
)^(3/2)-1/4*I*a^4/d/(a-I*a*tan(d*x+c))^2/(a+I*a*tan(d*x+c))^(3/2)-7/16*I*a^
3/d/(a-I*a*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used

= {3568, 44, 53, 65, 212}

$$\int \cos^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{3/2}} - \frac{7ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}} + \frac{35ia^2}{96d(a+ia\tan(c+dx))^{3/2}} - \frac{35i\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}d} + \frac{35ia}{64d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[Cos[c + d*x]^4*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((-35*I)/64)*Sqrt[a]*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])]
)/(Sqrt[2]*d) + (((35*I)/96)*a^2)/(d*(a + I*a*Tan[c + d*x])^(3/2)) - ((I/4)
*a^4)/(d*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(3/2)) - (((7*I)/1
6)*a^3)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(3/2)) + (((35*I)/
64)*a)/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(ia^5)\,\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^{5/2}}\,dx, x, ia\tan(c+dx)\right)}{d} \\
&= -\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{3/2}} \\
&\quad - \frac{(7ia^4)\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{5/2}}\,dx, x, ia\tan(c+dx)\right)}{8d} \\
&= -\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{3/2}} \\
&\quad - \frac{7ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}} \\
&\quad - \frac{(35ia^3)\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{5/2}}\,dx, x, ia\tan(c+dx)\right)}{32d} \\
&= \frac{35ia^2}{96d(a+ia\tan(c+dx))^{3/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{3/2}} \\
&\quad - \frac{7ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}} \\
&\quad - \frac{(35ia^2)\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{3/2}}\,dx, x, ia\tan(c+dx)\right)}{64d} \\
&= \frac{35ia^2}{96d(a+ia\tan(c+dx))^{3/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{3/2}} \\
&\quad - \frac{7ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}} + \frac{35ia}{64d\sqrt{a+ia\tan(c+dx)}} \\
&\quad - \frac{(35ia)\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{128d}
\end{aligned}$$

$$= \frac{35ia^2}{96d(a+ia\tan(c+dx))^{3/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{3/2}} - \frac{7ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}} + \frac{35ia}{64d\sqrt{a+ia\tan(c+dx)}} - \frac{(35ia)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+ia\tan(c+dx)}\right)}{64d}$$

$$= -\frac{35i\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}d} + \frac{35ia^2}{96d(a+ia\tan(c+dx))^{3/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{3/2}} - \frac{7ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}} + \frac{35ia}{64d\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.27

$$\int \cos^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{ia^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},3,-\frac{1}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{12d(a+ia\tan(c+dx))^{3/2}}$$

```
[In] Integrate[Cos[c + d*x]^4*Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] ((I/12)*a^2*Hypergeometric2F1[-3/2, 3, -1/2, (1 + I*Tan[c + d*x])/2])/(d*(a
 + I*a*Tan[c + d*x])^(3/2))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 410 vs. 2(154) = 308.

Time = 109.91 (sec) , antiderivative size = 411, normalized size of antiderivative = 2.13

method	result
default	$-\frac{i\sqrt{a(1+i\tan(dx+c))}\left(112i(\cos^3(dx+c))\sin(dx+c)+105i\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)+10\right.}{}$

```
[In] int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/384*I/d*(a*(1+I*tan(d*x+c)))^(1/2)*(112*I*cos(d*x+c)^3*sin(d*x+c)+105*I*
arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+105*I*(-cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-16*cos(d*x+c)^
4+105*I*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2
))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+210*I*cos(d*x+c)*sin(d*x+c)+105*(-cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c
)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-105*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-105*(-cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-70*cos(d*x+c)^
2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.42

$$\int \cos^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{\left(105\sqrt{\frac{1}{2}}d\sqrt{-\frac{a}{d^2}}e^{(3i\,dx+3i\,c)}\log\left(-4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{-\frac{a}{d^2}}-ae^{(i\,dx+i\,c)}\right)e^{(-i\,d}\right.\right.}{}$$

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/384*(105*sqrt(1/2)*d*sqrt(-a/d^2)*e^(3*I*d*x + 3*I*c)*log(-4*(sqrt(2)*sq
rt(1/2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*s
qrt(-a/d^2) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 105*sqrt(1/2)*d*sqrt(-
a/d^2)*e^(3*I*d*x + 3*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(-I*d*e^(2*I*d*x + 2*I
*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-a/d^2) - a*e^(I*d*x + I*
c))*e^(-I*d*x - I*c)) - sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-6*I*e^(
8*I*d*x + 8*I*c) - 45*I*e^(6*I*d*x + 6*I*c) + 41*I*e^(4*I*d*x + 4*I*c) + 88
*I*e^(2*I*d*x + 2*I*c) + 8*I))*e^(-3*I*d*x - 3*I*c)/d
```

Sympy [F]

$$\int \cos^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\cos^4\left(c+dx\right)dx$$

```
[In] integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*cos(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.91

$$\int \cos^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$
$$= \frac{i\left(105\sqrt{2}a^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+\frac{4\left(105\,(i\,a\tan(dx+c)+a)^3a^2-350\,(i\,a\tan(dx+c)+a)^2a^3+224\,(i\,a\tan(dx+c)+a)a^4+6\right.}{(i\,a\tan(dx+c)+a)^{\frac{7}{2}}-4\,(i\,a\tan(dx+c)+a)^{\frac{5}{2}}a+4\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a^2}\right.}{768\,ad}$$

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/768*I*(105*sqrt(2)*a^(3/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c)
+ a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(105*(I*a*tan(d*x
 + c) + a)^3*a^2 - 350*(I*a*tan(d*x + c) + a)^2*a^3 + 224*(I*a*tan(d*x + c)
 + a)*a^4 + 64*a^5)/((I*a*tan(d*x + c) + a)^(7/2) - 4*(I*a*tan(d*x + c) + a
)^(5/2)*a + 4*(I*a*tan(d*x + c) + a)^(3/2)*a^2))/(a*d)
```

Giac [F]

$$\int \cos^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{i\,a\tan(dx+c)+a}\cos(dx+c)^4\,dx$$

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*cos(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^4(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \cos(c+d\,x)^4\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}\,dx$$

```
[In] int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.285 $\int \cos^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 266

$$\int \cos^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{231i\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{512\sqrt{2}d} + \frac{231ia^3}{640d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{5/2}} - \frac{11ia^5}{48d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}} - \frac{33ia^4}{64d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}} + \frac{77ia^2}{256d(a+ia\tan(c+dx))^{3/2}} + \frac{231ia}{512d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] -231/1024*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*a^(1/2)/d
*2^(1/2)+231/512*I*a/d/(a+I*a*tan(d*x+c))^(1/2)+231/640*I*a^3/d/(a+I*a*tan(
d*x+c))^(5/2)-1/6*I*a^6/d/(a-I*a*tan(d*x+c))^3/(a+I*a*tan(d*x+c))^(5/2)-11/
48*I*a^5/d/(a-I*a*tan(d*x+c))^2/(a+I*a*tan(d*x+c))^(5/2)-33/64*I*a^4/d/(a-I
*a*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2)+77/256*I*a^2/d/(a+I*a*tan(d*x+c))^(
3/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3568, 44, 53, 65, 212}

$$\int \cos^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{5/2}} - \frac{11ia^5}{48d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}} - \frac{33ia^4}{64d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}} + \frac{231ia^3}{640d(a+ia\tan(c+dx))^{5/2}} + \frac{77ia^2}{256d(a+ia\tan(c+dx))^{3/2}} - \frac{231i\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{512\sqrt{2}d} + \frac{231ia}{512d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[Cos[c + d*x]^6*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((-231*I)/512)*Sqrt[a]*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a]
)])/(Sqrt[2]*d) + (((231*I)/640)*a^3)/(d*(a + I*a*Tan[c + d*x])^(5/2)) - ((
I/6)*a^6)/(d*(a - I*a*Tan[c + d*x])^3*(a + I*a*Tan[c + d*x])^(5/2)) - (((11
*I)/48)*a^5)/(d*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(5/2)) - ((
(33*I)/64)*a^4)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(5/2)) + (
((77*I)/256)*a^2)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((231*I)/512)*a)/(d*S
qrt[a + I*a*Tan[c + d*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
```

```
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(ia^7)\,\text{Subst}\left(\int \frac{1}{(a-x)^4(a+x)^{7/2}}\,dx, x, ia\tan(c+dx)\right)}{d} \\
&= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{5/2}} \\
&\quad - \frac{(11ia^6)\,\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^{7/2}}\,dx, x, ia\tan(c+dx)\right)}{12d} \\
&= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{5/2}} \\
&\quad - \frac{11ia^5}{48d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}} \\
&\quad - \frac{(33ia^5)\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{7/2}}\,dx, x, ia\tan(c+dx)\right)}{32d}
\end{aligned}$$

$$= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{5/2}} - \frac{11ia^5}{48d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}} - \frac{33ia^4}{64d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}} - \frac{(231ia^4)\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{7/2}}\,dx, x, ia\tan(c+dx)\right)}{128d}$$

$$= \frac{231ia^3}{640d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{5/2}} - \frac{11ia^5}{48d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}} - \frac{33ia^4}{64d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}} - \frac{(231ia^3)\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{5/2}}\,dx, x, ia\tan(c+dx)\right)}{256d}$$

$$= \frac{231ia^3}{640d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{5/2}} - \frac{11ia^5}{48d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}} - \frac{33ia^4}{64d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}} + \frac{77ia^2}{256d(a+ia\tan(c+dx))^{3/2}} - \frac{(231ia^2)\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{3/2}}\,dx, x, ia\tan(c+dx)\right)}{512d}$$

$$= \frac{231ia^3}{640d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{5/2}} - \frac{11ia^5}{48d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}} - \frac{33ia^4}{64d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}} + \frac{77ia^2}{256d(a+ia\tan(c+dx))^{3/2}} + \frac{231ia}{512d\sqrt{a+ia\tan(c+dx)}} - \frac{(231ia)\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{1024d}$$

$$= \frac{231ia^3}{640d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{5/2}} - \frac{11ia^5}{48d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}} - \frac{33ia^4}{64d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}} + \frac{77ia^2}{256d(a+ia\tan(c+dx))^{3/2}} + \frac{231ia}{512d\sqrt{a+ia\tan(c+dx)}} - \frac{(231ia)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+ia\tan(c+dx)}\right)}{512d}$$

$$= -\frac{231i\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{512\sqrt{2}d} + \frac{231ia^3}{640d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{5/2}} - \frac{11ia^5}{48d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}} - \frac{33ia^4}{64d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}} + \frac{77ia^2}{256d(a+ia\tan(c+dx))^{3/2}} + \frac{231ia}{512d\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.20

$$\int \cos^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{ia^3\,\text{Hypergeometric2F1}\left(-\frac{5}{2},4,-\frac{3}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{40d(a+ia\tan(c+dx))^{5/2}}$$

```
[In] Integrate[Cos[c + d*x]^6*Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] ((I/40)*a^3*Hypergeometric2F1[-5/2, 4, -3/2, (1 + I*Tan[c + d*x])/2])/(d*(a
 + I*a*Tan[c + d*x])^(5/2))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(214) = 428$.

Time = 108.03 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.65

method	result
default	$-\frac{i\sqrt{a(1+i\tan(dx+c))}\left(2816i(\cos^5(dx+c))\sin(dx+c)-256(\cos^6(dx+c))+3696i(\cos^3(dx+c))\sin(dx+c)+3465i\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\mathrm{arc}\right.}{}$

```
[In] int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15360*I/d*(a*(1+I*tan(d*x+c)))^(1/2)*(2816*I*cos(d*x+c)^5*sin(d*x+c)-256
*cos(d*x+c)^6+3696*I*cos(d*x+c)^3*sin(d*x+c)+3465*I*(-cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^
(1/2))*cos(d*x+c)+3465*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*
x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-528*cos(d*x+c)^4+3465*I*(-cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(
d*x+c)+1))^(1/2))+6930*I*cos(d*x+c)*sin(d*x+c)+3465*(-cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^
(1/2))*sin(d*x+c)-3465*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+
c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-3465*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-2310*cos(d*x+c)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.12

$$\int \cos^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx =$$

$$-\frac{\left(3465\sqrt{\frac{1}{2}}d\sqrt{-\frac{a}{d^2}}e^{(5i\,dx+5i\,c)}\log\left(-4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{-\frac{a}{d^2}}-ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx}\right.\right.}{}$$

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15360*(3465*sqrt(1/2)*d*sqrt(-a/d^2)*e^(5*I*d*x + 5*I*c)*log(-4*(sqrt(2)
*sqrt(1/2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)
)*sqrt(-a/d^2) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) - 3465*sqrt(1/2)*d*sq
rt(-a/d^2)*e^(5*I*d*x + 5*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(-I*d*e^(2*I*d*x +
 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-a/d^2) - a*e^(I*d*x
+ I*c))*e^(-I*d*x - I*c)) - sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-40*
I*e^(12*I*d*x + 12*I*c) - 350*I*e^(10*I*d*x + 10*I*c) - 1645*I*e^(8*I*d*x +
 8*I*c) + 1433*I*e^(6*I*d*x + 6*I*c) + 3184*I*e^(4*I*d*x + 4*I*c) + 464*I*e
^(2*I*d*x + 2*I*c) + 48*I))*e^(-5*I*d*x - 5*I*c)/d
```

Sympy [F]

$$\int \cos^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx=\int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\cos^6\left(c+dx\right)dx$$

```
[In] integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*cos(c + d*x)**6, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.86

$$\int \cos^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$
$$=\frac{i\left(3465\sqrt{2}a^{\frac{3}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+\frac{4\left(3465\,(i\,a\tan(dx+c)+a)^5a^2-18480\,(i\,a\tan(dx+c)+a)^4a^3+30492\,(i\,a\tan(dx+c\right.}{(i\,a\tan(dx+c)+a)^{\frac{11}{2}}-6\,(i\,a\tan(dx+c)+a)^{\frac{9}{2}}a+12\,(i}\right.}{30720\,ad}$$

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] 1/30720*I*(3465*sqrt(2)*a^(3/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x +
c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(3465*(I*a*tan
(d*x + c) + a)^5*a^2 - 18480*(I*a*tan(d*x + c) + a)^4*a^3 + 30492*(I*a*tan(
d*x + c) + a)^3*a^4 - 12672*(I*a*tan(d*x + c) + a)^2*a^5 - 2816*(I*a*tan(d*
x + c) + a)*a^6 - 1536*a^7)/((I*a*tan(d*x + c) + a)^(11/2) - 6*(I*a*tan(d*x
 + c) + a)^(9/2)*a + 12*(I*a*tan(d*x + c) + a)^(7/2)*a^2 - 8*(I*a*tan(d*x +
 c) + a)^(5/2)*a^3))/(a*d)
```

Giac [F]

$$\int \cos^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx=\int \sqrt{i\,a\tan\left(dx+c\right)+a}\cos\left(dx+c\right)^6\,dx$$

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*cos(d*x + c)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^6(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \cos\left(c+d\,x\right)^6\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

```
[In] int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.286 $\int \sec^7(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 147

$$\int \sec^7(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{256ia^4\sec^7(c+dx)}{3003d(a+ia\tan(c+dx))^{7/2}} + \frac{64ia^3\sec^7(c+dx)}{429d(a+ia\tan(c+dx))^{5/2}} + \frac{24ia^2\sec^7(c+dx)}{143d(a+ia\tan(c+dx))^{3/2}} + \frac{2ia\sec^7(c+dx)}{13d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 2/13*I*a*sec(d*x+c)^7/d/(a+I*a*tan(d*x+c))^(1/2)+256/3003*I*a^4*sec(d*x+c)^
7/d/(a+I*a*tan(d*x+c))^(7/2)+64/429*I*a^3*sec(d*x+c)^7/d/(a+I*a*tan(d*x+c))
^(5/2)+24/143*I*a^2*sec(d*x+c)^7/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3575, 3574}

$$\int \sec^7(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{256ia^4\sec^7(c+dx)}{3003d(a+ia\tan(c+dx))^{7/2}} + \frac{64ia^3\sec^7(c+dx)}{429d(a+ia\tan(c+dx))^{5/2}} + \frac{24ia^2\sec^7(c+dx)}{143d(a+ia\tan(c+dx))^{3/2}} + \frac{2ia\sec^7(c+dx)}{13d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[Sec[c + d*x]^7*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((((256*I)/3003)*a^4*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (((6
4*I)/429)*a^3*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((24*I)/1
43)*a^2*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((2*I)/13)*a*Se
c[c + d*x]^7)/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2ia\sec^7(c+dx)}{13d\sqrt{a+ia\tan(c+dx)}} + \frac{1}{13}(12a)\int \frac{\sec^7(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx \\
&= \frac{24ia^2\sec^7(c+dx)}{143d(a+ia\tan(c+dx))^{3/2}} + \frac{2ia\sec^7(c+dx)}{13d\sqrt{a+ia\tan(c+dx)}} \\
&\quad + \frac{1}{143}\left(96a^2\right)\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx \\
&= \frac{64ia^3\sec^7(c+dx)}{429d(a+ia\tan(c+dx))^{5/2}} + \frac{24ia^2\sec^7(c+dx)}{143d(a+ia\tan(c+dx))^{3/2}} \\
&\quad + \frac{2ia\sec^7(c+dx)}{13d\sqrt{a+ia\tan(c+dx)}} + \frac{1}{429}\left(128a^3\right)\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx \\
&= \frac{256ia^4\sec^7(c+dx)}{3003d(a+ia\tan(c+dx))^{7/2}} + \frac{64ia^3\sec^7(c+dx)}{429d(a+ia\tan(c+dx))^{5/2}} \\
&\quad + \frac{24ia^2\sec^7(c+dx)}{143d(a+ia\tan(c+dx))^{3/2}} + \frac{2ia\sec^7(c+dx)}{13d\sqrt{a+ia\tan(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.65

$$\int \sec^7(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$
$$=\frac{2\sec^6(c+dx)(390\cos(c+dx)+445\cos(3(c+dx))+7i(26\sin(c+dx)+59\sin(3(c+dx))))(i\cos(4(c+}{3003d}$$

```
[In] Integrate[Sec[c + d*x]^7*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (2*Sec[c + d*x]^6*(390*Cos[c + d*x] + 445*Cos[3*(c + d*x)] + (7*I)*(26*Sin[
c + d*x] + 59*Sin[3*(c + d*x)]))*(I*Cos[4*(c + d*x)] + Sin[4*(c + d*x)])*Sq
rt[a + I*a*Tan[c + d*x]])/(3003*d)
```

Maple [A] (verified)

Time = 8.63 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.79

method	result
default	$\frac{2\sqrt{a(1+i\tan(dx+c))}\,(1024i\cos(dx+c)+1024\sin(dx+c)-128i\sec(dx+c)+384\sec(dx+c)\tan(dx+c)-40i(\sec^3(dx+c))+280\tan(dx-}{3003d}$

```
[In] int(sec(d*x+c)^7*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3003/d*(a*(1+I*tan(d*x+c)))^(1/2)*(1024*I*cos(d*x+c)+1024*sin(d*x+c)-128*
I*sec(d*x+c)+384*sec(d*x+c)*tan(d*x+c)-40*I*sec(d*x+c)^3+280*tan(d*x+c)*sec
(d*x+c)^3-21*I*sec(d*x+c)^5+231*tan(d*x+c)*sec(d*x+c)^5)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.90

$$\int \sec^7(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx =$$
$$-\frac{128\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-429i\,e^{(6i\,dx+6i\,c)}-286i\,e^{(4i\,dx+4i\,c)}-104i\,e^{(2i\,dx+2i\,c)}-16i\right)}{3003\left(de^{(12i\,dx+12i\,c)}+6\,de^{(10i\,dx+10i\,c)}+15\,de^{(8i\,dx+8i\,c)}+20\,de^{(6i\,dx+6i\,c)}+15\,de^{(4i\,dx+4i\,c)}+6\,de^{(2i\,dx+2i\,c)}\right.}$$

```
[In] integrate(sec(d*x+c)^7*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -128/3003*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-429*I*e^(6*I*d*x + 6*
I*c) - 286*I*e^(4*I*d*x + 4*I*c) - 104*I*e^(2*I*d*x + 2*I*c) - 16*I)/(d*e^(
12*I*d*x + 12*I*c) + 6*d*e^(10*I*d*x + 10*I*c) + 15*d*e^(8*I*d*x + 8*I*c) +
 20*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*I*d*x + 4*I*c) + 6*d*e^(2*I*d*x + 2*I
*c) + d)
```

Sympy [F]

$$\int \sec^7(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\sec^7\left(c+dx\right)dx$$

```
[In] integrate(sec(d*x+c)**7*(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*sec(c + d*x)**7, x)
```

Maxima [F(-1)]

Timed out.

$$\int \sec^7(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate(sec(d*x+c)^7*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \sec^7(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{i\,a\tan\left(dx+c\right)+a}\,\sec\left(dx+c\right)^7\,dx$$

```
[In] integrate(sec(d*x+c)^7*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*sec(d*x + c)^7, x)
```

Mupad [B] (verification not implemented)

Time = 9.18 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.97

$$\int \sec^7(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{7\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^3} - \frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{3\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^4} + \frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,384\mathrm{i}}{11\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^5} - \frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{13\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^6}$$

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2)/cos(c + d*x)^7,x)
```

```
[Out] (exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i +
d*x*2i) + 1))^(1/2)*128i)/(7*d*(exp(c*2i + d*x*2i) + 1)^3) - (exp(- c*1i -
d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^
(1/2)*128i)/(3*d*(exp(c*2i + d*x*2i) + 1)^4) + (exp(- c*1i - d*x*1i)*(a - (
a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*384i)/(1
1*d*(exp(c*2i + d*x*2i) + 1)^5) - (exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i +
 d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*128i)/(13*d*(exp(c*2i
 + d*x*2i) + 1)^6)
```

3.287 $\int \sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 110

$$\int \sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{64ia^3\sec^5(c+dx)}{315d(a+ia\tan(c+dx))^{5/2}} + \frac{16ia^2\sec^5(c+dx)}{63d(a+ia\tan(c+dx))^{3/2}} + \frac{2ia\sec^5(c+dx)}{9d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 2/9*I*a*sec(d*x+c)^5/d/(a+I*a*tan(d*x+c))^(1/2)+64/315*I*a^3*sec(d*x+c)^5/d
/(a+I*a*tan(d*x+c))^(5/2)+16/63*I*a^2*sec(d*x+c)^5/d/(a+I*a*tan(d*x+c))^(3/
2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3575, 3574}

$$\int \sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{64ia^3\sec^5(c+dx)}{315d(a+ia\tan(c+dx))^{5/2}} + \frac{16ia^2\sec^5(c+dx)}{63d(a+ia\tan(c+dx))^{3/2}} + \frac{2ia\sec^5(c+dx)}{9d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[Sec[c + d*x]^5*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((64*I)/315)*a^3*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((16*
I)/63)*a^2*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((2*I)/9)*a*
Sec[c + d*x]^5)/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2ia\sec^5(c+dx)}{9d\sqrt{a+ia\tan(c+dx)}}+\frac{1}{9}(8a)\int\frac{\sec^5(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx\\
&= \frac{16ia^2\sec^5(c+dx)}{63d(a+ia\tan(c+dx))^{3/2}}+\frac{2ia\sec^5(c+dx)}{9d\sqrt{a+ia\tan(c+dx)}}\\
&\quad+\frac{1}{63}\left(32a^2\right)\int\frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx\\
&= \frac{64ia^3\sec^5(c+dx)}{315d(a+ia\tan(c+dx))^{5/2}}+\frac{16ia^2\sec^5(c+dx)}{63d(a+ia\tan(c+dx))^{3/2}}+\frac{2ia\sec^5(c+dx)}{9d\sqrt{a+ia\tan(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.70

$$\int\sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$
$$=\frac{2\sec^4(c+dx)(36+71\cos(2(c+dx))+55i\sin(2(c+dx)))(i\cos(3(c+dx))+\sin(3(c+dx)))\sqrt{a+ia\tan(c+dx)}}{315d}$$

```
[In] Integrate[Sec[c + d*x]^5*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (2*Sec[c + d*x]^4*(36 + 71*Cos[2*(c + d*x)] + (55*I)*Sin[2*(c + d*x)])*(I*C
os[3*(c + d*x)] + Sin[3*(c + d*x)])*Sqrt[a + I*a*Tan[c + d*x]])/(315*d)
```

Maple [A] (verified)

Time = 7.64 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.81

method	result
default	$\frac{2\sqrt{a(1+i\tan(dx+c))}\,(128i\cos(dx+c)+128\sin(dx+c)-16i\sec(dx+c)+48\sec(dx+c)\tan(dx+c)-5i(\sec^3(dx+c))+35\tan(dx+c)(\sec^3(}{315d}$

```
[In] int(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/315/d*(a*(1+I*tan(d*x+c)))^(1/2)*(128*I*cos(d*x+c)+128*sin(d*x+c)-16*I*se
c(d*x+c)+48*sec(d*x+c)*tan(d*x+c)-5*I*sec(d*x+c)^3+35*tan(d*x+c)*sec(d*x+c)
^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.88

$$\int \sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{32\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-63i\,e^{(4i\,dx+4i\,c)}-36i\,e^{(2i\,dx+2i\,c)}-8i\right)}{315\left(de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}+4\,de^{(2i\,dx+2i\,c)}+d\right)}$$

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -32/315*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-63*I*e^(4*I*d*x + 4*I*c
) - 36*I*e^(2*I*d*x + 2*I*c) - 8*I)/(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x
 + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\sec^5\left(c+dx\right)dx$$

```
[In] integrate(sec(d*x+c)**5*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*sec(c + d*x)**5, x)
```

Maxima [F(-1)]

Timed out.

$$\int \sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{i\,a\tan(dx+c)+a}\sec(dx+c)^5\,dx$$

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*sec(d*x + c)^5, x)
```

Mupad [B] (verification not implemented)

Time = 6.94 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.93

$$\int \sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{32\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,36\mathrm{i}+\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}\,63\mathrm{i}+8\mathrm{i}\right)}{315\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^4}$$

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2)/cos(c + d*x)^5,x)
[Out] (32*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i
 + d*x*2i) + 1))^(1/2)*(exp(c*2i + d*x*2i)*36i + exp(c*4i + d*x*4i)*63i + 8
i))/(315*d*(exp(c*2i + d*x*2i) + 1)^4)
```

3.288 $\int \sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 73

$$\int \sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx=\frac{8ia^2\sec^3(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}+\frac{2ia\sec^3(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 2/5*I*a*sec(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(1/2)+8/15*I*a^2*sec(d*x+c)^3/d/(
a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3575, 3574}

$$\int \sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx=\frac{8ia^2\sec^3(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}+\frac{2ia\sec^3(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[Sec[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((8*I)/15)*a^2*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((2*I)/
5)*a*Sec[c + d*x]^3)/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2ia\sec^3(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}}+\frac{1}{5}(4a)\int\frac{\sec^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx\\ &= \frac{8ia^2\sec^3(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}+\frac{2ia\sec^3(c+dx)}{5d\sqrt{a+ia\tan(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{2\sec(c+dx)(\cos(2(c+dx))-i\sin(2(c+dx)))(-7i+3\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{15d}$$

```
[In] Integrate[Sec[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] (-2*Sec[c + d*x]*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)])*(-7*I + 3*Tan[c +
d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(15*d)
```

Maple [A] (verified)

Time = 7.95 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{2\sqrt{a(1+i\tan(dx+c))}\,(8i\cos(dx+c)+8\sin(dx+c)-i\sec(dx+c)+3\sec(dx+c)\tan(dx+c))}{15d}$	62

```
[In] int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/15/d*(a*(1+I*tan(d*x+c)))^(1/2)*(8*I*cos(d*x+c)+8*sin(d*x+c)-I*sec(d*x+c)
+3*sec(d*x+c)*tan(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.85

$$\int \sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{8\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-5i\,e^{(2i\,dx+2i\,c)}-2i\right)}{15\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -8/15*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-5*I*e^(2*I*d*x + 2*I*c) -
 2*I)/(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\sec^3\left(c+dx\right)dx$$

```
[In] integrate(sec(d*x+c)**3*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*sec(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(57) = 114.

Time = 21.46 (sec) , antiderivative size = 222, normalized size of antiderivative = 3.04

$$\int \sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$

$$= \frac{}{15\left(\cos\left(2\,dx+2\,c\right)^2+\sin\left(2\,dx+2\,c\right)^2+2\,\cos\left(2\,dx+2\,c\right)+1\right)^{\frac{1}{4}}\left(\left(\cos\left(4\,dx+4\,c\right)+2\,\cos\left(2\,dx+2\,c\right)+i\right.\right.}$$

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 8/15*(5*I*sqrt(2)*cos(2*d*x + 2*c) - 5*sqrt(2)*sin(2*d*x + 2*c) + 2*I*sqrt(
2))*sqrt(a)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*((cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + I*sin(4*d*x + 4*c) + 2
*I*sin(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) + (I*cos(4*d*x + 4*c) + 2*I*cos(2*d*x + 2*c) - sin(4*d*x + 4*c) - 2*
sin(2*d*x + 2*c) + I)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)))*d)
```

Giac [F]

$$\int \sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{i\,a\tan\left(dx+c\right)+a}\sec\left(dx+c\right)^3\,dx$$

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*sec(d*x + c)^3, x)
```

Mupad [B] (verification not implemented)

Time = 6.65 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.21

$$\int \sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{8\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,5\mathrm{i}+2\mathrm{i}\right)\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}}{15\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^2}$$

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2)/cos(c + d*x)^3,x)
[Out] (8*exp(- c*1i - d*x*1i)*(exp(c*2i + d*x*2i)*5i + 2i)*(a - (a*(exp(c*2i + d*
x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2))/(15*d*(exp(c*2i + d*x*2
i) + 1)^2)
```

3.289 $\int \sec(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 31

$$\int \sec(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx=\frac{2ia\sec(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 2*I*a*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {3574}

$$\int \sec(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx=\frac{2ia\sec(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[Sec[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((2*I)*a*Sec[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\text{integral}=\frac{2ia\sec(c+dx)}{d\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$\int \sec(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{2(i\cos(c+dx)+\sin(c+dx))\sqrt{a+ia\tan(c+dx)}}{d}$$

```
[In] Integrate[Sec[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] (2*(I*Cos[c + d*x] + Sin[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Maple [A] (verified)

Time = 5.63 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

method	result	size
default	$\frac{2\sqrt{a(1+i\tan(dx+c))}\,(i\cos(dx+c)+\sin(dx+c))}{d}$	37

```
[In] int(sec(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/d*(a*(1+I*tan(d*x+c)))^(1/2)*(I*cos(d*x+c)+sin(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sec(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{2i\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{d}$$

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
[Out] 2*I*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/d
```

Sympy [F]

$$\int \sec(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\sec\left(c+dx\right)dx$$

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*sec(c + d*x), x)
```

Maxima [F]

$$\int \sec(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{i\,a\tan\left(dx+c\right)+a}\sec\left(dx+c\right)\,dx$$

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*sec(d*x + c), x)
```

Giac [F]

$$\int \sec(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{i\,a\tan\left(dx+c\right)+a}\sec\left(dx+c\right)\,dx$$

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*sec(d*x + c), x)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.97

$$\int \sec(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{2\left(\sin\left(c+d\,x\right)+\cos\left(c+d\,x\right)\,1\mathrm{i}\right)\sqrt{\frac{a\left(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i}\right)}{\cos(2\,c+2\,d\,x)+1}}}{d}$$

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2)/cos(c + d*x),x)
[Out] (2*(cos(c + d*x)*1i + sin(c + d*x))*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x
)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))/d
```

3.290 $\int \cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 83

$$\int \cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{i\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d} - \frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

```
[Out] 1/2*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*a^(1
/2)/d*2^(1/2)-I*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3571, 3570, 212}

$$\int \cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{i\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d} - \frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

```
[In] Int[Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (I*Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x
]])])/(Sqrt[2]*d) - (I*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3571

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}+\frac{1}{2}a\int\frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx\\
&= -\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}+\frac{(ia)\text{Subst}\left(\int\frac{1}{2-ax^2}\,dx,x,\frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}\\
&= \frac{i\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}d}-\frac{i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\int\cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{ie^{-i(c+dx)}\left(1+e^{2i(c+dx)}-\sqrt{1+e^{2i(c+dx)}}\,\text{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)\sqrt{a+ia\tan(c+dx)}}{2d}$$

```
[In] Integrate[Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] ((-1/2*I)*(1 + E^((2*I)*(c + d*x)) - Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[
Sqrt[1 + E^((2*I)*(c + d*x))]])*Sqrt[a + I*a*Tan[c + d*x]])/(d*E^(I*(c + d*
x)))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 363 vs. 2(68) = 136.

Time = 17.33 (sec) , antiderivative size = 364, normalized size of antiderivative = 4.39

method	result
default	$-\frac{i\sqrt{a(1+i\tan(dx+c))}\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\sin(dx+c)+\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\arctan\left(\sqrt{-\frac{\cos(}{\cos(d}}\right.\right.}{}$

```
[In] int(cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I/d*(a*(1+I*tan(d*x+c)))^(1/2)*((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arc
tanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+
c)+(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(
1/2))*cos(d*x+c)+I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(c
os(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)+(-cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-I*(-cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x
+c)+I*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))
*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+2*cos(d*x+c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(64) = 128.

Time = 0.25 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.22

$$\int \cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$
$$=\frac{\sqrt{2}d\sqrt{-\frac{a}{d^2}}\log\left(\frac{2\left(\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{-\frac{a}{d^2}}+i\,a\right)e^{(-i\,dx-i\,c)}}{d}\right)-\sqrt{2}d\sqrt{-\frac{a}{d^2}}\log\left(-\frac{2\left(\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{-\cdots}\right.}{}\right.}{4\,d}$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*d*sqrt(-a/d^2)*log(2*((d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2
*I*d*x + 2*I*c) + 1))*sqrt(-a/d^2) + I*a)*e^(-I*d*x - I*c)/d) - sqrt(2)*d*s
qrt(-a/d^2)*log(-2*((d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c)
 + 1))*sqrt(-a/d^2) - I*a)*e^(-I*d*x - I*c)/d) - 2*sqrt(2)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*(I*e^(2*I*d*x + 2*I*c) + I))/d
```

Sympy [F]

$$\int \cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\cos\left(c+dx\right)dx$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))*cos(c + d*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 774 vs. $2(64) = 128$.

Time = 0.41 (sec) , antiderivative size = 774, normalized size of antiderivative = 9.33

$$\int \cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] -1/8*(4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*(I*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) -
 sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a)
+ (2*sqrt(2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
, (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - 2*sqrt(2)*a
rctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) - I*sqrt(2)*log(sqrt(cos(2*
d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)) + 1) + I*sqrt(2)*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 - 2*(co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1))*sqrt(a))/d
```

Giac [F]

$$\int \cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{i\,a\tan\left(dx+c\right)+a}\cos\left(dx+c\right)\,dx$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*cos(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \cos\left(c+d\,x\right)\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

```
[In] int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.291 $\int \cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 154

$$\int \cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{5i\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{8\sqrt{2}d} + \frac{5ia\cos(c+dx)}{12d\sqrt{a+ia\tan(c+dx)}} - \frac{5i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{8d} - \frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

```
[Out] 5/16*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*a^(
1/2)/d*2^(1/2)+5/12*I*a*cos(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)-5/8*I*cos(d*x
+c)*(a+I*a*tan(d*x+c))^(1/2)/d-1/3*I*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2)/
d
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used

= {3578, 3583, 3571, 3570, 212}

$$\int \cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{5i\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{8\sqrt{2}d} - \frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d} - \frac{5i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{8d} + \frac{5ia\cos(c+dx)}{12d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((5*I)/8)*Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan
[c + d*x]])])/(Sqrt[2]*d) + (((5*I)/12)*a*Cos[c + d*x])/(d*Sqrt[a + I*a*Tan
[c + d*x]]) - (((5*I)/8)*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d - ((I/3
)*Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3571

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}+\frac{1}{6}(5a)\int\frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx\\
&= \frac{5ia\cos(c+dx)}{12d\sqrt{a+ia\tan(c+dx)}}-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\\
&\quad+\frac{5}{8}\int\cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx\\
&= \frac{5ia\cos(c+dx)}{12d\sqrt{a+ia\tan(c+dx)}}-\frac{5i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{8d}\\
&\quad-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}+\frac{1}{16}(5a)\int\frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx\\
&= \frac{5ia\cos(c+dx)}{12d\sqrt{a+ia\tan(c+dx)}}-\frac{5i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{8d}\\
&\quad-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}+\frac{(5ia)\text{Subst}\left(\int\frac{1}{2-ax^2}\,dx,x,\frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\right)}{8d}\\
&= \frac{5i\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{8\sqrt{2}d}+\frac{5ia\cos(c+dx)}{12d\sqrt{a+ia\tan(c+dx)}}\\
&\quad-\frac{5i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{8d}-\frac{i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.82

$$\int \cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx =$$

$$-\frac{ie^{-3i(c+dx)}\left(-3+11e^{2i(c+dx)}+16e^{4i(c+dx)}+2e^{6i(c+dx)}-15e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx}}\right.\right.}{48d}$$

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-1/48*I)*(-3 + 11*E^((2*I)*(c + d*x)) + 16*E^((4*I)*(c + d*x)) + 2*E^((6*
I)*(c + d*x)) - 15*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTan
h[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sqrt[a + I*a*Tan[c + d*x]])/(d*E^((3*I)*(
c + d*x)))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 393 vs. 2(123) = 246.

Time = 23.32 (sec) , antiderivative size = 394, normalized size of antiderivative = 2.56

method	result
default	$-\frac{i\sqrt{a(1+i\tan(dx+c))}\left(15i\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\cos(dx+c)-15i\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(\sqrt{-\right.\right.}{}$

```
[In] int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*I/d*(a*(1+I*tan(d*x+c)))^(1/2)*(15*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*co
s(d*x+c)-15*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d
*x+c)+1))^(1/2))*sin(d*x+c)+20*I*cos(d*x+c)^2*sin(d*x+c)+15*I*(-cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d
*x+c)+1))^(1/2))+15*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(
cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+15*(-cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+
c)-4*cos(d*x+c)^3+15*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)
/(cos(d*x+c)+1))^(1/2))+30*cos(d*x+c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(115) = 230.

Time = 0.28 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.59

$$\int \cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{\left(15\sqrt{\frac{1}{2}}d\sqrt{-\frac{a}{d^2}}e^{(2i\,dx+2i\,c)}\log\left(\frac{5\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{-\frac{a}{d^2}}+i\,a\right)e^{(-i\,dx-i\,c)}}{4\,d}\right)-15\sqrt{\frac{1}{2}}d\sqrt{-\frac{a}{d^2}}e^{(2i\,a}}{}$$

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/48*(15*sqrt(1/2)*d*sqrt(-a/d^2)*e^(2*I*d*x + 2*I*c)*log(5/4*(sqrt(2)*sqrt
(1/2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-a
/d^2) + I*a)*e^(-I*d*x - I*c)/d) - 15*sqrt(1/2)*d*sqrt(-a/d^2)*e^(2*I*d*x +
 2*I*c)*log(-5/4*(sqrt(2)*sqrt(1/2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(
2*I*d*x + 2*I*c) + 1))*sqrt(-a/d^2) - I*a)*e^(-I*d*x - I*c)/d) + sqrt(2)*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-2*I*e^(6*I*d*x + 6*I*c) - 16*I*e^(4*I*d*x
 + 4*I*c) - 11*I*e^(2*I*d*x + 2*I*c) + 3*I))*e^(-2*I*d*x - 2*I*c)/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 935 vs. 2(115) = 230.

Time = 0.48 (sec) , antiderivative size = 935, normalized size of antiderivative = 6.07

$$\int \cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/192*(8*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(3/4)*(I*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
```

```
 - sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a
) + 12*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*((-I*sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d*x + 2*c) + 4*I*sqrt(2)
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (sqrt(2)*cos(2
*d*x + 2*c) - I*sqrt(2)*sin(2*d*x + 2*c) - 4*sqrt(2))*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 15*(2*sqrt(2)*arctan2((cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1)) + 1) - 2*sqrt(2)*arctan2((cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1)) - 1) - I*sqrt(2)*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 +
 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + I*sqrt(2)*
log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x
 + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) + 1))*sqrt(a))/d
```

Giac [F]

$$\int \cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{i\,a\tan\left(dx+c\right)+a}\cos\left(dx+c\right)^3\,dx$$

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*cos(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \cos\left(c+d\,x\right)^3\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

```
[In] int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.292 $\int \cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 223

$$\int \cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{63i\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{128\sqrt{2}d} + \frac{21ia\cos(c+dx)}{64d\sqrt{a+ia\tan(c+dx)}} + \frac{9ia\cos^3(c+dx)}{40d\sqrt{a+ia\tan(c+dx)}} - \frac{63i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{128d} - \frac{21i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{80d} - \frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}$$

```
[Out] 63/256*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*a
^(1/2)/d*2^(1/2)+21/64*I*a*cos(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)+9/40*I*a*c
os(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(1/2)-63/128*I*cos(d*x+c)*(a+I*a*tan(d*x+c
))^(1/2)/d-21/80*I*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2)/d-1/5*I*cos(d*x+c)
^5*(a+I*a*tan(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3578, 3583, 3571, 3570, 212}

$$\int \cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{63i\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{128\sqrt{2}d} - \frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d} - \frac{21i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{80d} + \frac{9ia\cos^3(c+dx)}{40d\sqrt{a+ia\tan(c+dx)}} - \frac{63i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{128d} + \frac{21ia\cos(c+dx)}{64d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[Cos[c + d*x]^5*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((63*I)/128)*Sqrt[a]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*
Tan[c + d*x]])])/(Sqrt[2]*d) + (((21*I)/64)*a*Cos[c + d*x])/(d*Sqrt[a + I*a
*Tan[c + d*x]]) + (((9*I)/40)*a*Cos[c + d*x]^3)/(d*Sqrt[a + I*a*Tan[c + d*x
]]) - (((63*I)/128)*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d - (((21*I)/8
0)*Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d - ((I/5)*Cos[c + d*x]^5*Sqr
t[a + I*a*Tan[c + d*x]])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3571

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
```

```
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}+\frac{1}{10}(9a)\int\frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx\\
&= \frac{9ia\cos^3(c+dx)}{40d\sqrt{a+ia\tan(c+dx)}}-\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}\\
&\quad+\frac{63}{80}\int\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx\\
&= \frac{9ia\cos^3(c+dx)}{40d\sqrt{a+ia\tan(c+dx)}}-\frac{21i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{80d}\\
&\quad-\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}+\frac{1}{32}(21a)\int\frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx\\
&= \frac{21ia\cos(c+dx)}{64d\sqrt{a+ia\tan(c+dx)}}+\frac{9ia\cos^3(c+dx)}{40d\sqrt{a+ia\tan(c+dx)}}-\frac{21i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{80d}\\
&\quad-\frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}+\frac{63}{128}\int\cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx
\end{aligned}$$

$$= \frac{21ia\cos(c+dx)}{64d\sqrt{a+ia\tan(c+dx)}} + \frac{9ia\cos^3(c+dx)}{40d\sqrt{a+ia\tan(c+dx)}} - \frac{63i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{128d} - \frac{21i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{80d} - \frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d} + \frac{1}{256}(63a)\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$$

$$= \frac{21ia\cos(c+dx)}{64d\sqrt{a+ia\tan(c+dx)}} + \frac{9ia\cos^3(c+dx)}{40d\sqrt{a+ia\tan(c+dx)}} - \frac{63i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{128d} - \frac{21i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{80d} - \frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d} + \frac{(63ia)\text{Subst}\left(\int \frac{1}{2-ax^2}\,dx, x, \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\right)}{128d}$$

$$= \frac{63i\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{128\sqrt{2}d} + \frac{21ia\cos(c+dx)}{64d\sqrt{a+ia\tan(c+dx)}} + \frac{9ia\cos^3(c+dx)}{40d\sqrt{a+ia\tan(c+dx)}} - \frac{63i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{128d} - \frac{21i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{80d} - \frac{i\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{5d}$$

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.68

$$\int \cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{ie^{-5i(c+dx)}\left(-10-95e^{2i(c+dx)}+203e^{4i(c+dx)}+344e^{6i(c+dx)}+64e^{8i(c+dx)}+8e^{10i(c+dx)}-315e^{4i(c+dx)}\sqrt{1+\right.}{1280d}$$

```
[In] Integrate[Cos[c + d*x]^5*Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] ((-1/1280*I)*(-10 - 95*E^((2*I)*(c + d*x)) + 203*E^((4*I)*(c + d*x)) + 344*
E^((6*I)*(c + d*x)) + 64*E^((8*I)*(c + d*x)) + 8*E^((10*I)*(c + d*x)) - 315
*E^((4*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I
)*(c + d*x))]])*Sqrt[a + I*a*Tan[c + d*x]])/(d*E^((5*I)*(c + d*x)))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(180) = 360$.

Time = 22.48 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.89

method	result
default	$\frac{i\sqrt{a(1+i\tan(dx+c))}\left(-288i\sin(dx+c)(\cos^4(dx+c))+32(\cos^5(dx+c))+315i\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\arctan\left(\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\right)\sin(dx+c)-3\right.}{}$

```
[In] int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1280*I/d*(a*(1+I*tan(d*x+c)))^(1/2)*(-288*I*sin(d*x+c)*cos(d*x+c)^4+32*co
s(d*x+c)^5+315*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(co
s(d*x+c)+1))^(1/2))*sin(d*x+c)-315*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arc
tanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+
c)-420*I*sin(d*x+c)*cos(d*x+c)^2-315*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a
rctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-315*(-
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))
*cos(d*x+c)-315*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(
d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+84*cos(d*x+c)^3-31
5*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1
/2))-630*cos(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.20

$$\int \cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \frac{\left(315\sqrt{\frac{1}{2}}d\sqrt{-\frac{a}{d^2}}e^{(4i\,dx+4i\,c)}\log\left(\frac{63\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{-\frac{a}{d^2}}+i\,a\right)e^{(-i\,dx-i\,c)}}{64\,d}\right)-315\sqrt{\frac{1}{2}}d\sqrt{-\frac{a}{d^2}}e\right.}{}$$

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/1280*(315*sqrt(1/2)*d*sqrt(-a/d^2)*e^(4*I*d*x + 4*I*c)*log(63/64*(sqrt(2)
*sqrt(1/2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sq
rt(-a/d^2) + I*a)*e^(-I*d*x - I*c)/d) - 315*sqrt(1/2)*d*sqrt(-a/d^2)*e^(4*I
*d*x + 4*I*c)*log(-63/64*(sqrt(2)*sqrt(1/2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(-a/d^2) - I*a)*e^(-I*d*x - I*c)/d) + sq
rt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-8*I*e^(10*I*d*x + 10*I*c) - 64*I*
e^(8*I*d*x + 8*I*c) - 344*I*e^(6*I*d*x + 6*I*c) - 203*I*e^(4*I*d*x + 4*I*c)
 + 95*I*e^(2*I*d*x + 2*I*c) + 10*I))*e^(-4*I*d*x - 4*I*c)/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2220 vs. 2(168) = 336.

Time = 0.62 (sec) , antiderivative size = 2220, normalized size of antiderivative = 9.96

$$\int \cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5120*(20*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((-3*I*sqrt(2)*cos(4*d*x + 4*c) -
 3*sqrt(2)*sin(4*d*x + 4*c) - 8*I*sqrt(2))*cos(3/2*arctan2(sin(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c))) + 1)) + (3*sqrt(2)*cos(4*d*x + 4*c) - 3*I*sqrt(2)*sin(4*d*x
 + 4*c) + 8*sqrt(2))*sin(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))
*sqrt(a) + 4*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(8*(-I*sqrt(2)*cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 - I*sqrt(2)*sin(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c)))^2 - 2*I*sqrt(2)*cos(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c))) - I*sqrt(2))*cos(5/2*arctan2(sin(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))) + 1)) + 5*(5*I*sqrt(2)*cos(4*d*x + 4*c) + 20*I*sqrt(2)*cos(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 5*sqrt(2)*sin(4*d*x + 4*c) + 20
*sqrt(2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 48*I*sqrt(2
))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), co
s(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + 8*(sqrt(2)*cos(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sqrt(2)*sin(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c))) + sqrt(2))*sin(5/2*arctan2(sin(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4
```

*d*x + 4*c))) + 1)) - 5*(5*sqrt(2)*cos(4*d*x + 4*c) + 20*sqrt(2)*cos(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 5*I*sqrt(2)*sin(4*d*x + 4*c) -
 20*I*sqrt(2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 48*sqr
t(2))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))),
 cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) - 315*
(2*sqrt(2)*arctan2((cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2
+ sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*
c), cos(4*d*x + 4*c))) + 1)), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos
(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan
2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + 1) - 2*sqrt(2)*arctan2((cos(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)),
 (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c))) + 1)) - 1) - I*sqrt(2)*log(sqrt(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 +
 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)*cos(1/2*arctan
2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))^2 + sqrt(cos(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)*si
n(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))^2 + 2*(cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
)) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + 1) +
I*sqrt(2)*log(sqrt(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 +
 sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)*cos(1/2*arctan2(sin(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c))) + 1))^2 + sqrt(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos
(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)*sin(1/2*arctan2(sin(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c))) + 1))^2 - 2*(cos(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))

```
^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(
1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + 1))*sqrt(a))/d
```

Giac [F]

$$\int \cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{i\,a\tan\left(dx+c\right)+a}\cos\left(dx+c\right)^5\,dx$$

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*cos(d*x + c)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx = \int \cos\left(c+d\,x\right)^5\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

```
[In] int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.293 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 117

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{16i(a+ia\tan(c+dx))^{11/2}}{11a^4d} + \frac{24i(a+ia\tan(c+dx))^{13/2}}{13a^5d} - \frac{4i(a+ia\tan(c+dx))^{15/2}}{5a^6d} + \frac{2i(a+ia\tan(c+dx))^{17/2}}{17a^7d}$$

```
[Out] -16/11*I*(a+I*a*tan(d*x+c))^(11/2)/a^4/d+24/13*I*(a+I*a*tan(d*x+c))^(13/2)/
a^5/d-4/5*I*(a+I*a*tan(d*x+c))^(15/2)/a^6/d+2/17*I*(a+I*a*tan(d*x+c))^(17/2
)/a^7/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3568, 45}

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{2i(a+ia\tan(c+dx))^{17/2}}{17a^7d} - \frac{4i(a+ia\tan(c+dx))^{15/2}}{5a^6d} + \frac{24i(a+ia\tan(c+dx))^{13/2}}{13a^5d} - \frac{16i(a+ia\tan(c+dx))^{11/2}}{11a^4d}$$

```
[In] Int[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((-16*I)/11)*(a + I*a*Tan[c + d*x])^(11/2))/(a^4*d) + (((24*I)/13)*(a + I*
a*Tan[c + d*x])^(13/2))/(a^5*d) - (((4*I)/5)*(a + I*a*Tan[c + d*x])^(15/2))
/(a^6*d) + (((2*I)/17)*(a + I*a*Tan[c + d*x])^(17/2))/(a^7*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{i\text{Subst}\left(\int (a-x)^3(a+x)^{9/2}\,dx, x, ia\tan(c+dx)\right)}{a^7d}$$

$$= -\frac{i\text{Subst}\left(\int \left(8a^3(a+x)^{9/2} - 12a^2(a+x)^{11/2} + 6a(a+x)^{13/2} - (a+x)^{15/2}\right)\,dx, x, ia\tan(c+dx)}{a^7d}$$

$$= -\frac{16i(a+ia\tan(c+dx))^{11/2}}{11a^4d} + \frac{24i(a+ia\tan(c+dx))^{13/2}}{13a^5d} - \frac{4i(a+ia\tan(c+dx))^{15/2}}{5a^6d} + \frac{2i(a+ia\tan(c+dx))^{17/2}}{17a^7d}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.62

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{2a(1+i\tan(c+dx))^5\sqrt{a+ia\tan(c+dx)}(-1767i - 3641\tan(c+dx) + 2717i\tan^2(c+dx)}{12155d}$$

```
[In] Integrate[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^(3/2),x]
[Out] (2*a*(1 + I*Tan[c + d*x])^5*Sqrt[a + I*a*Tan[c + d*x]]*(-1767*I - 3641*Tan[
c + d*x] + (2717*I)*Tan[c + d*x]^2 + 715*Tan[c + d*x]^3))/(12155*d)
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{17}{2}}}{17}-\frac{2a(a+ia\tan(dx+c))^{\frac{15}{2}}}{5}+\frac{12a^2(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}-\frac{8a^3(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}\right)}{d\,a^7}$	82
default	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{17}{2}}}{17}-\frac{2a(a+ia\tan(dx+c))^{\frac{15}{2}}}{5}+\frac{12a^2(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}-\frac{8a^3(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}\right)}{d\,a^7}$	82

```
[In] int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/d/a^7*(1/17*(a+I*a*tan(d*x+c))^(17/2)-2/5*a*(a+I*a*tan(d*x+c))^(15/2)+1
2/13*a^2*(a+I*a*tan(d*x+c))^(13/2)-8/11*a^3*(a+I*a*tan(d*x+c))^(11/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.45

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{512\sqrt{2}\left(16i\,ae^{(17i\,dx+17i\,c)}+136i\,ae^{(15i\,dx+15i\,c)}+510i\,ae^{(13i\,dx+13i\,c)}+1105i\,ae^{(11i\,dx+11i\,c)}\right.}{12155\left(de^{(16i\,dx+16i\,c)}+8\,de^{(14i\,dx+14i\,c)}+28\,de^{(12i\,dx+12i\,c)}+56\,de^{(10i\,dx+10i\,c)}+70\,de^{(8i\,dx+8i\,c)}+56\,de^{(6i\,dx+6i\,c}\right.}$$

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -512/12155*sqrt(2)*(16*I*a*e^(17*I*d*x + 17*I*c) + 136*I*a*e^(15*I*d*x + 15
*I*c) + 510*I*a*e^(13*I*d*x + 13*I*c) + 1105*I*a*e^(11*I*d*x + 11*I*c))*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(16*I*d*x + 16*I*c) + 8*d*e^(14*I*d*x +
 14*I*c) + 28*d*e^(12*I*d*x + 12*I*c) + 56*d*e^(10*I*d*x + 10*I*c) + 70*d*e
^(8*I*d*x + 8*I*c) + 56*d*e^(6*I*d*x + 6*I*c) + 28*d*e^(4*I*d*x + 4*I*c) +
8*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.65

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{2i\left(715\,(i\,a\tan(dx+c)+a)^{\frac{17}{2}} - 4862\,(i\,a\tan(dx+c)+a)^{\frac{15}{2}}a + 11220\,(i\,a\tan(dx+c)+a)\right.}{12155\,a^7d}$$

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
[Out] 2/12155*I*(715*(I*a*tan(d*x + c) + a)^(17/2) - 4862*(I*a*tan(d*x + c) + a)^
(15/2)*a + 11220*(I*a*tan(d*x + c) + a)^(13/2)*a^2 - 8840*(I*a*tan(d*x + c)
 + a)^(11/2)*a^3)/(a^7*d)
```

Giac [F]

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^8\,dx$$

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*sec(d*x + c)^8, x)
```

Mupad [B] (verification not implemented)

Time = 17.49 (sec) , antiderivative size = 544, normalized size of antiderivative = 4.65

$$\begin{aligned}\int \sec^8(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx &= -\frac{a\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,8192i}{12155\,d} \\ &- \frac{a\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,4096i}{12155\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)} - \frac{a\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,3072i}{12155\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^2} \\ &- \frac{a\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,512i}{2431\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^3} + \frac{a\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,155136i}{2431\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^4} \\ &- \frac{a\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,2413568i}{12155\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^5} + \frac{a\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,270336i}{1105\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^6} \\ &- \frac{a\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,11776i}{85\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^7} + \frac{a\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,512i}{17\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^8}\end{aligned}$$

```
[In] int((a + a*tan(c + d*x)*1i)^(3/2)/cos(c + d*x)^8,x)

[Out] (a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)
*155136i)/(2431*d*(exp(c*2i + d*x*2i) + 1)^4) - (a*(a - (a*(exp(c*2i + d*x*
2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*4096i)/(12155*d*(exp(c*2i
+ d*x*2i) + 1)) - (a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d
*x*2i) + 1))^(1/2)*3072i)/(12155*d*(exp(c*2i + d*x*2i) + 1)^2) - (a*(a - (a
*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*512i)/(24
31*d*(exp(c*2i + d*x*2i) + 1)^3) - (a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*
1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*8192i)/(12155*d) - (a*(a - (a*(exp(c*2i
 + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*2413568i)/(12155*d*
(exp(c*2i + d*x*2i) + 1)^5) + (a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(
exp(c*2i + d*x*2i) + 1))^(1/2)*270336i)/(1105*d*(exp(c*2i + d*x*2i) + 1)^6)
 - (a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1
/2)*11776i)/(85*d*(exp(c*2i + d*x*2i) + 1)^7) + (a*(a - (a*(exp(c*2i + d*x*
2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*512i)/(17*d*(exp(c*2i + d*
x*2i) + 1)^8)
```

3.294 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{8i(a+ia\tan(c+dx))^{9/2}}{9a^3d} + \frac{8i(a+ia\tan(c+dx))^{11/2}}{11a^4d} - \frac{2i(a+ia\tan(c+dx))^{13/2}}{13a^5d}$$

```
[Out] -8/9*I*(a+I*a*tan(d*x+c))^(9/2)/a^3/d+8/11*I*(a+I*a*tan(d*x+c))^(11/2)/a^4/
d-2/13*I*(a+I*a*tan(d*x+c))^(13/2)/a^5/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3568, 45}

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{2i(a+ia\tan(c+dx))^{13/2}}{13a^5d} + \frac{8i(a+ia\tan(c+dx))^{11/2}}{11a^4d} - \frac{8i(a+ia\tan(c+dx))^{9/2}}{9a^3d}$$

```
[In] Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((-8*I)/9)*(a + I*a*Tan[c + d*x])^(9/2))/(a^3*d) + (((8*I)/11)*(a + I*a*Ta
n[c + d*x])^(11/2))/(a^4*d) - (((2*I)/13)*(a + I*a*Tan[c + d*x])^(13/2))/(a
^5*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)^2(a+x)^{7/2}\,dx, x, ia\tan(c+dx)\right)}{a^5d}\\
&= -\frac{i\text{Subst}\left(\int \left(4a^2(a+x)^{7/2}-4a(a+x)^{9/2}+(a+x)^{11/2}\right)\,dx, x, ia\tan(c+dx)\right)}{a^5d}\\
&= -\frac{8i(a+ia\tan(c+dx))^{9/2}}{9a^3d}+\frac{8i(a+ia\tan(c+dx))^{11/2}}{11a^4d}-\frac{2i(a+ia\tan(c+dx))^{13/2}}{13a^5d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.69

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx=\frac{2ia(-i+\tan(c+dx))^4\sqrt{a+ia\tan(c+dx)}(-203+270i\tan(c+dx)+99\tan^2(c+dx))}{1287d}$$

```
[In] Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^(3/2),x]
[Out] (((2*I)/1287)*a*(-I + Tan[c + d*x])^4*Sqrt[a + I*a*Tan[c + d*x]]*(-203 + (2
70*I)*Tan[c + d*x] + 99*Tan[c + d*x]^2))/d
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}+\frac{4a(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}-\frac{4a^2(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}\right)}{d\,a^5}$	63
default	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}+\frac{4a(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}-\frac{4a^2(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}\right)}{d\,a^5}$	63

```
[In] int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/d/a^5*(-1/13*(a+I*a*tan(d*x+c))^(13/2)+4/11*a*(a+I*a*tan(d*x+c))^(11/2)
-4/9*a^2*(a+I*a*tan(d*x+c))^(9/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(64) = 128$.

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.52

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx =$$

$$-\frac{128\sqrt{2}\left(8i\,ae^{(13i\,dx+13i\,c)}+52i\,ae^{(11i\,dx+11i\,c)}+143i\,ae^{(9i\,dx+9i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{1287\left(de^{(12i\,dx+12i\,c)}+6\,de^{(10i\,dx+10i\,c)}+15\,de^{(8i\,dx+8i\,c)}+20\,de^{(6i\,dx+6i\,c)}+15\,de^{(4i\,dx+4i\,c)}+6\,de^{(2i\,dx+2i\,c)}+d\right.}$$

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -128/1287*sqrt(2)*(8*I*a*e^(13*I*d*x + 13*I*c) + 52*I*a*e^(11*I*d*x + 11*I*
c) + 143*I*a*e^(9*I*d*x + 9*I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(1
2*I*d*x + 12*I*c) + 6*d*e^(10*I*d*x + 10*I*c) + 15*d*e^(8*I*d*x + 8*I*c) +
20*d*e^(6*I*d*x + 6*I*c) + 15*d*e^(4*I*d*x + 4*I*c) + 6*d*e^(2*I*d*x + 2*I*
c) + d)
```

Sympy [F]

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int (ia(\tan(c+dx)-i))^{\frac{3}{2}}\sec^6(c+dx)\,dx$$

```
[In] integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)*sec(c + d*x)**6, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.66

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx =$$

$$-\frac{2i\left(99\,(i\,a\tan(dx+c)+a)^{\frac{13}{2}}-468\,(i\,a\tan(dx+c)+a)^{\frac{11}{2}}a+572\,(i\,a\tan(dx+c)+a)^{\frac{9}{2}}a^2\right)}{1287\,a^5d}$$

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
[Out] -2/1287*I*(99*(I*a*tan(d*x + c) + a)^(13/2) - 468*(I*a*tan(d*x + c) + a)^(1
1/2)*a + 572*(I*a*tan(d*x + c) + a)^(9/2)*a^2)/(a^5*d)
```

Giac [F]

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int (i\,a\tan{(dx+c)}+a)^{\frac{3}{2}}\sec{(dx+c)}^6\,dx$$

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*sec(d*x + c)^6, x)
```

Mupad [B] (verification not implemented)

Time = 8.31 (sec) , antiderivative size = 420, normalized size of antiderivative = 4.77

$$\begin{aligned}\int \sec^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx &= -\frac{a\sqrt{a-\frac{a\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,1024\mathrm{i}}{1287\,d} \\ &- \frac{a\sqrt{a-\frac{a\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,512\mathrm{i}}{1287\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)} - \frac{a\sqrt{a-\frac{a\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{429\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2} \\ &+ \frac{a\sqrt{a-\frac{a\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,27136\mathrm{i}}{1287\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3} - \frac{a\sqrt{a-\frac{a\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,58624\mathrm{i}}{1287\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4} \\ &+ \frac{a\sqrt{a-\frac{a\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,5120\mathrm{i}}{143\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^5} - \frac{a\sqrt{a-\frac{a\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{13\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^6}\end{aligned}$$

```
[In] int((a + a*tan(c + d*x)*1i)^(3/2)/cos(c + d*x)^6,x)
[Out] (a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)
*27136i)/(1287*d*(exp(c*2i + d*x*2i) + 1)^3) - (a*(a - (a*(exp(c*2i + d*x*2
i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*512i)/(1287*d*(exp(c*2i + d
*x*2i) + 1)) - (a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*
2i) + 1))^(1/2)*128i)/(429*d*(exp(c*2i + d*x*2i) + 1)^2) - (a*(a - (a*(exp(
c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1024i)/(1287*d)
 - (a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1
/2)*58624i)/(1287*d*(exp(c*2i + d*x*2i) + 1)^4) + (a*(a - (a*(exp(c*2i + d*
x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*5120i)/(143*d*(exp(c*2i
+ d*x*2i) + 1)^5) - (a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i +
 d*x*2i) + 1))^(1/2)*128i)/(13*d*(exp(c*2i + d*x*2i) + 1)^6)
```

3.295 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 59

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{4i(a+ia\tan(c+dx))^{7/2}}{7a^2d}+\frac{2i(a+ia\tan(c+dx))^{9/2}}{9a^3d}$$

```
[Out] -4/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^2/d+2/9*I*(a+I*a*tan(d*x+c))^(9/2)/a^3/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3568, 45}

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{2i(a+ia\tan(c+dx))^{9/2}}{9a^3d} - \frac{4i(a+ia\tan(c+dx))^{7/2}}{7a^2d}$$

```
[In] Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((-4*I)/7)*(a + I*a*Tan[c + d*x])^(7/2))/(a^2*d) + (((2*I)/9)*(a + I*a*Tan
[c + d*x])^(9/2))/(a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)(a+x)^{5/2}\,dx, x, ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{i\text{Subst}\left(\int \left(2a(a+x)^{5/2}-(a+x)^{7/2}\right)\,dx, x, ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{4i(a+ia\tan(c+dx))^{7/2}}{7a^2d}+\frac{2i(a+ia\tan(c+dx))^{9/2}}{9a^3d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{2a(-1-i\tan(c+dx))^3(11i+7\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{63d}$$

```
[In] Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^(3/2),x]
[Out] (2*a*(-1 - I*Tan[c + d*x])^3*(11*I + 7*Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d
*x]])/(63*d)
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}-\frac{2a(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}\right)}{d\,a^3}$	44
default	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}-\frac{2a(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}\right)}{d\,a^3}$	44

```
[In] int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2*I/d/a^3*(1/9*(a+I*a*tan(d*x+c))^(9/2)-2/7*a*(a+I*a*tan(d*x+c))^(7/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(43) = 86$.

Time = 0.25 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.66

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{32\sqrt{2}\left(2i\,ae^{(9i\,dx+9i\,c)}+9i\,ae^{(7i\,dx+7i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{63\left(de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}+4\,de^{(2i\,dx+2i\,c)}+d\right)}$$

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -32/63*sqrt(2)*(2*I*a*e^(9*I*d*x + 9*I*c) + 9*I*a*e^(7*I*d*x + 7*I*c))*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I
*c) + 6*d*e^(4*I*d*x + 4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int (ia(\tan(c+dx)-i))^{\frac{3}{2}}\sec^4(c+dx)\,dx$$

```
[In] integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)*sec(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.68

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{2i\left(7\,(i\,a\tan(dx+c)+a)^{\frac{9}{2}}-18\,(i\,a\tan(dx+c)+a)^{\frac{7}{2}}a\right)}{63\,a^3d}$$

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/63*I*(7*(I*a*tan(d*x + c) + a)^(9/2) - 18*(I*a*tan(d*x + c) + a)^(7/2)*a)
/(a^3*d)
```

Giac [F]

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int (i\,a\tan\left(dx+c\right)+a)^{\frac{3}{2}}\sec\left(dx+c\right)^4\,dx$$

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*sec(d*x + c)^4, x)
```

Mupad [B] (verification not implemented)

Time = 7.04 (sec) , antiderivative size = 296, normalized size of antiderivative = 5.02

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{a\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{63\,d} - \frac{a\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{63\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)} + \frac{a\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,160\mathrm{i}}{21\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^2} - \frac{a\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,608\mathrm{i}}{63\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^3} + \frac{a\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{9\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^4}$$

```
[In] int((a + a*tan(c + d*x)*1i)^(3/2)/cos(c + d*x)^4,x)
[Out] (a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)
*160i)/(21*d*(exp(c*2i + d*x*2i) + 1)^2) - (a*(a - (a*(exp(c*2i + d*x*2i)*1
i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*32i)/(63*d*(exp(c*2i + d*x*2i)
+ 1)) - (a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1
))^(1/2)*64i)/(63*d) - (a*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2
i + d*x*2i) + 1))^(1/2)*608i)/(63*d*(exp(c*2i + d*x*2i) + 1)^3) + (a*(a - (
a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*32i)/(9*
d*(exp(c*2i + d*x*2i) + 1)^4)
```

3.296 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 29

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{2i(a+ia\tan(c+dx))^{5/2}}{5ad}$$

```
[Out] -2/5*I*(a+I*a*tan(d*x+c))^(5/2)/a/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3568, 32}

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{2i(a+ia\tan(c+dx))^{5/2}}{5ad}$$

```
[In] Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((-2*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(a*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{i\text{Subst}\left(\int (a+x)^{3/2}\,dx, x, ia\tan(c+dx)\right)}{ad}$$
$$= -\frac{2i(a+ia\tan(c+dx))^{5/2}}{5ad}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{2i(a+ia\tan(c+dx))^{5/2}}{5ad}$$

```
[In] Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^(3/2),x]
[Out] (((-2*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(a*d)
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{2i(a+ia\tan(dx+c))^{\frac{5}{2}}}{5ad}$	24
default	$-\frac{2i(a+ia\tan(dx+c))^{\frac{5}{2}}}{5ad}$	24

```
[In] int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/5*I*(a+I*a*tan(d*x+c))^(5/2)/a/d
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(21) = 42.

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.03

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{8i\sqrt{2}a\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(5i\,dx+5i\,c)}}{5\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
[Out] -8/5*I*sqrt(2)*a*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(5*I*d*x + 5*I*c)/(d*e
^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int (ia(\tan(c+dx)-i))^{\frac{3}{2}}\sec^2(c+dx)\,dx$$

```
[In] integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**(3/2),x)
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)*sec(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{2i\,(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}{5\,ad}$$

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
[Out] -2/5*I*(I*a*tan(d*x + c) + a)^(5/2)/(a*d)
```

Giac [F]

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^2\,dx$$

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*sec(d*x + c)^2, x)
```

Mupad [B] (verification not implemented)

Time = 1.52 (sec) , antiderivative size = 153, normalized size of antiderivative = 5.28

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{4\,a\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,(\cos(2\,c+2\,d\,x)\,7\mathrm{i}+\cos(4\,c+4\,d\,x)\,4\mathrm{i}+\cos(6\,c+6\,d\,x)\,1\mathrm{i}-5\,\sin(2\,c+2\,d\,x)-4\,\sin(4\,c+4\,d\,x)-\sin(6\,c+6\,d\,x)+4\mathrm{i})}{5\,d\,(15\,\cos(2\,c+2\,d\,x)+6\,\cos(4\,c+4\,d\,x)+\cos(6\,c+6\,d\,x)+10)}$$

```
[In] int((a + a*tan(c + d*x)*1i)^(3/2)/cos(c + d*x)^2,x)
[Out] -(4*a*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) +
 1))^(1/2)*(cos(2*c + 2*d*x)*7i + cos(4*c + 4*d*x)*4i + cos(6*c + 6*d*x)*1i
 - 5*sin(2*c + 2*d*x) - 4*sin(4*c + 4*d*x) - sin(6*c + 6*d*x) + 4i))/(5*d*(
15*cos(2*c + 2*d*x) + 6*cos(4*c + 4*d*x) + cos(6*c + 6*d*x) + 10))
```

3.297 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 93

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{ia^{3/2}\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}d} - \frac{ia^2\sqrt{a+ia\tan(c+dx)}}{2d(a-ia\tan(c+dx))}$$

```
[Out] -1/4*I*a^(3/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1
/2)-1/2*I*a^2*(a+I*a*tan(d*x+c))^(1/2)/d/(a-I*a*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3568, 44, 65, 212}

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{ia^{3/2}\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}d} - \frac{ia^2\sqrt{a+ia\tan(c+dx)}}{2d(a-ia\tan(c+dx))}$$

```
[In] Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-1/2*I)*a^(3/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(S
qrt[2]*d) - ((I/2)*a^2*Sqrt[a + I*a*Tan[c + d*x]])/(d*(a - I*a*Tan[c + d*x]
))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(ia^3)\,\text{Subst}\left(\int \frac{1}{(a-x)^2\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{d} \\
&= -\frac{ia^2\sqrt{a+ia\tan(c+dx)}}{2d(a-ia\tan(c+dx))} - \frac{(ia^2)\,\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{4d} \\
&= -\frac{ia^2\sqrt{a+ia\tan(c+dx)}}{2d(a-ia\tan(c+dx))} - \frac{(ia^2)\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+ia\tan(c+dx)}\right)}{2d} \\
&= -\frac{ia^{3/2}\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2\sqrt{2}d} - \frac{ia^2\sqrt{a+ia\tan(c+dx)}}{2d(a-ia\tan(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.99

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)(1-i\tan(c+dx))+2a\sqrt{a+ia\tan(c+dx)}}{4d(i+\tan(c+dx))}$$

```
[In] Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[2]*a^(3/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])]*(1 -
 I*Tan[c + d*x]) + 2*a*Sqrt[a + I*a*Tan[c + d*x]])/(4*d*(I + Tan[c + d*x]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 575 vs. 2(73) = 146.

Time = 15.41 (sec) , antiderivative size = 576, normalized size of antiderivative = 6.19

method	result
default	$-\frac{(-\tan(dx+c)+i)\sqrt{a(1+i\tan(dx+c))}\,a\cos(dx+c)\left(i\,\text{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\cos(dx+c)\sin(dx+\cdots}{\cdots}$

```
[In] int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(-tan(d*x+c)+I)*(a*(1+I*tan(d*x+c)))^(1/2)*a*cos(d*x+c)*(I*arctanh(s
in(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)-I*(-cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2+I*arctanh(sin(
d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*sin(d*x+c)-I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-
cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-arctanh(sin(d*x+c)/(cos(d*x+c)
+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
cos(d*x+c)^2-(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2))*cos(d*x+c)*sin(d*x+c)+I*cos(d*x+c)^2-(-cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))
^(1/2))*cos(d*x+c)-(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(
cos(d*x+c)+1))^(1/2))*sin(d*x+c)+I*cos(d*x+c)-sin(d*x+c)*cos(d*x+c))/(I*cos
(d*x+c)+I-sin(d*x+c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(68) = 136$.

Time = 0.24 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.62

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}d\log\left(-\frac{4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(ide^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{-\frac{a^3}{d^2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}-a^2e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{a}\right)-\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}d\log\left(-\frac{4\,(\cdots}{}\right.}{}$$

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(1/2)*sqrt(-a^3/d^2)*d*log(-4*(sqrt(2)*sqrt(1/2)*(I*d*e^(2*I*d*x
+ 2*I*c) + I*d)*sqrt(-a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)) - a^2*e^(I
*d*x + I*c))*e^(-I*d*x - I*c)/a) - sqrt(1/2)*sqrt(-a^3/d^2)*d*log(-4*(sqrt(
2)*sqrt(1/2)*(-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(-a^3/d^2)*sqrt(a/(e^(2*I
*d*x + 2*I*c) + 1)) - a^2*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a) - sqrt(2)*(-
I*a*e^(3*I*d*x + 3*I*c) - I*a*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1)))/d
```

Sympy [F]

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int (ia(\tan(c+dx)-i))^{\frac{3}{2}}\cos^2(c+dx)\,dx$$

```
[In] integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)*cos(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.05

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{i\left(\sqrt{2}a^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+\frac{16\sqrt{i\,a\tan(dx+c)+a}a^3}{4i\,a\tan(dx+c)-4\,a}\right)}{8\,ad}$$

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*I*(sqrt(2)*a^(5/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/
(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 16*sqrt(I*a*tan(d*x + c)
+ a)*a^3/(4*I*a*tan(d*x + c) - 4*a))/(a*d)
```

Giac [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int \cos\left(c+d\,x\right)^2\,\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^{3/2}\,dx$$

```
[In] int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.298 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 166

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{15ia^{3/2}\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}d} + \frac{15ia^2}{32d\sqrt{a+ia\tan(c+dx)}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}} - \frac{5ia^3}{16d(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] -15/64*I*a^(3/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^
(1/2)+15/32*I*a^2/d/(a+I*a*tan(d*x+c))^(1/2)-1/4*I*a^4/d/(a+I*a*tan(d*x+c))
^(1/2)/(a-I*a*tan(d*x+c))^2-5/16*I*a^3/d/(a+I*a*tan(d*x+c))^(1/2)/(a-I*a*ta
n(d*x+c))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3568, 44, 53, 65, 212}

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{15ia^{3/2}\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}d} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}} - \frac{5ia^3}{16d(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}} + \frac{15ia^2}{32d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((-15*I)/32)*a^(3/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])]
)/(Sqrt[2]*d) + (((15*I)/32)*a^2)/(d*Sqrt[a + I*a*Tan[c + d*x]]) - ((I/4)*a
^4)/(d*(a - I*a*Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]]) - (((5*I)/16)*a
^3)/(d*(a - I*a*Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(ia^5)\,\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^{3/2}}\,dx, x, ia\tan(c+dx)\right)}{d} \\
&= -\frac{ia^4}{4d(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}} \\
&\quad -\frac{(5ia^4)\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{3/2}}\,dx, x, ia\tan(c+dx)\right)}{8d} \\
&= -\frac{ia^4}{4d(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}} \\
&\quad -\frac{5ia^3}{16d(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}} \\
&\quad -\frac{(15ia^3)\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{3/2}}\,dx, x, ia\tan(c+dx)\right)}{32d} \\
&= \frac{15ia^2}{32d\sqrt{a+ia\tan(c+dx)}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}} \\
&\quad -\frac{5ia^3}{16d(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}} \\
&\quad -\frac{(15ia^2)\,\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{64d} \\
&= \frac{15ia^2}{32d\sqrt{a+ia\tan(c+dx)}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}} \\
&\quad -\frac{5ia^3}{16d(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}} \\
&\quad -\frac{(15ia^2)\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+ia\tan(c+dx)}\right)}{32d} \\
&= -\frac{15ia^{3/2}\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}d} + \frac{15ia^2}{32d\sqrt{a+ia\tan(c+dx)}} \\
&\quad -\frac{ia^4}{4d(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}} \\
&\quad -\frac{5ia^3}{16d(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.32

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{ia^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},3,\frac{1}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{4d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((I/4)*a^2*Hypergeometric2F1[-1/2, 3, 1/2, (1 + I*Tan[c + d*x])/2])/(d*Sqrt
[a + I*a*Tan[c + d*x]])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 658 vs. 2(133) = 266.

Time = 14.37 (sec) , antiderivative size = 659, normalized size of antiderivative = 3.97

method	result
default	$-\frac{(-\tan(dx+c)+i)\sqrt{a(1+i\tan(dx+c))}\,a\cos(dx+c)\left(30i\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)(\cos^2(dx+c))+30}{}$

```
[In] int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/64/d*(-tan(d*x+c)+I)*(a*(1+I*tan(d*x+c)))^(1/2)*a*cos(d*x+c)*(30*I*(-cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c
)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2+30*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*sin(d*x+c)+15*I*(-
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*
x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)+15*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+40*I*cos(d*x+c)^2
*sin(d*x+c)+30*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1
))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)-30*(-cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*co
s(d*x+c)^2-15*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(
d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+15*(-cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/
2))*sin(d*x+c)-15*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(c
os(d*x+c)+1))^(1/2))*cos(d*x+c)-24*cos(d*x+c)^3+15*(-cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+30*cos(d*x+c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(123) = 246.

Time = 0.26 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.73

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx =$$

$$-\frac{\left(15\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}de^{(i\,dx+i\,c)}\log\left(-\frac{4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{-\frac{a^3}{d^2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}-a^2e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{a}\right)-15\sqrt{\frac{1}{2}}\sqrt{}$$

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/64*(15*sqrt(1/2)*sqrt(-a^3/d^2)*d*e^(I*d*x + I*c)*log(-4*(sqrt(2)*sqrt(1
/2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(-a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1)) - a^2*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a) - 15*sqrt(1/2)*sqrt(-a
^3/d^2)*d*e^(I*d*x + I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(-I*d*e^(2*I*d*x + 2*I*
c) - I*d)*sqrt(-a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)) - a^2*e^(I*d*x +
 I*c))*e^(-I*d*x - I*c)/a) - sqrt(2)*(-2*I*a*e^(6*I*d*x + 6*I*c) - 11*I*a*e
^(4*I*d*x + 4*I*c) - I*a*e^(2*I*d*x + 2*I*c) + 8*I*a)*sqrt(a/(e^(2*I*d*x +
2*I*c) + 1)))*e^(-I*d*x - I*c)/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.95

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{i\left(15\sqrt{2}a^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+\frac{4\left(15\,(i\,a\tan(dx+c)+a)^2a^3-50\,(i\,a\tan(dx+c)+a)a^4+32\,a^5\right)}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}-4\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a+4\sqrt{i\,a\tan(dx+c)+a}a}\right.}{128\,ad}$$

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/128*I*(15*sqrt(2)*a^(5/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) +
 a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(15*(I*a*tan(d*x +
 c) + a)^2*a^3 - 50*(I*a*tan(d*x + c) + a)*a^4 + 32*a^5)/((I*a*tan(d*x + c)
 + a)^(5/2) - 4*(I*a*tan(d*x + c) + a)^(3/2)*a + 4*sqrt(I*a*tan(d*x + c) +
a)*a^2))/(a*d)
```

Giac [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int \cos\left(c+d\,x\right)^4\,{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}\,dx$$

```
[In] int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.299 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 239

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{105ia^{3/2}\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{256\sqrt{2}d} + \frac{35ia^3}{128d(a+ia\tan(c+dx))^{3/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{3/2}} - \frac{3ia^5}{16d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{3/2}} - \frac{21ia^4}{64d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}} + \frac{105ia^2}{256d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] -105/512*I*a^(3/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*
2^(1/2)+105/256*I*a^2/d/(a+I*a*tan(d*x+c))^(1/2)+35/128*I*a^3/d/(a+I*a*tan(
d*x+c))^(3/2)-1/6*I*a^6/d/(a-I*a*tan(d*x+c))^3/(a+I*a*tan(d*x+c))^(3/2)-3/1
6*I*a^5/d/(a-I*a*tan(d*x+c))^2/(a+I*a*tan(d*x+c))^(3/2)-21/64*I*a^4/d/(a-I*
a*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used

= {3568, 44, 53, 65, 212}

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{105ia^{3/2}\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{256\sqrt{2}d} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{3/2}} - \frac{3ia^5}{16d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{3/2}} - \frac{21ia^4}{64d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}} + \frac{35ia^3}{128d(a+ia\tan(c+dx))^{3/2}} + \frac{105ia^2}{256d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((-105*I)/256)*a^(3/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a]
)])/(Sqrt[2]*d) + (((35*I)/128)*a^3)/(d*(a + I*a*Tan[c + d*x])^(3/2)) - ((I
/6)*a^6)/(d*(a - I*a*Tan[c + d*x])^3*(a + I*a*Tan[c + d*x])^(3/2)) - (((3*I
)/16)*a^5)/(d*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(3/2)) - (((2
1*I)/64)*a^4)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(3/2)) + (((
105*I)/256)*a^2)/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(ia^7)\,\text{Subst}\left(\int \frac{1}{(a-x)^4(a+x)^{5/2}}\,dx, x, ia\tan(c+dx)\right)}{d} \\
&= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{3/2}} \\
&\quad -\frac{(3ia^6)\,\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^{5/2}}\,dx, x, ia\tan(c+dx)\right)}{4d} \\
&= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{3/2}} \\
&\quad -\frac{3ia^5}{16d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{3/2}} \\
&\quad -\frac{(21ia^5)\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{5/2}}\,dx, x, ia\tan(c+dx)\right)}{32d} \\
&= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{3/2}} \\
&\quad -\frac{3ia^5}{16d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{3/2}} \\
&\quad -\frac{21ia^4}{64d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}} \\
&\quad -\frac{(105ia^4)\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{5/2}}\,dx, x, ia\tan(c+dx)\right)}{128d}
\end{aligned}$$

$$=\frac{35ia^3}{128d(a+ia\tan(c+dx))^{3/2}}-\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{3/2}}-\frac{3ia^5}{16d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{3/2}}-\frac{21ia^4}{64d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}-\frac{(105ia^3)\,\text{Subst}\left(\int\frac{1}{(a-x)(a+x)^{3/2}}\,dx,x,ia\tan(c+dx)\right)}{256d}$$

$$=\frac{35ia^3}{128d(a+ia\tan(c+dx))^{3/2}}-\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{3/2}}-\frac{3ia^5}{16d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{3/2}}-\frac{21ia^4}{64d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}+\frac{105ia^2}{256d\sqrt{a+ia\tan(c+dx)}}-\frac{(105ia^2)\,\text{Subst}\left(\int\frac{1}{(a-x)\sqrt{a+x}}\,dx,x,ia\tan(c+dx)\right)}{512d}$$

$$=\frac{35ia^3}{128d(a+ia\tan(c+dx))^{3/2}}-\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{3/2}}-\frac{3ia^5}{16d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{3/2}}-\frac{21ia^4}{64d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}+\frac{105ia^2}{256d\sqrt{a+ia\tan(c+dx)}}-\frac{(105ia^2)\,\text{Subst}\left(\int\frac{1}{2a-x^2}\,dx,x,\sqrt{a+ia\tan(c+dx)}\right)}{256d}$$

$$=-\frac{105ia^{3/2}\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{256\sqrt{2}d}+\frac{35ia^3}{128d(a+ia\tan(c+dx))^{3/2}}-\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{3/2}}-\frac{3ia^5}{16d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{3/2}}-\frac{21ia^4}{64d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}+\frac{105ia^2}{256d\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.22

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx=\frac{ia^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},4,-\frac{1}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{24d(a+ia\tan(c+dx))^{3/2}}$$

```
[In] Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((I/24)*a^3*Hypergeometric2F1[-3/2, 4, -1/2, (1 + I*Tan[c + d*x])/2])/(d*(a
 + I*a*Tan[c + d*x])^(3/2))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 685 vs. 2(193) = 386.

Time = 14.05 (sec) , antiderivative size = 686, normalized size of antiderivative = 2.87

method	result
default	$-\frac{(-\tan(dx+c)+i)\sqrt{a(1+i\tan(dx+c))}\,a\cos(dx+c)\left(384i\sin(dx+c)(\cos^4(dx+c))+630i\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\text{arctanh}\left(\frac{\sin(d}{(\cos(dx+c)+1)\sqrt{}}\right.\right.}{}$

```
[In] int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/1536/d*(-tan(d*x+c)+I)*(a*(1+I*tan(d*x+c)))^(1/2)*a*cos(d*x+c)*(384*I*co
s(d*x+c)^4*sin(d*x+c)+630*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(
d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2+630*
I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1
/2))*cos(d*x+c)*sin(d*x+c)-128*cos(d*x+c)^5+315*I*(-cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1
/2))*cos(d*x+c)+315*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c
)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+840*I*cos(d*x+c)^2*sin(d*x+c)+630*arcta
nh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)-630*(-cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2-315*I*(-
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*
x+c)/(cos(d*x+c)+1))^(1/2))+315*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(
sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-31
5*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1
```

```
/2))*cos(d*x+c)-504*cos(d*x+c)^3+315*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arc
tan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+630*cos(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.30

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx =$$

$$-\frac{\left(315\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}de^{(3i\,dx+3i\,c)}\log\left(-\frac{4\left(\sqrt{2}\sqrt{\frac{1}{2}}(i\,de^{(2i\,dx+2i\,c)}+i\,d)\sqrt{-\frac{a^3}{d^2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}-a^2e^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c)}}{a}\right)-315\sqrt{\cdots}}{\cdots}$$

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/1536*(315*sqrt(1/2)*sqrt(-a^3/d^2)*d*e^(3*I*d*x + 3*I*c)*log(-4*(sqrt(2)
*sqrt(1/2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(-a^3/d^2)*sqrt(a/(e^(2*I*d*
x + 2*I*c) + 1)) - a^2*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a) - 315*sqrt(1/2)
*sqrt(-a^3/d^2)*d*e^(3*I*d*x + 3*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(-I*d*e^(2*
I*d*x + 2*I*c) - I*d)*sqrt(-a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)) - a^
2*e^(I*d*x + I*c))*e^(-I*d*x - I*c)/a) - sqrt(2)*(-8*I*a*e^(10*I*d*x + 10*I
*c) - 58*I*a*e^(8*I*d*x + 8*I*c) - 215*I*a*e^(6*I*d*x + 6*I*c) + 43*I*a*e^(
4*I*d*x + 4*I*c) + 224*I*a*e^(2*I*d*x + 2*I*c) + 16*I*a)*sqrt(a/(e^(2*I*d*x
 + 2*I*c) + 1)))*e^(-3*I*d*x - 3*I*c)/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.89

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{i\left(315\sqrt{2}a^{\frac{5}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)+\frac{4\left(315(ia\tan(dx+c)+a)^4a^3-1680(ia\tan(dx+c)+a)^3a^4+2772(i\cdots\right.}{(ia\tan(dx+c)+a)^{\frac{9}{2}}-6(ia\tan(dx+c)+a)^{\frac{7}{2}}a+12(ia\,t\cdots}\right.}{3072\,ad}$$

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3072*I*(315*sqrt(2)*a^(5/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c)
 + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(315*(I*a*tan(d*
x + c) + a)^4*a^3 - 1680*(I*a*tan(d*x + c) + a)^3*a^4 + 2772*(I*a*tan(d*x +
 c) + a)^2*a^5 - 1152*(I*a*tan(d*x + c) + a)*a^6 - 256*a^7)/((I*a*tan(d*x +
 c) + a)^(9/2) - 6*(I*a*tan(d*x + c) + a)^(7/2)*a + 12*(I*a*tan(d*x + c) +
a)^(5/2)*a^2 - 8*(I*a*tan(d*x + c) + a)^(3/2)*a^3))/(a*d)
```

Giac [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int \cos(c+d\,x)^6\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{3/2}\,dx$$

```
[In] int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.300 $\int \sec^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 147

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{256ia^4\sec^5(c+dx)}{1155d(a+ia\tan(c+dx))^{5/2}} + \frac{64ia^3\sec^5(c+dx)}{231d(a+ia\tan(c+dx))^{3/2}} + \frac{8ia^2\sec^5(c+dx)}{33d\sqrt{a+ia\tan(c+dx)}} + \frac{2ia\sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{11d}$$

```
[Out] 8/33*I*a^2*sec(d*x+c)^5/d/(a+I*a*tan(d*x+c))^(1/2)+2/11*I*a*sec(d*x+c)^5*(a
+I*a*tan(d*x+c))^(1/2)/d+256/1155*I*a^4*sec(d*x+c)^5/d/(a+I*a*tan(d*x+c))^(
5/2)+64/231*I*a^3*sec(d*x+c)^5/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3575, 3574}

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{256ia^4\sec^5(c+dx)}{1155d(a+ia\tan(c+dx))^{5/2}} + \frac{64ia^3\sec^5(c+dx)}{231d(a+ia\tan(c+dx))^{3/2}} + \frac{8ia^2\sec^5(c+dx)}{33d\sqrt{a+ia\tan(c+dx)}} + \frac{2ia\sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{11d}$$

```
[In] Int[Sec[c + d*x]^5*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((256*I)/1155)*a^4*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((6
4*I)/231)*a^3*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((8*I)/33
)*a^2*Sec[c + d*x]^5)/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((2*I)/11)*a*Sec[c
+ d*x]^5*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2ia\sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{11d} \\
&\quad + \frac{1}{11}(12a)\int \sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx \\
&= \frac{8ia^2\sec^5(c+dx)}{33d\sqrt{a+ia\tan(c+dx)}} + \frac{2ia\sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{11d} \\
&\quad + \frac{1}{33}\left(32a^2\right)\int \frac{\sec^5(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx \\
&= \frac{64ia^3\sec^5(c+dx)}{231d(a+ia\tan(c+dx))^{3/2}} + \frac{8ia^2\sec^5(c+dx)}{33d\sqrt{a+ia\tan(c+dx)}} \\
&\quad + \frac{2ia\sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{11d} + \frac{1}{231}\left(128a^3\right)\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx \\
&= \frac{256ia^4\sec^5(c+dx)}{1155d(a+ia\tan(c+dx))^{5/2}} + \frac{64ia^3\sec^5(c+dx)}{231d(a+ia\tan(c+dx))^{3/2}} \\
&\quad + \frac{8ia^2\sec^5(c+dx)}{33d\sqrt{a+ia\tan(c+dx)}} + \frac{2ia\sec^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{11d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.74

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{2a\sec^4(c+dx)(\cos(dx)-i\sin(dx))(i\cos(3c+2dx)+\sin(3c+2dx))(39+494\cos(2(c+dx))}{1155d}$$

```
[In] Integrate[Sec[c + d*x]^5*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (2*a*Sec[c + d*x]^4*(Cos[d*x] - I*Sin[d*x])*(I*Cos[3*c + 2*d*x] + Sin[3*c +
 2*d*x])*(39 + 494*Cos[2*(c + d*x)] + (215*I)*Sec[c + d*x]*Sin[3*(c + d*x)]
 + (110*I)*Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(1155*d)
```

Maple [A] (verified)

Time = 7.16 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.15

method	result
default	$\frac{2a\sqrt{a(1+i\tan(dx+c))}\,(-72i\sec(dx+c)+105i(\tan^2(dx+c))(\sec^3(dx+c))+1024i\cos(dx+c)(\sin^2(dx+c))+512\sin(dx+c)-384i\cos(dx}{}$

```
[In] int(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/1155/d*a*(a*(1+I*tan(d*x+c)))^(1/2)*(-72*I*sec(d*x+c)+105*I*tan(d*x+c)^2*
sec(d*x+c)^3+1024*I*cos(d*x+c)*sin(d*x+c)^2+512*sin(d*x+c)-384*I*cos(d*x+c)
-35*I*sec(d*x+c)^3+192*sec(d*x+c)*tan(d*x+c)+1024*I*cos(d*x+c)^3+128*I*sin(
d*x+c)*tan(d*x+c)+140*tan(d*x+c)*sec(d*x+c)^3+120*I*tan(d*x+c)^2*sec(d*x+c)
)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.85

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{64\sqrt{2}\left(-231i\,ae^{(6i\,dx+6i\,c)}-198i\,ae^{(4i\,dx+4i\,c)}-88i\,ae^{(2i\,dx+2i\,c)}-16i\,a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{1155\left(de^{(10i\,dx+10i\,c)}+5\,de^{(8i\,dx+8i\,c)}+10\,de^{(6i\,dx+6i\,c)}+10\,de^{(4i\,dx+4i\,c)}+5\,de^{(2i\,dx+2i\,c)}+d\right)}$$

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -64/1155*sqrt(2)*(-231*I*a*e^(6*I*d*x + 6*I*c) - 198*I*a*e^(4*I*d*x + 4*I*c
) - 88*I*a*e^(2*I*d*x + 2*I*c) - 16*I*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/
```

```
(d*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*I*d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*
c) + 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int (ia(\tan{(c+dx)}-i))^{\frac{3}{2}}\sec^5{(c+dx)}\,dx$$

```
[In] integrate(sec(d*x+c)**5*(a+I*a*tan(d*x+c))**(3/2),x)
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)*sec(c + d*x)**5, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 994 vs. $2(115) = 230$.

Time = 9.67 (sec) , antiderivative size = 994, normalized size of antiderivative = 6.76

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
[Out] 64/1155*(231*I*sqrt(2)*a*cos(6*d*x + 6*c) + 198*I*sqrt(2)*a*cos(4*d*x + 4*c
) + 88*I*sqrt(2)*a*cos(2*d*x + 2*c) - 231*sqrt(2)*a*sin(6*d*x + 6*c) - 198*
sqrt(2)*a*sin(4*d*x + 4*c) - 88*sqrt(2)*a*sin(2*d*x + 2*c) + 16*I*sqrt(2)*a
)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*
sqrt(a)/(((4*cos(2*d*x + 2*c)^3 + (4*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c)
^2 + 4*I*sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 6*(cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(2*
d*x + 2*c)^2 + (I*cos(2*d*x + 2*c)^2 + I*sin(2*d*x + 2*c)^2 + 2*I*cos(2*d*x
 + 2*c) + I)*sin(8*d*x + 8*c) + 4*(I*cos(2*d*x + 2*c)^2 + I*sin(2*d*x + 2*c
)^2 + 2*I*cos(2*d*x + 2*c) + I)*sin(6*d*x + 6*c) + 6*(I*cos(2*d*x + 2*c)^2
+ I*sin(2*d*x + 2*c)^2 + 2*I*cos(2*d*x + 2*c) + I)*sin(4*d*x + 4*c) + 4*(I*
cos(2*d*x + 2*c)^2 + 2*I*cos(2*d*x + 2*c) + I)*sin(2*d*x + 2*c) + 6*cos(2*d
*x + 2*c) + 1)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (
4*I*cos(2*d*x + 2*c)^3 + (4*I*cos(2*d*x + 2*c) + I)*sin(2*d*x + 2*c)^2 - 4*
sin(2*d*x + 2*c)^3 + (I*cos(2*d*x + 2*c)^2 + I*sin(2*d*x + 2*c)^2 + 2*I*cos
(2*d*x + 2*c) + I)*cos(8*d*x + 8*c) + 4*(I*cos(2*d*x + 2*c)^2 + I*sin(2*d*x
 + 2*c)^2 + 2*I*cos(2*d*x + 2*c) + I)*cos(6*d*x + 6*c) + 6*(I*cos(2*d*x + 2
*c)^2 + I*sin(2*d*x + 2*c)^2 + 2*I*cos(2*d*x + 2*c) + I)*cos(4*d*x + 4*c) +
```

```
 9*I*cos(2*d*x + 2*c)^2 - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)*sin(8*d*x + 8*c) - 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) - 6*(cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) - 4*(cos(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c) + 6*I*cos(2*d*x + 2*c)
 + I)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*d)
```

Giac [F]

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx=\int (i\,a\tan\left(dx+c\right)+a)^{\frac{3}{2}}\sec\left(dx+c\right)^5\,dx$$

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*sec(d*x + c)^5, x)
```

Mupad [B] (verification not implemented)

Time = 8.40 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.99

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx=\frac{a\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{5\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}-\frac{a\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,192\mathrm{i}}{7\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}+\frac{a\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{3\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4}-\frac{a\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{11\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^5}$$

```
[In] int((a + a*tan(c + d*x)*1i)^(3/2)/cos(c + d*x)^5,x)
```

```
[Out] (a*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i
+ d*x*2i) + 1))^(1/2)*64i)/(5*d*(exp(c*2i + d*x*2i) + 1)^2) - (a*exp(- c*1i
 - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1
))^(1/2)*192i)/(7*d*(exp(c*2i + d*x*2i) + 1)^3) + (a*exp(- c*1i - d*x*1i)*(
a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*64i
)/(3*d*(exp(c*2i + d*x*2i) + 1)^4) - (a*exp(- c*1i - d*x*1i)*(a - (a*(exp(c
*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*64i)/(11*d*(exp(
c*2i + d*x*2i) + 1)^5)
```

3.301 $\int \sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 110

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{64ia^3\sec^3(c+dx)}{105d(a+ia\tan(c+dx))^{3/2}} + \frac{16ia^2\sec^3(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}} + \frac{2ia\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}$$

```
[Out] 16/35*I*a^2*sec(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(1/2)+2/7*I*a*sec(d*x+c)^3*(a
+I*a*tan(d*x+c))^(1/2)/d+64/105*I*a^3*sec(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(3/
2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3575, 3574}

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{64ia^3\sec^3(c+dx)}{105d(a+ia\tan(c+dx))^{3/2}} + \frac{16ia^2\sec^3(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}} + \frac{2ia\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}$$

```
[In] Int[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((64*I)/105)*a^3*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((16*
I)/35)*a^2*Sec[c + d*x]^3)/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((2*I)/7)*a*Se
c[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\begin{aligned}
&\text{integral}\\
&= \frac{2ia\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d} + \frac{1}{7}(8a)\int \sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx\\
&= \frac{16ia^2\sec^3(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}} + \frac{2ia\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}\\
&\quad + \frac{1}{35}(32a^2)\int \frac{\sec^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx\\
&= \frac{64ia^3\sec^3(c+dx)}{105d(a+ia\tan(c+dx))^{3/2}} + \frac{16ia^2\sec^3(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}}\\
&\quad + \frac{2ia\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{7d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.83

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{2a\sec^3(c+dx)(\cos(dx)-i\sin(dx))(28+43\cos(2(c+dx))+27i\sin(2(c+dx)))(i\cos(2c+dx}{105d}$$

```
[In] Integrate[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (2*a*Sec[c + d*x]^3*(Cos[d*x] - I*Sin[d*x])*(28 + 43*Cos[2*(c + d*x)] + (27
*I)*Sin[2*(c + d*x)])*(I*Cos[2*c + d*x] + Sin[2*c + d*x])*Sqrt[a + I*a*Tan[
c + d*x]])/(105*d)
```

Maple [A] (verified)

Time = 7.28 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.12

method	result
default	$\frac{2a\sqrt{a(1+i\tan(dx+c))}\,(128i\cos(dx+c)(\sin^2(dx+c))+128i(\cos^3(dx+c))+16i\sin(dx+c)\tan(dx+c)-48i\cos(dx+c)+64\sin(dx+c)+1}{105d}$

```
[In] int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/105/d*a*(a*(1+I*tan(d*x+c)))^(1/2)*(128*I*cos(d*x+c)*sin(d*x+c)^2+128*I*c
os(d*x+c)^3+16*I*sin(d*x+c)*tan(d*x+c)-48*I*cos(d*x+c)+64*sin(d*x+c)+15*I*t
an(d*x+c)^2*sec(d*x+c)-9*I*sec(d*x+c)+24*sec(d*x+c)*tan(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.81

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{16\,\sqrt{2}\left(-35i\,ae^{(4i\,dx+4i\,c)}-28i\,ae^{(2i\,dx+2i\,c)}-8i\,a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{105\,\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)}$$

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -16/105*sqrt(2)*(-35*I*a*e^(4*I*d*x + 4*I*c) - 28*I*a*e^(2*I*d*x + 2*I*c) -
 8*I*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4
*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int (ia(\tan{(c+dx)}-i))^{\frac{3}{2}}\sec^3{(c+dx)}\,dx$$

```
[In] integrate(sec(d*x+c)**3*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)*sec(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 580 vs. 2(86) = 172.

Time = 0.61 (sec) , antiderivative size = 580, normalized size of antiderivative = 5.27

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{}{105\left(\left(2\cos\left(2\,dx+2\,c\right)^3+\left(2\cos\left(2\,dx+2\,c\right)+1\right)\sin\left(2\,dx+2\,c\right)^2+2i\,\sin\left(2\,dx+2\,c\right)^3+\left(\cos\right.\right.\right.}$$

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 16/105*(35*I*sqrt(2)*a*cos(4*d*x + 4*c) + 28*I*sqrt(2)*a*cos(2*d*x + 2*c) -
 35*sqrt(2)*a*sin(4*d*x + 4*c) - 28*sqrt(2)*a*sin(2*d*x + 2*c) + 8*I*sqrt(2
)*a)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/
4)*sqrt(a)/(((2*cos(2*d*x + 2*c)^3 + (2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2
*c)^2 + 2*I*sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 5*cos(2*d*x + 2*c)^2 + (I*cos(2
*d*x + 2*c)^2 + I*sin(2*d*x + 2*c)^2 + 2*I*cos(2*d*x + 2*c) + I)*sin(4*d*x
+ 4*c) + 2*(I*cos(2*d*x + 2*c)^2 + 2*I*cos(2*d*x + 2*c) + I)*sin(2*d*x + 2*
c) + 4*cos(2*d*x + 2*c) + 1)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)) + (2*I*cos(2*d*x + 2*c)^3 + (2*I*cos(2*d*x + 2*c) + I)*sin(2*d*x
 + 2*c)^2 - 2*sin(2*d*x + 2*c)^3 + (I*cos(2*d*x + 2*c)^2 + I*sin(2*d*x + 2*
c)^2 + 2*I*cos(2*d*x + 2*c) + I)*cos(4*d*x + 4*c) + 5*I*cos(2*d*x + 2*c)^2
- (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*
d*x + 4*c) - 2*(cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*
c) + 4*I*cos(2*d*x + 2*c) + I)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1)))*d)
```

Giac [F]

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int \left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^3\,dx$$

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*sec(d*x + c)^3, x)
```

Mupad [B] (verification not implemented)

Time = 6.53 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{16\,a\,e^{-c\,1i-d\,x\,1i}\,\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,\left(e^{c\,2i+d\,x\,2i}\,28i+e^{c\,4i+d\,x\,4i}\,35i+8i\right)}{105\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^3}$$

```
[In] int((a + a*tan(c + d*x)*1i)^(3/2)/cos(c + d*x)^3,x)
```

```
[Out] (16*a*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*
2i + d*x*2i) + 1))^(1/2)*(exp(c*2i + d*x*2i)*28i + exp(c*4i + d*x*4i)*35i +
 8i))/(105*d*(exp(c*2i + d*x*2i) + 1)^3)
```

3.302 $\int \sec(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 69

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx=\frac{8ia^2\sec(c+dx)}{3d\sqrt{a+ia\tan(c+dx)}}+\frac{2ia\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

```
[Out] 8/3*I*a^2*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)+2/3*I*a*sec(d*x+c)*(a+I*a*t
an(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3575, 3574}

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx=\frac{8ia^2\sec(c+dx)}{3d\sqrt{a+ia\tan(c+dx)}}+\frac{2ia\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}$$

```
[In] Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((8*I)/3)*a^2*Sec[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((2*I)/3)*a*
Sec[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
```

```
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2ia\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}+\frac{1}{3}(4a)\int\sec(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx\\ &= \frac{8ia^2\sec(c+dx)}{3d\sqrt{a+ia\tan(c+dx)}}+\frac{2ia\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int\sec(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{2a(\cos(c)-i\sin(c))(\cos(dx)-i\sin(dx))(-5i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{3d}$$

```
[In] Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (-2*a*(Cos[c] - I*Sin[c])*(Cos[d*x] - I*Sin[d*x])*(-5*I + Tan[c + d*x])*Sqr
t[a + I*a*Tan[c + d*x]])/(3*d)
```

Maple [A] (verified)

Time = 6.76 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.16

method	result	size
default	$\frac{2ia\sqrt{a(1+i\tan(dx+c))}\,(8(\sin^2(dx+c))\cos(dx+c)-4i\sin(dx+c)+8(\cos^3(dx+c))+\sin(dx+c)\tan(dx+c)-3\cos(dx+c))}{3d}$	80

```
[In] int(sec(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I/d*a*(a*(1+I*tan(d*x+c)))^(1/2)*(8*sin(d*x+c)^2*cos(d*x+c)-4*I*sin(d*x
+c)+8*cos(d*x+c)^3+sin(d*x+c)*tan(d*x+c)-3*cos(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.77

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{4\sqrt{2}\left(-3i\,ae^{(2i\,dx+2i\,c)}-2i\,a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{3\left(de^{(2i\,dx+2i\,c)}+d\right)}$$

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
[Out] -4/3*sqrt(2)*(-3*I*a*e^(2*I*d*x + 2*I*c) - 2*I*a)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int (ia(\tan{(c+dx)}-i))^{\frac{3}{2}}\sec{(c+dx)}\,dx$$

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**(3/2),x)
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)*sec(c + d*x), x)
```

Maxima [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int (i\,a\tan\left(dx+c\right)+a)^{\frac{3}{2}}\sec\left(dx+c\right)\,dx$$

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

Giac [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int (i\,a\tan\left(dx+c\right)+a)^{\frac{3}{2}}\sec\left(dx+c\right)\,dx$$

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

Mupad [B] (verification not implemented)

Time = 5.45 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.42

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{2\,a\,\sqrt{\frac{a\,\left(2\cos(c+d\,x)^2+\sin(2\,c+2\,d\,x)\,1\mathrm{i}\right)}{2\cos(c+d\,x)^2}}\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,8\mathrm{i}+\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)^2\,2\mathrm{i}+\sin\left(c+d\,x\right)+\sin\left(3\right.\right.}{3\,d\cos\left(c+d\,x\right)^2}$$

```
[In] int((a + a*tan(c + d*x)*1i)^(3/2)/cos(c + d*x),x)
```

```
[Out] (2*a*((a*(sin(2*c + 2*d*x)*1i + 2*cos(c + d*x)^2))/(2*cos(c + d*x)^2))^(1/2
)*(sin(c + d*x) + sin(3*c + 3*d*x) + cos(c/2 + (d*x)/2)^2*8i + cos((3*c)/2
+ (3*d*x)/2)^2*2i - 5i))/(3*d*cos(c + d*x)^2)
```

3.303 $\int \cos(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 31

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{2ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

```
[Out] -2*I*a*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {3574}

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{2ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

```
[In] Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-2*I)*a*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\text{integral} = -\frac{2ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{2ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

```
[In] Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^(3/2),x]
[Out] ((-2*I)*a*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Maple [A] (verified)

Time = 15.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.52

method	result	size
default	$-\frac{2i\sqrt{a(1+i\tan(dx+c))}\,a\left((\sin^2(dx+c))\cos(dx+c)+\cos^3(dx+c)\right)}{d}$	47

```
[In] int(cos(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] -2*I/d*(a*(1+I*tan(d*x+c)))^(1/2)*a*(sin(d*x+c)^2*cos(d*x+c)+cos(d*x+c)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{\sqrt{2}\left(-i\,ae^{(2i\,dx+2i\,c)}-i\,a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{d}$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
[Out] sqrt(2)*(-I*a*e^(2*I*d*x + 2*I*c) - I*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/
d
```

Sympy [F]

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int (ia(\tan(c+dx)-i))^{\frac{3}{2}}\cos(c+dx)\,dx$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**(3/2),x)
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)*cos(c + d*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(25) = 50$.

Time = 0.40 (sec) , antiderivative size = 201, normalized size of antiderivative = 6.48

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{2\left(i\,a^{\frac{3}{2}} - \frac{2i\,a^{\frac{3}{2}}\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{i\,a^{\frac{3}{2}}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)\left(-\frac{2i\sin(dx+c)}{\cos(dx+c)+1} + \frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - 1\right)^{\frac{3}{2}}}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)^{\frac{3}{2}}\left(-\frac{2i\sin(dx+c)}{\cos(dx+c)+1} - \frac{2i\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - 1\right)}$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
[Out] 2*(I*a^(3/2) - 2*I*a^(3/2)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + I*a^(3/2)*
sin(d*x + c)^4/(cos(d*x + c) + 1)^4)*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1)
+ sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(3/2)/(d*(sin(d*x + c)/(cos(d*x
+ c) + 1) + 1)^(3/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(3/2)*(-2*I*sin(
d*x + c)/(cos(d*x + c) + 1) - 2*I*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sin
(d*x + c)^4/(cos(d*x + c) + 1)^4 - 1))
```

Giac [F]

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)\,dx$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*cos(d*x + c), x)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.94

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{a\left(2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)\sqrt{\frac{a\left(2\cos(c+d\,x)^2+\sin(2\,c+2\,d\,x)\,1\mathrm{i}\right)}{2\cos(c+d\,x)^2}}\,2\mathrm{i}}{d}$$

```
[In] int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] -(a*(2*cos(c/2 + (d*x)/2)^2 - 1)*((a*(sin(2*c + 2*d*x)*1i + 2*cos(c + d*x)^
2))/(2*cos(c + d*x)^2))^(1/2)*2i)/d
```

3.304 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 122

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{ia^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{2\sqrt{2}d} - \frac{ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d} - \frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

```
[Out] 1/4*I*a^(3/2)*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/
2))/d*2^(1/2)-1/2*I*a*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d-1/3*I*cos(d*x+c
)^3*(a+I*a*tan(d*x+c))^(3/2)/d
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3571, 3570, 212}

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{ia^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{2\sqrt{2}d} - \frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d} - \frac{ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}$$

```
[In] Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((I/2)*a^(3/2)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c +
 d*x]])])/(Sqrt[2]*d) - ((I/2)*a*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d
 - ((I/3)*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2))/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3571

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}+\frac{1}{2}a\int\cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx\\
&= -\frac{ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}\\
&\quad -\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}+\frac{1}{4}a^2\int\frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx\\
&= -\frac{ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}-\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}\\
&\quad +\frac{(ia^2)\,\text{Subst}\left(\int\frac{1}{2-ax^2}\,dx,x,\frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\right)}{2d}\\
&= \frac{ia^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{2\sqrt{2}d}-\frac{ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{2d}\\
&\quad -\frac{i\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.83

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{iae^{-i(c+dx)}\left(4+5e^{2i(c+dx)}+e^{4i(c+dx)}-3\sqrt{1+e^{2i(c+dx)}}\,\text{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)\sqrt{a+ia\tan(c+dx)}}{12d}$$

```
[In] Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-1/12*I)*a*(4 + 5*E^((2*I)*(c + d*x)) + E^((4*I)*(c + d*x)) - 3*Sqrt[1 +
E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sqrt[a + I*a*T
an[c + d*x]])/(d*E^(I*(c + d*x)))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 647 vs. 2(97) = 194.

Time = 11.93 (sec) , antiderivative size = 648, normalized size of antiderivative = 5.31

method	result
default	$-\frac{i(\tan(dx+c)-i)\sqrt{a(1+i\tan(dx+c))}\,a\cos(dx+c)\left(6i\,\text{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\cos(dx+c)\sin(dx+c}{}$

```
[In] int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*I/d*(tan(d*x+c)-I)*(a*(1+I*tan(d*x+c)))^(1/2)*a*cos(d*x+c)*(6*I*(-cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c
)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*sin(d*x+c)+6*I*(-cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2+3*I*(-co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+
c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+3*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-6*arctanh(sin(d*x+c)
/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*cos(d*x+c)^2+6*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(
d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*sin(d*x+c)-3*I*(-cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+10*I*cos(d*x+c)^
2-3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-
cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)+3*(-cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+3*(-cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(
d*x+c)+1))^(1/2))+6*sin(d*x+c)*cos(d*x+c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(91) = 182.

Time = 0.25 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.82

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{3\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}d\log\left(\frac{\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{-\frac{a^3}{d^2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}+i\,a^2\right)e^{(-i\,dx-i\,c)}}{d}\right)-3\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}d\log\left(\right.$$

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*sqrt(1/2)*sqrt(-a^3/d^2)*d*log((sqrt(2)*sqrt(1/2)*(d*e^(2*I*d*x + 2
*I*c) + d)*sqrt(-a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)) + I*a^2)*e^(-I*
d*x - I*c)/d) - 3*sqrt(1/2)*sqrt(-a^3/d^2)*d*log(-(sqrt(2)*sqrt(1/2)*(d*e^(
2*I*d*x + 2*I*c) + d)*sqrt(-a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)) - I*
a^2)*e^(-I*d*x - I*c)/d) + sqrt(2)*(-I*a*e^(4*I*d*x + 4*I*c) - 5*I*a*e^(2*I
*d*x + 2*I*c) - 4*I*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 884 vs. 2(91) = 182.

Time = 0.83 (sec) , antiderivative size = 884, normalized size of antiderivative = 7.25

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/48*(4*(I*sqrt(2)*a*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)) - sqrt(2)*a*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*(
```

```
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)*sqr
t(a) + 12*(I*sqrt(2)*a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) - sqrt(2)*a*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sq
rt(a) + 3*(2*sqrt(2)*a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - 2
*sqrt(2)*a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)),
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) - I*sqrt(2)*a*l
og(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x
+ 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + 1) + I*sqrt(2)*a*log(sqrt(cos(2*d*x + 2*c)^2 + si
n(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1))^2 - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1))*s
qrt(a))/d
```

Giac [F]

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int (i\,a\tan\left(dx+c\right)+a)^{\frac{3}{2}}\cos\left(dx+c\right)^3\,dx$$

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*cos(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int \cos\left(c+d\,x\right)^3\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{3/2}\,dx$$

```
[In] int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.305 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 192

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{7ia^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{16\sqrt{2}d} + \frac{7ia^2\cos(c+dx)}{24d\sqrt{a+ia\tan(c+dx)}} - \frac{7ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{16d} - \frac{7ia\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{30d} - \frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

```
[Out] 7/32*I*a^(3/2)*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1
/2))/d*2^(1/2)+7/24*I*a^2*cos(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)-7/16*I*a*co
s(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d-7/30*I*a*cos(d*x+c)^3*(a+I*a*tan(d*x+c)
)^(1/2)/d-1/5*I*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2)/d
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3578, 3583, 3571, 3570, 212}

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{7ia^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{16\sqrt{2}d} + \frac{7ia^2\cos(c+dx)}{24d\sqrt{a+ia\tan(c+dx)}} - \frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d} - \frac{7ia\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{30d} - \frac{7ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{16d}$$

```
[In] Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((7*I)/16)*a^(3/2)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Ta
n[c + d*x]])])/(Sqrt[2]*d) + (((7*I)/24)*a^2*Cos[c + d*x])/(d*Sqrt[a + I*a*
Tan[c + d*x]]) - (((7*I)/16)*a*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d -
 (((7*I)/30)*a*Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d - ((I/5)*Cos[c
+ d*x]^5*(a + I*a*Tan[c + d*x])^(3/2))/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3571

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\text{integral} = -\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d} + \frac{1}{10}(7a)\int \cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$

$$= -\frac{7ia\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{30d} - \frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d} + \frac{1}{12}(7a^2)\int \frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$$

$$= \frac{7ia^2\cos(c+dx)}{24d\sqrt{a+ia\tan(c+dx)}} - \frac{7ia\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{30d} - \frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d} + \frac{1}{16}(7a)\int \cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$

$$= \frac{7ia^2\cos(c+dx)}{24d\sqrt{a+ia\tan(c+dx)}} - \frac{7ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{16d} - \frac{7ia\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{30d} - \frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d} + \frac{1}{32}(7a^2)\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$$

$$= \frac{7ia^2\cos(c+dx)}{24d\sqrt{a+ia\tan(c+dx)}} - \frac{7ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{16d} - \frac{7ia\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{30d} - \frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d} + \frac{(7ia^2)\,\text{Subst}\left(\int \frac{1}{2-ax^2}\,dx, x, \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\right)}{16d}$$

$$= \frac{7ia^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{16\sqrt{2}d} + \frac{7ia^2\cos(c+dx)}{24d\sqrt{a+ia\tan(c+dx)}} - \frac{7ia\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{16d} - \frac{7ia\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{30d} - \frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.83

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx =$$
$$-\frac{iae^{-3i(c+dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(-15+101e^{2i(c+dx)}+148e^{4i(c+dx)}+38e^{6i(c+dx)}+6e^{8i(c+dx)}-105e^{2i(c+dx)}\sqrt{1+e^{2i(c}}\right.}{240\sqrt{2}d}$$

```
[In] Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-1/240*I)*a*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(-15
+ 101*E^((2*I)*(c + d*x)) + 148*E^((4*I)*(c + d*x)) + 38*E^((6*I)*(c + d*x)
) + 6*E^((8*I)*(c + d*x)) - 105*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c +
d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/(Sqrt[2]*d*E^((3*I)*(c + d*
x)))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 675 vs. 2(155) = 310.

Time = 15.24 (sec) , antiderivative size = 676, normalized size of antiderivative = 3.52

method	result
default	$-\frac{(-\tan(dx+c)+i)\sqrt{a(1+i\tan(dx+c))}\,a\cos(dx+c)\left(210i\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)(\cos^2(dx+c))-2\right.}{}$

```
[In] int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/480/d*(-tan(d*x+c)+I)*(a*(1+I*tan(d*x+c)))^(1/2)*a*cos(d*x+c)*(210*I*(-c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x
+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2-210*I*(-cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*sin(d*x+c)+168*
I*cos(d*x+c)^3*sin(d*x+c)+105*I*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x
+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-10
5*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^
(1/2))*sin(d*x+c)+210*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d
*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)+2
10*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(
1/2))*cos(d*x+c)^2-72*cos(d*x+c)^4-105*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-210*
I*sin(d*x+c)*cos(d*x+c)+105*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(
d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+105*(-
```

```
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))
*cos(d*x+c)-105*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos
(d*x+c)+1))^(1/2))+350*cos(d*x+c)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.43

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \frac{\left(105\sqrt{\frac{1}{2}}\sqrt{-\frac{a^3}{d^2}}de^{(2i\,dx+2i\,c)}\log\left(\frac{7\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{-\frac{a^3}{d^2}}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}+i\,a^2\right)e^{(-i\,dx-i\,c)}}{8\,d}\right)-105$$

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/480*(105*sqrt(1/2)*sqrt(-a^3/d^2)*d*e^(2*I*d*x + 2*I*c)*log(7/8*(sqrt(2)*
sqrt(1/2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(-a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1)) + I*a^2)*e^(-I*d*x - I*c)/d) - 105*sqrt(1/2)*sqrt(-a^3/d^2)*d*e
^(2*I*d*x + 2*I*c)*log(-7/8*(sqrt(2)*sqrt(1/2)*(d*e^(2*I*d*x + 2*I*c) + d)*
sqrt(-a^3/d^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)) - I*a^2)*e^(-I*d*x - I*c)/
d) + sqrt(2)*(-6*I*a*e^(8*I*d*x + 8*I*c) - 38*I*a*e^(6*I*d*x + 6*I*c) - 148
*I*a*e^(4*I*d*x + 4*I*c) - 101*I*a*e^(2*I*d*x + 2*I*c) + 15*I*a)*sqrt(a/(e^
(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^5\,dx$$

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*cos(d*x + c)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx = \int \cos(c+d\,x)^5\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{3/2}\,dx$$

```
[In] int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.306 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 117

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{16i(a+ia\tan(c+dx))^{13/2}}{13a^4d} + \frac{8i(a+ia\tan(c+dx))^{15/2}}{5a^5d} - \frac{12i(a+ia\tan(c+dx))^{17/2}}{17a^6d} + \frac{2i(a+ia\tan(c+dx))^{19/2}}{19a^7d}$$

```
[Out] -16/13*I*(a+I*a*tan(d*x+c))^(13/2)/a^4/d+8/5*I*(a+I*a*tan(d*x+c))^(15/2)/a^
5/d-12/17*I*(a+I*a*tan(d*x+c))^(17/2)/a^6/d+2/19*I*(a+I*a*tan(d*x+c))^(19/2
)/a^7/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3568, 45}

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{2i(a+ia\tan(c+dx))^{19/2}}{19a^7d} - \frac{12i(a+ia\tan(c+dx))^{17/2}}{17a^6d} + \frac{8i(a+ia\tan(c+dx))^{15/2}}{5a^5d} - \frac{16i(a+ia\tan(c+dx))^{13/2}}{13a^4d}$$

```
[In] Int[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((-16*I)/13)*(a + I*a*Tan[c + d*x])^(13/2))/(a^4*d) + (((8*I)/5)*(a + I*a*
Tan[c + d*x])^(15/2))/(a^5*d) - (((12*I)/17)*(a + I*a*Tan[c + d*x])^(17/2))
/(a^6*d) + (((2*I)/19)*(a + I*a*Tan[c + d*x])^(19/2))/(a^7*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)^3(a+x)^{11/2}\,dx, x, ia\tan(c+dx)\right)}{a^7d}\\ &= -\frac{i\text{Subst}\left(\int \left(8a^3(a+x)^{11/2} - 12a^2(a+x)^{13/2} + 6a(a+x)^{15/2} - (a+x)^{17/2}\right)\,dx, x, ia\tan(c+dx)\right)}{a^7d}\\ &= -\frac{16i(a+ia\tan(c+dx))^{13/2}}{13a^4d} + \frac{8i(a+ia\tan(c+dx))^{15/2}}{5a^5d}\\ &\quad - \frac{12i(a+ia\tan(c+dx))^{17/2}}{17a^6d} + \frac{2i(a+ia\tan(c+dx))^{19/2}}{19a^7d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.62

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{2a^2(-i+\tan(c+dx))^6\sqrt{a+ia\tan(c+dx)}(-2429i - 5291\tan(c+dx) + 4095i\tan^2(c+dx) + 1105\tan^3(c+dx))}{20995d}$$

```
[In] Integrate[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (-2*a^2*(-I + Tan[c + d*x])^6*Sqrt[a + I*a*Tan[c + d*x]]*(-2429*I - 5291*Ta
n[c + d*x] + (4095*I)*Tan[c + d*x]^2 + 1105*Tan[c + d*x]^3))/(20995*d)
```

Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.70

$$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{19}{2}}}{19}-\frac{6a(a+ia\tan(dx+c))^{\frac{17}{2}}}{17}+\frac{4a^2(a+ia\tan(dx+c))^{\frac{15}{2}}}{5}-\frac{8a^3(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}\right)}{d\,a^7}$$

```
[In] int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] 2*I/d/a^7*(1/19*(a+I*a*tan(d*x+c))^(19/2)-6/17*a*(a+I*a*tan(d*x+c))^(17/2)+
4/5*a^2*(a+I*a*tan(d*x+c))^(15/2)-8/13*a^3*(a+I*a*tan(d*x+c))^(13/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(85) = 170.

Time = 0.29 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.62

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx =$$

$$-\frac{1024\,\sqrt{2}\left(16i\,a^2e^{(19i\,dx+19i\,c)}+152i\,a^2e^{(17i\,dx+17i\,c)}+646i\,a^2e^{(15i\,dx+15i\,c)}+161\right.}{20995\,\left(de^{(18i\,dx+18i\,c)}+9\,de^{(16i\,dx+16i\,c)}+36\,de^{(14i\,dx+14i\,c)}+84\,de^{(12i\,dx+12i\,c)}+126\,de^{(10i\,dx+10i\,c)}+126\,de^{(8i}\right.}$$

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1024/20995*sqrt(2)*(16*I*a^2*e^(19*I*d*x + 19*I*c) + 152*I*a^2*e^(17*I*d*x
 + 17*I*c) + 646*I*a^2*e^(15*I*d*x + 15*I*c) + 1615*I*a^2*e^(13*I*d*x + 13*
I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(18*I*d*x + 18*I*c) + 9*d*e^(1
6*I*d*x + 16*I*c) + 36*d*e^(14*I*d*x + 14*I*c) + 84*d*e^(12*I*d*x + 12*I*c)
 + 126*d*e^(10*I*d*x + 10*I*c) + 126*d*e^(8*I*d*x + 8*I*c) + 84*d*e^(6*I*d*
x + 6*I*c) + 36*d*e^(4*I*d*x + 4*I*c) + 9*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.65

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{2i\left(1105\,(i\,a\tan(dx+c)+a)^{\frac{19}{2}} - 7410\,(i\,a\tan(dx+c)+a)^{\frac{17}{2}}a + 16796\,(i\,a\tan(dx+c)+a)\right.}{20995\,a^7d}$$

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
[Out] 2/20995*I*(1105*(I*a*tan(d*x + c) + a)^(19/2) - 7410*(I*a*tan(d*x + c) + a)
^(17/2)*a + 16796*(I*a*tan(d*x + c) + a)^(15/2)*a^2 - 12920*(I*a*tan(d*x +
c) + a)^(13/2)*a^3)/(a^7*d)
```

Giac [F]

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^8\,dx$$

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*sec(d*x + c)^8, x)
```

Mupad [B] (verification not implemented)

Time = 16.99 (sec) , antiderivative size = 626, normalized size of antiderivative = 5.35

$$\begin{aligned}\int \sec^8(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = &-\frac{a^2\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,16384\mathrm{i}}{20995\,d} - \frac{a^2\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,8192\mathrm{i}}{20995\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}\\ &-\frac{a^2\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,6144\mathrm{i}}{20995\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2} - \frac{a^2\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,1024\mathrm{i}}{4199\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}\\ &+\frac{a^2\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,536576\mathrm{i}}{4199\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4} - \frac{a^2\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,10484736\mathrm{i}}{20995\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^5}\\ &+\frac{a^2\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,17262592\mathrm{i}}{20995\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^6} - \frac{a^2\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,1129472\mathrm{i}}{1615\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^7}\\ &+\frac{a^2\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,98304\mathrm{i}}{323\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^8} - \frac{a^2\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,1024\mathrm{i}}{19\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^9}\end{aligned}$$

```
[In] int((a + a*tan(c + d*x)*1i)^(5/2)/cos(c + d*x)^8,x)
```

```
[Out] (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/
2)*536576i)/(4199*d*(exp(c*2i + d*x*2i) + 1)^4) - (a^2*(a - (a*(exp(c*2i +
d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*8192i)/(20995*d*(exp(c
*2i + d*x*2i) + 1)) - (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*
2i + d*x*2i) + 1))^(1/2)*6144i)/(20995*d*(exp(c*2i + d*x*2i) + 1)^2) - (a^2
*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1
024i)/(4199*d*(exp(c*2i + d*x*2i) + 1)^3) - (a^2*(a - (a*(exp(c*2i + d*x*2i
)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*16384i)/(20995*d) - (a^2*(a
- (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*10484
736i)/(20995*d*(exp(c*2i + d*x*2i) + 1)^5) + (a^2*(a - (a*(exp(c*2i + d*x*2
i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*17262592i)/(20995*d*(exp(c*
2i + d*x*2i) + 1)^6) - (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c
*2i + d*x*2i) + 1))^(1/2)*1129472i)/(1615*d*(exp(c*2i + d*x*2i) + 1)^7) + (
a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2
)*98304i)/(323*d*(exp(c*2i + d*x*2i) + 1)^8) - (a^2*(a - (a*(exp(c*2i + d*x
*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1024i)/(19*d*(exp(c*2i +
d*x*2i) + 1)^9)
```

3.307 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx=-\frac{8i(a+ia\tan(c+dx))^{11/2}}{11a^3d}+\frac{8i(a+ia\tan(c+dx))^{13/2}}{13a^4d}-\frac{2i(a+ia\tan(c+dx))^{15/2}}{15a^5d}$$

```
[Out] -8/11*I*(a+I*a*tan(d*x+c))^(11/2)/a^3/d+8/13*I*(a+I*a*tan(d*x+c))^(13/2)/a^
4/d-2/15*I*(a+I*a*tan(d*x+c))^(15/2)/a^5/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3568, 45}

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx=-\frac{2i(a+ia\tan(c+dx))^{15/2}}{15a^5d}+\frac{8i(a+ia\tan(c+dx))^{13/2}}{13a^4d}-\frac{8i(a+ia\tan(c+dx))^{11/2}}{11a^3d}$$

```
[In] Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((-8*I)/11)*(a + I*a*Tan[c + d*x])^(11/2))/(a^3*d) + (((8*I)/13)*(a + I*a*
Tan[c + d*x])^(13/2))/(a^4*d) - (((2*I)/15)*(a + I*a*Tan[c + d*x])^(15/2))/
(a^5*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)^2(a+x)^{9/2}\,dx, x, ia\tan(c+dx)\right)}{a^5 d}\\ &= -\frac{i\text{Subst}\left(\int \left(4a^2(a+x)^{9/2}-4a(a+x)^{11/2}+(a+x)^{13/2}\right)\,dx, x, ia\tan(c+dx)\right)}{a^5 d}\\ &= -\frac{8i(a+ia\tan(c+dx))^{11/2}}{11a^3 d}+\frac{8i(a+ia\tan(c+dx))^{13/2}}{13a^4 d}-\frac{2i(a+ia\tan(c+dx))^{15/2}}{15a^5 d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.69

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{2a^2(-i+\tan(c+dx))^5\sqrt{a+ia\tan(c+dx)}(-263+374i\tan(c+dx)+143\tan^2(c+dx))}{2145d}$$

```
[In] Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^(5/2),x]
[Out] (-2*a^2*(-I + Tan[c + d*x])^5*Sqrt[a + I*a*Tan[c + d*x]]*(-263 + (374*I)*Ta
n[c + d*x] + 143*Tan[c + d*x]^2))/(2145*d)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{15}{2}}}{15}+\frac{4a(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}-\frac{4a^2(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}\right)}{d\,a^5}$$

```
[In] int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x)
[Out] 2*I/d/a^5*(-1/15*(a+I*a*tan(d*x+c))^(15/2)+4/13*a*(a+I*a*tan(d*x+c))^(13/2)
-4/11*a^2*(a+I*a*tan(d*x+c))^(11/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(64) = 128.

Time = 0.25 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.73

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx =$$

$$-\frac{256\,\sqrt{2}\left(8i\,a^2e^{(15i\,dx+15i\,c)}+60i\,a^2e^{(13i\,dx+13i\,c)}+195i\,a^2e^{(11i\,dx+11i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{2145\left(de^{(14i\,dx+14i\,c)}+7\,de^{(12i\,dx+12i\,c)}+21\,de^{(10i\,dx+10i\,c)}+35\,de^{(8i\,dx+8i\,c)}+35\,de^{(6i\,dx+6i\,c)}+21\,de^{(4i\,dx+4i\,c)}+\right.}$$

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -256/2145*sqrt(2)*(8*I*a^2*e^(15*I*d*x + 15*I*c) + 60*I*a^2*e^(13*I*d*x + 1
3*I*c) + 195*I*a^2*e^(11*I*d*x + 11*I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
/(d*e^(14*I*d*x + 14*I*c) + 7*d*e^(12*I*d*x + 12*I*c) + 21*d*e^(10*I*d*x +
10*I*c) + 35*d*e^(8*I*d*x + 8*I*c) + 35*d*e^(6*I*d*x + 6*I*c) + 21*d*e^(4*I
*d*x + 4*I*c) + 7*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.66

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx =$$

$$-\frac{2i\left(143\,(i\,a\tan\left(dx+c\right)+a)^{\frac{15}{2}}-660\,(i\,a\tan\left(dx+c\right)+a)^{\frac{13}{2}}a+780\,(i\,a\tan\left(dx+c\right)+a)^{\frac{11}{2}}a^2\right)}{2145\,a^5d}$$

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/2145*I*(143*(I*a*tan(d*x + c) + a)^(15/2) - 660*(I*a*tan(d*x + c) + a)^(
13/2)*a + 780*(I*a*tan(d*x + c) + a)^(11/2)*a^2)/(a^5*d)
```

Giac [F]

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^6\,dx$$

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*sec(d*x + c)^6, x)
```

Mupad [B] (verification not implemented)

Time = 12.64 (sec) , antiderivative size = 498, normalized size of antiderivative = 5.66

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{a^2\sqrt{a-\frac{a\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,2048\mathrm{i}}{2145\,d} - \frac{a^2\sqrt{a-\frac{a\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,1024\mathrm{i}}{2145\,d\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1)} - \frac{a^2\sqrt{a-\frac{a\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,256\mathrm{i}}{715\,d\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1)^2} + \frac{a^2\sqrt{a-\frac{a\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,18176\mathrm{i}}{429\,d\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1)^3} - \frac{a^2\sqrt{a-\frac{a\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,52736\mathrm{i}}{429\,d\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1)^4} + \frac{a^2\sqrt{a-\frac{a\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,103936\mathrm{i}}{715\,d\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1)^5} - \frac{a^2\sqrt{a-\frac{a\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,15616\mathrm{i}}{195\,d\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1)^6} + \frac{a^2\sqrt{a-\frac{a\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,256\mathrm{i}}{15\,d\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1)^7}$$

```
[In] int((a + a*tan(c + d*x)*1i)^(5/2)/cos(c + d*x)^6,x)
[Out] (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/
2)*18176i)/(429*d*(exp(c*2i + d*x*2i) + 1)^3) - (a^2*(a - (a*(exp(c*2i + d*
x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1024i)/(2145*d*(exp(c*2i
 + d*x*2i) + 1)) - (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i
+ d*x*2i) + 1))^(1/2)*256i)/(715*d*(exp(c*2i + d*x*2i) + 1)^2) - (a^2*(a -
(a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*2048i)/
(2145*d) - (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i
) + 1))^(1/2)*52736i)/(429*d*(exp(c*2i + d*x*2i) + 1)^4) + (a^2*(a - (a*(ex
p(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*103936i)/(715
*d*(exp(c*2i + d*x*2i) + 1)^5) - (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*
1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*15616i)/(195*d*(exp(c*2i + d*x*2i) + 1)
^6) + (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1
))^(1/2)*256i)/(15*d*(exp(c*2i + d*x*2i) + 1)^7)
```

3.308 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 59

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{4i(a+ia\tan(c+dx))^{9/2}}{9a^2d}+\frac{2i(a+ia\tan(c+dx))^{11/2}}{11a^3d}$$

```
[Out] -4/9*I*(a+I*a*tan(d*x+c))^(9/2)/a^2/d+2/11*I*(a+I*a*tan(d*x+c))^(11/2)/a^3/
d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3568, 45}

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{2i(a+ia\tan(c+dx))^{11/2}}{11a^3d} - \frac{4i(a+ia\tan(c+dx))^{9/2}}{9a^2d}$$

```
[In] Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((-4*I)/9)*(a + I*a*Tan[c + d*x])^(9/2))/(a^2*d) + (((2*I)/11)*(a + I*a*Ta
n[c + d*x])^(11/2))/(a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)(a+x)^{7/2}\,dx, x, ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{i\text{Subst}\left(\int \left(2a(a+x)^{7/2}-(a+x)^{9/2}\right)\,dx, x, ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{4i(a+ia\tan(c+dx))^{9/2}}{9a^2d}+\frac{2i(a+ia\tan(c+dx))^{11/2}}{11a^3d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{2a^2(-i+\tan(c+dx))^4(13i+9\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{99d}$$

```
[In] Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^(5/2),x]
[Out] (-2*a^2*(-I + Tan[c + d*x])^4*(13*I + 9*Tan[c + d*x])*Sqrt[a + I*a*Tan[c +
d*x]])/(99*d)
```

Maple [A] (verified)

Time = 185.67 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}-\frac{2a(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}\right)}{d\,a^3}$	44
default	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}-\frac{2a(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}\right)}{d\,a^3}$	44

```
[In] int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] 2*I/d/a^3*(1/11*(a+I*a*tan(d*x+c))^(11/2)-2/9*a*(a+I*a*tan(d*x+c))^(9/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(43) = 86.

Time = 0.25 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.93

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{64\sqrt{2}\left(2i\,a^2e^{(11i\,dx+11i\,c)}+11i\,a^2e^{(9i\,dx+9i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{99\left(de^{(10i\,dx+10i\,c)}+5\,de^{(8i\,dx+8i\,c)}+10\,de^{(6i\,dx+6i\,c)}+10\,de^{(4i\,dx+4i\,c)}+5\,de^{(2i\,dx+2i\,c)}+d\right)}$$

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
[Out] -64/99*sqrt(2)*(2*I*a^2*e^(11*I*d*x + 11*I*c) + 11*I*a^2*e^(9*I*d*x + 9*I*c
))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(10*I*d*x + 10*I*c) + 5*d*e^(8*I*
d*x + 8*I*c) + 10*d*e^(6*I*d*x + 6*I*c) + 10*d*e^(4*I*d*x + 4*I*c) + 5*d*e^
(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**(5/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.68

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{2i\left(9\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{11}{2}}-22\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{9}{2}}a\right)}{99\,a^3d}$$

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
[Out] 2/99*I*(9*(I*a*tan(d*x + c) + a)^(11/2) - 22*(I*a*tan(d*x + c) + a)^(9/2)*a
)/(a^3*d)
```

Giac [F]

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx=\int (i\,a\tan(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^4\,dx$$

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*sec(d*x + c)^4, x)
```

Mupad [B] (verification not implemented)

Time = 7.13 (sec) , antiderivative size = 370, normalized size of antiderivative = 6.27

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx=-\frac{a^2\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,128i}{99\,d}-\frac{a^2\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,64i}{99\,d\left(e^{c\,2i+d\,x\,2i}+1\right)}+\frac{a^2\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,512i}{33\,d\left(e^{c\,2i+d\,x\,2i}+1\right)^2}-\frac{a^2\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,2944i}{99\,d\left(e^{c\,2i+d\,x\,2i}+1\right)^3}+\frac{a^2\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,2176i}{99\,d\left(e^{c\,2i+d\,x\,2i}+1\right)^4}-\frac{a^2\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,64i}{11\,d\left(e^{c\,2i+d\,x\,2i}+1\right)^5}$$

```
[In] int((a + a*tan(c + d*x)*1i)^(5/2)/cos(c + d*x)^4,x)
```

```
[Out] (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/
2)*512i)/(33*d*(exp(c*2i + d*x*2i) + 1)^2) - (a^2*(a - (a*(exp(c*2i + d*x*2
i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*64i)/(99*d*(exp(c*2i + d*x*
2i) + 1)) - (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2
i) + 1))^(1/2)*128i)/(99*d) - (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)
/(exp(c*2i + d*x*2i) + 1))^(1/2)*2944i)/(99*d*(exp(c*2i + d*x*2i) + 1)^3) +
 (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1
/2)*2176i)/(99*d*(exp(c*2i + d*x*2i) + 1)^4) - (a^2*(a - (a*(exp(c*2i + d*x
*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*64i)/(11*d*(exp(c*2i + d*
x*2i) + 1)^5)
```

3.309 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 29

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx=-\frac{2i(a+ia\tan(c+dx))^{7/2}}{7ad}$$

```
[Out] -2/7*I*(a+I*a*tan(d*x+c))^(7/2)/a/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3568, 32}

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx=-\frac{2i(a+ia\tan(c+dx))^{7/2}}{7ad}$$

```
[In] Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((-2*I)/7)*(a + I*a*Tan[c + d*x])^(7/2))/(a*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int (a+x)^{5/2}\,dx, x, ia\tan(c+dx)\right)}{ad}\\ &= -\frac{2i(a+ia\tan(c+dx))^{7/2}}{7ad}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{2i(a+ia\tan(c+dx))^{7/2}}{7ad}$$

```
[In] Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^(5/2),x]
[Out] (((-2*I)/7)*(a + I*a*Tan[c + d*x])^(7/2))/(a*d)
```

Maple [A] (verified)

Time = 4.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{2i(a+ia\tan(dx+c))^{\frac{7}{2}}}{7ad}$	24
default	$-\frac{2i(a+ia\tan(dx+c))^{\frac{7}{2}}}{7ad}$	24

```
[In] int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/7*I*(a+I*a*tan(d*x+c))^(7/2)/a/d
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(21) = 42.

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.52

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{16i\sqrt{2}a^2\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(7i\,dx+7i\,c)}}{7\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)}$$

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
[Out] -16/7*I*sqrt(2)*a^2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(7*I*d*x + 7*I*c)/(
d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I*c) +
 d)
```

Sympy [F]

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \int (ia(\tan(c+dx)-i))^{\frac{5}{2}}\sec^2(c+dx)\,dx$$

```
[In] integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**(5/2),x)
[Out] Integral((I*a*(tan(c + d*x) - I))**(5/2)*sec(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{2i\,(i\,a\tan(dx+c)+a)^{\frac{7}{2}}}{7\,ad}$$

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
[Out] -2/7*I*(I*a*tan(d*x + c) + a)^(7/2)/(a*d)
```

Giac [F]

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^2\,dx$$

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*sec(d*x + c)^2, x)
```

Mupad [B] (verification not implemented)

Time = 7.43 (sec) , antiderivative size = 242, normalized size of antiderivative = 8.34

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{a^2\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,16\mathrm{i}}{7\,d}+\frac{a^2\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,48\mathrm{i}}{7\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}-\frac{a^2\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,48\mathrm{i}}{7\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}+\frac{a^2\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,16\mathrm{i}}{7\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}$$

```
[In] int((a + a*tan(c + d*x)*1i)^(5/2)/cos(c + d*x)^2,x)
```

```
[Out] (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/
2)*48i)/(7*d*(exp(c*2i + d*x*2i) + 1)) - (a^2*(a - (a*(exp(c*2i + d*x*2i)*1
i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*16i)/(7*d) - (a^2*(a - (a*(exp(
c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*48i)/(7*d*(exp(
c*2i + d*x*2i) + 1)^2) + (a^2*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp
(c*2i + d*x*2i) + 1))^(1/2)*16i)/(7*d*(exp(c*2i + d*x*2i) + 1)^3)
```

3.310 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 89

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{ia^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}d} - \frac{ia^3\sqrt{a+ia\tan(c+dx)}}{d(a-ia\tan(c+dx))}$$

```
[Out] 1/2*I*a^(5/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1/
2)-I*a^3*(a+I*a*tan(d*x+c))^(1/2)/d/(a-I*a*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3568, 43, 65, 212}

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{ia^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}d} - \frac{ia^3\sqrt{a+ia\tan(c+dx)}}{d(a-ia\tan(c+dx))}$$

```
[In] Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (I*a^(5/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2]*
d) - (I*a^3*Sqrt[a + I*a*Tan[c + d*x]])/(d*(a - I*a*Tan[c + d*x]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(ia^3)\,\text{Subst}\left(\int \frac{\sqrt{a+x}}{(a-x)^2}\,dx, x, ia\tan(c+dx)\right)}{d} \\
&= -\frac{ia^3\sqrt{a+ia\tan(c+dx)}}{d(a-ia\tan(c+dx))} + \frac{(ia^3)\,\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{2d} \\
&= -\frac{ia^3\sqrt{a+ia\tan(c+dx)}}{d(a-ia\tan(c+dx))} + \frac{(ia^3)\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+ia\tan(c+dx)}\right)}{d} \\
&= \frac{ia^{5/2}\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}d} - \frac{ia^3\sqrt{a+ia\tan(c+dx)}}{d(a-ia\tan(c+dx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.93

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{ia^{5/2}\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{2}d} + \frac{a^2\sqrt{a+ia\tan(c+dx)}}{d(i+\tan(c+dx))}$$

```
[In] Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (I*a^(5/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2]*
d) + (a^2*Sqrt[a + I*a*Tan[c + d*x]])/(d*(I + Tan[c + d*x]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 602 vs. 2(73) = 146.

Time = 18.58 (sec) , antiderivative size = 603, normalized size of antiderivative = 6.78

method	result
default	$\frac{i(\tan(dx+c)-i)^2\sqrt{a(1+i\tan(dx+c))}\,a^2(\cos^2(dx+c))\left(i\,\text{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\cos(dx+c)\sin(dx+\cdots}{}$

```
[In] int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] I/d*(tan(d*x+c)-I)^2*(a*(1+I*tan(d*x+c)))^(1/2)*a^2*cos(d*x+c)^2*(I*arctanh
(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)-I*(-cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2+I*arctanh(si
n(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*sin(d*x+c)-I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(
(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-arctanh(sin(d*x+c)/(cos(d*x+
c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*cos(d*x+c)^2-(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(
d*x+c)+1))^(1/2))*cos(d*x+c)*sin(d*x+c)-I*cos(d*x+c)^2-(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1
))^(1/2))*cos(d*x+c)-(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)
/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-I*cos(d*x+c)+sin(d*x+c)*cos(d*x+c))/(-2*
I*cos(d*x+c)^2+2*sin(d*x+c)*cos(d*x+c)-I*cos(d*x+c)+sin(d*x+c)+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(68) = 136.

Time = 0.25 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.65

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{\sqrt{2}\sqrt{-\frac{a^5}{d^2}}d\log\left(\frac{4\left(a^3e^{(i\,dx+i\,c)}-\sqrt{-\frac{a^5}{d^2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a^2}\right)-\sqrt{2}\sqrt{-\frac{a^5}{d^2}}d\log\left(\cdots\right.}{\cdots}$$

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*sqrt(-a^5/d^2)*d*log(4*(a^3*e^(I*d*x + I*c) - sqrt(-a^5/d^2)*(
I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x
 - I*c)/a^2) - sqrt(2)*sqrt(-a^5/d^2)*d*log(4*(a^3*e^(I*d*x + I*c) - sqrt(-
a^5/d^2)*(-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
)*e^(-I*d*x - I*c)/a^2) - 2*sqrt(2)*(I*a^2*e^(3*I*d*x + 3*I*c) + I*a^2*e^(I
*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.10

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{i\left(\sqrt{2}a^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)-\frac{8\,\sqrt{i\,a\tan(dx+c)+a}a^4}{2i\,a\tan(dx+c)-2\,a}\right)}{4\,ad}$$

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4*I*(sqrt(2)*a^(7/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))
/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) - 8*sqrt(I*a*tan(d*x + c)
+ a)*a^4/(2*I*a*tan(d*x + c) - 2*a))/(a*d)
```

Giac [F]

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \int (i\,a\tan\left(dx+c\right)+a)^{\frac{5}{2}}\cos\left(dx+c\right)^2\,dx$$

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \int \cos\left(c+d\,x\right)^2\,\left(a+a\tan(c+d\,x)\;1\mathrm{i}\right)^{5/2}\,dx$$

```
[In] int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(5/2),x)
[Out] int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.311 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 137

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{3ia^{5/2}\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}d} - \frac{ia^4\sqrt{a+ia\tan(c+dx)}}{4d(a-ia\tan(c+dx))^2} - \frac{3ia^3\sqrt{a+ia\tan(c+dx)}}{16d(a-ia\tan(c+dx))}$$

```
[Out] -3/32*I*a^(5/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(
1/2)-1/4*I*a^4*(a+I*a*tan(d*x+c))^(1/2)/d/(a-I*a*tan(d*x+c))^2-3/16*I*a^3*(
a+I*a*tan(d*x+c))^(1/2)/d/(a-I*a*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3568, 44, 65, 212}

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{3ia^{5/2}\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}d} - \frac{ia^4\sqrt{a+ia\tan(c+dx)}}{4d(a-ia\tan(c+dx))^2} - \frac{3ia^3\sqrt{a+ia\tan(c+dx)}}{16d(a-ia\tan(c+dx))}$$

```
[In] Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((-3*I)/16)*a^(5/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])
/(Sqrt[2]*d) - ((I/4)*a^4*Sqrt[a + I*a*Tan[c + d*x]])/(d*(a - I*a*Tan[c + d
*x])^2) - (((3*I)/16)*a^3*Sqrt[a + I*a*Tan[c + d*x]])/(d*(a - I*a*Tan[c + d
*x]))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(ia^5)\,\text{Subst}\left(\int \frac{1}{(a-x)^3\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{d}\\
&= -\frac{ia^4\sqrt{a+ia\tan(c+dx)}}{4d(a-ia\tan(c+dx))^2} - \frac{(3ia^4)\,\text{Subst}\left(\int \frac{1}{(a-x)^2\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{8d}\\
&= -\frac{ia^4\sqrt{a+ia\tan(c+dx)}}{4d(a-ia\tan(c+dx))^2} - \frac{3ia^3\sqrt{a+ia\tan(c+dx)}}{16d(a-ia\tan(c+dx))}\\
&\quad - \frac{(3ia^3)\,\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{32d}\\
&= -\frac{ia^4\sqrt{a+ia\tan(c+dx)}}{4d(a-ia\tan(c+dx))^2} - \frac{3ia^3\sqrt{a+ia\tan(c+dx)}}{16d(a-ia\tan(c+dx))}\\
&\quad - \frac{(3ia^3)\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+ia\tan(c+dx)}\right)}{16d}
\end{aligned}
$$

$$=-\frac{3ia^{5/2}\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}d}-\frac{ia^4\sqrt{a+ia\tan(c+dx)}}{4d(a-ia\tan(c+dx))^2}-\frac{3ia^3\sqrt{a+ia\tan(c+dx)}}{16d(a-ia\tan(c+dx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.39

$$\int\cos^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx=$$
$$-\frac{ia^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},\frac{1}{2}(1+i\tan(c+dx))\right)\sqrt{a+ia\tan(c+dx)}}{4d}$$

```
[In] Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-1/4*I)*a^2*Hypergeometric2F1[1/2, 3, 3/2, (1 + I*Tan[c + d*x])/2]*Sqrt[a
 + I*a*Tan[c + d*x]])/d
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 845 vs. 2(110) = 220.

Time = 123.88 (sec) , antiderivative size = 846, normalized size of antiderivative = 6.18

method	result	size
default	Expression too large to display	846

```
[In] int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d*(-tan(d*x+c)+I)^2*(a*(1+I*tan(d*x+c)))^(1/2)*a^2*cos(d*x+c)^2*(-3*I*
(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(
d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)+4*I*cos(d*x+c)^2*sin(d*x+c)+6*I*(-
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))
*cos(d*x+c)^2*sin(d*x+c)+6*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(
cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)+6*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)
+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2-6*cos(d*x+c)^3*(-cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+6*I*
(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2
))*cos(d*x+c)*sin(d*x+c)+6*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(
cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x
+c)-6*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2))*cos(d*x+c)^2+6*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*
```

```
x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^3-3*I*(-
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*
x+c)/(cos(d*x+c)+1))^(1/2))+3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-c
os(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-3*I*cos(d*x+c)*sin(d*x+c)+4*cos
(d*x+c)^3+3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2))+7*cos(d*x+c)^2+3*cos(d*x+c))/(-I*cos(d*x+c)+sin(d*x+c)-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(102) = 204$.

Time = 0.25 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.92

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx =$$

$$-\frac{3\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}d\log\left(\frac{4\left(a^3e^{(i\,dx+i\,c)}-\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a^2}\right)-3\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}d\log\left(\frac{4\left(a^3\right.}{}\right.}{}$$

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/32*(3*sqrt(1/2)*sqrt(-a^5/d^2)*d*log(4*(a^3*e^(I*d*x + I*c) - sqrt(2)*sq
rt(1/2)*sqrt(-a^5/d^2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1)))*e^(-I*d*x - I*c)/a^2) - 3*sqrt(1/2)*sqrt(-a^5/d^2)*d*log(4*(
a^3*e^(I*d*x + I*c) - sqrt(2)*sqrt(1/2)*sqrt(-a^5/d^2)*(-I*d*e^(2*I*d*x + 2
*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a^2) - sqr
t(2)*(-2*I*a^2*e^(5*I*d*x + 5*I*c) - 7*I*a^2*e^(3*I*d*x + 3*I*c) - 5*I*a^2*
e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.02

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{i\left(3\sqrt{2}a^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)+\frac{4\left(3\,(ia\tan(dx+c)+a)^{\frac{3}{2}}a^4-10\,\sqrt{ia\tan(dx+c)+a}a^5\right)}{(ia\tan(dx+c)+a)^2-4\,(ia\tan(dx+c)+a)a+4\,a^2}\right)}{64\,ad}$$

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
[Out] 1/64*I*(3*sqrt(2)*a^(7/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a
))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(3*(I*a*tan(d*x + c)
 + a)^(3/2)*a^4 - 10*sqrt(I*a*tan(d*x + c) + a)*a^5)/((I*a*tan(d*x + c) + a
)^2 - 4*(I*a*tan(d*x + c) + a)*a + 4*a^2))/(a*d)
```

Giac [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \int \cos\left(c+d\,x\right)^4\,\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)^{5/2} \,d x$$

```
[In] int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(5/2),x)
[Out] int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.312 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 210

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{35ia^{5/2}\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}d} + \frac{35ia^3}{128d\sqrt{a+ia\tan(c+dx)}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}} - \frac{7ia^5}{48d(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}} - \frac{35ia^4}{192d(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] -35/256*I*a^(5/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2
^(1/2)+35/128*I*a^3/d/(a+I*a*tan(d*x+c))^(1/2)-1/6*I*a^6/d/(a+I*a*tan(d*x+c
))^(1/2)/(a-I*a*tan(d*x+c))^3-7/48*I*a^5/d/(a+I*a*tan(d*x+c))^(1/2)/(a-I*a*
tan(d*x+c))^2-35/192*I*a^4/d/(a+I*a*tan(d*x+c))^(1/2)/(a-I*a*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used

$= \{3568, 44, 53, 65, 212\}$

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{35ia^{5/2}\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}d} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}} - \frac{7ia^5}{48d(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}} - \frac{35ia^4}{192d(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}} + \frac{35ia^3}{128d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((-35*I)/128)*a^(5/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])
])/(Sqrt[2]*d) + (((35*I)/128)*a^3)/(d*Sqrt[a + I*a*Tan[c + d*x]]) - ((I/6)
*a^6)/(d*(a - I*a*Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c + d*x]]) - (((7*I)/48)
*a^5)/(d*(a - I*a*Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]]) - (((35*I)/19
2)*a^4)/(d*(a - I*a*Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(ia^7)\,\text{Subst}\left(\int \frac{1}{(a-x)^4(a+x)^{3/2}}\,dx, x, ia\tan(c+dx)\right)}{d} \\
&= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}} \\
&\quad -\frac{(7ia^6)\,\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^{3/2}}\,dx, x, ia\tan(c+dx)\right)}{12d} \\
&= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}} \\
&\quad -\frac{7ia^5}{48d(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}} \\
&\quad -\frac{(35ia^5)\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{3/2}}\,dx, x, ia\tan(c+dx)\right)}{96d} \\
&= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}} \\
&\quad -\frac{7ia^5}{48d(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}} \\
&\quad -\frac{35ia^4}{192d(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}} \\
&\quad -\frac{(35ia^4)\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{3/2}}\,dx, x, ia\tan(c+dx)\right)}{128d}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{35ia^3}{128d\sqrt{a+ia\tan(c+dx)}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{7ia^5}{48d(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{35ia^4}{192d(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{(35ia^3)\operatorname{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{256d} \\
&= \frac{35ia^3}{128d\sqrt{a+ia\tan(c+dx)}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{7ia^5}{48d(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{35ia^4}{192d(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{(35ia^3)\operatorname{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+ia\tan(c+dx)}\right)}{128d} \\
&= -\frac{35ia^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}d} + \frac{35ia^3}{128d\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{ia^6}{6d(a-ia\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{7ia^5}{48d(a-ia\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{35ia^4}{192d(a-ia\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.25

$$
\int \cos^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{ia^3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},4,\frac{1}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{8d\sqrt{a+ia\tan(c+dx)}}
$$

```
[In] Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^(5/2),x]
[Out] ((I/8)*a^3*Hypergeometric2F1[-1/2, 4, 1/2, (1 + I*Tan[c + d*x])/2])/(d*Sqrt
[a + I*a*Tan[c + d*x]])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 932 vs. $2(170) = 340$.

Time = 5.97 (sec) , antiderivative size = 933, normalized size of antiderivative = 4.44

Expression too large to display

```
[In] int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] 1/768*I/d*(tan(d*x+c)-I)^2*(a*(1+I*tan(d*x+c)))^(1/2)*a^2*cos(d*x+c)^2*(420
*I*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3-315*I*(-cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^
(1/2))*cos(d*x+c)-105*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x
+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+420*arctanh(sin(d*x+c)/(cos(d*x+c)+1)
/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*(-cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)+420*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(
d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2*sin(d*x+c)-420*cos(d*x+c)^3*(-co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+2
10*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(
-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2+210*I*(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*sin(d*x
+c)+210*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2
))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)+448*I*cos(d*x+c
)^3*sin(d*x+c)-210*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(
cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2-320*cos(d*x+c)^4-105*I*(-cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c
)+1))^(1/2))-105*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos
(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+315*(-cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)
-210*I*sin(d*x+c)*cos(d*x+c)+105*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan(
(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+490*cos(d*x+c)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.47

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{\left(105\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}de^{(i\,dx+i\,c)}\log\left(\frac{4\left(a^3e^{(i\,dx+i\,c)}-\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a^2}\right)-105\sqrt{\frac{1}{2}}\sqrt{\cdots}}{\cdots}$$

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/768*(105*sqrt(1/2)*sqrt(-a^5/d^2)*d*e^(I*d*x + I*c)*log(4*(a^3*e^(I*d*x
+ I*c) - sqrt(2)*sqrt(1/2)*sqrt(-a^5/d^2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*s
qrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a^2) - 105*sqrt(1/2)*sqr
t(-a^5/d^2)*d*e^(I*d*x + I*c)*log(4*(a^3*e^(I*d*x + I*c) - sqrt(2)*sqrt(1/2
)*sqrt(-a^5/d^2)*(-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1)))*e^(-I*d*x - I*c)/a^2) - sqrt(2)*(-8*I*a^2*e^(8*I*d*x + 8*I*c) - 4
6*I*a^2*e^(6*I*d*x + 6*I*c) - 125*I*a^2*e^(4*I*d*x + 4*I*c) - 39*I*a^2*e^(2
*I*d*x + 2*I*c) + 48*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x -
I*c)/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**(5/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.92

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{i\left(105\sqrt{2}a^{\frac{7}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+\frac{4\left(105\,(i\,a\tan(dx+c)+a)^3a^4-560\,(i\,a\tan(dx+c)+a)^2a^5+924\,(i\,\cdots\right.}{(i\,a\tan(dx+c)+a)^{\frac{7}{2}}-6\,(i\,a\tan(dx+c)+a)^{\frac{5}{2}}a+12\,(i\,a\tan(dx+c)+a\cdots}\right.}{1536\,ad}$$

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
[Out] 1/1536*I*(105*sqrt(2)*a^(7/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c)
 + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(105*(I*a*tan(d*
x + c) + a)^3*a^4 - 560*(I*a*tan(d*x + c) + a)^2*a^5 + 924*(I*a*tan(d*x + c
) + a)*a^6 - 384*a^7)/((I*a*tan(d*x + c) + a)^(7/2) - 6*(I*a*tan(d*x + c) +
 a)^(5/2)*a + 12*(I*a*tan(d*x + c) + a)^(3/2)*a^2 - 8*sqrt(I*a*tan(d*x + c)
 + a)*a^3))/(a*d)
```

Giac [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \int \cos\left(c+d\,x\right)^6\,\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^{5/2}\,dx$$

```
[In] int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(5/2),x)
[Out] int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.313 $\int \sec^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 147

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{256ia^4\sec^3(c+dx)}{315d(a+ia\tan(c+dx))^{3/2}} + \frac{64ia^3\sec^3(c+dx)}{105d\sqrt{a+ia\tan(c+dx)}} + \frac{8ia^2\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{21d} + \frac{2ia\sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d}$$

```
[Out] 64/105*I*a^3*sec(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(1/2)+8/21*I*a^2*sec(d*x+c)^
3*(a+I*a*tan(d*x+c))^(1/2)/d+256/315*I*a^4*sec(d*x+c)^3/d/(a+I*a*tan(d*x+c)
)^(3/2)+2/9*I*a*sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2)/d
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3575, 3574}

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{256ia^4\sec^3(c+dx)}{315d(a+ia\tan(c+dx))^{3/2}} + \frac{64ia^3\sec^3(c+dx)}{105d\sqrt{a+ia\tan(c+dx)}} + \frac{8ia^2\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{21d} + \frac{2ia\sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d}$$

```
[In] Int[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((256*I)/315)*a^4*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((64
*I)/105)*a^3*Sec[c + d*x]^3)/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((8*I)/21)*a
^2*Sec[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d + (((2*I)/9)*a*Sec[c + d*x]
^3*(a + I*a*Tan[c + d*x])^(3/2))/d
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2ia\sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d} \\
&\quad+\frac{1}{3}(4a)\int\sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx \\
&= \frac{8ia^2\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{21d}+\frac{2ia\sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d} \\
&\quad+\frac{1}{21}\left(32a^2\right)\int\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx \\
&= \frac{64ia^3\sec^3(c+dx)}{105d\sqrt{a+ia\tan(c+dx)}}+\frac{8ia^2\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{21d} \\
&\quad+\frac{2ia\sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d}+\frac{1}{105}\left(128a^3\right)\int\frac{\sec^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx \\
&= \frac{256ia^4\sec^3(c+dx)}{315d(a+ia\tan(c+dx))^{3/2}}+\frac{64ia^3\sec^3(c+dx)}{105d\sqrt{a+ia\tan(c+dx)}} \\
&\quad+\frac{8ia^2\sec^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{21d}+\frac{2ia\sec^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{9d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.70

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{2a^2\sec^3(c+dx)(i\cos(2c)+\sin(2c))(77+242\cos(2(c+dx))+89i\sec(c+dx)\sin(3(c+dx))}{315d(\cos(dx)+i\sin(dx))^2}$$

```
[In] Integrate[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*Sec[c + d*x]^3*(I*Cos[2*c] + Sin[2*c])*(77 + 242*Cos[2*(c + d*x)] +
(89*I)*Sec[c + d*x]*Sin[3*(c + d*x)] + (54*I)*Tan[c + d*x])*Sqrt[a + I*a*Ta
n[c + d*x]])/(315*d*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [A] (verified)

Time = 41.25 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.52

method	result
default	$\frac{2i(-\tan(dx+c)+i)^2a^2\sqrt{a(1+i\tan(dx+c))}\,(256i(\cos^2(dx+c))\sin(dx+c)+128i\cos(dx+c)\sin(dx+c)-256(\cos^3(dx+c))+96i\sin(dx-}{315d(4(\cos^3(dx+c))+2(\cos^2(dx+c))+4i(\cos^2(dx+c))\sin(dx+c)-}$

```
[In] int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/315*I/d*(-tan(d*x+c)+I)^2*a^2*(a*(1+I*tan(d*x+c)))^(1/2)/(4*cos(d*x+c)^3+
2*cos(d*x+c)^2+4*I*cos(d*x+c)^2*sin(d*x+c)-3*cos(d*x+c)+2*I*cos(d*x+c)*sin(
d*x+c)-1-I*sin(d*x+c))*(256*I*cos(d*x+c)^2*sin(d*x+c)+128*I*cos(d*x+c)*sin(
d*x+c)-256*cos(d*x+c)^3+96*I*sin(d*x+c)-128*cos(d*x+c)^2-130*I*tan(d*x+c)+3
2*cos(d*x+c)-35*I*tan(d*x+c)*sec(d*x+c)-226-95*sec(d*x+c)+35*sec(d*x+c)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.82

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{32\sqrt{2}\left(-105i\,a^2e^{(6i\,dx+6i\,c)}-126i\,a^2e^{(4i\,dx+4i\,c)}-72i\,a^2e^{(2i\,dx+2i\,c)}-16i\,a^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{315\left(de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}+4\,de^{(2i\,dx+2i\,c)}+d\right)}$$

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -32/315*sqrt(2)*(-105*I*a^2*e^(6*I*d*x + 6*I*c) - 126*I*a^2*e^(4*I*d*x + 4*
I*c) - 72*I*a^2*e^(2*I*d*x + 2*I*c) - 16*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c)
```

```
 + 1))/(d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x +
4*I*c) + 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(sec(d*x+c)**3*(a+I*a*tan(d*x+c))**(5/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(115) = 230$.

Time = 243.69 (sec) , antiderivative size = 624, normalized size of antiderivative = 4.24

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{}{315\left(\cos\left(2\,dx+2\,c\right)^2+\sin\left(2\,dx+2\,c\right)^2+2\,\cos\left(2\,dx+2\,c\right)+1\right)^{\frac{1}{4}}\left(\left(2\,\cos\left(2\,dx+2\,c\right)^3+(2\,\cos\right.\right.}$$

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
[Out] 32/315*(105*I*sqrt(2)*a^2*cos(6*d*x + 6*c) + 126*I*sqrt(2)*a^2*cos(4*d*x +
4*c) + 72*I*sqrt(2)*a^2*cos(2*d*x + 2*c) - 105*sqrt(2)*a^2*sin(6*d*x + 6*c)
 - 126*sqrt(2)*a^2*sin(4*d*x + 4*c) - 72*sqrt(2)*a^2*sin(2*d*x + 2*c) + 16*
I*sqrt(2)*a^2)*sqrt(a)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*((2*cos(2*d*x + 2*c)^3 + (2*cos(2*d*x + 2*c) + 1)*sin
(2*d*x + 2*c)^2 + 2*I*sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 5*cos(2*d*x + 2*c)^2
+ (I*cos(2*d*x + 2*c)^2 + I*sin(2*d*x + 2*c)^2 + 2*I*cos(2*d*x + 2*c) + I)*
sin(4*d*x + 4*c) + 2*(I*cos(2*d*x + 2*c)^2 + 2*I*cos(2*d*x + 2*c) + I)*sin(
2*d*x + 2*c) + 4*cos(2*d*x + 2*c) + 1)*cos(5/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)) + (2*I*cos(2*d*x + 2*c)^3 + (2*I*cos(2*d*x + 2*c) + I)
*sin(2*d*x + 2*c)^2 - 2*sin(2*d*x + 2*c)^3 + (I*cos(2*d*x + 2*c)^2 + I*sin(
2*d*x + 2*c)^2 + 2*I*cos(2*d*x + 2*c) + I)*cos(4*d*x + 4*c) + 5*I*cos(2*d*x
 + 2*c)^2 - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)*sin(4*d*x + 4*c) - 2*(cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(
2*d*x + 2*c) + 4*I*cos(2*d*x + 2*c) + I)*sin(5/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1))*d)
```

Giac [F]

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^3\,dx$$

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*sec(d*x + c)^3, x)
```

Mupad [B] (verification not implemented)

Time = 8.78 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.05

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{a^2\,e^{-c\,1i-d\,x\,1i}\,\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,32i}{3\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)} - \frac{a^2\,e^{-c\,1i-d\,x\,1i}\,\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,96i}{5\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^2} + \frac{a^2\,e^{-c\,1i-d\,x\,1i}\,\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,96i}{7\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^3} - \frac{a^2\,e^{-c\,1i-d\,x\,1i}\,\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,32i}{9\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^4}$$

```
[In] int((a + a*tan(c + d*x)*1i)^(5/2)/cos(c + d*x)^3,x)
[Out] (a^2*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2
i + d*x*2i) + 1))^(1/2)*32i)/(3*d*(exp(c*2i + d*x*2i) + 1)) - (a^2*exp(- c*
1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) +
 1))^(1/2)*96i)/(5*d*(exp(c*2i + d*x*2i) + 1)^2) + (a^2*exp(- c*1i - d*x*1i
)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*
96i)/(7*d*(exp(c*2i + d*x*2i) + 1)^3) - (a^2*exp(- c*1i - d*x*1i)*(a - (a*(
exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*32i)/(9*d*(
exp(c*2i + d*x*2i) + 1)^4)
```

3.314 $\int \sec(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 104

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{64ia^3\sec(c+dx)}{15d\sqrt{a+ia\tan(c+dx)}} + \frac{16ia^2\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{15d} + \frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

```
[Out] 64/15*I*a^3*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)+16/15*I*a^2*sec(d*x+c)*(a
+I*a*tan(d*x+c))^(1/2)/d+2/5*I*a*sec(d*x+c)*(a+I*a*tan(d*x+c))^(3/2)/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3575, 3574}

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{64ia^3\sec(c+dx)}{15d\sqrt{a+ia\tan(c+dx)}} + \frac{16ia^2\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{15d} + \frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}$$

```
[In] Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((64*I)/15)*a^3*Sec[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((16*I)/15
)*a^2*Sec[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d + (((2*I)/5)*a*Sec[c + d*x
]*(a + I*a*Tan[c + d*x])^(3/2))/d
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
```

```
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d} \\
&\quad + \frac{1}{5}(8a)\int \sec(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx \\
&= \frac{16ia^2\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{15d} + \frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d} \\
&\quad + \frac{1}{15}\left(32a^2\right)\int \sec(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx \\
&= \frac{64ia^3\sec(c+dx)}{15d\sqrt{a+ia\tan(c+dx)}} + \frac{16ia^2\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{15d} \\
&\quad + \frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}{5d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{2a^2\sec^2(c+dx)(i\cos(c-dx)+\sin(c-dx))(20+23\cos(2(c+dx))+7i\sin(2(c+dx)))\sqrt{a+ia\tan(c+dx)}}{15d(\cos(dx)+i\sin(dx))^2}$$

```
[In] Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*Sec[c + d*x]^2*(I*Cos[c - d*x] + Sin[c - d*x])*(20 + 23*Cos[2*(c + d
*x)] + (7*I)*Sin[2*(c + d*x)])*Sqrt[a + I*a*Tan[c + d*x]])/(15*d*(Cos[d*x]
+ I*Sin[d*x])^2)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(86) = 172$.

Time = 7.42 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.75

method	result
default	$\frac{2i(-\tan(dx+c)+i)^2 a^2 \sqrt{a(1+i\tan(dx+c))}\,(32i(\cos^2(dx+c))\sin(dx+c)-14i\cos(dx+c)\sin(dx+c)-32(\cos^3(dx+c))-3i\sin(dx+c)-46 \cdots}{15d(4(\cos^3(dx+c))+2(\cos^2(dx+c))+4i(\cos^2(dx+c))\sin(dx+c)-3\cos(dx+c)+2i\cos(dx+c)\sin(dx+c)-1-i \cdots}$

```
[In] int(sec(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*I/d*(-tan(d*x+c)+I)^2*a^2*(a*(1+I*tan(d*x+c)))^(1/2)*(32*I*cos(d*x+c)^
2*sin(d*x+c)-14*I*cos(d*x+c)*sin(d*x+c)-32*cos(d*x+c)^3-3*I*sin(d*x+c)-46*c
os(d*x+c)^2-11*cos(d*x+c)+3)/(4*cos(d*x+c)^3+2*cos(d*x+c)^2+4*I*cos(d*x+c)^
2*sin(d*x+c)-3*cos(d*x+c)+2*I*cos(d*x+c)*sin(d*x+c)-1-I*sin(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.80

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{8\sqrt{2}\left(-15i\,a^2e^{(4i\,dx+4i\,c)}-20i\,a^2e^{(2i\,dx+2i\,c)}-8i\,a^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{15\left(de^{(4i\,dx+4i\,c)}+2\,de^{(2i\,dx+2i\,c)}+d\right)}$$

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -8/15*sqrt(2)*(-15*I*a^2*e^(4*I*d*x + 4*I*c) - 20*I*a^2*e^(2*I*d*x + 2*I*c)
 - 8*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(4*I*d*x + 4*I*c) + 2*d*
e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \int (ia(\tan(c+dx)-i))^{\frac{5}{2}}\sec(c+dx)\,dx$$

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(5/2)*sec(c + d*x), x)
```

Maxima [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)\,dx$$

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

Giac [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)\,dx$$

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

Mupad [B] (verification not implemented)

Time = 7.17 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.01

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{8\,a^2\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,20\mathrm{i}+\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}\,15\mathrm{i}+8\mathrm{i}\right)}{15\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}$$

```
[In] int((a + a*tan(c + d*x)*1i)^(5/2)/cos(c + d*x),x)
[Out] (8*a^2*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c
*2i + d*x*2i) + 1))^(1/2)*(exp(c*2i + d*x*2i)*20i + exp(c*4i + d*x*4i)*15i
+ 8i))/(15*d*(exp(c*2i + d*x*2i) + 1)^2)
```

3.315 $\int \cos(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{8ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}+\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{3/2}}{d}$$

```
[Out] -8*I*a^2*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d+2*I*a*cos(d*x+c)*(a+I*a*tan(
d*x+c))^(3/2)/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3575, 3574}

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{3/2}}{d}-\frac{8ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}$$

```
[In] Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-8*I)*a^2*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d + ((2*I)*a*Cos[c + d
*x]*(a + I*a*Tan[c + d*x])^(3/2))/d
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
```

```
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{3/2}}{d}+(4a)\int\cos(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx\\ &= -\frac{8ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{d}+\frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{3/2}}{d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.71

$$\int\cos(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{2ia^2(3\cos(c+dx)-i\sin(c+dx))\sqrt{a+ia\tan(c+dx)}}{d}$$

```
[In] Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^(5/2),x]
[Out] ((-2*I)*a^2*(3*Cos[c + d*x] - I*Sin[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Maple [A] (verified)

Time = 24.90 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.35

method	result
default	$-\frac{2(-\tan(dx+c)+i)^2\sqrt{a(1+i\tan(dx+c))}\,a^2(\cos^2(dx+c))(3i\cos(dx+c)+\sin(dx+c))(2i\cos(dx+c)\sin(dx+c)-2(\cos^2(dx+c))+1)}{d}$

```
[In] int(cos(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/d*(-tan(d*x+c)+I)^2*(a*(1+I*tan(d*x+c)))^(1/2)*a^2*cos(d*x+c)^2*(3*I*cos
(d*x+c)+sin(d*x+c))*(2*I*cos(d*x+c)*sin(d*x+c)-2*cos(d*x+c)^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{2\sqrt{2}\left(i\,a^2e^{(2i\,dx+2i\,c)}+2i\,a^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{d}$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
[Out] -2*sqrt(2)*(I*a^2*e^(2*I*d*x + 2*I*c) + 2*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**(5/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(53) = 106.

Time = 0.41 (sec) , antiderivative size = 331, normalized size of antiderivative = 5.09

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{2\left(-3i\,a^{\frac{5}{2}}-\frac{2\,a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}+\frac{9i\,a^{\frac{5}{2}}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{4\,a^{\frac{5}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{9i\,a^{\frac{5}{2}}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{2\,a^{\frac{5}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{3i\,a^{\frac{5}{2}}\,\mathrm{s}}{(\cos(d}\right.}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)^{\frac{5}{2}}\left(\frac{4i\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{5\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\right.}$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
[Out] 2*(-3*I*a^(5/2) - 2*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) + 9*I*a^(5/2)*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 + 4*a^(5/2)*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 - 9*I*a^(5/2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 2*a^(5/2)*sin(d
*x + c)^5/(cos(d*x + c) + 1)^5 + 3*I*a^(5/2)*sin(d*x + c)^6/(cos(d*x + c) +
 1)^6)*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 - 1)^(5/2)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(sin(d*x
```

```
+ c)/(cos(d*x + c) + 1) - 1)^(5/2)*(4*I*sin(d*x + c)/(cos(d*x + c) + 1) - 5
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 5*sin(d*x + c)^4/(cos(d*x + c) + 1)^
4 - 4*I*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + sin(d*x + c)^6/(cos(d*x + c)
+ 1)^6 + 1))
```

Giac [F]

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)\;dx$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*cos(d*x + c), x)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{2\,a^2\,(\sin(c+d\,x)+\cos(c+d\,x)\;3\mathrm{i})\;\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\;1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}}{d}$$

```
[In] int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] -(2*a^2*(cos(c + d*x)*3i + sin(c + d*x))*((a*(cos(2*c + 2*d*x) + sin(2*c +
2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))/d
```

3.316 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 35

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

```
[Out] -2/3*I*a*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2)/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {3574}

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

```
[In] Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((-2*I)/3)*a*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2))/d
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\text{integral} = -\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d}$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.97

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx=\frac{2a^2\cos^2(c+dx)(-i\cos(c+3dx)+\sin(c+3dx))\sqrt{a+ia\tan(c+dx)}}{3d(\cos(dx)+i\sin(dx))^2}$$

```
[In] Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*Cos[c + d*x]^2*((-I)*Cos[c + 3*d*x] + Sin[c + 3*d*x])*Sqrt[a + I*a*T
an[c + d*x]])/(3*d*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(29) = 58.

Time = 35.41 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.89

method	result	size
default	$-\frac{2i(-\tan(dx+c)+i)^2\sqrt{a(1+i\tan(dx+c))}\,(i\sin(dx+c)(\cos^4(dx+c))-(\cos^5(dx+c)))a^2}{3d}$	66

```
[In] int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I/d*(-tan(d*x+c)+I)^2*(a*(1+I*tan(d*x+c)))^(1/2)*(I*cos(d*x+c)^4*sin(d
*x+c)-cos(d*x+c)^5)*a^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(27) = 54.

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx=\frac{\sqrt{2}\left(-i\,a^2e^{(4i\,dx+4i\,c)}-2i\,a^2e^{(2i\,dx+2i\,c)}-i\,a^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{6\,d}$$

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(2)*(-I*a^2*e^(4*I*d*x + 4*I*c) - 2*I*a^2*e^(2*I*d*x + 2*I*c) - I*a
^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(27) = 54$.

Time = 0.46 (sec) , antiderivative size = 328, normalized size of antiderivative = 9.37

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{2\left(i\,a^{\frac{5}{2}} - \frac{4i\,a^{\frac{5}{2}}\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{6i\,a^{\frac{5}{2}}\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{4i\,a^{\frac{5}{2}}\sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{i\,a^{\frac{5}{2}}\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)\Big(}{-3\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)^{\frac{5}{2}}\left(\frac{2i\sin(dx+c)}{\cos(dx+c)+1} + \frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{6i\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{6i\sin(dx}{(\cos(dx+c}\right.}$$

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2*(I*a^(5/2) - 4*I*a^(5/2)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*I*a^(5/2
)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*I*a^(5/2)*sin(d*x + c)^6/(cos(d*x
 + c) + 1)^6 + I*a^(5/2)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*(-2*I*sin(d*x
 + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(5/2)/(
d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(sin(d*x + c)/(cos(d*x + c) +
 1) - 1)^(5/2)*(-6*I*sin(d*x + c)/(cos(d*x + c) + 1) - 6*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 - 18*I*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 18*I*sin(d*x
 + c)^5/(cos(d*x + c) + 1)^5 + 6*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 6*I*
sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8
 - 3))
```

Giac [F]

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^3\,dx$$

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*cos(d*x + c)^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.54

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx =$$
$$-\frac{a^2\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1i)}{\cos(2\,c+2\,d\,x)+1}}\,(-\sin(c+d\,x)-\sin(3\,c+3\,d\,x)+\cos(c+d\,x)\,3i+\cos(3\,c+3\,d\,x)\,1i}{6\,d}$$

```
[In] int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(5/2),x)
[Out] -(a^2*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) +
 1))^(1/2)*(cos(c + d*x)*3i - sin(c + d*x) + cos(3*c + 3*d*x)*1i - sin(3*c
+ 3*d*x)))/(6*d)
```

3.317 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 159

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{ia^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{4\sqrt{2}d} - \frac{ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d} - \frac{ia\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{6d} - \frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}$$

```
[Out] 1/8*I*a^(5/2)*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/
2))/d*2^(1/2)-1/4*I*a^2*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d-1/6*I*a*cos(d
*x+c)^3*(a+I*a*tan(d*x+c))^(3/2)/d-1/5*I*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(5
/2)/d
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3571, 3570, 212}

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{ia^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{4\sqrt{2}d} - \frac{ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d} - \frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d} - \frac{ia\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{6d}$$

```
[In] Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((I/4)*a^(5/2)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c +
 d*x]])])/(Sqrt[2]*d) - ((I/4)*a^2*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])
/d - ((I/6)*a*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2))/d - ((I/5)*Cos[c
 + d*x]^5*(a + I*a*Tan[c + d*x])^(5/2))/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3571

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}+\frac{1}{2}a\int\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx\\
&= -\frac{ia\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{6d}-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}\\
&\quad+\frac{1}{4}a^2\int\cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx\\
&= -\frac{ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d}-\frac{ia\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{6d}\\
&\quad-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}+\frac{1}{8}a^3\int\frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx\\
&= -\frac{ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d}-\frac{ia\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{6d}\\
&\quad-\frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}+\frac{(ia^3)\operatorname{Subst}\left(\int\frac{1}{2-ax^2}\,dx,x,\frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\right)}{4d}
\end{aligned}$$

$$= \frac{ia^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{4\sqrt{2}d} - \frac{ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d} - \frac{ia\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{6d} - \frac{i\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}$$

Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.74

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx =$$

$$-\frac{ia^2e^{-i(c+dx)}\left(23+34e^{2i(c+dx)}+14e^{4i(c+dx)}+3e^{6i(c+dx)}-15\sqrt{1+e^{2i(c+dx)}}\text{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)\sqrt{a+i}}{120d}$$

```
[In] Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-1/120*I)*a^2*(23 + 34*E^((2*I)*(c + d*x)) + 14*E^((4*I)*(c + d*x)) + 3*E
^((6*I)*(c + d*x)) - 15*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((
2*I)*(c + d*x))]])*Sqrt[a + I*a*Tan[c + d*x]])/(d*E^(I*(c + d*x)))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 915 vs. 2(128) = 256.

Time = 3.30 (sec) , antiderivative size = 916, normalized size of antiderivative = 5.76

Expression too large to display

```
[In] int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] 1/120/d*(tan(d*x+c)-I)^2*(a*(1+I*tan(d*x+c)))^(1/2)*a^2*cos(d*x+c)^2*(-45*I
*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/
2))*cos(d*x+c)+30*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(
cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*sin(d*x+c)+104
*I*cos(d*x+c)^3-15*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)
/(cos(d*x+c)+1))^(1/2))-60*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d
*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^3+60*(-
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))
*cos(d*x+c)^2*sin(d*x+c)-15*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(si
n(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+60*I
*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos
(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2*sin(d*x+c)+30*I*(-cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2
-30*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(
-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2+30*(-cos(d*x+c)/(cos(d*x+c)+
```

```
1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*sin(d*x+c)-
30*I*cos(d*x+c)+45*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(c
os(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-15*(-cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c
)+80*cos(d*x+c)^2*sin(d*x+c)+60*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan
((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^3+15*(-cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(120) = 240$.

Time = 0.27 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.53

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{15\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}\,d\log\left(\frac{\left(ia^3+\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{2\,d}\right)-15\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}\,d\log\cdots}{\cdots}$$

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/120*(15*sqrt(1/2)*sqrt(-a^5/d^2)*d*log(1/2*(I*a^3 + sqrt(2)*sqrt(1/2)*sqr
t(-a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*
e^(-I*d*x - I*c)/d) - 15*sqrt(1/2)*sqrt(-a^5/d^2)*d*log(1/2*(I*a^3 - sqrt(2
)*sqrt(1/2)*sqrt(-a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1)))*e^(-I*d*x - I*c)/d) + sqrt(2)*(-3*I*a^2*e^(6*I*d*x + 6*I*c)
- 14*I*a^2*e^(4*I*d*x + 4*I*c) - 34*I*a^2*e^(2*I*d*x + 2*I*c) - 23*I*a^2)*s
qrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1076 vs. 2(120) = 240.

Time = 0.47 (sec) , antiderivative size = 1076, normalized size of antiderivative = 6.77

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/480*(20*(I*sqrt(2)*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)) - sqrt(2)*a^2*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/
4)*sqrt(a) + 12*(5*I*sqrt(2)*a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)) - 5*sqrt(2)*a^2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1)) + (I*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + I*sqrt(2)*a^2*sin(2*d*x
+ 2*c)^2 + 2*I*sqrt(2)*a^2*cos(2*d*x + 2*c) + I*sqrt(2)*a^2)*cos(5/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - (sqrt(2)*a^2*cos(2*d*x + 2*c)^
2 + sqrt(2)*a^2*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(
2)*a^2)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*(cos(2*d*
x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 1
5*(2*sqrt(2)*a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - 2*sqrt(
2)*a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) - I*sqrt(2)*a^2*log
(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)) + 1) + I*sqrt(2)*a^2*log(sqrt(cos(2*d*x + 2*c)^2 + si
n(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1))^2 - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1))*s
qrt(a))/d
```

Giac [F]

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^5\,dx$$

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*cos(d*x + c)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \int \cos(c+d\,x)^5\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{5/2}\,dx$$

```
[In] int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(5/2),x)
[Out] int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.318 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 231

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{9ia^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{32\sqrt{2}d} + \frac{3ia^3\cos(c+dx)}{16d\sqrt{a+ia\tan(c+dx)}} - \frac{9ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{32d} - \frac{3ia^2\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{20d} - \frac{9ia\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{70d} - \frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

```
[Out] 9/64*I*a^(5/2)*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1
/2))/d*2^(1/2)+3/16*I*a^3*cos(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)-9/32*I*a^2*
cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d-3/20*I*a^2*cos(d*x+c)^3*(a+I*a*tan(d*
x+c))^(1/2)/d-9/70*I*a*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2)/d-1/7*I*cos(d*
x+c)^7*(a+I*a*tan(d*x+c))^(5/2)/d
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used

$= \{3578, 3583, 3571, 3570, 212\}$

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = \frac{9ia^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{32\sqrt{2}d} + \frac{3ia^3\cos(c+dx)}{16d\sqrt{a+ia\tan(c+dx)}} - \frac{3ia^2\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{20d} - \frac{9ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{32d} - \frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d} - \frac{9ia\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{70d}$$

```
[In] Int[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((9*I)/32)*a^(5/2)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Ta
n[c + d*x]])])/(Sqrt[2]*d) + (((3*I)/16)*a^3*Cos[c + d*x])/(d*Sqrt[a + I*a*
Tan[c + d*x]]) - (((9*I)/32)*a^2*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d
 - (((3*I)/20)*a^2*Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d - (((9*I)/7
0)*a*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(3/2))/d - ((I/7)*Cos[c + d*x]^7
*(a + I*a*Tan[c + d*x])^(5/2))/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3571

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
```

```
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d} \\
&\quad+\frac{1}{14}(9a)\int\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx \\
&= -\frac{9ia\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{70d}-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d} \\
&\quad+\frac{1}{20}\left(9a^2\right)\int\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx \\
&= -\frac{3ia^2\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{20d}-\frac{9ia\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{70d} \\
&\quad-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}+\frac{1}{8}\left(3a^3\right)\int\frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx \\
&= \frac{3ia^3\cos(c+dx)}{16d\sqrt{a+ia\tan(c+dx)}}-\frac{3ia^2\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{20d} \\
&\quad-\frac{9ia\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{70d}-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d} \\
&\quad+\frac{1}{32}\left(9a^2\right)\int\cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx \\
&= \frac{3ia^3\cos(c+dx)}{16d\sqrt{a+ia\tan(c+dx)}}-\frac{9ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{32d} \\
&\quad-\frac{3ia^2\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{20d}-\frac{9ia\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{70d} \\
&\quad-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}+\frac{1}{64}\left(9a^3\right)\int\frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx
\end{aligned}
$$

$$= \frac{3ia^3\cos(c+dx)}{16d\sqrt{a+ia\tan(c+dx)}} - \frac{9ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{32d} - \frac{3ia^2\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{20d} - \frac{9ia\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{70d} - \frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d} + \frac{(9ia^3)\,\text{Subst}\left(\int\frac{1}{2-ax^2}\,dx, x, \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\right)}{32d}$$

$$= \frac{9ia^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{32\sqrt{2}d} + \frac{3ia^3\cos(c+dx)}{16d\sqrt{a+ia\tan(c+dx)}} - \frac{9ia^2\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{32d} - \frac{3ia^2\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{20d} - \frac{9ia\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{70d} - \frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

Mathematica [A] (verified)

Time = 1.85 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.67

$$\int\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{ia^2e^{-3i(c+dx)}\left(-35+353e^{2i(c+dx)}+544e^{4i(c+dx)}+214e^{6i(c+dx)}+68e^{8i(c+dx)}+10e^{10i(c+dx)}-315e^{2i(c+dx)}\sqrt{1}\right.}{2240d}$$

```
[In] Integrate[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-1/2240*I)*a^2*(-35 + 353*E^((2*I)*(c + d*x)) + 544*E^((4*I)*(c + d*x)) +
 214*E^((6*I)*(c + d*x)) + 68*E^((8*I)*(c + d*x)) + 10*E^((10*I)*(c + d*x))
 - 315*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E
^((2*I)*(c + d*x))]])*Sqrt[a + I*a*Tan[c + d*x]])/(d*E^((3*I)*(c + d*x)))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 944 vs. 2(188) = 376.

Time = 3.72 (sec) , antiderivative size = 945, normalized size of antiderivative = 4.09

Expression too large to display

```
[In] int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] 1/2240*I/d*(-tan(d*x+c)+I)^2*(a*(1+I*tan(d*x+c)))^(1/2)*a^2*cos(d*x+c)^2*(-
945*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/
```

```
(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-1680*I*cos(d*x+c)^2*sin(d*x+
c)+315*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)
+1))^(1/2))*sin(d*x+c)-1260*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-c
os(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2*sin(d*x+c)+1260*I*(-cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos
(d*x+c)+1))^(1/2))*cos(d*x+c)^3+1260*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-co
s(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*(-cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)+1260*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan
((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-400*cos(d*x+c)^5+720*I*cos(d*x+c)^4*si
n(d*x+c)+630*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d
*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2-630*I*(-cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+
c)*sin(d*x+c)+630*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c
)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)+630*(
-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
)*cos(d*x+c)^2-315*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/
(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-315*(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1
))^(1/2))*sin(d*x+c)-945*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*
x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)+2184*cos(d*x+c)^3-315*(-cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-630*cos(d*x
+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.30

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{\left(315\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}de^{(2i\,dx+2i\,c)}\log\left(-\frac{9\left(-i\,a^3+\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{16\,d}\right)-315\sqrt{\frac{1}{2}}\sqrt{-\frac{a^5}{d^2}}\cdots\right.}{}$$

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/2240*(315*sqrt(1/2)*sqrt(-a^5/d^2)*d*e^(2*I*d*x + 2*I*c)*log(-9/16*(-I*a
^3 + sqrt(2)*sqrt(1/2)*sqrt(-a^5/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e
^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/d) - 315*sqrt(1/2)*sqrt(-a^5/d^2
)*d*e^(2*I*d*x + 2*I*c)*log(-9/16*(-I*a^3 - sqrt(2)*sqrt(1/2)*sqrt(-a^5/d^2
)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x
- I*c)/d) - sqrt(2)*(-10*I*a^2*e^(10*I*d*x + 10*I*c) - 68*I*a^2*e^(8*I*d*x
+ 8*I*c) - 214*I*a^2*e^(6*I*d*x + 6*I*c) - 544*I*a^2*e^(4*I*d*x + 4*I*c) -
353*I*a^2*e^(2*I*d*x + 2*I*c) + 35*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
)*e^(-2*I*d*x - 2*I*c)/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx=\text{Timed out}$$

```
[In] integrate(cos(d*x+c)**7*(a+I*a*tan(d*x+c))**(5/2),x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx=\text{Timed out}$$

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx=\int (i\,a\tan(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^7\,dx$$

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*cos(d*x + c)^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx=\int \cos(c+d\,x)^7\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{5/2}\,dx$$

```
[In] int(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^(5/2),x)
[Out] int(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.319 $\int \sec^8(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 117

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{16i(a+ia\tan(c+dx))^{15/2}}{15a^4d} + \frac{24i(a+ia\tan(c+dx))^{17/2}}{17a^5d} - \frac{12i(a+ia\tan(c+dx))^{19/2}}{19a^6d} + \frac{2i(a+ia\tan(c+dx))^{21/2}}{21a^7d}$$

```
[Out] -16/15*I*(a+I*a*tan(d*x+c))^(15/2)/a^4/d+24/17*I*(a+I*a*tan(d*x+c))^(17/2)/
a^5/d-12/19*I*(a+I*a*tan(d*x+c))^(19/2)/a^6/d+2/21*I*(a+I*a*tan(d*x+c))^(21
/2)/a^7/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3568, 45}

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{2i(a+ia\tan(c+dx))^{21/2}}{21a^7d} - \frac{12i(a+ia\tan(c+dx))^{19/2}}{19a^6d} + \frac{24i(a+ia\tan(c+dx))^{17/2}}{17a^5d} - \frac{16i(a+ia\tan(c+dx))^{15/2}}{15a^4d}$$

```
[In] Int[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((-16*I)/15)*(a + I*a*Tan[c + d*x])^(15/2))/(a^4*d) + (((24*I)/17)*(a + I*
a*Tan[c + d*x])^(17/2))/(a^5*d) - (((12*I)/19)*(a + I*a*Tan[c + d*x])^(19/2
))/(a^6*d) + (((2*I)/21)*(a + I*a*Tan[c + d*x])^(21/2))/(a^7*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{i\text{Subst}\left(\int (a-x)^3(a+x)^{13/2}\,dx, x, ia\tan(c+dx)\right)}{a^7d}$$

$$= -\frac{i\text{Subst}\left(\int \left(8a^3(a+x)^{13/2} - 12a^2(a+x)^{15/2} + 6a(a+x)^{17/2} - (a+x)^{19/2}\right)\,dx, x, ia\tan(c+dx}{a^7d}$$

$$= -\frac{16i(a+ia\tan(c+dx))^{15/2}}{15a^4d} + \frac{24i(a+ia\tan(c+dx))^{17/2}}{17a^5d} - \frac{12i(a+ia\tan(c+dx))^{19/2}}{19a^6d} + \frac{2i(a+ia\tan(c+dx))^{21/2}}{21a^7d}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.62

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{2a^3(-i+\tan(c+dx))^7\sqrt{a+ia\tan(c+dx)}(-3243+7365i\tan(c+dx)+5865\tan^2(c+dx)}{33915d}$$

```
[In] Integrate[Sec[c + d*x]^8*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (2*a^3*(-I + Tan[c + d*x])^7*Sqrt[a + I*a*Tan[c + d*x]]*(-3243 + (7365*I)*T
an[c + d*x] + 5865*Tan[c + d*x]^2 - (1615*I)*Tan[c + d*x]^3))/(33915*d)
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.70

$$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{21}{2}}}{21}-\frac{6a(a+ia\tan(dx+c))^{\frac{19}{2}}}{19}+\frac{12a^2(a+ia\tan(dx+c))^{\frac{17}{2}}}{17}-\frac{8a^3(a+ia\tan(dx+c))^{\frac{15}{2}}}{15}\right)}{d\,a^7}$$

```
[In] int(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] 2*I/d/a^7*(1/21*(a+I*a*tan(d*x+c))^(21/2)-6/19*a*(a+I*a*tan(d*x+c))^(19/2)+
12/17*a^2*(a+I*a*tan(d*x+c))^(17/2)-8/15*a^3*(a+I*a*tan(d*x+c))^(15/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(85) = 170.

Time = 0.30 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.73

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx =$$

$$-\frac{2048\sqrt{2}\left(16i\,a^3e^{(21i\,dx+21i\,c)}+168i\,a^3e^{(19i\,dx+19i\,c)}+798i\,a^3e^{(17i\,dx+}\right.}{33915\left(de^{(20i\,dx+20i\,c)}+10\,de^{(18i\,dx+18i\,c)}+45\,de^{(16i\,dx+16i\,c)}+120\,de^{(14i\,dx+14i\,c)}+210\,de^{(12i\,dx+12i\,c)}+252\,de^{(1}\right.}$$

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -2048/33915*sqrt(2)*(16*I*a^3*e^(21*I*d*x + 21*I*c) + 168*I*a^3*e^(19*I*d*x
 + 19*I*c) + 798*I*a^3*e^(17*I*d*x + 17*I*c) + 2261*I*a^3*e^(15*I*d*x + 15*
I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(20*I*d*x + 20*I*c) + 10*d*e^(
18*I*d*x + 18*I*c) + 45*d*e^(16*I*d*x + 16*I*c) + 120*d*e^(14*I*d*x + 14*I*
c) + 210*d*e^(12*I*d*x + 12*I*c) + 252*d*e^(10*I*d*x + 10*I*c) + 210*d*e^(8
*I*d*x + 8*I*c) + 120*d*e^(6*I*d*x + 6*I*c) + 45*d*e^(4*I*d*x + 4*I*c) + 10
*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate(sec(d*x+c)**8*(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.65

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{2i\left(1615\,(i\,a\tan(dx+c)+a)^{\frac{21}{2}} - 10710\,(i\,a\tan(dx+c)+a)^{\frac{19}{2}}a + 23940\,(i\,a\tan(dx+c)+\right.}{33915\,a^7d}$$

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 2/33915*I*(1615*(I*a*tan(d*x + c) + a)^(21/2) - 10710*(I*a*tan(d*x + c) + a
)^(19/2)*a + 23940*(I*a*tan(d*x + c) + a)^(17/2)*a^2 - 18088*(I*a*tan(d*x +
 c) + a)^(15/2)*a^3)/(a^7*d)
```

Giac [F]

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{7}{2}}\sec(dx+c)^8\,dx$$

```
[In] integrate(sec(d*x+c)^8*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(7/2)*sec(d*x + c)^8, x)
```

Mupad [B] (verification not implemented)

Time = 16.01 (sec) , antiderivative size = 690, normalized size of antiderivative = 5.90

$$\int \sec^8(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Too large to display}$$

```
[In] int((a + a*tan(c + d*x)*1i)^(7/2)/cos(c + d*x)^8,x)
```

```
[Out] (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/
2)*247808i)/(969*d*(exp(c*2i + d*x*2i) + 1)^4) - (a^3*(a - (a*(exp(c*2i + d
*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*16384i)/(33915*d*(exp(c
*2i + d*x*2i) + 1)) - (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*
2i + d*x*2i) + 1))^(1/2)*4096i)/(11305*d*(exp(c*2i + d*x*2i) + 1)^2) - (a^3
*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*2
048i)/(6783*d*(exp(c*2i + d*x*2i) + 1)^3) - (a^3*(a - (a*(exp(c*2i + d*x*2i
)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*32768i)/(33915*d) - (a^3*(a
- (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*19435
```

```
52i)/(1615*d*(exp(c*2i + d*x*2i) + 1)^5) + (a^3*(a - (a*(exp(c*2i + d*x*2i)
*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*12019712i)/(4845*d*(exp(c*2i
+ d*x*2i) + 1)^6) - (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i
 + d*x*2i) + 1))^(1/2)*95516672i)/(33915*d*(exp(c*2i + d*x*2i) + 1)^7) + (a
^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)
*4159488i)/(2261*d*(exp(c*2i + d*x*2i) + 1)^8) - (a^3*(a - (a*(exp(c*2i + d
*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*260096i)/(399*d*(exp(c*
2i + d*x*2i) + 1)^9) + (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c
*2i + d*x*2i) + 1))^(1/2)*2048i)/(21*d*(exp(c*2i + d*x*2i) + 1)^10)
```

3.320 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{8i(a+ia\tan(c+dx))^{13/2}}{13a^3d} + \frac{8i(a+ia\tan(c+dx))^{15/2}}{15a^4d} - \frac{2i(a+ia\tan(c+dx))^{17/2}}{17a^5d}$$

```
[Out] -8/13*I*(a+I*a*tan(d*x+c))^(13/2)/a^3/d+8/15*I*(a+I*a*tan(d*x+c))^(15/2)/a^
4/d-2/17*I*(a+I*a*tan(d*x+c))^(17/2)/a^5/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3568, 45}

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{2i(a+ia\tan(c+dx))^{17/2}}{17a^5d} + \frac{8i(a+ia\tan(c+dx))^{15/2}}{15a^4d} - \frac{8i(a+ia\tan(c+dx))^{13/2}}{13a^3d}$$

```
[In] Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((-8*I)/13)*(a + I*a*Tan[c + d*x])^(13/2))/(a^3*d) + (((8*I)/15)*(a + I*a*
Tan[c + d*x])^(15/2))/(a^4*d) - (((2*I)/17)*(a + I*a*Tan[c + d*x])^(17/2))/
(a^5*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)^2(a+x)^{11/2}\,dx, x, ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{i\text{Subst}\left(\int \left(4a^2(a+x)^{11/2}-4a(a+x)^{13/2}+(a+x)^{15/2}\right)\,dx, x, ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{8i(a+ia\tan(c+dx))^{13/2}}{13a^3d}+\frac{8i(a+ia\tan(c+dx))^{15/2}}{15a^4d}-\frac{2i(a+ia\tan(c+dx))^{17/2}}{17a^5d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{2a^3(-i+\tan(c+dx))^6\sqrt{a+ia\tan(c+dx)}(331i+494\tan(c+dx)-195i\tan^2(c+dx))}{3315d}$$

```
[In] Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^(7/2),x]
[Out] (2*a^3*(-I + Tan[c + d*x])^6*Sqrt[a + I*a*Tan[c + d*x]]*(331*I + 494*Tan[c
+ d*x] - (195*I)*Tan[c + d*x]^2))/(3315*d)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{17}{2}}}{17}+\frac{4a(a+ia\tan(dx+c))^{\frac{15}{2}}}{15}-\frac{4a^2(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}\right)}{d\,a^5}$$

```
[In] int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x)
[Out] 2*I/d/a^5*(-1/17*(a+I*a*tan(d*x+c))^(17/2)+4/15*a*(a+I*a*tan(d*x+c))^(15/2)
-4/13*a^2*(a+I*a*tan(d*x+c))^(13/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(64) = 128.

Time = 0.27 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.86

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx =$$

$$-\frac{512\sqrt{2}\left(8i\,a^3e^{(17i\,dx+17i\,c)}+68i\,a^3e^{(15i\,dx+15i\,c)}+255i\,a^3e^{(13i\,dx+13i\,c)}\right)\sqrt{\frac{\ }{e^{(2i\,d}}}}{3315\left(de^{(16i\,dx+16i\,c)}+8\,de^{(14i\,dx+14i\,c)}+28\,de^{(12i\,dx+12i\,c)}+56\,de^{(10i\,dx+10i\,c)}+70\,de^{(8i\,dx+8i\,c)}+56\,de^{(6i\,dx+6i}\right.}$$

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -512/3315*sqrt(2)*(8*I*a^3*e^(17*I*d*x + 17*I*c) + 68*I*a^3*e^(15*I*d*x + 1
5*I*c) + 255*I*a^3*e^(13*I*d*x + 13*I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
/(d*e^(16*I*d*x + 16*I*c) + 8*d*e^(14*I*d*x + 14*I*c) + 28*d*e^(12*I*d*x +
12*I*c) + 56*d*e^(10*I*d*x + 10*I*c) + 70*d*e^(8*I*d*x + 8*I*c) + 56*d*e^(6
*I*d*x + 6*I*c) + 28*d*e^(4*I*d*x + 4*I*c) + 8*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.66

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx =$$

$$-\frac{2i\left(195\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{17}{2}}-884\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{15}{2}}a+1020\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{13}{2}}a^2\right)}{3315\,a^5d}$$

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -2/3315*I*(195*(I*a*tan(d*x + c) + a)^(17/2) - 884*(I*a*tan(d*x + c) + a)^(
15/2)*a + 1020*(I*a*tan(d*x + c) + a)^(13/2)*a^2)/(a^5*d)
```

Giac [F]

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{7}{2}}\sec(dx+c)^6\,dx$$

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^(7/2)*sec(d*x + c)^6, x)
```

Mupad [B] (verification not implemented)

Time = 17.40 (sec) , antiderivative size = 562, normalized size of antiderivative = 6.39

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{a^3\sqrt{a-\frac{a\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,4096\mathrm{i}}{3315\,d} - \frac{a^3\sqrt{a-\frac{a\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,2048\mathrm{i}}{3315\,d\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1)} - \frac{a^3\sqrt{a-\frac{a\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,512\mathrm{i}}{1105\,d\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1)^2} + \frac{a^3\sqrt{a-\frac{a\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,56320\mathrm{i}}{663\,d\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1)^3} - \frac{a^3\sqrt{a-\frac{a\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,205312\mathrm{i}}{663\,d\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1)^4} + \frac{a^3\sqrt{a-\frac{a\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,540672\mathrm{i}}{1105\,d\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1)^5} - \frac{a^3\sqrt{a-\frac{a\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,1341952\mathrm{i}}{3315\,d\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1)^6} + \frac{a^3\sqrt{a-\frac{a\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,44032\mathrm{i}}{255\,d\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1)^7} - \frac{a^3\sqrt{a-\frac{a\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,512\mathrm{i}}{17\,d\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1)^8}$$

```
[In] int((a + a*tan(c + d*x)*1i)^(7/2)/cos(c + d*x)^6,x)
[Out] (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/
2)*56320i)/(663*d*(exp(c*2i + d*x*2i) + 1)^3) - (a^3*(a - (a*(exp(c*2i + d*
x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*2048i)/(3315*d*(exp(c*2i
 + d*x*2i) + 1)) - (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i
+ d*x*2i) + 1))^(1/2)*512i)/(1105*d*(exp(c*2i + d*x*2i) + 1)^2) - (a^3*(a -
 (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*4096i)
/(3315*d) - (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2
i) + 1))^(1/2)*205312i)/(663*d*(exp(c*2i + d*x*2i) + 1)^4) + (a^3*(a - (a*(
exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*540672i)/(1
105*d*(exp(c*2i + d*x*2i) + 1)^5) - (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1
i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1341952i)/(3315*d*(exp(c*2i + d*x*2i
) + 1)^6) + (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2
i) + 1))^(1/2)*44032i)/(255*d*(exp(c*2i + d*x*2i) + 1)^7) - (a^3*(a - (a*(e
xp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*512i)/(17*d*
(exp(c*2i + d*x*2i) + 1)^8)
```

3.321 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 59

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{4i(a+ia\tan(c+dx))^{11/2}}{11a^2d}+\frac{2i(a+ia\tan(c+dx))^{13/2}}{13a^3d}$$

```
[Out] -4/11*I*(a+I*a*tan(d*x+c))^(11/2)/a^2/d+2/13*I*(a+I*a*tan(d*x+c))^(13/2)/a^
3/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3568, 45}

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{2i(a+ia\tan(c+dx))^{13/2}}{13a^3d} - \frac{4i(a+ia\tan(c+dx))^{11/2}}{11a^2d}$$

```
[In] Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((-4*I)/11)*(a + I*a*Tan[c + d*x])^(11/2))/(a^2*d) + (((2*I)/13)*(a + I*a*
Tan[c + d*x])^(13/2))/(a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)(a+x)^{9/2}\,dx, x, ia\tan(c+dx)\right)}{a^3d} \\
&= -\frac{i\text{Subst}\left(\int \left(2a(a+x)^{9/2}-(a+x)^{11/2}\right)\,dx, x, ia\tan(c+dx)\right)}{a^3d} \\
&= -\frac{4i(a+ia\tan(c+dx))^{11/2}}{11a^2d}+\frac{2i(a+ia\tan(c+dx))^{13/2}}{13a^3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{2a^3(15-11i\tan(c+dx))(-i+\tan(c+dx))^5\sqrt{a+ia\tan(c+dx)}}{143d}$$

```
[In] Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^(7/2),x]
[Out] (2*a^3*(15 - (11*I)*Tan[c + d*x])*(-I + Tan[c + d*x])^5*Sqrt[a + I*a*Tan[c
+ d*x]])/(143*d)
```

Maple [A] (verified)

Time = 187.49 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}-\frac{2a(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}\right)}{d\,a^3}$	44
default	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}-\frac{2a(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}\right)}{d\,a^3}$	44

```
[In] int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
[Out] 2*I/d/a^3*(1/13*(a+I*a*tan(d*x+c))^(13/2)-2/11*a*(a+I*a*tan(d*x+c))^(11/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(43) = 86.

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.14

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{128\sqrt{2}\left(2i\,a^3e^{(13i\,dx+13i\,c)}+13i\,a^3e^{(11i\,dx+11i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{143\left(de^{(12i\,dx+12i\,c)}+6\,de^{(10i\,dx+10i\,c)}+15\,de^{(8i\,dx+8i\,c)}+20\,de^{(6i\,dx+6i\,c)}+15\,de^{(4i\,dx+4i\,c)}+6\,de^{(2i\,dx+2i\,c)}+d\right)}$$

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
[Out] -128/143*sqrt(2)*(2*I*a^3*e^(13*I*d*x + 13*I*c) + 13*I*a^3*e^(11*I*d*x + 11
*I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(12*I*d*x + 12*I*c) + 6*d*e^(
10*I*d*x + 10*I*c) + 15*d*e^(8*I*d*x + 8*I*c) + 20*d*e^(6*I*d*x + 6*I*c) +
15*d*e^(4*I*d*x + 4*I*c) + 6*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**(7/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.68

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{2i\left(11\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{13}{2}}-26\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{11}{2}}a\right)}{143\,a^3d}$$

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
[Out] 2/143*I*(11*(I*a*tan(d*x + c) + a)^(13/2) - 26*(I*a*tan(d*x + c) + a)^(11/2
)*a)/(a^3*d)
```

Giac [F]

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{7}{2}}\sec(dx+c)^4\,dx$$

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^(7/2)*sec(d*x + c)^4, x)
```

Mupad [B] (verification not implemented)

Time = 8.68 (sec) , antiderivative size = 434, normalized size of antiderivative = 7.36

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{a^3\sqrt{a-\frac{a\,(e^{c\,2i+d\,x\,2i}\,1i-i)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,256i}{143\,d} - \frac{a^3\sqrt{a-\frac{a\,(e^{c\,2i+d\,x\,2i}\,1i-i)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,128i}{143\,d\,(e^{c\,2i+d\,x\,2i}+1)} + \frac{a^3\sqrt{a-\frac{a\,(e^{c\,2i+d\,x\,2i}\,1i-i)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,4480i}{143\,d\,(e^{c\,2i+d\,x\,2i}+1)^2} - \frac{a^3\sqrt{a-\frac{a\,(e^{c\,2i+d\,x\,2i}\,1i-i)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,11520i}{143\,d\,(e^{c\,2i+d\,x\,2i}+1)^3} + \frac{a^3\sqrt{a-\frac{a\,(e^{c\,2i+d\,x\,2i}\,1i-i)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,12800i}{143\,d\,(e^{c\,2i+d\,x\,2i}+1)^4} - \frac{a^3\sqrt{a-\frac{a\,(e^{c\,2i+d\,x\,2i}\,1i-i)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,6784i}{143\,d\,(e^{c\,2i+d\,x\,2i}+1)^5} + \frac{a^3\sqrt{a-\frac{a\,(e^{c\,2i+d\,x\,2i}\,1i-i)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,128i}{13\,d\,(e^{c\,2i+d\,x\,2i}+1)^6}$$

```
[In] int((a + a*tan(c + d*x)*1i)^(7/2)/cos(c + d*x)^4,x)
[Out] (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/
2)*4480i)/(143*d*(exp(c*2i + d*x*2i) + 1)^2) - (a^3*(a - (a*(exp(c*2i + d*x
*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*128i)/(143*d*(exp(c*2i +
d*x*2i) + 1)) - (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d
*x*2i) + 1))^(1/2)*256i)/(143*d) - (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i
)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*11520i)/(143*d*(exp(c*2i + d*x*2i) +
1)^3) + (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) +
 1))^(1/2)*12800i)/(143*d*(exp(c*2i + d*x*2i) + 1)^4) - (a^3*(a - (a*(exp(c
*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*6784i)/(143*d*(e
xp(c*2i + d*x*2i) + 1)^5) + (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(
exp(c*2i + d*x*2i) + 1))^(1/2)*128i)/(13*d*(exp(c*2i + d*x*2i) + 1)^6)
```

3.322 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 29

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx=-\frac{2i(a+ia\tan(c+dx))^{9/2}}{9ad}$$

```
[Out] -2/9*I*(a+I*a*tan(d*x+c))^(9/2)/a/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3568, 32}

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx=-\frac{2i(a+ia\tan(c+dx))^{9/2}}{9ad}$$

```
[In] Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((-2*I)/9)*(a + I*a*Tan[c + d*x])^(9/2))/(a*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int (a+x)^{7/2}\,dx, x, ia\tan(c+dx)\right)}{ad}\\ &= -\frac{2i(a+ia\tan(c+dx))^{9/2}}{9ad}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{2i(a+ia\tan(c+dx))^{9/2}}{9ad}$$

```
[In] Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^(7/2),x]
[Out] (((-2*I)/9)*(a + I*a*Tan[c + d*x])^(9/2))/(a*d)
```

Maple [A] (verified)

Time = 4.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{2i(a+ia\tan(dx+c))^{\frac{9}{2}}}{9ad}$	24
default	$-\frac{2i(a+ia\tan(dx+c))^{\frac{9}{2}}}{9ad}$	24

```
[In] int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
[Out] -2/9*I*(a+I*a*tan(d*x+c))^(9/2)/a/d
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(21) = 42.

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.93

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{32i\sqrt{2}a^3\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(9i\,dx+9i\,c)}}{9\left(de^{(8i\,dx+8i\,c)}+4\,de^{(6i\,dx+6i\,c)}+6\,de^{(4i\,dx+4i\,c)}+4\,de^{(2i\,dx+2i\,c)}+d\right)}$$

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -32/9*I*sqrt(2)*a^3*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(9*I*d*x + 9*I*c)/(
d*e^(8*I*d*x + 8*I*c) + 4*d*e^(6*I*d*x + 6*I*c) + 6*d*e^(4*I*d*x + 4*I*c) +
 4*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**(7/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{2i\,(i\,a\tan\left(dx+c\right)+a)^{\frac{9}{2}}}{9\,ad}$$

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
[Out] -2/9*I*(I*a*tan(d*x + c) + a)^(9/2)/(a*d)
```

Giac [F]

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \int (i\,a\tan\left(dx+c\right)+a)^{\frac{7}{2}}\sec\left(dx+c\right)^2\,dx$$

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^(7/2)*sec(d*x + c)^2, x)
```

Mupad [B] (verification not implemented)

Time = 6.92 (sec) , antiderivative size = 306, normalized size of antiderivative = 10.55

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{a^3\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,32i}{9\,d} + \frac{a^3\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,128i}{9\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)} - \frac{a^3\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,64i}{3\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^2} + \frac{a^3\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,128i}{9\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^3} - \frac{a^3\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,32i}{9\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^4}$$

```
[In] int((a + a*tan(c + d*x)*1i)^(7/2)/cos(c + d*x)^2,x)
```

```
[Out] (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/
2)*128i)/(9*d*(exp(c*2i + d*x*2i) + 1)) - (a^3*(a - (a*(exp(c*2i + d*x*2i)*
1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*32i)/(9*d) - (a^3*(a - (a*(exp
(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*64i)/(3*d*(exp
(c*2i + d*x*2i) + 1)^2) + (a^3*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(ex
p(c*2i + d*x*2i) + 1))^(1/2)*128i)/(9*d*(exp(c*2i + d*x*2i) + 1)^3) - (a^3*
(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*32
i)/(9*d*(exp(c*2i + d*x*2i) + 1)^4)
```

3.323 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 116

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{3i\sqrt{2}a^{7/2}\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{3ia^3\sqrt{a+ia\tan(c+dx)}}{d} - \frac{ia^3(a+ia\tan(c+dx))^{3/2}}{d(a-ia\tan(c+dx))}$$

```
[Out] 3*I*a^(7/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)/d
-3*I*a^3*(a+I*a*tan(d*x+c))^(1/2)/d-I*a^3*(a+I*a*tan(d*x+c))^(3/2)/d/(a-I*a
*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3568, 43, 52, 65, 212}

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{3i\sqrt{2}a^{7/2}\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{ia^3(a+ia\tan(c+dx))^{3/2}}{d(a-ia\tan(c+dx))} - \frac{3ia^3\sqrt{a+ia\tan(c+dx)}}{d}$$

```
[In] Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((3*I)*Sqrt[2]*a^(7/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])
])/d - ((3*I)*a^3*Sqrt[a + I*a*Tan[c + d*x]])/d - (I*a^3*(a + I*a*Tan[c + d
*x])^(3/2))/(d*(a - I*a*Tan[c + d*x]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(ia^3)\operatorname{Subst}\left(\int \frac{(a+x)^{3/2}}{(a-x)^2}\,dx, x, ia\tan(c+dx)\right)}{d} \\
&= -\frac{ia^3(a+ia\tan(c+dx))^{3/2}}{d(a-ia\tan(c+dx))} + \frac{(3ia^3)\operatorname{Subst}\left(\int \frac{\sqrt{a+x}}{a-x}\,dx, x, ia\tan(c+dx)\right)}{2d}
\end{aligned}
$$

$$= -\frac{3ia^3\sqrt{a+ia\tan(c+dx)}}{d} - \frac{ia^3(a+ia\tan(c+dx))^{3/2}}{d(a-ia\tan(c+dx))} + \frac{(3ia^4)\operatorname{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{3ia^3\sqrt{a+ia\tan(c+dx)}}{d} - \frac{ia^3(a+ia\tan(c+dx))^{3/2}}{d(a-ia\tan(c+dx))} + \frac{(6ia^4)\operatorname{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+ia\tan(c+dx)}\right)}{d}$$

$$= \frac{3i\sqrt{2}a^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{d} - \frac{3ia^3\sqrt{a+ia\tan(c+dx)}}{d} - \frac{ia^3(a+ia\tan(c+dx))^{3/2}}{d(a-ia\tan(c+dx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.44

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{ia\,\text{Hypergeometric2F1}\left(2,\frac{5}{2},\frac{7}{2},\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^{5/2}}{10d}$$

```
[In] Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^(7/2),x]
[Out] ((-1/10*I)*a*Hypergeometric2F1[2, 5/2, 7/2, (1 + I*Tan[c + d*x])/2]*(a + I*
a*Tan[c + d*x])^(5/2))/d
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 640 vs. 2(96) = 192.

Time = 38.36 (sec) , antiderivative size = 641, normalized size of antiderivative = 5.53

method	result
default	$-\frac{2i(\tan(dx+c)-i)^3\sqrt{a(1+i\tan(dx+c))}\,a^3(\cos^3(dx+c))\left(3i\operatorname{arctanh}\left(\frac{\sin(dx+c)}{(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}\cos(dx+c)\ldots\right.}{\ldots}$

```
[In] int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
[Out] -2*I/d*(tan(d*x+c)-I)^3*(a*(1+I*tan(d*x+c)))^(1/2)*a^3*cos(d*x+c)^3*(3*I*ar
ctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d
```

```
*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)-3*I*(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2+3*I*(
-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d
*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-3*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-3*arctanh(sin(d*x
+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*cos(d*x+c)^2-3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-c
os(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*sin(d*x+c)-I*cos(d*x+c)^2-3*(-c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x
+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-2*I*cos(d*x+c)+sin(d*
x+c)*cos(d*x+c)-I-sin(d*x+c))/(4*cos(d*x+c)^3+2*cos(d*x+c)^2+4*I*cos(d*x+c)
^2*sin(d*x+c)-3*cos(d*x+c)+2*I*cos(d*x+c)*sin(d*x+c)-1-I*sin(d*x+c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(89) = 178.

Time = 0.26 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.03

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{3\sqrt{2}\sqrt{-\frac{a^7}{d^2}}d\log\left(\frac{4\left(a^4e^{(i\,dx+i\,c)}+\sqrt{-\frac{a^7}{d^2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a^3}\right)-3\sqrt{2}\sqrt{-\frac{a^7}{d^2}}d\log\left(\frac{4\left(a^4e^{(i\,dx+i\,c)}\right.}{}\right.}{2\,d}$$

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/2*(3*sqrt(2)*sqrt(-a^7/d^2)*d*log(4*(a^4*e^(I*d*x + I*c) + sqrt(-a^7/d^2
)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*
d*x - I*c)/a^3) - 3*sqrt(2)*sqrt(-a^7/d^2)*d*log(4*(a^4*e^(I*d*x + I*c) + s
qrt(-a^7/d^2)*(-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1)))*e^(-I*d*x - I*c)/a^3) + 2*sqrt(2)*(I*a^3*e^(3*I*d*x + 3*I*c) + 3*I*a
^3*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**(7/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.01

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{i\left(3\sqrt{2}a^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+4\sqrt{i\,a\tan(dx+c)+a}\,a^4-\frac{4\sqrt{i\,a\tan(dx+c)+a}\,a^5}{i\,a\tan(dx+c)-a}\right)}{2\,ad}$$

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
[Out] -1/2*I*(3*sqrt(2)*a^(9/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a
))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*sqrt(I*a*tan(d*x + c
) + a)*a^4 - 4*sqrt(I*a*tan(d*x + c) + a)*a^5/(I*a*tan(d*x + c) - a))/(a*d)
```

Giac [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \int \cos\left(c+d\,x\right)^2\,\left(a+a\tan(c+d\,x)\;1\mathrm{i}\right)^{7/2}\,dx$$

```
[In] int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(7/2),x)
[Out] int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(7/2), x)
```

3.324 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 137

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{ia^{7/2}\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}d} - \frac{ia^5\sqrt{a+ia\tan(c+dx)}}{2d(a-ia\tan(c+dx))^2} + \frac{ia^4\sqrt{a+ia\tan(c+dx)}}{8d(a-ia\tan(c+dx))}$$

```
[Out] 1/16*I*a^(7/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1
/2)-1/2*I*a^5*(a+I*a*tan(d*x+c))^(1/2)/d/(a-I*a*tan(d*x+c))^2+1/8*I*a^4*(a+
I*a*tan(d*x+c))^(1/2)/d/(a-I*a*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3568, 43, 44, 65, 212}

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{ia^{7/2}\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}d} - \frac{ia^5\sqrt{a+ia\tan(c+dx)}}{2d(a-ia\tan(c+dx))^2} + \frac{ia^4\sqrt{a+ia\tan(c+dx)}}{8d(a-ia\tan(c+dx))}$$

```
[In] Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((I/8)*a^(7/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt
[2]*d) - ((I/2)*a^5*Sqrt[a + I*a*Tan[c + d*x]])/(d*(a - I*a*Tan[c + d*x])^2
) + ((I/8)*a^4*Sqrt[a + I*a*Tan[c + d*x]])/(d*(a - I*a*Tan[c + d*x]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(ia^5)\,\text{Subst}\left(\int \frac{\sqrt{a+x}}{(a-x)^3}\,dx, x, ia\tan(c+dx)\right)}{d}\\ &= -\frac{ia^5\sqrt{a+ia\tan(c+dx)}}{2d(a-ia\tan(c+dx))^2}+\frac{(ia^5)\,\text{Subst}\left(\int \frac{1}{(a-x)^2\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{4d}\end{aligned}$$

$$= -\frac{ia^5\sqrt{a+ia\tan(c+dx)}}{2d(a-ia\tan(c+dx))^2} + \frac{ia^4\sqrt{a+ia\tan(c+dx)}}{8d(a-ia\tan(c+dx))} + \frac{(ia^4)\operatorname{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{16d}$$

$$= -\frac{ia^5\sqrt{a+ia\tan(c+dx)}}{2d(a-ia\tan(c+dx))^2} + \frac{ia^4\sqrt{a+ia\tan(c+dx)}}{8d(a-ia\tan(c+dx))} + \frac{(ia^4)\operatorname{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+ia\tan(c+dx)}\right)}{8d}$$

$$= \frac{ia^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}d} - \frac{ia^5\sqrt{a+ia\tan(c+dx)}}{2d(a-ia\tan(c+dx))^2} + \frac{ia^4\sqrt{a+ia\tan(c+dx)}}{8d(a-ia\tan(c+dx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.39

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{ia^2\operatorname{Hypergeometric2F1}\left(\frac{3}{2},3,\frac{5}{2},\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^{3/2}}{12d}$$

```
[In] Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((-1/12*I)*a^2*Hypergeometric2F1[3/2, 3, 5/2, (1 + I*Tan[c + d*x])/2]*(a +
I*a*Tan[c + d*x])^(3/2))/d
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 868 vs. 2(110) = 220.

Time = 123.98 (sec) , antiderivative size = 869, normalized size of antiderivative = 6.34

method	result	size
default	Expression too large to display	869

```
[In] int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d*(-tan(d*x+c)+I)^3*(a*(1+I*tan(d*x+c)))^(1/2)*a^3*cos(d*x+c)^3*(I*(-co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*c
os(d*x+c)-3*I*cos(d*x+c)^2+2*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(s
in(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*sin
```

```
(d*x+c)-4*I*cos(d*x+c)^3-2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d
*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^3-2*(-c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*
cos(d*x+c)^2*sin(d*x+c)+I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d
*x+c)/(cos(d*x+c)+1))^(1/2))+2*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh
(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2
*sin(d*x+c)-2*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2-2*(-cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*si
n(d*x+c)-2*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2))*cos(d*x+c)^2+I*cos(d*x+c)+(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos
(d*x+c)+4*cos(d*x+c)^2*sin(d*x+c)-2*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*ar
ctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^3+(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2))+sin(d*x+c)*cos(d*x+c))/(-2*I*cos(d*x+c)^2+2*sin(d*x+c)*cos(d*x+c)-
I*cos(d*x+c)+sin(d*x+c)+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(102) = 204$.

Time = 0.24 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.91

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}d\log\left(\frac{4\left(a^4e^{(i\,dx+i\,c)}-\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a^3}\right)-\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}d\,l}{}$$

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/16*(sqrt(1/2)*sqrt(-a^7/d^2)*d*log(4*(a^4*e^(I*d*x + I*c) - sqrt(2)*sqrt(
1/2)*sqrt(-a^7/d^2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*
I*c) + 1)))*e^(-I*d*x - I*c)/a^3) - sqrt(1/2)*sqrt(-a^7/d^2)*d*log(4*(a^4*e
^(I*d*x + I*c) - sqrt(2)*sqrt(1/2)*sqrt(-a^7/d^2)*(-I*d*e^(2*I*d*x + 2*I*c)
 - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a^3) + sqrt(2)*
(-2*I*a^3*e^(5*I*d*x + 5*I*c) - 3*I*a^3*e^(3*I*d*x + 3*I*c) - I*a^3*e^(I*d*
x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**(7/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.01

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{i\left(\sqrt{2}a^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+\frac{4\left((i\,a\tan(dx+c)+a)^{\frac{3}{2}}a^5+2\sqrt{i\,a\tan(dx+c)+a}a^6\right)}{(i\,a\tan(dx+c)+a)^2-4\,(i\,a\tan(dx+c)+a)a+4\,a^2}\right)}{32\,ad}$$

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
[Out] -1/32*I*(sqrt(2)*a^(9/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a)
)/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*((I*a*tan(d*x + c) +
a)^(3/2)*a^5 + 2*sqrt(I*a*tan(d*x + c) + a)*a^6)/((I*a*tan(d*x + c) + a)^2
- 4*(I*a*tan(d*x + c) + a)*a + 4*a^2))/(a*d)
```

Giac [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \int \cos\left(c+d\,x\right)^4\,\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^{7/2}\,dx$$

```
[In] int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(7/2),x)
[Out] int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(7/2), x)
```

3.325 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 181

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{5ia^{7/2}\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}d} - \frac{ia^6\sqrt{a+ia\tan(c+dx)}}{6d(a-ia\tan(c+dx))^3} - \frac{5ia^5\sqrt{a+ia\tan(c+dx)}}{48d(a-ia\tan(c+dx))^2} - \frac{5ia^4\sqrt{a+ia\tan(c+dx)}}{64d(a-ia\tan(c+dx))}$$

```
[Out] -5/128*I*a^(7/2)*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^
(1/2)-1/6*I*a^6*(a+I*a*tan(d*x+c))^(1/2)/d/(a-I*a*tan(d*x+c))^3-5/48*I*a^5*
(a+I*a*tan(d*x+c))^(1/2)/d/(a-I*a*tan(d*x+c))^2-5/64*I*a^4*(a+I*a*tan(d*x+c
))^(1/2)/d/(a-I*a*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3568, 44, 65, 212}

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{5ia^{7/2}\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}d} - \frac{ia^6\sqrt{a+ia\tan(c+dx)}}{6d(a-ia\tan(c+dx))^3} - \frac{5ia^5\sqrt{a+ia\tan(c+dx)}}{48d(a-ia\tan(c+dx))^2} - \frac{5ia^4\sqrt{a+ia\tan(c+dx)}}{64d(a-ia\tan(c+dx))}$$

```
[In] Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((-5*I)/64)*a^(7/2)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])
/(Sqrt[2]*d) - ((I/6)*a^6*Sqrt[a + I*a*Tan[c + d*x]])/(d*(a - I*a*Tan[c + d
*x])^3) - (((5*I)/48)*a^5*Sqrt[a + I*a*Tan[c + d*x]])/(d*(a - I*a*Tan[c + d
```

```
*x])^2) - (((5*I)/64)*a^4*Sqrt[a + I*a*Tan[c + d*x]])/(d*(a - I*a*Tan[c + d
*x]))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(ia^7)\,\text{Subst}\left(\int \frac{1}{(a-x)^4\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{d} \\
&= -\frac{ia^6\sqrt{a+ia\tan(c+dx)}}{6d(a-ia\tan(c+dx))^3} - \frac{(5ia^6)\,\text{Subst}\left(\int \frac{1}{(a-x)^3\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{12d} \\
&= -\frac{ia^6\sqrt{a+ia\tan(c+dx)}}{6d(a-ia\tan(c+dx))^3} - \frac{5ia^5\sqrt{a+ia\tan(c+dx)}}{48d(a-ia\tan(c+dx))^2} \\
&\quad - \frac{(5ia^5)\,\text{Subst}\left(\int \frac{1}{(a-x)^2\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{32d}
\end{aligned}$$

$$= -\frac{ia^6\sqrt{a+ia\tan(c+dx)}}{6d(a-ia\tan(c+dx))^3} - \frac{5ia^5\sqrt{a+ia\tan(c+dx)}}{48d(a-ia\tan(c+dx))^2} - \frac{5ia^4\sqrt{a+ia\tan(c+dx)}}{64d(a-ia\tan(c+dx))} - \frac{(5ia^4)\,\mathrm{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{128d}$$

$$= -\frac{ia^6\sqrt{a+ia\tan(c+dx)}}{6d(a-ia\tan(c+dx))^3} - \frac{5ia^5\sqrt{a+ia\tan(c+dx)}}{48d(a-ia\tan(c+dx))^2} - \frac{5ia^4\sqrt{a+ia\tan(c+dx)}}{64d(a-ia\tan(c+dx))} - \frac{(5ia^4)\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+ia\tan(c+dx)}\right)}{64d}$$

$$= -\frac{5ia^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}d} - \frac{ia^6\sqrt{a+ia\tan(c+dx)}}{6d(a-ia\tan(c+dx))^3} - \frac{5ia^5\sqrt{a+ia\tan(c+dx)}}{48d(a-ia\tan(c+dx))^2} - \frac{5ia^4\sqrt{a+ia\tan(c+dx)}}{64d(a-ia\tan(c+dx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.29

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{ia^3\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2},4,\frac{3}{2},\frac{1}{2}(1+i\tan(c+dx))\right)\sqrt{a+ia\tan(c+dx)}}{8d}$$

```
[In] Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^(7/2),x]
[Out] ((-1/8*I)*a^3*Hypergeometric2F1[1/2, 4, 3/2, (1 + I*Tan[c + d*x])/2]*Sqrt[a
 + I*a*Tan[c + d*x]])/d
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1135 vs. 2(147) = 294.

Time = 4.65 (sec) , antiderivative size = 1136, normalized size of antiderivative = 6.28

Expression too large to display

```
[In] int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x)
[Out] 1/192/d*(tan(d*x+c)-I)^3*(a*(1+I*tan(d*x+c)))^(1/2)*a^3*cos(d*x+c)^3*(45*I*
(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2
))*cos(d*x+c)-15*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(c
```

```
os(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*sin(d*x+c)+35*I
*cos(d*x+c)^2+60*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(c
os(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^3*sin(d*x+c)-60
*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos
(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^4-60*(-cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^3*sin(d*x+c)+8
2*I*cos(d*x+c)^3+32*I*cos(d*x+c)^4-15*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d
*x+c)-60*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+
1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^3-60*(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2*sin(
d*x+c)+60*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+
c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2*sin(d*x+c)+45*I*(-co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*c
os(d*x+c)^2-15*I*cos(d*x+c)+45*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+
c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2+1
5*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1
/2))*cos(d*x+c)*sin(d*x+c)-32*cos(d*x+c)^3*sin(d*x+c)-60*I*(-cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^4+
45*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-c
os(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)+15*(-cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+50*cos(d*x+c)^
2*sin(d*x+c)-60*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(c
os(d*x+c)+1))^(1/2))*cos(d*x+c)^3+15*sin(d*x+c)*cos(d*x+c))/(-I*cos(d*x+c)+
sin(d*x+c)-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(136) = 272$.

Time = 0.25 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.53

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{15\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}d\log\left(\frac{4\left(a^4e^{(i\,dx+i\,c)}-\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{a^3}\right)-15\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}d\log\left(\frac{4}{-}\right.}{}$$

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/384*(15*sqrt(1/2)*sqrt(-a^7/d^2)*d*log(4*(a^4*e^(I*d*x + I*c) - sqrt(2)*
sqrt(1/2)*sqrt(-a^7/d^2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x
 + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a^3) - 15*sqrt(1/2)*sqrt(-a^7/d^2)*d*log(
4*(a^4*e^(I*d*x + I*c) - sqrt(2)*sqrt(1/2)*sqrt(-a^7/d^2)*(-I*d*e^(2*I*d*x
```

```
+ 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/a^3) -
sqrt(2)*(-8*I*a^3*e^(7*I*d*x + 7*I*c) - 34*I*a^3*e^(5*I*d*x + 5*I*c) - 59*I
*a^3*e^(3*I*d*x + 3*I*c) - 33*I*a^3*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1)))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**(7/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.97

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{i\left(15\sqrt{2}a^{\frac{9}{2}}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+\frac{4\left(15\,(i\,a\tan(dx+c)+a)^{\frac{5}{2}}a^5-80\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a^6+132\,\sqrt{i\,a\tan(dx+c)+a}\,a^7\right)}{(i\,a\tan(dx+c)+a)^3-6\,(i\,a\tan(dx+c)+a)^2a+12\,(i\,a\tan(dx+c)+a)a^2-8\,a^3}\right)}{768\,ad}$$

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
[Out] 1/768*I*(15*sqrt(2)*a^(9/2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) +
 a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(15*(I*a*tan(d*x +
 c) + a)^(5/2)*a^5 - 80*(I*a*tan(d*x + c) + a)^(3/2)*a^6 + 132*sqrt(I*a*tan
(d*x + c) + a)*a^7)/((I*a*tan(d*x + c) + a)^3 - 6*(I*a*tan(d*x + c) + a)^2*
a + 12*(I*a*tan(d*x + c) + a)*a^2 - 8*a^3))/(a*d)
```

Giac [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx=\int \cos\left(c+d\,x\right)^6\,\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^{7/2}\,dx$$

```
[In] int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(7/2),x)
[Out] int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(7/2), x)
```

3.326 $\int \sec(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 139

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{256ia^4\sec(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}} + \frac{64ia^3\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{35d} + \frac{24ia^2\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}{35d} + \frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

```
[Out] 256/35*I*a^4*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)+64/35*I*a^3*sec(d*x+c)*(
a+I*a*tan(d*x+c))^(1/2)/d+24/35*I*a^2*sec(d*x+c)*(a+I*a*tan(d*x+c))^(3/2)/d
+2/7*I*a*sec(d*x+c)*(a+I*a*tan(d*x+c))^(5/2)/d
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3575, 3574}

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{256ia^4\sec(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}} + \frac{64ia^3\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{35d} + \frac{24ia^2\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}{35d} + \frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}$$

```
[In] Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((256*I)/35)*a^4*Sec[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((64*I)/3
5)*a^3*Sec[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d + (((24*I)/35)*a^2*Sec[c
+ d*x]*(a + I*a*Tan[c + d*x])^(3/2))/d + (((2*I)/7)*a*Sec[c + d*x]*(a + I*a
*Tan[c + d*x])^(5/2))/d
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d} \\
&\quad +\frac{1}{7}(12a)\int \sec(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx \\
&= \frac{24ia^2\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}{35d}+\frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d} \\
&\quad +\frac{1}{35}\left(96a^2\right)\int \sec(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx \\
&= \frac{64ia^3\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{35d}+\frac{24ia^2\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}{35d} \\
&\quad +\frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}+\frac{1}{35}\left(128a^3\right)\int \sec(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx \\
&= \frac{256ia^4\sec(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}}+\frac{64ia^3\sec(c+dx)\sqrt{a+ia\tan(c+dx)}}{35d} \\
&\quad +\frac{24ia^2\sec(c+dx)(a+ia\tan(c+dx))^{3/2}}{35d}+\frac{2ia\sec(c+dx)(a+ia\tan(c+dx))^{5/2}}{7d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.78

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{2a^3\sec^2(c+dx)(i\cos(c-2dx)+\sin(c-2dx))(75+102\cos(2(c+dx))+19i\sec(c+dx)\sin}{35d(\cos(dx)+i\sin(dx))^3}$$

```
[In] Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (2*a^3*Sec[c + d*x]^2*(I*Cos[c - 2*d*x] + Sin[c - 2*d*x])*(75 + 102*Cos[2*(
c + d*x)] + (19*I)*Sec[c + d*x]*Sin[3*(c + d*x)] + (14*I)*Tan[c + d*x])*Sqr
t[a + I*a*Tan[c + d*x]])/(35*d*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 234 vs. 2(115) = 230.

Time = 7.88 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.69

method	result
default	$\frac{2(-\tan(dx+c)+i)^3a^3\sqrt{a(1+i\tan(dx+c))}\,(128i(\cos^3(dx+c))\sin(dx+c)-76i(\cos^2(dx+c))\sin(dx+c)-128(\cos^4(dx+c))-22i\cos(dx}{35d(8(\cos^4(dx+c))+4(\cos^3(dx+c))+8i(\cos^3(dx+c))\sin(dx+c)-8(\cos^2(dx+c))+4i(\cos^2(dx+c))\sin(dx+}$

```
[In] int(sec(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/35/d*(-tan(d*x+c)+I)^3*a^3*(a*(1+I*tan(d*x+c)))^(1/2)*(128*I*cos(d*x+c)^3
*sin(d*x+c)-76*I*cos(d*x+c)^2*sin(d*x+c)-128*cos(d*x+c)^4-22*I*cos(d*x+c)*s
in(d*x+c)-204*cos(d*x+c)^3+5*I*sin(d*x+c)-54*cos(d*x+c)^2+27*cos(d*x+c)+5)/
(8*cos(d*x+c)^4+4*cos(d*x+c)^3+8*I*sin(d*x+c)*cos(d*x+c)^3-8*cos(d*x+c)^2+4
*I*cos(d*x+c)^2*sin(d*x+c)-3*cos(d*x+c)-4*I*sin(d*x+c)*cos(d*x+c)+1-I*sin(d
*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.78

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{16\sqrt{2}\left(-35i\,a^3e^{(6i\,dx+6i\,c)}-70i\,a^3e^{(4i\,dx+4i\,c)}-56i\,a^3e^{(2i\,dx+2i\,c)}-16i\,a^3\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{35\left(de^{(6i\,dx+6i\,c)}+3\,de^{(4i\,dx+4i\,c)}+3\,de^{(2i\,dx+2i\,c)}+d\right)}$$

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -16/35*sqrt(2)*(-35*I*a^3*e^(6*I*d*x + 6*I*c) - 70*I*a^3*e^(4*I*d*x + 4*I*c
) - 56*I*a^3*e^(2*I*d*x + 2*I*c) - 16*I*a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
1))/(d*e^(6*I*d*x + 6*I*c) + 3*d*e^(4*I*d*x + 4*I*c) + 3*d*e^(2*I*d*x + 2*I
*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \int (i\,a\tan\left(dx+c\right)+a)^{\frac{7}{2}}\sec\left(dx+c\right)\,dx$$

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(7/2)*sec(d*x + c), x)
```

Giac [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \int (i\,a\tan\left(dx+c\right)+a)^{\frac{7}{2}}\sec\left(dx+c\right)\,dx$$

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(7/2)*sec(d*x + c), x)
```

Mupad [B] (verification not implemented)

Time = 6.74 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.06

$$\int \sec(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{a^3\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,16\mathrm{i}}{d} - \frac{a^3\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,16\mathrm{i}}{d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)} + \frac{a^3\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,48\mathrm{i}}{5\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2} - \frac{a^3\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,16\mathrm{i}}{7\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}$$

```
[In] int((a + a*tan(c + d*x)*1i)^(7/2)/cos(c + d*x),x)
```

```
[Out] (a^3*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2
i + d*x*2i) + 1))^(1/2)*16i)/d - (a^3*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2
i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*16i)/(d*(exp(c*2i
+ d*x*2i) + 1)) + (a^3*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i
- 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*48i)/(5*d*(exp(c*2i + d*x*2i) + 1
)^2) - (a^3*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(
exp(c*2i + d*x*2i) + 1))^(1/2)*16i)/(7*d*(exp(c*2i + d*x*2i) + 1)^3)
```

3.327 $\int \cos(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 104

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{64ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d} + \frac{16ia^2\cos(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d} + \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{5/2}}{3d}$$

```
[Out] -64/3*I*a^3*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d+16/3*I*a^2*cos(d*x+c)*(a+
I*a*tan(d*x+c))^(3/2)/d+2/3*I*a*cos(d*x+c)*(a+I*a*tan(d*x+c))^(5/2)/d
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3575, 3574}

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{64ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d} + \frac{16ia^2\cos(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d} + \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{5/2}}{3d}$$

```
[In] Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((-64*I)/3)*a^3*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/d + (((16*I)/3)*a
^2*Cos[c + d*x]*(a + I*a*Tan[c + d*x])^(3/2))/d + (((2*I)/3)*a*Cos[c + d*x]
*(a + I*a*Tan[c + d*x])^(5/2))/d
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
```

```
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{5/2}}{3d} \\
&\quad + \frac{1}{3}(8a)\int\cos(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx \\
&= \frac{16ia^2\cos(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d} + \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{5/2}}{3d} \\
&\quad + \frac{1}{3}(32a^2)\int\cos(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx \\
&= -\frac{64ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{3d} + \frac{16ia^2\cos(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d} \\
&\quad + \frac{2ia\cos(c+dx)(a+ia\tan(c+dx))^{5/2}}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.57

$$\int\cos(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{2ia^3\sec(c+dx)(12+11\cos(2(c+dx))-5i\sin(2(c+dx)))\sqrt{a+ia\tan(c+dx)}}{3d}$$

```
[In] Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^(7/2),x]
[Out] (((-2*I)/3)*a^3*Sec[c + d*x]*(12 + 11*Cos[2*(c + d*x)] - (5*I)*Sin[2*(c + d
*x)])*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Maple [A] (verified)

Time = 29.97 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.17

method	result	size
default	$-\frac{2(-\tan(dx+c)+i)^{3}\sqrt{a(1+i\tan(dx+c))}\, a^{3}\left(10 i\left(\cos^{3}(dx+c)\right)\sin(dx+c)-22\left(\cos^{4}(dx+c)\right)-\left(\cos^{2}(dx+c)\right)\right)}{3d\left(4 i\left(\cos^{2}(dx+c)\right)\sin(dx+c)+4\left(\cos^{3}(dx+c)\right)-i\sin(dx+c)-3\cos(dx+c)\right)}$	122

```
[In] int(cos(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
[Out] -2/3/d*(-tan(d*x+c)+I)^3*(a*(1+I*tan(d*x+c)))^(1/2)*a^3/(4*I*cos(d*x+c)^2*s
in(d*x+c)+4*cos(d*x+c)^3-I*sin(d*x+c)-3*cos(d*x+c))*(10*I*cos(d*x+c)^3*sin(
d*x+c)-22*cos(d*x+c)^4-cos(d*x+c)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.68

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{4\sqrt{2}\left(3i\,a^{3}e^{(4i\,dx+4i\,c)}+12i\,a^{3}e^{(2i\,dx+2i\,c)}+8i\,a^{3}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{3\left(de^{(2i\,dx+2i\,c)}+d\right)}$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
[Out] -4/3*sqrt(2)*(3*I*a^3*e^(4*I*d*x + 4*I*c) + 12*I*a^3*e^(2*I*d*x + 2*I*c) +
8*I*a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**(7/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 418 vs. 2(80) = 160.

Time = 0.40 (sec) , antiderivative size = 418, normalized size of antiderivative = 4.02

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{2\left(23ia^{\frac{7}{2}} + \frac{20a^{\frac{7}{2}}\sin(dx+c)}{\cos(dx+c)+1} - \frac{88ia^{\frac{7}{2}}\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{60a^{\frac{7}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{130ia^{\frac{7}{2}}\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{60a^{\frac{7}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \cdots\right.}{-3d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)^{\frac{7}{2}}\left(\frac{6i\sin(dx+c)}{\cos(dx+c)+1} - \frac{14\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{14i\sin(dx+c)\cdots}{(\cos(dx+c)\cdots}\right.}$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 2*(23*I*a^(7/2) + 20*a^(7/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 88*I*a^(7/2)
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 60*a^(7/2)*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 130*I*a^(7/2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 60*a^(7/2)
*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 88*I*a^(7/2)*sin(d*x + c)^6/(cos(d*x
 + c) + 1)^6 - 20*a^(7/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 23*I*a^(7/2
)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1
) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(7/2)/(d*(sin(d*x + c)/(cos(d*
x + c) + 1) + 1)^(7/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(7/2)*(-18*I*s
in(d*x + c)/(cos(d*x + c) + 1) + 42*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 4
2*I*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 42*I*sin(d*x + c)^5/(cos(d*x + c)
 + 1)^5 - 42*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 18*I*sin(d*x + c)^7/(cos
(d*x + c) + 1)^7 + 3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 3))
```

Giac [F]

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \int (ia\tan(dx+c)+a)^{\frac{7}{2}}\cos(dx+c)\,dx$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(7/2)*cos(d*x + c), x)
```

Mupad [B] (verification not implemented)

Time = 5.74 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98

$$\int \cos(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx =$$

$$-\frac{2\,a^3\,\sqrt{\frac{a\,\left(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i}\right)}{\cos(2\,c+2\,d\,x)+1}}\,\left(5\,\sin\left(c+d\,x\right)+5\,\sin\left(3\,c+3\,d\,x\right)+\cos\left(c+d\,x\right)\,35\mathrm{i}+\cos\left(3\,c+3\,d\,x\right)\right.}{3\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

```
[In] int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(7/2),x)
```

```
[Out] -(2*a^3*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x)
 + 1))^(1/2)*(cos(c + d*x)*35i + 5*sin(c + d*x) + cos(3*c + 3*d*x)*11i + 5*
sin(3*c + 3*d*x)))/(3*d*(cos(2*c + 2*d*x) + 1))
```

3.328 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 71

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{8ia^2\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d} - \frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}}{d}$$

```
[Out] 8/3*I*a^2*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2)/d-2*I*a*cos(d*x+c)^3*(a+I*a
*tan(d*x+c))^(5/2)/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3575, 3574}

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{8ia^2\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d} - \frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}}{d}$$

```
[In] Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((8*I)/3)*a^2*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2))/d - ((2*I)*a*Co
s[c + d*x]^3*(a + I*a*Tan[c + d*x])^(5/2))/d
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
```

```
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\text{integral} = -\frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}}{d} - (4a)\int\cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx$$
$$= \frac{8ia^2\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{3d} - \frac{2ia\cos^3(c+dx)(a+ia\tan(c+dx))^{5/2}}{d}$$

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.21

$$\int\cos^3(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{2a^3\cos(c+dx)(i\cos(c+dx)+3\sin(c+dx))(\cos(c+4dx)+i\sin(c+4dx))\sqrt{a+ia\tan(c+dx)}}{3d(\cos(dx)+i\sin(dx))^3}$$

```
[In] Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(7/2),x]
[Out] (2*a^3*Cos[c + d*x]*(I*Cos[c + d*x] + 3*Sin[c + d*x])*(Cos[c + 4*d*x] + I*S
in[c + 4*d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(3*d*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [A] (verified)

Time = 34.54 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.24

method	result	size
default	$\frac{2(\tan(dx+c)-i)^3\sqrt{a(1+i\tan(dx+c))}\,a^3(\cos^4(dx+c))(3i\sin(dx+c)-\cos(dx+c))(2i\cos(dx+c)\sin(dx+c)-2(\cos^2(dx+c))+1)}{3d}$	88

```
[In] int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/d*(tan(d*x+c)-I)^3*(a*(1+I*tan(d*x+c)))^(1/2)*a^3*cos(d*x+c)^4*(3*I*sin
(d*x+c)-cos(d*x+c))*(2*I*cos(d*x+c)*sin(d*x+c)-2*cos(d*x+c)^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.83

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{\sqrt{2}\left(-i\,a^3e^{(4i\,dx+4i\,c)}+i\,a^3e^{(2i\,dx+2i\,c)}+2i\,a^3\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{3\,d}$$

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
[Out] 1/3*sqrt(2)*(-I*a^3*e^(4*I*d*x + 4*I*c) + I*a^3*e^(2*I*d*x + 2*I*c) + 2*I*a
^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**(7/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 504 vs. 2(57) = 114.

Time = 0.40 (sec) , antiderivative size = 504, normalized size of antiderivative = 7.10

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{2\left(-i\,a^{\frac{7}{2}}-\frac{6\,a^{\frac{7}{2}}\sin(dx+c)}{\cos(dx+c)+1}+\frac{5i\,a^{\frac{7}{2}}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{24\,a^{\frac{7}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{10i\,a^{\frac{7}{2}}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{36\,a^{\frac{7}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{10i\,a^{\frac{7}{2}}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\right.}{-3\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)^{\frac{7}{2}}\left(-\frac{4i\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{8i\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{14\,\sin(\right.}$$

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -2*(-I*a^(7/2) - 6*a^(7/2)*sin(d*x + c)/(cos(d*x + c) + 1) + 5*I*a^(7/2)*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 + 24*a^(7/2)*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 - 10*I*a^(7/2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 36*a^(7/2)*sin
(d*x + c)^5/(cos(d*x + c) + 1)^5 + 10*I*a^(7/2)*sin(d*x + c)^6/(cos(d*x + c
) + 1)^6 + 24*a^(7/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 5*I*a^(7/2)*sin
(d*x + c)^8/(cos(d*x + c) + 1)^8 - 6*a^(7/2)*sin(d*x + c)^9/(cos(d*x + c) +
 1)^9 + I*a^(7/2)*sin(d*x + c)^10/(cos(d*x + c) + 1)^10)*(-2*I*sin(d*x + c)
/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(7/2)/(d*(si
n(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(sin(d*x + c)/(cos(d*x + c) + 1) -
 1)^(7/2)*(12*I*sin(d*x + c)/(cos(d*x + c) + 1) - 9*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 24*I*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 42*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 - 42*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 24*I*sin(d
*x + c)^7/(cos(d*x + c) + 1)^7 - 9*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 12
*I*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 3*sin(d*x + c)^10/(cos(d*x + c) +
1)^10 + 3))
```

Giac [F]

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{7}{2}}\cos(dx+c)^3\;dx$$

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(7/2)*cos(d*x + c)^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.20

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{a^3\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,(\sin(c+d\,x)+\sin(3\,c+3\,d\,x)+\cos(c+d\,x)\,3\mathrm{i}-\cos(3\,c+}{3\,d}$$

```
[In] int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(7/2),x)
```

```
[Out] (a^3*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) +
1))^(1/2)*(cos(c + d*x)*3i + sin(c + d*x) - cos(3*c + 3*d*x)*1i + sin(3*c +
 3*d*x)))/(3*d)
```

3.329 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 35

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}$$

```
[Out] -2/5*I*a*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(5/2)/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {3574}

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}$$

```
[In] Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((-2*I)/5)*a*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(5/2))/d
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\text{integral} = -\frac{2ia\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{5d}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 73 vs. 2(35) = 70.

Time = 1.31 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.09

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{2a^3\cos^3(c+dx)(-i\cos(2c+5dx)+\sin(2c+5dx))\sqrt{a+ia\tan(c+dx)}}{5d(\cos(dx)+i\sin(dx))^3}$$

```
[In] Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (2*a^3*Cos[c + d*x]^3*((-I)*Cos[2*c + 5*d*x] + Sin[2*c + 5*d*x])*Sqrt[a + I
*a*Tan[c + d*x]])/(5*d*(Cos[d*x] + I*Sin[d*x])^3)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(29) = 58.

Time = 4.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.80

$$\frac{2(-\tan(dx+c)+i)^3\sqrt{a(1+i\tan(dx+c))}\,a^3(-i(\cos^6(dx+c))\sin(dx+c)+\cos^7(dx+c))}{5d}$$

```
[In] int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] 2/5/d*(-tan(d*x+c)+I)^3*(a*(1+I*tan(d*x+c)))^(1/2)*a^3*(-I*cos(d*x+c)^6*sin
(d*x+c)+cos(d*x+c)^7)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(27) = 54.

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.09

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{\sqrt{2}\left(-i\,a^3e^{(6i\,dx+6i\,c)}-3i\,a^3e^{(4i\,dx+4i\,c)}-3i\,a^3e^{(2i\,dx+2i\,c)}-i\,a^3\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{20\,d}$$

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(2)*(-I*a^3*e^(6*I*d*x + 6*I*c) - 3*I*a^3*e^(4*I*d*x + 4*I*c) - 3*
I*a^3*e^(2*I*d*x + 2*I*c) - I*a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(27) = 54$.

Time $= 0.69$ (sec) , antiderivative size $= 454$, normalized size of antiderivative $= 12.97$

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{2\left(i\,a^{\frac{7}{2}} - \frac{6i\,a^{\frac{7}{2}}\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{15i\,a^{\frac{7}{2}}\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{20i\,a^{\frac{7}{2}}\sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \cdots\right.}{-5\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)^{\frac{7}{2}}\left(\frac{2i\sin(dx+c)}{\cos(dx+c)+1} + \frac{4\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{10i\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{5\sin(d\cdots}{(\cos(dx+\cdots}\right.}$$

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 2*(I*a^(7/2) - 6*I*a^(7/2)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*I*a^(7/
2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 20*I*a^(7/2)*sin(d*x + c)^6/(cos(d
*x + c) + 1)^6 + 15*I*a^(7/2)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 6*I*a^(
7/2)*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + I*a^(7/2)*sin(d*x + c)^12/(cos
(d*x + c) + 1)^12)*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(
cos(d*x + c) + 1)^2 - 1)^(7/2)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/
2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(7/2)*(-10*I*sin(d*x + c)/(cos(d*x
 + c) + 1) - 20*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 50*I*sin(d*x + c)^3/(
cos(d*x + c) + 1)^3 - 25*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 100*I*sin(d*
x + c)^5/(cos(d*x + c) + 1)^5 - 100*I*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 +
 25*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 50*I*sin(d*x + c)^9/(cos(d*x + c)
 + 1)^9 + 20*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 10*I*sin(d*x + c)^11/(
cos(d*x + c) + 1)^11 + 5*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 5))
```

Giac [F]

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{7}{2}}\cos(dx+c)^5\,dx$$

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^(7/2)*cos(d*x + c)^5, x)
```

Mupad [B] (verification not implemented)

Time = 5.76 (sec) , antiderivative size = 112, normalized size of antiderivative = 3.20

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{a^3\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,(-2\,\sin(c+d\,x)-3\,\sin(3\,c+3\,d\,x)-\sin(5\,c+5\,d\,x)+\cos(c+d\,x)\,4\mathrm{i}}{20\,d}$$

```
[In] int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(7/2),x)
[Out] -(a^3*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) +
 1))^(1/2)*(cos(c + d*x)*4i - 2*sin(c + d*x) + cos(3*c + 3*d*x)*3i + cos(5*
c + 5*d*x)*1i - 3*sin(3*c + 3*d*x) - sin(5*c + 5*d*x)))/(20*d)
```

3.330 $\int \cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 196

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{ia^{7/2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{8\sqrt{2}d} - \frac{ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{8d} - \frac{ia^2\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{12d} - \frac{ia\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{10d} - \frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}}{7d}$$

```
[Out] 1/16*I*a^(7/2)*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1
/2))/d*2^(1/2)-1/8*I*a^3*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d-1/12*I*a^2*c
os(d*x+c)^3*(a+I*a*tan(d*x+c))^(3/2)/d-1/10*I*a*cos(d*x+c)^5*(a+I*a*tan(d*x
+c))^(5/2)/d-1/7*I*cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(7/2)/d
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3571, 3570, 212}

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{ia^{7/2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{8\sqrt{2}d} - \frac{ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{8d} - \frac{ia^2\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{12d} - \frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}}{7d} - \frac{ia\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{10d}$$

```
[In] Int[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((I/8)*a^(7/2)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c +
 d*x]])])/(Sqrt[2]*d) - ((I/8)*a^3*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])
/d - ((I/12)*a^2*Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^(3/2))/d - ((I/10)*a
*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(5/2))/d - ((I/7)*Cos[c + d*x]^7*(a
+ I*a*Tan[c + d*x])^(7/2))/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3571

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}}{7d}+\frac{1}{2}a\int\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx\\
&= -\frac{ia\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{10d}-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}}{7d}\\
&\quad+\frac{1}{4}a^2\int\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx\\
&= -\frac{ia^2\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{12d}-\frac{ia\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{10d}\\
&\quad-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}}{7d}+\frac{1}{8}a^3\int\cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx\\
&= -\frac{ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{8d}-\frac{ia^2\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{12d}\\
&\quad-\frac{ia\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{10d}\\
&\quad-\frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}}{7d}+\frac{1}{16}a^4\int\frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{8d} - \frac{ia^2\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{12d} \\
&\quad - \frac{ia\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{10d} - \frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}}{7d} \\
&\quad + \frac{(ia^4)\,\mathrm{Subst}\left(\int \frac{1}{2-ax^2}\,dx, x, \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\right)}{8d} \\
&= \frac{ia^{7/2}\mathrm{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{8\sqrt{2}d} - \frac{ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{8d} \\
&\quad - \frac{ia^2\cos^3(c+dx)(a+ia\tan(c+dx))^{3/2}}{12d} \\
&\quad - \frac{ia\cos^5(c+dx)(a+ia\tan(c+dx))^{5/2}}{10d} - \frac{i\cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}}{7d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.41 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.67

$$
\int \cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx =
$$

$$
-\frac{ia^3e^{-i(c+dx)}\left(176+298e^{2i(c+dx)}+188e^{4i(c+dx)}+81e^{6i(c+dx)}+15e^{8i(c+dx)}-105\sqrt{1+e^{2i(c+dx)}}\mathrm{arctanh}\left(\sqrt{1-\right.\right.}{1680d}
$$

```
[In] Integrate[Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((-1/1680*I)*a^3*(176 + 298*E^((2*I)*(c + d*x)) + 188*E^((4*I)*(c + d*x)) +
 81*E^((6*I)*(c + d*x)) + 15*E^((8*I)*(c + d*x)) - 105*Sqrt[1 + E^((2*I)*(c
 + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sqrt[a + I*a*Tan[c + d*x]
])/(d*E^(I*(c + d*x)))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1184 vs. 2(159) = 318.

Time = 3.88 (sec) , antiderivative size = 1185, normalized size of antiderivative = 6.05

Expression too large to display

```
[In] int(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] 1/1680*I/d*(tan(d*x+c)-I)^3*(a*(1+I*tan(d*x+c)))^(1/2)*a^3*cos(d*x+c)^3*(-3
15*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))
^(1/2))*cos(d*x+c)-420*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x
+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*sin(d*x+c
)-770*I*cos(d*x+c)^2+840*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d
```

```
*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^3*sin(d
*x+c)-840*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)
+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^4+840*(-cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^3*si
n(d*x+c)+105*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(
d*x+c)+1))^(1/2))+1528*I*cos(d*x+c)^4-105*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*s
in(d*x+c)-420*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*
x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^3+420*(-cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^
2*sin(d*x+c)+420*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(c
os(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2*sin(d*x+c)-84
0*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^
(1/2))*cos(d*x+c)^2+840*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos
(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2-420*(-co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*c
os(d*x+c)*sin(d*x+c)+1288*cos(d*x+c)^3*sin(d*x+c)+840*I*(-cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^4+420
*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(
1/2))*cos(d*x+c)^3+315*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c
)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-105*(-cos(d
*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(
d*x+c)-105*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c
)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-210*sin(d*x+c)*cos(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.32

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{105\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}d\log\left(\frac{\left(ia^4+\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{4\,d}\right)-105\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}d\,\mathrm{lo}}{}$$

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/1680*(105*sqrt(1/2)*sqrt(-a^7/d^2)*d*log(1/4*(I*a^4 + sqrt(2)*sqrt(1/2)*s
qrt(-a^7/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
)*e^(-I*d*x - I*c)/d) - 105*sqrt(1/2)*sqrt(-a^7/d^2)*d*log(1/4*(I*a^4 - sqr
t(2)*sqrt(1/2)*sqrt(-a^7/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*
x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/d) + sqrt(2)*(-15*I*a^3*e^(8*I*d*x + 8*I
*c) - 81*I*a^3*e^(6*I*d*x + 6*I*c) - 188*I*a^3*e^(4*I*d*x + 4*I*c) - 298*I*
a^3*e^(2*I*d*x + 2*I*c) - 176*I*a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**7*(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1253 vs. 2(149) = 298.

Time = 0.70 (sec) , antiderivative size = 1253, normalized size of antiderivative = 6.39

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -1/6720*(20*(7*I*sqrt(2)*a^3*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)) - 7*sqrt(2)*a^3*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)) + 3*(I*sqrt(2)*a^3*cos(2*d*x + 2*c)^2 + I*sqrt(2)*a^3*sin(2*d*x +
2*c)^2 + 2*I*sqrt(2)*a^3*cos(2*d*x + 2*c) + I*sqrt(2)*a^3)*cos(7/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 3*(sqrt(2)*a^3*cos(2*d*x + 2*c)^
2 + sqrt(2)*a^3*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^3*cos(2*d*x + 2*c) + sqrt(
2)*a^3)*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*(cos(2*d*
x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(3/4)*sqrt(a) + 8
4*(5*I*sqrt(2)*a^3*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
 - 5*sqrt(2)*a^3*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
 (I*sqrt(2)*a^3*cos(2*d*x + 2*c)^2 + I*sqrt(2)*a^3*sin(2*d*x + 2*c)^2 + 2*I
*sqrt(2)*a^3*cos(2*d*x + 2*c) + I*sqrt(2)*a^3)*cos(5/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)) - (sqrt(2)*a^3*cos(2*d*x + 2*c)^2 + sqrt(2)*a^
3*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*a^3*cos(2*d*x + 2*c) + sqrt(2)*a^3)*sin(5/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 105*(2*sqrt(2)*
a^3*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*
d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - 2*sqrt(2)*a^3*arctan
2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) - I*sqrt(2)*a^3*log(sqrt(cos(2*d
```

```
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1))^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)) + 1) + I*sqrt(2)*a^3*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 - 2*
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1))*sqrt(a))/d
```

Giac [F]

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{7}{2}}\cos(dx+c)^7\,dx$$

```
[In] integrate(cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^(7/2)*cos(d*x + c)^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^7(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \int \cos(c+d\,x)^7\,(a+a\tan(c+d\,x)\ 1\mathrm{i})^{7/2}\,dx$$

```
[In] int(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^(7/2),x)
[Out] int(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^(7/2), x)
```

3.331 $\int \cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 268

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{11ia^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{64\sqrt{2}d} + \frac{11ia^4\cos(c+dx)}{96d\sqrt{a+ia\tan(c+dx)}} - \frac{11ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{64d} - \frac{11ia^3\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{120d} - \frac{11ia^2\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{140d} - \frac{11ia\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{126d} - \frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d}$$

```
[Out] 11/128*I*a^(7/2)*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^
(1/2))/d*2^(1/2)+11/96*I*a^4*cos(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)-11/64*I*
a^3*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/d-11/120*I*a^3*cos(d*x+c)^3*(a+I*a*
tan(d*x+c))^(1/2)/d-11/140*I*a^2*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(3/2)/d-11
/126*I*a*cos(d*x+c)^7*(a+I*a*tan(d*x+c))^(5/2)/d-1/9*I*cos(d*x+c)^9*(a+I*a*
tan(d*x+c))^(7/2)/d
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3578, 3583, 3571, 3570, 212}

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{11ia^{7/2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{64\sqrt{2}d} + \frac{11ia^4\cos(c+dx)}{96d\sqrt{a+ia\tan(c+dx)}} - \frac{11ia^3\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{120d} - \frac{11ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{64d} - \frac{11ia^2\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{140d} - \frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d} - \frac{11ia\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{126d}$$

```
[In] Int[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((11*I)/64)*a^(7/2)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*T
an[c + d*x]])])/(Sqrt[2]*d) + (((11*I)/96)*a^4*Cos[c + d*x])/(d*Sqrt[a + I*
a*Tan[c + d*x]]) - (((11*I)/64)*a^3*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]]
)/d - (((11*I)/120)*a^3*Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d - (((1
1*I)/140)*a^2*Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^(3/2))/d - (((11*I)/126
)*a*Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^(5/2))/d - ((I/9)*Cos[c + d*x]^9*
(a + I*a*Tan[c + d*x])^(7/2))/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3571

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d} \\
&\quad+\frac{1}{18}(11a)\int\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx \\
&= -\frac{11ia\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{126d}-\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d} \\
&\quad+\frac{1}{28}\left(11a^2\right)\int\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx \\
&= -\frac{11ia^2\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{140d}-\frac{11ia\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{126d} \\
&\quad-\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d}+\frac{1}{40}\left(11a^3\right)\int\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx \\
&= -\frac{11ia^3\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{120d} \\
&\quad-\frac{11ia^2\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{140d} \\
&\quad-\frac{11ia\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{126d} \\
&\quad-\frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d}+\frac{1}{48}\left(11a^4\right)\int\frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx
\end{aligned}
$$

$$= \frac{11ia^4\cos(c+dx)}{96d\sqrt{a+ia\tan(c+dx)}} - \frac{11ia^3\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{120d} - \frac{11ia^2\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{140d} - \frac{11ia\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{126d} - \frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d} + \frac{1}{64}(11a^3)\int\cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$

$$= \frac{11ia^4\cos(c+dx)}{96d\sqrt{a+ia\tan(c+dx)}} - \frac{11ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{64d} - \frac{11ia^3\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{120d} - \frac{11ia^2\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{140d} - \frac{11ia\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{126d} - \frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d} + \frac{1}{128}(11a^4)\int\frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$$

$$= \frac{11ia^4\cos(c+dx)}{96d\sqrt{a+ia\tan(c+dx)}} - \frac{11ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{64d} - \frac{11ia^3\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{120d} - \frac{11ia^2\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{140d} - \frac{11ia\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{126d} - \frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d} + \frac{(11ia^4)\,\text{Subst}\left(\int\frac{1}{2-ax^2}\,dx, x, \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\right)}{64d}$$

$$= \frac{11ia^{7/2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{64\sqrt{2}d} + \frac{11ia^4\cos(c+dx)}{96d\sqrt{a+ia\tan(c+dx)}} - \frac{11ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{64d} - \frac{11ia^3\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{120d} - \frac{11ia^2\cos^5(c+dx)(a+ia\tan(c+dx))^{3/2}}{140d} - \frac{11ia\cos^7(c+dx)(a+ia\tan(c+dx))^{5/2}}{126d} - \frac{i\cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}}{9d}$$

Mathematica [A] (verified)

Time = 3.65 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.70

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx =$$

$$-\frac{ia^3e^{-3i(c+dx)}\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}\left(-315+4303e^{2i(c+dx)}+7034e^{4i(c+dx)}+3754e^{6i(c+dx)}+1798e^{8i(c+dx)}+530e^{10i(c+\right.}{20160\sqrt{2}d}$$

```
[In] Integrate[Cos[c + d*x]^9*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((-1/20160*I)*a^3*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*(
-315 + 4303*E^((2*I)*(c + d*x)) + 7034*E^((4*I)*(c + d*x)) + 3754*E^((6*I)*
(c + d*x)) + 1798*E^((8*I)*(c + d*x)) + 530*E^((10*I)*(c + d*x)) + 70*E^((1
2*I)*(c + d*x)) - 3465*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Ar
cTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/(Sqrt[2]*d*E^((3*I)*(c + d*x)))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1210 vs. 2(219) = 438.

Time = 4.77 (sec) , antiderivative size = 1211, normalized size of antiderivative = 4.52

Expression too large to display

```
[In] int(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] 1/40320/d*(tan(d*x+c)-I)^3*(a*(1+I*tan(d*x+c)))^(1/2)*a^3*cos(d*x+c)^3*(103
95*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(
-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-3465*I*(-cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+13860*I*
(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2
))*cos(d*x+c)^2*sin(d*x+c)+7840*cos(d*x+c)^6-27720*cos(d*x+c)^4*(-cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-13860*I*
(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(
d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^3-27720*cos(d*x+c)^3*arctanh(sin(d
*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)*sin(d*x+c)+27720*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arct
anh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c
)^2-13860*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+
c)/(cos(d*x+c)+1))^(1/2))-13860*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan
((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*sin(d*x+c)+42504*I*cos(d*x+
c)^3*sin(d*x+c)-13860*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d
*x+c)+1))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
-3465*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1
```

```
)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-50424*cos(d*x+c)^4+27720*(-cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c
)^2-27720*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+
c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^4+13860*arctanh(sin(d*
x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)+27720*I*(-cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^3*sin(d*x+c)+10
395*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^
(1/2))*cos(d*x+c)-6930*I*sin(d*x+c)*cos(d*x+c)-12320*I*cos(d*x+c)^5*sin(d*x
+c)+3465*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+
1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+25410*cos(d*x+c)^2-3465*(
-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.17

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{\left(3465\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}de^{(2i\,dx+2i\,c)}\log\left(-\frac{11\left(-i\,a^4+\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{32\,d}\right)-3465\sqrt{\frac{1}{2}}\sqrt{}\right.}{}$$

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/40320*(3465*sqrt(1/2)*sqrt(-a^7/d^2)*d*e^(2*I*d*x + 2*I*c)*log(-11/32*(-
I*a^4 + sqrt(2)*sqrt(1/2)*sqrt(-a^7/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a
/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/d) - 3465*sqrt(1/2)*sqrt(-a^7
/d^2)*d*e^(2*I*d*x + 2*I*c)*log(-11/32*(-I*a^4 - sqrt(2)*sqrt(1/2)*sqrt(-a^
7/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I
*d*x - I*c)/d) - sqrt(2)*(-70*I*a^3*e^(12*I*d*x + 12*I*c) - 530*I*a^3*e^(10
*I*d*x + 10*I*c) - 1798*I*a^3*e^(8*I*d*x + 8*I*c) - 3754*I*a^3*e^(6*I*d*x +
 6*I*c) - 7034*I*a^3*e^(4*I*d*x + 4*I*c) - 4303*I*a^3*e^(2*I*d*x + 2*I*c) +
 315*I*a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-2*I*d*x - 2*I*c)/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**9*(a+I*a*tan(d*x+c))**(7/2),x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{7}{2}}\cos(dx+c)^9\,dx$$

```
[In] integrate(cos(d*x+c)^9*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^(7/2)*cos(d*x + c)^9, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^9(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \int \cos(c+d\,x)^9\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{7/2}\,dx$$

```
[In] int(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^(7/2),x)
[Out] int(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^(7/2), x)
```

3.332 $\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 342

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{195ia^{7/2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{1024\sqrt{2}d} + \frac{65ia^4\cos(c+dx)}{512d\sqrt{a+ia\tan(c+dx)}} + \frac{39ia^4\cos^3(c+dx)}{448d\sqrt{a+ia\tan(c+dx)}} - \frac{195ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{1024d} - \frac{13ia^3\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{128d} - \frac{13ia^3\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{168d} - \frac{65ia^2\cos^7(c+dx)(a+ia\tan(c+dx))^{3/2}}{924d} - \frac{5ia\cos^9(c+dx)(a+ia\tan(c+dx))^{5/2}}{66d} - \frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}$$

```
[Out] 195/2048*I*a^(7/2)*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c)
)^(1/2))/d*2^(1/2)+65/512*I*a^4*cos(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)+39/44
8*I*a^4*cos(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(1/2)-195/1024*I*a^3*cos(d*x+c)*(
a+I*a*tan(d*x+c))^(1/2)/d-13/128*I*a^3*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2
)/d-13/168*I*a^3*cos(d*x+c)^5*(a+I*a*tan(d*x+c))^(1/2)/d-65/924*I*a^2*cos(d
*x+c)^7*(a+I*a*tan(d*x+c))^(3/2)/d-5/66*I*a*cos(d*x+c)^9*(a+I*a*tan(d*x+c))
^(5/2)/d-1/11*I*cos(d*x+c)^11*(a+I*a*tan(d*x+c))^(7/2)/d
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3578, 3583, 3571, 3570, 212}

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = \frac{195ia^{7/2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{1024\sqrt{2}d} + \frac{39ia^4\cos^3(c+dx)}{448d\sqrt{a+ia\tan(c+dx)}} + \frac{65ia^4\cos(c+dx)}{512d\sqrt{a+ia\tan(c+dx)}} - \frac{13ia^3\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{168d} - \frac{13ia^3\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{128d} - \frac{195ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{1024d} - \frac{65ia^2\cos^7(c+dx)(a+ia\tan(c+dx))^{3/2}}{924d} - \frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d} - \frac{5ia\cos^9(c+dx)(a+ia\tan(c+dx))^{5/2}}{66d}$$

```
[In] Int[Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((195*I)/1024)*a^(7/2)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*
a*Tan[c + d*x]])])/(Sqrt[2]*d) + (((65*I)/512)*a^4*Cos[c + d*x])/(d*Sqrt[a
+ I*a*Tan[c + d*x]]) + (((39*I)/448)*a^4*Cos[c + d*x]^3)/(d*Sqrt[a + I*a*Ta
n[c + d*x]]) - (((195*I)/1024)*a^3*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])
/d - (((13*I)/128)*a^3*Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/d - (((13
*I)/168)*a^3*Cos[c + d*x]^5*Sqrt[a + I*a*Tan[c + d*x]])/d - (((65*I)/924)*a
^2*Cos[c + d*x]^7*(a + I*a*Tan[c + d*x])^(3/2))/d - (((5*I)/66)*a*Cos[c + d
*x]^9*(a + I*a*Tan[c + d*x])^(5/2))/d - ((I/11)*Cos[c + d*x]^11*(a + I*a*Ta
n[c + d*x])^(7/2))/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3571

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d} \\
&\quad +\frac{1}{22}(15a)\int\cos^9(c+dx)(a+ia\tan(c+dx))^{5/2}\,dx \\
&= -\frac{5ia\cos^9(c+dx)(a+ia\tan(c+dx))^{5/2}}{66d}-\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d} \\
&\quad +\frac{1}{132}\left(65a^2\right)\int\cos^7(c+dx)(a+ia\tan(c+dx))^{3/2}\,dx \\
&= -\frac{65ia^2\cos^7(c+dx)(a+ia\tan(c+dx))^{3/2}}{924d} \\
&\quad -\frac{5ia\cos^9(c+dx)(a+ia\tan(c+dx))^{5/2}}{66d}-\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d} \\
&\quad +\frac{1}{168}\left(65a^3\right)\int\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx
\end{aligned}
$$

$$= -\frac{13ia^3\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{168d} - \frac{65ia^2\cos^7(c+dx)(a+ia\tan(c+dx))^{3/2}}{924d} - \frac{5ia\cos^9(c+dx)(a+ia\tan(c+dx))^{5/2}}{66d} - \frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d} + \frac{1}{112}(39a^4)\int\frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$= \frac{39ia^4\cos^3(c+dx)}{448d\sqrt{a+ia\tan(c+dx)}} - \frac{13ia^3\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{168d} - \frac{65ia^2\cos^7(c+dx)(a+ia\tan(c+dx))^{3/2}}{924d} - \frac{5ia\cos^9(c+dx)(a+ia\tan(c+dx))^{5/2}}{66d} - \frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d} + \frac{1}{128}(39a^3)\int\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx$$

$$= \frac{39ia^4\cos^3(c+dx)}{448d\sqrt{a+ia\tan(c+dx)}} - \frac{13ia^3\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{128d} - \frac{13ia^3\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{168d} - \frac{65ia^2\cos^7(c+dx)(a+ia\tan(c+dx))^{3/2}}{924d} - \frac{5ia\cos^9(c+dx)(a+ia\tan(c+dx))^{5/2}}{66d} - \frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d} + \frac{1}{256}(65a^4)\int\frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}dx$$

$$= \frac{65ia^4\cos(c+dx)}{512d\sqrt{a+ia\tan(c+dx)}} + \frac{39ia^4\cos^3(c+dx)}{448d\sqrt{a+ia\tan(c+dx)}} - \frac{13ia^3\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{128d} - \frac{13ia^3\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{168d} - \frac{65ia^2\cos^7(c+dx)(a+ia\tan(c+dx))^{3/2}}{924d} - \frac{5ia\cos^9(c+dx)(a+ia\tan(c+dx))^{5/2}}{66d} - \frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d} + \frac{(195a^3)\int\cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx}{1024}$$

$$=\frac{65ia^4\cos(c+dx)}{512d\sqrt{a+ia\tan(c+dx)}}+\frac{39ia^4\cos^3(c+dx)}{448d\sqrt{a+ia\tan(c+dx)}}-\frac{195ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{1024d}-\frac{13ia^3\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{128d}-\frac{13ia^3\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{168d}-\frac{65ia^2\cos^7(c+dx)(a+ia\tan(c+dx))^{3/2}}{924d}-\frac{5ia\cos^9(c+dx)(a+ia\tan(c+dx))^{5/2}}{66d}-\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}+\frac{(195a^4)\int\frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}dx}{2048}$$

$$=\frac{65ia^4\cos(c+dx)}{512d\sqrt{a+ia\tan(c+dx)}}+\frac{39ia^4\cos^3(c+dx)}{448d\sqrt{a+ia\tan(c+dx)}}-\frac{195ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{1024d}-\frac{13ia^3\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{128d}-\frac{13ia^3\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{168d}-\frac{65ia^2\cos^7(c+dx)(a+ia\tan(c+dx))^{3/2}}{924d}-\frac{5ia\cos^9(c+dx)(a+ia\tan(c+dx))^{5/2}}{66d}-\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}+\frac{(195ia^4)\,\text{Subst}\left(\int\frac{1}{2-ax^2}dx,x,\frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\right)}{1024d}$$

$$=\frac{195ia^{7/2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{1024\sqrt{2}d}+\frac{65ia^4\cos(c+dx)}{512d\sqrt{a+ia\tan(c+dx)}}+\frac{39ia^4\cos^3(c+dx)}{448d\sqrt{a+ia\tan(c+dx)}}-\frac{195ia^3\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{1024d}-\frac{13ia^3\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{128d}-\frac{13ia^3\cos^5(c+dx)\sqrt{a+ia\tan(c+dx)}}{168d}-\frac{65ia^2\cos^7(c+dx)(a+ia\tan(c+dx))^{3/2}}{924d}-\frac{5ia\cos^9(c+dx)(a+ia\tan(c+dx))^{5/2}}{66d}-\frac{i\cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}}{11d}$$

Mathematica [A] (verified)

Time = 6.02 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.57

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx =$$

$$-\frac{ia^3e^{-5i(c+dx)}\left(-462-7161e^{2i(c+dx)}+47413e^{4i(c+dx)}+78800e^{6i(c+dx)}+38512e^{8i(c+dx)}+19552e^{10i(c+dx)}+\right.}{}$$

```
[In] Integrate[Cos[c + d*x]^11*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((-1/473088*I)*a^3*(-462 - 7161*E^((2*I)*(c + d*x)) + 47413*E^((4*I)*(c + d
*x)) + 78800*E^((6*I)*(c + d*x)) + 38512*E^((8*I)*(c + d*x)) + 19552*E^((10
*I)*(c + d*x)) + 7184*E^((12*I)*(c + d*x)) + 1624*E^((14*I)*(c + d*x)) + 16
8*E^((16*I)*(c + d*x)) - 45045*E^((4*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d
*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sqrt[a + I*a*Tan[c + d*x]])/(
d*E^((5*I)*(c + d*x)))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1237 vs. 2(281) = 562.

Time = 6.02 (sec) , antiderivative size = 1238, normalized size of antiderivative = 3.62

Expression too large to display

```
[In] int(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] 1/473088/d*(tan(d*x+c)-I)^3*(a*(1+I*tan(d*x+c)))^(1/2)*a^3*cos(d*x+c)^3*(-3
60360*cos(d*x+c)^3*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+
c)+1))^(1/2))*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+37632*cos(d*x+c
)^8+101920*cos(d*x+c)^6+330330*cos(d*x+c)^2-655512*cos(d*x+c)^4+180180*arct
anh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*(-cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)+45045*(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2))*sin(d*x+c)+135135*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(
d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-180180*arctanh(sin(d*x+c)/(cos(d*x
+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*(-cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)+360360*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan
((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2-45045*(-cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-360360*cos(d*x+c
)^4*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^
(1/2))-180180*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(
d*x+c)/(cos(d*x+c)+1))^(1/2))+360360*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*a
rctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^3*sin(d*x+c)+180180*I*
(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2
```

```
))*cos(d*x+c)^2*sin(d*x+c)-180180*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arct
an((-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)*sin(d*x+c)-45045*I*(-cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)
/(cos(d*x+c)+1))^(1/2))-80640*I*cos(d*x+c)^7*sin(d*x+c)-160160*I*cos(d*x+c)
^5*sin(d*x+c)+552552*I*cos(d*x+c)^3*sin(d*x+c)-90090*I*sin(d*x+c)*cos(d*x+c
)-180180*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c
)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^3+360360*I*(-cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)*arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(
d*x+c)+1))^(1/2))*cos(d*x+c)^2+135135*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*
arctanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d
*x+c)-45045*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arctan((-cos(d*x+c)/(cos(d
*x+c)+1))^(1/2))*sin(d*x+c)-360360*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*arc
tanh(sin(d*x+c)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+
c)^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.00

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{\left(45045\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}de^{(4i\,dx+4i\,c)}\log\left(-\frac{195\left(-i\,a^4+\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a^7}{d^2}}\left(de^{(2i\,dx+2i\,c)}+d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\right)e^{(-i\,dx-i\,c)}}{512\,d}\right)-45045\sqrt{\frac{1}{2}}\cdots\right.}{\cdots}$$

```
[In] integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/473088*(45045*sqrt(1/2)*sqrt(-a^7/d^2)*d*e^(4*I*d*x + 4*I*c)*log(-195/51
2*(-I*a^4 + sqrt(2)*sqrt(1/2)*sqrt(-a^7/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-I*d*x - I*c)/d) - 45045*sqrt(1/2)*sqrt
(-a^7/d^2)*d*e^(4*I*d*x + 4*I*c)*log(-195/512*(-I*a^4 - sqrt(2)*sqrt(1/2)*s
qrt(-a^7/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
)*e^(-I*d*x - I*c)/d) - sqrt(2)*(-168*I*a^3*e^(16*I*d*x + 16*I*c) - 1624*I*
a^3*e^(14*I*d*x + 14*I*c) - 7184*I*a^3*e^(12*I*d*x + 12*I*c) - 19552*I*a^3*
e^(10*I*d*x + 10*I*c) - 38512*I*a^3*e^(8*I*d*x + 8*I*c) - 78800*I*a^3*e^(6*
I*d*x + 6*I*c) - 47413*I*a^3*e^(4*I*d*x + 4*I*c) + 7161*I*a^3*e^(2*I*d*x +
2*I*c) + 462*I*a^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))*e^(-4*I*d*x - 4*I*c)
/d
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx=\text{Timed out}$$

```
[In] integrate(cos(d*x+c)**11*(a+I*a*tan(d*x+c))**(7/2),x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx=\text{Timed out}$$

```
[In] integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx=\int (i\,a\tan(dx+c)+a)^{\frac{7}{2}}\cos(dx+c)^{11}\,dx$$

```
[In] integrate(cos(d*x+c)^11*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^(7/2)*cos(d*x + c)^11, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{11}(c+dx)(a+ia\tan(c+dx))^{7/2}\,dx=\int \cos(c+d\,x)^{11}\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{7/2}\,dx$$

```
[In] int(cos(c + d*x)^11*(a + a*tan(c + d*x)*1i)^(7/2),x)
[Out] int(cos(c + d*x)^11*(a + a*tan(c + d*x)*1i)^(7/2), x)
```

3.333 $\int \frac{\sec^8(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 117

$$\int \frac{\sec^8(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{16i(a+ia\tan(c+dx))^{7/2}}{7a^4d} + \frac{8i(a+ia\tan(c+dx))^{9/2}}{3a^5d} - \frac{12i(a+ia\tan(c+dx))^{11/2}}{11a^6d} + \frac{2i(a+ia\tan(c+dx))^{13/2}}{13a^7d}$$

```
[Out] -16/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^4/d+8/3*I*(a+I*a*tan(d*x+c))^(9/2)/a^5/d
-12/11*I*(a+I*a*tan(d*x+c))^(11/2)/a^6/d+2/13*I*(a+I*a*tan(d*x+c))^(13/2)/a
^7/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3568, 45}

$$\int \frac{\sec^8(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i(a+ia\tan(c+dx))^{13/2}}{13a^7d} - \frac{12i(a+ia\tan(c+dx))^{11/2}}{11a^6d} + \frac{8i(a+ia\tan(c+dx))^{9/2}}{3a^5d} - \frac{16i(a+ia\tan(c+dx))^{7/2}}{7a^4d}$$

```
[In] Int[Sec[c + d*x]^8/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((-16*I)/7)*(a + I*a*Tan[c + d*x])^(7/2))/(a^4*d) + (((8*I)/3)*(a + I*a*Ta
n[c + d*x])^(9/2))/(a^5*d) - (((12*I)/11)*(a + I*a*Tan[c + d*x])^(11/2))/(a
^6*d) + (((2*I)/13)*(a + I*a*Tan[c + d*x])^(13/2))/(a^7*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)^3(a+x)^{5/2}\,dx, x, ia\tan(c+dx)\right)}{a^7 d} \\ &= -\frac{i\text{Subst}\left(\int \left(8a^3(a+x)^{5/2}-12a^2(a+x)^{7/2}+6a(a+x)^{9/2}-(a+x)^{11/2}\right)\,dx, x, ia\tan(c+dx)\right)}{a^7 d} \\ &= -\frac{16i(a+ia\tan(c+dx))^{7/2}}{7a^4 d}+\frac{8i(a+ia\tan(c+dx))^{9/2}}{3a^5 d} \\ &\quad -\frac{12i(a+ia\tan(c+dx))^{11/2}}{11a^6 d}+\frac{2i(a+ia\tan(c+dx))^{13/2}}{13a^7 d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.62

$$\int \frac{\sec^8(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}(-835+1421i\tan(c+dx)+945\tan^2(c+dx)-231i\tan^3(c+$$
$$3003ad$$

```
[In] Integrate[Sec[c + d*x]^8/Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] (2*(-I + Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c + d*x]]*(-835 + (1421*I)*Tan[c
+ d*x] + 945*Tan[c + d*x]^2 - (231*I)*Tan[c + d*x]^3))/(3003*a*d)
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}-\frac{6a(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}+\frac{4a^2(a+ia\tan(dx+c))^{\frac{9}{2}}}{3}-\frac{8a^3(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}\right)}{d\,a^7}$	82
default	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}-\frac{6a(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}+\frac{4a^2(a+ia\tan(dx+c))^{\frac{9}{2}}}{3}-\frac{8a^3(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}\right)}{d\,a^7}$	82

```
[In] int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/d/a^7*(1/13*(a+I*a*tan(d*x+c))^(13/2)-6/11*a*(a+I*a*tan(d*x+c))^(11/2)+
4/3*a^2*(a+I*a*tan(d*x+c))^(9/2)-8/7*a^3*(a+I*a*tan(d*x+c))^(7/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.28

$$\int \frac{\sec^8(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{128\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(16i\,e^{(13i\,dx+13i\,c)}+104i\,e^{(11i\,dx+11i\,c)}+286i\,e^{(9i\,dx+9i\,c)}+429i\,e^{(7i\,dx+7i\,c)}\right)}{3003\left(ade^{(12i\,dx+12i\,c)}+6\,ade^{(10i\,dx+10i\,c)}+15\,ade^{(8i\,dx+8i\,c)}+20\,ade^{(6i\,dx+6i\,c)}+15\,ade^{(4i\,dx+4i\,c)}+6\,ade^{(2i\,d}\right.}$$

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -128/3003*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(16*I*e^(13*I*d*x + 13*
I*c) + 104*I*e^(11*I*d*x + 11*I*c) + 286*I*e^(9*I*d*x + 9*I*c) + 429*I*e^(7
*I*d*x + 7*I*c))/(a*d*e^(12*I*d*x + 12*I*c) + 6*a*d*e^(10*I*d*x + 10*I*c) +
 15*a*d*e^(8*I*d*x + 8*I*c) + 20*a*d*e^(6*I*d*x + 6*I*c) + 15*a*d*e^(4*I*d*
x + 4*I*c) + 6*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

Sympy [F]

$$\int \frac{\sec^8(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec^8(c+dx)}{\sqrt{ia\left(\tan(c+dx)-i\right)}}\,dx$$

```
[In] integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**8/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(85) = 170$.

Time = 0.24 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.54

$$\int \frac{\sec^8(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx =$$
$$-\frac{2i\left(15015\sqrt{ia\tan(dx+c)+a}-\frac{3003\left(3(ia\tan(dx+c)+a)^{\frac{5}{2}}-10(ia\tan(dx+c)+a)^{\frac{3}{2}}a+15\sqrt{ia\tan(dx+c)+a}\,a^2\right)}{a^2}+\frac{143\left(3\cdots\right.}{\cdots}\right.}{\cdots}$$

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/15015*I*(15015*sqrt(I*a*tan(d*x + c) + a) - 3003*(3*(I*a*tan(d*x + c) +
a)^(5/2) - 10*(I*a*tan(d*x + c) + a)^(3/2)*a + 15*sqrt(I*a*tan(d*x + c) + a
)*a^2)/a^2 + 143*(35*(I*a*tan(d*x + c) + a)^(9/2) - 180*(I*a*tan(d*x + c) +
 a)^(7/2)*a + 378*(I*a*tan(d*x + c) + a)^(5/2)*a^2 - 420*(I*a*tan(d*x + c)
+ a)^(3/2)*a^3 + 315*sqrt(I*a*tan(d*x + c) + a)*a^4)/a^4 - 5*(231*(I*a*tan(
d*x + c) + a)^(13/2) - 1638*(I*a*tan(d*x + c) + a)^(11/2)*a + 5005*(I*a*tan
(d*x + c) + a)^(9/2)*a^2 - 8580*(I*a*tan(d*x + c) + a)^(7/2)*a^3 + 9009*(I*
a*tan(d*x + c) + a)^(5/2)*a^4 - 6006*(I*a*tan(d*x + c) + a)^(3/2)*a^5 + 300
3*sqrt(I*a*tan(d*x + c) + a)*a^6)/a^6)/(a*d)
```

Giac [F]

$$\int \frac{\sec^8(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec(dx+c)^8}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^8/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [B] (verification not implemented)

Time = 9.74 (sec) , antiderivative size = 434, normalized size of antiderivative = 3.71

$$\int \frac{\sec^8(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,2048\mathrm{i}}{3003\,a\,d} - \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,1024\mathrm{i}}{3003\,a\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)} - \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,256\mathrm{i}}{1001\,a\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2} - \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,640\mathrm{i}}{3003\,a\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3} + \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,6784\mathrm{i}}{429\,a\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4} - \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,3456\mathrm{i}}{143\,a\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^5} + \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{13\,a\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^6}$$

```
[In] int(1/(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^(1/2)),x)
[Out] ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*6
784i)/(429*a*d*(exp(c*2i + d*x*2i) + 1)^4) - ((a - (a*(exp(c*2i + d*x*2i)*1
i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1024i)/(3003*a*d*(exp(c*2i + d*
x*2i) + 1)) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i)
 + 1))^(1/2)*256i)/(1001*a*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (a*(exp(c*
2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*640i)/(3003*a*d*(
exp(c*2i + d*x*2i) + 1)^3) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp
(c*2i + d*x*2i) + 1))^(1/2)*2048i)/(3003*a*d) - ((a - (a*(exp(c*2i + d*x*2i
)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*3456i)/(143*a*d*(exp(c*2i +
d*x*2i) + 1)^5) + ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x
*2i) + 1))^(1/2)*128i)/(13*a*d*(exp(c*2i + d*x*2i) + 1)^6)
```

3.334 $\int \frac{\sec^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \frac{\sec^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{8i(a+ia\tan(c+dx))^{5/2}}{5a^3d} + \frac{8i(a+ia\tan(c+dx))^{7/2}}{7a^4d} - \frac{2i(a+ia\tan(c+dx))^{9/2}}{9a^5d}$$

```
[Out] -8/5*I*(a+I*a*tan(d*x+c))^(5/2)/a^3/d+8/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^4/d-
2/9*I*(a+I*a*tan(d*x+c))^(9/2)/a^5/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3568, 45}

$$\int \frac{\sec^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{2i(a+ia\tan(c+dx))^{9/2}}{9a^5d} + \frac{8i(a+ia\tan(c+dx))^{7/2}}{7a^4d} - \frac{8i(a+ia\tan(c+dx))^{5/2}}{5a^3d}$$

```
[In] Int[Sec[c + d*x]^6/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((-8*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(a^3*d) + (((8*I)/7)*(a + I*a*Tan
[c + d*x])^(7/2))/(a^4*d) - (((2*I)/9)*(a + I*a*Tan[c + d*x])^(9/2))/(a^5*d
)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)^2(a+x)^{3/2}\,dx, x, ia\tan(c+dx)\right)}{a^5d}\\
&= -\frac{i\text{Subst}\left(\int \left(4a^2(a+x)^{3/2}-4a(a+x)^{5/2}+(a+x)^{7/2}\right)\,dx, x, ia\tan(c+dx)\right)}{a^5d}\\
&= -\frac{8i(a+ia\tan(c+dx))^{5/2}}{5a^3d}+\frac{8i(a+ia\tan(c+dx))^{7/2}}{7a^4d}-\frac{2i(a+ia\tan(c+dx))^{9/2}}{9a^5d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\int \frac{\sec^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}(107i+110\tan(c+dx)-35i\tan^2(c+dx))}{315ad}$$

```
[In] Integrate[Sec[c + d*x]^6/Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] (2*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]]*(107*I + 110*Tan[c + d*
x] - (35*I)*Tan[c + d*x]^2))/(315*a*d)
```

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}+\frac{4a(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{4a^{2}(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}\right)}{d\,a^{5}}$	63
default	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}+\frac{4a(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{4a^{2}(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}\right)}{d\,a^{5}}$	63

```
[In] int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*I/d/a^5*(-1/9*(a+I*a*tan(d*x+c))^(9/2)+4/7*a*(a+I*a*tan(d*x+c))^(7/2)-4/5
*a^2*(a+I*a*tan(d*x+c))^(5/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.28

$$\int \frac{\sec^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{32\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(8i\,e^{(9i\,dx+9i\,c)}+36i\,e^{(7i\,dx+7i\,c)}+63i\,e^{(5i\,dx+5i\,c)}\right)}{315\left(ade^{(8i\,dx+8i\,c)}+4\,ade^{(6i\,dx+6i\,c)}+6\,ade^{(4i\,dx+4i\,c)}+4\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
[Out] -32/315*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(8*I*e^(9*I*d*x + 9*I*c)
+ 36*I*e^(7*I*d*x + 7*I*c) + 63*I*e^(5*I*d*x + 5*I*c))/(a*d*e^(8*I*d*x + 8*
I*c) + 4*a*d*e^(6*I*d*x + 6*I*c) + 6*a*d*e^(4*I*d*x + 4*I*c) + 4*a*d*e^(2*I
*d*x + 2*I*c) + a*d)
```

Sympy [F]

$$\int \frac{\sec^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec^6\left(c+dx\right)}{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}\,dx$$

```
[In] integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Integral(sec(c + d*x)**6/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(64) = 128.

Time = 0.24 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.92

$$\int \frac{\sec^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{2i\left(315\sqrt{i\,a\tan\left(dx+c\right)+a}-\frac{42\left(3\,(i\,a\tan(dx+c)+a)^{\frac{5}{2}}-10\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a+15\,\sqrt{i\,a\tan(dx+c)+a}\,a^2\right)}{a^2}+\frac{35\,(i\,a\tan(dx-\cdots}{\cdots}\right.}{315\,ad}$$

[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")

[Out] -2/315*I*(315*sqrt(I*a*tan(d*x + c) + a) - 42*(3*(I*a*tan(d*x + c) + a)^(5/2) - 10*(I*a*tan(d*x + c) + a)^(3/2)*a + 15*sqrt(I*a*tan(d*x + c) + a)*a^2)/a^2 + (35*(I*a*tan(d*x + c) + a)^(9/2) - 180*(I*a*tan(d*x + c) + a)^(7/2)*a + 378*(I*a*tan(d*x + c) + a)^(5/2)*a^2 - 420*(I*a*tan(d*x + c) + a)^(3/2)*a^3 + 315*sqrt(I*a*tan(d*x + c) + a)*a^4)/a^4)/(a*d)

Giac [F]

$$\int \frac{\sec^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^6}{\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sec(d*x + c)^6/sqrt(I*a*tan(d*x + c) + a), x)

Mupad [B] (verification not implemented)

Time = 7.16 (sec) , antiderivative size = 306, normalized size of antiderivative = 3.48

$$\int \frac{\sec^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,256\mathrm{i}}{315\,a\,d} - \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{315\,a\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)} - \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{105\,a\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2} + \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,320\mathrm{i}}{63\,a\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3} - \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{9\,a\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4}$$

[In] int(1/(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(1/2)),x)

```
[Out] ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*3
20i)/(63*a*d*(exp(c*2i + d*x*2i) + 1)^3) - ((a - (a*(exp(c*2i + d*x*2i)*1i
- 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*128i)/(315*a*d*(exp(c*2i + d*x*2i
) + 1)) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1
))^(1/2)*32i)/(105*a*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (a*(exp(c*2i + d
*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*256i)/(315*a*d) - ((a -
 (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*32i)/(
9*a*d*(exp(c*2i + d*x*2i) + 1)^4)
```

3.335 $\int \frac{\sec^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 59

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{4i(a+ia\tan(c+dx))^{3/2}}{3a^2d} + \frac{2i(a+ia\tan(c+dx))^{5/2}}{5a^3d}$$

```
[Out] -4/3*I*(a+I*a*tan(d*x+c))^(3/2)/a^2/d+2/5*I*(a+I*a*tan(d*x+c))^(5/2)/a^3/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3568, 45}

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i(a+ia\tan(c+dx))^{5/2}}{5a^3d} - \frac{4i(a+ia\tan(c+dx))^{3/2}}{3a^2d}$$

```
[In] Int[Sec[c + d*x]^4/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((-4*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(a^2*d) + (((2*I)/5)*(a + I*a*Tan
[c + d*x])^(5/2))/(a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)\sqrt{a+x}\,dx, x, ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{i\text{Subst}\left(\int \left(2a\sqrt{a+x}-(a+x)^{3/2}\right)\,dx, x, ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{4i(a+ia\tan(c+dx))^{3/2}}{3a^2d}+\frac{2i(a+ia\tan(c+dx))^{5/2}}{5a^3d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2(7-3i\tan(c+dx))(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{15ad}$$

```
[In] Integrate[Sec[c + d*x]^4/Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] (2*(7 - (3*I)*Tan[c + d*x])*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
/(15*a*d)
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{2a(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}\right)}{d\,a^3}$	44
default	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{2a(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}\right)}{d\,a^3}$	44

```
[In] int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*I/d/a^3*(1/5*(a+I*a*tan(d*x+c))^(5/2)-2/3*a*(a+I*a*tan(d*x+c))^(3/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{8\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(2i\,e^{(5i\,dx+5i\,c)}+5i\,e^{(3i\,dx+3i\,c)}\right)}{15\left(ade^{(4i\,dx+4i\,c)}+2\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
[Out] -8/15*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(2*I*e^(5*I*d*x + 5*I*c) +
5*I*e^(3*I*d*x + 3*I*c))/(a*d*e^(4*I*d*x + 4*I*c) + 2*a*d*e^(2*I*d*x + 2*I*
c) + a*d)
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec^4(c+dx)}{\sqrt{ia\left(\tan(c+dx)-i\right)}}\,dx$$

```
[In] integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Integral(sec(c + d*x)**4/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.34

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{2i\left(15\sqrt{i\,a\tan(dx+c)+a}-\frac{3\,(i\,a\tan(dx+c)+a)^{\frac{5}{2}}-10\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a+15\sqrt{i\,a\tan(dx+c)+a}a^2}{a^2}\right)}{15\,ad}$$

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] -2/15*I*(15*sqrt(I*a*tan(d*x + c) + a) - (3*(I*a*tan(d*x + c) + a)^(5/2) -
10*(I*a*tan(d*x + c) + a)^(3/2)*a + 15*sqrt(I*a*tan(d*x + c) + a)*a^2)/a^2)
/(a*d)
```

Giac [F]

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec(dx+c)^4}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sec(d*x + c)^4/sqrt(I*a*tan(d*x + c) + a), x)

Mupad [B] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.63

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{8\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,(\cos(2\,c+2\,d\,x)\,27\mathrm{i}+\cos(4\,c+4\,d\,x)\,9\mathrm{i}+\cos(6\,c+6\,d\,x)\,1\mathrm{i}-5\sin}{15\,a\,d\,(15\cos(2\,c+2\,d\,x)+6\cos(4\,c+4\,d\,x)+\cos(6\,c+}$$

[In] int(1/(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(1/2)),x)

[Out] -(8*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1
))^(1/2)*(cos(2*c + 2*d*x)*27i + cos(4*c + 4*d*x)*9i + cos(6*c + 6*d*x)*1i
- 5*sin(2*c + 2*d*x) - 4*sin(4*c + 4*d*x) - sin(6*c + 6*d*x) + 19i))/(15*a*
d*(15*cos(2*c + 2*d*x) + 6*cos(4*c + 4*d*x) + cos(6*c + 6*d*x) + 10))

3.336 $\int \frac{\sec^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 27

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{2i\sqrt{a+ia\tan(c+dx)}}{ad}$$

```
[Out] -2*I*(a+I*a*tan(d*x+c))^(1/2)/a/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3568, 32}

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{2i\sqrt{a+ia\tan(c+dx)}}{ad}$$

```
[In] Int[Sec[c + d*x]^2/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-2*I)*Sqrt[a + I*a*Tan[c + d*x]])/(a*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
```

```
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{i\text{Subst}\left(\int \frac{1}{\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{ad} \\ &= -\frac{2i\sqrt{a+ia\tan(c+dx)}}{ad} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{2i\sqrt{a+ia\tan(c+dx)}}{ad}$$

```
[In] Integrate[Sec[c + d*x]^2/Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] ((-2*I)*Sqrt[a + I*a*Tan[c + d*x]])/(a*d)
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$-\frac{2i\sqrt{a+ia\tan(dx+c)}}{ad}$	24
default	$-\frac{2i\sqrt{a+ia\tan(dx+c)}}{ad}$	24

```
[In] int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*I*(a+I*a*tan(d*x+c))^(1/2)/a/d
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{2i\,\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{(i\,dx+i\,c)}}{ad}$$

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
[Out] -2*I*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c)/(a*d)
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec^2{(c+dx)}}{\sqrt{ia\,(\tan{(c+dx)}-i)}}\,dx$$

```
[In] integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Integral(sec(c + d*x)**2/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{2i\,\sqrt{i\,a\tan{(dx+c)}+a}}{ad}$$

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] -2*I*sqrt(I*a*tan(d*x + c) + a)/(a*d)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(21) = 42.

Time = 0.60 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.04

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{2i\,\sqrt{\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2i\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}}{ad}$$

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -2*I*sqrt((a*tan(1/2*d*x + 1/2*c)^2 - 2*I*a*tan(1/2*d*x + 1/2*c) - a)/(tan(
1/2*d*x + 1/2*c)^2 - 1))/(a*d)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.74

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{\sqrt{\frac{a\left(2\cos(c+d\,x)^2+\sin(2\,c+2\,d\,x)\,1\mathrm{i}\right)}{2\cos(c+d\,x)^2}}\,2\mathrm{i}}{a\,d}$$

```
[In] int(1/(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

```
[Out] -(((a*(sin(2*c + 2*d*x)*1i + 2*cos(c + d*x)^2))/(2*cos(c + d*x)^2))^(1/2)*2
i)/(a*d)
```

3.337 $\int \frac{\cos^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 146

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{5i\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}\sqrt{a}d}+\frac{5ia}{12d(a+ia\tan(c+dx))^{3/2}}-\frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}+\frac{5i}{8d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] -5/16*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1/2)/a^(
1/2)+5/8*I/d/(a+I*a*tan(d*x+c))^(1/2)+5/12*I*a/d/(a+I*a*tan(d*x+c))^(3/2)-1
/2*I*a^2/d/(a-I*a*tan(d*x+c))/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3568, 44, 53, 65, 212}

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}-\frac{5i\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}\sqrt{a}d}+\frac{5ia}{12d(a+ia\tan(c+dx))^{3/2}}+\frac{5i}{8d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[Cos[c + d*x]^2/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((-5*I)/8)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2]
*Sqrt[a]*d) + (((5*I)/12)*a)/(d*(a + I*a*Tan[c + d*x])^(3/2)) - ((I/2)*a^2)
/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(3/2)) + ((5*I)/8)/(d*Sqr
t[a + I*a*Tan[c + d*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(ia^3)\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{5/2}}\,dx, x, ia\tan(c+dx)\right)}{d}\\
&= -\frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}\\
&\quad -\frac{(5ia^2)\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{5/2}}\,dx, x, ia\tan(c+dx)\right)}{4d}\\
&= \frac{5ia}{12d(a+ia\tan(c+dx))^{3/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}\\
&\quad -\frac{(5ia)\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{3/2}}\,dx, x, ia\tan(c+dx)\right)}{8d}\\
&= \frac{5ia}{12d(a+ia\tan(c+dx))^{3/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}\\
&\quad + \frac{5i}{8d\sqrt{a+ia\tan(c+dx)}} - \frac{(5i)\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{16d}\\
&= \frac{5ia}{12d(a+ia\tan(c+dx))^{3/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}}\\
&\quad + \frac{5i}{8d\sqrt{a+ia\tan(c+dx)}} - \frac{(5i)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+ia\tan(c+dx)}\right)}{8d}\\
&= -\frac{5i\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{8\sqrt{2}\sqrt{a}d} + \frac{5ia}{12d(a+ia\tan(c+dx))^{3/2}}\\
&\quad - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{3/2}} + \frac{5i}{8d\sqrt{a+ia\tan(c+dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.35

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{ia\,\text{Hypergeometric2F1}\left(-\frac{3}{2}, 2, -\frac{1}{2}, \frac{1}{2}(1+i\tan(c+dx))\right)}{6d(a+ia\tan(c+dx))^{3/2}}$$

```
[In] Integrate[Cos[c + d*x]^2/Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] ((I/6)*a*Hypergeometric2F1[-3/2, 2, -1/2, (1 + I*Tan[c + d*x])/2])/(d*(a + 
I*a*Tan[c + d*x])^(3/2))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 405 vs. 2(114) = 228.

Time = 10.67 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.78

method	result
default	$-\frac{4i\left(\cos^3(dx+c)\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+4i\left(\cos^2(dx+c)\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-20\sin(dx+c)\left(\cos^2(dx+c)\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-15i\cos(dx+c)\,\mathrm{ar}}{}$

```
[In] int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48/d*(4*I*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+4*I*cos(d*x+c)
^2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-20*sin(d*x+c)*cos(d*x+c)^2*(-cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)-15*I*cos(d*x+c)*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x
+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-30*I*cos(d*x+c)*(-c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)-20*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d
*x+c)*sin(d*x+c)-15*I*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)
/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-30*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)+15*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(co
s(d*x+c)+1))^(1/2))*sin(d*x+c))/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(cos(d*x
+c)+1)/(a*(1+I*tan(d*x+c)))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(105) = 210.

Time = 0.25 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.86

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= \frac{\left(-15i\sqrt{\frac{1}{2}}ad\sqrt{\frac{1}{ad^2}}e^{(3i\,dx+3i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(ade^{(2i\,dx+2i\,c)}+ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c}\right.\right.}{}$$

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/48*(-15*I*sqrt(1/2)*a*d*sqrt(1/(a*d^2))*e^(3*I*d*x + 3*I*c)*log(4*(sqrt(2
)*sqrt(1/2)*(a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1
))*sqrt(1/(a*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + 15*I*sqrt(1/2)*
a*d*sqrt(1/(a*d^2))*e^(3*I*d*x + 3*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(a*d*e^(2
*I*d*x + 2*I*c) + a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a*d^2)) -
a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1))*(-3*I*e^(6*I*d*x + 6*I*c) + 11*I*e^(4*I*d*x + 4*I*c) + 16*I*e^(2*I*d*
x + 2*I*c) + 2*I))*e^(-3*I*d*x - 3*I*c)/(a*d)
```

Sympy [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\cos^2{(c+dx)}}{\sqrt{ia\,(\tan{(c+dx)}-i)}}\,dx$$

```
[In] integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Integral(cos(c + d*x)**2/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.95

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{i\left(15\sqrt{2}\sqrt{a}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+\frac{4\left(15\,(i\,a\tan(dx+c)+a)^2a-20\,(i\,a\tan(dx+c)+a)a^2-8\,a^3\right)}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}-2\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a}\right)}{96\,ad}$$

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] 1/96*I*(15*sqrt(2)*sqrt(a)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) +
a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(15*(I*a*tan(d*x +
c) + a)^2*a - 20*(I*a*tan(d*x + c) + a)*a^2 - 8*a^3)/((I*a*tan(d*x + c) + a
)^(5/2) - 2*(I*a*tan(d*x + c) + a)^(3/2)*a))/(a*d)
```

Giac [F]

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\cos{(dx+c)}^2}{\sqrt{i\,a\tan{(dx+c)}+a}}\,dx$$

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(cos(d*x + c)^2/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2}{\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}\,dx$$

```
[In] int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.338 $\int \frac{\cos^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 219

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{63i\,\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}\sqrt{a}d}+\frac{63ia^2}{160d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}} - \frac{9ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}} + \frac{21ia}{64d(a+ia\tan(c+dx))^{3/2}}+\frac{63i}{128d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] -63/256*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1/2)/a
^(1/2)+63/128*I/d/(a+I*a*tan(d*x+c))^(1/2)+63/160*I*a^2/d/(a+I*a*tan(d*x+c)
)^(5/2)-1/4*I*a^4/d/(a-I*a*tan(d*x+c))^2/(a+I*a*tan(d*x+c))^(5/2)-9/16*I*a^
3/d/(a-I*a*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2)+21/64*I*a/d/(a+I*a*tan(d*x+
c))^(3/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used

$= \{3568, 44, 53, 65, 212\}$

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}} - \frac{9ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}} + \frac{63ia^2}{160d(a+ia\tan(c+dx))^{5/2}} - \frac{63i\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}\sqrt{a}d} + \frac{21ia}{64d(a+ia\tan(c+dx))^{3/2}} + \frac{63i}{128d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[Cos[c + d*x]^4/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((-63*I)/128)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt
[2]*Sqrt[a]*d) + (((63*I)/160)*a^2)/(d*(a + I*a*Tan[c + d*x])^(5/2)) - ((I/
4)*a^4)/(d*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(5/2)) - (((9*I)
/16)*a^3)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(5/2)) + (((21*I
)/64)*a)/(d*(a + I*a*Tan[c + d*x])^(3/2)) + ((63*I)/128)/(d*Sqrt[a + I*a*Ta
n[c + d*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(ia^5)\,\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^{7/2}}\,dx, x, ia\tan(c+dx)\right)}{d}\\
&= -\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}}\\
&\quad-\frac{(9ia^4)\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{7/2}}\,dx, x, ia\tan(c+dx)\right)}{8d}\\
&= -\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}}\\
&\quad-\frac{9ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}}\\
&\quad-\frac{(63ia^3)\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{7/2}}\,dx, x, ia\tan(c+dx)\right)}{32d}\\
&= \frac{63ia^2}{160d(a+ia\tan(c+dx))^{5/2}}-\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}}\\
&\quad-\frac{9ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}}\\
&\quad-\frac{(63ia^2)\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{5/2}}\,dx, x, ia\tan(c+dx)\right)}{64d}\\
&= \frac{63ia^2}{160d(a+ia\tan(c+dx))^{5/2}}-\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}}\\
&\quad-\frac{9ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}}+\frac{21ia}{64d(a+ia\tan(c+dx))^{3/2}}\\
&\quad-\frac{(63ia)\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{3/2}}\,dx, x, ia\tan(c+dx)\right)}{128d}
\end{aligned}$$

$$= \frac{63ia^2}{160d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}} - \frac{9ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}} + \frac{21ia}{64d(a+ia\tan(c+dx))^{3/2}} + \frac{63i}{128d\sqrt{a+ia\tan(c+dx)}} - \frac{(63i)\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{256d}$$

$$= \frac{63ia^2}{160d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}} - \frac{9ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}} + \frac{21ia}{64d(a+ia\tan(c+dx))^{3/2}} + \frac{63i}{128d\sqrt{a+ia\tan(c+dx)}} - \frac{(63i)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+ia\tan(c+dx)}\right)}{128d}$$

$$= -\frac{63i\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{128\sqrt{2}\sqrt{a}d} + \frac{63ia^2}{160d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{5/2}} - \frac{9ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}} + \frac{21ia}{64d(a+ia\tan(c+dx))^{3/2}} + \frac{63i}{128d\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.24

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{ia^2\,\text{Hypergeometric2F1}\left(-\frac{5}{2}, 3, -\frac{3}{2}, \frac{1}{2}(1+i\tan(c+dx))\right)}{20d(a+ia\tan(c+dx))^{5/2}}$$

```
[In] Integrate[Cos[c + d*x]^4/Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] ((I/20)*a^2*Hypergeometric2F1[-5/2, 3, -3/2, (1 + I*Tan[c + d*x])/2])/(d*(a
 + I*a*Tan[c + d*x])^(5/2))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 539 vs. 2(174) = 348.

Time = 11.10 (sec) , antiderivative size = 540, normalized size of antiderivative = 2.47

method	result
default	$-\frac{32i(\cos^5(dx+c))\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+32i(\cos^4(dx+c))\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-288\sin(dx+c)(\cos^4(dx+c))\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+84i(\cos^3(dx+c}{}$

```
[In] int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/1280/d*(32*I*cos(d*x+c)^5*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+32*I*cos(d*
x+c)^4*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-288*sin(d*x+c)*cos(d*x+c)^4*(-cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)+84*I*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)-288*sin(d*x+c)*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+84*I
*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-420*sin(d*x+c)*cos(d*x+c)^
2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-630*I*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)-315*I*cos(d*x+c)*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(
d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-420*(-cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)*cos(d*x+c)*sin(d*x+c)-630*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-31
5*I*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos
(d*x+c)+1))^(1/2))+315*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1
)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c))/(-cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)/(cos(d*x+c)+1)/(a*(1+I*tan(d*x+c)))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.34

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\left(-315i\sqrt{\frac{1}{2}}ad\sqrt{\frac{1}{ad^2}}e^{(5i\,dx+5i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(ade^{(2i\,dx+2i\,c)}+ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,dx-i\,c}\right.\right.}{}$$

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/1280*(-315*I*sqrt(1/2)*a*d*sqrt(1/(a*d^2))*e^(5*I*d*x + 5*I*c)*log(4*(sqr
t(2)*sqrt(1/2)*(a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1))*sqrt(1/(a*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + 315*I*sqrt(1
/2)*a*d*sqrt(1/(a*d^2))*e^(5*I*d*x + 5*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(a*d*
```

```
e^(2*I*d*x + 2*I*c) + a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a*d^2)
) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*(-10*I*e^(10*I*d*x + 10*I*c) - 95*I*e^(8*I*d*x + 8*I*c) + 203*I*e
^(6*I*d*x + 6*I*c) + 344*I*e^(4*I*d*x + 4*I*c) + 64*I*e^(2*I*d*x + 2*I*c) +
 8*I))*e^(-5*I*d*x - 5*I*c)/(a*d)
```

Sympy [F]

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\cos^4(c+dx)}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

```
[In] integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Integral(cos(c + d*x)**4/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.88

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{i\left(315\sqrt{2}\sqrt{a}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{ia\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{ia\tan(dx+c)+a}}\right)+\frac{4\left(315\,(ia\tan(dx+c)+a)^4a-1050\,(ia\tan(dx+c)+a)^3a^2+672\,(ia\tan(dx+c)+a)^2a\right.}{(ia\tan(dx+c)+a)^{\frac{9}{2}}-4\,(ia\tan(dx+c)+a)^{\frac{7}{2}}a+4\,(ia\tan(d\right.}}{2560\,ad}$$

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] 1/2560*I*(315*sqrt(2)*sqrt(a)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c)
 + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(315*(I*a*tan(d*
x + c) + a)^4*a - 1050*(I*a*tan(d*x + c) + a)^3*a^2 + 672*(I*a*tan(d*x + c)
 + a)^2*a^3 + 192*(I*a*tan(d*x + c) + a)*a^4 + 128*a^5)/((I*a*tan(d*x + c)
+ a)^(9/2) - 4*(I*a*tan(d*x + c) + a)^(7/2)*a + 4*(I*a*tan(d*x + c) + a)^(5
/2)*a^2))/(a*d)
```

Giac [F]

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\cos(dx+c)^4}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(cos(d*x + c)^4/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\cos(c+d\,x)^4}{\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}\,dx$$

```
[In] int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.339 $\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 292

$$\begin{aligned}\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx =& -\frac{429i\,\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{1024\sqrt{2}\sqrt{a}d}+\frac{429ia^3}{896d(a+ia\tan(c+dx))^{7/2}}\\ &-\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{7/2}}\\ &-\frac{13ia^5}{48d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}}\\ &-\frac{143ia^4}{192d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}}\\ &+\frac{429ia^2}{1280d(a+ia\tan(c+dx))^{5/2}}\\ &+\frac{143ia}{512d(a+ia\tan(c+dx))^{3/2}}+\frac{429i}{1024d\sqrt{a+ia\tan(c+dx)}}\end{aligned}$$

```
[Out] -429/2048*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/d*2^(1/2)
/a^(1/2)+429/1024*I/d/(a+I*a*tan(d*x+c))^(1/2)+429/896*I*a^3/d/(a+I*a*tan(d
*x+c))^(7/2)-1/6*I*a^6/d/(a-I*a*tan(d*x+c))^3/(a+I*a*tan(d*x+c))^(7/2)-13/4
8*I*a^5/d/(a-I*a*tan(d*x+c))^2/(a+I*a*tan(d*x+c))^(7/2)-143/192*I*a^4/d/(a-
I*a*tan(d*x+c))/(a+I*a*tan(d*x+c))^(7/2)+429/1280*I*a^2/d/(a+I*a*tan(d*x+c)
)^(5/2)+143/512*I*a/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3568, 44, 53, 65, 212}

$$\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{7/2}} - \frac{13ia^5}{48d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}} - \frac{143ia^4}{192d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}} + \frac{429ia^3}{896d(a+ia\tan(c+dx))^{7/2}} + \frac{429ia^2}{1280d(a+ia\tan(c+dx))^{5/2}} - \frac{429i\,\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{1024\sqrt{2}\sqrt{a}d} + \frac{143ia}{512d(a+ia\tan(c+dx))^{3/2}} + \frac{429i}{1024d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[Cos[c + d*x]^6/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((-429*I)/1024)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sq
rt[2]*Sqrt[a]*d) + (((429*I)/896)*a^3)/(d*(a + I*a*Tan[c + d*x])^(7/2)) - (
(I/6)*a^6)/(d*(a - I*a*Tan[c + d*x])^3*(a + I*a*Tan[c + d*x])^(7/2)) - (((1
3*I)/48)*a^5)/(d*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(7/2)) - (
((143*I)/192)*a^4)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(7/2))
+ (((429*I)/1280)*a^2)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((143*I)/512)*a)
/(d*(a + I*a*Tan[c + d*x])^(3/2)) + ((429*I)/1024)/(d*Sqrt[a + I*a*Tan[c +
d*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(ia^7)\,\text{Subst}\left(\int \frac{1}{(a-x)^4(a+x)^{9/2}}\,dx, x, ia\tan(c+dx)\right)}{d}\\
&= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{7/2}}\\
&\quad -\frac{(13ia^6)\,\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^{9/2}}\,dx, x, ia\tan(c+dx)\right)}{12d}\\
&= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{7/2}}\\
&\quad -\frac{13ia^5}{48d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}}\\
&\quad -\frac{(143ia^5)\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{9/2}}\,dx, x, ia\tan(c+dx)\right)}{96d}
\end{aligned}$$

$$=-\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{7/2}}-\frac{13ia^5}{48d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}}-\frac{143ia^4}{192d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}}-\frac{(429ia^4)\,\text{Subst}\left(\int\frac{1}{(a-x)(a+x)^{9/2}}\,dx,x,ia\tan(c+dx)\right)}{128d}$$

$$=\frac{429ia^3}{896d(a+ia\tan(c+dx))^{7/2}}-\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{7/2}}-\frac{13ia^5}{48d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}}-\frac{143ia^4}{192d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}}-\frac{(429ia^3)\,\text{Subst}\left(\int\frac{1}{(a-x)(a+x)^{7/2}}\,dx,x,ia\tan(c+dx)\right)}{256d}$$

$$=\frac{429ia^3}{896d(a+ia\tan(c+dx))^{7/2}}-\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{7/2}}-\frac{13ia^5}{48d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}}-\frac{143ia^4}{192d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}}+\frac{429ia^2}{1280d(a+ia\tan(c+dx))^{5/2}}-\frac{(429ia^2)\,\text{Subst}\left(\int\frac{1}{(a-x)(a+x)^{5/2}}\,dx,x,ia\tan(c+dx)\right)}{512d}$$

$$=\frac{429ia^3}{896d(a+ia\tan(c+dx))^{7/2}}-\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{7/2}}-\frac{13ia^5}{48d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}}-\frac{143ia^4}{192d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}}+\frac{429ia^2}{1280d(a+ia\tan(c+dx))^{5/2}}+\frac{143ia}{512d(a+ia\tan(c+dx))^{3/2}}-\frac{(429ia)\text{Subst}\left(\int\frac{1}{(a-x)(a+x)^{3/2}}\,dx,x,ia\tan(c+dx)\right)}{1024d}$$

$$
\begin{aligned}
&= \frac{429ia^3}{896d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{7/2}} \\
&\quad - \frac{13ia^5}{48d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}} \\
&\quad - \frac{143ia^4}{192d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}} \\
&\quad + \frac{429ia^2}{1280d(a+ia\tan(c+dx))^{5/2}} + \frac{143ia}{512d(a+ia\tan(c+dx))^{3/2}} \\
&\quad + \frac{429i}{1024d\sqrt{a+ia\tan(c+dx)}} - \frac{(429i)\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{2048d} \\
&= \frac{429ia^3}{896d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{7/2}} \\
&\quad - \frac{13ia^5}{48d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}} \\
&\quad - \frac{143ia^4}{192d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}} \\
&\quad + \frac{429ia^2}{1280d(a+ia\tan(c+dx))^{5/2}} + \frac{143ia}{512d(a+ia\tan(c+dx))^{3/2}} \\
&\quad + \frac{429i}{1024d\sqrt{a+ia\tan(c+dx)}} - \frac{(429i)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+ia\tan(c+dx)}\right)}{1024d} \\
&= -\frac{429i\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{1024\sqrt{2}\sqrt{a}d} + \frac{429ia^3}{896d(a+ia\tan(c+dx))^{7/2}} \\
&\quad - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{7/2}} \\
&\quad - \frac{13ia^5}{48d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}} \\
&\quad - \frac{143ia^4}{192d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}} + \frac{429ia^2}{1280d(a+ia\tan(c+dx))^{5/2}} \\
&\quad + \frac{143ia}{512d(a+ia\tan(c+dx))^{3/2}} + \frac{429i}{1024d\sqrt{a+ia\tan(c+dx)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.36 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.18

$$\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{ia^3\,\text{Hypergeometric2F1}\left(-\frac{7}{2},4,-\frac{5}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{56d(a+ia\tan(c+dx))^{7/2}}$$

```
[In] Integrate[Cos[c + d*x]^6/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((I/56)*a^3*Hypergeometric2F1[-7/2, 4, -5/2, (1 + I*Tan[c + d*x])/2])/(d*(a
 + I*a*Tan[c + d*x])^(7/2))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 673 vs. 2(234) = 468.

Time = 9.44 (sec) , antiderivative size = 674, normalized size of antiderivative = 2.31

method	result
default	$-\frac{-90090 i \cos(dx+c)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-45045 i \arctan\left(\frac{\cos(dx+c)+1+i\sin(dx+c)}{2(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)-33280\sin(dx+c)\left(\cos^6(dx+c)\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)}}}{}$

```
[In] int(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/215040/d*(-90090*I*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-45045*I
*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2))-33280*sin(d*x+c)*cos(d*x+c)^6*(-cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)+12012*I*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-33280*sin(d*x
+c)*cos(d*x+c)^5*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-90090*I*(-cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)-41184*sin(d*x+c)*cos(d*x+c)^4*(-cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)+4576*I*cos(d*x+c)^5*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-41184*sin(
d*x+c)*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+4576*I*cos(d*x+c)^4*
(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-60060*sin(d*x+c)*cos(d*x+c)^2*(-cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)+2560*I*cos(d*x+c)^7*(-cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)+2560*I*cos(d*x+c)^6*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-60060*(-cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)-45045*I*cos(d*x+c)*arctan(1
/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^
(1/2))+12012*I*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+45045*arctan
(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2))*sin(d*x+c))/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)+1)/(a*(
1+I*tan(d*x+c)))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.08

$$\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\left(-45045i\sqrt{\frac{1}{2}}ad\sqrt{\frac{1}{ad^2}}e^{(7i\,dx+7i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(ade^{(2i\,dx+2i\,c)}+ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}+ae^{(i\,dx+i\,c)}\right)e^{(-i\,d}\right.\right.}{}$$

```
[In] integrate(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/215040*(-45045*I*sqrt(1/2)*a*d*sqrt(1/(a*d^2))*e^(7*I*d*x + 7*I*c)*log(4*
(sqrt(2)*sqrt(1/2)*(a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*sqrt(1/(a*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + 45045*I*
sqrt(1/2)*a*d*sqrt(1/(a*d^2))*e^(7*I*d*x + 7*I*c)*log(-4*(sqrt(2)*sqrt(1/2)
*(a*d*e^(2*I*d*x + 2*I*c) + a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(
a*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2)*sqrt(a/(e^(2*I*d*x
 + 2*I*c) + 1))*(-280*I*e^(14*I*d*x + 14*I*c) - 2870*I*e^(12*I*d*x + 12*I*c
) - 16345*I*e^(10*I*d*x + 10*I*c) + 27029*I*e^(8*I*d*x + 8*I*c) + 49792*I*e
^(6*I*d*x + 6*I*c) + 11072*I*e^(4*I*d*x + 4*I*c) + 2304*I*e^(2*I*d*x + 2*I*
c) + 240*I))*e^(-7*I*d*x - 7*I*c)/(a*d)
```

Sympy [F]

$$\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\cos^6(c+dx)}{\sqrt{ia\left(\tan(c+dx)-i\right)}}\,dx$$

```
[In] integrate(cos(d*x+c)**6/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**6/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.84

$$\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{i\left(45045\sqrt{2}\sqrt{a}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)+\frac{4\left(45045\,(i\,a\tan(dx+c)+a)^6a-240240\,(i\,a\tan(dx+c)+a)^5a^2+396396\,(i\,a\tan(\right.}{(i\,a\tan(dx+c)+a)^{\frac{13}{2}}-6\,(i\,a\tan}\right.}{430080\,ad}$$

```
[In] integrate(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/430080*I*(45045*sqrt(2)*sqrt(a)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x
+ c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a))) + 4*(45045*(I*a*
tan(d*x + c) + a)^6*a - 240240*(I*a*tan(d*x + c) + a)^5*a^2 + 396396*(I*a*t
an(d*x + c) + a)^4*a^3 - 164736*(I*a*tan(d*x + c) + a)^3*a^4 - 36608*(I*a*t
an(d*x + c) + a)^2*a^5 - 19968*(I*a*tan(d*x + c) + a)*a^6 - 15360*a^7)/((I*
a*tan(d*x + c) + a)^(13/2) - 6*(I*a*tan(d*x + c) + a)^(11/2)*a + 12*(I*a*ta
n(d*x + c) + a)^(9/2)*a^2 - 8*(I*a*tan(d*x + c) + a)^(7/2)*a^3))/(a*d)
```

Giac [F]

$$\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\cos\left(dx+c\right)^6}{\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

```
[In] integrate(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^6/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^6(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\cos\left(c+d\,x\right)^6}{\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

```
[In] int(cos(c + d*x)^6/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] int(cos(c + d*x)^6/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.340 $\int \frac{\sec^9(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 147

$$\int \frac{\sec^9(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{256ia^4\sec^9(c+dx)}{6435d(a+ia\tan(c+dx))^{9/2}} + \frac{64ia^3\sec^9(c+dx)}{715d(a+ia\tan(c+dx))^{7/2}} + \frac{8ia^2\sec^9(c+dx)}{65d(a+ia\tan(c+dx))^{5/2}} + \frac{2ia\sec^9(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] 256/6435*I*a^4*sec(d*x+c)^9/d/(a+I*a*tan(d*x+c))^(9/2)+64/715*I*a^3*sec(d*x
+c)^9/d/(a+I*a*tan(d*x+c))^(7/2)+8/65*I*a^2*sec(d*x+c)^9/d/(a+I*a*tan(d*x+c
))^(5/2)+2/15*I*a*sec(d*x+c)^9/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3575, 3574}

$$\int \frac{\sec^9(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{256ia^4\sec^9(c+dx)}{6435d(a+ia\tan(c+dx))^{9/2}} + \frac{64ia^3\sec^9(c+dx)}{715d(a+ia\tan(c+dx))^{7/2}} + \frac{8ia^2\sec^9(c+dx)}{65d(a+ia\tan(c+dx))^{5/2}} + \frac{2ia\sec^9(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}$$

```
[In] Int[Sec[c + d*x]^9/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((256*I)/6435)*a^4*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(9/2)) + (((6
4*I)/715)*a^3*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (((8*I)/65
)*a^2*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((2*I)/15)*a*Sec[
c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2ia\sec^9(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}+\frac{1}{5}(4a)\int\frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx\\
&= \frac{8ia^2\sec^9(c+dx)}{65d(a+ia\tan(c+dx))^{5/2}}+\frac{2ia\sec^9(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}\\
&\quad+\frac{1}{65}\left(32a^2\right)\int\frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx\\
&= \frac{64ia^3\sec^9(c+dx)}{715d(a+ia\tan(c+dx))^{7/2}}+\frac{8ia^2\sec^9(c+dx)}{65d(a+ia\tan(c+dx))^{5/2}}\\
&\quad+\frac{2ia\sec^9(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}+\frac{1}{715}\left(128a^3\right)\int\frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx\\
&= \frac{256ia^4\sec^9(c+dx)}{6435d(a+ia\tan(c+dx))^{9/2}}+\frac{64ia^3\sec^9(c+dx)}{715d(a+ia\tan(c+dx))^{7/2}}\\
&\quad+\frac{8ia^2\sec^9(c+dx)}{65d(a+ia\tan(c+dx))^{5/2}}+\frac{2ia\sec^9(c+dx)}{15d(a+ia\tan(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.65

$$\int\frac{\sec^9(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$=\frac{2\sec^8(c+dx)(510\cos(c+dx)+731\cos(3(c+dx))+3i(90\sin(c+dx)+233\sin(3(c+dx))))(i\cos(4(c+}{6435d\sqrt{a+ia\tan(c+dx)}}$$

[In] Integrate[Sec[c + d*x]^9/Sqrt[a + I*a*Tan[c + d*x]],x]

[Out] (2*Sec[c + d*x]^8*(510*Cos[c + d*x] + 731*Cos[3*(c + d*x)] + (3*I)*(90*Sin[
c + d*x] + 233*Sin[3*(c + d*x)]))*(I*Cos[4*(c + d*x)] + Sin[4*(c + d*x)]))/
(6435*d*Sqrt[a + I*a*Tan[c + d*x]])

Maple [A] (verified)

Time = 9.33 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.86

method	result
default	$\frac{\frac{2048i\sec(dx+c)}{6435}+\frac{2048\sec(dx+c)\tan(dx+c)}{6435}+\frac{256i\left(\sec^{3}(dx+c)\right)}{6435}+\frac{256\tan(dx+c)\left(\sec^{3}(dx+c)\right)}{1287}+\frac{112i\left(\sec^{5}(dx+c)\right)}{6435}+\frac{112\tan(dx+c)\left(\sec^{5}(dx+c)\right)}{715}}{d\sqrt{a(1+i\tan(dx+c))}}$

[In] int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/6435/d/(a*(1+I*tan(d*x+c)))^(1/2)*(1024*I*sec(d*x+c)+1024*sec(d*x+c)*tan(
d*x+c)+128*I*sec(d*x+c)^3+640*tan(d*x+c)*sec(d*x+c)^3+56*I*sec(d*x+c)^5+504
*tan(d*x+c)*sec(d*x+c)^5+33*I*sec(d*x+c)^7+429*tan(d*x+c)*sec(d*x+c)^7)

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.04

$$\int \frac{\sec^9(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx =$$
$$-\frac{256\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-715i\,e^{(6i\,dx+6i\,c)}-390i\,e^{(4i\,dx+4i\,c)}-120i\,e^{(2i\,dx+2i\,c)}-\right.}{6435\left(ade^{(14i\,dx+14i\,c)}+7\,ade^{(12i\,dx+12i\,c)}+21\,ade^{(10i\,dx+10i\,c)}+35\,ade^{(8i\,dx+8i\,c)}+35\,ade^{(6i\,dx+6i\,c)}+21\,ad\right.}$$

[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")

[Out] -256/6435*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-715*I*e^(6*I*d*x + 6*
I*c) - 390*I*e^(4*I*d*x + 4*I*c) - 120*I*e^(2*I*d*x + 2*I*c) - 16*I)/(a*d*e
^(14*I*d*x + 14*I*c) + 7*a*d*e^(12*I*d*x + 12*I*c) + 21*a*d*e^(10*I*d*x + 1
0*I*c) + 35*a*d*e^(8*I*d*x + 8*I*c) + 35*a*d*e^(6*I*d*x + 6*I*c) + 21*a*d*e
^(4*I*d*x + 4*I*c) + 7*a*d*e^(2*I*d*x + 2*I*c) + a*d)

Sympy [F]

$$\int \frac{\sec^9(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec^9(c+dx)}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

```
[In] integrate(sec(d*x+c)**9/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**9/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 608 vs. 2(115) = 230.

Time = 0.43 (sec) , antiderivative size = 608, normalized size of antiderivative = 4.14

$$\int \frac{\sec^9(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx =$$
$$-\frac{2\left(-1241i\sqrt{a}-\frac{5194\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}+\frac{6090i\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2490\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14430i\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{33618\sqrt{a}\sin(dx+c}{(\cos(dx+c)+1)^5}\right.}{6435\left(a-\frac{8a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right.}$$

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/6435*(-1241*I*sqrt(a) - 5194*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) + 6
090*I*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 2490*sqrt(a)*sin(d*x +
c)^3/(cos(d*x + c) + 1)^3 - 14430*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) +
1)^4 - 33618*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 13442*I*sqrt(a)*
sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 18590*sqrt(a)*sin(d*x + c)^7/(cos(d*x
 + c) + 1)^7 - 18590*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 13442*I*
sqrt(a)*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 33618*sqrt(a)*sin(d*x + c)^
11/(cos(d*x + c) + 1)^11 + 14430*I*sqrt(a)*sin(d*x + c)^12/(cos(d*x + c) +
1)^12 + 2490*sqrt(a)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - 6090*I*sqrt(a)
*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 5194*sqrt(a)*sin(d*x + c)^15/(cos(
d*x + c) + 1)^15 + 1241*I*sqrt(a)*sin(d*x + c)^16/(cos(d*x + c) + 1)^16)*sq
rt(sin(d*x + c)/(cos(d*x + c) + 1) + 1)*sqrt(sin(d*x + c)/(cos(d*x + c) + 1
) - 1)/((a - 8*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 28*a*sin(d*x + c)^4/
(cos(d*x + c) + 1)^4 - 56*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a*sin(
d*x + c)^8/(cos(d*x + c) + 1)^8 - 56*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^1
0 + 28*a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 8*a*sin(d*x + c)^14/(cos(d
*x + c) + 1)^14 + a*sin(d*x + c)^16/(cos(d*x + c) + 1)^16)*d*sqrt(-2*I*sin(
d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1))
```

Giac [F]

$$\int \frac{\sec^9(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^9}{\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^9/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [B] (verification not implemented)

Time = 10.28 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.05

$$\int \frac{\sec^9(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{e^{-c\,1i-d\,x\,1i}\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,256i}{9\,a\,d\left(e^{c\,2i+d\,x\,2i}+1\right)^4} - \frac{e^{-c\,1i-d\,x\,1i}\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,768i}{11\,a\,d\left(e^{c\,2i+d\,x\,2i}+1\right)^5} + \frac{e^{-c\,1i-d\,x\,1i}\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,768i}{13\,a\,d\left(e^{c\,2i+d\,x\,2i}+1\right)^6} - \frac{e^{-c\,1i-d\,x\,1i}\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,256i}{15\,a\,d\left(e^{c\,2i+d\,x\,2i}+1\right)^7}$$

```
[In] int(1/(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^(1/2)),x)
[Out] (exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i +
d*x*2i) + 1))^(1/2)*256i)/(9*a*d*(exp(c*2i + d*x*2i) + 1)^4) - (exp(- c*1i
- d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1)
)^(1/2)*768i)/(11*a*d*(exp(c*2i + d*x*2i) + 1)^5) + (exp(- c*1i - d*x*1i)*(
a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*768
i)/(13*a*d*(exp(c*2i + d*x*2i) + 1)^6) - (exp(- c*1i - d*x*1i)*(a - (a*(exp
(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*256i)/(15*a*d*
(exp(c*2i + d*x*2i) + 1)^7)
```

3.341 $\int \frac{\sec^7(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 110

$$\int \frac{\sec^7(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{64ia^3\sec^7(c+dx)}{693d(a+ia\tan(c+dx))^{7/2}} + \frac{16ia^2\sec^7(c+dx)}{99d(a+ia\tan(c+dx))^{5/2}} + \frac{2ia\sec^7(c+dx)}{11d(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] 64/693*I*a^3*sec(d*x+c)^7/d/(a+I*a*tan(d*x+c))^(7/2)+16/99*I*a^2*sec(d*x+c)
^7/d/(a+I*a*tan(d*x+c))^(5/2)+2/11*I*a*sec(d*x+c)^7/d/(a+I*a*tan(d*x+c))^(3
/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3575, 3574}

$$\int \frac{\sec^7(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{64ia^3\sec^7(c+dx)}{693d(a+ia\tan(c+dx))^{7/2}} + \frac{16ia^2\sec^7(c+dx)}{99d(a+ia\tan(c+dx))^{5/2}} + \frac{2ia\sec^7(c+dx)}{11d(a+ia\tan(c+dx))^{3/2}}$$

```
[In] Int[Sec[c + d*x]^7/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((64*I)/693)*a^3*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (((16*
I)/99)*a^2*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((2*I)/11)*a
*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2ia\sec^7(c+dx)}{11d(a+ia\tan(c+dx))^{3/2}}+\frac{1}{11}(8a)\int\frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx\\
&= \frac{16ia^2\sec^7(c+dx)}{99d(a+ia\tan(c+dx))^{5/2}}+\frac{2ia\sec^7(c+dx)}{11d(a+ia\tan(c+dx))^{3/2}}\\
&\quad+\frac{1}{99}\left(32a^2\right)\int\frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx\\
&= \frac{64ia^3\sec^7(c+dx)}{693d(a+ia\tan(c+dx))^{7/2}}+\frac{16ia^2\sec^7(c+dx)}{99d(a+ia\tan(c+dx))^{5/2}}+\frac{2ia\sec^7(c+dx)}{11d(a+ia\tan(c+dx))^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.70

$$
\int\frac{\sec^7(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx
=\frac{2\sec^6(c+dx)(44+107\cos(2(c+dx))+91i\sin(2(c+dx)))(i\cos(3(c+dx))+\sin(3(c+dx)))}{693d\sqrt{a+ia\tan(c+dx)}}
$$

```
[In] Integrate[Sec[c + d*x]^7/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (2*Sec[c + d*x]^6*(44 + 107*Cos[2*(c + d*x)] + (91*I)*Sin[2*(c + d*x)])*(I*
Cos[3*(c + d*x)] + Sin[3*(c + d*x)]))/(693*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] (verified)

Time = 6.94 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{\frac{256 i \sec(dx+c)}{693}+\frac{256 \sec(dx+c) \tan(dx+c)}{693}+\frac{32 i \left(\sec^{3}(dx+c)\right)}{693}+\frac{160 \tan(dx+c) \left(\sec^{3}(dx+c)\right)}{693}+\frac{2 i \left(\sec^{5}(dx+c)\right)}{99}+\frac{2 \tan(dx+c) \left(\sec^{5}(dx+c)\right)}{11}}{d \sqrt{a(1+i \tan(dx+c))}}$	99

```
[In] int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/693/d/(a*(1+I*tan(d*x+c)))^(1/2)*(128*I*sec(d*x+c)+128*sec(d*x+c)*tan(d*x
+c)+16*I*sec(d*x+c)^3+80*tan(d*x+c)*sec(d*x+c)^3+7*I*sec(d*x+c)^5+63*tan(d*
x+c)*sec(d*x+c)^5)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05

$$\int \frac{\sec^7(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{64\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-99i\,e^{(4i\,dx+4i\,c)}-44i\,e^{(2i\,dx+2i\,c)}-8i\right)}{693\left(ade^{(10i\,dx+10i\,c)}+5\,ade^{(8i\,dx+8i\,c)}+10\,ade^{(6i\,dx+6i\,c)}+10\,ade^{(4i\,dx+4i\,c)}+5\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -64/693*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-99*I*e^(4*I*d*x + 4*I*c
) - 44*I*e^(2*I*d*x + 2*I*c) - 8*I)/(a*d*e^(10*I*d*x + 10*I*c) + 5*a*d*e^(8
*I*d*x + 8*I*c) + 10*a*d*e^(6*I*d*x + 6*I*c) + 10*a*d*e^(4*I*d*x + 4*I*c) +
 5*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

Sympy [F]

$$\int \frac{\sec^7(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec^7(c+dx)}{\sqrt{ia\left(\tan(c+dx)-i\right)}}\,dx$$

```
[In] integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**7/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(86) = 172$.

Time = 0.38 (sec) , antiderivative size = 474, normalized size of antiderivative = 4.31

$$\int \frac{\sec^7(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{2\left(-151i\sqrt{a} - \frac{542\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1} + \frac{484i\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{22\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{627i\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{1452\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - 1\right.}{693\left(a - \frac{6a\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{15a\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{20a\sin(dx+c)}{(\cos(dx+c)+1)}\right.}$$

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/693*(-151*I*sqrt(a) - 542*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) + 484*
I*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 22*sqrt(a)*sin(d*x + c)^3/(
cos(d*x + c) + 1)^3 - 627*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 1
452*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1452*sqrt(a)*sin(d*x + c)
^7/(cos(d*x + c) + 1)^7 + 627*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8
 - 22*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 484*I*sqrt(a)*sin(d*x +
 c)^10/(cos(d*x + c) + 1)^10 - 542*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) +
1)^11 + 151*I*sqrt(a)*sin(d*x + c)^12/(cos(d*x + c) + 1)^12)*sqrt(sin(d*x +
 c)/(cos(d*x + c) + 1) + 1)*sqrt(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/((a -
 6*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a*sin(d*x + c)^4/(cos(d*x + c
) + 1)^4 - 20*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 15*a*sin(d*x + c)^8/(
cos(d*x + c) + 1)^8 - 6*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + a*sin(d*x
 + c)^12/(cos(d*x + c) + 1)^12)*d*sqrt(-2*I*sin(d*x + c)/(cos(d*x + c) + 1)
 + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1))
```

Giac [F]

$$\int \frac{\sec^7(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec(dx+c)^7}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^7/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [B] (verification not implemented)

Time = 6.63 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

$$\int \frac{\sec^7(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{64\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,44\mathrm{i}+\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}\,99\mathrm{i}+8\mathrm{i}\right)}{693\,a\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^5}$$

```
[In] int(1/(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

```
[Out] (64*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i
 + d*x*2i) + 1))^(1/2)*(exp(c*2i + d*x*2i)*44i + exp(c*4i + d*x*4i)*99i + 8
i))/(693*a*d*(exp(c*2i + d*x*2i) + 1)^5)
```

3.342 $\int \frac{\sec^5(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 73

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{8ia^2\sec^5(c+dx)}{35d(a+ia\tan(c+dx))^{5/2}} + \frac{2ia\sec^5(c+dx)}{7d(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] 8/35*I*a^2*sec(d*x+c)^5/d/(a+I*a*tan(d*x+c))^(5/2)+2/7*I*a*sec(d*x+c)^5/d/(
a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3575, 3574}

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{8ia^2\sec^5(c+dx)}{35d(a+ia\tan(c+dx))^{5/2}} + \frac{2ia\sec^5(c+dx)}{7d(a+ia\tan(c+dx))^{3/2}}$$

```
[In] Int[Sec[c + d*x]^5/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((8*I)/35)*a^2*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((2*I)/
7)*a*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(3/2))
```

<u>Rule 3574</u>

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2ia\sec^5(c+dx)}{7d(a+ia\tan(c+dx))^{3/2}}+\frac{1}{7}(4a)\int\frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx\\ &= \frac{8ia^2\sec^5(c+dx)}{35d(a+ia\tan(c+dx))^{5/2}}+\frac{2ia\sec^5(c+dx)}{7d(a+ia\tan(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

$$\int\frac{\sec^5(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{2\sec^3(c+dx)(\cos(2(c+dx))-i\sin(2(c+dx)))(-9i+5\tan(c+dx))}{35d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[Sec[c + d*x]^5/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (-2*Sec[c + d*x]^3*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)])*(-9*I + 5*Tan[c
+ d*x]))/(35*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] (verified)

Time = 7.52 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.99

method	result	size
default	$\frac{\frac{16i\sec(dx+c)}{35}+\frac{16\sec(dx+c)\tan(dx+c)}{35}+\frac{2i\left(\sec^3(dx+c)\right)}{35}+\frac{2\tan(dx+c)\left(\sec^3(dx+c)\right)}{7}}{d\sqrt{a(1+i\tan(dx+c))}}$	72

```
[In] int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/35/d/(a*(1+I*tan(d*x+c)))^(1/2)*(8*I*sec(d*x+c)+8*sec(d*x+c)*tan(d*x+c)+I
*sec(d*x+c)^3+5*tan(d*x+c)*sec(d*x+c)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{16\,\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-7i\,e^{(2i\,dx+2i\,c)}-2i\right)}{35\,\left(ade^{(6i\,dx+6i\,c)}+3\,ade^{(4i\,dx+4i\,c)}+3\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -16/35*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-7*I*e^(2*I*d*x + 2*I*c)
- 2*I)/(a*d*e^(6*I*d*x + 6*I*c) + 3*a*d*e^(4*I*d*x + 4*I*c) + 3*a*d*e^(2*I*
d*x + 2*I*c) + a*d)
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec^5(c+dx)}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

```
[In] integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**5/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 340 vs. 2(57) = 114.

Time = 0.35 (sec) , antiderivative size = 340, normalized size of antiderivative = 4.66

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{2\left(-9i\sqrt{a}-\frac{26\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}+\frac{14i\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{14\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{14i\sqrt{a}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{26\sqrt{a}\sin(\cdots}{(\cos(dx+\cdots}\cdots\right.}{35\left(a-\frac{4a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d\sqrt{-\frac{2\cdots}{c\cdots}}\cdots}$$

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/35*(-9*I*sqrt(a) - 26*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) + 14*I*sqr
t(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 14*sqrt(a)*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 - 14*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 14*I*sqrt
(a)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 26*sqrt(a)*sin(d*x + c)^7/(cos(d*
x + c) + 1)^7 + 9*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*sqrt(sin(d
```

```
*x + c)/(cos(d*x + c) + 1) + 1)*sqrt(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/(
(a - 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*sin(d*x + c)^4/(cos(d*x
+ c) + 1)^4 - 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a*sin(d*x + c)^8/(c
os(d*x + c) + 1)^8)*d*sqrt(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 - 1))
```

Giac [F]

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)^5}{\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^5/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [B] (verification not implemented)

Time = 10.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

$$\int \frac{\sec^5(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{16\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,7\mathrm{i}+2\mathrm{i}\right)\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}}{35\,a\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^3}$$

```
[In] int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

```
[Out] (16*exp(- c*1i - d*x*1i)*(exp(c*2i + d*x*2i)*7i + 2i)*(a - (a*(exp(c*2i + d
*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2))/(35*a*d*(exp(c*2i + d*
x*2i) + 1)^3)
```

3.343 $\int \frac{\sec^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 35

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2ia\sec^3(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] 2/3*I*a*sec(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {3574}

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2ia\sec^3(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

```
[In] Int[Sec[c + d*x]^3/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((2*I)/3)*a*Sec[c + d*x]^3)/(d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\text{integral} = \frac{2ia\sec^3(c+dx)}{3d(a+ia\tan(c+dx))^{3/2}}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2\sec(c+dx)(i+\tan(c+dx))}{3d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[Sec[c + d*x]^3/Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] (2*Sec[c + d*x]*(I + Tan[c + d*x]))/(3*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] (verified)

Time = 7.64 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

method	result	size
default	$\frac{\frac{2i\sec(dx+c)}{3}+\frac{2\sec(dx+c)\tan(dx+c)}{3}}{d\sqrt{a(1+i\tan(dx+c))}}$	44

```
[In] int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3/d/(a*(1+I*tan(d*x+c)))^(1/2)*(I*sec(d*x+c)+sec(d*x+c)*tan(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{4i\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{3\,(ade^{(2i\,dx+2i\,c)}+ad)}$$

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
[Out] 4/3*I*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(a*d*e^(2*I*d*x + 2*I*c) +
a*d)
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec^3(c+dx)}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

```
[In] integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Integral(sec(c + d*x)**3/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(27) = 54.

Time = 0.33 (sec) , antiderivative size = 206, normalized size of antiderivative = 5.89

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{2\left(-i\sqrt{a}-\frac{2\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{i\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)\sqrt{\frac{\sin(dx+c)}{\cos(dx+c)+1}+1}\sqrt{\frac{\sin(dx+c)}{\cos(dx+c)+1}-1}}{3\left(a-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)d\sqrt{-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}}$$

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] -2/3*(-I*sqrt(a) - 2*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 2*sqrt(a)*si
n(d*x + c)^3/(cos(d*x + c) + 1)^3 + I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c)
+ 1)^4)*sqrt(sin(d*x + c)/(cos(d*x + c) + 1) + 1)*sqrt(sin(d*x + c)/(cos(d*
x + c) + 1) - 1)/((a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4)*d*sqrt(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) +
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1))
```

Giac [F]

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec(dx+c)^3}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^3/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [B] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.80

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$=\frac{2\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1i)}{\cos(2\,c+2\,d\,x)+1}}\,(\sin\left(c+d\,x\right)+\sin\left(3\,c+3\,d\,x\right)+\cos\left(c+d\,x\right)\,1i+\cos\left(3\,c+3\,d\,x\right)\,1i)}{3\,a\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

```
[In] int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

```
[Out] (2*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1)
)^(1/2)*(cos(c + d*x)*1i + sin(c + d*x) + cos(3*c + 3*d*x)*1i + sin(3*c + 3
*d*x)))/(3*a*d*(cos(2*c + 2*d*x) + 1))
```

3.344 $\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 52

$$\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{i\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}d}$$

```
[Out] I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*2^(1/2)/
d/a^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3570, 212}

$$\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{i\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}d}$$

```
[In] Int[Sec[c + d*x]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (I*Sqrt[2]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x
]])])/(Sqrt[a]*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{(2i)\text{Subst}\left(\int \frac{1}{2-ax^2}\,dx, x, \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\right)}{d}$$
$$= \frac{i\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{a}d}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.35

$$\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2ie^{i(c+dx)}\text{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)}{d\sqrt{1+e^{2i(c+dx)}}\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[Sec[c + d*x]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((2*I)*E^(I*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])/(d*Sqrt[1 +
E^((2*I)*(c + d*x))]*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(41) = 82.

Time = 7.04 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.35

method	result	size
default	$\frac{(i\cos(dx+c)+i-\sin(dx+c))\arctan\left(\frac{i\sin(dx+c)-\cos(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)}{d(\cos(dx+c)+1)\sqrt{a(1+i\tan(dx+c))}\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}$	122

```
[In] int(sec(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(I*cos(d*x+c)+I-sin(d*x+c))*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos
(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))/(cos(d*x+c)+1)/(a*(1+I*tan(d
*x+c)))^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(39) = 78.

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.87

$$\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= -\frac{1}{2}i\sqrt{2}\sqrt{\frac{1}{ad^2}}\log\left(-\frac{4\left(\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}-i\right)e^{(-i\,dx-i\,c)}}{d}\right)$$
$$+\frac{1}{2}i\sqrt{2}\sqrt{\frac{1}{ad^2}}\log\left(-\frac{4\left(\left(-i\,de^{(2i\,dx+2i\,c)}-i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}-i\right)e^{(-i\,dx-i\,c)}}{d}\right)$$

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*I*sqrt(2)*sqrt(1/(a*d^2))*log(-4*((I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a*d^2)) - I)*e^(-I*d*x - I*c)/d) + 1/
2*I*sqrt(2)*sqrt(1/(a*d^2))*log(-4*((-I*d*e^(2*I*d*x + 2*I*c) - I*d)*sqrt(a
/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a*d^2)) - I)*e^(-I*d*x - I*c)/d)
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec(c+dx)}{\sqrt{ia\left(\tan(c+dx)-i\right)}}\,dx$$

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [F]

$$\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec(dx+c)}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/sqrt(I*a*tan(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sec\left(dx+c\right)}{\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}\,dx$$

```
[In] int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
[Out] int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(1/2)), x)
```

3.345 $\int \frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 122

$$\int \frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{3i\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{4\sqrt{2}\sqrt{a}d}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}-\frac{3i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4ad}$$

```
[Out] 3/8*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/d*2^
(1/2)/a^(1/2)+1/2*I*cos(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)-3/4*I*cos(d*x+c)*
(a+I*a*tan(d*x+c))^(1/2)/a/d
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3583, 3571, 3570, 212}

$$\int \frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{3i\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{4\sqrt{2}\sqrt{a}d}-\frac{3i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4ad}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[Cos[c + d*x]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((3*I)/4)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x
]])])/(Sqrt[2]*Sqrt[a]*d) + ((I/2)*Cos[c + d*x])/(d*Sqrt[a + I*a*Tan[c + d*
x]]) - (((3*I)/4)*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/(a*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3571

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\text{integral} = \frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}} + \frac{3\int\cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx}{4a}$$

$$= \frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}} - \frac{3i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4ad} + \frac{3}{8}\int\frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$$

$$= \frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}} - \frac{3i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4ad} + \frac{(3i)\text{Subst}\left(\int\frac{1}{2-ax^2}\,dx, x, \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\right)}{4d}$$

$$=\frac{3i\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{4\sqrt{2}\sqrt{a}d}+\frac{i\cos(c+dx)}{2d\sqrt{a+ia\tan(c+dx)}}-\frac{3i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4ad}$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.79

$$\int\frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$=\frac{\sec(c+dx)\left(3i\sqrt{1+e^{2i(c+dx)}}\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)-i(1+\cos(2(c+dx))+3i\sin(2(c+dx)))\right)}{8d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[Cos[c + d*x]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (Sec[c + d*x]*((3*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I
)*(c + d*x))]] - I*(1 + Cos[2*(c + d*x)] + (3*I)*Sin[2*(c + d*x)])))/(8*d*S
qrt[a + I*a*Tan[c + d*x]])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 349 vs. 2(97) = 194.

Time = 9.74 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.87

method	result
default	$-\frac{2i(\cos^2(dx+c))\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-3i\cos(dx+c)\arctan\left(\frac{i\sin(dx+c)-\cos(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+2i\cos(dx+c)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-6\sqrt{-\frac{c}{\cos}}\cdots}{8d\sqrt{-\frac{\cos}{\cos(}}\cdots}$

```
[In] int(cos(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/d*(2*I*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-3*I*cos(d*x+c)*
arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2))+2*I*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-6*(-cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)-3*I*arctan(1/2*(I*sin(d*x+
c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+3*arcta
n(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1
))^(1/2))*sin(d*x+c)-6*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))/(-cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)+1)/(a*(1+I*tan(d*x+c)))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(91) = 182.

Time = 0.25 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.01

$$\int \frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\left(-3i\sqrt{\frac{1}{2}}ad\sqrt{\frac{1}{ad^2}}e^{(2i\,dx+2i\,c)}\log\left(-\frac{3\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}-i\right)e^{(-i\,dx-i\,c)}}{2\,d}\right)+3i\sqrt{\frac{1}{2}}ad\sqrt{\frac{1}{ad^2}}e\cdots\right.}{\cdots}$$

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(-3*I*sqrt(1/2)*a*d*sqrt(1/(a*d^2))*e^(2*I*d*x + 2*I*c)*log(-3/2*(sqrt(
2)*sqrt(1/2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
1))*sqrt(1/(a*d^2)) - I)*e^(-I*d*x - I*c)/d) + 3*I*sqrt(1/2)*a*d*sqrt(1/(a*
d^2))*e^(2*I*d*x + 2*I*c)*log(-3/2*(sqrt(2)*sqrt(1/2)*(-I*d*e^(2*I*d*x + 2*
I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a*d^2)) - I)*e^(-I*d*
x - I*c)/d) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-2*I*e^(4*I*d*x +
4*I*c) - I*e^(2*I*d*x + 2*I*c) + I))*e^(-2*I*d*x - 2*I*c)/(a*d)
```

Sympy [F]

$$\int \frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\cos(c+dx)}{\sqrt{ia\left(\tan(c+dx)-i\right)}}\,dx$$

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 837 vs. 2(91) = 182.

Time = 0.44 (sec) , antiderivative size = 837, normalized size of antiderivative = 6.86

$$\int \frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/32*(4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*((-I*sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d*x + 2*c) + 2*I*sqrt(
2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (sqrt(2)*cos
(2*d*x + 2*c) - I*sqrt(2)*sin(2*d*x + 2*c) - 2*sqrt(2))*sin(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 3*(2*sqrt(2)*arctan2((cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + si
n(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)) + 1) - 2*sqrt(2)*arctan2((cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)) - 1) - I*sqrt(2)*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2
+ 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + I*sqrt(2)
*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*
x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 - 2*(cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)) + 1))*sqrt(a))/(a*d)
```

Giac [F]

$$\int \frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{\cos\left(dx+c\right)}{\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(cos(d*x + c)/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{\cos\left(c+d\,x\right)}{\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

```
[In] int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.346 $\int \frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 193

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{35i\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{64\sqrt{2}\sqrt{a}d} + \frac{35i\cos(c+dx)}{96d\sqrt{a+ia\tan(c+dx)}} + \frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}} - \frac{35i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{64ad} - \frac{7i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{24ad}$$

```
[Out] 35/128*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/d
*2^(1/2)/a^(1/2)+35/96*I*cos(d*x+c)/d/(a+I*a*tan(d*x+c))^(1/2)+1/4*I*cos(d*
x+c)^3/d/(a+I*a*tan(d*x+c))^(1/2)-35/64*I*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/
2)/a/d-7/24*I*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2)/a/d
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used

= {3583, 3578, 3571, 3570, 212}

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{35i\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{64\sqrt{2}\sqrt{a}d} - \frac{7i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{24ad} + \frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}} - \frac{35i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{64ad} + \frac{35i\cos(c+dx)}{96d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[Cos[c + d*x]^3/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((35*I)/64)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d
*x]])])/(Sqrt[2]*Sqrt[a]*d) + (((35*I)/96)*Cos[c + d*x])/(d*Sqrt[a + I*a*Ta
n[c + d*x]]) + ((I/4)*Cos[c + d*x]^3)/(d*Sqrt[a + I*a*Tan[c + d*x]]) - (((3
5*I)/64)*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/(a*d) - (((7*I)/24)*Cos[c
 + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/(a*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3571

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
```

```
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}+\frac{7\int\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx}{8a}\\
&= \frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}-\frac{7i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{24ad}\\
&\quad+\frac{35}{48}\int\frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx\\
&= \frac{35i\cos(c+dx)}{96d\sqrt{a+ia\tan(c+dx)}}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\\
&\quad-\frac{7i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{24ad}+\frac{35\int\cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx}{64a}\\
&= \frac{35i\cos(c+dx)}{96d\sqrt{a+ia\tan(c+dx)}}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\\
&\quad-\frac{35i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{64ad}-\frac{7i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{24ad}\\
&\quad+\frac{35}{128}\int\frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx\\
&= \frac{35i\cos(c+dx)}{96d\sqrt{a+ia\tan(c+dx)}}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}-\frac{35i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{64ad}\\
&\quad-\frac{7i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{24ad}+\frac{(35i)\text{Subst}\left(\int\frac{1}{2-ax^2}\,dx,x,\frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\right)}{64d}\\
&= \frac{35i\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{64\sqrt{2}\sqrt{a}d}+\frac{35i\cos(c+dx)}{96d\sqrt{a+ia\tan(c+dx)}}+\frac{i\cos^3(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}}\\
&\quad-\frac{35i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{64ad}-\frac{7i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{24ad}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.61

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sec(c+dx)\left(-41i+105i\sqrt{1+e^{2i(c+dx)}}\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)-43i\cos(2(c+dx))-2i\cos(4(c+dx))\right.}{384d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[Cos[c + d*x]^3/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (Sec[c + d*x]*(-41*I + (105*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1
 + E^((2*I)*(c + d*x))]] - (43*I)*Cos[2*(c + d*x)] - (2*I)*Cos[4*(c + d*x)]
 + 133*Sin[2*(c + d*x)] + 14*Sin[4*(c + d*x)]))/(384*d*Sqrt[a + I*a*Tan[c +
 d*x]])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 483 vs. 2(156) = 312.

Time = 11.70 (sec) , antiderivative size = 484, normalized size of antiderivative = 2.51

method	result
default	$\frac{-16i(\cos^4(dx+c))\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+112\sin(dx+c)(\cos^3(dx+c))\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-16i(\cos^3(dx+c))\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+112\sin(dx+c)}{}$

```
[In] int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/384/d*(-16*I*cos(d*x+c)^4*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+112*sin(d*x+
c)*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-16*I*cos(d*x+c)^3*(-cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)+112*sin(d*x+c)*cos(d*x+c)^2*(-cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)-70*I*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+210*(
-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)-70*I*cos(d*x+c)*(-c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)+105*I*cos(d*x+c)*arctan(1/2*(I*sin(d*x+c)-c
os(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+210*sin(d*x
+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-105*arctan(1/2*(I*sin(d*x+c)-cos(d*x
+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+105*I*
arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)))/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)+1)/(a*(1+I*t
an(d*x+c)))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.38

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\left(-105i\sqrt{\frac{1}{2}}ad\sqrt{\frac{1}{ad^2}}e^{(4i\,dx+4i\,c)}\log\left(-\frac{35\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,de^{(2i\,dx+2i\,c)}+i\,d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{ad^2}}-i\right)e^{(-i\,dx-i\,c)}}{32\,d}\right)+105i\sqrt{\frac{1}{2}}ad\cdots\right.}{}$$

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/384*(-105*I*sqrt(1/2)*a*d*sqrt(1/(a*d^2))*e^(4*I*d*x + 4*I*c)*log(-35/32*
(sqrt(2)*sqrt(1/2)*(I*d*e^(2*I*d*x + 2*I*c) + I*d)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*sqrt(1/(a*d^2)) - I)*e^(-I*d*x - I*c)/d) + 105*I*sqrt(1/2)*a*d*sq
rt(1/(a*d^2))*e^(4*I*d*x + 4*I*c)*log(-35/32*(sqrt(2)*sqrt(1/2)*(-I*d*e^(2*
I*d*x + 2*I*c) - I*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a*d^2)) - I
)*e^(-I*d*x - I*c)/d) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-8*I*e^(
8*I*d*x + 8*I*c) - 88*I*e^(6*I*d*x + 6*I*c) - 41*I*e^(4*I*d*x + 4*I*c) + 45
*I*e^(2*I*d*x + 2*I*c) + 6*I))*e^(-4*I*d*x - 4*I*c)/(a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1938 vs. 2(146) = 292.

Time = 0.52 (sec) , antiderivative size = 1938, normalized size of antiderivative = 10.04

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/1536*(4*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((-3*I*sqrt(2)*cos(4*d*x + 4*c) -
 3*sqrt(2)*sin(4*d*x + 4*c) + 8*I*sqrt(2))*cos(3/2*arctan2(sin(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c))) + 1)) + (3*sqrt(2)*cos(4*d*x + 4*c) - 3*I*sqrt(2)*sin(4*d*x
 + 4*c) - 8*sqrt(2))*sin(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))
*sqrt(a) + 12*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin
(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*((-I*sqrt(2)*cos(4*d*x + 4*c)
- 12*I*sqrt(2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - sqrt(
2)*sin(4*d*x + 4*c) - 12*sqrt(2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c))) + 24*I*sqrt(2))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
+ 1)) + (sqrt(2)*cos(4*d*x + 4*c) + 12*sqrt(2)*cos(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c))) - I*sqrt(2)*sin(4*d*x + 4*c) - 12*I*sqrt(2)*sin(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 24*sqrt(2))*sin(1/2*arctan
2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) + 105*(2*sqrt(2)*arctan2((
cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
)) + 1)), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c))) + 1)) + 1) - 2*sqrt(2)*arctan2((cos(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)
*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)), (cos(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
 + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - 1) - I*
sqrt(2)*log(sqrt(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + s
in(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)*cos(1/2*arctan2(sin(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c))) + 1))^2 + sqrt(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)*sin(1/2*arctan2(sin(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x +
```

```
4*c), cos(4*d*x + 4*c))) + 1))^2 + 2*(cos(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2
 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/
2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + 1) + I*sqrt(2)*log(sqrt(c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c))) + 1)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)
)^2 + sqrt(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c))) + 1)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c))) + 1))^2 - 2*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2
 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c))) + 1)) + 1))*sqrt(a))/(a*d)
```

Giac [F]

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{\cos\left(dx+c\right)^3}{\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(cos(d*x + c)^3/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{\cos\left(c+d\,x\right)^3}{\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}\,dx$$

```
[In] int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.347 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 117

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{16i(a+ia\tan(c+dx))^{5/2}}{5a^4d} + \frac{24i(a+ia\tan(c+dx))^{7/2}}{7a^5d} - \frac{4i(a+ia\tan(c+dx))^{9/2}}{3a^6d} + \frac{2i(a+ia\tan(c+dx))^{11/2}}{11a^7d}$$

```
[Out] -16/5*I*(a+I*a*tan(d*x+c))^(5/2)/a^4/d+24/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^5/
d-4/3*I*(a+I*a*tan(d*x+c))^(9/2)/a^6/d+2/11*I*(a+I*a*tan(d*x+c))^(11/2)/a^7
/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3568, 45}

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2i(a+ia\tan(c+dx))^{11/2}}{11a^7d} - \frac{4i(a+ia\tan(c+dx))^{9/2}}{3a^6d} + \frac{24i(a+ia\tan(c+dx))^{7/2}}{7a^5d} - \frac{16i(a+ia\tan(c+dx))^{5/2}}{5a^4d}$$

```
[In] Int[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((-16*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(a^4*d) + (((24*I)/7)*(a + I*a*T
an[c + d*x])^(7/2))/(a^5*d) - (((4*I)/3)*(a + I*a*Tan[c + d*x])^(9/2))/(a^6
*d) + (((2*I)/11)*(a + I*a*Tan[c + d*x])^(11/2))/(a^7*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)^3(a+x)^{3/2}\,dx, x, ia\tan(c+dx)\right)}{a^7d}\\ &= -\frac{i\text{Subst}\left(\int \left(8a^3(a+x)^{3/2}-12a^2(a+x)^{5/2}+6a(a+x)^{7/2}-(a+x)^{9/2}\right)\,dx, x, ia\tan(c+dx)\right)}{a^7d}\\ &= -\frac{16i(a+ia\tan(c+dx))^{5/2}}{5a^4d}+\frac{24i(a+ia\tan(c+dx))^{7/2}}{7a^5d}\\ &\quad -\frac{4i(a+ia\tan(c+dx))^{9/2}}{3a^6d}+\frac{2i(a+ia\tan(c+dx))^{11/2}}{11a^7d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.62

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{2(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}(-533i-755\tan(c+dx)+455i\tan^2(c+dx)+105\tan^3(c+dx}{1155a^2d}$$

```
[In] Integrate[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (-2*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]]*(-533*I - 755*Tan[c +
d*x] + (455*I)*Tan[c + d*x]^2 + 105*Tan[c + d*x]^3))/(1155*a^2*d)
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}-\frac{2a(a+ia\tan(dx+c))^{\frac{9}{2}}}{3}+\frac{12a^2(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{8a^3(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}\right)}{d\,a^7}$	82
default	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}-\frac{2a(a+ia\tan(dx+c))^{\frac{9}{2}}}{3}+\frac{12a^2(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{8a^3(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}\right)}{d\,a^7}$	82

```
[In] int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2*I/d/a^7*(1/11*(a+I*a*tan(d*x+c))^(11/2)-2/3*a*(a+I*a*tan(d*x+c))^(9/2)+12
/7*a^2*(a+I*a*tan(d*x+c))^(7/2)-8/5*a^3*(a+I*a*tan(d*x+c))^(5/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.27

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{64\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(16i\,e^{(11i\,dx+11i\,c)}+88i\,e^{(9i\,dx+9i\,c)}+198i\,e^{(7i\,dx+7i\,c)}+231i\,e^{(5i\,dx+5i\,c)}\right)}{1155\left(a^2de^{(10i\,dx+10i\,c)}+5\,a^2de^{(8i\,dx+8i\,c)}+10\,a^2de^{(6i\,dx+6i\,c)}+10\,a^2de^{(4i\,dx+4i\,c)}+5\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
[Out] -64/1155*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(16*I*e^(11*I*d*x + 11*I
*c) + 88*I*e^(9*I*d*x + 9*I*c) + 198*I*e^(7*I*d*x + 7*I*c) + 231*I*e^(5*I*d
*x + 5*I*c))/(a^2*d*e^(10*I*d*x + 10*I*c) + 5*a^2*d*e^(8*I*d*x + 8*I*c) + 1
0*a^2*d*e^(6*I*d*x + 6*I*c) + 10*a^2*d*e^(4*I*d*x + 4*I*c) + 5*a^2*d*e^(2*I
*d*x + 2*I*c) + a^2*d)
```

Sympy [F]

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec^8(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c))**(3/2),x)
[Out] Integral(sec(c + d*x)**8/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.65

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2i\left(105\,(i\,a\tan\left(dx+c\right)+a)^{\frac{11}{2}}-770\,(i\,a\tan\left(dx+c\right)+a)^{\frac{9}{2}}a+1980\,(i\,a\tan\right.}{1155\,a^7d}$$

[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")

[Out] 2/1155*I*(105*(I*a*tan(d*x + c) + a)^(11/2) - 770*(I*a*tan(d*x + c) + a)^(9/2)*a + 1980*(I*a*tan(d*x + c) + a)^(7/2)*a^2 - 1848*(I*a*tan(d*x + c) + a)^(5/2)*a^3)/(a^7*d)

Giac [F]

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(dx+c\right)^8}{(i\,a\tan\left(dx+c\right)+a)^{\frac{3}{2}}}\,dx$$

[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate(sec(d*x + c)^8/(I*a*tan(d*x + c) + a)^(3/2), x)

Mupad [B] (verification not implemented)

Time = 8.73 (sec) , antiderivative size = 370, normalized size of antiderivative = 3.16

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{\sqrt{a-\frac{a\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,1024\mathrm{i}}{1155\,a^2\,d} - \frac{\sqrt{a-\frac{a\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,512\mathrm{i}}{1155\,a^2\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)} - \frac{\sqrt{a-\frac{a\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{385\,a^2\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2} - \frac{\sqrt{a-\frac{a\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{231\,a^2\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3} + \frac{\sqrt{a-\frac{a\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,256\mathrm{i}}{33\,a^2\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4} - \frac{\sqrt{a-\frac{a\,(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{11\,a^2\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^5}$$

[In] int(1/(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^(3/2)),x)

[Out] ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*256i)/(33*a^2*d*(exp(c*2i + d*x*2i) + 1)^4) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*512i)/(1155*a^2*d*(exp(c*2i + d*x*2i) + 1)) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i

```
) + 1))^(1/2)*128i)/(385*a^2*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (a*(exp(
c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*64i)/(231*a^2*d
*(exp(c*2i + d*x*2i) + 1)^3) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(e
xp(c*2i + d*x*2i) + 1))^(1/2)*1024i)/(1155*a^2*d) - ((a - (a*(exp(c*2i + d*
x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*64i)/(11*a^2*d*(exp(c*2i
 + d*x*2i) + 1)^5)
```

3.348 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{8i(a+ia\tan(c+dx))^{3/2}}{3a^3d} + \frac{8i(a+ia\tan(c+dx))^{5/2}}{5a^4d} - \frac{2i(a+ia\tan(c+dx))^{7/2}}{7a^5d}$$

```
[Out] -8/3*I*(a+I*a*tan(d*x+c))^(3/2)/a^3/d+8/5*I*(a+I*a*tan(d*x+c))^(5/2)/a^4/d-
2/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^5/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3568, 45}

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{2i(a+ia\tan(c+dx))^{7/2}}{7a^5d} + \frac{8i(a+ia\tan(c+dx))^{5/2}}{5a^4d} - \frac{8i(a+ia\tan(c+dx))^{3/2}}{3a^3d}$$

```
[In] Int[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((-8*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(a^3*d) + (((8*I)/5)*(a + I*a*Tan
[c + d*x])^(5/2))/(a^4*d) - (((2*I)/7)*(a + I*a*Tan[c + d*x])^(7/2))/(a^5*d
)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)^2\sqrt{a+x}\,dx, x, ia\tan(c+dx)\right)}{a^5d}\\
&= -\frac{i\text{Subst}\left(\int \left(4a^2\sqrt{a+x}-4a(a+x)^{3/2}+(a+x)^{5/2}\right)\,dx, x, ia\tan(c+dx)\right)}{a^5d}\\
&= -\frac{8i(a+ia\tan(c+dx))^{3/2}}{3a^3d}+\frac{8i(a+ia\tan(c+dx))^{5/2}}{5a^4d}-\frac{2i(a+ia\tan(c+dx))^{7/2}}{7a^5d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.67

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{2(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}(-71+54i\tan(c+dx)+15\tan^2(c+dx))}{105a^2d}$$

```
[In] Integrate[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^(3/2),x]
[Out] (-2*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]]*(-71 + (54*I)*Tan[c + d*
x] + 15*Tan[c + d*x]^2))/(105*a^2*d)
```

Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}+\frac{4a(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{4a^2(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}\right)}{d\,a^5}$	63
default	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}+\frac{4a(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{4a^2(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}\right)}{d\,a^5}$	63

```
[In] int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/d/a^5*(-1/7*(a+I*a*tan(d*x+c))^(7/2)+4/5*a*(a+I*a*tan(d*x+c))^(5/2)-4/3
*a^2*(a+I*a*tan(d*x+c))^(3/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.23

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{16\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(8i\,e^{(7i\,dx+7i\,c)}+28i\,e^{(5i\,dx+5i\,c)}+35i\,e^{(3i\,dx+3i\,c)}\right)}{105\left(a^2de^{(6i\,dx+6i\,c)}+3\,a^2de^{(4i\,dx+4i\,c)}+3\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -16/105*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(8*I*e^(7*I*d*x + 7*I*c)
+ 28*I*e^(5*I*d*x + 5*I*c) + 35*I*e^(3*I*d*x + 3*I*c))/(a^2*d*e^(6*I*d*x +
6*I*c) + 3*a^2*d*e^(4*I*d*x + 4*I*c) + 3*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F]

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec^6(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**6/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.66

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{2i\left(15\,(i\,a\tan(dx+c)+a)^{\frac{7}{2}}-84\,(i\,a\tan(dx+c)+a)^{\frac{5}{2}}a+140\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a^2\right)}{105\,a^5d}$$

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
[Out] -2/105*I*(15*(I*a*tan(d*x + c) + a)^(7/2) - 84*(I*a*tan(d*x + c) + a)^(5/2)
*a + 140*(I*a*tan(d*x + c) + a)^(3/2)*a^2)/(a^5*d)
```

Giac [F]

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^6}{(i\,a\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^6/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 7.57 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.75

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{105\,a^2\,d} - \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{105\,a^2\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)} - \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,16\mathrm{i}}{35\,a^2\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2} + \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,16\mathrm{i}}{7\,a^2\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}$$

```
[In] int(1/(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(3/2)),x)
[Out] ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1
6i)/(7*a^2*d*(exp(c*2i + d*x*2i) + 1)^3) - ((a - (a*(exp(c*2i + d*x*2i)*1i
- 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*64i)/(105*a^2*d*(exp(c*2i + d*x*2
i) + 1)) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) +
1))^(1/2)*16i)/(35*a^2*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (a*(exp(c*2i +
 d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*128i)/(105*a^2*d)
```

3.349 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 57

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{4i\sqrt{a+ia\tan(c+dx)}}{a^2d}+\frac{2i(a+ia\tan(c+dx))^{3/2}}{3a^3d}$$

```
[Out] -4*I*(a+I*a*tan(d*x+c))^(1/2)/a^2/d+2/3*I*(a+I*a*tan(d*x+c))^(3/2)/a^3/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3568, 45}

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2i(a+ia\tan(c+dx))^{3/2}}{3a^3d}-\frac{4i\sqrt{a+ia\tan(c+dx)}}{a^2d}$$

```
[In] Int[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-4*I)*Sqrt[a + I*a*Tan[c + d*x]])/(a^2*d) + (((2*I)/3)*(a + I*a*Tan[c + d
*x])^(3/2))/(a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int \frac{a-x}{\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{a^3 d}\\ &= -\frac{i\text{Subst}\left(\int \left(\frac{2a}{\sqrt{a+x}}-\sqrt{a+x}\right)\,dx, x, ia\tan(c+dx)\right)}{a^3 d}\\ &= -\frac{4i\sqrt{a+ia\tan(c+dx)}}{a^2 d}+\frac{2i(a+ia\tan(c+dx))^{3/2}}{3a^3 d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.65

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{2(5i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{3a^2 d}$$

```
[In] Integrate[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^(3/2),x]
[Out] (-2*(5*I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(3*a^2*d)
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-2a\sqrt{a+ia\tan(dx+c)}\right)}{d\,a^3}$	44
default	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-2a\sqrt{a+ia\tan(dx+c)}\right)}{d\,a^3}$	44

```
[In] int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2*I/d/a^3*(1/3*(a+I*a*tan(d*x+c))^(3/2)-2*a*(a+I*a*tan(d*x+c))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.18

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{4\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(2i\,e^{(3i\,dx+3i\,c)}+3i\,e^{(i\,dx+i\,c)}\right)}{3\left(a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
[Out] -4/3*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(2*I*e^(3*I*d*x + 3*I*c) + 3
*I*e^(I*d*x + I*c))/(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec^4(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c))**(3/2),x)
[Out] Integral(sec(c + d*x)**4/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.67

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2i\left((i\,a\tan(dx+c)+a)^{\frac{3}{2}}-6\sqrt{i\,a\tan(dx+c)+a}\,a\right)}{3\,a^3d}$$

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
[Out] 2/3*I*((I*a*tan(d*x + c) + a)^(3/2) - 6*sqrt(I*a*tan(d*x + c) + a)*a)/(a^3*
d)
```

Giac [F]

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx=\int \frac{\sec\left(dx+c\right)^4}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^4/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 4.71 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.49

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx=-\frac{2\left(\cos\left(2\,c+2\,d\,x\right)\,5\mathrm{i}+\sin\left(2\,c+2\,d\,x\right)+5\mathrm{i}\right)\,\sqrt{\frac{a\left(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i}\right)}{\cos(2\,c+2\,d\,x)+1}}}{3\,a^2\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

```
[In] int(1/(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(3/2)),x)
[Out] -(2*(cos(2*c + 2*d*x)*5i + sin(2*c + 2*d*x) + 5i)*((a*(cos(2*c + 2*d*x) + s
in(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))/(3*a^2*d*(cos(2*c +
 2*d*x) + 1))
```

3.350 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 27

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2i}{ad\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 2*I/a/d/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3568, 32}

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2i}{ad\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (2*I)/(a*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
```

```
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{i\text{Subst}\left(\int \frac{1}{(a+x)^{3/2}}\,dx, x, ia\tan(c+dx)\right)}{ad}$$
$$= \frac{2i}{ad\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2i}{ad\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^(3/2),x]
[Out] (2*I)/(a*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{2i}{ad\sqrt{a+ia\tan(dx+c)}}$	24
default	$\frac{2i}{ad\sqrt{a+ia\tan(dx+c)}}$	24

```
[In] int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2*I/a/d/(a+I*a*tan(d*x+c))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(21) = 42.

Time = 0.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.81

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-i\,dx-i\,c)}}{a^2d}$$

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
[Out] sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(I*e^(2*I*d*x + 2*I*c) + I)*e^(-I
*d*x - I*c)/(a^2*d)
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec^2(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c))**(3/2),x)
[Out] Integral(sec(c + d*x)**2/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2i}{\sqrt{i\,a\tan(dx+c)+a}ad}$$

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
[Out] 2*I/(sqrt(I*a*tan(d*x + c) + a)*a*d)
```

Giac [F]

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(dx+c\right)^2}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^2/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.48

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\left(\cos\left(c+d\,x\right)^2\,2\mathrm{i}+\sin\left(2\,c+2\,d\,x\right)\right)\,\sqrt{\frac{a\,\left(2\cos\left(c+d\,x\right)^2+\sin\left(2\,c+2\,d\,x\right)\,1\mathrm{i}\right)}{2\cos\left(c+d\,x\right)^2}}}{a^2\,d}$$

```
[In] int(1/(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(3/2)),x)
[Out] ((sin(2*c + 2*d*x) + cos(c + d*x)^2*2i)*((a*(sin(2*c + 2*d*x)*1i + 2*cos(c
+ d*x)^2))/(2*cos(c + d*x)^2))^(1/2))/(a^2*d)
```

3.351 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}} dx$

Optimal result

Integrand size = 26, antiderivative size = 175

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}} dx = -\frac{7i\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}a^{3/2}d} + \frac{7ia}{20d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}} + \frac{7i}{24d(a+ia\tan(c+dx))^{3/2}} + \frac{7i}{16ad\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] -7/32*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(3/2)/d*2^(
1/2)+7/16*I/a/d/(a+I*a*tan(d*x+c))^(1/2)+7/20*I*a/d/(a+I*a*tan(d*x+c))^(5/2
)-1/2*I*a^2/d/(a-I*a*tan(d*x+c))/(a+I*a*tan(d*x+c))^(5/2)+7/24*I/d/(a+I*a*t
an(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3568, 44, 53, 65, 212}

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}} dx = -\frac{7i\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}a^{3/2}d} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}} + \frac{7ia}{20d(a+ia\tan(c+dx))^{5/2}} + \frac{7i}{24d(a+ia\tan(c+dx))^{3/2}} + \frac{7i}{16ad\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((-7*I)/16)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2
]*a^(3/2)*d) + (((7*I)/20)*a)/(d*(a + I*a*Tan[c + d*x])^(5/2)) - ((I/2)*a^2
)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(5/2)) + ((7*I)/24)/(d*(
a + I*a*Tan[c + d*x])^(3/2)) + ((7*I)/16)/(a*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{(ia^3)\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{7/2}}\,dx, x, ia\tan(c+dx)\right)}{d}$$
$$= -\frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}} - \frac{(7ia^2)\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{7/2}}\,dx, x, ia\tan(c+dx)\right)}{4d}$$
$$= \frac{7ia}{20d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}} - \frac{(7ia)\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{5/2}}\,dx, x, ia\tan(c+dx)\right)}{8d}$$
$$= \frac{7ia}{20d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}} + \frac{7i}{24d(a+ia\tan(c+dx))^{3/2}} - \frac{(7i)\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{3/2}}\,dx, x, ia\tan(c+dx)\right)}{16d}$$
$$= \frac{7ia}{20d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}} + \frac{7i}{24d(a+ia\tan(c+dx))^{3/2}} + \frac{7i}{16ad\sqrt{a+ia\tan(c+dx)}} - \frac{(7i)\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{32ad}$$
$$= \frac{7ia}{20d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}} + \frac{7i}{24d(a+ia\tan(c+dx))^{3/2}} + \frac{7i}{16ad\sqrt{a+ia\tan(c+dx)}} - \frac{(7i)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+ia\tan(c+dx)}\right)}{16ad}$$
$$= -\frac{7i\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{16\sqrt{2}a^{3/2}d} + \frac{7ia}{20d(a+ia\tan(c+dx))^{5/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{5/2}} + \frac{7i}{24d(a+ia\tan(c+dx))^{3/2}} + \frac{7i}{16ad\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.29

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{ia\,\text{Hypergeometric2F1}\left(-\frac{5}{2},2,-\frac{3}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{10d(a+ia\tan(c+dx))^{5/2}}$$

```
[In] Integrate[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^(3/2),x]
[Out] ((I/10)*a*Hypergeometric2F1[-5/2, 2, -3/2, (1 + I*Tan[c + d*x])/2])/(d*(a +
 I*a*Tan[c + d*x])^(5/2))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 594 vs. 2(137) = 274.

Time = 10.01 (sec) , antiderivative size = 595, normalized size of antiderivative = 3.40

method	result
default	$-72i(\cos^3(dx+c))\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+168\sin(dx+c)(\cos^2(dx+c))\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-72i(\cos^2(dx+c))\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+168\sqrt{-\frac{\cos(d}{\cos(dx}}$

```
[In] int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/480/d/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)+1)/(a*(1+I*tan(d*x+c
)))^(1/2)/(1+I*tan(d*x+c))/a*(-72*I*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)+168*sin(d*x+c)*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-72*
I*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+168*(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)+210*I*cos(d*x+c)*arctan(1/2*(cos(d*x+c)
+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+350*I*c
os(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-210*arctan(1/2*(cos(d*x+c)+1+I
*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-
210*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+105*I*arctan(1/2*(cos(d*x
+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+350*
I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-105*tan(d*x+c)*arctan(1/2*(cos(d*x+c)+
1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-210*tan(
d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-105*I*sec(d*x+c)*arctan(1/2*(cos(
d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(126) = 252.

Time = 0.26 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.68

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\left(-105i\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{1}{a^3d^2}}e^{(5i\,dx+5i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^2de^{(2i\,dx+2i\,c)}+a^2d\right)\sqrt{\frac{a}{e^{(2i\,dx+}}}\right.\right.\right.}{}$$

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/480*(-105*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(5*I*d*x + 5*I*c)*log(4*(
sqrt(2)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d*x +
2*I*c) + 1))*sqrt(1/(a^3*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + 105
*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(5*I*d*x + 5*I*c)*log(-4*(sqrt(2)*sq
rt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1
))*sqrt(1/(a^3*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2)*sqrt(
a/(e^(2*I*d*x + 2*I*c) + 1))*(-15*I*e^(8*I*d*x + 8*I*c) + 101*I*e^(6*I*d*x
+ 6*I*c) + 148*I*e^(4*I*d*x + 4*I*c) + 38*I*e^(2*I*d*x + 2*I*c) + 6*I))*e^(
-5*I*d*x - 5*I*c)/(a^2*d)
```

Sympy [F]

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos^2(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.87

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{i\left(\frac{105\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)}{\sqrt{a}}+\frac{4\left(105\,(i\,a\tan(dx+c)+a)^3-140\,(i\,a\tan(dx+c)+a)^2a-56}{(i\,a\tan(dx+c)+a)^{\frac{7}{2}}-2\,(i\,a\tan(dx+}\right.}{960\,ad}$$

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/960*I*(105*sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(s
qrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/sqrt(a) + 4*(105*(I*a*tan(d*x
 + c) + a)^3 - 140*(I*a*tan(d*x + c) + a)^2*a - 56*(I*a*tan(d*x + c) + a)*a
^2 - 48*a^3)/((I*a*tan(d*x + c) + a)^(7/2) - 2*(I*a*tan(d*x + c) + a)^(5/2)
*a))/(a*d)
```

Giac [F]

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(dx+c\right)^2}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(cos(d*x + c)^2/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{3/2}}\,dx$$

```
[In] int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.352 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}} dx$

Optimal result

Integrand size = 26, antiderivative size = 248

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx=-\frac{99i\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{256\sqrt{2}a^{3/2}d}+\frac{99ia^2}{224d(a+ia\tan(c+dx))^{7/2}}-\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}}-\frac{11ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}}+\frac{99ia}{320d(a+ia\tan(c+dx))^{5/2}}+\frac{33i}{128d(a+ia\tan(c+dx))^{3/2}}+\frac{99i}{256ad\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] -99/512*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(3/2)/d*2
^(1/2)+99/256*I/a/d/(a+I*a*tan(d*x+c))^(1/2)+99/224*I*a^2/d/(a+I*a*tan(d*x+
c))^(7/2)-1/4*I*a^4/d/(a-I*a*tan(d*x+c))^2/(a+I*a*tan(d*x+c))^(7/2)-11/16*I
*a^3/d/(a-I*a*tan(d*x+c))/(a+I*a*tan(d*x+c))^(7/2)+99/320*I*a/d/(a+I*a*tan(
d*x+c))^(5/2)+33/128*I/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used

= {3568, 44, 53, 65, 212}

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{99i\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{256\sqrt{2}a^{3/2}d} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}} - \frac{11ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}} + \frac{99ia^2}{224d(a+ia\tan(c+dx))^{7/2}} + \frac{99ia}{320d(a+ia\tan(c+dx))^{5/2}} + \frac{33i}{128d(a+ia\tan(c+dx))^{3/2}} + \frac{99i}{256ad\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((-99*I)/256)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt
[2]*a^(3/2)*d) + (((99*I)/224)*a^2)/(d*(a + I*a*Tan[c + d*x])^(7/2)) - ((I/
4)*a^4)/(d*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(7/2)) - (((11*I
)/16)*a^3)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(7/2)) + (((99*
I)/320)*a)/(d*(a + I*a*Tan[c + d*x])^(5/2)) + ((33*I)/128)/(d*(a + I*a*Tan[
c + d*x])^(3/2)) + ((99*I)/256)/(a*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(ia^5)\,\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^{9/2}}\,dx, x, ia\tan(c+dx)\right)}{d} \\
&= -\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}} - \frac{(11ia^4)\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{9/2}}\,dx, x, ia\tan(c+dx)\right)}{8d} \\
&= -\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}} - \frac{11ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}} - \frac{(99ia^3)\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{9/2}}\,dx, x, ia\tan(c+dx)\right)}{32d} \\
&= \frac{99ia^2}{224d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}} - \frac{11ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}} - \frac{(99ia^2)\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{7/2}}\,dx, x, ia\tan(c+dx)\right)}{64d}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{99ia^2}{224d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}} \\
&\quad - \frac{11ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}} + \frac{99ia}{320d(a+ia\tan(c+dx))^{5/2}} \\
&\quad - \frac{(99ia)\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{5/2}}\,dx, x, ia\tan(c+dx)\right)}{128d} \\
&= \frac{99ia^2}{224d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}} \\
&\quad - \frac{11ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}} + \frac{99ia}{320d(a+ia\tan(c+dx))^{5/2}} \\
&\quad + \frac{33i}{128d(a+ia\tan(c+dx))^{3/2}} - \frac{(99i)\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{3/2}}\,dx, x, ia\tan(c+dx)\right)}{256d} \\
&= \frac{99ia^2}{224d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}} \\
&\quad - \frac{11ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}} \\
&\quad + \frac{99ia}{320d(a+ia\tan(c+dx))^{5/2}} + \frac{33i}{128d(a+ia\tan(c+dx))^{3/2}} \\
&\quad + \frac{99i}{256ad\sqrt{a+ia\tan(c+dx)}} - \frac{(99i)\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{512ad} \\
&= \frac{99ia^2}{224d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}} \\
&\quad - \frac{11ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}} + \frac{99ia}{320d(a+ia\tan(c+dx))^{5/2}} \\
&\quad + \frac{33i}{128d(a+ia\tan(c+dx))^{3/2}} + \frac{99i}{256ad\sqrt{a+ia\tan(c+dx)}} \\
&\quad - \frac{(99i)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+ia\tan(c+dx)}\right)}{256ad} \\
&= -\frac{99i\,\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{256\sqrt{2}a^{3/2}d} + \frac{99ia^2}{224d(a+ia\tan(c+dx))^{7/2}} \\
&\quad - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{7/2}} \\
&\quad - \frac{11ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}} + \frac{99ia}{320d(a+ia\tan(c+dx))^{5/2}} \\
&\quad + \frac{33i}{128d(a+ia\tan(c+dx))^{3/2}} + \frac{99i}{256ad\sqrt{a+ia\tan(c+dx)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.34 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.21

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{ia^2\,\text{Hypergeometric2F1}\left(-\frac{7}{2},3,-\frac{5}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{28d(a+ia\tan(c+dx))^{7/2}}$$

```
[In] Integrate[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((I/28)*a^2*Hypergeometric2F1[-7/2, 3, -5/2, (1 + I*Tan[c + d*x])/2])/(d*(a
 + I*a*Tan[c + d*x])^(7/2))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 728 vs. 2(197) = 394.

Time = 8.62 (sec) , antiderivative size = 729, normalized size of antiderivative = 2.94

method	result
default	$-\frac{-11550i\cos(dx+c)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-3465i\arctan\left(\frac{\cos(dx+c)+1+i\sin(dx+c)}{2(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)-3520\sin(dx+c)\left(\cos^4(dx+c)\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}{}$

```
[In] int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/17920/d/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)+1)/(a*(1+I*tan(d*
x+c)))^(1/2)/(1+I*tan(d*x+c))/a*(-11550*I*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)-3465*I*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(
-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-3520*sin(d*x+c)*cos(d*x+c)^4*(-cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)+3465*I*sec(d*x+c)*arctan(1/2*(cos(d*x+c)+1+I*sin(d*
x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-3520*sin(d*x+c)*co
s(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+2376*I*cos(d*x+c)^2*(-cos(d*x
+c)/(cos(d*x+c)+1))^(1/2)-5544*sin(d*x+c)*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)-11550*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+960*I*cos(d*x+c)^
5*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-5544*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2
)*cos(d*x+c)*sin(d*x+c)+960*I*cos(d*x+c)^4*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)-6930*I*cos(d*x+c)*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/
(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+6930*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)+6930*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-co
s(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+2376*I*cos(d*x+c)^3*(-cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)+6930*tan(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+3
465*tan(d*x+c)*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.27

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\left(-3465i\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{1}{a^3d^2}}e^{(7i\,dx+7i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^2de^{(2i\,dx+2i\,c)}+a^2d\right)\sqrt{\frac{}{e^{(2i}}}\right.\right.\right.}{}$$

[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")

[Out] 1/17920*(-3465*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(7*I*d*x + 7*I*c)*log(
4*(sqrt(2)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d*x
 + 2*I*c) + 1))*sqrt(1/(a^3*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) +
3465*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(7*I*d*x + 7*I*c)*log(-4*(sqrt(2
)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c)
 + 1))*sqrt(1/(a^3*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2)*s
qrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-70*I*e^(12*I*d*x + 12*I*c) - 805*I*e^(10
*I*d*x + 10*I*c) + 2833*I*e^(8*I*d*x + 8*I*c) + 4584*I*e^(6*I*d*x + 6*I*c)
+ 1304*I*e^(4*I*d*x + 4*I*c) + 328*I*e^(2*I*d*x + 2*I*c) + 40*I))*e^(-7*I*d
*x - 7*I*c)/(a^2*d)

Sympy [F]

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos^4(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

[In] integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c))**(3/2),x)

[Out] Integral(cos(c + d*x)**4/(I*a*(tan(c + d*x) - I))**(3/2), x)

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.83

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{i\left(\frac{3465\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)}{\sqrt{a}}+\frac{4\left(3465\,(i\,a\tan(dx+c)+a)^5-11550\,(i\,a\tan(dx+c)+a)\right.}{(i\,a\tan(dx+c}\right.}{3584}$$

[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")

```
[Out] 1/35840*I*(3465*sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))
/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/sqrt(a) + 4*(3465*(I*a*tan
(d*x + c) + a)^5 - 11550*(I*a*tan(d*x + c) + a)^4*a + 7392*(I*a*tan(d*x + c
) + a)^3*a^2 + 2112*(I*a*tan(d*x + c) + a)^2*a^3 + 1408*(I*a*tan(d*x + c) +
 a)*a^4 + 1280*a^5)/((I*a*tan(d*x + c) + a)^(11/2) - 4*(I*a*tan(d*x + c) +
a)^(9/2)*a + 4*(I*a*tan(d*x + c) + a)^(7/2)*a^2))/(a*d)
```

Giac [F]

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx=\int \frac{\cos\left(dx+c\right)^4}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(cos(d*x + c)^4/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx=\int \frac{\cos\left(c+d\,x\right)^4}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{3/2}}\,dx$$

```
[In] int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.353 $\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 321

$$\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{715i\,\mathrm{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2048\sqrt{2}a^{3/2}d} + \frac{715ia^3}{1152d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{9/2}} - \frac{5ia^5}{16d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}} - \frac{65ia^4}{64d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}} + \frac{715ia^2}{1792d(a+ia\tan(c+dx))^{7/2}} + \frac{143ia}{512d(a+ia\tan(c+dx))^{5/2}} + \frac{715i}{3072d(a+ia\tan(c+dx))^{3/2}} + \frac{715i}{2048ad\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] -715/4096*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(3/2)/d
*2^(1/2)+715/2048*I/a/d/(a+I*a*tan(d*x+c))^(1/2)+715/1152*I*a^3/d/(a+I*a*ta
n(d*x+c))^(9/2)-1/6*I*a^6/d/(a-I*a*tan(d*x+c))^3/(a+I*a*tan(d*x+c))^(9/2)-5
/16*I*a^5/d/(a-I*a*tan(d*x+c))^2/(a+I*a*tan(d*x+c))^(9/2)-65/64*I*a^4/d/(a-
I*a*tan(d*x+c))/(a+I*a*tan(d*x+c))^(9/2)+715/1792*I*a^2/d/(a+I*a*tan(d*x+c)
)^(7/2)+143/512*I*a/d/(a+I*a*tan(d*x+c))^(5/2)+715/3072*I/d/(a+I*a*tan(d*x+
c))^(3/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3568, 44, 53, 65, 212}

$$\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{715i\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2048\sqrt{2}a^{3/2}d} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{9/2}} - \frac{5ia^5}{16d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}} - \frac{65ia^4}{64d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}} + \frac{715ia^3}{1152d(a+ia\tan(c+dx))^{9/2}} + \frac{715ia^2}{1792d(a+ia\tan(c+dx))^{7/2}} + \frac{143ia}{512d(a+ia\tan(c+dx))^{5/2}} + \frac{715i}{3072d(a+ia\tan(c+dx))^{3/2}} + \frac{715i}{2048ad\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[Cos[c + d*x]^6/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((-715*I)/2048)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sq
rt[2]*a^(3/2)*d) + (((715*I)/1152)*a^3)/(d*(a + I*a*Tan[c + d*x])^(9/2)) -
((I/6)*a^6)/(d*(a - I*a*Tan[c + d*x])^3*(a + I*a*Tan[c + d*x])^(9/2)) - (((
5*I)/16)*a^5)/(d*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(9/2)) - (
((65*I)/64)*a^4)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(9/2)) +
(((715*I)/1792)*a^2)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (((143*I)/512)*a)/(
d*(a + I*a*Tan[c + d*x])^(5/2)) + ((715*I)/3072)/(d*(a + I*a*Tan[c + d*x])^
(3/2)) + ((715*I)/2048)/(a*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(ia^7)\,\text{Subst}\left(\int \frac{1}{(a-x)^4(a+x)^{11/2}}\,dx, x, ia\tan(c+dx)\right)}{d} \\
&= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{9/2}} \\
&\quad - \frac{(5ia^6)\,\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^{11/2}}\,dx, x, ia\tan(c+dx)\right)}{4d} \\
&= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{9/2}} \\
&\quad - \frac{5ia^5}{16d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}} \\
&\quad - \frac{(65ia^5)\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{11/2}}\,dx, x, ia\tan(c+dx)\right)}{32d}
\end{aligned}
$$

$$= -\frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{9/2}} - \frac{5ia^5}{16d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}} - \frac{65ia^4}{64d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}} - \frac{(715ia^4)\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{11/2}}\,dx, x, ia\tan(c+dx)\right)}{128d}$$

$$= \frac{715ia^3}{1152d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{9/2}} - \frac{5ia^5}{16d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}} - \frac{65ia^4}{64d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}} - \frac{(715ia^3)\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{9/2}}\,dx, x, ia\tan(c+dx)\right)}{256d}$$

$$= \frac{715ia^3}{1152d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{9/2}} - \frac{5ia^5}{16d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}} - \frac{65ia^4}{64d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}} + \frac{715ia^2}{1792d(a+ia\tan(c+dx))^{7/2}} - \frac{(715ia^2)\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{7/2}}\,dx, x, ia\tan(c+dx)\right)}{512d}$$

$$= \frac{715ia^3}{1152d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{9/2}} - \frac{5ia^5}{16d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}} - \frac{65ia^4}{64d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}} + \frac{715ia^2}{1792d(a+ia\tan(c+dx))^{7/2}} + \frac{143ia}{512d(a+ia\tan(c+dx))^{5/2}} - \frac{(715ia)\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{5/2}}\,dx, x, ia\tan(c+dx)\right)}{1024d}$$

$$= \frac{715ia^3}{1152d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{9/2}} - \frac{5ia^5}{16d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}} - \frac{65ia^4}{64d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}} + \frac{715ia^2}{1792d(a+ia\tan(c+dx))^{7/2}} + \frac{143ia}{512d(a+ia\tan(c+dx))^{5/2}} + \frac{715i}{3072d(a+ia\tan(c+dx))^{3/2}} - \frac{(715i)\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{3/2}}\,dx, x, ia\tan(c+dx)\right)}{2048d}$$

$$= \frac{715ia^3}{1152d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{9/2}} - \frac{5ia^5}{16d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}} - \frac{65ia^4}{64d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}} + \frac{715ia^2}{1792d(a+ia\tan(c+dx))^{7/2}} + \frac{143ia}{512d(a+ia\tan(c+dx))^{5/2}} + \frac{715i}{3072d(a+ia\tan(c+dx))^{3/2}} + \frac{715i}{2048ad\sqrt{a+ia\tan(c+dx)}} - \frac{(715i)\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{4096ad}$$

$$= \frac{715ia^3}{1152d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{9/2}} - \frac{5ia^5}{16d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}} - \frac{65ia^4}{64d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}} + \frac{715ia^2}{1792d(a+ia\tan(c+dx))^{7/2}} + \frac{143ia}{512d(a+ia\tan(c+dx))^{5/2}} + \frac{715i}{3072d(a+ia\tan(c+dx))^{3/2}} + \frac{715i}{2048ad\sqrt{a+ia\tan(c+dx)}} - \frac{(715i)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+ia\tan(c+dx)}\right)}{2048ad}$$

$$= -\frac{715i\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{2048\sqrt{2}a^{3/2}d} + \frac{715ia^3}{1152d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^6}{6d(a-ia\tan(c+dx))^3(a+ia\tan(c+dx))^{9/2}} - \frac{5ia^5}{16d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}} - \frac{65ia^4}{64d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}} + \frac{715ia^2}{1792d(a+ia\tan(c+dx))^{7/2}} + \frac{143ia}{512d(a+ia\tan(c+dx))^{5/2}} + \frac{715i}{3072d(a+ia\tan(c+dx))^{3/2}} + \frac{715i}{2048ad\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.53 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.17

$$\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{ia^3\,\text{Hypergeometric2F1}\left(-\frac{9}{2},4,-\frac{7}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{72d(a+ia\tan(c+dx))^{9/2}}$$

```
[In] Integrate[Cos[c + d*x]^6/(a + I*a*Tan[c + d*x])^(3/2),x]
[Out] ((I/72)*a^3*Hypergeometric2F1[-9/2, 4, -7/2, (1 + I*Tan[c + d*x])/2])/(d*(a
 + I*a*Tan[c + d*x])^(9/2))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 810 vs. 2(257) = 514.

Time = 9.31 (sec) , antiderivative size = 811, normalized size of antiderivative = 2.53

method	result	size
default	Expression too large to display	811

```
[In] int(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/258048/d*(90090*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+28672*sin
(d*x+c)*cos(d*x+c)^8*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+45045*I*arctan(1/2*
(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/
2))+28672*sin(d*x+c)*cos(d*x+c)^7*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-12012*
I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2+33280*sin(d*x+c)*cos(d*x+
```

```
c)^6*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+28672*I*(-cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)*cos(d*x+c)^8+33280*sin(d*x+c)*cos(d*x+c)^5*(-cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)+90090*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+41184*sin(d*x+c)*cos(
d*x+c)^4*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-4576*I*cos(d*x+c)^5*(-cos(d*x+c
)/(cos(d*x+c)+1))^(1/2)+41184*sin(d*x+c)*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)-4576*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^4+60060*
sin(d*x+c)*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+28672*I*cos(d*x+
c)^9*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+60060*(-cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)*cos(d*x+c)*sin(d*x+c)-2560*I*cos(d*x+c)^7*(-cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)-2560*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^6-45045*arctan
(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2))*sin(d*x+c)+45045*I*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x
+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-12012*I*cos(d*x+c)^3*
(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))/(a*(1+I*tan(d*x+c)))^(1/2)/(cos(d*x+c)+
1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.05

$$\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\left(-45045i\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{1}{a^3d^2}}e^{(9i\,dx+9i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^2de^{(2i\,dx+2i\,c)}+a^2d\right)\sqrt{\frac{}{e^{(2}}}\right.\right.\right.}{}$$

```
[In] integrate(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/258048*(-45045*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(9*I*d*x + 9*I*c)*lo
g(4*(sqrt(2)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*sqrt(1/(a^3*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c))
+ 45045*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(9*I*d*x + 9*I*c)*log(-4*(sqr
t(2)*sqrt(1/2)*(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*sqrt(1/(a^3*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2
)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-168*I*e^(16*I*d*x + 16*I*c) - 1974*I*
e^(14*I*d*x + 14*I*c) - 13209*I*e^(12*I*d*x + 12*I*c) + 33301*I*e^(10*I*d*x
 + 10*I*c) + 57632*I*e^(8*I*d*x + 8*I*c) + 17344*I*e^(6*I*d*x + 6*I*c) + 54
40*I*e^(4*I*d*x + 4*I*c) + 1136*I*e^(2*I*d*x + 2*I*c) + 112*I))*e^(-9*I*d*x
 - 9*I*c)/(a^2*d)
```

Sympy [F]

$$\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos^6{(c+dx)}}{(ia(\tan{(c+dx)}-i))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)**6/(a+I*a*tan(d*x+c))**(3/2),x)
[Out] Integral(cos(c + d*x)**6/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.81

$$\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{i\left(\frac{45045\sqrt{2}\log\left(-\frac{\sqrt{2}\sqrt{a}-\sqrt{i\,a\tan(dx+c)+a}}{\sqrt{2}\sqrt{a}+\sqrt{i\,a\tan(dx+c)+a}}\right)}{\sqrt{a}} + \frac{4\left(45045\,(i\,a\tan(dx+c)+a)^7-240240\,(i\,a\tan(dx+c)+\cdots\right.}{\cdots}\right.}{\cdots}$$

```
[In] integrate(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
[Out] 1/516096*I*(45045*sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a
))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/sqrt(a) + 4*(45045*(I*a*
tan(d*x + c) + a)^7 - 240240*(I*a*tan(d*x + c) + a)^6*a + 396396*(I*a*tan(d
*x + c) + a)^5*a^2 - 164736*(I*a*tan(d*x + c) + a)^4*a^3 - 36608*(I*a*tan(d
*x + c) + a)^3*a^4 - 19968*(I*a*tan(d*x + c) + a)^2*a^5 - 15360*(I*a*tan(d*
x + c) + a)*a^6 - 14336*a^7)/((I*a*tan(d*x + c) + a)^(15/2) - 6*(I*a*tan(d*
x + c) + a)^(13/2)*a + 12*(I*a*tan(d*x + c) + a)^(11/2)*a^2 - 8*(I*a*tan(d*
x + c) + a)^(9/2)*a^3))/(a*d)
```

Giac [F]

$$\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos{(dx+c)}^6}{(i\,a\tan{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)^6/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(cos(d*x + c)^6/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^6(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^6}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{3/2}}\,dx$$

```
[In] int(cos(c + d*x)^6/(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] int(cos(c + d*x)^6/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.354 $\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 147

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{256ia^4\sec^{11}(c+dx)}{12155d(a+ia\tan(c+dx))^{11/2}} + \frac{64ia^3\sec^{11}(c+dx)}{1105d(a+ia\tan(c+dx))^{9/2}} + \frac{8ia^2\sec^{11}(c+dx)}{85d(a+ia\tan(c+dx))^{7/2}} + \frac{2ia\sec^{11}(c+dx)}{17d(a+ia\tan(c+dx))^{5/2}}$$

```
[Out] 256/12155*I*a^4*sec(d*x+c)^11/d/(a+I*a*tan(d*x+c))^(11/2)+64/1105*I*a^3*sec
(d*x+c)^11/d/(a+I*a*tan(d*x+c))^(9/2)+8/85*I*a^2*sec(d*x+c)^11/d/(a+I*a*tan
(d*x+c))^(7/2)+2/17*I*a*sec(d*x+c)^11/d/(a+I*a*tan(d*x+c))^(5/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3575, 3574}

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{256ia^4\sec^{11}(c+dx)}{12155d(a+ia\tan(c+dx))^{11/2}} + \frac{64ia^3\sec^{11}(c+dx)}{1105d(a+ia\tan(c+dx))^{9/2}} + \frac{8ia^2\sec^{11}(c+dx)}{85d(a+ia\tan(c+dx))^{7/2}} + \frac{2ia\sec^{11}(c+dx)}{17d(a+ia\tan(c+dx))^{5/2}}$$

```
[In] Int[Sec[c + d*x]^11/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((256*I)/12155)*a^4*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(11/2)) + (
((64*I)/1105)*a^3*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(9/2)) + (((8*
I)/85)*a^2*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (((2*I)/17)*
a*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(5/2))
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2ia\sec^{11}(c+dx)}{17d(a+ia\tan(c+dx))^{5/2}}+\frac{1}{17}(12a)\int\frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx\\
&= \frac{8ia^2\sec^{11}(c+dx)}{85d(a+ia\tan(c+dx))^{7/2}}+\frac{2ia\sec^{11}(c+dx)}{17d(a+ia\tan(c+dx))^{5/2}}\\
&\quad+\frac{1}{85}\left(32a^2\right)\int\frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx\\
&= \frac{64ia^3\sec^{11}(c+dx)}{1105d(a+ia\tan(c+dx))^{9/2}}+\frac{8ia^2\sec^{11}(c+dx)}{85d(a+ia\tan(c+dx))^{7/2}}\\
&\quad+\frac{2ia\sec^{11}(c+dx)}{17d(a+ia\tan(c+dx))^{5/2}}+\frac{(128a^3)\int\frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{9/2}}\,dx}{1105}\\
&= \frac{256ia^4\sec^{11}(c+dx)}{12155d(a+ia\tan(c+dx))^{11/2}}+\frac{64ia^3\sec^{11}(c+dx)}{1105d(a+ia\tan(c+dx))^{9/2}}\\
&\quad+\frac{8ia^2\sec^{11}(c+dx)}{85d(a+ia\tan(c+dx))^{7/2}}+\frac{2ia\sec^{11}(c+dx)}{17d(a+ia\tan(c+dx))^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.90 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.73

$$
\int\frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx=\frac{2\sec^9(c+dx)(i\cos(4(c+dx))+\sin(4(c+dx)))(475i-2242i\cos(2(c+dx
}{12155ad(-i+\tan(c+dx))\sqrt{a+
$$

```
[In] Integrate[Sec[c + d*x]^11/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (2*Sec[c + d*x]^9*(I*Cos[4*(c + d*x)] + Sin[4*(c + d*x)])*(475*I - (2242*I)
*Cos[2*(c + d*x)] + 1089*Sec[c + d*x]*Sin[3*(c + d*x)] + 374*Tan[c + d*x]))
/(12155*a*d*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [F(-1)]

Timed out.

$$\int \frac{\sec^{11}(dx+c)}{(a+ia\tan(dx+c))^{\frac{3}{2}}}dx$$

[In] int(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(3/2),x)

[Out] int(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(3/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.25

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{512\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-1105i\,e^{(6i\,dx+6i\,c)}-510i\,e^{(4i\,dx+4i\,c)}-136i\,e\cdots\right.}{12155\left(a^2de^{(16i\,dx+16i\,c)}+8\,a^2de^{(14i\,dx+14i\,c)}+28\,a^2de^{(12i\,dx+12i\,c)}+56\,a^2de^{(10i\,dx+10i\,c)}+70\,a^2de^{(8i\,dx+8i\,c)}+56\cdots\right.}$$

[In] integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")

[Out] -512/12155*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-1105*I*e^(6*I*d*x + 6*I*c) - 510*I*e^(4*I*d*x + 4*I*c) - 136*I*e^(2*I*d*x + 2*I*c) - 16*I)/(a^2 *d*e^(16*I*d*x + 16*I*c) + 8*a^2*d*e^(14*I*d*x + 14*I*c) + 28*a^2*d*e^(12*I *d*x + 12*I*c) + 56*a^2*d*e^(10*I*d*x + 10*I*c) + 70*a^2*d*e^(8*I*d*x + 8*I *c) + 56*a^2*d*e^(6*I*d*x + 6*I*c) + 28*a^2*d*e^(4*I*d*x + 4*I*c) + 8*a^2*d *e^(2*I*d*x + 2*I*c) + a^2*d)

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Timed out}$$

[In] integrate(sec(d*x+c)**11/(a+I*a*tan(d*x+c))**(3/2),x)

[Out] Timed out

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 764 vs. $2(115) = 230$.

Time = 0.48 (sec) , antiderivative size = 764, normalized size of antiderivative = 5.20

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/12155*(-1767*I*sqrt(a) - 6854*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) +
2088*I*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 16438*sqrt(a)*sin(d*x
+ c)^3/(cos(d*x + c) + 1)^3 - 5661*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) +
 1)^4 - 56984*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 13328*I*sqrt(a)
*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 129336*sqrt(a)*sin(d*x + c)^7/(cos(d
*x + c) + 1)^7 + 7514*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 15646
8*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 156468*sqrt(a)*sin(d*x + c)
^11/(cos(d*x + c) + 1)^11 - 7514*I*sqrt(a)*sin(d*x + c)^12/(cos(d*x + c) +
1)^12 - 129336*sqrt(a)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 + 13328*I*sqrt
(a)*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 56984*sqrt(a)*sin(d*x + c)^15/(
cos(d*x + c) + 1)^15 + 5661*I*sqrt(a)*sin(d*x + c)^16/(cos(d*x + c) + 1)^16
 - 16438*sqrt(a)*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 - 2088*I*sqrt(a)*sin
(d*x + c)^18/(cos(d*x + c) + 1)^18 - 6854*sqrt(a)*sin(d*x + c)^19/(cos(d*x
+ c) + 1)^19 + 1767*I*sqrt(a)*sin(d*x + c)^20/(cos(d*x + c) + 1)^20)*(sin(d
*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)
^(3/2)/((a^2 - 10*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 45*a^2*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4 - 120*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 +
 210*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 252*a^2*sin(d*x + c)^10/(cos
(d*x + c) + 1)^10 + 210*a^2*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 120*a^2
*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + 45*a^2*sin(d*x + c)^16/(cos(d*x +
c) + 1)^16 - 10*a^2*sin(d*x + c)^18/(cos(d*x + c) + 1)^18 + a^2*sin(d*x + c
)^20/(cos(d*x + c) + 1)^20)*d*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(3/2))
```

Giac [F]

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{11}}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^11/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 11.26 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.05

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{e^{-c\,1i-d\,x\,1i}\,\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,512i}{11\,a^2\,d\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^5} - \frac{e^{-c\,1i-d\,x\,1i}\,\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,1536i}{13\,a^2\,d\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^6} + \frac{e^{-c\,1i-d\,x\,1i}\,\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,512i}{5\,a^2\,d\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^7} - \frac{e^{-c\,1i-d\,x\,1i}\,\sqrt{a-\frac{a\,\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)\,1i}{e^{c\,2i+d\,x\,2i}+1}}\,512i}{17\,a^2\,d\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^8}$$

```
[In] int(1/(cos(c + d*x)^11*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

```
[Out] (exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i +
d*x*2i) + 1))^(1/2)*512i)/(11*a^2*d*(exp(c*2i + d*x*2i) + 1)^5) - (exp(- c*
1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) +
 1))^(1/2)*1536i)/(13*a^2*d*(exp(c*2i + d*x*2i) + 1)^6) + (exp(- c*1i - d*x
*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/
2)*512i)/(5*a^2*d*(exp(c*2i + d*x*2i) + 1)^7) - (exp(- c*1i - d*x*1i)*(a -
(a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*512i)/(
17*a^2*d*(exp(c*2i + d*x*2i) + 1)^8)
```

3.355 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 110

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{64ia^3\sec^9(c+dx)}{1287d(a+ia\tan(c+dx))^{9/2}} + \frac{16ia^2\sec^9(c+dx)}{143d(a+ia\tan(c+dx))^{7/2}} + \frac{2ia\sec^9(c+dx)}{13d(a+ia\tan(c+dx))^{5/2}}$$

```
[Out] 64/1287*I*a^3*sec(d*x+c)^9/d/(a+I*a*tan(d*x+c))^(9/2)+16/143*I*a^2*sec(d*x+c)^9/d/(a+I*a*tan(d*x+c))^(7/2)+2/13*I*a*sec(d*x+c)^9/d/(a+I*a*tan(d*x+c))^(5/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3575, 3574}

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{64ia^3\sec^9(c+dx)}{1287d(a+ia\tan(c+dx))^{9/2}} + \frac{16ia^2\sec^9(c+dx)}{143d(a+ia\tan(c+dx))^{7/2}} + \frac{2ia\sec^9(c+dx)}{13d(a+ia\tan(c+dx))^{5/2}}$$

```
[In] Int[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((64*I)/1287)*a^3*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(9/2)) + (((16*I)/143)*a^2*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (((2*I)/13)*a*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(5/2))
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2ia\sec^9(c+dx)}{13d(a+ia\tan(c+dx))^{5/2}}+\frac{1}{13}(8a)\int\frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx\\ &= \frac{16ia^2\sec^9(c+dx)}{143d(a+ia\tan(c+dx))^{7/2}}+\frac{2ia\sec^9(c+dx)}{13d(a+ia\tan(c+dx))^{5/2}}\\ &\quad+\frac{1}{143}(32a^2)\int\frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx\\ &= \frac{64ia^3\sec^9(c+dx)}{1287d(a+ia\tan(c+dx))^{9/2}}+\frac{16ia^2\sec^9(c+dx)}{143d(a+ia\tan(c+dx))^{7/2}}+\frac{2ia\sec^9(c+dx)}{13d(a+ia\tan(c+dx))^{5/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

$$\int\frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx=\frac{2\sec^8(c+dx)(52+151\cos(2(c+dx))+135i\sin(2(c+dx)))(\cos(3(c+dx))}{1287ad(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (2*Sec[c + d*x]^8*(52 + 151*Cos[2*(c + d*x)] + (135*I)*Sin[2*(c + d*x)])*(C
os[3*(c + d*x)] - I*Sin[3*(c + d*x)]))/(1287*a*d*(-I + Tan[c + d*x])*Sqrt[a
 + I*a*Tan[c + d*x]])
```

Maple [F(-1)]

Timed out.

$$\int \frac{\sec^9(dx+c)}{(a+ia\tan(dx+c))^{\frac{3}{2}}}dx$$

[In] int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(3/2),x)

[Out] int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(3/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.30

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{128\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-143i\,e^{(4i\,dx+4i\,c)}-52i\,e^{(2i\,dx+2i\,c)}-8i\right)}{1287\,(a^2de^{(12i\,dx+12i\,c)}+6\,a^2de^{(10i\,dx+10i\,c)}+15\,a^2de^{(8i\,dx+8i\,c)}+20\,a^2de^{(6i\,dx+6i\,c)}+15\,a^2de^{(4i\,dx+4i\,c)}+6\,a^2d$$

[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")

[Out] -128/1287*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-143*I*e^(4*I*d*x + 4*I*c) - 52*I*e^(2*I*d*x + 2*I*c) - 8*I)/(a^2*d*e^(12*I*d*x + 12*I*c) + 6*a^2*d*e^(10*I*d*x + 10*I*c) + 15*a^2*d*e^(8*I*d*x + 8*I*c) + 20*a^2*d*e^(6*I*d*x + 6*I*c) + 15*a^2*d*e^(4*I*d*x + 4*I*c) + 6*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)

Sympy [F]

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec^9(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

[In] integrate(sec(d*x+c)**9/(a+I*a*tan(d*x+c))**(3/2),x)

[Out] Integral(sec(c + d*x)**9/(I*a*(tan(c + d*x) - I))**(3/2), x)

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 626 vs. 2(86) = 172.

Time = 0.43 (sec) , antiderivative size = 626, normalized size of antiderivative = 5.69

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx =$$

$$-\frac{2\left(-203i\sqrt{a}-\frac{678\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{2i\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{1802\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{26i\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{3614\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{858i\sqrt{a}\ldots}{(\cos(d\ldots}\right.}{1287\left(a^2-\frac{8a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{28a^2\sin(dx\ldots}{(\cos(dx+c)\ldots}\right.}$$

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/1287*(-203*I*sqrt(a) - 678*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 2*I
*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1802*sqrt(a)*sin(d*x + c)^3/
(cos(d*x + c) + 1)^3 - 26*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 3
614*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 858*I*sqrt(a)*sin(d*x + c
)^6/(cos(d*x + c) + 1)^6 - 6578*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7
 - 6578*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 858*I*sqrt(a)*sin(d*x
 + c)^10/(cos(d*x + c) + 1)^10 - 3614*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c)
 + 1)^11 + 26*I*sqrt(a)*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 1802*sqrt(a
)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 + 2*I*sqrt(a)*sin(d*x + c)^14/(cos(
d*x + c) + 1)^14 - 678*sqrt(a)*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 + 203*
I*sqrt(a)*sin(d*x + c)^16/(cos(d*x + c) + 1)^16)*(sin(d*x + c)/(cos(d*x + c
) + 1) + 1)^(3/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(3/2)/((a^2 - 8*a^2
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 28*a^2*sin(d*x + c)^4/(cos(d*x + c)
+ 1)^4 - 56*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a^2*sin(d*x + c)^8
/(cos(d*x + c) + 1)^8 - 56*a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 28*a
^2*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 8*a^2*sin(d*x + c)^14/(cos(d*x +
 c) + 1)^14 + a^2*sin(d*x + c)^16/(cos(d*x + c) + 1)^16)*d*(-2*I*sin(d*x +
c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(3/2))
```

Giac [F]

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^9}{(i\,a\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^9/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 9.55 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{128\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,52\mathrm{i}+\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}\,143\mathrm{i}+8\mathrm{i}\right)}{1287\,a^2\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^6}$$

```
[In] int(1/(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

```
[Out] (128*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2
i + d*x*2i) + 1))^(1/2)*(exp(c*2i + d*x*2i)*52i + exp(c*4i + d*x*4i)*143i +
 8i))/(1287*a^2*d*(exp(c*2i + d*x*2i) + 1)^6)
```

3.356 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{3/2}} dx$

Optimal result

Integrand size = 26, antiderivative size = 73

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{3/2}} dx = \frac{8ia^2\sec^7(c+dx)}{63d(a+ia\tan(c+dx))^{7/2}} + \frac{2ia\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^{5/2}}$$

```
[Out] 8/63*I*a^2*sec(d*x+c)^7/d/(a+I*a*tan(d*x+c))^(7/2)+2/9*I*a*sec(d*x+c)^7/d/(
a+I*a*tan(d*x+c))^(5/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3575, 3574}

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{3/2}} dx = \frac{8ia^2\sec^7(c+dx)}{63d(a+ia\tan(c+dx))^{7/2}} + \frac{2ia\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^{5/2}}$$

```
[In] Int[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((8*I)/63)*a^2*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + (((2*I)/
9)*a*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(5/2))
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2ia\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^{5/2}} + \frac{1}{9}(4a)\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx \\ &= \frac{8ia^2\sec^7(c+dx)}{63d(a+ia\tan(c+dx))^{7/2}} + \frac{2ia\sec^7(c+dx)}{9d(a+ia\tan(c+dx))^{5/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.10

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2\sec^5(c+dx)(i\cos(2(c+dx))+\sin(2(c+dx)))(-11i+7\tan(c+dx))}{63ad(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^(3/2),x]
[Out] (2*Sec[c + d*x]^5*(I*Cos[2*(c + d*x)] + Sin[2*(c + d*x)])*(-11*I + 7*Tan[c
+ d*x]))/(63*a*d*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [F(-1)]

Timed out.

$$\int \frac{\sec^7(dx+c)}{(a+ia\tan(dx+c))^{\frac{3}{2}}}dx$$

```
[In] int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(3/2),x)
[Out] int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.40

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{32\,\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-9i\,e^{(2i\,dx+2i\,c)}-2i\right)}{63\,\left(a^2de^{(8i\,dx+8i\,c)}+4\,a^2de^{(6i\,dx+6i\,c)}+6\,a^2de^{(4i\,dx+4i\,c)}+4\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -32/63*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-9*I*e^(2*I*d*x + 2*I*c)
- 2*I)/(a^2*d*e^(8*I*d*x + 8*I*c) + 4*a^2*d*e^(6*I*d*x + 6*I*c) + 6*a^2*d*e
^(4*I*d*x + 4*I*c) + 4*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F]

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec^7\left(c+dx\right)}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**7/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 488 vs. 2(57) = 114.

Time = 0.37 (sec) , antiderivative size = 488, normalized size of antiderivative = 6.68

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{2\left(-11i\,\sqrt{a}-\frac{30\,\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{12i\,\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{86\,\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{9i\,\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{108\,\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{108\,\sqrt{a}\sin(dx-}{(\cos(dx+c)+}\right.}{63\left(a^2-\frac{6\,a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15\,a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{20\,a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15\,a^2}{(\cos(}\right.}$$

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/63*(-11*I*sqrt(a) - 30*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 12*I*sq
rt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 86*sqrt(a)*sin(d*x + c)^3/(cos(
```

```
d*x + c) + 1)^3 + 9*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 108*sqr
t(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 108*sqrt(a)*sin(d*x + c)^7/(cos(
d*x + c) + 1)^7 - 9*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 86*sqrt
(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 12*I*sqrt(a)*sin(d*x + c)^10/(cos
(d*x + c) + 1)^10 - 30*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 11*I
*sqrt(a)*sin(d*x + c)^12/(cos(d*x + c) + 1)^12)*(sin(d*x + c)/(cos(d*x + c)
 + 1) + 1)^(3/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(3/2)/((a^2 - 6*a^2*
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a^2*sin(d*x + c)^4/(cos(d*x + c) +
 1)^4 - 20*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 15*a^2*sin(d*x + c)^8/
(cos(d*x + c) + 1)^8 - 6*a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + a^2*si
n(d*x + c)^12/(cos(d*x + c) + 1)^12)*d*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1
) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(3/2))
```

Giac [F]

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(dx+c\right)^7}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^7/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 7.74 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{32\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,9\mathrm{i}+2\mathrm{i}\right)\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}}{63\,a^2\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^4}$$

```
[In] int(1/(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^(3/2)),x)
[Out] (32*exp(- c*1i - d*x*1i)*(exp(c*2i + d*x*2i)*9i + 2i)*(a - (a*(exp(c*2i + d
*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2))/(63*a^2*d*(exp(c*2i +
d*x*2i) + 1)^4)
```

3.357 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 35

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2ia\sec^5(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

```
[Out] 2/5*I*a*sec(d*x+c)^5/d/(a+I*a*tan(d*x+c))^(5/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {3574}

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2ia\sec^5(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

```
[In] Int[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^(3/2),x]
[Out] (((2*I)/5)*a*Sec[c + d*x]^5)/(d*(a + I*a*Tan[c + d*x])^(5/2))
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\text{integral} = \frac{2ia\sec^5(c+dx)}{5d(a+ia\tan(c+dx))^{5/2}}$$

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2\sec^3(c+dx)(1-i\tan(c+dx))}{5ad(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^(3/2),x]
[Out] (2*Sec[c + d*x]^3*(1 - I*Tan[c + d*x]))/(5*a*d*(-I + Tan[c + d*x])*Sqrt[a +
 I*a*Tan[c + d*x]])
```

Maple [F(-1)]

Timed out.

$$\int \frac{\sec^5(dx+c)}{(a+ia\tan(dx+c))^{\frac{3}{2}}}dx$$

```
[In] int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(3/2),x)
[Out] int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(3/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(27) = 54.

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{8i\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{5\left(a^2de^{(4i\,dx+4i\,c)}+2\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
[Out] 8/5*I*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(a^2*d*e^(4*I*d*x + 4*I*c)
+ 2*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec^5(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**5/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(27) = 54$.

Time = 0.34 (sec) , antiderivative size = 350, normalized size of antiderivative = 10.00

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx =$$

$$-\frac{2\left(-i\sqrt{a}-\frac{2\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{2i\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{6\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{6\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{2i\sqrt{a}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{2\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\cdots\right.}{5\left(a^2-\frac{4a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d\left(-\frac{2i\sin(dx+c)}{\cos(dx+c)+\cdots}\right.}$$

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/5*(-I*sqrt(a) - 2*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 2*I*sqrt(a)*
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 6*sqrt(a)*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 - 6*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2*I*sqrt(a)*sin(
d*x + c)^6/(cos(d*x + c) + 1)^6 - 2*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) +
1)^7 + I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*(sin(d*x + c)/(cos(d*
x + c) + 1) + 1)^(3/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(3/2)/((a^2 -
4*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^2*sin(d*x + c)^4/(cos(d*x +
 c) + 1)^4 - 4*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^2*sin(d*x + c)^8
/(cos(d*x + c) + 1)^8)*d*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 - 1)^(3/2))
```

Giac [F]

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec\left(dx+c\right)^5}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate(sec(d*x + c)^5/(I*a*tan(d*x + c) + a)^(3/2), x)

Mupad [B] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 139, normalized size of antiderivative = 3.97

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\left(\cos\left(d\,x\right)-\sin\left(d\,x\right)\,1{}\mathrm{i}\right)\,\left(\cos\left(c\right)-\sin\left(c\right)\,1{}\mathrm{i}\right)\,\sqrt{\frac{a\,\left(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1{}\mathrm{i}\right)}{\cos(2\,c+2\,d\,x)+1}}}{5\,a^2\,d\,\left(4\,\cos\left(2\,c+\right.\right.}$$

[In] int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(3/2)),x)

[Out] ((cos(d*x) - sin(d*x)*1i)*(cos(c) - sin(c)*1i)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2)*(2*cos(2*c + 2*d*x) + cos(4*c + 4*d*x) - sin(2*c + 2*d*x)*2i - sin(4*c + 4*d*x)*1i + 1)*4i)/(5*a^2*d*(4*cos(2*c + 2*d*x) + cos(4*c + 4*d*x) + 3))

3.358 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 86

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2i\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d} - \frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 2*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*2^(1/2
)/a^(3/2)/d-2*I*sec(d*x+c)/a/d/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3572, 3570, 212}

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2i\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d} - \frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((2*I)*Sqrt[2]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c +
 d*x]])])/(a^(3/2)*d) - ((2*I)*Sec[c + d*x])/(a*d*Sqrt[a + I*a*Tan[c + d*x]
])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3572

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e
+ f*x])^(n + 1)/(b*f*(m - 2))), x] + Dist[2*(d^2/a), Int[(d*Sec[e + f*x])^(
m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 + b^2, 0] && EqQ[m/2 + n, 0] && LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}}+\frac{2\int\frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{a}\\
&= -\frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}}+\frac{(4i)\text{Subst}\left(\int\frac{1}{2-ax^2}\,dx,x,\frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\right)}{ad}\\
&= \frac{2i\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{3/2}d}-\frac{2i\sec(c+dx)}{ad\sqrt{a+ia\tan(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.17

$$
\int\frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx=\frac{8e^{3i(c+dx)}\left(-1+\sqrt{1+e^{2i(c+dx)}}\text{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)}{ad\left(1+e^{2i(c+dx)}\right)^2(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}
$$

```
[In] Integrate[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (8*E^((3*I)*(c + d*x))*(-1 + Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 +
 E^((2*I)*(c + d*x))]]))/(a*d*(1 + E^((2*I)*(c + d*x)))^2*(-I + Tan[c + d*x
])*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(71) = 142.

Time = 8.94 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.59

method	result
default	$-\frac{2(-\csc(dx+c)+\cot(dx+c)+i)^{3}\left(-\sqrt{2}\,\arctan\left(\frac{(i(\csc(dx+c)-\cot(dx+c))-1)\sqrt{2}}{2\sqrt{\left(\csc^{2}(dx+c)\right)(1-\cos(dx+c))^{2}-1}}\right)\sqrt{(\csc^{2}(dx+c))(1-\cos(dx+c))^{2}-1}+i(\csc(dx+c)}{d\left(-\frac{a\left(2i(\csc(dx+c)-\cot(dx+c))-\left(\csc^{2}(dx+c)\right)(1-\cos(dx+c))^{2}+1\right)}{\left(\csc^{2}(dx+c)\right)(1-\cos(dx+c))^{2}-1}\right)^{\frac{3}{2}}\left((\csc^{2}(dx+c))(1-\cos(dx+c))^{2}-1\right)^{2}}$

```
[In] int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*(-csc(d*x+c)+cot(d*x+c)+I)^3*(-2^(1/2)*arctan(1/2*(I*(csc(d*x+c)-cot(d
*x+c))-1)*2^(1/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)^(1/2))*(csc(d*x+c)^2*(1
-cos(d*x+c))^2-1)^(1/2)+I*(csc(d*x+c)-cot(d*x+c))-1)/(-a*(2*I*(csc(d*x+c)-c
ot(d*x+c))-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-
1))^(3/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(67) = 134.

Time = 0.24 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.28

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{-i\sqrt{2}a^2d\sqrt{\frac{1}{a^3d^2}}\log\left(-\frac{8\left(\left(i\,ade^{(2i\,dx+2i\,c)}+i\,ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^3d^2}}-i\right)e^{(-i\,dx-i\,c)}}{ad}\right)+}{}$$

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*a^2*d*sqrt(1/(a^3*d^2))*log(-8*((I*a*d*e^(2*I*d*x + 2*I*c) + I*
a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^3*d^2)) - I)*e^(-I*d*x - I
*c)/(a*d)) + I*sqrt(2)*a^2*d*sqrt(1/(a^3*d^2))*log(-8*((-I*a*d*e^(2*I*d*x +
 2*I*c) - I*a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^3*d^2)) - I)*e
^(-I*d*x - I*c)/(a*d)) - 2*I*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)))/(a^
2*d)
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec^3(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 813 vs. $2(67) = 134$.

Time = 0.45 (sec) , antiderivative size = 813, normalized size of antiderivative = 9.45

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*(2*sqrt(2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - 2*sqrt(2)
*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x
 + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) - I*sqrt(2)*log(sqrt(cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1))^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)) + 1) + I*sqrt(2)*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 - 2*(
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1))*sqrt(a) + 4*(I*
sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sqrt(2)*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a))/((cos(2*
d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*a^2*d)
```

Giac [F]

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)^3}{(i\,a\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^3/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos(c+d\,x)^3\,(a+a\tan(c+d\,x)\ 1\mathrm{i})^{3/2}}\,dx$$

```
[In] int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(3/2)),x)
[Out] int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(3/2)), x)
```

3.359 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{i\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] 1/4*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/a^(3
/2)/d*2^(1/2)+1/2*I*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3583, 3570, 212}

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{i\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}$$

```
[In] Int[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((I/2)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])]
)/(Sqrt[2]*a^(3/2)*d) + ((I/2)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])^(3/2
))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}} + \frac{\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{4a}\\ &= \frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}} + \frac{i\text{Subst}\left(\int \frac{1}{2-ax^2}\,dx, x, \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\right)}{2ad}\\ &= \frac{i\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{i\sec(c+dx)}{2d(a+ia\tan(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.09

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\left(2+\frac{2e^{2i(c+dx)}\text{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)}{\sqrt{1+e^{2i(c+dx)}}}\right)\sec(c+dx)}{4ad(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^(3/2),x]
[Out] ((2 + (2*E^((2*I)*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])/Sqrt[1
 + E^((2*I)*(c + d*x))])*Sec[c + d*x])/(4*a*d*(-I + Tan[c + d*x])*Sqrt[a +
I*a*Tan[c + d*x]])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 415 vs. 2(68) = 136.

Time = 8.67 (sec) , antiderivative size = 416, normalized size of antiderivative = 4.78

method	result
default	$-\frac{i\left(2i\cos(dx+c)\arctan\left(\frac{i\sin(dx+c)-\cos(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+i\arctan\left(\frac{i\sin(dx+c)-\cos(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+2i\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-2\operatorname{arct}}{4d(\text{ta}}$

[In] int(sec(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/4*I/d/(tan(d*x+c)-I)/(a*(1+I*tan(d*x+c)))^(1/2)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/a/(cos(d*x+c)+1)*(2*I*cos(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+I*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+2*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-2*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-I*sec(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+2*I*sec(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-tan(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)))

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(64) = 128.

Time = 0.26 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.83

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\left(i\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{1}{a^3d^2}}e^{(2i\,dx+2i\,c)}\log\left(\frac{\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,ade^{(2i\,dx+2i\,c)}+i\,ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^3d^2}}+i\right.}{ad}\right.\right.}{}$$

[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")

[Out] 1/4*(I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(2*I*d*x + 2*I*c)*log((sqrt(2)*sqrt(1/2)*(I*a*d*e^(2*I*d*x + 2*I*c) + I*a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^3*d^2)) + I)*e^(-I*d*x - I*c)/(a*d)) - I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(2*I*d*x + 2*I*c)*log((sqrt(2)*sqrt(1/2)*(-I*a*d*e^(2*I*d*x + 2*I*c) - I*a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^3*d^2)) + I)*e^(-I*d*x - I*c)/(a*d)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(I*e^(2*I*d*x + 2*I*c) + I))*e^(-2*I*d*x - 2*I*c)/(a^2*d)

Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))**(3/2),x)
[Out] Integral(sec(c + d*x)/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)}{(i\,a\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sec(dx+c)}{(i\,a\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{1}{\cos(c+d\,x)\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{3/2}}\,dx$$

```
[In] int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(3/2)),x)
[Out] int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(3/2)), x)
```

3.360 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 157

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{15i\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{32\sqrt{2}a^{3/2}d} + \frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}} + \frac{5i\cos(c+dx)}{16ad\sqrt{a+ia\tan(c+dx)}} - \frac{15i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{32a^2d}$$

```
[Out] 15/64*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/a^
(3/2)/d*2^(1/2)+5/16*I*cos(d*x+c)/a/d/(a+I*a*tan(d*x+c))^(1/2)-15/32*I*cos(
d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/a^2/d+1/4*I*cos(d*x+c)/d/(a+I*a*tan(d*x+c))
^(3/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3583, 3571, 3570, 212}

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{15i\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{32\sqrt{2}a^{3/2}d} - \frac{15i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{32a^2d} + \frac{5i\cos(c+dx)}{16ad\sqrt{a+ia\tan(c+dx)}} + \frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}}$$

```
[In] Int[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((15*I)/32)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d
*x]])])/(Sqrt[2]*a^(3/2)*d) + ((I/4)*Cos[c + d*x])/(d*(a + I*a*Tan[c + d*x]
)^(3/2)) + (((5*I)/16)*Cos[c + d*x])/(a*d*Sqrt[a + I*a*Tan[c + d*x]]) - (((
15*I)/32)*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/(a^2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3571

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}} + \frac{5\int \frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{8a} \\
&= \frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}} + \frac{5i\cos(c+dx)}{16ad\sqrt{a+ia\tan(c+dx)}} \\
&\quad + \frac{15\int \cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx}{32a^2}
\end{aligned}
$$

$$= \frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}} + \frac{5i\cos(c+dx)}{16ad\sqrt{a+ia\tan(c+dx)}} - \frac{15i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{32a^2d} + \frac{15\int\frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}dx}{64a}$$

$$= \frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}} + \frac{5i\cos(c+dx)}{16ad\sqrt{a+ia\tan(c+dx)}} - \frac{15i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{32a^2d} + \frac{(15i)\text{Subst}\left(\int\frac{1}{2-ax^2}dx, x, \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\right)}{32ad}$$

$$= \frac{15i\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{32\sqrt{2}a^{3/2}d} + \frac{i\cos(c+dx)}{4d(a+ia\tan(c+dx))^{3/2}} + \frac{5i\cos(c+dx)}{16ad\sqrt{a+ia\tan(c+dx)}} - \frac{15i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{32a^2d}$$

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.76

$$\int\frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{3/2}}dx = \frac{\sec(c+dx)\left(\frac{30e^{2i(c+dx)}\text{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)}{\sqrt{1+e^{2i(c+dx)}}} - 2(-9+6\cos(2(c+dx))+10\right.}{64ad(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (Sec[c + d*x]*((30*E^((2*I)*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))
]]])/Sqrt[1 + E^((2*I)*(c + d*x))] - 2*(-9 + 6*Cos[2*(c + d*x)] + (10*I)*Sin
[2*(c + d*x)])))/(64*a*d*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 538 vs. 2(126) = 252.

Time = 9.95 (sec) , antiderivative size = 539, normalized size of antiderivative = 3.43

method	result
default	$-\frac{24i(\cos^2(dx+c))\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-30i\cos(dx+c)\arctan\left(\frac{i\sin(dx+c)-\cos(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+24i\cos(dx+c)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-40\sqrt{}}{}$

```
[In] int(cos(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/64/d/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)+1)/(a*(1+I*tan(d*x+c
)))^(1/2)/(1+I*tan(d*x+c))/a*(24*I*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)-30*I*cos(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+
1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+24*I*cos(d*x+c)*(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)-40*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)-
15*I*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(co
s(d*x+c)+1))^(1/2))-30*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+30*arctan(1/2*(
I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2
))*sin(d*x+c)-40*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+15*I*sec(d*x
+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos
(d*x+c)+1))^(1/2))-30*I*sec(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+15*ta
n(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)
/(cos(d*x+c)+1))^(1/2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 270 vs. 2(118) = 236.

Time = 0.24 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.72

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\left(-15i\,\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{1}{a^3d^2}}e^{(4i\,dx+4i\,c)}\log\left(-\frac{15\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,ade^{(2i\,dx+2i\,c)}+i\,ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\cdots\right.}{16\,ad}\right.\right.}{\cdots}$$

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/64*(-15*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(4*I*d*x + 4*I*c)*log(-15/1
6*(sqrt(2)*sqrt(1/2)*(I*a*d*e^(2*I*d*x + 2*I*c) + I*a*d)*sqrt(a/(e^(2*I*d*x
 + 2*I*c) + 1))*sqrt(1/(a^3*d^2)) - I)*e^(-I*d*x - I*c)/(a*d)) + 15*I*sqrt(
1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(4*I*d*x + 4*I*c)*log(-15/16*(sqrt(2)*sqrt(1
/2)*(-I*a*d*e^(2*I*d*x + 2*I*c) - I*a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*
sqrt(1/(a^3*d^2)) - I)*e^(-I*d*x - I*c)/(a*d)) + sqrt(2)*sqrt(a/(e^(2*I*d*x
 + 2*I*c) + 1))*(-8*I*e^(6*I*d*x + 6*I*c) + I*e^(4*I*d*x + 4*I*c) + 11*I*e^
(2*I*d*x + 2*I*c) + 2*I))*e^(-4*I*d*x - 4*I*c)/(a^2*d)
```

Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+dx\right)}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1821 vs. 2(118) = 236.

Time = 0.55 (sec) , antiderivative size = 1821, normalized size of antiderivative = 11.60

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/256*(36*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((-I*sqrt(2)*cos(4*d*x + 4*c) - s
qrt(2)*sin(4*d*x + 4*c))*cos(3/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
1)) + (sqrt(2)*cos(4*d*x + 4*c) - I*sqrt(2)*sin(4*d*x + 4*c))*sin(3/2*arcta
n2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) + 4*(cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
 1)^(1/4)*((7*I*sqrt(2)*cos(4*d*x + 4*c) + 7*sqrt(2)*sin(4*d*x + 4*c) + 8*I
*sqrt(2))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - (7*sqrt(2
)*cos(4*d*x + 4*c) - 7*I*sqrt(2)*sin(4*d*x + 4*c) + 8*sqrt(2))*sin(1/2*arct
an2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) + 15*(2*sqrt(2)*arctan2(
(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
))) + 1)), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))) + 1)) + 1) - 2*sqrt(2)*arctan2((cos(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4
)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos
(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)), (cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - 1) - I
*sqrt(2)*log(sqrt(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 +
```

```
sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)*cos(1/2*arctan2(sin(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))) + 1))^2 + sqrt(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)*sin(1/2*arctan2(sin(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c))) + 1))^2 + 2*(cos(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^
2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1
/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) + 1) + I*sqrt(2)*log(sqrt(
cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))) + 1)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1
))^2 + sqrt(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c))) + 1)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))) + 1))^2 - 2*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^
2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c))) + 1)) + 1))*sqrt(a))/(a^2*d)
```

Giac [F]

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos(dx+c)}{(i\,a\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(cos(d*x + c)/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{3/2}}\,dx$$

```
[In] int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.361 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 233

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{105i\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{256\sqrt{2}a^{3/2}d} + \frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}} + \frac{35i\cos(c+dx)}{128ad\sqrt{a+ia\tan(c+dx)}} + \frac{3i\cos^3(c+dx)}{16ad\sqrt{a+ia\tan(c+dx)}} - \frac{105i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{256a^2d} - \frac{7i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{32a^2d}$$

```
[Out] 105/512*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/
a^(3/2)/d*2^(1/2)+35/128*I*cos(d*x+c)/a/d/(a+I*a*tan(d*x+c))^(1/2)+3/16*I*c
os(d*x+c)^3/a/d/(a+I*a*tan(d*x+c))^(1/2)-105/256*I*cos(d*x+c)*(a+I*a*tan(d*
x+c))^(1/2)/a^2/d-7/32*I*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2)/a^2/d+1/6*I*
cos(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3583, 3578, 3571, 3570, 212}

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{105i\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{256\sqrt{2}a^{3/2}d} - \frac{7i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{32a^2d} - \frac{105i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{256a^2d} + \frac{3i\cos^3(c+dx)}{16ad\sqrt{a+ia\tan(c+dx)}} + \frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}} + \frac{35i\cos(c+dx)}{128ad\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((105*I)/256)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c +
 d*x]])])/(Sqrt[2]*a^(3/2)*d) + ((I/6)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[c +
d*x])^(3/2)) + (((35*I)/128)*Cos[c + d*x])/(a*d*Sqrt[a + I*a*Tan[c + d*x]])
 + (((3*I)/16)*Cos[c + d*x]^3)/(a*d*Sqrt[a + I*a*Tan[c + d*x]]) - (((105*I)
/256)*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/(a^2*d) - (((7*I)/32)*Cos[c
+ d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/(a^2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3571

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}+\frac{3\int \frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{4a} \\
&= \frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}+\frac{3i\cos^3(c+dx)}{16ad\sqrt{a+ia\tan(c+dx)}} \\
&\quad+\frac{21\int \cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx}{32a^2} \\
&= \frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}+\frac{3i\cos^3(c+dx)}{16ad\sqrt{a+ia\tan(c+dx)}} \\
&\quad-\frac{7i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{32a^2d}+\frac{35\int \frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{64a} \\
&= \frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}+\frac{35i\cos(c+dx)}{128ad\sqrt{a+ia\tan(c+dx)}}+\frac{3i\cos^3(c+dx)}{16ad\sqrt{a+ia\tan(c+dx)}} \\
&\quad-\frac{7i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{32a^2d}+\frac{105\int \cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx}{256a^2} \\
&= \frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}+\frac{35i\cos(c+dx)}{128ad\sqrt{a+ia\tan(c+dx)}} \\
&\quad+\frac{3i\cos^3(c+dx)}{16ad\sqrt{a+ia\tan(c+dx)}}-\frac{105i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{256a^2d} \\
&\quad-\frac{7i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{32a^2d}+\frac{105\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{512a} \\
&= \frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}}+\frac{35i\cos(c+dx)}{128ad\sqrt{a+ia\tan(c+dx)}}+\frac{3i\cos^3(c+dx)}{16ad\sqrt{a+ia\tan(c+dx)}} \\
&\quad-\frac{105i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{256a^2d}-\frac{7i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{32a^2d} \\
&\quad+\frac{(105i)\text{Subst}\left(\int \frac{1}{2-ax^2}\,dx,x,\frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\right)}{256ad} \\
&= \frac{105i\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{256\sqrt{2}a^{3/2}d}+\frac{i\cos^3(c+dx)}{6d(a+ia\tan(c+dx))^{3/2}} \\
&\quad+\frac{35i\cos(c+dx)}{128ad\sqrt{a+ia\tan(c+dx)}}+\frac{3i\cos^3(c+dx)}{16ad\sqrt{a+ia\tan(c+dx)}} \\
&\quad-\frac{105i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{256a^2d}-\frac{7i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{32a^2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.62

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sec(c+dx)\left(\frac{630e^{2i(c+dx)}\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)}{\sqrt{1+e^{2i(c+dx)}}} - 2(158\cos(2(c+dx)) + 8\cos(\right.}{1536ad(-i+\tan(c+dx))\sqrt{a}}$$

```
[In] Integrate[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (Sec[c + d*x]*((630*E^((2*I)*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x)
)]])/Sqrt[1 + E^((2*I)*(c + d*x))] - 2*(158*Cos[2*(c + d*x)] + 8*Cos[4*(c +
 d*x)] + (3*I)*(55*I + 86*Sin[2*(c + d*x)] + 8*Sin[4*(c + d*x)]))))/(1536*a
*d*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 672 vs. 2(190) = 380.

Time = 10.02 (sec) , antiderivative size = 673, normalized size of antiderivative = 2.89

method	result
default	$-\frac{128i(\cos^4(dx+c))\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+128i(\cos^3(dx+c))\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-384\sin(dx+c)(\cos^3(dx+c))\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+504i(\cos^2(}{}$

```
[In] int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/1536/d/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)+1)/(1+I*tan(d*x+c)
)/(a*(1+I*tan(d*x+c)))^(1/2)/a*(128*I*cos(d*x+c)^4*(-cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)+128*I*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-384*sin(d*
x+c)*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+504*I*cos(d*x+c)^2*(-c
os(d*x+c)/(cos(d*x+c)+1))^(1/2)-384*sin(d*x+c)*cos(d*x+c)^2*(-cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)+504*I*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-630
*I*cos(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d
*x+c)/(cos(d*x+c)+1))^(1/2))-840*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x
+c)*sin(d*x+c)-630*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-315*I*arctan(1/2*(I
*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
)-840*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+630*arctan(1/2*(I*sin(d
*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(
d*x+c)-630*I*sec(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+315*I*sec(d*x+c)
*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2))+315*tan(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(
d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.25

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\left(-315i\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{1}{a^3d^2}}e^{(6i\,dx+6i\,c)}\log\left(-\frac{105\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,ade^{(2i\,dx+2i\,c)}+i\,ad\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+\cdots}}\right.}{128\,ad}\right.\right.}{\cdots}$$

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/1536*(-315*I*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(6*I*d*x + 6*I*c)*log(-1
05/128*(sqrt(2)*sqrt(1/2)*(I*a*d*e^(2*I*d*x + 2*I*c) + I*a*d)*sqrt(a/(e^(2*
I*d*x + 2*I*c) + 1))*sqrt(1/(a^3*d^2)) - I)*e^(-I*d*x - I*c)/(a*d)) + 315*I
*sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2))*e^(6*I*d*x + 6*I*c)*log(-105/128*(sqrt(2
)*sqrt(1/2)*(-I*a*d*e^(2*I*d*x + 2*I*c) - I*a*d)*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))*sqrt(1/(a^3*d^2)) - I)*e^(-I*d*x - I*c)/(a*d)) + sqrt(2)*sqrt(a/(e^
(2*I*d*x + 2*I*c) + 1))*(-16*I*e^(10*I*d*x + 10*I*c) - 224*I*e^(8*I*d*x + 8
*I*c) - 43*I*e^(6*I*d*x + 6*I*c) + 215*I*e^(4*I*d*x + 4*I*c) + 58*I*e^(2*I*
d*x + 2*I*c) + 8*I))*e^(-6*I*d*x - 6*I*c)/(a^2*d)
```

Sympy [F]

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos^3\left(c+dx\right)}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**3/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2632 vs. 2(178) = 356.

Time = 0.53 (sec) , antiderivative size = 2632, normalized size of antiderivative = 11.30

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/6144*(8*(cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(1/
3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin(6*
```

```
d*x + 6*c), cos(6*d*x + 6*c))) + 1)^(3/4)*((-4*I*sqrt(2)*cos(6*d*x + 6*c) -
 45*I*sqrt(2)*cos(2/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) - 4*sqrt
(2)*sin(6*d*x + 6*c) - 45*sqrt(2)*sin(2/3*arctan2(sin(6*d*x + 6*c), cos(6*d
*x + 6*c))) + 8*I*sqrt(2))*cos(3/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c)
, cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))
+ 1)) + (4*sqrt(2)*cos(6*d*x + 6*c) + 45*sqrt(2)*cos(2/3*arctan2(sin(6*d*x
+ 6*c), cos(6*d*x + 6*c))) - 4*I*sqrt(2)*sin(6*d*x + 6*c) - 45*I*sqrt(2)*si
n(2/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) - 8*sqrt(2))*sin(3/2*arc
tan2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(
sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)))*sqrt(a) + 12*(cos(1/3*arctan2(s
in(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(1/3*arctan2(sin(6*d*x + 6*c), c
os(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))
) + 1)^(1/4)*(((-I*sqrt(2)*cos(6*d*x + 6*c) - sqrt(2)*sin(6*d*x + 6*c))*cos
(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + (-I*sqrt(2)*cos(6*d*x
 + 6*c) - sqrt(2)*sin(6*d*x + 6*c))*sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6
*d*x + 6*c)))^2 + 2*(-I*sqrt(2)*cos(6*d*x + 6*c) - sqrt(2)*sin(6*d*x + 6*c)
)*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) - I*sqrt(2)*cos(6*d*
x + 6*c) - sqrt(2)*sin(6*d*x + 6*c))*cos(5/2*arctan2(sin(1/3*arctan2(sin(6*
d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x
 + 6*c))) + 1)) + (I*sqrt(2)*cos(6*d*x + 6*c) + 18*I*sqrt(2)*cos(2/3*arctan
2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) - 24*I*sqrt(2)*cos(1/3*arctan2(sin(6
*d*x + 6*c), cos(6*d*x + 6*c))) + sqrt(2)*sin(6*d*x + 6*c) + 18*sqrt(2)*sin
(2/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) - 24*sqrt(2)*sin(1/3*arct
an2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 64*I*sqrt(2))*cos(1/2*arctan2(si
n(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d
*x + 6*c), cos(6*d*x + 6*c))) + 1)) + ((sqrt(2)*cos(6*d*x + 6*c) - I*sqrt(2
)*sin(6*d*x + 6*c))*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2
+ (sqrt(2)*cos(6*d*x + 6*c) - I*sqrt(2)*sin(6*d*x + 6*c))*sin(1/3*arctan2(s
in(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*(sqrt(2)*cos(6*d*x + 6*c) - I*sqr
t(2)*sin(6*d*x + 6*c))*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))
 + sqrt(2)*cos(6*d*x + 6*c) - I*sqrt(2)*sin(6*d*x + 6*c))*sin(5/2*arctan2(s
in(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*
d*x + 6*c), cos(6*d*x + 6*c))) + 1)) - (sqrt(2)*cos(6*d*x + 6*c) + 18*sqrt(
2)*cos(2/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) - 24*sqrt(2)*cos(1/
3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) - I*sqrt(2)*sin(6*d*x + 6*c)
 - 18*I*sqrt(2)*sin(2/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 24*I
*sqrt(2)*sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 64*sqrt(2))
*sin(1/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(
1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)))*sqrt(a) + 315*(2*sq
rt(2)*arctan2((cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin
(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin
(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/3*arctan
2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), c
os(6*d*x + 6*c))) + 1)), (cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c
)))^2 + sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*
```

```
arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)^(1/4)*cos(1/2*arctan2(sin
(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*
x + 6*c), cos(6*d*x + 6*c))) + 1)) + 1) - 2*sqrt(2)*arctan2((cos(1/3*arctan
2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(1/3*arctan2(sin(6*d*x + 6*c)
, cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*
c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x
 + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)), (cos
(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(1/3*arctan2(sin(6
*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin(6*d*x + 6*c), cos
(6*d*x + 6*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c)
, cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))
+ 1)) - 1) - I*sqrt(2)*log(sqrt(cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x
 + 6*c)))^2 + sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*co
s(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)*cos(1/2*arctan2(sin
(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*
x + 6*c), cos(6*d*x + 6*c))) + 1))^2 + sqrt(cos(1/3*arctan2(sin(6*d*x + 6*c
), cos(6*d*x + 6*c)))^2 + sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c
)))^2 + 2*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)*sin(1/2
*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arct
an2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1))^2 + 2*(cos(1/3*arctan2(sin(6
*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6
*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) +
1)^(1/4)*cos(1/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)
)), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)) + 1) + I*sqr
t(2)*log(sqrt(cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(
1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin(
6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)*cos(1/2*arctan2(sin(1/3*arctan2(sin(6
*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*
x + 6*c))) + 1))^2 + sqrt(cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c
)))^2 + sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*
arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)*sin(1/2*arctan2(sin(1/3*a
rctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*
c), cos(6*d*x + 6*c))) + 1))^2 - 2*(cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6
*d*x + 6*c)))^2 + sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 +
2*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)^(1/4)*cos(1/2*a
rctan2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan
2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)) + 1))*sqrt(a))/(a^2*d)
```

Giac [F]

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(dx+c\right)^3}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(cos(d*x + c)^3/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^3}{\left(a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)^{3/2}}\,dx$$

```
[In] int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.362 $\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 146

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{32i(a+ia\tan(c+dx))^{5/2}}{5a^5d}+\frac{64i(a+ia\tan(c+dx))^{7/2}}{7a^6d} -\frac{16i(a+ia\tan(c+dx))^{9/2}}{3a^7d}+\frac{16i(a+ia\tan(c+dx))^{11/2}}{11a^8d}-\frac{2i(a+ia\tan(c+dx))^{13/2}}{13a^9d}$$

```
[Out] -32/5*I*(a+I*a*tan(d*x+c))^(5/2)/a^5/d+64/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^6/
d-16/3*I*(a+I*a*tan(d*x+c))^(9/2)/a^7/d+16/11*I*(a+I*a*tan(d*x+c))^(11/2)/a
^8/d-2/13*I*(a+I*a*tan(d*x+c))^(13/2)/a^9/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3568, 45}

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{2i(a+ia\tan(c+dx))^{13/2}}{13a^9d} +\frac{16i(a+ia\tan(c+dx))^{11/2}}{11a^8d}-\frac{16i(a+ia\tan(c+dx))^{9/2}}{3a^7d} +\frac{64i(a+ia\tan(c+dx))^{7/2}}{7a^6d}-\frac{32i(a+ia\tan(c+dx))^{5/2}}{5a^5d}$$

```
[In] Int[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((-32*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(a^5*d) + (((64*I)/7)*(a + I*a*T
an[c + d*x])^(7/2))/(a^6*d) - (((16*I)/3)*(a + I*a*Tan[c + d*x])^(9/2))/(a^
```

```
7*d) + (((16*I)/11)*(a + I*a*Tan[c + d*x])^(11/2))/(a^8*d) - (((2*I)/13)*(a
 + I*a*Tan[c + d*x])^(13/2))/(a^9*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{i\text{Subst}\left(\int (a-x)^4(a+x)^{3/2}\,dx, x, ia\tan(c+dx)\right)}{a^9d}$$

$$= -\frac{i\text{Subst}\left(\int \left(16a^4(a+x)^{3/2} - 32a^3(a+x)^{5/2} + 24a^2(a+x)^{7/2} - 8a(a+x)^{9/2} + (a+x)^{11/2}\right)\,dx,\right.}{a^9d}$$

$$= -\frac{32i(a+ia\tan(c+dx))^{5/2}}{5a^5d} + \frac{64i(a+ia\tan(c+dx))^{7/2}}{7a^6d} - \frac{16i(a+ia\tan(c+dx))^{9/2}}{3a^7d} + \frac{16i(a+ia\tan(c+dx))^{11/2}}{11a^8d} - \frac{2i(a+ia\tan(c+dx))^{13/2}}{13a^9d}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.58

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}(9683i+16700\tan(c+dx)-142}{15015a^3d}$$

```
[In] Integrate[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (2*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]]*(9683*I + 16700*Tan[c +
 d*x] - (14210*I)*Tan[c + d*x]^2 - 6300*Tan[c + d*x]^3 + (1155*I)*Tan[c + d
*x]^4))/(15015*a^3*d)
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.69

method	result
derivativedivides	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}+\frac{8a(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}-\frac{8a^2(a+ia\tan(dx+c))^{\frac{9}{2}}}{3}+\frac{32a^3(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{16a^4(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}\right)}{d\,a^9}$
default	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{13}{2}}}{13}+\frac{8a(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}-\frac{8a^2(a+ia\tan(dx+c))^{\frac{9}{2}}}{3}+\frac{32a^3(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{16a^4(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}\right)}{d\,a^9}$

```
[In] int(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/d/a^9*(-1/13*(a+I*a*tan(d*x+c))^(13/2)+8/11*a*(a+I*a*tan(d*x+c))^(11/2)
-8/3*a^2*(a+I*a*tan(d*x+c))^(9/2)+32/7*a^3*(a+I*a*tan(d*x+c))^(7/2)-16/5*a^
4*(a+I*a*tan(d*x+c))^(5/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.20

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{128\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(128i\,e^{(13i\,dx+13i\,c)}+832i\,e^{(11i\,dx+11i\,c)}+2288i\,e^{(9i\,dx+9i\,c)}+3432i\,e^{(7i\,dx+7i\,c)}+3003\right.}{15015\left(a^3de^{(12i\,dx+12i\,c)}+6\,a^3de^{(10i\,dx+10i\,c)}+15\,a^3de^{(8i\,dx+8i\,c)}+20\,a^3de^{(6i\,dx+6i\,c)}+15\,a^3de^{(4i\,dx+4i\,c)}+6\,a^3d\right.}$$

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -128/15015*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(128*I*e^(13*I*d*x + 1
3*I*c) + 832*I*e^(11*I*d*x + 11*I*c) + 2288*I*e^(9*I*d*x + 9*I*c) + 3432*I*
e^(7*I*d*x + 7*I*c) + 3003*I*e^(5*I*d*x + 5*I*c))/(a^3*d*e^(12*I*d*x + 12*I
*c) + 6*a^3*d*e^(10*I*d*x + 10*I*c) + 15*a^3*d*e^(8*I*d*x + 8*I*c) + 20*a^3
*d*e^(6*I*d*x + 6*I*c) + 15*a^3*d*e^(4*I*d*x + 4*I*c) + 6*a^3*d*e^(2*I*d*x
+ 2*I*c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate(sec(d*x+c)**10/(a+I*a*tan(d*x+c))**(5/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.64

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx=$$
$$-\frac{2i\left(1155\,(i\,a\tan\left(dx+c\right)+a)^{\frac{13}{2}}-10920\,(i\,a\tan\left(dx+c\right)+a)^{\frac{11}{2}}a+40040\,(i\,a\tan\left(dx+c\right)+a)^{\frac{9}{2}}a^2-686}{15015\,a^9d}$$

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
[Out] -2/15015*I*(1155*(I*a*tan(d*x + c) + a)^(13/2) - 10920*(I*a*tan(d*x + c) +
a)^(11/2)*a + 40040*(I*a*tan(d*x + c) + a)^(9/2)*a^2 - 68640*(I*a*tan(d*x +
 c) + a)^(7/2)*a^3 + 48048*(I*a*tan(d*x + c) + a)^(5/2)*a^4)/(a^9*d)
```

Giac [F]

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx=\int \frac{\sec\left(dx+c\right)^{10}}{(i\,a\tan\left(dx+c\right)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^10/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 10.15 (sec) , antiderivative size = 434, normalized size of antiderivative = 2.97

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,16384i}{15015\,a^3\,d} - \frac{\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,8192i}{15015\,a^3\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)} - \frac{\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,2048i}{5005\,a^3\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^2} - \frac{\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,1024i}{3003\,a^3\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^3} - \frac{\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,128i}{429\,a^3\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^4} + \frac{\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,1792i}{143\,a^3\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^5} - \frac{\sqrt{a-\frac{a\left(e^{c\,2i+d\,x\,2i}\,1i-i\right)1i}{e^{c\,2i+d\,x\,2i}+1}}\,128i}{13\,a^3\,d\,\left(e^{c\,2i+d\,x\,2i}+1\right)^6}$$

```
[In] int(1/(cos(c + d*x)^10*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

```
[Out] ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1
792i)/(143*a^3*d*(exp(c*2i + d*x*2i) + 1)^5) - ((a - (a*(exp(c*2i + d*x*2i)
*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*8192i)/(15015*a^3*d*(exp(c*2i
 + d*x*2i) + 1)) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*
x*2i) + 1))^(1/2)*2048i)/(5005*a^3*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (a
*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1024i)/(3
003*a^3*d*(exp(c*2i + d*x*2i) + 1)^3) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1
i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*128i)/(429*a^3*d*(exp(c*2i + d*x*2i)
 + 1)^4) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) +
1))^(1/2)*16384i)/(15015*a^3*d) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)
/(exp(c*2i + d*x*2i) + 1))^(1/2)*128i)/(13*a^3*d*(exp(c*2i + d*x*2i) + 1)^6
)
```

3.363 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 117

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{16i(a+ia\tan(c+dx))^{3/2}}{3a^4d} + \frac{24i(a+ia\tan(c+dx))^{5/2}}{5a^5d} - \frac{12i(a+ia\tan(c+dx))^{7/2}}{7a^6d} + \frac{2i(a+ia\tan(c+dx))^{9/2}}{9a^7d}$$

```
[Out] -16/3*I*(a+I*a*tan(d*x+c))^(3/2)/a^4/d+24/5*I*(a+I*a*tan(d*x+c))^(5/2)/a^5/
d-12/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^6/d+2/9*I*(a+I*a*tan(d*x+c))^(9/2)/a^7/
d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3568, 45}

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i(a+ia\tan(c+dx))^{9/2}}{9a^7d} - \frac{12i(a+ia\tan(c+dx))^{7/2}}{7a^6d} + \frac{24i(a+ia\tan(c+dx))^{5/2}}{5a^5d} - \frac{16i(a+ia\tan(c+dx))^{3/2}}{3a^4d}$$

```
[In] Int[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((-16*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(a^4*d) + (((24*I)/5)*(a + I*a*T
an[c + d*x])^(5/2))/(a^5*d) - (((12*I)/7)*(a + I*a*Tan[c + d*x])^(7/2))/(a^
6*d) + (((2*I)/9)*(a + I*a*Tan[c + d*x])^(9/2))/(a^7*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)^3\sqrt{a+x}\,dx, x, ia\tan(c+dx)\right)}{a^7d}\\ &= -\frac{i\text{Subst}\left(\int \left(8a^3\sqrt{a+x}-12a^2(a+x)^{3/2}+6a(a+x)^{5/2}-(a+x)^{7/2}\right)\,dx, x, ia\tan(c+dx)\right)}{a^7d}\\ &= -\frac{16i(a+ia\tan(c+dx))^{3/2}}{3a^4d}+\frac{24i(a+ia\tan(c+dx))^{5/2}}{5a^5d}\\ &\quad -\frac{12i(a+ia\tan(c+dx))^{7/2}}{7a^6d}+\frac{2i(a+ia\tan(c+dx))^{9/2}}{9a^7d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.62

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2(1+i\tan(c+dx))\sqrt{a+ia\tan(c+dx)}(-319i-321\tan(c+dx)+165i\tan$$

```
[In] Integrate[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^(5/2),x]
[Out] (2*(1 + I*Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]]*(-319*I - 321*Tan[c + d*
x] + (165*I)*Tan[c + d*x]^2 + 35*Tan[c + d*x]^3))/(315*a^3*d)
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}-\frac{6a(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}+\frac{12a^2(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{8a^3(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}\right)}{d\,a^7}$	82
default	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}-\frac{6a(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}+\frac{12a^2(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{8a^3(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}\right)}{d\,a^7}$	82

```
[In] int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] 2*I/d/a^7*(1/9*(a+I*a*tan(d*x+c))^(9/2)-6/7*a*(a+I*a*tan(d*x+c))^(7/2)+12/5
*a^2*(a+I*a*tan(d*x+c))^(5/2)-8/3*a^3*(a+I*a*tan(d*x+c))^(3/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.15

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{32\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(16i\,e^{(9i\,dx+9i\,c)}+72i\,e^{(7i\,dx+7i\,c)}+126i\,e^{(5i\,dx+5i\,c)}+105i\,e^{(3i\,dx+3i\,c)}\right)}{315\,(a^3de^{(8i\,dx+8i\,c)}+4\,a^3de^{(6i\,dx+6i\,c)}+6\,a^3de^{(4i\,dx+4i\,c)}+4\,a^3de^{(2i\,dx+2i\,c)}+a^3d)}$$

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
[Out] -32/315*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(16*I*e^(9*I*d*x + 9*I*c)
 + 72*I*e^(7*I*d*x + 7*I*c) + 126*I*e^(5*I*d*x + 5*I*c) + 105*I*e^(3*I*d*x
+ 3*I*c))/(a^3*d*e^(8*I*d*x + 8*I*c) + 4*a^3*d*e^(6*I*d*x + 6*I*c) + 6*a^3*
d*e^(4*I*d*x + 4*I*c) + 4*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

Sympy [F]

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec^8(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c))**(5/2),x)
[Out] Integral(sec(c + d*x)**8/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.65

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i\left(35\,(i\,a\tan\left(dx+c\right)+a)^{\frac{9}{2}} - 270\,(i\,a\tan\left(dx+c\right)+a)^{\frac{7}{2}}a + 756\,(i\,a\tan\left(dx\right.\right.}{315\,a^7d}$$

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
[Out] 2/315*I*(35*(I*a*tan(d*x + c) + a)^(9/2) - 270*(I*a*tan(d*x + c) + a)^(7/2)
*a + 756*(I*a*tan(d*x + c) + a)^(5/2)*a^2 - 840*(I*a*tan(d*x + c) + a)^(3/2
)*a^3)/(a^7*d)
```

Giac [F]

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)^8}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^8/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 7.33 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.62

$$\begin{aligned}\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx &= -\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,512\mathrm{i}}{315\,a^3\,d} \\ &\quad -\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,256\mathrm{i}}{315\,a^3\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)} - \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{105\,a^3\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2} \\ &\quad -\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{63\,a^3\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3} + \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,32\mathrm{i}}{9\,a^3\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4}\end{aligned}$$

```
[In] int(1/(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^(5/2)),x)
[Out] ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*3
2i)/(9*a^3*d*(exp(c*2i + d*x*2i) + 1)^4) - ((a - (a*(exp(c*2i + d*x*2i)*1i
- 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*256i)/(315*a^3*d*(exp(c*2i + d*x*
2i) + 1)) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) +
 1))^(1/2)*64i)/(105*a^3*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (a*(exp(c*2i
 + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*32i)/(63*a^3*d*(exp
(c*2i + d*x*2i) + 1)^3) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*
2i + d*x*2i) + 1))^(1/2)*512i)/(315*a^3*d)
```

3.364 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{5/2}} dx$

Optimal result

Integrand size = 26, antiderivative size = 86

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{5/2}} dx = -\frac{8i\sqrt{a+ia\tan(c+dx)}}{a^3d} + \frac{8i(a+ia\tan(c+dx))^{3/2}}{3a^4d} - \frac{2i(a+ia\tan(c+dx))^{5/2}}{5a^5d}$$

```
[Out] -8*I*(a+I*a*tan(d*x+c))^(1/2)/a^3/d+8/3*I*(a+I*a*tan(d*x+c))^(3/2)/a^4/d-2/
5*I*(a+I*a*tan(d*x+c))^(5/2)/a^5/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3568, 45}

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{5/2}} dx = -\frac{2i(a+ia\tan(c+dx))^{5/2}}{5a^5d} + \frac{8i(a+ia\tan(c+dx))^{3/2}}{3a^4d} - \frac{8i\sqrt{a+ia\tan(c+dx)}}{a^3d}$$

```
[In] Int[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-8*I)*Sqrt[a + I*a*Tan[c + d*x]])/(a^3*d) + (((8*I)/3)*(a + I*a*Tan[c + d
*x])^(3/2))/(a^4*d) - (((2*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(a^5*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int \frac{(a-x)^2}{\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{a^5 d}\\ &= -\frac{i\text{Subst}\left(\int \left(\frac{4a^2}{\sqrt{a+x}} - 4a\sqrt{a+x} + (a+x)^{3/2}\right)\,dx, x, ia\tan(c+dx)\right)}{a^5 d}\\ &= -\frac{8i\sqrt{a+ia\tan(c+dx)}}{a^3 d} + \frac{8i(a+ia\tan(c+dx))^{3/2}}{3a^4 d} - \frac{2i(a+ia\tan(c+dx))^{5/2}}{5a^5 d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.59

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i\sqrt{a+ia\tan(c+dx)}(-43+14i\tan(c+dx)+3\tan^2(c+dx))}{15a^3 d}$$

```
[In] Integrate[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^(5/2),x]
[Out] (((2*I)/15)*Sqrt[a + I*a*Tan[c + d*x]]*(-43 + (14*I)*Tan[c + d*x] + 3*Tan[c
 + d*x]^2))/(a^3*d)
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}+\frac{4a(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-4a^2\sqrt{a+ia\tan(dx+c)}\right)}{d\,a^5}$	63
default	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}+\frac{4a(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}-4a^2\sqrt{a+ia\tan(dx+c)}\right)}{d\,a^5}$	63

```
[In] int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/d/a^5*(-1/5*(a+I*a*tan(d*x+c))^(5/2)+4/3*a*(a+I*a*tan(d*x+c))^(3/2)-4*a
^2*(a+I*a*tan(d*x+c))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.08

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{8\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(8i\,e^{(5i\,dx+5i\,c)}+20i\,e^{(3i\,dx+3i\,c)}+15i\,e^{(i\,dx+i\,c)}\right)}{15\left(a^3de^{(4i\,dx+4i\,c)}+2\,a^3de^{(2i\,dx+2i\,c)}+a^3d\right)}$$

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -8/15*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(8*I*e^(5*I*d*x + 5*I*c) +
20*I*e^(3*I*d*x + 3*I*c) + 15*I*e^(I*d*x + I*c))/(a^3*d*e^(4*I*d*x + 4*I*c)
 + 2*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

Sympy [F]

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec^6\left(c+dx\right)}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**6/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.67

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{2i\left(3\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}-20\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}a+60\sqrt{i\,a\tan\left(dx+c\right)+a}a^2\right)}{15\,a^5d}$$

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/15*I*(3*(I*a*tan(d*x + c) + a)^(5/2) - 20*(I*a*tan(d*x + c) + a)^(3/2)*a
 + 60*sqrt(I*a*tan(d*x + c) + a)*a^2)/(a^5*d)
```

Giac [F]

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^6}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate(sec(d*x + c)^6/(I*a*tan(d*x + c) + a)^(5/2), x)

Mupad [B] (verification not implemented)

Time = 1.37 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.80

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{4\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,(\cos(2\,c+2\,d\,x)\,321\mathrm{i}+\cos(4\,c+4\,d\,x)\,132\mathrm{i}+\cos(6\,c+6\,d\,x)\,23\mathrm{i}+35\,\mathrm{si}}{15\,a^3\,d\,(15\,\cos(2\,c+2\,d\,x)+6\,\cos(4\,c+4\,d\,x)+\cos(6\,c}$$

[In] int(1/(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(5/2)),x)

[Out] -(4*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1
))^(1/2)*(cos(2*c + 2*d*x)*321i + cos(4*c + 4*d*x)*132i + cos(6*c + 6*d*x)*
23i + 35*sin(2*c + 2*d*x) + 28*sin(4*c + 4*d*x) + 7*sin(6*c + 6*d*x) + 212i
))/(15*a^3*d*(15*cos(2*c + 2*d*x) + 6*cos(4*c + 4*d*x) + cos(6*c + 6*d*x) +
 10))

3.365 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 55

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{4i}{a^2d\sqrt{a+ia\tan(c+dx)}} + \frac{2i\sqrt{a+ia\tan(c+dx)}}{a^3d}$$

```
[Out] 4*I/a^2/d/(a+I*a*tan(d*x+c))^(1/2)+2*I*(a+I*a*tan(d*x+c))^(1/2)/a^3/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3568, 45}

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i\sqrt{a+ia\tan(c+dx)}}{a^3d} + \frac{4i}{a^2d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (4*I)/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]]) + ((2*I)*Sqrt[a + I*a*Tan[c + d*x]
])/(a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{i\text{Subst}\left(\int \frac{a-x}{(a+x)^{3/2}}\,dx, x, ia\tan(c+dx)\right)}{a^3d} \\
&= -\frac{i\text{Subst}\left(\int \left(\frac{2a}{(a+x)^{3/2}} - \frac{1}{\sqrt{a+x}}\right)\,dx, x, ia\tan(c+dx)\right)}{a^3d} \\
&= \frac{4i}{a^2d\sqrt{a+ia\tan(c+dx)}} + \frac{2i\sqrt{a+ia\tan(c+dx)}}{a^3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{6i-2\tan(c+dx)}{a^2d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^(5/2),x]
[Out] (6*I - 2*Tan[c + d*x])/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{2i\left(\sqrt{a+ia\tan(dx+c)}+\frac{2a}{\sqrt{a+ia\tan(dx+c)}}\right)}{d\,a^3}$	42
default	$\frac{2i\left(\sqrt{a+ia\tan(dx+c)}+\frac{2a}{\sqrt{a+ia\tan(dx+c)}}\right)}{d\,a^3}$	42

```
[In] int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] 2*I/d/a^3*((a+I*a*tan(d*x+c))^(1/2)+2*a/(a+I*a*tan(d*x+c))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{2\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-2i\,e^{(2i\,dx+2i\,c)}-i\right)e^{(-i\,dx-i\,c)}}{a^3d}$$

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
[Out] -2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-2*I*e^(2*I*d*x + 2*I*c) - I)
*e^(-I*d*x - I*c)/(a^3*d)
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec^4(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c))**(5/2),x)
[Out] Integral(sec(c + d*x)**4/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i\left(\frac{\sqrt{i\,a\tan(dx+c)+a}}{a^2}+\frac{2}{\sqrt{i\,a\tan(dx+c)+a}\,a}\right)}{ad}$$

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
[Out] 2*I*(sqrt(I*a*tan(d*x + c) + a)/a^2 + 2/(sqrt(I*a*tan(d*x + c) + a)*a))/(a*
d)
```

Giac [F]

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^4}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate(sec(d*x + c)^4/(I*a*tan(d*x + c) + a)^(5/2), x)

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.31

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2\left(\cos(2\,c+2\,d\,x)\ 1\mathrm{i}+\sin(2\,c+2\,d\,x)+2\mathrm{i}\right)\sqrt{\frac{a\left(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\ 1\mathrm{i}\right)}{\cos(2\,c+2\,d\,x)+1}}}{a^3\,d}$$

[In] int(1/(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(5/2)),x)

[Out] (2*(cos(2*c + 2*d*x)*1i + sin(2*c + 2*d*x) + 2i)*((a*(cos(2*c + 2*d*x) + si
n(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))/(a^3*d)

3.366 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 29

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i}{3ad(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] 2/3*I/a/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3568, 32}

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i}{3ad(a+ia\tan(c+dx))^{3/2}}$$

```
[In] Int[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((2*I)/3)/(a*d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
```

```
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{i\text{Subst}\left(\int \frac{1}{(a+x)^{5/2}}\,dx, x, ia\tan(c+dx)\right)}{ad}$$
$$= \frac{2i}{3ad(a+ia\tan(c+dx))^{3/2}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i}{3ad(a+ia\tan(c+dx))^{3/2}}$$

```
[In] Integrate[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^(5/2),x]
[Out] ((2*I)/3)/(a*d*(a + I*a*Tan[c + d*x])^(3/2))
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2i}{3ad(a+ia\tan(dx+c))^{\frac{3}{2}}}$	24
default	$\frac{2i}{3ad(a+ia\tan(dx+c))^{\frac{3}{2}}}$	24

```
[In] int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/3*I/a/d/(a+I*a*tan(d*x+c))^(3/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(21) = 42.

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.10

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(i\,e^{(4i\,dx+4i\,c)}+2i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-3i\,dx-3i\,c)}}{6\,a^3d}$$

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
[Out] 1/6*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(I*e^(4*I*d*x + 4*I*c) + 2*I*
e^(2*I*d*x + 2*I*c) + I)*e^(-3*I*d*x - 3*I*c)/(a^3*d)
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec^2(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c))**(5/2),x)
[Out] Integral(sec(c + d*x)**2/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i}{3\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}ad}$$

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
[Out] 2/3*I/((I*a*tan(d*x + c) + a)^(3/2)*a*d)
```

Giac [F]

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)^2}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^2/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 4.59 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2\mathrm{i}}{3\,a\,d\,{\left(a+a\,\tan\left(c+d\,x\right)\,1\mathrm{i}\right)}^{3/2}}$$

```
[In] int(1/(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(5/2)),x)
[Out] 2i/(3*a*d*(a + a*tan(c + d*x)*1i)^(3/2))
```

3.367 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 204

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{9i\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}a^{5/2}d}+\frac{9ia}{28d(a+ia\tan(c+dx))^{7/2}} -\frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}}+\frac{9i}{40d(a+ia\tan(c+dx))^{5/2}} +\frac{3i}{16ad(a+ia\tan(c+dx))^{3/2}}+\frac{9i}{32a^2d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] -9/64*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(5/2)/d*2^(
1/2)+9/32*I/a^2/d/(a+I*a*tan(d*x+c))^(1/2)+9/28*I*a/d/(a+I*a*tan(d*x+c))^(7
/2)-1/2*I*a^2/d/(a-I*a*tan(d*x+c))/(a+I*a*tan(d*x+c))^(7/2)+9/40*I/d/(a+I*a
*tan(d*x+c))^(5/2)+3/16*I/a/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used

= {3568, 44, 53, 65, 212}

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{9i\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}a^{5/2}d} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}} + \frac{9i}{32a^2d\sqrt{a+ia\tan(c+dx)}} + \frac{9ia}{28d(a+ia\tan(c+dx))^{7/2}} + \frac{9i}{40d(a+ia\tan(c+dx))^{5/2}} + \frac{3i}{16ad(a+ia\tan(c+dx))^{3/2}}$$

```
[In] Int[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((-9*I)/32)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[2
]*a^(5/2)*d) + (((9*I)/28)*a)/(d*(a + I*a*Tan[c + d*x])^(7/2)) - ((I/2)*a^2
)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(7/2)) + ((9*I)/40)/(d*(
a + I*a*Tan[c + d*x])^(5/2)) + ((3*I)/16)/(a*d*(a + I*a*Tan[c + d*x])^(3/2)
) + ((9*I)/32)/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(ia^3)\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{9/2}}\,dx, x, ia\tan(c+dx)\right)}{d}\\
&= -\frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}}\\
&\quad -\frac{(9ia^2)\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{9/2}}\,dx, x, ia\tan(c+dx)\right)}{4d}\\
&= \frac{9ia}{28d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}}\\
&\quad -\frac{(9ia)\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{7/2}}\,dx, x, ia\tan(c+dx)\right)}{8d}\\
&= \frac{9ia}{28d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}}\\
&\quad +\frac{9i}{40d(a+ia\tan(c+dx))^{5/2}} - \frac{(9i)\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{5/2}}\,dx, x, ia\tan(c+dx)\right)}{16d}\\
&= \frac{9ia}{28d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}}\\
&\quad +\frac{9i}{40d(a+ia\tan(c+dx))^{5/2}} + \frac{3i}{16ad(a+ia\tan(c+dx))^{3/2}}\\
&\quad -\frac{(9i)\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{3/2}}\,dx, x, ia\tan(c+dx)\right)}{32ad}\\
&= \frac{9ia}{28d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}}\\
&\quad +\frac{9i}{40d(a+ia\tan(c+dx))^{5/2}} + \frac{3i}{16ad(a+ia\tan(c+dx))^{3/2}}\\
&\quad +\frac{9i}{32a^2d\sqrt{a+ia\tan(c+dx)}} - \frac{(9i)\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{64a^2d}
\end{aligned}$$

$$= \frac{9ia}{28d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}} + \frac{9i}{40d(a+ia\tan(c+dx))^{5/2}} + \frac{3i}{16ad(a+ia\tan(c+dx))^{3/2}} + \frac{9i}{32a^2d\sqrt{a+ia\tan(c+dx)}} - \frac{(9i)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+ia\tan(c+dx)}\right)}{32a^2d}$$

$$= -\frac{9i\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{32\sqrt{2}a^{5/2}d} + \frac{9ia}{28d(a+ia\tan(c+dx))^{7/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{7/2}} + \frac{9i}{40d(a+ia\tan(c+dx))^{5/2}} + \frac{3i}{16ad(a+ia\tan(c+dx))^{3/2}} + \frac{9i}{32a^2d\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.25

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{ia\,\text{Hypergeometric2F1}\left(-\frac{7}{2}, 2, -\frac{5}{2}, \frac{1}{2}(1+i\tan(c+dx))\right)}{14d(a+ia\tan(c+dx))^{7/2}}$$

```
[In] Integrate[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^(5/2),x]
[Out] ((I/14)*a*Hypergeometric2F1[-7/2, 2, -5/2, (1 + I*Tan[c + d*x])/2])/(d*(a +
 I*a*Tan[c + d*x])^(7/2))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 779 vs. 2(160) = 320.

Time = 10.47 (sec) , antiderivative size = 780, normalized size of antiderivative = 3.82

method	result
default	$\frac{2184i\cos(dx+c)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+720\sin(dx+c)\left(\cos^2(dx+c)\right)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+630i\arctan\left(\frac{\cos(dx+c)+1+i\sin(dx+c)}{2(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+720\sqrt{}}{}$

```
[In] int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/2240/d/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)+1)/(a*(1+I*tan(d*x+
c)))^(1/2)/(1+I*tan(d*x+c))^2/a^2*(2184*I*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+
```

```
c)+1))^(1/2)+720*sin(d*x+c)*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
+630*I*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(
cos(d*x+c)+1))^(1/2))+720*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin
(d*x+c)-400*I*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+2184*I*(-cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)-1260*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(c
os(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-1680*sin(d*x+c)
*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-630*I*sec(d*x+c)^2*(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2)-945*I*sec(d*x+c)*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos
(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-630*tan(d*x+c)*arctan(1/2*(c
os(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
)-1680*tan(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-630*I*sec(d*x+c)*(-cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)+1260*I*cos(d*x+c)*arctan(1/2*(cos(d*x+c)+1+I*
sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+315*tan(d*x+
c)*sec(d*x+c)*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d
*x+c)/(cos(d*x+c)+1))^(1/2))-400*I*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2)-315*I*sec(d*x+c)^2*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+
c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(147) = 294$.

Time = 0.25 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.50

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\left(-315i\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{1}{a^5d^2}}e^{(7i\,dx+7i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^3de^{(2i\,dx+2i\,c)}+a^3d\right)\sqrt{\frac{a}{e^{(2i\,dx}}}\right.\right.\right.}{}$$

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/2240*(-315*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(7*I*d*x + 7*I*c)*log(4*
(sqrt(2)*sqrt(1/2)*(a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1))*sqrt(1/(a^5*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + 31
5*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(7*I*d*x + 7*I*c)*log(-4*(sqrt(2)*s
qrt(1/2)*(a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
1))*sqrt(1/(a^5*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2)*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1))*(-35*I*e^(10*I*d*x + 10*I*c) + 353*I*e^(8*I*d
*x + 8*I*c) + 544*I*e^(6*I*d*x + 6*I*c) + 214*I*e^(4*I*d*x + 4*I*c) + 68*I*
e^(2*I*d*x + 2*I*c) + 10*I))*e^(-7*I*d*x - 7*I*c)/(a^3*d)
```

Sympy [F]

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx=\int \frac{\cos^2{(c+dx)}}{(ia(\tan{(c+dx)}-i))^{\frac{5}{2}}}\,dx$$

[In] integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c))**(5/2),x)

[Out] Integral(cos(c + d*x)**2/(I*a*(tan(c + d*x) - I))**(5/2), x)

Maxima [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.86

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx=\frac{i\left(\frac{4\left(315\,(i\,a\tan(dx+c)+a)^4-420\,(i\,a\tan(dx+c)+a)^3a-168\,(i\,a\tan(dx+c)+a)^2a^2-144\,(i\,a\tan(dx+c)-\right.}{(i\,a\tan(dx+c)+a)^{\frac{9}{2}}a-2\,(i\,a\tan(dx+c)+a)^{\frac{7}{2}}a^2}\right.}{4480\,ad}$$

[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")

[Out] 1/4480*I*(4*(315*(I*a*tan(d*x + c) + a)^4 - 420*(I*a*tan(d*x + c) + a)^3*a - 168*(I*a*tan(d*x + c) + a)^2*a^2 - 144*(I*a*tan(d*x + c) + a)*a^3 - 160*a^4)/((I*a*tan(d*x + c) + a)^(9/2)*a - 2*(I*a*tan(d*x + c) + a)^(7/2)*a^2) + 315*sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/a^(3/2))/(a*d)

Giac [F]

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx=\int \frac{\cos{(dx+c)}^2}{(i\,a\tan{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate(cos(d*x + c)^2/(I*a*tan(d*x + c) + a)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{5/2}}\,dx$$

```
[In] int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(5/2),x)
[Out] int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.368 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 277

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{143i\,\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{512\sqrt{2}a^{5/2}d} + \frac{143ia^2}{288d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}} - \frac{13ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}} + \frac{143ia}{448d(a+ia\tan(c+dx))^{7/2}} + \frac{143i}{640d(a+ia\tan(c+dx))^{5/2}} + \frac{143i}{768ad(a+ia\tan(c+dx))^{3/2}} + \frac{143i}{512a^2d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] -143/1024*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(5/2)/d
*2^(1/2)+143/512*I/a^2/d/(a+I*a*tan(d*x+c))^(1/2)+143/288*I*a^2/d/(a+I*a*ta
n(d*x+c))^(9/2)-1/4*I*a^4/d/(a-I*a*tan(d*x+c))^2/(a+I*a*tan(d*x+c))^(9/2)-1
3/16*I*a^3/d/(a-I*a*tan(d*x+c))/(a+I*a*tan(d*x+c))^(9/2)+143/448*I*a/d/(a+I
*a*tan(d*x+c))^(7/2)+143/640*I/d/(a+I*a*tan(d*x+c))^(5/2)+143/768*I/a/d/(a+
I*a*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3568, 44, 53, 65, 212}

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{143i\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{512\sqrt{2}a^{5/2}d} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}} - \frac{13ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}} + \frac{143ia^2}{288d(a+ia\tan(c+dx))^{9/2}} + \frac{143i}{512a^2d\sqrt{a+ia\tan(c+dx)}} + \frac{143ia}{448d(a+ia\tan(c+dx))^{7/2}} + \frac{143i}{640d(a+ia\tan(c+dx))^{5/2}} + \frac{143i}{768ad(a+ia\tan(c+dx))^{3/2}}$$

```
[In] Int[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((-143*I)/512)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqr
t[2]*a^(5/2)*d) + (((143*I)/288)*a^2)/(d*(a + I*a*Tan[c + d*x])^(9/2)) - ((
I/4)*a^4)/(d*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(9/2)) - (((13
*I)/16)*a^3)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(9/2)) + (((1
43*I)/448)*a)/(d*(a + I*a*Tan[c + d*x])^(7/2)) + ((143*I)/640)/(d*(a + I*a*
Tan[c + d*x])^(5/2)) + ((143*I)/768)/(a*d*(a + I*a*Tan[c + d*x])^(3/2)) + (
(143*I)/512)/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(ia^5)\,\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^{11/2}}\,dx, x, ia\tan(c+dx)\right)}{d}\\
&= -\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}}\\
&\quad -\frac{(13ia^4)\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{11/2}}\,dx, x, ia\tan(c+dx)\right)}{8d}\\
&= -\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}}\\
&\quad -\frac{13ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}}\\
&\quad -\frac{(143ia^3)\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{11/2}}\,dx, x, ia\tan(c+dx)\right)}{32d}\\
&= \frac{143ia^2}{288d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}}\\
&\quad -\frac{13ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}}\\
&\quad -\frac{(143ia^2)\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{9/2}}\,dx, x, ia\tan(c+dx)\right)}{64d}
\end{aligned}$$

$$= \frac{143ia^2}{288d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}} - \frac{13ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}} + \frac{143ia}{448d(a+ia\tan(c+dx))^{7/2}} - \frac{(143ia)\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{7/2}}\,dx, x, ia\tan(c+dx)\right)}{128d}$$

$$= \frac{143ia^2}{288d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}} - \frac{13ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}} + \frac{143ia}{448d(a+ia\tan(c+dx))^{7/2}} + \frac{143i}{640d(a+ia\tan(c+dx))^{5/2}} - \frac{(143i)\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{5/2}}\,dx, x, ia\tan(c+dx)\right)}{256d}$$

$$= \frac{143ia^2}{288d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}} - \frac{13ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}} + \frac{143ia}{448d(a+ia\tan(c+dx))^{7/2}} + \frac{143i}{640d(a+ia\tan(c+dx))^{5/2}} + \frac{143i}{768ad(a+ia\tan(c+dx))^{3/2}} - \frac{(143i)\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{3/2}}\,dx, x, ia\tan(c+dx)\right)}{512ad}$$

$$= \frac{143ia^2}{288d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}} - \frac{13ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}} + \frac{143ia}{448d(a+ia\tan(c+dx))^{7/2}} + \frac{143i}{640d(a+ia\tan(c+dx))^{5/2}} + \frac{143i}{768ad(a+ia\tan(c+dx))^{3/2}} + \frac{143i}{512a^2d\sqrt{a+ia\tan(c+dx)}} - \frac{(143i)\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{1024a^2d}$$

$$= \frac{143ia^2}{288d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}} - \frac{13ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}} + \frac{143ia}{448d(a+ia\tan(c+dx))^{7/2}} + \frac{143i}{640d(a+ia\tan(c+dx))^{5/2}} + \frac{143i}{768ad(a+ia\tan(c+dx))^{3/2}} + \frac{143i}{512a^2d\sqrt{a+ia\tan(c+dx)}} - \frac{(143i)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+ia\tan(c+dx)}\right)}{512a^2d}$$

$$= -\frac{143i\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{512\sqrt{2}a^{5/2}d} + \frac{143ia^2}{288d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{9/2}} - \frac{13ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}} + \frac{143ia}{448d(a+ia\tan(c+dx))^{7/2}} + \frac{143i}{640d(a+ia\tan(c+dx))^{5/2}} + \frac{143i}{768ad(a+ia\tan(c+dx))^{3/2}} + \frac{143i}{512a^2d\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.49 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.19

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{ia^2 \operatorname{Hypergeometric2F1}\left(-\frac{9}{2},3,-\frac{7}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{36d(a+ia\tan(c+dx))^{9/2}}$$

```
[In] Integrate[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^(5/2),x]
[Out] ((I/36)*a^2*Hypergeometric2F1[-9/2, 3, -7/2, (1 + I*Tan[c + d*x])/2])/(d*(a
 + I*a*Tan[c + d*x])^(9/2))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 913 vs. 2(220) = 440.

Time = 9.16 (sec) , antiderivative size = 914, normalized size of antiderivative = 3.30

method	result	size
default	Expression too large to display	914

```
[In] int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/322560/d/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)+1)/(1+I*tan(d*x+c
))^2/(a*(1+I*tan(d*x+c)))^(1/2)/a^2*(312312*I*cos(d*x+c)*(-cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)+90090*I*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)
+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+58240*sin(d*x+c)*cos(d*x+c)^4*(-cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)-57200*I*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)+58240*sin(d*x+c)*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
+312312*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+102960*sin(d*x+c)*cos(d*x+c)^2
*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-22400*I*cos(d*x+c)^5*(-cos(d*x+c)/(cos(
```

```
d*x+c)+1))^(1/2)-90090*I*sec(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+10
2960*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)-135135*I*sec(
d*x+c)*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(
cos(d*x+c)+1))^(1/2))-22400*I*cos(d*x+c)^4*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)-180180*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c
)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-240240*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)-90090*I*sec(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+180180*
I*cos(d*x+c)*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*
x+c)/(cos(d*x+c)+1))^(1/2))-90090*tan(d*x+c)*arctan(1/2*(cos(d*x+c)+1+I*sin
(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-240240*tan(d*x+
c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-57200*I*cos(d*x+c)^3*(-cos(d*x+c)/(co
s(d*x+c)+1))^(1/2)-45045*I*sec(d*x+c)^2*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+
c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+45045*tan(d*x+c)*sec
(d*x+c)*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/
(cos(d*x+c)+1))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.18

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\left(-45045i\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{1}{a^5d^2}}e^{(9i\,dx+9i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^3de^{(2i\,dx+2i\,c)}+a^3d\right)\sqrt{\frac{}{e^{(2}}}\right.\right.\right.}{}$$

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/322560*(-45045*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(9*I*d*x + 9*I*c)*lo
g(4*(sqrt(2)*sqrt(1/2)*(a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*sqrt(1/(a^5*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c))
+ 45045*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(9*I*d*x + 9*I*c)*log(-4*(sqr
t(2)*sqrt(1/2)*(a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*sqrt(1/(a^5*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2
)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-630*I*e^(14*I*d*x + 14*I*c) - 8505*I*
e^(12*I*d*x + 12*I*c) + 42709*I*e^(10*I*d*x + 10*I*c) + 69392*I*e^(8*I*d*x
+ 8*I*c) + 26752*I*e^(6*I*d*x + 6*I*c) + 10144*I*e^(4*I*d*x + 4*I*c) + 2480
*I*e^(2*I*d*x + 2*I*c) + 280*I))*e^(-9*I*d*x - 9*I*c)/(a^3*d)
```

Sympy [F]

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\cos^4{(c+dx)}}{(ia(\tan{(c+dx)}-i))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c))**(5/2),x)
[Out] Integral(cos(c + d*x)**4/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.83

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{i\left(\frac{4\left(45045\,(i\,a\tan(dx+c)+a)^6-150150\,(i\,a\tan(dx+c)+a)^5a+96096\,(i\,a\tan(dx+c)+a)^4a^2+27456\,(i\,a\,t\cdots\right.}{(i\,a\tan(dx+c)+a)^{\frac{13}{2}}a-4\,(i\,a\tan(dx+c)+a)\cdots}\cdots\right.}{\cdots}$$

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
[Out] 1/645120*I*(4*(45045*(I*a*tan(d*x + c) + a)^6 - 150150*(I*a*tan(d*x + c) +
a)^5*a + 96096*(I*a*tan(d*x + c) + a)^4*a^2 + 27456*(I*a*tan(d*x + c) + a)^
3*a^3 + 18304*(I*a*tan(d*x + c) + a)^2*a^4 + 16640*(I*a*tan(d*x + c) + a)*a
^5 + 17920*a^6)/((I*a*tan(d*x + c) + a)^(13/2)*a - 4*(I*a*tan(d*x + c) + a)
^(11/2)*a^2 + 4*(I*a*tan(d*x + c) + a)^(9/2)*a^3) + 45045*sqrt(2)*log(-(sqr
t(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(
d*x + c) + a)))/a^(3/2))/(a*d)
```

Giac [F]

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\cos{(dx+c)}^4}{(i\,a\tan{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(cos(d*x + c)^4/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^4}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{5/2}}\,dx$$

```
[In] int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^(5/2),x)
[Out] int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.369 $\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 147

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{256ia^4\sec^{13}(c+dx)}{20995d(a+ia\tan(c+dx))^{13/2}} + \frac{64ia^3\sec^{13}(c+dx)}{1615d(a+ia\tan(c+dx))^{11/2}} + \frac{24ia^2\sec^{13}(c+dx)}{323d(a+ia\tan(c+dx))^{9/2}} + \frac{2ia\sec^{13}(c+dx)}{19d(a+ia\tan(c+dx))^{7/2}}$$

```
[Out] 256/20995*I*a^4*sec(d*x+c)^13/d/(a+I*a*tan(d*x+c))^(13/2)+64/1615*I*a^3*sec
(d*x+c)^13/d/(a+I*a*tan(d*x+c))^(11/2)+24/323*I*a^2*sec(d*x+c)^13/d/(a+I*a*
tan(d*x+c))^(9/2)+2/19*I*a*sec(d*x+c)^13/d/(a+I*a*tan(d*x+c))^(7/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3575, 3574}

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{256ia^4\sec^{13}(c+dx)}{20995d(a+ia\tan(c+dx))^{13/2}} + \frac{64ia^3\sec^{13}(c+dx)}{1615d(a+ia\tan(c+dx))^{11/2}} + \frac{24ia^2\sec^{13}(c+dx)}{323d(a+ia\tan(c+dx))^{9/2}} + \frac{2ia\sec^{13}(c+dx)}{19d(a+ia\tan(c+dx))^{7/2}}$$

```
[In] Int[Sec[c + d*x]^13/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((256*I)/20995)*a^4*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(13/2)) + (
((64*I)/1615)*a^3*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(11/2)) + (((2
4*I)/323)*a^2*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(9/2)) + (((2*I)/1
9)*a*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(7/2))
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2ia\sec^{13}(c+dx)}{19d(a+ia\tan(c+dx))^{7/2}}+\frac{1}{19}(12a)\int\frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx \\
&= \frac{24ia^2\sec^{13}(c+dx)}{323d(a+ia\tan(c+dx))^{9/2}}+\frac{2ia\sec^{13}(c+dx)}{19d(a+ia\tan(c+dx))^{7/2}} \\
&\quad+\frac{1}{323}\left(96a^2\right)\int\frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{9/2}}\,dx \\
&= \frac{64ia^3\sec^{13}(c+dx)}{1615d(a+ia\tan(c+dx))^{11/2}}+\frac{24ia^2\sec^{13}(c+dx)}{323d(a+ia\tan(c+dx))^{9/2}} \\
&\quad+\frac{2ia\sec^{13}(c+dx)}{19d(a+ia\tan(c+dx))^{7/2}}+\frac{\left(128a^3\right)\int\frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{11/2}}\,dx}{1615} \\
&= \frac{256ia^4\sec^{13}(c+dx)}{20995d(a+ia\tan(c+dx))^{13/2}}+\frac{64ia^3\sec^{13}(c+dx)}{1615d(a+ia\tan(c+dx))^{11/2}} \\
&\quad+\frac{24ia^2\sec^{13}(c+dx)}{323d(a+ia\tan(c+dx))^{9/2}}+\frac{2ia\sec^{13}(c+dx)}{19d(a+ia\tan(c+dx))^{7/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.99 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.76

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\sec^{12}(c+dx)(798\cos(c+dx)+1631\cos(3(c+dx))+13i(38\sin(c+dx)+1}{20995a^2d(-i+\tan(c+dx))^2\sqrt{a-}}$$

```
[In] Integrate[Sec[c + d*x]^13/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (Sec[c + d*x]^12*(798*Cos[c + d*x] + 1631*Cos[3*(c + d*x)] + (13*I)*(38*Sin
[c + d*x] + 123*Sin[3*(c + d*x)]))*((-2*I)*Cos[4*(c + d*x)] - 2*Sin[4*(c +
d*x)]))/(20995*a^2*d*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [F(-1)]

Timed out.

$$\int \frac{\sec^{13}(dx+c)}{(a+ia\tan(dx+c))^{\frac{5}{2}}}dx$$

```
[In] int(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] int(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(5/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.35

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx =$$

$$-\frac{1024\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-1615i\,e^{(6i\,dx+6i\,c)}-646i\,e^{(4i\,dx-}\right.}{20995\left(a^3de^{(18i\,dx+18i\,c)}+9\,a^3de^{(16i\,dx+16i\,c)}+36\,a^3de^{(14i\,dx+14i\,c)}+84\,a^3de^{(12i\,dx+12i\,c)}+126\,a^3de^{(10i\,dx+10i\,c)}+\right.}$$

```
[In] integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1024/20995*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-1615*I*e^(6*I*d*x +
 6*I*c) - 646*I*e^(4*I*d*x + 4*I*c) - 152*I*e^(2*I*d*x + 2*I*c) - 16*I)/(a^
3*d*e^(18*I*d*x + 18*I*c) + 9*a^3*d*e^(16*I*d*x + 16*I*c) + 36*a^3*d*e^(14*
I*d*x + 14*I*c) + 84*a^3*d*e^(12*I*d*x + 12*I*c) + 126*a^3*d*e^(10*I*d*x +
10*I*c) + 126*a^3*d*e^(8*I*d*x + 8*I*c) + 84*a^3*d*e^(6*I*d*x + 6*I*c) + 36
*a^3*d*e^(4*I*d*x + 4*I*c) + 9*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sec(d*x+c)**13/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 902 vs. 2(115) = 230.

Time = 1.05 (sec) , antiderivative size = 902, normalized size of antiderivative = 6.14

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/20995*(-2429*I*sqrt(a) - 8850*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) -
5122*I*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 45190*sqrt(a)*sin(d*x
+ c)^3/(cos(d*x + c) + 1)^3 - 12924*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c)
+ 1)^4 - 152478*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 40470*I*sqrt(
a)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 397594*sqrt(a)*sin(d*x + c)^7/(cos
(d*x + c) + 1)^7 - 50065*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 72
2228*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 19380*I*sqrt(a)*sin(d*x
+ c)^10/(cos(d*x + c) + 1)^10 - 936700*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c
) + 1)^11 - 936700*sqrt(a)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 + 19380*I*
sqrt(a)*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 722228*sqrt(a)*sin(d*x + c)
^15/(cos(d*x + c) + 1)^15 + 50065*I*sqrt(a)*sin(d*x + c)^16/(cos(d*x + c) +
 1)^16 - 397594*sqrt(a)*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 + 40470*I*sqr
t(a)*sin(d*x + c)^18/(cos(d*x + c) + 1)^18 - 152478*sqrt(a)*sin(d*x + c)^19
/(cos(d*x + c) + 1)^19 + 12924*I*sqrt(a)*sin(d*x + c)^20/(cos(d*x + c) + 1)
^20 - 45190*sqrt(a)*sin(d*x + c)^21/(cos(d*x + c) + 1)^21 + 5122*I*sqrt(a)*
sin(d*x + c)^22/(cos(d*x + c) + 1)^22 - 8850*sqrt(a)*sin(d*x + c)^23/(cos(d
*x + c) + 1)^23 + 2429*I*sqrt(a)*sin(d*x + c)^24/(cos(d*x + c) + 1)^24)*(si
n(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(sin(d*x + c)/(cos(d*x + c) + 1) -
 1)^(5/2)/((a^3 - 12*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 66*a^3*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4 - 220*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6 + 495*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 792*a^3*sin(d*x + c)^10/(
cos(d*x + c) + 1)^10 + 924*a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 792*
a^3*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + 495*a^3*sin(d*x + c)^16/(cos(d*
x + c) + 1)^16 - 220*a^3*sin(d*x + c)^18/(cos(d*x + c) + 1)^18 + 66*a^3*sin
```

```
(d*x + c)^20/(cos(d*x + c) + 1)^20 - 12*a^3*sin(d*x + c)^22/(cos(d*x + c) +
 1)^22 + a^3*sin(d*x + c)^24/(cos(d*x + c) + 1)^24)*d*(-2*I*sin(d*x + c)/(c
os(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(5/2))
```

Giac [F]

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{13}}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^13/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 12.75 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.05

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,1024\mathrm{i}}{13\,a^3\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^6} - \frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,1024\mathrm{i}}{5\,a^3\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^7} + \frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,3072\mathrm{i}}{17\,a^3\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^8} - \frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,1024\mathrm{i}}{19\,a^3\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^9}$$

```
[In] int(1/(cos(c + d*x)^13*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

```
[Out] (exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i +
d*x*2i) + 1))^(1/2)*1024i)/(13*a^3*d*(exp(c*2i + d*x*2i) + 1)^6) - (exp(- c
*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i)
+ 1))^(1/2)*1024i)/(5*a^3*d*(exp(c*2i + d*x*2i) + 1)^7) + (exp(- c*1i - d*x
*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/
2)*3072i)/(17*a^3*d*(exp(c*2i + d*x*2i) + 1)^8) - (exp(- c*1i - d*x*1i)*(a
- (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*1024i
)/(19*a^3*d*(exp(c*2i + d*x*2i) + 1)^9)
```

3.370 $\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 110

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{64ia^3\sec^{11}(c+dx)}{2145d(a+ia\tan(c+dx))^{11/2}} + \frac{16ia^2\sec^{11}(c+dx)}{195d(a+ia\tan(c+dx))^{9/2}} + \frac{2ia\sec^{11}(c+dx)}{15d(a+ia\tan(c+dx))^{7/2}}$$

```
[Out] 64/2145*I*a^3*sec(d*x+c)^11/d/(a+I*a*tan(d*x+c))^(11/2)+16/195*I*a^2*sec(d*
x+c)^11/d/(a+I*a*tan(d*x+c))^(9/2)+2/15*I*a*sec(d*x+c)^11/d/(a+I*a*tan(d*x+
c))^(7/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3575, 3574}

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{64ia^3\sec^{11}(c+dx)}{2145d(a+ia\tan(c+dx))^{11/2}} + \frac{16ia^2\sec^{11}(c+dx)}{195d(a+ia\tan(c+dx))^{9/2}} + \frac{2ia\sec^{11}(c+dx)}{15d(a+ia\tan(c+dx))^{7/2}}$$

```
[In] Int[Sec[c + d*x]^11/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((64*I)/2145)*a^3*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(11/2)) + (((
16*I)/195)*a^2*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(9/2)) + (((2*I)/
15)*a*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(7/2))
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2ia\sec^{11}(c+dx)}{15d(a+ia\tan(c+dx))^{7/2}}+\frac{1}{15}(8a)\int\frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx\\
&= \frac{16ia^2\sec^{11}(c+dx)}{195d(a+ia\tan(c+dx))^{9/2}}+\frac{2ia\sec^{11}(c+dx)}{15d(a+ia\tan(c+dx))^{7/2}}\\
&\quad+\frac{1}{195}\left(32a^2\right)\int\frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{9/2}}\,dx\\
&= \frac{64ia^3\sec^{11}(c+dx)}{2145d(a+ia\tan(c+dx))^{11/2}}+\frac{16ia^2\sec^{11}(c+dx)}{195d(a+ia\tan(c+dx))^{9/2}}+\frac{2ia\sec^{11}(c+dx)}{15d(a+ia\tan(c+dx))^{7/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.74 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.85

$$\int\frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx=\frac{\sec^{10}(c+dx)(60+203\cos(2(c+dx))+187i\sin(2(c+dx)))(-2i\cos(3(c+d}{2145a^2d(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[Sec[c + d*x]^11/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (Sec[c + d*x]^10*(60 + 203*Cos[2*(c + d*x)] + (187*I)*Sin[2*(c + d*x)])*((-
2*I)*Cos[3*(c + d*x)] - 2*Sin[3*(c + d*x)]))/(2145*a^2*d*(-I + Tan[c + d*x]
)^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [F(-1)]

Timed out.

$$\int \frac{\sec^{11}(dx+c)}{(a+ia\tan(dx+c))^{\frac{5}{2}}}dx$$

```
[In] int(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(5/2),x)
[Out] int(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(5/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.44

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{256\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-195i\,e^{(4i\,dx+4i\,c)}-60i\,e^{(2i\,dx+2i\,c)}-8i\right)}{2145\,(a^3de^{(14i\,dx+14i\,c)}+7\,a^3de^{(12i\,dx+12i\,c)}+21\,a^3de^{(10i\,dx+10i\,c)}+35\,a^3de^{(8i\,dx+8i\,c)}+35\,a^3de^{(6i\,dx+6i\,c)}+21\,a}$$

```
[In] integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
[Out] -256/2145*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-195*I*e^(4*I*d*x + 4*
I*c) - 60*I*e^(2*I*d*x + 2*I*c) - 8*I)/(a^3*d*e^(14*I*d*x + 14*I*c) + 7*a^3
*d*e^(12*I*d*x + 12*I*c) + 21*a^3*d*e^(10*I*d*x + 10*I*c) + 35*a^3*d*e^(8*I
*d*x + 8*I*c) + 35*a^3*d*e^(6*I*d*x + 6*I*c) + 21*a^3*d*e^(4*I*d*x + 4*I*c)
 + 7*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sec(d*x+c)**11/(a+I*a*tan(d*x+c))**(5/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 764 vs. $2(86) = 172$.

Time = 0.48 (sec) , antiderivative size = 764, normalized size of antiderivative = 6.95

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/2145*(-263*I*sqrt(a) - 830*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 760
*I*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4270*sqrt(a)*sin(d*x + c)^
3/(cos(d*x + c) + 1)^3 - 1085*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4
 - 11576*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 2000*I*sqrt(a)*sin(d
*x + c)^6/(cos(d*x + c) + 1)^6 - 23000*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c)
 + 1)^7 - 2470*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 33540*sqrt(a
)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 33540*sqrt(a)*sin(d*x + c)^11/(cos(
d*x + c) + 1)^11 + 2470*I*sqrt(a)*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 2
3000*sqrt(a)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 + 2000*I*sqrt(a)*sin(d*x
 + c)^14/(cos(d*x + c) + 1)^14 - 11576*sqrt(a)*sin(d*x + c)^15/(cos(d*x + c
) + 1)^15 + 1085*I*sqrt(a)*sin(d*x + c)^16/(cos(d*x + c) + 1)^16 - 4270*sqr
t(a)*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 + 760*I*sqrt(a)*sin(d*x + c)^18/
(cos(d*x + c) + 1)^18 - 830*sqrt(a)*sin(d*x + c)^19/(cos(d*x + c) + 1)^19 +
 263*I*sqrt(a)*sin(d*x + c)^20/(cos(d*x + c) + 1)^20)*(sin(d*x + c)/(cos(d*
x + c) + 1) + 1)^(5/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(5/2)/((a^3 -
10*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 45*a^3*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 - 120*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 210*a^3*sin(d*
x + c)^8/(cos(d*x + c) + 1)^8 - 252*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^
10 + 210*a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 120*a^3*sin(d*x + c)^1
4/(cos(d*x + c) + 1)^14 + 45*a^3*sin(d*x + c)^16/(cos(d*x + c) + 1)^16 - 10
*a^3*sin(d*x + c)^18/(cos(d*x + c) + 1)^18 + a^3*sin(d*x + c)^20/(cos(d*x +
 c) + 1)^20)*d*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 - 1)^(5/2))
```

Giac [F]

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^{11}}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^11/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 9.81 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{256\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,60\mathrm{i}+\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}\,195\mathrm{i}+8\mathrm{i}\right)}{2145\,a^3\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^7}$$

```
[In] int(1/(cos(c + d*x)^11*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

```
[Out] (256*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2
i + d*x*2i) + 1))^(1/2)*(exp(c*2i + d*x*2i)*60i + exp(c*4i + d*x*4i)*195i +
 8i))/(2145*a^3*d*(exp(c*2i + d*x*2i) + 1)^7)
```

3.371 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 73

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx=\frac{8ia^2\sec^9(c+dx)}{99d(a+ia\tan(c+dx))^{9/2}}+\frac{2ia\sec^9(c+dx)}{11d(a+ia\tan(c+dx))^{7/2}}$$

```
[Out] 8/99*I*a^2*sec(d*x+c)^9/d/(a+I*a*tan(d*x+c))^(9/2)+2/11*I*a*sec(d*x+c)^9/d/
(a+I*a*tan(d*x+c))^(7/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3575, 3574}

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx=\frac{8ia^2\sec^9(c+dx)}{99d(a+ia\tan(c+dx))^{9/2}}+\frac{2ia\sec^9(c+dx)}{11d(a+ia\tan(c+dx))^{7/2}}$$

```
[In] Int[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((8*I)/99)*a^2*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(9/2)) + (((2*I)/
11)*a*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(7/2))
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2ia\sec^9(c+dx)}{11d(a+ia\tan(c+dx))^{7/2}}+\frac{1}{11}(4a)\int\frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx\\ &= \frac{8ia^2\sec^9(c+dx)}{99d(a+ia\tan(c+dx))^{9/2}}+\frac{2ia\sec^9(c+dx)}{11d(a+ia\tan(c+dx))^{7/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.10

$$\int\frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx=\frac{2\sec^7(c+dx)(\cos(2(c+dx))-i\sin(2(c+dx)))(-13i+9\tan(c+dx))}{99a^2d(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^(5/2),x]
[Out] (2*Sec[c + d*x]^7*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)])*(-13*I + 9*Tan[c
+ d*x]))/(99*a^2*d*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [F(-1)]

Timed out.

$$\int\frac{\sec^9(dx+c)}{(a+ia\tan(dx+c))^{\frac{5}{2}}}dx$$

```
[In] int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(5/2),x)
[Out] int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(5/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(57) = 114.

Time = 0.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.60

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{64\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-11i\,e^{(2i\,dx+2i\,c)}-2i\right)}{99\left(a^3de^{(10i\,dx+10i\,c)}+5\,a^3de^{(8i\,dx+8i\,c)}+10\,a^3de^{(6i\,dx+6i\,c)}+10\,a^3de^{(4i\,dx+4i\,c)}+5\,a^3de^{(2i\,dx+2i\,c)}+a^3d\right)}$$

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -64/99*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-11*I*e^(2*I*d*x + 2*I*c)
 - 2*I)/(a^3*d*e^(10*I*d*x + 10*I*c) + 5*a^3*d*e^(8*I*d*x + 8*I*c) + 10*a^3
*d*e^(6*I*d*x + 6*I*c) + 10*a^3*d*e^(4*I*d*x + 4*I*c) + 5*a^3*d*e^(2*I*d*x
+ 2*I*c) + a^3*d)
```

Sympy [F]

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec^9\left(c+dx\right)}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)**9/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**9/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 626 vs. 2(57) = 114.

Time = 0.43 (sec) , antiderivative size = 626, normalized size of antiderivative = 8.58

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{2\left(-13i\sqrt{a}-\frac{34\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{46i\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{174\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{54i\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{394\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{22i\sqrt{a}\sin(d}{(\cos(dx+c)}\right.}{99\left(a^3-\frac{8a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{28a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\right.}$$

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/99*(-13*I*sqrt(a) - 34*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 46*I*sq
rt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 174*sqrt(a)*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3 - 54*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 394*s
qrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 22*I*sqrt(a)*sin(d*x + c)^6/(c
os(d*x + c) + 1)^6 - 550*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 550*
sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 22*I*sqrt(a)*sin(d*x + c)^10/
(cos(d*x + c) + 1)^10 - 394*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 +
 54*I*sqrt(a)*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 174*sqrt(a)*sin(d*x +
 c)^13/(cos(d*x + c) + 1)^13 + 46*I*sqrt(a)*sin(d*x + c)^14/(cos(d*x + c) +
 1)^14 - 34*sqrt(a)*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 + 13*I*sqrt(a)*si
n(d*x + c)^16/(cos(d*x + c) + 1)^16)*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^
(5/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(5/2)/((a^3 - 8*a^3*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + 28*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 56*
a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a^3*sin(d*x + c)^8/(cos(d*x +
c) + 1)^8 - 56*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 28*a^3*sin(d*x +
 c)^12/(cos(d*x + c) + 1)^12 - 8*a^3*sin(d*x + c)^14/(cos(d*x + c) + 1)^14
+ a^3*sin(d*x + c)^16/(cos(d*x + c) + 1)^16)*d*(-2*I*sin(d*x + c)/(cos(d*x
+ c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(5/2))
```

Giac [F]

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)^9}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^9/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 7.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{64\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,11\mathrm{i}+2\mathrm{i}\right)\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}}{99\,a^3\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^5}$$

```
[In] int(1/(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

```
[Out] (64*exp(- c*1i - d*x*1i)*(exp(c*2i + d*x*2i)*11i + 2i)*(a - (a*(exp(c*2i +
d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2))/(99*a^3*d*(exp(c*2i +
 d*x*2i) + 1)^5)
```

3.372 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 35

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2ia\sec^7(c+dx)}{7d(a+ia\tan(c+dx))^{7/2}}$$

```
[Out] 2/7*I*a*sec(d*x+c)^7/d/(a+I*a*tan(d*x+c))^(7/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {3574}

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2ia\sec^7(c+dx)}{7d(a+ia\tan(c+dx))^{7/2}}$$

```
[In] Int[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((2*I)/7)*a*Sec[c + d*x]^7)/(d*(a + I*a*Tan[c + d*x])^(7/2))
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\text{integral} = \frac{2ia\sec^7(c+dx)}{7d(a+ia\tan(c+dx))^{7/2}}$$

Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.63

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{2\sec^5(c+dx)(i+\tan(c+dx))}{7a^2d(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (-2*Sec[c + d*x]^5*(I + Tan[c + d*x]))/(7*a^2*d*(-I + Tan[c + d*x])^2*Sqrt[
a + I*a*Tan[c + d*x]])
```

Maple [F(-1)]

Timed out.

$$\int \frac{\sec^7(dx+c)}{(a+ia\tan(dx+c))^{\frac{5}{2}}}dx$$

```
[In] int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(5/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(27) = 54.

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.11

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{16i\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{7\left(a^3de^{(6i\,dx+6i\,c)}+3\,a^3de^{(4i\,dx+4i\,c)}+3\,a^3de^{(2i\,dx+2i\,c)}+a^3d\right)}$$

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 16/7*I*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(a^3*d*e^(6*I*d*x + 6*I*c)
 + 3*a^3*d*e^(4*I*d*x + 4*I*c) + 3*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

Sympy [F]

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec^7(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c))**(5/2),x)
[Out] Integral(sec(c + d*x)**7/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 488 vs. 2(27) = 54.

Time = 0.38 (sec) , antiderivative size = 488, normalized size of antiderivative = 13.94

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx =$$
$$-\frac{2\left(-i\sqrt{a}-\frac{2\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{4i\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{10\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{5i\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{20\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{20\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\cdots}{7\left(a^3-\frac{6a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{20a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a^3\sin(dx\cdots}{(\cos(dx+c)\cdots}\cdots}$$

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
[Out] -2/7*(-I*sqrt(a) - 2*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 4*I*sqrt(a)*
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 10*sqrt(a)*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 - 5*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 20*sqrt(a)*si
n(d*x + c)^5/(cos(d*x + c) + 1)^5 - 20*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c)
 + 1)^7 + 5*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 10*sqrt(a)*sin(
d*x + c)^9/(cos(d*x + c) + 1)^9 + 4*I*sqrt(a)*sin(d*x + c)^10/(cos(d*x + c)
 + 1)^10 - 2*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + I*sqrt(a)*sin(
d*x + c)^12/(cos(d*x + c) + 1)^12)*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5
/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(5/2)/((a^3 - 6*a^3*sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 + 15*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 20*a^
3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 15*a^3*sin(d*x + c)^8/(cos(d*x + c)
 + 1)^8 - 6*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + a^3*sin(d*x + c)^12
/(cos(d*x + c) + 1)^12)*d*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 - 1)^(5/2))
```

Giac [F]

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)^7}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^7/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.43

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\mathrm{e}^{-c\,4\mathrm{i}-d\,x\,4\mathrm{i}}\,\sqrt{a+\frac{a\,\sin(c+d\,x)\,1\mathrm{i}}{\cos(c+d\,x)}}\,2\mathrm{i}}{7\,a^3\,d\cos\left(c+d\,x\right)^3}$$

```
[In] int(1/(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^(5/2)),x)
[Out] (exp(- c*4i - d*x*4i)*(a + (a*sin(c + d*x)*1i)/cos(c + d*x))^(1/2)*2i)/(7*a
^3*d*cos(c + d*x)^3)
```

3.373 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 123

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{4i\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d} - \frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}} - \frac{4i\sec(c+dx)}{a^2d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 4*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*2^(1/2
)/a^(5/2)/d-4*I*sec(d*x+c)/a^2/d/(a+I*a*tan(d*x+c))^(1/2)-2/3*I*sec(d*x+c)^
3/a/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3572, 3570, 212}

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{4i\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d} - \frac{4i\sec(c+dx)}{a^2d\sqrt{a+ia\tan(c+dx)}} - \frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}}$$

```
[In] Int[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((4*I)*Sqrt[2]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c +
 d*x]])])/(a^(5/2)*d) - (((2*I)/3)*Sec[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*
x])^(3/2)) - ((4*I)*Sec[c + d*x])/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3572

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e
+ f*x])^(n + 1)/(b*f*(m - 2))), x] + Dist[2*(d^2/a), Int[(d*Sec[e + f*x])^(
m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 + b^2, 0] && EqQ[m/2 + n, 0] && LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}}+\frac{2\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx}{a}\\
&= -\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}}-\frac{4i\sec(c+dx)}{a^2d\sqrt{a+ia\tan(c+dx)}}+\frac{4\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{a^2}\\
&= -\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}}-\frac{4i\sec(c+dx)}{a^2d\sqrt{a+ia\tan(c+dx)}}\\
&\quad+\frac{(8i)\text{Subst}\left(\int \frac{1}{2-ax^2}\,dx,x,\frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^2d}\\
&= \frac{4i\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{5/2}d}-\frac{2i\sec^3(c+dx)}{3ad(a+ia\tan(c+dx))^{3/2}}-\frac{4i\sec(c+dx)}{a^2d\sqrt{a+ia\tan(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.67

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{2\sec(c+dx)\left(7i-6i\sqrt{1+e^{2i(c+dx)}}\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)+\tan(c+dx)\right)}{3a^2d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (-2*Sec[c + d*x]*(7*I - (6*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1
+ E^((2*I)*(c + d*x))]] + Tan[c + d*x]))/(3*a^2*d*Sqrt[a + I*a*Tan[c + d*x]
])
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(102) = 204.

Time = 9.84 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.18

method	result
default	$\frac{2(-\csc(dx+c)+\cot(dx+c)+i)^5\left(6\sqrt{2}\,\arctan\left(\frac{(i(\csc(dx+c)-\cot(dx+c))-1)\sqrt{2}}{2\sqrt{\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2-1}}\right)\left(\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2-1\right)^{\frac{3}{2}}-7i(\csc^3(dx+}{3d\left(-\frac{a\left(2i(\csc(dx+c)-\cot(dx+c))-\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2+1\right)}{\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2-1}\right)^{\frac{5}{2}}\left(\left(\csc^2(d\right.}$

```
[In] int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/d*(-csc(d*x+c)+cot(d*x+c)+I)^5*(6*2^(1/2)*arctan(1/2*(I*(csc(d*x+c)-cot
(d*x+c))-1)*2^(1/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)^(1/2))*(csc(d*x+c)^2*
(1-cos(d*x+c))^2-1)^(3/2)-7*I*csc(d*x+c)^3*(1-cos(d*x+c))^3+9*I*(csc(d*x+c)
-cot(d*x+c))+9*csc(d*x+c)^2*(1-cos(d*x+c))^2-7)/(-a*(2*I*(csc(d*x+c)-cot(d*
x+c))-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1))^(
5/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 270 vs. 2(96) = 192.

Time = 0.25 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.20

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{2\left(3\sqrt{2}\left(ia^3de^{(2i\,dx+2i\,c)}+i\,a^3d\right)\sqrt{\frac{1}{a^5d^2}}\log\left(-\frac{16\left(\left(i\,a^2de^{(2i\,dx+2i\,c)}+i\,a^2d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^5d^2}}-i\right)e^{(-i\,dx-i\,c)}}{a^2d}\right)+3\sqrt{}\right.}{}$$

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(3*sqrt(2)*(I*a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)*sqrt(1/(a^5*d^2))*l
og(-16*((I*a^2*d*e^(2*I*d*x + 2*I*c) + I*a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c)
 + 1))*sqrt(1/(a^5*d^2)) - I)*e^(-I*d*x - I*c)/(a^2*d)) + 3*sqrt(2)*(-I*a^3
*d*e^(2*I*d*x + 2*I*c) - I*a^3*d)*sqrt(1/(a^5*d^2))*log(-16*((-I*a^2*d*e^(2
*I*d*x + 2*I*c) - I*a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^5*d^
2)) - I)*e^(-I*d*x - I*c)/(a^2*d)) + 2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1))*(3*I*e^(2*I*d*x + 2*I*c) + 4*I))/(a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec^5(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**5/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1074 vs. 2(96) = 192.

Time = 0.48 (sec) , antiderivative size = 1074, normalized size of antiderivative = 8.73

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*((-3*I*sqrt(2)*cos(2*d*x + 2*c) + 3*sqrt(2)*sin(2*d*x + 2*c) - 4*I*sqr
t(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - (3*sqrt(2)
*cos(2*d*x + 2*c) + 3*I*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt(2))*sin(3/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) - 3*(2*(sqrt(2)*cos(2*
d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sq
rt(2))*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - 2*(sqrt(2)*cos(2*
d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sq
rt(2))*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) - (I*sqrt(2)*cos(2*
d*x + 2*c)^2 + I*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*I*sqrt(2)*cos(2*d*x + 2*c)
+ I*sqrt(2))*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + s
qrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + 2*(cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - (-I*sqrt(2)*cos(2*d*x + 2*c)^2
 - I*sqrt(2)*sin(2*d*x + 2*c)^2 - 2*I*sqrt(2)*cos(2*d*x + 2*c) - I*sqrt(2))
*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*
x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 - 2*(cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)) + 1))*sqrt(a))/((a^3*cos(2*d*x + 2*c)^2 + a^3*sin
(2*d*x + 2*c)^2 + 2*a^3*cos(2*d*x + 2*c) + a^3)*d)
```

Giac [F]

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec\left(dx+c\right)^5}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^5/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^5\,\left(a+a\,\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{5/2}}\,dx$$

```
[In] int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(5/2)),x)
[Out] int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(5/2)), x)
```

3.374 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 86

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{i\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}a^{5/2}d} + \frac{i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] -1/2*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/a^(
5/2)/d*2^(1/2)+I*sec(d*x+c)/a/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3582, 3583, 3570, 212}

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}} - \frac{i\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}a^{5/2}d}$$

```
[In] Int[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-I)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]])])
/(Sqrt[2]*a^(5/2)*d) + (I*Sec[c + d*x])/(a*d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}} - \frac{2\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx}{a} \\
&= \frac{i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}} - \frac{\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{2a^2} \\
&= \frac{i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}} - \frac{i\text{Subst}\left(\int \frac{1}{2-ax^2}\,dx, x, \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^2d} \\
&= -\frac{i\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{\sqrt{2}a^{5/2}d} + \frac{i\sec(c+dx)}{ad(a+ia\tan(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.73

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{ie^{-\frac{1}{2}i(2c+dx)}\left(-1-e^{2i(c+dx)}+e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)}{2a^2d(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c}}$$

```
[In] Integrate[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((I/2)*(-1 - E^((2*I)*(c + d*x)) + E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c
 + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])*Sec[c + d*x]^3*(Cos[c + (
d*x)/2] + I*Sin[c + (d*x)/2]))/(a^2*d*E^((I/2)*(2*c + d*x))*(-I + Tan[c + d
*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(71) = 142.

Time = 9.18 (sec) , antiderivative size = 396, normalized size of antiderivative = 4.60

method	result
default	$\frac{\left(\sqrt{2}\arctan\left(\frac{(i(\csc(dx+c)-\cot(dx+c))-1)\sqrt{2}}{2\sqrt{\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2-1}}\right)+2i\arctan\left(\frac{(i(\csc(dx+c)-\cot(dx+c))-1)\sqrt{2}}{2\sqrt{\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2-1}}\right)\sqrt{2}\,(\csc(dx+c)-\cot(dx+c))-(\csc^2(}{2d\left((\csc^2(dx+c))\right.}$

```
[In] int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(2^(1/2)*arctan(1/2*(I*(csc(d*x+c)-cot(d*x+c))-1)*2^(1/2)/(csc(d*x+c)
^2*(1-cos(d*x+c))^2-1)^(1/2))+2*I*arctan(1/2*(I*(csc(d*x+c)-cot(d*x+c))-1)*
2^(1/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)^(1/2))*2^(1/2)*(csc(d*x+c)-cot(d*
x+c))-csc(d*x+c)^2*arctan(1/2*(I*(csc(d*x+c)-cot(d*x+c))-1)*2^(1/2)/(csc(d*
x+c)^2*(1-cos(d*x+c))^2-1)^(1/2))*2^(1/2)*(1-cos(d*x+c))^2-2*(csc(d*x+c)^2*
(1-cos(d*x+c))^2-1)^(1/2)+2*I*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)^(1/2)*(csc(
d*x+c)-cot(d*x+c)))*(-csc(d*x+c)+cot(d*x+c)+I)^3/(csc(d*x+c)^2*(1-cos(d*x+c
))^2-1)^(5/2)/(-a*(2*I*(csc(d*x+c)-cot(d*x+c))-csc(d*x+c)^2*(1-cos(d*x+c))^
2+1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1))^(5/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(67) = 134$.

Time = 0.25 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.85

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\left(i\sqrt{2}a^3d\sqrt{\frac{1}{a^5d^2}}e^{(2i\,dx+2i\,c)}\log\left(\frac{2\left(\left(i\,a^2de^{(2i\,dx+2i\,c)}+i\,a^2d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^5d^2}}-i\right)e^{(\cdots}}{a^2d}\right.\right.}{\cdots}$$

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(I*sqrt(2)*a^3*d*sqrt(1/(a^5*d^2))*e^(2*I*d*x + 2*I*c)*log(2*((I*a^2*d*
e^(2*I*d*x + 2*I*c) + I*a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^
5*d^2)) - I)*e^(-I*d*x - I*c)/(a^2*d)) - I*sqrt(2)*a^3*d*sqrt(1/(a^5*d^2))*
e^(2*I*d*x + 2*I*c)*log(2*((-I*a^2*d*e^(2*I*d*x + 2*I*c) - I*a^2*d)*sqrt(a/
(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^5*d^2)) - I)*e^(-I*d*x - I*c)/(a^2*d))
 - 2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-I*e^(2*I*d*x + 2*I*c) - I)
)*e^(-2*I*d*x - 2*I*c)/(a^3*d)
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec^3(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 827 vs. $2(67) = 134$.

Time = 0.70 (sec) , antiderivative size = 827, normalized size of antiderivative = 9.62

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/8*(4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*((-I*sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d*x + 2*c))*cos(1/2*arc
```

```
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (sqrt(2)*cos(2*d*x + 2*c) -
 I*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)))*sqrt(a) - (2*sqrt(2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
) + 1) - 2*sqrt(2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1) - I*sqr
t(2)*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + 2*(cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1)) + 1) + I*sqrt(2)*log(sqrt(cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1))^2 - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1
))*sqrt(a))/(a^3*d)
```

Giac [F]

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)^3}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^3/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos(c+d\,x)^3\,(a+a\tan(c+d\,x)\,1\mathrm{i})^{5/2}}\,dx$$

```
[In] int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(5/2)),x)
[Out] int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(5/2)), x)
```

3.375 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 122

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{3i\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}} + \frac{3i\sec(c+dx)}{16ad(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] 3/32*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/a^(
5/2)/d*2^(1/2)+1/4*I*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^(5/2)+3/16*I*sec(d*x+c
)/a/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3583, 3570, 212}

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{3i\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{3i\sec(c+dx)}{16ad(a+ia\tan(c+dx))^{3/2}} + \frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}}$$

```
[In] Int[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((3*I)/16)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*
x]])])/(Sqrt[2]*a^(5/2)*d) + ((I/4)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])
^(5/2)) + (((3*I)/16)*Sec[c + d*x])/(a*d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}} + \frac{3\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx}{8a} \\
&= \frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}} + \frac{3i\sec(c+dx)}{16ad(a+ia\tan(c+dx))^{3/2}} + \frac{3\int \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{32a^2} \\
&= \frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}} + \frac{3i\sec(c+dx)}{16ad(a+ia\tan(c+dx))^{3/2}} \\
&\quad + \frac{(3i)\text{Subst}\left(\int \frac{1}{2-ax^2}\,dx, x, \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\right)}{16a^2d} \\
&= \frac{3i\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{i\sec(c+dx)}{4d(a+ia\tan(c+dx))^{5/2}} + \frac{3i\sec(c+dx)}{16ad(a+ia\tan(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.99

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx =$$
$$-\frac{i\sec^3(c+dx)\left(7+3e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)+7\cos(2(c+dx))+3i\sin(2(c+dx))\right.}{32a^2d(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-1/32*I)*Sec[c + d*x]^3*(7 + 3*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c +
 d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]] + 7*Cos[2*(c + d*x)] + (3*I)
*Sin[2*(c + d*x)]))/(a^2*d*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]]
)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 605 vs. 2(97) = 194.

Time = 9.23 (sec) , antiderivative size = 606, normalized size of antiderivative = 4.97

method	result
default	$-\frac{i\left(12i\arctan\left(\frac{i\sin(dx+c)-\cos(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\sin(dx+c)+6i\tan(dx+c)\arctan\left(\frac{i\sin(dx+c)-\cos(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+6i\tan(dx+c)\right.}{}$

```
[In] int(sec(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/32*I/d/(tan(d*x+c)-I)^2/(a*(1+I*tan(d*x+c)))^(1/2)/(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)/a^2/(cos(d*x+c)+1)*(12*I*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-
1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+6*I*tan(d*
x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(co
s(d*x+c)+1))^(1/2))+6*I*tan(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+12*co
s(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)
/(cos(d*x+c)+1))^(1/2))-3*I*tan(d*x+c)*sec(d*x+c)*arctan(1/2*(I*sin(d*x+c)-
cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+6*I*tan(d*
x+c)*sec(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+6*arctan(1/2*(I*sin(d*x+
c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+14*(-co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)-9*sec(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*
x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+14*sec(d*x+c)*(-
cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-3*sec(d*x+c)^2*arctan(1/2*(I*sin(d*x+c)-co
s(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(91) = 182.

Time = 0.26 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.19

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\left(-3i\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{1}{a^5d^2}}e^{(4i\,dx+4i\,c)}\log\left(-\frac{3\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,a^2de^{(2i\,dx+2i\,c)}+i\,a^2d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\cdots}\right.}{8\,a^2d}\right.\right.}{\cdots}$$

[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")

[Out] 1/32*(-3*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(4*I*d*x + 4*I*c)*log(-3/8*(sqrt(2)*sqrt(1/2)*(I*a^2*d*e^(2*I*d*x + 2*I*c) + I*a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^5*d^2)) - I)*e^(-I*d*x - I*c)/(a^2*d)) + 3*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(4*I*d*x + 4*I*c)*log(-3/8*(sqrt(2)*sqrt(1/2)*(-I*a^2*d*e^(2*I*d*x + 2*I*c) - I*a^2*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^5*d^2)) - I)*e^(-I*d*x - I*c)/(a^2*d)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(5*I*e^(4*I*d*x + 4*I*c) + 7*I*e^(2*I*d*x + 2*I*c) + 2*I))*e^(-4*I*d*x - 4*I*c)/(a^3*d)

Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{5}{2}}}\,dx$$

[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))**(5/2),x)

[Out] Integral(sec(c + d*x)/(I*a*(tan(c + d*x) - I))**(5/2), x)

Maxima [F]

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate(sec(d*x + c)/(I*a*tan(d*x + c) + a)^(5/2), x)

Giac [F]

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sec(dx+c)}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{1}{\cos(c+d\,x)\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{5/2}}\,dx$$

```
[In] int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(5/2)),x)
[Out] int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(5/2)), x)
```

3.376 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 192

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{35i\,\mathrm{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{128\sqrt{2}a^{5/2}d} + \frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}} + \frac{7i\cos(c+dx)}{48ad(a+ia\tan(c+dx))^{3/2}} + \frac{35i\cos(c+dx)}{192a^2d\sqrt{a+ia\tan(c+dx)}} - \frac{35i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{128a^3d}$$

```
[Out] 35/256*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/a
^(5/2)/d*2^(1/2)+35/192*I*cos(d*x+c)/a^2/d/(a+I*a*tan(d*x+c))^(1/2)-35/128*
I*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/a^3/d+1/6*I*cos(d*x+c)/d/(a+I*a*tan(d
*x+c))^(5/2)+7/48*I*cos(d*x+c)/a/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3583, 3571, 3570, 212}

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{35i\,\mathrm{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{128\sqrt{2}a^{5/2}d} - \frac{35i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{128a^3d} + \frac{35i\cos(c+dx)}{192a^2d\sqrt{a+ia\tan(c+dx)}} + \frac{7i\cos(c+dx)}{48ad(a+ia\tan(c+dx))^{3/2}} + \frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}$$

```
[In] Int[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((35*I)/128)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c +
d*x]])])/(Sqrt[2]*a^(5/2)*d) + ((I/6)*Cos[c + d*x])/(d*(a + I*a*Tan[c + d*x
])^(5/2)) + (((7*I)/48)*Cos[c + d*x])/(a*d*(a + I*a*Tan[c + d*x])^(3/2)) +
(((35*I)/192)*Cos[c + d*x])/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]]) - (((35*I)/1
28)*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/(a^3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3571

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}+\frac{7\int\frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx}{12a}\\ &= \frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}}+\frac{7i\cos(c+dx)}{48ad(a+ia\tan(c+dx))^{3/2}}+\frac{35\int\frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{96a^2}\end{aligned}$$

$$= \frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}} + \frac{7i\cos(c+dx)}{48ad(a+ia\tan(c+dx))^{3/2}} + \frac{35i\cos(c+dx)}{192a^2d\sqrt{a+ia\tan(c+dx)}} + \frac{35\int\cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx}{128a^3}$$

$$= \frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}} + \frac{7i\cos(c+dx)}{48ad(a+ia\tan(c+dx))^{3/2}} + \frac{35i\cos(c+dx)}{192a^2d\sqrt{a+ia\tan(c+dx)}} - \frac{35i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{128a^3d} + \frac{35\int\frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}dx}{256a^2}$$

$$= \frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}} + \frac{7i\cos(c+dx)}{48ad(a+ia\tan(c+dx))^{3/2}} + \frac{35i\cos(c+dx)}{192a^2d\sqrt{a+ia\tan(c+dx)}} - \frac{35i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{128a^3d} + \frac{(35i)\text{Subst}\left(\int\frac{1}{2-ax^2}dx, x, \frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\right)}{128a^2d}$$

$$= \frac{35i\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{128\sqrt{2}a^{5/2}d} + \frac{i\cos(c+dx)}{6d(a+ia\tan(c+dx))^{5/2}} + \frac{7i\cos(c+dx)}{48ad(a+ia\tan(c+dx))^{3/2}} + \frac{35i\cos(c+dx)}{192a^2d\sqrt{a+ia\tan(c+dx)}} - \frac{35i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{128a^3d}$$

Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.74

$$\int\frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{5/2}}dx = \frac{i\sec^3(c+dx)\left(-125-105e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)-\right.}{768a^2d(-i+\tan(c+d}$$

```
[In] Integrate[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((I/768)*Sec[c + d*x]^3*(-125 - 105*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(
c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]] - 85*Cos[2*(c + d*x)] + 4
0*Cos[4*(c + d*x)] + (7*I)*Sin[2*(c + d*x)] + (56*I)*Sin[4*(c + d*x)]))/(a^
2*d*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 729 vs. $2(155) = 310$.

Time = 10.36 (sec) , antiderivative size = 730, normalized size of antiderivative = 3.80

method	result
default	$-\frac{320i(\cos^2(dx+c))\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}+320i\cos(dx+c)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}-420i\cos(dx+c)\arctan\left(\frac{i\sin(dx+c)-\cos(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)-4}{}$

```
[In] int(cos(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/768/d/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)+1)/(a*(1+I*tan(d*x+
c)))^(1/2)/(1+I*tan(d*x+c))^2/a^2*(320*I*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)+320*I*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-420*I*cos
(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/
(cos(d*x+c)+1))^(1/2))-448*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*si
n(d*x+c)-490*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-210*I*arctan(1/2*(I*sin(d
*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-448*
sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+420*arctan(1/2*(I*sin(d*x+c)-
cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)
-490*I*sec(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+315*I*sec(d*x+c)*arcta
n(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1
))^(1/2))+210*tan(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+210*tan(d*x+c)*
arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2))+105*I*sec(d*x+c)^2*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(c
os(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+210*tan(d*x+c)*sec(d*x+c)*
(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-105*tan(d*x+c)*sec(d*x+c)*arctan(1/2*(I*
sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))
)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.51

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\left(-105i\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{1}{a^5d^2}}e^{(6i\,dx+6i\,c)}\log\left(-\frac{35\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,a^2de^{(2i\,dx+2i\,c)}+i\,a^2d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i}}}\right.}{64\,a^2d}\right.\right.}{}$$

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/768*(-105*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(6*I*d*x + 6*I*c)*log(-35
/64*(sqrt(2)*sqrt(1/2)*(I*a^2*d*e^(2*I*d*x + 2*I*c) + I*a^2*d)*sqrt(a/(e^(2
*I*d*x + 2*I*c) + 1))*sqrt(1/(a^5*d^2)) - I)*e^(-I*d*x - I*c)/(a^2*d)) + 10
5*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(6*I*d*x + 6*I*c)*log(-35/64*(sqrt(
2)*sqrt(1/2)*(-I*a^2*d*e^(2*I*d*x + 2*I*c) - I*a^2*d)*sqrt(a/(e^(2*I*d*x +
2*I*c) + 1))*sqrt(1/(a^5*d^2)) - I)*e^(-I*d*x - I*c)/(a^2*d)) + sqrt(2)*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*(-48*I*e^(8*I*d*x + 8*I*c) + 39*I*e^(6*I*d*x
 + 6*I*c) + 125*I*e^(4*I*d*x + 4*I*c) + 46*I*e^(2*I*d*x + 2*I*c) + 8*I))*e^
(-6*I*d*x - 6*I*c)/(a^3*d)
```

Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\cos{(c+dx)}}{(ia(\tan{(c+dx)}-i))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(cos(c + d*x)/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2297 vs. 2(145) = 290.

Time = 0.51 (sec) , antiderivative size = 2297, normalized size of antiderivative = 11.96

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3072*(544*(cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(1
/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin(6
*d*x + 6*c), cos(6*d*x + 6*c))) + 1)^(3/4)*((-I*sqrt(2)*cos(6*d*x + 6*c) -
sqrt(2)*sin(6*d*x + 6*c))*cos(3/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c),
 cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) +
 1)) + (sqrt(2)*cos(6*d*x + 6*c) - I*sqrt(2)*sin(6*d*x + 6*c))*sin(3/2*arct
an2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(s
in(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)))*sqrt(a) + 12*(cos(1/3*arctan2(si
n(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(1/3*arctan2(sin(6*d*x + 6*c), co
s(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))
 + 1)^(1/4)*(29*((I*sqrt(2)*cos(6*d*x + 6*c) + sqrt(2)*sin(6*d*x + 6*c))*co
s(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + (I*sqrt(2)*cos(6*d*x
 + 6*c) + sqrt(2)*sin(6*d*x + 6*c))*sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6
```

*d*x + 6*c)))^2 + 2*(I*sqrt(2)*cos(6*d*x + 6*c) + sqrt(2)*sin(6*d*x + 6*c))
*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + I*sqrt(2)*cos(6*d*x
 + 6*c) + sqrt(2)*sin(6*d*x + 6*c))*cos(5/2*arctan2(sin(1/3*arctan2(sin(6*d
*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x
+ 6*c))) + 1)) + (19*I*sqrt(2)*cos(6*d*x + 6*c) + 19*sqrt(2)*sin(6*d*x + 6*
c) - 16*I*sqrt(2))*cos(1/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*
d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)) -
29*((sqrt(2)*cos(6*d*x + 6*c) - I*sqrt(2)*sin(6*d*x + 6*c))*cos(1/3*arctan2
(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + (sqrt(2)*cos(6*d*x + 6*c) - I*sqr
t(2)*sin(6*d*x + 6*c))*sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))
^2 + 2*(sqrt(2)*cos(6*d*x + 6*c) - I*sqrt(2)*sin(6*d*x + 6*c))*cos(1/3*arct
an2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + sqrt(2)*cos(6*d*x + 6*c) - I*sqr
t(2)*sin(6*d*x + 6*c))*sin(5/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c), co
s(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)
) - (19*sqrt(2)*cos(6*d*x + 6*c) - 19*I*sqrt(2)*sin(6*d*x + 6*c) - 16*sqrt(
2))*sin(1/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), c
os(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)))*sqrt(a) - 105*(2
*sqrt(2)*arctan2((cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 +
sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(
sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/3*arc
tan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c)
, cos(6*d*x + 6*c))) + 1)), (cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x +
6*c)))^2 + sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1
/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)^(1/4)*cos(1/2*arctan2(
sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6
*d*x + 6*c), cos(6*d*x + 6*c))) + 1)) + 1) - 2*sqrt(2)*arctan2((cos(1/3*arc
tan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(1/3*arctan2(sin(6*d*x + 6
*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x +
 6*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*
d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)), (
cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(1/3*arctan2(si
n(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin(6*d*x + 6*c),
cos(6*d*x + 6*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6
*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)
)) + 1)) - 1) - I*sqrt(2)*log(sqrt(cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*
d*x + 6*c)))^2 + sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2
*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)*cos(1/2*arctan2(
sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6
*d*x + 6*c), cos(6*d*x + 6*c))) + 1))^2 + sqrt(cos(1/3*arctan2(sin(6*d*x +
6*c), cos(6*d*x + 6*c)))^2 + sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x +
6*c)))^2 + 2*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)*sin(
1/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*a
rctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1))^2 + 2*(cos(1/3*arctan2(si
n(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + sin(1/3*arctan2(sin(6*d*x + 6*c), co
s(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))

```
 + 1)^(1/4)*cos(1/2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6
*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)) + 1) + I*
sqrt(2)*log(sqrt(cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + s
in(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1/3*arctan2(s
in(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)*cos(1/2*arctan2(sin(1/3*arctan2(si
n(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6
*d*x + 6*c))) + 1))^2 + sqrt(cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x +
6*c)))^2 + sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2 + 2*cos(1
/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)*sin(1/2*arctan2(sin(1/
3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arctan2(sin(6*d*x +
 6*c), cos(6*d*x + 6*c))) + 1))^2 - 2*(cos(1/3*arctan2(sin(6*d*x + 6*c), co
s(6*d*x + 6*c)))^2 + sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c)))^2
 + 2*cos(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)^(1/4)*cos(1/
2*arctan2(sin(1/3*arctan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))), cos(1/3*arc
tan2(sin(6*d*x + 6*c), cos(6*d*x + 6*c))) + 1)) + 1))*sqrt(a))/(a^3*d)
```

Giac [F]

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx=\int \frac{\cos\left(dx+c\right)}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(cos(d*x + c)/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx=\int \frac{\cos\left(c+d\,x\right)}{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}}\,dx$$

```
[In] int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^(5/2),x)
[Out] int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.377 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 270

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{1155i\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{4096\sqrt{2}a^{5/2}d} + \frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}} + \frac{11i\cos^3(c+dx)}{96ad(a+ia\tan(c+dx))^{3/2}} + \frac{385i\cos(c+dx)}{2048a^2d\sqrt{a+ia\tan(c+dx)}} + \frac{33i\cos^3(c+dx)}{256a^2d\sqrt{a+ia\tan(c+dx)}} - \frac{1155i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4096a^3d} - \frac{77i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{512a^3d}$$

```
[Out] 1155/8192*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
)/a^(5/2)/d*2^(1/2)+385/2048*I*cos(d*x+c)/a^2/d/(a+I*a*tan(d*x+c))^(1/2)+33
/256*I*cos(d*x+c)^3/a^2/d/(a+I*a*tan(d*x+c))^(1/2)-1155/4096*I*cos(d*x+c)*(
a+I*a*tan(d*x+c))^(1/2)/a^3/d-77/512*I*cos(d*x+c)^3*(a+I*a*tan(d*x+c))^(1/2
)/a^3/d+1/8*I*cos(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(5/2)+11/96*I*cos(d*x+c)^3/
a/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used

$= \{3583, 3578, 3571, 3570, 212\}$

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{1155i\,\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{4096\sqrt{2}a^{5/2}d} - \frac{77i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{512a^3d} - \frac{1155i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4096a^3d} + \frac{33i\cos^3(c+dx)}{256a^2d\sqrt{a+ia\tan(c+dx)}} + \frac{385i\cos(c+dx)}{2048a^2d\sqrt{a+ia\tan(c+dx)}} + \frac{11i\cos^3(c+dx)}{96ad(a+ia\tan(c+dx))^{3/2}} + \frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}$$

```
[In] Int[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((1155*I)/4096)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c
 + d*x]])])/(Sqrt[2]*a^(5/2)*d) + ((I/8)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[c
+ d*x])^(5/2)) + (((11*I)/96)*Cos[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*x])^(
3/2)) + (((385*I)/2048)*Cos[c + d*x])/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]]) +
(((33*I)/256)*Cos[c + d*x]^3)/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]]) - (((1155*
I)/4096)*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/(a^3*d) - (((77*I)/512)*C
os[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/(a^3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3571

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
```

```
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}} + \frac{11\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx}{16a} \\
&= \frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}} + \frac{11i\cos^3(c+dx)}{96ad(a+ia\tan(c+dx))^{3/2}} + \frac{33\int \frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{64a^2} \\
&= \frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}} + \frac{11i\cos^3(c+dx)}{96ad(a+ia\tan(c+dx))^{3/2}} \\
&\quad + \frac{33i\cos^3(c+dx)}{256a^2d\sqrt{a+ia\tan(c+dx)}} + \frac{231\int \cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx}{512a^3} \\
&= \frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}} + \frac{11i\cos^3(c+dx)}{96ad(a+ia\tan(c+dx))^{3/2}} \\
&\quad + \frac{33i\cos^3(c+dx)}{256a^2d\sqrt{a+ia\tan(c+dx)}} \\
&\quad - \frac{77i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{512a^3d} + \frac{385\int \frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{1024a^2} \\
&= \frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}} + \frac{11i\cos^3(c+dx)}{96ad(a+ia\tan(c+dx))^{3/2}} \\
&\quad + \frac{385i\cos(c+dx)}{2048a^2d\sqrt{a+ia\tan(c+dx)}} + \frac{33i\cos^3(c+dx)}{256a^2d\sqrt{a+ia\tan(c+dx)}} \\
&\quad - \frac{77i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{512a^3d} \\
&\quad + \frac{1155\int \cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx}{4096a^3}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}+\frac{11i\cos^3(c+dx)}{96ad(a+ia\tan(c+dx))^{3/2}} \\
&\quad+\frac{385i\cos(c+dx)}{2048a^2d\sqrt{a+ia\tan(c+dx)}}+\frac{33i\cos^3(c+dx)}{256a^2d\sqrt{a+ia\tan(c+dx)}} \\
&\quad-\frac{1155i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4096a^3d} \\
&\quad-\frac{77i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{512a^3d}+\frac{1155\int\frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{8192a^2} \\
&= \frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}}+\frac{11i\cos^3(c+dx)}{96ad(a+ia\tan(c+dx))^{3/2}} \\
&\quad+\frac{385i\cos(c+dx)}{2048a^2d\sqrt{a+ia\tan(c+dx)}}+\frac{33i\cos^3(c+dx)}{256a^2d\sqrt{a+ia\tan(c+dx)}} \\
&\quad-\frac{1155i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4096a^3d}-\frac{77i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{512a^3d} \\
&\quad+\frac{(1155i)\text{Subst}\left(\int\frac{1}{2-ax^2}\,dx,x,\frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\right)}{4096a^2d} \\
&= \frac{1155i\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{4096\sqrt{2}a^{5/2}d}+\frac{i\cos^3(c+dx)}{8d(a+ia\tan(c+dx))^{5/2}} \\
&\quad+\frac{11i\cos^3(c+dx)}{96ad(a+ia\tan(c+dx))^{3/2}}+\frac{385i\cos(c+dx)}{2048a^2d\sqrt{a+ia\tan(c+dx)}} \\
&\quad+\frac{33i\cos^3(c+dx)}{256a^2d\sqrt{a+ia\tan(c+dx)}}-\frac{1155i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{4096a^3d} \\
&\quad-\frac{77i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{512a^3d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.93 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.61

$$
\int\frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx=\frac{i\sec^3(c+dx)\left(-3325-3465e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right.}{}
$$

```
[In] Integrate[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((I/24576)*Sec[c + d*x]^3*(-3325 - 3465*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*
I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]] - 1605*Cos[2*(c + d*x
)] + 1800*Cos[4*(c + d*x)] + 80*Cos[6*(c + d*x)] + (1111*I)*Sin[2*(c + d*x)
] + (2552*I)*Sin[4*(c + d*x)] + (176*I)*Sin[6*(c + d*x)]))/(a^2*d*(-I + Tan
[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 863 vs. 2(221) = 442.

Time = 9.11 (sec) , antiderivative size = 864, normalized size of antiderivative = 3.20

method	result	size
default	Expression too large to display	864

```
[In] int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24576/d/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)+1)/(a*(1+I*tan(d*x
+c)))^(1/2)/(1+I*tan(d*x+c))^2/a^2*(-10560*I*cos(d*x+c)*(-cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)+5632*sin(d*x+c)*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(
1/2)-10560*I*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+5632*sin(d*x+c
)*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+13860*I*cos(d*x+c)*arctan
(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2))+14784*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)+161
70*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-2560*I*cos(d*x+c)^4*(-cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)+14784*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-138
60*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(
d*x+c)+1))^(1/2))*sin(d*x+c)+6930*I*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/
(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+16170*I*sec(d*x+c)*(-cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)-6930*tan(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^
(1/2)-6930*tan(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)
/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-2560*I*cos(d*x+c)^3*(-cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)-10395*I*sec(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/
(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-6930*tan(d*x+c)*sec(d*x+
c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+3465*tan(d*x+c)*sec(d*x+c)*arctan(1/2
*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1
/2))-3465*I*sec(d*x+c)^2*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)
+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.15

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\left(-3465i\,\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{1}{a^5d^2}}e^{(8i\,dx+8i\,c)}\log\left(-\frac{1155\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,a^2de^{(2i\,dx+2i\,c)}+i\,a^2d\right)\sqrt{\frac{}{e^{(2i\,dx}}}\right.}{2048\,a^2d}\right.\right.}{}$$

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24576*(-3465*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(8*I*d*x + 8*I*c)*log(
-1155/2048*(sqrt(2)*sqrt(1/2)*(I*a^2*d*e^(2*I*d*x + 2*I*c) + I*a^2*d)*sqrt(
a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^5*d^2)) - I)*e^(-I*d*x - I*c)/(a^2*d
)) + 3465*I*sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2))*e^(8*I*d*x + 8*I*c)*log(-1155
/2048*(sqrt(2)*sqrt(1/2)*(-I*a^2*d*e^(2*I*d*x + 2*I*c) - I*a^2*d)*sqrt(a/(e
^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^5*d^2)) - I)*e^(-I*d*x - I*c)/(a^2*d)) +
 sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-128*I*e^(12*I*d*x + 12*I*c) -
2176*I*e^(10*I*d*x + 10*I*c) + 247*I*e^(8*I*d*x + 8*I*c) + 3325*I*e^(6*I*d*
x + 6*I*c) + 1358*I*e^(4*I*d*x + 4*I*c) + 376*I*e^(2*I*d*x + 2*I*c) + 48*I)
)*e^(-8*I*d*x - 8*I*c)/(a^3*d)
```

Sympy [F]

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\cos^3{(c+dx)}}{(ia(\tan{(c+dx)}-i))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(cos(c + d*x)**3/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3789 vs. 2(207) = 414.

Time = 0.60 (sec) , antiderivative size = 3789, normalized size of antiderivative = 14.03

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/98304*(4*(cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(1
/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin(8
*d*x + 8*c), cos(8*d*x + 8*c))) + 1)^(3/4)*(15*((-I*sqrt(2)*cos(8*d*x + 8*c
) - sqrt(2)*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x +
 8*c)))^2 + (-I*sqrt(2)*cos(8*d*x + 8*c) - sqrt(2)*sin(8*d*x + 8*c))*sin(1/
4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*(-I*sqrt(2)*cos(8*d*x
+ 8*c) - sqrt(2)*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*
d*x + 8*c))) - I*sqrt(2)*cos(8*d*x + 8*c) - sqrt(2)*sin(8*d*x + 8*c))*cos(7
/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*ar
ctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)) + (55*I*sqrt(2)*cos(8*d*x
+ 8*c) + 960*I*sqrt(2)*cos(3/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))
 - 1296*I*sqrt(2)*cos(1/2*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 55
```

*sqrt(2)*sin(8*d*x + 8*c) + 960*sqrt(2)*sin(3/4*arctan2(sin(8*d*x + 8*c), c
os(8*d*x + 8*c))) - 1296*sqrt(2)*sin(1/2*arctan2(sin(8*d*x + 8*c), cos(8*d*
x + 8*c))) + 128*I*sqrt(2))*cos(3/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c
), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))
 + 1)) + 15*((sqrt(2)*cos(8*d*x + 8*c) - I*sqrt(2)*sin(8*d*x + 8*c))*cos(1/
4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + (sqrt(2)*cos(8*d*x + 8*c
) - I*sqrt(2)*sin(8*d*x + 8*c))*sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x
 + 8*c)))^2 + 2*(sqrt(2)*cos(8*d*x + 8*c) - I*sqrt(2)*sin(8*d*x + 8*c))*cos
(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + sqrt(2)*cos(8*d*x + 8*c
) - I*sqrt(2)*sin(8*d*x + 8*c))*sin(7/2*arctan2(sin(1/4*arctan2(sin(8*d*x +
 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*
c))) + 1)) - (55*sqrt(2)*cos(8*d*x + 8*c) + 960*sqrt(2)*cos(3/4*arctan2(sin
(8*d*x + 8*c), cos(8*d*x + 8*c))) - 1296*sqrt(2)*cos(1/2*arctan2(sin(8*d*x
+ 8*c), cos(8*d*x + 8*c))) - 55*I*sqrt(2)*sin(8*d*x + 8*c) - 960*I*sqrt(2)*
sin(3/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1296*I*sqrt(2)*sin(1
/2*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 128*sqrt(2))*sin(3/2*arct
an2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(s
in(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)))*sqrt(a) + 4*(cos(1/4*arctan2(sin
(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(1/4*arctan2(sin(8*d*x + 8*c), cos
(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))
+ 1)^(1/4)*((73*(-I*sqrt(2)*cos(8*d*x + 8*c) - sqrt(2)*sin(8*d*x + 8*c))*co
s(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 73*(-I*sqrt(2)*cos(8
*d*x + 8*c) - sqrt(2)*sin(8*d*x + 8*c))*sin(1/4*arctan2(sin(8*d*x + 8*c), c
os(8*d*x + 8*c)))^2 + 792*(-I*sqrt(2)*cos(1/4*arctan2(sin(8*d*x + 8*c), cos
(8*d*x + 8*c)))^2 - I*sqrt(2)*sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x +
 8*c)))^2 - 2*I*sqrt(2)*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))
) - I*sqrt(2))*cos(3/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 146*(
-I*sqrt(2)*cos(8*d*x + 8*c) - sqrt(2)*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin
(8*d*x + 8*c), cos(8*d*x + 8*c))) - 792*(sqrt(2)*cos(1/4*arctan2(sin(8*d*x
+ 8*c), cos(8*d*x + 8*c)))^2 + sqrt(2)*sin(1/4*arctan2(sin(8*d*x + 8*c), co
s(8*d*x + 8*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x
+ 8*c))) + sqrt(2))*sin(3/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) -
73*I*sqrt(2)*cos(8*d*x + 8*c) - 73*sqrt(2)*sin(8*d*x + 8*c))*cos(5/2*arctan
2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin
(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)) + 3*(-5*I*sqrt(2)*cos(8*d*x + 8*c)
- 120*I*sqrt(2)*cos(3/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 336*
I*sqrt(2)*cos(1/2*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) - 64*I*sqrt(
2)*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) - 5*sqrt(2)*sin(8*d
*x + 8*c) - 120*sqrt(2)*sin(3/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))
) + 336*sqrt(2)*sin(1/2*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) - 64*s
qrt(2)*sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 640*I*sqrt(2)
)*cos(1/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos
(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)) + (73*(sqrt(2)*cos(
8*d*x + 8*c) - I*sqrt(2)*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c)
, cos(8*d*x + 8*c)))^2 + 73*(sqrt(2)*cos(8*d*x + 8*c) - I*sqrt(2)*sin(8*d*x

+ 8*c))*sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 792*(sqrt
(2)*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sqrt(2)*sin(1/
4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*sqrt(2)*cos(1/4*arctan
2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + sqrt(2))*cos(3/4*arctan2(sin(8*d*x
 + 8*c), cos(8*d*x + 8*c))) + 146*(sqrt(2)*cos(8*d*x + 8*c) - I*sqrt(2)*sin
(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 792*(
-I*sqrt(2)*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 - I*sqrt(
2)*sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 - 2*I*sqrt(2)*cos
(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) - I*sqrt(2))*sin(3/4*arct
an2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 73*sqrt(2)*cos(8*d*x + 8*c) - 73
*I*sqrt(2)*sin(8*d*x + 8*c))*sin(5/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*
c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))
) + 1)) + 3*(5*sqrt(2)*cos(8*d*x + 8*c) + 120*sqrt(2)*cos(3/4*arctan2(sin(8
*d*x + 8*c), cos(8*d*x + 8*c))) - 336*sqrt(2)*cos(1/2*arctan2(sin(8*d*x + 8
*c), cos(8*d*x + 8*c))) + 64*sqrt(2)*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(
8*d*x + 8*c))) - 5*I*sqrt(2)*sin(8*d*x + 8*c) - 120*I*sqrt(2)*sin(3/4*arcta
n2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 336*I*sqrt(2)*sin(1/2*arctan2(sin
(8*d*x + 8*c), cos(8*d*x + 8*c))) - 64*I*sqrt(2)*sin(1/4*arctan2(sin(8*d*x
 + 8*c), cos(8*d*x + 8*c))) - 640*sqrt(2))*sin(1/2*arctan2(sin(1/4*arctan2(s
in(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(
8*d*x + 8*c))) + 1)))*sqrt(a) + 3465*(2*sqrt(2)*arctan2((cos(1/4*arctan2(si
n(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(1/4*arctan2(sin(8*d*x + 8*c), co
s(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))
 + 1)^(1/4)*sin(1/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8
*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)), (cos(1/4
*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(1/4*arctan2(sin(8*d*x
 + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d
*x + 8*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), co
s(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)
) + 1) - 2*sqrt(2)*arctan2((cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8
*c)))^2 + sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/
4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)^(1/4)*sin(1/2*arctan2(s
in(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*
d*x + 8*c), cos(8*d*x + 8*c))) + 1)), (cos(1/4*arctan2(sin(8*d*x + 8*c), co
s(8*d*x + 8*c)))^2 + sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2
 + 2*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)^(1/4)*cos(1/
2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arc
tan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)) - 1) - I*sqrt(2)*log(sqrt(c
os(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(1/4*arctan2(sin
(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin(8*d*x + 8*c), c
os(8*d*x + 8*c))) + 1)*cos(1/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), co
s(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)
)^2 + sqrt(cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(1/4
*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin(8*d
*x + 8*c), cos(8*d*x + 8*c))) + 1)*sin(1/2*arctan2(sin(1/4*arctan2(sin(8*d*

```
x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x +
 8*c))) + 1))^2 + 2*(cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2
 + sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4*arcta
n2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/4*
arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8
*c), cos(8*d*x + 8*c))) + 1)) + 1) + I*sqrt(2)*log(sqrt(cos(1/4*arctan2(sin
(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(1/4*arctan2(sin(8*d*x + 8*c), cos
(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))
+ 1)*cos(1/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))),
cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1))^2 + sqrt(cos(1/4
*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(1/4*arctan2(sin(8*d*x
 + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d
*x + 8*c))) + 1)*sin(1/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*
x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1))^2 -
2*(cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(1/4*arctan2
(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin(8*d*x + 8*c
), cos(8*d*x + 8*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/4*arctan2(sin(8*d*x
+ 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8
*c))) + 1)) + 1))*sqrt(a))/(a^3*d)
```

Giac [F]

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx=\int \frac{\cos\left(dx+c\right)^3}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(cos(d*x + c)^3/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx=\int \frac{\cos\left(c+d\,x\right)^3}{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)^{5/2}}\,dx$$

```
[In] int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^(5/2),x)
[Out] int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.378 $\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 146

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{32i(a+ia\tan(c+dx))^{3/2}}{3a^5d}+\frac{64i(a+ia\tan(c+dx))^{5/2}}{5a^6d} - \frac{48i(a+ia\tan(c+dx))^{7/2}}{7a^7d}+\frac{16i(a+ia\tan(c+dx))^{9/2}}{9a^8d}-\frac{2i(a+ia\tan(c+dx))^{11/2}}{11a^9d}$$

```
[Out] -32/3*I*(a+I*a*tan(d*x+c))^(3/2)/a^5/d+64/5*I*(a+I*a*tan(d*x+c))^(5/2)/a^6/
d-48/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^7/d+16/9*I*(a+I*a*tan(d*x+c))^(9/2)/a^8
/d-2/11*I*(a+I*a*tan(d*x+c))^(11/2)/a^9/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3568, 45}

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{2i(a+ia\tan(c+dx))^{11/2}}{11a^9d}+\frac{16i(a+ia\tan(c+dx))^{9/2}}{9a^8d} - \frac{48i(a+ia\tan(c+dx))^{7/2}}{7a^7d}+\frac{64i(a+ia\tan(c+dx))^{5/2}}{5a^6d}-\frac{32i(a+ia\tan(c+dx))^{3/2}}{3a^5d}$$

```
[In] Int[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((-32*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(a^5*d) + (((64*I)/5)*(a + I*a*T
an[c + d*x])^(5/2))/(a^6*d) - (((48*I)/7)*(a + I*a*Tan[c + d*x])^(7/2))/(a^
7*d) + (((16*I)/9)*(a + I*a*Tan[c + d*x])^(9/2))/(a^8*d) - (((2*I)/11)*(a +
 I*a*Tan[c + d*x])^(11/2))/(a^9*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{i\text{Subst}\left(\int (a-x)^4\sqrt{a+x}\,dx, x, ia\tan(c+dx)\right)}{a^9 d}$$

$$= -\frac{i\text{Subst}\left(\int \left(16a^4\sqrt{a+x} - 32a^3(a+x)^{3/2} + 24a^2(a+x)^{5/2} - 8a(a+x)^{7/2} + (a+x)^{9/2}\right)\,dx, x, i}{a^9 d}$$

$$= -\frac{32i(a+ia\tan(c+dx))^{3/2}}{3a^5 d} + \frac{64i(a+ia\tan(c+dx))^{5/2}}{5a^6 d} - \frac{48i(a+ia\tan(c+dx))^{7/2}}{7a^7 d} + \frac{16i(a+ia\tan(c+dx))^{9/2}}{9a^8 d} - \frac{2i(a+ia\tan(c+dx))^{11/2}}{11a^9 d}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.55

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{2(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}(5419-6396i\tan(c+dx)-4530}{3465a^4 d}$$

```
[In] Integrate[Sec[c + d*x]^10/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (2*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]]*(5419 - (6396*I)*Tan[c +
d*x] - 4530*Tan[c + d*x]^2 + (1820*I)*Tan[c + d*x]^3 + 315*Tan[c + d*x]^4))
/(3465*a^4*d)
```

Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.69

method	result
derivativedivides	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}+\frac{8a(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}-\frac{24a^2(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}+\frac{32a^3(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{16a^4(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}\right.}{d\,a^9}$
default	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{11}{2}}}{11}+\frac{8a(a+ia\tan(dx+c))^{\frac{9}{2}}}{9}-\frac{24a^2(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}+\frac{32a^3(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}-\frac{16a^4(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}\right.}{d\,a^9}$

```
[In] int(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/d/a^9*(-1/11*(a+I*a*tan(d*x+c))^(11/2)+8/9*a*(a+I*a*tan(d*x+c))^(9/2)-2
4/7*a^2*(a+I*a*tan(d*x+c))^(7/2)+32/5*a^3*(a+I*a*tan(d*x+c))^(5/2)-16/3*a^4
*(a+I*a*tan(d*x+c))^(3/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.10

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx =$$
$$-\frac{64\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(128i\,e^{(11i\,dx+11i\,c)}+704i\,e^{(9i\,dx+9i\,c)}+1584i\,e^{(7i\,dx+7i\,c)}+1848i\,e^{(5i\,dx+5i\,c)}+1155i\,e^{(3i\,dx+\cdots}\right.}{3465\left(a^4de^{(10i\,dx+10i\,c)}+5\,a^4de^{(8i\,dx+8i\,c)}+10\,a^4de^{(6i\,dx+6i\,c)}+10\,a^4de^{(4i\,dx+4i\,c)}+5\,a^4de^{(2i\,dx+2i\,c)}+a^4d\right)}$$

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -64/3465*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(128*I*e^(11*I*d*x + 11*
I*c) + 704*I*e^(9*I*d*x + 9*I*c) + 1584*I*e^(7*I*d*x + 7*I*c) + 1848*I*e^(5
*I*d*x + 5*I*c) + 1155*I*e^(3*I*d*x + 3*I*c))/(a^4*d*e^(10*I*d*x + 10*I*c)
+ 5*a^4*d*e^(8*I*d*x + 8*I*c) + 10*a^4*d*e^(6*I*d*x + 6*I*c) + 10*a^4*d*e^(
4*I*d*x + 4*I*c) + 5*a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sec(d*x+c)**10/(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.64

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{2i\left(315\,(i\,a\tan\left(dx+c\right)+a)^{\frac{11}{2}} - 3080\,(i\,a\tan\left(dx+c\right)+a)^{\frac{9}{2}}a + 11880\,(i\,a\tan\left(dx+c\right)+a)^{\frac{7}{2}}a^2 - 22176\right.}{3465\,a^9d}$$

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
[Out] -2/3465*I*(315*(I*a*tan(d*x + c) + a)^(11/2) - 3080*(I*a*tan(d*x + c) + a)^
(9/2)*a + 11880*(I*a*tan(d*x + c) + a)^(7/2)*a^2 - 22176*(I*a*tan(d*x + c)
+ a)^(5/2)*a^3 + 18480*(I*a*tan(d*x + c) + a)^(3/2)*a^4)/(a^9*d)
```

Giac [F]

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\sec\left(dx+c\right)^{10}}{(i\,a\tan\left(dx+c\right)+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)^10/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^10/(I*a*tan(d*x + c) + a)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 370, normalized size of antiderivative = 2.53

$$\int \frac{\sec^{10}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,8192\mathrm{i}}{3465\,a^4\,d} - \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,4096\mathrm{i}}{3465\,a^4\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)} - \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,1024\mathrm{i}}{1155\,a^4\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2} - \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,512\mathrm{i}}{693\,a^4\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3} - \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{99\,a^4\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^4} + \frac{\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,64\mathrm{i}}{11\,a^4\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^5}$$

```
[In] int(1/(cos(c + d*x)^10*(a + a*tan(c + d*x)*1i)^(7/2)),x)
[Out] ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*6
4i)/(11*a^4*d*(exp(c*2i + d*x*2i) + 1)^5) - ((a - (a*(exp(c*2i + d*x*2i)*1i
```

```
 - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*4096i)/(3465*a^4*d*(exp(c*2i + d
*x*2i) + 1)) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i
) + 1))^(1/2)*1024i)/(1155*a^4*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (a*(ex
p(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*512i)/(693*a^
4*d*(exp(c*2i + d*x*2i) + 1)^3) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)
/(exp(c*2i + d*x*2i) + 1))^(1/2)*64i)/(99*a^4*d*(exp(c*2i + d*x*2i) + 1)^4)
 - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2
)*8192i)/(3465*a^4*d)
```

3.379 $\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 113

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{16i\sqrt{a+ia\tan(c+dx)}}{a^4d} + \frac{8i(a+ia\tan(c+dx))^{3/2}}{a^5d} - \frac{12i(a+ia\tan(c+dx))^{5/2}}{5a^6d} + \frac{2i(a+ia\tan(c+dx))^{7/2}}{7a^7d}$$

```
[Out] -16*I*(a+I*a*tan(d*x+c))^(1/2)/a^4/d+8*I*(a+I*a*tan(d*x+c))^(3/2)/a^5/d-12/
5*I*(a+I*a*tan(d*x+c))^(5/2)/a^6/d+2/7*I*(a+I*a*tan(d*x+c))^(7/2)/a^7/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3568, 45}

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{2i(a+ia\tan(c+dx))^{7/2}}{7a^7d} - \frac{12i(a+ia\tan(c+dx))^{5/2}}{5a^6d} + \frac{8i(a+ia\tan(c+dx))^{3/2}}{a^5d} - \frac{16i\sqrt{a+ia\tan(c+dx)}}{a^4d}$$

```
[In] Int[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((-16*I)*Sqrt[a + I*a*Tan[c + d*x]])/(a^4*d) + ((8*I)*(a + I*a*Tan[c + d*x]
)^(3/2))/(a^5*d) - (((12*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(a^6*d) + (((2
*I)/7)*(a + I*a*Tan[c + d*x])^(7/2))/(a^7*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{i\text{Subst}\left(\int \frac{(a-x)^3}{\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{a^7 d}$$

$$= -\frac{i\text{Subst}\left(\int \left(\frac{8a^3}{\sqrt{a+x}} - 12a^2\sqrt{a+x} + 6a(a+x)^{3/2} - (a+x)^{5/2}\right) dx, x, ia\tan(c+dx)\right)}{a^7 d}$$

$$= -\frac{16i\sqrt{a+ia\tan(c+dx)}}{a^4 d} + \frac{8i(a+ia\tan(c+dx))^{3/2}}{a^5 d} - \frac{12i(a+ia\tan(c+dx))^{5/2}}{5a^6 d} + \frac{2i(a+ia\tan(c+dx))^{7/2}}{7a^7 d}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.54

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{2\sqrt{a+ia\tan(c+dx)}(-177i - 71\tan(c+dx) + 27i\tan^2(c+dx) + 5\tan^3(c}{35a^4 d}$$

```
[In] Integrate[Sec[c + d*x]^8/(a + I*a*Tan[c + d*x])^(7/2),x]
[Out] (2*Sqrt[a + I*a*Tan[c + d*x]]*(-177*I - 71*Tan[c + d*x] + (27*I)*Tan[c + d*
x]^2 + 5*Tan[c + d*x]^3))/(35*a^4*d)
```

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{6a(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}+4a^{2}(a+ia\tan(dx+c))^{\frac{3}{2}}-8a^{3}\sqrt{a+ia\tan(dx+c)}\right)}{d\,a^{7}}$	82
default	$\frac{2i\left(\frac{(a+ia\tan(dx+c))^{\frac{7}{2}}}{7}-\frac{6a(a+ia\tan(dx+c))^{\frac{5}{2}}}{5}+4a^{2}(a+ia\tan(dx+c))^{\frac{3}{2}}-8a^{3}\sqrt{a+ia\tan(dx+c)}\right)}{d\,a^{7}}$	82

```
[In] int(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
[Out] 2*I/d/a^7*(1/7*(a+I*a*tan(d*x+c))^(7/2)-6/5*a*(a+I*a*tan(d*x+c))^(5/2)+4*a^
2*(a+I*a*tan(d*x+c))^(3/2)-8*a^3*(a+I*a*tan(d*x+c))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.05

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{16\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(16i\,e^{(7i\,dx+7i\,c)}+56i\,e^{(5i\,dx+5i\,c)}+70i\,e^{(3i\,dx+3i\,c)}+35i\,e^{(i\,dx+i\,c)}\right)}{35\left(a^4de^{(6i\,dx+6i\,c)}+3\,a^4de^{(4i\,dx+4i\,c)}+3\,a^4de^{(2i\,dx+2i\,c)}+a^4d\right)}$$

```
[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
[Out] -16/35*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(16*I*e^(7*I*d*x + 7*I*c)
+ 56*I*e^(5*I*d*x + 5*I*c) + 70*I*e^(3*I*d*x + 3*I*c) + 35*I*e^(I*d*x + I*c
))/(a^4*d*e^(6*I*d*x + 6*I*c) + 3*a^4*d*e^(4*I*d*x + 4*I*c) + 3*a^4*d*e^(2*
I*d*x + 2*I*c) + a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sec(d*x+c)**8/(a+I*a*tan(d*x+c))**(7/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.67

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{2i\left(5\,(i\,a\tan\left(dx+c\right)+a)^{\frac{7}{2}}-42\,(i\,a\tan\left(dx+c\right)+a)^{\frac{5}{2}}a+140\,(i\,a\tan\left(dx+\right.\right.}{35\,a^7d}$$

[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")

[Out] 2/35*I*(5*(I*a*tan(d*x + c) + a)^(7/2) - 42*(I*a*tan(d*x + c) + a)^(5/2)*a + 140*(I*a*tan(d*x + c) + a)^(3/2)*a^2 - 280*sqrt(I*a*tan(d*x + c) + a)*a^3)/(a^7*d)

Giac [F]

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\sec\left(dx+c\right)^8}{(i\,a\tan\left(dx+c\right)+a)^{\frac{7}{2}}}\,dx$$

[In] integrate(sec(d*x+c)^8/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")

[Out] integrate(sec(d*x + c)^8/(I*a*tan(d*x + c) + a)^(7/2), x)

Mupad [B] (verification not implemented)

Time = 7.49 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.14

$$\int \frac{\sec^8(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,256\mathrm{i}}{35\,a^4\,d} - \frac{\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,128\mathrm{i}}{35\,a^4\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)} - \frac{\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,96\mathrm{i}}{35\,a^4\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2} - \frac{\sqrt{a-\frac{a\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,16\mathrm{i}}{7\,a^4\,d\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}$$

[In] int(1/(cos(c + d*x)^8*(a + a*tan(c + d*x)*1i)^(7/2)),x)

[Out] - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*256i)/(35*a^4*d) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*128i)/(35*a^4*d*(exp(c*2i + d*x*2i) + 1)) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*96i)/(35*a^4*d*(exp(c*2i + d*x*2i) + 1)^2) - ((a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*16i)/(7*a^4*d*(exp(c*2i + d*x*2i) + 1)^3)

3.380 $\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 84

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{8i}{a^3d\sqrt{a+ia\tan(c+dx)}} + \frac{8i\sqrt{a+ia\tan(c+dx)}}{a^4d} - \frac{2i(a+ia\tan(c+dx))^{3/2}}{3a^5d}$$

```
[Out] 8*I/a^3/d/(a+I*a*tan(d*x+c))^(1/2)+8*I*(a+I*a*tan(d*x+c))^(1/2)/a^4/d-2/3*I
*(a+I*a*tan(d*x+c))^(3/2)/a^5/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3568, 45}

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{2i(a+ia\tan(c+dx))^{3/2}}{3a^5d} + \frac{8i\sqrt{a+ia\tan(c+dx)}}{a^4d} + \frac{8i}{a^3d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (8*I)/(a^3*d*Sqrt[a + I*a*Tan[c + d*x]]) + ((8*I)*Sqrt[a + I*a*Tan[c + d*x]
])/(a^4*d) - (((2*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(a^5*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{i\text{Subst}\left(\int \frac{(a-x)^2}{(a+x)^{3/2}}\,dx, x, ia\tan(c+dx)\right)}{a^5 d}\\
&= -\frac{i\text{Subst}\left(\int \left(\frac{4a^2}{(a+x)^{3/2}} - \frac{4a}{\sqrt{a+x}} + \sqrt{a+x}\right)dx, x, ia\tan(c+dx)\right)}{a^5 d}\\
&= \frac{8i}{a^3 d\sqrt{a+ia\tan(c+dx)}} + \frac{8i\sqrt{a+ia\tan(c+dx)}}{a^4 d} - \frac{2i(a+ia\tan(c+dx))^{3/2}}{3a^5 d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.58

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{2i(23+10i\tan(c+dx)+\tan^2(c+dx))}{3a^3 d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[Sec[c + d*x]^6/(a + I*a*Tan[c + d*x])^(7/2),x]
[Out] (((2*I)/3)*(23 + (10*I)*Tan[c + d*x] + Tan[c + d*x]^2))/(a^3*d*Sqrt[a + I*a
*Tan[c + d*x]])
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+4a\sqrt{a+ia\tan(dx+c)}+\frac{4a^2}{\sqrt{a+ia\tan(dx+c)}}\right)}{d\,a^5}$	63
default	$\frac{2i\left(-\frac{(a+ia\tan(dx+c))^{\frac{3}{2}}}{3}+4a\sqrt{a+ia\tan(dx+c)}+\frac{4a^2}{\sqrt{a+ia\tan(dx+c)}}\right)}{d\,a^5}$	63

```
[In] int(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I/d/a^5*(-1/3*(a+I*a*tan(d*x+c))^(3/2)+4*a*(a+I*a*tan(d*x+c))^(1/2)+4*a^2
/(a+I*a*tan(d*x+c))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{4\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-8i\,e^{(4i\,dx+4i\,c)}-12i\,e^{(2i\,dx+2i\,c)}-3i\right)}{3\left(a^4de^{(3i\,dx+3i\,c)}+a^4de^{(i\,dx+i\,c)}\right)}$$

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -4/3*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-8*I*e^(4*I*d*x + 4*I*c) -
12*I*e^(2*I*d*x + 2*I*c) - 3*I)/(a^4*d*e^(3*I*d*x + 3*I*c) + a^4*d*e^(I*d*x
 + I*c))
```

Sympy [F]

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\sec^6{(c+dx)}}{(ia(\tan{(c+dx)}-i))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)**6/(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Integral(sec(c + d*x)**6/(I*a*(tan(c + d*x) - I))**(7/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.74

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{2i\left(\frac{12}{\sqrt{i\,a\tan(dx+c)+a}a^2}-\frac{(i\,a\tan(dx+c)+a)^{\frac{3}{2}}-12\sqrt{i\,a\tan(dx+c)+a}a}{a^4}\right)}{3\,ad}$$

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 2/3*I*(12/(sqrt(I*a*tan(d*x + c) + a)*a^2) - ((I*a*tan(d*x + c) + a)^(3/2)
- 12*sqrt(I*a*tan(d*x + c) + a)*a)/a^4)/(a*d)
```

Giac [F]

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\sec(dx+c)^6}{(i\,a\tan(dx+c)+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)^6/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^6/(I*a*tan(d*x + c) + a)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.31

$$\int \frac{\sec^6(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{2\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,(\cos(2\,c+2\,d\,x)\,23\mathrm{i}+\cos(4\,c+4\,d\,x)\,3\mathrm{i}+ \ldots}{3\,a^4\,d\,(\cos(2\,c+2\,d\,x)+1)}$$

```
[In] int(1/(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^(7/2)),x)
```

```
[Out] (2*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1)
)^(1/2)*(cos(2*c + 2*d*x)*23i + cos(4*c + 4*d*x)*3i + 7*sin(2*c + 2*d*x) +
3*sin(4*c + 4*d*x) + 20i))/(3*a^4*d*(cos(2*c + 2*d*x) + 1))
```

3.381 $\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 57

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=\frac{4i}{3a^2d(a+ia\tan(c+dx))^{3/2}}-\frac{2i}{a^3d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] -2*I/a^3/d/(a+I*a*tan(d*x+c))^(1/2)+4/3*I/a^2/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3568, 45}

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=\frac{4i}{3a^2d(a+ia\tan(c+dx))^{3/2}}-\frac{2i}{a^3d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((4*I)/3)/(a^2*d*(a + I*a*Tan[c + d*x])^(3/2)) - (2*I)/(a^3*d*Sqrt[a + I*a*
Tan[c + d*x]])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{i\text{Subst}\left(\int \frac{a-x}{(a+x)^{5/2}}\,dx, x, ia\tan(c+dx)\right)}{a^3d} \\
&= -\frac{i\text{Subst}\left(\int \left(\frac{2a}{(a+x)^{5/2}} - \frac{1}{(a+x)^{3/2}}\right)\,dx, x, ia\tan(c+dx)\right)}{a^3d} \\
&= \frac{4i}{3a^2d(a+ia\tan(c+dx))^{3/2}} - \frac{2i}{a^3d\sqrt{a+ia\tan(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{i\left(-\frac{4a}{3(a+ia\tan(c+dx))^{3/2}} + \frac{2}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^3d}$$

```
[In] Integrate[Sec[c + d*x]^4/(a + I*a*Tan[c + d*x])^(7/2),x]
[Out] ((-I)*((-4*a)/(3*(a + I*a*Tan[c + d*x])^(3/2)) + 2/Sqrt[a + I*a*Tan[c + d*x
]]))/(a^3*d)
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{2i\left(-\frac{1}{\sqrt{a+ia\tan(dx+c)}}+\frac{2a}{3(a+ia\tan(dx+c))^{\frac{3}{2}}}\right)}{d\,a^3}$	44
default	$\frac{2i\left(-\frac{1}{\sqrt{a+ia\tan(dx+c)}}+\frac{2a}{3(a+ia\tan(dx+c))^{\frac{3}{2}}}\right)}{d\,a^3}$	44

```
[In] int(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
[Out] 2*I/d/a^3*(-1/(a+I*a*tan(d*x+c))^(1/2)+2/3*a/(a+I*a*tan(d*x+c))^(3/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-2i\,e^{(4i\,dx+4i\,c)}-i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-3i\,dx-3i\,c)}}{3\,a^4d}$$

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-2*I*e^(4*I*d*x + 4*I*c) - I
*e^(2*I*d*x + 2*I*c) + I)*e^(-3*I*d*x - 3*I*c)/(a^4*d)
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\sec^4(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)**4/(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/(I*a*(tan(c + d*x) - I))**(7/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.56

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{2i\,(3i\,a\tan(dx+c)+a)}{3\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}a^3d}$$

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -2/3*I*(3*I*a*tan(d*x + c) + a)/((I*a*tan(d*x + c) + a)^(3/2)*a^3*d)
```

Giac [F]

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\sec\left(dx+c\right)^4}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^4/(I*a*tan(d*x + c) + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^4\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{7/2}}\,dx$$

```
[In] int(1/(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(7/2)),x)
[Out] int(1/(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^(7/2)), x)
```

3.382 $\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 29

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{2i}{5ad(a+ia\tan(c+dx))^{5/2}}$$

```
[Out] 2/5*I/a/d/(a+I*a*tan(d*x+c))^(5/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3568, 32}

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{2i}{5ad(a+ia\tan(c+dx))^{5/2}}$$

```
[In] Int[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((2*I)/5)/(a*d*(a + I*a*Tan[c + d*x])^(5/2))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
```

```
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{i\text{Subst}\left(\int \frac{1}{(a+x)^{7/2}}\,dx, x, ia\tan(c+dx)\right)}{ad}$$
$$= \frac{2i}{5ad(a+ia\tan(c+dx))^{5/2}}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{2i}{5ad(a+ia\tan(c+dx))^{5/2}}$$

```
[In] Integrate[Sec[c + d*x]^2/(a + I*a*Tan[c + d*x])^(7/2),x]
[Out] ((2*I)/5)/(a*d*(a + I*a*Tan[c + d*x])^(5/2))
```

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2i}{5ad(a+ia\tan(dx+c))^{\frac{5}{2}}}$	24
default	$\frac{2i}{5ad(a+ia\tan(dx+c))^{\frac{5}{2}}}$	24

```
[In] int(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
[Out] 2/5*I/a/d/(a+I*a*tan(d*x+c))^(5/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(21) = 42$.

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.48

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(i\,e^{(6i\,dx+6i\,c)}+3i\,e^{(4i\,dx+4i\,c)}+3i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-5i\,dx-5i\,c)}}{20\,a^4d}$$

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
[Out] 1/20*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(I*e^(6*I*d*x + 6*I*c) + 3*I
*e^(4*I*d*x + 4*I*c) + 3*I*e^(2*I*d*x + 2*I*c) + I)*e^(-5*I*d*x - 5*I*c)/(a
^4*d)
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\sec^2(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)**2/(a+I*a*tan(d*x+c))**(7/2),x)
[Out] Integral(sec(c + d*x)**2/(I*a*(tan(c + d*x) - I))**(7/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{2i}{5\,(i\,a\tan(dx+c)+a)^{\frac{5}{2}}ad}$$

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
[Out] 2/5*I/((I*a*tan(d*x + c) + a)^(5/2)*a*d)
```

Giac [F]

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\sec\left(dx+c\right)^2}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^2/(I*a*tan(d*x + c) + a)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 4.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{\sec^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{2\mathrm{i}}{5\,a\,d\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{5/2}}$$

```
[In] int(1/(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^(7/2)),x)
[Out] 2i/(5*a*d*(a + a*tan(c + d*x)*1i)^(5/2))
```

3.383 $\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 233

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = & -\frac{11i\,\mathrm{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}a^{7/2}d} \\ & + \frac{11ia}{36d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}} \\ & + \frac{11i}{56d(a+ia\tan(c+dx))^{7/2}} + \frac{11i}{80ad(a+ia\tan(c+dx))^{5/2}} \\ & + \frac{11i}{96a^2d(a+ia\tan(c+dx))^{3/2}} + \frac{11i}{64a^3d\sqrt{a+ia\tan(c+dx)}}\end{aligned}$$

```
[Out] -11/128*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(7/2)/d*2
^(1/2)+11/64*I/a^3/d/(a+I*a*tan(d*x+c))^(1/2)+11/36*I*a/d/(a+I*a*tan(d*x+c)
)^(9/2)-1/2*I*a^2/d/(a-I*a*tan(d*x+c))/(a+I*a*tan(d*x+c))^(9/2)+11/56*I/d/(
a+I*a*tan(d*x+c))^(7/2)+11/80*I/a/d/(a+I*a*tan(d*x+c))^(5/2)+11/96*I/a^2/d/
(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used

$= \{3568, 44, 53, 65, 212\}$

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{11i\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}a^{7/2}d} + \frac{11i}{64a^3d\sqrt{a+ia\tan(c+dx)}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}} + \frac{11i}{96a^2d(a+ia\tan(c+dx))^{3/2}} + \frac{11ia}{36d(a+ia\tan(c+dx))^{9/2}} + \frac{11i}{56d(a+ia\tan(c+dx))^{7/2}} + \frac{11i}{80ad(a+ia\tan(c+dx))^{5/2}}$$

```
[In] Int[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((-11*I)/64)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sqrt[
2]*a^(7/2)*d) + (((11*I)/36)*a)/(d*(a + I*a*Tan[c + d*x])^(9/2)) - ((I/2)*a
^2)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(9/2)) + ((11*I)/56)/(
d*(a + I*a*Tan[c + d*x])^(7/2)) + ((11*I)/80)/(a*d*(a + I*a*Tan[c + d*x])^(
5/2)) + ((11*I)/96)/(a^2*d*(a + I*a*Tan[c + d*x])^(3/2)) + ((11*I)/64)/(a^3
*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(ia^3)\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{11/2}}\,dx, x, ia\tan(c+dx)\right)}{d} \\
&= -\frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}} \\
&\quad -\frac{(11ia^2)\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{11/2}}\,dx, x, ia\tan(c+dx)\right)}{4d} \\
&= \frac{11ia}{36d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}} \\
&\quad -\frac{(11ia)\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{9/2}}\,dx, x, ia\tan(c+dx)\right)}{8d} \\
&= \frac{11ia}{36d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}} \\
&\quad + \frac{11i}{56d(a+ia\tan(c+dx))^{7/2}} - \frac{(11i)\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{7/2}}\,dx, x, ia\tan(c+dx)\right)}{16d} \\
&= \frac{11ia}{36d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}} \\
&\quad + \frac{11i}{56d(a+ia\tan(c+dx))^{7/2}} + \frac{11i}{80ad(a+ia\tan(c+dx))^{5/2}} \\
&\quad - \frac{(11i)\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{5/2}}\,dx, x, ia\tan(c+dx)\right)}{32ad} \\
&= \frac{11ia}{36d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}} \\
&\quad + \frac{11i}{56d(a+ia\tan(c+dx))^{7/2}} + \frac{11i}{80ad(a+ia\tan(c+dx))^{5/2}} \\
&\quad + \frac{11i}{96a^2d(a+ia\tan(c+dx))^{3/2}} - \frac{(11i)\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{3/2}}\,dx, x, ia\tan(c+dx)\right)}{64a^2d}
\end{aligned}
$$

$$= \frac{11ia}{36d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}} + \frac{11i}{56d(a+ia\tan(c+dx))^{7/2}} + \frac{11i}{80ad(a+ia\tan(c+dx))^{5/2}} + \frac{11i}{96a^2d(a+ia\tan(c+dx))^{3/2}} + \frac{11i}{64a^3d\sqrt{a+ia\tan(c+dx)}} - \frac{(11i)\text{Subst}\left(\int \frac{1}{(a-x)\sqrt{a+x}}\,dx, x, ia\tan(c+dx)\right)}{128a^3d}$$

$$= \frac{11ia}{36d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}} + \frac{11i}{56d(a+ia\tan(c+dx))^{7/2}} + \frac{11i}{80ad(a+ia\tan(c+dx))^{5/2}} + \frac{11i}{96a^2d(a+ia\tan(c+dx))^{3/2}} + \frac{11i}{64a^3d\sqrt{a+ia\tan(c+dx)}} - \frac{(11i)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \sqrt{a+ia\tan(c+dx)}\right)}{64a^3d}$$

$$= -\frac{11i\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{64\sqrt{2}a^{7/2}d} + \frac{11ia}{36d(a+ia\tan(c+dx))^{9/2}} - \frac{ia^2}{2d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{9/2}} + \frac{11i}{56d(a+ia\tan(c+dx))^{7/2}} + \frac{11i}{80ad(a+ia\tan(c+dx))^{5/2}} + \frac{11i}{96a^2d(a+ia\tan(c+dx))^{3/2}} + \frac{11i}{64a^3d\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.44 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.22

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{ia\,\text{Hypergeometric2F1}\left(-\frac{9}{2}, 2, -\frac{7}{2}, \frac{1}{2}(1+i\tan(c+dx))\right)}{18d(a+ia\tan(c+dx))^{9/2}}$$

```
[In] Integrate[Cos[c + d*x]^2/(a + I*a*Tan[c + d*x])^(7/2),x]
[Out] ((I/18)*a*Hypergeometric2F1[-9/2, 2, -7/2, (1 + I*Tan[c + d*x])/2])/(d*(a +
 I*a*Tan[c + d*x])^(9/2))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 976 vs. 2(183) = 366.

Time = 10.36 (sec) , antiderivative size = 977, normalized size of antiderivative = 4.19

method	result	size
default	Expression too large to display	977

```
[In] int(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/40320/d/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)+1)/(1+I*tan(d*x+c
))^3/(a*(1+I*tan(d*x+c)))^(1/2)/a^3*(-50424*I*cos(d*x+c)*(-cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)-13860*I*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)
+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-12320*sin(d*x+c)*cos(d*x+c)^2*(-cos
(d*x+c)/(cos(d*x+c)+1))^(1/2)+7840*I*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+
1))^(1/2)-3465*I*sec(d*x+c)^3*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d
*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-12320*(-cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)*cos(d*x+c)*sin(d*x+c)-50424*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
+25410*I*sec(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+42504*sin(d*x+c)*(
-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+27720*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+
c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+27720*I*s
ec(d*x+c)*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c
)/(cos(d*x+c)+1))^(1/2))+25410*I*sec(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1
/2)+42504*tan(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+13860*tan(d*x+c)*ar
ctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c
)+1))^(1/2))-27720*I*cos(d*x+c)*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos
(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+7840*I*cos(d*x+c)^3*(-cos(d*
x+c)/(cos(d*x+c)+1))^(1/2)-6930*tan(d*x+c)*sec(d*x+c)*(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)-13860*tan(d*x+c)*sec(d*x+c)*arctan(1/2*(cos(d*x+c)+1+I*sin(d*
x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+10395*I*sec(d*x+c)
^2*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(
d*x+c)+1))^(1/2))-6930*tan(d*x+c)*sec(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))
^(1/2)-3465*tan(d*x+c)*sec(d*x+c)^2*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/
(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.36

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{\left(-3465i\,\sqrt{\frac{1}{2}}a^4d\sqrt{\frac{1}{a^7d^2}}e^{(9i\,dx+9i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^4de^{(2i\,dx+2i\,c)}+a^4d\right)\sqrt{\frac{}{e^{(2i}}}\right.\right.\right.}{}$$

[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")

[Out] 1/40320*(-3465*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(9*I*d*x + 9*I*c)*log(4*(sqrt(2)*sqrt(1/2)*(a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + 3465*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(9*I*d*x + 9*I*c)*log(-4*(sqrt(2)*sqrt(1/2)*(a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-315*I*e^(12*I*d*x + 12*I*c) + 4303*I*e^(10*I*d*x + 10*I*c) + 7034*I*e^(8*I*d*x + 8*I*c) + 3754*I*e^(6*I*d*x + 6*I*c) + 1798*I*e^(4*I*d*x + 4*I*c) + 530*I*e^(2*I*d*x + 2*I*c) + 70*I))*e^(-9*I*d*x - 9*I*c)/(a^4*d)

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**2/(a+I*a*tan(d*x+c))**(7/2),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.84

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{i\left(\frac{4\left(3465\,(i\,a\tan(dx+c)+a)^5-4620\,(i\,a\tan(dx+c)+a)^4a-1848\,(i\,a\tan(dx+c)+a)^3a^2-1584\,(i\,a\tan(dx \ldots\right.}{(i\,a\tan(dx+c)+a)^{\frac{11}{2}}a^2-2\,(i\,a\tan(dx+c)+a)^{\frac{9}{2}}a^3 \ldots}\right.}{80640\,a \ldots}$$

[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")

[Out] 1/80640*I*(4*(3465*(I*a*tan(d*x + c) + a)^5 - 4620*(I*a*tan(d*x + c) + a)^4*a - 1848*(I*a*tan(d*x + c) + a)^3*a^2 - 1584*(I*a*tan(d*x + c) + a)^2*a^3 - 1760*(I*a*tan(d*x + c) + a)*a^4 - 2240*a^5)/((I*a*tan(d*x + c) + a)^(11/2)*a^2 - 2*(I*a*tan(d*x + c) + a)^(9/2)*a^3) + 3465*sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/a^(5/2))/(a*d)

Giac [F]

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\cos\left(dx+c\right)^2}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)^2/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate(cos(d*x + c)^2/(I*a*tan(d*x + c) + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^2}{\left(a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)^{7/2}}\,dx$$

```
[In] int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(7/2),x)
[Out] int(cos(c + d*x)^2/(a + a*tan(c + d*x)*1i)^(7/2), x)
```

3.384 $\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 306

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{195i\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{1024\sqrt{2}a^{7/2}d} + \frac{195ia^2}{352d(a+ia\tan(c+dx))^{11/2}} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{11/2}} - \frac{15ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{11/2}} + \frac{65ia}{192d(a+ia\tan(c+dx))^{9/2}} + \frac{195i}{896d(a+ia\tan(c+dx))^{7/2}} + \frac{39i}{256ad(a+ia\tan(c+dx))^{5/2}} + \frac{65i}{512a^2d(a+ia\tan(c+dx))^{3/2}} + \frac{195i}{1024a^3d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] -195/2048*I*arctanh(1/2*(a+I*a*tan(d*x+c))^(1/2)*2^(1/2)/a^(1/2))/a^(7/2)/d
*2^(1/2)+195/1024*I/a^3/d/(a+I*a*tan(d*x+c))^(1/2)+195/352*I*a^2/d/(a+I*a*t
an(d*x+c))^(11/2)-1/4*I*a^4/d/(a-I*a*tan(d*x+c))^2/(a+I*a*tan(d*x+c))^(11/2
)-15/16*I*a^3/d/(a-I*a*tan(d*x+c))/(a+I*a*tan(d*x+c))^(11/2)+65/192*I*a/d/(
a+I*a*tan(d*x+c))^(9/2)+195/896*I/d/(a+I*a*tan(d*x+c))^(7/2)+39/256*I/a/d/(
a+I*a*tan(d*x+c))^(5/2)+65/512*I/a^2/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3568, 44, 53, 65, 212}

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{195i\operatorname{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{1024\sqrt{2}a^{7/2}d} - \frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{11/2}} - \frac{15ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{11/2}} + \frac{195i}{1024a^3d\sqrt{a+ia\tan(c+dx)}} + \frac{195ia^2}{352d(a+ia\tan(c+dx))^{11/2}} + \frac{65i}{512a^2d(a+ia\tan(c+dx))^{3/2}} + \frac{65ia}{192d(a+ia\tan(c+dx))^{9/2}} + \frac{195i}{896d(a+ia\tan(c+dx))^{7/2}} + \frac{39i}{256ad(a+ia\tan(c+dx))^{5/2}}$$

```
[In] Int[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((-195*I)/1024)*ArcTanh[Sqrt[a + I*a*Tan[c + d*x]]/(Sqrt[2]*Sqrt[a])])/(Sq
rt[2]*a^(7/2)*d) + (((195*I)/352)*a^2)/(d*(a + I*a*Tan[c + d*x])^(11/2)) -
((I/4)*a^4)/(d*(a - I*a*Tan[c + d*x])^2*(a + I*a*Tan[c + d*x])^(11/2)) - ((
(15*I)/16)*a^3)/(d*(a - I*a*Tan[c + d*x])*(a + I*a*Tan[c + d*x])^(11/2)) +
(((65*I)/192)*a)/(d*(a + I*a*Tan[c + d*x])^(9/2)) + ((195*I)/896)/(d*(a + I
*a*Tan[c + d*x])^(7/2)) + ((39*I)/256)/(a*d*(a + I*a*Tan[c + d*x])^(5/2)) +
 ((65*I)/512)/(a^2*d*(a + I*a*Tan[c + d*x])^(3/2)) + ((195*I)/1024)/(a^3*d*
Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{(ia^5)\,\text{Subst}\left(\int \frac{1}{(a-x)^3(a+x)^{13/2}}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= -\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{11/2}} - \frac{(15ia^4)\,\text{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^{13/2}}\,dx, x, ia\tan(c+dx)\right)}{8d}$$

$$= -\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{11/2}} - \frac{15ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{11/2}} - \frac{(195ia^3)\,\text{Subst}\left(\int \frac{1}{(a-x)(a+x)^{13/2}}\,dx, x, ia\tan(c+dx)\right)}{32d}$$

$$=\frac{195ia^2}{352d(a+ia\tan(c+dx))^{11/2}}-\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{11/2}}-\frac{15ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{11/2}}-\frac{(195ia^2)\operatorname{Subst}\left(\int\frac{1}{(a-x)(a+x)^{11/2}}\,dx,x,ia\tan(c+dx)\right)}{64d}$$

$$=\frac{195ia^2}{352d(a+ia\tan(c+dx))^{11/2}}-\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{11/2}}-\frac{15ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{11/2}}+\frac{65ia}{192d(a+ia\tan(c+dx))^{9/2}}-\frac{(195ia)\operatorname{Subst}\left(\int\frac{1}{(a-x)(a+x)^{9/2}}\,dx,x,ia\tan(c+dx)\right)}{128d}$$

$$=\frac{195ia^2}{352d(a+ia\tan(c+dx))^{11/2}}-\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{11/2}}-\frac{15ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{11/2}}+\frac{65ia}{192d(a+ia\tan(c+dx))^{9/2}}+\frac{195i}{896d(a+ia\tan(c+dx))^{7/2}}-\frac{(195i)\operatorname{Subst}\left(\int\frac{1}{(a-x)(a+x)^{7/2}}\,dx,x,ia\tan(c+dx)\right)}{256d}$$

$$=\frac{195ia^2}{352d(a+ia\tan(c+dx))^{11/2}}-\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{11/2}}-\frac{15ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{11/2}}+\frac{65ia}{192d(a+ia\tan(c+dx))^{9/2}}+\frac{195i}{896d(a+ia\tan(c+dx))^{7/2}}+\frac{39i}{256ad(a+ia\tan(c+dx))^{5/2}}-\frac{(195i)\operatorname{Subst}\left(\int\frac{1}{(a-x)(a+x)^{5/2}}\,dx,x,ia\tan(c+dx)\right)}{512ad}$$

$$=\frac{195ia^2}{352d(a+ia\tan(c+dx))^{11/2}}-\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{11/2}}-\frac{15ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{11/2}}+\frac{65ia}{192d(a+ia\tan(c+dx))^{9/2}}+\frac{195i}{896d(a+ia\tan(c+dx))^{7/2}}+\frac{39i}{256ad(a+ia\tan(c+dx))^{5/2}}+\frac{65i}{512a^2d(a+ia\tan(c+dx))^{3/2}}-\frac{(195i)\operatorname{Subst}\left(\int\frac{1}{(a-x)(a+x)^{3/2}}\,dx,x,ia\tan(c+dx)\right)}{1024a^2d}$$

$$=\frac{195ia^2}{352d(a+ia\tan(c+dx))^{11/2}}-\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{11/2}}-\frac{15ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{11/2}}+\frac{65ia}{192d(a+ia\tan(c+dx))^{9/2}}+\frac{195i}{896d(a+ia\tan(c+dx))^{7/2}}+\frac{39i}{256ad(a+ia\tan(c+dx))^{5/2}}+\frac{65i}{512a^2d(a+ia\tan(c+dx))^{3/2}}+\frac{195i}{1024a^3d\sqrt{a+ia\tan(c+dx)}}-\frac{(195i)\text{Subst}\left(\int\frac{1}{(a-x)\sqrt{a+x}}\,dx,x,ia\tan(c+dx)\right)}{2048a^3d}$$

$$=\frac{195ia^2}{352d(a+ia\tan(c+dx))^{11/2}}-\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{11/2}}-\frac{15ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{11/2}}+\frac{65ia}{192d(a+ia\tan(c+dx))^{9/2}}+\frac{195i}{896d(a+ia\tan(c+dx))^{7/2}}+\frac{39i}{256ad(a+ia\tan(c+dx))^{5/2}}+\frac{65i}{512a^2d(a+ia\tan(c+dx))^{3/2}}+\frac{195i}{1024a^3d\sqrt{a+ia\tan(c+dx)}}-\frac{(195i)\text{Subst}\left(\int\frac{1}{2a-x^2}\,dx,x,\sqrt{a+ia\tan(c+dx)}\right)}{1024a^3d}$$

$$=-\frac{195i\text{arctanh}\left(\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{2}\sqrt{a}}\right)}{1024\sqrt{2}a^{7/2}d}+\frac{195ia^2}{352d(a+ia\tan(c+dx))^{11/2}}-\frac{ia^4}{4d(a-ia\tan(c+dx))^2(a+ia\tan(c+dx))^{11/2}}-\frac{15ia^3}{16d(a-ia\tan(c+dx))(a+ia\tan(c+dx))^{11/2}}+\frac{65ia}{192d(a+ia\tan(c+dx))^{9/2}}+\frac{195i}{896d(a+ia\tan(c+dx))^{7/2}}+\frac{39i}{256ad(a+ia\tan(c+dx))^{5/2}}+\frac{65i}{512a^2d(a+ia\tan(c+dx))^{3/2}}+\frac{195i}{1024a^3d\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.72 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.17

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{ia^2 \text{Hypergeometric2F1}\left(-\frac{11}{2},3,-\frac{9}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{44d(a+ia\tan(c+dx))^{11/2}}$$

```
[In] Integrate[Cos[c + d*x]^4/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((I/44)*a^2*Hypergeometric2F1[-11/2, 3, -9/2, (1 + I*Tan[c + d*x])/2])/(d*(
a + I*a*Tan[c + d*x])^(11/2))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1110 vs. 2(243) = 486.

Time = 9.97 (sec) , antiderivative size = 1111, normalized size of antiderivative = 3.63

method	result	size
default	Expression too large to display	1111

```
[In] int(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/473088/d/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)+1)/(a*(1+I*tan(d*
x+c)))^(1/2)/(1+I*tan(d*x+c))^3/a^3*(-360360*arctan(1/2*(cos(d*x+c)+1+I*sin
(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+1801
80*tan(d*x+c)*sec(d*x+c)*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)
+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+655512*I*(-cos(d*x+c)/(cos(d*x+c)+1
))^(1/2)+180180*I*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-c
os(d*x+c)/(cos(d*x+c)+1))^(1/2))+90090*tan(d*x+c)*sec(d*x+c)*(-cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)-180180*tan(d*x+c)*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c
))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+45045*I*sec(d*x+c)^3*
arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2))-37632*I*cos(d*x+c)^5*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-3763
2*I*cos(d*x+c)^4*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-101920*I*cos(d*x+c)^3*(
-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-101920*I*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d
*x+c)+1))^(1/2)+360360*I*cos(d*x+c)*arctan(1/2*(cos(d*x+c)+1+I*sin(d*x+c))/
(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+655512*I*cos(d*x+c)*(-co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)-360360*I*sec(d*x+c)*arctan(1/2*(cos(d*x+c)+1
+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-330330*I*
sec(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-135135*I*sec(d*x+c)^2*arctan(
1/2*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))
^(1/2))-552552*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-552552*tan(d*x
```

```
+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+90090*tan(d*x+c)*sec(d*x+c)^2*(-cos(
d*x+c)/(cos(d*x+c)+1))^(1/2)+45045*tan(d*x+c)*sec(d*x+c)^2*arctan(1/2*(cos(
d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+8
0640*sin(d*x+c)*cos(d*x+c)^4*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+80640*sin(d
*x+c)*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+160160*sin(d*x+c)*cos
(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+160160*(-cos(d*x+c)/(cos(d*x+c
)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)-330330*I*sec(d*x+c)^2*(-cos(d*x+c)/(cos(d
*x+c)+1))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.10

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{\left(-45045i\sqrt{\frac{1}{2}}a^4d\sqrt{\frac{1}{a^7d^2}}e^{(11i\,dx+11i\,c)}\log\left(4\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(a^4de^{(2i\,dx+2i\,c)}+a^4d\right)\sqrt{\frac{a}{e^{(2}}}\right.\right.\right.}{}$$

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/473088*(-45045*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(11*I*d*x + 11*I*c)*
log(4*(sqrt(2)*sqrt(1/2)*(a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)*sqrt(a/(e^(2*I
*d*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) + a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)
) + 45045*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(11*I*d*x + 11*I*c)*log(-4*
(sqrt(2)*sqrt(1/2)*(a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1))*sqrt(1/(a^7*d^2)) - a*e^(I*d*x + I*c))*e^(-I*d*x - I*c)) + sq
rt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-462*I*e^(16*I*d*x + 16*I*c) - 716
1*I*e^(14*I*d*x + 14*I*c) + 47413*I*e^(12*I*d*x + 12*I*c) + 78800*I*e^(10*I
*d*x + 10*I*c) + 38512*I*e^(8*I*d*x + 8*I*c) + 19552*I*e^(6*I*d*x + 6*I*c)
+ 7184*I*e^(4*I*d*x + 4*I*c) + 1624*I*e^(2*I*d*x + 2*I*c) + 168*I))*e^(-11*
I*d*x - 11*I*c)/(a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**4/(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.81

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{i\left(\frac{4\left(45045\,(i\,a\tan(dx+c)+a)^7-150150\,(i\,a\tan(dx+c)+a)^6a+96096\,(i\,a\tan(dx+c)+a)^5a^2+27456\,(i\,\cdots\right.}{(i\,a\tan(dx+c)+a)^{\frac{15}{2}}a^2-4\,(\cdots}\right.}{}$$

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 1/946176*I*(4*(45045*(I*a*tan(d*x + c) + a)^7 - 150150*(I*a*tan(d*x + c) +
a)^6*a + 96096*(I*a*tan(d*x + c) + a)^5*a^2 + 27456*(I*a*tan(d*x + c) + a)^
4*a^3 + 18304*(I*a*tan(d*x + c) + a)^3*a^4 + 16640*(I*a*tan(d*x + c) + a)^2
*a^5 + 17920*(I*a*tan(d*x + c) + a)*a^6 + 21504*a^7)/((I*a*tan(d*x + c) + a
)^(15/2)*a^2 - 4*(I*a*tan(d*x + c) + a)^(13/2)*a^3 + 4*(I*a*tan(d*x + c) +
a)^(11/2)*a^4) + 45045*sqrt(2)*log(-(sqrt(2)*sqrt(a) - sqrt(I*a*tan(d*x + c
) + a))/(sqrt(2)*sqrt(a) + sqrt(I*a*tan(d*x + c) + a)))/a^(5/2))/(a*d)
```

Giac [F]

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\cos\left(dx+c\right)^4}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)^4/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^4/(I*a*tan(d*x + c) + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)^4}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{7/2}}\,dx$$

```
[In] int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^(7/2),x)
```

```
[Out] int(cos(c + d*x)^4/(a + a*tan(c + d*x)*1i)^(7/2), x)
```

3.385 $\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 110

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{64ia^3\sec^{13}(c+dx)}{3315d(a+ia\tan(c+dx))^{13/2}} + \frac{16ia^2\sec^{13}(c+dx)}{255d(a+ia\tan(c+dx))^{11/2}} + \frac{2ia\sec^{13}(c+dx)}{17d(a+ia\tan(c+dx))^{9/2}}$$

```
[Out] 64/3315*I*a^3*sec(d*x+c)^13/d/(a+I*a*tan(d*x+c))^(13/2)+16/255*I*a^2*sec(d*
x+c)^13/d/(a+I*a*tan(d*x+c))^(11/2)+2/17*I*a*sec(d*x+c)^13/d/(a+I*a*tan(d*x
+c))^(9/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3575, 3574}

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{64ia^3\sec^{13}(c+dx)}{3315d(a+ia\tan(c+dx))^{13/2}} + \frac{16ia^2\sec^{13}(c+dx)}{255d(a+ia\tan(c+dx))^{11/2}} + \frac{2ia\sec^{13}(c+dx)}{17d(a+ia\tan(c+dx))^{9/2}}$$

```
[In] Int[Sec[c + d*x]^13/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((64*I)/3315)*a^3*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(13/2)) + (((
16*I)/255)*a^2*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(11/2)) + (((2*I)
/17)*a*Sec[c + d*x]^13)/(d*(a + I*a*Tan[c + d*x])^(9/2))
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2ia\sec^{13}(c+dx)}{17d(a+ia\tan(c+dx))^{9/2}}+\frac{1}{17}(8a)\int\frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{9/2}}\,dx\\
&= \frac{16ia^2\sec^{13}(c+dx)}{255d(a+ia\tan(c+dx))^{11/2}}+\frac{2ia\sec^{13}(c+dx)}{17d(a+ia\tan(c+dx))^{9/2}}\\
&\quad+\frac{1}{255}\left(32a^2\right)\int\frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{11/2}}\,dx\\
&= \frac{64ia^3\sec^{13}(c+dx)}{3315d(a+ia\tan(c+dx))^{13/2}}+\frac{16ia^2\sec^{13}(c+dx)}{255d(a+ia\tan(c+dx))^{11/2}}+\frac{2ia\sec^{13}(c+dx)}{17d(a+ia\tan(c+dx))^{9/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.00 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

$$\int\frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{2\sec^{12}(c+dx)(68+263\cos(2(c+dx))+247i\sin(2(c+dx)))(\cos(3(c+dx))-i\sin(3(c+dx)))}{3315a^3d(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[Sec[c + d*x]^13/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (-2*Sec[c + d*x]^12*(68 + 263*Cos[2*(c + d*x)] + (247*I)*Sin[2*(c + d*x)])*
(Cos[3*(c + d*x)] - I*Sin[3*(c + d*x)]))/(3315*a^3*d*(-I + Tan[c + d*x])^3*
Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [F(-1)]

Timed out.

$$\int \frac{\sec^{13}(dx+c)}{(a+ia\tan(dx+c))^{\frac{7}{2}}}dx$$

```
[In] int(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] int(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(7/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(86) = 172$.

Time = 0.32 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.57

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{512\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-255i\,e^{(4i\,dx+4i\,c)}-68i\,e^{(2i\,dx+2i\,c)}\right.}{3315\left(a^4de^{(16i\,dx+16i\,c)}+8\,a^4de^{(14i\,dx+14i\,c)}+28\,a^4de^{(12i\,dx+12i\,c)}+56\,a^4de^{(10i\,dx+10i\,c)}+70\,a^4de^{(8i\,dx+8i\,c)}+56\right.}$$

```
[In] integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -512/3315*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-255*I*e^(4*I*d*x + 4*
I*c) - 68*I*e^(2*I*d*x + 2*I*c) - 8*I)/(a^4*d*e^(16*I*d*x + 16*I*c) + 8*a^4
*d*e^(14*I*d*x + 14*I*c) + 28*a^4*d*e^(12*I*d*x + 12*I*c) + 56*a^4*d*e^(10*
I*d*x + 10*I*c) + 70*a^4*d*e^(8*I*d*x + 8*I*c) + 56*a^4*d*e^(6*I*d*x + 6*I*
c) + 28*a^4*d*e^(4*I*d*x + 4*I*c) + 8*a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sec(d*x+c)**13/(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 902 vs. 2(86) = 172.

Time = 0.57 (sec) , antiderivative size = 902, normalized size of antiderivative = 8.20

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -2/3315*(-331*I*sqrt(a) - 998*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 183
8*I*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 7522*sqrt(a)*sin(d*x + c)
^3/(cos(d*x + c) + 1)^3 - 4836*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^
4 - 27882*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 8954*I*sqrt(a)*sin(
d*x + c)^6/(cos(d*x + c) + 1)^6 - 68926*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c
) + 1)^7 - 12631*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 125052*sqr
t(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 10540*I*sqrt(a)*sin(d*x + c)^10/
(cos(d*x + c) + 1)^10 - 168980*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) + 1)^1
1 - 168980*sqrt(a)*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 + 10540*I*sqrt(a)*
sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 125052*sqrt(a)*sin(d*x + c)^15/(cos
(d*x + c) + 1)^15 + 12631*I*sqrt(a)*sin(d*x + c)^16/(cos(d*x + c) + 1)^16 -
 68926*sqrt(a)*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 + 8954*I*sqrt(a)*sin(d
*x + c)^18/(cos(d*x + c) + 1)^18 - 27882*sqrt(a)*sin(d*x + c)^19/(cos(d*x +
 c) + 1)^19 + 4836*I*sqrt(a)*sin(d*x + c)^20/(cos(d*x + c) + 1)^20 - 7522*s
qrt(a)*sin(d*x + c)^21/(cos(d*x + c) + 1)^21 + 1838*I*sqrt(a)*sin(d*x + c)^
22/(cos(d*x + c) + 1)^22 - 998*sqrt(a)*sin(d*x + c)^23/(cos(d*x + c) + 1)^2
3 + 331*I*sqrt(a)*sin(d*x + c)^24/(cos(d*x + c) + 1)^24)*(sin(d*x + c)/(cos
(d*x + c) + 1) + 1)^(7/2)*(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(7/2)/((a^4
 - 12*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 66*a^4*sin(d*x + c)^4/(cos(
d*x + c) + 1)^4 - 220*a^4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 495*a^4*sin
(d*x + c)^8/(cos(d*x + c) + 1)^8 - 792*a^4*sin(d*x + c)^10/(cos(d*x + c) +
1)^10 + 924*a^4*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 792*a^4*sin(d*x + c
)^14/(cos(d*x + c) + 1)^14 + 495*a^4*sin(d*x + c)^16/(cos(d*x + c) + 1)^16
- 220*a^4*sin(d*x + c)^18/(cos(d*x + c) + 1)^18 + 66*a^4*sin(d*x + c)^20/(c
os(d*x + c) + 1)^20 - 12*a^4*sin(d*x + c)^22/(cos(d*x + c) + 1)^22 + a^4*si
n(d*x + c)^24/(cos(d*x + c) + 1)^24)*d*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1
) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(7/2))
```

Giac [F]

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\sec(dx+c)^{13}}{(i\,a\tan(dx+c)+a)^{\frac{7}{2}}}\,dx$$

[In] integrate(sec(d*x+c)^13/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")

[Out] integrate(sec(d*x + c)^13/(I*a*tan(d*x + c) + a)^(7/2), x)

Mupad [B] (verification not implemented)

Time = 10.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

$$\int \frac{\sec^{13}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{512\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,68\mathrm{i}+\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}\,255\mathrm{i}+8\mathrm{i}\right)}{3315\,a^4\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^8}$$

[In] int(1/(cos(c + d*x)^13*(a + a*tan(c + d*x)*1i)^(7/2)),x)

[Out] (512*exp(- c*1i - d*x*1i)*(a - (a*(exp(c*2i + d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2)*(exp(c*2i + d*x*2i)*68i + exp(c*4i + d*x*4i)*255i + 8i))/(3315*a^4*d*(exp(c*2i + d*x*2i) + 1)^8)

3.386 $\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 73

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{8ia^2\sec^{11}(c+dx)}{143d(a+ia\tan(c+dx))^{11/2}} + \frac{2ia\sec^{11}(c+dx)}{13d(a+ia\tan(c+dx))^{9/2}}$$

```
[Out] 8/143*I*a^2*sec(d*x+c)^11/d/(a+I*a*tan(d*x+c))^(11/2)+2/13*I*a*sec(d*x+c)^1
1/d/(a+I*a*tan(d*x+c))^(9/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3575, 3574}

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{8ia^2\sec^{11}(c+dx)}{143d(a+ia\tan(c+dx))^{11/2}} + \frac{2ia\sec^{11}(c+dx)}{13d(a+ia\tan(c+dx))^{9/2}}$$

```
[In] Int[Sec[c + d*x]^11/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((8*I)/143)*a^2*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(11/2)) + (((2*
I)/13)*a*Sec[c + d*x]^11)/(d*(a + I*a*Tan[c + d*x])^(9/2))
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] && !Integ
erQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2ia\sec^{11}(c+dx)}{13d(a+ia\tan(c+dx))^{9/2}}+\frac{1}{13}(4a)\int\frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{9/2}}\,dx\\ &= \frac{8ia^2\sec^{11}(c+dx)}{143d(a+ia\tan(c+dx))^{11/2}}+\frac{2ia\sec^{11}(c+dx)}{13d(a+ia\tan(c+dx))^{9/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.66 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.12

$$\int\frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{2i\sec^9(c+dx)(\cos(2(c+dx))-i\sin(2(c+dx)))(-15i+11\tan(c+dx))}{143a^3d(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[Sec[c + d*x]^11/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((-2*I)/143)*Sec[c + d*x]^9*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)])*(-15*I
 + 11*Tan[c + d*x]))/(a^3*d*(-I + Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c + d*x]
])
```

Maple [F(-1)]

Timed out.

$$\int\frac{\sec^{11}(dx+c)}{(a+ia\tan(dx+c))^{\frac{7}{2}}}dx$$

```
[In] int(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] int(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(7/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(57) = 114.

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.81

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx =$$

$$-\frac{128\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-13i\,e^{(2i\,dx+2i\,c)}-2i\right)}{143\left(a^4de^{(12i\,dx+12i\,c)}+6\,a^4de^{(10i\,dx+10i\,c)}+15\,a^4de^{(8i\,dx+8i\,c)}+20\,a^4de^{(6i\,dx+6i\,c)}+15\,a^4de^{(4i\,dx+4i\,c)}+6\,a^4de\right.}$$

```
[In] integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -128/143*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-13*I*e^(2*I*d*x + 2*I*
c) - 2*I)/(a^4*d*e^(12*I*d*x + 12*I*c) + 6*a^4*d*e^(10*I*d*x + 10*I*c) + 15
*a^4*d*e^(8*I*d*x + 8*I*c) + 20*a^4*d*e^(6*I*d*x + 6*I*c) + 15*a^4*d*e^(4*I
*d*x + 4*I*c) + 6*a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sec(d*x+c)**11/(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 764 vs. 2(57) = 114.

Time = 0.47 (sec) , antiderivative size = 764, normalized size of antiderivative = 10.47

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -2/143*(-15*I*sqrt(a) - 38*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 88*I*s
qrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 278*sqrt(a)*sin(d*x + c)^3/(co
s(d*x + c) + 1)^3 - 213*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 920
*sqrt(a)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 272*I*sqrt(a)*sin(d*x + c)^6
/(cos(d*x + c) + 1)^6 - 1848*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 -
```

```
182*I*sqrt(a)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 2548*sqrt(a)*sin(d*x +
c)^9/(cos(d*x + c) + 1)^9 - 2548*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) + 1)
^11 + 182*I*sqrt(a)*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 1848*sqrt(a)*si
n(d*x + c)^13/(cos(d*x + c) + 1)^13 + 272*I*sqrt(a)*sin(d*x + c)^14/(cos(d*
x + c) + 1)^14 - 920*sqrt(a)*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 + 213*I*
sqrt(a)*sin(d*x + c)^16/(cos(d*x + c) + 1)^16 - 278*sqrt(a)*sin(d*x + c)^17
/(cos(d*x + c) + 1)^17 + 88*I*sqrt(a)*sin(d*x + c)^18/(cos(d*x + c) + 1)^18
 - 38*sqrt(a)*sin(d*x + c)^19/(cos(d*x + c) + 1)^19 + 15*I*sqrt(a)*sin(d*x
+ c)^20/(cos(d*x + c) + 1)^20)*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*
(sin(d*x + c)/(cos(d*x + c) + 1) - 1)^(7/2)/((a^4 - 10*a^4*sin(d*x + c)^2/(
cos(d*x + c) + 1)^2 + 45*a^4*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 120*a^4*
sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 210*a^4*sin(d*x + c)^8/(cos(d*x + c)
+ 1)^8 - 252*a^4*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 210*a^4*sin(d*x +
c)^12/(cos(d*x + c) + 1)^12 - 120*a^4*sin(d*x + c)^14/(cos(d*x + c) + 1)^14
 + 45*a^4*sin(d*x + c)^16/(cos(d*x + c) + 1)^16 - 10*a^4*sin(d*x + c)^18/(c
os(d*x + c) + 1)^18 + a^4*sin(d*x + c)^20/(cos(d*x + c) + 1)^20)*d*(-2*I*si
n(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(7
/2))
```

Giac [F]

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=\int \frac{\sec\left(dx+c\right)^{11}}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)^11/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^11/(I*a*tan(d*x + c) + a)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 7.62 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

$$\int \frac{\sec^{11}(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=\frac{128\,\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,13\mathrm{i}+2\mathrm{i}\right)\,\sqrt{a-\frac{a\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,1\mathrm{i}}{\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1}}}{143\,a^4\,d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)^6}$$

```
[In] int(1/(cos(c + d*x)^11*(a + a*tan(c + d*x)*1i)^(7/2)),x)
```

```
[Out] (128*exp(- c*1i - d*x*1i)*(exp(c*2i + d*x*2i)*13i + 2i)*(a - (a*(exp(c*2i +
 d*x*2i)*1i - 1i)*1i)/(exp(c*2i + d*x*2i) + 1))^(1/2))/(143*a^4*d*(exp(c*2i
 + d*x*2i) + 1)^6)
```

3.387 $\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 35

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{2ia\sec^9(c+dx)}{9d(a+ia\tan(c+dx))^{9/2}}$$

```
[Out] 2/9*I*a*sec(d*x+c)^9/d/(a+I*a*tan(d*x+c))^(9/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {3574}

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{2ia\sec^9(c+dx)}{9d(a+ia\tan(c+dx))^{9/2}}$$

```
[In] Int[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((2*I)/9)*a*Sec[c + d*x]^9)/(d*(a + I*a*Tan[c + d*x])^(9/2))
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\text{integral} = \frac{2ia\sec^9(c+dx)}{9d(a+ia\tan(c+dx))^{9/2}}$$

Mathematica [A] (verified)

Time = 1.47 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{2i\sec^7(c+dx)(i+\tan(c+dx))}{9a^3d(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[Sec[c + d*x]^9/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((2*I)/9)*Sec[c + d*x]^7*(I + Tan[c + d*x]))/(a^3*d*(-I + Tan[c + d*x])^3*
Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [F(-1)]

Timed out.

$$\int \frac{\sec^9(dx+c)}{(a+ia\tan(dx+c))^{\frac{7}{2}}}dx$$

```
[In] int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] int(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(7/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(27) = 54$.

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.54

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{32i\sqrt{2}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{9\left(a^4de^{(8i\,dx+8i\,c)}+4\,a^4de^{(6i\,dx+6i\,c)}+6\,a^4de^{(4i\,dx+4i\,c)}+4\,a^4de^{(2i\,dx+2i\,c)}+a^4d\right)}$$

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 32/9*I*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))/(a^4*d*e^(8*I*d*x + 8*I*c)
 + 4*a^4*d*e^(6*I*d*x + 6*I*c) + 6*a^4*d*e^(4*I*d*x + 4*I*c) + 4*a^4*d*e^(2
*I*d*x + 2*I*c) + a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sec(d*x+c)**9/(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 626 vs. $2(27) = 54$.

Time = 0.72 (sec) , antiderivative size = 626, normalized size of antiderivative = 17.89

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx =$$

$$-\frac{2\left(-i\sqrt{a}-\frac{2\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{6i\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{14\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14i\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{42\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{14i\sqrt{a}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\cdots\right.}{9\left(a^4-\frac{8a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{28a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{56a^4\cdots}{(\cos(\cdots}\right.}$$

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -2/9*(-I*sqrt(a) - 2*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 6*I*sqrt(a)*
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 14*sqrt(a)*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 - 14*I*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 42*sqrt(a)*s
in(d*x + c)^5/(cos(d*x + c) + 1)^5 - 14*I*sqrt(a)*sin(d*x + c)^6/(cos(d*x +
 c) + 1)^6 - 70*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 70*sqrt(a)*si
n(d*x + c)^9/(cos(d*x + c) + 1)^9 + 14*I*sqrt(a)*sin(d*x + c)^10/(cos(d*x +
 c) + 1)^10 - 42*sqrt(a)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 14*I*sqrt(
a)*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 14*sqrt(a)*sin(d*x + c)^13/(cos(
d*x + c) + 1)^13 + 6*I*sqrt(a)*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 2*sq
rt(a)*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 + I*sqrt(a)*sin(d*x + c)^16/(co
s(d*x + c) + 1)^16)*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(sin(d*x +
c)/(cos(d*x + c) + 1) - 1)^(7/2)/((a^4 - 8*a^4*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 28*a^4*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 56*a^4*sin(d*x + c)^
6/(cos(d*x + c) + 1)^6 + 70*a^4*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 56*a^
4*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 28*a^4*sin(d*x + c)^12/(cos(d*x +
 c) + 1)^12 - 8*a^4*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + a^4*sin(d*x + c
)^16/(cos(d*x + c) + 1)^16)*d*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(7/2))
```

Giac [F]

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\sec(dx+c)^9}{(i\,a\tan(dx+c)+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)^9/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^9/(I*a*tan(d*x + c) + a)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 7.38 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.43

$$\int \frac{\sec^9(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{e^{-c\,5i-d\,x\,5i}\,\sqrt{a+\frac{a\,\sin(c+d\,x)\,1i}{\cos(c+d\,x)}}\,2i}{9\,a^4\,d\cos\left(c+d\,x\right)^4}$$

```
[In] int(1/(cos(c + d*x)^9*(a + a*tan(c + d*x)*1i)^(7/2)),x)
[Out] (exp(- c*5i - d*x*5i)*(a + (a*sin(c + d*x)*1i)/cos(c + d*x))^(1/2)*2i)/(9*a
^4*d*cos(c + d*x)^4)
```

3.388 $\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 160

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{8i\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{7/2}d} - \frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^{5/2}} - \frac{4i\sec^3(c+dx)}{3a^2d(a+ia\tan(c+dx))^{3/2}} - \frac{8i\sec(c+dx)}{a^3d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 8*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*2^(1/2
)/a^(7/2)/d-8*I*sec(d*x+c)/a^3/d/(a+I*a*tan(d*x+c))^(1/2)-2/5*I*sec(d*x+c)^
5/a/d/(a+I*a*tan(d*x+c))^(5/2)-4/3*I*sec(d*x+c)^3/a^2/d/(a+I*a*tan(d*x+c))^
(3/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3572, 3570, 212}

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{8i\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{7/2}d} - \frac{8i\sec(c+dx)}{a^3d\sqrt{a+ia\tan(c+dx)}} - \frac{4i\sec^3(c+dx)}{3a^2d(a+ia\tan(c+dx))^{3/2}} - \frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^{5/2}}$$

```
[In] Int[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((8*I)*Sqrt[2]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c +
 d*x]])])/(a^(7/2)*d) - (((2*I)/5)*Sec[c + d*x]^5)/(a*d*(a + I*a*Tan[c + d*
```

```
x])^(5/2)) - (((4*I)/3)*Sec[c + d*x]^3)/(a^2*d*(a + I*a*Tan[c + d*x])^(3/2)
) - ((8*I)*Sec[c + d*x])/(a^3*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3572

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e
+ f*x])^(n + 1)/(b*f*(m - 2))), x] + Dist[2*(d^2/a), Int[(d*Sec[e + f*x])^(
m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 + b^2, 0] && EqQ[m/2 + n, 0] && LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^{5/2}}+\frac{2\int\frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx}{a}\\
&= -\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^{5/2}}-\frac{4i\sec^3(c+dx)}{3a^2d(a+ia\tan(c+dx))^{3/2}}+\frac{4\int\frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx}{a^2}\\
&= -\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^{5/2}}-\frac{4i\sec^3(c+dx)}{3a^2d(a+ia\tan(c+dx))^{3/2}}\\
&\quad-\frac{8i\sec(c+dx)}{a^3d\sqrt{a+ia\tan(c+dx)}}+\frac{8\int\frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{a^3}\\
&= -\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^{5/2}}-\frac{4i\sec^3(c+dx)}{3a^2d(a+ia\tan(c+dx))^{3/2}}\\
&\quad-\frac{8i\sec(c+dx)}{a^3d\sqrt{a+ia\tan(c+dx)}}+\frac{(16i)\text{Subst}\left(\int\frac{1}{2-ax^2}\,dx,x,\frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^3d}\\
&= \frac{8i\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{7/2}d}-\frac{2i\sec^5(c+dx)}{5ad(a+ia\tan(c+dx))^{5/2}}\\
&\quad-\frac{4i\sec^3(c+dx)}{3a^2d(a+ia\tan(c+dx))^{3/2}}-\frac{8i\sec(c+dx)}{a^3d\sqrt{a+ia\tan(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.05 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.81

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{128e^{7i(c+dx)}\left(-23-35e^{2i(c+dx)}-15e^{4i(c+dx)}+15\left(1+e^{2i(c+dx)}\right)^{5/2}\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)}{15a^3d\left(1+e^{2i(c+dx)}\right)^6(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[Sec[c + d*x]^7/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (-128*E^((7*I)*(c + d*x))*(-23 - 35*E^((2*I)*(c + d*x)) - 15*E^((4*I)*(c +
d*x)) + 15*(1 + E^((2*I)*(c + d*x)))^(5/2)*ArcTanh[Sqrt[1 + E^((2*I)*(c + d
*x))]]))/(15*a^3*d*(1 + E^((2*I)*(c + d*x)))^6*(-I + Tan[c + d*x])^3*Sqrt[a
 + I*a*Tan[c + d*x]])
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 312 vs. 2(133) = 266.

Time = 10.79 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.96

method	result
default	$-\frac{2(-\csc(dx+c)+\cot(dx+c)+i)^7\left(-60\sqrt{2}\arctan\left(\frac{(i(\csc(dx+c)-\cot(dx+c))-1)\sqrt{2}}{2\sqrt{\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2-1}}\right)\left(\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2-1\right)^{\frac{5}{2}}+73i(\cdots}{15d\left(-\frac{a\left(2i(\csc(dx+c)-\cot(dx+c)\cdots}{\left(\csc^2(dx+c\cdots}\right.}$

```
[In] int(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15/d*(-csc(d*x+c)+cot(d*x+c)+I)^7*(-60*2^(1/2)*arctan(1/2*(I*(csc(d*x+c)
-cot(d*x+c))-1)*2^(1/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)^(1/2))*(csc(d*x+c
)^2*(1-cos(d*x+c))^2-1)^(5/2)+73*I*csc(d*x+c)^5*(1-cos(d*x+c))^5-190*I*csc(
d*x+c)^3*(1-cos(d*x+c))^3-105*csc(d*x+c)^4*(1-cos(d*x+c))^4+105*I*(csc(d*x+
c)-cot(d*x+c))+190*csc(d*x+c)^2*(1-cos(d*x+c))^2-73)/(-a*(2*I*(csc(d*x+c)-c
ot(d*x+c))-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-
1))^(7/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)^6
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(125) = 250.

Time = 0.26 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.04

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx =$$

$$-\frac{4\left(15\sqrt{2}\left(i\,a^4de^{(4i\,dx+4i\,c)}+2i\,a^4de^{(2i\,dx+2i\,c)}+i\,a^4d\right)\sqrt{\frac{1}{a^7d^2}}\log\left(-\frac{32\left(\left(i\,a^3de^{(2i\,dx+2i\,c)}+i\,a^3d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^7d^2}}\right.}{a^3d}\right.\right.}{}$$

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -4/15*(15*sqrt(2)*(I*a^4*d*e^(4*I*d*x + 4*I*c) + 2*I*a^4*d*e^(2*I*d*x + 2*I
*c) + I*a^4*d)*sqrt(1/(a^7*d^2))*log(-32*((I*a^3*d*e^(2*I*d*x + 2*I*c) + I*
a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) - I)*e^(-I*d*x -
 I*c)/(a^3*d)) + 15*sqrt(2)*(-I*a^4*d*e^(4*I*d*x + 4*I*c) - 2*I*a^4*d*e^(2*
I*d*x + 2*I*c) - I*a^4*d)*sqrt(1/(a^7*d^2))*log(-32*((-I*a^3*d*e^(2*I*d*x +
 2*I*c) - I*a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) - I)
*e^(-I*d*x - I*c)/(a^3*d)) + 2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(1
5*I*e^(4*I*d*x + 4*I*c) + 35*I*e^(2*I*d*x + 2*I*c) + 23*I))/(a^4*d*e^(4*I*d
*x + 4*I*c) + 2*a^4*d*e^(2*I*d*x + 2*I*c) + a^4*d)
```

Sympy [F]

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\sec^7{(c+dx)}}{(ia(\tan{(c+dx)}-i))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)**7/(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Integral(sec(c + d*x)**7/(I*a*(tan(c + d*x) - I))**(7/2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1164 vs. 2(125) = 250.

Time = 0.72 (sec) , antiderivative size = 1164, normalized size of antiderivative = 7.28

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -2/15*(15*(2*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*s
qrt(2)*cos(2*d*x + 2*c) + sqrt(2))*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)) + 1) - 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*s
qrt(2)*cos(2*d*x + 2*c) + sqrt(2))*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)) - 1) + (-I*sqrt(2)*cos(2*d*x + 2*c)^2 - I*sqrt(2)*sin(2*d*x + 2*c)^2 -
2*I*sqrt(2)*cos(2*d*x + 2*c) - I*sqrt(2))*log(sqrt(cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
+ 1))^2 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + (
I*sqrt(2)*cos(2*d*x + 2*c)^2 + I*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*I*sqrt(2)*c
os(2*d*x + 2*c) + I*sqrt(2))*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 - 2*(
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1))*(cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 4*((15*
I*sqrt(2)*cos(4*d*x + 4*c) + 35*I*sqrt(2)*cos(2*d*x + 2*c) - 15*sqrt(2)*sin
(4*d*x + 4*c) - 35*sqrt(2)*sin(2*d*x + 2*c) + 23*I*sqrt(2))*cos(5/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (15*sqrt(2)*cos(4*d*x + 4*c) +
35*sqrt(2)*cos(2*d*x + 2*c) + 15*I*sqrt(2)*sin(4*d*x + 4*c) + 35*I*sqrt(2)*
sin(2*d*x + 2*c) + 23*sqrt(2))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c) + 1)))*sqrt(a))/((a^4*cos(2*d*x + 2*c)^2 + a^4*sin(2*d*x + 2*c)^2 +
2*a^4*cos(2*d*x + 2*c) + a^4)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*d)
```

Giac [F]

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=\int \frac{\sec\left(dx+c\right)^7}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)^7/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^7/(I*a*tan(d*x + c) + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^7(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^7\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{7/2}}\,dx$$

```
[In] int(1/(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^(7/2)),x)
[Out] int(1/(cos(c + d*x)^7*(a + a*tan(c + d*x)*1i)^(7/2)), x)
```

3.389 $\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 121

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{3i\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{7/2}d} - \frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^{5/2}} + \frac{6i\sec(c+dx)}{a^2d(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] -3*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))*2^(1/
2)/a^(7/2)/d-2*I*sec(d*x+c)^3/a/d/(a+I*a*tan(d*x+c))^(5/2)+6*I*sec(d*x+c)/a
^2/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3582, 3583, 3570, 212}

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{3i\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{7/2}d} + \frac{6i\sec(c+dx)}{a^2d(a+ia\tan(c+dx))^{3/2}} - \frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^{5/2}}$$

```
[In] Int[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((-3*I)*Sqrt[2]*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c
+ d*x]])])/(a^(7/2)*d) - ((2*I)*Sec[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*x])
^(5/2)) + ((6*I)*Sec[c + d*x])/(a^2*d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^{5/2}}+\frac{6\int\frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx}{a}\\
&=-\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^{5/2}}+\frac{12i\sec(c+dx)}{a^2d(a+ia\tan(c+dx))^{3/2}}-\frac{12\int\frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx}{a^2}\\
&=-\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^{5/2}}+\frac{6i\sec(c+dx)}{a^2d(a+ia\tan(c+dx))^{3/2}}-\frac{3\int\frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{a^3}\\
&=-\frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^{5/2}}+\frac{6i\sec(c+dx)}{a^2d(a+ia\tan(c+dx))^{3/2}}\\
&\quad-\frac{(6i)\text{Subst}\left(\int\frac{1}{2-ax^2}\,dx,x,\frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\right)}{a^3d}
\end{aligned}
$$

$$= -\frac{3i\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{a^{7/2}d} - \frac{2i\sec^3(c+dx)}{ad(a+ia\tan(c+dx))^{5/2}} + \frac{6i\sec(c+dx)}{a^2d(a+ia\tan(c+dx))^{3/2}}$$

Mathematica [A] (verified)

Time = 1.78 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.04

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{16e^{5i(c+dx)}\left(-1-3e^{2i(c+dx)}+3e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right.\right.}{a^3d\left(1+e^{2i(c+dx)}\right)^4(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[Sec[c + d*x]^5/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (16*E^((5*I)*(c + d*x))*(-1 - 3*E^((2*I)*(c + d*x)) + 3*E^((2*I)*(c + d*x))
*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/(a^
3*d*(1 + E^((2*I)*(c + d*x)))^4*(-I + Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c +
d*x]])
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 424 vs. 2(102) = 204.

Time = 10.22 (sec) , antiderivative size = 425, normalized size of antiderivative = 3.51

method	result
default	$\frac{\left(3\sqrt{2}\arctan\left(\frac{(i(\csc(dx+c)-\cot(dx+c))-1)\sqrt{2}}{2\sqrt{\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2-1}}\right)\sqrt{(\csc^2(dx+c))(1-\cos(dx+c))^2-1}+6i\sqrt{2}\arctan\left(\frac{(i(\csc(dx+c)-\cot(dx+c))-1)\sqrt{}}{2\sqrt{\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2}}\right.\right.}{}$

```
[In] int(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(3*2^(1/2)*arctan(1/2*(I*(csc(d*x+c)-cot(d*x+c))-1)*2^(1/2)/(csc(d*x+c)
^2*(1-cos(d*x+c))^2-1)^(1/2))*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)^(1/2)+6*I*2
^(1/2)*arctan(1/2*(I*(csc(d*x+c)-cot(d*x+c))-1)*2^(1/2)/(csc(d*x+c)^2*(1-co
s(d*x+c))^2-1)^(1/2))*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)^(1/2)*(csc(d*x+c)-c
ot(d*x+c))-3*csc(d*x+c)^2*2^(1/2)*arctan(1/2*(I*(csc(d*x+c)-cot(d*x+c))-1)*
2^(1/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)^(1/2))*(csc(d*x+c)^2*(1-cos(d*x+c
))^2-1)^(1/2)*(1-cos(d*x+c))^2+4+4*I*csc(d*x+c)^3*(1-cos(d*x+c))^3)*(-csc(d
*x+c)+cot(d*x+c)+I)^5/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)^4/(-a*(2*I*(csc(d*x
+c)-cot(d*x+c))-csc(d*x+c)^2*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^2*(1-cos(d*x+c
))^2-1))^(7/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(96) = 192.

Time = 0.25 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.02

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{\left(-3i\sqrt{2}a^4d\sqrt{\frac{1}{a^7d^2}}e^{(2i\,dx+2i\,c)}\log\left(-\frac{12\left(\left(i\,a^3de^{(2i\,dx+2i\,c)}+i\,a^3d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{1}{a^7d^2}}\right.}{a^3d}\right.\right.}{}$$

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/2*(-3*I*sqrt(2)*a^4*d*sqrt(1/(a^7*d^2))*e^(2*I*d*x + 2*I*c)*log(-12*((I*a
^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(
1/(a^7*d^2)) + I)*e^(-I*d*x - I*c)/(a^3*d)) + 3*I*sqrt(2)*a^4*d*sqrt(1/(a^7
*d^2))*e^(2*I*d*x + 2*I*c)*log(-12*((-I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a^3*d
)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) + I)*e^(-I*d*x - I*c)
/(a^3*d)) - 2*sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-3*I*e^(2*I*d*x +
2*I*c) - I))*e^(-2*I*d*x - 2*I*c)/(a^4*d)
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\sec^5\left(c+dx\right)}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)**5/(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Integral(sec(c + d*x)**5/(I*a*(tan(c + d*x) - I))**(7/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Giac [F]

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\sec\left(dx+c\right)^5}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)^5/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^5/(I*a*tan(d*x + c) + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^5(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)^5\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{7/2}}\,dx$$

```
[In] int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(7/2)),x)
[Out] int(1/(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^(7/2)), x)
```

3.390 $\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 125

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{i\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{8\sqrt{2}a^{7/2}d} + \frac{i\sec(c+dx)}{2ad(a+ia\tan(c+dx))^{5/2}} - \frac{i\sec(c+dx)}{8a^2d(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] -1/16*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/a^
(7/2)/d*2^(1/2)+1/2*I*sec(d*x+c)/a/d/(a+I*a*tan(d*x+c))^(5/2)-1/8*I*sec(d*x
+c)/a^2/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3582, 3583, 3570, 212}

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{i\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{8\sqrt{2}a^{7/2}d} - \frac{i\sec(c+dx)}{8a^2d(a+ia\tan(c+dx))^{3/2}} + \frac{i\sec(c+dx)}{2ad(a+ia\tan(c+dx))^{5/2}}$$

```
[In] Int[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((-1/8*I)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*x]
])])/(Sqrt[2]*a^(7/2)*d) + ((I/2)*Sec[c + d*x])/(a*d*(a + I*a*Tan[c + d*x])
^(5/2)) - ((I/8)*Sec[c + d*x])/(a^2*d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2i\sec(c+dx)}{3ad(a+ia\tan(c+dx))^{5/2}}-\frac{2\int\frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx}{3a}\\
&= \frac{i\sec(c+dx)}{2ad(a+ia\tan(c+dx))^{5/2}}-\frac{\int\frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx}{4a^2}\\
&= \frac{i\sec(c+dx)}{2ad(a+ia\tan(c+dx))^{5/2}}-\frac{i\sec(c+dx)}{8a^2d(a+ia\tan(c+dx))^{3/2}}-\frac{\int\frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{16a^3}\\
&= \frac{i\sec(c+dx)}{2ad(a+ia\tan(c+dx))^{5/2}}-\frac{i\sec(c+dx)}{8a^2d(a+ia\tan(c+dx))^{3/2}}\\
&\quad-\frac{i\text{Subst}\left(\int\frac{1}{2-ax^2}\,dx,x,\frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\right)}{8a^3d}
\end{aligned}
$$

$$=-\frac{i\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{8\sqrt{2}a^{7/2}d}+\frac{i\sec(c+dx)}{2ad(a+ia\tan(c+dx))^{5/2}}-\frac{i\sec(c+dx)}{8a^2d(a+ia\tan(c+dx))^{3/2}}$$

Mathematica [A] (verified)

Time = 1.86 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.96

$$\int\frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=\frac{i\sec^3(c+dx)\left(-3+e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\text{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)-3\cos(2\right.}{16a^3d(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[Sec[c + d*x]^3/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((I/16)*Sec[c + d*x]^3*(-3 + E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x
))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]] - 3*Cos[2*(c + d*x)] + I*Sin[2*(
c + d*x)]))/(a^3*d*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 794 vs. 2(100) = 200.

Time = 10.65 (sec) , antiderivative size = 795, normalized size of antiderivative = 6.36

method	result
default	$\frac{8i\arctan\left(\frac{i\sin(dx+c)-\cos(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\sin(dx+c)+4i\tan(dx+c)\arctan\left(\frac{i\sin(dx+c)-\cos(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)-4i\tan(dx+c)\sqrt{-\frac{\cos}{\cos(}}}{}$

```
[In] int(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d/(tan(d*x+c)-I)^3/(a*(1+I*tan(d*x+c)))^(1/2)/(-cos(d*x+c)/(cos(d*x+c)
+1))^(1/2)/a^3/(cos(d*x+c)+1)*(8*I*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(
cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)+4*I*tan(d*x+c)
*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2))-4*I*tan(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+8*cos(d*x
+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos
(d*x+c)+1))^(1/2))-4*I*tan(d*x+c)*sec(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d
*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-4*I*tan(d*x+c)*
sec(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+4*arctan(1/2*(I*sin(d*x+c)-co
s(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-4*(-cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)-I*tan(d*x+c)*sec(d*x+c)^2*arctan(1/2*(I*sin(d*x+c)
-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-8*sec(d*x
+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos
(d*x+c)+1))^(1/2))-4*sec(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-3*sec(d*
```

```
x+c)^2*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(
cos(d*x+c)+1))^(1/2))-2*sec(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+sec
(d*x+c)^3*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c
)/(cos(d*x+c)+1))^(1/2))-2*sec(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(94) = 188.

Time = 0.26 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.14

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{\left(-i\sqrt{\frac{1}{2}}a^4d\sqrt{\frac{1}{a^7d^2}}e^{(4i\,dx+4i\,c)}\log\left(-\frac{\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,a^3de^{(2i\,dx+2i\,c)}+i\,a^3d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\cdots}\right.}{4\,a^3d}\right.\right.}{\cdots}$$

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/16*(-I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(4*I*d*x + 4*I*c)*log(-1/4*(sq
rt(2)*sqrt(1/2)*(I*a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1))*sqrt(1/(a^7*d^2)) + I)*e^(-I*d*x - I*c)/(a^3*d)) + I*sqrt(1/
2)*a^4*d*sqrt(1/(a^7*d^2))*e^(4*I*d*x + 4*I*c)*log(-1/4*(sqrt(2)*sqrt(1/2)*
(-I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*
sqrt(1/(a^7*d^2)) + I)*e^(-I*d*x - I*c)/(a^3*d)) + sqrt(2)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*(I*e^(4*I*d*x + 4*I*c) + 3*I*e^(2*I*d*x + 2*I*c) + 2*I))*
e^(-4*I*d*x - 4*I*c)/(a^4*d)
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\sec^3(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)**3/(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/(I*a*(tan(c + d*x) - I))**(7/2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 977 vs. $2(94) = 188$.

Time = 0.46 (sec) , antiderivative size = 977, normalized size of antiderivative = 7.82

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -1/64*(4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(3/4)*((-I*sqrt(2)*cos(4*d*x + 4*c) - sqrt(2)*sin(4*d*x + 4*c))*cos(3/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (sqrt(2)*cos(4*d*x + 4*c)
- I*sqrt(2)*sin(4*d*x + 4*c))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)))*sqrt(a) + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*((-I*sqrt(2)*cos(4*d*x + 4*c) - sqrt(2)*sin(4*d*x +
4*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (sqrt(2)*c
os(4*d*x + 4*c) - I*sqrt(2)*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) - (2*sqrt(2)*arctan2((cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)) + 1) - 2*sqrt(2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) - 1) - I*sqrt(2)*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + 2*(cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + I*sqrt(2)*log(sqrt(co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))^2 - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)) + 1))*sqrt(a))/(a^4*d)
```

Giac [F]

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=\int \frac{\sec\left(dx+c\right)^3}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^3/(I*a*tan(d*x + c) + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx=\int \frac{1}{\cos\left(c+d\,x\right)^3\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{7/2}}\,dx$$

```
[In] int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(7/2)),x)
[Out] int(1/(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^(7/2)), x)
```

3.391 $\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 157

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{5i\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} + \frac{i\sec(c+dx)}{6d(a+ia\tan(c+dx))^{7/2}} + \frac{5i\sec(c+dx)}{48ad(a+ia\tan(c+dx))^{5/2}} + \frac{5i\sec(c+dx)}{64a^2d(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] 5/128*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))/a^
(7/2)/d*2^(1/2)+1/6*I*sec(d*x+c)/d/(a+I*a*tan(d*x+c))^(7/2)+5/48*I*sec(d*x+
c)/a/d/(a+I*a*tan(d*x+c))^(5/2)+5/64*I*sec(d*x+c)/a^2/d/(a+I*a*tan(d*x+c))^
(3/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3583, 3570, 212}

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{5i\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} + \frac{5i\sec(c+dx)}{64a^2d(a+ia\tan(c+dx))^{3/2}} + \frac{5i\sec(c+dx)}{48ad(a+ia\tan(c+dx))^{5/2}} + \frac{i\sec(c+dx)}{6d(a+ia\tan(c+dx))^{7/2}}$$

```
[In] Int[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((5*I)/64)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c + d*
x]])])/(Sqrt[2]*a^(7/2)*d) + ((I/6)*Sec[c + d*x])/(d*(a + I*a*Tan[c + d*x])
^(7/2)) + (((5*I)/48)*Sec[c + d*x])/(a*d*(a + I*a*Tan[c + d*x])^(5/2)) + ((
(5*I)/64)*Sec[c + d*x])/(a^2*d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{i\sec(c+dx)}{6d(a+ia\tan(c+dx))^{7/2}}+\frac{5\int\frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx}{12a}\\
&= \frac{i\sec(c+dx)}{6d(a+ia\tan(c+dx))^{7/2}}+\frac{5i\sec(c+dx)}{48ad(a+ia\tan(c+dx))^{5/2}}+\frac{5\int\frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx}{32a^2}\\
&= \frac{i\sec(c+dx)}{6d(a+ia\tan(c+dx))^{7/2}}+\frac{5i\sec(c+dx)}{48ad(a+ia\tan(c+dx))^{5/2}}\\
&\quad+\frac{5i\sec(c+dx)}{64a^2d(a+ia\tan(c+dx))^{3/2}}+\frac{5\int\frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{128a^3}\\
&= \frac{i\sec(c+dx)}{6d(a+ia\tan(c+dx))^{7/2}}+\frac{5i\sec(c+dx)}{48ad(a+ia\tan(c+dx))^{5/2}}\\
&\quad+\frac{5i\sec(c+dx)}{64a^2d(a+ia\tan(c+dx))^{3/2}}+\frac{(5i)\text{Subst}\left(\int\frac{1}{2-ax^2}\,dx,x,\frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\right)}{64a^3d}
\end{aligned}$$

$$= \frac{5i\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} + \frac{i\sec(c+dx)}{6d(a+ia\tan(c+dx))^{7/2}} + \frac{5i\sec(c+dx)}{48ad(a+ia\tan(c+dx))^{5/2}} + \frac{5i\sec(c+dx)}{64a^2d(a+ia\tan(c+dx))^{3/2}}$$

Mathematica [A] (verified)

Time = 1.88 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.76

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{\sec^3(c+dx)\left(52+\frac{30e^{4i(c+dx)}\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)}{\sqrt{1+e^{2i(c+dx)}}}+82\cos(2(c+dx))+50i\sin(2(c+dx))\right)}{384a^3d(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[Sec[c + d*x]/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] -1/384*(Sec[c + d*x]^3*(52 + (30*E^((4*I)*(c + d*x))*ArcTanh[Sqrt[1 + E^((2
*I)*(c + d*x))]])/Sqrt[1 + E^((2*I)*(c + d*x))] + 82*Cos[2*(c + d*x)] + (50
*I)*Sin[2*(c + d*x)]))/(a^3*d*(-I + Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c + d*
x]])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 797 vs. 2(126) = 252.

Time = 9.87 (sec) , antiderivative size = 798, normalized size of antiderivative = 5.08

method	result
default	$\frac{120i\arctan\left(\frac{i\sin(dx+c)-\cos(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)\sin(dx+c)+60i\tan(dx+c)\arctan\left(\frac{i\sin(dx+c)-\cos(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{-\frac{\cos(dx+c)}{\cos(dx+c)+1}}}\right)+100i\tan(dx+c)\sqrt{}}{}$

```
[In] int(sec(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/384/d/(-tan(d*x+c)+I)^3/(a*(1+I*tan(d*x+c)))^(1/2)/(-cos(d*x+c)/(cos(d*x+
c)+1))^(1/2)/a^3/(cos(d*x+c)+1)*(120*I*sin(d*x+c)*arctan(1/2*(I*sin(d*x+c)-
cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+60*I*tan(d
*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(c
os(d*x+c)+1))^(1/2))+100*I*tan(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+12
0*cos(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*
x+c)/(cos(d*x+c)+1))^(1/2))-60*I*tan(d*x+c)*sec(d*x+c)*arctan(1/2*(I*sin(d*
x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+100*I
```

```
*tan(d*x+c)*sec(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+60*arctan(1/2*(I*
sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))
+164*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-15*I*tan(d*x+c)*sec(d*x+c)^2*arctan
(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1)
)^(1/2))-120*sec(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+
1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+164*sec(d*x+c)*(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)-45*sec(d*x+c)^2*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d
*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-30*sec(d*x+c)^2*(-cos(d*x+c)/(
cos(d*x+c)+1))^(1/2)+15*sec(d*x+c)^3*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)
/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-30*sec(d*x+c)^3*(-cos(d
*x+c)/(cos(d*x+c)+1))^(1/2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(118) = 236$.

Time = 0.25 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.77

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{\left(-15i\sqrt{\frac{1}{2}}a^4d\sqrt{\frac{1}{a^7d^2}}e^{(6i\,dx+6i\,c)}\log\left(-\frac{5\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,a^3de^{(2i\,dx+2i\,c)}+i\,a^3d\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}\dots}}\right.}{32\,a^3d}\right.\right.}{\dots}$$

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/384*(-15*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(6*I*d*x + 6*I*c)*log(-5/3
2*(sqrt(2)*sqrt(1/2)*(I*a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)*sqrt(a/(e^(2*I
*d*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) - I)*e^(-I*d*x - I*c)/(a^3*d)) + 15*I
*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(6*I*d*x + 6*I*c)*log(-5/32*(sqrt(2)*s
qrt(1/2)*(-I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a^3*d)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))*sqrt(1/(a^7*d^2)) - I)*e^(-I*d*x - I*c)/(a^3*d)) + sqrt(2)*sqrt(a/
(e^(2*I*d*x + 2*I*c) + 1))*(33*I*e^(6*I*d*x + 6*I*c) + 59*I*e^(4*I*d*x + 4*
I*c) + 34*I*e^(2*I*d*x + 2*I*c) + 8*I))*e^(-6*I*d*x - 6*I*c)/(a^4*d)
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\sec(c+dx)}{(ia(\tan(c+dx)-i))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Integral(sec(c + d*x)/(I*a*(tan(c + d*x) - I))**(7/2), x)
```

Maxima [F]

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\sec\left(dx+c\right)}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)/(I*a*tan(d*x + c) + a)^(7/2), x)
```

Giac [F]

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\sec\left(dx+c\right)}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(sec(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)/(I*a*tan(d*x + c) + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{1}{\cos\left(c+d\,x\right)\,\left(a+a\,\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{7/2}}\,dx$$

```
[In] int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(7/2)),x)
[Out] int(1/(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^(7/2)), x)
```

3.392 $\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 227

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{315i\,\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{2048\sqrt{2}a^{7/2}d} + \frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}} + \frac{3i\cos(c+dx)}{32ad(a+ia\tan(c+dx))^{5/2}} + \frac{21i\cos(c+dx)}{256a^2d(a+ia\tan(c+dx))^{3/2}} + \frac{105i\cos(c+dx)}{1024a^3d\sqrt{a+ia\tan(c+dx)}} - \frac{315i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{2048a^4d}$$

```
[Out] 315/4096*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2))
/a^(7/2)/d*2^(1/2)+105/1024*I*cos(d*x+c)/a^3/d/(a+I*a*tan(d*x+c))^(1/2)-315
/2048*I*cos(d*x+c)*(a+I*a*tan(d*x+c))^(1/2)/a^4/d+1/8*I*cos(d*x+c)/d/(a+I*a
*tan(d*x+c))^(7/2)+3/32*I*cos(d*x+c)/a/d/(a+I*a*tan(d*x+c))^(5/2)+21/256*I*
cos(d*x+c)/a^2/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used

$= \{3583, 3571, 3570, 212\}$

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{315i\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{2048\sqrt{2}a^{7/2}d} - \frac{315i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{2048a^4d} + \frac{105i\cos(c+dx)}{1024a^3d\sqrt{a+ia\tan(c+dx)}} + \frac{21i\cos(c+dx)}{256a^2d(a+ia\tan(c+dx))^{3/2}} + \frac{3i\cos(c+dx)}{32ad(a+ia\tan(c+dx))^{5/2}} + \frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}}$$

```
[In] Int[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((315*I)/2048)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[c
+ d*x]])])/(Sqrt[2]*a^(7/2)*d) + ((I/8)*Cos[c + d*x])/(d*(a + I*a*Tan[c + d
*x])^(7/2)) + (((3*I)/32)*Cos[c + d*x])/(a*d*(a + I*a*Tan[c + d*x])^(5/2))
+ (((21*I)/256)*Cos[c + d*x])/(a^2*d*(a + I*a*Tan[c + d*x])^(3/2)) + (((105
*I)/1024)*Cos[c + d*x])/(a^3*d*Sqrt[a + I*a*Tan[c + d*x]]) - (((315*I)/2048
)*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/(a^4*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3571

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
```

```
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}}+\frac{9\int\frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx}{16a}\\
&= \frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}}+\frac{3i\cos(c+dx)}{32ad(a+ia\tan(c+dx))^{5/2}}+\frac{21\int\frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx}{64a^2}\\
&= \frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}}+\frac{3i\cos(c+dx)}{32ad(a+ia\tan(c+dx))^{5/2}}\\
&\quad+\frac{21i\cos(c+dx)}{256a^2d(a+ia\tan(c+dx))^{3/2}}+\frac{105\int\frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{512a^3}\\
&= \frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}}+\frac{3i\cos(c+dx)}{32ad(a+ia\tan(c+dx))^{5/2}}+\frac{21i\cos(c+dx)}{256a^2d(a+ia\tan(c+dx))^{3/2}}\\
&\quad+\frac{105i\cos(c+dx)}{1024a^3d\sqrt{a+ia\tan(c+dx)}}+\frac{315\int\cos(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx}{2048a^4}\\
&= \frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}}+\frac{3i\cos(c+dx)}{32ad(a+ia\tan(c+dx))^{5/2}}\\
&\quad+\frac{21i\cos(c+dx)}{256a^2d(a+ia\tan(c+dx))^{3/2}}+\frac{105i\cos(c+dx)}{1024a^3d\sqrt{a+ia\tan(c+dx)}}\\
&\quad-\frac{315i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{2048a^4d}+\frac{315\int\frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{4096a^3}\\
&= \frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}}+\frac{3i\cos(c+dx)}{32ad(a+ia\tan(c+dx))^{5/2}}\\
&\quad+\frac{21i\cos(c+dx)}{256a^2d(a+ia\tan(c+dx))^{3/2}}+\frac{105i\cos(c+dx)}{1024a^3d\sqrt{a+ia\tan(c+dx)}}\\
&\quad-\frac{315i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{2048a^4d}\\
&\quad+\frac{(315i)\text{Subst}\left(\int\frac{1}{2-ax^2}\,dx,x,\frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\right)}{2048a^3d}\\
&= \frac{315i\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{2048\sqrt{2}a^{7/2}d}+\frac{i\cos(c+dx)}{8d(a+ia\tan(c+dx))^{7/2}}\\
&\quad+\frac{3i\cos(c+dx)}{32ad(a+ia\tan(c+dx))^{5/2}}+\frac{21i\cos(c+dx)}{256a^2d(a+ia\tan(c+dx))^{3/2}}\\
&\quad+\frac{105i\cos(c+dx)}{1024a^3d\sqrt{a+ia\tan(c+dx)}}-\frac{315i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{2048a^4d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.32 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.62

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx =$$
$$-\frac{\sec^3(c+dx)\left(420+\frac{630e^{4i(c+dx)}\operatorname{arctanh}\left(\sqrt{1+e^{2i(c+dx)}}\right)}{\sqrt{1+e^{2i(c+dx)}}}+826\cos(2(c+dx))-224\cos(4(c+dx))+474i\sin(2\right.}{4096a^3d(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[Cos[c + d*x]/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] -1/4096*(Sec[c + d*x]^3*(420 + (630*E^((4*I)*(c + d*x))*ArcTanh[Sqrt[1 + E^
((2*I)*(c + d*x))]])/Sqrt[1 + E^((2*I)*(c + d*x))] + 826*Cos[2*(c + d*x)] -
 224*Cos[4*(c + d*x)] + (474*I)*Sin[2*(c + d*x)] - (288*I)*Sin[4*(c + d*x)]
))/(a^3*d*(-I + Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 922 vs. 2(184) = 368.

Time = 12.80 (sec) , antiderivative size = 923, normalized size of antiderivative = 4.07

method	result	size
default	Expression too large to display	923

```
[In] int(cos(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4096/d/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)+1)/(a*(1+I*tan(d*x
+c)))^(1/2)/(1+I*tan(d*x+c))^3/a^3*(1792*I*cos(d*x+c)*(-cos(d*x+c)/(cos(d*x
+c)+1))^(1/2)+1792*I*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-2520*I
*cos(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x
+c)/(cos(d*x+c)+1))^(1/2))-2304*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)*cos(d*x+
c)*sin(d*x+c)+630*I*sec(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-315*I*s
ec(d*x+c)^3*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x
+c)/(cos(d*x+c)+1))^(1/2))-2304*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/
2)+2520*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/
(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-3444*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
+630*I*sec(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+2100*tan(d*x+c)*(-co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)+1260*tan(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos
(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-3444*I*sec(d*
x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-1260*I*arctan(1/2*(I*sin(d*x+c)-cos
(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+2100*tan(d*x+
c)*sec(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-1260*tan(d*x+c)*sec(d*x+c)
```

```
*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*
x+c)+1))^(1/2))+2520*I*sec(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(c
os(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+945*I*sec(d*x+c)^2*arctan(
1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))
^(1/2))-315*tan(d*x+c)*sec(d*x+c)^2*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/
(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.32

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{\left(-315i\sqrt{\frac{1}{2}}a^4d\sqrt{\frac{1}{a^7d^2}}e^{(8i\,dx+8i\,c)}\log\left(-\frac{315\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,a^3de^{(2i\,dx+2i\,c)}+i\,a^3d\right)\sqrt{\frac{a}{e^{(2i\,dx+2}}}\right.}{1024\,a^3d}\right.\right.}{}$$

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/4096*(-315*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(8*I*d*x + 8*I*c)*log(-3
15/1024*(sqrt(2)*sqrt(1/2)*(I*a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)*sqrt(a/(
e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) - I)*e^(-I*d*x - I*c)/(a^3*d))
+ 315*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(8*I*d*x + 8*I*c)*log(-315/1024
*(sqrt(2)*sqrt(1/2)*(-I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a^3*d)*sqrt(a/(e^(2*I
*d*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) - I)*e^(-I*d*x - I*c)/(a^3*d)) + sqrt
(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-128*I*e^(10*I*d*x + 10*I*c) + 197*I
*e^(8*I*d*x + 8*I*c) + 535*I*e^(6*I*d*x + 6*I*c) + 298*I*e^(4*I*d*x + 4*I*c
) + 104*I*e^(2*I*d*x + 2*I*c) + 16*I))*e^(-8*I*d*x - 8*I*c)/(a^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2779 vs. $2(172) = 344$.

Time = 0.49 (sec) , antiderivative size = 2779, normalized size of antiderivative = 12.24

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -1/16384*(4*(cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(1
/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin(8
*d*x + 8*c), cos(8*d*x + 8*c))) + 1)^(3/4)*(325*((-I*sqrt(2)*cos(8*d*x + 8*
c) - sqrt(2)*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x
+ 8*c)))^2 + (-I*sqrt(2)*cos(8*d*x + 8*c) - sqrt(2)*sin(8*d*x + 8*c))*sin(1
/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*(-I*sqrt(2)*cos(8*d*x
 + 8*c) - sqrt(2)*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8
*d*x + 8*c))) - I*sqrt(2)*cos(8*d*x + 8*c) - sqrt(2)*sin(8*d*x + 8*c))*cos(
7/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*a
rctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)) + 643*(-I*sqrt(2)*cos(8*d
*x + 8*c) - sqrt(2)*sin(8*d*x + 8*c))*cos(3/2*arctan2(sin(1/4*arctan2(sin(8
*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*
x + 8*c))) + 1)) + 325*((sqrt(2)*cos(8*d*x + 8*c) - I*sqrt(2)*sin(8*d*x + 8
*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + (sqrt(2)*cos(
8*d*x + 8*c) - I*sqrt(2)*sin(8*d*x + 8*c))*sin(1/4*arctan2(sin(8*d*x + 8*c)
, cos(8*d*x + 8*c)))^2 + 2*(sqrt(2)*cos(8*d*x + 8*c) - I*sqrt(2)*sin(8*d*x
+ 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + sqrt(2)*cos(
8*d*x + 8*c) - I*sqrt(2)*sin(8*d*x + 8*c))*sin(7/2*arctan2(sin(1/4*arctan2(
sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos
(8*d*x + 8*c))) + 1)) + 643*(sqrt(2)*cos(8*d*x + 8*c) - I*sqrt(2)*sin(8*d*x
 + 8*c))*sin(3/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)
)), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)))*sqrt(a) + 4
*(cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(1/4*arctan2(
sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin(8*d*x + 8*c)
, cos(8*d*x + 8*c))) + 1)^(1/4)*(765*((I*sqrt(2)*cos(8*d*x + 8*c) + sqrt(2)
*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 +
 (I*sqrt(2)*cos(8*d*x + 8*c) + sqrt(2)*sin(8*d*x + 8*c))*sin(1/4*arctan2(si
n(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*(I*sqrt(2)*cos(8*d*x + 8*c) + sqrt
(2)*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))
+ I*sqrt(2)*cos(8*d*x + 8*c) + sqrt(2)*sin(8*d*x + 8*c))*cos(5/2*arctan2(si
n(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d
*x + 8*c), cos(8*d*x + 8*c))) + 1)) + (187*I*sqrt(2)*cos(8*d*x + 8*c) + 187
*sqrt(2)*sin(8*d*x + 8*c) + 128*I*sqrt(2))*cos(1/2*arctan2(sin(1/4*arctan2(
sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos
```

(8*d*x + 8*c))) + 1)) - 765*((sqrt(2)*cos(8*d*x + 8*c) - I*sqrt(2)*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + (sqrt(2)*cos(8*d*x + 8*c) - I*sqrt(2)*sin(8*d*x + 8*c))*sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*(sqrt(2)*cos(8*d*x + 8*c) - I*sqrt(2)*sin(8*d*x + 8*c))*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + sqrt(2)*cos(8*d*x + 8*c) - I*sqrt(2)*sin(8*d*x + 8*c))*sin(5/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)) - (187*sqrt(2)*cos(8*d*x + 8*c) - 187*I*sqrt(2)*sin(8*d*x + 8*c) + 128*sqrt(2))*sin(1/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)))*sqrt(a) + 315*(2*sqrt(2)*arctan2((cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)), (cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)) + 1) - 2*sqrt(2)*arctan2((cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)), (cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)) - 1) - I*sqrt(2)*log(sqrt(cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)*cos(1/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1))^2 + sqrt(cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)*sin(1/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1))^2 + 2*(cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)) + 1) + I*sqrt(2)*log(sqrt(cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1)*cos(1/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1))^2 + sqrt(cos(1/4*arctan2(si

```
n(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(1/4*arctan2(sin(8*d*x + 8*c), co
s(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))
 + 1)*sin(1/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))),
 cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1))^2 - 2*(cos(1/4*
arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c)))^2 + sin(1/4*arctan2(sin(8*d*x
+ 8*c), cos(8*d*x + 8*c)))^2 + 2*cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*
x + 8*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/4*arctan2(sin(8*d*x + 8*c), cos
(8*d*x + 8*c))), cos(1/4*arctan2(sin(8*d*x + 8*c), cos(8*d*x + 8*c))) + 1))
 + 1))*sqrt(a))/(a^4*d)
```

Giac [F]

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\cos\left(dx+c\right)}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate(cos(d*x + c)/(I*a*tan(d*x + c) + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\cos\left(c+d\,x\right)}{{\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{7/2}}\,dx$$

```
[In] int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^(7/2),x)
[Out] int(cos(c + d*x)/(a + a*tan(c + d*x)*1i)^(7/2), x)
```

3.393 $\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 307

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx &= \frac{3003i\,\text{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{16384\sqrt{2}a^{7/2}d} \\ &+ \frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}} + \frac{13i\cos^3(c+dx)}{160ad(a+ia\tan(c+dx))^{5/2}} \\ &+ \frac{143i\cos^3(c+dx)}{1920a^2d(a+ia\tan(c+dx))^{3/2}} + \frac{1001i\cos(c+dx)}{8192a^3d\sqrt{a+ia\tan(c+dx)}} \\ &+ \frac{429i\cos^3(c+dx)}{5120a^3d\sqrt{a+ia\tan(c+dx)}} - \frac{3003i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{16384a^4d} \\ &- \frac{1001i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{10240a^4d}\end{aligned}$$

```
[Out] 3003/32768*I*arctanh(1/2*sec(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*tan(d*x+c))^(1/2
))/a^(7/2)/d*2^(1/2)+1001/8192*I*cos(d*x+c)/a^3/d/(a+I*a*tan(d*x+c))^(1/2)+
429/5120*I*cos(d*x+c)^3/a^3/d/(a+I*a*tan(d*x+c))^(1/2)-3003/16384*I*cos(d*x
+c)*(a+I*a*tan(d*x+c))^(1/2)/a^4/d-1001/10240*I*cos(d*x+c)^3*(a+I*a*tan(d*x
+c))^(1/2)/a^4/d+1/10*I*cos(d*x+c)^3/d/(a+I*a*tan(d*x+c))^(7/2)+13/160*I*co
s(d*x+c)^3/a/d/(a+I*a*tan(d*x+c))^(5/2)+143/1920*I*cos(d*x+c)^3/a^2/d/(a+I*
a*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3583, 3578, 3571, 3570, 212}

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{3003i\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{16384\sqrt{2}a^{7/2}d} - \frac{1001i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{10240a^4d} - \frac{3003i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{16384a^4d} + \frac{429i\cos^3(c+dx)}{5120a^3d\sqrt{a+ia\tan(c+dx)}} + \frac{1001i\cos(c+dx)}{8192a^3d\sqrt{a+ia\tan(c+dx)}} + \frac{143i\cos^3(c+dx)}{1920a^2d(a+ia\tan(c+dx))^{3/2}} + \frac{13i\cos^3(c+dx)}{160ad(a+ia\tan(c+dx))^{5/2}} + \frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}$$

```
[In] Int[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (((3003*I)/16384)*ArcTanh[(Sqrt[a]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Tan[
c + d*x]])]])/(Sqrt[2]*a^(7/2)*d) + ((I/10)*Cos[c + d*x]^3)/(d*(a + I*a*Tan[
c + d*x])^(7/2)) + (((13*I)/160)*Cos[c + d*x]^3)/(a*d*(a + I*a*Tan[c + d*x]
)^(5/2)) + (((143*I)/1920)*Cos[c + d*x]^3)/(a^2*d*(a + I*a*Tan[c + d*x])^(3
/2)) + (((1001*I)/8192)*Cos[c + d*x])/(a^3*d*Sqrt[a + I*a*Tan[c + d*x]]) +
(((429*I)/5120)*Cos[c + d*x]^3)/(a^3*d*Sqrt[a + I*a*Tan[c + d*x]]) - (((300
3*I)/16384)*Cos[c + d*x]*Sqrt[a + I*a*Tan[c + d*x]])/(a^4*d) - (((1001*I)/1
0240)*Cos[c + d*x]^3*Sqrt[a + I*a*Tan[c + d*x]])/(a^4*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3570

```
Int[sec[(e_.) + (f_.)*(x_)]/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_S
ymbol] :> Dist[-2*(a/(b*f)), Subst[Int[1/(2 - a*x^2), x], x, Sec[e + f*x]/S
qrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3571

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] + Dist[a/(2*d^2), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e +
```

```
f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] &&
EqQ[m/2 + n, 0] && GtQ[n, 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}+\frac{13\int\frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{5/2}}\,dx}{20a}\\
&= \frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}+\frac{13i\cos^3(c+dx)}{160ad(a+ia\tan(c+dx))^{5/2}}+\frac{143\int\frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{3/2}}\,dx}{320a^2}\\
&= \frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}+\frac{13i\cos^3(c+dx)}{160ad(a+ia\tan(c+dx))^{5/2}}\\
&\quad+\frac{143i\cos^3(c+dx)}{1920a^2d(a+ia\tan(c+dx))^{3/2}}+\frac{429\int\frac{\cos^3(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\,dx}{1280a^3}\\
&= \frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}+\frac{13i\cos^3(c+dx)}{160ad(a+ia\tan(c+dx))^{5/2}}\\
&\quad+\frac{143i\cos^3(c+dx)}{1920a^2d(a+ia\tan(c+dx))^{3/2}}+\frac{429i\cos^3(c+dx)}{5120a^3d\sqrt{a+ia\tan(c+dx)}}\\
&\quad+\frac{3003\int\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}\,dx}{10240a^4}
\end{aligned}
$$

$$=\frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}+\frac{13i\cos^3(c+dx)}{160ad(a+ia\tan(c+dx))^{5/2}}+\frac{143i\cos^3(c+dx)}{1920a^2d(a+ia\tan(c+dx))^{3/2}}+\frac{429i\cos^3(c+dx)}{5120a^3d\sqrt{a+ia\tan(c+dx)}}-\frac{1001i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{10240a^4d}+\frac{1001\int\frac{\cos(c+dx)}{\sqrt{a+ia\tan(c+dx)}}dx}{4096a^3}$$

$$=\frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}+\frac{13i\cos^3(c+dx)}{160ad(a+ia\tan(c+dx))^{5/2}}+\frac{143i\cos^3(c+dx)}{1920a^2d(a+ia\tan(c+dx))^{3/2}}+\frac{1001i\cos(c+dx)}{8192a^3d\sqrt{a+ia\tan(c+dx)}}+\frac{429i\cos^3(c+dx)}{5120a^3d\sqrt{a+ia\tan(c+dx)}}-\frac{1001i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{10240a^4d}+\frac{3003\int\cos(c+dx)\sqrt{a+ia\tan(c+dx)}dx}{16384a^4}$$

$$=\frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}+\frac{13i\cos^3(c+dx)}{160ad(a+ia\tan(c+dx))^{5/2}}+\frac{143i\cos^3(c+dx)}{1920a^2d(a+ia\tan(c+dx))^{3/2}}+\frac{1001i\cos(c+dx)}{8192a^3d\sqrt{a+ia\tan(c+dx)}}+\frac{429i\cos^3(c+dx)}{5120a^3d\sqrt{a+ia\tan(c+dx)}}-\frac{3003i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{16384a^4d}-\frac{1001i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{10240a^4d}+\frac{3003\int\frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}dx}{32768a^3}$$

$$=\frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}}+\frac{13i\cos^3(c+dx)}{160ad(a+ia\tan(c+dx))^{5/2}}+\frac{143i\cos^3(c+dx)}{1920a^2d(a+ia\tan(c+dx))^{3/2}}+\frac{1001i\cos(c+dx)}{8192a^3d\sqrt{a+ia\tan(c+dx)}}+\frac{429i\cos^3(c+dx)}{5120a^3d\sqrt{a+ia\tan(c+dx)}}-\frac{3003i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{16384a^4d}-\frac{1001i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{10240a^4d}+\frac{(3003i)\text{Subst}\left(\int\frac{1}{2-ax^2}dx,x,\frac{\sec(c+dx)}{\sqrt{a+ia\tan(c+dx)}}\right)}{16384a^3d}$$

$$= \frac{3003i\operatorname{arctanh}\left(\frac{\sqrt{a}\sec(c+dx)}{\sqrt{2}\sqrt{a+ia\tan(c+dx)}}\right)}{16384\sqrt{2}a^{7/2}d} + \frac{i\cos^3(c+dx)}{10d(a+ia\tan(c+dx))^{7/2}} + \frac{13i\cos^3(c+dx)}{160ad(a+ia\tan(c+dx))^{5/2}} + \frac{143i\cos^3(c+dx)}{1920a^2d(a+ia\tan(c+dx))^{3/2}} + \frac{1001i\cos(c+dx)}{8192a^3d\sqrt{a+ia\tan(c+dx)}} + \frac{429i\cos^3(c+dx)}{5120a^3d\sqrt{a+ia\tan(c+dx)}} - \frac{3003i\cos(c+dx)\sqrt{a+ia\tan(c+dx)}}{16384a^4d} - \frac{1001i\cos^3(c+dx)\sqrt{a+ia\tan(c+dx)}}{10240a^4d}$$

Mathematica [A] (verified)

Time = 2.97 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.57

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = -\frac{\left(42140 + 20048e^{-2i(c+dx)} + 71190e^{2i(c+dx)} + 5856e^{-4i(c+dx)} - 48640e^{4i(c+dx)} + 768e^{-6i(c+dx)} - 2560e^{6i(c+d}\right.}{491520a^3d(-i+\tan(c+dx))^3\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[Cos[c + d*x]^3/(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] -1/491520*((42140 + 20048/E^((2*I)*(c + d*x)) + 71190*E^((2*I)*(c + d*x)) +
 5856/E^((4*I)*(c + d*x)) - 48640*E^((4*I)*(c + d*x)) + 768/E^((6*I)*(c + d
*x)) - 2560*E^((6*I)*(c + d*x)) + (90090*E^((4*I)*(c + d*x))*ArcTanh[Sqrt[1
 + E^((2*I)*(c + d*x))]])/Sqrt[1 + E^((2*I)*(c + d*x))])*Sec[c + d*x]^3)/(a
^3*d*(-I + Tan[c + d*x])^3*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1056 vs. 2(252) = 504.

Time = 11.08 (sec) , antiderivative size = 1057, normalized size of antiderivative = 3.44

method	result	size
default	Expression too large to display	1057

```
[In] int(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/491520/d/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)+1)/(a*(1+I*tan(d*
x+c)))^(1/2)/(1+I*tan(d*x+c))^3/a^3*(-256256*I*cos(d*x+c)*(-cos(d*x+c)/(cos
(d*x+c)+1))^(1/2)-256256*I*cos(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+
106496*sin(d*x+c)*cos(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+360360*I*
cos(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+
```

```
c)/(cos(d*x+c)+1))^(1/2))+106496*sin(d*x+c)*cos(d*x+c)^2*(-cos(d*x+c)/(cos(
d*x+c)+1))^(1/2)-90090*I*sec(d*x+c)^3*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+45
045*I*sec(d*x+c)^3*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-
cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+329472*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)
*cos(d*x+c)*sin(d*x+c)+492492*I*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-90090*I*
sec(d*x+c)^2*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-360360*arctan(1/2*(I*sin(d*
x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))*sin(d
*x+c)+329472*sin(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-57344*I*cos(d*x+
c)^4*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+492492*I*sec(d*x+c)*(-cos(d*x+c)/(c
os(d*x+c)+1))^(1/2)-180180*tan(d*x+c)*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1
)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-300300*tan(d*x+c)*(-co
s(d*x+c)/(cos(d*x+c)+1))^(1/2)+180180*I*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)
-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))-57344*I*cos(d*x+c)^3
*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)+180180*tan(d*x+c)*sec(d*x+c)*arctan(1/2
*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1
/2))-300300*tan(d*x+c)*sec(d*x+c)*(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)-135135
*I*sec(d*x+c)^2*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos
(d*x+c)/(cos(d*x+c)+1))^(1/2))-360360*I*sec(d*x+c)*arctan(1/2*(I*sin(d*x+c)
-cos(d*x+c)-1)/(cos(d*x+c)+1)/(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2))+45045*tan
(d*x+c)*sec(d*x+c)^2*arctan(1/2*(I*sin(d*x+c)-cos(d*x+c)-1)/(cos(d*x+c)+1)/
(-cos(d*x+c)/(cos(d*x+c)+1))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.05

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \frac{\left(-45045i\sqrt{\frac{1}{2}}a^4d\sqrt{\frac{1}{a^7d^2}}e^{(10i\,dx+10i\,c)}\log\left(-\frac{3003\left(\sqrt{2}\sqrt{\frac{1}{2}}\left(i\,a^3de^{(2i\,dx+2i\,c)}+i\,a^3d\right)\sqrt{\frac{}{e^{(2i}}}\right.}{8192\,a^3d}\right.\right.}{}$$

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/491520*(-45045*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(10*I*d*x + 10*I*c)*
log(-3003/8192*(sqrt(2)*sqrt(1/2)*(I*a^3*d*e^(2*I*d*x + 2*I*c) + I*a^3*d)*s
qrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) - I)*e^(-I*d*x - I*c)/(a
^3*d)) + 45045*I*sqrt(1/2)*a^4*d*sqrt(1/(a^7*d^2))*e^(10*I*d*x + 10*I*c)*lo
g(-3003/8192*(sqrt(2)*sqrt(1/2)*(-I*a^3*d*e^(2*I*d*x + 2*I*c) - I*a^3*d)*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(1/(a^7*d^2)) - I)*e^(-I*d*x - I*c)/(a^
3*d)) + sqrt(2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(-1280*I*e^(14*I*d*x + 14
*I*c) - 25600*I*e^(12*I*d*x + 12*I*c) + 11275*I*e^(10*I*d*x + 10*I*c) + 566
65*I*e^(8*I*d*x + 8*I*c) + 31094*I*e^(6*I*d*x + 6*I*c) + 12952*I*e^(4*I*d*x
 + 4*I*c) + 3312*I*e^(2*I*d*x + 2*I*c) + 384*I))*e^(-10*I*d*x - 10*I*c)/(a^
4*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**3/(a+I*a*tan(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5821 vs. 2(236) = 472.

Time = 0.62 (sec) , antiderivative size = 5821, normalized size of antiderivative = 18.96

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] -1/1966080*(40*(cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2
+ sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 2*cos(1/5*ar
ctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 1)^(3/4)*((79*(-I*sqrt(2)*
cos(10*d*x + 10*c) - sqrt(2)*sin(10*d*x + 10*c))*cos(1/5*arctan2(sin(10*d*x
 + 10*c), cos(10*d*x + 10*c)))^2 + 79*(-I*sqrt(2)*cos(10*d*x + 10*c) - sqrt
(2)*sin(10*d*x + 10*c))*sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10
*c)))^2 + 837*(-I*sqrt(2)*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x +
10*c)))^2 - I*sqrt(2)*sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c
)))^2 - 2*I*sqrt(2)*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))
) - I*sqrt(2))*cos(4/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 1
58*(-I*sqrt(2)*cos(10*d*x + 10*c) - sqrt(2)*sin(10*d*x + 10*c))*cos(1/5*arc
tan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) - 837*(sqrt(2)*cos(1/5*arctan
2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + sqrt(2)*sin(1/5*arctan2(sin(
10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 2*sqrt(2)*cos(1/5*arctan2(sin(10*d
*x + 10*c), cos(10*d*x + 10*c))) + sqrt(2))*sin(4/5*arctan2(sin(10*d*x + 10
*c), cos(10*d*x + 10*c))) - 79*I*sqrt(2)*cos(10*d*x + 10*c) - 79*sqrt(2)*si
n(10*d*x + 10*c))*cos(7/2*arctan2(sin(1/5*arctan2(sin(10*d*x + 10*c), cos(1
0*d*x + 10*c))), cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) +
 1)) + (-49*I*sqrt(2)*cos(10*d*x + 10*c) - 1155*I*sqrt(2)*cos(4/5*arctan2(s
in(10*d*x + 10*c), cos(10*d*x + 10*c))) + 3264*I*sqrt(2)*cos(3/5*arctan2(si
n(10*d*x + 10*c), cos(10*d*x + 10*c))) - 624*I*sqrt(2)*cos(2/5*arctan2(sin(
10*d*x + 10*c), cos(10*d*x + 10*c))) - 49*sqrt(2)*sin(10*d*x + 10*c) - 1155
*sqrt(2)*sin(4/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 3264*sq
rt(2)*sin(3/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) - 624*sqrt(2
```

)*sin(2/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 128*I*sqrt(2))
*cos(3/2*arctan2(sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))),
cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 1)) + (79*(sqrt(
2)*cos(10*d*x + 10*c) - I*sqrt(2)*sin(10*d*x + 10*c))*cos(1/5*arctan2(sin(1
0*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 79*(sqrt(2)*cos(10*d*x + 10*c) - I*
sqrt(2)*sin(10*d*x + 10*c))*sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x
+ 10*c)))^2 + 837*(sqrt(2)*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x +
 10*c)))^2 + sqrt(2)*sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)
))^2 + 2*sqrt(2)*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) +
 sqrt(2))*cos(4/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 158*(s
qrt(2)*cos(10*d*x + 10*c) - I*sqrt(2)*sin(10*d*x + 10*c))*cos(1/5*arctan2(s
in(10*d*x + 10*c), cos(10*d*x + 10*c))) + 837*(-I*sqrt(2)*cos(1/5*arctan2(s
in(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 - I*sqrt(2)*sin(1/5*arctan2(sin(1
0*d*x + 10*c), cos(10*d*x + 10*c)))^2 - 2*I*sqrt(2)*cos(1/5*arctan2(sin(10*
d*x + 10*c), cos(10*d*x + 10*c))) - I*sqrt(2))*sin(4/5*arctan2(sin(10*d*x +
 10*c), cos(10*d*x + 10*c))) + 79*sqrt(2)*cos(10*d*x + 10*c) - 79*I*sqrt(2)
*sin(10*d*x + 10*c))*sin(7/2*arctan2(sin(1/5*arctan2(sin(10*d*x + 10*c), co
s(10*d*x + 10*c))), cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))
) + 1)) + (49*sqrt(2)*cos(10*d*x + 10*c) + 1155*sqrt(2)*cos(4/5*arctan2(sin
(10*d*x + 10*c), cos(10*d*x + 10*c))) - 3264*sqrt(2)*cos(3/5*arctan2(sin(10
*d*x + 10*c), cos(10*d*x + 10*c))) + 624*sqrt(2)*cos(2/5*arctan2(sin(10*d*x
 + 10*c), cos(10*d*x + 10*c))) - 49*I*sqrt(2)*sin(10*d*x + 10*c) - 1155*I*s
qrt(2)*sin(4/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 3264*I*sq
rt(2)*sin(3/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) - 624*I*sqrt
(2)*sin(2/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) - 128*sqrt(2))
*sin(3/2*arctan2(sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))),
cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 1)))*sqrt(a) + 4
*(cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + sin(1/5*arct
an2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 2*cos(1/5*arctan2(sin(10*d
*x + 10*c), cos(10*d*x + 10*c))) + 1)^(1/4)*(105*((-I*sqrt(2)*cos(10*d*x +
10*c) - sqrt(2)*sin(10*d*x + 10*c))*cos(1/5*arctan2(sin(10*d*x + 10*c), cos
(10*d*x + 10*c)))^4 + (-I*sqrt(2)*cos(10*d*x + 10*c) - sqrt(2)*sin(10*d*x +
 10*c))*sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^4 + 4*(-I*
sqrt(2)*cos(10*d*x + 10*c) - sqrt(2)*sin(10*d*x + 10*c))*cos(1/5*arctan2(si
n(10*d*x + 10*c), cos(10*d*x + 10*c)))^3 + 6*(-I*sqrt(2)*cos(10*d*x + 10*c)
 - sqrt(2)*sin(10*d*x + 10*c))*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d
*x + 10*c)))^2 + 2*((-I*sqrt(2)*cos(10*d*x + 10*c) - sqrt(2)*sin(10*d*x + 1
0*c))*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 2*(-I*sq
rt(2)*cos(10*d*x + 10*c) - sqrt(2)*sin(10*d*x + 10*c))*cos(1/5*arctan2(sin(
10*d*x + 10*c), cos(10*d*x + 10*c))) - I*sqrt(2)*cos(10*d*x + 10*c) - sqrt(
2)*sin(10*d*x + 10*c))*sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*
c)))^2 + 4*(-I*sqrt(2)*cos(10*d*x + 10*c) - sqrt(2)*sin(10*d*x + 10*c))*cos
(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) - I*sqrt(2)*cos(10*d*
x + 10*c) - sqrt(2)*sin(10*d*x + 10*c))*cos(9/2*arctan2(sin(1/5*arctan2(sin
(10*d*x + 10*c), cos(10*d*x + 10*c))), cos(1/5*arctan2(sin(10*d*x + 10*c),

cos(10*d*x + 10*c))) + 1)) + 2*(448*(I*sqrt(2)*cos(10*d*x + 10*c) + sqrt(2)
*sin(10*d*x + 10*c))*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)
))^2 + 448*(I*sqrt(2)*cos(10*d*x + 10*c) + sqrt(2)*sin(10*d*x + 10*c))*sin(
1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 7665*(I*sqrt(2)*co
s(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + I*sqrt(2)*sin(1/
5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 2*I*sqrt(2)*cos(1/5*
arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + I*sqrt(2))*cos(4/5*arcta
n2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 10440*(-I*sqrt(2)*cos(1/5*arc
tan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 - I*sqrt(2)*sin(1/5*arctan2
(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 - 2*I*sqrt(2)*cos(1/5*arctan2(s
in(10*d*x + 10*c), cos(10*d*x + 10*c))) - I*sqrt(2))*cos(3/5*arctan2(sin(10
*d*x + 10*c), cos(10*d*x + 10*c))) + 896*(I*sqrt(2)*cos(10*d*x + 10*c) + sq
rt(2)*sin(10*d*x + 10*c))*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x +
10*c))) + 7665*(sqrt(2)*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10
*c)))^2 + sqrt(2)*sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^
2 + 2*sqrt(2)*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + sq
rt(2))*sin(4/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) - 10440*(sq
rt(2)*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + sqrt(2)*
sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 2*sqrt(2)*cos(
1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + sqrt(2))*sin(3/5*arc
tan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 448*I*sqrt(2)*cos(10*d*x +
10*c) + 448*sqrt(2)*sin(10*d*x + 10*c))*cos(5/2*arctan2(sin(1/5*arctan2(sin
(10*d*x + 10*c), cos(10*d*x + 10*c))), cos(1/5*arctan2(sin(10*d*x + 10*c),
cos(10*d*x + 10*c))) + 1)) + 15*(7*I*sqrt(2)*cos(10*d*x + 10*c) + 210*I*sqr
t(2)*cos(4/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) - 912*I*sqrt(
2)*cos(3/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 352*I*sqrt(2)
*cos(2/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 7*sqrt(2)*sin(1
0*d*x + 10*c) + 210*sqrt(2)*sin(4/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x
+ 10*c))) - 912*sqrt(2)*sin(3/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10
*c))) + 352*sqrt(2)*sin(2/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))
) + 1536*I*sqrt(2))*cos(1/2*arctan2(sin(1/5*arctan2(sin(10*d*x + 10*c), cos
(10*d*x + 10*c))), cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))
 + 1)) + 105*((sqrt(2)*cos(10*d*x + 10*c) - I*sqrt(2)*sin(10*d*x + 10*c))*c
os(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^4 + (sqrt(2)*cos(10
*d*x + 10*c) - I*sqrt(2)*sin(10*d*x + 10*c))*sin(1/5*arctan2(sin(10*d*x + 1
0*c), cos(10*d*x + 10*c)))^4 + 4*(sqrt(2)*cos(10*d*x + 10*c) - I*sqrt(2)*si
n(10*d*x + 10*c))*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^
3 + 6*(sqrt(2)*cos(10*d*x + 10*c) - I*sqrt(2)*sin(10*d*x + 10*c))*cos(1/5*a
rctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 2*((sqrt(2)*cos(10*d*x
+ 10*c) - I*sqrt(2)*sin(10*d*x + 10*c))*cos(1/5*arctan2(sin(10*d*x + 10*c),
 cos(10*d*x + 10*c)))^2 + 2*(sqrt(2)*cos(10*d*x + 10*c) - I*sqrt(2)*sin(10*
d*x + 10*c))*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + sqr
t(2)*cos(10*d*x + 10*c) - I*sqrt(2)*sin(10*d*x + 10*c))*sin(1/5*arctan2(sin
(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 4*(sqrt(2)*cos(10*d*x + 10*c) - I
*sqrt(2)*sin(10*d*x + 10*c))*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x

```
 + 10*c))) + sqrt(2)*cos(10*d*x + 10*c) - I*sqrt(2)*sin(10*d*x + 10*c))*sin
(9/2*arctan2(sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))), cos(
1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 1)) - 2*(448*(sqrt(2
)*cos(10*d*x + 10*c) - I*sqrt(2)*sin(10*d*x + 10*c))*cos(1/5*arctan2(sin(10
*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 448*(sqrt(2)*cos(10*d*x + 10*c) - I*
sqrt(2)*sin(10*d*x + 10*c))*sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x
+ 10*c)))^2 + 7665*(sqrt(2)*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x
+ 10*c)))^2 + sqrt(2)*sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c
)))^2 + 2*sqrt(2)*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))
+ sqrt(2))*cos(4/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) - 10440
*(sqrt(2)*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + sqrt
(2)*sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 2*sqrt(2)*
cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + sqrt(2))*cos(3/5
*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 896*(sqrt(2)*cos(10*d*x
 + 10*c) - I*sqrt(2)*sin(10*d*x + 10*c))*cos(1/5*arctan2(sin(10*d*x + 10*c)
, cos(10*d*x + 10*c))) - 7665*(I*sqrt(2)*cos(1/5*arctan2(sin(10*d*x + 10*c)
, cos(10*d*x + 10*c)))^2 + I*sqrt(2)*sin(1/5*arctan2(sin(10*d*x + 10*c), co
s(10*d*x + 10*c)))^2 + 2*I*sqrt(2)*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(
10*d*x + 10*c))) + I*sqrt(2))*sin(4/5*arctan2(sin(10*d*x + 10*c), cos(10*d*
x + 10*c))) - 10440*(-I*sqrt(2)*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*
d*x + 10*c)))^2 - I*sqrt(2)*sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x
+ 10*c)))^2 - 2*I*sqrt(2)*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x +
10*c))) - I*sqrt(2))*sin(3/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)
)) + 448*sqrt(2)*cos(10*d*x + 10*c) - 448*I*sqrt(2)*sin(10*d*x + 10*c))*sin
(5/2*arctan2(sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))), cos(
1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 1)) - 15*(7*sqrt(2)*
cos(10*d*x + 10*c) + 210*sqrt(2)*cos(4/5*arctan2(sin(10*d*x + 10*c), cos(10
*d*x + 10*c))) - 912*sqrt(2)*cos(3/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x
 + 10*c))) + 352*sqrt(2)*cos(2/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 1
0*c))) - 7*I*sqrt(2)*sin(10*d*x + 10*c) - 210*I*sqrt(2)*sin(4/5*arctan2(sin
(10*d*x + 10*c), cos(10*d*x + 10*c))) + 912*I*sqrt(2)*sin(3/5*arctan2(sin(1
0*d*x + 10*c), cos(10*d*x + 10*c))) - 352*I*sqrt(2)*sin(2/5*arctan2(sin(10*
d*x + 10*c), cos(10*d*x + 10*c))) + 1536*sqrt(2))*sin(1/2*arctan2(sin(1/5*a
rctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))), cos(1/5*arctan2(sin(10*d*x
 + 10*c), cos(10*d*x + 10*c))) + 1)))*sqrt(a) + 45045*(2*sqrt(2)*arctan2((c
os(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + sin(1/5*arctan2
(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 2*cos(1/5*arctan2(sin(10*d*x
+ 10*c), cos(10*d*x + 10*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/5*arctan2(si
n(10*d*x + 10*c), cos(10*d*x + 10*c))), cos(1/5*arctan2(sin(10*d*x + 10*c),
 cos(10*d*x + 10*c))) + 1)), (cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*
x + 10*c)))^2 + sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2
+ 2*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 1)^(1/4)*cos
(1/2*arctan2(sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))), cos(
1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 1)) + 1) - 2*sqrt(2)
*arctan2((cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + sin(
```

```
1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 2*cos(1/5*arctan2(
sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/5
*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))), cos(1/5*arctan2(sin(10*d
*x + 10*c), cos(10*d*x + 10*c))) + 1)), (cos(1/5*arctan2(sin(10*d*x + 10*c)
, cos(10*d*x + 10*c)))^2 + sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x +
 10*c)))^2 + 2*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 1
)^(1/4)*cos(1/2*arctan2(sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10
*c))), cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 1)) - 1)
- I*sqrt(2)*log(sqrt(cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)
))^2 + sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 2*cos(1
/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 1)*cos(1/2*arctan2(si
n(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))), cos(1/5*arctan2(sin
(10*d*x + 10*c), cos(10*d*x + 10*c))) + 1))^2 + sqrt(cos(1/5*arctan2(sin(10
*d*x + 10*c), cos(10*d*x + 10*c)))^2 + sin(1/5*arctan2(sin(10*d*x + 10*c),
cos(10*d*x + 10*c)))^2 + 2*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x +
 10*c))) + 1)*sin(1/2*arctan2(sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*
x + 10*c))), cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 1))
^2 + 2*(cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + sin(1/
5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 2*cos(1/5*arctan2(si
n(10*d*x + 10*c), cos(10*d*x + 10*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/5*a
rctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))), cos(1/5*arctan2(sin(10*d*x
 + 10*c), cos(10*d*x + 10*c))) + 1)) + 1) + I*sqrt(2)*log(sqrt(cos(1/5*arct
an2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + sin(1/5*arctan2(sin(10*d*x
 + 10*c), cos(10*d*x + 10*c)))^2 + 2*cos(1/5*arctan2(sin(10*d*x + 10*c), co
s(10*d*x + 10*c))) + 1)*cos(1/2*arctan2(sin(1/5*arctan2(sin(10*d*x + 10*c),
 cos(10*d*x + 10*c))), cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*
c))) + 1))^2 + sqrt(cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))
)^2 + sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c)))^2 + 2*cos(1/
5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) + 1)*sin(1/2*arctan2(sin
(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))), cos(1/5*arctan2(sin(
10*d*x + 10*c), cos(10*d*x + 10*c))) + 1))^2 - 2*(cos(1/5*arctan2(sin(10*d*
x + 10*c), cos(10*d*x + 10*c)))^2 + sin(1/5*arctan2(sin(10*d*x + 10*c), cos
(10*d*x + 10*c)))^2 + 2*cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10
*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/5*arctan2(sin(10*d*x + 10*c), cos(10
*d*x + 10*c))), cos(1/5*arctan2(sin(10*d*x + 10*c), cos(10*d*x + 10*c))) +
1)) + 1))*sqrt(a))/(a^4*d)
```

Giac [F]

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\cos(dx+c)^3}{(i\,a\tan(dx+c)+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(cos(d*x+c)^3/(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate(cos(d*x + c)^3/(I*a*tan(d*x + c) + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a+ia\tan(c+dx))^{7/2}}\,dx = \int \frac{\cos(c+d\,x)^3}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^{7/2}}\,dx$$

```
[In] int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^(7/2),x)
[Out] int(cos(c + d*x)^3/(a + a*tan(c + d*x)*1i)^(7/2), x)
```

3.394 $\int (e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 524

$$\int (e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}} - \frac{ia^{3/2}e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{ia^{3/2}e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{ia^{3/2}e^{3/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{2\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{ia^{3/2}e^{3/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{2\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] I*a*(e*sec(d*x+c))^(3/2)/d/(a+I*a*tan(d*x+c))^(1/2)-1/2*I*a^(3/2)*e^(3/2)*a
rctan(1-2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/
2))*sec(d*x+c)/d*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+
1/2*I*a^(3/2)*e^(3/2)*arctan(1+2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(
1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)/d*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a
+I*a*tan(d*x+c))^(1/2)+1/4*I*a^(3/2)*e^(3/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(
a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a-I*a*tan(d*x+c)))
*sec(d*x+c)/d*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-1/4
*I*a^(3/2)*e^(3/2)*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e
*sec(d*x+c))^(1/2)+cos(d*x+c)*(a-I*a*tan(d*x+c)))*sec(d*x+c)/d*2^(1/2)/(a-I
*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3579, 3580, 3576, 303, 1176, 631, 210, 1179, 642}

$$\int (e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx =$$
$$-\frac{ia^{3/2}e^{3/2}\sec(c+dx)\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$+\frac{ia^{3/2}e^{3/2}\sec(c+dx)\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$+\frac{ia^{3/2}e^{3/2}\sec(c+dx)\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{2\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$-\frac{ia^{3/2}e^{3/2}\sec(c+dx)\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{2\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$+\frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (I*a*(e*Sec[c + d*x])^(3/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) - (I*a^(3/2)*e^
(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt
[e*Sec[c + d*x]])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt
[a + I*a*Tan[c + d*x]]) + (I*a^(3/2)*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e]*Sq
rt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(Sq
rt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + ((I/2)*a^(
3/2)*e^(3/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/S
qrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/(S
qrt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - ((I/2)*a^
(3/2)*e^(3/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/
Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/(
Sqrt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
```

```
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3580

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Dist[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]])), Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}+\frac{1}{2}a\int\frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx\\
&= \frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}+\frac{(ae\sec(c+dx))\int\sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}\,dx}{2\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\
&= \frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}+\frac{\left(2ia^2e^3\sec(c+dx)\right)\text{Subst}\left(\int\frac{x^2}{a^2+e^2x^4}\,dx,x,\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\
&= \frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}-\frac{\left(ia^2e^2\sec(c+dx)\right)\text{Subst}\left(\int\frac{a-ex^2}{a^2+e^2x^4}\,dx,x,\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\
&\quad+\frac{\left(ia^2e^2\sec(c+dx)\right)\text{Subst}\left(\int\frac{a+ex^2}{a^2+e^2x^4}\,dx,x,\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\
&= \frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}}\\
&\quad+\frac{\left(ia^2e\sec(c+dx)\right)\text{Subst}\left(\int\frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx,x,\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{2d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\
&\quad+\frac{\left(ia^2e\sec(c+dx)\right)\text{Subst}\left(\int\frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx,x,\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{2d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\
&\quad+\frac{\left(ia^{3/2}e^{3/2}\sec(c+dx)\right)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}+2x}{-\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx,x,\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{2\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\
&\quad+\frac{\left(ia^{3/2}e^{3/2}\sec(c+dx)\right)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}-2x}{-\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx,x,\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{2\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}} \\
&+ \frac{ia^{3/2}e^{3/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{2\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{ia^{3/2}e^{3/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{2\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&+ \frac{\left(ia^{3/2}e^{3/2}\sec(c+dx)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{\left(ia^{3/2}e^{3/2}\sec(c+dx)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{ia(e\sec(c+dx))^{3/2}}{d\sqrt{a+ia\tan(c+dx)}} - \frac{ia^{3/2}e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&+ \frac{ia^{3/2}e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&+ \frac{ia^{3/2}e^{3/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{2\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{ia^{3/2}e^{3/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{2\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.12 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.71

$$
\int(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{e\sqrt{e\sec(c+dx)}(\cos(c)-i\sin(c))\left(\text{arctanh}\left(\frac{\sqrt{1+i\cos(c)-\sin(c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}}{\sqrt{-1+i\cos(c)+\sin(c)}\sqrt{i+\tan\left(\frac{dx}{2}\right)}}\right.\right.}{}
$$

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (e*Sqrt[e*Sec[c + d*x]]*(Cos[c] - I*Sin[c])*(ArcTanh[(Sqrt[1 + I*Cos[c] - S
in[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(
d*x)/2]])]*Cos[c + d*x]*Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[1 + I*Cos[c] - Si
n[c]]*Sqrt[I + Tan[(d*x)/2]] + Sqrt[-1 + I*Cos[c] + Sin[c]]*(Sqrt[-1 - I*Co
```

```
s[c] - Sin[c]]*(I*Cos[d*x] + Sin[d*x])*Sqrt[I - Tan[(d*x)/2]] - ArcTanh[(Sq
rt[1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin
[c]]*Sqrt[I + Tan[(d*x)/2]])]*Cos[c + d*x]*Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt
[I + Tan[(d*x)/2]]))*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[-1 - I*Cos[c] - Si
n[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])
```

Maple [A] (verified)

Time = 11.39 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.62

method	result
default	$-\frac{i\sqrt{a(1+i\tan(dx+c))}\,\sqrt{e\sec(dx+c)}\left(2i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\cos(dx+c)-i\,\text{arctanh}\left(\frac{\cos(dx+c)+\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)\cos(dx+c)-i\,\text{arctanh}}{}$

```
[In] int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*I/d*(a*(1+I*tan(d*x+c)))^(1/2)*(e*sec(d*x+c))^(1/2)*(2*I*(1/(cos(d*x+c
)+1))^(1/2)*cos(d*x+c)-I*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+
1)/(1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-I*arctanh(1/2*(cos(d*x+c)-sin(d*x+c
)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)+2*I*(1/(cos(d*x+c)
+1))^(1/2)+2*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+arctanh(1/2*(cos(d*x+c)+si
n(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-arctanh(1/2
*(cos(d*x+c)-sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*cos(d*x
+c))*(I*cos(d*x+c)+I+sin(d*x+c))*e/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 418, normalized size of antiderivative = 0.80

$$\int (e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{4i\,e\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+\sqrt{\frac{i\,ae^3}{d^2}}d\log\left(\frac{2\left(\left(ee^{(2i\,dx+2i\,c)}+e\right)\right.}{}\right.}{}$$

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(4*I*e*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) +
1))*e^(1/2*I*d*x + 1/2*I*c) + sqrt(I*a*e^3/d^2)*d*log(2*((e*e^(2*I*d*x + 2*
I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1
))*e^(1/2*I*d*x + 1/2*I*c) + I*sqrt(I*a*e^3/d^2)*d)/e) - sqrt(I*a*e^3/d^2)*
```

```
d*log(2*((e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt
(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - I*sqrt(I*a*e^3/d^2)
*d)/e) + sqrt(-I*a*e^3/d^2)*d*log(2*((e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^
(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/
2*I*c) + I*sqrt(-I*a*e^3/d^2)*d)/e) - sqrt(-I*a*e^3/d^2)*d*log(2*((e*e^(2*I
*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2
*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - I*sqrt(-I*a*e^3/d^2)*d)/e))/d
```

Sympy [F]

$$\int (e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx = \int (e\sec{(c+dx)})^{\frac{3}{2}}\sqrt{ia(\tan{(c+dx)}-i)}\,dx$$

```
[In] integrate((e*sec(d*x+c))**(3/2)*(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Integral((e*sec(c + d*x))**(3/2)*sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1870 vs. $2(396) = 792$.

Time = 0.48 (sec) , antiderivative size = 1870, normalized size of antiderivative = 3.57

$$\int (e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
[Out] -8*(2*(sqrt(2)*e*cos(2*d*x + 2*c) + I*sqrt(2)*e*sin(2*d*x + 2*c) + sqrt(2)*
e)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1
, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 2*(sq
rt(2)*e*cos(2*d*x + 2*c) + I*sqrt(2)*e*sin(2*d*x + 2*c) + sqrt(2)*e)*arctan
2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, -sqrt(2
)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 2*(sqrt(2)*e*
cos(2*d*x + 2*c) + I*sqrt(2)*e*sin(2*d*x + 2*c) + sqrt(2)*e)*arctan2(sqrt(2
)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 2*(sqrt(2)*e*cos(2*d*x
 + 2*c) + I*sqrt(2)*e*sin(2*d*x + 2*c) + sqrt(2)*e)*arctan2(sqrt(2)*cos(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 2*(-I*sqrt(2)*e*cos(2*d*x + 2*
c) + sqrt(2)*e*sin(2*d*x + 2*c) - I*sqrt(2)*e)*arctan2(sqrt(2)*sin(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c)
```

```
, cos(2*d*x + 2*c))), sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 2*(I*
sqrt(2)*e*cos(2*d*x + 2*c) - sqrt(2)*e*sin(2*d*x + 2*c) + I*sqrt(2)*e)*arct
an2(-sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), -sqrt(2)*cos(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 1) - 16*e*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + (sqrt(2)*e*cos(2*d*x + 2*c) + I*sqrt(2)*e*sin(2*d*x + 2*c) + sqrt(2)*
e)*log(2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(2)*cos(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 1) - (sqrt(2)*e*cos(2*d*x + 2*c) + I*sqrt(2)*e*sin(2*d
*x + 2*c) + sqrt(2)*e)*log(-2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*(s
qrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1)*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - (-I*sqrt(2)*e*cos(2*d*x + 2*
c) + sqrt(2)*e*sin(2*d*x + 2*c) - I*sqrt(2)*e)*log(2*cos(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2
) - (I*sqrt(2)*e*cos(2*d*x + 2*c) - sqrt(2)*e*sin(2*d*x + 2*c) + I*sqrt(2)*
e)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 2) - (-I*sqrt(2)*e*cos(2*d*x + 2*c) + sqrt(2)
*e*sin(2*d*x + 2*c) - I*sqrt(2)*e)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*s
qrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - (I*sqrt(
2)*e*cos(2*d*x + 2*c) - sqrt(2)*e*sin(2*d*x + 2*c) + I*sqrt(2)*e)*log(2*cos
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + 2) - 16*I*e*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))))*sqrt(a)*sqrt(e)/(d*(-64*I*cos(2*d*x + 2*c) + 64*sin(2*d*x + 2*c) -
 64*I))
```

Giac [F]

$$\int (e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx = \int (e\sec(dx+c))^{\frac{3}{2}}\sqrt{ia\tan(dx+c)+a}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
[Out] integrate((e*sec(d*x + c))^(3/2)*sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}\,dx$$

```
[In] int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.395 $\int \sqrt{e \sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 323

$$\int \sqrt{e \sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{i\sqrt{2}\sqrt{a}\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{d} - \frac{i\sqrt{2}\sqrt{a}\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{d} - \frac{i\sqrt{a}\sqrt{e}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{\sqrt{2}d} + \frac{i\sqrt{a}\sqrt{e}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{\sqrt{2}d}$$

```
[Out] -1/2*I*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))
^(1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))*a^(1/2)*e^(1/2)/d*2^(1/2)+1/2*I*ln(a+
2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d
*x+c)*(a+I*a*tan(d*x+c)))*a^(1/2)*e^(1/2)/d*2^(1/2)+I*arctan(1-2^(1/2)*e^(1
/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*2^(1/2)*a^(1/2)*
e^(1/2)/d-I*arctan(1+2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*se
c(d*x+c))^(1/2))*2^(1/2)*a^(1/2)*e^(1/2)/d
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {3576, 303, 1176, 631, 210, 1179, 642}

$$
\begin{aligned}
&\int \sqrt{e \sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx \\
&= \frac{i\sqrt{2}\sqrt{a}\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{d} \\
&\quad - \frac{i\sqrt{2}\sqrt{a}\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{d} \\
&\quad - \frac{i\sqrt{a}\sqrt{e}\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{\sqrt{2}d} \\
&\quad + \frac{i\sqrt{a}\sqrt{e}\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{\sqrt{2}d}
\end{aligned}
$$

```
[In] Int[Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (I*Sqrt[2]*Sqrt[a]*Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c +
 d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]])/d - (I*Sqrt[2]*Sqrt[a]*Sqrt[e]*Arc
Tan[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c
+ d*x]])]])/d - (I*Sqrt[a]*Sqrt[e]*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a +
 I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c + d*
x])])/(Sqrt[2]*d) + (I*Sqrt[a]*Sqrt[e]*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqr
t[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c
 + d*x])])/(Sqrt[2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(4iae^2)\,\text{Subst}\left(\int \frac{x^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d}$$

$$= \frac{(2iae)\text{Subst}\left(\int \frac{a-ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d} - \frac{(2iae)\text{Subst}\left(\int \frac{a+ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d}$$

$$= -\frac{(ia)\text{Subst}\left(\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d} - \frac{(ia)\text{Subst}\left(\int \frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d} - \frac{\left(i\sqrt{a}\sqrt{e}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}+2x}{-\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d} - \frac{\left(i\sqrt{a}\sqrt{e}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}-2x}{-\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d}$$

$$= -\frac{i\sqrt{a}\sqrt{e}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{\sqrt{2}d} + \frac{i\sqrt{a}\sqrt{e}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{\sqrt{2}d} - \frac{\left(i\sqrt{2}\sqrt{a}\sqrt{e}\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{d} + \frac{\left(i\sqrt{2}\sqrt{a}\sqrt{e}\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{d}$$

$$= \frac{i\sqrt{2}\sqrt{a}\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{d} - \frac{i\sqrt{2}\sqrt{a}\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{d} - \frac{i\sqrt{a}\sqrt{e}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{\sqrt{2}d} + \frac{i\sqrt{a}\sqrt{e}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{\sqrt{2}d}$$

Mathematica [A] (verified)

Time = 1.90 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.86

$$\int \sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx =$$

$$-\frac{2e\left(\text{arctanh}\left(\frac{\sqrt{1-i\cos(c)+\sin(c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}}{\sqrt{-1-i\cos(c)-\sin(c)}\sqrt{i+\tan\left(\frac{dx}{2}\right)}}\right)\sqrt{-1-i\cos(c)-\sin(c)}\sqrt{1+i\cos(c)-\sin(c)}-\text{arctanh}\left(\right.\right.}{d\sqrt{e\sec(c+dx)}\sqrt{1+}}$$

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (-2*e*(ArcTanh[(Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-
1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1 - I*Cos[c] - Sin[c]
]*Sqrt[1 + I*Cos[c] - Sin[c]] - ArcTanh[(Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I
 - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sq
rt[1 - I*Cos[c] + Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]])*Sqrt[I + Tan[(d*x)/
2]]*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[e*Sec[c + d*x]]*Sqrt[1 + Cos[2*c] +
 I*Sin[2*c]]*Sqrt[I - Tan[(d*x)/2]])
```

Maple [A] (verified)

Time = 11.34 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.49

method	result
default	$\frac{(-1+i)\sqrt{e\sec(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}\left(i\,\mathrm{arctanh}\left(\frac{-\cos(dx+c)+\sin(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)-\mathrm{arctanh}\left(\frac{\cos(dx+c)+\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)\right)\mathrm{co}}{d(-i\cos(dx+c)+\sin(dx+c)-i)\sqrt{\frac{1}{\cos(dx+c)+1}}}$

```
[In] int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1+I)/d*(e*sec(d*x+c))^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)*(I*arctanh(1/2*(-c
os(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))-arctanh(1/
2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2)))*cos(d
*x+c)/(-I*cos(d*x+c)+sin(d*x+c)-I)/(1/(cos(d*x+c)+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.00

$$\int \sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx$$
$$= \frac{1}{2}\sqrt{\frac{4i\,ae}{d^2}}\log\left(2\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)} + d\sqrt{\frac{4i\,ae}{d^2}}\right)$$
$$- \frac{1}{2}\sqrt{\frac{4i\,ae}{d^2}}\log\left(2\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)} - d\sqrt{\frac{4i\,ae}{d^2}}\right)$$
$$- \frac{1}{2}\sqrt{-\frac{4i\,ae}{d^2}}\log\left(2\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)} + d\sqrt{-\frac{4i\,ae}{d^2}}\right)$$
$$+ \frac{1}{2}\sqrt{-\frac{4i\,ae}{d^2}}\log\left(2\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(e^{(2i\,dx+2i\,c)}+1\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)} - d\sqrt{-\frac{4i\,ae}{d^2}}\right)$$

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/2*sqrt(4*I*a*e/d^2)*log(2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*
I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(1/2*I*d*x + 1/2*I*c) + d*
sqrt(4*I*a*e/d^2)) - 1/2*sqrt(4*I*a*e/d^2)*log(2*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(1/2*
I*d*x + 1/2*I*c) - d*sqrt(4*I*a*e/d^2)) - 1/2*sqrt(-4*I*a*e/d^2)*log(2*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x
 + 2*I*c) + 1)*e^(1/2*I*d*x + 1/2*I*c) + d*sqrt(-4*I*a*e/d^2)) + 1/2*sqrt(-
4*I*a*e/d^2)*log(2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2
*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(1/2*I*d*x + 1/2*I*c) - d*sqrt(-4*I
*a*e/d^2))
```

Sympy [F]

$$\int \sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{e\sec{(c+dx)}}\sqrt{ia\left(\tan{(c+dx)}-i\right)}\,dx$$

```
[In] integrate((e*sec(d*x+c))**(1/2)*(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Integral(sqrt(e*sec(c + d*x))*sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1400 vs. 2(239) = 478.

Time = 0.48 (sec) , antiderivative size = 1400, normalized size of antiderivative = 4.33

$$\int \sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
[Out] 1/4*(-2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + 1, sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + 1) - 2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1, -sqrt(2)*sin(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 2*I*sqrt(2)*arctan2(sqrt(2)*
cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 1, sqrt(2)*s
in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 2*I*sqrt
(2)*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) - 1, -sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) + 1) - 2*sqrt(2)*arctan2(sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c))) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))), sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)
 + 2*sqrt(2)*arctan2(-sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))), -sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
+ cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + I*sqr
t(2)*log(2*sqrt(2)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c)))*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*(sqrt
(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(2
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(2/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*cos(1/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c
```

```
), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) + 1) - I*sqrt(2)*log(-2*sqrt(2)*sin(2/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) - 2*(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) - 1)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^
2 + 2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(
2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + sqrt(2)*log(2*cos(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - sqrt(2)*log(2*cos(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + sqrt(2)*log(2*cos(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - sqrt(2)*log(2*cos(1/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2))*sqrt(a)*sqrt(e)/d
```

Giac [F]

$$\int \sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx=\int \sqrt{e\sec\left(dx+c\right)}\sqrt{i\,a\tan\left(dx+c\right)+a}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(e*sec(d*x + c))*sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

```
[In] int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.396 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 36

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx=-\frac{2i\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

```
[Out] -2*I*(a+I*a*tan(d*x+c))^(1/2)/d/(e*sec(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {3569}

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx=-\frac{2i\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

```
[In] Int[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[e*Sec[c + d*x]],x]
[Out] ((-2*I)*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[e*Sec[c + d*x]])
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\text{integral}=-\frac{2i\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx=-\frac{2i\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

```
[In] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[e*Sec[c + d*x]],x]
[Out] ((-2*I)*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[e*Sec[c + d*x]])
```

Maple [A] (verified)

Time = 8.49 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{2i\sqrt{a(1+i\tan(dx+c))}}{d\sqrt{e\sec(dx+c)}}$	32
risch	$-\frac{2i\sqrt{\frac{a\,\mathrm{e}^{2i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}{\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}$	59

```
[In] int((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*I/d*(a*(1+I*tan(d*x+c)))^(1/2)/(e*sec(d*x+c))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(28) = 56.

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.78

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx=\frac{2\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-i\,e^{(2i\,dx+2i\,c)}-i\right)e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{de}$$

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2),x, algorithm="frica
s")
[Out] 2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-I*e
^(2*I*d*x + 2*I*c) - I)*e^(1/2*I*d*x + 1/2*I*c)/(d*e)
```

Sympy [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx = \int \frac{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}{\sqrt{e\sec\left(c+dx\right)}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2)/(e*sec(d*x+c))**(1/2),x)
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))/sqrt(e*sec(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(28) = 56$.

Time = 0.32 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.11

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx = -\frac{2i\sqrt{a}\sqrt{-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}}{d\sqrt{e}\sqrt{-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}}$$

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2),x, algorithm="maxim
a")
[Out] -2*I*sqrt(a)*sqrt(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 - 1)/(d*sqrt(e)*sqrt(-sin(d*x + c)^2/(cos(d*x + c) + 1)^2
 - 1))
```

Giac [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx = \int \frac{\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{e\sec\left(dx+c\right)}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2),x, algorithm="giac"
)
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)/sqrt(e*sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx = \int \frac{\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}{\sqrt{\frac{e}{\cos(c+d\,x)}}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2)/(e/cos(c + d*x))^(1/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^(1/2)/(e/cos(c + d*x))^(1/2), x)
```

3.397 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 81

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx = \frac{4ia\sqrt{e\sec(c+dx)}}{3de^2\sqrt{a+ia\tan(c+dx)}} - \frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}$$

```
[Out] 4/3*I*a*(e*sec(d*x+c))^(1/2)/d/e^2/(a+I*a*tan(d*x+c))^(1/2)-2/3*I*(a+I*a*ta
n(d*x+c))^(1/2)/d/(e*sec(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3578, 3569}

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx = \frac{4ia\sqrt{e\sec(c+dx)}}{3de^2\sqrt{a+ia\tan(c+dx)}} - \frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}$$

```
[In] Int[Sqrt[a + I*a*Tan[c + d*x]]/(e*Sec[c + d*x])^(3/2),x]
```

```
[Out] (((4*I)/3)*a*Sqrt[e*Sec[c + d*x]])/(d*e^2*Sqrt[a + I*a*Tan[c + d*x]]) - (((
2*I)/3)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Sec[c + d*x])^(3/2))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}+\frac{(2a)\int\frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{3e^2}\\ &= \frac{4ia\sqrt{e\sec(c+dx)}}{3de^2\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.59

$$\int\frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx=\frac{2(i+2\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{3d(e\sec(c+dx))^{3/2}}$$

```
[In] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/(e*Sec[c + d*x])^(3/2),x]
[Out] (2*(I + 2*Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(3*d*(e*Sec[c + d*x])^(
3/2))
```

Maple [A] (verified)

Time = 10.64 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.64

method	result	size
default	$\frac{2(i\cos(dx+c)+2\sin(dx+c))\sqrt{a(1+i\tan(dx+c))}}{3d\sqrt{e\sec(dx+c)}\,e}$	52
risch	$-\frac{i\sqrt{\frac{a\,e^{2i(dx+c)}}{e^{2i(dx+c)}+1}}\,(-2\cos(dx+c)+4i\sin(dx+c))}{3e\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	80

```
[In] int((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/3/d*(I*cos(d*x+c)+2*sin(d*x+c))*(a*(1+I*tan(d*x+c)))^(1/2)/(e*sec(d*x+c))
^(1/2)/e
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-i\,e^{(4i\,dx+4i\,c)}+2i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{3\,de^2}$$

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/3*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-I
*e^(4*I*d*x + 4*I*c) + 2*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(-1/2*I*d*x - 1/2*I
*c)/(d*e^2)
```

Sympy [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}{\left(e\sec\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2)/(e*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))/(e*sec(c + d*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(-i\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3i\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3\sin\left(\frac{1}{2}\,dx\right.\right.}{3\,de^{\frac{3}{2}}}$$

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(3/2),x, algorithm="maxim
a")
```

```
[Out] 1/3*sqrt(a)*(-I*cos(3/2*d*x + 3/2*c) + 3*I*cos(1/2*d*x + 1/2*c) + sin(3/2*d
*x + 3/2*c) + 3*sin(1/2*d*x + 1/2*c))/(d*e^(3/2))
```

Giac [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx=\int \frac{\sqrt{i\,a\tan\left(dx+c\right)+a}}{\left(e\sec\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(3/2),x, algorithm="giac"
)

[Out] integrate(sqrt(I*a*tan(d*x + c) + a)/(e*sec(d*x + c))^(3/2), x)

Mupad [B] (verification not implemented)

Time = 5.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx=\frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,\left(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i}\right)}{\cos(2\,c+2\,d\,x)+1}}\,\left(\cos\left(2\,c+2\,d\,x\right)\,1\mathrm{i}+2\,\sin\left(2\,c+2\,d\right.\right.}{3\,d\,e^2}$$

[In] int((a + a*tan(c + d*x)*1i)^(1/2)/(e/cos(c + d*x))^(3/2),x)

[Out] ((e/cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(
cos(2*c + 2*d*x) + 1))^(1/2)*(cos(2*c + 2*d*x)*1i + 2*sin(2*c + 2*d*x) + 1i
))/(3*d*e^2)

3.398 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 122

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx = \frac{8ia}{15de^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}} - \frac{16i\sqrt{a+ia\tan(c+dx)}}{15de^2\sqrt{e\sec(c+dx)}}$$

```
[Out] 8/15*I*a/d/e^2/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-2/5*I*(a+I*a*t
an(d*x+c))^(1/2)/d/(e*sec(d*x+c))^(5/2)-16/15*I*(a+I*a*tan(d*x+c))^(1/2)/d/
e^2/(e*sec(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3578, 3583, 3569}

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx = \frac{8ia}{15de^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{16i\sqrt{a+ia\tan(c+dx)}}{15de^2\sqrt{e\sec(c+dx)}} - \frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}$$

```
[In] Int[Sqrt[a + I*a*Tan[c + d*x]]/(e*Sec[c + d*x])^(5/2),x]
```

```
[Out] (((8*I)/15)*a)/(d*e^2*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - ((
(2*I)/5)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Sec[c + d*x])^(5/2)) - (((16*I)/
15)*Sqrt[a + I*a*Tan[c + d*x]])/(d*e^2*Sqrt[e*Sec[c + d*x]])
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}+\frac{(4a)\int\frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx}{5e^2}\\
&= \frac{8ia}{15de^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}+\frac{8\int\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx}{15e^2}\\
&= \frac{8ia}{15de^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\
&\quad-\frac{2i\sqrt{a+ia\tan(c+dx)}}{5d(e\sec(c+dx))^{5/2}}-\frac{16i\sqrt{a+ia\tan(c+dx)}}{15de^2\sqrt{e\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx = \frac{i(-15+\cos(2(c+dx))-4i\sin(2(c+dx)))\sqrt{a+ia\tan(c+dx)}}{15de^2\sqrt{e\sec(c+dx)}}$$

```
[In] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/(e*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((I/15)*(-15 + Cos[2*(c + d*x)] - (4*I)*Sin[2*(c + d*x)])*Sqrt[a + I*a*Tan[
c + d*x]])/(d*e^2*Sqrt[e*Sec[c + d*x]])
```

Maple [A] (verified)

Time = 10.52 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.51

method	result	size
default	$\frac{2\left(i\left(\cos^2(dx+c)\right)+4\sin(dx+c)\cos(dx+c)-8i\right)\sqrt{a(1+i\tan(dx+c))}}{15d\sqrt{e\sec(dx+c)}\,e^2}$	62
risch	$-\frac{i\sqrt{\frac{a\,e^{2i(dx+c)}}{e^{2i(dx+c)}+1}}\,(30-2\cos(2dx+2c)+8i\sin(2dx+2c))}{30e^2\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	87

```
[In] int((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/d*(I*cos(d*x+c)^2+4*sin(d*x+c)*cos(d*x+c)-8*I)*(a*(1+I*tan(d*x+c)))^(1
/2)/(e*sec(d*x+c))^(1/2)/e^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-3i\,e^{(6i\,dx+6i\,c)}-33i\,e^{(4i\,dx+4i\,c)}-25i\,e^{(2i\,dx+2i\,c)}+\right.}{30\,de^3}$$

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/30*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-
3*I*e^(6*I*d*x + 6*I*c) - 33*I*e^(4*I*d*x + 4*I*c) - 25*I*e^(2*I*d*x + 2*I*
c) + 5*I)*e^(-3/2*I*d*x - 3/2*I*c)/(d*e^3)
```

Sympy [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{ia\,(\tan{(c+dx)}-i)}}{(e\sec{(c+dx)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2)/(e*sec(d*x+c))**(5/2),x)
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))/(e*sec(c + d*x))**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{a}\left(5i\,\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)-3i\,\cos\left(\frac{5}{3}\,\arctan\left(\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right),\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\right)\right)\right.}{}$$

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(5/2),x, algorithm="maxim
a")
[Out] 1/30*sqrt(a)*(5*I*cos(3/2*d*x + 3/2*c) - 3*I*cos(5/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) - 30*I*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + 5*sin(3/2*d*x + 3/2*c) + 3*sin(5/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 30*sin(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))))/(d*e^(5/2))
```

Giac [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{i\,a\tan{(dx+c)}+a}}{(e\sec{(dx+c)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(5/2),x, algorithm="giac"
)
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)/(e*sec(d*x + c))^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 5.80 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx = \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,(4\,\sin\left(c+d\,x\right)+4\,\sin\left(3\,c+3\,d\,x\right)}{30\,d\,e^3}$$

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2)/(e/cos(c + d*x))^(5/2),x)
```

```
[Out] ((e/cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(
cos(2*c + 2*d*x) + 1))^(1/2)*(4*sin(c + d*x) - cos(c + d*x)*29i + cos(3*c +
 3*d*x)*1i + 4*sin(3*c + 3*d*x)))/(30*d*e^3)
```

3.399 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 164

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}\,dx = \frac{12ia}{35de^2(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} + \frac{32ia\sqrt{e\sec(c+dx)}}{35de^4\sqrt{a+ia\tan(c+dx)}} - \frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}} - \frac{16i\sqrt{a+ia\tan(c+dx)}}{35de^2(e\sec(c+dx))^{3/2}}$$

```
[Out] 12/35*I*a/d/e^2/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2)+32/35*I*a*(e*
sec(d*x+c))^(1/2)/d/e^4/(a+I*a*tan(d*x+c))^(1/2)-2/7*I*(a+I*a*tan(d*x+c))^(
1/2)/d/(e*sec(d*x+c))^(7/2)-16/35*I*(a+I*a*tan(d*x+c))^(1/2)/d/e^2/(e*sec(d
*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3578, 3583, 3569}

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}\,dx = \frac{32ia\sqrt{e\sec(c+dx)}}{35de^4\sqrt{a+ia\tan(c+dx)}} + \frac{12ia}{35de^2\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}} - \frac{16i\sqrt{a+ia\tan(c+dx)}}{35de^2(e\sec(c+dx))^{3/2}} - \frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}$$

```
[In] Int[Sqrt[a + I*a*Tan[c + d*x]]/(e*Sec[c + d*x])^(7/2),x]
```

```
[Out] (((12*I)/35)*a)/(d*e^2*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) +
 (((32*I)/35)*a*Sqrt[e*Sec[c + d*x]])/(d*e^4*Sqrt[a + I*a*Tan[c + d*x]]) -
(((2*I)/7)*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Sec[c + d*x])^(7/2)) - (((16*I
)/35)*Sqrt[a + I*a*Tan[c + d*x]])/(d*e^2*(e*Sec[c + d*x])^(3/2))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}+\frac{(6a)\int\frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx}{7e^2}\\
&= \frac{12ia}{35de^2(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\\
&\quad -\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}+\frac{24\int\frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx}{35e^2}\\
&= \frac{12ia}{35de^2(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}-\frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}}\\
&\quad -\frac{16i\sqrt{a+ia\tan(c+dx)}}{35de^2(e\sec(c+dx))^{3/2}}+\frac{(16a)\int\frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{35e^4}
\end{aligned}
$$

$$= \frac{12ia}{35de^2(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} + \frac{32ia\sqrt{e\sec(c+dx)}}{35de^4\sqrt{a+ia\tan(c+dx)}} - \frac{2i\sqrt{a+ia\tan(c+dx)}}{7d(e\sec(c+dx))^{7/2}} - \frac{16i\sqrt{a+ia\tan(c+dx)}}{35de^2(e\sec(c+dx))^{3/2}}$$

Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}\,dx = \frac{(35i\cos(c+dx)+i\cos(3(c+dx))+70\sin(c+dx)+6\sin(3(c+dx)))\sqrt{a+ia}}{70de^3\sqrt{e\sec(c+dx)}}$$

```
[In] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/(e*Sec[c + d*x])^(7/2),x]
```

```
[Out] (((35*I)*Cos[c + d*x] + I*Cos[3*(c + d*x)] + 70*Sin[c + d*x] + 6*Sin[3*(c +
 d*x)])*Sqrt[a + I*a*Tan[c + d*x]])/(70*d*e^3*Sqrt[e*Sec[c + d*x]])
```

Maple [A] (verified)

Time = 10.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.49

method	result	size
default	$-\frac{2i\sqrt{a(1+i\tan(dx+c))}\,(6i(\cos^2(dx+c))\sin(dx+c)-(\cos^3(dx+c))+16i\sin(dx+c)-8\cos(dx+c))}{35d\sqrt{e\sec(dx+c)}\,e^3}$	80

```
[In] int((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/35*I/d*(a*(1+I*tan(d*x+c)))^(1/2)*(6*I*cos(d*x+c)^2*sin(d*x+c)-cos(d*x+c
)^3+16*I*sin(d*x+c)-8*cos(d*x+c))/(e*sec(d*x+c))^(1/2)/e^3
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-5i\,e^{(8i\,dx+8i\,c)}-40i\,e^{(6i\,dx+6i\,c)}+70i\,e^{(4i\,dx+4i\,c)}+1\right.}{140\,de^4}$$

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(7/2),x, algorithm="frica
s")
```

```
[Out] 1/140*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(
-5*I*e^(8*I*d*x + 8*I*c) - 40*I*e^(6*I*d*x + 6*I*c) + 70*I*e^(4*I*d*x + 4*I
*c) + 112*I*e^(2*I*d*x + 2*I*c) + 7*I)*e^(-5/2*I*d*x - 5/2*I*c)/(d*e^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2)/(e*sec(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}\,dx = \frac{\sqrt{a}\left(7i\,\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)-5i\,\cos\left(\frac{7}{5}\,\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\right)\right.}{}$$

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(7/2),x, algorithm="maxim
a")
```

```
[Out] 1/140*sqrt(a)*(7*I*cos(5/2*d*x + 5/2*c) - 5*I*cos(7/5*arctan2(sin(5/2*d*x +
 5/2*c), cos(5/2*d*x + 5/2*c))) - 35*I*cos(3/5*arctan2(sin(5/2*d*x + 5/2*c)
, cos(5/2*d*x + 5/2*c))) + 105*I*cos(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(
5/2*d*x + 5/2*c))) + 7*sin(5/2*d*x + 5/2*c) + 5*sin(7/5*arctan2(sin(5/2*d*x
 + 5/2*c), cos(5/2*d*x + 5/2*c))) + 35*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c)
, cos(5/2*d*x + 5/2*c))) + 105*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/
2*d*x + 5/2*c))))/(d*e^(7/2))
```

Giac [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}\,dx = \int \frac{\sqrt{i\,a\tan\left(dx+c\right)+a}}{\left(e\sec\left(dx+c\right)\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(7/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)/(e*sec(d*x + c))^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 5.78 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}\,dx = \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,\left(\cos\left(2\,c+2\,d\,x\right)\,36\mathrm{i}+\cos\left(4\,c+4\,d\,x\right.\right.}{140\,d\,e^4}$$

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2)/(e/cos(c + d*x))^(7/2),x)
```

```
[Out] ((e/cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(
cos(2*c + 2*d*x) + 1))^(1/2)*(cos(2*c + 2*d*x)*36i + cos(4*c + 4*d*x)*1i +
76*sin(2*c + 2*d*x) + 6*sin(4*c + 4*d*x) + 35i))/(140*d*e^4)
```

3.400 $\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 453

$$\begin{aligned}\int (e\sec(c+dx))^{5/2}(a&+ia\tan(c+dx))^{3/2}\,dx = \frac{7ia^{3/2}e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}d}\\
&-\frac{7ia^{3/2}e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}d}\\
&-\frac{7ia^{3/2}e^{5/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{16\sqrt{2}d}\\
&+\frac{7ia^{3/2}e^{5/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{16\sqrt{2}d}\\
&+\frac{7ia^2(e\sec(c+dx))^{5/2}}{12d\sqrt{a+ia\tan(c+dx)}}-\frac{7iae^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{8d}\\
&+\frac{ia(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}{3d}\end{aligned}$$

```
[Out] 7/16*I*a^(3/2)*e^(5/2)*arctan(1-2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^
(1/2)/(e*sec(d*x+c))^(1/2))/d*2^(1/2)-7/16*I*a^(3/2)*e^(5/2)*arctan(1+2^(1/
2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))/d*2^(1/2)
-7/32*I*a^(3/2)*e^(5/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/
2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))/d*2^(1/2)+7/32*I*a^(
3/2)*e^(5/2)*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(d
*x+c))^(1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))/d*2^(1/2)+7/12*I*a^2*(e*sec(d*x
+c))^(5/2)/d/(a+I*a*tan(d*x+c))^(1/2)+1/3*I*a*(e*sec(d*x+c))^(5/2)*(a+I*a*t
```

```
an(d*x+c))^(1/2)/d-7/8*I*a*e^2*(e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2
)/d
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3579, 3582, 3576, 303, 1176, 631, 210, 1179, 642}

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}\,dx=\frac{7ia^{3/2}e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}d}-\frac{7ia^{3/2}e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}d}-\frac{7ia^{3/2}e^{5/2}\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{16\sqrt{2}d}+\frac{7ia^{3/2}e^{5/2}\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{16\sqrt{2}d}+\frac{7ia^2(e\sec(c+dx))^{5/2}}{12d\sqrt{a+ia\tan(c+dx)}}-\frac{7iae^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{8d}+\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}{3d}$$

```
[In] Int[(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((7*I)/8)*a^(3/2)*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c +
 d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])])/(Sqrt[2]*d) - (((7*I)/8)*a^(3/2)*e
^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqr
t[e*Sec[c + d*x]])])/(Sqrt[2]*d) - (((7*I)/16)*a^(3/2)*e^(5/2)*Log[a - (Sqr
t[2]*Sqrt[a]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos
[c + d*x]*(a + I*a*Tan[c + d*x])])/(Sqrt[2]*d) + (((7*I)/16)*a^(3/2)*e^(5/2
)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c
 + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c + d*x])])/(Sqrt[2]*d) + (((7*I)/12)*
a^2*(e*Sec[c + d*x])^(5/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) - (((7*I)/8)*a*e
^2*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d + ((I/3)*a*(e*Sec[c +
 d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{ia(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}{3d} \\
&\quad +\frac{1}{6}(7a)\int(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}\,dx \\
&= \frac{7ia^2(e\sec(c+dx))^{5/2}}{12d\sqrt{a+ia\tan(c+dx)}}+\frac{ia(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}{3d} \\
&\quad +\frac{1}{8}\left(7a^2\right)\int\frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx \\
&= \frac{7ia^2(e\sec(c+dx))^{5/2}}{12d\sqrt{a+ia\tan(c+dx)}}-\frac{7iae^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{8d} \\
&\quad +\frac{ia(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}{3d} \\
&\quad +\frac{1}{16}\left(7ae^2\right)\int\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx \\
&= \frac{7ia^2(e\sec(c+dx))^{5/2}}{12d\sqrt{a+ia\tan(c+dx)}}-\frac{7iae^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{8d} \\
&\quad +\frac{ia(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}{3d} \\
&\quad -\frac{\left(7ia^2e^4\right)\text{Subst}\left(\int\frac{x^2}{a^2+e^2x^4}\,dx,x,\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{4d}
\end{aligned}$$

$$= \frac{7ia^2(e\sec(c+dx))^{5/2}}{12d\sqrt{a+ia\tan(c+dx)}} - \frac{7iae^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{8d} + \frac{ia(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}{3d} + \frac{(7ia^2e^3)\operatorname{Subst}\left(\int \frac{a-ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{8d} - \frac{(7ia^2e^3)\operatorname{Subst}\left(\int \frac{a+ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{8d}$$

$$= \frac{7ia^2(e\sec(c+dx))^{5/2}}{12d\sqrt{a+ia\tan(c+dx)}} - \frac{7iae^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{8d} + \frac{ia(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}{3d} - \frac{(7ia^2e^2)\operatorname{Subst}\left(\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{16d} - \frac{(7ia^2e^2)\operatorname{Subst}\left(\int \frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{16d} - \frac{(7ia^{3/2}e^{5/2})\operatorname{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}+2x}{-\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{16\sqrt{2}d} - \frac{(7ia^{3/2}e^{5/2})\operatorname{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}-2x}{-\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{16\sqrt{2}d}$$

$$= -\frac{7ia^{3/2}e^{5/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{16\sqrt{2}d} + \frac{7ia^{3/2}e^{5/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{16\sqrt{2}d} + \frac{7ia^2(e\sec(c+dx))^{5/2}}{12d\sqrt{a+ia\tan(c+dx)}} - \frac{7iae^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{8d} + \frac{ia(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}{3d} - \frac{(7ia^{3/2}e^{5/2})\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}d} + \frac{(7ia^{3/2}e^{5/2})\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}d}$$

$$
\begin{aligned}
&= \frac{7ia^{3/2}e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}d} \\
&- \frac{7ia^{3/2}e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}d} \\
&- \frac{7ia^{3/2}e^{5/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{16\sqrt{2}d} \\
&+ \frac{7ia^{3/2}e^{5/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{16\sqrt{2}d} \\
&+ \frac{7ia^2(e\sec(c+dx))^{5/2}}{12d\sqrt{a+ia\tan(c+dx)}} - \frac{7iae^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{8d} \\
&+ \frac{ia(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}{3d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 4.19 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.83

$$
\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}\,dx =
$$

$$
-\frac{a(e\sec(c+dx))^{5/2}\left(2i\sqrt{1+\cos(2c)+i\sin(2c)}(-9+7\cos(2c+2dx)+14i\sin(2c+2dx))\sqrt{i-\tan\left(\frac{dx}{2}\right)} \cdots\right.}{}
$$

```
[In] Integrate[(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] -1/96*(a*(e*Sec[c + d*x])^(5/2)*((2*I)*Sqrt[1 + Cos[2*c] + I*Sin[2*c]]*(-9
+ 7*Cos[2*c + 2*d*x] + (14*I)*Sin[2*c + 2*d*x])*Sqrt[I - Tan[(d*x)/2]] + 84
*ArcTanh[(Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 - I*
Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Cos[c + d*x]^3*Sqrt[-1 - I*Cos[c]
 - Sin[c]]*Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]] - 84*ArcTanh[
(Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] +
Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Cos[c + d*x]^3*Sqrt[1 - I*Cos[c] + Sin[c]]
*Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])*Sqrt[a + I*a*Tan[c +
d*x]])/(d*Sqrt[1 + Cos[2*c] + I*Sin[2*c]]*Sqrt[I - Tan[(d*x)/2]])
```

Maple [A] (verified)

Time = 9.60 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.16

method	result
default	$\frac{\left(-\frac{1}{48}+\frac{i}{48}\right)\sec(dx+c)(-\tan(dx+c)+i)\sqrt{a(1+i\tan(dx+c))}\,\sqrt{e\sec(dx+c)}\,a\,e^2\left(-8\sqrt{\frac{1}{\cos(dx+c)+1}}+8i\sqrt{\frac{1}{\cos(dx+c)+1}}+21\sin(dx+c)(\right.}{}$

```
[In] int((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/48+1/48*I)/d*sec(d*x+c)*(-tan(d*x+c)+I)*(a*(1+I*tan(d*x+c)))^(1/2)*(e*s
ec(d*x+c))^(1/2)*a*e^2*(-8*(1/(cos(d*x+c)+1))^(1/2)+8*I*(1/(cos(d*x+c)+1))^
(1/2)+21*sin(d*x+c)*cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2)-7*I*(1/(cos(d*x+c
)+1))^(1/2)*cos(d*x+c)^2+14*sin(d*x+c)*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)-
8*I*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+21*cos(d*x+c)^3*arctanh(1/2*(-cos(d
*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))+14*I*(1/(cos(d
*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)-21*I*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x
+c)^3+21*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3+7*(1/(cos(d*x+c)+1))^(1/2)*c
os(d*x+c)^2+21*I*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*sin(d*x+c)+22*I*(1/(
cos(d*x+c)+1))^(1/2)*cos(d*x+c)-22*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-8*si
n(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+21*I*cos(d*x+c)^3*arctanh(1/2*(cos(d*x+c)
+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2)))/(-2*I*cos(d*x+c)^2
+2*sin(d*x+c)*cos(d*x+c)-I*cos(d*x+c)+sin(d*x+c)+I)/(1/(cos(d*x+c)+1))^(1/2
)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 644, normalized size of antiderivative = 1.42

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}\,dx = \frac{\left(-21i\,ae^2e^{(5i\,dx+5i\,c)}+18i\,ae^2e^{(3i\,dx+3i\,c)}+7i\,ae^2e^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+}}}{}$$

```
[In] integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/12*((-21*I*a*e^2*e^(5*I*d*x + 5*I*c) + 18*I*a*e^2*e^(3*I*d*x + 3*I*c) + 7
*I*a*e^2*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*
d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 6*sqrt(49/64*I*a^3*e^5/d^2)*(d
*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(2/7*(7*(a*e^2*e^(2*
```

```
I*d*x + 2*I*c) + a*e^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*
x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 8*sqrt(49/64*I*a^3*e^5/d^2)*d)/(
a*e^2)) - 6*sqrt(49/64*I*a^3*e^5/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d
*x + 2*I*c) + d)*log(2/7*(7*(a*e^2*e^(2*I*d*x + 2*I*c) + a*e^2)*sqrt(a/(e^(
2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2
*I*c) - 8*sqrt(49/64*I*a^3*e^5/d^2)*d)/(a*e^2)) - 6*sqrt(-49/64*I*a^3*e^5/d
^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(2/7*(7*(a*e^2
*e^(2*I*d*x + 2*I*c) + a*e^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(
2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 8*sqrt(-49/64*I*a^3*e^5/d^
2)*d)/(a*e^2)) + 6*sqrt(-49/64*I*a^3*e^5/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*
e^(2*I*d*x + 2*I*c) + d)*log(2/7*(7*(a*e^2*e^(2*I*d*x + 2*I*c) + a*e^2)*sqr
t(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d
*x + 1/2*I*c) - 8*sqrt(-49/64*I*a^3*e^5/d^2)*d)/(a*e^2)))/(d*e^(4*I*d*x + 4
*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate((e*sec(d*x+c))**(5/2)*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3005 vs. $2(331) = 662$.

Time = 0.62 (sec) , antiderivative size = 3005, normalized size of antiderivative = 6.63

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -192*(336*a*e^2*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 288
*a*e^2*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 112*a*e^2*cos
(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 336*I*a*e^2*sin(11/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 288*I*a*e^2*sin(7/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) - 112*I*a*e^2*sin(3/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 42*(sqrt(2)*a*e^2*cos(6*d*x + 6*c) + 3*sqrt(2)
*a*e^2*cos(4*d*x + 4*c) + 3*sqrt(2)*a*e^2*cos(2*d*x + 2*c) + I*sqrt(2)*a*e^
2*sin(6*d*x + 6*c) + 3*I*sqrt(2)*a*e^2*sin(4*d*x + 4*c) + 3*I*sqrt(2)*a*e^2
```

*sin(2*d*x + 2*c) + sqrt(2)*a*e^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 42*(sqrt(2)*a*e^2*cos(6*d*x + 6*c) + 3*sqrt(2)*a*e^2*cos(4*d*x + 4*c) + 3*sqrt(2)*a*e^2*cos(2*d*x + 2*c) + I*sqrt(2)*a*e^2*sin(6*d*x + 6*c) + 3*I*sqrt(2)*a*e^2*sin(4*d*x + 4*c) + 3*I*sqrt(2)*a*e^2*sin(2*d*x + 2*c) + sqrt(2)*a*e^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 42*(sqrt(2)*a*e^2*cos(6*d*x + 6*c) + 3*sqrt(2)*a*e^2*cos(4*d*x + 4*c) + 3*sqrt(2)*a*e^2*cos(2*d*x + 2*c) + I*sqrt(2)*a*e^2*sin(6*d*x + 6*c) + 3*I*sqrt(2)*a*e^2*sin(4*d*x + 4*c) + 3*I*sqrt(2)*a*e^2*sin(2*d*x + 2*c) + sqrt(2)*a*e^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 42*(sqrt(2)*a*e^2*cos(6*d*x + 6*c) + 3*sqrt(2)*a*e^2*cos(4*d*x + 4*c) + 3*sqrt(2)*a*e^2*cos(2*d*x + 2*c) + I*sqrt(2)*a*e^2*sin(6*d*x + 6*c) + 3*I*sqrt(2)*a*e^2*sin(4*d*x + 4*c) + 3*I*sqrt(2)*a*e^2*sin(2*d*x + 2*c) + sqrt(2)*a*e^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 42*(-I*sqrt(2)*a*e^2*cos(6*d*x + 6*c) - 3*I*sqrt(2)*a*e^2*cos(4*d*x + 4*c) - 3*I*sqrt(2)*a*e^2*cos(2*d*x + 2*c) + sqrt(2)*a*e^2*sin(6*d*x + 6*c) + 3*sqrt(2)*a*e^2*sin(4*d*x + 4*c) + 3*sqrt(2)*a*e^2*sin(2*d*x + 2*c) - I*sqrt(2)*a*e^2)*arctan2(sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 42*(I*sqrt(2)*a*e^2*cos(6*d*x + 6*c) + 3*I*sqrt(2)*a*e^2*cos(4*d*x + 4*c) + 3*I*sqrt(2)*a*e^2*cos(2*d*x + 2*c) - sqrt(2)*a*e^2*sin(6*d*x + 6*c) - 3*sqrt(2)*a*e^2*sin(4*d*x + 4*c) - 3*sqrt(2)*a*e^2*sin(2*d*x + 2*c) + I*sqrt(2)*a*e^2)*arctan2(-sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), -sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 21*(sqrt(2)*a*e^2*cos(6*d*x + 6*c) + 3*sqrt(2)*a*e^2*cos(4*d*x + 4*c) + 3*sqrt(2)*a*e^2*cos(2*d*x + 2*c) + I*sqrt(2)*a*e^2*sin(6*d*x + 6*c) + 3*I*sqrt(2)*a*e^2*sin(4*d*x + 4*c) + 3*I*sqrt(2)*a*e^2*sin(2*d*x + 2*c) + sqrt(2)*a*e^2)*log(2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 21*(sqrt(2)*a*e^2*cos(6*d*x + 6*c) + 3*sqrt(2)*a*e^2*cos(4*d*x + 4*c) + 3*sqrt(2)*a*e^2*cos(2*d*x + 2*c) + I*sqrt(2)*a*e^2*sin(6*d*x + 6*c) + 3*I*sqrt(2)*a*e^2*sin(4*d*x + 4*c) + 3*I*sqrt(2)*a*e^2*sin(2*d*x + 2*c) + sqrt(2)*a*e^2)*log(-2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c),

```
cos(2*d*x + 2*c))) - 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) - 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
- 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 21*
(I*sqrt(2)*a*e^2*cos(6*d*x + 6*c) + 3*I*sqrt(2)*a*e^2*cos(4*d*x + 4*c) + 3*
I*sqrt(2)*a*e^2*cos(2*d*x + 2*c) - sqrt(2)*a*e^2*sin(6*d*x + 6*c) - 3*sqrt(
2)*a*e^2*sin(4*d*x + 4*c) - 3*sqrt(2)*a*e^2*sin(2*d*x + 2*c) + I*sqrt(2)*a*
e^2)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 2) + 21*(-I*sqrt(2)*a*e^2*cos(6*d*x + 6*c)
- 3*I*sqrt(2)*a*e^2*cos(4*d*x + 4*c) - 3*I*sqrt(2)*a*e^2*cos(2*d*x + 2*c) +
 sqrt(2)*a*e^2*sin(6*d*x + 6*c) + 3*sqrt(2)*a*e^2*sin(4*d*x + 4*c) + 3*sqrt
(2)*a*e^2*sin(2*d*x + 2*c) - I*sqrt(2)*a*e^2)*log(2*cos(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2)
 + 21*(I*sqrt(2)*a*e^2*cos(6*d*x + 6*c) + 3*I*sqrt(2)*a*e^2*cos(4*d*x + 4*c
) + 3*I*sqrt(2)*a*e^2*cos(2*d*x + 2*c) - sqrt(2)*a*e^2*sin(6*d*x + 6*c) - 3
*sqrt(2)*a*e^2*sin(4*d*x + 4*c) - 3*sqrt(2)*a*e^2*sin(2*d*x + 2*c) + I*sqrt
(2)*a*e^2)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2
*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 21*(-I*sqrt(2)*a*e^2*cos(6*d*x +
 6*c) - 3*I*sqrt(2)*a*e^2*cos(4*d*x + 4*c) - 3*I*sqrt(2)*a*e^2*cos(2*d*x +
2*c) + sqrt(2)*a*e^2*sin(6*d*x + 6*c) + 3*sqrt(2)*a*e^2*sin(4*d*x + 4*c) +
3*sqrt(2)*a*e^2*sin(2*d*x + 2*c) - I*sqrt(2)*a*e^2)*log(2*cos(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 2))*sqrt(a)*sqrt(e)/(d*(-36864*I*cos(6*d*x + 6*c) - 110592*I*cos(4*d*x
+ 4*c) - 110592*I*cos(2*d*x + 2*c) + 36864*sin(6*d*x + 6*c) + 110592*sin(4*
d*x + 4*c) + 110592*sin(2*d*x + 2*c) - 36864*I))
```

Giac [F]

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}\,dx=\int (e\sec(dx+c))^{\frac{5}{2}}\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((e*sec(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}\,dx = \int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^{5/2}\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^{3/2}\,dx$$

```
[In] int((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] int((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.401 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 571

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\,dx=\frac{5ia^2(e\sec(c+dx))^{3/2}}{4d\sqrt{a+ia\tan(c+dx)}}$$
$$-\frac{5ia^{5/2}e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{4\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$+\frac{5ia^{5/2}e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{4\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$+\frac{5ia^{5/2}e^{3/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$-\frac{5ia^{5/2}e^{3/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$+\frac{ia(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{2d}$$

```
[Out] 5/4*I*a^2*(e*sec(d*x+c))^(3/2)/d/(a+I*a*tan(d*x+c))^(1/2)-5/8*I*a^(5/2)*e^(
3/2)*arctan(1-2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c
))^(1/2))*sec(d*x+c)/d*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^
(1/2)+5/8*I*a^(5/2)*e^(3/2)*arctan(1+2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/
2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)/d*2^(1/2)/(a-I*a*tan(d*x+c))^(1
/2)/(a+I*a*tan(d*x+c))^(1/2)+5/16*I*a^(5/2)*e^(3/2)*ln(a-2^(1/2)*a^(1/2)*e^
(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a-I*a*tan(d
*x+c)))*sec(d*x+c)/d*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1
/2)-5/16*I*a^(5/2)*e^(3/2)*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^
(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a-I*a*tan(d*x+c)))*sec(d*x+c)/d*2^(1
```

```
/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+1/2*I*a*(e*sec(d*x+c)
)^(3/2)*(a+I*a*tan(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 571, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3579, 3580, 3576, 303, 1176, 631, 210, 1179, 642}

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\,dx = -\frac{5ia^{5/2}e^{3/2}\sec(c+dx)\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$+\frac{5ia^{5/2}e^{3/2}\sec(c+dx)\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$+\frac{5ia^{5/2}e^{3/2}\sec(c+dx)\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$-\frac{5ia^{5/2}e^{3/2}\sec(c+dx)\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$+\frac{5ia^2(e\sec(c+dx))^{3/2}}{4d\sqrt{a+ia\tan(c+dx)}}+\frac{ia\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{2d}$$

```
[In] Int[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((((5*I)/4)*a^2*(e*Sec[c + d*x])^(3/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) - (((
5*I)/4)*a^(5/2)*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*
x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[a - I*a*
Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + (((5*I)/4)*a^(5/2)*e^(3/2)*ArcT
an[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c +
 d*x]])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*T
an[c + d*x]]) + (((5*I)/8)*a^(5/2)*e^(3/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]
*Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*T
an[c + d*x])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a +
I*a*Tan[c + d*x]]) - (((5*I)/8)*a^(5/2)*e^(3/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sq
rt[e]*Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a -
I*a*Tan[c + d*x])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt
[a + I*a*Tan[c + d*x]]) + ((I/2)*a*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[
c + d*x]])/d
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3580

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Dist[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]])), Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{ia(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{2d} \\
&\quad + \frac{1}{4}(5a)\int(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx \\
&= \frac{5ia^2(e\sec(c+dx))^{3/2}}{4d\sqrt{a+ia\tan(c+dx)}} + \frac{ia(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{2d} \\
&\quad + \frac{1}{8}(5a^2)\int\frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx \\
&= \frac{5ia^2(e\sec(c+dx))^{3/2}}{4d\sqrt{a+ia\tan(c+dx)}} + \frac{ia(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{2d} \\
&\quad + \frac{(5a^2e\sec(c+dx))\int\sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}\,dx}{8\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&= \frac{5ia^2(e\sec(c+dx))^{3/2}}{4d\sqrt{a+ia\tan(c+dx)}} + \frac{ia(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{2d} \\
&\quad + \frac{(5ia^3e^3\sec(c+dx))\,\text{Subst}\left(\int\frac{x^2}{a^2+e^2x^4}\,dx,x,\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{2d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&= \frac{5ia^2(e\sec(c+dx))^{3/2}}{4d\sqrt{a+ia\tan(c+dx)}} + \frac{ia(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{2d} \\
&\quad - \frac{(5ia^3e^2\sec(c+dx))\,\text{Subst}\left(\int\frac{a-ex^2}{a^2+e^2x^4}\,dx,x,\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{4d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&\quad + \frac{(5ia^3e^2\sec(c+dx))\,\text{Subst}\left(\int\frac{a+ex^2}{a^2+e^2x^4}\,dx,x,\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{4d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}
\end{aligned}
$$

$$= \frac{5ia^2(e\sec(c+dx))^{3/2}}{4d\sqrt{a+ia\tan(c+dx)}} + \frac{ia(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{2d} + \frac{(5ia^3e\sec(c+dx))\operatorname{Subst}\left(\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{8d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{(5ia^3e\sec(c+dx))\operatorname{Subst}\left(\int \frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{8d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{(5ia^{5/2}e^{3/2}\sec(c+dx))\operatorname{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}+2x}{-\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{(5ia^{5/2}e^{3/2}\sec(c+dx))\operatorname{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}-2x}{-\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{5ia^2(e\sec(c+dx))^{3/2}}{4d\sqrt{a+ia\tan(c+dx)}} + \frac{5ia^{5/2}e^{3/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{5ia^{5/2}e^{3/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{ia(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{2d} + \frac{(5ia^{5/2}e^{3/2}\sec(c+dx))\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{(5ia^{5/2}e^{3/2}\sec(c+dx))\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$
\begin{aligned}
&= \frac{5ia^2(e\sec(c+dx))^{3/2}}{4d\sqrt{a+ia\tan(c+dx)}} - \frac{5ia^{5/2}e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{4\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&+ \frac{5ia^{5/2}e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{4\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&+ \frac{5ia^{5/2}e^{3/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{5ia^{5/2}e^{3/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&+ \frac{ia(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{2d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 3.66 (sec) , antiderivative size = 375, normalized size of antiderivative = 0.66

$$
\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\,dx = \frac{\cos^3(c+dx)(e\sec(c+dx))^{3/2}(\cos(dx)-i\sin(dx))\left(2\sec^2(c+dx)(i\cos(c)+\sin(c)\right.}{}
$$

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (Cos[c + d*x]^3*(e*Sec[c + d*x])^(3/2)*(Cos[d*x] - I*Sin[d*x])*(2*Sec[c + d
*x]^2*(I*Cos[c] + Sin[c]) + 5*Sec[c + d*x]*(I*Cos[2*c + d*x] + Sin[2*c + d*
x]) + (5*(ArcTanh[(Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqr
t[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1 - I*Cos[c] - Sin
[c]]*Sqrt[1 + I*Cos[c] - Sin[c]] - ArcTanh[(Sqrt[1 - I*Cos[c] + Sin[c]]*Sqr
t[I - Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]
*Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]])*(Cos[2*c] - I*Si
n[2*c])*Sqrt[I + Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[-1 + I*C
os[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]]))*(a + I*a*Tan[c + d*x])^(3/2))/(4*d
)
```

Maple [A] (verified)

Time = 10.54 (sec) , antiderivative size = 422, normalized size of antiderivative = 0.74

method	result
default	$\frac{\left(\frac{1}{16}+\frac{i}{16}\right)(-\tan(dx+c)+i)\sqrt{a(1+i\tan(dx+c))}\,\sqrt{e\sec(dx+c)}\left(5i\left(\cos^2(dx+c)\right)\operatorname{arctanh}\left(\frac{\cos(dx+c)+\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)-5i\sqrt{\frac{1}{\cos(dx-}}\right.}{}$

```
[In] int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/16+1/16*I)/d*(-tan(d*x+c)+I)*(a*(1+I*tan(d*x+c)))^(1/2)*(e*sec(d*x+c))^(
1/2)*(5*I*cos(d*x+c)^2*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)
/(1/(cos(d*x+c)+1))^(1/2))-5*I*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2+5*I*(1
/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)*sin(d*x+c)-7*I*(1/(cos(d*x+c)+1))^(1/2)*c
os(d*x+c)-2*I*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)-5*cos(d*x+c)^2*arctanh(1/
2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))-5*(1/
(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2-5*sin(d*x+c)*cos(d*x+c)*(1/(cos(d*x+c)+1
))^(1/2)-2*I*(1/(cos(d*x+c)+1))^(1/2)-7*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)
+2*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)-2*(1/(cos(d*x+c)+1))^(1/2))*(2*I*cos
(d*x+c)^2+2*sin(d*x+c)*cos(d*x+c)+I*cos(d*x+c)+sin(d*x+c)-I)*a*e/(cos(d*x+c
)+1)/(1/(cos(d*x+c)+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 538, normalized size of antiderivative = 0.94

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\,dx = \frac{\left(9i\,aee^{(2i\,dx+2i\,c)}+5i\,ae\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+\sqrt{\frac{25i\,a^3e^3}{16\,d^2}}\left(de^{(2i\,dx+2i}\right.}{}$$

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/2*((9*I*a*e*e^(2*I*d*x + 2*I*c) + 5*I*a*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + sqrt(25/16*
I*a^3*e^3/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(2/5*(5*(a*e*e^(2*I*d*x + 2*I
*c) + a*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) +
1))*e^(1/2*I*d*x + 1/2*I*c) + 4*I*sqrt(25/16*I*a^3*e^3/d^2)*d)/(a*e)) - sqr
t(25/16*I*a^3*e^3/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(2/5*(5*(a*e*e^(2*I*d
*x + 2*I*c) + a*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2
```

```
*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - 4*I*sqrt(25/16*I*a^3*e^3/d^2)*d)/(a*e
)) + sqrt(-25/16*I*a^3*e^3/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(2/5*(5*(a*e
*e^(2*I*d*x + 2*I*c) + a*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*
I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 4*I*sqrt(-25/16*I*a^3*e^3/d^
2)*d)/(a*e)) - sqrt(-25/16*I*a^3*e^3/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(2
/5*(5*(a*e*e^(2*I*d*x + 2*I*c) + a*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqr
t(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - 4*I*sqrt(-25/16*I*
a^3*e^3/d^2)*d)/(a*e)))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate((e*sec(d*x+c))**(3/2)*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2367 vs. $2(427) = 854$.

Time = 0.53 (sec) , antiderivative size = 2367, normalized size of antiderivative = 4.15

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxim
a")
```

```
[Out] 32*(144*a*e*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 80*a*e*c
os(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 144*I*a*e*sin(5/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 80*I*a*e*sin(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) - 10*(sqrt(2)*a*e*cos(4*d*x + 4*c) + 2*sqrt(
2)*a*e*cos(2*d*x + 2*c) + I*sqrt(2)*a*e*sin(4*d*x + 4*c) + 2*I*sqrt(2)*a*e*
sin(2*d*x + 2*c) + sqrt(2)*a*e)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 1) - 10*(sqrt(2)*a*e*cos(4*d*x + 4*c) + 2*sqrt(2)*a*e*co
s(2*d*x + 2*c) + I*sqrt(2)*a*e*sin(4*d*x + 4*c) + 2*I*sqrt(2)*a*e*sin(2*d*x
 + 2*c) + sqrt(2)*a*e)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 1) - 10*(sqrt(2)*a*e*cos(4*d*x + 4*c) + 2*sqrt(2)*a*e*cos(2*d*x
+ 2*c) + I*sqrt(2)*a*e*sin(4*d*x + 4*c) + 2*I*sqrt(2)*a*e*sin(2*d*x + 2*c)
+ sqrt(2)*a*e)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
```

+ 2*c))) - 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 1) - 10*(sqrt(2)*a*e*cos(4*d*x + 4*c) + 2*sqrt(2)*a*e*cos(2*d*x + 2*c) +
I*sqrt(2)*a*e*sin(4*d*x + 4*c) + 2*I*sqrt(2)*a*e*sin(2*d*x + 2*c) + sqrt(2)
*a*e)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
- 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 1
0*(-I*sqrt(2)*a*e*cos(4*d*x + 4*c) - 2*I*sqrt(2)*a*e*cos(2*d*x + 2*c) + sqr
t(2)*a*e*sin(4*d*x + 4*c) + 2*sqrt(2)*a*e*sin(2*d*x + 2*c) - I*sqrt(2)*a*e)
*arctan2(sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), sqrt(2)*cos(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 1) + 10*(I*sqrt(2)*a*e*cos(4*d*x + 4*c) + 2*I*sqrt(2)*a*
e*cos(2*d*x + 2*c) - sqrt(2)*a*e*sin(4*d*x + 4*c) - 2*sqrt(2)*a*e*sin(2*d*x
 + 2*c) + I*sqrt(2)*a*e)*arctan2(-sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))),
 -sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 5*(sqrt(2)*a*e*cos(4*d*x
+ 4*c) + 2*sqrt(2)*a*e*cos(2*d*x + 2*c) + I*sqrt(2)*a*e*sin(4*d*x + 4*c) +
2*I*sqrt(2)*a*e*sin(2*d*x + 2*c) + sqrt(2)*a*e)*log(2*sqrt(2)*sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 5*(s
qrt(2)*a*e*cos(4*d*x + 4*c) + 2*sqrt(2)*a*e*cos(2*d*x + 2*c) + I*sqrt(2)*a*
e*sin(4*d*x + 4*c) + 2*I*sqrt(2)*a*e*sin(2*d*x + 2*c) + sqrt(2)*a*e)*log(-2
*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*(sqrt(2)*cos(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) - 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*
cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 1) + 5*(-I*sqrt(2)*a*e*cos(4*d*x + 4*c) - 2*I*sqrt(2)*a*e*cos(
2*d*x + 2*c) + sqrt(2)*a*e*sin(4*d*x + 4*c) + 2*sqrt(2)*a*e*sin(2*d*x + 2*c
) - I*sqrt(2)*a*e)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)
*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 5*(I*sqrt(2)*a*e*cos(4*d
*x + 4*c) + 2*I*sqrt(2)*a*e*cos(2*d*x + 2*c) - sqrt(2)*a*e*sin(4*d*x + 4*c)
 - 2*sqrt(2)*a*e*sin(2*d*x + 2*c) + I*sqrt(2)*a*e)*log(2*cos(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))

```
+ 2) + 5*(-I*sqrt(2)*a*e*cos(4*d*x + 4*c) - 2*I*sqrt(2)*a*e*cos(2*d*x + 2*
c) + sqrt(2)*a*e*sin(4*d*x + 4*c) + 2*sqrt(2)*a*e*sin(2*d*x + 2*c) - I*sqrt
(2)*a*e)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*s
in(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 5*(I*sqrt(2)*a*e*cos(4*d*x + 4*c)
+ 2*I*sqrt(2)*a*e*cos(2*d*x + 2*c) - sqrt(2)*a*e*sin(4*d*x + 4*c) - 2*sqrt(
2)*a*e*sin(2*d*x + 2*c) + I*sqrt(2)*a*e)*log(2*cos(1/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2))*sqr
t(a)*sqrt(e)/(d*(-1024*I*cos(4*d*x + 4*c) - 2048*I*cos(2*d*x + 2*c) + 1024*
sin(4*d*x + 4*c) + 2048*sin(2*d*x + 2*c) - 1024*I))
```

Giac [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\,dx=\int (e\sec(dx+c))^{\frac{3}{2}}\,(i\,a\tan(dx+c)+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac"
)
[Out] integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\,dx=\int \left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{3/2}\,dx$$

```
[In] int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.402 $\int \sqrt{e \sec(c+dx)}(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 364

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}\,dx = \frac{3ia^{3/2}\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d} - \frac{3ia^{3/2}\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d} - \frac{3ia^{3/2}\sqrt{e}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{2\sqrt{2}d} + \frac{3ia^{3/2}\sqrt{e}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{2\sqrt{2}d} + \frac{ia\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}$$

```
[Out] 3/2*I*a^(3/2)*arctan(1-2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*
sec(d*x+c))^(1/2))*e^(1/2)/d*2^(1/2)-3/2*I*a^(3/2)*arctan(1+2^(1/2)*e^(1/2)
*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*e^(1/2)/d*2^(1/2)-3
/4*I*a^(3/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(d
*x+c))^(1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))*e^(1/2)/d*2^(1/2)+3/4*I*a^(3/2)
*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)
+cos(d*x+c)*(a+I*a*tan(d*x+c)))*e^(1/2)/d*2^(1/2)+I*a*(e*sec(d*x+c))^(1/2)*
(a+I*a*tan(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3579, 3576, 303, 1176, 631, 210, 1179, 642}

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}\,dx = \frac{3ia^{3/2}\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d} - \frac{3ia^{3/2}\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d} - \frac{3ia^{3/2}\sqrt{e}\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{2\sqrt{2}d} + \frac{3ia^{3/2}\sqrt{e}\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{2\sqrt{2}d} + \frac{ia\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{d}$$

```
[In] Int[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((3*I)*a^(3/2)*Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x
]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]])/(Sqrt[2]*d) - ((3*I)*a^(3/2)*Sqrt[e]*A
rcTan[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[
c + d*x]])]])/(Sqrt[2]*d) - (((3*I)/2)*a^(3/2)*Sqrt[e]*Log[a - (Sqrt[2]*Sqrt
[a]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]
*(a + I*a*Tan[c + d*x])]])/(Sqrt[2]*d) + (((3*I)/2)*a^(3/2)*Sqrt[e]*Log[a +
(Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] +
 Cos[c + d*x]*(a + I*a*Tan[c + d*x])]])/(Sqrt[2]*d) + (I*a*Sqrt[e*Sec[c + d*
x]]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{ia\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d} \\
&\quad + \frac{1}{2}(3a)\int \sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx \\
&= \frac{ia\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d} - \frac{(6ia^2e^2)\,\text{Subst}\left(\int \frac{x^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d} \\
&= \frac{ia\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d} \\
&\quad + \frac{(3ia^2e)\,\text{Subst}\left(\int \frac{a-ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d} \\
&\quad - \frac{(3ia^2e)\,\text{Subst}\left(\int \frac{a+ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d} \\
&= \frac{ia\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d} \\
&\quad - \frac{(3ia^2)\,\text{Subst}\left(\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{2d} \\
&\quad - \frac{(3ia^2)\,\text{Subst}\left(\int \frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{2d} \\
&\quad - \frac{(3ia^{3/2}\sqrt{e})\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}+2x}{-\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{2\sqrt{2}d} \\
&\quad - \frac{(3ia^{3/2}\sqrt{e})\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}-2x}{-\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{2\sqrt{2}d} \\
&= -\frac{3ia^{3/2}\sqrt{e}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{2\sqrt{2}d} \\
&\quad + \frac{3ia^{3/2}\sqrt{e}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{2\sqrt{2}d} \\
&\quad + \frac{ia\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d} \\
&\quad - \frac{(3ia^{3/2}\sqrt{e})\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d} \\
&\quad + \frac{(3ia^{3/2}\sqrt{e})\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d}
\end{aligned}
$$

$$= \frac{3ia^{3/2}\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d} - \frac{3ia^{3/2}\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d} - \frac{3ia^{3/2}\sqrt{e}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{2\sqrt{2}d} + \frac{3ia^{3/2}\sqrt{e}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{2\sqrt{2}d} + \frac{ia\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}$$

Mathematica [A] (verified)

Time = 3.27 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.93

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}\,dx = \frac{ae\left(i\sec(c+dx)\sqrt{1+\cos(2c)+i\sin(2c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}-3\text{arctanh}\left(\frac{\sqrt{1-i\cos(c)+\sin(c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}}{\sqrt{-1-i\cos(c)-\sin(c)}\sqrt{i+\tan\left(\frac{dx}{2}\right)}}\right.\right.}{}$$

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (a*e*(I*Sec[c + d*x]*Sqrt[1 + Cos[2*c] + I*Sin[2*c]]*Sqrt[I - Tan[(d*x)/2]]
 - 3*ArcTanh[(Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1
- I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1 - I*Cos[c] - Sin[c]]*
Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]] + 3*ArcTanh[(Sqrt[1 + I*
Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt
[I + Tan[(d*x)/2]])]*Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c
]]*Sqrt[I + Tan[(d*x)/2]])*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[e*Sec[c + d*
x]]*Sqrt[1 + Cos[2*c] + I*Sin[2*c]]*Sqrt[I - Tan[(d*x)/2]])
```

Maple [A] (verified)

Time = 10.48 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.99

method	result
default	$-\frac{(-\tan(dx+c)+i)\sqrt{a(1+i\tan(dx+c))}\,\sqrt{e\sec(dx+c)}\,a\cos(dx+c)\left(3i\cos(dx+c)\operatorname{arctanh}\left(\frac{-\cos(dx+c)+\sin(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)-3i\operatorname{arcta}\right.}{}$

[In] int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/2/d*(-tan(d*x+c)+I)*(a*(1+I*tan(d*x+c)))^(1/2)*(e*sec(d*x+c))^(1/2)*a*co
s(d*x+c)*(3*I*cos(d*x+c)*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)
+1)/(1/(cos(d*x+c)+1))^(1/2))-3*I*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(co
s(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-2*I*(1/(cos(d*x+c)+1))^(1/
2)*cos(d*x+c)-3*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(c
os(d*x+c)+1))^(1/2))*cos(d*x+c)-3*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(co
s(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-2*I*(1/(cos(d*x+c)+1))^(1/
2)+2*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2))/(2*I*cos(d*x+c)^2+I*cos(d*x+c)-2*
sin(d*x+c)*cos(d*x+c)-I-sin(d*x+c))/(1/(cos(d*x+c)+1))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.15

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}\,dx = \frac{4i\,a\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{3}{2}i\,dx+\frac{3}{2}i\,c\right)}+\sqrt{\frac{9i\,a^3e}{d^2}}d\log\left(\frac{2\left(3\left(ae^{(2i\,dx+2i\,c)}+a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i}}}\right.}{3a}\right.}{}$$

[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="frica
s")

[Out] 1/2*(4*I*a*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) +
1))*e^(3/2*I*d*x + 3/2*I*c) + sqrt(9*I*a^3*e/d^2)*d*log(2/3*(3*(a*e^(2*I*d*
x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*
c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + sqrt(9*I*a^3*e/d^2)*d)/a) - sqrt(9*I*a^3
*e/d^2)*d*log(2/3*(3*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - sqrt(9*
I*a^3*e/d^2)*d)/a) - sqrt(-9*I*a^3*e/d^2)*d*log(2/3*(3*(a*e^(2*I*d*x + 2*I*
c) + a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))
*e^(1/2*I*d*x + 1/2*I*c) + sqrt(-9*I*a^3*e/d^2)*d)/a) + sqrt(-9*I*a^3*e/d^2

```
)*d*log(2/3*(3*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)
)*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - sqrt(-9*I*a^3
*e/d^2)*d)/a))/d
```

Sympy [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}\,dx = \int \sqrt{e\sec{(c+dx)}}(ia(\tan{(c+dx)}-i))^{\frac{3}{2}}\,dx$$

```
[In] integrate((e*sec(d*x+c))**(1/2)*(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(sqrt(e*sec(c + d*x))*(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1871 vs. 2(268) = 536.

Time = 0.49 (sec) , antiderivative size = 1871, normalized size of antiderivative = 5.14

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -8*(6*(sqrt(2)*a*cos(2*d*x + 2*c) + I*sqrt(2)*a*sin(2*d*x + 2*c) + sqrt(2)*
a)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1
, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 6*(sq
rt(2)*a*cos(2*d*x + 2*c) + I*sqrt(2)*a*sin(2*d*x + 2*c) + sqrt(2)*a)*arctan
2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, -sqrt(2
)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 6*(sqrt(2)*a*
cos(2*d*x + 2*c) + I*sqrt(2)*a*sin(2*d*x + 2*c) + sqrt(2)*a)*arctan2(sqrt(2
)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 6*(sqrt(2)*a*cos(2*d*x
 + 2*c) + I*sqrt(2)*a*sin(2*d*x + 2*c) + sqrt(2)*a)*arctan2(sqrt(2)*cos(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 6*(-I*sqrt(2)*a*cos(2*d*x + 2*
c) + sqrt(2)*a*sin(2*d*x + 2*c) - I*sqrt(2)*a)*arctan2(sqrt(2)*sin(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))), sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 6*(I*
sqrt(2)*a*cos(2*d*x + 2*c) - sqrt(2)*a*sin(2*d*x + 2*c) + I*sqrt(2)*a)*arct
an2(-sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2
```

```
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), -sqrt(2)*cos(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 1) - 16*a*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) - 3*(sqrt(2)*a*cos(2*d*x + 2*c) + I*sqrt(2)*a*sin(2*d*x + 2*c) + sqrt(2
)*a)*log(2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(2)*cos(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 1) + 3*(sqrt(2)*a*cos(2*d*x + 2*c) + I*sqrt(2)*a*sin
(2*d*x + 2*c) + sqrt(2)*a)*log(-2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1)*cos(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*s
in(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 3*(I*sqrt(2)*a*cos(2*d*x
 + 2*c) - sqrt(2)*a*sin(2*d*x + 2*c) + I*sqrt(2)*a)*log(2*cos(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 2) + 3*(-I*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a*sin(2*d*x + 2*c) - I*
sqrt(2)*a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2
*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 3*(I*sqrt(2)*a*cos(2*d*x + 2*c)
- sqrt(2)*a*sin(2*d*x + 2*c) + I*sqrt(2)*a)*log(2*cos(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) +
 3*(-I*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a*sin(2*d*x + 2*c) - I*sqrt(2)*
a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 2) - 16*I*a*sin(3/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))))*sqrt(a)*sqrt(e)/(d*(-64*I*cos(2*d*x + 2*c) + 64*sin(2*
d*x + 2*c) - 64*I))
```

Giac [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}\,dx = \int \sqrt{e\sec\left(dx+c\right)}(i\,a\tan\left(dx+c\right)+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}\,dx = \int \sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^{3/2}\,dx$$

```
[In] int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.403 $\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{e\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 520

$$\begin{aligned}\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{e\sec(c+dx)}}\,dx &= \frac{i\sqrt{2}a^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\
&\quad-\frac{i\sqrt{2}a^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\
&\quad-\frac{ia^{5/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\
&\quad+\frac{ia^{5/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\
&\quad-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}\end{aligned}$$

```
[Out] -1/2*I*a^(5/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec
(d*x+c))^(1/2)+cos(d*x+c)*(a-I*a*tan(d*x+c)))*sec(d*x+c)/d*2^(1/2)/e^(1/2)/
(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+1/2*I*a^(5/2)*ln(a+2^(1/2
)*a^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*
(a-I*a*tan(d*x+c)))*sec(d*x+c)/d*2^(1/2)/e^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(
a+I*a*tan(d*x+c))^(1/2)+I*a^(5/2)*arctan(1-2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c
))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)*2^(1/2)/d/e^(1/2)/(a-I*a*
tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-I*a^(5/2)*arctan(1+2^(1/2)*e^(1/
2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)*2^(1/2
)/d/e^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-4*I*a*(a+I*a*
tan(d*x+c))^(1/2)/d/(e*sec(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3577, 3580, 3576, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{e\sec(c+dx)}}\,dx=\frac{i\sqrt{2}a^{5/2}\sec(c+dx)\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{i\sqrt{2}a^{5/2}\sec(c+dx)\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{ia^{5/2}\sec(c+dx)\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{ia^{5/2}\sec(c+dx)\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^(3/2)/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] (I*Sqrt[2]*a^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/
(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(d*Sqrt[e]*Sqrt[a - I*a*Tan[c
 + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (I*Sqrt[2]*a^(5/2)*ArcTan[1 + (Sqrt[
2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[
c + d*x])/(d*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
 - (I*a^(5/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/
Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/(
Sqrt[2]*d*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) +
(I*a^(5/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/Sqr
t[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/(Sqr
t[2]*d*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - ((4
*I)*a*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[e*Sec[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x], x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) || (EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] && LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
```

```
& IntegerQ[2*m]
```

Rule 3580

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Dist[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]])), Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}} - \frac{a^2\int\frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{e^2}\\
&= -\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}} - \frac{(a^2\sec(c+dx))\int\sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}\,dx}{e\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\
&= -\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}} - \frac{(4ia^3e\sec(c+dx))\,\text{Subst}\left(\int\frac{x^2}{a^2+e^2x^4}\,dx,x,\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\
&= -\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}\\
&\quad + \frac{(2ia^3\sec(c+dx))\,\text{Subst}\left(\int\frac{a-ex^2}{a^2+e^2x^4}\,dx,x,\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\
&\quad - \frac{(2ia^3\sec(c+dx))\,\text{Subst}\left(\int\frac{a+ex^2}{a^2+e^2x^4}\,dx,x,\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\
&= -\frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}\\
&\quad - \frac{(ia^3\sec(c+dx))\,\text{Subst}\left(\int\frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx,x,\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{de\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\
&\quad - \frac{(ia^3\sec(c+dx))\,\text{Subst}\left(\int\frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx,x,\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{de\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\
&\quad - \frac{\left(ia^{5/2}\sec(c+dx)\right)\,\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}+2x}{-\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx,x,\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\
&\quad - \frac{\left(ia^{5/2}\sec(c+dx)\right)\,\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}-2x}{-\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx,x,\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}
\end{aligned}$$

$$= -\frac{ia^{5/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{ia^{5/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}} - \frac{\left(i\sqrt{2}a^{5/2}\sec(c+dx)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{\left(i\sqrt{2}a^{5/2}\sec(c+dx)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{i\sqrt{2}a^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{i\sqrt{2}a^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{ia^{5/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{ia^{5/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{4ia\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}$$

Mathematica [A] (verified)

Time = 2.83 (sec) , antiderivative size = 401, normalized size of antiderivative = 0.77

$$\int\frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{e\sec(c+dx)}}\,dx = \frac{2ae(\cos(dx)-i\sin(dx))\left(\text{arctanh}\left(\frac{\sqrt{1+i\cos(c)-\sin(c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}}{\sqrt{-1+i\cos(c)+\sin(c)}\sqrt{i+\tan\left(\frac{dx}{2}\right)}}\right)\sqrt{-1-i}\right.}{}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(3/2)/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] (2*a*e*(Cos[d*x] - I*Sin[d*x])*(ArcTanh[(Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I
 - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sq
```

```
rt[-1 - I*Cos[c] - Sin[c]]*Sqrt[1 + I*Cos[c] - Sin[c]]*((-I)*Cos[2*c] - Sin
[2*c])*Sqrt[I + Tan[(d*x)/2]] + Sqrt[-1 + I*Cos[c] + Sin[c]]*(2*Sqrt[-1 - I
*Cos[c] - Sin[c]]*(Cos[c] - I*Sin[c])*Sqrt[I - Tan[(d*x)/2]] + ArcTanh[(Sqr
t[1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[
c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[1 - I*Cos[c] + Sin[c]]*(I*Cos[2*c] + Sin[
2*c])*Sqrt[I + Tan[(d*x)/2]]))*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x
]])/(d*(e*Sec[c + d*x])^(3/2)*Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[-1 + I*Cos[
c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])
```

Maple [A] (warning: unable to verify)

Time = 10.45 (sec) , antiderivative size = 502, normalized size of antiderivative = 0.97

method	result
default	$-\frac{\left(i\sqrt{2}\,\operatorname{arctanh}\left(\frac{(-\cot(dx+c)+\csc(dx+c)-1)\sqrt{2}}{2\sqrt{\left(\csc^2(dx+c)\right)(1-\cos(dx+c))^2+1}}\right)\sqrt{(\csc^2(dx+c))(1-\cos(dx+c))^2+1}+i\sqrt{2}\,\operatorname{arctanh}\left(\frac{(-\cot(dx+c)+\csc(dx+c)+1)}{2\sqrt{\left(\csc^2(dx+c)\right)(1-\cos(dx+c)}}\right.\right.}{}$

```
[In] int((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(I*2^(1/2)*arctanh(1/2*(-cot(d*x+c)+csc(d*x+c)-1)*2^(1/2)/(csc(d*x+c
)^2*(1-cos(d*x+c))^2+1)^(1/2))*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)+I*2^
(1/2)*arctanh(1/2*(-cot(d*x+c)+csc(d*x+c)+1)*2^(1/2)/(csc(d*x+c)^2*(1-cos(d
*x+c))^2+1)^(1/2))*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)-2^(1/2)*arctanh(
1/2*(-cot(d*x+c)+csc(d*x+c)-1)*2^(1/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1
/2))*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)+2^(1/2)*arctanh(1/2*(-cot(d*x+
c)+csc(d*x+c)+1)*2^(1/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2))*(csc(d*x+
c)^2*(1-cos(d*x+c))^2+1)^(1/2)-8*I*(csc(d*x+c)-cot(d*x+c))-8)*(csc(d*x+c)^2
*(1-cos(d*x+c))^2-1)*(-a*(2*I*(csc(d*x+c)-cot(d*x+c))-csc(d*x+c)^2*(1-cos(d
*x+c))^2+1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1))^(3/2)/(-csc(d*x+c)+cot(d*x+c
)+I)^3/(-e*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2
-1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 460, normalized size of antiderivative = 0.88

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{e\sec(c+dx)}}\,dx =$$

$$-\frac{de\sqrt{\frac{4i\,a^3}{d^2e}}\log\left(\frac{2\left(ae^{(2i\,dx+2i\,c)}+a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+i\,de\sqrt{\frac{4i\,a^3}{d^2e}}}{a}\right)-de\sqrt{\frac{4i\,a^3}{d^2e}}\log\left(\frac{2\left(ae^{(2i\,dx+2i\,c)}+a\right.}{}\right.}{}$$

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] -1/2*(d*e*sqrt(4*I*a^3/(d^2*e))*log((2*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt(a/(
e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x +
1/2*I*c) + I*d*e*sqrt(4*I*a^3/(d^2*e)))/a) - d*e*sqrt(4*I*a^3/(d^2*e))*log(
(2*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^
(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - I*d*e*sqrt(4*I*a^3/(d^2*e
)))/a) + d*e*sqrt(-4*I*a^3/(d^2*e))*log((2*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*
x + 1/2*I*c) + I*d*e*sqrt(-4*I*a^3/(d^2*e)))/a) - d*e*sqrt(-4*I*a^3/(d^2*e)
)*log((2*(a*e^(2*I*d*x + 2*I*c) + a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt
(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - I*d*e*sqrt(-4*I*a^3
/(d^2*e)))/a) + 8*(I*a*e^(2*I*d*x + 2*I*c) + I*a)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*e)
```

Sympy [F]

$$\int \frac{(a + ia\tan(c + dx))^{3/2}}{\sqrt{e\sec(c + dx)}}\,dx = \int \frac{(ia(\tan{(c + dx)} - i))^{\frac{3}{2}}}{\sqrt{e\sec{(c + dx)}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))**(3/2)/(e*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)/sqrt(e*sec(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1462 vs. $2(396) = 792$.

Time = 0.50 (sec) , antiderivative size = 1462, normalized size of antiderivative = 2.81

$$\int \frac{(a + ia\tan(c + dx))^{3/2}}{\sqrt{e\sec(c + dx)}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] 1/4*(2*I*sqrt(2)*a*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 1) + 2*I*sqrt(2)*a*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 1) + 2*I*sqrt(2)*a*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), co
```

```
s(2*d*x + 2*c))) + 1) + 2*I*sqrt(2)*a*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + 1) - 2*sqrt(2)*a*arctan2(sqrt(2)*sin(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))), sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 2*sqrt(2)
*a*arctan2(-sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), -sqrt(2)*cos(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 1) + I*sqrt(2)*a*log(2*sqrt(2)*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(
2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - I*sqrt(2)*a*
log(-2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*(sqrt(2)*cos(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 1) - sqrt(2)*a*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sq
rt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + sqrt(2)*a
*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + 2) - sqrt(2)*a*log(2*cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) +
sqrt(2)*a*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*
sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 16*I*a*cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 16*a*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))))*sqrt(a)/(d*sqrt(e))
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{e\sec(c+dx)}}\,dx = \int \frac{(i\,a\tan\left(dx+c\right)+a)^{\frac{3}{2}}}{\sqrt{e\sec\left(dx+c\right)}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(1/2),x, algorithm="giac"
)
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)/sqrt(e*sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{e\sec(c+dx)}}\,dx = \int \frac{(a+a\tan(c+d\,x)\;1\mathrm{i})^{3/2}}{\sqrt{\frac{e}{\cos(c+d\,x)}}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^(3/2)/(e/cos(c + d*x))^(1/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^(3/2)/(e/cos(c + d*x))^(1/2), x)
```

3.404 $\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 38

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{2i(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

```
[Out] -2/3*I*(a+I*a*tan(d*x+c))^(3/2)/d/(e*sec(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {3569}

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{2i(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^(3/2)/(e*Sec[c + d*x])^(3/2),x]
[Out] (((-2*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(d*(e*Sec[c + d*x])^(3/2))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\text{integral} = -\frac{2i(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{2i(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(3/2)/(e*Sec[c + d*x])^(3/2),x]
[Out] (((-2*I)/3)*(a + I*a*Tan[c + d*x])^(3/2))/(d*(e*Sec[c + d*x])^(3/2))
```

Maple [A] (verified)

Time = 9.75 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.34

method	result	size
default	$\frac{2a\sqrt{a(1+i\tan(dx+c))}\,(-i\cos(dx+c)+\sin(dx+c))}{3d\sqrt{e\sec(dx+c)}\,e}$	51
risch	$-\frac{2ia\sqrt{\frac{a\,\mathrm{e}^{2i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,\mathrm{e}^{i(dx+c)}}{3e\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}$	72

```
[In] int((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/3/d*a*(a*(1+I*tan(d*x+c)))^(1/2)/(e*sec(d*x+c))^(1/2)/e*(-I*cos(d*x+c)+si
n(d*x+c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(28) = 56.

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.00

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{3/2}}\,dx = \frac{2\left(-i\,ae^{(3i\,dx+3i\,c)}-i\,ae^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{3\,de^{2}}$$

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(3/2),x, algorithm="frica
s")
[Out] 2/3*(-I*a*e^(3*I*d*x + 3*I*c) - I*a*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/(d*e^
2)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{(ia(\tan{(c+dx)}-i))^{\frac{3}{2}}}{(e\sec{(c+dx)})^{\frac{3}{2}}}\,dx$$

[In] integrate((a+I*a*tan(d*x+c))**(3/2)/(e*sec(d*x+c))**(3/2),x)

[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)/(e*sec(c + d*x))**(3/2), x)

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(28) = 56.

Time = 0.35 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.00

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{2i\,a^{\frac{3}{2}}\left(-\frac{2i\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{3}{2}}}{3\,de^{\frac{3}{2}}\left(-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{3}{2}}}$$

[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] -2/3*I*a^(3/2)*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(3/2)/(d*e^(3/2)*(-sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(3/2))

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{(i\,a\tan{(dx+c)}+a)^{\frac{3}{2}}}{(e\sec{(dx+c)})^{\frac{3}{2}}}\,dx$$

[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)/(e*sec(d*x + c))^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{3/2}}\,dx=\int \frac{\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^{3/2}}{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^(3/2)/(e/cos(c + d*x))^(3/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^(3/2)/(e/cos(c + d*x))^(3/2), x)
```

3.405 $\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 81

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{5/2}}\,dx=-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{5de^2\sqrt{e\sec(c+dx)}}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{5d(e\sec(c+dx))^{5/2}}$$

```
[Out] -4/5*I*a*(a+I*a*tan(d*x+c))^(1/2)/d/e^2/(e*sec(d*x+c))^(1/2)-2/5*I*(a+I*a*t
an(d*x+c))^(3/2)/d/(e*sec(d*x+c))^(5/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3578, 3569}

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{5/2}}\,dx=-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{5de^2\sqrt{e\sec(c+dx)}}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{5d(e\sec(c+dx))^{5/2}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^(3/2)/(e*Sec[c + d*x])^(5/2),x]
```

```
[Out] (((-4*I)/5)*a*Sqrt[a + I*a*Tan[c + d*x]])/(d*e^2*Sqrt[e*Sec[c + d*x]]) - ((
(2*I)/5)*(a + I*a*Tan[c + d*x])^(3/2))/(d*(e*Sec[c + d*x])^(5/2))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2i(a+ia\tan(c+dx))^{3/2}}{5d(e\sec(c+dx))^{5/2}}+\frac{(2a)\int\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx}{5e^2}\\ &= -\frac{4ia\sqrt{a+ia\tan(c+dx)}}{5de^2\sqrt{e\sec(c+dx)}}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{5d(e\sec(c+dx))^{5/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.04

$$\int\frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{5/2}}\,dx=-\frac{2a(\cos(dx)-i\sin(dx))(\cos(c+2dx)+i\sin(c+2dx))(3i+2\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{5de(e\sec(c+dx))^{3/2}}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(3/2)/(e*Sec[c + d*x])^(5/2),x]
[Out] (-2*a*(Cos[d*x] - I*Sin[d*x])*(Cos[c + 2*d*x] + I*Sin[c + 2*d*x])*(3*I + 2*
Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(5*d*e*(e*Sec[c + d*x])^(3/2))
```

Maple [A] (verified)

Time = 9.54 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{2\cos(dx+c)(\tan(dx+c)-i)a\sqrt{a(1+i\tan(dx+c))}\,(2i\sin(dx+c)-3\cos(dx+c))}{5d\sqrt{e\sec(dx+c)}\,e^2}$	68
risch	$-\frac{ia\sqrt{\frac{a\,e^{2i(dx+c)}}{e^{2i(dx+c)}+1}}\,(e^{2i(dx+c)}+5)}{5e^2\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	74

```
[In] int((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5/d*cos(d*x+c)*(tan(d*x+c)-I)*a*(a*(1+I*tan(d*x+c)))^(1/2)*(2*I*sin(d*x+
c)-3*cos(d*x+c))/(e*sec(d*x+c))^(1/2)/e^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{5/2}}\,dx = \frac{\left(-i\,ae^{(4i\,dx+4i\,c)}-6i\,ae^{(2i\,dx+2i\,c)}-5i\,a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{5\,de^{3}}$$

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/5*(-I*a*e^(4*I*d*x + 4*I*c) - 6*I*a*e^(2*I*d*x + 2*I*c) - 5*I*a)*sqrt(a/(
e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x +
1/2*I*c)/(d*e^3)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{(ia(\tan{(c+dx)}-i))^{\frac{3}{2}}}{(e\sec{(c+dx)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))**(3/2)/(e*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**(3/2)/(e*sec(c + d*x))**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.73

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{5/2}}\,dx = \frac{\left(-i\,a\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)-5i\,a\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\,a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{5\,de^{\frac{5}{2}}}$$

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(5/2),x, algorithm="maxim
a")
```

```
[Out] 1/5*(-I*a*cos(5/2*d*x + 5/2*c) - 5*I*a*cos(1/2*d*x + 1/2*c) + a*sin(5/2*d*x
 + 5/2*c) + 5*a*sin(1/2*d*x + 1/2*c))*sqrt(a)/(d*e^(5/2))
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{5/2}}\,dx=\int \frac{(i\,a\tan\left(dx+c\right)+a)^{\frac{3}{2}}}{(e\sec\left(dx+c\right))^{\frac{5}{2}}}\,dx$$

[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(5/2),x, algorithm="giac"
)

[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)/(e*sec(d*x + c))^(5/2), x)

Mupad [B] (verification not implemented)

Time = 5.66 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.26

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{a\,\sqrt{\frac{e}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,\left(-\sin\left(c+d\,x\right)-\sin\left(3\,c+3\,d\,x\right)+\cos\left(c+d\,x\right)\,11\mathrm{i}+\cos\left(3\,c\right.\right.}{10\,d\,e^3}$$

[In] int((a + a*tan(c + d*x)*1i)^(3/2)/(e/cos(c + d*x))^(5/2),x)

[Out] -(a*(e/cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1)
)/(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(c + d*x)*11i - sin(c + d*x) + cos(3*c
+ 3*d*x)*1i - sin(3*c + 3*d*x)))/(10*d*e^3)

3.406 $\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 125

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{7/2}}\,dx = \frac{16ia^2\sqrt{e\sec(c+dx)}}{21de^4\sqrt{a+ia\tan(c+dx)}} - \frac{8ia\sqrt{a+ia\tan(c+dx)}}{21de^2(e\sec(c+dx))^{3/2}} - \frac{2i(a+ia\tan(c+dx))^{3/2}}{7d(e\sec(c+dx))^{7/2}}$$

```
[Out] 16/21*I*a^2*(e*sec(d*x+c))^(1/2)/d/e^4/(a+I*a*tan(d*x+c))^(1/2)-8/21*I*a*(a
+I*a*tan(d*x+c))^(1/2)/d/e^2/(e*sec(d*x+c))^(3/2)-2/7*I*(a+I*a*tan(d*x+c))^
(3/2)/d/(e*sec(d*x+c))^(7/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3578, 3569}

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{7/2}}\,dx = \frac{16ia^2\sqrt{e\sec(c+dx)}}{21de^4\sqrt{a+ia\tan(c+dx)}} - \frac{8ia\sqrt{a+ia\tan(c+dx)}}{21de^2(e\sec(c+dx))^{3/2}} - \frac{2i(a+ia\tan(c+dx))^{3/2}}{7d(e\sec(c+dx))^{7/2}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^(3/2)/(e*Sec[c + d*x])^(7/2),x]
```

```
[Out] (((16*I)/21)*a^2*Sqrt[e*Sec[c + d*x]])/(d*e^4*Sqrt[a + I*a*Tan[c + d*x]]) -
 (((8*I)/21)*a*Sqrt[a + I*a*Tan[c + d*x]])/(d*e^2*(e*Sec[c + d*x])^(3/2)) -
 (((2*I)/7)*(a + I*a*Tan[c + d*x])^(3/2))/(d*(e*Sec[c + d*x])^(7/2))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2i(a+ia\tan(c+dx))^{3/2}}{7d(e\sec(c+dx))^{7/2}}+\frac{(4a)\int\frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx}{7e^2}\\
&= -\frac{8ia\sqrt{a+ia\tan(c+dx)}}{21de^2(e\sec(c+dx))^{3/2}}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{7d(e\sec(c+dx))^{7/2}}+\frac{(8a^2)\int\frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{21e^4}\\
&= \frac{16ia^2\sqrt{e\sec(c+dx)}}{21de^4\sqrt{a+ia\tan(c+dx)}}-\frac{8ia\sqrt{a+ia\tan(c+dx)}}{21de^2(e\sec(c+dx))^{3/2}}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{7d(e\sec(c+dx))^{7/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.78

$$\int\frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{7/2}}\,dx=\frac{a(\cos(dx)-i\sin(dx))(-7i+9i\cos(2(c+dx))+12\sin(2(c+dx)))(\cos(c+}{21de^3\sqrt{e\sec(c+dx)}}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(3/2)/(e*Sec[c + d*x])^(7/2),x]
```

```
[Out] (a*(Cos[d*x] - I*Sin[d*x])*(-7*I + (9*I)*Cos[2*(c + d*x)] + 12*Sin[2*(c + d
*x)])*(Cos[c + 2*d*x] + I*Sin[c + 2*d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(21*d
*e^3*Sqrt[e*Sec[c + d*x]])
```

Maple [A] (verified)

Time = 9.70 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.62

method	result	size
default	$-\frac{2\cos(dx+c)(\tan(dx+c)-i)a\sqrt{a(1+i\tan(dx+c))}\,(-12i\cos(dx+c)\sin(dx+c)+9(\cos^2(dx+c))-8)}{21d\sqrt{e\sec(dx+c)}\,e^3}$	77
risch	$-\frac{ia\sqrt{\frac{a\,e^{2i(dx+c)}}{e^{2i(dx+c)}+1}}\,(3\,e^{3i(dx+c)}-7\cos(dx+c)+35i\sin(dx+c))}{42e^3\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	92

```
[In] int((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21/d*cos(d*x+c)*(tan(d*x+c)-I)*a*(a*(1+I*tan(d*x+c)))^(1/2)*(-12*I*sin(d
*x+c)*cos(d*x+c)+9*cos(d*x+c)^2-8)/(e*sec(d*x+c))^(1/2)/e^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.73

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{7/2}}\,dx = \frac{\left(-3i\,ae^{(6i\,dx+6i\,c)}-17i\,ae^{(4i\,dx+4i\,c)}+7i\,ae^{(2i\,dx+2i\,c)}+21i\,a\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(}}}}{42\,de^4}$$

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(7/2),x, algorithm="frica
s")
```

```
[Out] 1/42*(-3*I*a*e^(6*I*d*x + 6*I*c) - 17*I*a*e^(4*I*d*x + 4*I*c) + 7*I*a*e^(2*
I*d*x + 2*I*c) + 21*I*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d
*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/(d*e^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+I*a*tan(d*x+c))**(3/2)/(e*sec(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.67

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{7/2}}\,dx = \frac{\left(-3i\,a\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)-14i\,a\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+21i\,a\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a}{42\,de^{\frac{7}{2}}}$$

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(7/2),x, algorithm="maxim
a")
```

```
[Out] 1/42*(-3*I*a*cos(7/2*d*x + 7/2*c) - 14*I*a*cos(3/2*d*x + 3/2*c) + 21*I*a*co
s(1/2*d*x + 1/2*c) + 3*a*sin(7/2*d*x + 7/2*c) + 14*a*sin(3/2*d*x + 3/2*c) +
 21*a*sin(1/2*d*x + 1/2*c))*sqrt(a)/(d*e^(7/2))
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{7/2}}\,dx = \int \frac{(i\,a\tan\left(dx+c\right)+a)^{\frac{3}{2}}}{(e\sec\left(dx+c\right))^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(7/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)/(e*sec(d*x + c))^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 5.85 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.88

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{7/2}}\,dx = \frac{a\,\sqrt{\frac{e}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,\left(\cos\left(2\,c+2\,d\,x\right)\,4\mathrm{i}-\cos\left(4\,c+\right.\right.}{84\,d\,e^4}$$

```
[In] int((a + a*tan(c + d*x)*1i)^(3/2)/(e/cos(c + d*x))^(7/2),x)
```

```
[Out] (a*(e/cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))
/(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(2*c + 2*d*x)*4i - cos(4*c + 4*d*x)*3i +
 38*sin(2*c + 2*d*x) + 3*sin(4*c + 4*d*x) + 7i))/(84*d*e^4)
```

3.407 $\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{9/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 167

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{9/2}}\,dx = \frac{16ia^2}{45de^4\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{4ia\sqrt{a+ia\tan(c+dx)}}{15de^2(e\sec(c+dx))^{5/2}} - \frac{32ia\sqrt{a+ia\tan(c+dx)}}{45de^4\sqrt{e\sec(c+dx)}} - \frac{2i(a+ia\tan(c+dx))^{3/2}}{9d(e\sec(c+dx))^{9/2}}$$

```
[Out] 16/45*I*a^2/d/e^4/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-4/15*I*a*(a
+I*a*tan(d*x+c))^(1/2)/d/e^2/(e*sec(d*x+c))^(5/2)-32/45*I*a*(a+I*a*tan(d*x+
c))^(1/2)/d/e^4/(e*sec(d*x+c))^(1/2)-2/9*I*(a+I*a*tan(d*x+c))^(3/2)/d/(e*se
c(d*x+c))^(9/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3578, 3583, 3569}

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{9/2}}\,dx = \frac{16ia^2}{45de^4\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{32ia\sqrt{a+ia\tan(c+dx)}}{45de^4\sqrt{e\sec(c+dx)}} - \frac{4ia\sqrt{a+ia\tan(c+dx)}}{15de^2(e\sec(c+dx))^{5/2}} - \frac{2i(a+ia\tan(c+dx))^{3/2}}{9d(e\sec(c+dx))^{9/2}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^(3/2)/(e*Sec[c + d*x])^(9/2),x]
```

```
[Out] (((16*I)/45)*a^2)/(d*e^4*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) -
 (((4*I)/15)*a*Sqrt[a + I*a*Tan[c + d*x]])/(d*e^2*(e*Sec[c + d*x])^(5/2)) -
```

```
(((32*I)/45)*a*Sqrt[a + I*a*Tan[c + d*x]])/(d*e^4*Sqrt[e*Sec[c + d*x]]) -
(((2*I)/9)*(a + I*a*Tan[c + d*x])^(3/2))/(d*(e*Sec[c + d*x])^(9/2))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\text{integral} = -\frac{2i(a+ia\tan(c+dx))^{3/2}}{9d(e\sec(c+dx))^{9/2}}+\frac{(2a)\int\frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx}{3e^2}$$

$$=-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{15de^2(e\sec(c+dx))^{5/2}}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{9d(e\sec(c+dx))^{9/2}}+\frac{(8a^2)\int\frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx}{15e^4}$$

$$=\frac{16ia^2}{45de^4\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{15de^2(e\sec(c+dx))^{5/2}}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{9d(e\sec(c+dx))^{9/2}}+\frac{(16a)\int\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx}{45e^4}$$

$$=\frac{16ia^2}{45de^4\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{4ia\sqrt{a+ia\tan(c+dx)}}{15de^2(e\sec(c+dx))^{5/2}}-\frac{32ia\sqrt{a+ia\tan(c+dx)}}{45de^4\sqrt{e\sec(c+dx)}}-\frac{2i(a+ia\tan(c+dx))^{3/2}}{9d(e\sec(c+dx))^{9/2}}$$

Mathematica [A] (verified)

Time = 1.58 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.68

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{9/2}}\,dx = \frac{a(\cos(dx)-i\sin(dx))(-81i\cos(c+dx)+5i\cos(3(c+dx))-54\sin(c+dx)}{90de^4\sqrt{e\sec}}$$

[In] Integrate[(a + I*a*Tan[c + d*x])^(3/2)/(e*Sec[c + d*x])^(9/2),x]

[Out] (a*(Cos[d*x] - I*Sin[d*x])*((-81*I)*Cos[c + d*x] + (5*I)*Cos[3*(c + d*x)] - 54*Sin[c + d*x] + 10*Sin[3*(c + d*x)])*(Cos[c + 2*d*x] + I*Sin[c + 2*d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(90*d*e^4*Sqrt[e*Sec[c + d*x]])

Maple [A] (verified)

Time = 9.89 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.57

method	result	size
default	$\frac{2\cos(dx+c)(\tan(dx+c)-i)a\sqrt{a(1+i\tan(dx+c))}\,(10i(\cos^2(dx+c))\sin(dx+c)-5(\cos^3(dx+c))-16i\sin(dx+c)+24\cos(dx+c))}{45d\sqrt{e\sec(dx+c)}\,e^4}$	95
risch	$-\frac{ia\sqrt{\frac{a\,e^{2i(dx+c)}}{e^{2i(dx+c)}+1}}\,(5\,e^{4i(dx+c)}+135+12\cos(2dx+2c)+42i\sin(2dx+2c))}{180e^4\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	99

[In] int((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(9/2),x,method=_RETURNVERBOSE)

[Out] 2/45/d*cos(d*x+c)*(tan(d*x+c)-I)*a*(a*(1+I*tan(d*x+c)))^(1/2)*(10*I*cos(d*x+c)^2*sin(d*x+c)-5*cos(d*x+c)^3-16*I*sin(d*x+c)+24*cos(d*x+c))/(e*sec(d*x+c))^(1/2)/e^4

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.62

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{9/2}}\,dx = \frac{\left(-5i\,ae^{(8i\,dx+8i\,c)}-32i\,ae^{(6i\,dx+6i\,c)}-162i\,ae^{(4i\,dx+4i\,c)}-120i\,ae^{(2i\,dx+2i\,c)}+15\right.}{180\,de^5}$$

[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(9/2),x, algorithm="fricas")

[Out] 1/180*(-5*I*a*e^(8*I*d*x + 8*I*c) - 32*I*a*e^(6*I*d*x + 6*I*c) - 162*I*a*e^(4*I*d*x + 4*I*c) - 120*I*a*e^(2*I*d*x + 2*I*c) + 15*I*a)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(-3/2*I*d*x - 3/2*I*c)/(d*e^5)

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+I*a*tan(d*x+c))**(3/2)/(e*sec(d*x+c))**(9/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.96

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{9/2}}\,dx = \frac{\left(-5i\,a\cos\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)+15i\,a\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)-27i\,a\cos\left(\frac{5}{3}\,\arctan\left(\sin\left(\frac{3}{2}\right.\right.\right.}{}$$

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(9/2),x, algorithm="maxim
a")
```

```
[Out] 1/180*(-5*I*a*cos(9/2*d*x + 9/2*c) + 15*I*a*cos(3/2*d*x + 3/2*c) - 27*I*a*c
os(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 135*I*a*cos(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 5*a*sin(9/2*d*x +
 9/2*c) + 15*a*sin(3/2*d*x + 3/2*c) + 27*a*sin(5/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c))) + 135*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))))*sqrt(a)/(d*e^(9/2))
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{9/2}}\,dx = \int \frac{(i\,a\tan\left(dx+c\right)+a)^{\frac{3}{2}}}{(e\sec\left(dx+c\right))^{\frac{9}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^(3/2)/(e*sec(d*x+c))^(9/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)/(e*sec(d*x + c))^(9/2), x)
```

Mupad [B] (verification not implemented)

Time = 6.54 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.75

$$\int \frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{9/2}}\,dx = -\frac{a\sqrt{\frac{e}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,\left(-42\,\sin\left(c+d\,x\right)-47\,\sin\left(3\,c+3\,d\,x\right)-5\,\sin\left(5\,c+5\,d\,x\right)+c\right.}{360\,d\,e^5}$$

```
[In] int((a + a*tan(c + d*x)*1i)^(3/2)/(e/cos(c + d*x))^(9/2),x)
```

```
[Out] -(a*(e/cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1)
)/(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(c + d*x)*282i - 42*sin(c + d*x) + cos(
3*c + 3*d*x)*17i + cos(5*c + 5*d*x)*5i - 47*sin(3*c + 3*d*x) - 5*sin(5*c +
5*d*x)))/(360*d*e^5)
```

3.408 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 612

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}\,dx=\frac{15ia^3(e\sec(c+dx))^{3/2}}{8d\sqrt{a+ia\tan(c+dx)}}$$
$$-\frac{15ia^{7/2}e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$+\frac{15ia^{7/2}e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$+\frac{15ia^{7/2}e^{3/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{16\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$-\frac{15ia^{7/2}e^{3/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{16\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$+\frac{3ia^2(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{4d}+\frac{ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}{3d}$$

```
[Out] 15/8*I*a^3*(e*sec(d*x+c))^(3/2)/d/(a+I*a*tan(d*x+c))^(1/2)-15/16*I*a^(7/2)*
e^(3/2)*arctan(1-2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*
x+c))^(1/2))*sec(d*x+c)/d*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c
))^(1/2)+15/16*I*a^(7/2)*e^(3/2)*arctan(1+2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c)
)^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)/d*2^(1/2)/(a-I*a*tan(d*x+c
))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+15/32*I*a^(7/2)*e^(3/2)*ln(a-2^(1/2)*a^(1
/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a-I*a
*tan(d*x+c)))*sec(d*x+c)/d*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+
c))^(1/2)-15/32*I*a^(7/2)*e^(3/2)*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a-I*a*tan(d
*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a-I*a*tan(d*x+c)))*sec(d*x+c)
```

```
/d*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+3/4*I*a^2*(e*s
ec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2)/d+1/3*I*a*(e*sec(d*x+c))^(3/2)*(a
+I*a*tan(d*x+c))^(3/2)/d
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 612, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3579, 3580, 3576, 303, 1176, 631, 210, 1179, 642}

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{15ia^{7/2}e^{3/2}\sec(c+dx)\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{15ia^{7/2}e^{3/2}\sec(c+dx)\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{15ia^{7/2}e^{3/2}\sec(c+dx)\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{16\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{15ia^{7/2}e^{3/2}\sec(c+dx)\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{16\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{15ia^3(e\sec(c+dx))^{3/2}}{8d\sqrt{a+ia\tan(c+dx)}} + \frac{3ia^2\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}{4d} + \frac{ia(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}{3d}$$

```
[In] Int[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((15*I)/8)*a^3*(e*Sec[c + d*x])^(3/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) - ((
(15*I)/8)*a^(7/2)*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c +
d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[a - I*
a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + (((15*I)/8)*a^(7/2)*e^(3/2)*A
rcTan[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[
c + d*x]])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*
a*Tan[c + d*x]]) + (((15*I)/16)*a^(7/2)*e^(3/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sq
rt[e]*Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a -
I*a*Tan[c + d*x])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt
[a + I*a*Tan[c + d*x]]) - (((15*I)/16)*a^(7/2)*e^(3/2)*Log[a + (Sqrt[2]*Sqr
t[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x
]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[a - I*a*Tan[c + d*x
]]*Sqrt[a + I*a*Tan[c + d*x]]) + (((3*I)/4)*a^2*(e*Sec[c + d*x])^(3/2)*Sqrt
```

```
[a + I*a*Tan[c + d*x]])/d + ((I/3)*a*(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c
+ d*x])^(3/2))/d
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
```

```
x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3580

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Dist[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]])), Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}{3d} \\
&\quad+\frac{1}{2}(3a)\int(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\,dx \\
&= \frac{3ia^2(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{4d} \\
&\quad+\frac{ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}{3d} \\
&\quad+\frac{1}{8}\left(15a^2\right)\int(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx \\
&= \frac{15ia^3(e\sec(c+dx))^{3/2}}{8d\sqrt{a+ia\tan(c+dx)}}+\frac{3ia^2(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{4d} \\
&\quad+\frac{ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}{3d} \\
&\quad+\frac{1}{16}\left(15a^3\right)\int\frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx \\
&= \frac{15ia^3(e\sec(c+dx))^{3/2}}{8d\sqrt{a+ia\tan(c+dx)}}+\frac{3ia^2(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{4d} \\
&\quad+\frac{ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}{3d} \\
&\quad+\frac{\left(15a^3e\sec(c+dx)\right)\int\sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}\,dx}{16\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}
\end{aligned}$$

$$= \frac{15ia^3(e\sec(c+dx))^{3/2}}{8d\sqrt{a+ia\tan(c+dx)}} + \frac{3ia^2(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{4d} + \frac{ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}{3d} + \frac{(15ia^4e^3\sec(c+dx))\,\text{Subst}\left(\int \frac{x^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{4d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{15ia^3(e\sec(c+dx))^{3/2}}{8d\sqrt{a+ia\tan(c+dx)}} + \frac{3ia^2(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{4d} + \frac{ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}{3d} - \frac{(15ia^4e^2\sec(c+dx))\,\text{Subst}\left(\int \frac{a-ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{8d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{(15ia^4e^2\sec(c+dx))\,\text{Subst}\left(\int \frac{a+ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{8d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{15ia^3(e\sec(c+dx))^{3/2}}{8d\sqrt{a+ia\tan(c+dx)}} + \frac{3ia^2(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{4d} + \frac{ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}{3d} + \frac{(15ia^4e\sec(c+dx))\,\text{Subst}\left(\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{16d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{(15ia^4e\sec(c+dx))\,\text{Subst}\left(\int \frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{16d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{(15ia^{7/2}e^{3/2}\sec(c+dx))\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}+2x}{-\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{16\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{(15ia^{7/2}e^{3/2}\sec(c+dx))\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}-2x}{-\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{16\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$
\begin{aligned}
&= \frac{15ia^3(e\sec(c+dx))^{3/2}}{8d\sqrt{a+ia\tan(c+dx)}} \\
&+ \frac{15ia^{7/2}e^{3/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{16\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{15ia^{7/2}e^{3/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{16\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&+ \frac{3ia^2(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{4d} \\
&+ \frac{ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}{3d} \\
&+ \frac{\left(15ia^{7/2}e^{3/2}\sec(c+dx)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{\left(15ia^{7/2}e^{3/2}\sec(c+dx)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{15ia^3(e\sec(c+dx))^{3/2}}{8d\sqrt{a+ia\tan(c+dx)}} - \frac{15ia^{7/2}e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&+ \frac{15ia^{7/2}e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&+ \frac{15ia^{7/2}e^{3/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{16\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{15ia^{7/2}e^{3/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{16\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&+ \frac{3ia^2(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{4d} \\
&+ \frac{ia(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}{3d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 3.20 (sec) , antiderivative size = 386, normalized size of antiderivative = 0.63

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}\,dx = \frac{\cos^4(c+dx)(e\sec(c+dx))^{3/2}\left(\frac{1}{6}\sec^3(c+dx)(63+79\cos(2(c+dx))+34i\sin(2(c$$

[In] Integrate[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(5/2),x]

[Out] (Cos[c + d*x]^4*(e*Sec[c + d*x])^(3/2)*((Sec[c + d*x]^3*(63 + 79*Cos[2*(c + d*x)] + (34*I)*Sin[2*(c + d*x)])*(I*Cos[3*c + d*x] + Sin[3*c + d*x]))/6 + (15*(ArcTanh[(Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[1 + I*Cos[c] - Sin[c]] - ArcTanh[(Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]])*(Cos[3*c] - I*Sin[3*c])*Sqrt[I + Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]]))*(a + I*a*Tan[c + d*x])^(5/2))/(8*d*(Cos[d*x] + I*Sin[d*x])^2)

Maple [A] (verified)

Time = 9.46 (sec) , antiderivative size = 556, normalized size of antiderivative = 0.91

method	result
default	$\frac{(-\frac{1}{96}-\frac{i}{96})(-\tan(dx+c)+i)^2\sqrt{a(1+i\tan(dx+c))}\sqrt{e\sec(dx+c)}\left(-45\sin(dx+c)(\cos^2(dx+c))\sqrt{\frac{1}{\cos(dx+c)+1}}+34\sin(dx+c)\cos(dx\right.}{}$

[In] int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)

[Out] (-1/96-1/96*I)/d*(-tan(d*x+c)+I)^2*(a*(1+I*tan(d*x+c)))^(1/2)*(e*sec(d*x+c))^(1/2)*(-45*sin(d*x+c)*cos(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2)+34*sin(d*x+c)*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+45*I*cos(d*x+c)^3*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))+8*I*(1/(cos(d*x+c)+1))^(1/2)-79*I*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2+45*cos(d*x+c)^3*arctanh(1/2*(cos(d*x+c)-sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))-45*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3-79*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2-26*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+8*sin(d*x+c)*(1/(cos(d*x+c)+

```
1))^(1/2)-8*I*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)-34*I*(1/(cos(d*x+c)+1))^(
1/2)*cos(d*x+c)*sin(d*x+c)-45*I*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^3+45*I*
(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2*sin(d*x+c)-26*I*(1/(cos(d*x+c)+1))^(1
/2)*cos(d*x+c)+8*(1/(cos(d*x+c)+1))^(1/2))*(4*I*cos(d*x+c)^2*sin(d*x+c)+2*I
*cos(d*x+c)*sin(d*x+c)-4*cos(d*x+c)^3-I*sin(d*x+c)-2*cos(d*x+c)^2+3*cos(d*x
+c)+1)*a^2*e/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 635, normalized size of antiderivative = 1.04

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}\,dx = \frac{\left(113i\,a^2ee^{(4i\,dx+4i\,c)}+126i\,a^2ee^{(2i\,dx+2i\,c)}+45i\,a^2e\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\right.}}{}$$

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/12*((113*I*a^2*e*e^(4*I*d*x + 4*I*c) + 126*I*a^2*e*e^(2*I*d*x + 2*I*c) +
45*I*a^2*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) +
 1))*e^(1/2*I*d*x + 1/2*I*c) + 6*sqrt(225/64*I*a^5*e^3/d^2)*(d*e^(4*I*d*x +
 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(2/15*(15*(a^2*e*e^(2*I*d*x + 2*I
*c) + a^2*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c)
+ 1))*e^(1/2*I*d*x + 1/2*I*c) + 8*I*sqrt(225/64*I*a^5*e^3/d^2)*d)/(a^2*e))
- 6*sqrt(225/64*I*a^5*e^3/d^2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*
I*c) + d)*log(2/15*(15*(a^2*e*e^(2*I*d*x + 2*I*c) + a^2*e)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)
 - 8*I*sqrt(225/64*I*a^5*e^3/d^2)*d)/(a^2*e)) + 6*sqrt(-225/64*I*a^5*e^3/d^
2)*(d*e^(4*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)*log(2/15*(15*(a^2*
e*e^(2*I*d*x + 2*I*c) + a^2*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^
(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 8*I*sqrt(-225/64*I*a^5*e^
3/d^2)*d)/(a^2*e)) - 6*sqrt(-225/64*I*a^5*e^3/d^2)*(d*e^(4*I*d*x + 4*I*c) +
 2*d*e^(2*I*d*x + 2*I*c) + d)*log(2/15*(15*(a^2*e*e^(2*I*d*x + 2*I*c) + a^2
*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(
1/2*I*d*x + 1/2*I*c) - 8*I*sqrt(-225/64*I*a^5*e^3/d^2)*d)/(a^2*e)))/(d*e^(4
*I*d*x + 4*I*c) + 2*d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate((e*sec(d*x+c))**(3/2)*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3005 vs. $2(458) = 916$.

Time = 0.65 (sec) , antiderivative size = 3005, normalized size of antiderivative = 4.91

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxim
a")
```

```
[Out] 192*(1808*a^2*e*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2016
*a^2*e*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 720*a^2*e*cos
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1808*I*a^2*e*sin(9/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2016*I*a^2*e*sin(5/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 720*I*a^2*e*sin(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) - 90*(sqrt(2)*a^2*e*cos(6*d*x + 6*c) + 3*sqrt(2
)*a^2*e*cos(4*d*x + 4*c) + 3*sqrt(2)*a^2*e*cos(2*d*x + 2*c) + I*sqrt(2)*a^2
*e*sin(6*d*x + 6*c) + 3*I*sqrt(2)*a^2*e*sin(4*d*x + 4*c) + 3*I*sqrt(2)*a^2*
e*sin(2*d*x + 2*c) + sqrt(2)*a^2*e)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 1) - 90*(sqrt(2)*a^2*e*cos(6*d*x + 6*c) + 3*sqrt(2)*
a^2*e*cos(4*d*x + 4*c) + 3*sqrt(2)*a^2*e*cos(2*d*x + 2*c) + I*sqrt(2)*a^2*e
*sin(6*d*x + 6*c) + 3*I*sqrt(2)*a^2*e*sin(4*d*x + 4*c) + 3*I*sqrt(2)*a^2*e*
sin(2*d*x + 2*c) + sqrt(2)*a^2*e)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 1) - 90*(sqrt(2)*a^2*e*cos(6*d*x + 6*c) + 3*sqrt(2)*a
^2*e*cos(4*d*x + 4*c) + 3*sqrt(2)*a^2*e*cos(2*d*x + 2*c) + I*sqrt(2)*a^2*e*
sin(6*d*x + 6*c) + 3*I*sqrt(2)*a^2*e*sin(4*d*x + 4*c) + 3*I*sqrt(2)*a^2*e*s
in(2*d*x + 2*c) + sqrt(2)*a^2*e)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 1) - 90*(sqrt(2)*a^2*e*cos(6*d*x + 6*c) + 3*sqrt(2)*a^2
*e*cos(4*d*x + 4*c) + 3*sqrt(2)*a^2*e*cos(2*d*x + 2*c) + I*sqrt(2)*a^2*e*si
n(6*d*x + 6*c) + 3*I*sqrt(2)*a^2*e*sin(4*d*x + 4*c) + 3*I*sqrt(2)*a^2*e*sin
```

```
(2*d*x + 2*c) + sqrt(2)*a^2*e)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 1) + 90*(-I*sqrt(2)*a^2*e*cos(6*d*x + 6*c) - 3*I*sqrt(2)
*a^2*e*cos(4*d*x + 4*c) - 3*I*sqrt(2)*a^2*e*cos(2*d*x + 2*c) + sqrt(2)*a^2*
e*sin(6*d*x + 6*c) + 3*sqrt(2)*a^2*e*sin(4*d*x + 4*c) + 3*sqrt(2)*a^2*e*sin
(2*d*x + 2*c) - I*sqrt(2)*a^2*e)*arctan2(sqrt(2)*sin(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))), sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 90*(I*sqrt(2)*a^2*e
*cos(6*d*x + 6*c) + 3*I*sqrt(2)*a^2*e*cos(4*d*x + 4*c) + 3*I*sqrt(2)*a^2*e*
cos(2*d*x + 2*c) - sqrt(2)*a^2*e*sin(6*d*x + 6*c) - 3*sqrt(2)*a^2*e*sin(4*d
*x + 4*c) - 3*sqrt(2)*a^2*e*sin(2*d*x + 2*c) + I*sqrt(2)*a^2*e)*arctan2(-sq
rt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), -sqrt(2)*cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 1) - 45*(sqrt(2)*a^2*e*cos(6*d*x + 6*c) + 3*sqrt(2)*a^2*e*cos(4*d*
x + 4*c) + 3*sqrt(2)*a^2*e*cos(2*d*x + 2*c) + I*sqrt(2)*a^2*e*sin(6*d*x + 6
*c) + 3*I*sqrt(2)*a^2*e*sin(4*d*x + 4*c) + 3*I*sqrt(2)*a^2*e*sin(2*d*x + 2*
c) + sqrt(2)*a^2*e)*log(2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(
2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 45*(sqrt(2)*a^2*e*cos(6*d*x + 6*
c) + 3*sqrt(2)*a^2*e*cos(4*d*x + 4*c) + 3*sqrt(2)*a^2*e*cos(2*d*x + 2*c) +
I*sqrt(2)*a^2*e*sin(6*d*x + 6*c) + 3*I*sqrt(2)*a^2*e*sin(4*d*x + 4*c) + 3*I
*sqrt(2)*a^2*e*sin(2*d*x + 2*c) + sqrt(2)*a^2*e)*log(-2*sqrt(2)*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) - 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 45
*(-I*sqrt(2)*a^2*e*cos(6*d*x + 6*c) - 3*I*sqrt(2)*a^2*e*cos(4*d*x + 4*c) -
3*I*sqrt(2)*a^2*e*cos(2*d*x + 2*c) + sqrt(2)*a^2*e*sin(6*d*x + 6*c) + 3*sqr
t(2)*a^2*e*sin(4*d*x + 4*c) + 3*sqrt(2)*a^2*e*sin(2*d*x + 2*c) - I*sqrt(2)*
a^2*e)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 45*(I*sqrt(2)*a^2*e*cos(6*d*x + 6*c)
 + 3*I*sqrt(2)*a^2*e*cos(4*d*x + 4*c) + 3*I*sqrt(2)*a^2*e*cos(2*d*x + 2*c)
- sqrt(2)*a^2*e*sin(6*d*x + 6*c) - 3*sqrt(2)*a^2*e*sin(4*d*x + 4*c) - 3*sqr
```

```
t(2)*a^2*e*sin(2*d*x + 2*c) + I*sqrt(2)*a^2*e)*log(2*cos(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2
) + 45*(-I*sqrt(2)*a^2*e*cos(6*d*x + 6*c) - 3*I*sqrt(2)*a^2*e*cos(4*d*x + 4
*c) - 3*I*sqrt(2)*a^2*e*cos(2*d*x + 2*c) + sqrt(2)*a^2*e*sin(6*d*x + 6*c) +
 3*sqrt(2)*a^2*e*sin(4*d*x + 4*c) + 3*sqrt(2)*a^2*e*sin(2*d*x + 2*c) - I*sq
rt(2)*a^2*e)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 45*(I*sqrt(2)*a^2*e*cos(6*d*x
+ 6*c) + 3*I*sqrt(2)*a^2*e*cos(4*d*x + 4*c) + 3*I*sqrt(2)*a^2*e*cos(2*d*x +
 2*c) - sqrt(2)*a^2*e*sin(6*d*x + 6*c) - 3*sqrt(2)*a^2*e*sin(4*d*x + 4*c) -
 3*sqrt(2)*a^2*e*sin(2*d*x + 2*c) + I*sqrt(2)*a^2*e)*log(2*cos(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 2))*sqrt(a)*sqrt(e)/(d*(-36864*I*cos(6*d*x + 6*c) - 110592*I*cos(4*d*x
 + 4*c) - 110592*I*cos(2*d*x + 2*c) + 36864*sin(6*d*x + 6*c) + 110592*sin(4
*d*x + 4*c) + 110592*sin(2*d*x + 2*c) - 36864*I))
```

Giac [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}\,dx=\int (e\sec(dx+c))^{\frac{3}{2}}\,(i\,a\tan(dx+c)+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac"
)
[Out] integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}\,dx=\int \left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}\,(a+a\tan(c+d\,x)\;1\mathrm{i})^{5/2}\,dx$$

```
[In] int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(5/2),x)
[Out] int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.409 $\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 411

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}\,dx = \frac{21ia^{5/2}\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d} - \frac{21ia^{5/2}\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d} - \frac{21ia^{5/2}\sqrt{e}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{8\sqrt{2}d} + \frac{21ia^{5/2}\sqrt{e}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{8\sqrt{2}d} + \frac{7ia^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{4d} + \frac{ia\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}{2d}$$

```
[Out] 21/8*I*a^(5/2)*arctan(1-2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e
*sec(d*x+c))^(1/2))*e^(1/2)/d*2^(1/2)-21/8*I*a^(5/2)*arctan(1+2^(1/2)*e^(1/
2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*e^(1/2)/d*2^(1/2)
-21/16*I*a^(5/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*s
ec(d*x+c))^(1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))*e^(1/2)/d*2^(1/2)+21/16*I*a
^(5/2)*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))
^(1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))*e^(1/2)/d*2^(1/2)+7/4*I*a^2*(e*sec(d*
x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/d+1/2*I*a*(e*sec(d*x+c))^(1/2)*(a+I*a*
tan(d*x+c))^(3/2)/d
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3579, 3576, 303, 1176, 631, 210, 1179, 642}

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}\,dx = \frac{21ia^{5/2}\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d} - \frac{21ia^{5/2}\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d} - \frac{21ia^{5/2}\sqrt{e}\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{8\sqrt{2}d} + \frac{21ia^{5/2}\sqrt{e}\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{8\sqrt{2}d} + \frac{7ia^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{4d} + \frac{ia(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}{2d}$$

```
[In] Int[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((21*I)/4)*a^(5/2)*Sqrt[e]*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c
+ d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]])/(Sqrt[2]*d) - (((21*I)/4)*a^(5/2)
*Sqrt[e]*ArcTan[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*S
qrt[e*Sec[c + d*x]])]])/(Sqrt[2]*d) - (((21*I)/8)*a^(5/2)*Sqrt[e]*Log[a - (S
qrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + C
os[c + d*x]*(a + I*a*Tan[c + d*x])]])/(Sqrt[2]*d) + (((21*I)/8)*a^(5/2)*Sqrt
[e]*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec
[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c + d*x])]])/(Sqrt[2]*d) + (((7*I)/4)
*a^2*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d + ((I/2)*a*Sqrt[e*S
ec[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2))/d
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rubi steps

$$\text{integral} = \frac{ia\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}{2d} + \frac{1}{4}(7a)\int\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}\,dx$$

$$= \frac{7ia^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{4d} + \frac{ia\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}{2d} + \frac{1}{8}(21a^2)\int\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx$$

$$= \frac{7ia^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{4d} + \frac{ia\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}{2d} - \frac{(21ia^3e^2)\,\text{Subst}\left(\int\frac{x^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{2d}$$

$$= \frac{7ia^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{4d} + \frac{ia\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}{2d} + \frac{(21ia^3e)\,\text{Subst}\left(\int\frac{a-ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{4d} - \frac{(21ia^3e)\,\text{Subst}\left(\int\frac{a+ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{4d}$$

$$= \frac{7ia^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{4d} + \frac{ia\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}{2d} - \frac{(21ia^3)\,\text{Subst}\left(\int\frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{8d} - \frac{(21ia^3)\,\text{Subst}\left(\int\frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{8d} - \frac{(21ia^{5/2}\sqrt{e})\,\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}+2x}{-\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}d} - \frac{(21ia^{5/2}\sqrt{e})\,\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}-2x}{-\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}d}$$

$$= -\frac{21ia^{5/2}\sqrt{e}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{8\sqrt{2}d} + \frac{21ia^{5/2}\sqrt{e}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{8\sqrt{2}d} + \frac{7ia^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{4d} + \frac{ia\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}{2d} - \frac{\left(21ia^{5/2}\sqrt{e}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d} + \frac{\left(21ia^{5/2}\sqrt{e}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d}$$

$$= \frac{21ia^{5/2}\sqrt{e}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d} - \frac{21ia^{5/2}\sqrt{e}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d} - \frac{21ia^{5/2}\sqrt{e}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{8\sqrt{2}d} + \frac{21ia^{5/2}\sqrt{e}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{8\sqrt{2}d} + \frac{7ia^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{4d} + \frac{ia\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}{2d}$$

Mathematica [A] (verified)

Time = 4.09 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.94

$$\int\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}\,dx =$$

$$-\frac{a^2\sqrt{e\sec(c+dx)}(\cos(2dx)+i\sin(2dx))\sqrt{a+ia\tan(c+dx)}\left(21\text{arctanh}\left(\frac{\sqrt{1-i\cos(c)+\sin(c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}}{\sqrt{-1-i\cos(c)-\sin(c)}\sqrt{i+\tan\left(\frac{dx}{2}\right)}}\right)\text{co}\right.}{}$$

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] -1/4*(a^2*Sqrt[e*Sec[c + d*x]]*(Cos[2*d*x] + I*Sin[2*d*x])*Sqrt[a + I*a*Tan
[c + d*x]]*(21*ArcTanh[(Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])
/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Cos[c + d*x]*Sqrt[-
1 - I*Cos[c] - Sin[c]]*Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]] -
```

```
21*ArcTanh[(Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 +
 I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Cos[c + d*x]*Sqrt[1 - I*Cos[c]
 + Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]] + Sqrt[1 + C
os[2*c] + I*Sin[2*c]]*Sqrt[I - Tan[(d*x)/2]]*(-9*I + 2*Tan[c + d*x])))/(d*S
qrt[1 + Cos[2*c] + I*Sin[2*c]]*(Cos[d*x] + I*Sin[d*x])^2*Sqrt[I - Tan[(d*x)
/2]])
```

Maple [A] (verified)

Time = 10.24 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.09

method	result
default	$\frac{\left(\frac{1}{8}-\frac{i}{8}\right)(\tan(dx+c)-i)^2\sqrt{e\sec(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}\,a^2\cos(dx+c)\left(11i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\cos(dx+c)\sin(dx+c)-11i\sqrt{\frac{1}{\cos(dx+c)+}}\right.}{}$

```
[In] int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/8-1/8*I)/d*(tan(d*x+c)-I)^2*(e*sec(d*x+c))^(1/2)*(a*(1+I*tan(d*x+c)))^(1
/2)*a^2*cos(d*x+c)*(11*I*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)*cos(d*x+c)-11*
I*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2-21*I*arctanh(1/2*(cos(d*x+c)+sin(d*
x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)^2+11*sin(d*x+c)
*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+2*I*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c
)+11*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2-9*I*(1/(cos(d*x+c)+1))^(1/2)*cos
(d*x+c)-21*cos(d*x+c)^2*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+
1)/(1/(cos(d*x+c)+1))^(1/2))+2*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+9*(1/(co
s(d*x+c)+1))^(1/2)*cos(d*x+c)+2*I*(1/(cos(d*x+c)+1))^(1/2)-2*(1/(cos(d*x+c)
+1))^(1/2))/(1/(cos(d*x+c)+1))^(1/2)/(4*cos(d*x+c)^3+2*cos(d*x+c)^2+4*I*cos
(d*x+c)^2*sin(d*x+c)-3*cos(d*x+c)+2*I*cos(d*x+c)*sin(d*x+c)-1-I*sin(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.28

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}\,dx = \frac{\left(11i\,a^2e^{(3i\,dx+3i\,c)}+7i\,a^2e^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+\sqrt{\frac{441i\,a^5e}{16\,d^2}}\left(de^{(2i\,dx+2}\right.}{}$$

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/2*((11*I*a^2*e^(3*I*d*x + 3*I*c) + 7*I*a^2*e^(I*d*x + I*c))*sqrt(a/(e^(2*
I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I
*c) + sqrt(441/16*I*a^5*e/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(2/21*(21*(a^
2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2
*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 4*sqrt(441/16*I*a^5*e/d^2)*
d)/a^2) - sqrt(441/16*I*a^5*e/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(2/21*(21
*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(
e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - 4*sqrt(441/16*I*a^5*e/d
^2)*d)/a^2) - sqrt(-441/16*I*a^5*e/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*log(2/2
1*(21*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqr
t(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + 4*sqrt(-441/16*I*a
^5*e/d^2)*d)/a^2) + sqrt(-441/16*I*a^5*e/d^2)*(d*e^(2*I*d*x + 2*I*c) + d)*l
og(2/21*(21*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1
))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - 4*sqrt(-441/
16*I*a^5*e/d^2)*d)/a^2))/(d*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate((e*sec(d*x+c))**(1/2)*(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2431 vs. 2(299) = 598.

Time = 0.87 (sec) , antiderivative size = 2431, normalized size of antiderivative = 5.91

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxim
a")
```

```
[Out] 32*(176*a^2*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 112*a^2*
cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 176*I*a^2*sin(7/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 112*I*a^2*sin(3/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) - 42*(sqrt(2)*a^2*cos(4*d*x + 4*c) + 2*sqr
t(2)*a^2*cos(2*d*x + 2*c) + I*sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*I*sqrt(2)*a^
2*sin(2*d*x + 2*c) + sqrt(2)*a^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c),
```

cos(2*d*x + 2*c))) + 1) - 42*(sqrt(2)*a^2*cos(4*d*x + 4*c) + 2*sqrt(2)*a^2*
cos(2*d*x + 2*c) + I*sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*I*sqrt(2)*a^2*sin(2*d
*x + 2*c) + sqrt(2)*a^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 1) - 42*(sqrt(2)*a^2*cos(4*d*x + 4*c) + 2*sqrt(2)*a^2*cos(2*d*
x + 2*c) + I*sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*I*sqrt(2)*a^2*sin(2*d*x + 2*c
) + sqrt(2)*a^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 1) - 42*(sqrt(2)*a^2*cos(4*d*x + 4*c) + 2*sqrt(2)*a^2*cos(2*d*x + 2*c)
+ I*sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*I*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(
2)*a^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) - 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) -
42*(-I*sqrt(2)*a^2*cos(4*d*x + 4*c) - 2*I*sqrt(2)*a^2*cos(2*d*x + 2*c) + s
qrt(2)*a^2*sin(4*d*x + 4*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) - I*sqrt(2)*a^
2)*arctan2(sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), sqrt(2)*cos(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 1) - 42*(I*sqrt(2)*a^2*cos(4*d*x + 4*c) + 2*I*sqrt(2)*
a^2*cos(2*d*x + 2*c) - sqrt(2)*a^2*sin(4*d*x + 4*c) - 2*sqrt(2)*a^2*sin(2*d
*x + 2*c) + I*sqrt(2)*a^2)*arctan2(-sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
), -sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 21*(sqrt(2)*a^2*cos(4*d
*x + 4*c) + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + I*sqrt(2)*a^2*sin(4*d*x + 4*c)
+ 2*I*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2)*log(2*sqrt(2)*sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 2
1*(sqrt(2)*a^2*cos(4*d*x + 4*c) + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + I*sqrt(2
)*a^2*sin(4*d*x + 4*c) + 2*I*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2)*lo
g(-2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*(sqrt(2)*cos(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
+ 2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 1) - 21*(I*sqrt(2)*a^2*cos(4*d*x + 4*c) + 2*I*sqrt(2)*a^2*
cos(2*d*x + 2*c) - sqrt(2)*a^2*sin(4*d*x + 4*c) - 2*sqrt(2)*a^2*sin(2*d*x +
2*c) + I*sqrt(2)*a^2)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqr

```
t(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 21*(-I*sqrt(2)*a^2*c
os(4*d*x + 4*c) - 2*I*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2*sin(4*d*x
+ 4*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c) - I*sqrt(2)*a^2)*log(2*cos(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 2) - 21*(I*sqrt(2)*a^2*cos(4*d*x + 4*c) + 2*I*sqrt(2)*a^2*cos(2*d*
x + 2*c) - sqrt(2)*a^2*sin(4*d*x + 4*c) - 2*sqrt(2)*a^2*sin(2*d*x + 2*c) +
I*sqrt(2)*a^2)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 21*(-I*sqrt(2)*a^2*cos(4*d*x
 + 4*c) - 2*I*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2*sin(4*d*x + 4*c) +
 2*sqrt(2)*a^2*sin(2*d*x + 2*c) - I*sqrt(2)*a^2)*log(2*cos(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 2))*sqrt(a)*sqrt(e)/(d*(-1024*I*cos(4*d*x + 4*c) - 2048*I*cos(2*d*x + 2*c)
 + 1024*sin(4*d*x + 4*c) + 2048*sin(2*d*x + 2*c) - 1024*I))
```

Giac [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}\,dx = \int \sqrt{e\sec\left(dx+c\right)}(i\,a\tan\left(dx+c\right)+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac"
)
[Out] integrate(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}\,dx = \int \sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,{\left(a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{5/2}\,dx$$

```
[In] int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(5/2),x)
[Out] int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.410 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sqrt{e\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 563

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sqrt{e\sec(c+dx)}}\,dx = \frac{5ia^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$-\frac{5ia^{7/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$-\frac{5ia^{7/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{2\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$+\frac{5ia^{7/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{2\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$-\frac{10ia^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}+\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}$$

```
[Out] 5/2*I*a^(7/2)*arctan(1-2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*
sec(d*x+c))^(1/2))*sec(d*x+c)/d*2^(1/2)/e^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a
+I*a*tan(d*x+c))^(1/2)-5/2*I*a^(7/2)*arctan(1+2^(1/2)*e^(1/2)*(a-I*a*tan(d*
x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)/d*2^(1/2)/e^(1/2)/(a-I
*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-5/4*I*a^(7/2)*ln(a-2^(1/2)*a^
(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a-I
*a*tan(d*x+c)))*sec(d*x+c)/d*2^(1/2)/e^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*
a*tan(d*x+c))^(1/2)+5/4*I*a^(7/2)*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a-I*a*tan(d
*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a-I*a*tan(d*x+c)))*sec(d*x+c)
/d*2^(1/2)/e^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-10*I*a
^2*(a+I*a*tan(d*x+c))^(1/2)/d/(e*sec(d*x+c))^(1/2)+I*a*(a+I*a*tan(d*x+c))^(
3/2)/d/(e*sec(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 563, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3579, 3577, 3580, 3576, 303, 1176, 631, 210, 1179, 642}

$$\begin{aligned}\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sqrt{e\sec(c+dx)}}\,dx &= \frac{5ia^{7/2}\sec(c+dx)\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\ &- \frac{5ia^{7/2}\sec(c+dx)\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\ &- \frac{5ia^{7/2}\sec(c+dx)\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{2\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\ &+ \frac{5ia^{7/2}\sec(c+dx)\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{2\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\ &- \frac{10ia^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}} + \frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}\end{aligned}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^(5/2)/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] ((5*I)*a^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqr
t[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[e]*Sqrt[a - I*a*T
an[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - ((5*I)*a^(7/2)*ArcTan[1 + (Sqrt[
2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[
c + d*x])/(Sqrt[2]*d*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c
+ d*x]]) - (((5*I)/2)*a^(7/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a
*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]
*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*T
an[c + d*x]]) + (((5*I)/2)*a^(7/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a
- I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d
*x])]*Sec[c + d*x])/(Sqrt[2]*d*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a +
I*a*Tan[c + d*x]]) - ((10*I)*a^2*Sqrt[a + I*a*Tan[c + d*x]])/(d*Sqrt[e*Sec[
c + d*x]]) + (I*a*(a + I*a*Tan[c + d*x])^(3/2))/(d*Sqrt[e*Sec[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
```

```
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3580

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Dist[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]])), Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}+\frac{1}{2}(5a)\int\frac{(a+ia\tan(c+dx))^{3/2}}{\sqrt{e\sec(c+dx)}}\,dx\\
&= -\frac{10ia^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}+\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}-\frac{(5a^3)\int\frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{2e^2}\\
&= -\frac{10ia^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}+\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}\\
&\quad-\frac{(5a^3\sec(c+dx))\int\sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}\,dx}{2e\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\
&= -\frac{10ia^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}+\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}\\
&\quad-\frac{(10ia^4e\sec(c+dx))\operatorname{Subst}\left(\int\frac{x^2}{a^2+e^2x^4}\,dx,x,\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\
&= -\frac{10ia^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}}+\frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}\\
&\quad+\frac{(5ia^4\sec(c+dx))\operatorname{Subst}\left(\int\frac{a-ex^2}{a^2+e^2x^4}\,dx,x,\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\
&\quad-\frac{(5ia^4\sec(c+dx))\operatorname{Subst}\left(\int\frac{a+ex^2}{a^2+e^2x^4}\,dx,x,\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}
\end{aligned}
$$

$$= -\frac{10ia^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}} + \frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}$$
$$- \frac{(5ia^4\sec(c+dx))\,\text{Subst}\left(\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{2de\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$- \frac{(5ia^4\sec(c+dx))\,\text{Subst}\left(\int \frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{2de\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$- \frac{(5ia^{7/2}\sec(c+dx))\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}+2x}{-\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{2\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$- \frac{(5ia^{7/2}\sec(c+dx))\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}-2x}{-\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{2\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$=$$
$$-\frac{5ia^{7/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{2\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$+\frac{5ia^{7/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{2\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$-\frac{10ia^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}} + \frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}$$
$$-\frac{(5ia^{7/2}\sec(c+dx))\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$+\frac{(5ia^{7/2}\sec(c+dx))\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$
= \frac{5ia^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}
- \frac{5ia^{7/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}
- \frac{5ia^{7/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{2\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}
+ \frac{5ia^{7/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{2\sqrt{2}d\sqrt{e}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}
- \frac{10ia^2\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\sec(c+dx)}} + \frac{ia(a+ia\tan(c+dx))^{3/2}}{d\sqrt{e\sec(c+dx)}}
$$

Mathematica [A] (verified)

Time = 3.41 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.64

$$
\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sqrt{e\sec(c+dx)}}\,dx = \frac{e^2(a+ia\tan(c+dx))^{5/2}\left(\frac{5\operatorname{arctanh}\left(\frac{\sqrt{1-i\cos(c)+\sin(c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}}{\sqrt{-1-i\cos(c)-\sin(c)}\sqrt{i+\tan\left(\frac{dx}{2}\right)}}\right)\sqrt{1-i\cos(c)+\sin(c)}}{\sqrt{-1-i\cos(c)-\sin(c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}}\right.}{}
$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(5/2)/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] (e^2*(a + I*a*Tan[c + d*x])^(5/2)*((5*ArcTanh[(Sqrt[1 - I*Cos[c] + Sin[c]]*
Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]
])]*Sqrt[1 - I*Cos[c] + Sin[c]]*(Cos[3*c] - I*Sin[3*c])*Sqrt[I + Tan[(d*x)/
2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]]) - (5*ArcTanh[(Sq
rt[1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin
[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[1 + I*Cos[c] - Sin[c]]*(Cos[3*c] - I*Sin
[3*c])*Sqrt[I + Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(
d*x)/2]]) - (Cos[2*c] - I*Sin[2*c])*(9*I + Tan[c + d*x])))/(d*(e*Sec[c + d*
x])^(5/2)*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [A] (verified)

Time = 10.55 (sec) , antiderivative size = 661, normalized size of antiderivative = 1.17

method	result
default	$-\frac{(\tan(dx+c)-i)^2\sqrt{a(1+i\tan(dx+c))}\,a^2\cos(dx+c)\left(-5i\,\mathrm{arctanh}\left(\frac{-\cos(dx+c)+\sin(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)\sqrt{\frac{1}{\cos(dx+c)+1}}\,(\cos^2(dx+c))+5\cdots\right.}{}$

```
[In] int((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(tan(d*x+c)-I)^2*(a*(1+I*tan(d*x+c)))^(1/2)*a^2*cos(d*x+c)*(-5*I*arc
tanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2)
)*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)^2+5*I*(1/(cos(d*x+c)+1))^(1/2)*arctan
h(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*co
s(d*x+c)^2-5*cos(d*x+c)^2*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c
)+1)/(1/(cos(d*x+c)+1))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)-5*cos(d*x+c)^2*arct
anh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*
(1/(cos(d*x+c)+1))^(1/2)-5*I*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*
x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+5*I*(
1/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1
)/(1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-16*I*cos(d*x+c)^2-5*cos(d*x+c)*arcta
nh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*
(1/(cos(d*x+c)+1))^(1/2)-5*cos(d*x+c)*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)
/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)+16*sin(d
*x+c)*cos(d*x+c)-18*I*cos(d*x+c)-2*sin(d*x+c)-2*I)/(e*sec(d*x+c))^(1/2)/(4*
cos(d*x+c)^3+2*cos(d*x+c)^2+4*I*cos(d*x+c)^2*sin(d*x+c)-3*cos(d*x+c)+2*I*co
s(d*x+c)*sin(d*x+c)-1-I*sin(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 484, normalized size of antiderivative = 0.86

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sqrt{e\sec(c+dx)}}\,dx =$$

$$-\frac{\sqrt{\frac{25i\,a^5}{d^2e}}de\log\left(\frac{2\left(5\left(a^2e^{(2i\,dx+2i\,c)}+a^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+i\sqrt{\frac{25i\,a^5}{d^2e}}de\right)}{5\,a^2}\right)-\sqrt{\frac{25i\,a^5}{d^2e}}de\log\left(\frac{2\left(5\left(\cdots\right.\right.}{}\right.}{}$$

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] -1/2*(sqrt(25*I*a^5/(d^2*e))*d*e*log(2/5*(5*(a^2*e^(2*I*d*x + 2*I*c) + a^2)
*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2
```

```
*I*d*x + 1/2*I*c) + I*sqrt(25*I*a^5/(d^2*e))*d*e)/a^2) - sqrt(25*I*a^5/(d^2
*e))*d*e*log(2/5*(5*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*
I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - I*sq
rt(25*I*a^5/(d^2*e))*d*e)/a^2) + sqrt(-25*I*a^5/(d^2*e))*d*e*log(2/5*(5*(a^
2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2
*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + I*sqrt(-25*I*a^5/(d^2*e))*d
*e)/a^2) - sqrt(-25*I*a^5/(d^2*e))*d*e*log(2/5*(5*(a^2*e^(2*I*d*x + 2*I*c)
+ a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*
e^(1/2*I*d*x + 1/2*I*c) - I*sqrt(-25*I*a^5/(d^2*e))*d*e)/a^2) + 4*(4*I*a^2*
e^(2*I*d*x + 2*I*c) + 5*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^
(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*e)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sqrt{e\sec(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate((a+I*a*tan(d*x+c))**(5/2)/(e*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2013 vs. $2(427) = 854$.

Time = 1.02 (sec) , antiderivative size = 2013, normalized size of antiderivative = 3.58

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{\sqrt{e\sec(c+dx)}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] 8*(10*(sqrt(2)*a^2*cos(2*d*x + 2*c) + I*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt
(2)*a^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) +
 10*(sqrt(2)*a^2*cos(2*d*x + 2*c) + I*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2
)*a^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) +
10*(sqrt(2)*a^2*cos(2*d*x + 2*c) + I*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)
*a^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
- 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 10
*(sqrt(2)*a^2*cos(2*d*x + 2*c) + I*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a
```

```
^2)*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 10*
(-I*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2*sin(2*d*x + 2*c) - I*sqrt(2)
*a^2)*arctan2(sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), sqrt(2)*cos(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 1) - 10*(I*sqrt(2)*a^2*cos(2*d*x + 2*c) - sqrt(2)*a
^2*sin(2*d*x + 2*c) + I*sqrt(2)*a^2)*arctan2(-sqrt(2)*sin(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))), -sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 16*(4*a^2*cos
(2*d*x + 2*c) + 4*I*a^2*sin(2*d*x + 2*c) + 5*a^2)*cos(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 5*(sqrt(2)*a^2*cos(2*d*x + 2*c) + I*sqrt(2)*a
^2*sin(2*d*x + 2*c) + sqrt(2)*a^2)*log(2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*co
s(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 5*(sqrt(2)*a^2*co
s(2*d*x + 2*c) + I*sqrt(2)*a^2*sin(2*d*x + 2*c) + sqrt(2)*a^2)*log(-2*sqrt(
2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) - 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 1) - 5*(-I*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2*sin(2*d*x + 2*
c) - I*sqrt(2)*a^2)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2
)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 5*(I*sqrt(2)*a^2*cos(2*
d*x + 2*c) - sqrt(2)*a^2*sin(2*d*x + 2*c) + I*sqrt(2)*a^2)*log(2*cos(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 2) - 5*(-I*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2*sin(2*d*x
+ 2*c) - I*sqrt(2)*a^2)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sq
rt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 5*(I*sqrt(2)*a^2*co
s(2*d*x + 2*c) - sqrt(2)*a^2*sin(2*d*x + 2*c) + I*sqrt(2)*a^2)*log(2*cos(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*
```

```
d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 2) - 16*(4*I*a^2*cos(2*d*x + 2*c) - 4*a^2*sin(2*d*x + 2*c) +
 5*I*a^2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sqrt(a)*sqr
t(e)/((-64*I*e*cos(2*d*x + 2*c) + 64*e*sin(2*d*x + 2*c) - 64*I*e)*d)
```

Giac [F]

$$\int \frac{(a + ia\tan(c + dx))^{5/2}}{\sqrt{e\sec(c + dx)}}\,dx = \int \frac{(i\,a\tan\left(dx + c\right) + a)^{\frac{5}{2}}}{\sqrt{e\sec\left(dx + c\right)}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(1/2),x, algorithm="giac"
)
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)/sqrt(e*sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + ia\tan(c + dx))^{5/2}}{\sqrt{e\sec(c + dx)}}\,dx = \int \frac{\left(a + a\tan(c + d\,x)\ 1\mathrm{i}\right)^{5/2}}{\sqrt{\frac{e}{\cos(c + d\,x)}}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^(5/2)/(e/cos(c + d*x))^(1/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^(5/2)/(e/cos(c + d*x))^(1/2), x)
```

3.411 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 362

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{i\sqrt{2}a^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{de^{3/2}} + \frac{i\sqrt{2}a^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{de^{3/2}} + \frac{ia^{5/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{\sqrt{2}de^{3/2}} - \frac{ia^{5/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{\sqrt{2}de^{3/2}} - \frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

```
[Out] 1/2*I*a^(5/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(
d*x+c))^(1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))/d/e^(3/2)*2^(1/2)-1/2*I*a^(5/2
)*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2
)+cos(d*x+c)*(a+I*a*tan(d*x+c)))/d/e^(3/2)*2^(1/2)-I*a^(5/2)*arctan(1-2^(1/
2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*2^(1/2)/d
/e^(3/2)+I*a^(5/2)*arctan(1+2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2
)/(e*sec(d*x+c))^(1/2))*2^(1/2)/d/e^(3/2)-4/3*I*a*(a+I*a*tan(d*x+c))^(3/2)/
d/(e*sec(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3577, 3576, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{i\sqrt{2}a^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{de^{3/2}} + \frac{i\sqrt{2}a^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{de^{3/2}} + \frac{ia^{5/2}\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{\sqrt{2}de^{3/2}} - \frac{ia^{5/2}\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{\sqrt{2}de^{3/2}} - \frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((-I)*Sqrt[2]*a^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]
])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]])/(d*e^(3/2)) + (I*Sqrt[2]*a^(5/2)*ArcTan
[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d
*x]])]])/(d*e^(3/2)) + (I*a^(5/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a +
I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c + d*x
])])/(Sqrt[2]*d*e^(3/2)) - (I*a^(5/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt
[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c
+ d*x])])/(Sqrt[2]*d*e^(3/2)) - (((4*I)/3)*a*(a + I*a*Tan[c + d*x])^(3/2))/
(d*(e*Sec[c + d*x])^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3577

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(
n - 1)/(f*m)), x] - Dist[b^2*((m + 2*n - 2)/(d^2*m)), Int[(d*Sec[e + f*x])^
(m + 2)*(a + b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, d, e, f}, x] &
& EqQ[a^2 + b^2, 0] && GtQ[n, 1] && ((IGtQ[n/2, 0] && ILtQ[m - 1/2, 0]) ||
(EqQ[n, 2] && LtQ[m, 0]) || (LeQ[m, -1] && GtQ[m + n, 0]) || (ILtQ[m, 0] &&
 LtQ[m/2 + n - 1, 0] && IntegerQ[n]) || (EqQ[n, 3/2] && EqQ[m, -2^(-1)])) &
& IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}} - \frac{a^2\int\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx}{e^2}\\
&= -\frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}} + \frac{(4ia^3)\,\text{Subst}\left(\int\frac{x^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d}\\
&= -\frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}} - \frac{(2ia^3)\,\text{Subst}\left(\int\frac{a-ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{de}\\
&\quad + \frac{(2ia^3)\,\text{Subst}\left(\int\frac{a+ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{de}\\
&= -\frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}} + \frac{(ia^3)\,\text{Subst}\left(\int\frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{de^2}\\
&\quad + \frac{(ia^3)\,\text{Subst}\left(\int\frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{de^2}\\
&\quad + \frac{(ia^{5/2})\,\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}+2x}{-\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}de^{3/2}}\\
&\quad + \frac{(ia^{5/2})\,\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}-2x}{-\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}de^{3/2}}\\
&= \frac{ia^{5/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{\sqrt{2}de^{3/2}}\\
&\quad - \frac{ia^{5/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{\sqrt{2}de^{3/2}}\\
&\quad - \frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}\\
&\quad + \frac{(i\sqrt{2}a^{5/2})\,\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{de^{3/2}}\\
&\quad - \frac{(i\sqrt{2}a^{5/2})\,\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{de^{3/2}}
\end{aligned}$$

$$= -\frac{i\sqrt{2}a^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{de^{3/2}} + \frac{i\sqrt{2}a^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{de^{3/2}} + \frac{ia^{5/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{\sqrt{2}de^{3/2}} - \frac{ia^{5/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{\sqrt{2}de^{3/2}} - \frac{4ia(a+ia\tan(c+dx))^{3/2}}{3d(e\sec(c+dx))^{3/2}}$$

Mathematica [A] (verified)

Time = 3.81 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.95

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{3/2}}\,dx = \frac{e\left(-\frac{4}{3}i\cos(dx)(\cos(c)-i\sin(c))+\frac{4}{3}(\cos(c)-i\sin(c))\sin(dx)+\frac{2\left(\text{arctanh}\ldots\right.}{\ldots}\right.}{\ldots}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(3/2),x]
```

```
[Out] (e*(((-4*I)/3)*Cos[d*x]*(Cos[c] - I*Sin[c]) + (4*(Cos[c] - I*Sin[c])*Sin[d*
x])/3 + (2*(ArcTanh[(Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(S
qrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1 - I*Cos[c] - S
in[c]]*Sqrt[1 + I*Cos[c] - Sin[c]] - ArcTanh[(Sqrt[1 + I*Cos[c] - Sin[c]]*S
qrt[I - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]]
)]*Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]])*(Cos[2*c] - I*
Sin[2*c])*Sqrt[I + Tan[(d*x)/2]])/(Sqrt[1 + Cos[2*c] + I*Sin[2*c]]*Sqrt[I -
 Tan[(d*x)/2]]))*(a + I*a*Tan[c + d*x])^(5/2))/(d*(e*Sec[c + d*x])^(5/2)*(C
os[d*x] + I*Sin[d*x])^2)
```

Maple [A] (warning: unable to verify)

Time = 10.55 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.30

method	result
default	$\frac{\left(\frac{1}{6}+\frac{i}{6}\right)\left(\cos^2(dx+c)\right)(\tan(dx+c)-i)^2\sqrt{a(1+i\tan(dx+c))}\,a^2\left(-4i\sin(dx+c)-4i\cos(dx+c)-3\sqrt{\frac{1}{\cos(dx+c)+1}}\,\text{arctanh}\left(\frac{\cos(dx+c)\ldots}{2(\cos(dx+c)+\ldots}\right.\right.}{\ldots}$

```
[In] int((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/6+1/6*I)/d*cos(d*x+c)^2*(tan(d*x+c)-I)^2*(a*(1+I*tan(d*x+c)))^(1/2)*a^2*
(-4*I*sin(d*x+c)-4*I*cos(d*x+c)-3*(1/(cos(d*x+c)+1))^(1/2)*arctanh(1/2*(cos
(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))+3*I*(1/(cos(
d*x+c)+1))^(1/2)*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(
cos(d*x+c)+1))^(1/2))+4*sin(d*x+c)-4*cos(d*x+c)+3*sin(d*x+c)*arctanh(1/2*(-
cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*(1/(cos(d
*x+c)+1))^(1/2)-3*cos(d*x+c)*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x
+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)+3*I*arctanh(1/2*(
-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*(1/(cos(
d*x+c)+1))^(1/2)*cos(d*x+c)+3*I*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(
d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c))*(2
*I*cos(d*x+c)*sin(d*x+c)-2*cos(d*x+c)^2+1)/e/(e*sec(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.40

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{3\,de^2\sqrt{\frac{4i\,a^5}{d^2e^3}}\log\left(\frac{de^2\sqrt{\frac{4i\,a^5}{d^2e^3}}+2\left(a^2e^{(2i\,dx+2i\,c)}+a^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{a^2}\right)-3\,de^2\sqrt{\frac{4i\,a^5}{d^2e^3}}\log\left(-\frac{de^2\sqrt{4 \cdots}}{\cdots}\right.}{\cdots}$$

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(3/2),x, algorithm="frica
s")
```

```
[Out] -1/6*(3*d*e^2*sqrt(4*I*a^5/(d^2*e^3))*log((d*e^2*sqrt(4*I*a^5/(d^2*e^3)) +
2*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/
(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/a^2) - 3*d*e^2*sqrt(4*I
*a^5/(d^2*e^3))*log(-(d*e^2*sqrt(4*I*a^5/(d^2*e^3)) - 2*(a^2*e^(2*I*d*x + 2
*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c)
+ 1))*e^(1/2*I*d*x + 1/2*I*c))/a^2) - 3*d*e^2*sqrt(-4*I*a^5/(d^2*e^3))*log(
(d*e^2*sqrt(-4*I*a^5/(d^2*e^3)) + 2*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/
(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x +
 1/2*I*c))/a^2) + 3*d*e^2*sqrt(-4*I*a^5/(d^2*e^3))*log(-(d*e^2*sqrt(-4*I*a^
5/(d^2*e^3)) - 2*(a^2*e^(2*I*d*x + 2*I*c) + a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/a^2) + 8
*(I*a^2*e^(3*I*d*x + 3*I*c) + I*a^2*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/(d*e
^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+I*a*tan(d*x+c))**(5/2)/(e*sec(d*x+c))**(3/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1492 vs. $2(268) = 536$.

Time = 0.50 (sec) , antiderivative size = 1492, normalized size of antiderivative = 4.12

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(3/2),x, algorithm="maxim
a")
[Out] -1/12*(-6*I*sqrt(2)*a^2*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 1) - 6*I*sqrt(2)*a^2*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 1) - 6*I*sqrt(2)*a^2*arctan2(sqrt(2)*cos(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + 1) - 6*I*sqrt(2)*a^2*arctan2(sqrt(2)*cos(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 6*sqrt(2)*a^2*arctan2(sqrt(2)*sin
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))), sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1
) + 6*sqrt(2)*a^2*arctan2(-sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), -sqrt(
2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 3*I*sqrt(2)*a^2*log(2*sqrt(2)*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
```

```
+ 1) - 3*I*sqrt(2)*a^2*log(-2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*(s
qrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1)*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 3*sqrt(2)*a^2*log(2*cos(1/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 2) - 3*sqrt(2)*a^2*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt
(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 3*sqrt(2)*a
^2*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 2) - 3*sqrt(2)*a^2*log(2*cos(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 2) + 16*I*a^2*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 16*a^
2*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sqrt(a)/(d*e^(3/2))
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{3/2}}\,dx=\int \frac{(i\,a\tan{(dx+c)}+a)^{\frac{5}{2}}}{(e\sec{(dx+c)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)/(e*sec(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + ia\tan(c + dx))^{5/2}}{(e\sec(c + dx))^{3/2}}\,dx = \int \frac{\left(a + a\tan(c + d\,x)\ 1\mathrm{i}\right)^{5/2}}{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^(5/2)/(e/cos(c + d*x))^(3/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^(5/2)/(e/cos(c + d*x))^(3/2), x)
```

3.412 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 38

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{2i(a+ia\tan(c+dx))^{5/2}}{5d(e\sec(c+dx))^{5/2}}$$

```
[Out] -2/5*I*(a+I*a*tan(d*x+c))^(5/2)/d/(e*sec(d*x+c))^(5/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {3569}

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{2i(a+ia\tan(c+dx))^{5/2}}{5d(e\sec(c+dx))^{5/2}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(5/2),x]
[Out] (((-2*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(d*(e*Sec[c + d*x])^(5/2))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\text{integral} = -\frac{2i(a+ia\tan(c+dx))^{5/2}}{5d(e\sec(c+dx))^{5/2}}$$

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{5/2}}\,dx=-\frac{2i(a+ia\tan(c+dx))^{5/2}}{5d(e\sec(c+dx))^{5/2}}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(5/2),x]
[Out] (((-2*I)/5)*(a + I*a*Tan[c + d*x])^(5/2))/(d*(e*Sec[c + d*x])^(5/2))
```

Maple [A] (verified)

Time = 9.62 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.50

method	result	size
default	$\frac{2i(\cos^2(dx+c))\sqrt{a(1+i\tan(dx+c))}\,a^2(\tan(dx+c)-i)^2}{5d\sqrt{e\sec(dx+c)}\,e^2}$	57
risch	$-\frac{2ia^2\sqrt{\frac{a\,e^{2i(dx+c)}}{e^{2i(dx+c)}+1}}\,e^{2i(dx+c)}}{5e^2\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	74

```
[In] int((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/5*I/d*cos(d*x+c)^2*(a*(1+I*tan(d*x+c)))^(1/2)*a^2*(tan(d*x+c)-I)^2/(e*sec
(d*x+c))^(1/2)/e^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(28) = 56.

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.11

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{5/2}}\,dx=\frac{2\left(-i\,a^2e^{(4i\,dx+4i\,c)}-i\,a^2e^{(2i\,dx+2i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{5\,de^3}$$

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2),x, algorithm="frica
s")
[Out] 2/5*(-I*a^2*e^(4*I*d*x + 4*I*c) - I*a^2*e^(2*I*d*x + 2*I*c))*sqrt(a/(e^(2*I
*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*
c)/(d*e^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+I*a*tan(d*x+c))**(5/2)/(e*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(28) = 56.

Time = 0.34 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.00

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{2i\,a^{\frac{5}{2}}\left(-\frac{2i\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{5}{2}}}{5\,de^{\frac{5}{2}}\left(-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{5}{2}}}$$

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2),x, algorithm="maxim
a")
[Out] -2/5*I*a^(5/2)*(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 - 1)^(5/2)/(d*e^(5/2)*(-sin(d*x + c)^2/(cos(d*x + c) + 1)^2
 - 1)^(5/2))
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}{(e\sec(dx+c))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2),x, algorithm="giac"
)
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)/(e*sec(d*x + c))^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 5.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.74

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{5/2}}\,dx =$$

$$-\frac{a^2\sqrt{\frac{e}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,\left(-\sin\left(c+d\,x\right)-\sin\left(3\,c+3\,d\,x\right)+\cos\left(c+d\,x\right)\,1\mathrm{i}+\cos\left(3\,c\right.\right.}{5\,d\,e^3}$$

```
[In] int((a + a*tan(c + d*x)*1i)^(5/2)/(e/cos(c + d*x))^(5/2),x)
```

```
[Out] -(a^2*(e/cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i +
1))/(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(c + d*x)*1i - sin(c + d*x) + cos(3*c
 + 3*d*x)*1i - sin(3*c + 3*d*x)))/(5*d*e^3)
```

3.413 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 81

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{7/2}}\,dx = -\frac{4ia(a+ia\tan(c+dx))^{3/2}}{21de^2(e\sec(c+dx))^{3/2}} - \frac{2i(a+ia\tan(c+dx))^{5/2}}{7d(e\sec(c+dx))^{7/2}}$$

```
[Out] -4/21*I*a*(a+I*a*tan(d*x+c))^(3/2)/d/e^2/(e*sec(d*x+c))^(3/2)-2/7*I*(a+I*a*
tan(d*x+c))^(5/2)/d/(e*sec(d*x+c))^(7/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3578, 3569}

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{7/2}}\,dx = -\frac{4ia(a+ia\tan(c+dx))^{3/2}}{21de^2(e\sec(c+dx))^{3/2}} - \frac{2i(a+ia\tan(c+dx))^{5/2}}{7d(e\sec(c+dx))^{7/2}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(7/2),x]
```

```
[Out] (((-4*I)/21)*a*(a + I*a*Tan[c + d*x])^(3/2))/(d*e^2*(e*Sec[c + d*x])^(3/2))
 - (((2*I)/7)*(a + I*a*Tan[c + d*x])^(5/2))/(d*(e*Sec[c + d*x])^(7/2))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2i(a+ia\tan(c+dx))^{5/2}}{7d(e\sec(c+dx))^{7/2}}+\frac{(2a)\int\frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{3/2}}\,dx}{7e^2}\\ &= -\frac{4ia(a+ia\tan(c+dx))^{3/2}}{21de^2(e\sec(c+dx))^{3/2}}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{7d(e\sec(c+dx))^{7/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.14

$$\int\frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{7/2}}\,dx = -\frac{2a^2(\cos(2(c+2dx))+i\sin(2(c+2dx)))(5i+2\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}{21de^2(e\sec(c+dx))^{3/2}(\cos(dx)+i\sin(dx))^2}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(7/2),x]
```

```
[Out] (-2*a^2*(Cos[2*(c + 2*d*x)] + I*Sin[2*(c + 2*d*x)])*(5*I + 2*Tan[c + d*x])*
Sqrt[a + I*a*Tan[c + d*x]])/(21*d*e^2*(e*Sec[c + d*x])^(3/2)*(Cos[d*x] + I*
Sin[d*x])^2)
```

Maple [A] (verified)

Time = 9.87 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{2(\tan(dx+c)-i)^2a^2\sqrt{a(1+i\tan(dx+c))}\,(5i(\cos^3(dx+c))+2(\cos^2(dx+c))\sin(dx+c))}{21d\sqrt{e\sec(dx+c)}\,e^3}$	76
risch	$-\frac{ia^2\sqrt{\frac{a\,e^{2i(dx+c)}}{e^{2i(dx+c)}+1}}\,(3\,e^{3i(dx+c)}+7\,e^{i(dx+c)})}{21e^3\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	88

```
[In] int((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21/d*(tan(d*x+c)-I)^2*a^2*(a*(1+I*tan(d*x+c)))^(1/2)/(e*sec(d*x+c))^(1/2)
/e^3*(5*I*cos(d*x+c)^3+2*cos(d*x+c)^2*sin(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.16

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{7/2}}\,dx = \frac{\left(-3i\,a^2e^{(5i\,dx+5i\,c)}-10i\,a^2e^{(3i\,dx+3i\,c)}-7i\,a^2e^{(i\,dx+i\,c)}\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c}}}}{21\,de^4}$$

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(7/2),x, algorithm="frica
s")
```

```
[Out] 1/21*(-3*I*a^2*e^(5*I*d*x + 5*I*c) - 10*I*a^2*e^(3*I*d*x + 3*I*c) - 7*I*a^2
*e^(I*d*x + I*c))*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*
I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/(d*e^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+I*a*tan(d*x+c))**(5/2)/(e*sec(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.16

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{7/2}}\,dx = \frac{\left(-7i\,a^2\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)-3i\,a^2\cos\left(\frac{7}{3}\,\arctan\left(\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right),\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\right.\right.\right.\right.}{}$$

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(7/2),x, algorithm="maxim
a")
```

```
[Out] 1/21*(-7*I*a^2*cos(3/2*d*x + 3/2*c) - 3*I*a^2*cos(7/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) + 7*a^2*sin(3/2*d*x + 3/2*c) + 3*a^2*sin(7/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sqrt(a)/(d*e^(7/2))
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{7/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}{(e\sec(dx+c))^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(7/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)/(e*sec(d*x + c))^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 5.99 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.38

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{7/2}}\,dx = -\frac{a^2\sqrt{\frac{e}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,(\cos(2\,c+2\,d\,x)\,10\mathrm{i}+\cos(4\,c+4\,d\,x)\,3\mathrm{i}-10\,\sin(2\,c+2\,d}{42\,d\,e^4}$$

```
[In] int((a + a*tan(c + d*x)*1i)^(5/2)/(e/cos(c + d*x))^(7/2),x)
```

```
[Out] -(a^2*(e/cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i +
1))/(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(2*c + 2*d*x)*10i + cos(4*c + 4*d*x)*
3i - 10*sin(2*c + 2*d*x) - 3*sin(4*c + 4*d*x) + 7i))/(42*d*e^4)
```

3.414 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{9/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 125

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{9/2}}\,dx = -\frac{16ia^2\sqrt{a+ia\tan(c+dx)}}{45de^4\sqrt{e\sec(c+dx)}} - \frac{8ia(a+ia\tan(c+dx))^{3/2}}{45de^2(e\sec(c+dx))^{5/2}} - \frac{2i(a+ia\tan(c+dx))^{5/2}}{9d(e\sec(c+dx))^{9/2}}$$

```
[Out] -16/45*I*a^2*(a+I*a*tan(d*x+c))^(1/2)/d/e^4/(e*sec(d*x+c))^(1/2)-8/45*I*a*(
a+I*a*tan(d*x+c))^(3/2)/d/e^2/(e*sec(d*x+c))^(5/2)-2/9*I*(a+I*a*tan(d*x+c))
^(5/2)/d/(e*sec(d*x+c))^(9/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3578, 3569}

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{9/2}}\,dx = -\frac{16ia^2\sqrt{a+ia\tan(c+dx)}}{45de^4\sqrt{e\sec(c+dx)}} - \frac{8ia(a+ia\tan(c+dx))^{3/2}}{45de^2(e\sec(c+dx))^{5/2}} - \frac{2i(a+ia\tan(c+dx))^{5/2}}{9d(e\sec(c+dx))^{9/2}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(9/2),x]
```

```
[Out] (((-16*I)/45)*a^2*Sqrt[a + I*a*Tan[c + d*x]])/(d*e^4*Sqrt[e*Sec[c + d*x]])
- (((8*I)/45)*a*(a + I*a*Tan[c + d*x])^(3/2))/(d*e^2*(e*Sec[c + d*x])^(5/2)
) - (((2*I)/9)*(a + I*a*Tan[c + d*x])^(5/2))/(d*(e*Sec[c + d*x])^(9/2))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2i(a+ia\tan(c+dx))^{5/2}}{9d(e\sec(c+dx))^{9/2}}+\frac{(4a)\int\frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{5/2}}\,dx}{9e^2}\\
&= -\frac{8ia(a+ia\tan(c+dx))^{3/2}}{45de^2(e\sec(c+dx))^{5/2}}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{9d(e\sec(c+dx))^{9/2}}+\frac{(8a^2)\int\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx}{45e^4}\\
&= -\frac{16ia^2\sqrt{a+ia\tan(c+dx)}}{45de^4\sqrt{e\sec(c+dx)}}-\frac{8ia(a+ia\tan(c+dx))^{3/2}}{45de^2(e\sec(c+dx))^{5/2}}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{9d(e\sec(c+dx))^{9/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.46 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.83

$$\int\frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{9/2}}\,dx=\frac{a^2(9+25\cos(2(c+dx))-20i\sin(2(c+dx)))(-i\cos(2(c+2dx))+\sin(2(c}{45de^4\sqrt{e\sec(c+dx)}(\cos(dx)+i\sin(dx))^2}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(9/2),x]
```

```
[Out] (a^2*(9 + 25*Cos[2*(c + d*x)] - (20*I)*Sin[2*(c + d*x)])*((-I)*Cos[2*(c + 2
*d*x)] + Sin[2*(c + 2*d*x)])*Sqrt[a + I*a*Tan[c + d*x]])/(45*d*e^4*Sqrt[e*S
ec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [A] (verified)

Time = 9.88 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{2(\tan(dx+c)-i)^2 a^2 \sqrt{a(1+i\tan(dx+c))}\,(25i(\cos^4(dx+c))+20(\cos^3(dx+c))\sin(dx+c)-8i(\cos^2(dx+c)))}{45d\sqrt{e\sec(dx+c)}\,e^4}$	87
risch	$-\frac{ia^2\sqrt{\frac{a\,e^{2i(dx+c)}}{e^{2i(dx+c)}+1}}\,(5\,e^{4i(dx+c)}+18\,e^{2i(dx+c)}+45)}{90e^4\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	89

[In] int((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(9/2),x,method=_RETURNVERBOSE)

[Out] 2/45/d*(tan(d*x+c)-I)^2*a^2*(a*(1+I*tan(d*x+c)))^(1/2)/(e*sec(d*x+c))^(1/2)/e^4*(25*I*cos(d*x+c)^4+20*cos(d*x+c)^3*sin(d*x+c)-8*I*cos(d*x+c)^2)

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.79

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{9/2}}\,dx = \frac{\left(-5i\,a^2e^{(6i\,dx+6i\,c)}-23i\,a^2e^{(4i\,dx+4i\,c)}-63i\,a^2e^{(2i\,dx+2i\,c)}-45i\,a^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+}}}{90\,de^5}$$

[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(9/2),x, algorithm="fricas")

[Out] 1/90*(-5*I*a^2*e^(6*I*d*x + 6*I*c) - 23*I*a^2*e^(4*I*d*x + 4*I*c) - 63*I*a^2*e^(2*I*d*x + 2*I*c) - 45*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/(d*e^5)

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{9/2}}\,dx = \text{Timed out}$$

[In] integrate((a+I*a*tan(d*x+c))**(5/2)/(e*sec(d*x+c))**(9/2),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.77

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{9/2}}\,dx = \frac{\left(-5i\,a^2\cos\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)-18i\,a^2\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)-45i\,a^2\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\right.}{90\,de}$$

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(9/2),x, algorithm="maxim
a")
```

```
[Out] 1/90*(-5*I*a^2*cos(9/2*d*x + 9/2*c) - 18*I*a^2*cos(5/2*d*x + 5/2*c) - 45*I*
a^2*cos(1/2*d*x + 1/2*c) + 5*a^2*sin(9/2*d*x + 9/2*c) + 18*a^2*sin(5/2*d*x
+ 5/2*c) + 45*a^2*sin(1/2*d*x + 1/2*c))*sqrt(a)/(d*e^(9/2))
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{9/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}{(e\sec(dx+c))^{\frac{9}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(9/2),x, algorithm="giac"
)
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)/(e*sec(d*x + c))^(9/2), x)
```

Mupad [B] (verification not implemented)

Time = 6.67 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.02

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{9/2}}\,dx =$$
$$-\frac{a^2\sqrt{\frac{e}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1{\rm i})}{\cos(2\,c+2\,d\,x)+1}}\,\left(-18\,\sin(c+d\,x)-23\,\sin(3\,c+3\,d\,x)-5\,\sin(5\,c+5\,d\,x)-\right.}{180\,d\,e^5}$$

```
[In] int((a + a*tan(c + d*x)*1i)^(5/2)/(e/cos(c + d*x))^(9/2),x)
```

```
[Out] -(a^2*(e/cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i +
1))/(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(c + d*x)*108i - 18*sin(c + d*x) + co
s(3*c + 3*d*x)*23i + cos(5*c + 5*d*x)*5i - 23*sin(3*c + 3*d*x) - 5*sin(5*c
+ 5*d*x)))/(180*d*e^5)
```

3.415 $\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{11/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 169

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{11/2}}\,dx = \frac{32ia^3\sqrt{e\sec(c+dx)}}{77de^6\sqrt{a+ia\tan(c+dx)}} - \frac{16ia^2\sqrt{a+ia\tan(c+dx)}}{77de^4(e\sec(c+dx))^{3/2}} - \frac{12ia(a+ia\tan(c+dx))^{3/2}}{77de^2(e\sec(c+dx))^{7/2}} - \frac{2i(a+ia\tan(c+dx))^{5/2}}{11d(e\sec(c+dx))^{11/2}}$$

```
[Out] 32/77*I*a^3*(e*sec(d*x+c))^(1/2)/d/e^6/(a+I*a*tan(d*x+c))^(1/2)-16/77*I*a^2
*(a+I*a*tan(d*x+c))^(1/2)/d/e^4/(e*sec(d*x+c))^(3/2)-12/77*I*a*(a+I*a*tan(d
*x+c))^(3/2)/d/e^2/(e*sec(d*x+c))^(7/2)-2/11*I*(a+I*a*tan(d*x+c))^(5/2)/d/(
e*sec(d*x+c))^(11/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3578, 3569}

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{11/2}}\,dx = \frac{32ia^3\sqrt{e\sec(c+dx)}}{77de^6\sqrt{a+ia\tan(c+dx)}} - \frac{16ia^2\sqrt{a+ia\tan(c+dx)}}{77de^4(e\sec(c+dx))^{3/2}} - \frac{12ia(a+ia\tan(c+dx))^{3/2}}{77de^2(e\sec(c+dx))^{7/2}} - \frac{2i(a+ia\tan(c+dx))^{5/2}}{11d(e\sec(c+dx))^{11/2}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(11/2),x]
```

```
[Out] (((32*I)/77)*a^3*Sqrt[e*Sec[c + d*x]])/(d*e^6*Sqrt[a + I*a*Tan[c + d*x]]) -
 (((16*I)/77)*a^2*Sqrt[a + I*a*Tan[c + d*x]])/(d*e^4*(e*Sec[c + d*x])^(3/2)
```

```
) - (((12*I)/77)*a*(a + I*a*Tan[c + d*x])^(3/2))/(d*e^2*(e*Sec[c + d*x])^(7
/2)) - (((2*I)/11)*(a + I*a*Tan[c + d*x])^(5/2))/(d*(e*Sec[c + d*x])^(11/2)
)
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2i(a+ia\tan(c+dx))^{5/2}}{11d(e\sec(c+dx))^{11/2}}+\frac{(6a)\int\frac{(a+ia\tan(c+dx))^{3/2}}{(e\sec(c+dx))^{7/2}}\,dx}{11e^2}\\
&= -\frac{12ia(a+ia\tan(c+dx))^{3/2}}{77de^2(e\sec(c+dx))^{7/2}}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{11d(e\sec(c+dx))^{11/2}}+\frac{(24a^2)\int\frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx}{77e^4}\\
&= -\frac{16ia^2\sqrt{a+ia\tan(c+dx)}}{77de^4(e\sec(c+dx))^{3/2}}-\frac{12ia(a+ia\tan(c+dx))^{3/2}}{77de^2(e\sec(c+dx))^{7/2}}\\
&\quad -\frac{2i(a+ia\tan(c+dx))^{5/2}}{11d(e\sec(c+dx))^{11/2}}+\frac{(16a^3)\int\frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{77e^6}\\
&= \frac{32ia^3\sqrt{e\sec(c+dx)}}{77de^6\sqrt{a+ia\tan(c+dx)}}-\frac{16ia^2\sqrt{a+ia\tan(c+dx)}}{77de^4(e\sec(c+dx))^{3/2}}\\
&\quad -\frac{12ia(a+ia\tan(c+dx))^{3/2}}{77de^2(e\sec(c+dx))^{7/2}}-\frac{2i(a+ia\tan(c+dx))^{5/2}}{11d(e\sec(c+dx))^{11/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.72

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{11/2}}\,dx = \frac{a^2(-55i\cos(c+dx)+35i\cos(3(c+dx))-22\sin(c+dx)+42\sin(3(c+dx)}{154de^5\sqrt{e\sec(c+dx)}(\cos(dx)}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^(5/2)/(e*Sec[c + d*x])^(11/2),x]
```

```
[Out] (a^2*((-55*I)*Cos[c + d*x] + (35*I)*Cos[3*(c + d*x)] - 22*Sin[c + d*x] + 42
*Sin[3*(c + d*x)])*(Cos[2*(c + 2*d*x)] + I*Sin[2*(c + 2*d*x)])*Sqrt[a + I*a
*Tan[c + d*x]])/(154*d*e^5*Sqrt[e*Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])^2)
```

Maple [A] (verified)

Time = 9.82 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.57

method	result
risch	$-\frac{ia^2\sqrt{\frac{a\,e^{2i(dx+c)}}{e^{2i(dx+c)}+1}}\,(7\,e^{5i(dx+c)}+33\,e^{3i(dx+c)}+154i\sin(dx+c))}{308e^5\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$
default	$\frac{2i(\tan(dx+c)-i)^2a^2\sqrt{a(1+i\tan(dx+c))}\,(42i\sin(dx+c)(\cos^4(dx+c))-35(\cos^5(dx+c))-16i(\cos^2(dx+c))\sin(dx+c)+40(\cos^3(dx+c)}{77d\sqrt{e\sec(dx+c)}\,e^5}$

```
[In] int((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/308*I*a^2/e^5/(e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^(1/2)*(a*exp(2*I*(
d*x+c))/(exp(2*I*(d*x+c))+1))^(1/2)/d*(7*exp(5*I*(d*x+c))+33*exp(3*I*(d*x+c
))+154*I*sin(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.59

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{11/2}}\,dx = \frac{\left(-7i\,a^2e^{(8i\,dx+8i\,c)}-40i\,a^2e^{(6i\,dx+6i\,c)}-110i\,a^2e^{(4i\,dx+4i\,c)}+77i\,a^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}}}}{308\,de^6}$$

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(11/2),x, algorithm="fric
as")
```

```
[Out] 1/308*(-7*I*a^2*e^(8*I*d*x + 8*I*c) - 40*I*a^2*e^(6*I*d*x + 6*I*c) - 110*I*
a^2*e^(4*I*d*x + 4*I*c) + 77*I*a^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(
e/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/(d*e^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{11/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+I*a*tan(d*x+c))**(5/2)/(e*sec(d*x+c))**(11/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.84 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.73

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{11/2}}\,dx = \frac{\left(-7i\,a^2\cos\left(\frac{11}{2}\,dx+\frac{11}{2}\,c\right)-33i\,a^2\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)-77i\,a^2\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\right.}{}$$

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(11/2),x, algorithm="maxi
ma")
[Out] 1/308*(-7*I*a^2*cos(11/2*d*x + 11/2*c) - 33*I*a^2*cos(7/2*d*x + 7/2*c) - 77
*I*a^2*cos(3/2*d*x + 3/2*c) + 77*I*a^2*cos(1/2*d*x + 1/2*c) + 7*a^2*sin(11/
2*d*x + 11/2*c) + 33*a^2*sin(7/2*d*x + 7/2*c) + 77*a^2*sin(3/2*d*x + 3/2*c)
 + 77*a^2*sin(1/2*d*x + 1/2*c))*sqrt(a)/(d*e^(11/2))
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{11/2}}\,dx = \int \frac{(i\,a\tan\left(dx+c\right)+a)^{\frac{5}{2}}}{(e\sec\left(dx+c\right))^{\frac{11}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^(5/2)/(e*sec(d*x+c))^(11/2),x, algorithm="giac
")
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)/(e*sec(d*x + c))^(11/2), x)
```

Mupad [B] (verification not implemented)

Time = 6.80 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.79

$$\int \frac{(a+ia\tan(c+dx))^{5/2}}{(e\sec(c+dx))^{11/2}}\,dx =$$

$$-\frac{a^2\sqrt{\frac{e}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}\,\left(-187\,\sin\left(2\,c+2\,d\,x\right)-40\,\sin\left(4\,c+4\,d\,x\right)-7\,\sin\left(6\,c+6\,d\,x\right.\right.}{616\,d\,e^6}$$

```
[In] int((a + a*tan(c + d*x)*1i)^(5/2)/(e/cos(c + d*x))^(11/2),x)
```

```
[Out] -(a^2*(e/cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i +
1))/(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(2*c + 2*d*x)*33i + cos(4*c + 4*d*x)*
40i + cos(6*c + 6*d*x)*7i - 187*sin(2*c + 2*d*x) - 40*sin(4*c + 4*d*x) - 7*
sin(6*c + 6*d*x)))/(616*d*e^6)
```

3.416 $\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 369

$$\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{ie^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d} - \frac{ie^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d} - \frac{ie^{5/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{2\sqrt{2}\sqrt{a}d} + \frac{ie^{5/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{2\sqrt{2}\sqrt{a}d} - \frac{ie^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{ad}$$

```
[Out] 1/2*I*e^(5/2)*arctan(1-2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*
sec(d*x+c))^(1/2))/d*2^(1/2)/a^(1/2)-1/2*I*e^(5/2)*arctan(1+2^(1/2)*e^(1/2)
*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))/d*2^(1/2)/a^(1/2)-1
/4*I*e^(5/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(d
*x+c))^(1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))/d*2^(1/2)/a^(1/2)+1/4*I*e^(5/2)
*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)
+cos(d*x+c)*(a+I*a*tan(d*x+c)))/d*2^(1/2)/a^(1/2)-I*e^2*(e*sec(d*x+c))^(1/2
)*(a+I*a*tan(d*x+c))^(1/2)/a/d
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3582, 3576, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{ie^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d} - \frac{ie^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d} - \frac{ie^{5/2}\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{2\sqrt{2}\sqrt{a}d} + \frac{ie^{5/2}\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{2\sqrt{2}\sqrt{a}d} - \frac{ie^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{ad}$$

```
[In] Int[(e*Sec[c + d*x])^(5/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (I*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]
*Sqrt[e*Sec[c + d*x]])]])/(Sqrt[2]*Sqrt[a]*d) - (I*e^(5/2)*ArcTan[1 + (Sqrt[
2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]])/(Sq
rt[2]*Sqrt[a]*d) - ((I/2)*e^(5/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a +
 I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c + d*
x])]])/(Sqrt[2]*Sqrt[a]*d) + ((I/2)*e^(5/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]
*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*T
an[c + d*x])]])/(Sqrt[2]*Sqrt[a]*d) - (I*e^2*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I
*a*Tan[c + d*x]])/(a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\text{integral} = -\frac{ie^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{ad} + \frac{e^2\int\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx}{2a}$$

$$= -\frac{ie^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{ad} - \frac{(2ie^4)\,\text{Subst}\left(\int\frac{x^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d}$$

$$= -\frac{ie^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{ad} + \frac{(ie^3)\,\text{Subst}\left(\int\frac{a-ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d} - \frac{(ie^3)\,\text{Subst}\left(\int\frac{a+ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d}$$

$$= -\frac{ie^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{ad} - \frac{(ie^2)\,\text{Subst}\left(\int\frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{2d} - \frac{(ie^2)\,\text{Subst}\left(\int\frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{2d} - \frac{(ie^{5/2})\,\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}+2x}{-\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{2\sqrt{2}\sqrt{a}d} - \frac{(ie^{5/2})\,\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}-2x}{-\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{2\sqrt{2}\sqrt{a}d}$$

$$= -\frac{ie^{5/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{2\sqrt{2}\sqrt{a}d} + \frac{ie^{5/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{2\sqrt{2}\sqrt{a}d} - \frac{ie^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{ad} - \frac{(ie^{5/2})\operatorname{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d} + \frac{(ie^{5/2})\operatorname{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d}$$

$$= \frac{ie^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d} - \frac{ie^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d} - \frac{ie^{5/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{2\sqrt{2}\sqrt{a}d} + \frac{ie^{5/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{2\sqrt{2}\sqrt{a}d} - \frac{ie^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{ad}$$

Mathematica [A] (verified)

Time = 3.78 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.95

$$\int\frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{e^3\left(\sec(c+dx)\sqrt{1+\cos(2c)+i\sin(2c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}-i\operatorname{arctanh}\left(\frac{\sqrt{1-i\cos(c}}{\sqrt{-1-i\cos(}}\right.\right.}{}$$

```
[In] Integrate[(e*Sec[c + d*x])^(5/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] (e^3*(Sec[c + d*x]*Sqrt[1 + Cos[2*c] + I*Sin[2*c]]*Sqrt[I - Tan[(d*x)/2]] -
 I*ArcTanh[(Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 -
I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1 - I*Cos[c] - Sin[c]]*Sq
rt[1 + I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]] + I*ArcTanh[(Sqrt[1 + I*Co
s[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I
 + Tan[(d*x)/2]])]*Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]]
*Sqrt[I + Tan[(d*x)/2]])*(-I + Tan[c + d*x]))/(d*Sqrt[e*Sec[c + d*x]]*Sqrt[
1 + Cos[2*c] + I*Sin[2*c]]*Sqrt[I - Tan[(d*x)/2]]*Sqrt[a + I*a*Tan[c + d*x]
])
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 764 vs. 2(285) = 570.

Time = 15.95 (sec) , antiderivative size = 765, normalized size of antiderivative = 2.07

method	result	size
default	Expression too large to display	765

```
[In] int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*(-e*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2
-1))^(5/2)*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)*(-csc(d*x+c)+cot(d*x+c)+I)*(I*
csc(d*x+c)^2*arctanh(1/2*(-cot(d*x+c)+csc(d*x+c)+1)*2^(1/2)/(csc(d*x+c)^2*(
1-cos(d*x+c))^2+1)^(1/2))*2^(1/2)*(1-cos(d*x+c))^2+I*csc(d*x+c)^2*arctanh(1
/2*(-cot(d*x+c)+csc(d*x+c)-1)*2^(1/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/
2))*2^(1/2)*(1-cos(d*x+c))^2-csc(d*x+c)^2*arctanh(1/2*(-cot(d*x+c)+csc(d*x+
c)+1)*2^(1/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2))*2^(1/2)*(1-cos(d*x+c
))^2+csc(d*x+c)^2*arctanh(1/2*(-cot(d*x+c)+csc(d*x+c)-1)*2^(1/2)/(csc(d*x+c
)^2*(1-cos(d*x+c))^2+1)^(1/2))*2^(1/2)*(1-cos(d*x+c))^2-I*arctanh(1/2*(-cot
(d*x+c)+csc(d*x+c)+1)*2^(1/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2))*2^(1
/2)-I*arctanh(1/2*(-cot(d*x+c)+csc(d*x+c)-1)*2^(1/2)/(csc(d*x+c)^2*(1-cos(d
*x+c))^2+1)^(1/2))*2^(1/2)-4*I*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2)*(csc
(d*x+c)-cot(d*x+c))+2^(1/2)*arctanh(1/2*(-cot(d*x+c)+csc(d*x+c)+1)*2^(1/2)/
(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2))-2^(1/2)*arctanh(1/2*(-cot(d*x+c)+c
sc(d*x+c)-1)*2^(1/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2))-4*(csc(d*x+c)
^2*(1-cos(d*x+c))^2+1)^(1/2))/(-a*(2*I*(csc(d*x+c)-cot(d*x+c))-csc(d*x+c)^2
*(1-cos(d*x+c))^2+1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1))^(1/2)/(csc(d*x+c)^2
*(1-cos(d*x+c))^2+1)^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.25

$$\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{-4i\,e^2\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{3}{2}i\,dx+\frac{3}{2}i\,c\right)}+\sqrt{\frac{i\,e^5}{ad^2}}ad\log\left(\frac{2\left(\left(e^2e^{(2i\,dx+2i\,c)}+e^2\right)\right.}{}\right.}{}$$

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(-4*I*e^2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c)
 + 1))*e^(3/2*I*d*x + 3/2*I*c) + sqrt(I*e^5/(a*d^2))*a*d*log(2*((e^2*e^(2*I
```

```
*d*x + 2*I*c) + e^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x +
 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + sqrt(I*e^5/(a*d^2))*a*d)/e^2) - sqr
t(I*e^5/(a*d^2))*a*d*log(2*((e^2*e^(2*I*d*x + 2*I*c) + e^2)*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c
) - sqrt(I*e^5/(a*d^2))*a*d)/e^2) - sqrt(-I*e^5/(a*d^2))*a*d*log(2*((e^2*e^
(2*I*d*x + 2*I*c) + e^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d
*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) + sqrt(-I*e^5/(a*d^2))*a*d)/e^2)
+ sqrt(-I*e^5/(a*d^2))*a*d*log(2*((e^2*e^(2*I*d*x + 2*I*c) + e^2)*sqrt(a/(e
^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1
/2*I*c) - sqrt(-I*e^5/(a*d^2))*a*d)/e^2))/(a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate((e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2258 vs. $2(273) = 546$.

Time = 0.78 (sec) , antiderivative size = 2258, normalized size of antiderivative = 6.12

$$\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Too large to display}$$

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -8*(16*e^2*cos(3/2*d*x + 3/2*c) + 16*I*e^2*sin(3/2*d*x + 3/2*c) + 2*(sqrt(2
)*e^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + I*sqrt
(2)*e^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt
(2)*e^2)*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + 1, sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + 1) + 2*(sqrt(2)*e^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) + I*sqrt(2)*e^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + sqrt(2)*e^2)*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1, -sqrt(2)*sin(1/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 2*(sqrt(2)*e^2*cos(4/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + I*sqrt(2)*e^2*sin(4/3*arctan
```

2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*e^2)*arctan2(sqrt(
2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 1, sqrt(2
)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 2*(sq
rt(2)*e^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + I*
sqrt(2)*e^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
sqrt(2)*e^2)*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) - 1, -sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 1) + 2*(-I*sqrt(2)*e^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + sqrt(2)*e^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) - I*sqrt(2)*e^2)*arctan2(sqrt(2)*sin(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sin(2/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))), sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) + cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + 1) + 2*(I*sqrt(2)*e^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) - sqrt(2)*e^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + I*sqrt(2)*e^2)*arctan2(-sqrt(2)*sin(1/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sin(2/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))), -sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))) + cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) + 1) - (sqrt(2)*e^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + I*sqrt(2)*e^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + sqrt(2)*e^2)*log(2*sqrt(2)*sin(2/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(1/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + 2*(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 1)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c)))^2 + 2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
 sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + (sqrt(2)*e^2*c
os(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + I*sqrt(2)*e^2
*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*e^2
)*log(-2*sqrt(2)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
))*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*(sqrt(2
)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 1)*cos(2/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(2/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*cos(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + 1) + (I*sqrt(2)*e^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) - sqrt(2)*e^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + I*sqrt(2)*e^2)*log(2*cos(1/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c)

```
, cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) + 2) + (-I*sqrt(2)*e^2*cos(4/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*e^2*sin(4/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) - I*sqrt(2)*e^2)*log(2*cos(1/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + (I*sqrt(2)*e^2*cos(4/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - sqrt(2)*e^2*sin(4/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + I*sqrt(2)*e^2)*log(2*cos(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + (-I*sqrt(2)*e^2*cos(4/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*e^2*sin(4/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - I*sqrt(2)*e^2)*log(2*cos(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2))*sqrt(a)*sqrt(e)/((-64*I
*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 64*a*sin(
4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 64*I*a)*d)
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\sec(dx+c))^{\frac{5}{2}}}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((e*sec(d*x + c))^(5/2)/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}\,dx$$

```
[In] int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.417 $\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 483

$$\begin{aligned}
\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx &= -\frac{i\sqrt{2}\sqrt{a}e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&+\frac{i\sqrt{2}\sqrt{a}e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&+\frac{i\sqrt{a}e^{3/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&-\frac{i\sqrt{a}e^{3/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}
\end{aligned}$$

```
[Out] 1/2*I*e^(3/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(
d*x+c))^(1/2)+cos(d*x+c)*(a-I*a*tan(d*x+c)))*sec(d*x+c)*a^(1/2)/d*2^(1/2)/(
a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-1/2*I*e^(3/2)*ln(a+2^(1/2)
*a^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(
a-I*a*tan(d*x+c)))*sec(d*x+c)*a^(1/2)/d*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a
+I*a*tan(d*x+c))^(1/2)-I*e^(3/2)*arctan(1-2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c)
)^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)*2^(1/2)*a^(1/2)/d/(a-I*a*t
an(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+I*e^(3/2)*arctan(1+2^(1/2)*e^(1/2
)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)*2^(1/2)
*a^(1/2)/d/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3580, 3576, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{i\sqrt{2}\sqrt{a}e^{3/2}\sec(c+dx)\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{i\sqrt{2}\sqrt{a}e^{3/2}\sec(c+dx)\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{i\sqrt{a}e^{3/2}\sec(c+dx)\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{i\sqrt{a}e^{3/2}\sec(c+dx)\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[(e*Sec[c + d*x])^(3/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-I)*Sqrt[2]*Sqrt[a]*e^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[
c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(d*Sqrt[a - I*a*Ta
n[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + (I*Sqrt[2]*Sqrt[a]*e^(3/2)*ArcTan
[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d
*x]])]*Sec[c + d*x])/(d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x
]]) + (I*Sqrt[a]*e^(3/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[
c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[
c + d*x])/(Sqrt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
 - (I*Sqrt[a]*e^(3/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c +
 d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c +
 d*x])/(Sqrt[2]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3580

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Dist[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]])), Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{(e\sec(c+dx))\int\sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}\,dx}{\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{(4iae^3\sec(c+dx))\,\text{Subst}\left(\int \frac{x^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= -\frac{(2iae^2\sec(c+dx))\,\text{Subst}\left(\int \frac{a-ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{(2iae^2\sec(c+dx))\,\text{Subst}\left(\int \frac{a+ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{(iae\sec(c+dx))\text{Subst}\left(\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{(iae\sec(c+dx))\text{Subst}\left(\int \frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{\left(i\sqrt{a}e^{3/2}\sec(c+dx)\right)\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}+2x}{-\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{\left(i\sqrt{a}e^{3/2}\sec(c+dx)\right)\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}-2x}{-\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{i\sqrt{a}e^{3/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{i\sqrt{a}e^{3/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{\left(i\sqrt{2}\sqrt{a}e^{3/2}\sec(c+dx)\right)\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{\left(i\sqrt{2}\sqrt{a}e^{3/2}\sec(c+dx)\right)\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= -\frac{i\sqrt{2}\sqrt{a}e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{i\sqrt{2}\sqrt{a}e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{i\sqrt{a}e^{3/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{i\sqrt{a}e^{3/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A] (verified)

Time = 1.85 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.63

$$\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2e\sqrt{e\sec(c+dx)}\left(\text{arctanh}\left(\frac{\sqrt{1+i\cos(c)-\sin(c)}\sqrt{i-\tan\left(\frac{dx}{2}\right)}}{\sqrt{-1+i\cos(c)+\sin(c)}\sqrt{i+\tan\left(\frac{dx}{2}\right)}}\right)\sqrt{-1-i\cos(c)-\mathrm{si}}\right.}{d\sqrt{-}}$$

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (2*e*Sqrt[e*Sec[c + d*x]]*(ArcTanh[(Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I - Ta
n[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1
 - I*Cos[c] - Sin[c]]*Sqrt[1 + I*Cos[c] - Sin[c]] - ArcTanh[(Sqrt[1 - I*Cos
[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I
+ Tan[(d*x)/2]])]*Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]])
*(Cos[d*x] + I*Sin[d*x])*Sqrt[I + Tan[(d*x)/2]])/(d*Sqrt[-1 - I*Cos[c] - Si
n[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]]*Sqrt[a + I*a*Tan[
c + d*x]])
```

Maple [A] (verified)

Time = 14.69 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.33

method	result
default	$\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\left(i\,\text{arctanh}\left(\frac{-\cos(dx+c)+\sin(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)+\text{arctanh}\left(\frac{\cos(dx+c)+\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)\right)\sqrt{e\sec(dx+c)}\,e(\cos(dx+c)+1+i\sin(}{d(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}\,\sqrt{a(1+i\tan(dx+c))}}$

```
[In] int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/2-1/2*I)/d*(I*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(
cos(d*x+c)+1))^(1/2))+arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/
(1/(cos(d*x+c)+1))^(1/2)))*(e*sec(d*x+c))^(1/2)*e*(cos(d*x+c)+1+I*sin(d*x+c
))/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2)/(a*(1+I*tan(d*x+c)))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 385, normalized size of antiderivative = 0.80

$$\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{1}{2}\sqrt{\frac{4i\,e^3}{ad^2}}\log\left(\frac{2\left(ee^{(2i\,dx+2i\,c)}+e\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+i\,a\right.}{e}$$

$$-\frac{1}{2}\sqrt{\frac{4i\,e^3}{ad^2}}\log\left(\frac{2\left(ee^{(2i\,dx+2i\,c)}+e\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}-i\,ad\sqrt{\frac{4i\,e^3}{ad^2}}}{e}\right)$$

$$+\frac{1}{2}\sqrt{-\frac{4i\,e^3}{ad^2}}\log\left(\frac{2\left(ee^{(2i\,dx+2i\,c)}+e\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+i\,ad\sqrt{-\frac{4i\,e^3}{ad^2}}}{e}\right)$$

$$-\frac{1}{2}\sqrt{-\frac{4i\,e^3}{ad^2}}\log\left(\frac{2\left(ee^{(2i\,dx+2i\,c)}+e\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}-i\,ad\sqrt{-\frac{4i\,e^3}{ad^2}}}{e}\right)$$

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/2*sqrt(4*I*e^3/(a*d^2))*log((2*(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I
*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*
c) + I*a*d*sqrt(4*I*e^3/(a*d^2)))/e) - 1/2*sqrt(4*I*e^3/(a*d^2))*log((2*(e*
e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d
*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - I*a*d*sqrt(4*I*e^3/(a*d^2)))/e)
 + 1/2*sqrt(-4*I*e^3/(a*d^2))*log((2*(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^
(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/
2*I*c) + I*a*d*sqrt(-4*I*e^3/(a*d^2)))/e) - 1/2*sqrt(-4*I*e^3/(a*d^2))*log(
(2*(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^
(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - I*a*d*sqrt(-4*I*e^3/(a*d^
2)))/e)
```

Sympy [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\sec{(c+dx)})^{\frac{3}{2}}}{\sqrt{ia\,(\tan{(c+dx)}-i)}}\,dx$$

```
[In] integrate((e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Integral((e*sec(c + d*x))**(3/2)/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.47 (sec) , antiderivative size = 726, normalized size of antiderivative = 1.50

$$\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Too large to display}$$

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
[Out] -1/4*(2*I*sqrt(2)*e*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, sqrt(2)*sin(1
/2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*e*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) +
 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*e*arctan2(sqrt(2)*cos(
1/2*d*x + 1/2*c) - 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*e*arc
tan2(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) -
 2*sqrt(2)*e*arctan2(sqrt(2)*sin(1/2*d*x + 1/2*c) + sin(d*x + c), sqrt(2)*c
os(1/2*d*x + 1/2*c) + cos(d*x + c) + 1) + 2*sqrt(2)*e*arctan2(-sqrt(2)*sin(
1/2*d*x + 1/2*c) + sin(d*x + c), -sqrt(2)*cos(1/2*d*x + 1/2*c) + cos(d*x +
c) + 1) + I*sqrt(2)*e*log(2*sqrt(2)*sin(d*x + c)*sin(1/2*d*x + 1/2*c) + 2*(
sqrt(2)*cos(1/2*d*x + 1/2*c) + 1)*cos(d*x + c) + cos(d*x + c)^2 + 2*cos(1/2
*d*x + 1/2*c)^2 + sin(d*x + c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) + 1) - I*sqrt(2)*e*log(-2*sqrt(2)*sin(d*x + c)*sin(1/2*d*
x + 1/2*c) - 2*(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1)*cos(d*x + c) + cos(d*x +
c)^2 + 2*cos(1/2*d*x + 1/2*c)^2 + sin(d*x + c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 1) - sqrt(2)*e*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*e*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2) - sqrt(2)*e*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
 + 2) + sqrt(2)*e*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sqrt
(e)/(sqrt(a)*d)
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\sec(dx+c))^{\frac{3}{2}}}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
[Out] integrate((e*sec(d*x + c))^(3/2)/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{a+a\tan(c+d\,x)\ 1\mathrm{i}}}\,dx$$

```
[In] int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.418 $\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 36

$$\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i\sqrt{e\sec(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 2*I*(e*sec(d*x+c))^(1/2)/d/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {3569}

$$\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i\sqrt{e\sec(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[Sqrt[e*Sec[c + d*x]]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((2*I)*Sqrt[e*Sec[c + d*x]])/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\text{integral} = \frac{2i\sqrt{e\sec(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i\sqrt{e\sec(c+dx)}}{d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]/Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] ((2*I)*Sqrt[e*Sec[c + d*x]])/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] (verified)

Time = 11.60 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{2i\sqrt{e\sec(dx+c)}}{d\sqrt{a(1+i\tan(dx+c))}}$	32
risch	$\frac{2i\sqrt{\frac{e\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}{\sqrt{\frac{a\,e^{2i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	59

```
[In] int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*I/d*(e*sec(d*x+c))^(1/2)/(a*(1+I*tan(d*x+c)))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(28) = 56.

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.78

$$\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(i\,e^{(2i\,dx+2i\,c)}+i\right)e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{ad}$$

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
[Out] 2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(I*e^
(2*I*d*x + 2*I*c) + I)*e^(-1/2*I*d*x - 1/2*I*c)/(a*d)
```

Sympy [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sqrt{e\sec{(c+dx)}}}{\sqrt{ia\left(\tan{(c+dx)}-i\right)}}\,dx$$

```
[In] integrate((e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Integral(sqrt(e*sec(c + d*x))/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(28) = 56$.

Time = 0.35 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.11

$$\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i\sqrt{e}\sqrt{-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}}{\sqrt{a}d\sqrt{-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}}$$

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
[Out] 2*I*sqrt(e)*sqrt(-sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)/(sqrt(a)*d*sqrt(
-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2
- 1))
```

Giac [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sqrt{e\sec{(dx+c)}}}{\sqrt{i\,a\tan{(dx+c)}+a}}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
[Out] integrate(sqrt(e*sec(d*x + c))/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [B] (verification not implemented)

Time = 5.70 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{e \sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,2\mathrm{i}}{d\,\sqrt{a+\frac{a\,\sin(c+d\,x)\,1\mathrm{i}}{\cos(c+d\,x)}}}$$

```
[In] int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] ((e/cos(c + d*x))^(1/2)*2i)/(d*(a + (a*sin(c + d*x)*1i)/cos(c + d*x))^(1/2)
)
```

3.419 $\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 80

$$\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i}{3d\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{4i\sqrt{a+ia\tan(c+dx)}}{3ad\sqrt{e\sec(c+dx)}}$$

```
[Out] 2/3*I/d/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-4/3*I*(a+I*a*tan(d*x+
c))^(1/2)/a/d/(e*sec(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3583, 3569}

$$\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i}{3d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}} - \frac{4i\sqrt{a+ia\tan(c+dx)}}{3ad\sqrt{e\sec(c+dx)}}$$

```
[In] Int[1/(Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] ((2*I)/3)/(d*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((4*I)/3)*
Sqrt[a + I*a*Tan[c + d*x]])/(a*d*Sqrt[e*Sec[c + d*x]])
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
```

```
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2i}{3d\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{2\int\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx}{3a}\\ &= \frac{2i}{3d\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{4i\sqrt{a+ia\tan(c+dx)}}{3ad\sqrt{e\sec(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.60

$$\int\frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{-2i+4\tan(c+dx)}{3d\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[1/(Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]),x]
[Out] (-2*I + 4*Tan[c + d*x])/(3*d*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]
])
```

Maple [A] (verified)

Time = 10.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.52

method	result	size
default	$-\frac{2(i-2\tan(dx+c))}{3d\sqrt{e\sec(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}}$	42
risch	$-\frac{i\left(3\,\mathrm{e}^{2i(dx+c)}-1\right)}{3\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\left(\mathrm{e}^{2i(dx+c)}+1\right)\sqrt{\frac{a\,\mathrm{e}^{2i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}$	85

```
[In] int(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -2/3/d/(e*sec(d*x+c))^(1/2)/(a*(1+I*tan(d*x+c)))^(1/2)*(I-2*tan(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-3i\,e^{(4i\,dx+4i\,c)}-2i\,e^{(2i\,dx+2i\,c)}+i\right)e^{\left(-\frac{3}{2}i\,dx-\frac{3}{2}i\,c\right)}}{3\,ade}$$

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/3*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-3
*I*e^(4*I*d*x + 4*I*c) - 2*I*e^(2*I*d*x + 2*I*c) + I)*e^(-3/2*I*d*x - 3/2*I
*c)/(a*d*e)
```

Sympy [F]

$$\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{\sqrt{e\sec\left(c+dx\right)}\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}\,dx$$

```
[In] integrate(1/(e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(e*sec(c + d*x))*sqrt(I*a*(tan(c + d*x) - I))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{i\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)-3i\cos\left(\frac{1}{3}\arctan\left(\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right),\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\right)\right)+\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3\sin\left(\frac{1}{3}\,\mathrm{arct}\right.}{3\sqrt{a}d\sqrt{e}}$$

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="max
ima")
```

```
[Out] 1/3*(I*cos(3/2*d*x + 3/2*c) - 3*I*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + sin(3/2*d*x + 3/2*c) + 3*sin(1/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))))/(sqrt(a)*d*sqrt(e))
```

Giac [F]

$$\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{\sqrt{e\sec\left(dx+c\right)}\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/(sqrt(e*sec(d*x + c))*sqrt(I*a*tan(d*x + c) + a)), x)
```

Mupad [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{2\sqrt{\frac{e}{\cos(c+d\,x)}}\,\left(-2\,\sin\left(c+d\,x\right)+\cos\left(c+d\,x\right)\,1\mathrm{i}\right)}{3\,d\,e\,\sqrt{\frac{a\,\left(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i}\right)}{\cos(2\,c+2\,d\,x)+1}}}$$

```
[In] int(1/((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

```
[Out] -(2*(e/cos(c + d*x))^(1/2)*(cos(c + d*x)*1i - 2*sin(c + d*x)))/(3*d*e*((a*(
cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))
```

3.420 $\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 121

$$\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx=\frac{2i}{5d(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}+\frac{16i\sqrt{e\sec(c+dx)}}{15de^2\sqrt{a+ia\tan(c+dx)}}-\frac{8i\sqrt{a+ia\tan(c+dx)}}{15ad(e\sec(c+dx))^{3/2}}$$

```
[Out] 2/5*I/d/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2)+16/15*I*(e*sec(d*x+c)
)^(1/2)/d/e^2/(a+I*a*tan(d*x+c))^(1/2)-8/15*I*(a+I*a*tan(d*x+c))^(1/2)/a/d/
(e*sec(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3583, 3578, 3569}

$$\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx=\frac{16i\sqrt{e\sec(c+dx)}}{15de^2\sqrt{a+ia\tan(c+dx)}}-\frac{8i\sqrt{a+ia\tan(c+dx)}}{15ad(e\sec(c+dx))^{3/2}}+\frac{2i}{5d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}$$

```
[In] Int[1/((e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] ((2*I)/5)/(d*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (((16*I)/
15)*Sqrt[e*Sec[c + d*x]])/(d*e^2*Sqrt[a + I*a*Tan[c + d*x]]) - (((8*I)/15)*
Sqrt[a + I*a*Tan[c + d*x]])/(a*d*(e*Sec[c + d*x])^(3/2))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2i}{5d(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}+\frac{4\int\frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx}{5a}\\
&= \frac{2i}{5d(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}-\frac{8i\sqrt{a+ia\tan(c+dx)}}{15ad(e\sec(c+dx))^{3/2}}+\frac{8\int\frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{15e^2}\\
&= \frac{2i}{5d(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\\
&\quad+\frac{16i\sqrt{e\sec(c+dx)}}{15de^2\sqrt{a+ia\tan(c+dx)}}-\frac{8i\sqrt{a+ia\tan(c+dx)}}{15ad(e\sec(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.56

$$\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{i\sec^2(c+dx)(-15+\cos(2(c+dx))+4i\sin(2(c+dx)))}{15d(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[1/((e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
[Out] ((-1/15*I)*Sec[c + d*x]^2*(-15 + Cos[2*(c + d*x)] + (4*I)*Sin[2*(c + d*x)])
)/(d*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] (verified)

Time = 10.35 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.50

method	result	size
default	$-\frac{2(i\cos(dx+c)-4\sin(dx+c)-8i\sec(dx+c))}{15d\sqrt{e\sec(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}\,e}$	61

```
[In] int(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE
)
[Out] -2/15/d/(e*sec(d*x+c))^(1/2)/(a*(1+I*tan(d*x+c)))^(1/2)/e*(I*cos(d*x+c)-4*s
in(d*x+c)-8*I*sec(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74

$$\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-5i\,e^{(6i\,dx+6i\,c)}+25i\,e^{(4i\,dx+4i\,c)}+\right.}{30\,ade^2}$$

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fri
cas")
[Out] 1/30*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-
5*I*e^(6*I*d*x + 6*I*c) + 25*I*e^(4*I*d*x + 4*I*c) + 33*I*e^(2*I*d*x + 2*I*
c) + 3*I)*e^(-5/2*I*d*x - 5/2*I*c)/(a*d*e^2)
```

Sympy [F]

$$\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{(e\sec{(c+dx)})^{\frac{3}{2}}\sqrt{ia\left(\tan{(c+dx)}-i\right)}}\,dx$$

```
[In] integrate(1/(e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Integral(1/((e*sec(c + d*x))**(3/2)*sqrt(I*a*(tan(c + d*x) - I))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07

$$\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{3i\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)-5i\cos\left(\frac{3}{5}\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\left(\right.\right.\right.}{}$$

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="max
ima")
[Out] 1/30*(3*I*cos(5/2*d*x + 5/2*c) - 5*I*cos(3/5*arctan2(sin(5/2*d*x + 5/2*c),
cos(5/2*d*x + 5/2*c))) + 30*I*cos(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2
*d*x + 5/2*c))) + 3*sin(5/2*d*x + 5/2*c) + 5*sin(3/5*arctan2(sin(5/2*d*x +
5/2*c), cos(5/2*d*x + 5/2*c))) + 30*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), c
os(5/2*d*x + 5/2*c))))/(sqrt(a)*d*e^(3/2))
```

Giac [F]

$$\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{(e\sec{(dx+c)})^{\frac{3}{2}}\sqrt{i\,a\tan{(dx+c)}+a}}\,dx$$

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gia
c")
[Out] integrate(1/((e*sec(d*x + c))^(3/2)*sqrt(I*a*tan(d*x + c) + a)), x)
```

Mupad [B] (verification not implemented)

Time = 4.56 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.71

$$\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\left(4\,\sin\left(2\,c+2\,d\,x\right)-\cos\left(2\,c+2\,d\,x\right)\,1\mathrm{i}+15\mathrm{i}\right)}{15\,d\,e^2\,\sqrt{\frac{a\,\left(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i}\right)}{\cos(2\,c+2\,d\,x)+1}}}$$

```
[In] int(1/((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

```
[Out] ((e/cos(c + d*x))^(1/2)*(4*sin(2*c + 2*d*x) - cos(2*c + 2*d*x)*1i + 15i))/(
15*d*e^2*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x
) + 1))^(1/2))
```

3.421 $\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 165

$$\begin{aligned}\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx &= \frac{2i}{7d(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\\ &+ \frac{16i}{35de^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\ &- \frac{12i\sqrt{a+ia\tan(c+dx)}}{35ad(e\sec(c+dx))^{5/2}} - \frac{32i\sqrt{a+ia\tan(c+dx)}}{35ade^2\sqrt{e\sec(c+dx)}}\end{aligned}$$

```
[Out] 2/7*I/d/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2)+16/35*I/d/e^2/(e*sec(
d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-12/35*I*(a+I*a*tan(d*x+c))^(1/2)/a/d
/(e*sec(d*x+c))^(5/2)-32/35*I*(a+I*a*tan(d*x+c))^(1/2)/a/d/e^2/(e*sec(d*x+c
))^(1/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3583, 3578, 3569}

$$\begin{aligned}\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx &=\\ &-\frac{32i\sqrt{a+ia\tan(c+dx)}}{35ade^2\sqrt{e\sec(c+dx)}} + \frac{16i}{35de^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}\\ &-\frac{12i\sqrt{a+ia\tan(c+dx)}}{35ad(e\sec(c+dx))^{5/2}} + \frac{2i}{7d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}}\end{aligned}$$

```
[In] Int[1/((e*Sec[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] ((2*I)/7)/(d*(e*Sec[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]) + ((16*I)/3
5)/(d*e^2*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((12*I)/35)*S
qrt[a + I*a*Tan[c + d*x]])/(a*d*(e*Sec[c + d*x])^(5/2)) - (((32*I)/35)*Sqrt
[a + I*a*Tan[c + d*x]])/(a*d*e^2*Sqrt[e*Sec[c + d*x]])
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2i}{7d(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}+\frac{6\int\frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx}{7a}\\
&= \frac{2i}{7d(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\\
&\quad-\frac{12i\sqrt{a+ia\tan(c+dx)}}{35ad(e\sec(c+dx))^{5/2}}+\frac{24\int\frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx}{35e^2}\\
&= \frac{2i}{7d(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\\
&\quad+\frac{16i}{35de^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\
&\quad-\frac{12i\sqrt{a+ia\tan(c+dx)}}{35ad(e\sec(c+dx))^{5/2}}+\frac{16\int\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx}{35ae^2}
\end{aligned}
$$

$$= \frac{2i}{7d(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}} + \frac{16i}{35de^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{12i\sqrt{a+ia\tan(c+dx)}}{35ad(e\sec(c+dx))^{5/2}} - \frac{32i\sqrt{a+ia\tan(c+dx)}}{35ade^2\sqrt{e\sec(c+dx)}}$$

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.48

$$\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{i(17+\cos(2(c+dx))+3i\sec(c+dx)\sin(3(c+dx))+35i\tan(c+dx))}{35de^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[1/((e*Sec[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] ((-1/35*I)*(17 + Cos[2*(c + d*x)] + (3*I)*Sec[c + d*x]*Sin[3*(c + d*x)] + (
35*I)*Tan[c + d*x]))/(d*e^2*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]
)
```

Maple [A] (verified)

Time = 10.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.42

method	result	size
default	$-\frac{2(i(\cos^2(dx+c))-6\sin(dx+c)\cos(dx+c)+8i-16\tan(dx+c))}{35d\sqrt{a(1+i\tan(dx+c))}\,\sqrt{e\sec(dx+c)}\,e^2}$	70

```
[In] int(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -2/35/d/(a*(1+I*tan(d*x+c)))^(1/2)/(e*sec(d*x+c))^(1/2)/e^2*(I*cos(d*x+c)^2
-6*sin(d*x+c)*cos(d*x+c)+8*I-16*tan(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.61

$$\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-7i\,e^{(8i\,dx+8i\,c)} - 112i\,e^{(6i\,dx+6i\,c)} - \cdots}{140\,ade^3}$$

```
[In] integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/140*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(
-7*I*e^(8*I*d*x + 8*I*c) - 112*I*e^(6*I*d*x + 6*I*c) - 70*I*e^(4*I*d*x + 4*
I*c) + 40*I*e^(2*I*d*x + 2*I*c) + 5*I)*e^(-7/2*I*d*x - 7/2*I*c)/(a*d*e^3)
```

Sympy [F]

$$\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{(e\sec{(c+dx)})^{\frac{5}{2}}\sqrt{ia\left(\tan{(c+dx)}-i\right)}}\,dx$$

```
[In] integrate(1/(e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/((e*sec(c + d*x))**(5/2)*sqrt(I*a*(tan(c + d*x) - I))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.59 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.08

$$\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{5i\,\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right) - 7i\,\cos\left(\frac{5}{7}\,\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\left(\frac{7}{2}\cdots\right.\right.\right.}{\cdots}$$

```
[In] integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="max
ima")
```

```
[Out] 1/140*(5*I*cos(7/2*d*x + 7/2*c) - 7*I*cos(5/7*arctan2(sin(7/2*d*x + 7/2*c),
 cos(7/2*d*x + 7/2*c))) + 35*I*cos(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/
2*d*x + 7/2*c))) - 105*I*cos(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x
+ 7/2*c))) + 5*sin(7/2*d*x + 7/2*c) + 7*sin(5/7*arctan2(sin(7/2*d*x + 7/2*c
), cos(7/2*d*x + 7/2*c))) + 35*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/
2*d*x + 7/2*c))) + 105*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x +
7/2*c))))/(sqrt(a)*d*e^(5/2))
```

Giac [F]

$$\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{(e\sec(dx+c))^{\frac{5}{2}}\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

```
[In] integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gia
c")
[Out] integrate(1/((e*sec(d*x + c))^(5/2)*sqrt(I*a*tan(d*x + c) + a)), x)
```

Mupad [B] (verification not implemented)

Time = 5.35 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.61

$$\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\left(-\sin\left(c+d\,x\right)-\frac{3\,\sin(3\,c+3\,d\,x)}{35}+\frac{\cos(c+d\,x)\,1\mathrm{i}}{2}+\frac{\cos(3\,c+3\,d\,x)\,1\mathrm{i}}{70}\right)}{d\,e^3\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}}$$

```
[In] int(1/((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
[Out] -((e/cos(c + d*x))^(1/2)*((cos(c + d*x)*1i)/2 - sin(c + d*x) + (cos(3*c + 3
*d*x)*1i)/70 - (3*sin(3*c + 3*d*x))/35))/(d*e^3*((a*(cos(2*c + 2*d*x) + sin
(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))
```

3.422 $\int \frac{1}{(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 206

$$\begin{aligned}\int \frac{1}{(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx &= \frac{2i}{9d(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\\&+\frac{32i}{105de^2(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}+\frac{256i\sqrt{e\sec(c+dx)}}{315de^4\sqrt{a+ia\tan(c+dx)}}\\&-\frac{16i\sqrt{a+ia\tan(c+dx)}}{63ad(e\sec(c+dx))^{7/2}}-\frac{128i\sqrt{a+ia\tan(c+dx)}}{315ade^2(e\sec(c+dx))^{3/2}}\end{aligned}$$

```
[Out] 2/9*I/d/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2)+32/105*I/d/e^2/(e*sec
(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2)+256/315*I*(e*sec(d*x+c))^(1/2)/d/e^
4/(a+I*a*tan(d*x+c))^(1/2)-16/63*I*(a+I*a*tan(d*x+c))^(1/2)/a/d/(e*sec(d*x+
c))^(7/2)-128/315*I*(a+I*a*tan(d*x+c))^(1/2)/a/d/e^2/(e*sec(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3583, 3578, 3569}

$$\begin{aligned}\int \frac{1}{(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx &= \frac{256i\sqrt{e\sec(c+dx)}}{315de^4\sqrt{a+ia\tan(c+dx)}}\\&-\frac{128i\sqrt{a+ia\tan(c+dx)}}{315ade^2(e\sec(c+dx))^{3/2}}+\frac{32i}{105de^2\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}}\\&-\frac{16i\sqrt{a+ia\tan(c+dx)}}{63ad(e\sec(c+dx))^{7/2}}+\frac{2i}{9d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{7/2}}\end{aligned}$$

```
[In] Int[1/((e*Sec[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] ((2*I)/9)/(d*(e*Sec[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]) + ((32*I)/1
05)/(d*e^2*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (((256*I)/3
15)*Sqrt[e*Sec[c + d*x]])/(d*e^4*Sqrt[a + I*a*Tan[c + d*x]]) - (((16*I)/63)
*Sqrt[a + I*a*Tan[c + d*x]])/(a*d*(e*Sec[c + d*x])^(7/2)) - (((128*I)/315)*
Sqrt[a + I*a*Tan[c + d*x]])/(a*d*e^2*(e*Sec[c + d*x])^(3/2))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\text{integral} = \frac{2i}{9d(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} + \frac{8\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}\,dx}{9a}$$

$$= \frac{2i}{9d(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} - \frac{16i\sqrt{a+ia\tan(c+dx)}}{63ad(e\sec(c+dx))^{7/2}} + \frac{16\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx}{21e^2}$$

$$
\begin{aligned}
&= \frac{2i}{9d(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} \\
&\quad + \frac{32i}{105de^2(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} \\
&\quad - \frac{16i\sqrt{a+ia\tan(c+dx)}}{63ad(e\sec(c+dx))^{7/2}} + \frac{64\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}dx}{105ae^2} \\
&= \frac{2i}{9d(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} + \frac{32i}{105de^2(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} \\
&\quad - \frac{16i\sqrt{a+ia\tan(c+dx)}}{63ad(e\sec(c+dx))^{7/2}} - \frac{128i\sqrt{a+ia\tan(c+dx)}}{315ade^2(e\sec(c+dx))^{3/2}} + \frac{128\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}dx}{315e^4} \\
&= \frac{2i}{9d(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} + \frac{32i}{105de^2(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} \\
&\quad + \frac{256i\sqrt{e\sec(c+dx)}}{315de^4\sqrt{a+ia\tan(c+dx)}} - \frac{16i\sqrt{a+ia\tan(c+dx)}}{63ad(e\sec(c+dx))^{7/2}} - \frac{128i\sqrt{a+ia\tan(c+dx)}}{315ade^2(e\sec(c+dx))^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.42

$$
\int \frac{1}{(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}dx = \frac{\sqrt{e\sec(c+dx)}(945i - 84i\cos(2(c+dx)) - 5i\cos(4(c+dx))}{1260de^4\sqrt{a+ia\tan(c+}}
$$

```
[In] Integrate[1/((e*Sec[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] (Sqrt[e*Sec[c + d*x]]*(945*I - (84*I)*Cos[2*(c + d*x)] - (5*I)*Cos[4*(c + d
*x)] + 336*Sin[2*(c + d*x)] + 40*Sin[4*(c + d*x)]))/(1260*d*e^4*Sqrt[a + I*
a*Tan[c + d*x]])
```

Maple [A] (verified)

Time = 10.60 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.43

method	result	size
default	$\frac{-\frac{2i\left(\cos^{3}(dx+c)\right)}{63}+\frac{16\left(\cos^{2}(dx+c)\right)\sin(dx+c)}{63}-\frac{32i\cos(dx+c)}{315}+\frac{128\sin(dx+c)}{315}+\frac{256i\sec(dx+c)}{315}}{d\sqrt{e\sec(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}\,e^{3}}$	88

```
[In] int(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 2/315/d/(e*sec(d*x+c))^(1/2)/(a*(1+I*tan(d*x+c)))^(1/2)/e^3*(-5*I*cos(d*x+c
)^3+40*cos(d*x+c)^2*sin(d*x+c)-16*I*cos(d*x+c)+64*sin(d*x+c)+128*I*sec(d*x+
c))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.54

$$\int \frac{1}{(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-45i\,e^{(10i\,dx+10i\,c)}-465i\,e^{(8i\,dx+8}\right.}{}$$

```
[In] integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/2520*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*
(-45*I*e^(10*I*d*x + 10*I*c) - 465*I*e^(8*I*d*x + 8*I*c) + 1470*I*e^(6*I*d*
x + 6*I*c) + 2142*I*e^(4*I*d*x + 4*I*c) + 287*I*e^(2*I*d*x + 2*I*c) + 35*I)
*e^(-9/2*I*d*x - 9/2*I*c)/(a*d*e^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.80 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.10

$$\int \frac{1}{(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{35i\cos\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)-45i\cos\left(\frac{7}{9}\arctan\left(\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right),\text{co}\right.\right.}{}$$

```
[In] integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="max
ima")
```

```
[Out] 1/2520*(35*I*cos(9/2*d*x + 9/2*c) - 45*I*cos(7/9*arctan2(sin(9/2*d*x + 9/2*
c), cos(9/2*d*x + 9/2*c))) + 252*I*cos(5/9*arctan2(sin(9/2*d*x + 9/2*c), co
```

```
s(9/2*d*x + 9/2*c))) - 420*I*cos(1/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*
d*x + 9/2*c))) + 1890*I*cos(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x +
 9/2*c))) + 35*sin(9/2*d*x + 9/2*c) + 45*sin(7/9*arctan2(sin(9/2*d*x + 9/2*
c), cos(9/2*d*x + 9/2*c))) + 252*sin(5/9*arctan2(sin(9/2*d*x + 9/2*c), cos(
9/2*d*x + 9/2*c))) + 420*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
+ 9/2*c))) + 1890*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c
))))/(sqrt(a)*d*e^(7/2))
```

Giac [F]

$$\int \frac{1}{(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{(e\sec(dx+c))^{\frac{7}{2}}\sqrt{ia\tan(dx+c)+a}}\,dx$$

```
[In] integrate(1/(e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((e*sec(d*x + c))^(7/2)*sqrt(I*a*tan(d*x + c) + a)), x)
```

Mupad [B] (verification not implemented)

Time = 4.94 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.53

$$\int \frac{1}{(e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,(336\,\sin(2\,c+2\,d\,x)-\cos(4\,c+4\,d\,x)\,5\mathrm{i}-\cos(2}{1260\,d\,e^4\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin}{\cos(2\,c+2\,d\,x)}}}$$

```
[In] int(1/((e/cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

```
[Out] ((e/cos(c + d*x))^(1/2)*(336*sin(2*c + 2*d*x) - cos(4*c + 4*d*x)*5i - cos(2
*c + 2*d*x)*84i + 40*sin(4*c + 4*d*x) + 945i))/(1260*d*e^4*((a*(cos(2*c + 2
*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))
```

3.423 $\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 529

$$\begin{aligned}\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx &= -\frac{ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}}\\
&-\frac{3ie^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{\sqrt{2}\sqrt{a}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\
&+\frac{3ie^{7/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{\sqrt{2}\sqrt{a}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\
&+\frac{3ie^{7/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{2\sqrt{2}\sqrt{a}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\
&-\frac{3ie^{7/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{2\sqrt{2}\sqrt{a}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\end{aligned}$$

```
[Out] -I*e^2*(e*sec(d*x+c))^(3/2)/a/d/(a+I*a*tan(d*x+c))^(1/2)-3/2*I*e^(7/2)*arct
an(1-2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))
*sec(d*x+c)/d*2^(1/2)/a^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(
1/2)+3/2*I*e^(7/2)*arctan(1+2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2
)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)/d*2^(1/2)/a^(1/2)/(a-I*a*tan(d*x+c))^(1/
2)/(a+I*a*tan(d*x+c))^(1/2)+3/4*I*e^(7/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a-I
*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a-I*a*tan(d*x+c)))*se
c(d*x+c)/d*2^(1/2)/a^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2
)-3/4*I*e^(7/2)*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*se
c(d*x+c))^(1/2)+cos(d*x+c)*(a-I*a*tan(d*x+c)))*sec(d*x+c)/d*2^(1/2)/a^(1/2)
/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3581, 3579, 3580, 3576, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{3ie^{7/2}\sec(c+dx)\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$+\frac{3ie^{7/2}\sec(c+dx)\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$+\frac{3ie^{7/2}\sec(c+dx)\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{2\sqrt{2}\sqrt{a}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$-\frac{3ie^{7/2}\sec(c+dx)\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{2\sqrt{2}\sqrt{a}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$-\frac{ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-I)*e^2*(e*Sec[c + d*x])^(3/2))/(a*d*Sqrt[a + I*a*Tan[c + d*x]]) - ((3*I)
*e^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*S
qrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(Sqrt[2]*Sqrt[a]*d*Sqrt[a - I*a*Tan[c +
 d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + ((3*I)*e^(7/2)*ArcTan[1 + (Sqrt[2]*Sqr
t[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*
x])/(Sqrt[2]*Sqrt[a]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]
]) + (((3*I)/2)*e^(7/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c
 + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c
 + d*x])/(Sqrt[2]*Sqrt[a]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c +
 d*x]]) - (((3*I)/2)*e^(7/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*
Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*
Sec[c + d*x])/(Sqrt[2]*Sqrt[a]*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Ta
n[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3580

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Dist[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]])), Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rubi steps

$$\text{integral} = -\frac{4ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}} + \frac{(3e^2)\int(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx}{a^2}$$

$$= -\frac{ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}} + \frac{(3e^2)\int\frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{2a}$$

$$= -\frac{ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}} + \frac{(3e^3\sec(c+dx))\int\sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}\,dx}{2a\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= -\frac{ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}} + \frac{(6ie^5\sec(c+dx))\,\text{Subst}\left(\int\frac{x^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= -\frac{ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}} - \frac{(3ie^4\sec(c+dx))\,\text{Subst}\left(\int\frac{a-ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{(3ie^4\sec(c+dx))\,\text{Subst}\left(\int\frac{a+ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= -\frac{ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}} + \frac{(3ie^3\sec(c+dx))\,\text{Subst}\left(\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{2d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{(3ie^3\sec(c+dx))\,\text{Subst}\left(\int \frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{2d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{(3ie^{7/2}\sec(c+dx))\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}+2x}{-\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{2\sqrt{2}\sqrt{a}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{(3ie^{7/2}\sec(c+dx))\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}-2x}{-\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{2\sqrt{2}\sqrt{a}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= -\frac{ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}} + \frac{3ie^{7/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{2\sqrt{2}\sqrt{a}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{3ie^{7/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{2\sqrt{2}\sqrt{a}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{(3ie^{7/2}\sec(c+dx))\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{(3ie^{7/2}\sec(c+dx))\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= -\frac{ie^2(e\sec(c+dx))^{3/2}}{ad\sqrt{a+ia\tan(c+dx)}} - \frac{3ie^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{\sqrt{2}\sqrt{a}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{3ie^{7/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{\sqrt{2}\sqrt{a}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{3ie^{7/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{2\sqrt{2}\sqrt{a}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{3ie^{7/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{2\sqrt{2}\sqrt{a}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A] (verified)

Time = 3.66 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.64

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{e(e\sec(c+dx))^{5/2}\left(-i\cos(c+dx)+\sin(c+dx)+\frac{3\cos(c+dx)(\cos(c)+i\sin(c))\left(\mathrm{ar}\right.}{}\right.}{}$$

```
[In] Integrate[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (e*(e*Sec[c + d*x])^(5/2)*((-I)*Cos[c + d*x] + Sin[c + d*x] + (3*Cos[c + d*
x]*(Cos[c] + I*Sin[c])*(ArcTanh[(Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(
d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1 -
I*Cos[c] - Sin[c]]*Sqrt[1 + I*Cos[c] - Sin[c]] - ArcTanh[(Sqrt[1 - I*Cos[c]
 + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I + T
an[(d*x)/2]])]*Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]])*(C
os[d*x] + I*Sin[d*x])^2*Sqrt[I + Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c
]]*Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])))/(d*(a + I*a*Tan[c
 + d*x])^(3/2))
```

Maple [A] (verified)

Time = 16.85 (sec) , antiderivative size = 712, normalized size of antiderivative = 1.35

method	result
default	$\frac{\left(\frac{1}{4}-\frac{i}{4}\right)\sqrt{e\sec(dx+c)}\,e^3\left(6i\,\mathrm{arctanh}\left(\frac{\cos(dx+c)+\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)\cos(dx+c)+2i\tan(dx+c)\sec(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}-3i\sec(dx+c\right.}{}$

```
[In] int((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/4-1/4*I)/d*(e*sec(d*x+c))^(1/2)*e^3/(-tan(d*x+c)+I)/a/(a*(1+I*tan(d*x+c)
))^(1/2)/(1/(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)+1)*(6*I*cos(d*x+c)*arctanh(1/
2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))+2*I*ta
n(d*x+c)*sec(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)-3*I*sec(d*x+c)*arctanh(1/2*(co
s(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))+2*I*tan(d*x
+c)*(1/(cos(d*x+c)+1))^(1/2)+2*I*(1/(cos(d*x+c)+1))^(1/2)+2*I*sec(d*x+c)*(1
/(cos(d*x+c)+1))^(1/2)-6*sin(d*x+c)*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(
cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))-6*arctanh(1/2*(-cos(d*x+c)+sin(d*x+
c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-3*I*tan(d*x+c)*ar
ctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2
))-6*I*sin(d*x+c)*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/
```

```
(cos(d*x+c)+1))^(1/2))-2*tan(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+2*(1/(cos(d*x+
c)+1))^(1/2)+3*I*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(c
os(d*x+c)+1))^(1/2))-3*tan(d*x+c)*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(co
s(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))-3*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)
-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))-2*tan(d*x+c)*sec(d*x+c)*(1/(co
s(d*x+c)+1))^(1/2)+2*sec(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+3*sec(d*x+c)*arcta
nh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 483, normalized size of antiderivative = 0.91

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{-4i\,e^3\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}-\sqrt{\frac{9i\,e^7}{a^3d^2}}\,a^2d\log\left(-\frac{2\left(i\sqrt{\frac{9i\,e^7}{a^3d^2}}\,a^2\right.}{}\right.}{}$$

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(-4*I*e^3*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c)
 + 1))*e^(1/2*I*d*x + 1/2*I*c) - sqrt(9*I*e^7/(a^3*d^2))*a^2*d*log(-2/3*(I*
sqrt(9*I*e^7/(a^3*d^2))*a^2*d - 3*(e^3*e^(2*I*d*x + 2*I*c) + e^3)*sqrt(a/(e
^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1
/2*I*c))/e^3) + sqrt(9*I*e^7/(a^3*d^2))*a^2*d*log(-2/3*(-I*sqrt(9*I*e^7/(a^
3*d^2))*a^2*d - 3*(e^3*e^(2*I*d*x + 2*I*c) + e^3)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/e^3) -
sqrt(-9*I*e^7/(a^3*d^2))*a^2*d*log(-2/3*(I*sqrt(-9*I*e^7/(a^3*d^2))*a^2*d -
 3*(e^3*e^(2*I*d*x + 2*I*c) + e^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e
/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/e^3) + sqrt(-9*I*e^7/(
a^3*d^2))*a^2*d*log(-2/3*(-I*sqrt(-9*I*e^7/(a^3*d^2))*a^2*d - 3*(e^3*e^(2*I
*d*x + 2*I*c) + e^3)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x +
 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/e^3))/(a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1817 vs. $2(401) = 802$.

Time = 0.51 (sec) , antiderivative size = 1817, normalized size of antiderivative = 3.43

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -8*(6*sqrt(2)*e^3*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, sqrt(2)*sin(1/2
*d*x + 1/2*c) + 1) + 6*sqrt(2)*e^3*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1
, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 6*sqrt(2)*e^3*arctan2(sqrt(2)*cos(1/
2*d*x + 1/2*c) - 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 6*sqrt(2)*e^3*arcta
n2(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 3
*I*sqrt(2)*e^3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*
sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*I*sq
rt(2)*e^3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(
2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*I*sqrt(2)
*e^3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*co
s(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*I*sqrt(2)*e^3*
log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2
*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 16*e^3*cos(1/2*d*x +
1/2*c) + 16*I*e^3*sin(1/2*d*x + 1/2*c) - 6*(-I*sqrt(2)*e^3*cos(2*d*x + 2*c)
 + sqrt(2)*e^3*sin(2*d*x + 2*c) - I*sqrt(2)*e^3)*arctan2(sqrt(2)*sin(1/2*d*
x + 1/2*c) + sin(d*x + c), sqrt(2)*cos(1/2*d*x + 1/2*c) + cos(d*x + c) + 1)
 - 6*(I*sqrt(2)*e^3*cos(2*d*x + 2*c) - sqrt(2)*e^3*sin(2*d*x + 2*c) + I*sqr
t(2)*e^3)*arctan2(-sqrt(2)*sin(1/2*d*x + 1/2*c) + sin(d*x + c), -sqrt(2)*co
s(1/2*d*x + 1/2*c) + cos(d*x + c) + 1) + 3*(2*sqrt(2)*e^3*arctan2(sqrt(2)*c
os(1/2*d*x + 1/2*c) + 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 2*sqrt(2)*e^3*
arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1
) + 2*sqrt(2)*e^3*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1, sqrt(2)*sin(1/2
*d*x + 1/2*c) + 1) + 2*sqrt(2)*e^3*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1
, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + I*sqrt(2)*e^3*log(2*cos(1/2*d*x + 1/
2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt
(2)*sin(1/2*d*x + 1/2*c) + 2) - I*sqrt(2)*e^3*log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2) + I*sqrt(2)*e^3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*
x + 1/2*c) + 2) - I*sqrt(2)*e^3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2
*c) + 2))*cos(2*d*x + 2*c) + 3*(sqrt(2)*e^3*cos(2*d*x + 2*c) + I*sqrt(2)*e^
3*sin(2*d*x + 2*c) + sqrt(2)*e^3)*log(2*sqrt(2)*sin(d*x + c)*sin(1/2*d*x +
```

```
1/2*c) + 2*(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1)*cos(d*x + c) + cos(d*x + c)^2
 + 2*cos(1/2*d*x + 1/2*c)^2 + sin(d*x + c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2
*sqrt(2)*cos(1/2*d*x + 1/2*c) + 1) - 3*(sqrt(2)*e^3*cos(2*d*x + 2*c) + I*sq
rt(2)*e^3*sin(2*d*x + 2*c) + sqrt(2)*e^3)*log(-2*sqrt(2)*sin(d*x + c)*sin(1
/2*d*x + 1/2*c) - 2*(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1)*cos(d*x + c) + cos(d
*x + c)^2 + 2*cos(1/2*d*x + 1/2*c)^2 + sin(d*x + c)^2 + 2*sin(1/2*d*x + 1/2
*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 1) - 3*(-2*I*sqrt(2)*e^3*arctan2(s
qrt(2)*cos(1/2*d*x + 1/2*c) + 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) - 2*I*sq
rt(2)*e^3*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, -sqrt(2)*sin(1/2*d*x +
1/2*c) + 1) - 2*I*sqrt(2)*e^3*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1, sqr
t(2)*sin(1/2*d*x + 1/2*c) + 1) - 2*I*sqrt(2)*e^3*arctan2(sqrt(2)*cos(1/2*d*
x + 1/2*c) - 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + sqrt(2)*e^3*log(2*cos(
1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*e^3*log(2*cos(1/2*d*x +
 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*s
qrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*e^3*log(2*cos(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*si
n(1/2*d*x + 1/2*c) + 2) - sqrt(2)*e^3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
 + 1/2*c) + 2))*sin(2*d*x + 2*c))*sqrt(a)*sqrt(e)/((-64*I*a^2*cos(2*d*x + 2
*c) + 64*a^2*sin(2*d*x + 2*c) - 64*I*a^2)*d)
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{(e\sec(dx+c))^{\frac{7}{2}}}{(i\,a\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac"
)
[Out] integrate((e*sec(d*x + c))^(7/2)/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}}{(a+a\tan(c+d\,x)\;1\mathrm{i})^{3/2}}\,dx$$

```
[In] int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.424 $\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 365

$$\begin{aligned}\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx &= -\frac{i\sqrt{2}e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{a^{3/2}d}\\ &+\frac{i\sqrt{2}e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{a^{3/2}d}\\ &+\frac{ie^{5/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{\sqrt{2}a^{3/2}d}\\ &-\frac{ie^{5/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{\sqrt{2}a^{3/2}d}\\ &+\frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}}\end{aligned}$$

```
[Out] 1/2*I*e^(5/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(
d*x+c))^(1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))/a^(3/2)/d*2^(1/2)-1/2*I*e^(5/2
)*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2
)+cos(d*x+c)*(a+I*a*tan(d*x+c)))/a^(3/2)/d*2^(1/2)-I*e^(5/2)*arctan(1-2^(1/
2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*2^(1/2)/a
^(3/2)/d+I*e^(5/2)*arctan(1+2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2
)/(e*sec(d*x+c))^(1/2))*2^(1/2)/a^(3/2)/d+4*I*e^2*(e*sec(d*x+c))^(1/2)/a/d/
(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3581, 3576, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{i\sqrt{2}e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{i\sqrt{2}e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{ie^{5/2}\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{\sqrt{2}a^{3/2}d} - \frac{ie^{5/2}\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{\sqrt{2}a^{3/2}d} + \frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-I)*Sqrt[2]*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]
])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]])/(a^(3/2)*d) + (I*Sqrt[2]*e^(5/2)*ArcTan
[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d
*x]])]])/(a^(3/2)*d) + (I*e^(5/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a +
I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c + d*x
])]])/(Sqrt[2]*a^(3/2)*d) - (I*e^(5/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt
[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c
+ d*x])]])/(Sqrt[2]*a^(3/2)*d) + ((4*I)*e^2*Sqrt[e*Sec[c + d*x]])/(a*d*Sqrt[
a + I*a*Tan[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rubi steps

$$\text{integral} = \frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}} - \frac{e^2\int\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx}{a^2}$$

$$= \frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}} + \frac{(4ie^4)\,\text{Subst}\left(\int\frac{x^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{ad}$$

$$= \frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}} - \frac{(2ie^3)\,\text{Subst}\left(\int\frac{a-ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{ad} + \frac{(2ie^3)\,\text{Subst}\left(\int\frac{a+ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{ad}$$

$$= \frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}} + \frac{(ie^2)\,\text{Subst}\left(\int\frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{ad} + \frac{(ie^2)\,\text{Subst}\left(\int\frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{ad} + \frac{(ie^{5/2})\,\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}+2x}{-\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}a^{3/2}d} + \frac{(ie^{5/2})\,\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}-2x}{-\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}a^{3/2}d}$$

$$= \frac{ie^{5/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{\sqrt{2}a^{3/2}d} - \frac{ie^{5/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{\sqrt{2}a^{3/2}d} + \frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}} + \frac{(i\sqrt{2}e^{5/2})\,\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{a^{3/2}d} - \frac{(i\sqrt{2}e^{5/2})\,\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{a^{3/2}d}$$

$$= -\frac{i\sqrt{2}e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{i\sqrt{2}e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{ie^{5/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{\sqrt{2}a^{3/2}d} - \frac{ie^{5/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{\sqrt{2}a^{3/2}d} + \frac{4ie^2\sqrt{e\sec(c+dx)}}{ad\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A] (verified)

Time = 4.03 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.93

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{e(e\sec(c+dx))^{3/2}(\cos(dx)+i\sin(dx))^2\left(\cos(dx)(4i\cos(c)-4\sin(c))+4(c\right.}{}$$

```
[In] Integrate[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (e*(e*Sec[c + d*x])^(3/2)*(Cos[d*x] + I*Sin[d*x])^2*(Cos[d*x]*((4*I)*Cos[c]
 - 4*Sin[c]) + 4*(Cos[c] + I*Sin[c])*Sin[d*x] + (2*(ArcTanh[(Sqrt[1 - I*Cos
[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I
+ Tan[(d*x)/2]])]*Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[1 + I*Cos[c] - Sin[c]]
- ArcTanh[(Sqrt[1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 + I
*Cos[c] + Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt
[-1 + I*Cos[c] + Sin[c]])*(Cos[2*c] + I*Sin[2*c])*Sqrt[I + Tan[(d*x)/2]])/(
Sqrt[1 + Cos[2*c] + I*Sin[2*c]]*Sqrt[I - Tan[(d*x)/2]])))/(d*(a + I*a*Tan[c
 + d*x])^(3/2))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1089 vs. 2(285) = 570.

Time = 16.18 (sec) , antiderivative size = 1090, normalized size of antiderivative = 2.99

method	result	size
default	Expression too large to display	1090

```
[In] int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/2/d*e^2*(e*sec(d*x+c))^(1/2)/(tan(d*x+c)-I)/a/(a*(1+I*tan(d*x+c)))^(1/2)/
(1/(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)+1)*(8*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x
+c)+2*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+
1))^(1/2))*cos(d*x+c)-2*sin(d*x+c)*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(
cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))-2*arctanh(1/2*(cos(d*x+c)+sin(d*x+c
)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-2*sin(d*x+c)*arcta
nh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))+8
*(1/(cos(d*x+c)+1))^(1/2)+2*I*cos(d*x+c)*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)
+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))+arctanh(1/2*(-cos(d*x+c)+sin(d
*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))-tan(d*x+c)*arctanh(1/2*(-
cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))-arctanh(1
/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))-tan(d
*x+c)*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1
))^(1/2))-I*sec(d*x+c)*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1
)/(1/(cos(d*x+c)+1))^(1/2))+I*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d
*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))+I*tan(d*x+c)*arctanh(1/2*(-cos(d*x+c)+si
n(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))-I*tan(d*x+c)*arctanh(1
/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))+2*I*a
rctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/
2))*sin(d*x+c)-sec(d*x+c)*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c
)+1)/(1/(cos(d*x+c)+1))^(1/2))+sec(d*x+c)*arctanh(1/2*(cos(d*x+c)+sin(d*x+c
)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))+8*I*tan(d*x+c)*(1/(cos(d*x+c)
+1))^(1/2)+8*I*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)-2*I*arctanh(1/2*(cos(d*x
+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-I*sec
(d*x+c)*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)
+1))^(1/2))+2*I*cos(d*x+c)*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+
c)+1)/(1/(cos(d*x+c)+1))^(1/2))+I*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(co
s(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 539, normalized size of antiderivative = 1.48

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx =$$

$$-\frac{\left(a^2 d\sqrt{\frac{4i\,e^5}{a^3d^2}}e^{(i\,dx+i\,c)}\log\left(\frac{a^2 d\sqrt{\frac{4i\,e^5}{a^3d^2}}+2\left(e^2e^{(2i\,dx+2i\,c)}+e^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{e^2}\right)-a^2 d\sqrt{\frac{4i\,e^5}{a^3d^2}}e^{(i\,dx}}{}$$

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="frica
s")
```

```
[Out] -1/2*(a^2*d*sqrt(4*I*e^5/(a^3*d^2))*e^(I*d*x + I*c)*log((a^2*d*sqrt(4*I*e^5
/(a^3*d^2)) + 2*(e^2*e^(2*I*d*x + 2*I*c) + e^2)*sqrt(a/(e^(2*I*d*x + 2*I*c)
 + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/e^2) - a^
2*d*sqrt(4*I*e^5/(a^3*d^2))*e^(I*d*x + I*c)*log(-(a^2*d*sqrt(4*I*e^5/(a^3*d
^2)) - 2*(e^2*e^(2*I*d*x + 2*I*c) + e^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*
sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/e^2) - a^2*d*sqr
t(-4*I*e^5/(a^3*d^2))*e^(I*d*x + I*c)*log((a^2*d*sqrt(-4*I*e^5/(a^3*d^2)) +
 2*(e^2*e^(2*I*d*x + 2*I*c) + e^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e
/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/e^2) + a^2*d*sqrt(-4*I
*e^5/(a^3*d^2))*e^(I*d*x + I*c)*log(-(a^2*d*sqrt(-4*I*e^5/(a^3*d^2)) - 2*(e
^2*e^(2*I*d*x + 2*I*c) + e^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(
2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/e^2) + 8*(-I*e^2*e^(2*I*d*x
 + 2*I*c) - I*e^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2
*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-I*d*x - I*c)/(a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 778 vs. 2(273) = 546.

Time = 0.47 (sec) , antiderivative size = 778, normalized size of antiderivative = 2.13

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -1/4*(2*I*sqrt(2)*e^2*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, sqrt(2)*sin
(1/2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*e^2*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*
c) + 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*e^2*arctan2(sqrt(2
)*cos(1/2*d*x + 1/2*c) - 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)
*e^2*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1, -sqrt(2)*sin(1/2*d*x + 1/2*c
```

```
) + 1) + 2*sqrt(2)*e^2*arctan2(sqrt(2)*sin(1/2*d*x + 1/2*c) + sin(d*x + c),
 sqrt(2)*cos(1/2*d*x + 1/2*c) + cos(d*x + c) + 1) - 2*sqrt(2)*e^2*arctan2(-
sqrt(2)*sin(1/2*d*x + 1/2*c) + sin(d*x + c), -sqrt(2)*cos(1/2*d*x + 1/2*c)
+ cos(d*x + c) + 1) + I*sqrt(2)*e^2*log(2*sqrt(2)*sin(d*x + c)*sin(1/2*d*x
+ 1/2*c) + 2*(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1)*cos(d*x + c) + cos(d*x + c)
^2 + 2*cos(1/2*d*x + 1/2*c)^2 + sin(d*x + c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 1) - I*sqrt(2)*e^2*log(-2*sqrt(2)*sin(d*x
 + c)*sin(1/2*d*x + 1/2*c) - 2*(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1)*cos(d*x +
 c) + cos(d*x + c)^2 + 2*cos(1/2*d*x + 1/2*c)^2 + sin(d*x + c)^2 + 2*sin(1/
2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 1) + sqrt(2)*e^2*log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*e^2*log(2*cos(1/2*d
*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) -
 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*e^2*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*e^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2
*d*x + 1/2*c) + 2) - 16*I*e^2*cos(1/2*d*x + 1/2*c) - 16*e^2*sin(1/2*d*x + 1
/2*c))*sqrt(e)/(a^(3/2)*d)
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx=\int \frac{(e\sec\left(dx+c\right))^{\frac{5}{2}}}{(i\,a\tan\left(dx+c\right)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac"
)
[Out] integrate((e*sec(d*x + c))^(5/2)/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx=\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}}{(a+a\tan\left(c+d\,x\right)\,1\mathrm{i})^{3/2}}\,dx$$

```
[In] int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.425 $\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 38

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2i(e\sec(c+dx))^{3/2}}{3d(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] 2/3*I*(e*sec(d*x+c))^(3/2)/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {3569}

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2i(e\sec(c+dx))^{3/2}}{3d(a+ia\tan(c+dx))^{3/2}}$$

```
[In] Int[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((2*I)/3)*(e*Sec[c + d*x])^(3/2))/(d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\text{integral} = \frac{2i(e\sec(c+dx))^{3/2}}{3d(a+ia\tan(c+dx))^{3/2}}$$

Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx=\frac{2i(e\sec(c+dx))^{3/2}}{3d(a+ia\tan(c+dx))^{3/2}}$$

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^(3/2),x]
[Out] (((2*I)/3)*(e*Sec[c + d*x])^(3/2))/(d*(a + I*a*Tan[c + d*x])^(3/2))
```

Maple [A] (verified)

Time = 11.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.45

method	result	size
default	$\frac{2i\sec(dx+c)e\sqrt{e\sec(dx+c)}}{3d(1+i\tan(dx+c))a\sqrt{a(1+i\tan(dx+c))}}$	55
risch	$\frac{2ie\sqrt{\frac{e\,\mathrm{e}^{i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,\mathrm{e}^{-i(dx+c)}}{3a\sqrt{\frac{a\,\mathrm{e}^{2i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}$	72

```
[In] int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/3*I/d*sec(d*x+c)*e*(e*sec(d*x+c))^(1/2)/(1+I*tan(d*x+c))/a/(a*(1+I*tan(d*
x+c)))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(28) = 56.

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx=\frac{2\left(i\,ee^{(2i\,dx+2i\,c)}+i\,e\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{3}{2}i\,dx-\frac{3}{2}i\,c\right)}}{3\,a^2d}$$

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="frica
s")
[Out] 2/3*(I*e*e^(2*I*d*x + 2*I*c) + I*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(
e/(e^(2*I*d*x + 2*I*c) + 1))*e^(-3/2*I*d*x - 3/2*I*c)/(a^2*d)
```

Sympy [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{(e\sec{(c+dx)})^{\frac{3}{2}}}{(ia(\tan{(c+dx)}-i))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral((e*sec(c + d*x))**(3/2)/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(28) = 56$.

Time = 0.38 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.00

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2i\,e^{\frac{3}{2}}\left(-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{3}{2}}}{3\,a^{\frac{3}{2}}d\left(-\frac{2i\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{3}{2}}}$$

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxim
a")
```

```
[Out] 2/3*I*e^(3/2)*(-sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(3/2)/(a^(3/2)*d*(
-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2
- 1)^(3/2))
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{(e\sec{(dx+c)})^{\frac{3}{2}}}{(i\,a\tan{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((e*sec(d*x + c))^(3/2)/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{3/2}}\,dx$$

```
[In] int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.426 $\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 80

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2i\sqrt{e\sec(c+dx)}}{5d(a+ia\tan(c+dx))^{3/2}} + \frac{4i\sqrt{e\sec(c+dx)}}{5ad\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 4/5*I*(e*sec(d*x+c))^(1/2)/a/d/(a+I*a*tan(d*x+c))^(1/2)+2/5*I*(e*sec(d*x+c)
)^(1/2)/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3583, 3569}

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{4i\sqrt{e\sec(c+dx)}}{5ad\sqrt{a+ia\tan(c+dx)}} + \frac{2i\sqrt{e\sec(c+dx)}}{5d(a+ia\tan(c+dx))^{3/2}}$$

```
[In] Int[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (((2*I)/5)*Sqrt[e*Sec[c + d*x]])/(d*(a + I*a*Tan[c + d*x])^(3/2)) + (((4*I)
/5)*Sqrt[e*Sec[c + d*x]])/(a*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2i\sqrt{e\sec(c+dx)}}{5d(a+ia\tan(c+dx))^{3/2}}+\frac{2\int\frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{5a}\\ &= \frac{2i\sqrt{e\sec(c+dx)}}{5d(a+ia\tan(c+dx))^{3/2}}+\frac{4i\sqrt{e\sec(c+dx)}}{5ad\sqrt{a+ia\tan(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.79

$$\int\frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx=\frac{2\sqrt{e\sec(c+dx)}(3+2i\tan(c+dx))}{5ad(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^(3/2),x]
[Out] (2*Sqrt[e*Sec[c + d*x]]*(3 + (2*I)*Tan[c + d*x]))/(5*a*d*(-I + Tan[c + d*x]
)*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] (verified)

Time = 14.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{2i\sqrt{e\sec(dx+c)}\,(2i\tan(dx+c)+3)}{5d(1+i\tan(dx+c))a\sqrt{a(1+i\tan(dx+c))}}$	59

```
[In] int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/5*I/d*(e*sec(d*x+c))^(1/2)/(1+I*tan(d*x+c))/a/(a*(1+I*tan(d*x+c)))^(1/2)*
(2*I*tan(d*x+c)+3)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(5i\,e^{(4i\,dx+4i\,c)}+6i\,e^{(2i\,dx+2i\,c)}+i\right)e^{\left(-\frac{5}{2}i\,dx-\frac{5}{2}i\,c\right)}}{5\,a^2d}$$

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/5*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(5*
I*e^(4*I*d*x + 4*I*c) + 6*I*e^(2*I*d*x + 2*I*c) + I)*e^(-5/2*I*d*x - 5/2*I*
c)/(a^2*d)
```

Sympy [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{e\sec\left(c+dx\right)}}{\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(sqrt(e*sec(c + d*x))/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{e}\left(i\,\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5i\,\cos\left(\frac{1}{5}\,\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\right)\right)}{5\,a^{\frac{3}{2}}}$$

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxim
a")
```

```
[Out] 1/5*sqrt(e)*(I*cos(5/2*d*x + 5/2*c) + 5*I*cos(1/5*arctan2(sin(5/2*d*x + 5/2
*c), cos(5/2*d*x + 5/2*c))) + sin(5/2*d*x + 5/2*c) + 5*sin(1/5*arctan2(sin(
5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))/(a^(3/2)*d)
```

Giac [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{e\sec\left(dx+c\right)}}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(e*sec(d*x + c))/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 4.84 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\left(\cos\left(2\,c+2\,d\,x\right)\,1\mathrm{i}+\sin\left(2\,c+2\,d\,x\right)+5\mathrm{i}\right)}{5\,a\,d\,\sqrt{\frac{a\,\left(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i}\right)}{\cos(2\,c+2\,d\,x)+1}}}$$

```
[In] int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^(3/2),x)
```

```
[Out] ((e/cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*1i + sin(2*c + 2*d*x) + 5i))/(5*a
*d*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1)
)^(1/2))
```

3.427 $\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 121

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2i}{7d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}} + \frac{8i}{21ad\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{16i\sqrt{a+ia\tan(c+dx)}}{21a^2d\sqrt{e\sec(c+dx)}}$$

```
[Out] 8/21*I/a/d/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-16/21*I*(a+I*a*tan
(d*x+c))^(1/2)/a^2/d/(e*sec(d*x+c))^(1/2)+2/7*I/d/(e*sec(d*x+c))^(1/2)/(a+I
*a*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3583, 3569}

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{16i\sqrt{a+ia\tan(c+dx)}}{21a^2d\sqrt{e\sec(c+dx)}} + \frac{8i}{21ad\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{2i}{7d(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}}$$

```
[In] Int[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

```
[Out] ((2*I)/7)/(d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2)) + ((8*I)/21
)/(a*d*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((16*I)/21)*Sqrt
[a + I*a*Tan[c + d*x]])/(a^2*d*Sqrt[e*Sec[c + d*x]])
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2i}{7d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}+\frac{4\int\frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx}{7a}\\
&= \frac{2i}{7d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}\\
&\quad+\frac{8i}{21ad\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{8\int\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx}{21a^2}\\
&= \frac{2i}{7d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}\\
&\quad+\frac{8i}{21ad\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{16i\sqrt{a+ia\tan(c+dx)}}{21a^2d\sqrt{e\sec(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.69

$$\int\frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx=\\-\frac{\sec^2(c+dx)(-7+9\cos(2(c+dx))+12i\sin(2(c+dx)))}{21ad\sqrt{e\sec(c+dx)}(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2)),x]
[Out] -1/21*(Sec[c + d*x]^2*(-7 + 9*Cos[2*(c + d*x)] + (12*I)*Sin[2*(c + d*x)]))/
(a*d*Sqrt[e*Sec[c + d*x]]*(-I + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] (verified)

Time = 9.74 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.57

method	result	size
default	$-\frac{2\left(9 i-12 \tan \left(d x+c\right)-8 i \left(\sec ^{2}\left(d x+c\right)\right)\right)}{21 d \left(1+i \tan \left(d x+c\right)\right) \sqrt{e \sec \left(d x+c\right)}\, \sqrt{a \left(1+i \tan \left(d x+c\right)\right)}\, a}$	69

```
[In] int(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -2/21/d/(1+I*tan(d*x+c))/(e*sec(d*x+c))^(1/2)/(a*(1+I*tan(d*x+c)))^(1/2)/a*
(9*I-12*tan(d*x+c)-8*I*sec(d*x+c)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-21i\,e^{(6i\,dx+6i\,c)} - 7i\,e^{(4i\,dx+4i\,c)} + \right.}{42\,a^2de}$$

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fri
cas")
```

```
[Out] 1/42*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(-
21*I*e^(6*I*d*x + 6*I*c) - 7*I*e^(4*I*d*x + 4*I*c) + 17*I*e^(2*I*d*x + 2*I*
c) + 3*I)*e^(-7/2*I*d*x - 7/2*I*c)/(a^2*d*e)
```

Sympy [F]

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{1}{\sqrt{e\sec{\left(c+dx \right)}}\left(ia\left(\tan{\left(c+dx \right)}-i\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(1/(sqrt(e*sec(c + d*x))*(I*a*(tan(c + d*x) - I))**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{3i\,\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+14i\,\cos\left(\frac{3}{7}\,\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\right.\right.}{}$$

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="max
ima")
```

```
[Out] 1/42*(3*I*cos(7/2*d*x + 7/2*c) + 14*I*cos(3/7*arctan2(sin(7/2*d*x + 7/2*c),
 cos(7/2*d*x + 7/2*c))) - 21*I*cos(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/
2*d*x + 7/2*c))) + 3*sin(7/2*d*x + 7/2*c) + 14*sin(3/7*arctan2(sin(7/2*d*x
+ 7/2*c), cos(7/2*d*x + 7/2*c))) + 21*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c),
 cos(7/2*d*x + 7/2*c))))/(a^(3/2)*d*sqrt(e))
```

Giac [F]

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{1}{\sqrt{e\sec\left(dx+c\right)}\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^(3/2)), x)
```

Mupad [B] (verification not implemented)

Time = 4.87 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\left(35\,\sin\left(c+d\,x\right)+3\,\sin\left(3\,c+3\,d\,x\right)-\cos\left(c+d\right.\right.}{42\,a\,d\,e\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)}{\cos(2\,c+2\,d\,x)+1}}}$$

```
[In] int(1/((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

```
[Out] ((e/cos(c + d*x))^(1/2)*(35*sin(c + d*x) - cos(c + d*x)*7i + cos(3*c + 3*d*
x)*3i + 3*sin(3*c + 3*d*x)))/(42*a*d*e*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*
d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))
```

3.428 $\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 165

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2i}{9d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}} + \frac{4i}{15ad(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} + \frac{32i\sqrt{e\sec(c+dx)}}{45ade^2\sqrt{a+ia\tan(c+dx)}} - \frac{16i\sqrt{a+ia\tan(c+dx)}}{45a^2d(e\sec(c+dx))^{3/2}}$$

```
[Out] 4/15*I/a/d/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2)+32/45*I*(e*sec(d*x
+c))^(1/2)/a/d/e^2/(a+I*a*tan(d*x+c))^(1/2)-16/45*I*(a+I*a*tan(d*x+c))^(1/2
)/a^2/d/(e*sec(d*x+c))^(3/2)+2/9*I/d/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c)
)^(3/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3583, 3578, 3569}

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{16i\sqrt{a+ia\tan(c+dx)}}{45a^2d(e\sec(c+dx))^{3/2}} + \frac{32i\sqrt{e\sec(c+dx)}}{45ade^2\sqrt{a+ia\tan(c+dx)}} + \frac{4i}{15ad\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}} + \frac{2i}{9d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

```
[In] Int[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

```
[Out] ((2*I)/9)/(d*(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(3/2)) + ((4*I)/
15)/(a*d*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (((32*I)/45)*
Sqrt[e*Sec[c + d*x]])/(a*d*e^2*Sqrt[a + I*a*Tan[c + d*x]]) - (((16*I)/45)*S
qrt[a + I*a*Tan[c + d*x]])/(a^2*d*(e*Sec[c + d*x])^(3/2))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2i}{9d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}+\frac{2\int\frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx}{3a}\\
&= \frac{2i}{9d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}\\
&\quad+\frac{4i}{15ad(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}+\frac{8\int\frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx}{15a^2}\\
&= \frac{2i}{9d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}\\
&\quad+\frac{4i}{15ad(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\\
&\quad-\frac{16i\sqrt{a+ia\tan(c+dx)}}{45a^2d(e\sec(c+dx))^{3/2}}+\frac{16\int\frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{45ae^2}
\end{aligned}$$

$$= \frac{2i}{9d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}} + \frac{4i}{15ad(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} + \frac{32i\sqrt{e\sec(c+dx)}}{45ade^2\sqrt{a+ia\tan(c+dx)}} - \frac{16i\sqrt{a+ia\tan(c+dx)}}{45a^2d(e\sec(c+dx))^{3/2}}$$

Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.61

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{\sec^3(c+dx)(-81\cos(c+dx)+5\cos(3(c+dx))-54i\sin(c+dx)+10i\sin(3(c+dx)))}{90ad(e\sec(c+dx))^{3/2}(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

```
[Out] -1/90*(Sec[c + d*x]^3*(-81*Cos[c + d*x] + 5*Cos[3*(c + d*x)] - (54*I)*Sin[c
 + d*x] + (10*I)*Sin[3*(c + d*x)]))/(a*d*(e*Sec[c + d*x])^(3/2)*(-I + Tan[c
 + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] (verified)

Time = 9.50 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.55

method	result	size
default	$-\frac{2(5i\cos(dx+c)-10\sin(dx+c)-24i\sec(dx+c)+16\sec(dx+c)\tan(dx+c))}{45d(1+i\tan(dx+c))\sqrt{a(1+i\tan(dx+c))}\,\sqrt{e\sec(dx+c)}\,ae}$	91

```
[In] int(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -2/45/d/(1+I*tan(d*x+c))/(a*(1+I*tan(d*x+c)))^(1/2)/(e*sec(d*x+c))^(1/2)/a/
e*(5*I*cos(d*x+c)-10*sin(d*x+c)-24*I*sec(d*x+c)+16*sec(d*x+c)*tan(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.61

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-15i\,e^{(8i\,dx+8i\,c)}+120i\,e^{(6i\,dx+\cdots}\right.}{180\,a^{2}\cdots}$$

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fri
cas")
```

```
[Out] 1/180*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(
-15*I*e^(8*I*d*x + 8*I*c) + 120*I*e^(6*I*d*x + 6*I*c) + 162*I*e^(4*I*d*x +
4*I*c) + 32*I*e^(2*I*d*x + 2*I*c) + 5*I)*e^(-9/2*I*d*x - 9/2*I*c)/(a^2*d*e^
2)
```

Sympy [F]

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{1}{(e\sec{(c+dx)})^{\frac{3}{2}}\,(ia\,(\tan{(c+dx)}-i))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**(3/2),x)
```

```
[Out] Integral(1/((e*sec(c + d*x))**(3/2)*(I*a*(tan(c + d*x) - I))**(3/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.08

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{5i\,\cos\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)+27i\,\cos\left(\frac{5}{9}\,\arctan\left(\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right),\cdots\right.\right.}{\cdots}$$

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="max
ima")
```

```
[Out] 1/180*(5*I*cos(9/2*d*x + 9/2*c) + 27*I*cos(5/9*arctan2(sin(9/2*d*x + 9/2*c)
, cos(9/2*d*x + 9/2*c))) - 15*I*cos(1/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9
/2*d*x + 9/2*c))) + 135*I*cos(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
 + 9/2*c))) + 5*sin(9/2*d*x + 9/2*c) + 27*sin(5/9*arctan2(sin(9/2*d*x + 9/2
*c), cos(9/2*d*x + 9/2*c))) + 15*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c), cos(
9/2*d*x + 9/2*c))) + 135*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
+ 9/2*c))))/(a^(3/2)*d*e^(3/2))
```

Giac [F]

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(e\sec\left(dx+c\right)\right)^{\frac{3}{2}}\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="gia
c")
[Out] integrate(1/((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^(3/2)), x)
```

Mupad [B] (verification not implemented)

Time = 5.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.68

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\left(\cos\left(2\,c+2\,d\,x\right)\,12\mathrm{i}+\cos\left(4\,c+4\,d\,x\right)\,5\mathrm{i}+42\,\mathrm{si}\right.}{180\,a\,d\,e^2\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\mathrm{si}}{\cos(2\,c+2\,d}}}$$

```
[In] int(1/((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(3/2)),x)
[Out] ((e/cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*12i + cos(4*c + 4*d*x)*5i + 42*si
n(2*c + 2*d*x) + 5*sin(4*c + 4*d*x) + 135i))/(180*a*d*e^2*((a*(cos(2*c + 2*
d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))
```

3.429 $\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 209

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{2i}{11d(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}} + \frac{16i}{77ad(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}} + \frac{128i}{385ade^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{96i\sqrt{a+ia\tan(c+dx)}}{385a^2d(e\sec(c+dx))^{5/2}} - \frac{256i\sqrt{a+ia\tan(c+dx)}}{385a^2de^2\sqrt{e\sec(c+dx)}}$$

```
[Out] 16/77*I/a/d/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2)+128/385*I/a/d/e^2
/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-96/385*I*(a+I*a*tan(d*x+c))^
(1/2)/a^2/d/(e*sec(d*x+c))^(5/2)-256/385*I*(a+I*a*tan(d*x+c))^(1/2)/a^2/d/e
^2/(e*sec(d*x+c))^(1/2)+2/11*I/d/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3
/2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3583, 3578, 3569}

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{256i\sqrt{a+ia\tan(c+dx)}}{385a^2de^2\sqrt{e\sec(c+dx)}} - \frac{96i\sqrt{a+ia\tan(c+dx)}}{385a^2d(e\sec(c+dx))^{5/2}} + \frac{128i}{385ade^2\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{16i}{77ad\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{5/2}} + \frac{2i}{11d(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{5/2}}$$

```
[In] Int[1/((e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

```
[Out] ((2*I)/11)/(d*(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^(3/2)) + ((16*I
)/77)/(a*d*(e*Sec[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]) + ((128*I)/38
5)/(a*d*e^2*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((96*I)/385
)*Sqrt[a + I*a*Tan[c + d*x]])/(a^2*d*(e*Sec[c + d*x])^(5/2)) - (((256*I)/38
5)*Sqrt[a + I*a*Tan[c + d*x]])/(a^2*d*e^2*Sqrt[e*Sec[c + d*x]])
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
```

]

Rubi steps

$$\text{integral} = \frac{2i}{11d(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}} + \frac{8\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx}{11a}$$

$$= \frac{2i}{11d(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}} + \frac{16i}{77ad(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}} + \frac{48\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx}{77a^2}$$

$$= \frac{2i}{11d(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}} + \frac{16i}{77ad(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}} - \frac{96i\sqrt{a+ia\tan(c+dx)}}{385a^2d(e\sec(c+dx))^{5/2}} + \frac{192\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx}{385ae^2}$$

$$= \frac{2i}{11d(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}} + \frac{16i}{77ad(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}} + \frac{128i}{385ade^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{96i\sqrt{a+ia\tan(c+dx)}}{385a^2d(e\sec(c+dx))^{5/2}} + \frac{128\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx}{385a^2e^2}$$

$$= \frac{2i}{11d(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}} + \frac{16i}{77ad(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}} + \frac{128i}{385ade^2\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{96i\sqrt{a+ia\tan(c+dx)}}{385a^2d(e\sec(c+dx))^{5/2}} - \frac{256i\sqrt{a+ia\tan(c+dx)}}{385a^2de^2\sqrt{e\sec(c+dx)}}$$

Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.48

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{(e\sec(c+dx))^{3/2}(-385+660\cos(2(c+dx))+21\cos(4(c+dx))+880i\sin(2(c+dx))+56i\sin(4(c+dx)}{1540ade^4(-i+\tan(c+dx))\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[1/((e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^(3/2)),x]
```

```
[Out] -1/1540*((e*Sec[c + d*x])^(3/2)*(-385 + 660*Cos[2*(c + d*x)] + 21*Cos[4*(c
+ d*x)] + (880*I)*Sin[2*(c + d*x)] + (56*I)*Sin[4*(c + d*x)]))/(a*d*e^4*(-I
 + Tan[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] (verified)

Time = 9.83 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.46

method	result	size
default	$-\frac{2\left(21i\left(\cos^2(dx+c)\right)-56\sin(dx+c)\cos(dx+c)+144i-192\tan(dx+c)-128i\left(\sec^2(dx+c)\right)\right)}{385d\sqrt{a(1+i\tan(dx+c))}\,\sqrt{e\sec(dx+c)}\,(1+i\tan(dx+c))a\,e^2}$	97

```
[In] int(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -2/385/d/(a*(1+I*tan(d*x+c)))^(1/2)/(e*sec(d*x+c))^(1/2)/(1+I*tan(d*x+c))/a
/e^2*(21*I*cos(d*x+c)^2-56*sin(d*x+c)*cos(d*x+c)+144*I-192*tan(d*x+c)-128*I
*sec(d*x+c)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.53

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-77i\,e^{(10i\,dx+10i\,c)}-1617i\,e^{(8i\,dx}\right.}{}$$

```
[In] integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fri
cas")
```

```
[Out] 1/3080*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*
(-77*I*e^(10*I*d*x + 10*I*c) - 1617*I*e^(8*I*d*x + 8*I*c) - 770*I*e^(6*I*d*
x + 6*I*c) + 990*I*e^(4*I*d*x + 4*I*c) + 255*I*e^(2*I*d*x + 2*I*c) + 35*I)*
e^(-11/2*I*d*x - 11/2*I*c)/(a^2*d*e^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**(3/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.76 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.08

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{35i\cos\left(\frac{11}{2}dx+\frac{11}{2}c\right)+220i\cos\left(\frac{7}{11}\arctan\left(\sin\left(\frac{11}{2}dx+\right.\right.\right.}{}$$

```
[In] integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="max
ima")
[Out] 1/3080*(35*I*cos(11/2*d*x + 11/2*c) + 220*I*cos(7/11*arctan2(sin(11/2*d*x +
 11/2*c), cos(11/2*d*x + 11/2*c))) - 77*I*cos(5/11*arctan2(sin(11/2*d*x + 1
1/2*c), cos(11/2*d*x + 11/2*c))) + 770*I*cos(3/11*arctan2(sin(11/2*d*x + 11
/2*c), cos(11/2*d*x + 11/2*c))) - 1540*I*cos(1/11*arctan2(sin(11/2*d*x + 11
/2*c), cos(11/2*d*x + 11/2*c))) + 35*sin(11/2*d*x + 11/2*c) + 220*sin(7/11*
arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 77*sin(5/11*arct
an2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 770*sin(3/11*arctan2
(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 1540*sin(1/11*arctan2(s
in(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))))/(a^(3/2)*d*e^(5/2))
```

Giac [F]

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{1}{(e\sec(dx+c))^{\frac{5}{2}}(ia\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="gia
c")
[Out] integrate(1/((e*sec(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a)^(3/2)), x)
```

Mupad [B] (verification not implemented)

Time = 5.43 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.61

$$\int \frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{3/2}}\,dx=\frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\left(2310\,\sin\left(c+d\,x\right)+297\,\sin\left(3\,c+3\,d\,x\right)+35\,\mathrm{si}\right.}{3080\,a\,d\,e^3}$$

```
[In] int(1/((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^(3/2)),x)
```

```
[Out] ((e/cos(c + d*x))^(1/2)*(2310*sin(c + d*x) - cos(c + d*x)*770i + cos(3*c +
3*d*x)*143i + cos(5*c + 5*d*x)*35i + 297*sin(3*c + 3*d*x) + 35*sin(5*c + 5*
d*x)))/(3080*a*d*e^3*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos
(2*c + 2*d*x) + 1))^(1/2))
```

3.430 $\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 411

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{5ie^{9/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}a^{5/2}d} + \frac{5ie^{9/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}a^{5/2}d} + \frac{5ie^{9/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{2\sqrt{2}a^{5/2}d} - \frac{5ie^{9/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{2\sqrt{2}a^{5/2}d} + \frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}} + \frac{5ie^4\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{a^3d}$$

```
[Out] -5/2*I*e^(9/2)*arctan(1-2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e
*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)+5/2*I*e^(9/2)*arctan(1+2^(1/2)*e^(1/2
)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)+
5/4*I*e^(9/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(
d*x+c))^(1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))/a^(5/2)/d*2^(1/2)-5/4*I*e^(9/2
)*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2
)+cos(d*x+c)*(a+I*a*tan(d*x+c)))/a^(5/2)/d*2^(1/2)+5*I*e^4*(e*sec(d*x+c))^(
1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^3/d+4*I*e^2*(e*sec(d*x+c))^(5/2)/a/d/(a+I*a
*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3581, 3582, 3576, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{5ie^{9/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}a^{5/2}d} + \frac{5ie^{9/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}a^{5/2}d} + \frac{5ie^{9/2}\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{2\sqrt{2}a^{5/2}d} - \frac{5ie^{9/2}\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{2\sqrt{2}a^{5/2}d} + \frac{5ie^4\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}}{a^3d} + \frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}}$$

```
[In] Int[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-5*I)*e^(9/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sq
rt[a]*Sqrt[e*Sec[c + d*x]])])/(Sqrt[2]*a^(5/2)*d) + ((5*I)*e^(9/2)*ArcTan[1
 + (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x
]])])/(Sqrt[2]*a^(5/2)*d) + (((5*I)/2)*e^(9/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqr
t[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I
*a*Tan[c + d*x])])/(Sqrt[2]*a^(5/2)*d) - (((5*I)/2)*e^(9/2)*Log[a + (Sqrt[2
]*Sqrt[a]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c
+ d*x]*(a + I*a*Tan[c + d*x])])/(Sqrt[2]*a^(5/2)*d) + ((4*I)*e^2*(e*Sec[c +
 d*x])^(5/2))/(a*d*(a + I*a*Tan[c + d*x])^(3/2)) + ((5*I)*e^4*Sqrt[e*Sec[c
+ d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/(a^3*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\text{integral} = \frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}} - \frac{(5e^2)\int\frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{a^2}$$

$$= \frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}} + \frac{5ie^4\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{a^3d} - \frac{(5e^4)\int\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx}{2a^3}$$

$$= \frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}} + \frac{5ie^4\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{a^3d} + \frac{(10ie^6)\,\text{Subst}\left(\int\frac{x^2}{a^2+e^2x^4}\,dx,x,\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{a^2d}$$

$$= \frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}} + \frac{5ie^4\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{a^3d} - \frac{(5ie^5)\,\text{Subst}\left(\int\frac{a-ex^2}{a^2+e^2x^4}\,dx,x,\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{a^2d} + \frac{(5ie^5)\,\text{Subst}\left(\int\frac{a+ex^2}{a^2+e^2x^4}\,dx,x,\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{a^2d}$$

$$= \frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}} + \frac{5ie^4\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{a^3d} + \frac{(5ie^4)\,\text{Subst}\left(\int\frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx,x,\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{2a^2d} + \frac{(5ie^4)\,\text{Subst}\left(\int\frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx,x,\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{2a^2d} + \frac{(5ie^{9/2})\,\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}+2x}{-\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx,x,\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{2\sqrt{2}a^{5/2}d} + \frac{(5ie^{9/2})\,\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}-2x}{-\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx,x,\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{2\sqrt{2}a^{5/2}d}$$

$$= \frac{5ie^{9/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{2\sqrt{2}a^{5/2}d}$$
$$-\frac{5ie^{9/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{2\sqrt{2}a^{5/2}d}$$
$$+\frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}}+\frac{5ie^4\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{a^3d}$$
$$+\frac{(5ie^{9/2})\,\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}a^{5/2}d}$$
$$-\frac{(5ie^{9/2})\,\mathrm{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}a^{5/2}d}$$

$$= -\frac{5ie^{9/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}a^{5/2}d}+\frac{5ie^{9/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}a^{5/2}d}$$
$$+\frac{5ie^{9/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{2\sqrt{2}a^{5/2}d}$$
$$-\frac{5ie^{9/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{2\sqrt{2}a^{5/2}d}$$
$$+\frac{4ie^2(e\sec(c+dx))^{5/2}}{ad(a+ia\tan(c+dx))^{3/2}}+\frac{5ie^4\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}{a^3d}$$

Mathematica [A] (verified)

Time = 5.50 (sec) , antiderivative size = 370, normalized size of antiderivative = 0.90

$$\int\frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{e^2(e\sec(c+dx))^{5/2}(\cos(dx)+i\sin(dx))^3\left(\cos(dx)(8i\cos(2c)-8\sin(2c))\right.}{}$$

```
[In] Integrate[(e*Sec[c + d*x])^(9/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (e^2*(e*Sec[c + d*x])^(5/2)*(Cos[d*x] + I*Sin[d*x])^3*(Cos[d*x]*((8*I)*Cos[
2*c] - 8*Sin[2*c]) + Sec[c + d*x]*(I*Cos[3*c] - Sin[3*c]) + 8*(Cos[2*c] + I
*Sin[2*c])*Sin[d*x] + (5*(ArcTanh[(Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan
[(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1
- I*Cos[c] - Sin[c]]*Sqrt[1 + I*Cos[c] - Sin[c]] - ArcTanh[(Sqrt[1 + I*Cos[
c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[I +
 Tan[(d*x)/2]])]*Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]])*
(Cos[3*c] + I*Sin[3*c])*Sqrt[I + Tan[(d*x)/2]])/(Sqrt[1 + Cos[2*c] + I*Sin[
2*c]]*Sqrt[I - Tan[(d*x)/2]])))/(d*(a + I*a*Tan[c + d*x])^(5/2))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1611 vs. 2(321) = 642.

Time = 15.10 (sec) , antiderivative size = 1612, normalized size of antiderivative = 3.92

method	result	size
default	Expression too large to display	1612

```
[In] int((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*I/d*(e*sec(d*x+c))^(1/2)*e^4/(tan(d*x+c)-I)^2/a^2/(a*(1+I*tan(d*x+c)))
^(1/2)/(1/(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)+1)*(-20*arctanh(1/2*(cos(d*x+c)
+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)+80*(1/(c
os(d*x+c)+1))^(1/2)*cos(d*x+c)+10*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(c
os(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))+20*arctanh(1/2*(-cos(d*x+c)+sin(d*x+
c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-20*sin(d*x+c)*arc
tanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2)
)-10*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1)
)^(1/2))-4*I*tan(d*x+c)*sec(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)-4*I*tan(d*x+c)*
sec(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2)-5*I*tan(d*x+c)*sec(d*x+c)*arctanh(1/2
*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))+5*I*ta
n(d*x+c)*sec(d*x+c)*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1
/(cos(d*x+c)+1))^(1/2))-10*tan(d*x+c)*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1
)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))+15*sec(d*x+c)*arctanh(1/2*(cos(d
*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))+10*I*arctanh(1
/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))+10*I
*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1
/2))-5*sec(d*x+c)^2*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(
1/(cos(d*x+c)+1))^(1/2))+5*sec(d*x+c)^2*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+
1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))-44*sec(d*x+c)^2*(1/(cos(d*x+c)+
1))^(1/2)+80*(1/(cos(d*x+c)+1))^(1/2)-20*I*arctanh(1/2*(cos(d*x+c)+sin(d*x+
c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-15*I*sec(d*x+c)*a
rctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/
2))-15*I*sec(d*x+c)*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1
/(cos(d*x+c)+1))^(1/2))+5*tan(d*x+c)*sec(d*x+c)*arctanh(1/2*(-cos(d*x+c)+si
n(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))+5*tan(d*x+c)*sec(d*x+c
)*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(
1/2))+10*I*tan(d*x+c)*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)
/(1/(cos(d*x+c)+1))^(1/2))-10*I*tan(d*x+c)*arctanh(1/2*(cos(d*x+c)+sin(d*x+
c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))+80*I*tan(d*x+c)*(1/(cos(d*x+
c)+1))^(1/2)-5*I*sec(d*x+c)^2*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d
*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))-5*I*sec(d*x+c)^2*arctanh(1/2*(cos(d*x+c)
+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))+20*I*cos(d*x+c)*arc
tanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2)
```

```
)+20*I*cos(d*x+c)*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(
cos(d*x+c)+1))^(1/2))+80*I*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+20*I*arctanh
(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*si
n(d*x+c)-20*sin(d*x+c)*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)
/(1/(cos(d*x+c)+1))^(1/2))-10*tan(d*x+c)*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)
+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))-44*sec(d*x+c)*(1/(cos(d*x+c)+1
))^(1/2)-15*sec(d*x+c)*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1
)/(1/(cos(d*x+c)+1))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.32

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{\left(\sqrt{\frac{25ie^9}{a^5d^2}}a^3de^{(idx+ic)}\log\left(\frac{2\left(\sqrt{\frac{25ie^9}{a^5d^2}}a^3d+5\left(e^4e^{(2idx+2ic)}+e^4\right)\sqrt{\frac{a}{e^{(2idx+2ic)}+1}}\sqrt{\frac{e}{e^{(2idx+2ic)}+1}}e^{\left(\frac{1}{2}idx+\frac{1}{2}ic\right)}\right)}{5e^4}\right)-\sqrt{\frac{25ie^9}{a^5d^2}}a^3}{}$$

```
[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="frica
s")
```

```
[Out] -1/2*(sqrt(25*I*e^9/(a^5*d^2))*a^3*d*e^(I*d*x + I*c)*log(2/5*(sqrt(25*I*e^9
/(a^5*d^2))*a^3*d + 5*(e^4*e^(2*I*d*x + 2*I*c) + e^4)*sqrt(a/(e^(2*I*d*x +
2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/e^4
) - sqrt(25*I*e^9/(a^5*d^2))*a^3*d*e^(I*d*x + I*c)*log(-2/5*(sqrt(25*I*e^9/
(a^5*d^2))*a^3*d - 5*(e^4*e^(2*I*d*x + 2*I*c) + e^4)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/e^4)
 - sqrt(-25*I*e^9/(a^5*d^2))*a^3*d*e^(I*d*x + I*c)*log(2/5*(sqrt(-25*I*e^9/
(a^5*d^2))*a^3*d + 5*(e^4*e^(2*I*d*x + 2*I*c) + e^4)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/e^4)
 + sqrt(-25*I*e^9/(a^5*d^2))*a^3*d*e^(I*d*x + I*c)*log(-2/5*(sqrt(-25*I*e^9
/(a^5*d^2))*a^3*d - 5*(e^4*e^(2*I*d*x + 2*I*c) + e^4)*sqrt(a/(e^(2*I*d*x +
2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/e^4
) + 4*(-5*I*e^4*e^(2*I*d*x + 2*I*c) - 4*I*e^4)*sqrt(a/(e^(2*I*d*x + 2*I*c)
+ 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-I*d*x
- I*c)/(a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((e*sec(d*x+c))**(9/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2449 vs. $2(307) = 614$.

Time = 0.92 (sec) , antiderivative size = 2449, normalized size of antiderivative = 5.96

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxim
a")
```

```
[Out] -(64*e^4*cos(1/2*d*x + 1/2*c)^2 + 64*e^4*sin(1/2*d*x + 1/2*c)^2 + 16*e^4 -
10*(-I*sqrt(2)*e^4*cos(3/2*d*x + 3/2*c) - I*sqrt(2)*e^4*cos(1/2*d*x + 1/2*c
) - sqrt(2)*e^4*sin(3/2*d*x + 3/2*c) + sqrt(2)*e^4*sin(1/2*d*x + 1/2*c))*ar
ctan2(sqrt(2)*sin(1/2*d*x + 1/2*c) + sin(d*x + c), sqrt(2)*cos(1/2*d*x + 1/
2*c) + cos(d*x + c) + 1) - 10*(I*sqrt(2)*e^4*cos(3/2*d*x + 3/2*c) + I*sqrt(
2)*e^4*cos(1/2*d*x + 1/2*c) + sqrt(2)*e^4*sin(3/2*d*x + 3/2*c) - sqrt(2)*e^
4*sin(1/2*d*x + 1/2*c))*arctan2(-sqrt(2)*sin(1/2*d*x + 1/2*c) + sin(d*x + c
), -sqrt(2)*cos(1/2*d*x + 1/2*c) + cos(d*x + c) + 1) - (10*sqrt(2)*e^4*arct
an2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 1
0*sqrt(2)*e^4*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, -sqrt(2)*sin(1/2*d*
x + 1/2*c) + 1) + 10*sqrt(2)*e^4*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1,
sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 10*sqrt(2)*e^4*arctan2(sqrt(2)*cos(1/2*
d*x + 1/2*c) - 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) - 5*I*sqrt(2)*e^4*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 5*I*sqrt(2)*e^4*log(2*cos
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/
2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 5*I*sqrt(2)*e^4*log(2*cos(1/2*
d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c)
+ 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 5*I*sqrt(2)*e^4*log(2*cos(1/2*d*x +
 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*s
qrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 64*e^4*cos(1/2*d*x + 1/2*c) + 64*I*e^4*s
in(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) - 5*(2*sqrt(2)*e^4*arctan2(sqrt(2
)*cos(1/2*d*x + 1/2*c) + 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 2*sqrt(2)*e
```

```
^4*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, -sqrt(2)*sin(1/2*d*x + 1/2*c)
+ 1) + 2*sqrt(2)*e^4*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1, sqrt(2)*sin(
1/2*d*x + 1/2*c) + 1) + 2*sqrt(2)*e^4*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c)
- 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) - I*sqrt(2)*e^4*log(2*cos(1/2*d*x +
 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*s
qrt(2)*sin(1/2*d*x + 1/2*c) + 2) + I*sqrt(2)*e^4*log(2*cos(1/2*d*x + 1/2*c)
^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*
sin(1/2*d*x + 1/2*c) + 2) - I*sqrt(2)*e^4*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2
*d*x + 1/2*c) + 2) + I*sqrt(2)*e^4*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x +
1/2*c) + 2))*cos(1/2*d*x + 1/2*c) - 5*(sqrt(2)*e^4*cos(3/2*d*x + 3/2*c) + s
qrt(2)*e^4*cos(1/2*d*x + 1/2*c) - I*sqrt(2)*e^4*sin(3/2*d*x + 3/2*c) + I*sq
rt(2)*e^4*sin(1/2*d*x + 1/2*c))*log(2*sqrt(2)*sin(d*x + c)*sin(1/2*d*x + 1/
2*c) + 2*(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1)*cos(d*x + c) + cos(d*x + c)^2 +
 2*cos(1/2*d*x + 1/2*c)^2 + sin(d*x + c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*s
qrt(2)*cos(1/2*d*x + 1/2*c) + 1) + 5*(sqrt(2)*e^4*cos(3/2*d*x + 3/2*c) + sq
rt(2)*e^4*cos(1/2*d*x + 1/2*c) - I*sqrt(2)*e^4*sin(3/2*d*x + 3/2*c) + I*sqr
t(2)*e^4*sin(1/2*d*x + 1/2*c))*log(-2*sqrt(2)*sin(d*x + c)*sin(1/2*d*x + 1/
2*c) - 2*(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1)*cos(d*x + c) + cos(d*x + c)^2 +
 2*cos(1/2*d*x + 1/2*c)^2 + sin(d*x + c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*s
qrt(2)*cos(1/2*d*x + 1/2*c) + 1) + (10*I*sqrt(2)*e^4*arctan2(sqrt(2)*cos(1/
2*d*x + 1/2*c) + 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 10*I*sqrt(2)*e^4*ar
ctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1)
+ 10*I*sqrt(2)*e^4*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1, sqrt(2)*sin(1/
2*d*x + 1/2*c) + 1) + 10*I*sqrt(2)*e^4*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c)
 - 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 5*sqrt(2)*e^4*log(2*cos(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*
sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 5*sqrt(2)*e^4*log(2*cos(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)
*sin(1/2*d*x + 1/2*c) + 2) + 5*sqrt(2)*e^4*log(2*cos(1/2*d*x + 1/2*c)^2 + 2
*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/
2*d*x + 1/2*c) + 2) - 5*sqrt(2)*e^4*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x +
 1/2*c) + 2) - 64*I*e^4*cos(1/2*d*x + 1/2*c) - 64*e^4*sin(1/2*d*x + 1/2*c))
*sin(3/2*d*x + 3/2*c) - 5*(2*I*sqrt(2)*e^4*arctan2(sqrt(2)*cos(1/2*d*x + 1/
2*c) + 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*e^4*arctan2(sqrt(
2)*cos(1/2*d*x + 1/2*c) + 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 2*I*sqrt(
2)*e^4*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1, sqrt(2)*sin(1/2*d*x + 1/2*
c) + 1) + 2*I*sqrt(2)*e^4*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) - 1, -sqrt(2
)*sin(1/2*d*x + 1/2*c) + 1) + sqrt(2)*e^4*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2
*d*x + 1/2*c) + 2) - sqrt(2)*e^4*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/
2*c) + 2) + sqrt(2)*e^4*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
```

```
c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
 - sqrt(2)*e^4*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*
sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(1/2
*d*x + 1/2*c))*sqrt(a)*sqrt(e)/((8*I*a^3*cos(3/2*d*x + 3/2*c) + 8*I*a^3*cos
(1/2*d*x + 1/2*c) + 8*a^3*sin(3/2*d*x + 3/2*c) - 8*a^3*sin(1/2*d*x + 1/2*c)
)*d)
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{(e\sec(dx+c))^{\frac{9}{2}}}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac"
)
[Out] integrate((e*sec(d*x + c))^(9/2)/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{9/2}}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^{5/2}}\,dx$$

```
[In] int((e/cos(c + d*x))^(9/2)/(a + a*tan(c + d*x)*1i)^(5/2),x)
[Out] int((e/cos(c + d*x))^(9/2)/(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.431 $\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 527

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}} + \frac{i\sqrt{2}e^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{a^{3/2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{i\sqrt{2}e^{7/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{a^{3/2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{ie^{7/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{\sqrt{2}a^{3/2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{ie^{7/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{\sqrt{2}a^{3/2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] -1/2*I*e^(7/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec
(d*x+c))^(1/2)+cos(d*x+c)*(a-I*a*tan(d*x+c)))*sec(d*x+c)/a^(3/2)/d*2^(1/2)/
(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+1/2*I*e^(7/2)*ln(a+2^(1/2
)*a^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*
(a-I*a*tan(d*x+c)))*sec(d*x+c)/a^(3/2)/d*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(
a+I*a*tan(d*x+c))^(1/2)+I*e^(7/2)*arctan(1-2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c
))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)*2^(1/2)/a^(3/2)/d/(a-I*a*
tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-I*e^(7/2)*arctan(1+2^(1/2)*e^(1/
2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*x+c)*2^(1/2
)/a^(3/2)/d/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+4/3*I*e^2*(e*
sec(d*x+c))^(3/2)/a/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3581, 3580, 3576, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{i\sqrt{2}e^{7/2}\sec(c+dx)\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{a^{3/2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{i\sqrt{2}e^{7/2}\sec(c+dx)\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{a^{3/2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{ie^{7/2}\sec(c+dx)\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{\sqrt{2}a^{3/2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{ie^{7/2}\sec(c+dx)\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{\sqrt{2}a^{3/2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}}$$

```
[In] Int[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((4*I)/3)*e^2*(e*Sec[c + d*x])^(3/2))/(a*d*(a + I*a*Tan[c + d*x])^(3/2)) +
 (I*Sqrt[2]*e^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])
/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(a^(3/2)*d*Sqrt[a - I*a*Tan[
c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (I*Sqrt[2]*e^(7/2)*ArcTan[1 + (Sqrt
[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec
[c + d*x])/(a^(3/2)*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]
) - (I*e^(7/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])
/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/
(Sqrt[2]*a^(3/2)*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) +
 (I*e^(7/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/Sq
rt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/(Sq
rt[2]*a^(3/2)*d*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3580

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Dist[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]])), Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}} - \frac{e^2\int\frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{a^2} \\
&= \frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}} - \frac{(e^3\sec(c+dx))\int\sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}\,dx}{a^2\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&= \frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}} - \frac{(4ie^5\sec(c+dx))\,\text{Subst}\left(\int\frac{x^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{ad\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&= \frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}} \\
&\quad + \frac{(2ie^4\sec(c+dx))\,\text{Subst}\left(\int\frac{a-ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{ad\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&\quad - \frac{(2ie^4\sec(c+dx))\,\text{Subst}\left(\int\frac{a+ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{ad\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&= \frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}} \\
&\quad - \frac{(ie^3\sec(c+dx))\,\text{Subst}\left(\int\frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{ad\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&\quad - \frac{(ie^3\sec(c+dx))\,\text{Subst}\left(\int\frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{ad\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&\quad - \frac{\left(ie^{7/2}\sec(c+dx)\right)\,\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}+2x}{-\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}a^{3/2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&\quad - \frac{\left(ie^{7/2}\sec(c+dx)\right)\,\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}-2x}{-\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}a^{3/2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}
\end{aligned}$$

$$= \frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}} - \frac{ie^{7/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{\sqrt{2}a^{3/2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{ie^{7/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{\sqrt{2}a^{3/2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{\left(i\sqrt{2}e^{7/2}\sec(c+dx)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{a^{3/2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{\left(i\sqrt{2}e^{7/2}\sec(c+dx)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{a^{3/2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{4ie^2(e\sec(c+dx))^{3/2}}{3ad(a+ia\tan(c+dx))^{3/2}} + \frac{i\sqrt{2}e^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{a^{3/2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{i\sqrt{2}e^{7/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{a^{3/2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{ie^{7/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{\sqrt{2}a^{3/2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{ie^{7/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{\sqrt{2}a^{3/2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A] (warning: unable to verify)

Time = 4.27 (sec) , antiderivative size = 357, normalized size of antiderivative = 0.68

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{e(e\sec(c+dx))^{5/2}(\cos(dx)+i\sin(dx))^3\left(\frac{4}{3}i\cos(2dx)(\cos(c)+i\sin(c))+\frac{4}{3}\right.}{}$$

```
[In] Integrate[(e*Sec[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (e*(e*Sec[c + d*x])^(5/2)*(Cos[d*x] + I*Sin[d*x])^3*(((4*I)/3)*Cos[2*d*x]*(
Cos[c] + I*Sin[c]) + (4*(Cos[c] + I*Sin[c])*Sin[2*d*x])/3 - (2*(ArcTanh[(Sq
rt[1 + I*Cos[c] - Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/(Sqrt[-1 + I*Cos[c] + Sin
[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[1 + I*Cos[c
] - Sin[c]] - ArcTanh[(Sqrt[1 - I*Cos[c] + Sin[c]]*Sqrt[I - Tan[(d*x)/2]])/
(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[I + Tan[(d*x)/2]])]*Sqrt[1 - I*Cos[c] +
```

```
Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]])*(Cos[2*c] + I*Sin[2*c])*Sqrt[I + Tan[
(d*x)/2]])/(Sqrt[-1 - I*Cos[c] - Sin[c]]*Sqrt[-1 + I*Cos[c] + Sin[c]]*Sqrt[
I - Tan[(d*x)/2]])))/(d*(a + I*a*Tan[c + d*x])^(5/2))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 919 vs. 2(421) = 842.

Time = 15.14 (sec) , antiderivative size = 920, normalized size of antiderivative = 1.75

method	result	size
default	Expression too large to display	920

```
[In] int((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/6+1/6*I)/d*e^3*(e*sec(d*x+c))^(1/2)/(tan(d*x+c)-I)^2/a^2/(a*(1+I*tan(d*x
+c)))^(1/2)/(1/(cos(d*x+c)+1))^(1/2)/(cos(d*x+c)+1)*(-12*I*cos(d*x+c)*arcta
nh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))+3
*I*sec(d*x+c)^2*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(co
s(d*x+c)+1))^(1/2))-4*I*tan(d*x+c)*sec(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+6*I*
tan(d*x+c)*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*
x+c)+1))^(1/2))+12*I*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/
(1/(cos(d*x+c)+1))^(1/2))*sin(d*x+c)-3*I*tan(d*x+c)*sec(d*x+c)*arctanh(1/2*
(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))+12*arct
anh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))
*cos(d*x+c)+12*sin(d*x+c)*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)
+1)/(1/(cos(d*x+c)+1))^(1/2))-4*I*tan(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)-4*I*(
1/(cos(d*x+c)+1))^(1/2)-4*I*sec(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+9*I*sec(d*x
+c)*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))
^(1/2))+6*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x
+c)+1))^(1/2))+6*tan(d*x+c)*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+
c)+1)/(1/(cos(d*x+c)+1))^(1/2))-4*(1/(cos(d*x+c)+1))^(1/2)+4*tan(d*x+c)*(1/
(cos(d*x+c)+1))^(1/2)-6*I*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)
+1)/(1/(cos(d*x+c)+1))^(1/2))-9*sec(d*x+c)*arctanh(1/2*(-cos(d*x+c)+sin(d*x
+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))-3*tan(d*x+c)*sec(d*x+c)*arc
tanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))
-4*sec(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+4*tan(d*x+c)*sec(d*x+c)*(1/(cos(d*x+
c)+1))^(1/2)-3*sec(d*x+c)^2*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x
+c)+1)/(1/(cos(d*x+c)+1))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.03

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{\left(3\,a^3d\sqrt{\frac{4i\,e^7}{a^5d^2}}e^{(2i\,dx+2i\,c)}\log\left(\frac{i\,a^3d\sqrt{\frac{4i\,e^7}{a^5d^2}}+2\left(e^3e^{(2i\,dx+2i\,c)}+e^3\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}\,i\,dx+\frac{1}{2}\,i\,c\right)}}{e^3}\right)-3\,a^3d\sqrt{\frac{4i\,e}{a^5d}}\right.}{}$$

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="frica
s")
```

```
[Out] -1/6*(3*a^3*d*sqrt(4*I*e^7/(a^5*d^2))*e^(2*I*d*x + 2*I*c)*log((I*a^3*d*sqrt
(4*I*e^7/(a^5*d^2)) + 2*(e^3*e^(2*I*d*x + 2*I*c) + e^3)*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/e
^3) - 3*a^3*d*sqrt(4*I*e^7/(a^5*d^2))*e^(2*I*d*x + 2*I*c)*log((-I*a^3*d*sqr
t(4*I*e^7/(a^5*d^2)) + 2*(e^3*e^(2*I*d*x + 2*I*c) + e^3)*sqrt(a/(e^(2*I*d*x
 + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))/
e^3) + 3*a^3*d*sqrt(-4*I*e^7/(a^5*d^2))*e^(2*I*d*x + 2*I*c)*log((I*a^3*d*sq
rt(-4*I*e^7/(a^5*d^2)) + 2*(e^3*e^(2*I*d*x + 2*I*c) + e^3)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)
)/e^3) - 3*a^3*d*sqrt(-4*I*e^7/(a^5*d^2))*e^(2*I*d*x + 2*I*c)*log((-I*a^3*d
*sqrt(-4*I*e^7/(a^5*d^2)) + 2*(e^3*e^(2*I*d*x + 2*I*c) + e^3)*sqrt(a/(e^(2*
I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I
*c))/e^3) + 8*(-I*e^3*e^(2*I*d*x + 2*I*c) - I*e^3)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c))*e^(-2*
I*d*x - 2*I*c)/(a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((e*sec(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1466 vs. 2(401) = 802.

Time = 0.67 (sec) , antiderivative size = 1466, normalized size of antiderivative = 2.78

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxim
a")
```

```
[Out] 1/12*(6*I*sqrt(2)*e^3*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + 1, sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) + 1) + 6*I*sqrt(2)*e^3*arctan2(sqrt(2)*cos(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1, -sqrt(2)*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 6*I*sqrt(2)*e^3*arct
an2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) -
1, sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1
) + 6*I*sqrt(2)*e^3*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) - 1, -sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) + 1) - 6*sqrt(2)*e^3*arctan2(sqrt(2)*sin(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sin(2/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))), sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) + cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + 1) + 6*sqrt(2)*e^3*arctan2(-sqrt(2)*sin(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sin(2/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))), -sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + 1) + 3*I*sqrt(2)*e^3*log(2*sqrt(2)*sin(2/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) + 2*(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + 1)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
 + 2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(2
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 3*I*sqrt(2)*e^3*log(
-2*sqrt(2)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*(sqrt(2)*cos(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 1)*cos(2/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(2/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*cos(1/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c)))^2 + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
```

```
3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))) + 1) - 3*sqrt(2)*e^3*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) + 2) + 3*sqrt(2)*e^3*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + 2) - 3*sqrt(2)*e^3*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + 2) + 3*sqrt(2)*e^3*log(2*cos(1/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) + 2) + 16*I*e^3*cos(3/2*d*x + 3/2*c) + 16*e^3*sin(3/2
*d*x + 3/2*c))*sqrt(e)/(a^(5/2)*d)
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx=\int \frac{(e\sec(dx+c))^{\frac{7}{2}}}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac"
)
[Out] integrate((e*sec(d*x + c))^(7/2)/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx=\int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/2}}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^{5/2}}\,dx$$

```
[In] int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^(5/2),x)
[Out] int((e/cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.432 $\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 38

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i(e\sec(c+dx))^{5/2}}{5d(a+ia\tan(c+dx))^{5/2}}$$

```
[Out] 2/5*I*(e*sec(d*x+c))^(5/2)/d/(a+I*a*tan(d*x+c))^(5/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {3569}

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i(e\sec(c+dx))^{5/2}}{5d(a+ia\tan(c+dx))^{5/2}}$$

```
[In] Int[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((2*I)/5)*(e*Sec[c + d*x])^(5/2))/(d*(a + I*a*Tan[c + d*x])^(5/2))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\text{integral} = \frac{2i(e\sec(c+dx))^{5/2}}{5d(a+ia\tan(c+dx))^{5/2}}$$

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i(e\sec(c+dx))^{5/2}}{5d(a+ia\tan(c+dx))^{5/2}}$$

```
[In] Integrate[(e*Sec[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
[Out] (((2*I)/5)*(e*Sec[c + d*x])^(5/2))/(d*(a + I*a*Tan[c + d*x])^(5/2))
```

Maple [A] (verified)

Time = 11.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.55

method	result	size
default	$\frac{2i(\sec^2(dx+c))e^2\sqrt{e\sec(dx+c)}}{5d(1+i\tan(dx+c))^2a^2\sqrt{a(1+i\tan(dx+c))}}$	59

```
[In] int((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/5*I/d*sec(d*x+c)^2*e^2*(e*sec(d*x+c))^(1/2)/(1+I*tan(d*x+c))^2/a^2/(a*(1+
I*tan(d*x+c)))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(28) = 56.

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.87

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2\left(i\,e^2e^{(2i\,dx+2i\,c)}+i\,e^2\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{5}{2}i\,dx-\frac{5}{2}i\,c\right)}}{5\,a^3d}$$

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="frica
s")
[Out] 2/5*(I*e^2*e^(2*I*d*x + 2*I*c) + I*e^2)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*s
qrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(-5/2*I*d*x - 5/2*I*c)/(a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((e*sec(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**(5/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(28) = 56.

Time = 0.34 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.00

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i\,e^{\frac{5}{2}}\left(-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{5}{2}}}{5\,a^{\frac{5}{2}}d\left(-\frac{2i\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)^{\frac{5}{2}}}$$

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxim
a")
[Out] 2/5*I*e^(5/2)*(-sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)^(5/2)/(a^(5/2)*d*(
-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2
- 1)^(5/2))
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{(e\sec(dx+c))^{\frac{5}{2}}}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac"
)
[Out] integrate((e*sec(d*x + c))^(5/2)/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^{5/2}}\,dx$$

```
[In] int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^(5/2),x)
[Out] int((e/cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.433 $\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 80

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i(e\sec(c+dx))^{3/2}}{7d(a+ia\tan(c+dx))^{5/2}} + \frac{4i(e\sec(c+dx))^{3/2}}{21ad(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] 2/7*I*(e*sec(d*x+c))^(3/2)/d/(a+I*a*tan(d*x+c))^(5/2)+4/21*I*(e*sec(d*x+c))
^(3/2)/a/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3583, 3569}

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{4i(e\sec(c+dx))^{3/2}}{21ad(a+ia\tan(c+dx))^{3/2}} + \frac{2i(e\sec(c+dx))^{3/2}}{7d(a+ia\tan(c+dx))^{5/2}}$$

```
[In] Int[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((2*I)/7)*(e*Sec[c + d*x])^(3/2))/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((4*
I)/21)*(e*Sec[c + d*x])^(3/2))/(a*d*(a + I*a*Tan[c + d*x])^(3/2))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2i(e\sec(c+dx))^{3/2}}{7d(a+ia\tan(c+dx))^{5/2}} + \frac{2\int\frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{3/2}}\,dx}{7a}\\ &= \frac{2i(e\sec(c+dx))^{3/2}}{7d(a+ia\tan(c+dx))^{5/2}} + \frac{4i(e\sec(c+dx))^{3/2}}{21ad(a+ia\tan(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.79

$$\int\frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2(e\sec(c+dx))^{3/2}(-5i+2\tan(c+dx))}{21a^2d(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^(5/2),x]
[Out] (2*(e*Sec[c + d*x])^(3/2)*(-5*I + 2*Tan[c + d*x]))/(21*a^2*d*(-I + Tan[c +
d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] (verified)

Time = 14.33 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{2i\sqrt{e\sec(dx+c)}\,e(2i\tan(dx+c)\sec(dx+c)+5\sec(dx+c))}{21d(1+i\tan(dx+c))^2a^2\sqrt{a(1+i\tan(dx+c))}}$	73

```
[In] int((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/21*I/d*(e*sec(d*x+c))^(1/2)*e/(1+I*tan(d*x+c))^2/a^2/(a*(1+I*tan(d*x+c)))
^(1/2)*(2*I*sec(d*x+c)*tan(d*x+c)+5*sec(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\left(7i\,ee^{(4i\,dx+4i\,c)}+10i\,ee^{(2i\,dx+2i\,c)}+3i\,e\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{7}{2}i\,dx-\frac{7}{2}i\right.}}{21\,a^3d}$$

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/21*(7*I*e*e^(4*I*d*x + 4*I*c) + 10*I*e*e^(2*I*d*x + 2*I*c) + 3*I*e)*sqrt(
a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(-7/2*I*d*
x - 7/2*I*c)/(a^3*d)
```

Sympy [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{(e\sec{(c+dx)})^{\frac{3}{2}}}{(ia\,(\tan{(c+dx)}-i))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral((e*sec(c + d*x))**(3/2)/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\left(3i\,e\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+7i\,e\cos\left(\frac{3}{7}\,\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right)\right)}{2}$$

```
[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxim
a")
```

```
[Out] 1/21*(3*I*e*cos(7/2*d*x + 7/2*c) + 7*I*e*cos(3/7*arctan2(sin(7/2*d*x + 7/2*
c), cos(7/2*d*x + 7/2*c))) + 3*e*sin(7/2*d*x + 7/2*c) + 7*e*sin(3/7*arctan2
(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*sqrt(e)/(a^(5/2)*d)
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{(e\sec(dx+c))^{\frac{3}{2}}}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

[In] integrate((e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac"
)

[Out] integrate((e*sec(d*x + c))^(3/2)/(I*a*tan(d*x + c) + a)^(5/2), x)

Mupad [B] (verification not implemented)

Time = 4.77 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.28

$$\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{e\sqrt{\frac{e}{\cos(c+d\,x)}}\,(7\,\sin(c+d\,x)+3\,\sin(3\,c+3\,d\,x)+\cos(c+d\,x)\,7\mathrm{i}+\cos(3\,c}{21\,a^2\,d\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}}}$$

[In] int((e/cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^(5/2),x)

[Out] (e*(e/cos(c + d*x))^(1/2)*(cos(c + d*x)*7i + 7*sin(c + d*x) + cos(3*c + 3*d
*x)*3i + 3*sin(3*c + 3*d*x)))/(21*a^2*d*((a*(cos(2*c + 2*d*x) + sin(2*c + 2
*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))

3.434 $\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 121

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i\sqrt{e\sec(c+dx)}}{9d(a+ia\tan(c+dx))^{5/2}} + \frac{8i\sqrt{e\sec(c+dx)}}{45ad(a+ia\tan(c+dx))^{3/2}} + \frac{16i\sqrt{e\sec(c+dx)}}{45a^2d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 16/45*I*(e*sec(d*x+c))^(1/2)/a^2/d/(a+I*a*tan(d*x+c))^(1/2)+2/9*I*(e*sec(d*
x+c))^(1/2)/d/(a+I*a*tan(d*x+c))^(5/2)+8/45*I*(e*sec(d*x+c))^(1/2)/a/d/(a+I
*a*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3583, 3569}

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{16i\sqrt{e\sec(c+dx)}}{45a^2d\sqrt{a+ia\tan(c+dx)}} + \frac{8i\sqrt{e\sec(c+dx)}}{45ad(a+ia\tan(c+dx))^{3/2}} + \frac{2i\sqrt{e\sec(c+dx)}}{9d(a+ia\tan(c+dx))^{5/2}}$$

```
[In] Int[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (((2*I)/9)*Sqrt[e*Sec[c + d*x]])/(d*(a + I*a*Tan[c + d*x])^(5/2)) + (((8*I)
/45)*Sqrt[e*Sec[c + d*x]])/(a*d*(a + I*a*Tan[c + d*x])^(3/2)) + (((16*I)/45
)*Sqrt[e*Sec[c + d*x]])/(a^2*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2i\sqrt{e\sec(c+dx)}}{9d(a+ia\tan(c+dx))^{5/2}}+\frac{4\int\frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{3/2}}\,dx}{9a}\\
&= \frac{2i\sqrt{e\sec(c+dx)}}{9d(a+ia\tan(c+dx))^{5/2}}+\frac{8i\sqrt{e\sec(c+dx)}}{45ad(a+ia\tan(c+dx))^{3/2}}+\frac{8\int\frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{45a^2}\\
&= \frac{2i\sqrt{e\sec(c+dx)}}{9d(a+ia\tan(c+dx))^{5/2}}+\frac{8i\sqrt{e\sec(c+dx)}}{45ad(a+ia\tan(c+dx))^{3/2}}+\frac{16i\sqrt{e\sec(c+dx)}}{45a^2d\sqrt{a+ia\tan(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.70

$$\int\frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{i\sec^2(c+dx)\sqrt{e\sec(c+dx)}(9+25\cos(2(c+dx))+20i\sin(2(c+dx)))}{45a^2d(-i+\tan(c+dx))^2\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-1/45*I)*Sec[c + d*x]^2*Sqrt[e*Sec[c + d*x]]*(9 + 25*Cos[2*(c + d*x)] + (
20*I)*Sin[2*(c + d*x)]))/(a^2*d*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c +
d*x]])
```

Maple [A] (verified)

Time = 13.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.57

method	result	size
default	$\frac{2i\sqrt{e\sec(dx+c)}\,(20i\tan(dx+c)+25-8(\sec^2(dx+c)))}{45d(1+i\tan(dx+c))^2\sqrt{a(1+i\tan(dx+c))}\,a^2}$	69

```
[In] int((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/45*I/d*(e*sec(d*x+c))^(1/2)/(1+I*tan(d*x+c))^2/(a*(1+I*tan(d*x+c)))^(1/2)
/a^2*(20*I*tan(d*x+c)+25-8*sec(d*x+c)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(45i\,e^{(6i\,dx+6i\,c)}+63i\,e^{(4i\,dx+4i\,c)}+23i\,e^{(2i\,dx+2i\,c)}+\right.}{90\,a^3d}$$

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="frica
s")
[Out] 1/90*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(4
5*I*e^(6*I*d*x + 6*I*c) + 63*I*e^(4*I*d*x + 4*I*c) + 23*I*e^(2*I*d*x + 2*I*
c) + 5*I)*e^(-9/2*I*d*x - 9/2*I*c)/(a^3*d)
```

Sympy [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{e\sec{(c+dx)}}}{(ia\,(\tan{(c+dx)}-i))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**(5/2),x)
[Out] Integral(sqrt(e*sec(c + d*x))/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\sqrt{e}\left(5i\,\cos\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)+18i\,\cos\left(\frac{5}{9}\,\arctan\left(\sin\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right),\cos\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right.\right.\right.\right.}{}$$

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxim
a")
```

```
[Out] 1/90*sqrt(e)*(5*I*cos(9/2*d*x + 9/2*c) + 18*I*cos(5/9*arctan2(sin(9/2*d*x +
 9/2*c), cos(9/2*d*x + 9/2*c))) + 45*I*cos(1/9*arctan2(sin(9/2*d*x + 9/2*c)
, cos(9/2*d*x + 9/2*c))) + 5*sin(9/2*d*x + 9/2*c) + 18*sin(5/9*arctan2(sin(
9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 45*sin(1/9*arctan2(sin(9/2*d*x +
 9/2*c), cos(9/2*d*x + 9/2*c))))/(a^(5/2)*d)
```

Giac [F]

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{e\sec\left(dx+c\right)}}{\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(e*sec(d*x + c))/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 5.34 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\left(\cos\left(2\,c+2\,d\,x\right)\,18\mathrm{i}+\cos\left(4\,c+4\,d\,x\right)\,5\mathrm{i}+18\,\sin\left(2\,c+2\,d\,x\right)+\right.}{90\,a^2\,d\,\sqrt{\frac{a\,\left(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i}\right)}{\cos(2\,c+2\,d\,x)+1}}}$$

```
[In] int((e/cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^(5/2),x)
```

```
[Out] ((e/cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*18i + cos(4*c + 4*d*x)*5i + 18*si
n(2*c + 2*d*x) + 5*sin(4*c + 4*d*x) + 45i))/(90*a^2*d*((a*(cos(2*c + 2*d*x)
 + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))
```

3.435 $\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 162

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i}{11d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}} + \frac{12i}{77ad\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}} + \frac{16i}{77a^2d\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{32i\sqrt{a+ia\tan(c+dx)}}{77a^3d\sqrt{e\sec(c+dx)}}$$

```
[Out] 16/77*I/a^2/d/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-32/77*I*(a+I*a*
tan(d*x+c))^(1/2)/a^3/d/(e*sec(d*x+c))^(1/2)+2/11*I/d/(e*sec(d*x+c))^(1/2)/
(a+I*a*tan(d*x+c))^(5/2)+12/77*I/a/d/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c)
)^(3/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3583, 3569}

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{32i\sqrt{a+ia\tan(c+dx)}}{77a^3d\sqrt{e\sec(c+dx)}} + \frac{16i}{77a^2d\sqrt{a+ia\tan(c+dx)}\sqrt{e\sec(c+dx)}} + \frac{12i}{77ad(a+ia\tan(c+dx))^{3/2}\sqrt{e\sec(c+dx)}} + \frac{2i}{11d(a+ia\tan(c+dx))^{5/2}\sqrt{e\sec(c+dx)}}$$

```
[In] Int[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2)),x]
```

```
[Out] ((2*I)/11)/(d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2)) + ((12*I)/
77)/(a*d*Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(3/2)) + ((16*I)/77)/(
a^2*d*Sqrt[e*Sec[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((32*I)/77)*Sqrt[
a + I*a*Tan[c + d*x]])/(a^3*d*Sqrt[e*Sec[c + d*x]])
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2i}{11d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}}+\frac{6\int\frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}\,dx}{11a}\\
&= \frac{2i}{11d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}}\\
&\quad+\frac{12i}{77ad\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}+\frac{24\int\frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx}{77a^2}\\
&= \frac{2i}{11d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}}\\
&\quad+\frac{12i}{77ad\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}\\
&\quad+\frac{16i}{77a^2d\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{16\int\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx}{77a^3}\\
&= \frac{2i}{11d\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}}\\
&\quad+\frac{12i}{77ad\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{3/2}}\\
&\quad+\frac{16i}{77a^2d\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}-\frac{32i\sqrt{a+ia\tan(c+dx)}}{77a^3d\sqrt{e\sec(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.63

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{i\sec^3(c+dx)(-55\cos(c+dx)+35\cos(3(c+dx))-22i\sin(c$$
$$\overline{154a^2d\sqrt{e\sec(c+dx)}(-i+\tan(c+dx))^2\sqrt{a+}}$$

```
[In] Integrate[1/(Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^(5/2)),x]
```

```
[Out] ((I/154)*Sec[c + d*x]^3*(-55*Cos[c + d*x] + 35*Cos[3*(c + d*x)] - (22*I)*Si
n[c + d*x] + (42*I)*Sin[3*(c + d*x)]))/(a^2*d*Sqrt[e*Sec[c + d*x]]*(-I + Ta
n[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] (verified)

Time = 10.02 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.52

method	result	size
default	$\frac{-\frac{10i}{11}+\frac{12\tan(dx+c)}{11}+\frac{80i\left(\sec^2(dx+c)\right)}{77}-\frac{32\left(\sec^2(dx+c)\right)\tan(dx+c)}{77}}{d\sqrt{a(1+i\tan(dx+c))}\,\sqrt{e\sec(dx+c)}\,(1+i\tan(dx+c))^2a^2}$	85

```
[In] int(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE
)
```

```
[Out] 2/77/d/(a*(1+I*tan(d*x+c)))^(1/2)/(e*sec(d*x+c))^(1/2)/(1+I*tan(d*x+c))^2/a
^2*(-35*I+42*tan(d*x+c)+40*I*sec(d*x+c)^2-16*sec(d*x+c)^2*tan(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.55

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-77i\,e^{(8i\,dx+8i\,c)}+110i\,e^{(4i\,dx+4i\,c)}\right.}{308\,a^3de}$$

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fri
cas")
```

```
[Out] 1/308*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(
-77*I*e^(8*I*d*x + 8*I*c) + 110*I*e^(4*I*d*x + 4*I*c) + 40*I*e^(2*I*d*x + 2
*I*c) + 7*I)*e^(-11/2*I*d*x - 11/2*I*c)/(a^3*d*e)
```

Sympy [F]

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{1}{\sqrt{e\sec\left(c+dx\right)}\left(ia\left(\tan\left(c+dx\right)-i\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(e*sec(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**(5/2),x)
```

```
[Out] Integral(1/(sqrt(e*sec(c + d*x))*(I*a*(tan(c + d*x) - I))**(5/2)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.77 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{7i\,\cos\left(\frac{11}{2}\,dx+\frac{11}{2}\,c\right)+33i\,\cos\left(\frac{7}{11}\,\arctan\left(\sin\left(\frac{11}{2}\,dx+\frac{11}{2}\,c\right)\right.\right.}{}$$

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="max
ima")
```

```
[Out] 1/308*(7*I*cos(11/2*d*x + 11/2*c) + 33*I*cos(7/11*arctan2(sin(11/2*d*x + 11
/2*c), cos(11/2*d*x + 11/2*c))) + 77*I*cos(3/11*arctan2(sin(11/2*d*x + 11/2
*c), cos(11/2*d*x + 11/2*c))) - 77*I*cos(1/11*arctan2(sin(11/2*d*x + 11/2*c
), cos(11/2*d*x + 11/2*c))) + 7*sin(11/2*d*x + 11/2*c) + 33*sin(7/11*arctan
2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 77*sin(3/11*arctan2(si
n(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 77*sin(1/11*arctan2(sin(11
/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))))/(a^(5/2)*d*sqrt(e))
```

Giac [F]

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{1}{\sqrt{e\sec\left(dx+c\right)}(i\,a\tan\left(dx+c\right)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(e*sec(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^(5/2)), x)
```

Mupad [B] (verification not implemented)

Time = 5.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.73

$$\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\left(154\,\sin\left(c+d\,x\right)+33\,\sin\left(3\,c+3\,d\,x\right)+7\,\sin\left(5\,c\right.\right.}{308\,a^2\,d\,e\,\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)-}{\cos(2\,c}}}$$

```
[In] int(1/((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(5/2)),x)
```

```
[Out] ((e/cos(c + d*x))^(1/2)*(154*sin(c + d*x) + cos(3*c + 3*d*x)*33i + cos(5*c
+ 5*d*x)*7i + 33*sin(3*c + 3*d*x) + 7*sin(5*c + 5*d*x)))/(308*a^2*d*e*((a*(
cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))
```

3.436 $\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 206

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{2i}{13d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}} + \frac{16i}{117ad(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}} + \frac{32i}{195a^2d(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} + \frac{256i\sqrt{e\sec(c+dx)}}{585a^2de^2\sqrt{a+ia\tan(c+dx)}} - \frac{128i\sqrt{a+ia\tan(c+dx)}}{585a^3d(e\sec(c+dx))^{3/2}}$$

```
[Out] 32/195*I/a^2/d/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2)+256/585*I*(e*s
ec(d*x+c))^(1/2)/a^2/d/e^2/(a+I*a*tan(d*x+c))^(1/2)-128/585*I*(a+I*a*tan(d*
x+c))^(1/2)/a^3/d/(e*sec(d*x+c))^(3/2)+2/13*I/d/(e*sec(d*x+c))^(3/2)/(a+I*a
*tan(d*x+c))^(5/2)+16/117*I/a/d/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(3/
2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used

$= \{3583, 3578, 3569\}$

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{128i\sqrt{a+ia\tan(c+dx)}}{585a^3d(e\sec(c+dx))^{3/2}} + \frac{256i\sqrt{e\sec(c+dx)}}{585a^2de^2\sqrt{a+ia\tan(c+dx)}} + \frac{32i}{195a^2d\sqrt{a+ia\tan(c+dx)}(e\sec(c+dx))^{3/2}} + \frac{16i}{117ad(a+ia\tan(c+dx))^{3/2}(e\sec(c+dx))^{3/2}} + \frac{2i}{13d(a+ia\tan(c+dx))^{5/2}(e\sec(c+dx))^{3/2}}$$

```
[In] Int[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(5/2)),x]
```

```
[Out] ((2*I)/13)/(d*(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(5/2)) + ((16*I
)/117)/(a*d*(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(3/2)) + ((32*I)/
195)/(a^2*d*(e*Sec[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (((256*I)/
585)*Sqrt[e*Sec[c + d*x]])/(a^2*d*e^2*Sqrt[a + I*a*Tan[c + d*x]]) - (((128*
I)/585)*Sqrt[a + I*a*Tan[c + d*x]])/(a^3*d*(e*Sec[c + d*x])^(3/2))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\text{integral} = \frac{2i}{13d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}} + \frac{8\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}\,dx}{13a}$$

$$= \frac{2i}{13d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}} + \frac{16i}{117ad(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}} + \frac{16\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx}{39a^2}$$

$$= \frac{2i}{13d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}} + \frac{16i}{117ad(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}} + \frac{32i}{195a^2d(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} + \frac{64\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx}{195a^3}$$

$$= \frac{2i}{13d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}} + \frac{16i}{117ad(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}} + \frac{32i}{195a^2d(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} - \frac{128i\sqrt{a+ia\tan(c+dx)}}{585a^3d(e\sec(c+dx))^{3/2}} + \frac{128\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{585a^2e^2}$$

$$= \frac{2i}{13d(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}} + \frac{16i}{117ad(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}} + \frac{32i}{195a^2d(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} + \frac{256i\sqrt{e\sec(c+dx)}}{585a^2de^2\sqrt{a+ia\tan(c+dx)}} - \frac{128i\sqrt{a+ia\tan(c+dx)}}{585a^3d(e\sec(c+dx))^{3/2}}$$

Mathematica [A] (verified)

Time = 1.57 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.52

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\sec^4(c+dx)(-351i-1300i\cos(2(c+dx))+75i\cos(4(c+\ldots}{2340a^2d(e\sec(c+dx))^{3/2}(-i+\tan(c\ldots}$$

```
[In] Integrate[1/((e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^(5/2)),x]
```

```
[Out] (Sec[c + d*x]^4*(-351*I - (1300*I)*Cos[2*(c + d*x)] + (75*I)*Cos[4*(c + d*x
)] + 1040*Sin[2*(c + d*x)] - 120*Sin[4*(c + d*x)]))/(2340*a^2*d*(e*Sec[c +
d*x])^(3/2)*(-I + Tan[c + d*x])^2*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] (verified)

Time = 8.52 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.50

method	result	size
default	$-\frac{2(75i\cos(dx+c)-120\sin(dx+c)-400i\sec(dx+c)+320\sec(dx+c)\tan(dx+c)+128i(\sec^3(dx+c)))}{585d\sqrt{a(1+i\tan(dx+c))}\,(1+i\tan(dx+c))^2\sqrt{e\sec(dx+c)}\,a^2e}$	102

```
[In] int(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -2/585/d/(a*(1+I*tan(d*x+c)))^(1/2)/(1+I*tan(d*x+c))^2/(e*sec(d*x+c))^(1/2)
/a^2/e*(75*I*cos(d*x+c)-120*sin(d*x+c)-400*I*sec(d*x+c)+320*sec(d*x+c)*tan(
d*x+c)+128*I*sec(d*x+c)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.54

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\sqrt{\frac{e}{e^{(2i\,dx+2i\,c)}+1}}\left(-195i\,e^{(10i\,dx+10i\,c)}+2145i\,e^{(8i\,d\ldots}\right.}{\ldots}$$

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fri
cas")
```

```
[Out] 1/4680*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*
(-195*I*e^(10*I*d*x + 10*I*c) + 2145*I*e^(8*I*d*x + 8*I*c) + 3042*I*e^(6*I*
d*x + 6*I*c) + 962*I*e^(4*I*d*x + 4*I*c) + 305*I*e^(2*I*d*x + 2*I*c) + 45*I
)*e^(-13/2*I*d*x - 13/2*I*c)/(a^3*d*e^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*sec(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**(5/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.64 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.10

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{45i\,\cos\left(\frac{13}{2}\,dx+\frac{13}{2}\,c\right)+260i\,\cos\left(\frac{9}{13}\,\arctan\left(\sin\left(\frac{13}{2}\,dx+\right.\right.\right.}{}$$

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="max
ima")
[Out] 1/4680*(45*I*cos(13/2*d*x + 13/2*c) + 260*I*cos(9/13*arctan2(sin(13/2*d*x +
 13/2*c), cos(13/2*d*x + 13/2*c))) + 702*I*cos(5/13*arctan2(sin(13/2*d*x +
13/2*c), cos(13/2*d*x + 13/2*c))) - 195*I*cos(3/13*arctan2(sin(13/2*d*x + 1
3/2*c), cos(13/2*d*x + 13/2*c))) + 2340*I*cos(1/13*arctan2(sin(13/2*d*x + 1
3/2*c), cos(13/2*d*x + 13/2*c))) + 45*sin(13/2*d*x + 13/2*c) + 260*sin(9/13
*arctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c))) + 702*sin(5/13*ar
ctan2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c))) + 195*sin(3/13*arcta
n2(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c))) + 2340*sin(1/13*arctan2
(sin(13/2*d*x + 13/2*c), cos(13/2*d*x + 13/2*c))))/(a^(5/2)*d*e^(3/2))
```

Giac [F]

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{1}{(e\sec\left(dx+c\right))^{\frac{3}{2}}\left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(e*sec(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="gia
c")
[Out] integrate(1/((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^(5/2)), x)
```

Mupad [B] (verification not implemented)

Time = 5.95 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.66

$$\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{\sqrt{\frac{e}{\cos(c+d\,x)}}\,\left(\cos\left(2\,c+2\,d\,x\right)\,507\mathrm{i}+\cos\left(4\,c+4\,d\,x\right)\,260\mathrm{i}+\mathrm{c}\right.}{468\mathrm{0}}$$

```
[In] int(1/((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(5/2)),x)
[Out] ((e/cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*507i + cos(4*c + 4*d*x)*260i + co
s(6*c + 6*d*x)*45i + 897*sin(2*c + 2*d*x) + 260*sin(4*c + 4*d*x) + 45*sin(6
*c + 6*d*x) + 2340i))/(4680*a^2*d*e^2*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d
*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))
```

3.437 $\int \frac{(e\sec(c+dx))^{7/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 86

$$\int \frac{(e\sec(c+dx))^{7/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{3i2^{2/3}a\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{7}{6},\frac{13}{6},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{7/3}\sqrt[3]{\ldots}}{7d(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] 3/7*I*2^(2/3)*a*hypergeom([1/3, 7/6],[13/6],1/2-1/2*I*tan(d*x+c))*(e*sec(d*
x+c))^(7/3)*(1+I*tan(d*x+c))^(1/3)/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3586, 3604, 72, 71}

$$\int \frac{(e\sec(c+dx))^{7/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{3i2^{2/3}a\sqrt[3]{1+i\tan(c+dx)}(e\sec(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{7}{6},\frac{13}{6},\frac{1}{2}(1\ldots\right)}{7d(a+ia\tan(c+dx))^{3/2}}$$

```
[In] Int[(e*Sec[c + d*x])^(7/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((3*I)/7)*2^(2/3)*a*Hypergeometric2F1[1/3, 7/6, 13/6, (1 - I*Tan[c + d*x])
/2]*(e*Sec[c + d*x])^(7/3)*(1 + I*Tan[c + d*x])^(1/3))/(d*(a + I*a*Tan[c +
d*x])^(3/2))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(e\sec(c+dx))^{7/3}\int(a-ia\tan(c+dx))^{7/6}(a+ia\tan(c+dx))^{2/3}\,dx}{(a-ia\tan(c+dx))^{7/6}(a+ia\tan(c+dx))^{7/6}}\\
&= \frac{\left(a^2(e\sec(c+dx))^{7/3}\right)\text{Subst}\left(\int\frac{\sqrt[6]{a-iax}}{\sqrt[3]{a+iax}}\,dx,x,\tan(c+dx)\right)}{d(a-ia\tan(c+dx))^{7/6}(a+ia\tan(c+dx))^{7/6}}\\
&= \frac{\left(a^2(e\sec(c+dx))^{7/3}\sqrt[3]{\frac{a+ia\tan(c+dx)}{a}}\right)\text{Subst}\left(\int\frac{\sqrt[6]{a-iax}}{\sqrt[3]{\frac{1}{2}+\frac{ix}{2}}}\,dx,x,\tan(c+dx)\right)}{\sqrt[3]{2}d(a-ia\tan(c+dx))^{7/6}(a+ia\tan(c+dx))^{3/2}}\\
&= \frac{3i2^{2/3}a\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{7}{6},\frac{13}{6},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{7/3}\sqrt[3]{1+i\tan(c+dx)}}{7d(a+ia\tan(c+dx))^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.69 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.37

$$\int \frac{(e\sec(c+dx))^{7/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{3i\sqrt[3]{2}ee^{i(c+dx)}\left(\frac{ee^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{4/3}\left(4+\left(1+e^{2i(c+dx)}\right)^{5/6}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{6},\frac{5}{3},-e^{2i(c+dx)}\right)\right)}{5d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[(e*Sec[c + d*x])^(7/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((-3*I)/5)*2^(1/3)*e*E^(I*(c + d*x))*((e*E^(I*(c + d*x)))/(1 + E^((2*I)*(c
 + d*x))))^(4/3)*(4 + (1 + E^((2*I)*(c + d*x)))^(5/6)*Hypergeometric2F1[2/3
, 5/6, 5/3, -E^((2*I)*(c + d*x))]))/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [F]

$$\int \frac{(e\sec(dx+c))^{\frac{7}{3}}}{\sqrt{a+ia\tan(dx+c)}}dx$$

```
[In] int((e*sec(d*x+c))^(7/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] int((e*sec(d*x+c))^(7/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int \frac{(e\sec(c+dx))^{7/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\sec(dx+c))^{\frac{7}{3}}}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(7/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/5*(-6*I*2^(5/6)*e^2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e/(e^(2*I*d*x + 2*
I*c) + 1))^(1/3)*e^(4/3*I*d*x + 4/3*I*c) + 5*a*d*integral(-2/5*I*2^(5/6)*e^
2*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(
1/3*I*d*x + 1/3*I*c)/(a*d), x))/(a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate((e*sec(d*x+c))**(7/3)/(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(e\sec(c+dx))^{7/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\sec(dx+c))^{\frac{7}{3}}}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(7/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
[Out] integrate((e*sec(d*x + c))^(7/3)/sqrt(I*a*tan(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{7/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\sec(dx+c))^{\frac{7}{3}}}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(7/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
[Out] integrate((e*sec(d*x + c))^(7/3)/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{7/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{7/3}}{\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}\,dx$$

```
[In] int((e/cos(c + d*x))^(7/3)/(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] int((e/cos(c + d*x))^(7/3)/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.438 $\int \frac{(e\sec(c+dx))^{5/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 86

$$\int \frac{(e\sec(c+dx))^{5/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{3i\sqrt[3]{2}a\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{5/3}(1}{5d(a+ia\tan(c+dx))^{3/2}}$$

```
[Out] 3/5*I*2^(1/3)*a*hypergeom([2/3, 5/6],[11/6],1/2-1/2*I*tan(d*x+c))*(e*sec(d*
x+c))^(5/3)*(1+I*tan(d*x+c))^(2/3)/d/(a+I*a*tan(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3586, 3604, 72, 71}

$$\int \frac{(e\sec(c+dx))^{5/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{3i\sqrt[3]{2}a(1+i\tan(c+dx))^{2/3}(e\sec(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{6},\frac{11}{6},\frac{1}{2}(\right.}{5d(a+ia\tan(c+dx))^{3/2}}$$

```
[In] Int[(e*Sec[c + d*x])^(5/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((3*I)/5)*2^(1/3)*a*Hypergeometric2F1[2/3, 5/6, 11/6, (1 - I*Tan[c + d*x])
/2]*(e*Sec[c + d*x])^(5/3)*(1 + I*Tan[c + d*x])^(2/3))/(d*(a + I*a*Tan[c +
d*x])^(3/2))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{(e\sec(c+dx))^{5/3}\int (a-ia\tan(c+dx))^{5/6}\sqrt[3]{a+ia\tan(c+dx)}\,dx}{(a-ia\tan(c+dx))^{5/6}(a+ia\tan(c+dx))^{5/6}}$$

$$= \frac{\left(a^2(e\sec(c+dx))^{5/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[6]{a-iax}(a+iax)^{2/3}}\,dx, x, \tan(c+dx)\right)}{d(a-ia\tan(c+dx))^{5/6}(a+ia\tan(c+dx))^{5/6}}$$

$$= \frac{\left(a^2(e\sec(c+dx))^{5/3}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{2/3}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{1}{2}+\frac{ix}{2}\right)^{2/3}\sqrt[6]{a-iax}}\,dx, x, \tan(c+dx)\right)}{2^{2/3}d(a-ia\tan(c+dx))^{5/6}(a+ia\tan(c+dx))^{3/2}}$$

$$= \frac{3i\sqrt[3]{2}a\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{6},\frac{11}{6},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{5/3}(1+i\tan(c+dx))^{2/3}}{5d(a+ia\tan(c+dx))^{3/2}}$$

Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.35

$$\int \frac{(e\sec(c+dx))^{5/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{3i2^{2/3}ee^{i(c+dx)}\left(\frac{ee^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{2/3}\left(-2+\sqrt[6]{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{3},\right.\right.}{d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[(e*Sec[c + d*x])^(5/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((3*I)*2^(2/3)*e*E^(I*(c + d*x))*((e*E^(I*(c + d*x)))/(1 + E^((2*I)*(c + d*
x))))^(2/3)*(-2 + (1 + E^((2*I)*(c + d*x)))^(1/6)*Hypergeometric2F1[1/6, 1/
3, 4/3, -E^((2*I)*(c + d*x))]))/(d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [F]

$$\int \frac{(e\sec(dx+c))^{\frac{5}{3}}}{\sqrt{a+ia\tan(dx+c)}}dx$$

```
[In] int((e*sec(d*x+c))^(5/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] int((e*sec(d*x+c))^(5/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int \frac{(e\sec(c+dx))^{5/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\sec(dx+c))^{\frac{5}{3}}}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(5/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] -(6*2^(1/6)*(I*e*e^(2*I*d*x + 2*I*c) + I*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1
))*(e/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*e^(2/3*I*d*x + 2/3*I*c) - a*d*integr
al(2^(1/6)*(I*e*e^(2*I*d*x + 2*I*c) + I*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1)
)*(e/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*e^(-4/3*I*d*x - 4/3*I*c)/(a*d), x))/(
a*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{5/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate((e*sec(d*x+c))**(5/3)/(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(e\sec(c+dx))^{5/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\sec(dx+c))^{\frac{5}{3}}}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(5/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
[Out] integrate((e*sec(d*x + c))^(5/3)/sqrt(I*a*tan(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{5/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\sec(dx+c))^{\frac{5}{3}}}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(5/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
[Out] integrate((e*sec(d*x + c))^(5/3)/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{5/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/3}}{\sqrt{a+a\tan(c+d\,x)\ 1\mathrm{i}}}\,dx$$

```
[In] int((e/cos(c + d*x))^(5/3)/(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] int((e/cos(c + d*x))^(5/3)/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.439 $\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 85

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{3i\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{7}{6},\frac{4}{3},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{2/3}\sqrt[6]{1+i}}{2\sqrt[6]{2}d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 3/4*I*hypergeom([1/3, 7/6],[4/3],1/2-1/2*I*tan(d*x+c))*(e*sec(d*x+c))^(2/3)
*(1+I*tan(d*x+c))^(1/6)*2^(5/6)/d/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3586, 3604, 72, 71}

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{3i\sqrt[6]{1+i\tan(c+dx)}(e\sec(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{7}{6},\frac{4}{3},\frac{1}{2}(1-i\tan\right.}{2\sqrt[6]{2}d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[(e*Sec[c + d*x])^(2/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((3*I)/2)*Hypergeometric2F1[1/3, 7/6, 4/3, (1 - I*Tan[c + d*x])/2]*(e*Sec[
c + d*x])^(2/3)*(1 + I*Tan[c + d*x])^(1/6))/(2^(1/6)*d*Sqrt[a + I*a*Tan[c +
 d*x]])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{(e\sec(c+dx))^{2/3}\int \frac{\sqrt[3]{a-ia\tan(c+dx)}}{\sqrt[6]{a+ia\tan(c+dx)}}\,dx}{\sqrt[3]{a-ia\tan(c+dx)}\sqrt[3]{a+ia\tan(c+dx)}}$$

$$= \frac{\left(a^2(e\sec(c+dx))^{2/3}\right)\text{Subst}\left(\int \frac{1}{(a-iax)^{2/3}(a+iax)^{7/6}}\,dx, x, \tan(c+dx)\right)}{d\sqrt[3]{a-ia\tan(c+dx)}\sqrt[3]{a+ia\tan(c+dx)}}$$

$$= \frac{\left(a(e\sec(c+dx))^{2/3}\sqrt[6]{\frac{a+ia\tan(c+dx)}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{1}{2}+\frac{ix}{2}\right)^{7/6}(a-iax)^{2/3}}\,dx, x, \tan(c+dx)\right)}{2\sqrt[6]{2}d\sqrt[3]{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{3i\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{7}{6},\frac{4}{3},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{2/3}\sqrt[6]{1+i\tan(c+dx)}}{2\sqrt[6]{2}d\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.36

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{3i\sqrt[6]{2}\left(\frac{ee^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{2/3}\sqrt[6]{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{6},\frac{5}{6},-e^{2i(c+dx)}\right)}{d\sqrt{\frac{ae^{2i(c+dx)}}{1+e^{2i(c+dx)}}}}$$

```
[In] Integrate[(e*Sec[c + d*x])^(2/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((3*I)*2^(1/6)*((e*E^(I*(c + d*x)))/(1 + E^((2*I)*(c + d*x))))^(2/3)*(1 + E
^((2*I)*(c + d*x)))^(1/6)*Hypergeometric2F1[-1/6, 1/6, 5/6, -E^((2*I)*(c +
d*x))])/(d*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))])
```

Maple [F]

$$\int \frac{(e\sec(dx+c))^{\frac{2}{3}}}{\sqrt{a+ia\tan(dx+c)}}dx$$

```
[In] int((e*sec(d*x+c))^(2/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] int((e*sec(d*x+c))^(2/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\sec(dx+c))^{\frac{2}{3}}}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(2/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] -(3*2^(1/6)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e/(e^(2*I*d*x + 2*I*c) + 1))
^(2/3)*(-I*e^(4*I*d*x + 4*I*c) - 2*I*e^(2*I*d*x + 2*I*c) - I)*e^(2/3*I*d*x
+ 2/3*I*c) - (a*d*e^(3*I*d*x + 3*I*c) - 2*a*d*e^(2*I*d*x + 2*I*c) + a*d*e^(
I*d*x + I*c))*integral(2^(1/6)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e/(e^(2*I
*d*x + 2*I*c) + 1))^(2/3)*(I*e^(4*I*d*x + 4*I*c) + 7*I*e^(3*I*d*x + 3*I*c)
+ 5*I*e^(2*I*d*x + 2*I*c) + 7*I*e^(I*d*x + I*c) + 4*I)*e^(2/3*I*d*x + 2/3*I
*c)/(a*d*e^(4*I*d*x + 4*I*c) - 3*a*d*e^(3*I*d*x + 3*I*c) + 3*a*d*e^(2*I*d*x
 + 2*I*c) - a*d*e^(I*d*x + I*c)), x))/(a*d*e^(3*I*d*x + 3*I*c) - 2*a*d*e^(2
*I*d*x + 2*I*c) + a*d*e^(I*d*x + I*c))
```

Sympy [F]

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\sec{(c+dx)})^{\frac{2}{3}}}{\sqrt{ia(\tan{(c+dx)}-i)}}\,dx$$

```
[In] integrate((e*sec(d*x+c))**(2/3)/(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Integral((e*sec(c + d*x))**(2/3)/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [F]

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\sec{(dx+c)})^{\frac{2}{3}}}{\sqrt{i\,a\tan{(dx+c)}+a}}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(2/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
[Out] integrate((e*sec(d*x + c))^(2/3)/sqrt(I*a*tan(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\sec{(dx+c)})^{\frac{2}{3}}}{\sqrt{i\,a\tan{(dx+c)}+a}}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(2/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
[Out] integrate((e*sec(d*x + c))^(2/3)/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^{2/3}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{2/3}}{\sqrt{a+a\tan{(c+d\,x)}\,1\mathrm{i}}}\,dx$$

```
[In] int((e/cos(c + d*x))^(2/3)/(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] int((e/cos(c + d*x))^(2/3)/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.440 $\int \frac{\sqrt[3]{e \sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 83

$$\int \frac{\sqrt[3]{e \sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{3i\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{4}{3},\frac{7}{6},\frac{1}{2}(1-i\tan(c+dx))\right)\sqrt[3]{e\sec(c+dx)}\sqrt[3]{1+i\tan(c+dx)}}{\sqrt[3]{2}d\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 3/2*I*hypergeom([1/6, 4/3],[7/6],1/2-1/2*I*tan(d*x+c))*(e*sec(d*x+c))^(1/3)
*(1+I*tan(d*x+c))^(1/3)*2^(2/3)/d/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3586, 3604, 72, 71}

$$\int \frac{\sqrt[3]{e \sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{3i\sqrt[3]{1+i\tan(c+dx)}\sqrt[3]{e\sec(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{4}{3},\frac{7}{6},\frac{1}{2}(1-i\tan(c+dx))\right)}{\sqrt[3]{2}d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[(e*Sec[c + d*x])^(1/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((3*I)*Hypergeometric2F1[1/6, 4/3, 7/6, (1 - I*Tan[c + d*x])/2]*(e*Sec[c +
d*x])^(1/3)*(1 + I*Tan[c + d*x])^(1/3))/(2^(1/3)*d*Sqrt[a + I*a*Tan[c + d*x
]])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt[3]{e\sec(c+dx)}\int\frac{\sqrt[6]{a-ia\tan(c+dx)}}{\sqrt[3]{a+ia\tan(c+dx)}}\,dx}{\sqrt[6]{a-ia\tan(c+dx)}\sqrt[6]{a+ia\tan(c+dx)}}\\
&= \frac{\left(a^2\sqrt[3]{e\sec(c+dx)}\right)\operatorname{Subst}\left(\int\frac{1}{(a-iax)^{5/6}(a+iax)^{4/3}}\,dx,x,\tan(c+dx)\right)}{d\sqrt[6]{a-ia\tan(c+dx)}\sqrt[6]{a+ia\tan(c+dx)}}\\
&= \frac{\left(a\sqrt[3]{e\sec(c+dx)}\sqrt[3]{\frac{a+ia\tan(c+dx)}{a}}\right)\operatorname{Subst}\left(\int\frac{1}{\left(\frac{1}{2}+\frac{ix}{2}\right)^{4/3}(a-iax)^{5/6}}\,dx,x,\tan(c+dx)\right)}{2\sqrt[3]{2}d\sqrt[6]{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}\\
&= \frac{3i\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{4}{3},\frac{7}{6},\frac{1}{2}(1-i\tan(c+dx))\right)\sqrt[3]{e\sec(c+dx)}\sqrt[3]{1+i\tan(c+dx)}}{\sqrt[3]{2}d\sqrt{a+ia\tan(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{3\left(8i-\frac{2ie^{2i(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{6},\frac{5}{3},-e^{2i(c+dx)}\right)}{\sqrt[6]{1+e^{2i(c+dx)}}}\right)\sqrt[3]{e\sec(c+dx)}}{16d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[(e*Sec[c + d*x])^(1/3)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (3*(8*I - ((2*I)*E^((2*I)*(c + d*x))*Hypergeometric2F1[2/3, 5/6, 5/3, -E^((
2*I)*(c + d*x))])/(1 + E^((2*I)*(c + d*x)))^(1/6))*(e*Sec[c + d*x])^(1/3))/
(16*d*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [F]

$$\int \frac{(e\sec(dx+c))^{\frac{1}{3}}}{\sqrt{a+ia\tan(dx+c)}}dx$$

```
[In] int((e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] int((e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\sec(dx+c))^{\frac{1}{3}}}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/4*(4*a*d*e^(I*d*x + I*c)*integral(-1/4*I*2^(5/6)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*(e/(e^(2*I*d*x + 2*I*c) + 1))^(1/3)*e^(1/3*I*d*x + 1/3*I*c)/(a*d)
, x) - 3*2^(5/6)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e/(e^(2*I*d*x + 2*I*c)
+ 1))^(1/3)*(-I*e^(2*I*d*x + 2*I*c) - I)*e^(1/3*I*d*x + 1/3*I*c))*e^(-I*d*x
 - I*c)/(a*d)
```

Sympy [F]

$$\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sqrt[3]{e\sec{(c+dx)}}}{\sqrt{ia\left(\tan{(c+dx)}-i\right)}}\,dx$$

```
[In] integrate((e*sec(d*x+c))**(1/3)/(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Integral((e*sec(c + d*x))**(1/3)/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\left(e\sec\left(dx+c\right)\right)^{\frac{1}{3}}}{\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
[Out] integrate((e*sec(d*x + c))^(1/3)/sqrt(I*a*tan(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\left(e\sec\left(dx+c\right)\right)^{\frac{1}{3}}}{\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

```
[In] integrate((e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
[Out] integrate((e*sec(d*x + c))^(1/3)/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^{1/3}}{\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

```
[In] int((e/cos(c + d*x))^(1/3)/(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] int((e/cos(c + d*x))^(1/3)/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.441 $\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 83

$$\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{3i\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{5}{3},\frac{5}{6},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{2/3}}{2^{2/3}d\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] -3/2*I*hypergeom([-1/6, 5/3],[5/6],1/2-1/2*I*tan(d*x+c))*(1+I*tan(d*x+c))^(
2/3)*2^(1/3)/d/(e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3586, 3604, 72, 71}

$$\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{3i(1+i\tan(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{5}{3},\frac{5}{6},\frac{1}{2}(1-i\tan(c+dx))\right)}{2^{2/3}d\sqrt{a+ia\tan(c+dx)}\sqrt[3]{e\sec(c+dx)}}$$

```
[In] Int[1/((e*Sec[c + d*x])^(1/3)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] ((-3*I)*Hypergeometric2F1[-1/6, 5/3, 5/6, (1 - I*Tan[c + d*x])/2]*(1 + I*Ta
n[c + d*x])^(2/3))/(2^(2/3)*d*(e*Sec[c + d*x])^(1/3)*Sqrt[a + I*a*Tan[c + d
*x]])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

integral

$$=\frac{\left(\sqrt[6]{a-ia\tan(c+dx)}\sqrt[6]{a+ia\tan(c+dx)}\right)\int\frac{1}{\sqrt[6]{a-ia\tan(c+dx)}(a+ia\tan(c+dx))^{2/3}}dx}{\sqrt[3]{e\sec(c+dx)}}$$

$$=\frac{\left(a^2\sqrt[6]{a-ia\tan(c+dx)}\sqrt[6]{a+ia\tan(c+dx)}\right)\text{Subst}\left(\int\frac{1}{(a-iax)^{7/6}(a+iax)^{5/3}}dx,x,\tan(c+dx)\right)}{d\sqrt[3]{e\sec(c+dx)}}$$

$$=\frac{\left(a\sqrt[6]{a-ia\tan(c+dx)}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{2/3}\right)\text{Subst}\left(\int\frac{1}{\left(\frac{1}{2}+\frac{ix}{2}\right)^{5/3}(a-iax)^{7/6}}dx,x,\tan(c+dx)\right)}{2\,2^{2/3}d\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$=-\frac{3i\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{5}{3},\frac{5}{6},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{2/3}}{2^{2/3}d\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.14

$$\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{12i-\frac{30ie^{2i(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{3},\frac{4}{3},-e^{2i(c+dx)}\right)}{\left(1+e^{2i(c+dx)}\right)^{5/6}}}{16d\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[1/((e*Sec[c + d*x])^(1/3)*Sqrt[a + I*a*Tan[c + d*x]]),x]
[Out] (12*I - ((30*I)*E^((2*I)*(c + d*x))*Hypergeometric2F1[1/6, 1/3, 4/3, -E^((2
*I)*(c + d*x))])/(1 + E^((2*I)*(c + d*x)))^(5/6))/(16*d*(e*Sec[c + d*x])^(1
/3)*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [F]

$$\int \frac{1}{\left(e\sec\left(dx+c\right)\right)^{\frac{1}{3}}\sqrt{a+ia\tan\left(dx+c\right)}}dx$$

```
[In] int(1/(e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x)
[Out] int(1/(e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{\left(e\sec\left(dx+c\right)\right)^{\frac{1}{3}}\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

```
[In] integrate(1/(e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fri
cas")
[Out] -1/8*(3*2^(1/6)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e/(e^(2*I*d*x + 2*I*c) +
 1))^(2/3)*(4*I*e^(6*I*d*x + 6*I*c) + 9*I*e^(4*I*d*x + 4*I*c) + 6*I*e^(2*I*
d*x + 2*I*c) + I)*e^(2/3*I*d*x + 2/3*I*c) - 8*(a*d*e*e^(4*I*d*x + 4*I*c) -
a*d*e*e^(2*I*d*x + 2*I*c))*integral(-15/16*2^(1/6)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*(e/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*(3*I*e^(4*I*d*x + 4*I*c) + 4*
I*e^(2*I*d*x + 2*I*c) + I)*e^(2/3*I*d*x + 2/3*I*c)/(a*d*e*e^(6*I*d*x + 6*I*
c) - 2*a*d*e*e^(4*I*d*x + 4*I*c) + a*d*e*e^(2*I*d*x + 2*I*c)), x))/(a*d*e*e
^(4*I*d*x + 4*I*c) - a*d*e*e^(2*I*d*x + 2*I*c))
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{\sqrt[3]{e\sec{(c+dx)}}\sqrt{ia\left(\tan{(c+dx)}-i\right)}}\,dx$$

```
[In] integrate(1/(e*sec(d*x+c))**(1/3)/(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Integral(1/((e*sec(c + d*x))**(1/3)*sqrt(I*a*(tan(c + d*x) - I))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{\left(e\sec\left(dx+c\right)\right)^{\frac{1}{3}}\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

```
[In] integrate(1/(e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="max
ima")
[Out] integrate(1/((e*sec(d*x + c))^(1/3)*sqrt(I*a*tan(d*x + c) + a)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{\left(e\sec\left(dx+c\right)\right)^{\frac{1}{3}}\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

```
[In] integrate(1/(e*sec(d*x+c))^(1/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gia
c")
[Out] integrate(1/((e*sec(d*x + c))^(1/3)*sqrt(I*a*tan(d*x + c) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{{\left(\frac{e}{\cos\left(c+d\,x\right)}\right)}^{1/3}\,\sqrt{a+a\,\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

```
[In] int(1/((e/cos(c + d*x))^(1/3)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
[Out] int(1/((e/cos(c + d*x))^(1/3)*(a + a*tan(c + d*x)*1i)^(1/2)), x)
```

3.442 $\int \frac{1}{(e\sec(c+dx))^{4/3}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 88

$$\int \frac{1}{(e\sec(c+dx))^{4/3}\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{3i\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{13}{6},\frac{1}{3},\frac{1}{2}(1-i\tan(c+dx))\right)\sqrt[6]{1+i\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{8\sqrt[6]{2}ad(e\sec(c+dx))^{4/3}}$$

```
[Out] -3/16*I*hypergeom([-2/3, 13/6],[1/3],1/2-1/2*I*tan(d*x+c))*(a+I*a*tan(d*x+c
))^(1/2)*(1+I*tan(d*x+c))^(1/6)*2^(5/6)/a/d/(e*sec(d*x+c))^(4/3)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3586, 3604, 72, 71}

$$\int \frac{1}{(e\sec(c+dx))^{4/3}\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{3i\sqrt[6]{1+i\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{13}{6},\frac{1}{3},\frac{1}{2}(1-i\tan(c+dx))\right)}{8\sqrt[6]{2}ad(e\sec(c+dx))^{4/3}}$$

```
[In] Int[1/((e*Sec[c + d*x])^(4/3)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] (((-3*I)/8)*Hypergeometric2F1[-2/3, 13/6, 1/3, (1 - I*Tan[c + d*x])/2]*(1 +
 I*Tan[c + d*x])^(1/6)*Sqrt[a + I*a*Tan[c + d*x]])/(2^(1/6)*a*d*(e*Sec[c +
d*x])^(4/3))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

integral

$$= \frac{\left((a-ia\tan(c+dx))^{2/3}(a+ia\tan(c+dx))^{2/3}\right)\int \frac{1}{(a-ia\tan(c+dx))^{2/3}(a+ia\tan(c+dx))^{7/6}}\,dx}{(e\sec(c+dx))^{4/3}}$$

$$= \frac{\left(a^2(a-ia\tan(c+dx))^{2/3}(a+ia\tan(c+dx))^{2/3}\right)\text{Subst}\left(\int \frac{1}{(a-iax)^{5/3}(a+iax)^{13/6}}\,dx, x, \tan(c+dx)\right)}{d(e\sec(c+dx))^{4/3}}$$

$$= \frac{\left((a-ia\tan(c+dx))^{2/3}\sqrt{a+ia\tan(c+dx)}\sqrt[6]{\frac{a+ia\tan(c+dx)}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{1}{2}+\frac{ix}{2}\right)^{13/6}(a-iax)^{5/3}}\,dx, x, \tan\right.}{4\sqrt[6]{2}d(e\sec(c+dx))^{4/3}}$$

$$= -\frac{3i\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{13}{6},\frac{1}{3},\frac{1}{2}(1-i\tan(c+dx))\right)\sqrt[6]{1+i\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}{8\sqrt[6]{2}ad(e\sec(c+dx))^{4/3}}$$

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.27

$$\int \frac{1}{(e\sec(c+dx))^{4/3}\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{3i\sec^2(c+dx)\left(3+3\cos(2(c+dx))-55\sqrt[6]{1+e^{2i(c+dx)}}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{6},\frac{5}{6},-e^{2i(c+dx)}\right)+11i\sin\right.}{112d(e\sec(c+dx))^{4/3}\sqrt{a+ia\tan(c+dx)}}$$

[In] Integrate[1/((e*Sec[c + d*x])^(4/3)*Sqrt[a + I*a*Tan[c + d*x]]),x]

[Out] (((-3*I)/112)*Sec[c + d*x]^2*(3 + 3*Cos[2*(c + d*x)] - 55*(1 + E^((2*I)*(c + d*x)))^(1/6)*Hypergeometric2F1[-1/6, 1/6, 5/6, -E^((2*I)*(c + d*x))] + (11*I)*Sin[2*(c + d*x)]))/(d*(e*Sec[c + d*x])^(4/3)*Sqrt[a + I*a*Tan[c + d*x]])

Maple [F]

$$\int \frac{1}{(e\sec(dx+c))^{\frac{4}{3}}\sqrt{a+ia\tan(dx+c)}}dx$$

[In] int(1/(e*sec(d*x+c))^(4/3)/(a+I*a*tan(d*x+c))^(1/2),x)

[Out] int(1/(e*sec(d*x+c))^(4/3)/(a+I*a*tan(d*x+c))^(1/2),x)

Fricas [F]

$$\int \frac{1}{(e\sec(c+dx))^{4/3}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{(e\sec(dx+c))^{\frac{4}{3}}\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

[In] integrate(1/(e*sec(d*x+c))^(4/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")

[Out] -1/112*(3*2^(1/6)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*(7*I*e^(8*I*d*x + 8*I*c) - 14*I*e^(7*I*d*x + 7*I*c) - 38*I*e^(6*I*d*x + 6*I*c) - 20*I*e^(5*I*d*x + 5*I*c) - 101*I*e^(4*I*d*x + 4*I*c) + 2*I*e^(3*I*d*x + 3*I*c) - 60*I*e^(2*I*d*x + 2*I*c) + 8*I*e^(I*d*x + I*c) - 4*I)*e^(2/3*I*d*x + 2/3*I*c) - 112*(a*d*e^2*e^(5*I*d*x + 5*I*c) - 2*a*d*e^2*e^(4*I*d*x + 4*I*c) + a*d*e^2*e^(3*I*d*x + 3*I*c))*integral(-55/112*2^(1/6)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e/(e^(2*I*d*x + 2*I*c) + 1))^(2/3)*(-I*e^(4*I*d*x + 4*I*c) - 7*I*e^(3*I*d*x + 3*I*c) - 5*I*e^(2*I*d*x + 2*I*c) - 7*I*e^(I*d*x + I*c) - 4*I)*e^(2/3*I*d*x + 2/3*I*c)/(a*d*e^2*e^(4*I*d*x + 4*I*c) - 3*a*d*e^2*e^(3*I*d*x + 3*I*c) + 3*a*d*e^2*e^(2*I*d*x + 2*I*c) - a*d*e^2*e^(I*d*x + I*c)), x))/(a*d*e^2*e^(5*I*d*x + 5*I*c) - 2*a*d*e^2*e^(4*I*d*x + 4*I*c) + a*d*e^2*e^(3*I*d*x + 3*I*c))

Sympy [F]

$$\int \frac{1}{(e\sec(c+dx))^{4/3}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{(e\sec{(c+dx)})^{\frac{4}{3}}\sqrt{ia\,(\tan{(c+dx)}-i)}}\,dx$$

```
[In] integrate(1/(e*sec(d*x+c))**(4/3)/(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Integral(1/((e*sec(c + d*x))**(4/3)*sqrt(I*a*(tan(c + d*x) - I))), x)
```

Maxima [F]

$$\int \frac{1}{(e\sec(c+dx))^{4/3}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{(e\sec{(dx+c)})^{\frac{4}{3}}\sqrt{i\,a\tan{(dx+c)}+a}}\,dx$$

```
[In] integrate(1/(e*sec(d*x+c))^(4/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="max
ima")
[Out] integrate(1/((e*sec(d*x + c))^(4/3)*sqrt(I*a*tan(d*x + c) + a)), x)
```

Giac [F]

$$\int \frac{1}{(e\sec(c+dx))^{4/3}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{(e\sec{(dx+c)})^{\frac{4}{3}}\sqrt{i\,a\tan{(dx+c)}+a}}\,dx$$

```
[In] integrate(1/(e*sec(d*x+c))^(4/3)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gia
c")
[Out] integrate(1/((e*sec(d*x + c))^(4/3)*sqrt(I*a*tan(d*x + c) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\sec(c+dx))^{4/3}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{{\left(\frac{e}{\cos(c+d\,x)}\right)}^{4/3}\,\sqrt{a+a\tan{(c+d\,x)}\,1{}\mathrm{i}}}\,dx$$

```
[In] int(1/((e/cos(c + d*x))^(4/3)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
[Out] int(1/((e/cos(c + d*x))^(4/3)*(a + a*tan(c + d*x)*1i)^(1/2)), x)
```

3.443 $\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{7/3}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 437

$$\begin{aligned}\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{7/3}}\,dx &= \frac{i(d\sec(e+fx))^{2/3}}{4f(a+ia\tan(e+fx))^{7/3}} \\ &- \frac{5x(d\sec(e+fx))^{2/3}}{72\ 2^{2/3}a^{5/3}\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} \\ &+ \frac{5i\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}}{\sqrt{3}\sqrt[3]{a}}\right)(d\sec(e+fx))^{2/3}}{12\ 2^{2/3}\sqrt{3}a^{5/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} \\ &- \frac{5i\log(\cos(e+fx))(d\sec(e+fx))^{2/3}}{72\ 2^{2/3}a^{5/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} \\ &- \frac{5i\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a-ia\tan(e+fx)}\right)(d\sec(e+fx))^{2/3}}{24\ 2^{2/3}a^{5/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} \\ &+ \frac{5i(d\sec(e+fx))^{2/3}}{24f\sqrt[3]{a+ia\tan(e+fx)}\,(a^2+ia^2\tan(e+fx))}\end{aligned}$$

```
[Out] 1/4*I*(d*sec(f*x+e))^(2/3)/f/(a+I*a*tan(f*x+e))^(7/3)-5/144*x*(d*sec(f*x+e)
)^(2/3)*2^(1/3)/a^(5/3)/(a-I*a*tan(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^(1/3)-5
/144*I*ln(cos(f*x+e))*(d*sec(f*x+e))^(2/3)*2^(1/3)/a^(5/3)/f/(a-I*a*tan(f*x
+e))^(1/3)/(a+I*a*tan(f*x+e))^(1/3)-5/48*I*ln(2^(1/3)*a^(1/3)-(a-I*a*tan(f*
x+e))^(1/3))*(d*sec(f*x+e))^(2/3)*2^(1/3)/a^(5/3)/f/(a-I*a*tan(f*x+e))^(1/3
)/(a+I*a*tan(f*x+e))^(1/3)+5/72*I*arctan(1/3*(a^(1/3)+2^(2/3)*(a-I*a*tan(f*
x+e))^(1/3))/a^(1/3)*3^(1/2))*(d*sec(f*x+e))^(2/3)*2^(1/3)/a^(5/3)/f*3^(1/2
)/(a-I*a*tan(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^(1/3)+5/24*I*(d*sec(f*x+e))^(
2/3)/f/(a+I*a*tan(f*x+e))^(1/3)/(a^2+I*a^2*tan(f*x+e))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3586, 3603, 3568, 44, 59, 631, 210, 31}

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{7/3}}\,dx = \frac{5i(d\sec(e+fx))^{2/3}\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{12\ 2^{2/3}\sqrt{3}a^{5/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} - \frac{5x(d\sec(e+fx))^{2/3}}{72\ 2^{2/3}a^{5/3}\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} - \frac{5i(d\sec(e+fx))^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a-ia\tan(e+fx)}\right)}{24\ 2^{2/3}a^{5/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} - \frac{5i(d\sec(e+fx))^{2/3}\log(\cos(e+fx))}{72\ 2^{2/3}a^{5/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} + \frac{5i(d\sec(e+fx))^{2/3}}{24f\sqrt[3]{a+ia\tan(e+fx)}\,(a^2+ia^2\tan(e+fx))} + \frac{i(d\sec(e+fx))^{2/3}}{4f(a+ia\tan(e+fx))^{7/3}}$$

```
[In] Int[(d*Sec[e + f*x])^(2/3)/(a + I*a*Tan[e + f*x])^(7/3),x]
```

```
[Out] ((I/4)*(d*Sec[e + f*x])^(2/3))/(f*(a + I*a*Tan[e + f*x])^(7/3)) - (5*x*(d*S
ec[e + f*x])^(2/3))/(72*2^(2/3)*a^(5/3)*(a - I*a*Tan[e + f*x])^(1/3)*(a + I
*a*Tan[e + f*x])^(1/3)) + (((5*I)/12)*ArcTan[(a^(1/3) + 2^(2/3)*(a - I*a*Ta
n[e + f*x])^(1/3))/(Sqrt[3]*a^(1/3))]*(d*Sec[e + f*x])^(2/3))/(2^(2/3)*Sqrt
[3]*a^(5/3)*f*(a - I*a*Tan[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3)) -
(((5*I)/72)*Log[Cos[e + f*x]]*(d*Sec[e + f*x])^(2/3))/(2^(2/3)*a^(5/3)*f*(a
 - I*a*Tan[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3)) - (((5*I)/24)*Log[
2^(1/3)*a^(1/3) - (a - I*a*Tan[e + f*x])^(1/3)]*(d*Sec[e + f*x])^(2/3))/(2^
(2/3)*a^(5/3)*f*(a - I*a*Tan[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3))
+ (((5*I)/24)*(d*Sec[e + f*x])^(2/3))/(f*(a + I*a*Tan[e + f*x])^(1/3)*(a^2
+ I*a^2*Tan[e + f*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !Int
egerQ[n] && LtQ[n, 0]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\text{integral}=\frac{(d\sec(e+fx))^{2/3}\int\frac{\sqrt[3]{a-ia\tan(e+fx)}}{(a+ia\tan(e+fx))^2}\,dx}{\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$=\frac{(d\sec(e+fx))^{2/3}\int\cos^4(e+fx)(a-ia\tan(e+fx))^{7/3}\,dx}{a^4\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$=\frac{\left(ia(d\sec(e+fx))^{2/3}\right)\text{Subst}\left(\int\frac{1}{(a-x)^3(a+x)^{2/3}}\,dx,x,-ia\tan(e+fx)\right)}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$=\frac{i(d\sec(e+fx))^{2/3}}{4f(a+ia\tan(e+fx))^{7/3}}+\frac{\left(5i(d\sec(e+fx))^{2/3}\right)\text{Subst}\left(\int\frac{1}{(a-x)^2(a+x)^{2/3}}\,dx,x,-ia\tan(e+fx)\right)}{12f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$=\frac{i(d\sec(e+fx))^{2/3}}{4f(a+ia\tan(e+fx))^{7/3}}+\frac{5i(d\sec(e+fx))^{2/3}}{24af(a+ia\tan(e+fx))^{4/3}}+\frac{\left(5i(d\sec(e+fx))^{2/3}\right)\text{Subst}\left(\int\frac{1}{(a-x)(a+x)^{2/3}}\,dx,x,-ia\tan(e+fx)\right)}{36af\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$=\frac{i(d\sec(e+fx))^{2/3}}{4f(a+ia\tan(e+fx))^{7/3}}+\frac{5i(d\sec(e+fx))^{2/3}}{24af(a+ia\tan(e+fx))^{4/3}}-\frac{5x(d\sec(e+fx))^{2/3}}{72\ 2^{2/3}a^{5/3}\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}-\frac{5i\log(\cos(e+fx))(d\sec(e+fx))^{2/3}}{72\ 2^{2/3}a^{5/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}+\frac{\left(5i(d\sec(e+fx))^{2/3}\right)\text{Subst}\left(\int\frac{1}{\sqrt[3]{2}\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a-ia\tan(e+fx)}\right)}{24\ 2^{2/3}a^{5/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}+\frac{\left(5i(d\sec(e+fx))^{2/3}\right)\text{Subst}\left(\int\frac{1}{2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}x+x^2}\,dx,x,\sqrt[3]{a-ia\tan(e+fx)}\right)}{24\sqrt[3]{2}a^{4/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$= \frac{i(d\sec(e+fx))^{2/3}}{4f(a+ia\tan(e+fx))^{7/3}} + \frac{5i(d\sec(e+fx))^{2/3}}{24af(a+ia\tan(e+fx))^{4/3}} - \frac{5x(d\sec(e+fx))^{2/3}}{72\ 2^{2/3}a^{5/3}\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} - \frac{5i\log(\cos(e+fx))(d\sec(e+fx))^{2/3}}{72\ 2^{2/3}a^{5/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} - \frac{5i\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a-ia\tan(e+fx)}\right)(d\sec(e+fx))^{2/3}}{24\ 2^{2/3}a^{5/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} - \frac{\left(5i(d\sec(e+fx))^{2/3}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+\frac{2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}}{\sqrt[3]{a}}\right)}{12\ 2^{2/3}a^{5/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$= \frac{i(d\sec(e+fx))^{2/3}}{4f(a+ia\tan(e+fx))^{7/3}} + \frac{5i(d\sec(e+fx))^{2/3}}{24af(a+ia\tan(e+fx))^{4/3}} - \frac{5x(d\sec(e+fx))^{2/3}}{72\ 2^{2/3}a^{5/3}\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} + \frac{5i\arctan\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)(d\sec(e+fx))^{2/3}}{12\ 2^{2/3}\sqrt{3}a^{5/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} - \frac{5i\log(\cos(e+fx))(d\sec(e+fx))^{2/3}}{72\ 2^{2/3}a^{5/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} - \frac{5i\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a-ia\tan(e+fx)}\right)(d\sec(e+fx))^{2/3}}{24\ 2^{2/3}a^{5/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

Mathematica [A] (verified)

Time = 2.44 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.55

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{7/3}}\,dx = \frac{e^{-2i(e+fx)}\left(9i+33ie^{2i(e+fx)}+24ie^{4i(e+fx)}-10e^{4i(e+fx)}\sqrt[3]{1+e^{2i(e+fx)}}fx-\right.}{}$$

```
[In] Integrate[(d*Sec[e + f*x])^(2/3)/(a + I*a*Tan[e + f*x])^(7/3),x]
```

```
[Out] ((9*I + (33*I)*E^((2*I)*(e + f*x)) + (24*I)*E^((4*I)*(e + f*x)) - 10*E^((4*
I)*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^(1/3)*f*x - (10*I)*Sqrt[3]*E^((4*I)
*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^(1/3)*ArcTan[(1 + 2*(1 + E^((2*I)*(e
+ f*x)))^(1/3))/Sqrt[3]] - (15*I)*E^((4*I)*(e + f*x))*(1 + E^((2*I)*(e + f*
x)))^(1/3)*Log[1 - (1 + E^((2*I)*(e + f*x)))^(1/3)])*Sec[e + f*x]^2*(d*Sec[
e + f*x])^(2/3))/(144*E^((2*I)*(e + f*x))*f*(a + I*a*Tan[e + f*x])^(7/3))
```

Maple [F]

$$\int \frac{(d\sec(fx+e))^{\frac{2}{3}}}{(a+ia\tan(fx+e))^{\frac{7}{3}}}dx$$

```
[In] int((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(7/3),x)
```

```
[Out] int((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(7/3),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 528, normalized size of antiderivative = 1.21

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{7/3}}\,dx = \frac{\left(48\,a^3 f\left(\frac{125 i\,d^2}{186624\,a^7 f^3}\right)^{\frac{1}{3}} e^{(6i\,fx+6i\,e)}\log\left(-\frac{2}{5}\left(72i\,a^3 f\left(\frac{125i\,d^2}{186624\,a^7 f^3}\right)^{\frac{1}{3}} e^{(2i\,fx+2i\,e)} - \right.\right.\right.}{}$$

```
[In] integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(7/3),x, algorithm="frica
s")
```

```
[Out] 1/48*(48*a^3*f*(125/186624*I*d^2/(a^7*f^3))^(1/3)*e^(6*I*f*x + 6*I*e)*log(-
2/5*(72*I*a^3*f*(125/186624*I*d^2/(a^7*f^3))^(1/3)*e^(2*I*f*x + 2*I*e) - 5*
2^(1/3)*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(
2/3)*(e^(2*I*f*x + 2*I*e) + 1)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)) +
 2^(1/3)*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^
(2/3)*(8*I*e^(6*I*f*x + 6*I*e) + 19*I*e^(4*I*f*x + 4*I*e) + 14*I*e^(2*I*f*x
 + 2*I*e) + 3*I)*e^(2*I*f*x + 2*I*e) - 24*(-I*sqrt(3)*a^3*f + a^3*f)*(125/1
86624*I*d^2/(a^7*f^3))^(1/3)*e^(6*I*f*x + 6*I*e)*log(2/5*(5*2^(1/3)*(a/(e^(
2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(e^(2*I*f*
x + 2*I*e) + 1)*e^(2*I*f*x + 2*I*e) + 36*(sqrt(3)*a^3*f + I*a^3*f)*(125/186
624*I*d^2/(a^7*f^3))^(1/3)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)) - 24*
(I*sqrt(3)*a^3*f + a^3*f)*(125/186624*I*d^2/(a^7*f^3))^(1/3)*e^(6*I*f*x + 6
*I*e)*log(2/5*(5*2^(1/3)*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x
 + 2*I*e) + 1))^(2/3)*(e^(2*I*f*x + 2*I*e) + 1)*e^(2*I*f*x + 2*I*e) - 36*(s
qrt(3)*a^3*f - I*a^3*f)*(125/186624*I*d^2/(a^7*f^3))^(1/3)*e^(2*I*f*x + 2*I
*e))*e^(-2*I*f*x - 2*I*e)))*e^(-6*I*f*x - 6*I*e)/(a^3*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{7/3}}\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))**(2/3)/(a+I*a*tan(f*x+e))**(7/3),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3902 vs. 2(324) = 648.

Time = 0.55 (sec) , antiderivative size = 3902, normalized size of antiderivative = 8.93

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{7/3}}\,dx = \text{Too large to display}$$

```
[In] integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(7/3),x, algorithm="maxim
a")
```

```
[Out] 1/288*(48*(cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2
*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f
*x + 4*e), cos(4*f*x + 4*e))) + 1)^(5/6)*((I*2^(1/3)*cos(4*f*x + 4*e) + 2^(
1/3)*sin(4*f*x + 4*e))*cos(5/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), co
s(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)
) - (2^(1/3)*cos(4*f*x + 4*e) - I*2^(1/3)*sin(4*f*x + 4*e))*sin(5/3*arctan2
(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(
4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)))*d^(2/3) + 30*(cos(1/2*arctan2(sin(4
*f*x + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4
*f*x + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) +
1)^(1/3)*((-I*2^(1/3)*cos(4*f*x + 4*e) - 2^(1/3)*sin(4*f*x + 4*e))*cos(2/3*
arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arcta
n2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)) + (2^(1/3)*cos(4*f*x + 4*e) -
 I*2^(1/3)*sin(4*f*x + 4*e))*sin(2/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*
e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))
) + 1)))*d^(2/3) + 5*(-2*I*sqrt(3)*2^(1/3)*arctan2(2/3*sqrt(3)*(cos(1/2*arc
tan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4*f*x + 4
*e), cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x +
 4*e))) + 1)^(1/6)*cos(1/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*
f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)) +
1/3*sqrt(3), 1/3*sqrt(3)*(2*(cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x +
4*e)))^2 + sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + 2*cos(1
/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)^(1/6)*sin(1/3*arctan2(
```

```
sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4
*f*x + 4*e), cos(4*f*x + 4*e))) + 1)) + sqrt(3))) - 2*I*sqrt(3)*2^(1/3)*arc
tan2(2/3*sqrt(3)*(cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 +
sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arctan2(
sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)^(1/6)*cos(1/3*arctan2(sin(1/2*arc
tan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e)
, cos(4*f*x + 4*e))) + 1)) + 1/3*sqrt(3), -1/3*sqrt(3)*(2*(cos(1/2*arctan2(
sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4*f*x + 4*e),
cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)
)) + 1)^(1/6)*sin(1/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x +
 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)) - sqrt(
3))) + sqrt(3)*2^(1/3)*log(4/3*(cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x
 + 4*e)))^2 + sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + 2*co
s(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)^(1/3)*(cos(1/3*arct
an2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(s
in(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1))^2 + sin(1/3*arctan2(sin(1/2*arcta
n2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e),
cos(4*f*x + 4*e))) + 1))^2) + 4/3*(cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*
f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + 2
*cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)^(1/6)*(sqrt(3)*s
in(1/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/
2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)) + cos(1/3*arctan2(sin(
1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x
 + 4*e), cos(4*f*x + 4*e))) + 1))) + 4/3) - sqrt(3)*2^(1/3)*log(4/3*(cos(1/
2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4*f*
x + 4*e), cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*
f*x + 4*e))) + 1)^(1/3)*(cos(1/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e),
cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) +
1))^2 + sin(1/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))
), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1))^2) - 4/3*(cos
(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4
*f*x + 4*e), cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*x + 4*e), cos
(4*f*x + 4*e))) + 1)^(1/6)*(sqrt(3)*sin(1/3*arctan2(sin(1/2*arctan2(sin(4*f
*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x
+ 4*e))) + 1)) - cos(1/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*
x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1))) + 4
/3) - 2*2^(1/3)*arctan2((cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)
))^2 + sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + 2*cos(1/2*a
rctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)^(1/3)*sin(2/3*arctan2(sin(
1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x
 + 4*e), cos(4*f*x + 4*e))) + 1)) + (cos(1/2*arctan2(sin(4*f*x + 4*e), cos(
4*f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 +
 2*cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)^(1/6)*sin(1/3*
arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arcta
n2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)), (cos(1/2*arctan2(sin(4*f*x +
```

```
 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x +
 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)^(1/
3)*cos(2/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), co
s(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)) + (cos(1/2*arctan2
(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4*f*x + 4*e),
 cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e
))) + 1)^(1/6)*cos(1/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x
+ 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)) + 1) +
 4*2^(1/3)*arctan2((cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2
+ sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arctan
2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)^(1/6)*sin(1/3*arctan2(sin(1/2*a
rctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*
e), cos(4*f*x + 4*e))) + 1)), (cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x
+ 4*e)))^2 + sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + 2*cos
(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)^(1/6)*cos(1/3*arctan
2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin
(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)) - 1) - 2*I*2^(1/3)*log((cos(1/2*arc
tan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4*f*x + 4
*e), cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x +
 4*e))) + 1)^(1/3)*cos(1/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*
f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1))^2
+ (cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*arctan2
(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*x + 4*e
), cos(4*f*x + 4*e))) + 1)^(1/3)*sin(1/3*arctan2(sin(1/2*arctan2(sin(4*f*x
+ 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4
*e))) + 1))^2 - 2*(cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 +
 sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arctan2
(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)^(1/6)*cos(1/3*arctan2(sin(1/2*ar
ctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e
), cos(4*f*x + 4*e))) + 1)) + 1) + I*2^(1/3)*log((cos(1/2*arctan2(sin(4*f*x
 + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x
 + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)^(
2/3)*(cos(2/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))),
 cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1))^2 + sin(2/3*arc
tan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(
sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1))^2) + (cos(1/2*arctan2(sin(4*f*x
+ 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x
+ 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)^(1
/3)*(cos(1/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))),
cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1))^2 + sin(1/3*arct
an2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(s
in(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1))^2) + 2*(cos(1/2*arctan2(sin(4*f*x
 + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x
 + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)^(
1/3)*((cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + sin(1/2*arc
```

```
tan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arctan2(sin(4*f*x +
 4*e), cos(4*f*x + 4*e))) + 1)^(1/6)*(cos(2/3*arctan2(sin(1/2*arctan2(sin(4
*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*
x + 4*e))) + 1))*cos(1/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*
x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)) + si
n(2/3*arctan2(sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2
*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1))*sin(1/3*arctan2(sin(1/2
*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x +
4*e), cos(4*f*x + 4*e))) + 1))) + cos(2/3*arctan2(sin(1/2*arctan2(sin(4*f*x
 + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x +
4*e))) + 1))) + 2*(cos(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 +
 sin(1/2*arctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e)))^2 + 2*cos(1/2*arctan2
(sin(4*f*x + 4*e), cos(4*f*x + 4*e))) + 1)^(1/6)*cos(1/3*arctan2(sin(1/2*ar
ctan2(sin(4*f*x + 4*e), cos(4*f*x + 4*e))), cos(1/2*arctan2(sin(4*f*x + 4*e
), cos(4*f*x + 4*e))) + 1)) + 1))*d^(2/3))/(a^(7/3)*f)
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{7/3}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{2}{3}}}{(i\,a\tan(fx+e)+a)^{\frac{7}{3}}}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(7/3),x, algorithm="giac"
)
```

```
[Out] integrate((d*sec(f*x + e))^(2/3)/(I*a*tan(f*x + e) + a)^(7/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{7/3}}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{2/3}}{(a+a\tan(e+f\,x)\ 1\mathrm{i})^{7/3}}\,dx$$

```
[In] int((d/cos(e + f*x))^(2/3)/(a + a*tan(e + f*x)*1i)^(7/3),x)
```

```
[Out] int((d/cos(e + f*x))^(2/3)/(a + a*tan(e + f*x)*1i)^(7/3), x)
```

3.444 $\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{4/3}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 378

$$
\begin{aligned}
\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{4/3}}\,dx &= \frac{i(d\sec(e+fx))^{2/3}}{2f(a+ia\tan(e+fx))^{4/3}} \\
&- \frac{x(d\sec(e+fx))^{2/3}}{6\ 2^{2/3}a^{2/3}\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} \\
&+ \frac{i\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}}{\sqrt{3}\sqrt[3]{a}}\right)(d\sec(e+fx))^{2/3}}{2^{2/3}\sqrt{3}a^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} \\
&- \frac{i\log(\cos(e+fx))(d\sec(e+fx))^{2/3}}{6\ 2^{2/3}a^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} \\
&- \frac{i\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a-ia\tan(e+fx)}\right)(d\sec(e+fx))^{2/3}}{2\ 2^{2/3}a^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}
\end{aligned}
$$

```
[Out] 1/2*I*(d*sec(f*x+e))^(2/3)/f/(a+I*a*tan(f*x+e))^(4/3)-1/12*x*(d*sec(f*x+e))
^(2/3)*2^(1/3)/a^(2/3)/(a-I*a*tan(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^(1/3)-1/
12*I*ln(cos(f*x+e))*(d*sec(f*x+e))^(2/3)*2^(1/3)/a^(2/3)/f/(a-I*a*tan(f*x+e
))^(1/3)/(a+I*a*tan(f*x+e))^(1/3)-1/4*I*ln(2^(1/3)*a^(1/3)-(a-I*a*tan(f*x+e
))^(1/3))*(d*sec(f*x+e))^(2/3)*2^(1/3)/a^(2/3)/f/(a-I*a*tan(f*x+e))^(1/3)/(
a+I*a*tan(f*x+e))^(1/3)+1/6*I*arctan(1/3*(a^(1/3)+2^(2/3)*(a-I*a*tan(f*x+e)
)^(1/3))/a^(1/3)*3^(1/2))*(d*sec(f*x+e))^(2/3)*2^(1/3)/a^(2/3)/f*3^(1/2)/(a
-I*a*tan(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^(1/3)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3586, 3603, 3568, 44, 59, 631, 210, 31}

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{4/3}}\,dx = \frac{i(d\sec(e+fx))^{2/3}\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{2^{2/3}\sqrt{3}a^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} - \frac{x(d\sec(e+fx))^{2/3}}{6\ 2^{2/3}a^{2/3}\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} - \frac{i(d\sec(e+fx))^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a-ia\tan(e+fx)}\right)}{2\ 2^{2/3}a^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} - \frac{i(d\sec(e+fx))^{2/3}\log(\cos(e+fx))}{6\ 2^{2/3}a^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} + \frac{i(d\sec(e+fx))^{2/3}}{2f(a+ia\tan(e+fx))^{4/3}}$$

```
[In] Int[(d*Sec[e + f*x])^(2/3)/(a + I*a*Tan[e + f*x])^(4/3),x]
```

```
[Out] ((I/2)*(d*Sec[e + f*x])^(2/3))/(f*(a + I*a*Tan[e + f*x])^(4/3)) - (x*(d*Sec
[e + f*x])^(2/3))/(6*2^(2/3)*a^(2/3)*(a - I*a*Tan[e + f*x])^(1/3)*(a + I*a*
Tan[e + f*x])^(1/3)) + (I*ArcTan[(a^(1/3) + 2^(2/3)*(a - I*a*Tan[e + f*x])^
(1/3))/(Sqrt[3]*a^(1/3))]*(d*Sec[e + f*x])^(2/3))/(2^(2/3)*Sqrt[3]*a^(2/3)*
f*(a - I*a*Tan[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3)) - ((I/6)*Log[C
os[e + f*x]]*(d*Sec[e + f*x])^(2/3))/(2^(2/3)*a^(2/3)*f*(a - I*a*Tan[e + f*
x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3)) - ((I/2)*Log[2^(1/3)*a^(1/3) - (a -
 I*a*Tan[e + f*x])^(1/3)]*(d*Sec[e + f*x])^(2/3))/(2^(2/3)*a^(2/3)*f*(a - I
*a*Tan[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
```

```
x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\text{integral} = \frac{(d\sec(e+fx))^{2/3}\int\frac{\sqrt[3]{a-ia\tan(e+fx)}}{a+ia\tan(e+fx)}\,dx}{\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$= \frac{(d\sec(e+fx))^{2/3}\int\cos^2(e+fx)(a-ia\tan(e+fx))^{4/3}\,dx}{a^2\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$= \frac{\left(ia(d\sec(e+fx))^{2/3}\right)\text{Subst}\left(\int\frac{1}{(a-x)^2(a+x)^{2/3}}\,dx,x,-ia\tan(e+fx)\right)}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$= \frac{i(d\sec(e+fx))^{2/3}}{2f(a+ia\tan(e+fx))^{4/3}} + \frac{\left(i(d\sec(e+fx))^{2/3}\right)\text{Subst}\left(\int\frac{1}{(a-x)(a+x)^{2/3}}\,dx,x,-ia\tan(e+fx)\right)}{3f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$= \frac{i(d\sec(e+fx))^{2/3}}{2f(a+ia\tan(e+fx))^{4/3}} - \frac{x(d\sec(e+fx))^{2/3}}{6\ 2^{2/3}a^{2/3}\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} - \frac{i\log(\cos(e+fx))(d\sec(e+fx))^{2/3}}{6\ 2^{2/3}a^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} + \frac{\left(i(d\sec(e+fx))^{2/3}\right)\text{Subst}\left(\int\frac{1}{\sqrt[3]{2}\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a-ia\tan(e+fx)}\right)}{2\ 2^{2/3}a^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} + \frac{\left(i(d\sec(e+fx))^{2/3}\right)\text{Subst}\left(\int\frac{1}{2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}x+x^2}\,dx,x,\sqrt[3]{a-ia\tan(e+fx)}\right)}{2\sqrt[3]{2}\sqrt[3]{a}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$= \frac{i(d\sec(e+fx))^{2/3}}{2f(a+ia\tan(e+fx))^{4/3}} - \frac{x(d\sec(e+fx))^{2/3}}{6\ 2^{2/3}a^{2/3}\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} - \frac{i\log(\cos(e+fx))(d\sec(e+fx))^{2/3}}{6\ 2^{2/3}a^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} - \frac{i\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a-ia\tan(e+fx)}\right)(d\sec(e+fx))^{2/3}}{2\ 2^{2/3}a^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} - \frac{\left(i(d\sec(e+fx))^{2/3}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+\frac{2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}}{\sqrt[3]{a}}\right)}{2^{2/3}a^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$= \frac{i(d\sec(e+fx))^{2/3}}{2f(a+ia\tan(e+fx))^{4/3}} - \frac{x(d\sec(e+fx))^{2/3}}{6\ 2^{2/3}a^{2/3}\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$
$$+ \frac{i\arctan\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)(d\sec(e+fx))^{2/3}}{2^{2/3}\sqrt{3}a^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$
$$- \frac{i\log(\cos(e+fx))(d\sec(e+fx))^{2/3}}{6\ 2^{2/3}a^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$
$$- \frac{i\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a-ia\tan(e+fx)}\right)(d\sec(e+fx))^{2/3}}{2\ 2^{2/3}a^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

Mathematica [A] (verified)

Time = 1.94 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.58

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{4/3}}\,dx = \frac{e^{-i(e+fx)}\left(3i+3ie^{2i(e+fx)}-2e^{2i(e+fx)}\sqrt[3]{1+e^{2i(e+fx)}}fx-2i\sqrt{3}e^{2i(e+fx)}\sqrt[3]{1\right.}{}$$

```
[In] Integrate[(d*Sec[e + f*x])^(2/3)/(a + I*a*Tan[e + f*x])^(4/3),x]
```

```
[Out] ((3*I + (3*I)*E^((2*I)*(e + f*x)) - 2*E^((2*I)*(e + f*x))*(1 + E^((2*I)*(e
+ f*x)))^(1/3)*f*x - (2*I)*Sqrt[3]*E^((2*I)*(e + f*x))*(1 + E^((2*I)*(e + f
*x)))^(1/3)*ArcTan[(1 + 2*(1 + E^((2*I)*(e + f*x)))^(1/3))/Sqrt[3]] - (3*I)
*E^((2*I)*(e + f*x))*(1 + E^((2*I)*(e + f*x)))^(1/3)*Log[1 - (1 + E^((2*I)*
(e + f*x)))^(1/3)])*(d*Sec[e + f*x])^(5/3))/(12*d*E^(I*(e + f*x))*f*(a + I*
a*Tan[e + f*x])^(4/3))
```

Maple [F]

$$\int \frac{(d\sec(fx+e))^{\frac{2}{3}}}{(a+ia\tan(fx+e))^{\frac{4}{3}}}dx$$

```
[In] int((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(4/3),x)
```

```
[Out] int((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(4/3),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.36

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{4/3}}\,dx = \frac{\left(4\,a^2 f\left(\frac{i\,d^2}{108\,a^4 f^3}\right)^{\frac{1}{3}} e^{(4i\,fx+4i\,e)}\log\left(-2\left(6i\,a^2 f\left(\frac{i\,d^2}{108\,a^4 f^3}\right)^{\frac{1}{3}} e^{(2i\,fx+2i\,e)} - 2^{\frac{1}{3}}\left(\frac{}{e^{(2i}}\right.\right.\right.\right.}{}$$

```
[In] integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(4/3),x, algorithm="frica
s")
```

```
[Out] 1/4*(4*a^2*f*(1/108*I*d^2/(a^4*f^3))^(1/3)*e^(4*I*f*x + 4*I*e)*log(-2*(6*I*
a^2*f*(1/108*I*d^2/(a^4*f^3))^(1/3)*e^(2*I*f*x + 2*I*e) - 2^(1/3)*(a/(e^(2*
I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(e^(2*I*f*x
+ 2*I*e) + 1)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)) + 2^(1/3)*(a/(e^(2
*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(I*e^(4*I*f
*x + 4*I*e) + 2*I*e^(2*I*f*x + 2*I*e) + I)*e^(2*I*f*x + 2*I*e) - 2*(-I*sqrt
(3)*a^2*f + a^2*f)*(1/108*I*d^2/(a^4*f^3))^(1/3)*e^(4*I*f*x + 4*I*e)*log(2*
(2^(1/3)*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^
(2/3)*(e^(2*I*f*x + 2*I*e) + 1)*e^(2*I*f*x + 2*I*e) + 3*(sqrt(3)*a^2*f + I*
a^2*f)*(1/108*I*d^2/(a^4*f^3))^(1/3)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I
*e)) - 2*(I*sqrt(3)*a^2*f + a^2*f)*(1/108*I*d^2/(a^4*f^3))^(1/3)*e^(4*I*f*x
 + 4*I*e)*log(2*(2^(1/3)*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x
 + 2*I*e) + 1))^(2/3)*(e^(2*I*f*x + 2*I*e) + 1)*e^(2*I*f*x + 2*I*e) - 3*(sq
rt(3)*a^2*f - I*a^2*f)*(1/108*I*d^2/(a^4*f^3))^(1/3)*e^(2*I*f*x + 2*I*e))*e
^(-2*I*f*x - 2*I*e)))*e^(-4*I*f*x - 4*I*e)/(a^2*f)
```

Sympy [F]

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{4/3}}\,dx = \int \frac{(d\sec{(e+fx)})^{\frac{2}{3}}}{(ia\,(\tan{(e+fx)}-i))^{\frac{4}{3}}}\,dx$$

```
[In] integrate((d*sec(f*x+e))**(2/3)/(a+I*a*tan(f*x+e))**(4/3),x)
```

```
[Out] Integral((d*sec(e + f*x))**(2/3)/(I*a*(tan(e + f*x) - I))**(4/3), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1906 vs. 2(279) = 558.

Time = 0.44 (sec) , antiderivative size = 1906, normalized size of antiderivative = 5.04

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{4/3}}\,dx = \text{Too large to display}$$

```
[In] integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(4/3),x, algorithm="maxim
a")
```

```
[Out] -1/24*(6*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)
^(1/3)*((-I*2^(1/3)*cos(2*f*x + 2*e) - 2^(1/3)*sin(2*f*x + 2*e))*cos(2/3*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + (2^(1/3)*cos(2*f*x + 2*e)
- I*2^(1/3)*sin(2*f*x + 2*e))*sin(2/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e) + 1)))*d^(2/3) - (-2*I*sqrt(3)*2^(1/3)*arctan2(2/3*sqrt(3)*(cos(2*f*x
 + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*cos(1/3*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 1/3*sqrt(3), 1/3*sqrt(3)*(2*
(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*si
n(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + sqrt(3))) - 2*I*sq
rt(3)*2^(1/3)*arctan2(2/3*sqrt(3)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
+ 2*cos(2*f*x + 2*e) + 1)^(1/6)*cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x
 + 2*e) + 1)) + 1/3*sqrt(3), -1/3*sqrt(3)*(2*(cos(2*f*x + 2*e)^2 + sin(2*f*
x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*sin(1/3*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e) + 1)) - sqrt(3))) + sqrt(3)*2^(1/3)*log(4/3*(cos(2*f*x +
 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/3)*(cos(1/3*arcta
n2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + sin(1/3*arctan2(sin(2*f*x +
 2*e), cos(2*f*x + 2*e) + 1))^2) + 4/3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*
e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*(sqrt(3)*sin(1/3*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e) + 1)) + cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e) + 1))) + 4/3) - sqrt(3)*2^(1/3)*log(4/3*(cos(2*f*x + 2*e)^2 + sin(2*f
*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/3)*(cos(1/3*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e) + 1))^2 + sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x
+ 2*e) + 1))^2) - 4/3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*
x + 2*e) + 1)^(1/6)*(sqrt(3)*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e) + 1)) - cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))) + 4/
3) - 2*2^(1/3)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f
*x + 2*e) + 1)^(1/3)*sin(2/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1
)) + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/
6)*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2
*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/3)*cos(2/3*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + (cos(2*f*x + 2*e)^2 + sin(2*f*x
+ 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*cos(1/3*arctan2(sin(2*f*x + 2*e),
cos(2*f*x + 2*e) + 1)) + 1) + 4*2^(1/3)*arctan2((cos(2*f*x + 2*e)^2 + sin(2
```

```
*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*sin(1/3*arctan2(sin(2*f*x + 2
*e), cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*c
os(2*f*x + 2*e) + 1)^(1/6)*cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e) + 1)) - 1) - 2*I*2^(1/3)*log((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 +
2*cos(2*f*x + 2*e) + 1)^(1/3)*cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e) + 1))^2 + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*
e) + 1)^(1/3)*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 -
2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*
cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 1) + I*2^(1/3)*l
og((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(2/3)
*(cos(2/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + sin(2/3*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2) + (cos(2*f*x + 2*e)^2 + sin
(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/3)*(cos(1/3*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e) + 1))^2 + sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*
f*x + 2*e) + 1))^2) + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*
f*x + 2*e) + 1)^(1/3)*((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f
*x + 2*e) + 1)^(1/6)*(cos(2/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
1))*cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + sin(2/3*arct
an2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*sin(1/3*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e) + 1))) + cos(2/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e) + 1))) + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2
*e) + 1)^(1/6)*cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 1
))*d^(2/3))/(a^(4/3)*f)
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{4/3}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{2}{3}}}{(i\,a\tan(fx+e)+a)^{\frac{4}{3}}}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(4/3),x, algorithm="giac"
)
[Out] integrate((d*sec(f*x + e))^(2/3)/(I*a*tan(f*x + e) + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{2/3}}{(a+ia\tan(e+fx))^{4/3}}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{2/3}}{(a+a\tan(e+f\,x)\ 1\mathrm{i})^{4/3}}\,dx$$

```
[In] int((d/cos(e + f*x))^(2/3)/(a + a*tan(e + f*x)*1i)^(4/3),x)
[Out] int((d/cos(e + f*x))^(2/3)/(a + a*tan(e + f*x)*1i)^(4/3), x)
```

3.445 $\int \frac{(d\sec(e+fx))^{2/3}}{\sqrt[3]{a+ia\tan(e+fx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 340

$$\begin{aligned}\int \frac{(d\sec(e+fx))^{2/3}}{\sqrt[3]{a+ia\tan(e+fx)}}\,dx &= -\frac{\sqrt[3]{a}x(d\sec(e+fx))^{2/3}}{2\,2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}\\
&+\frac{i\sqrt{3}\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}}{\sqrt{3}\sqrt[3]{a}}\right)(d\sec(e+fx))^{2/3}}{2^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}\\
&-\frac{i\sqrt[3]{a}\log(\cos(e+fx))(d\sec(e+fx))^{2/3}}{2\,2^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}\\
&-\frac{3i\sqrt[3]{a}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a-ia\tan(e+fx)}\right)(d\sec(e+fx))^{2/3}}{2\,2^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}\end{aligned}$$

```
[Out] -1/4*a^(1/3)*x*(d*sec(f*x+e))^(2/3)*2^(1/3)/(a-I*a*tan(f*x+e))^(1/3)/(a+I*a
*tan(f*x+e))^(1/3)-1/4*I*a^(1/3)*ln(cos(f*x+e))*(d*sec(f*x+e))^(2/3)*2^(1/3
)/f/(a-I*a*tan(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^(1/3)-3/4*I*a^(1/3)*ln(2^(1
/3)*a^(1/3)-(a-I*a*tan(f*x+e))^(1/3))*(d*sec(f*x+e))^(2/3)*2^(1/3)/f/(a-I*a
*tan(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^(1/3)+1/2*I*a^(1/3)*arctan(1/3*(a^(1/
3)+2^(2/3)*(a-I*a*tan(f*x+e))^(1/3))/a^(1/3)*3^(1/2))*(d*sec(f*x+e))^(2/3)*
3^(1/2)*2^(1/3)/f/(a-I*a*tan(f*x+e))^(1/3)/(a+I*a*tan(f*x+e))^(1/3)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3573, 3562, 59, 631, 210, 31}

$$\int \frac{(d\sec(e+fx))^{2/3}}{\sqrt[3]{a+ia\tan(e+fx)}}\,dx = \frac{i\sqrt{3}\sqrt[3]{a}(d\sec(e+fx))^{2/3}\arctan\left(\frac{\sqrt[3]{a}+2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}}{\sqrt{3}\sqrt[3]{a}}\right)}{2^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} - \frac{\sqrt[3]{a}x(d\sec(e+fx))^{2/3}}{2\,2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} - \frac{3i\sqrt[3]{a}(d\sec(e+fx))^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a-ia\tan(e+fx)}\right)}{2\,2^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} - \frac{i\sqrt[3]{a}(d\sec(e+fx))^{2/3}\log(\cos(e+fx))}{2\,2^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

```
[In] Int[(d*Sec[e + f*x])^(2/3)/(a + I*a*Tan[e + f*x])^(1/3),x]
```

```
[Out] -1/2*(a^(1/3)*x*(d*Sec[e + f*x])^(2/3))/(2^(2/3)*(a - I*a*Tan[e + f*x])^(1/
3)*(a + I*a*Tan[e + f*x])^(1/3)) + (I*Sqrt[3]*a^(1/3)*ArcTan[(a^(1/3) + 2^(
2/3)*(a - I*a*Tan[e + f*x])^(1/3))/(Sqrt[3]*a^(1/3))]*(d*Sec[e + f*x])^(2/3
))/(2^(2/3)*f*(a - I*a*Tan[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3)) -
((I/2)*a^(1/3)*Log[Cos[e + f*x]]*(d*Sec[e + f*x])^(2/3))/(2^(2/3)*f*(a - I*
a*Tan[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3)) - (((3*I)/2)*a^(1/3)*Lo
g[2^(1/3)*a^(1/3) - (a - I*a*Tan[e + f*x])^(1/3)]*(d*Sec[e + f*x])^(2/3))/(
2^(2/3)*f*(a - I*a*Tan[e + f*x])^(1/3)*(a + I*a*Tan[e + f*x])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3573

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Dist[(a/d)^(2*IntPart[n])*(a + b*Tan[e + f*x])^Frac
Part[n]*((a - b*Tan[e + f*x])^FracPart[n]/(d*Sec[e + f*x])^(2*FracPart[n]))
, Int[1/(a - b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] &
& EqQ[a^2 + b^2, 0] && EqQ[Simplify[m/2 + n], 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(d\sec(e+fx))^{2/3}\int\sqrt[3]{a-ia\tan(e+fx)}\,dx}{\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}\\
&= \frac{\left(ia(d\sec(e+fx))^{2/3}\right)\text{Subst}\left(\int\frac{1}{(a-x)(a+x)^{2/3}}\,dx,x,-ia\tan(e+fx)\right)}{f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}\\
&= -\frac{\sqrt[3]{a}x(d\sec(e+fx))^{2/3}}{2\ 2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}\\
&\quad-\frac{i\sqrt[3]{a}\log(\cos(e+fx))(d\sec(e+fx))^{2/3}}{2\ 2^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}\\
&\quad+\frac{\left(3i\sqrt[3]{a}(d\sec(e+fx))^{2/3}\right)\text{Subst}\left(\int\frac{1}{\sqrt[3]{2}\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a-ia\tan(e+fx)}\right)}{2\ 2^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}\\
&\quad+\frac{\left(3ia^{2/3}(d\sec(e+fx))^{2/3}\right)\text{Subst}\left(\int\frac{1}{2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}x+x^2}\,dx,x,\sqrt[3]{a-ia\tan(e+fx)}\right)}{2\sqrt[3]{2}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}
\end{aligned}
$$

$$= -\frac{\sqrt[3]{a}x(d\sec(e+fx))^{2/3}}{2\ 2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} - \frac{i\sqrt[3]{a}\log(\cos(e+fx))(d\sec(e+fx))^{2/3}}{2\ 2^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} - \frac{3i\sqrt[3]{a}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a-ia\tan(e+fx)}\right)(d\sec(e+fx))^{2/3}}{2\ 2^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} - \frac{\left(3i\sqrt[3]{a}(d\sec(e+fx))^{2/3}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}dx, x, 1+\frac{2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}}{\sqrt[3]{a}}\right)}{2^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

$$= -\frac{\sqrt[3]{a}x(d\sec(e+fx))^{2/3}}{2\ 2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} + \frac{i\sqrt{3}\sqrt[3]{a}\arctan\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a-ia\tan(e+fx)}}{\sqrt[3]{a}}}{\sqrt{3}}\right)(d\sec(e+fx))^{2/3}}{2^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} - \frac{i\sqrt[3]{a}\log(\cos(e+fx))(d\sec(e+fx))^{2/3}}{2\ 2^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}} - \frac{3i\sqrt[3]{a}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a-ia\tan(e+fx)}\right)(d\sec(e+fx))^{2/3}}{2\ 2^{2/3}f\sqrt[3]{a-ia\tan(e+fx)}\sqrt[3]{a+ia\tan(e+fx)}}$$

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.47

$$\int\frac{(d\sec(e+fx))^{2/3}}{\sqrt[3]{a+ia\tan(e+fx)}}\,dx = -\frac{\left(\frac{de^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{2/3}\sqrt[3]{1+e^{2i(e+fx)}}\left(2fx+2i\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1+e^{2i(e+fx)}}}{\sqrt{3}}\right)+3i\log\left(1-\sqrt[3]{1+e^{2i(e+fx)}}\right)\right)}{2\ 2^{2/3}\sqrt[3]{\frac{ae^{2i(e+fx)}}{1+e^{2i(e+fx)}}}f}$$

```
[In] Integrate[(d*Sec[e + f*x])^(2/3)/(a + I*a*Tan[e + f*x])^(1/3),x]
```

```
[Out] -1/2*(((d*E^(I*(e + f*x)))/(1 + E^((2*I)*(e + f*x))))^(2/3)*(1 + E^((2*I)*(
e + f*x)))^(1/3)*(2*f*x + (2*I)*Sqrt[3]*ArcTan[(1 + 2*(1 + E^((2*I)*(e + f*
x)))^(1/3))/Sqrt[3]] + (3*I)*Log[1 - (1 + E^((2*I)*(e + f*x)))^(1/3)]))/(2^
(2/3)*((a*E^((2*I)*(e + f*x)))/(1 + E^((2*I)*(e + f*x))))^(1/3)*f)
```

Maple [F]

$$\int \frac{(d\sec(fx+e))^{\frac{2}{3}}}{(a+ia\tan(fx+e))^{\frac{1}{3}}}dx$$

[In] int((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(1/3),x)

[Out] int((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(1/3),x)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.08

$$\int \frac{(d\sec(e+fx))^{2/3}}{\sqrt[3]{a+ia\tan(e+fx)}}\,dx = \frac{1}{2}\left(i\sqrt{3}-1\right)\left(\frac{i\,d^2}{4\,af^3}\right)^{\frac{1}{3}}\log\left(2\left(2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}\left(\frac{d}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}(e\right.\right.$$
$$+\frac{1}{2}\left(-i\sqrt{3}-1\right)\left(\frac{i\,d^2}{4\,af^3}\right)^{\frac{1}{3}}\log\left(2\left(2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}\left(\frac{d}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}\left(e^{(2i\,fx+2i\,e)}+1\right)e^{(2i\,fx+2i\,e)}-\right.\right.$$
$$+\left(\frac{i\,d^2}{4\,af^3}\right)^{\frac{1}{3}}\log\left(2\left(2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}\left(\frac{d}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}\left(e^{(2i\,fx+2i\,e)}+1\right)e^{(2i\,fx+2i\,e)}-2i\,af\left(\frac{i\,d^2}{4\,af^3}\right)\right.\right.$$

[In] integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(1/3),x, algorithm="frica
s")

[Out] 1/2*(I*sqrt(3) - 1)*(1/4*I*d^2/(a*f^3))^(1/3)*log(2*(2^(1/3)*(a/(e^(2*I*f*x
 + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(e^(2*I*f*x + 2*I
*e) + 1)*e^(2*I*f*x + 2*I*e) + (sqrt(3)*a*f + I*a*f)*(1/4*I*d^2/(a*f^3))^(1
/3)*e^(2*I*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e)) + 1/2*(-I*sqrt(3) - 1)*(1/4*
I*d^2/(a*f^3))^(1/3)*log(2*(2^(1/3)*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/
(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(e^(2*I*f*x + 2*I*e) + 1)*e^(2*I*f*x + 2*I
*e) - (sqrt(3)*a*f - I*a*f)*(1/4*I*d^2/(a*f^3))^(1/3)*e^(2*I*f*x + 2*I*e))*
e^(-2*I*f*x - 2*I*e)) + (1/4*I*d^2/(a*f^3))^(1/3)*log(2*(2^(1/3)*(a/(e^(2*I
*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(e^(2*I*f*x +
 2*I*e) + 1)*e^(2*I*f*x + 2*I*e) - 2*I*a*f*(1/4*I*d^2/(a*f^3))^(1/3)*e^(2*I
*f*x + 2*I*e))*e^(-2*I*f*x - 2*I*e))

Sympy [F]

$$\int \frac{(d\sec(e+fx))^{2/3}}{\sqrt[3]{a+ia\tan(e+fx)}}\,dx = \int \frac{(d\sec{(e+fx)})^{\frac{2}{3}}}{\sqrt[3]{ia\,(\tan{(e+fx)}-i)}}\,dx$$

```
[In] integrate((d*sec(f*x+e))**(2/3)/(a+I*a*tan(f*x+e))**(1/3),x)
[Out] Integral((d*sec(e + f*x))**(2/3)/(I*a*(tan(e + f*x) - I))**(1/3), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1753 vs. $2(251) = 502$.

Time = 0.47 (sec) , antiderivative size = 1753, normalized size of antiderivative = 5.16

$$\int \frac{(d\sec(e+fx))^{2/3}}{\sqrt[3]{a+ia\tan(e+fx)}}\,dx = \text{Too large to display}$$

```
[In] integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(1/3),x, algorithm="maxim
a")
[Out] 1/8*(-2*I*sqrt(3)*2^(1/3)*arctan2(2/3*sqrt(3)*(cos(2*f*x + 2*e)^2 + sin(2*f
*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*cos(1/3*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e) + 1)) + 1/3*sqrt(3), 1/3*sqrt(3)*(2*(cos(2*f*x + 2*e)^2
 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*sin(1/3*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + sqrt(3))) - 2*I*sqrt(3)*2^(1/3)*arcta
n2(2/3*sqrt(3)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e
) + 1)^(1/6)*cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 1/3
*sqrt(3), -1/3*sqrt(3)*(2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(
2*f*x + 2*e) + 1)^(1/6)*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e)
+ 1)) - sqrt(3))) + sqrt(3)*2^(1/3)*log(4/3*(cos(2*f*x + 2*e)^2 + sin(2*f*x
 + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/3)*(cos(1/3*arctan2(sin(2*f*x + 2*e)
, cos(2*f*x + 2*e) + 1))^2 + sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e) + 1))^2) + 4/3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x
+ 2*e) + 1)^(1/6)*(sqrt(3)*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e) + 1)) + cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))) + 4/3)
 - sqrt(3)*2^(1/3)*log(4/3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos
(2*f*x + 2*e) + 1)^(1/3)*(cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
) + 1))^2 + sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2) - 4
/3*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)
*(sqrt(3)*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - cos(1/
3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))) + 4/3) - 2*2^(1/3)*arct
an2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/3
```

```
)*sin(2/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + (cos(2*f*x + 2
*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*sin(1/3*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x +
 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/3)*cos(2/3*arctan2(sin(2*f*x + 2*e), c
os(2*f*x + 2*e) + 1)) + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*
f*x + 2*e) + 1)^(1/6)*cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
1)) + 1) + 4*2^(1/3)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*c
os(2*f*x + 2*e) + 1)^(1/6)*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e) + 1)), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1
)^(1/6)*cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 1) - 2*I
*2^(1/3)*log((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e)
+ 1)^(1/3)*cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + (co
s(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/3)*sin(1
/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 - 2*(cos(2*f*x + 2*e)
^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*cos(1/3*arctan2(sin
(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 1) + I*2^(1/3)*log((cos(2*f*x + 2*e
)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(2/3)*(cos(2/3*arctan2(s
in(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + sin(2/3*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e) + 1))^2) + (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2
*cos(2*f*x + 2*e) + 1)^(1/3)*(cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
 2*e) + 1))^2 + sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2)
 + 2*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/
3)*((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6
)*(cos(2/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))*cos(1/3*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + sin(2/3*arctan2(sin(2*f*x + 2*e
), cos(2*f*x + 2*e) + 1))*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
) + 1))) + cos(2/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))) + 2*(c
os(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)*cos(
1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 1))*d^(2/3)/(a^(1/3)
*f)
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^{2/3}}{\sqrt[3]{a+ia\tan(e+fx)}}\,dx = \int \frac{(d\sec(fx+e))^{\frac{2}{3}}}{(i\,a\tan(fx+e)+a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(2/3)/(a+I*a*tan(f*x+e))^(1/3),x, algorithm="giac"
)
```

```
[Out] integrate((d*sec(f*x + e))^(2/3)/(I*a*tan(f*x + e) + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{2/3}}{\sqrt[3]{a+ia\tan(e+fx)}}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{2/3}}{\left(a+a\tan\left(e+f\,x\right)\,1\mathrm{i}\right)^{1/3}}\,dx$$

```
[In] int((d/cos(e + f*x))^(2/3)/(a + a*tan(e + f*x)*1i)^(1/3),x)
[Out] int((d/cos(e + f*x))^(2/3)/(a + a*tan(e + f*x)*1i)^(1/3), x)
```

3.446 $\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 37

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}\,dx = \frac{3ia(d\sec(e+fx))^{2/3}}{f\sqrt[3]{a+ia\tan(e+fx)}}$$

```
[Out] 3*I*a*(d*sec(f*x+e))^(2/3)/f/(a+I*a*tan(f*x+e))^(1/3)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {3574}

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}\,dx = \frac{3ia(d\sec(e+fx))^{2/3}}{f\sqrt[3]{a+ia\tan(e+fx)}}$$

```
[In] Int[(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(2/3),x]
[Out] ((3*I)*a*(d*Sec[e + f*x])^(2/3))/(f*(a + I*a*Tan[e + f*x])^(1/3))
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\text{integral} = \frac{3ia(d\sec(e+fx))^{2/3}}{f\sqrt[3]{a+ia\tan(e+fx)}}$$

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}\,dx=\frac{3d^2(i+\tan(e+fx))(a+ia\tan(e+fx))^{2/3}}{f(d\sec(e+fx))^{4/3}}$$

```
[In] Integrate[(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(2/3),x]
```

```
[Out] (3*d^2*(I + Tan[e + f*x])*(a + I*a*Tan[e + f*x])^(2/3))/(f*(d*Sec[e + f*x])
^(4/3))
```

Maple [F]

$$\int (d\sec(fx+e))^{\frac{2}{3}}(a+ia\tan(fx+e))^{\frac{2}{3}}\,dx$$

```
[In] int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(2/3),x)
```

```
[Out] int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(2/3),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.49

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}\,dx = -\frac{3\cdot 2^{\frac{1}{3}}\left(\frac{a}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}\left(\frac{d}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}\left(-i\,e^{(2i\,fx+2i\,e)}-i\right)}{f}$$

```
[In] integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(2/3),x, algorithm="frica
s")
```

```
[Out] -3*2^(1/3)*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1)
)^(2/3)*(-I*e^(2*I*f*x + 2*I*e) - I)/f
```

Sympy [F]

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}\,dx = \int (d\sec{(e+fx)})^{\frac{2}{3}}\,(ia(\tan{(e+fx)}-i))^{\frac{2}{3}}\,dx$$

```
[In] integrate((d*sec(f*x+e))**(2/3)*(a+I*a*tan(f*x+e))**(2/3),x)
[Out] Integral((d*sec(e + f*x))**(2/3)*(I*a*(tan(e + f*x) - I))**(2/3), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(29) = 58.

Time = 0.62 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.89

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}\,dx =$$

$$-\frac{3\left(-i\cdot 2^{\frac{1}{3}}\cos\left(\frac{1}{3}\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)+1\right)\right)-2^{\frac{1}{3}}\sin\left(\frac{1}{3}\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\right.\right.\right.\right.}{\left(\cos\left(2\,fx+2\,e\right)^{2}+\sin\left(2\,fx+2\,e\right)^{2}+2\,\cos\left(2\,fx+2\,e\right)+1\right)^{\frac{1}{6}}f}$$

```
[In] integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(2/3),x, algorithm="maxim
a")
[Out] -3*(-I*2^(1/3)*cos(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 2
^(1/3)*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*a^(2/3)*d^
(2/3)/((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(
1/6)*f)
```

Giac [F]

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}\,dx = \int (d\sec{(fx+e)})^{\frac{2}{3}}\,(i\,a\tan{(fx+e)}+a)^{\frac{2}{3}}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(2/3),x, algorithm="giac"
)
[Out] integrate((d*sec(f*x + e))^(2/3)*(I*a*tan(f*x + e) + a)^(2/3), x)
```

Mupad [B] (verification not implemented)

Time = 5.79 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.19

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}\,dx=\frac{3\left(\frac{d}{\cos(e+f\,x)}\right)^{2/3}\,\left(\cos\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}+\sin\left(2\,e+2\,f\,x\right)+1{}\mathrm{i}\right)\,\left(\frac{a\,\left(\cos(2\,e+2\,f\,x)+1+\sin(2\,e+2\,f\,x)\,1{}\mathrm{i}\right)}{\cos(2\,e+2\,f\,x)+1}\right)^{2/3}}{2\,f}$$

```
[In] int((d/cos(e + f*x))^(2/3)*(a + a*tan(e + f*x)*1i)^(2/3),x)
```

```
[Out] (3*(d/cos(e + f*x))^(2/3)*(cos(2*e + 2*f*x)*1i + sin(2*e + 2*f*x) + 1i)*((a
*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i + 1))/(cos(2*e + 2*f*x) + 1))^(2/3
))/(2*f)
```

3.447 $\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 81

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}\,dx = \frac{9ia^2(d\sec(e+fx))^{2/3}}{2f\sqrt[3]{a+ia\tan(e+fx)}} + \frac{3ia(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}}{4f}$$

```
[Out] 9/2*I*a^2*(d*sec(f*x+e))^(2/3)/f/(a+I*a*tan(f*x+e))^(1/3)+3/4*I*a*(d*sec(f*
x+e))^(2/3)*(a+I*a*tan(f*x+e))^(2/3)/f
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3575, 3574}

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}\,dx = \frac{9ia^2(d\sec(e+fx))^{2/3}}{2f\sqrt[3]{a+ia\tan(e+fx)}} + \frac{3ia(a+ia\tan(e+fx))^{2/3}(d\sec(e+fx))^{2/3}}{4f}$$

```
[In] Int[(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(5/3),x]
```

```
[Out] (((9*I)/2)*a^2*(d*Sec[e + f*x])^(2/3))/(f*(a + I*a*Tan[e + f*x])^(1/3)) + (
((3*I)/4)*a*(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(2/3))/f
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
```

```
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{3ia(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}}{4f} \\ &\quad+\frac{1}{2}(3a)\int(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}\,dx \\ &= \frac{9ia^2(d\sec(e+fx))^{2/3}}{2f\sqrt[3]{a+ia\tan(e+fx)}}+\frac{3ia(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}}{4f}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}\,dx = -\frac{3ad(\cos(e)-i\sin(e))(\cos(fx)-i\sin(fx))(-7i+\tan(e+fx))(a+ia\tan(e+fx))^{2/3}}{4f\sqrt[3]{d\sec(e+fx)}}$$

```
[In] Integrate[(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(5/3),x]
[Out] (-3*a*d*(Cos[e] - I*Sin[e])*(Cos[f*x] - I*Sin[f*x])*(-7*I + Tan[e + f*x])*(
a + I*a*Tan[e + f*x])^(2/3))/(4*f*(d*Sec[e + f*x])^(1/3))
```

Maple [F]

$$\int \left(d \sec \left(f x +e \right)\right)^{\frac{2}{3}} \left(a +i a \tan \left(f x +e \right)\right)^{\frac{5}{3}}d x$$

```
[In] int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(5/3),x)
[Out] int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(5/3),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.72

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}\,dx = -\frac{3\cdot 2^{\frac{1}{3}}\left(-4i\,ae^{(2i\,fx+2i\,e)}-3i\,a\right)\left(\frac{a}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}\left(\frac{d}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}}{2\,f}$$

```
[In] integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(5/3),x, algorithm="frica
s")
[Out] -3/2*2^(1/3)*(-4*I*a*e^(2*I*f*x + 2*I*e) - 3*I*a)*(a/(e^(2*I*f*x + 2*I*e) +
 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)/f
```

Sympy [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))**(2/3)*(a+I*a*tan(f*x+e))**(5/3),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(61) = 122.

Time = 0.58 (sec) , antiderivative size = 316, normalized size of antiderivative = 3.90

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}\,dx = \frac{3\left(\left(\left(-i\cdot 2^{\frac{1}{3}}a\cos\left(\frac{4}{3}\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)+1\right)\right)-2^{\frac{1}{3}}a\sin\left(\frac{4}{3}\operatorname{arc}\right.\right.\right.\right.}{}$$

```
[In] integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(5/3),x, algorithm="maxim
a")
```

```
[Out] 3/2*((-I*2^(1/3)*a*cos(4/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))
 - 2^(1/3)*a*sin(4/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*sqrt
(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*a^(2/3)*
d^(2/3) + 4*((I*2^(1/3)*a*cos(2*f*x + 2*e)^2 + I*2^(1/3)*a*sin(2*f*x + 2*e)
^2 + 2*I*2^(1/3)*a*cos(2*f*x + 2*e) + I*2^(1/3)*a)*cos(1/3*arctan2(sin(2*f*
x + 2*e), cos(2*f*x + 2*e) + 1)) + (2^(1/3)*a*cos(2*f*x + 2*e)^2 + 2^(1/3)*
a*sin(2*f*x + 2*e)^2 + 2*2^(1/3)*a*cos(2*f*x + 2*e) + 2^(1/3)*a)*sin(1/3*ar
ctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*a^(2/3)*d^(2/3))/((cos(2*f*
x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(7/6)*f)
```

Giac [F]

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}\,dx = \int (d\sec\left(fx+e\right))^{\frac{2}{3}}\left(i\,a\tan\left(fx+e\right)+a\right)^{\frac{5}{3}}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(5/3),x, algorithm="giac"
)
```

```
[Out] integrate((d*sec(f*x + e))^(2/3)*(I*a*tan(f*x + e) + a)^(5/3), x)
```

Mupad [B] (verification not implemented)

Time = 5.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.11

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}\,dx = \frac{3\,a\,\left(\frac{d}{2\cos\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1}\right)^{2/3}\,\left(\cos\left(e+f\,x\right)^2\,6{}\mathrm{i}+3\,\sin\left(2\,e+2\,f\,x\right)+1{}\mathrm{i}\right)\,\left(\frac{a\,\left(2\cos(e+f\,x)^2+\mathrm{si}\ldots\right.}{2\cos(e+\ldots}\right.}{4\,f}$$

```
[In] int((d/cos(e + f*x))^(2/3)*(a + a*tan(e + f*x)*1i)^(5/3),x)
```

```
[Out] (3*a*(d/(2*cos(e/2 + (f*x)/2)^2 - 1))^(2/3)*(3*sin(2*e + 2*f*x) + cos(e + f
*x)^2*6i + 1i)*((a*(sin(2*e + 2*f*x)*1i + 2*cos(e + f*x)^2))/(2*cos(e + f*x
)^2))^(2/3))/(4*f)
```

3.448 $\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 122

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}\,dx = \frac{54ia^3(d\sec(e+fx))^{2/3}}{7f\sqrt[3]{a+ia\tan(e+fx)}} + \frac{9ia^2(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}}{7f} + \frac{3ia(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}}{7f}$$

```
[Out] 54/7*I*a^3*(d*sec(f*x+e))^(2/3)/f/(a+I*a*tan(f*x+e))^(1/3)+9/7*I*a^2*(d*sec
(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(2/3)/f+3/7*I*a*(d*sec(f*x+e))^(2/3)*(a+I
*a*tan(f*x+e))^(5/3)/f
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3575, 3574}

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}\,dx = \frac{54ia^3(d\sec(e+fx))^{2/3}}{7f\sqrt[3]{a+ia\tan(e+fx)}} + \frac{9ia^2(a+ia\tan(e+fx))^{2/3}(d\sec(e+fx))^{2/3}}{7f} + \frac{3ia(a+ia\tan(e+fx))^{5/3}(d\sec(e+fx))^{2/3}}{7f}$$

```
[In] Int[(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(8/3),x]
```

```
[Out] (((54*I)/7)*a^3*(d*Sec[e + f*x])^(2/3))/(f*(a + I*a*Tan[e + f*x])^(1/3)) +
(((9*I)/7)*a^2*(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(2/3))/f + (((
3*I)/7)*a*(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(5/3))/f
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3ia(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}}{7f} \\
&\quad + \frac{1}{7}(12a)\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}\,dx \\
&= \frac{9ia^2(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}}{7f} \\
&\quad + \frac{3ia(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}}{7f} \\
&\quad + \frac{1}{7}\left(18a^2\right)\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}\,dx \\
&= \frac{54ia^3(d\sec(e+fx))^{2/3}}{7f\sqrt[3]{a+ia\tan(e+fx)}} + \frac{9ia^2(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}}{7f} \\
&\quad + \frac{3ia(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}}{7f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.82

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}\,dx = \frac{3a^2(d\sec(e+fx))^{5/3}(i\cos(e-fx)+\sin(e-fx))(21+23\cos(2(e+fx))+5i\sin(}{14df(\cos(fx)+i\sin(fx))^2}$$

```
[In] Integrate[(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(8/3),x]
```

```
[Out] (3*a^2*(d*Sec[e + f*x])^(5/3)*(I*Cos[e - f*x] + Sin[e - f*x])*(21 + 23*Cos[
2*(e + f*x)] + (5*I)*Sin[2*(e + f*x)])*(a + I*a*Tan[e + f*x])^(2/3))/(14*d*
f*(Cos[f*x] + I*Sin[f*x])^2)
```

Maple [F]

$$\int (d\sec(fx+e))^{\frac{2}{3}}(a+ia\tan(fx+e))^{\frac{8}{3}}\,dx$$

```
[In] int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(8/3),x)
```

```
[Out] int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(8/3),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.88

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}\,dx = -\frac{6\cdot 2^{\frac{1}{3}}\left(-14i\,a^2e^{(4i\,fx+4i\,e)}-21i\,a^2e^{(2i\,fx+2i\,e)}-9i\,a^2\right)\left(\frac{a}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}\left(\frac{d}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}e^{(2i\,fx+2i\,e)}}{7\left(fe^{(4i\,fx+4i\,e)}+fe^{(2i\,fx+2i\,e)}\right)}$$

```
[In] integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(8/3),x, algorithm="frica
s")
```

```
[Out] -6/7*2^(1/3)*(-14*I*a^2*e^(4*I*f*x + 4*I*e) - 21*I*a^2*e^(2*I*f*x + 2*I*e)
- 9*I*a^2)*(a/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1)
)^(2/3)*e^(2*I*f*x + 2*I*e)/(f*e^(4*I*f*x + 4*I*e) + f*e^(2*I*f*x + 2*I*e))
```

Sympy [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))**(2/3)*(a+I*a*tan(f*x+e))**(8/3),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(92) = 184$.

Time = 0.89 (sec) , antiderivative size = 402, normalized size of antiderivative = 3.30

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}\,dx = \frac{6\left(7\left(-i\cdot 2^{\frac{1}{3}}a^2\cos\left(\frac{4}{3}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2e\right)+1\right)\right)-2^{\frac{1}{3}}a^2\sin\left(\frac{4}{3}\text{ a}\right.\right.\right.}{}$$

```
[In] integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(8/3),x, algorithm="maxim
a")
```

```
[Out] 6/7*(7*(-I*2^(1/3)*a^2*cos(4/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) +
 1)) - 2^(1/3)*a^2*sin(4/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))
)*sqrt(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*a^
(2/3)*d^(2/3) + 2*(I*2^(1/3)*a^2*cos(7/3*arctan2(sin(2*f*x + 2*e), cos(2*f*
x + 2*e) + 1)) + 2^(1/3)*a^2*sin(7/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x +
2*e) + 1)) + 7*(I*2^(1/3)*a^2*cos(2*f*x + 2*e)^2 + I*2^(1/3)*a^2*sin(2*f*x
+ 2*e)^2 + 2*I*2^(1/3)*a^2*cos(2*f*x + 2*e) + I*2^(1/3)*a^2)*cos(1/3*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 7*(2^(1/3)*a^2*cos(2*f*x + 2*e
)^2 + 2^(1/3)*a^2*sin(2*f*x + 2*e)^2 + 2*2^(1/3)*a^2*cos(2*f*x + 2*e) + 2^(
1/3)*a^2)*sin(1/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*a^(2/3)
*d^(2/3))/((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
1)^(7/6)*f)
```

Giac [F]

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}\,dx=\int (d\sec{(fx+e)})^{\frac{2}{3}}\,(i\,a\tan{(fx+e)}+a)^{\frac{8}{3}}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(8/3),x, algorithm="giac"
)
```

```
[Out] integrate((d*sec(f*x + e))^(2/3)*(I*a*tan(f*x + e) + a)^(8/3), x)
```

Mupad [B] (verification not implemented)

Time = 6.36 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}\,dx=\frac{3\,a^2\,\left(\frac{d}{\cos(e+f\,x)}\right)^{2/3}\,\left(\frac{a\,(\cos(2\,e+2\,f\,x)+1+\sin(2\,e+2\,f\,x)\,1\mathrm{i})}{\cos(2\,e+2\,f\,x)+1}\right)^{2/3}\,(\cos{(2\,e+2\,f\,x)}\,44\mathrm{i}+\cos{(4\,e}}{14\,f\,\left(\cos{(2\,e+2\,f\,x)}+\right.}$$

```
[In] int((d/cos(e + f*x))^(2/3)*(a + a*tan(e + f*x)*1i)^(8/3),x)
```

```
[Out] (3*a^2*(d/cos(e + f*x))^(2/3)*((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i +
 1))/(cos(2*e + 2*f*x) + 1))^(2/3)*(cos(2*e + 2*f*x)*44i + cos(4*e + 4*f*x)
*9i + 16*sin(2*e + 2*f*x) + 9*sin(4*e + 4*f*x) + 35i))/(14*f*(cos(2*e + 2*f
*x) + 1))
```

3.449 $\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{11/3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 163

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{11/3}\,dx = \frac{486ia^4(d\sec(e+fx))^{2/3}}{35f\sqrt[3]{a+ia\tan(e+fx)}} + \frac{81ia^3(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}}{35f} + \frac{27ia^2(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}}{35f} + \frac{3ia(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}}{10f}$$

```
[Out] 486/35*I*a^4*(d*sec(f*x+e))^(2/3)/f/(a+I*a*tan(f*x+e))^(1/3)+81/35*I*a^3*(d
*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(2/3)/f+27/35*I*a^2*(d*sec(f*x+e))^(2
/3)*(a+I*a*tan(f*x+e))^(5/3)/f+3/10*I*a*(d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x
+e))^(8/3)/f
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used

$= \{3575, 3574\}$

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{11/3}\,dx = \frac{486ia^4(d\sec(e+fx))^{2/3}}{35f\sqrt[3]{a+ia\tan(e+fx)}} + \frac{81ia^3(a+ia\tan(e+fx))^{2/3}(d\sec(e+fx))^{2/3}}{35f} + \frac{27ia^2(a+ia\tan(e+fx))^{5/3}(d\sec(e+fx))^{2/3}}{35f} + \frac{3ia(a+ia\tan(e+fx))^{8/3}(d\sec(e+fx))^{2/3}}{10f}$$

```
[In] Int[(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(11/3),x]
```

```
[Out] (((486*I)/35)*a^4*(d*Sec[e + f*x])^(2/3))/(f*(a + I*a*Tan[e + f*x])^(1/3))
+ (((81*I)/35)*a^3*(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(2/3))/f +
 (((27*I)/35)*a^2*(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(5/3))/f +
(((3*I)/10)*a*(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(8/3))/f
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\text{integral} = \frac{3ia(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}}{10f} + \frac{1}{5}(9a)\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}\,dx$$

$$= \frac{27ia^2(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}}{35f} + \frac{3ia(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}}{10f} + \frac{1}{35}(108a^2)\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}\,dx$$

$$
\begin{aligned}
&= \frac{81ia^3(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}}{35f} \\
&\quad + \frac{27ia^2(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}}{35f} \\
&\quad + \frac{3ia(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}}{10f} \\
&\quad + \frac{1}{35}(162a^3)\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}\,dx \\
&= \frac{486ia^4(d\sec(e+fx))^{2/3}}{35f\sqrt[3]{a+ia\tan(e+fx)}} + \frac{81ia^3(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{2/3}}{35f} \\
&\quad + \frac{27ia^2(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{5/3}}{35f} \\
&\quad + \frac{3ia(d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{8/3}}{10f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.04 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.71

$$
\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{11/3}\,dx = \frac{3a^3(d\sec(e+fx))^{5/3}(i\cos(e-2fx)+\sin(e-2fx))(364+442\cos(2(e+fx))+59}{140df(\cos(fx)+i\,\mathrm{si}}
$$

```
[In] Integrate[(d*Sec[e + f*x])^(2/3)*(a + I*a*Tan[e + f*x])^(11/3),x]
```

```
[Out] (3*a^3*(d*Sec[e + f*x])^(5/3)*(I*Cos[e - 2*f*x] + Sin[e - 2*f*x])*(364 + 44
2*Cos[2*(e + f*x)] + (59*I)*Sec[e + f*x]*Sin[3*(e + f*x)] + (45*I)*Tan[e +
f*x])*(a + I*a*Tan[e + f*x])^(2/3))/(140*d*f*(Cos[f*x] + I*Sin[f*x])^3)
```

Maple [F]

$$
\int (d\sec(fx+e))^{\frac{2}{3}}(a+ia\tan(fx+e))^{\frac{11}{3}}\,dx
$$

```
[In] int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(11/3),x)
```

```
[Out] int((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(11/3),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.82

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{11/3}\,dx = -\frac{6\cdot 2^{\frac{1}{3}}\left(-140i\,a^3e^{(6i\,fx+6i\,e)}-315i\,a^3e^{(4i\,fx+4i\,e)}-270i\,a^3e^{(2i\,fx+2i\,e)}-81i\,a^3\right)\left(\frac{a}{e^{(2i\,fx+2i\,e)}+1}\right)^{\frac{2}{3}}\left(\frac{d}{e^{(2i\,fx+2i\,e)}+1}\right)}{35\left(fe^{(6i\,fx+6i\,e)}+2\,fe^{(4i\,fx+4i\,e)}+fe^{(2i\,fx+2i\,e)}\right)}$$

```
[In] integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(11/3),x, algorithm="fric
as")
```

```
[Out] -6/35*2^(1/3)*(-140*I*a^3*e^(6*I*f*x + 6*I*e) - 315*I*a^3*e^(4*I*f*x + 4*I*
e) - 270*I*a^3*e^(2*I*f*x + 2*I*e) - 81*I*a^3)*(a/(e^(2*I*f*x + 2*I*e) + 1)
)^(2/3)*(d/(e^(2*I*f*x + 2*I*e) + 1))^(2/3)*e^(2*I*f*x + 2*I*e)/(f*e^(6*I*f
*x + 6*I*e) + 2*f*e^(4*I*f*x + 4*I*e) + f*e^(2*I*f*x + 2*I*e))
```

Sympy [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{11/3}\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))**(2/3)*(a+I*a*tan(f*x+e))**(11/3),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 983 vs. 2(123) = 246.

Time = 0.40 (sec) , antiderivative size = 983, normalized size of antiderivative = 6.03

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{11/3}\,dx = \text{Too large to display}$$

```
[In] integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(11/3),x, algorithm="maxi
ma")
```

```
[Out] 6/35*(7*(-2*I*2^(1/3)*a^3*cos(10/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e) + 1)) - 2*2^(1/3)*a^3*sin(10/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e
) + 1)) + 15*(-I*2^(1/3)*a^3*cos(2*f*x + 2*e)^2 - I*2^(1/3)*a^3*sin(2*f*x +
```

```
2*e)^2 - 2*I*2^(1/3)*a^3*cos(2*f*x + 2*e) - I*2^(1/3)*a^3)*cos(4/3*arctan2
(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) - 15*(2^(1/3)*a^3*cos(2*f*x + 2*e
)^2 + 2^(1/3)*a^3*sin(2*f*x + 2*e)^2 + 2*2^(1/3)*a^3*cos(2*f*x + 2*e) + 2^(
1/3)*a^3)*sin(4/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))*sqrt(co
s(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*a^(2/3)*d^(
2/3) + 20*(3*(I*2^(1/3)*a^3*cos(2*f*x + 2*e)^2 + I*2^(1/3)*a^3*sin(2*f*x +
2*e)^2 + 2*I*2^(1/3)*a^3*cos(2*f*x + 2*e) + I*2^(1/3)*a^3)*cos(7/3*arctan2(
sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 7*(I*2^(1/3)*a^3*cos(2*f*x + 2*e
)^4 + I*2^(1/3)*a^3*sin(2*f*x + 2*e)^4 + 4*I*2^(1/3)*a^3*cos(2*f*x + 2*e)^3
 + 6*I*2^(1/3)*a^3*cos(2*f*x + 2*e)^2 + 4*I*2^(1/3)*a^3*cos(2*f*x + 2*e) +
I*2^(1/3)*a^3 + 2*(I*2^(1/3)*a^3*cos(2*f*x + 2*e)^2 + 2*I*2^(1/3)*a^3*cos(2
*f*x + 2*e) + I*2^(1/3)*a^3)*sin(2*f*x + 2*e)^2)*cos(1/3*arctan2(sin(2*f*x
+ 2*e), cos(2*f*x + 2*e) + 1)) + 3*(2^(1/3)*a^3*cos(2*f*x + 2*e)^2 + 2^(1/3
)*a^3*sin(2*f*x + 2*e)^2 + 2*2^(1/3)*a^3*cos(2*f*x + 2*e) + 2^(1/3)*a^3)*si
n(7/3*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 7*(2^(1/3)*a^3*cos
(2*f*x + 2*e)^4 + 2^(1/3)*a^3*sin(2*f*x + 2*e)^4 + 4*2^(1/3)*a^3*cos(2*f*x
+ 2*e)^3 + 6*2^(1/3)*a^3*cos(2*f*x + 2*e)^2 + 4*2^(1/3)*a^3*cos(2*f*x + 2*e
) + 2^(1/3)*a^3 + 2*(2^(1/3)*a^3*cos(2*f*x + 2*e)^2 + 2*2^(1/3)*a^3*cos(2*f
*x + 2*e) + 2^(1/3)*a^3)*sin(2*f*x + 2*e)^2)*sin(1/3*arctan2(sin(2*f*x + 2*
e), cos(2*f*x + 2*e) + 1)))*a^(2/3)*d^(2/3))/((cos(2*f*x + 2*e)^4 + sin(2*f
*x + 2*e)^4 + 4*cos(2*f*x + 2*e)^3 + 2*(cos(2*f*x + 2*e)^2 + 2*cos(2*f*x +
2*e) + 1)*sin(2*f*x + 2*e)^2 + 6*cos(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) +
1)*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/6)
*f)
```

Giac [F]

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{11/3}\,dx=\int (d\sec(fx+e))^{\frac{2}{3}}\,(i\,a\tan(fx+e)+a)^{\frac{11}{3}}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(2/3)*(a+I*a*tan(f*x+e))^(11/3),x, algorithm="giac
")
[Out] integrate((d*sec(f*x + e))^(2/3)*(I*a*tan(f*x + e) + a)^(11/3), x)
```

Mupad [B] (verification not implemented)

Time = 8.95 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.86

$$\int (d\sec(e+fx))^{2/3}(a+ia\tan(e+fx))^{11/3}\,dx = \frac{\left(-\frac{d}{2\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^2-1}\right)^{2/3}\left(2\sin\left(2e+2fx\right)^2+\sin\left(4e+4fx\right)\,1\mathrm{i}-1\right)\left(\frac{a^3\left(a-\frac{a\,\sin(e}{2\sin\left(\frac{e}{2}\right.}\right.}{}\right.}{}$$

```
[In] int((d/cos(e + f*x))^(2/3)*(a + a*tan(e + f*x)*1i)^(11/3),x)
```

```
[Out] ((-d/(2*sin(e/2 + (f*x)/2)^2 - 1))^(2/3)*(sin(4*e + 4*f*x)*1i + 2*sin(2*e +
 2*f*x)^2 - 1)*((a^3*(a - (a*sin(e + f*x)*1i)/(2*sin(e/2 + (f*x)/2)^2 - 1))
^(2/3)*243i)/(35*f) + (a^3*(a - (a*sin(e + f*x)*1i)/(2*sin(e/2 + (f*x)/2)^2
 - 1))^(2/3)*(sin(2*e + 2*f*x)*1i - 2*sin(e + f*x)^2 + 1)*162i)/(7*f) + (a^
3*(a - (a*sin(e + f*x)*1i)/(2*sin(e/2 + (f*x)/2)^2 - 1))^(2/3)*(sin(4*e + 4
*f*x)*1i - 2*sin(2*e + 2*f*x)^2 + 1)*27i)/f + (a^3*(a - (a*sin(e + f*x)*1i)
/(2*sin(e/2 + (f*x)/2)^2 - 1))^(2/3)*(sin(6*e + 6*f*x)*1i - 2*sin(3*e + 3*f
*x)^2 + 1)*12i)/f))/(4*(sin(e + f*x)^2 - 1))
```

3.450 $\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^5\,dx$

Optimal result

Integrand size = 26, antiderivative size = 86

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^5\,dx = \frac{i2^{5+\frac{m}{2}}a^5\,\text{Hypergeometric2F1}\left(-4-\frac{m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{-m/2}}{dm}$$

```
[Out] I*2^(5+1/2*m)*a^5*hypergeom([1/2*m, -4-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x+c
))*(e*sec(d*x+c))^m/d/m/((1+I*tan(d*x+c))^(1/2*m))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3586, 3604, 72, 71}

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^5\,dx = \frac{ia^5 2^{\frac{m}{2}+5}(1+i\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\,\text{Hypergeometric2F1}\left(-\frac{m}{2}-4,\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{dm}$$

```
[In] Int[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (I*2^(5 + m/2)*a^5*Hypergeometric2F1[-4 - m/2, m/2, (2 + m)/2, (1 - I*Tan[c
 + d*x])/2]*(e*Sec[c + d*x])^m)/(d*m*(1 + I*Tan[c + d*x])^(m/2))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
```

```
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \left((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\int(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{5+\frac{m}{2}}\,dx$$

$$= \frac{\left(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\text{Subst}\left(\int(a-iax)^{-1+\frac{m}{2}}(a+}{d}$$

$$= \frac{\left(2^{4+\frac{m}{2}}a^6(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{-m/2}\right)\text{Subst}\left(\int\left(\frac{1}{2}+\frac{ix}{2}\right)^{4+\frac{m}{2}}(a}{d}$$

$$= \frac{i2^{5+\frac{m}{2}}a^5\,\text{Hypergeometric2F1}\left(-4-\frac{m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c}{dm}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 210 vs. 2(86) = 172.

Time = 6.42 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.44

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^5\,dx$$
$$= \frac{a^5(e\sec(c+dx))^m\left(\frac{i(16(8+6m+m^2)-12m(4+m)\sec^2(c+dx)+m(2+m)\sec^4(c+dx))}{8+6m+m^2}+5\cot(c+dx)\,\text{Hypergeometric2F1}\,(\right.}{}$$

```
[In] Integrate[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^5,x]
```

```
[Out] (a^5*(e*Sec[c + d*x])^m*((I*(16*(8 + 6*m + m^2) - 12*m*(4 + m)*Sec[c + d*x]
^2 + m*(2 + m)*Sec[c + d*x]^4))/(8 + 6*m + m^2) + 5*Cot[c + d*x]*Hypergeome
tric2F1[-3/2, m/2, (2 + m)/2, Sec[c + d*x]^2]*Sqrt[-Tan[c + d*x]^2] + 10*Co
t[c + d*x]*Hypergeometric2F1[-1/2, m/2, (2 + m)/2, Sec[c + d*x]^2]*Sqrt[-Ta
n[c + d*x]^2] + Cot[c + d*x]*Hypergeometric2F1[1/2, m/2, (2 + m)/2, Sec[c +
 d*x]^2]*Sqrt[-Tan[c + d*x]^2]))/(d*m)
```

Maple [F]

$$\int (e\sec(dx+c))^m (a+ia\tan(dx+c))^5\,dx$$

```
[In] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^5,x)
```

```
[Out] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^5,x)
```

Fricas [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^5\,dx = \int (i\,a\tan(dx+c)+a)^5(e\sec(dx+c))^m\,dx$$

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] integral(32*a^5*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*e^(10*I*d
*x + 10*I*c)/(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*I*d*x
 + 6*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^5\,dx = ia^5\left(\int(-i(e\sec(c+dx))^m)\,dx + \int 5(e\sec(c+dx))^m\tan(c+dx)\,dx + \int\left(-10(e\sec(c+dx))^m\tan^3(c+dx)\right)dx + \int(e\sec(c+dx))^m\tan^5(c+dx)\,dx + \int 10i(e\sec(c+dx))^m\tan^2(c+dx)\,dx + \int\left(-5i(e\sec(c+dx))^m\tan^4(c+dx)\right)dx\right)$$

```
[In] integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c))**5,x)
[Out] I*a**5*(Integral(-I*(e*sec(c + d*x))**m, x) + Integral(5*(e*sec(c + d*x))**
m*tan(c + d*x), x) + Integral(-10*(e*sec(c + d*x))**m*tan(c + d*x)**3, x) +
 Integral((e*sec(c + d*x))**m*tan(c + d*x)**5, x) + Integral(10*I*(e*sec(c
+ d*x))**m*tan(c + d*x)**2, x) + Integral(-5*I*(e*sec(c + d*x))**m*tan(c +
d*x)**4, x))
```

Maxima [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^5\,dx = \int (i\,a\tan(dx+c)+a)^5(e\sec(dx+c))^m\,dx$$

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^5,x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^5*(e*sec(d*x + c))^m, x)
```

Giac [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^5\,dx = \int (i\,a\tan(dx+c)+a)^5(e\sec(dx+c))^m\,dx$$

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^5,x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^5*(e*sec(d*x + c))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^5\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^m (a+a\tan(c+d\,x)\ 1\mathrm{i})^5\,dx$$

```
[In] int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^5,x)
[Out] int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^5, x)
```

3.451 $\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 26, antiderivative size = 86

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^3\,dx = \frac{i2^{3+\frac{m}{2}}a^3\,\text{Hypergeometric2F1}\left(-2-\frac{m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{\cdot}}{dm}$$

```
[Out] I*2^(3+1/2*m)*a^3*hypergeom([1/2*m, -2-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x+c
))*(e*sec(d*x+c))^m/d/m/((1+I*tan(d*x+c))^(1/2*m))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3586, 3604, 72, 71}

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^3\,dx = \frac{ia^3 2^{\frac{m}{2}+3}(1+i\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\,\text{Hypergeometric2F1}\left(-\frac{m}{2}-2,\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+d\cdot$$
$$\overline{\qquad\qquad dm \qquad\qquad}$$

```
[In] Int[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (I*2^(3 + m/2)*a^3*Hypergeometric2F1[-2 - m/2, m/2, (2 + m)/2, (1 - I*Tan[c
 + d*x])/2]*(e*Sec[c + d*x])^m)/(d*m*(1 + I*Tan[c + d*x])^(m/2))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
```

```
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \left((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\int(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{3+\frac{m}{2}}\,dx$$

$$=\frac{\left(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\text{Subst}\left(\int(a-iax)^{-1+\frac{m}{2}}(a+}{d}$$

$$=\frac{\left(2^{2+\frac{m}{2}}a^4(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{-m/2}\right)\text{Subst}\left(\int\left(\frac{1}{2}+\frac{ix}{2}\right)^{2+\frac{m}{2}}(a-}{d}$$

$$=\frac{i2^{3+\frac{m}{2}}a^3\,\text{Hypergeometric2F1}\left(-2-\frac{m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c-}{dm}$$

Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.71

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^3\,dx =$$

$$-\frac{ia^3(e\sec(c+dx))^m\left(-3i(2+m)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m}{2},\frac{2+m}{2},\sec^2(c+dx)\right)\tan(c+dx)-i(2+m\right.}{dm(2+m}$$

```
[In] Integrate[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] ((-I)*a^3*(e*Sec[c + d*x])^m*((-3*I)*(2 + m)*Hypergeometric2F1[-1/2, m/2, (
2 + m)/2, Sec[c + d*x]^2]*Tan[c + d*x] - I*(2 + m)*Hypergeometric2F1[1/2, m
/2, (2 + m)/2, Sec[c + d*x]^2]*Tan[c + d*x] + (-8 - 4*m + m*Sec[c + d*x]^2)
*Sqrt[-Tan[c + d*x]^2]))/(d*m*(2 + m)*Sqrt[-Tan[c + d*x]^2])
```

Maple [F]

$$\int \left(e\sec\left(dx+c\right)\right)^{m}\left(a+ia\tan\left(dx+c\right)\right)^{3}dx$$

```
[In] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^3,x)
```

Fricas [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^3\,dx = \int \left(i\,a\tan\left(dx+c\right)+a\right)^{3}\left(e\sec\left(dx+c\right)\right)^{m}\,dx$$

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(8*a^3*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*e^(6*I*d*x
 + 6*I*c)/(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I
*c) + 1), x)
```

Sympy [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^3\,dx = -ia^3\left(\int i(e\sec{(c+dx)})^m\,dx + \int \left(-3(e\sec{(c+dx)})^m\tan{(c+dx)}\right)dx + \int (e\sec{(c+dx)})^m\tan^3{(c+dx)}\,dx + \int \left(-3i(e\sec{(c+dx)})^m\tan^2{(c+dx)}\right)dx\right)$$

```
[In] integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c))**3,x)
[Out] -I*a**3*(Integral(I*(e*sec(c + d*x))**m, x) + Integral(-3*(e*sec(c + d*x))*
*m*tan(c + d*x), x) + Integral((e*sec(c + d*x))**m*tan(c + d*x)**3, x) + In
tegral(-3*I*(e*sec(c + d*x))**m*tan(c + d*x)**2, x))
```

Maxima [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^3\,dx = \int (i\,a\tan{(dx+c)}+a)^3(e\sec{(dx+c)})^m\,dx$$

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^3*(e*sec(d*x + c))^m, x)
```

Giac [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^3\,dx = \int (i\,a\tan{(dx+c)}+a)^3(e\sec{(dx+c)})^m\,dx$$

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^3*(e*sec(d*x + c))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^3\,dx = \int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^m\,\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^3\,dx$$

```
[In] int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^3,x)
[Out] int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^3, x)
```

3.452 $\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 26, antiderivative size = 86

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^2\,dx = \frac{i2^{2+\frac{m}{2}}a^2\,\text{Hypergeometric2F1}\left(-1-\frac{m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{-m}}{dm}$$

```
[Out] I*2^(2+1/2*m)*a^2*hypergeom([1/2*m, -1-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x+c
))*(e*sec(d*x+c))^m/d/m/((1+I*tan(d*x+c))^(1/2*m))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3586, 3604, 72, 71}

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^2\,dx = \frac{ia^2 2^{\frac{m}{2}+2}(1+i\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\,\text{Hypergeometric2F1}\left(-\frac{m}{2}-1,\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+dx\right.}{dm}$$

```
[In] Int[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (I*2^(2 + m/2)*a^2*Hypergeometric2F1[-1 - m/2, m/2, (2 + m)/2, (1 - I*Tan[c
 + d*x])/2]*(e*Sec[c + d*x])^m)/(d*m*(1 + I*Tan[c + d*x])^(m/2))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
```

```
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \left((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\int(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{2+\frac{m}{2}}\,dx$$

$$= \frac{\left(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\operatorname{Subst}\left(\int(a-iax)^{-1+\frac{m}{2}}(a+\cdots\right.}{d}$$

$$= \frac{\left(2^{1+\frac{m}{2}}a^3(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{-m/2}\right)\operatorname{Subst}\left(\int\left(\frac{1}{2}+\frac{ix}{2}\right)^{1+\frac{m}{2}}(a\cdots\right.}{d}$$

$$= \frac{i2^{2+\frac{m}{2}}a^2\,\text{Hypergeometric2F1}\left(-1-\frac{m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c\cdots}{dm}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.33

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^2\,dx = \frac{a^2(e\sec(c+dx))^m\left(2i+\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m}{2},\frac{2+m}{2},\sec^2(c+dx)\right)\sqrt{-\tan^2(c+dx)}+\text{c}\right.}{dm}$$

```
[In] Integrate[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*(e*Sec[c + d*x])^m*(2*I + Cot[c + d*x]*Hypergeometric2F1[-1/2, m/2, (2
 + m)/2, Sec[c + d*x]^2]*Sqrt[-Tan[c + d*x]^2] + Cot[c + d*x]*Hypergeometri
c2F1[1/2, m/2, (2 + m)/2, Sec[c + d*x]^2]*Sqrt[-Tan[c + d*x]^2]))/(d*m)
```

Maple [F]

$$\int (e\sec(dx+c))^m\,(a+ia\tan(dx+c))^2\,dx$$

```
[In] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x)
```

Fricas [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^2\,dx = \int (i\,a\tan(dx+c)+a)^2(e\sec(dx+c))^m\,dx$$

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(4*a^2*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*e^(4*I*d*x
 + 4*I*c)/(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^2\,dx = -a^2\left(\int\left(-(e\sec(c+dx))^m\right)dx + \int (e\sec(c+dx))^m\tan^2(c+dx)\,dx + \int\left(-2i(e\sec(c+dx))^m\tan(c+dx)\right)dx\right)$$

```
[In] integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] -a**2*(Integral(-(e*sec(c + d*x))**m, x) + Integral((e*sec(c + d*x))**m*tan
(c + d*x)**2, x) + Integral(-2*I*(e*sec(c + d*x))**m*tan(c + d*x), x))
```

Maxima [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^2\,dx = \int (i\,a\tan(dx+c)+a)^2(e\sec(dx+c))^m\,dx$$

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^2*(e*sec(d*x + c))^m, x)
```

Giac [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^2\,dx = \int (i\,a\tan(dx+c)+a)^2(e\sec(dx+c))^m\,dx$$

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^2*(e*sec(d*x + c))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^2\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^m(a+a\tan(c+d\,x)\,1\mathrm{i})^2\,dx$$

```
[In] int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^2,x)
[Out] int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^2, x)
```

3.453 $\int (e\sec(c+dx))^m(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))\,dx = \frac{i2^{1+\frac{m}{2}}a\,\text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{-m/2}}{dm}$$

```
[Out] I*2^(1+1/2*m)*a*hypergeom([-1/2*m, 1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x+c))*(
e*sec(d*x+c))^m/d/m/((1+I*tan(d*x+c))^(1/2*m))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3586, 3604, 72, 71}

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))\,dx = \frac{ia2^{\frac{m}{2}+1}(1+i\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\,\text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{dm}$$

```
[In] Int[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (I*2^(1 + m/2)*a*Hypergeometric2F1[-1/2*m, m/2, (2 + m)/2, (1 - I*Tan[c + d
*x])/2]*(e*Sec[c + d*x])^m)/(d*m*(1 + I*Tan[c + d*x])^(m/2))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
```

```
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \left((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\int(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{1+\frac{m}{2}}\,dx$$

$$= \frac{\left(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\text{Subst}\left(\int(a-iax)^{-1+\frac{m}{2}}(a\right.}{d}$$

$$= \frac{\left(2^{m/2}a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{-m/2}\right)\text{Subst}\left(\int\left(\frac{1}{2}+\frac{ix}{2}\right)^{m/2}(a\right.}{d}$$

$$= \frac{i2^{1+\frac{m}{2}}a\,\text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+d}{dm}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.82

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))\,dx = \frac{a(e\sec(c+dx))^m\left(i+\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m}{2},\frac{2+m}{2},\sec^2(c+dx)\right)\sqrt{-\tan^2(c+dx)}\right)}{dm}$$

```
[In] Integrate[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x]),x]
[Out] (a*(e*Sec[c + d*x])^m*(I + Cot[c + d*x]*Hypergeometric2F1[1/2, m/2, (2 + m)
/2, Sec[c + d*x]^2]*Sqrt[-Tan[c + d*x]^2]))/(d*m)
```

Maple [F]

$$\int (e\sec(dx+c))^m\,(a+ia\tan(dx+c))\,dx$$

```
[In] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c)),x)
[Out] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c)),x)
```

Fricas [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))\,dx = \int (i\,a\tan(dx+c)+a)(e\sec(dx+c))^m\,dx$$

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
[Out] integral(2*a*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*e^(2*I*d*x +
 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))\,dx = ia\left(\int (-i(e\sec(c+dx))^m)\,dx + \int (e\sec(c+dx))^m\tan(c+dx)\,dx\right)$$

```
[In] integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c)),x)
[Out] I*a*(Integral(-I*(e*sec(c + d*x))**m, x) + Integral((e*sec(c + d*x))**m*tan
(c + d*x), x))
```

Maxima [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))\,dx = \int (i\,a\tan(dx+c)+a)(e\sec(dx+c))^m\,dx$$

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)*(e*sec(d*x + c))^m, x)
```

Giac [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))\,dx = \int (i\,a\tan(dx+c)+a)(e\sec(dx+c))^m\,dx$$

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)*(e*sec(d*x + c))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^m (a+a\tan(c+d\,x)\ 1\mathrm{i})\,dx$$

```
[In] int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i),x)
[Out] int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i), x)
```

3.454 $\int \frac{(e\sec(c+dx))^m}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 86

$$\int \frac{(e\sec(c+dx))^m}{a+ia\tan(c+dx)}\,dx = \frac{i2^{-1+\frac{m}{2}}\,\text{Hypergeometric2F1}\left(2-\frac{m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{-m/2}}{adm}$$

```
[Out] I*2^(-1+1/2*m)*hypergeom([1/2*m, 2-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x+c))*(
e*sec(d*x+c))^m/a/d/m/((1+I*tan(d*x+c))^(1/2*m))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3586, 3604, 72, 71}

$$\int \frac{(e\sec(c+dx))^m}{a+ia\tan(c+dx)}\,dx = \frac{i2^{\frac{m}{2}-1}(1+i\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\,\text{Hypergeometric2F1}\left(2-\frac{m}{2},\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{adm}$$

```
[In] Int[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (I*2^(-1 + m/2)*Hypergeometric2F1[2 - m/2, m/2, (2 + m)/2, (1 - I*Tan[c + d
*x])/2]*(e*Sec[c + d*x])^m)/(a*d*m*(1 + I*Tan[c + d*x])^(m/2))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \left((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\int (a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{-1+\frac{m}{2}}\,dx$$

$$= \frac{\left(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\operatorname{Subst}\left(\int (a-iax)^{-1+\frac{m}{2}}(a+}{d}$$

$$= \frac{\left(2^{-2+\frac{m}{2}}(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{-m/2}\right)\operatorname{Subst}\left(\int\left(\frac{1}{2}+\frac{ix}{2}\right)^{-2+\frac{m}{2}}(a}{d}$$

$$= \frac{i2^{-1+\frac{m}{2}}\,\text{Hypergeometric2F1}\left(2-\frac{m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+}{adm}$$

Mathematica [A] (verified)

Time = 2.05 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.77

$$\int \frac{(e\sec(c+dx))^m}{a+ia\tan(c+dx)}\,dx = -\frac{i2^{-1+m}e^{-i(c+2dx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^m\left(1+e^{2i(c+dx)}\right)^m \text{Hypergeometric2F1}\left(\frac{1}{2}(-2+m),-1+m,\frac{m}{2},-e^{2i(c+dx)}\right)}{d(-2+m)(a+ia\tan(c+dx))}$$

```
[In] Integrate[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x]),x]
[Out] ((-I)*2^(-1 + m)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^m*(1 + E^((2*I
)*(c + d*x)))^m*Hypergeometric2F1[(-2 + m)/2, -1 + m, m/2, -E^((2*I)*(c + d
*x))]*Sec[c + d*x]^(1 - m)*(e*Sec[c + d*x])^m*(Cos[d*x] + I*Sin[d*x]))/(d*E
^(I*(c + 2*d*x))*(-2 + m)*(a + I*a*Tan[c + d*x]))
```

Maple [F]

$$\int \frac{(e\sec(dx+c))^m}{a+ia\tan(dx+c)}dx$$

```
[In] int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c)),x)
[Out] int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c)),x)
```

Fricas [F]

$$\int \frac{(e\sec(c+dx))^m}{a+ia\tan(c+dx)}\,dx = \int \frac{(e\sec(dx+c))^m}{i\,a\tan(dx+c)+a}\,dx$$

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c)),x, algorithm="fricas")
[Out] integral(1/2*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*(e^(2*I*d*x
+ 2*I*c) + 1)*e^(-2*I*d*x - 2*I*c)/a, x)
```

Sympy [F]

$$\int \frac{(e\sec(c+dx))^m}{a+ia\tan(c+dx)}\,dx = -\frac{i\int \frac{(e\sec(c+dx))^m}{\tan(c+dx)-i}\,dx}{a}$$

```
[In] integrate((e*sec(d*x+c))**m/(a+I*a*tan(d*x+c)),x)
[Out] -I*Integral((e*sec(c + d*x))**m/(tan(c + d*x) - I), x)/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^m}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c)),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^m}{a+ia\tan(c+dx)}\,dx = \int \frac{(e\sec(dx+c))^m}{i\,a\tan(dx+c)+a}\,dx$$

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^m/(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^m}{a+ia\tan(c+dx)}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^m}{a+a\tan(c+d\,x)\ 1\mathrm{i}}\,dx$$

```
[In] int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i),x)
[Out] int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i), x)
```

3.455 $\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 86

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = \frac{i2^{-2+\frac{m}{2}}\,\text{Hypergeometric2F1}\left(3-\frac{m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{-m/2}}{a^2dm}$$

```
[Out] I*2^(-2+1/2*m)*hypergeom([1/2*m, 3-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x+c))*(
e*sec(d*x+c))^m/a^2/d/m/((1+I*tan(d*x+c))^(1/2*m))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3586, 3604, 72, 71}

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = \frac{i2^{\frac{m}{2}-2}(1+i\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\,\text{Hypergeometric2F1}\left(3-\frac{m}{2},\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{a^2dm}$$

```
[In] Int[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (I*2^(-2 + m/2)*Hypergeometric2F1[3 - m/2, m/2, (2 + m)/2, (1 - I*Tan[c + d
*x])/2]*(e*Sec[c + d*x])^m)/(a^2*d*m*(1 + I*Tan[c + d*x])^(m/2))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \left((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\int(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{-2+\frac{m}{2}}\,dx$$

$$= \frac{\left(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\text{Subst}\left(\int(a-iax)^{-1+\frac{m}{2}}(a\cdots\right.}{d}$$

$$= \frac{\left(2^{-3+\frac{m}{2}}(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{-m/2}\right)\text{Subst}\left(\int\left(\frac{1}{2}+\frac{ix}{2}\right)^{-3+\frac{m}{2}}(a\cdots\right.}{ad}$$

$$= \frac{i2^{-2+\frac{m}{2}}\text{Hypergeometric2F1}\left(3-\frac{m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+\cdots}{a^2dm}$$

Mathematica [A] (verified)

Time = 1.97 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.81

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = -\frac{i2^{-2+m}e^{-2i(c+2dx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^m\left(1+e^{2i(c+dx)}\right)^m \text{Hypergeometric2F1}\left(\frac{1}{2}(-4+m),-2+m,\frac{1}{2}(-2+m),-\right.}{d(-4+m)(a+ia\tan(c+dx))^2}$$

```
[In] Integrate[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((-I)*2^(-2 + m)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^m*(1 + E^((2*I
)*(c + d*x)))^m*Hypergeometric2F1[(-4 + m)/2, -2 + m, (-2 + m)/2, -E^((2*I)
*(c + d*x))]*Sec[c + d*x]^(2 - m)*(e*Sec[c + d*x])^m*(Cos[d*x] + I*Sin[d*x]
)^2)/(d*E^((2*I)*(c + 2*d*x))*(-4 + m)*(a + I*a*Tan[c + d*x])^2)
```

Maple [F]

$$\int \frac{(e\sec(dx+c))^m}{(a+ia\tan(dx+c))^2}dx$$

```
[In] int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x)
```

Fricas [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\sec(dx+c))^m}{(i\,a\tan(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(1/4*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*(e^(4*I*d*x
+ 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1)*e^(-4*I*d*x - 4*I*c)/a^2, x)
```

Sympy [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{(e\sec(c+dx))^m}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

```
[In] integrate((e*sec(d*x+c))**m/(a+I*a*tan(d*x+c))**2,x)
[Out] -Integral((e*sec(c + d*x))**m/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)/
a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\sec(dx+c))^m}{(i\,a\tan(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^m/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^m}{(a+a\tan(c+d\,x)\,1\mathrm{i})^2}\,dx$$

```
[In] int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^2,x)
[Out] int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^2, x)
```

3.456 $\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 86

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^3}\,dx = \frac{i2^{-3+\frac{m}{2}}\text{Hypergeometric2F1}\left(4-\frac{m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{-m/2}}{a^3dm}$$

```
[Out] I*2^(-3+1/2*m)*hypergeom([1/2*m, 4-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x+c))*(
e*sec(d*x+c))^m/a^3/d/m/((1+I*tan(d*x+c))^(1/2*m))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3586, 3604, 72, 71}

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^3}\,dx = \frac{i2^{\frac{m}{2}-3}(1+i\tan(c+dx))^{-m/2}(e\sec(c+dx))^m\,\text{Hypergeometric2F1}\left(4-\frac{m}{2},\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{a^3dm}$$

```
[In] Int[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (I*2^(-3 + m/2)*Hypergeometric2F1[4 - m/2, m/2, (2 + m)/2, (1 - I*Tan[c + d
*x])/2]*(e*Sec[c + d*x])^m)/(a^3*d*m*(1 + I*Tan[c + d*x])^(m/2))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \left((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\int(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{-3+\frac{m}{2}}\,dx$$

$$=\frac{\left(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\text{Subst}\left(\int(a-iax)^{-1+\frac{m}{2}}(a\right.}{d}$$

$$=\frac{\left(2^{-4+\frac{m}{2}}(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{-m/2}\right)\text{Subst}\left(\int\left(\frac{1}{2}+\frac{ix}{2}\right)^{-4+\frac{m}{2}}(a\right.}{a^2d}$$

$$=\frac{i2^{-3+\frac{m}{2}}\text{Hypergeometric2F1}\left(4-\frac{m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+\right.}{a^3dm}$$

Mathematica [A] (verified)

Time = 2.09 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.78

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^3}\,dx$$
$$= \frac{2^{-3+m}e^{-3i(c+2dx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^m\left(1+e^{2i(c+dx)}\right)^m \text{Hypergeometric2F1}\left(\frac{1}{2}(-6+m),-3+m,\frac{1}{2}(-4+m),-e^{2i}\right.}{a^3d(-6+m)(-i+\tan(c+dx))^3}$$

```
[In] Integrate[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x])^3,x]
```

```
[Out] (2^(-3 + m)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^m*(1 + E^((2*I)*(c
+ d*x)))^m*Hypergeometric2F1[(-6 + m)/2, -3 + m, (-4 + m)/2, -E^((2*I)*(c +
 d*x))]*Sec[c + d*x]^(3 - m)*(e*Sec[c + d*x])^m*(Cos[d*x] + I*Sin[d*x])^3)/
(a^3*d*E^((3*I)*(c + 2*d*x))*(-6 + m)*(-I + Tan[c + d*x])^3)
```

Maple [F]

$$\int \frac{(e\sec(dx+c))^m}{(a+ia\tan(dx+c))^3}dx$$

```
[In] int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^3,x)
```

```
[Out] int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^3,x)
```

Fricas [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{(e\sec(dx+c))^m}{(i\,a\tan(dx+c)+a)^3}\,dx$$

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(1/8*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*(e^(6*I*d*x
+ 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I*c) + 1)*e^(-6*I*d*x -
 6*I*c)/a^3, x)
```

Sympy [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^3}\,dx = \frac{i\int \frac{(e\sec(c+dx))^m}{\tan^3(c+dx)-3i\tan^2(c+dx)-3\tan(c+dx)+i}\,dx}{a^3}$$

```
[In] integrate((e*sec(d*x+c))**m/(a+I*a*tan(d*x+c))**3,x)
```

```
[Out] I*Integral((e*sec(c + d*x))**m/(tan(c + d*x)**3 - 3*I*tan(c + d*x)**2 - 3*t
an(c + d*x) + I), x)/a**3
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{(e\sec(dx+c))^m}{(i\,a\tan(dx+c)+a)^3}\,dx$$

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^m/(I*a*tan(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^3}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^m}{(a+a\tan(c+d\,x)\ 1\mathrm{i})^3}\,dx$$

```
[In] int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^3,x)
```

```
[Out] int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^3, x)
```

3.457 $\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 109

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{7/2}\,dx = \frac{i2^{\frac{7+m}{2}}a^3\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-5-m),\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i}{dm}$$

```
[Out] I*2^(7/2+1/2*m)*a^3*hypergeom([1/2*m, -5/2-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d
*x+c))*(e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2)*(1+I*tan(d*x+c))^(-1/2-1/2
*m)/d/m
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3586, 3604, 72, 71}

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{7/2}\,dx = \frac{ia^3 2^{\frac{m+7}{2}}\sqrt{a+ia\tan(c+dx)}(1+i\tan(c+dx))^{\frac{1}{2}(-m-1)}(e\sec(c+dx))^m\,\text{Hypergeometric2F1}\left(\frac{1}{2}\right.}{dm}$$

```
[In] Int[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] (I*2^((7 + m)/2)*a^3*Hypergeometric2F1[(-5 - m)/2, m/2, (2 + m)/2, (1 - I*T
an[c + d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^((-1 - m)/2)*Sqrt[a
 + I*a*Tan[c + d*x]])/(d*m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \left((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\int(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{\frac{7}{2}+\frac{m}{2}}\,dx$$

$$=\frac{\left(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\operatorname{Subst}\left(\int(a-iax)^{-1+\frac{m}{2}}(a\right.}{d}$$

$$=\frac{\left(2^{\frac{5}{2}+\frac{m}{2}}a^4(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\sqrt{a+ia\tan(c+dx)}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{-\frac{1}{2}-\frac{m}{2}}\right)\operatorname{Su}}{d}$$

$$=\frac{i2^{\frac{7+m}{2}}a^3\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-5-m),\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan}{dm}$$

Mathematica [A] (verified)

Time = 3.29 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.71

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{7/2}\,dx = -\frac{i2^{\frac{7}{2}+m}e^{3i(c+2dx)}\sqrt{e^{idx}}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{1}{2}+m}\left(1+e^{2i(c+dx)}\right)^{\frac{1}{2}+m}\text{Hypergeometric2F1}\left(\frac{7}{2}+m,\frac{7+m}{2},\frac{9+m}{2},-e^{2i(c+dx)}\right)}{d(7+m)(\cos(dx)+i\sin(dx))^{7/2}}$$

```
[In] Integrate[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(7/2),x]
```

```
[Out] ((-I)*2^(7/2 + m)*E^((3*I)*(c + 2*d*x))*Sqrt[E^(I*d*x)]*(E^(I*(c + d*x))/(1
 + E^((2*I)*(c + d*x))))^(1/2 + m)*(1 + E^((2*I)*(c + d*x)))^(1/2 + m)*Hype
rgeometric2F1[7/2 + m, (7 + m)/2, (9 + m)/2, -E^((2*I)*(c + d*x))]*Sec[c +
d*x]^(-7/2 - m)*(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(7/2))/(d*(7 + m)
*(Cos[d*x] + I*Sin[d*x])^(7/2))
```

Maple [F]

$$\int (e\sec(dx+c))^m (a+ia\tan(dx+c))^{\frac{7}{2}}\,dx$$

```
[In] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(7/2),x)
```

```
[Out] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(7/2),x)
```

Fricas [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{7/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{7}{2}}(e\sec(dx+c))^m\,dx$$

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(8*sqrt(2)*a^3*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(7*I*d*x + 7*I*c)/(e^(6*I*d*x + 6*I*c) +
3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c))**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{7/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{7}{2}}(e\sec(dx+c))^m\,dx$$

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^(7/2)*(e*sec(d*x + c))^m, x)
```

Giac [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{7/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{7}{2}}(e\sec(dx+c))^m\,dx$$

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^(7/2)*(e*sec(d*x + c))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{7/2}\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^m(a+a\tan(c+d\,x)\;1\mathrm{i})^{7/2}\,dx$$

```
[In] int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^(7/2),x)
[Out] int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^(7/2), x)
```

3.458 $\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 109

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{5/2}\,dx = \frac{i2^{\frac{5+m}{2}}a^2\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-3-m),\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i}{dm}$$

```
[Out] I*2^(5/2+1/2*m)*a^2*hypergeom([1/2*m, -3/2-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d
*x+c))*(e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2)*(1+I*tan(d*x+c))^(-1/2-1/2
*m)/d/m
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3586, 3604, 72, 71}

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{5/2}\,dx = \frac{ia^2 2^{\frac{m+5}{2}}\sqrt{a+ia\tan(c+dx)}(1+i\tan(c+dx))^{\frac{1}{2}(-m-1)}(e\sec(c+dx))^m\,\text{Hypergeometric2F1}\left(\frac{1}{2}\right.}{dm}$$

```
[In] Int[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (I*2^((5 + m)/2)*a^2*Hypergeometric2F1[(-3 - m)/2, m/2, (2 + m)/2, (1 - I*T
an[c + d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^((-1 - m)/2)*Sqrt[a
 + I*a*Tan[c + d*x]])/(d*m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \left((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\int (a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{\frac{5}{2}+\frac{m}{2}}\,dx$$

$$= \frac{\left(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\text{Subst}\left(\int (a-iax)^{-1+\frac{m}{2}}(a$$

$$= \frac{\left(2^{\frac{3}{2}+\frac{m}{2}}a^3(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\sqrt{a+ia\tan(c+dx)}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{-\frac{1}{2}-\frac{m}{2}}\right)\text{Su}$$

$$= \frac{i2^{\frac{5+m}{2}}a^2\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-3-m),\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan}{dm}$$

Mathematica [A] (verified)

Time = 5.21 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.71

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{5/2}\,dx = -\frac{i2^{\frac{5}{2}+m}e^{2i(c+2dx)}\sqrt{e^{idx}}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{1}{2}+m}\left(1+e^{2i(c+dx)}\right)^{\frac{1}{2}+m}\text{Hypergeometric2F1}\left(\frac{5}{2}+m,\frac{5+m}{2},\frac{7+m}{2},-e^{2i(c+dx)}\right)}{d(5+m)(\cos(dx)+i\sin(dx))^{5/2}}$$

```
[In] Integrate[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-I)*2^(5/2 + m)*E^((2*I)*(c + 2*d*x))*Sqrt[E^(I*d*x)]*(E^(I*(c + d*x))/(1
 + E^((2*I)*(c + d*x))))^(1/2 + m)*(1 + E^((2*I)*(c + d*x)))^(1/2 + m)*Hype
rgeometric2F1[5/2 + m, (5 + m)/2, (7 + m)/2, -E^((2*I)*(c + d*x))]*Sec[c +
d*x]^(-5/2 - m)*(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(5/2))/(d*(5 + m)
*(Cos[d*x] + I*Sin[d*x])^(5/2))
```

Maple [F]

$$\int (e\sec(dx+c))^m (a+ia\tan(dx+c))^{\frac{5}{2}}\,dx$$

```
[In] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(5/2),x)
```

Fricas [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{5/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{5}{2}}(e\sec(dx+c))^m\,dx$$

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(4*sqrt(2)*a^2*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(5*I*d*x + 5*I*c)/(e^(4*I*d*x + 4*I*c) +
2*e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{5/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{5}{2}}(e\sec(dx+c))^m\,dx$$

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*(e*sec(d*x + c))^m, x)
```

Giac [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{5/2}\,dx = \int (i\,a\tan(dx+c)+a)^{\frac{5}{2}}(e\sec(dx+c))^m\,dx$$

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^(5/2)*(e*sec(d*x + c))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{5/2}\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^m(a+a\tan(c+d\,x)\ 1\mathrm{i})^{5/2}\,dx$$

```
[In] int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^(5/2),x)
[Out] int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.459 $\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 107

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{3/2}\,dx = \frac{i2^{\frac{3+m}{2}}a\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-1-m),\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{\frac{1}{2}(-1-m)}\sqrt{a+ia\tan(c+dx)}}{dm}$$

```
[Out] I*2^(3/2+1/2*m)*a*hypergeom([1/2*m, -1/2-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x
+c))*(e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2)*(1+I*tan(d*x+c))^(-1/2-1/2*m
)/d/m
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3586, 3604, 72, 71}

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{3/2}\,dx = \frac{ia2^{\frac{m+3}{2}}\sqrt{a+ia\tan(c+dx)}(1+i\tan(c+dx))^{\frac{1}{2}(-m-1)}(e\sec(c+dx))^m\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-m-1),\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{dm}$$

```
[In] Int[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (I*2^((3 + m)/2)*a*Hypergeometric2F1[(-1 - m)/2, m/2, (2 + m)/2, (1 - I*Tan
[c + d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^((-1 - m)/2)*Sqrt[a +
 I*a*Tan[c + d*x]])/(d*m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \left((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\int(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{\frac{3}{2}+\frac{m}{2}}\,dx$$

$$=\frac{\left(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\text{Subst}\left(\int(a-iax)^{-1+\frac{m}{2}}(a$$

$$=\frac{\left(2^{\frac{1}{2}+\frac{m}{2}}a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\sqrt{a+ia\tan(c+dx)}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{-\frac{1}{2}-\frac{m}{2}}\right)\text{Su}$$

$$=\frac{i2^{\frac{3+m}{2}}a\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-1-m),\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan}{dm}$$

Mathematica [A] (verified)

Time = 2.74 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.74

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{3/2}\,dx =$$

$$-\frac{i2^{\frac{3}{2}+m}e^{i(c+2dx)}\sqrt{e^{idx}}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{1}{2}+m}\left(1+e^{2i(c+dx)}\right)^{\frac{1}{2}+m}\text{Hypergeometric2F1}\left(\frac{3}{2}+m,\frac{3+m}{2},\frac{5+m}{2},-e^{2i(c+dx)}\right)\,\mathrm{s}}{d(3+m)(\cos(dx)+i\sin(dx))^{3/2}}$$

```
[In] Integrate[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-I)*2^(3/2 + m)*E^(I*(c + 2*d*x))*Sqrt[E^(I*d*x)]*(E^(I*(c + d*x))/(1 + E
^((2*I)*(c + d*x))))^(1/2 + m)*(1 + E^((2*I)*(c + d*x)))^(1/2 + m)*Hypergeo
metric2F1[3/2 + m, (3 + m)/2, (5 + m)/2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]
^(-3/2 - m)*(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^(3/2))/(d*(3 + m)*(Co
s[d*x] + I*Sin[d*x])^(3/2))
```

Maple [F]

$$\int \left(e\sec\left(dx+c\right)\right)^{m}\left(a+ia\tan\left(dx+c\right)\right)^{\frac{3}{2}}dx$$

```
[In] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(3/2),x)
```

Fricas [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{3/2}\,dx = \int \left(i\,a\tan\left(dx+c\right)+a\right)^{\frac{3}{2}}\left(e\sec\left(dx+c\right)\right)^{m}\,dx$$

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(2*sqrt(2)*a*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(3*I*d*x + 3*I*c)/(e^(2*I*d*x + 2*I*c) + 1)
, x)
```

Sympy [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{3/2}\,dx = \int (e\sec{(c+dx)})^m\,(ia(\tan{(c+dx)}-i))^{\frac{3}{2}}\,dx$$

```
[In] integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c))**(3/2),x)
[Out] Integral((e*sec(c + d*x))**m*(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Maxima [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{3/2}\,dx = \int (i\,a\tan{(dx+c)}+a)^{\frac{3}{2}}(e\sec{(dx+c)})^m\,dx$$

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*(e*sec(d*x + c))^m, x)
```

Giac [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{3/2}\,dx = \int (i\,a\tan{(dx+c)}+a)^{\frac{3}{2}}(e\sec{(dx+c)})^m\,dx$$

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^(3/2)*(e*sec(d*x + c))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^{3/2}\,dx = \int \left(\frac{e}{\cos{(c+d\,x)}}\right)^m\,(a+a\tan(c+d\,x)\;1\mathrm{i})^{3/2}\,dx$$

```
[In] int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.460 $\int (e\sec(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 107

$$\int (e\sec(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx = \frac{i2^{\frac{1+m}{2}}a\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{\frac{1-m}{2}}}{dm\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] I*2^(1/2+1/2*m)*a*hypergeom([1/2*m, 1/2-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x+
c))*(e*sec(d*x+c))^m*(1+I*tan(d*x+c))^(1/2-1/2*m)/d/m/(a+I*a*tan(d*x+c))^(1
/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3586, 3604, 72, 71}

$$\int (e\sec(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx = \frac{ia2^{\frac{m+1}{2}}(1+i\tan(c+dx))^{\frac{1-m}{2}}(e\sec(c+dx))^m\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{dm\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[(e*Sec[c + d*x])^m*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (I*2^((1 + m)/2)*a*Hypergeometric2F1[(1 - m)/2, m/2, (2 + m)/2, (1 - I*Tan[
c + d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^((1 - m)/2))/(d*m*Sqrt
[a + I*a*Tan[c + d*x]])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \left((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\int(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{\frac{1}{2}+\frac{m}{2}}\,dx$$

$$=\frac{\left(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\text{Subst}\left(\int(a-iax)^{-1+\frac{m}{2}}(a$$
$$d$$

$$=\frac{\left(2^{-\frac{1}{2}+\frac{m}{2}}a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{\frac{1}{2}-\frac{m}{2}}\right)\text{Subst}\left(\int\left(\frac{1}{2}+\frac{ix}{2}\right)^{-\frac{1}{2}+\frac{m}{2}}\right.}{d\sqrt{a+ia\tan(c+dx)}}$$

$$=\frac{i2^{\frac{1+m}{2}}a\,\text{Hypergeometric2F1}\left(\frac{1-m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx)}{dm\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A] (verified)

Time = 1.75 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.63

$$\int (e\sec(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx =$$

$$-\frac{i2^{\frac{1}{2}+m}\sqrt{e^{idx}}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{1}{2}+m}\left(1+e^{2i(c+dx)}\right)^{\frac{1}{2}+m}\text{Hypergeometric2F1}\left(\frac{1}{2}+m,\frac{1+m}{2},\frac{3+m}{2},-e^{2i(c+dx)}\right)\sec^{-\frac{1}{2}}}{d(1+m)\sqrt{\cos(dx)+i\sin(dx)}}$$

```
[In] Integrate[(e*Sec[c + d*x])^m*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-I)*2^(1/2 + m)*Sqrt[E^(I*d*x)]*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))
))^(1/2 + m)*(1 + E^((2*I)*(c + d*x)))^(1/2 + m)*Hypergeometric2F1[1/2 + m,
 (1 + m)/2, (3 + m)/2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(-1/2 - m)*(e*Sec
[c + d*x])^m*Sqrt[a + I*a*Tan[c + d*x]])/(d*(1 + m)*Sqrt[Cos[d*x] + I*Sin[d
*x]])
```

Maple [F]

$$\int (e\sec(dx+c))^m\sqrt{a+ia\tan(dx+c)}dx$$

```
[In] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int (e\sec(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{i\,a\tan(dx+c)+a}(e\sec(dx+c))^m\,dx$$

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2)*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*sqrt(a/(
e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c), x)
```

Sympy [F]

$$\int (e\sec(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx = \int (e\sec(c+dx))^m\sqrt{ia(\tan(c+dx)-i)}\,dx$$

```
[In] integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Integral((e*sec(c + d*x))**m*sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [F]

$$\int (e\sec(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{i\,a\tan(dx+c)+a}(e\sec(dx+c))^m\,dx$$

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*(e*sec(d*x + c))^m, x)
```

Giac [F]

$$\int (e\sec(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{i\,a\tan(dx+c)+a}(e\sec(dx+c))^m\,dx$$

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*(e*sec(d*x + c))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^m\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}\,dx$$

```
[In] int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.461 $\int \frac{(e\sec(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 106

$$\int \frac{(e\sec(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{i2^{\frac{1}{2}(-1+m)}\,\text{Hypergeometric2F1}\left(\frac{3-m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{\frac{1-m}{2}}}{dm\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] I*2^(-1/2+1/2*m)*hypergeom([1/2*m, 3/2-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x+c
))*(e*sec(d*x+c))^m*(1+I*tan(d*x+c))^(1/2-1/2*m)/d/m/(a+I*a*tan(d*x+c))^(1/
2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3586, 3604, 72, 71}

$$\int \frac{(e\sec(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{i2^{\frac{m-1}{2}}(1+i\tan(c+dx))^{\frac{1-m}{2}}(e\sec(c+dx))^m\,\text{Hypergeometric2F1}\left(\frac{3-m}{2},\frac{m}{2},\frac{m+2}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{dm\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[(e*Sec[c + d*x])^m/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (I*2^((-1 + m)/2)*Hypergeometric2F1[(3 - m)/2, m/2, (2 + m)/2, (1 - I*Tan[c
 + d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^((1 - m)/2))/(d*m*Sqrt[
a + I*a*Tan[c + d*x]])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \left((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\int(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{-\frac{1}{2}+\frac{m}{2}}\,dx$$

$$= \frac{\left(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\text{Subst}\left(\int(a-iax)^{-1+\frac{m}{2}}(a$$

$$d$$

$$= \frac{\left(2^{-\frac{3}{2}+\frac{m}{2}}a(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{\frac{1}{2}-\frac{m}{2}}\right)\text{Subst}\left(\int\left(\frac{1}{2}+\frac{ix}{2}\right)^{-\frac{3}{2}+\frac{m}{2}}\right.}{d\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{i2^{\frac{1}{2}(-1+m)}\,\text{Hypergeometric2F1}\left(\frac{3-m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+}{dm\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A] (verified)

Time = 2.49 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.64

$$\int \frac{(e\sec(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{i2^{-\frac{1}{2}+m}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-\frac{1}{2}+m}\left(1+e^{2i(c+dx)}\right)^{-\frac{1}{2}+m}\text{Hypergeometric2F1}\left(\frac{1}{2}(-1+m),-\frac{1}{2}+m,\frac{1+m}{2},-e^{2i(c+dx)}\right)}{d\sqrt{e^{idx}}(-1+m)\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[(e*Sec[c + d*x])^m/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-I)*2^(-1/2 + m)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(-1/2 + m)*(
1 + E^((2*I)*(c + d*x)))^(-1/2 + m)*Hypergeometric2F1[(-1 + m)/2, -1/2 + m,
 (1 + m)/2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(1/2 - m)*(e*Sec[c + d*x])^m
*Sqrt[Cos[d*x] + I*Sin[d*x]])/(d*Sqrt[E^(I*d*x)]*(-1 + m)*Sqrt[a + I*a*Tan[
c + d*x]])
```

Maple [F]

$$\int \frac{(e\sec(dx+c))^m}{\sqrt{a+ia\tan(dx+c)}}dx$$

```
[In] int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int \frac{(e\sec(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\sec(dx+c))^m}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/2*sqrt(2)*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(-I*d*x - I*c)/a,
 x)
```

Sympy [F]

$$\int \frac{(e\sec(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\sec{(c+dx)})^m}{\sqrt{ia(\tan{(c+dx)}-i)}}\,dx$$

```
[In] integrate((e*sec(d*x+c))**m/(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Integral((e*sec(c + d*x))**m/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [F]

$$\int \frac{(e\sec(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\sec{(dx+c)})^m}{\sqrt{i\,a\tan{(dx+c)}+a}}\,dx$$

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((e*sec(d*x + c))^m/sqrt(I*a*tan(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\sec{(dx+c)})^m}{\sqrt{i\,a\tan{(dx+c)}+a}}\,dx$$

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^m/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^m}{\sqrt{a+a\tan{(c+d\,x)}\,1\mathrm{i}}}\,dx$$

```
[In] int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.462 $\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 109

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{i2^{\frac{1}{2}(-3+m)}\,\text{Hypergeometric2F1}\left(\frac{5-m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+}{adm\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] I*2^(-3/2+1/2*m)*hypergeom([1/2*m, 5/2-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x+c
))*(e*sec(d*x+c))^m*(1+I*tan(d*x+c))^(1/2-1/2*m)/a/d/m/(a+I*a*tan(d*x+c))^(
1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3586, 3604, 72, 71}

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{3/2}}\,dx = \frac{i2^{\frac{m-3}{2}}(1+i\tan(c+dx))^{\frac{1-m}{2}}(e\sec(c+dx))^m\,\text{Hypergeometric2F1}\left(\frac{5-m}{2},\frac{m}{2},\frac{m}{}\right.}{adm\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] (I*2^((-3 + m)/2)*Hypergeometric2F1[(5 - m)/2, m/2, (2 + m)/2, (1 - I*Tan[c
 + d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^((1 - m)/2))/(a*d*m*Sqr
t[a + I*a*Tan[c + d*x]])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
```

```
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \left((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\int (a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{-\frac{3}{2}+\frac{m}{2}}\,dx$$

$$= \frac{\left(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\text{Subst}\left(\int (a-iax)^{-1+\frac{m}{2}}(a$$

$$d$$

$$= \frac{\left(2^{-\frac{5}{2}+\frac{m}{2}}(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{\frac{1}{2}-\frac{m}{2}}\right)\text{Subst}\left(\int\left(\frac{1}{2}+\frac{ix}{2}\right)^{-\frac{5}{2}+\frac{m}{2}}(a$$

$$d\sqrt{a+ia\tan(c+dx)}$$

$$= \frac{i2^{\frac{1}{2}(-3+m)}\,\text{Hypergeometric2F1}\left(\frac{5-m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+$$

$$adm\sqrt{a+ia\tan(c+dx)}$$

Mathematica [A] (verified)

Time = 3.14 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.63

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{3/2}}\,dx = -\frac{i2^{-\frac{3}{2}+m}e^{-2i(c+2dx)}\sqrt{e^{idx}}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{1}{2}+m}\left(1+e^{2i(c+dx)}\right)^3\text{Hypergeometric2F1}\left(1,1-\frac{m}{2},\frac{1}{2}(-1+m),-e^{2i(c+d}\right.}{d(-3+m)(a+ia\tan(c+dx))^{3/2}}$$

```
[In] Integrate[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x])^(3/2),x]
```

```
[Out] ((-I)*2^(-3/2 + m)*Sqrt[E^(I*d*x)]*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)
)))^(1/2 + m)*(1 + E^((2*I)*(c + d*x)))^3*Hypergeometric2F1[1, 1 - m/2, (-1
 + m)/2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(3/2 - m)*(e*Sec[c + d*x])^m*(C
os[d*x] + I*Sin[d*x])^(3/2))/(d*E^((2*I)*(c + 2*d*x))*(-3 + m)*(a + I*a*Tan
[c + d*x])^(3/2))
```

Maple [F]

$$\int \frac{(e\sec(dx+c))^m}{(a+ia\tan(dx+c))^{\frac{3}{2}}}dx$$

```
[In] int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(3/2),x)
```

```
[Out] int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(3/2),x)
```

Fricas [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{(e\sec(dx+c))^m}{(i\,a\tan(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(1/4*sqrt(2)*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c)
+ 1)*e^(-3*I*d*x - 3*I*c)/a^2, x)
```

Sympy [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{(e\sec{(c+dx)})^m}{(ia(\tan{(c+dx)}-i))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*sec(d*x+c))**m/(a+I*a*tan(d*x+c))**(3/2),x)
[Out] Integral((e*sec(c + d*x))**m/(I*a*(tan(c + d*x) - I))**(3/2), x)
```

Maxima [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{(e\sec{(dx+c)})^m}{(i\,a\tan{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((e*sec(d*x + c))^m/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{(e\sec{(dx+c)})^m}{(i\,a\tan{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^m/(I*a*tan(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{3/2}}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^m}{(a+a\tan{(c+d\,x)}\ 1\mathrm{i})^{3/2}}\,dx$$

```
[In] int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^(3/2),x)
[Out] int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^(3/2), x)
```

3.463 $\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 109

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{i2^{\frac{1}{2}(-5+m)}\,\text{Hypergeometric2F1}\left(\frac{7-m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+}{a^2dm\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] I*2^(-5/2+1/2*m)*hypergeom([1/2*m, 7/2-1/2*m],[1+1/2*m],1/2-1/2*I*tan(d*x+c
))*(e*sec(d*x+c))^m*(1+I*tan(d*x+c))^(1/2-1/2*m)/a^2/d/m/(a+I*a*tan(d*x+c))
^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3586, 3604, 72, 71}

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{5/2}}\,dx = \frac{i2^{\frac{m-5}{2}}(1+i\tan(c+dx))^{\frac{1-m}{2}}(e\sec(c+dx))^m\,\text{Hypergeometric2F1}\left(\frac{7-m}{2},\frac{m}{2},\frac{m}{}\right.}{a^2dm\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] (I*2^((-5 + m)/2)*Hypergeometric2F1[(7 - m)/2, m/2, (2 + m)/2, (1 - I*Tan[c
 + d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^((1 - m)/2))/(a^2*d*m*S
qrt[a + I*a*Tan[c + d*x]])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
```

```
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \left((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\int(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{-\frac{5}{2}+\frac{m}{2}}\,dx$$

$$= \frac{\left(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\text{Subst}\left(\int(a-iax)^{-1+\frac{m}{2}}(a\right.}{d}$$

$$= \frac{\left(2^{-\frac{7}{2}+\frac{m}{2}}(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{\frac{1}{2}-\frac{m}{2}}\right)\text{Subst}\left(\int\left(\frac{1}{2}+\frac{ix}{2}\right)^{-\frac{7}{2}+\frac{m}{2}}(\right.}{ad\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{i2^{\frac{1}{2}(-5+m)}\text{Hypergeometric2F1}\left(\frac{7-m}{2},\frac{m}{2},\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+\right.}{a^2dm\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A] (verified)

Time = 5.07 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.63

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{5/2}}\,dx = -\frac{i2^{-\frac{5}{2}+m}e^{-3i(c+2dx)}\sqrt{e^{idx}}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{1}{2}+m}\left(1+e^{2i(c+dx)}\right)^4 \text{Hypergeometric2F1}\left(1,1-\frac{m}{2},\frac{1}{2}(-3+m),-e^{2i(c+d}\right.}{d(-5+m)(a+ia\tan(c+dx))^{5/2}}$$

```
[In] Integrate[(e*Sec[c + d*x])^m/(a + I*a*Tan[c + d*x])^(5/2),x]
```

```
[Out] ((-I)*2^(-5/2 + m)*Sqrt[E^(I*d*x)]*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)
)))^(1/2 + m)*(1 + E^((2*I)*(c + d*x)))^4*Hypergeometric2F1[1, 1 - m/2, (-3
 + m)/2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(5/2 - m)*(e*Sec[c + d*x])^m*(C
os[d*x] + I*Sin[d*x])^(5/2))/(d*E^((3*I)*(c + 2*d*x))*(-5 + m)*(a + I*a*Tan
[c + d*x])^(5/2))
```

Maple [F]

$$\int \frac{(e\sec(dx+c))^m}{(a+ia\tan(dx+c))^{\frac{5}{2}}}dx$$

```
[In] int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(5/2),x)
```

```
[Out] int((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(5/2),x)
```

Fricas [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{(e\sec(dx+c))^m}{(i\,a\tan(dx+c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(1/8*sqrt(2)*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c)
+ 3*e^(2*I*d*x + 2*I*c) + 1)*e^(-5*I*d*x - 5*I*c)/a^3, x)
```

Sympy [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{(e\sec{(c+dx)})^m}{(ia(\tan{(c+dx)}-i))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((e*sec(d*x+c))**m/(a+I*a*tan(d*x+c))**(5/2),x)
[Out] Integral((e*sec(c + d*x))**m/(I*a*(tan(c + d*x) - I))**(5/2), x)
```

Maxima [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{(e\sec{(dx+c)})^m}{(i\,a\tan{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((e*sec(d*x + c))^m/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{(e\sec{(dx+c)})^m}{(i\,a\tan{(dx+c)}+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((e*sec(d*x+c))^m/(a+I*a*tan(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^m/(I*a*tan(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\sec(c+dx))^m}{(a+ia\tan(c+dx))^{5/2}}\,dx = \int \frac{\left(\frac{e}{\cos(c+d\,x)}\right)^m}{(a+a\tan{(c+d\,x)}\,1\mathrm{i})^{5/2}}\,dx$$

```
[In] int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^(5/2),x)
[Out] int((e/cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^(5/2), x)
```

3.464 $\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 26, antiderivative size = 105

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^n\,dx = \frac{i2^{\frac{m}{2}+n}\operatorname{Hypergeometric2F1}\left(\frac{m}{2},1-\frac{m}{2}-n,\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(c+dx))^{-}}{dm}$$

```
[Out] I*2^(1/2*m+n)*hypergeom([1/2*m, 1-1/2*m-n],[1+1/2*m],1/2-1/2*I*tan(d*x+c))*
(e*sec(d*x+c))^m*(1+I*tan(d*x+c))^(-1/2*m-n)*(a+I*a*tan(d*x+c))^n/d/m
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3586, 3604, 72, 71}

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^n\,dx = \frac{i2^{\frac{m}{2}+n}(a+ia\tan(c+dx))^n(e\sec(c+dx))^m(1+i\tan(c+dx))^{-\frac{m}{2}-n}\operatorname{Hypergeometric2F1}\left(\frac{m}{2},-\frac{m}{2}-n+1,\right.}{dm}$$

```
[In] Int[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (I*2^(m/2 + n)*Hypergeometric2F1[m/2, 1 - m/2 - n, (2 + m)/2, (1 - I*Tan[c
+ d*x])/2]*(e*Sec[c + d*x])^m*(1 + I*Tan[c + d*x])^(-1/2*m - n)*(a + I*a*Ta
n[c + d*x])^n)/(d*m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \left((e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\int(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{\frac{m}{2}+n}\,dx$$

$$= \frac{\left(a^2(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-m/2}\right)\text{Subst}\left(\int(a-iax)^{-1+\frac{m}{2}}(a\right.}{d}$$

$$= \frac{\left(2^{-1+\frac{m}{2}+n}a(e\sec(c+dx))^m(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^n\left(\frac{a+ia\tan(c+dx)}{a}\right)^{-\frac{m}{2}-n}\right)}{d}$$

$$= \frac{i2^{\frac{m}{2}+n}\text{Hypergeometric2F1}\left(\frac{m}{2},1-\frac{m}{2}-n,\frac{2+m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^m(1+i\tan(}{dm}$$

Mathematica [A] (verified)

Time = 10.02 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.57

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{m+n}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{m+n}\left(1+e^{2i(c+dx)}\right)^{m+n}\text{Hypergeometric2F1}\left(\frac{m}{2}+n,m+n,1+\frac{m}{2}+n,-e^{2i(c+\cdots}\right.}{d(m+2n)}$$

```
[In] Integrate[(e*Sec[c + d*x])^m*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(m + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(
m + n)*(1 + E^((2*I)*(c + d*x)))^(m + n)*Hypergeometric2F1[m/2 + n, m + n,
1 + m/2 + n, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(-m - n)*(e*Sec[c + d*x])^m
*(a + I*a*Tan[c + d*x])^n)/(d*(m + 2*n)*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F]

$$\int \left(e\sec\left(dx+c\right)\right)^{m}\left(a+ia\tan\left(dx+c\right)\right)^{n}dx$$

```
[In] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^n\,dx = \int \left(e\sec\left(dx+c\right)\right)^{m}\left(i\,a\tan\left(dx+c\right)+a\right)^{n}\,dx$$

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^m*e^(I*d*n*x + I*c
*n + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1)) + n*log(a/e)), x)
```

Sympy [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^n\,dx = \int \left(e\sec\left(c+dx\right)\right)^{m}\left(ia(\tan\left(c+dx\right)-i)\right)^{n}\,dx$$

```
[In] integrate((e*sec(d*x+c))**m*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Integral((e*sec(c + d*x))**m*(I*a*(tan(c + d*x) - I))**n, x)
```

Maxima [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^m\,(i\,a\tan(dx+c)+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((e*sec(d*x + c))^m*(I*a*tan(d*x + c) + a)^n, x)
```

Giac [F]

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^m\,(i\,a\tan(dx+c)+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^m*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^m(a+ia\tan(c+dx))^n\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^m\,(a+a\tan(c+d\,x)\,1\mathrm{i})^n\,dx$$

```
[In] int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^n,x)
[Out] int((e/cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^n, x)
```

3.465 $\int \sec^6(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 24, antiderivative size = 97

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{4i(a+ia\tan(c+dx))^{3+n}}{a^3d(3+n)} + \frac{4i(a+ia\tan(c+dx))^{4+n}}{a^4d(4+n)} - \frac{i(a+ia\tan(c+dx))^{5+n}}{a^5d(5+n)}$$

```
[Out] -4*I*(a+I*a*tan(d*x+c))^(3+n)/a^3/d/(3+n)+4*I*(a+I*a*tan(d*x+c))^(4+n)/a^4/
d/(4+n)-I*(a+I*a*tan(d*x+c))^(5+n)/a^5/d/(5+n)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{i(a+ia\tan(c+dx))^{n+5}}{a^5d(n+5)} + \frac{4i(a+ia\tan(c+dx))^{n+4}}{a^4d(n+4)} - \frac{4i(a+ia\tan(c+dx))^{n+3}}{a^3d(n+3)}$$

```
[In] Int[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-4*I)*(a + I*a*Tan[c + d*x])^(3 + n))/(a^3*d*(3 + n)) + ((4*I)*(a + I*a*T
an[c + d*x])^(4 + n))/(a^4*d*(4 + n)) - (I*(a + I*a*Tan[c + d*x])^(5 + n))/
(a^5*d*(5 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)^2(a+x)^{2+n}\,dx, x, ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{i\text{Subst}\left(\int \left(4a^2(a+x)^{2+n}-4a(a+x)^{3+n}+(a+x)^{4+n}\right)\,dx, x, ia\tan(c+dx)\right)}{a^5d}\\ &= -\frac{4i(a+ia\tan(c+dx))^{3+n}}{a^3d(3+n)}+\frac{4i(a+ia\tan(c+dx))^{4+n}}{a^4d(4+n)}-\frac{i(a+ia\tan(c+dx))^{5+n}}{a^5d(5+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{i(a+ia\tan(c+dx))^{3+n}\left(\frac{4a^2}{3+n}-\frac{4a(a+ia\tan(c+dx))}{4+n}+\frac{(a+ia\tan(c+dx))^2}{5+n}\right)}{a^5d}$$

```
[In] Integrate[Sec[c + d*x]^6*(a + I*a*Tan[c + d*x])^n,x]
[Out] ((-I)*(a + I*a*Tan[c + d*x])^(3 + n)*((4*a^2)/(3 + n) - (4*a*(a + I*a*Tan[c
 + d*x]))/(4 + n) + (a + I*a*Tan[c + d*x])^2/(5 + n)))/(a^5*d)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(91) = 182.

Time = 1.83 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.80

method	result
derivativedivides	$\frac{(\tan^5(dx+c))e^{n\ln(a+ia\tan(dx+c))}}{d(5+n)} + \frac{(n^2+15n+60)\tan(dx+c)e^{n\ln(a+ia\tan(dx+c))}}{d(3+n)(4+n)(5+n)} - \frac{in(\tan^4(dx+c))e^{n\ln(a+ia\tan(d}}{(nd+4d)(5+n)}$
default	$\frac{(\tan^5(dx+c))e^{n\ln(a+ia\tan(dx+c))}}{d(5+n)} + \frac{(n^2+15n+60)\tan(dx+c)e^{n\ln(a+ia\tan(dx+c))}}{d(3+n)(4+n)(5+n)} - \frac{in(\tan^4(dx+c))e^{n\ln(a+ia\tan(d}}{(nd+4d)(5+n)}$
risch	Expression too large to display

```
[In] int(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/(5+n)*tan(d*x+c)^5*exp(n*ln(a+I*a*tan(d*x+c)))+(n^2+15*n+60)/d/(3+n)/(4
+n)/(5+n)*tan(d*x+c)*exp(n*ln(a+I*a*tan(d*x+c)))-I*n/(d*n+4*d)/(5+n)*tan(d*
x+c)^4*exp(n*ln(a+I*a*tan(d*x+c)))+2*(n^2+11*n+20)/d/(3+n)/(4+n)/(5+n)*tan(
d*x+c)^3*exp(n*ln(a+I*a*tan(d*x+c)))-I*(n^2+11*n+32)/d/(3+n)/(4+n)/(5+n)*ex
p(n*ln(a+I*a*tan(d*x+c)))-2*I*n*(n+7)/d/(3+n)/(4+n)/(5+n)*tan(d*x+c)^2*exp(
n*ln(a+I*a*tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 247 vs. 2(85) = 170.

Time = 0.25 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.55

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^n\,dx =$$

$$-\frac{32\,(2\,(i\,n+5i)e^{(8i\,dx+8i}}{dn^3+12\,dn^2+47\,dn+(dn^3+12\,dn^2+47\,dn+60\,d)e^{(10i\,dx+10i\,c)}+5\,(dn^3+12\,dn^2+47\,dn+60\,d)e^{(8i\,d}}$$

```
[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] -32*(2*(I*n + 5*I)*e^(8*I*d*x + 8*I*c) + (I*n^2 + 9*I*n + 20*I)*e^(6*I*d*x
 + 6*I*c) + 2*I*e^(10*I*d*x + 10*I*c))*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x
 + 2*I*c) + 1))^n/(d*n^3 + 12*d*n^2 + 47*d*n + (d*n^3 + 12*d*n^2 + 47*d*n +
60*d)*e^(10*I*d*x + 10*I*c) + 5*(d*n^3 + 12*d*n^2 + 47*d*n + 60*d)*e^(8*I*d
*x + 8*I*c) + 10*(d*n^3 + 12*d*n^2 + 47*d*n + 60*d)*e^(6*I*d*x + 6*I*c) + 1
0*(d*n^3 + 12*d*n^2 + 47*d*n + 60*d)*e^(4*I*d*x + 4*I*c) + 5*(d*n^3 + 12*d*
n^2 + 47*d*n + 60*d)*e^(2*I*d*x + 2*I*c) + 60*d)
```

Sympy [F]

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (ia(\tan(c+dx)-i))^n \sec^6(c+dx)\,dx$$

[In] integrate(sec(d*x+c)**6*(a+I*a*tan(d*x+c))**n,x)

[Out] Integral((I*a*(tan(c + d*x) - I))**n*sec(c + d*x)**6, x)

Maxima [F]

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n \sec(dx+c)^6\,dx$$

[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")

[Out] integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c)^6, x)

Giac [F]

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n \sec(dx+c)^6\,dx$$

[In] integrate(sec(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")

[Out] integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c)^6, x)

Mupad [B] (verification not implemented)

Time = 9.52 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.73

$$\int \sec^6(c+dx)(a+ia\tan(c+dx))^n\,dx = \frac{e^{-c\,5i-d\,x\,5i}\left(a+\frac{a\sin(c+d\,x)\,1i}{\cos(c+d\,x)}\right)^n\left(\frac{64\,e^{c\,10i+d\,x\,10i}}{d\,(n^3\,1i+n^2\,12i+n\,47i+60i)}+\frac{e^{c\,6i+d\,x\,6i}\,(32\,n^2+288\,n+640)}{d\,(n^3\,1i+n^2\,12i+n\,47i+60i)}+\frac{e^{c\,8i+d\,x\,8i}\,(64\,n+320)}{d\,(n^3\,1i+n^2\,12i+n\,47i+60i)}\right)}{32\cos(c+d\,x)^5}$$

[In] int((a + a*tan(c + d*x)*1i)^n/cos(c + d*x)^6,x)

[Out] (exp(- c*5i - d*x*5i)*(a + (a*sin(c + d*x)*1i)/cos(c + d*x))^n*((64*exp(c*1
0i + d*x*10i))/(d*(n*47i + n^2*12i + n^3*1i + 60i)) + (exp(c*6i + d*x*6i)*(
288*n + 32*n^2 + 640))/(d*(n*47i + n^2*12i + n^3*1i + 60i)) + (exp(c*8i + d
*x*8i)*(64*n + 320))/(d*(n*47i + n^2*12i + n^3*1i + 60i))))/(32*cos(c + d*x
)^5)

3.466 $\int \sec^4(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 24, antiderivative size = 65

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{2i(a+ia\tan(c+dx))^{2+n}}{a^2d(2+n)} + \frac{i(a+ia\tan(c+dx))^{3+n}}{a^3d(3+n)}$$

```
[Out] -2*I*(a+I*a*tan(d*x+c))^(2+n)/a^2/d/(2+n)+I*(a+I*a*tan(d*x+c))^(3+n)/a^3/d/
(3+n)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 45}

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^n\,dx = \frac{i(a+ia\tan(c+dx))^{n+3}}{a^3d(n+3)} - \frac{2i(a+ia\tan(c+dx))^{n+2}}{a^2d(n+2)}$$

```
[In] Int[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-2*I)*(a + I*a*Tan[c + d*x])^(2 + n))/(a^2*d*(2 + n)) + (I*(a + I*a*Tan[c
 + d*x])^(3 + n))/(a^3*d*(3 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{Subst}\left(\int (a-x)(a+x)^{1+n}\,dx, x, ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{i\text{Subst}\left(\int \left(2a(a+x)^{1+n}-(a+x)^{2+n}\right)\,dx, x, ia\tan(c+dx)\right)}{a^3d}\\ &= -\frac{2i(a+ia\tan(c+dx))^{2+n}}{a^2d(2+n)}+\frac{i(a+ia\tan(c+dx))^{3+n}}{a^3d(3+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{i\left(\frac{2a(a+ia\tan(c+dx))^{2+n}}{2+n}-\frac{(a+ia\tan(c+dx))^{3+n}}{3+n}\right)}{a^3d}$$

```
[In] Integrate[Sec[c + d*x]^4*(a + I*a*Tan[c + d*x])^n,x]
[Out] ((-I)*((2*a*(a + I*a*Tan[c + d*x])^(2 + n))/(2 + n) - (a + I*a*Tan[c + d*x]
)^(3 + n)/(3 + n)))/(a^3*d)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(61) = 122.

Time = 1.01 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.31

method	result
derivativedivides	$\frac{(\tan^3(dx+c))e^{n\ln(a+ia\tan(dx+c))}}{d(3+n)}+\frac{(n+6)\tan(dx+c)e^{n\ln(a+ia\tan(dx+c))}}{d(2+n)(3+n)}-\frac{i(4+n)e^{n\ln(a+ia\tan(dx+c))}}{d(2+n)(3+n)}-\frac{in(\tan$
default	$\frac{(\tan^3(dx+c))e^{n\ln(a+ia\tan(dx+c))}}{d(3+n)}+\frac{(n+6)\tan(dx+c)e^{n\ln(a+ia\tan(dx+c))}}{d(2+n)(3+n)}-\frac{i(4+n)e^{n\ln(a+ia\tan(dx+c))}}{d(2+n)(3+n)}-\frac{in(\tan$
risch	Expression too large to display

```
[In] int(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/(3+n)*tan(d*x+c)^3*exp(n*ln(a+I*a*tan(d*x+c)))+(n+6)/d/(2+n)/(3+n)*tan(
d*x+c)*exp(n*ln(a+I*a*tan(d*x+c)))-I*(4+n)/d/(2+n)/(3+n)*exp(n*ln(a+I*a*tan
(d*x+c)))-I*n/d/(2+n)/(3+n)*tan(d*x+c)^2*exp(n*ln(a+I*a*tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(57) = 114$.

Time = 0.26 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.18

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{8\left((i\,n+3i)e^{(4i\,dx+4i\,c)}+i\,e^{(6i\,dx+6i\,c)}\right)\left(\frac{2\,ae^{(2i\,dx+2i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^n}{dn^2+5\,dn+(dn^2+5\,dn+6\,d)e^{(6i\,dx+6i\,c)}+3\,(dn^2+5\,dn+6\,d)e^{(4i\,dx+4i\,c)}+3\,(dn^2+5\,dn+6\,d)e^{(2i\,dx+}}$$

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] -8*((I*n + 3*I)*e^(4*I*d*x + 4*I*c) + I*e^(6*I*d*x + 6*I*c))*(2*a*e^(2*I*d*
x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n/(d*n^2 + 5*d*n + (d*n^2 + 5*d*n + 6
*d)*e^(6*I*d*x + 6*I*c) + 3*(d*n^2 + 5*d*n + 6*d)*e^(4*I*d*x + 4*I*c) + 3*(
d*n^2 + 5*d*n + 6*d)*e^(2*I*d*x + 2*I*c) + 6*d)
```

Sympy [F]

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (ia(\tan(c+dx)-i))^n\sec^4(c+dx)\,dx$$

```
[In] integrate(sec(d*x+c)**4*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**n*sec(c + d*x)**4, x)
```

Maxima [F]

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\sec(dx+c)^4\,dx$$

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c)^4, x)
```

Giac [F]

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\sec(dx+c)^4\,dx$$

```
[In] integrate(sec(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c)^4, x)
```

Mupad [B] (verification not implemented)

Time = 2.82 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.32

$$\int \sec^4(c+dx)(a+ia\tan(c+dx))^n\,dx =$$
$$-\frac{4\left(\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}\right)^n(n\,3\mathrm{i}+\cos(2\,c+2\,d\,x)\,15\mathrm{i}+\cos(4\,c+4\,d\,x)\,6\mathrm{i}+\cos(6\,c+6\,d\,x)\,1\mathrm{i}}{d\,(n^2+5\,n+6}$$

```
[In] int((a + a*tan(c + d*x)*1i)^n/cos(c + d*x)^4,x)
[Out] -(4*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1
))^n*(n*3i + cos(2*c + 2*d*x)*15i + cos(4*c + 4*d*x)*6i + cos(6*c + 6*d*x)*
1i - 9*sin(2*c + 2*d*x) - 6*sin(4*c + 4*d*x) - sin(6*c + 6*d*x) + n*cos(2*c
 + 2*d*x)*4i + n*cos(4*c + 4*d*x)*1i - 2*n*sin(2*c + 2*d*x) - n*sin(4*c + 4
*d*x) + 10i))/(d*(5*n + n^2 + 6)*(15*cos(2*c + 2*d*x) + 6*cos(4*c + 4*d*x)
+ cos(6*c + 6*d*x) + 10))
```

3.467 $\int \sec^2(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 24, antiderivative size = 32

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^n\,dx=-\frac{i(a+ia\tan(c+dx))^{1+n}}{ad(1+n)}$$

```
[Out] -I*(a+I*a*tan(d*x+c))^(1+n)/a/d/(1+n)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 32}

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^n\,dx=-\frac{i(a+ia\tan(c+dx))^{n+1}}{ad(n+1)}$$

```
[In] Int[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^n,x]
[Out] ((-I)*(a + I*a*Tan[c + d*x])^(1 + n))/(a*d*(1 + n))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{i\text{Subst}\left(\int (a+x)^n\,dx, x, ia\tan(c+dx)\right)}{ad}$$
$$= -\frac{i(a+ia\tan(c+dx))^{1+n}}{ad(1+n)}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{i(a+ia\tan(c+dx))^{1+n}}{ad(1+n)}$$

```
[In] Integrate[Sec[c + d*x]^2*(a + I*a*Tan[c + d*x])^n,x]
[Out] ((-I)*(a + I*a*Tan[c + d*x])^(1 + n))/(a*d*(1 + n))
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

method	result
derivativedivides	$-\frac{i(a+ia\tan(dx+c))^{1+n}}{ad(1+n)}$
default	$-\frac{i(a+ia\tan(dx+c))^{1+n}}{ad(1+n)}$
risch	$-\frac{2ia^n\left(e^{2i(dx+c)}+1\right)^{-n}\left(e^{i(dx+c)}\right)^{2n}2^n e^{\frac{i\left(-\operatorname{csgn}\left(\frac{ie^{2i(dx+c)}}{e^{2i(dx+c)}+1}\right)^3\pi n+\operatorname{csgn}\left(\frac{ie^{2i(dx+c)}}{e^{2i(dx+c)}+1}\right)^2\pi\operatorname{csgn}\left(\frac{i}{e^{2i(dx+c)}+1}\right)n+\operatorname{csgn}\left(\right.}{}}}{}$

```
[In] int(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x,method=_RETURNVERBOSE)
[Out] -I*(a+I*a*tan(d*x+c))^(1+n)/a/d/(1+n)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(28) = 56$.

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.88

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{2i\left(\frac{2\,ae^{(2i\,dx+2i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^n e^{(2i\,dx+2i\,c)}}{dn+(dn+d)e^{(2i\,dx+2i\,c)}+d}$$

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] -2*I*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*e^(2*I*d*x + 2*I
*c)/(d*n + (d*n + d)*e^(2*I*d*x + 2*I*c) + d)
```

Sympy [F]

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (ia(\tan{(c+dx)}-i))^n\sec^2{(c+dx)}\,dx$$

```
[In] integrate(sec(d*x+c)**2*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**n*sec(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{i\,(i\,a\tan{(dx+c)}+a)^{n+1}}{ad(n+1)}$$

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] -I*(I*a*tan(d*x + c) + a)^(n + 1)/(a*d*(n + 1))
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(28) = 56.

Time = 0.73 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.94

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{i\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2i\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}\right)^{n+1}}{ad(n+1)}$$

```
[In] integrate(sec(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] -I*((a*tan(1/2*d*x + 1/2*c)^2 - 2*I*a*tan(1/2*d*x + 1/2*c) - a)/(tan(1/2*d*
x + 1/2*c)^2 - 1))^(n + 1)/(a*d*(n + 1))
```

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.25

$$\int \sec^2(c+dx)(a+ia\tan(c+dx))^n\,dx = \frac{2\left(\cos\left(2\,d\,x\right)+\sin\left(2\,d\,x\right)\,1\mathrm{i}\right)\,\left(\cos\left(2\,c\right)+\sin\left(2\,c\right)\,1\mathrm{i}\right)\,{\left(\frac{a\,\left(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i}\right)}{\cos(2\,c+2\,d\,x)+1}\right)}^n}{d\,\left(n+1\right)\,\left(\cos\left(2\,c+2\,d\,x\right)\,1\mathrm{i}-\sin\left(2\,c+2\,d\,x\right)+1\mathrm{i}\right)}$$

```
[In] int((a + a*tan(c + d*x)*1i)^n/cos(c + d*x)^2,x)
```

```
[Out] (2*(cos(2*d*x) + sin(2*d*x)*1i)*(cos(2*c) + sin(2*c)*1i)*((a*(cos(2*c + 2*d
*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^n)/(d*(n + 1)*(cos(
2*c + 2*d*x)*1i - sin(2*c + 2*d*x) + 1i))
```

3.468 $\int \cos^2(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 24, antiderivative size = 56

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^n\,dx = \frac{ia\,\text{Hypergeometric2F1}\left(2,-1+n,n,\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^{-1+n}}{4d(1-n)}$$

```
[Out] 1/4*I*a*hypergeom([2, -1+n],[n],1/2+1/2*I*tan(d*x+c))*(a+I*a*tan(d*x+c))^(-
1+n)/d/(1-n)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 70}

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^n\,dx = \frac{ia(a+ia\tan(c+dx))^{n-1}\,\text{Hypergeometric2F1}\left(2,n-1,n,\frac{1}{2}(i\tan(c+dx)+1)\right)}{4d(1-n)}$$

```
[In] Int[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((I/4)*a*Hypergeometric2F1[2, -1 + n, n, (1 + I*Tan[c + d*x])/2]*(a + I*a*T
an[c + d*x])^(-1 + n))/(d*(1 - n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
```

```
+ 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{(ia^3)\,\text{Subst}\left(\int \frac{(a+x)^{-2+n}}{(a-x)^2}\,dx, x, ia\tan(c+dx)\right)}{d}$$
$$= \frac{ia\,\text{Hypergeometric2F1}\left(2, -1+n, n, \frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^{-1+n}}{4d(1-n)}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^n\,dx$$
$$= -\frac{ia\,\text{Hypergeometric2F1}\left(2, -1+n, n, \frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^{-1+n}}{4d(-1+n)}$$

```
[In] Integrate[Cos[c + d*x]^2*(a + I*a*Tan[c + d*x])^n,x]
[Out] ((-1/4*I)*a*Hypergeometric2F1[2, -1 + n, n, (1 + I*Tan[c + d*x])/2]*(a + I*
a*Tan[c + d*x])^(-1 + n))/(d*(-1 + n))
```

Maple [F]

$$\int \left(\cos^2(dx+c)\right)(a+ia\tan(dx+c))^n\,dx$$

```
[In] int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x)
[Out] int(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\cos(dx+c)^2\,dx$$

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(1/4*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(e^(4*I*
d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1)*e^(-2*I*d*x - 2*I*c), x)
```

Sympy [F]

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (ia(\tan(c+dx)-i))^n\cos^2(c+dx)\,dx$$

```
[In] integrate(cos(d*x+c)**2*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**n*cos(c + d*x)**2, x)
```

Maxima [F]

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\cos(dx+c)^2\,dx$$

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^2, x)
```

Giac [F]

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\cos(dx+c)^2\,dx$$

```
[In] integrate(cos(d*x+c)^2*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+ia\tan(c+dx))^n\,dx = \int \cos\left(c+d\,x\right)^2\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^n\,dx$$

```
[In] int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^n,x)
[Out] int(cos(c + d*x)^2*(a + a*tan(c + d*x)*1i)^n, x)
```

3.469 $\int \cos^4(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 24, antiderivative size = 60

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^n\,dx = \frac{ia^2\,\text{Hypergeometric2F1}\left(3,-2+n,-1+n,\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^{-2+n}}{8d(2-n)}$$

```
[Out] 1/8*I*a^2*hypergeom([3, -2+n],[-1+n],1/2+1/2*I*tan(d*x+c))*(a+I*a*tan(d*x+c
))^(-2+n)/d/(2-n)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3568, 70}

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^n\,dx = \frac{ia^2(a+ia\tan(c+dx))^{n-2}\,\text{Hypergeometric2F1}\left(3,n-2,n-1,\frac{1}{2}(i\tan(c+dx)+1)\right)}{8d(2-n)}$$

```
[In] Int[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((I/8)*a^2*Hypergeometric2F1[3, -2 + n, -1 + n, (1 + I*Tan[c + d*x])/2]*(a
+ I*a*Tan[c + d*x])^(-2 + n))/(d*(2 - n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
```

```
+ 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{(ia^5)\,\text{Subst}\left(\int \frac{(a+x)^{-3+n}}{(a-x)^3}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= \frac{ia^2\,\text{Hypergeometric2F1}\left(3,-2+n,-1+n,\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^{-2+n}}{8d(2-n)}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{ia^2\,\text{Hypergeometric2F1}\left(3,-2+n,-1+n,\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^{-2+n}}{8d(-2+n)}$$

```
[In] Integrate[Cos[c + d*x]^4*(a + I*a*Tan[c + d*x])^n,x]
[Out] ((-1/8*I)*a^2*Hypergeometric2F1[3, -2 + n, -1 + n, (1 + I*Tan[c + d*x])/2]*
(a + I*a*Tan[c + d*x])^(-2 + n))/(d*(-2 + n))
```

Maple [F]

$$\int \left(\cos^4(dx+c)\right)(a+ia\tan(dx+c))^n\,dx$$

```
[In] int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x)
[Out] int(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\cos(dx+c)^4\,dx$$

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
[Out] integral(1/16*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(e^(8*I
*d*x + 8*I*c) + 4*e^(6*I*d*x + 6*I*c) + 6*e^(4*I*d*x + 4*I*c) + 4*e^(2*I*d*
x + 2*I*c) + 1)*e^(-4*I*d*x - 4*I*c), x)
```

Sympy [F]

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (ia(\tan(c+dx)-i))^n\cos^4(c+dx)\,dx$$

```
[In] integrate(cos(d*x+c)**4*(a+I*a*tan(d*x+c))**n,x)
[Out] Integral((I*a*(tan(c + d*x) - I))**n*cos(c + d*x)**4, x)
```

Maxima [F]

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\cos(dx+c)^4\,dx$$

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^4, x)
```

Giac [F]

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\cos(dx+c)^4\,dx$$

```
[In] integrate(cos(d*x+c)^4*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+ia\tan(c+dx))^n\,dx = \int \cos\left(c+d\,x\right)^4\,\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^n\,dx$$

```
[In] int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^n,x)
[Out] int(cos(c + d*x)^4*(a + a*tan(c + d*x)*1i)^n, x)
```

3.470 $\int \cos^6(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 24, antiderivative size = 60

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^n\,dx = \frac{ia^3 \text{Hypergeometric2F1}\left(4,-3+n,-2+n,\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^{-3+n}}{16d(3-n)}$$

```
[Out] 1/16*I*a^3*hypergeom([4, -3+n],[-2+n],1/2+1/2*I*tan(d*x+c))*(a+I*a*tan(d*x+
c))^(-3+n)/d/(3-n)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3568, 70}

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^n\,dx = \frac{ia^3(a+ia\tan(c+dx))^{n-3} \text{Hypergeometric2F1}\left(4,n-3,n-2,\frac{1}{2}(i\tan(c+dx)+1)\right)}{16d(3-n)}$$

```
[In] Int[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((I/16)*a^3*Hypergeometric2F1[4, -3 + n, -2 + n, (1 + I*Tan[c + d*x])/2]*(a
 + I*a*Tan[c + d*x])^(-3 + n))/(d*(3 - n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
```

```
+ 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{(ia^7)\,\text{Subst}\left(\int \frac{(a+x)^{-4+n}}{(a-x)^4}\,dx, x, ia\tan(c+dx)\right)}{d}$$

$$= \frac{ia^3\,\text{Hypergeometric2F1}\left(4,-3+n,-2+n,\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^{-3+n}}{16d(3-n)}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{ia^3\,\text{Hypergeometric2F1}\left(4,-3+n,-2+n,\frac{1}{2}(1+i\tan(c+dx))\right)(a+ia\tan(c+dx))^{-3+n}}{16d(-3+n)}$$

```
[In] Integrate[Cos[c + d*x]^6*(a + I*a*Tan[c + d*x])^n,x]
[Out] ((-1/16*I)*a^3*Hypergeometric2F1[4, -3 + n, -2 + n, (1 + I*Tan[c + d*x])/2]
*(a + I*a*Tan[c + d*x])^(-3 + n))/(d*(-3 + n))
```

Maple [F]

$$\int \left(\cos^6(dx+c)\right)(a+ia\tan(dx+c))^n\,dx$$

```
[In] int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x)
[Out] int(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\cos(dx+c)^6\,dx$$

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(1/64*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(e^(12*
I*d*x + 12*I*c) + 6*e^(10*I*d*x + 10*I*c) + 15*e^(8*I*d*x + 8*I*c) + 20*e^(
6*I*d*x + 6*I*c) + 15*e^(4*I*d*x + 4*I*c) + 6*e^(2*I*d*x + 2*I*c) + 1)*e^(-
6*I*d*x - 6*I*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^n\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**6*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\cos(dx+c)^6\,dx$$

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^6, x)
```

Giac [F]

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\cos(dx+c)^6\,dx$$

```
[In] integrate(cos(d*x+c)^6*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^6(c+dx)(a+ia\tan(c+dx))^n\,dx = \int \cos\left(c+d\,x\right)^6\,\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^n\,dx$$

```
[In] int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^n,x)
[Out] int(cos(c + d*x)^6*(a + a*tan(c + d*x)*1i)^n, x)
```

3.471 $\int \sec^5(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 24, antiderivative size = 94

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^n\,dx = \frac{i2^{\frac{5}{2}+n}a^2\,\text{Hypergeometric2F1}\left(\frac{5}{2},-\frac{3}{2}-n,\frac{7}{2},\frac{1}{2}(1-i\tan(c+dx))\right)\sec^5(c+dx)(1+i\tan(c+dx))^{-\frac{1}{2}-n}(a+}{5d}$$

```
[Out] 1/5*I*2^(5/2+n)*a^2*hypergeom([5/2, -3/2-n],[7/2],1/2-1/2*I*tan(d*x+c))*sec
(d*x+c)^5*(1+I*tan(d*x+c))^(-1/2-n)*(a+I*a*tan(d*x+c))^(-2+n)/d
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3586, 3604, 72, 71}

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^n\,dx = \frac{ia^2 2^{n+\frac{5}{2}}\sec^5(c+dx)(1+i\tan(c+dx))^{-n-\frac{1}{2}}(a+ia\tan(c+dx))^{n-2}\,\text{Hypergeometric2F1}\left(\frac{5}{2},-n-\frac{3}{2},\frac{7}{2},\frac{1}{2}(1}{5d}$$

```
[In] Int[Sec[c + d*x]^5*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((I/5)*2^(5/2 + n)*a^2*Hypergeometric2F1[5/2, -3/2 - n, 7/2, (1 - I*Tan[c +
 d*x])/2]*Sec[c + d*x]^5*(1 + I*Tan[c + d*x])^(-1/2 - n)*(a + I*a*Tan[c + d
*x])^(-2 + n))/d
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\sec^5(c+dx)\int(a-ia\tan(c+dx))^{5/2}(a+ia\tan(c+dx))^{\frac{5}{2}+n}\,dx}{(a-ia\tan(c+dx))^{5/2}(a+ia\tan(c+dx))^{5/2}}$$

$$= \frac{(a^2\sec^5(c+dx))\,\text{Subst}\left(\int(a-iax)^{3/2}(a+iax)^{\frac{3}{2}+n}\,dx,x,\tan(c+dx)\right)}{d(a-ia\tan(c+dx))^{5/2}(a+ia\tan(c+dx))^{5/2}}$$

$$= \frac{\left(2^{\frac{3}{2}+n}a^3\sec^5(c+dx)(a+ia\tan(c+dx))^{-2+n}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{-\frac{1}{2}-n}\right)\text{Subst}\left(\int\left(\frac{1}{2}+\frac{ix}{2}\right)^{\frac{3}{2}+n}(a-i}{d(a-ia\tan(c+dx))^{5/2}}$$

$$= \frac{i2^{\frac{5}{2}+n}a^2\,\text{Hypergeometric2F1}\left(\frac{5}{2},-\frac{3}{2}-n,\frac{7}{2},\frac{1}{2}(1-i\tan(c+dx))\right)\sec^5(c+dx)(1+i\tan(c+dx))^{-}}{5d}$$

Mathematica [A] (verified)

Time = 14.72 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.59

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{5+n}e^{5i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n \text{Hypergeometric2F1}\left(-\frac{3}{2},1,\frac{7}{2}+n,-e^{2i(c+dx)}\right)\sec^{-n}(c+dx)(\cos(dx)\,-}{d\left(1+e^{2i(c+dx)}\right)^4(5+2n)}$$

```
[In] Integrate[Sec[c + d*x]^5*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(5 + n)*E^((5*I)*(c + d*x))*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^(
(2*I)*(c + d*x))))^n*Hypergeometric2F1[-3/2, 1, 7/2 + n, -E^((2*I)*(c + d*x
))]*(a + I*a*Tan[c + d*x])^n)/(d*(1 + E^((2*I)*(c + d*x)))^4*(5 + 2*n)*Sec[
c + d*x]^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F]

$$\int \left(\sec^5(dx+c)\right)(a+ia\tan(dx+c))^n\,dx$$

```
[In] int(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\sec(dx+c)^5\,dx$$

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(32*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*e^(5*I*d*
x + 5*I*c)/(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*I*d*x +
 6*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (ia(\tan{(c+dx)}-i))^n \sec^5{(c+dx)}\,dx$$

```
[In] integrate(sec(d*x+c)**5*(a+I*a*tan(d*x+c))**n,x)
[Out] Integral((I*a*(tan(c + d*x) - I))**n*sec(c + d*x)**5, x)
```

Maxima [F]

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan{(dx+c)}+a)^n \sec{(dx+c)}^5\,dx$$

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c)^5, x)
```

Giac [F]

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan{(dx+c)}+a)^n \sec{(dx+c)}^5\,dx$$

```
[In] integrate(sec(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^5(c+dx)(a+ia\tan(c+dx))^n\,dx = \int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^n}{\cos{(c+d\,x)}^5}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^n/cos(c + d*x)^5,x)
[Out] int((a + a*tan(c + d*x)*1i)^n/cos(c + d*x)^5, x)
```

3.472 $\int \sec^3(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 24, antiderivative size = 92

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^n\,dx = \frac{i2^{\frac{3}{2}+n}a\,\text{Hypergeometric2F1}\left(\frac{3}{2},-\frac{1}{2}-n,\frac{5}{2},\frac{1}{2}(1-i\tan(c+dx))\right)\sec^3(c+dx)(1+i\tan(c+dx))^{-\frac{1}{2}-n}(a+i$$
$$3d$$

```
[Out] 1/3*I*2^(3/2+n)*a*hypergeom([3/2, -1/2-n],[5/2],1/2-1/2*I*tan(d*x+c))*sec(d
*x+c)^3*(1+I*tan(d*x+c))^(-1/2-n)*(a+I*a*tan(d*x+c))^(-1+n)/d
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3586, 3604, 72, 71}

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^n\,dx = \frac{ia2^{n+\frac{3}{2}}\sec^3(c+dx)(1+i\tan(c+dx))^{-n-\frac{1}{2}}(a+ia\tan(c+dx))^{n-1}\,\text{Hypergeometric2F1}\left(\frac{3}{2},-n-\frac{1}{2},\frac{5}{2},\frac{1}{2}(1$$
$$3d$$

```
[In] Int[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((I/3)*2^(3/2 + n)*a*Hypergeometric2F1[3/2, -1/2 - n, 5/2, (1 - I*Tan[c + d
*x])/2]*Sec[c + d*x]^3*(1 + I*Tan[c + d*x])^(-1/2 - n)*(a + I*a*Tan[c + d*x
])^(-1 + n))/d
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\sec^3(c+dx)\int(a-ia\tan(c+dx))^{3/2}(a+ia\tan(c+dx))^{\frac{3}{2}+n}\,dx}{(a-ia\tan(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}$$

$$= \frac{(a^2\sec^3(c+dx))\,\text{Subst}\left(\int\sqrt{a-iax}(a+iax)^{\frac{1}{2}+n}\,dx,x,\tan(c+dx)\right)}{d(a-ia\tan(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}}$$

$$= \frac{\left(2^{\frac{1}{2}+n}a^2\sec^3(c+dx)(a+ia\tan(c+dx))^{-1+n}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{-\frac{1}{2}-n}\right)\text{Subst}\left(\int\left(\frac{1}{2}+\frac{ix}{2}\right)^{\frac{1}{2}+n}\sqrt{a-\cdots}\right.}{d(a-ia\tan(c+dx))^{3/2}}$$

$$= \frac{i2^{\frac{3}{2}+n}a\,\text{Hypergeometric2F1}\left(\frac{3}{2},-\frac{1}{2}-n,\frac{5}{2},\frac{1}{2}(1-i\tan(c+dx))\right)\sec^3(c+dx)(1+i\tan(c+dx))^{-\cdots}}{3d}$$

Mathematica [A] (verified)

Time = 13.91 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.62

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^n\,dx =$$
$$-\frac{i2^{3+n}e^{3i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n \text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{5}{2}+n,-e^{2i(c+dx)}\right)\sec^{-n}(c+dx)(\cos(dx)}{d\left(1+e^{2i(c+dx)}\right)^2(3+2n)}$$

```
[In] Integrate[Sec[c + d*x]^3*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(3 + n)*E^((3*I)*(c + d*x))*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^(
(2*I)*(c + d*x))))^n*Hypergeometric2F1[-1/2, 1, 5/2 + n, -E^((2*I)*(c + d*x
))]*(a + I*a*Tan[c + d*x])^n)/(d*(1 + E^((2*I)*(c + d*x)))^2*(3 + 2*n)*Sec[
c + d*x]^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F]

$$\int \left(\sec^3(dx+c)\right)(a+ia\tan(dx+c))^n\,dx$$

```
[In] int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n \sec(dx+c)^3\,dx$$

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(8*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*e^(3*I*d*x
 + 3*I*c)/(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I
*c) + 1), x)
```

Sympy [F]

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (ia(\tan(c+dx)-i))^n \sec^3(c+dx)\,dx$$

```
[In] integrate(sec(d*x+c)**3*(a+I*a*tan(d*x+c))**n,x)
[Out] Integral((I*a*(tan(c + d*x) - I))**n*sec(c + d*x)**3, x)
```

Maxima [F]

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n \sec(dx+c)^3\,dx$$

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c)^3, x)
```

Giac [F]

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n \sec(dx+c)^3\,dx$$

```
[In] integrate(sec(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(c+dx)(a+ia\tan(c+dx))^n\,dx = \int \frac{(a+a\tan(c+d\,x)\;1\mathrm{i})^n}{\cos(c+d\,x)^3}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^n/cos(c + d*x)^3,x)
[Out] int((a + a*tan(c + d*x)*1i)^n/cos(c + d*x)^3, x)
```

3.473 $\int \sec(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 22, antiderivative size = 88

$$\int \sec(c+dx)(a+ia\tan(c+dx))^n\,dx = \frac{i2^{\frac{1}{2}+n}a\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-n,\frac{3}{2},\frac{1}{2}(1-i\tan(c+dx))\right)\sec(c+dx)(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+ia\tan}{d}$$

```
[Out] I*2^(1/2+n)*a*hypergeom([1/2, 1/2-n],[3/2],1/2-1/2*I*tan(d*x+c))*sec(d*x+c)
*(1+I*tan(d*x+c))^(1/2-n)*(a+I*a*tan(d*x+c))^(-1+n)/d
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3586, 3604, 72, 71}

$$\int \sec(c+dx)(a+ia\tan(c+dx))^n\,dx = \frac{ia2^{n+\frac{1}{2}}\sec(c+dx)(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+ia\tan(c+dx))^{n-1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-n,\frac{3}{2},\frac{1}{2}(1-i\,t}{d}$$

```
[In] Int[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (I*2^(1/2 + n)*a*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1 - I*Tan[c + d*x])/
2]*Sec[c + d*x]*(1 + I*Tan[c + d*x])^(1/2 - n)*(a + I*a*Tan[c + d*x])^(-1 +
 n))/d
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\sec(c+dx)\int\sqrt{a-ia\tan(c+dx)}(a+ia\tan(c+dx))^{\frac{1}{2}+n}\,dx}{\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{\left(a^2\sec(c+dx)\right)\text{Subst}\left(\int\frac{(a+iax)^{-\frac{1}{2}+n}}{\sqrt{a-iax}}\,dx,x,\tan(c+dx)\right)}{d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{\left(2^{-\frac{1}{2}+n}a^2\sec(c+dx)(a+ia\tan(c+dx))^{-1+n}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{\frac{1}{2}-n}\right)\text{Subst}\left(\int\frac{\left(\frac{1}{2}+\frac{ix}{2}\right)^{-\frac{1}{2}+n}}{\sqrt{a-iax}}\,dx,x,\text{ta}\right.}{d\sqrt{a-ia\tan(c+dx)}}$$

$$= \frac{i2^{\frac{1}{2}+n}a\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-n,\frac{3}{2},\frac{1}{2}(1-i\tan(c+dx))\right)\sec(c+dx)(1+i\tan(c+dx))^{\frac{1}{2}-n}(}{d}$$

Mathematica [A] (verified)

Time = 9.96 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.66

$$\int \sec(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{1+n}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{1+n}\left(1+e^{2i(c+dx)}\right)^{1+n}\text{Hypergeometric2F1}\left(\frac{1}{2}+n,1+n,\frac{3}{2}+n,-e^{2i(c+dx)}\right)\sec^{-}}{d(1+2n)}$$

```
[In] Integrate[Sec[c + d*x]*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(1 + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(
1 + n)*(1 + E^((2*I)*(c + d*x)))^(1 + n)*Hypergeometric2F1[1/2 + n, 1 + n,
3/2 + n, -E^((2*I)*(c + d*x))]*(a + I*a*Tan[c + d*x])^n)/(d*(1 + 2*n)*Sec[c
 + d*x]^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F]

$$\int \sec(dx+c)(a+ia\tan(dx+c))^n\,dx$$

```
[In] int(sec(d*x+c)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int(sec(d*x+c)*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\sec(dx+c)\,dx$$

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(2*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*e^(I*d*x +
 I*c)/(e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (ia(\tan(c+dx)-i))^n\sec(c+dx)\,dx$$

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**n*sec(c + d*x), x)
```

Maxima [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan\left(dx+c\right)+a)^n\sec\left(dx+c\right)\,dx$$

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c), x)
```

Giac [F]

$$\int \sec(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan\left(dx+c\right)+a)^n\sec\left(dx+c\right)\,dx$$

```
[In] integrate(sec(d*x+c)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^n*sec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(c+dx)(a+ia\tan(c+dx))^n\,dx = \int \frac{(a+a\tan(c+d\,x)\;1\mathrm{i})^n}{\cos\left(c+d\,x\right)}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^n/cos(c + d*x),x)
[Out] int((a + a*tan(c + d*x)*1i)^n/cos(c + d*x), x)
```

3.474 $\int \cos(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 22, antiderivative size = 85

$$\int \cos(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{-\frac{1}{2}+n}\cos(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{2}-n,\frac{1}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+$$

```
[Out] -I*2^(-1/2+n)*cos(d*x+c)*hypergeom([-1/2, 3/2-n],[1/2],1/2-1/2*I*tan(d*x+c)
)*(1+I*tan(d*x+c))^(1/2-n)*(a+I*a*tan(d*x+c))^n/d
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3586, 3604, 72, 71}

$$\int \cos(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{n-\frac{1}{2}}\cos(c+dx)(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+ia\tan(c+dx))^n\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{2}-n,\frac{1}{2},\frac{1}{2}(1-i$$

```
[In] Int[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(-1/2 + n)*Cos[c + d*x]*Hypergeometric2F1[-1/2, 3/2 - n, 1/2, (1 -
I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(1/2 - n)*(a + I*a*Tan[c + d*x])^n)
/d
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \left(\cos(c + dx)\sqrt{a - ia\tan(c+dx)}\sqrt{a + ia\tan(c+dx)}\right)\int \frac{(a + ia\tan(c+dx))^{-\frac{1}{2}+n}}{\sqrt{a - ia\tan(c+dx)}}\,dx$$

$$= \frac{\left(a^2\cos(c+dx)\sqrt{a - ia\tan(c+dx)}\sqrt{a + ia\tan(c+dx)}\right)\text{Subst}\left(\int \frac{(a+iax)^{-\frac{3}{2}+n}}{(a-iax)^{3/2}}\,dx, x, \tan(c+dx\right.}{d}$$

$$= \frac{\left(2^{-\frac{3}{2}+n}a\cos(c+dx)\sqrt{a - ia\tan(c+dx)}(a + ia\tan(c+dx))^n\left(\frac{a+ia\tan(c+dx)}{a}\right)^{\frac{1}{2}-n}\right)\text{Subst}\left(\int \frac{(\frac{1}{2}+}{(a}\right.}{d}$$

$$= -\frac{i2^{-\frac{1}{2}+n}\cos(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2}, \frac{3}{2}-n, \frac{1}{2}, \frac{1}{2}(1 - i\tan(c+dx))\right)(1 + i\tan(c+dx)}{d}$$

Mathematica [A] (verified)

Time = 13.43 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.72

$$\int \cos(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{-1+n}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-1+n}\left(1+e^{2i(c+dx)}\right)^{-1+n}\text{Hypergeometric2F1}\left(-1+n,-\frac{1}{2}+n,\frac{1}{2}+n,-e^{2i(c+d}\right.}{d(-1+2n)}$$

```
[In] Integrate[Cos[c + d*x]*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(-1 + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^
(-1 + n)*(1 + E^((2*I)*(c + d*x)))^(-1 + n)*Hypergeometric2F1[-1 + n, -1/2
+ n, 1/2 + n, -E^((2*I)*(c + d*x))]*(a + I*a*Tan[c + d*x])^n)/(d*(-1 + 2*n)
*Sec[c + d*x]^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F]

$$\int \cos(dx+c)\,(a+ia\tan(dx+c))^n\,dx$$

```
[In] int(cos(d*x+c)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int(cos(d*x+c)*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int \cos(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\cos(dx+c)\,dx$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(1/2*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(e^(2*I*
d*x + 2*I*c) + 1)*e^(-I*d*x - I*c), x)
```

Sympy [F]

$$\int \cos(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (ia(\tan(c+dx)-i))^n\cos(c+dx)\,dx$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Integral((I*a*(tan(c + d*x) - I))**n*cos(c + d*x), x)
```

Maxima [F]

$$\int \cos(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan\left(dx+c\right)+a)^n\cos\left(dx+c\right)\,dx$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c), x)
```

Giac [F]

$$\int \cos(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan\left(dx+c\right)+a)^n\cos\left(dx+c\right)\,dx$$

```
[In] integrate(cos(d*x+c)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+ia\tan(c+dx))^n\,dx = \int \cos\left(c+d\,x\right)\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)^n\,dx$$

```
[In] int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^n,x)
[Out] int(cos(c + d*x)*(a + a*tan(c + d*x)*1i)^n, x)
```

3.475 $\int \cos^3(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 24, antiderivative size = 94

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{-\frac{3}{2}+n}\cos^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{5}{2}-n,-\frac{1}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{\frac{1}{2}-n}(a$$
$$\overline{\qquad 3ad \qquad}$$

```
[Out] -1/3*I*2^(-3/2+n)*cos(d*x+c)^3*hypergeom([-3/2, 5/2-n],[-1/2],1/2-1/2*I*tan
(d*x+c))*(1+I*tan(d*x+c))^(1/2-n)*(a+I*a*tan(d*x+c))^(1+n)/a/d
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3586, 3604, 72, 71}

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{n-\frac{3}{2}}\cos^3(c+dx)(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+ia\tan(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{5}{2}-n,-\frac{1}{2},\frac{1}{2}(\right.}{3ad}$$

```
[In] Int[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-1/3*I)*2^(-3/2 + n)*Cos[c + d*x]^3*Hypergeometric2F1[-3/2, 5/2 - n, -1/2
, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(1/2 - n)*(a + I*a*Tan[c + d
*x])^(1 + n))/(a*d)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \left(\cos^3(c+dx)(a - ia\tan(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\right)\int \frac{(a+ia\tan(c+dx))^{-\frac{3}{2}+n}}{(a-ia\tan(c+dx))^{3/2}}\,dx$$

$$= \frac{\left(a^2\cos^3(c+dx)(a-ia\tan(c+dx))^{3/2}(a+ia\tan(c+dx))^{3/2}\right)\text{Subst}\left(\int \frac{(a+iax)^{-\frac{5}{2}+n}}{(a-iax)^{5/2}}\,dx, x, \tan(c+\right.}{d}$$

$$= \frac{\left(2^{-\frac{5}{2}+n}\cos^3(c+dx)(a-ia\tan(c+dx))^{3/2}(a+ia\tan(c+dx))^{1+n}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{\frac{1}{2}-n}\right)\text{Subst}\left(\int\right.}{d}$$

$$= -\frac{i2^{-\frac{3}{2}+n}\cos^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{5}{2}-n,-\frac{1}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+d}{3ad}$$

Mathematica [A] (verified)

Time = 14.22 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.59

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{-3+n}e^{-3i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n\left(1+e^{2i(c+dx)}\right)^4\text{Hypergeometric2F1}\left(1,\frac{5}{2},-\frac{1}{2}+n,-e^{2i(c+dx)}\right)\sec^{-n}}{d(-3+2n)}$$

```
[In] Integrate[Cos[c + d*x]^3*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(-3 + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^
n*(1 + E^((2*I)*(c + d*x)))^4*Hypergeometric2F1[1, 5/2, -1/2 + n, -E^((2*I)
*(c + d*x))]*(a + I*a*Tan[c + d*x])^n)/(d*E^((3*I)*(c + d*x))*(-3 + 2*n)*Se
c[c + d*x]^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F]

$$\int \left(\cos^3(dx+c)\right)(a+ia\tan(dx+c))^n\,dx$$

```
[In] int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\cos(dx+c)^3\,dx$$

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(1/8*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(e^(6*I*
d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I*c) + 1)*e^(-3*I*d
*x - 3*I*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^n\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**3*(a+I*a*tan(d*x+c))**n,x)
[Out] Timed out
```

Maxima [F]

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\cos(dx+c)^3\,dx$$

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^3, x)
```

Giac [F]

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\cos(dx+c)^3\,dx$$

```
[In] integrate(cos(d*x+c)^3*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+ia\tan(c+dx))^n\,dx = \int \cos(c+d\,x)^3\,(a+a\tan(c+d\,x)\,1\mathrm{i})^n\,dx$$

```
[In] int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^n,x)
[Out] int(cos(c + d*x)^3*(a + a*tan(c + d*x)*1i)^n, x)
```

3.476 $\int \cos^5(c+dx)(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 24, antiderivative size = 94

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{-\frac{5}{2}+n}\cos^5(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{7}{2}-n,-\frac{3}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{\frac{1}{2}-n}(a}{5a^2d}$$

```
[Out] -1/5*I*2^(-5/2+n)*cos(d*x+c)^5*hypergeom([-5/2, 7/2-n],[-3/2],1/2-1/2*I*tan
(d*x+c))*(1+I*tan(d*x+c))^(1/2-n)*(a+I*a*tan(d*x+c))^(2+n)/a^2/d
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3586, 3604, 72, 71}

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{n-\frac{5}{2}}\cos^5(c+dx)(1+i\tan(c+dx))^{\frac{1}{2}-n}(a+ia\tan(c+dx))^{n+2}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{7}{2}-n,-\frac{3}{2},\frac{1}{2}(\right.}{5a^2d}$$

```
[In] Int[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-1/5*I)*2^(-5/2 + n)*Cos[c + d*x]^5*Hypergeometric2F1[-5/2, 7/2 - n, -3/2
, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(1/2 - n)*(a + I*a*Tan[c + d
*x])^(2 + n))/(a^2*d)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \left(\cos^5(c+dx)(a - ia\tan(c+dx))^{5/2}(a+ia\tan(c+dx))^{5/2}\right)\int \frac{(a+ia\tan(c+dx))^{-\frac{5}{2}+n}}{(a-ia\tan(c+dx))^{5/2}}\,dx$$

$$= \frac{\left(a^2\cos^5(c+dx)(a-ia\tan(c+dx))^{5/2}(a+ia\tan(c+dx))^{5/2}\right)\text{Subst}\left(\int \frac{(a+iax)^{-\frac{7}{2}+n}}{(a-iax)^{7/2}}\,dx, x, \tan(c+\right.}{d}$$

$$= \frac{\left(2^{-\frac{7}{2}+n}\cos^5(c+dx)(a-ia\tan(c+dx))^{5/2}(a+ia\tan(c+dx))^{2+n}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{\frac{1}{2}-n}\right)\text{Subst}\left(\int\right.}{ad}$$

$$= -\frac{i2^{-\frac{5}{2}+n}\cos^5(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{7}{2}-n,-\frac{3}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+d}{5a^2d}$$

Mathematica [A] (warning: unable to verify)

Time = 15.68 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.59

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{-5+n}e^{-5i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n\left(1+e^{2i(c+dx)}\right)^6\text{Hypergeometric2F1}\left(1,\frac{7}{2},-\frac{3}{2}+n,-e^{2i(c+dx)}\right)\sec^{-n}}{d(-5+2n)}$$

```
[In] Integrate[Cos[c + d*x]^5*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(-5 + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^
n*(1 + E^((2*I)*(c + d*x)))^6*Hypergeometric2F1[1, 7/2, -3/2 + n, -E^((2*I)
*(c + d*x))]*(a + I*a*Tan[c + d*x])^n)/(d*E^((5*I)*(c + d*x))*(-5 + 2*n)*Se
c[c + d*x]^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F]

$$\int \left(\cos^5(dx+c)\right)(a+ia\tan(dx+c))^n\,dx$$

```
[In] int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\cos(dx+c)^5\,dx$$

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(1/32*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*(e^(10*
I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*I*d*x + 6*I*c) + 10*e^(4*
I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*e^(-5*I*d*x - 5*I*c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^n\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**5*(a+I*a*tan(d*x+c))**n,x)
[Out] Timed out
```

Maxima [F]

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\cos(dx+c)^5\,dx$$

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^5, x)
```

Giac [F]

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^n\,dx = \int (i\,a\tan(dx+c)+a)^n\cos(dx+c)^5\,dx$$

```
[In] integrate(cos(d*x+c)^5*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^n*cos(d*x + c)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^5(c+dx)(a+ia\tan(c+dx))^n\,dx = \int \cos(c+d\,x)^5\,(a+a\tan(c+d\,x)\ 1\mathrm{i})^n\,dx$$

```
[In] int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^n,x)
[Out] int(cos(c + d*x)^5*(a + a*tan(c + d*x)*1i)^n, x)
```

3.477 $\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 28, antiderivative size = 96

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^n\,dx = \frac{i2^{\frac{9}{4}+n}a\,\text{Hypergeometric2F1}\left(\frac{5}{4},-\frac{1}{4}-n,\frac{9}{4},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{5/2}(1+ \cdots}{5d}$$

```
[Out] 1/5*I*2^(9/4+n)*a*hypergeom([5/4, -1/4-n],[9/4],1/2-1/2*I*tan(d*x+c))*(e*se
c(d*x+c))^(5/2)*(1+I*tan(d*x+c))^(-1/4-n)*(a+I*a*tan(d*x+c))^(-1+n)/d
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3586, 3604, 72, 71}

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^n\,dx = \frac{ia2^{n+\frac{9}{4}}(e\sec(c+dx))^{5/2}(1+i\tan(c+dx))^{-n-\frac{1}{4}}(a+ia\tan(c+dx))^{n-1}\,\text{Hypergeometr} \cdots}{5d}$$

```
[In] Int[(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((I/5)*2^(9/4 + n)*a*Hypergeometric2F1[5/4, -1/4 - n, 9/4, (1 - I*Tan[c + d
*x])/2]*(e*Sec[c + d*x])^(5/2)*(1 + I*Tan[c + d*x])^(-1/4 - n)*(a + I*a*Tan
[c + d*x])^(-1 + n))/d
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{(e\sec(c+dx))^{5/2}\int(a-ia\tan(c+dx))^{5/4}(a+ia\tan(c+dx))^{\frac{5}{4}+n}\,dx}{(a-ia\tan(c+dx))^{5/4}(a+ia\tan(c+dx))^{5/4}}$$

$$= \frac{\left(a^2(e\sec(c+dx))^{5/2}\right)\text{Subst}\left(\int\sqrt[4]{a-iax}(a+iax)^{\frac{1}{4}+n}\,dx,x,\tan(c+dx)\right)}{d(a-ia\tan(c+dx))^{5/4}(a+ia\tan(c+dx))^{5/4}}$$

$$= \frac{\left(2^{\frac{1}{4}+n}a^2(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^{-1+n}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{-\frac{1}{4}-n}\right)\text{Subst}\left(\int\left(\frac{1}{2}+\frac{ix}{2}\right)^{\frac{1}{4}+n}\sqrt[4]{\right.}{d(a-ia\tan(c+dx))^{5/4}}$$

$$= \frac{i2^{\frac{9}{4}+n}a\,\text{Hypergeometric2F1}\left(\frac{5}{4},-\frac{1}{4}-n,\frac{9}{4},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{5/2}(1+i\tan(c+d}{5d}$$

Mathematica [A] (verified)

Time = 10.90 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.89

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^n\,dx =$$

$$-\frac{i2^{\frac{7}{2}+n}e^{2i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{1}{2}+n}\left(1+e^{2i(c+dx)}\right)^{\frac{1}{2}+n}\text{Hypergeometric2F1}\left(\frac{5}{4}+n,\frac{5}{2}+n,\frac{9}{4}+n,-e^{2i(c+dx)}\right.}{d(5+4n)}$$

```
[In] Integrate[(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(7/2 + n)*E^((2*I)*(c + d*x))*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E
^((2*I)*(c + d*x))))^(1/2 + n)*(1 + E^((2*I)*(c + d*x)))^(1/2 + n)*Hypergeo
metric2F1[5/4 + n, 5/2 + n, 9/4 + n, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(-5
/2 - n)*(e*Sec[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^n)/(d*(5 + 4*n)*(Cos[
d*x] + I*Sin[d*x])^n)
```

Maple [F]

$$\int (e\sec(dx+c))^{\frac{5}{2}}(a+ia\tan(dx+c))^n\,dx$$

```
[In] int((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^{\frac{5}{2}}(i\,a\tan(dx+c)+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(4*sqrt(2)*e^2*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^
n*sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(5/2*I*d*x + 5/2*I*c)/(e^(4*I*d*x + 4
*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^n\,dx = \text{Timed out}$$

```
[In] integrate((e*sec(d*x+c))**(5/2)*(a+I*a*tan(d*x+c))**n,x)
[Out] Timed out
```

Maxima [F]

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^{\frac{5}{2}}(i\,a\tan(dx+c)+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((e*sec(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a)^n, x)
```

Giac [F]

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^{\frac{5}{2}}(i\,a\tan(dx+c)+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^n\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}(a+a\tan(c+d\,x)\;1\mathrm{i})^n\,dx$$

```
[In] int((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^n,x)
[Out] int((e/cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^n, x)
```

3.478 $\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 28, antiderivative size = 96

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^n\,dx = \frac{i2^{\frac{7}{4}+n}a\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1}{4}-n,\frac{7}{4},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{3/2}(1+i\tan(c+dx)) \cdots}{3d}$$

```
[Out] 1/3*I*2^(7/4+n)*a*hypergeom([3/4, 1/4-n],[7/4],1/2-1/2*I*tan(d*x+c))*(e*sec
(d*x+c))^(3/2)*(1+I*tan(d*x+c))^(1/4-n)*(a+I*a*tan(d*x+c))^(-1+n)/d
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3586, 3604, 72, 71}

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^n\,dx = \frac{ia2^{n+\frac{7}{4}}(e\sec(c+dx))^{3/2}(1+i\tan(c+dx))^{\frac{1}{4}-n}(a+ia\tan(c+dx))^{n-1}\,\text{Hypergeometric} \cdots}{3d}$$

```
[In] Int[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((I/3)*2^(7/4 + n)*a*Hypergeometric2F1[3/4, 1/4 - n, 7/4, (1 - I*Tan[c + d*
x])/2]*(e*Sec[c + d*x])^(3/2)*(1 + I*Tan[c + d*x])^(1/4 - n)*(a + I*a*Tan[c
 + d*x])^(-1 + n))/d
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{(e\sec(c+dx))^{3/2}\int(a-ia\tan(c+dx))^{3/4}(a+ia\tan(c+dx))^{\frac{3}{4}+n}\,dx}{(a-ia\tan(c+dx))^{3/4}(a+ia\tan(c+dx))^{3/4}}$$

$$= \frac{\left(a^2(e\sec(c+dx))^{3/2}\right)\operatorname{Subst}\left(\int\frac{(a+iax)^{-\frac{1}{4}+n}}{\sqrt[4]{a-iax}}\,dx, x, \tan(c+dx)\right)}{d(a-ia\tan(c+dx))^{3/4}(a+ia\tan(c+dx))^{3/4}}$$

$$= \frac{\left(2^{-\frac{1}{4}+n}a^2(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^{-1+n}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{\frac{1}{4}-n}\right)\operatorname{Subst}\left(\int\frac{\left(\frac{1}{2}+\frac{ix}{2}\right)^{-\frac{1}{4}+n}}{\sqrt[4]{a-iax}}\,dx\right.}{d(a-ia\tan(c+dx))^{3/4}}$$

$$= \frac{i2^{\frac{7}{4}+n}a\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1}{4}-n,\frac{7}{4},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{3/2}(1+i\tan(c+dx)}{3d}$$

Mathematica [A] (verified)

Time = 11.04 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.77

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{\frac{5}{2}+n}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{3}{2}+n}\left(1+e^{2i(c+dx)}\right)^{\frac{3}{2}+n}\text{Hypergeometric2F1}\left(\frac{3}{4}+n,\frac{3}{2}+n,\frac{7}{4}+n,-e^{2i(c+dx)}\right)\sec^{-\frac{3}{2}-}}{d(3+4n)}$$

```
[In] Integrate[(e*Sec[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(5/2 + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))
^(3/2 + n)*(1 + E^((2*I)*(c + d*x)))^(3/2 + n)*Hypergeometric2F1[3/4 + n, 3
/2 + n, 7/4 + n, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(-3/2 - n)*(e*Sec[c + d
*x])^(3/2)*(a + I*a*Tan[c + d*x])^n)/(d*(3 + 4*n)*(Cos[d*x] + I*Sin[d*x])^n
)
```

Maple [F]

$$\int (e\sec(dx+c))^{\frac{3}{2}}(a+ia\tan(dx+c))^n\,dx$$

```
[In] int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^{\frac{3}{2}}(i\,a\tan(dx+c)+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(2*sqrt(2)*e*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*
sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c)/(e^(2*I*d*x + 2*I
*c) + 1), x)
```

Sympy [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^n\,dx = \int (e\sec{(c+dx)})^{\frac{3}{2}}\,(ia(\tan{(c+dx)}-i))^n\,dx$$

```
[In] integrate((e*sec(d*x+c))**(3/2)*(a+I*a*tan(d*x+c))**n,x)
[Out] Integral((e*sec(c + d*x))**(3/2)*(I*a*(tan(c + d*x) - I))**n, x)
```

Maxima [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^n\,dx = \int (e\sec{(dx+c)})^{\frac{3}{2}}\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^n, x)
```

Giac [F]

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^n\,dx = \int (e\sec{(dx+c)})^{\frac{3}{2}}\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^n\,dx = \int \left(\frac{e}{\cos{(c+d\,x)}}\right)^{3/2}\,(a+a\tan(c+d\,x)\;1\mathrm{i})^n\,dx$$

```
[In] int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^n,x)
[Out] int((e/cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^n, x)
```

3.479 $\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 28, antiderivative size = 94

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^n\,dx = \frac{i2^{\frac{5}{4}+n}a\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4}-n,\frac{5}{4},\frac{1}{2}(1-i\tan(c+dx))\right)\sqrt{e\sec(c+dx)}(1+i\tan(c+dx))^{\frac{3}{4}-n}(a+ia\ldots}{d}$$

```
[Out] I*2^(5/4+n)*a*hypergeom([1/4, 3/4-n],[5/4],1/2-1/2*I*tan(d*x+c))*(e*sec(d*x
+c))^(1/2)*(1+I*tan(d*x+c))^(3/4-n)*(a+I*a*tan(d*x+c))^(-1+n)/d
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3586, 3604, 72, 71}

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^n\,dx = \frac{ia2^{n+\frac{5}{4}}\sqrt{e\sec(c+dx)}(1+i\tan(c+dx))^{\frac{3}{4}-n}(a+ia\tan(c+dx))^{n-1}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4}-n,\frac{5}{4},\frac{1}{2}(1\ldots}{d}$$

```
[In] Int[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (I*2^(5/4 + n)*a*Hypergeometric2F1[1/4, 3/4 - n, 5/4, (1 - I*Tan[c + d*x])/
2]*Sqrt[e*Sec[c + d*x]]*(1 + I*Tan[c + d*x])^(3/4 - n)*(a + I*a*Tan[c + d*x
])^(-1 + n))/d
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{e\sec(c+dx)}\int \sqrt[4]{a-ia\tan(c+dx)}(a+ia\tan(c+dx))^{\frac{1}{4}+n}\,dx}{\sqrt[4]{a-ia\tan(c+dx)}\sqrt[4]{a+ia\tan(c+dx)}}$$

$$= \frac{\left(a^2\sqrt{e\sec(c+dx)}\right)\text{Subst}\left(\int \frac{(a+iax)^{-\frac{3}{4}+n}}{(a-iax)^{3/4}}\,dx, x, \tan(c+dx)\right)}{d\sqrt[4]{a-ia\tan(c+dx)}\sqrt[4]{a+ia\tan(c+dx)}}$$

$$= \frac{\left(2^{-\frac{3}{4}+n}a^2\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^{-1+n}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{\frac{3}{4}-n}\right)\text{Subst}\left(\int \frac{\left(\frac{1}{2}+\frac{ix}{2}\right)^{-\frac{3}{4}+n}}{(a-iax)^{3/4}}\,dx, x\right.}{d\sqrt[4]{a-ia\tan(c+dx)}}$$

$$= \frac{i2^{\frac{5}{4}+n}a\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4}-n,\frac{5}{4},\frac{1}{2}(1-i\tan(c+dx))\right)\sqrt{e\sec(c+dx)}(1+i\tan(c+dx))^{\frac{3}{4}}}{d}$$

Mathematica [A] (verified)

Time = 10.43 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.81

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^n\,dx =$$
$$-\frac{i2^{\frac{3}{2}+n}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{1}{2}+n}\left(1+e^{2i(c+dx)}\right)^{\frac{1}{2}+n}\text{Hypergeometric2F1}\left(\frac{1}{4}+n,\frac{1}{2}+n,\frac{5}{4}+n,-e^{2i(c+dx)}\right)\sec}{d(1+4n)}$$

```
[In] Integrate[Sqrt[e*Sec[c + d*x]]*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(3/2 + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))
^(1/2 + n)*(1 + E^((2*I)*(c + d*x)))^(1/2 + n)*Hypergeometric2F1[1/4 + n, 1
/2 + n, 5/4 + n, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(-1/2 - n)*Sqrt[e*Sec[c
 + d*x]]*(a + I*a*Tan[c + d*x])^n)/(d*(1 + 4*n)*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F]

$$\int \sqrt{e\sec(dx+c)}\,(a+ia\tan(dx+c))^n\,dx$$

```
[In] int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^n\,dx = \int \sqrt{e\sec(dx+c)}(i\,a\tan(dx+c)+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral(sqrt(2)*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*sqrt
(e/(e^(2*I*d*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c), x)
```

Sympy [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^n\,dx = \int \sqrt{e\sec(c+dx)}(ia(\tan(c+dx)-i))^n\,dx$$

```
[In] integrate((e*sec(d*x+c))**(1/2)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Integral(sqrt(e*sec(c + d*x))*(I*a*(tan(c + d*x) - I))**n, x)
```

Maxima [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^n\,dx = \int \sqrt{e\sec\left(dx+c\right)}(i\,a\tan\left(dx+c\right)+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^n, x)
```

Giac [F]

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^n\,dx = \int \sqrt{e\sec\left(dx+c\right)}(i\,a\tan\left(dx+c\right)+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate(sqrt(e*sec(d*x + c))*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^n\,dx = \int \sqrt{\frac{e}{\cos\left(c+d\,x\right)}}\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^n\,dx$$

```
[In] int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^n,x)
[Out] int((e/cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^n, x)
```

3.480 $\int \frac{(a+ia\tan(c+dx))^n}{\sqrt{e\sec(c+dx)}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 91

$$\int \frac{(a+ia\tan(c+dx))^n}{\sqrt{e\sec(c+dx)}}\,dx = -\frac{i2^{\frac{3}{4}+n}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{5}{4}-n,\frac{3}{4},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{\frac{1}{4}-n}(a+ia\tan(c+dx}{d\sqrt{e\sec(c+dx)}}$$

```
[Out] -I*2^(3/4+n)*hypergeom([-1/4, 5/4-n],[3/4],1/2-1/2*I*tan(d*x+c))*(1+I*tan(d
*x+c))^(1/4-n)*(a+I*a*tan(d*x+c))^n/d/(e*sec(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3586, 3604, 72, 71}

$$\int \frac{(a+ia\tan(c+dx))^n}{\sqrt{e\sec(c+dx)}}\,dx = -\frac{i2^{n+\frac{3}{4}}(1+i\tan(c+dx))^{\frac{1}{4}-n}(a+ia\tan(c+dx))^n\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{5}{4}-n,\frac{3}{4},\frac{1}{2}(1-i\tan(c+dx\right.}{d\sqrt{e\sec(c+dx)}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^n/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] ((-I)*2^(3/4 + n)*Hypergeometric2F1[-1/4, 5/4 - n, 3/4, (1 - I*Tan[c + d*x]
)/2]*(1 + I*Tan[c + d*x])^(1/4 - n)*(a + I*a*Tan[c + d*x])^n)/(d*Sqrt[e*Sec
[c + d*x]])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\left(\sqrt[4]{a-ia\tan(c+dx)}\sqrt[4]{a+ia\tan(c+dx)}\right)\int\frac{(a+ia\tan(c+dx))^{-\frac{1}{4}+n}}{\sqrt[4]{a-ia\tan(c+dx)}}\,dx}{\sqrt{e\sec(c+dx)}}$$

$$= \frac{\left(a^2\sqrt[4]{a-ia\tan(c+dx)}\sqrt[4]{a+ia\tan(c+dx)}\right)\text{Subst}\left(\int\frac{(a+iax)^{-\frac{5}{4}+n}}{(a-iax)^{5/4}}\,dx,x,\tan(c+dx)\right)}{d\sqrt{e\sec(c+dx)}}$$

$$= \frac{\left(2^{-\frac{5}{4}+n}a\sqrt[4]{a-ia\tan(c+dx)}(a+ia\tan(c+dx))^n\left(\frac{a+ia\tan(c+dx)}{a}\right)^{\frac{1}{4}-n}\right)\text{Subst}\left(\int\frac{\left(\frac{1}{2}+\frac{ix}{2}\right)^{-\frac{5}{4}+n}}{(a-iax)^{5/4}}\,dx,\right.}{d\sqrt{e\sec(c+dx)}}$$

$$= -\frac{i2^{\frac{3}{4}+n}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{5}{4}-n,\frac{3}{4},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{\frac{1}{4}-n}(a+ia\,\text{ta}}{d\sqrt{e\sec(c+dx)}}$$

Mathematica [A] (verified)

Time = 11.19 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.57

$$\int \frac{(a+ia\tan(c+dx))^n}{\sqrt{e\sec(c+dx)}}\,dx = -\frac{i2^{\frac{1}{2}+n}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-\frac{1}{2}+n}\left(1+e^{2i(c+dx)}\right)^{-\frac{1}{2}+n}\text{Hypergeometric2F1}\left(-\frac{1}{2}+n,-\frac{1}{4}+n,\frac{3}{4}+n,-e^{2i(c+dx)}\right)\sec^{\frac{1}{2}}}{d(-1+4n)\sqrt{e\sec(c+dx)}}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^n/Sqrt[e*Sec[c + d*x]],x]
```

```
[Out] ((-I)*2^(1/2 + n)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(-1/2 + n)*(1
 + E^((2*I)*(c + d*x)))^(-1/2 + n)*Hypergeometric2F1[-1/2 + n, -1/4 + n, 3/
4 + n, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(1/2 - n)*(a + I*a*Tan[c + d*x])^
n)/(d*(-1 + 4*n)*Sqrt[e*Sec[c + d*x]])
```

Maple [F]

$$\int \frac{(a+ia\tan(dx+c))^n}{\sqrt{e\sec(dx+c)}}dx$$

```
[In] int((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(1/2),x)
```

```
[Out] int((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+ia\tan(c+dx))^n}{\sqrt{e\sec(c+dx)}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^n}{\sqrt{e\sec(dx+c)}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/2*sqrt(2)*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*
sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(-1/2*I*d*x -
 1/2*I*c)/e, x)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^n}{\sqrt{e\sec(c+dx)}}\,dx=\int \frac{(ia(\tan{(c+dx)}-i))^n}{\sqrt{e\sec{(c+dx)}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))**n/(e*sec(d*x+c))**(1/2),x)
[Out] Integral((I*a*(tan(c + d*x) - I))**n/sqrt(e*sec(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^n}{\sqrt{e\sec(c+dx)}}\,dx=\int \frac{(i\,a\tan{(dx+c)}+a)^n}{\sqrt{e\sec{(dx+c)}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^n/sqrt(e*sec(d*x + c)), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^n}{\sqrt{e\sec(c+dx)}}\,dx=\int \frac{(i\,a\tan{(dx+c)}+a)^n}{\sqrt{e\sec{(dx+c)}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^n/sqrt(e*sec(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^n}{\sqrt{e\sec(c+dx)}}\,dx=\int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^n}{\sqrt{\frac{e}{\cos(c+d\,x)}}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(1/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(1/2), x)
```

3.481 $\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 93

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{i2^{\frac{1}{4}+n}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{7}{4}-n,\frac{1}{4},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{\frac{3}{4}-n}(a+ia\tan(c+dx))^n}{3d(e\sec(c+dx))^{3/2}}$$

```
[Out] -1/3*I*2^(1/4+n)*hypergeom([-3/4, 7/4-n],[1/4],1/2-1/2*I*tan(d*x+c))*(1+I*t
an(d*x+c))^(3/4-n)*(a+I*a*tan(d*x+c))^n/d/(e*sec(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3586, 3604, 72, 71}

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{3/2}}\,dx = -\frac{i2^{n+\frac{1}{4}}(1+i\tan(c+dx))^{\frac{3}{4}-n}(a+ia\tan(c+dx))^n\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{7}{4}-n,\frac{1}{4},\frac{1}{2}(1-i\tan(c+dx))\right)}{3d(e\sec(c+dx))^{3/2}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^n/(e*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((-1/3*I)*2^(1/4 + n)*Hypergeometric2F1[-3/4, 7/4 - n, 1/4, (1 - I*Tan[c +
d*x])/2]*(1 + I*Tan[c + d*x])^(3/4 - n)*(a + I*a*Tan[c + d*x])^n)/(d*(e*Sec
[c + d*x])^(3/2))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\left((a-ia\tan(c+dx))^{3/4}(a+ia\tan(c+dx))^{3/4}\right)\int \frac{(a+ia\tan(c+dx))^{-\frac{3}{4}+n}}{(a-ia\tan(c+dx))^{3/4}}\,dx}{(e\sec(c+dx))^{3/2}}$$

$$= \frac{\left(a^2(a-ia\tan(c+dx))^{3/4}(a+ia\tan(c+dx))^{3/4}\right)\text{Subst}\left(\int \frac{(a+iax)^{-\frac{7}{4}+n}}{(a-iax)^{7/4}}\,dx, x, \tan(c+dx)\right)}{d(e\sec(c+dx))^{3/2}}$$

$$= \frac{\left(2^{-\frac{7}{4}+n}a(a-ia\tan(c+dx))^{3/4}(a+ia\tan(c+dx))^{n}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{\frac{3}{4}-n}\right)\text{Subst}\left(\int \frac{\left(\frac{1}{2}+\frac{ix}{2}\right)^{-\frac{7}{4}+n}}{(a-iax)^{7/4}}\,d\right.}{d(e\sec(c+dx))^{3/2}}$$

$$= -\frac{i2^{\frac{1}{4}+n}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{7}{4}-n,\frac{1}{4},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{\frac{3}{4}-n}(a+ia\,\text{ta}}{3d(e\sec(c+dx))^{3/2}}$$

Mathematica [A] (verified)

Time = 12.15 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.54

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{3/2}}\,dx =$$
$$-\frac{i2^{-\frac{1}{2}+n}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-\frac{3}{2}+n}\left(1+e^{2i(c+dx)}\right)^{-\frac{3}{2}+n}\text{Hypergeometric2F1}\left(-\frac{3}{2}+n,-\frac{3}{4}+n,\frac{1}{4}+n,-e^{2i(c+dx)}\right)\sec^{\frac{3}{2}-}}{d(-3+4n)(e\sec(c+dx))^{3/2}}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^n/(e*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((-I)*2^(-1/2 + n)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(-3/2 + n)*(
1 + E^((2*I)*(c + d*x)))^(-3/2 + n)*Hypergeometric2F1[-3/2 + n, -3/4 + n, 1
/4 + n, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(3/2 - n)*(a + I*a*Tan[c + d*x])
^n)/(d*(-3 + 4*n)*(e*Sec[c + d*x])^(3/2))
```

Maple [F]

$$\int \frac{(a+ia\tan(dx+c))^n}{(e\sec(dx+c))^{\frac{3}{2}}}dx$$

```
[In] int((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(3/2),x)
```

```
[Out] int((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(3/2),x)
```

Fricas [F]

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^n}{(e\sec(dx+c))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(1/4*sqrt(2)*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*
sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I
*c) + 1)*e^(-3/2*I*d*x - 3/2*I*c)/e^2, x)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{(ia(\tan(c+dx)-i))^n}{(e\sec(c+dx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))**n/(e*sec(d*x+c))**(3/2),x)
[Out] Integral((I*a*(tan(c + d*x) - I))**n/(e*sec(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^n}{(e\sec(dx+c))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^n/(e*sec(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^n}{(e\sec(dx+c))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^n/(e*sec(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{3/2}}\,dx = \int \frac{(a+a\tan(c+d\,x)\;1\mathrm{i})^n}{\left(\frac{e}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(3/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(3/2), x)
```

3.482 $\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 98

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{i2^{-\frac{1}{4}+n}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{9}{4}-n,-\frac{1}{4},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{\frac{1}{4}-n}(a+ia\tan(c+dx}{5ad(e\sec(c+dx))^{5/2}}$$

```
[Out] -1/5*I*2^(-1/4+n)*hypergeom([-5/4, 9/4-n],[-1/4],1/2-1/2*I*tan(d*x+c))*(1+I
*tan(d*x+c))^(1/4-n)*(a+I*a*tan(d*x+c))^(1+n)/a/d/(e*sec(d*x+c))^(5/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3586, 3604, 72, 71}

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{i2^{n-\frac{1}{4}}(1+i\tan(c+dx))^{\frac{1}{4}-n}(a+ia\tan(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{9}{4}-n,-\frac{1}{4},\frac{1}{2}(1-i\tan(c+d}{5ad(e\sec(c+dx))^{5/2}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^n/(e*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((-1/5*I)*2^(-1/4 + n)*Hypergeometric2F1[-5/4, 9/4 - n, -1/4, (1 - I*Tan[c
+ d*x])/2]*(1 + I*Tan[c + d*x])^(1/4 - n)*(a + I*a*Tan[c + d*x])^(1 + n))/(
a*d*(e*Sec[c + d*x])^(5/2))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\left((a-ia\tan(c+dx))^{5/4}(a+ia\tan(c+dx))^{5/4}\right)\int \frac{(a+ia\tan(c+dx))^{-\frac{5}{4}+n}}{(a-ia\tan(c+dx))^{5/4}}\,dx}{(e\sec(c+dx))^{5/2}}$$

$$= \frac{\left(a^2(a-ia\tan(c+dx))^{5/4}(a+ia\tan(c+dx))^{5/4}\right)\text{Subst}\left(\int \frac{(a+iax)^{-\frac{9}{4}+n}}{(a-iax)^{9/4}}\,dx, x, \tan(c+dx)\right)}{d(e\sec(c+dx))^{5/2}}$$

$$= \frac{\left(2^{-\frac{9}{4}+n}(a-ia\tan(c+dx))^{5/4}(a+ia\tan(c+dx))^{1+n}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{\frac{1}{4}-n}\right)\text{Subst}\left(\int \frac{\left(\frac{1}{2}+\frac{ix}{2}\right)^{-\frac{9}{4}+n}}{(a-iax)^{9/4}}\,d\right.}{d(e\sec(c+dx))^{5/2}}$$

$$= -\frac{i2^{-\frac{1}{4}+n}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{9}{4}-n,-\frac{1}{4},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{\frac{1}{4}-n}(a+i}{5ad(e\sec(c+dx))^{5/2}}$$

Mathematica [A] (verified)

Time = 12.48 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.60

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{5/2}}\,dx = -\frac{i2^{-\frac{3}{2}+n}e^{-3i(c+dx)}\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{\frac{1}{2}+n}\left(1+e^{2i(c+dx)}\right)^{\frac{1}{2}+n}\text{Hypergeometric2F1}\left(-\frac{5}{2}+n,-\frac{5}{4}+n,-\frac{1}{4}+n,-e^{2i(c+dx)}\right)}{de^2(-5+4n)\sqrt{e\sec(c+dx)}}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^n/(e*Sec[c + d*x])^(5/2),x]
[Out] ((-I)*2^(-3/2 + n)*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(1/2 + n)*(1
 + E^((2*I)*(c + d*x)))^(1/2 + n)*Hypergeometric2F1[-5/2 + n, -5/4 + n, -1/
4 + n, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(1/2 - n)*(a + I*a*Tan[c + d*x])^
n)/(d*e^2*E^((3*I)*(c + d*x))*(-5 + 4*n)*Sqrt[e*Sec[c + d*x]])
```

Maple [F]

$$\int \frac{(a+ia\tan(dx+c))^n}{(e\sec(dx+c))^{\frac{5}{2}}}dx$$

```
[In] int((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(5/2),x)
[Out] int((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(5/2),x)
```

Fricas [F]

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^n}{(e\sec(dx+c))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(5/2),x, algorithm="fricas")
[Out] integral(1/8*sqrt(2)*(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*
sqrt(e/(e^(2*I*d*x + 2*I*c) + 1))*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I
*c) + 3*e^(2*I*d*x + 2*I*c) + 1)*e^(-5/2*I*d*x - 5/2*I*c)/e^3, x)
```

Sympy [F]

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{(ia(\tan(c+dx)-i))^n}{(e\sec(c+dx))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))**n/(e*sec(d*x+c))**(5/2),x)
[Out] Integral((I*a*(tan(c + d*x) - I))**n/(e*sec(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^n}{(e\sec(dx+c))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^n/(e*sec(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{(i\,a\tan(dx+c)+a)^n}{(e\sec(dx+c))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^n/(e*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^n/(e*sec(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+ia\tan(c+dx))^n}{(e\sec(c+dx))^{5/2}}\,dx = \int \frac{(a+a\tan(c+d\,x)\,1\mathrm{i})^n}{\left(\frac{e}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(5/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(5/2), x)
```

3.483 $\int (e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 30, antiderivative size = 269

$$\begin{aligned}\int (e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n\,dx &= \frac{i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n}{d(4-n)} \\ &\quad+\frac{4i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^{1+n}}{ad\,(8-6n+n^2)} \\ &\quad-\frac{12i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^{2+n}}{a^2d(2-n)(4-n)n} \\ &\quad+\frac{24i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^{3+n}}{a^3d(4-n)n\,(4-n^2)} \\ &\quad-\frac{24i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^{4+n}}{a^4dn\,(64-20n^2+n^4)}\end{aligned}$$

```
[Out] I*(e*sec(d*x+c))^(-4-n)*(a+I*a*tan(d*x+c))^n/d/(4-n)+4*I*(e*sec(d*x+c))^(-4
-n)*(a+I*a*tan(d*x+c))^(1+n)/a/d/(n^2-6*n+8)-12*I*(e*sec(d*x+c))^(-4-n)*(a+
I*a*tan(d*x+c))^(2+n)/a^2/d/(2-n)/(4-n)/n+24*I*(e*sec(d*x+c))^(-4-n)*(a+I*a
*tan(d*x+c))^(3+n)/a^3/d/(4-n)/n/(-n^2+4)-24*I*(e*sec(d*x+c))^(-4-n)*(a+I*a
*tan(d*x+c))^(4+n)/a^4/d/n/(n^4-20*n^2+64)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3585, 3569}

$$\begin{aligned}
&\int (e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n\,dx\\
&=-\frac{24i(a+ia\tan(c+dx))^{n+4}(e\sec(c+dx))^{-n-4}}{a^4dn\,(n^4-20n^2+64)}\\
&\quad+\frac{24i(a+ia\tan(c+dx))^{n+3}(e\sec(c+dx))^{-n-4}}{a^3d(4-n)n\,(4-n^2)}\\
&\quad-\frac{12i(a+ia\tan(c+dx))^{n+2}(e\sec(c+dx))^{-n-4}}{a^2d(2-n)(4-n)n}\\
&\quad+\frac{4i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-n-4}}{ad\,(n^2-6n+8)}\\
&\quad+\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-4}}{d(4-n)}
\end{aligned}$$

```
[In] Int[(e*Sec[c + d*x])^(-4 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (I*(e*Sec[c + d*x])^(-4 - n)*(a + I*a*Tan[c + d*x])^n)/(d*(4 - n)) + ((4*I)
*(e*Sec[c + d*x])^(-4 - n)*(a + I*a*Tan[c + d*x])^(1 + n))/(a*d*(8 - 6*n +
n^2)) - ((12*I)*(e*Sec[c + d*x])^(-4 - n)*(a + I*a*Tan[c + d*x])^(2 + n))/(
a^2*d*(2 - n)*(4 - n)*n) + ((24*I)*(e*Sec[c + d*x])^(-4 - n)*(a + I*a*Tan[c
 + d*x])^(3 + n))/(a^3*d*(4 - n)*n*(4 - n^2)) - ((24*I)*(e*Sec[c + d*x])^(-
4 - n)*(a + I*a*Tan[c + d*x])^(4 + n))/(a^4*d*n*(64 - 20*n^2 + n^4))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3585

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 + b^2, 0] && ILtQ[Simplify[m + n], 0] && NeQ[m + 2*n, 0]
```

Rubi steps

$$\text{integral} = \frac{i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n}{d(4-n)} + \frac{4\int(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^{1+n}\,dx}{a(4-n)}$$

$$= \frac{i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n}{d(4-n)} + \frac{4i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^{1+n}}{ad\,(8-6n+n^2)} + \frac{12\int(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^{2+n}\,dx}{a^2(2-n)(4-n)}$$

$$= \frac{i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n}{d(4-n)} + \frac{4i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^{1+n}}{ad\,(8-6n+n^2)} - \frac{12i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^{2+n}}{a^2d(2-n)(4-n)n} - \frac{24\int(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^{3+n}\,dx}{a^3(2-n)(4-n)n}$$

$$= \frac{i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n}{d(4-n)} + \frac{4i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^{1+n}}{ad\,(8-6n+n^2)} - \frac{12i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^{2+n}}{a^2d(2-n)(4-n)n} + \frac{24i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^{3+n}}{a^3d(2-n)(4-n)n(2+n)} + \frac{24\int(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^{4+n}\,dx}{a^4(2-n)(4-n)n(2+n)}$$

$$= \frac{i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n}{d(4-n)} + \frac{4i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^{1+n}}{ad\,(8-6n+n^2)} - \frac{12i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^{2+n}}{a^2d(2-n)(4-n)n} + \frac{24i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^{3+n}}{a^3d(2-n)(4-n)n(2+n)} - \frac{24i(e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^{4+n}}{a^4dn\,(64-20n^2+n^4)}$$

Mathematica [A] (verified)

Time = 1.89 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.61

$$\int (e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n\,dx = -\frac{i(e\sec(c+dx))^{-n}\left(192-60n^2+3n^4+4n^2(-16+n^2)\cos(2(c+dx))+n^2(-4+n^2)\cos(4(c+dx))+\right.}{8de^4(-4+n)(-}$$

[In] Integrate[(e*Sec[c + d*x])^(-4 - n)*(a + I*a*Tan[c + d*x])^n,x]

[Out] ((-1/8*I)*(192 - 60*n^2 + 3*n^4 + 4*n^2*(-16 + n^2)*Cos[2*(c + d*x)] + n^2*(-4 + n^2)*Cos[4*(c + d*x)] + (128*I)*n*Sin[2*(c + d*x)] - (8*I)*n^3*Sin[2*(c + d*x)] + (16*I)*n*Sin[4*(c + d*x)] - (4*I)*n^3*Sin[4*(c + d*x)])*(a + I*a*Tan[c + d*x])^n)/(d*e^4*(-4 + n)*(-2 + n)*n*(2 + n)*(4 + n)*(e*Sec[c + d*x])^n)

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 6.43 (sec) , antiderivative size = 4331, normalized size of antiderivative = 16.10

method	result	size
risch	Expression too large to display	4331

[In] int((e*sec(d*x+c))^(-4-n)*(a+I*a*tan(d*x+c))^n,x,method=_RETURNVERBOSE)

[Out] -1/16*I/(n-4)/d*a^n/(e^n)/e^4*exp(I*(d*x+c))^n*exp(-1/2*I*(-csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*csgn(I*a)*Pi*n+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*a)*Pi*n+csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3*Pi*n-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*Pi*n-n*Pi*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3+n*Pi*csgn(I*e)*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+n*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-n*Pi*csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))+Pi*csgn(I*exp(2*I*(d*x+c)))^3*n-2*Pi*csgn(I*exp(2*I*(d*x+c)))^2*csgn(I*exp(I*(d*x+c)))*n+Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d*x+c)))^2*n-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*Pi*csgn(I*exp(2*I*(d*x+c)))*n+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(2*I*(d*x+c)))*n-n*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))+n*Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^3*Pi*n-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*n+n*Pi*

csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-
n*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3+8*d*x+8*c))-1/16*I/(4+n)
/d*a^n/(e^n)/e^4*exp(I*(d*x+c))^n*exp(1/2*I*(-csgn(I*a*exp(2*I*(d*x+c))/(ex
p(2*I*(d*x+c))+1))^3*Pi*n+csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2
*csgn(I*a)*Pi*n+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a*exp(
2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*Pi*n-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2
I(d*x+c)))*csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*a)*Pi*n
-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^3*Pi*n+csgn(I/(exp(2*I*(d*x+
c))+1)*exp(2*I*(d*x+c)))^2*Pi*csgn(I*exp(2*I*(d*x+c)))*n+csgn(I/(exp(2*I*(d
*x+c))+1)*exp(2*I*(d*x+c)))^2*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*n-csgn(I/(exp
(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*e
xp(2*I*(d*x+c)))*n+n*Pi*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3-n*P
i*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e*exp(I*(d*x+c))/(exp(
2*I*(d*x+c))+1))^2-n*Pi*csgn(I*e)*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))
+1))^2+n*Pi*csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e*
exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))+n*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d
*x+c))+1))^3-n*Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*
x+c))+1))^2-n*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c))/(exp(2*
I*(d*x+c))+1))^2+n*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c)))*c
sgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))-Pi*csgn(I*exp(2*I*(d*x+c)))^3*n+
2*Pi*csgn(I*exp(2*I*(d*x+c)))^2*csgn(I*exp(I*(d*x+c)))*n-Pi*csgn(I*exp(2*I*
(d*x+c)))*csgn(I*exp(I*(d*x+c)))^2*n+8*d*x+8*c))-3/8*I/d/n*a^n/(e^n)/e^4*ex
p(I*(d*x+c))^n*exp(1/2*I*n*Pi*(-csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))
+1))^3+csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*csgn(I*a)+csgn(I/(
exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d
*x+c))+1))^2-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a*exp(2*I
*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*a)-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2
I(d*x+c)))^3+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*csgn(I*exp(2
I(d*x+c)))+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*csgn(I/(exp(2*
I*(d*x+c))+1))-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I/(exp(2*
I*(d*x+c))+1))*csgn(I*exp(2*I*(d*x+c)))+csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d
*x+c))+1))^3-csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e*exp(I*(d*
x+c))/(exp(2*I*(d*x+c))+1))^2-csgn(I*e)*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d
*x+c))+1))^2+csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e
exp(I(d*x+c))/(exp(2*I*(d*x+c))+1))+csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c
))+1))^3-csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))
^2-csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))
^2+csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c)
)/(exp(2*I*(d*x+c))+1))-csgn(I*exp(2*I*(d*x+c)))^3+2*csgn(I*exp(2*I*(d*x+c)
))^2*csgn(I*exp(I*(d*x+c)))-csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d*x+c)))
^2))-1/4*I/(n-2)/d*a^n/(e^n)/e^4*exp(I*(d*x+c))^n*exp(-1/2*I*(csgn(I*a*exp(
2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3*Pi*n-csgn(I*a*exp(2*I*(d*x+c))/(exp(2*
I*(d*x+c))+1))^2*csgn(I*a)*Pi*n-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)
))*csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*Pi*n+csgn(I/(exp(2*I*(
d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1)

```
)*csgn(I*a)*Pi*n+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^3*Pi*n-csgn(
I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*Pi*csgn(I*exp(2*I*(d*x+c)))*n-cs
gn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*Pi*csgn(I/(exp(2*I*(d*x+c))+1
))*n+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*Pi*csgn(I/(exp(2*I*(d*x+
c))+1))*csgn(I*exp(2*I*(d*x+c)))*n-n*Pi*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d
*x+c))+1))^3+n*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e*exp(
I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+n*Pi*csgn(I*e)*csgn(I*e*exp(I*(d*x+c))/(
exp(2*I*(d*x+c))+1))^2-n*Pi*csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c
))+1))*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))-n*Pi*csgn(I*exp(I*(d*x
+c))/(exp(2*I*(d*x+c))+1))^3+n*Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+
c))/(exp(2*I*(d*x+c))+1))^2+n*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*
(d*x+c))/(exp(2*I*(d*x+c))+1))^2-n*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*e
xp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))+Pi*csgn(I*exp(2*
I*(d*x+c)))^3*n-2*Pi*csgn(I*exp(2*I*(d*x+c)))^2*csgn(I*exp(I*(d*x+c)))*n+Pi
*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d*x+c)))^2*n+4*d*x+4*c))-1/4*I/(2+n
)/d*a^n/(e^n)/e^4*exp(I*(d*x+c))^n*exp(1/2*I*(-csgn(I*a*exp(2*I*(d*x+c))/(e
xp(2*I*(d*x+c))+1))^3*Pi*n+csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^
2*csgn(I*a)*Pi*n+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a*exp
(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*Pi*n-csgn(I/(exp(2*I*(d*x+c))+1)*exp(
2*I*(d*x+c)))*csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*a)*Pi*
n-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^3*Pi*n+csgn(I/(exp(2*I*(d*x
+c))+1)*exp(2*I*(d*x+c)))^2*Pi*csgn(I*exp(2*I*(d*x+c)))*n+csgn(I/(exp(2*I*(
d*x+c))+1)*exp(2*I*(d*x+c)))^2*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*n-csgn(I/(ex
p(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*
exp(2*I*(d*x+c)))*n+n*Pi*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3-n*
Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e*exp(I*(d*x+c))/(exp
(2*I*(d*x+c))+1))^2-n*Pi*csgn(I*e)*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c)
)+1))^2+n*Pi*csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e
*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))+n*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(
d*x+c))+1))^3-n*Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d
*x+c))+1))^2-n*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c))/(exp(2
*I*(d*x+c))+1))^2+n*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c)))*
csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))-Pi*csgn(I*exp(2*I*(d*x+c)))^3*n
+2*Pi*csgn(I*exp(2*I*(d*x+c)))^2*csgn(I*exp(I*(d*x+c)))*n-Pi*csgn(I*exp(2*I
*(d*x+c)))*csgn(I*exp(I*(d*x+c)))^2*n+4*d*x+4*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.25

$$\int (e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n\,dx$$
$$= \frac{(-i\,n^4-4i\,n^3+4i\,n^2+(-i\,n^4+4i\,n^3+4i\,n^2-16i\,n)e^{(8i\,dx+8i\,c)}-4\,(i\,n^4-2i\,n^3-16i\,n^2+32i\,n)e^{(6i\,dx+}}{dn^5-20\,dn^3+64\,dn+(dn^5-20\,dn^3+64\,dn)e^{(8i\,dx+8i\,c)}+4\,(d}$$

```
[In] integrate((e*sec(d*x+c))^(-4-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
[Out] (-I*n^4 - 4*I*n^3 + 4*I*n^2 + (-I*n^4 + 4*I*n^3 + 4*I*n^2 - 16*I*n)*e^(8*I*
d*x + 8*I*c) - 4*(I*n^4 - 2*I*n^3 - 16*I*n^2 + 32*I*n)*e^(6*I*d*x + 6*I*c)
- 6*(I*n^4 - 20*I*n^2 + 64*I)*e^(4*I*d*x + 4*I*c) - 4*(I*n^4 + 2*I*n^3 - 16
*I*n^2 - 32*I*n)*e^(2*I*d*x + 2*I*c) + 16*I*n)*(2*e*e^(I*d*x + I*c)/(e^(2*I
*d*x + 2*I*c) + 1))^(-n - 4)*e^(I*d*n*x + I*c*n + n*log(2*e*e^(I*d*x + I*c)
/(e^(2*I*d*x + 2*I*c) + 1)) + n*log(a/e))/(d*n^5 - 20*d*n^3 + 64*d*n + (d*n
^5 - 20*d*n^3 + 64*d*n)*e^(8*I*d*x + 8*I*c) + 4*(d*n^5 - 20*d*n^3 + 64*d*n)
*e^(6*I*d*x + 6*I*c) + 6*(d*n^5 - 20*d*n^3 + 64*d*n)*e^(4*I*d*x + 4*I*c) +
4*(d*n^5 - 20*d*n^3 + 64*d*n)*e^(2*I*d*x + 2*I*c))
```

Sympy [F]

$$\int (e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int (e\sec{(c+dx)})^{-n-4}\,(ia(\tan{(c+dx)}-i))^n\;dx$$

```
[In] integrate((e*sec(d*x+c))**(-4-n)*(a+I*a*tan(d*x+c))**n,x)
[Out] Integral((e*sec(c + d*x))**(-n - 4)*(I*a*(tan(c + d*x) - I))**n, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.82 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.62

$$\int (e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n\,dx$$
$$= \frac{(-i\,a^nn^4+4i\,a^nn^3+4i\,a^nn^2-16i\,a^nn)\cos((dx+c)(n+4))-4\,(i\,a^nn^4-2i\,a^nn^3-16i\,a^nn^2+32i\,a^nn)\,c}{}$$

```
[In] integrate((e*sec(d*x+c))^(-4-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] 1/16*((-I*a^n*n^4 + 4*I*a^n*n^3 + 4*I*a^n*n^2 - 16*I*a^n*n)*cos((d*x + c)*(
n + 4)) - 4*(I*a^n*n^4 - 2*I*a^n*n^3 - 16*I*a^n*n^2 + 32*I*a^n*n)*cos((d*x
+ c)*(n + 2)) - 4*(I*a^n*n^4 + 2*I*a^n*n^3 - 16*I*a^n*n^2 - 32*I*a^n*n)*cos
((d*x + c)*(n - 2)) + (-I*a^n*n^4 - 4*I*a^n*n^3 + 4*I*a^n*n^2 + 16*I*a^n*n)
*cos((d*x + c)*(n - 4)) - 6*(I*a^n*n^4 - 20*I*a^n*n^2 + 64*I*a^n)*cos((d*x
+ c)*n) + (a^n*n^4 - 4*a^n*n^3 - 4*a^n*n^2 + 16*a^n*n)*sin((d*x + c)*(n + 4
)) + 4*(a^n*n^4 - 2*a^n*n^3 - 16*a^n*n^2 + 32*a^n*n)*sin((d*x + c)*(n + 2))
 + 4*(a^n*n^4 + 2*a^n*n^3 - 16*a^n*n^2 - 32*a^n*n)*sin((d*x + c)*(n - 2)) +
 (a^n*n^4 + 4*a^n*n^3 - 4*a^n*n^2 - 16*a^n*n)*sin((d*x + c)*(n - 4)) + 6*(a
^n*n^4 - 20*a^n*n^2 + 64*a^n)*sin((d*x + c)*n))/((e^(n + 4)*n^5 - 20*e^(n +
 4)*n^3 + 64*e^(n + 4)*n)*d)
```

Giac [F]

$$\int (e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^{-n-4}(ia\tan(dx+c)+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(-4-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(-n - 4)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [B] (verification not implemented)

Time = 11.48 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.90

$$\int (e\sec(c+dx))^{-4-n}(a+ia\tan(c+dx))^n\,dx = \frac{\left(2\sin(2c+2dx)^2+\sin(4c+4dx)\,1\mathrm{i}-1\right)\left(\frac{\left(a-\frac{a\sin(c+dx)\,1\mathrm{i}}{2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1}\right)^n(-n^3-4n^2+4n+16)}{d\,(n^4\,1\mathrm{i}-n^2\,20\mathrm{i}+64\mathrm{i})}+\frac{4\left(a-\frac{a\sin(c+dx)\,1\mathrm{i}}{2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1}\right)^n}{}\right.}{}$$

```
[In] int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(n + 4),x)
```

```
[Out] ((sin(4*c + 4*d*x)*1i + 2*sin(2*c + 2*d*x)^2 - 1)*(((a - (a*sin(c + d*x)*1i
)/(2*sin(c/2 + (d*x)/2)^2 - 1))^n*(4*n - 4*n^2 - n^3 + 16))/(d*(n^4*1i - n^
2*20i + 64i)) + (4*(a - (a*sin(c + d*x)*1i)/(2*sin(c/2 + (d*x)/2)^2 - 1))^n
*(sin(2*c + 2*d*x)*1i - 2*sin(c + d*x)^2 + 1)*(16*n - 2*n^2 - n^3 + 32))/(d
```

```
*(n^4*1i - n^2*20i + 64i)) + ((a - (a*sin(c + d*x)*1i)/(2*sin(c/2 + (d*x)/2
)^2 - 1))^n*(sin(8*c + 8*d*x)*1i - 2*sin(4*c + 4*d*x)^2 + 1)*(4*n + 4*n^2 -
 n^3 - 16))/(d*(n^4*1i - n^2*20i + 64i)) + (4*(a - (a*sin(c + d*x)*1i)/(2*s
in(c/2 + (d*x)/2)^2 - 1))^n*(sin(6*c + 6*d*x)*1i - 2*sin(3*c + 3*d*x)^2 + 1
)*(16*n + 2*n^2 - n^3 - 32))/(d*(n^4*1i - n^2*20i + 64i)) - ((a - (a*sin(c
+ d*x)*1i)/(2*sin(c/2 + (d*x)/2)^2 - 1))^n*(sin(4*c + 4*d*x)*1i - 2*sin(2*c
 + 2*d*x)^2 + 1)*(6*n^4 - 120*n^2 + 384))/(d*n*(n^4*1i - n^2*20i + 64i))))/
(16*(-e/(2*sin(c/2 + (d*x)/2)^2 - 1))^(n + 4)*(sin(c + d*x)^2 - 1)^2)
```

3.484 $\int (e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 30, antiderivative size = 205

$$\int (e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n\,dx = \frac{i(e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n}{d(3-n)} + \frac{3i(e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^{1+n}}{ad\,(3-4n+n^2)} - \frac{6i(e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^{2+n}}{a^2d(3-n)\,(1-n^2)} + \frac{6i(e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^{3+n}}{a^3d\,(9-10n^2+n^4)}$$

```
[Out] I*(e*sec(d*x+c))^(-3-n)*(a+I*a*tan(d*x+c))^n/d/(3-n)+3*I*(e*sec(d*x+c))^(-3
-n)*(a+I*a*tan(d*x+c))^(1+n)/a/d/(n^2-4*n+3)-6*I*(e*sec(d*x+c))^(-3-n)*(a+I
*a*tan(d*x+c))^(2+n)/a^2/d/(3-n)/(-n^2+1)+6*I*(e*sec(d*x+c))^(-3-n)*(a+I*a*
tan(d*x+c))^(3+n)/a^3/d/(n^4-10*n^2+9)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used

$= \{3585, 3569\}$

$$\int (e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n\,dx = \frac{6i(a+ia\tan(c+dx))^{n+3}(e\sec(c+dx))^{-n-3}}{a^3d\,(n^4-10n^2+9)} - \frac{6i(a+ia\tan(c+dx))^{n+2}(e\sec(c+dx))^{-n-3}}{a^2d(3-n)\,(1-n^2)} + \frac{3i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-n-3}}{ad\,(n^2-4n+3)} + \frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-3}}{d(3-n)}$$

```
[In] Int[(e*Sec[c + d*x])^(-3 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (I*(e*Sec[c + d*x])^(-3 - n)*(a + I*a*Tan[c + d*x])^n)/(d*(3 - n)) + ((3*I)
*(e*Sec[c + d*x])^(-3 - n)*(a + I*a*Tan[c + d*x])^(1 + n))/(a*d*(3 - 4*n +
n^2)) - ((6*I)*(e*Sec[c + d*x])^(-3 - n)*(a + I*a*Tan[c + d*x])^(2 + n))/(a
^2*d*(3 - n)*(1 - n^2)) + ((6*I)*(e*Sec[c + d*x])^(-3 - n)*(a + I*a*Tan[c +
 d*x])^(3 + n))/(a^3*d*(9 - 10*n^2 + n^4))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3585

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 + b^2, 0] && ILtQ[Simplify[m + n], 0] && NeQ[m + 2*n, 0]
```

Rubi steps

$$\text{integral} = \frac{i(e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n}{d(3-n)} + \frac{3\int (e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^{1+n}\,dx}{a(3-n)}$$

$$= \frac{i(e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n}{d(3-n)} + \frac{3i(e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^{1+n}}{ad(3-4n+n^2)} + \frac{6\int(e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^{2+n}\,dx}{a^2(1-n)(3-n)}$$

$$= \frac{i(e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n}{d(3-n)} + \frac{3i(e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^{1+n}}{ad(3-4n+n^2)} - \frac{6i(e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^{2+n}}{a^2d(1-n)(3-n)(1+n)} - \frac{6\int(e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^{3+n}\,dx}{a^3(1-n)(3-n)(1+n)}$$

$$= \frac{i(e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n}{d(3-n)} + \frac{3i(e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^{1+n}}{ad(3-4n+n^2)} - \frac{6i(e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^{2+n}}{a^2d(1-n)(3-n)(1+n)} + \frac{6i(e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^{3+n}}{a^3d(9-10n^2+n^4)}$$

Mathematica [A] (verified)

Time = 1.89 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.58

$$\int (e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n\,dx$$

$$= \frac{(e\sec(c+dx))^{-n}\left(-3in(-9+n^2)\cos(c+dx)-in(-1+n^2)\cos(3(c+dx))-6(-5+n^2+(-1+n^2)\cos\right.}{4de^3(-3+n)(-1+n)(1+n)(3+n)}$$

```
[In] Integrate[(e*Sec[c + d*x])^(-3 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (((-3*I)*n*(-9 + n^2)*Cos[c + d*x] - I*n*(-1 + n^2)*Cos[3*(c + d*x)] - 6*(-
5 + n^2 + (-1 + n^2)*Cos[2*(c + d*x)])*Sin[c + d*x])*(a + I*a*Tan[c + d*x])
^n)/(4*d*e^3*(-3 + n)*(-1 + n)*(1 + n)*(3 + n)*(e*Sec[c + d*x])^n)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 6.88 (sec) , antiderivative size = 4982, normalized size of antiderivative = 24.30

method	result	size
risch	Expression too large to display	4982

```
[In] int((e*sec(d*x+c))^(-3-n)*(a+I*a*tan(d*x+c))^n,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*I/(-3+n)/d/(e^n)*exp(I*(d*x+c))^n/e^3*a^n*exp(-1/2*I*(6*c-3*Pi*csgn(I*
e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3-csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I
*(d*x+c))+1))^2*csgn(I*a)*Pi*n-2*Pi*csgn(I*exp(2*I*(d*x+c)))^2*csgn(I*exp(I
*(d*x+c)))*n+Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d*x+c)))^2*n+6*d*x-n
*Pi*csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e*exp(I*(d
*x+c))/(exp(2*I*(d*x+c))+1))+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^
3*Pi*n+csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3*Pi*n+Pi*csgn(I*exp
(2*I*(d*x+c)))^3*n-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*Pi*csgn(
I/(exp(2*I*(d*x+c))+1))*n-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*P
i*csgn(I*exp(2*I*(d*x+c)))*n-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*
csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*Pi*n+3*Pi*csgn(I*e)*csgn(
I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+3*Pi*csgn(I/(exp(2*I*(d*x+c))+1)
)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+3*Pi*csgn(I*exp(I*(d*x+c)))
*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-n*Pi*csgn(I*exp(I*(d*x+c))/(
exp(2*I*(d*x+c))+1))^3-3*Pi*csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c
))+1))*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))+3*Pi*csgn(I*exp(I*(d*x
+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-
n*Pi*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3+n*Pi*csgn(I*exp(I*(d*x
+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+n*Pi*csgn(I*e)*csgn(I*e
*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-3*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*c
sgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))-3*Pi*csgn
(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3+n*Pi*csgn(I*exp(I*(d*x+c))/(exp(2
*I*(d*x+c))+1))*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+csgn(I/(exp
(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+
c))+1))*csgn(I*a)*Pi*n+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*Pi*csg
n(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(2*I*(d*x+c)))*n-n*Pi*csgn(I/(exp(2*I*(
d*x+c))+1))*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+
1))+n*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c
))+1))^2))-1/8*I/(3+n)/d/(e^n)*exp(I*(d*x+c))^n/e^3*a^n*exp(1/2*I*(6*c+3*Pi
*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3+csgn(I*a*exp(2*I*(d*x+c))/
(exp(2*I*(d*x+c))+1))^2*csgn(I*a)*Pi*n+2*Pi*csgn(I*exp(2*I*(d*x+c)))^2*csgn
(I*exp(I*(d*x+c)))*n-Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d*x+c)))^2*n
+6*d*x+n*Pi*csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e*
exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d
*x+c)))^3*Pi*n-csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3*Pi*n-Pi*cs
```

gn(I*exp(2*I*(d*x+c)))^3*n+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*
Pi*csgn(I/(exp(2*I*(d*x+c))+1))*n+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+
c)))^2*Pi*csgn(I*exp(2*I*(d*x+c)))*n+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d
*x+c)))*csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*Pi*n-3*Pi*csgn(I*
e)*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-3*Pi*csgn(I/(exp(2*I*(d*
x+c))+1))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-3*Pi*csgn(I*exp(I*(
d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+n*Pi*csgn(I*exp(I*(d
*x+c))/(exp(2*I*(d*x+c))+1))^3+3*Pi*csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*
I*(d*x+c))+1))*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))-3*Pi*csgn(I*ex
p(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c)
)+1))^2+n*Pi*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3-n*Pi*csgn(I*ex
p(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-n*Pi*csgn(I*e)*
csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+3*Pi*csgn(I/(exp(2*I*(d*x+c
))+1))*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))+3
*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3-n*Pi*csgn(I*exp(I*(d*x+c)
)/(exp(2*I*(d*x+c))+1))*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-csg
n(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a*exp(2*I*(d*x+c))/(exp(2
I(d*x+c))+1))*csgn(I*a)*Pi*n-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c))
)*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(2*I*(d*x+c)))*n+n*Pi*csgn(I/(e
xp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(
d*x+c))+1))-n*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c))/(exp(2*
I*(d*x+c))+1))^2))-3/8*I/(-1+n)/d/(e^n)*exp(I*(d*x+c))^n/e^3*a^n*exp(-1/2*I
*(2*c-3*Pi*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3-csgn(I*a*exp(2*I
*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*csgn(I*a)*Pi*n-2*Pi*csgn(I*exp(2*I*(d*x+c
)))^2*csgn(I*exp(I*(d*x+c)))*n+Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d*
x+c)))^2*n+2*d*x-n*Pi*csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))
*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))+csgn(I/(exp(2*I*(d*x+c))+1)*
exp(2*I*(d*x+c)))^3*Pi*n+csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3*
Pi*n+Pi*csgn(I*exp(2*I*(d*x+c)))^3*n-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d
*x+c)))^2*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*n-csgn(I/(exp(2*I*(d*x+c))+1)*exp
(2*I*(d*x+c)))^2*Pi*csgn(I*exp(2*I*(d*x+c)))*n-csgn(I/(exp(2*I*(d*x+c))+1)*
exp(2*I*(d*x+c)))*csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*Pi*n+3*
Pi*csgn(I*e)*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+3*Pi*csgn(I/(e
xp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+3*Pi*csgn
(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-n*Pi*csgn(
I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3-3*Pi*csgn(I*e)*csgn(I*exp(I*(d*x+c
))/(exp(2*I*(d*x+c))+1))*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))+3*Pi
*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e*exp(I*(d*x+c))/(exp(2
I(d*x+c))+1))^2-n*Pi*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3+n*Pi
*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+n*Pi*
csgn(I*e)*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-3*Pi*csgn(I/(exp(
2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x
+c))+1))-3*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3+n*Pi*csgn(I*exp
(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))
+1))^2+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a*exp(2*I*(d*x+

```
c))/(exp(2*I*(d*x+c))+1))*csgn(I*a)*Pi*n+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*
I*(d*x+c)))*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(2*I*(d*x+c)))*n-n*Pi
*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/
(exp(2*I*(d*x+c))+1))+n*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c
))/(exp(2*I*(d*x+c))+1))^2))-3/8*I/(1+n)/d/(e^n)*exp(I*(d*x+c))^n/e^3*a^n*e
xp(1/2*I*(2*c+3*Pi*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3+csgn(I*a
*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*csgn(I*a)*Pi*n+2*Pi*csgn(I*exp(2*
I*(d*x+c)))^2*csgn(I*exp(I*(d*x+c)))*n-Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*e
xp(I*(d*x+c)))^2*n+2*d*x+n*Pi*csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x
+c))+1))*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))-csgn(I/(exp(2*I*(d*x
+c))+1)*exp(2*I*(d*x+c)))^3*Pi*n-csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c)
)+1))^3*Pi*n-Pi*csgn(I*exp(2*I*(d*x+c)))^3*n+csgn(I/(exp(2*I*(d*x+c))+1)*ex
p(2*I*(d*x+c)))^2*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*n+csgn(I/(exp(2*I*(d*x+c)
)+1)*exp(2*I*(d*x+c)))^2*Pi*csgn(I*exp(2*I*(d*x+c)))*n+csgn(I/(exp(2*I*(d*x
+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2
*Pi*n-3*Pi*csgn(I*e)*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-3*Pi*c
sgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-3
*Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+n*
Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3+3*Pi*csgn(I*e)*csgn(I*exp(
I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+
1))-3*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e*exp(I*(d*x+c)
)/(exp(2*I*(d*x+c))+1))^2+n*Pi*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1)
)^3-n*Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))
^2-n*Pi*csgn(I*e)*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+3*Pi*csgn
(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(
2*I*(d*x+c))+1))+3*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3-n*Pi*cs
gn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*
(d*x+c))+1))^2-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a*exp(2
*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*a)*Pi*n-csgn(I/(exp(2*I*(d*x+c))+1
)*exp(2*I*(d*x+c)))*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(2*I*(d*x+c))
)*n+n*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(
d*x+c))/(exp(2*I*(d*x+c))+1))-n*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(
I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.29

$$\int (e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n\,dx$$

$$= \frac{(-i\,n^3 - 3i\,n^2 + (-i\,n^3 + 3i\,n^2 + i\,n - 3i)e^{(6i\,dx+6i\,c)} - 3\,(i\,n^3 - i\,n^2 - 9i\,n + 9i)e^{(4i\,dx+4i\,c)} - 3\,(i\,n^3 + i\,n^2}{dn^4 - 10\,dn^2 + (dn^4 - 10\,dn^2 + 9\,d)e^{(6i\,dx+6i\,c)} + 3\,(dn^4 - 10\,dn^2 + 9}$$

```
[In] integrate((e*sec(d*x+c))^(-3-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] (-I*n^3 - 3*I*n^2 + (-I*n^3 + 3*I*n^2 + I*n - 3*I)*e^(6*I*d*x + 6*I*c) - 3*
(I*n^3 - I*n^2 - 9*I*n + 9*I)*e^(4*I*d*x + 4*I*c) - 3*(I*n^3 + I*n^2 - 9*I*
n - 9*I)*e^(2*I*d*x + 2*I*c) + I*n + 3*I)*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x
+ 2*I*c) + 1))^(-n - 3)*e^(I*d*n*x + I*c*n + n*log(2*e*e^(I*d*x + I*c)/(e^(
2*I*d*x + 2*I*c) + 1)) + n*log(a/e))/(d*n^4 - 10*d*n^2 + (d*n^4 - 10*d*n^2
+ 9*d)*e^(6*I*d*x + 6*I*c) + 3*(d*n^4 - 10*d*n^2 + 9*d)*e^(4*I*d*x + 4*I*c)
 + 3*(d*n^4 - 10*d*n^2 + 9*d)*e^(2*I*d*x + 2*I*c) + 9*d)
```

Sympy [F]

$$\int (e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(c+dx))^{-n-3}\,(ia(\tan(c+dx)-i))^n\,dx$$

```
[In] integrate((e*sec(d*x+c))**(-3-n)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Integral((e*sec(c + d*x))**(-n - 3)*(I*a*(tan(c + d*x) - I))**n, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.69

$$\int (e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n\,dx = \frac{(-i\,a^n n^3+3i\,a^n n^2+i\,a^n n-3i\,a^n)\cos((dx+c)(n+3))-3\,(i\,a^n n^3-i\,a^n n^2-9i\,a^n n+9i\,a^n)\cos((dx+$$

```
[In] integrate((e*sec(d*x+c))^(-3-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] 1/8*((-I*a^n*n^3 + 3*I*a^n*n^2 + I*a^n*n - 3*I*a^n)*cos((d*x + c)*(n + 3))
- 3*(I*a^n*n^3 - I*a^n*n^2 - 9*I*a^n*n + 9*I*a^n)*cos((d*x + c)*(n + 1)) -
3*(I*a^n*n^3 + I*a^n*n^2 - 9*I*a^n*n - 9*I*a^n)*cos((d*x + c)*(n - 1)) + (-
I*a^n*n^3 - 3*I*a^n*n^2 + I*a^n*n + 3*I*a^n)*cos((d*x + c)*(n - 3)) + (a^n*
n^3 - 3*a^n*n^2 - a^n*n + 3*a^n)*sin((d*x + c)*(n + 3)) + 3*(a^n*n^3 - a^n*
n^2 - 9*a^n*n + 9*a^n)*sin((d*x + c)*(n + 1)) + 3*(a^n*n^3 + a^n*n^2 - 9*a^
n*n - 9*a^n)*sin((d*x + c)*(n - 1)) + (a^n*n^3 + 3*a^n*n^2 - a^n*n - 3*a^n)
*sin((d*x + c)*(n - 3)))/((e^(n + 3)*n^4 - 10*e^(n + 3)*n^2 + 9*e^(n + 3))*
d)
```

Giac [F]

$$\int (e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int \left(e\sec\left(dx+c\right)\right)^{-n-3}\left(i\,a\tan\left(dx+c\right)+a\right)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(-3-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(-n - 3)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [B] (verification not implemented)

Time = 11.06 (sec) , antiderivative size = 425, normalized size of antiderivative = 2.07

$$\int (e\sec(c+dx))^{-3-n}(a+ia\tan(c+dx))^n\,dx =$$

$$-\frac{\left(2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)\left(2\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)^2+\sin\left(3\,c+3\,d\,x\right)\,1\mathrm{i}-1\right)\left(\frac{\left(a-\frac{a\,\sin(c+d\,x)\,1\mathrm{i}}{2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1}\right)^n\left(-n^3-3\,n^2+n+3\right)}{d\,\left(n^4\,1\mathrm{i}-n^2\,10\mathrm{i}+9\mathrm{i}\right)}\right.}{}$$

```
[In] int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(n + 3),x)
```

```
[Out] -((2*sin(c/2 + (d*x)/2)^2 - 1)*(sin(3*c + 3*d*x)*1i + 2*sin((3*c)/2 + (3*d*
x)/2)^2 - 1)*(((a - (a*sin(c + d*x)*1i)/(2*sin(c/2 + (d*x)/2)^2 - 1))^n*(n
- 3*n^2 - n^3 + 3))/(d*(n^4*1i - n^2*10i + 9i)) + ((a - (a*sin(c + d*x)*1i)
/(2*sin(c/2 + (d*x)/2)^2 - 1))^n*(sin(6*c + 6*d*x)*1i - 2*sin(3*c + 3*d*x)^
2 + 1)*(n + 3*n^2 - n^3 - 3))/(d*(n^4*1i - n^2*10i + 9i)) + ((a - (a*sin(c
+ d*x)*1i)/(2*sin(c/2 + (d*x)/2)^2 - 1))^n*(sin(2*c + 2*d*x)*1i - 2*sin(c +
 d*x)^2 + 1)*(27*n - 3*n^2 - 3*n^3 + 27))/(d*(n^4*1i - n^2*10i + 9i)) + ((a
 - (a*sin(c + d*x)*1i)/(2*sin(c/2 + (d*x)/2)^2 - 1))^n*(sin(4*c + 4*d*x)*1i
 - 2*sin(2*c + 2*d*x)^2 + 1)*(27*n + 3*n^2 - 3*n^3 - 27))/(d*(n^4*1i - n^2*
10i + 9i))))/(8*(-e/(2*sin(c/2 + (d*x)/2)^2 - 1))^(n + 3)*(sin(c + d*x)^2 -
 1)^2)
```

3.485 $\int (e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 30, antiderivative size = 148

$$\int (e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n\,dx = \frac{i(e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n}{d(2-n)} - \frac{2i(e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^{1+n}}{ad(2-n)n} + \frac{2i(e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^{2+n}}{a^2dn\,(4-n^2)}$$

```
[Out] I*(e*sec(d*x+c))^(-2-n)*(a+I*a*tan(d*x+c))^n/d/(2-n)-2*I*(e*sec(d*x+c))^(-2
-n)*(a+I*a*tan(d*x+c))^(1+n)/a/d/(2-n)/n+2*I*(e*sec(d*x+c))^(-2-n)*(a+I*a*t
an(d*x+c))^(2+n)/a^2/d/n/(-n^2+4)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3585, 3569}

$$\int (e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n\,dx = \frac{2i(a+ia\tan(c+dx))^{n+2}(e\sec(c+dx))^{-n-2}}{a^2dn\,(4-n^2)} + \frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-2}}{d(2-n)} - \frac{2i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-n-2}}{ad(2-n)n}$$

```
[In] Int[(e*Sec[c + d*x])^(-2 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (I*(e*Sec[c + d*x])^(-2 - n)*(a + I*a*Tan[c + d*x])^n)/(d*(2 - n)) - ((2*I)
*(e*Sec[c + d*x])^(-2 - n)*(a + I*a*Tan[c + d*x])^(1 + n))/(a*d*(2 - n)*n)
+ ((2*I)*(e*Sec[c + d*x])^(-2 - n)*(a + I*a*Tan[c + d*x])^(2 + n))/(a^2*d*n
*(4 - n^2))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3585

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 + b^2, 0] && ILtQ[Simplify[m + n], 0] && NeQ[m + 2*n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{i(e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n}{d(2-n)} \\
&\quad + \frac{2\int(e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^{1+n}\,dx}{a(2-n)} \\
&= \frac{i(e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n}{d(2-n)} \\
&\quad - \frac{2i(e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^{1+n}}{ad(2-n)n} \\
&\quad - \frac{2\int(e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^{2+n}\,dx}{a^2(2-n)n} \\
&= \frac{i(e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n}{d(2-n)} \\
&\quad - \frac{2i(e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^{1+n}}{ad(2-n)n} \\
&\quad + \frac{2i(e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^{2+n}}{a^2dn\,(4-n^2)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.55

$$\int (e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n\,dx = -\frac{i(e\sec(c+dx))^{-n}\left(-4+n^2+n^2\cos(2(c+dx))-2in\sin(2(c+dx))\right)(a+ia\tan(c+dx))^n}{2de^2(-2+n)n(2+n)}$$

```
[In] Integrate[(e*Sec[c + d*x])^(-2 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-1/2*I)*(-4 + n^2 + n^2*Cos[2*(c + d*x)] - (2*I)*n*Sin[2*(c + d*x)])*(a +
 I*a*Tan[c + d*x])^n)/(d*e^2*(-2 + n)*n*(2 + n)*(e*Sec[c + d*x])^n)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 6.42 (sec) , antiderivative size = 2581, normalized size of antiderivative = 17.44

method	result	size
risch	Expression too large to display	2581

```
[In] int((e*sec(d*x+c))^(-n-2)*(a+I*a*tan(d*x+c))^n,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*I/(n-2)/d*exp(I*(d*x+c))^n/(e^n)/e^2*a^n*exp(-1/2*I*(csgn(I*a*exp(2*I*
(d*x+c))/(exp(2*I*(d*x+c))+1))^3*Pi*n-csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d
*x+c))+1))^2*csgn(I*a)*Pi*n-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*c
sgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*Pi*n+csgn(I/(exp(2*I*(d*x+
c))+1)*exp(2*I*(d*x+c)))*csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*cs
gn(I*a)*Pi*n+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^3*Pi*n-csgn(I/(e
xp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*Pi*csgn(I*exp(2*I*(d*x+c)))*n-csgn(I
/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*n
+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*Pi*csgn(I/(exp(2*I*(d*x+c))+
1))*csgn(I*exp(2*I*(d*x+c)))*n-n*Pi*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c
))+1))^3+n*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e*exp(I*(d
*x+c))/(exp(2*I*(d*x+c))+1))^2+n*Pi*csgn(I*e)*csgn(I*e*exp(I*(d*x+c))/(exp(
2*I*(d*x+c))+1))^2-n*Pi*csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1
))*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))-n*Pi*csgn(I*exp(I*(d*x+c))
/(exp(2*I*(d*x+c))+1))^3+n*Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/
(exp(2*I*(d*x+c))+1))^2+n*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x
+c))/(exp(2*I*(d*x+c))+1))^2-n*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I
*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))+Pi*csgn(I*exp(2*I*(d
*x+c)))^3*n-2*Pi*csgn(I*exp(2*I*(d*x+c)))^2*csgn(I*exp(I*(d*x+c)))*n+Pi*csg
n(I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d*x+c)))^2*n+4*d*x+4*c))-1/4*I/(2+n)/d*
exp(I*(d*x+c))^n/(e^n)/e^2*a^n*exp(1/2*I*(-csgn(I*a*exp(2*I*(d*x+c))/(exp(2
```

I(d*x+c))+1))^3*Pi*n+csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*cs
gn(I*a)*Pi*n+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a*exp(2*I
*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*Pi*n-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*
(d*x+c)))*csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*a)*Pi*n-cs
gn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^3*Pi*n+csgn(I/(exp(2*I*(d*x+c))
+1)*exp(2*I*(d*x+c)))^2*Pi*csgn(I*exp(2*I*(d*x+c)))*n+csgn(I/(exp(2*I*(d*x+
c))+1)*exp(2*I*(d*x+c)))^2*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*n-csgn(I/(exp(2*
I*(d*x+c))+1)*exp(2*I*(d*x+c)))*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(
2*I*(d*x+c)))*n+n*Pi*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3-n*Pi*c
sgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e*exp(I*(d*x+c))/(exp(2*I
*(d*x+c))+1))^2-n*Pi*csgn(I*e)*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1)
)^2+n*Pi*csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e*exp
(I*(d*x+c))/(exp(2*I*(d*x+c))+1))+n*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+
c))+1))^3-n*Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c
))+1))^2-n*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(
d*x+c))+1))^2+n*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c)))*csgn
(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))-Pi*csgn(I*exp(2*I*(d*x+c)))^3*n+2*P
i*csgn(I*exp(2*I*(d*x+c)))^2*csgn(I*exp(I*(d*x+c)))*n-Pi*csgn(I*exp(2*I*(d*
x+c)))*csgn(I*exp(I*(d*x+c)))^2*n+4*d*x+4*c))-1/2*I/d/n*exp(I*(d*x+c))^n/(e
^n)/e^2*a^n*exp(1/2*I*n*Pi*(-csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1)
)^3+csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*csgn(I*a)+csgn(I/(exp
(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+
c))+1))^2-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a*exp(2*I*(d
*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*a)-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*
(d*x+c)))^3+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*csgn(I*exp(2*I*
(d*x+c)))+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*csgn(I/(exp(2*I*(
d*x+c))+1))-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I/(exp(2*I*(
d*x+c))+1))*csgn(I*exp(2*I*(d*x+c)))+csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+
c))+1))^3-csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e*exp(I*(d*x+c
))/(exp(2*I*(d*x+c))+1))^2-csgn(I*e)*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+
c))+1))^2+csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e*ex
p(I*(d*x+c))/(exp(2*I*(d*x+c))+1))+csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+
1))^3-csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-
csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+
csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(
exp(2*I*(d*x+c))+1))-csgn(I*exp(2*I*(d*x+c)))^3+2*csgn(I*exp(2*I*(d*x+c)))^
2*csgn(I*exp(I*(d*x+c)))-csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d*x+c)))^2)
)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.20

$$\int (e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n\,dx$$
$$= \frac{(-i\,n^2+(-i\,n^2+2i\,n)e^{(4i\,dx+4i\,c)}-2\,(i\,n^2-4i)e^{(2i\,dx+2i\,c)}-2i\,n)\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{-n-2}e^{\left(i\,dnx+i\,cn+n\log\left(\frac{2\,ee^{(}}{e^{(2i\,d}}\right.\right.}}{dn^3-4\,dn+(dn^3-4\,dn)e^{(4i\,dx+4i\,c)}+2\,(dn^3-4\,dn)e^{(2i\,dx+2i\,c)}}$$

```
[In] integrate((e*sec(d*x+c))^(-2-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
[Out] (-I*n^2 + (-I*n^2 + 2*I*n)*e^(4*I*d*x + 4*I*c) - 2*(I*n^2 - 4*I)*e^(2*I*d*x
 + 2*I*c) - 2*I*n)*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-n - 2)
*e^(I*d*n*x + I*c*n + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))
+ n*log(a/e))/(d*n^3 - 4*d*n + (d*n^3 - 4*d*n)*e^(4*I*d*x + 4*I*c) + 2*(d*n
^3 - 4*d*n)*e^(2*I*d*x + 2*I*c))
```

Sympy [F]

$$\int (e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n\,dx$$
$$= \int (e\sec(c+dx))^{-n-2}\,(ia(\tan(c+dx)-i))^n\,dx$$

```
[In] integrate((e*sec(d*x+c))**(-2-n)*(a+I*a*tan(d*x+c))**n,x)
[Out] Integral((e*sec(c + d*x))**(-n - 2)*(I*a*(tan(c + d*x) - I))**n, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.18

$$\int (e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n\,dx$$
$$= \frac{(-i\,a^n n^2+2i\,a^n n)\cos((dx+c)(n+2))+(-i\,a^n n^2-2i\,a^n n)\cos((dx+c)(n-2))-2\,(i\,a^n n^2-4i\,a^n)\,c}{4}$$

```
[In] integrate((e*sec(d*x+c))^(-2-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
[Out] 1/4*((-I*a^n*n^2 + 2*I*a^n*n)*cos((d*x + c)*(n + 2)) + (-I*a^n*n^2 - 2*I*a^
n*n)*cos((d*x + c)*(n - 2)) - 2*(I*a^n*n^2 - 4*I*a^n)*cos((d*x + c)*n) + (a
^n*n^2 - 2*a^n*n)*sin((d*x + c)*(n + 2)) + (a^n*n^2 + 2*a^n*n)*sin((d*x + c
)*(n - 2)) + 2*(a^n*n^2 - 4*a^n)*sin((d*x + c)*n))/((e^(n + 2)*n^3 - 4*e^(n
 + 2)*n)*d)
```

Giac [F]

$$\int (e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int (e\sec(dx+c))^{-n-2}(i\,a\tan(dx+c)+a)^n\,dx$$

[In] integrate((e*sec(d*x+c))^(-2-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")

[Out] integrate((e*sec(d*x + c))^(-n - 2)*(I*a*tan(d*x + c) + a)^n, x)

Mupad [B] (verification not implemented)

Time = 10.53 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.53

$$\int (e\sec(c+dx))^{-2-n}(a+ia\tan(c+dx))^n\,dx$$
$$=\frac{(\cos(2c+2dx)-\sin(2c+2dx)\ 1\mathrm{i})\left(\frac{\left(a+\frac{a\ \sin(c+dx)\ 1\mathrm{i}}{\cos(c+dx)}\right)^n(n+2)}{d\,(n^2\ 1\mathrm{i}-4\mathrm{i})}+\frac{(\cos(4c+4dx)+\sin(4c+4dx)\ 1\mathrm{i})\left(a+\frac{a\ \sin(c+dx)\ 1\mathrm{i}}{\cos(c+dx)}\right)^n(n-}{d\,(n^2\ 1\mathrm{i}-4\mathrm{i})}\right.}{4\left(\frac{\cos(2c+2dx)}{2}+\frac{1}{2}\right)\left(\frac{e}{\cos(c+dx)}\right)^{n+2}}$$

[In] int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(n + 2),x)

[Out] ((cos(2*c + 2*d*x) - sin(2*c + 2*d*x)*1i)*(((a + (a*sin(c + d*x)*1i)/cos(c + d*x))^n*(n + 2))/(d*(n^2*1i - 4i)) + ((cos(4*c + 4*d*x) + sin(4*c + 4*d*x)*1i)*(a + (a*sin(c + d*x)*1i)/cos(c + d*x))^n*(n - 2))/(d*(n^2*1i - 4i)) + ((cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i)*(2*n^2 - 8)*(a + (a*sin(c + d*x)*1i)/cos(c + d*x))^n)/(d*n*(n^2*1i - 4i))))/(4*(cos(2*c + 2*d*x)/2 + 1/2)*(e/cos(c + d*x))^(n + 2))

3.486 $\int (e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 30, antiderivative size = 94

$$\int (e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n\,dx = \frac{i(e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n}{d(1-n)} - \frac{i(e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^{1+n}}{ad\,(1-n^2)}$$

```
[Out] I*(e*sec(d*x+c))^(-1-n)*(a+I*a*tan(d*x+c))^n/d/(1-n)-I*(e*sec(d*x+c))^(-1-n
)*(a+I*a*tan(d*x+c))^(1+n)/a/d/(-n^2+1)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3585, 3569}

$$\int (e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n\,dx = \frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n-1}}{d(1-n)} - \frac{i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-n-1}}{ad\,(1-n^2)}$$

```
[In] Int[(e*Sec[c + d*x])^(-1 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (I*(e*Sec[c + d*x])^(-1 - n)*(a + I*a*Tan[c + d*x])^n)/(d*(1 - n)) - (I*(e*
Sec[c + d*x])^(-1 - n)*(a + I*a*Tan[c + d*x])^(1 + n))/(a*d*(1 - n^2))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
```

```
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3585

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 + b^2, 0] && ILtQ[Simplify[m + n], 0] && NeQ[m + 2*n, 0]
```

Rubi steps

$$\text{integral} = \frac{i(e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n}{d(1-n)} + \frac{\int (e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^{1+n}\,dx}{a(1-n)}$$

$$= \frac{i(e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n}{d(1-n)} - \frac{i(e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^{1+n}}{ad\,(1-n^2)}$$

Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.62

$$\int (e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n\,dx = -\frac{i(e\sec(c+dx))^{-1-n}(n-i\tan(c+dx))(a+ia\tan(c+dx))^n}{d(-1+n)(1+n)}$$

```
[In] Integrate[(e*Sec[c + d*x])^(-1 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*(e*Sec[c + d*x])^(-1 - n)*(n - I*Tan[c + d*x])*(a + I*a*Tan[c + d*x])
^n)/(d*(-1 + n)*(1 + n))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 7.10 (sec) , antiderivative size = 2484, normalized size of antiderivative = 26.43

method	result	size
risch	Expression too large to display	2484

```
[In] int((e*sec(d*x+c))^(-1-n)*(a+I*a*tan(d*x+c))^n,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I/(1+n)/d*e^(-n)*a^n/e*exp(I*(d*x+c))^n*exp(1/2*I*(2*c+Pi*csgn(I*e*exp
(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3+csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x
+c))+1))^2*csgn(I*a)*Pi*n+2*Pi*csgn(I*exp(2*I*(d*x+c)))^2*csgn(I*exp(I*(d*x
+c)))*n-Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d*x+c)))^2*n+2*d*x+n*Pi*c
sgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e*exp(I*(d*x+c)
)/(exp(2*I*(d*x+c))+1))-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^3*Pi*
n-csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3*Pi*n-Pi*csgn(I*exp(2*I*
(d*x+c)))^3*n+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*Pi*csgn(I/(ex
p(2*I*(d*x+c))+1))*n+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*Pi*csg
n(I*exp(2*I*(d*x+c)))*n+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(
I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*Pi*n-Pi*csgn(I*e)*csgn(I*e*exp
(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*
exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*exp
(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+n*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*
x+c))+1))^3+Pi*csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I
*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))-Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(
d*x+c))+1))*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+n*Pi*csgn(I*e*e
xp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3-n*Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*ex
p(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-n*Pi*csgn(I*e)*csgn(I*e*exp(I*(d*x+c))
/(exp(2*I*(d*x+c))+1))^2+Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+
c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))+Pi*csgn(I*exp(I*(d*x+c))/(
exp(2*I*(d*x+c))+1))^3-n*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csg
n(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-csgn(I/(exp(2*I*(d*x+c))+1)*ex
p(2*I*(d*x+c)))*csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*a)*P
i*n-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*Pi*csgn(I/(exp(2*I*(d*x+c
))+1))*csgn(I*exp(2*I*(d*x+c)))*n+n*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*
exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))-n*Pi*csgn(I/(ex
p(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2))-1/2*I/(-
1+n)/d*e^(-n)*a^n/e*exp(I*(d*x+c))^n*exp(-1/2*I*(2*c-Pi*csgn(I*e*exp(I*(d*x
+c))/(exp(2*I*(d*x+c))+1))^3-csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1)
)^2*csgn(I*a)*Pi*n-2*Pi*csgn(I*exp(2*I*(d*x+c)))^2*csgn(I*exp(I*(d*x+c)))*n
+Pi*csgn(I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d*x+c)))^2*n+2*d*x-n*Pi*csgn(I*e
)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e*exp(I*(d*x+c))/(exp(
2*I*(d*x+c))+1))+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^3*Pi*n+csgn(
I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3*Pi*n+Pi*csgn(I*exp(2*I*(d*x+c)
))^3*n-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*Pi*csgn(I/(exp(2*I*(
d*x+c))+1))*n-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^2*Pi*csgn(I*exp
(2*I*(d*x+c)))*n-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a*exp
(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2*Pi*n+Pi*csgn(I*e)*csgn(I*e*exp(I*(d*x
+c))/(exp(2*I*(d*x+c))+1))^2+Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(
d*x+c))/(exp(2*I*(d*x+c))+1))^2+Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x
+c))/(exp(2*I*(d*x+c))+1))^2-n*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1
))^3-Pi*csgn(I*e)*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e*exp(
```

```
I*(d*x+c))/(exp(2*I*(d*x+c))+1))+Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))
+1))*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-n*Pi*csgn(I*e*exp(I*(d
*x+c))/(exp(2*I*(d*x+c))+1))^3+n*Pi*csgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*
x+c))/(exp(2*I*(d*x+c))+1))^2+n*Pi*csgn(I*e)*csgn(I*e*exp(I*(d*x+c))/(exp(2
*I*(d*x+c))+1))^2-Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(d*x+c)))*cs
gn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))-Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I
*(d*x+c))+1))^3+n*Pi*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e*e
xp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(
d*x+c)))*csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*a)*Pi*n+csg
n(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*
csgn(I*exp(2*I*(d*x+c)))*n-n*Pi*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(I*(
d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))+n*Pi*csgn(I/(exp(2*I*(
d*x+c))+1))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.37

$$\int (e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n\,dx = \frac{\left((-i\,n+i)e^{(2i\,dx+2i\,c)}-i\,n-i\right)\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{-n-1}e^{\left(i\,dnx+i\,cn+n\log\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)+n\log\left(\frac{a}{e}\right)\right)}}{dn^2+(dn^2-d)e^{(2i\,dx+2i\,c)}-d}$$

```
[In] integrate((e*sec(d*x+c))^(-1-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] ((-I*n + I)*e^(2*I*d*x + 2*I*c) - I*n - I)*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x
 + 2*I*c) + 1))^(-n - 1)*e^(I*d*n*x + I*c*n + n*log(2*e*e^(I*d*x + I*c)/(e^
(2*I*d*x + 2*I*c) + 1)) + n*log(a/e))/(d*n^2 + (d*n^2 - d)*e^(2*I*d*x + 2*I
*c) - d)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(71) = 142.

Time = 0.51 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.62

$$\int (e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n\,dx$$
$$= \begin{cases} x(e\sec{(c)})^{-n-1}\,(ia\tan{(c)}+a)^n & \text{for } d=0 \\ \frac{dx\tan{(c+dx)}}{2ad\tan{(c+dx)}-2iad} - \frac{idx}{2ad\tan{(c+dx)}-2iad} + \frac{1}{2ad\tan{(c+dx)}-2iad} & \text{for } n=-1 \\ \frac{\frac{ax\tan^2{(c+dx)}}{2\sec^2{(c+dx)}}+\frac{ax}{2\sec^2{(c+dx)}}+\frac{a\tan{(c+dx)}}{2d\sec^2{(c+dx)}}-\frac{ia}{2d\sec^2{(c+dx)}}}{e^2} & \text{for } n=1 \\ -\frac{in(e\sec{(c+dx)})^{-n-1}(ia\tan{(c+dx)}+a)^n}{dn^2-d} - \frac{(e\sec{(c+dx)})^{-n-1}(ia\tan{(c+dx)}+a)^n\tan{(c+dx)}}{dn^2-d} & \text{otherwise} \end{cases}$$

```
[In] integrate((e*sec(d*x+c))**(-1-n)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Piecewise((x*(e*sec(c))**(-n - 1)*(I*a*tan(c) + a)**n, Eq(d, 0)), (d*x*tan(
c + d*x)/(2*a*d*tan(c + d*x) - 2*I*a*d) - I*d*x/(2*a*d*tan(c + d*x) - 2*I*a
*d) + 1/(2*a*d*tan(c + d*x) - 2*I*a*d), Eq(n, -1)), ((a*x*tan(c + d*x)**2/(
2*sec(c + d*x)**2) + a*x/(2*sec(c + d*x)**2) + a*tan(c + d*x)/(2*d*sec(c +
d*x)**2) - I*a/(2*d*sec(c + d*x)**2))/e**2, Eq(n, 1)), (-I*n*(e*sec(c + d*x
))**(-n - 1)*(I*a*tan(c + d*x) + a)**n/(d*n**2 - d) - (e*sec(c + d*x))**(-n
 - 1)*(I*a*tan(c + d*x) + a)**n*tan(c + d*x)/(d*n**2 - d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.20

$$\int (e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n\,dx$$
$$= \frac{(-i\,a^n n + i\,a^n)\cos\left((dx+c)(n+1)\right) + (-i\,a^n n - i\,a^n)\cos\left((dx+c)(n-1)\right) + (a^n n - a^n)\sin\left((dx+c)(n\right.}{2\,(e^{n+1}n^2 - e^{n+1})d}$$

```
[In] integrate((e*sec(d*x+c))^(-1-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] 1/2*((-I*a^n*n + I*a^n)*cos((d*x + c)*(n + 1)) + (-I*a^n*n - I*a^n)*cos((d*
x + c)*(n - 1)) + (a^n*n - a^n)*sin((d*x + c)*(n + 1)) + (a^n*n + a^n)*sin(
(d*x + c)*(n - 1)))/((e^(n + 1)*n^2 - e^(n + 1))*d)
```

Giac [F]

$$\int (e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec{(dx+c)})^{-n-1}\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(-1-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(-n - 1)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [B] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.29

$$\int (e\sec(c+dx))^{-1-n}(a+ia\tan(c+dx))^n\,dx = -\frac{\left(\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}\right)^n\,(\sin{(c+d\,x)}+\sin{(3\,c+3\,d\,x)}+n\,\cos{(c+d\,x)}\,3\mathrm{i}+n\,\cos{(3\,c+3\,d\,x)}}{2\,d\,e\,\left(\cos{(2\,c+2\,d\,x)}+1\right)\,\left(n^2-1\right)\,\left(\frac{e}{\cos(c+d\,x)}\right)^n}$$

```
[In] int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(n + 1),x)
[Out] -(((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))
^n*(sin(c + d*x) + sin(3*c + 3*d*x) + n*cos(c + d*x)*3i + n*cos(3*c + 3*d*x
)*1i))/(2*d*e*(cos(2*c + 2*d*x) + 1)*(n^2 - 1)*(e/cos(c + d*x))^n)
```

3.487 $\int (e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 28, antiderivative size = 37

$$\int (e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n\,dx = -\frac{i(e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n}{dn}$$

```
[Out] -I*(a+I*a*tan(d*x+c))^n/d/n/((e*sec(d*x+c))^n)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {3569}

$$\int (e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n\,dx = -\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-n}}{dn}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^n/(e*Sec[c + d*x])^n,x]
[Out] ((-I)*(a + I*a*Tan[c + d*x])^n)/(d*n*(e*Sec[c + d*x])^n)
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\text{integral} = -\frac{i(e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n}{dn}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int (e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n\,dx = -\frac{i(e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n}{dn}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^n/(e*Sec[c + d*x])^n,x]
```

```
[Out] ((-I)*(a + I*a*Tan[c + d*x])^n)/(d*n*(e*Sec[c + d*x])^n)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.52 (sec) , antiderivative size = 842, normalized size of antiderivative = 22.76

method	result	size
risch	Expression too large to display	842

```
[In] int((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^n),x,method=_RETURNVERBOSE)
```

```
[Out] -I*exp(I*(d*x+c))^(2*n)*a^n/(exp(I*(d*x+c))^n)/(e^n)/n/d*exp(1/2*I*n*Pi*(-c
sgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3+csgn(I*a*exp(2*I*(d*x+c))/
(exp(2*I*(d*x+c))+1))^2*csgn(I*a)+csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+
c)))*csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-csgn(I/(exp(2*I*(d*x
+c))+1)*exp(2*I*(d*x+c)))*csgn(I*a*exp(2*I*(d*x+c))/(exp(2*I*(d*x+c))+1))*c
sgn(I*a)-csgn(I/(exp(2*I*(d*x+c))+1)*exp(2*I*(d*x+c)))^3+csgn(I/(exp(2*I*(d
*x+c))+1)*exp(2*I*(d*x+c)))^2*csgn(I*exp(2*I*(d*x+c)))+csgn(I/(exp(2*I*(d*x
+c))+1)*exp(2*I*(d*x+c)))^2*csgn(I/(exp(2*I*(d*x+c))+1))-csgn(I/(exp(2*I*(d
*x+c))+1)*exp(2*I*(d*x+c)))*csgn(I/(exp(2*I*(d*x+c))+1))*csgn(I*exp(2*I*(d*
x+c)))+csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3-csgn(I*exp(I*(d*x+c)
)/(exp(2*I*(d*x+c))+1))*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-csg
n(I*e)*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+csgn(I*e)*csgn(I*exp
(I*(d*x+c))/(exp(2*I*(d*x+c))+1))*csgn(I*e*exp(I*(d*x+c))/(exp(2*I*(d*x+c))
+1))+csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^3-csgn(I*exp(I*(d*x+c)))*c
sgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2-csgn(I/(exp(2*I*(d*x+c))+1))*c
sgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))^2+csgn(I/(exp(2*I*(d*x+c))+1))*c
sgn(I*exp(I*(d*x+c)))*csgn(I*exp(I*(d*x+c))/(exp(2*I*(d*x+c))+1))-csgn(I*ex
p(2*I*(d*x+c)))^3+2*csgn(I*exp(2*I*(d*x+c)))^2*csgn(I*exp(I*(d*x+c)))-csgn(
I*exp(2*I*(d*x+c)))*csgn(I*exp(I*(d*x+c)))^2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(33) = 66.

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.27

$$\int (e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n\,dx = -\frac{i\,e^{\left(i\,dnx+i\,cn+n\log\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)+n\log\left(\frac{a}{e}\right)\right)}}{dn\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^n}$$

```
[In] integrate((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^n),x, algorithm="fricas")
[Out] -I*e^(I*d*n*x + I*c*n + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1)
) + n*log(a/e))/(d*n*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n)
```

Sympy [A] (verification not implemented)

Time = 3.86 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.32

$$\int (e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n\,dx = \begin{cases} x & \text{for } d=0\wedge n=0 \\ x(e\sec(c))^{-n}(ia\tan(c)+a)^n & \text{for } d=0 \\ x & \text{for } n=0 \\ -\frac{i(e\sec(c+dx))^{-n}(ia\tan(c+dx)+a)^n}{dn} & \text{otherwise} \end{cases}$$

```
[In] integrate((a+I*a*tan(d*x+c))**n/((e*sec(d*x+c))**n),x)
[Out] Piecewise((x, Eq(d, 0) & Eq(n, 0)), (x*(I*a*tan(c) + a)**n/(e*sec(c))**n, E
q(d, 0)), (x, Eq(n, 0)), (-I*(I*a*tan(c + d*x) + a)**n/(d*n*(e*sec(c + d*x)
)**n), True))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(33) = 66.

Time = 0.33 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.32

$$\int (e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n\,dx = -\frac{i\,a^n e^{\left(n\log\left(-\frac{2i\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)-n\log\left(-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)\right)}}{de^n n}$$

```
[In] integrate((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^n),x, algorithm="maxima")
[Out] -I*a^n*e^(n*log(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 - 1) - n*log(-sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1))/(d*
e^n*n)
```

Giac [F]

$$\int (e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n\,dx = \int \frac{(i\,a\tan(dx+c)+a)^n}{(e\sec(dx+c))^n}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^n),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^n/(e*sec(d*x + c))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{-n}(a+ia\tan(c+dx))^n\,dx = \int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^n}{\left(\frac{e}{\cos(c+d\,x)}\right)^n}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^n,x)
[Out] int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^n, x)
```

3.488 $\int (e\sec(c+dx))^{1-n}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 30, antiderivative size = 118

$$\int (e\sec(c+dx))^{1-n}(a+ia\tan(c+dx))^n\,dx$$
$$= \frac{i2^{\frac{1+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{1-n}{2},\frac{3-n}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{1-n}(1+i\tan(c+dx))^{\frac{1}{2}(-1-}}{d(1-n)}$$

```
[Out] I*2^(1/2+1/2*n)*hypergeom([1/2-1/2*n, 1/2-1/2*n],[3/2-1/2*n],1/2-1/2*I*tan(
d*x+c))*(e*sec(d*x+c))^(1-n)*(1+I*tan(d*x+c))^(-1/2-1/2*n)*(a+I*a*tan(d*x+c
))^n/d/(1-n)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3586, 3604, 72, 71}

$$\int (e\sec(c+dx))^{1-n}(a+ia\tan(c+dx))^n\,dx$$
$$= \frac{i2^{\frac{n+1}{2}}(1+i\tan(c+dx))^{\frac{1}{2}(-n-1)}(a+ia\tan(c+dx))^n(e\sec(c+dx))^{1-n}\,\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{1-n}{2},\frac{3-}{2}\right.}{d(1-n)}$$

```
[In] Int[(e*Sec[c + d*x])^(1 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (I*2^((1 + n)/2)*Hypergeometric2F1[(1 - n)/2, (1 - n)/2, (3 - n)/2, (1 - I*
Tan[c + d*x])/2]*(e*Sec[c + d*x])^(1 - n)*(1 + I*Tan[c + d*x])^((-1 - n)/2)
*(a + I*a*Tan[c + d*x])^n)/(d*(1 - n))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \left((e\sec(c+dx))^{1-n}(a-ia\tan(c+dx))^{\frac{1}{2}(-1+n)}(a+ia\tan(c+dx))^{\frac{1}{2}(-1+n)}\right)\int (a-ia\tan(c+dx))^{\frac{1-n}{2}}(a+ia\tan(c+dx))^{\frac{1-n}{2}+n}\,dx$$

$$=\frac{\left(a^2(e\sec(c+dx))^{1-n}(a-ia\tan(c+dx))^{\frac{1}{2}(-1+n)}(a+ia\tan(c+dx))^{\frac{1}{2}(-1+n)}\right)\text{Subst}\left(\int (a-iax)^{-1}\right.}{d}$$

$$=\frac{\left(2^{-\frac{1}{2}+\frac{n}{2}}a(e\sec(c+dx))^{1-n}(a-ia\tan(c+dx))^{\frac{1}{2}(-1+n)}(a+ia\tan(c+dx))^{\frac{1}{2}+\frac{1}{2}(-1+n)+\frac{n}{2}}\left(\frac{a+ia\tan(c+}{a}\right.\right.}{d}$$

$$=\frac{i2^{\frac{1+n}{2}}\,\text{Hypergeometric2F1}\left(\frac{1-n}{2},\frac{1-n}{2},\frac{3-n}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{1-n}(1+i\tan(c+dx}{d(1-n)}$$

Mathematica [A] (verified)

Time = 5.84 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.74

$$\int (e\sec(c+dx))^{1-n}(a+ia\tan(c+dx))^n\,dx = -\frac{e(\text{Hypergeometric2F1}(1,n,1+n,i\cos(c+dx)-\sin(c+dx))-\text{Hypergeometric2F1}(1,n,1+n,-i\cos(c+dx)+\sin(c+dx))...}{dn}$$

```
[In] Integrate[(e*Sec[c + d*x])^(1 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] -((e*(Hypergeometric2F1[1, n, 1 + n, I*Cos[c + d*x] - Sin[c + d*x]] - Hyper
geometric2F1[1, n, 1 + n, (-I)*Cos[c + d*x] + Sin[c + d*x]])*(a + I*a*Tan[c
 + d*x])^n)/(d*n*(e*Sec[c + d*x])^n))
```

Maple [F]

$$\int (e\sec(dx+c))^{1-n}\,(a+ia\tan(dx+c))^n\,dx$$

```
[In] int((e*sec(d*x+c))^(1-n)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^(1-n)*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int (e\sec(c+dx))^{1-n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^{-n+1}\,(ia\tan(dx+c)+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(1-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-n + 1)*e^(I*d*n*
x + I*c*n + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1)) + n*log(a/
e)), x)
```

Sympy [F]

$$\int (e\sec(c+dx))^{1-n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(c+dx))^{1-n}\,(ia(\tan(c+dx)-i))^n\,dx$$

```
[In] integrate((e*sec(d*x+c))**(1-n)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Integral((e*sec(c + d*x))**(1 - n)*(I*a*(tan(c + d*x) - I))**n, x)
```

Maxima [F]

$$\int (e\sec(c+dx))^{1-n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^{-n+1}(i\,a\tan(dx+c)+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(1-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] -2*(a^n*e*cos(c*n + (d*n + d)*x + c) + I*a^n*e*sin(c*n + (d*n + d)*x + c) -
 2*(I*a^n*d*e^(n + 1)*n - I*a^n*d*e^(n + 1) + (I*a^n*d*e^(n + 1)*n - I*a^n*
d*e^(n + 1))*cos(2*d*x + 2*c) - (a^n*d*e^(n + 1)*n - a^n*d*e^(n + 1))*sin(2
*d*x + 2*c))*integrate(((cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*cos(c*n
 + (d*n + d)*x + c) + (sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(c*n + (d*
n + d)*x + c))/((e^n*n - e^n)*cos(4*d*x + 4*c)^2 + 4*(e^n*n - e^n)*cos(2*d*
x + 2*c)^2 + (e^n*n - e^n)*sin(4*d*x + 4*c)^2 + 4*(e^n*n - e^n)*sin(4*d*x +
 4*c)*sin(2*d*x + 2*c) + 4*(e^n*n - e^n)*sin(2*d*x + 2*c)^2 + e^n*n + 2*(e^
n*n + 2*(e^n*n - e^n)*cos(2*d*x + 2*c) - e^n)*cos(4*d*x + 4*c) + 4*(e^n*n -
 e^n)*cos(2*d*x + 2*c) - e^n), x) + 2*(a^n*d*e^(n + 1)*n - a^n*d*e^(n + 1)
+ (a^n*d*e^(n + 1)*n - a^n*d*e^(n + 1))*cos(2*d*x + 2*c) - (-I*a^n*d*e^(n +
 1)*n + I*a^n*d*e^(n + 1))*sin(2*d*x + 2*c))*integrate(-((sin(4*d*x + 4*c)
+ 2*sin(2*d*x + 2*c))*cos(c*n + (d*n + d)*x + c) - (cos(4*d*x + 4*c) + 2*co
s(2*d*x + 2*c) + 1)*sin(c*n + (d*n + d)*x + c))/((e^n*n - e^n)*cos(4*d*x +
4*c)^2 + 4*(e^n*n - e^n)*cos(2*d*x + 2*c)^2 + (e^n*n - e^n)*sin(4*d*x + 4*c
)^2 + 4*(e^n*n - e^n)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*(e^n*n - e^n)*s
in(2*d*x + 2*c)^2 + e^n*n + 2*(e^n*n + 2*(e^n*n - e^n)*cos(2*d*x + 2*c) - e
^n)*cos(4*d*x + 4*c) + 4*(e^n*n - e^n)*cos(2*d*x + 2*c) - e^n), x))/(-I*d*e
^n*n + I*d*e^n + (-I*d*e^n*n + I*d*e^n)*cos(2*d*x + 2*c) + (d*e^n*n - d*e^n
)*sin(2*d*x + 2*c))
```

Giac [F]

$$\int (e\sec(c+dx))^{1-n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^{-n+1}(i\,a\tan(dx+c)+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(1-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(-n + 1)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{1-n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int \left(\frac{e}{\cos\left(c+d\,x\right)}\right)^{1-n}\left(a+a\tan(c+d\,x)\ 1\mathrm{i}\right)^n\,dx$$

```
[In] int((e/cos(c + d*x))^(1 - n)*(a + a*tan(c + d*x)*1i)^n,x)
[Out] int((e/cos(c + d*x))^(1 - n)*(a + a*tan(c + d*x)*1i)^n, x)
```

3.489 $\int (e\sec(c+dx))^{2-n}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 30, antiderivative size = 113

$$\int (e\sec(c+dx))^{2-n}(a+ia\tan(c+dx))^n\,dx$$
$$=\frac{i2^{1+\frac{n}{2}}a\,\text{Hypergeometric2F1}\left(\frac{2-n}{2},-\frac{n}{2},\frac{4-n}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{2-n}(1+i\tan(c+dx))^{-n/2}(}{d(2-n)}$$

```
[Out] I*2^(1+1/2*n)*a*hypergeom([-1/2*n, 1-1/2*n],[2-1/2*n],1/2-1/2*I*tan(d*x+c))
*(e*sec(d*x+c))^(2-n)*(a+I*a*tan(d*x+c))^(-1+n)/d/(2-n)/((1+I*tan(d*x+c))^(
1/2*n))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3586, 3604, 72, 71}

$$\int (e\sec(c+dx))^{2-n}(a+ia\tan(c+dx))^n\,dx$$
$$=\frac{ia2^{\frac{n}{2}+1}(1+i\tan(c+dx))^{-n/2}(a+ia\tan(c+dx))^{n-1}(e\sec(c+dx))^{2-n}\,\text{Hypergeometric2F1}\left(\frac{2-n}{2},-\frac{n}{2},\frac{4-n}{2}\right.}{d(2-n)}$$

```
[In] Int[(e*Sec[c + d*x])^(2 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (I*2^(1 + n/2)*a*Hypergeometric2F1[(2 - n)/2, -1/2*n, (4 - n)/2, (1 - I*Tan
[c + d*x])/2]*(e*Sec[c + d*x])^(2 - n)*(a + I*a*Tan[c + d*x])^(-1 + n))/(d*
(2 - n)*(1 + I*Tan[c + d*x])^(n/2))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \left((e\sec(c+dx))^{2-n}(a-ia\tan(c+dx))^{\frac{1}{2}(-2+n)}(a+ia\tan(c+dx))^{\frac{1}{2}(-2+n)}\right)\int(a-ia\tan(c+dx))^{\frac{2-n}{2}}(a+ia\tan(c+dx))^{\frac{2-n}{2}+n}\,dx$$

$$= \frac{\left(a^2(e\sec(c+dx))^{2-n}(a-ia\tan(c+dx))^{\frac{1}{2}(-2+n)}(a+ia\tan(c+dx))^{\frac{1}{2}(-2+n)}\right)\text{Subst}\left(\int(a-iax)^{\cdots}\right.}{d}$$

$$= \frac{\left(2^{n/2}a^2(e\sec(c+dx))^{2-n}(a-ia\tan(c+dx))^{\frac{1}{2}(-2+n)}(a+ia\tan(c+dx))^{\frac{1}{2}(-2+n)+\frac{n}{2}}\left(\frac{a+ia\tan(c+dx)}{a}\cdots\right.\right.}{d}$$

$$= \frac{i2^{1+\frac{n}{2}}a\,\text{Hypergeometric2F1}\left(\frac{2-n}{2},-\frac{n}{2},\frac{4-n}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{2-n}(1+i\tan(c+\cdots}{d(2-n)}$$

Mathematica [A] (verified)

Time = 14.18 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.99

$$\int (e\sec(c+dx))^{2-n}(a+ia\tan(c+dx))^n\,dx = \frac{4e^2\,\text{Hypergeometric2F1}\left(2,1-\frac{n}{2},2-\frac{n}{2},-\cos(2(c+dx))+i\sin(2(c+dx))\right)(e\sec(c+dx))^{-n}(\cos(2c)-i}{d(-2+n)(-1-i\tan(dx))}$$

```
[In] Integrate[(e*Sec[c + d*x])^(2 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (4*e^2*Hypergeometric2F1[2, 1 - n/2, 2 - n/2, -Cos[2*(c + d*x)] + I*Sin[2*(
c + d*x)]]*(Cos[2*c] - I*Sin[2*c])*(I + Tan[d*x])*(a + I*a*Tan[c + d*x])^n)
/(d*(-2 + n)*(e*Sec[c + d*x])^n*(-1 - I*Tan[d*x]))
```

Maple [F]

$$\int \left(e\sec\left(dx+c\right)\right)^{2-n}\left(a+ia\tan\left(dx+c\right)\right)^{n}dx$$

```
[In] int((e*sec(d*x+c))^(2-n)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^(2-n)*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int (e\sec(c+dx))^{2-n}(a+ia\tan(c+dx))^n\,dx = \int \left(e\sec\left(dx+c\right)\right)^{-n+2}\left(i\,a\tan\left(dx+c\right)+a\right)^{n}dx$$

```
[In] integrate((e*sec(d*x+c))^(2-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] 1/2*((2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-n + 2)*(I*e^(2*I*d*x
 + 2*I*c) + I)*e^(I*d*n*x + I*c*n + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x +
 2*I*c) + 1)) + n*log(a/e)) + 2*d*e^(2*I*d*x + 2*I*c)*integral(1/2*(n*e^(2*
I*d*x + 2*I*c) + n)*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-n + 2
)*e^(I*d*n*x + I*c*n - 2*I*d*x + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*
I*c) + 1)) + n*log(a/e) - 2*I*c), x))*e^(-2*I*d*x - 2*I*c)/d
```

Sympy [F]

$$\int (e\sec(c+dx))^{2-n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec{(c+dx)})^{2-n}\,(ia(\tan{(c+dx)}-i))^n\;dx$$

```
[In] integrate((e*sec(d*x+c))**(2-n)*(a+I*a*tan(d*x+c))**n,x)
[Out] Integral((e*sec(c + d*x))**(2 - n)*(I*a*(tan(c + d*x) - I))**n, x)
```

Maxima [F]

$$\int (e\sec(c+dx))^{2-n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec{(dx+c)})^{-n+2}\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(2-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
[Out] 4*(4*a^n*e^2*cos(d*n*x + c*n) + 4*I*a^n*e^2*sin(d*n*x + c*n) - (a^n*e^2*n -
 4*a^n*e^2)*cos(c*n + (d*n + 2*d)*x + 2*c) - 4*(I*a^n*d*e^(n + 2)*n^3 - 6*I
*a^n*d*e^(n + 2)*n^2 + 8*I*a^n*d*e^(n + 2)*n + (I*a^n*d*e^(n + 2)*n^3 - 6*I
*a^n*d*e^(n + 2)*n^2 + 8*I*a^n*d*e^(n + 2)*n)*cos(4*d*x + 4*c) + 2*(I*a^n*d
*e^(n + 2)*n^3 - 6*I*a^n*d*e^(n + 2)*n^2 + 8*I*a^n*d*e^(n + 2)*n)*cos(2*d*x
 + 2*c) - (a^n*d*e^(n + 2)*n^3 - 6*a^n*d*e^(n + 2)*n^2 + 8*a^n*d*e^(n + 2)*
n)*sin(4*d*x + 4*c) - 2*(a^n*d*e^(n + 2)*n^3 - 6*a^n*d*e^(n + 2)*n^2 + 8*a^
n*d*e^(n + 2)*n)*sin(2*d*x + 2*c))*integrate(((cos(6*d*x + 6*c) + 3*cos(4*d
*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(d*n*x + c*n) + (sin(6*d*x + 6*c) +
3*sin(4*d*x + 4*c) + 3*sin(2*d*x + 2*c))*sin(d*n*x + c*n))/(e^n*n^2 + (e^n*
n^2 - 6*e^n*n + 8*e^n)*cos(6*d*x + 6*c)^2 + 9*(e^n*n^2 - 6*e^n*n + 8*e^n)*c
os(4*d*x + 4*c)^2 + 9*(e^n*n^2 - 6*e^n*n + 8*e^n)*cos(2*d*x + 2*c)^2 + (e^n
*n^2 - 6*e^n*n + 8*e^n)*sin(6*d*x + 6*c)^2 + 9*(e^n*n^2 - 6*e^n*n + 8*e^n)*
sin(4*d*x + 4*c)^2 + 18*(e^n*n^2 - 6*e^n*n + 8*e^n)*sin(4*d*x + 4*c)*sin(2*
d*x + 2*c) + 9*(e^n*n^2 - 6*e^n*n + 8*e^n)*sin(2*d*x + 2*c)^2 - 6*e^n*n + 2
*(e^n*n^2 - 6*e^n*n + 3*(e^n*n^2 - 6*e^n*n + 8*e^n)*cos(4*d*x + 4*c) + 3*(e
^n*n^2 - 6*e^n*n + 8*e^n)*cos(2*d*x + 2*c) + 8*e^n)*cos(6*d*x + 6*c) + 6*(e
^n*n^2 - 6*e^n*n + 3*(e^n*n^2 - 6*e^n*n + 8*e^n)*cos(2*d*x + 2*c) + 8*e^n)*
cos(4*d*x + 4*c) + 6*(e^n*n^2 - 6*e^n*n + 8*e^n)*cos(2*d*x + 2*c) + 6*((e^n
*n^2 - 6*e^n*n + 8*e^n)*sin(4*d*x + 4*c) + (e^n*n^2 - 6*e^n*n + 8*e^n)*sin(
2*d*x + 2*c))*sin(6*d*x + 6*c) + 8*e^n), x) + 4*(a^n*d*e^(n + 2)*n^3 - 6*a^
n*d*e^(n + 2)*n^2 + 8*a^n*d*e^(n + 2)*n + (a^n*d*e^(n + 2)*n^3 - 6*a^n*d*e^
(n + 2)*n^2 + 8*a^n*d*e^(n + 2)*n)*cos(4*d*x + 4*c) + 2*(a^n*d*e^(n + 2)*n^
3 - 6*a^n*d*e^(n + 2)*n^2 + 8*a^n*d*e^(n + 2)*n)*cos(2*d*x + 2*c) - (-I*a^n
*d*e^(n + 2)*n^3 + 6*I*a^n*d*e^(n + 2)*n^2 - 8*I*a^n*d*e^(n + 2)*n)*sin(4*d
*x + 4*c) - 2*(-I*a^n*d*e^(n + 2)*n^3 + 6*I*a^n*d*e^(n + 2)*n^2 - 8*I*a^n*d
*e^(n + 2)*n)*sin(2*d*x + 2*c))*integrate(-((sin(6*d*x + 6*c) + 3*sin(4*d*x
```

```
 + 4*c) + 3*sin(2*d*x + 2*c))*cos(d*n*x + c*n) - (cos(6*d*x + 6*c) + 3*cos(
4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*sin(d*n*x + c*n))/(e^n*n^2 + (e^n*n^
2 - 6*e^n*n + 8*e^n)*cos(6*d*x + 6*c)^2 + 9*(e^n*n^2 - 6*e^n*n + 8*e^n)*cos
(4*d*x + 4*c)^2 + 9*(e^n*n^2 - 6*e^n*n + 8*e^n)*cos(2*d*x + 2*c)^2 + (e^n*n
^2 - 6*e^n*n + 8*e^n)*sin(6*d*x + 6*c)^2 + 9*(e^n*n^2 - 6*e^n*n + 8*e^n)*si
n(4*d*x + 4*c)^2 + 18*(e^n*n^2 - 6*e^n*n + 8*e^n)*sin(4*d*x + 4*c)*sin(2*d*
x + 2*c) + 9*(e^n*n^2 - 6*e^n*n + 8*e^n)*sin(2*d*x + 2*c)^2 - 6*e^n*n + 2*(
e^n*n^2 - 6*e^n*n + 3*(e^n*n^2 - 6*e^n*n + 8*e^n)*cos(4*d*x + 4*c) + 3*(e^n
*n^2 - 6*e^n*n + 8*e^n)*cos(2*d*x + 2*c) + 8*e^n)*cos(6*d*x + 6*c) + 6*(e^n
*n^2 - 6*e^n*n + 3*(e^n*n^2 - 6*e^n*n + 8*e^n)*cos(2*d*x + 2*c) + 8*e^n)*co
s(4*d*x + 4*c) + 6*(e^n*n^2 - 6*e^n*n + 8*e^n)*cos(2*d*x + 2*c) + 6*((e^n*n
^2 - 6*e^n*n + 8*e^n)*sin(4*d*x + 4*c) + (e^n*n^2 - 6*e^n*n + 8*e^n)*sin(2*
d*x + 2*c))*sin(6*d*x + 6*c) + 8*e^n), x) - (I*a^n*e^2*n - 4*I*a^n*e^2)*sin
(c*n + (d*n + 2*d)*x + 2*c))/(-I*d*e^n*n^2 + 6*I*d*e^n*n - 8*I*d*e^n + (-I*
d*e^n*n^2 + 6*I*d*e^n*n - 8*I*d*e^n)*cos(4*d*x + 4*c) - 2*(I*d*e^n*n^2 - 6*
I*d*e^n*n + 8*I*d*e^n)*cos(2*d*x + 2*c) + (d*e^n*n^2 - 6*d*e^n*n + 8*d*e^n)
*sin(4*d*x + 4*c) + 2*(d*e^n*n^2 - 6*d*e^n*n + 8*d*e^n)*sin(2*d*x + 2*c))
```

Giac [F]

$$\int (e\sec(c+dx))^{2-n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^{-n+2}\,(i\,a\tan(dx+c)+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(2-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(-n + 2)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{2-n}(a+ia\tan(c+dx))^n\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^{2-n}\,(a+a\tan(c+d\,x)\;1\mathrm{i})^n\,dx$$

```
[In] int((e/cos(c + d*x))^(2 - n)*(a + a*tan(c + d*x)*1i)^n,x)
[Out] int((e/cos(c + d*x))^(2 - n)*(a + a*tan(c + d*x)*1i)^n, x)
```

3.490 $\int (e\sec(c+dx))^{3-n}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 30, antiderivative size = 121

$$\int (e\sec(c+dx))^{3-n}(a+ia\tan(c+dx))^n\,dx = \frac{i2^{\frac{3+n}{2}}a\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-1-n),\frac{3-n}{2},\frac{5-n}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{3-n}(1+i\tan(c+a$$

$$d(3-n)$$

```
[Out] I*2^(3/2+1/2*n)*a*hypergeom([3/2-1/2*n, -1/2-1/2*n],[5/2-1/2*n],1/2-1/2*I*t
an(d*x+c))*(e*sec(d*x+c))^(3-n)*(1+I*tan(d*x+c))^(-1/2-1/2*n)*(a+I*a*tan(d*
x+c))^(-1+n)/d/(3-n)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3586, 3604, 72, 71}

$$\int (e\sec(c+dx))^{3-n}(a+ia\tan(c+dx))^n\,dx = \frac{ia2^{\frac{n+3}{2}}(1+i\tan(c+dx))^{\frac{1}{2}(-n-1)}(a+ia\tan(c+dx))^{n-1}(e\sec(c+dx))^{3-n}\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-n-\right.$$

$$d(3-n)$$

```
[In] Int[(e*Sec[c + d*x])^(3 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (I*2^((3 + n)/2)*a*Hypergeometric2F1[(-1 - n)/2, (3 - n)/2, (5 - n)/2, (1 -
 I*Tan[c + d*x])/2]*(e*Sec[c + d*x])^(3 - n)*(1 + I*Tan[c + d*x])^((-1 - n)
/2)*(a + I*a*Tan[c + d*x])^(-1 + n))/(d*(3 - n))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \left((e\sec(c+dx))^{3-n}(a-ia\tan(c+dx))^{\frac{1}{2}(-3+n)}(a+ia\tan(c+dx))^{\frac{1}{2}(-3+n)}\right)\int(a-ia\tan(c+dx))^{\frac{3-n}{2}}(a+ia\tan(c+dx))^{\frac{3-n}{2}+n}\,dx$$

$$=\frac{\left(a^2(e\sec(c+dx))^{3-n}(a-ia\tan(c+dx))^{\frac{1}{2}(-3+n)}(a+ia\tan(c+dx))^{\frac{1}{2}(-3+n)}\right)\text{Subst}\left(\int(a-iax)^{-1}\right.}{d}$$

$$=\frac{\left(2^{\frac{1}{2}+\frac{n}{2}}a^2(e\sec(c+dx))^{3-n}(a-ia\tan(c+dx))^{\frac{1}{2}(-3+n)}(a+ia\tan(c+dx))^{\frac{1}{2}+\frac{1}{2}(-3+n)+\frac{n}{2}}\left(\frac{a+ia\tan(c+}{a}\right.\right.}{d}$$

$$=\frac{i2^{\frac{3+n}{2}}a\,\text{Hypergeometric2F1}\left(\frac{1}{2}(-1-n),\frac{3-n}{2},\frac{5-n}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{3-n}(1+i\tan}{d(3-n)}$$

Mathematica [A] (verified)

Time = 17.56 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.96

$$\int (e\sec(c+dx))^{3-n}(a+ia\tan(c+dx))^n\,dx$$
$$=\frac{8e^3\,\text{Hypergeometric2F1}\left(3,\frac{3-n}{2},\frac{5-n}{2},-\cos(2(c+dx))+i\sin(2(c+dx))\right)\sec(dx)(e\sec(c+dx))^{-n}(i+\text{ta}}{d(-3+n)(\cos(c)+i\sin(c))^3(-i+\tan(dx))^2}$$

```
[In] Integrate[(e*Sec[c + d*x])^(3 - n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (8*e^3*Hypergeometric2F1[3, (3 - n)/2, (5 - n)/2, -Cos[2*(c + d*x)] + I*Sin
[2*(c + d*x)]]*Sec[d*x]*(I + Tan[d*x])*(a + I*a*Tan[c + d*x])^n)/(d*(-3 + n
)*(e*Sec[c + d*x])^n*(Cos[c] + I*Sin[c])^3*(-I + Tan[d*x])^2)
```

Maple [F]

$$\int (e\sec(dx+c))^{3-n}(a+ia\tan(dx+c))^n\,dx$$

```
[In] int((e*sec(d*x+c))^(3-n)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^(3-n)*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int (e\sec(c+dx))^{3-n}(a+ia\tan(c+dx))^n\,dx=\int (e\sec(dx+c))^{-n+3}(i\,a\tan(dx+c)+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(3-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] 1/8*(((-I*n - I)*e^(4*I*d*x + 4*I*c) - 2*I*n*e^(2*I*d*x + 2*I*c) - I*n + I)
*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-n + 3)*e^(I*d*n*x + I*c*
n + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1)) + n*log(a/e)) + 8*
d*e^(2*I*d*x + 2*I*c)*integral(-1/8*(n^2 + (n^2 - 1)*e^(4*I*d*x + 4*I*c) +
2*(n^2 - 1)*e^(2*I*d*x + 2*I*c) - 1)*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I
*c) + 1))^(-n + 3)*e^(I*d*n*x + I*c*n - 2*I*d*x + n*log(2*e*e^(I*d*x + I*c)
/(e^(2*I*d*x + 2*I*c) + 1)) + n*log(a/e) - 2*I*c), x))*e^(-2*I*d*x - 2*I*c)
/d
```

Sympy [F]

$$\int (e\sec(c+dx))^{3-n}(a+ia\tan(c+dx))^n\,dx=\int (e\sec{(c+dx)})^{3-n}\,(ia(\tan{(c+dx)}-i))^n\,dx$$

```
[In] integrate((e*sec(d*x+c))**(3-n)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Integral((e*sec(c + d*x))**(3 - n)*(I*a*(tan(c + d*x) - I))**n, x)
```

Maxima [F]

$$\int (e\sec(c+dx))^{3-n}(a+ia\tan(c+dx))^n\,dx=\int (e\sec{(dx+c)})^{-n+3}\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(3-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] 8*(6*a^n*e^3*cos(c*n + (d*n + d)*x + c) + 6*I*a^n*e^3*sin(c*n + (d*n + d)*x
 + c) - (a^n*e^3*n - 5*a^n*e^3)*cos(c*n + (d*n + 3*d)*x + 3*c) - 6*((I*a^n*
d*e^(n + 3)*n^3 - 7*I*a^n*d*e^(n + 3)*n^2 + 7*I*a^n*d*e^(n + 3)*n + 15*I*a^
n*d*e^(n + 3))*cos(c*n) + ((I*a^n*d*e^(n + 3)*n^3 - 7*I*a^n*d*e^(n + 3)*n^2
 + 7*I*a^n*d*e^(n + 3)*n + 15*I*a^n*d*e^(n + 3))*cos(c*n) - (a^n*d*e^(n + 3
)*n^3 - 7*a^n*d*e^(n + 3)*n^2 + 7*a^n*d*e^(n + 3)*n + 15*a^n*d*e^(n + 3))*s
in(c*n))*cos(6*d*x + 6*c) + 3*((I*a^n*d*e^(n + 3)*n^3 - 7*I*a^n*d*e^(n + 3)
*n^2 + 7*I*a^n*d*e^(n + 3)*n + 15*I*a^n*d*e^(n + 3))*cos(c*n) - (a^n*d*e^(n
 + 3)*n^3 - 7*a^n*d*e^(n + 3)*n^2 + 7*a^n*d*e^(n + 3)*n + 15*a^n*d*e^(n + 3
))*sin(c*n))*cos(4*d*x + 4*c) + 3*((I*a^n*d*e^(n + 3)*n^3 - 7*I*a^n*d*e^(n
+ 3)*n^2 + 7*I*a^n*d*e^(n + 3)*n + 15*I*a^n*d*e^(n + 3))*cos(c*n) - (a^n*d*
e^(n + 3)*n^3 - 7*a^n*d*e^(n + 3)*n^2 + 7*a^n*d*e^(n + 3)*n + 15*a^n*d*e^(n
 + 3))*sin(c*n))*cos(2*d*x + 2*c) - (a^n*d*e^(n + 3)*n^3 - 7*a^n*d*e^(n + 3
)*n^2 + 7*a^n*d*e^(n + 3)*n + 15*a^n*d*e^(n + 3))*sin(c*n) - ((a^n*d*e^(n +
 3)*n^3 - 7*a^n*d*e^(n + 3)*n^2 + 7*a^n*d*e^(n + 3)*n + 15*a^n*d*e^(n + 3))
*cos(c*n) - (-I*a^n*d*e^(n + 3)*n^3 + 7*I*a^n*d*e^(n + 3)*n^2 - 7*I*a^n*d*e
^(n + 3)*n - 15*I*a^n*d*e^(n + 3))*sin(c*n))*sin(6*d*x + 6*c) - 3*((a^n*d*e
^(n + 3)*n^3 - 7*a^n*d*e^(n + 3)*n^2 + 7*a^n*d*e^(n + 3)*n + 15*a^n*d*e^(n
+ 3))*cos(c*n) - (-I*a^n*d*e^(n + 3)*n^3 + 7*I*a^n*d*e^(n + 3)*n^2 - 7*I*a^
n*d*e^(n + 3)*n - 15*I*a^n*d*e^(n + 3))*sin(c*n))*sin(4*d*x + 4*c) - 3*((a^
n*d*e^(n + 3)*n^3 - 7*a^n*d*e^(n + 3)*n^2 + 7*a^n*d*e^(n + 3)*n + 15*a^n*d*
e^(n + 3))*cos(c*n) - (-I*a^n*d*e^(n + 3)*n^3 + 7*I*a^n*d*e^(n + 3)*n^2 - 7
*I*a^n*d*e^(n + 3)*n - 15*I*a^n*d*e^(n + 3))*sin(c*n))*sin(2*d*x + 2*c))*in
tegrate(((cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*co
s(2*d*x + 2*c) + 1)*cos((d*n + d)*x + c) + (sin(8*d*x + 8*c) + 4*sin(6*d*x
+ 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*sin((d*n + d)*x + c))/(e^
n*n^2 + (e^n*n^2 - 8*e^n*n + 15*e^n)*cos(8*d*x + 8*c)^2 + 16*(e^n*n^2 - 8*e
```

```
^n*n + 15*e^n)*cos(6*d*x + 6*c)^2 + 36*(e^n*n^2 - 8*e^n*n + 15*e^n)*cos(4*d
*x + 4*c)^2 + 16*(e^n*n^2 - 8*e^n*n + 15*e^n)*cos(2*d*x + 2*c)^2 + (e^n*n^2
 - 8*e^n*n + 15*e^n)*sin(8*d*x + 8*c)^2 + 16*(e^n*n^2 - 8*e^n*n + 15*e^n)*s
in(6*d*x + 6*c)^2 + 36*(e^n*n^2 - 8*e^n*n + 15*e^n)*sin(4*d*x + 4*c)^2 + 48
*(e^n*n^2 - 8*e^n*n + 15*e^n)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(e^n*n
^2 - 8*e^n*n + 15*e^n)*sin(2*d*x + 2*c)^2 - 8*e^n*n + 2*(e^n*n^2 - 8*e^n*n
+ 4*(e^n*n^2 - 8*e^n*n + 15*e^n)*cos(6*d*x + 6*c) + 6*(e^n*n^2 - 8*e^n*n +
15*e^n)*cos(4*d*x + 4*c) + 4*(e^n*n^2 - 8*e^n*n + 15*e^n)*cos(2*d*x + 2*c)
+ 15*e^n)*cos(8*d*x + 8*c) + 8*(e^n*n^2 - 8*e^n*n + 6*(e^n*n^2 - 8*e^n*n +
15*e^n)*cos(4*d*x + 4*c) + 4*(e^n*n^2 - 8*e^n*n + 15*e^n)*cos(2*d*x + 2*c)
+ 15*e^n)*cos(6*d*x + 6*c) + 12*(e^n*n^2 - 8*e^n*n + 4*(e^n*n^2 - 8*e^n*n +
 15*e^n)*cos(2*d*x + 2*c) + 15*e^n)*cos(4*d*x + 4*c) + 8*(e^n*n^2 - 8*e^n*n
 + 15*e^n)*cos(2*d*x + 2*c) + 4*(2*(e^n*n^2 - 8*e^n*n + 15*e^n)*sin(6*d*x +
 6*c) + 3*(e^n*n^2 - 8*e^n*n + 15*e^n)*sin(4*d*x + 4*c) + 2*(e^n*n^2 - 8*e^
n*n + 15*e^n)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*(e^n*n^2 - 8*e^n*n
 + 15*e^n)*sin(4*d*x + 4*c) + 2*(e^n*n^2 - 8*e^n*n + 15*e^n)*sin(2*d*x + 2*
c))*sin(6*d*x + 6*c) + 15*e^n), x) + 6*((a^n*d*e^(n + 3)*n^3 - 7*a^n*d*e^(n
 + 3)*n^2 + 7*a^n*d*e^(n + 3)*n + 15*a^n*d*e^(n + 3))*cos(c*n) + ((a^n*d*e^
(n + 3)*n^3 - 7*a^n*d*e^(n + 3)*n^2 + 7*a^n*d*e^(n + 3)*n + 15*a^n*d*e^(n +
 3))*cos(c*n) - (-I*a^n*d*e^(n + 3)*n^3 + 7*I*a^n*d*e^(n + 3)*n^2 - 7*I*a^n
*d*e^(n + 3)*n - 15*I*a^n*d*e^(n + 3))*sin(c*n))*cos(6*d*x + 6*c) + 3*((a^n
*d*e^(n + 3)*n^3 - 7*a^n*d*e^(n + 3)*n^2 + 7*a^n*d*e^(n + 3)*n + 15*a^n*d*e
^(n + 3))*cos(c*n) - (-I*a^n*d*e^(n + 3)*n^3 + 7*I*a^n*d*e^(n + 3)*n^2 - 7*
I*a^n*d*e^(n + 3)*n - 15*I*a^n*d*e^(n + 3))*sin(c*n))*cos(4*d*x + 4*c) + 3*
((a^n*d*e^(n + 3)*n^3 - 7*a^n*d*e^(n + 3)*n^2 + 7*a^n*d*e^(n + 3)*n + 15*a^
n*d*e^(n + 3))*cos(c*n) - (-I*a^n*d*e^(n + 3)*n^3 + 7*I*a^n*d*e^(n + 3)*n^2
 - 7*I*a^n*d*e^(n + 3)*n - 15*I*a^n*d*e^(n + 3))*sin(c*n))*cos(2*d*x + 2*c)
 - (-I*a^n*d*e^(n + 3)*n^3 + 7*I*a^n*d*e^(n + 3)*n^2 - 7*I*a^n*d*e^(n + 3)*
n - 15*I*a^n*d*e^(n + 3))*sin(c*n) - ((-I*a^n*d*e^(n + 3)*n^3 + 7*I*a^n*d*e
^(n + 3)*n^2 - 7*I*a^n*d*e^(n + 3)*n - 15*I*a^n*d*e^(n + 3))*cos(c*n) + (a^
n*d*e^(n + 3)*n^3 - 7*a^n*d*e^(n + 3)*n^2 + 7*a^n*d*e^(n + 3)*n + 15*a^n*d*
e^(n + 3))*sin(c*n))*sin(6*d*x + 6*c) - 3*((-I*a^n*d*e^(n + 3)*n^3 + 7*I*a^
n*d*e^(n + 3)*n^2 - 7*I*a^n*d*e^(n + 3)*n - 15*I*a^n*d*e^(n + 3))*cos(c*n)
+ (a^n*d*e^(n + 3)*n^3 - 7*a^n*d*e^(n + 3)*n^2 + 7*a^n*d*e^(n + 3)*n + 15*a
^n*d*e^(n + 3))*sin(c*n))*sin(4*d*x + 4*c) - 3*((-I*a^n*d*e^(n + 3)*n^3 + 7
*I*a^n*d*e^(n + 3)*n^2 - 7*I*a^n*d*e^(n + 3)*n - 15*I*a^n*d*e^(n + 3))*cos(
c*n) + (a^n*d*e^(n + 3)*n^3 - 7*a^n*d*e^(n + 3)*n^2 + 7*a^n*d*e^(n + 3)*n +
 15*a^n*d*e^(n + 3))*sin(c*n))*sin(2*d*x + 2*c))*integrate(-((sin(8*d*x + 8
*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos((d*
n + d)*x + c) - (cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c)
 + 4*cos(2*d*x + 2*c) + 1)*sin((d*n + d)*x + c))/(e^n*n^2 + (e^n*n^2 - 8*e^
n*n + 15*e^n)*cos(8*d*x + 8*c)^2 + 16*(e^n*n^2 - 8*e^n*n + 15*e^n)*cos(6*d*
x + 6*c)^2 + 36*(e^n*n^2 - 8*e^n*n + 15*e^n)*cos(4*d*x + 4*c)^2 + 16*(e^n*n
^2 - 8*e^n*n + 15*e^n)*cos(2*d*x + 2*c)^2 + (e^n*n^2 - 8*e^n*n + 15*e^n)*si
n(8*d*x + 8*c)^2 + 16*(e^n*n^2 - 8*e^n*n + 15*e^n)*sin(6*d*x + 6*c)^2 + 36*
```

```
(e^n*n^2 - 8*e^n*n + 15*e^n)*sin(4*d*x + 4*c)^2 + 48*(e^n*n^2 - 8*e^n*n + 1
5*e^n)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(e^n*n^2 - 8*e^n*n + 15*e^n)*
sin(2*d*x + 2*c)^2 - 8*e^n*n + 2*(e^n*n^2 - 8*e^n*n + 4*(e^n*n^2 - 8*e^n*n
+ 15*e^n)*cos(6*d*x + 6*c) + 6*(e^n*n^2 - 8*e^n*n + 15*e^n)*cos(4*d*x + 4*c
) + 4*(e^n*n^2 - 8*e^n*n + 15*e^n)*cos(2*d*x + 2*c) + 15*e^n)*cos(8*d*x + 8
*c) + 8*(e^n*n^2 - 8*e^n*n + 6*(e^n*n^2 - 8*e^n*n + 15*e^n)*cos(4*d*x + 4*c
) + 4*(e^n*n^2 - 8*e^n*n + 15*e^n)*cos(2*d*x + 2*c) + 15*e^n)*cos(6*d*x + 6
*c) + 12*(e^n*n^2 - 8*e^n*n + 4*(e^n*n^2 - 8*e^n*n + 15*e^n)*cos(2*d*x + 2*
c) + 15*e^n)*cos(4*d*x + 4*c) + 8*(e^n*n^2 - 8*e^n*n + 15*e^n)*cos(2*d*x +
2*c) + 4*(2*(e^n*n^2 - 8*e^n*n + 15*e^n)*sin(6*d*x + 6*c) + 3*(e^n*n^2 - 8*
e^n*n + 15*e^n)*sin(4*d*x + 4*c) + 2*(e^n*n^2 - 8*e^n*n + 15*e^n)*sin(2*d*x
 + 2*c))*sin(8*d*x + 8*c) + 16*(3*(e^n*n^2 - 8*e^n*n + 15*e^n)*sin(4*d*x +
4*c) + 2*(e^n*n^2 - 8*e^n*n + 15*e^n)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) +
15*e^n), x) - (I*a^n*e^3*n - 5*I*a^n*e^3)*sin(c*n + (d*n + 3*d)*x + 3*c))/(
-I*d*e^n*n^2 + 8*I*d*e^n*n - 15*I*d*e^n + (-I*d*e^n*n^2 + 8*I*d*e^n*n - 15*
I*d*e^n)*cos(6*d*x + 6*c) - 3*(I*d*e^n*n^2 - 8*I*d*e^n*n + 15*I*d*e^n)*cos(
4*d*x + 4*c) - 3*(I*d*e^n*n^2 - 8*I*d*e^n*n + 15*I*d*e^n)*cos(2*d*x + 2*c)
+ (d*e^n*n^2 - 8*d*e^n*n + 15*d*e^n)*sin(6*d*x + 6*c) + 3*(d*e^n*n^2 - 8*d*
e^n*n + 15*d*e^n)*sin(4*d*x + 4*c) + 3*(d*e^n*n^2 - 8*d*e^n*n + 15*d*e^n)*s
in(2*d*x + 2*c))
```

Giac [F]

$$\int (e\sec(c+dx))^{3-n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^{-n+3}(i\,a\tan(dx+c)+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(3-n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(-n + 3)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{3-n}(a+ia\tan(c+dx))^n\,dx = \int \left(\frac{e}{\cos(c+d\,x)}\right)^{3-n}(a+a\tan(c+d\,x)\;1\mathrm{i})^n\,dx$$

```
[In] int((e/cos(c + d*x))^(3 - n)*(a + a*tan(c + d*x)*1i)^n,x)
[Out] int((e/cos(c + d*x))^(3 - n)*(a + a*tan(c + d*x)*1i)^n, x)
```

3.491 $\int (e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 30, antiderivative size = 156

$$\int (e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^n\,dx = \frac{8ia^3(e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^{-3+n}}{d(5-n)\,(12-7n+n^2)} + \frac{4ia^2(e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^{-2+n}}{d\,(20-9n+n^2)} + \frac{ia(e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^{-1+n}}{d(5-n)}$$

```
[Out] 8*I*a^3*(e*sec(d*x+c))^(6-2*n)*(a+I*a*tan(d*x+c))^(-3+n)/d/(-n^3+12*n^2-47*
n+60)+4*I*a^2*(e*sec(d*x+c))^(6-2*n)*(a+I*a*tan(d*x+c))^(-2+n)/d/(n^2-9*n+2
0)+I*a*(e*sec(d*x+c))^(6-2*n)*(a+I*a*tan(d*x+c))^(-1+n)/d/(5-n)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3575, 3574}

$$\int (e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^n\,dx = \frac{8ia^3(a+ia\tan(c+dx))^{n-3}(e\sec(c+dx))^{6-2n}}{d(5-n)\,(n^2-7n+12)} + \frac{4ia^2(a+ia\tan(c+dx))^{n-2}(e\sec(c+dx))^{6-2n}}{d\,(n^2-9n+20)} + \frac{ia(a+ia\tan(c+dx))^{n-1}(e\sec(c+dx))^{6-2n}}{d(5-n)}$$

```
[In] Int[(e*Sec[c + d*x])^(6 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((8*I)*a^3*(e*Sec[c + d*x])^(6 - 2*n)*(a + I*a*Tan[c + d*x])^(-3 + n))/(d*(
5 - n)*(12 - 7*n + n^2)) + ((4*I)*a^2*(e*Sec[c + d*x])^(6 - 2*n)*(a + I*a*T
an[c + d*x])^(-2 + n))/(d*(20 - 9*n + n^2)) + (I*a*(e*Sec[c + d*x])^(6 - 2*
n)*(a + I*a*Tan[c + d*x])^(-1 + n))/(d*(5 - n))
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{ia(e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^{-1+n}}{d(5-n)} \\
&\quad + \frac{(4a)\int(e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^{-1+n}\,dx}{5-n} \\
&= \frac{4ia^2(e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^{-2+n}}{d\left(20-9n+n^2\right)} \\
&\quad + \frac{ia(e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^{-1+n}}{d(5-n)} \\
&\quad + \frac{(8a^2)\int(e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^{-2+n}\,dx}{20-9n+n^2} \\
&= \frac{8ia^3(e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^{-3+n}}{d(3-n)\left(20-9n+n^2\right)} \\
&\quad + \frac{4ia^2(e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^{-2+n}}{d\left(20-9n+n^2\right)} \\
&\quad + \frac{ia(e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^{-1+n}}{d(5-n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 3.58 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.78

$$\int (e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^n\,dx = -\frac{e^6\sec^5(c+dx)(e\sec(c+dx))^{-2n}\left(-2(-5+n)+(22-9n+n^2)\cos(2(c+dx))+i(18-9n+n^2)\sin(2$$
$$d(-5+n)(-4+n)(-3+n)$$

```
[In] Integrate[(e*Sec[c + d*x])^(6 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] -((e^6*Sec[c + d*x]^5*(-2*(-5 + n) + (22 - 9*n + n^2)*Cos[2*(c + d*x)] + I*
(18 - 9*n + n^2)*Sin[2*(c + d*x)])*(I*Cos[3*(c + d*x)] + Sin[3*(c + d*x)])*
(a + I*a*Tan[c + d*x])^n)/(d*(-5 + n)*(-4 + n)*(-3 + n)*(e*Sec[c + d*x])^(2
*n)))
```

Maple [F]

$$\int (e\sec(dx+c))^{6-2n}(a+ia\tan(dx+c))^n\,dx$$

```
[In] int((e*sec(d*x+c))^(6-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^(6-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.06

$$\int (e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^n\,dx$$
$$= \frac{\left((-i\,n^2+9i\,n-20i)e^{(6i\,dx+6i\,c)}+(-i\,n^2+11i\,n-30i)e^{(4i\,dx+4i\,c)}-2\,(-i\,n+6i)e^{(2i\,dx+2i\,c)}-2i\right)\left(\frac{2\,ee^{(i\,dx}}{e^{(2i\,dx+2}}\right.}{2\,(dn^3-12\,dn^2+47\,dn-60\,d)}$$

```
[In] integrate((e*sec(d*x+c))^(6-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas"
)
```

```
[Out] 1/2*((-I*n^2 + 9*I*n - 20*I)*e^(6*I*d*x + 6*I*c) + (-I*n^2 + 11*I*n - 30*I)
*e^(4*I*d*x + 4*I*c) - 2*(-I*n + 6*I)*e^(2*I*d*x + 2*I*c) - 2*I)*(2*e*e^(I*
d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n + 6)*e^(I*d*n*x + I*c*n - 6*I*d
*x + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1)) + n*log(a/e) - 6*
I*c)/(d*n^3 - 12*d*n^2 + 47*d*n - 60*d)
```

Sympy [F]

$$\int (e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec{(c+dx)})^{6-2n}\,(ia(\tan{(c+dx)}-i))^n\;dx$$

```
[In] integrate((e*sec(d*x+c))**(6-2*n)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Integral((e*sec(c + d*x))**(6 - 2*n)*(I*a*(tan(c + d*x) - I))**n, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1067 vs. 2(140) = 280.

Time = 1.37 (sec) , antiderivative size = 1067, normalized size of antiderivative = 6.84

$$\int (e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^n\,dx = \text{Too large to display}$$

```
[In] integrate((e*sec(d*x+c))^(6-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima"
)
```

```
[Out] -32*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/2*n)*a^n*e^6*cos(n*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 2*I
*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/2*n)
*a^n*e^6*sin(n*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (a^n*e^6*
n^2 - 9*a^n*e^6*n + 20*a^n*e^6)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/2*n)*cos(4*d*x + n*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1) + 4*c) - 2*(a^n*e^6*n - 5*a^n*e^6)*(cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/2*n)*cos(2*d*x + n*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1) + 2*c) - (-I*a^n*e^6*n^2 + 9*I*a^n
*e^6*n - 20*I*a^n*e^6)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/2*n)*sin(4*d*x + n*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1) + 4*c) - 2*(I*a^n*e^6*n - 5*I*a^n*e^6)*(cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/2*n)*sin(2*d*x + n*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1) + 2*c))/(((-I*e^(2*n)*n^3 + 12*I*e^(2*n
)*n^2 - 47*I*e^(2*n)*n + 60*I*e^(2*n))*2^n*cos(10*d*x + 10*c) - 5*(I*e^(2*n
)*n^3 - 12*I*e^(2*n)*n^2 + 47*I*e^(2*n)*n - 60*I*e^(2*n))*2^n*cos(8*d*x + 8
*c) - 10*(I*e^(2*n)*n^3 - 12*I*e^(2*n)*n^2 + 47*I*e^(2*n)*n - 60*I*e^(2*n))
*2^n*cos(6*d*x + 6*c) - 10*(I*e^(2*n)*n^3 - 12*I*e^(2*n)*n^2 + 47*I*e^(2*n)
*n - 60*I*e^(2*n))*2^n*cos(4*d*x + 4*c) - 5*(I*e^(2*n)*n^3 - 12*I*e^(2*n)*n
^2 + 47*I*e^(2*n)*n - 60*I*e^(2*n))*2^n*cos(2*d*x + 2*c) + (e^(2*n)*n^3 - 1
2*e^(2*n)*n^2 + 47*e^(2*n)*n - 60*e^(2*n))*2^n*sin(10*d*x + 10*c) + 5*(e^(2
*n)*n^3 - 12*e^(2*n)*n^2 + 47*e^(2*n)*n - 60*e^(2*n))*2^n*sin(8*d*x + 8*c)
+ 10*(e^(2*n)*n^3 - 12*e^(2*n)*n^2 + 47*e^(2*n)*n - 60*e^(2*n))*2^n*sin(6*d
```

```
*x + 6*c) + 10*(e^(2*n)*n^3 - 12*e^(2*n)*n^2 + 47*e^(2*n)*n - 60*e^(2*n))*2
^n*sin(4*d*x + 4*c) + 5*(e^(2*n)*n^3 - 12*e^(2*n)*n^2 + 47*e^(2*n)*n - 60*e
^(2*n))*2^n*sin(2*d*x + 2*c) + (-I*e^(2*n)*n^3 + 12*I*e^(2*n)*n^2 - 47*I*e^
(2*n)*n + 60*I*e^(2*n))*2^n)*d)
```

Giac [F]

$$\int (e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^{-2n+6}(ia\tan(dx+c)+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(6-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(-2*n + 6)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [B] (verification not implemented)

Time = 11.81 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.04

$$\int (e\sec(c+dx))^{6-2n}(a+ia\tan(c+dx))^n\,dx = (\cos(6c+6dx) - \sin(6c+6dx)\ 1\mathrm{i})\left(\frac{e}{\cos(c+dx)}\right)^{6-2n}\left(\frac{\left(a+\frac{a\sin(c+dx)\,1\mathrm{i}}{\cos(c+dx)}\right)^n}{d\,(n^3\,1\mathrm{i}-n^2\,12\mathrm{i}+n\,47\mathrm{i}-60\mathrm{i})} - \frac{(2n-12)\,(\cos(2c+2dx)+\sin(2c+2dx)\ 1\mathrm{i})\left(a+\frac{a\sin(c+dx)\,1\mathrm{i}}{\cos(c+dx)}\right)^n}{2d\,(n^3\,1\mathrm{i}-n^2\,12\mathrm{i}+n\,47\mathrm{i}-60\mathrm{i})} + \frac{(\cos(6c+6dx)+\sin(6c+6dx)\ 1\mathrm{i})\left(a+\frac{a\sin(c+dx)\,1\mathrm{i}}{\cos(c+dx)}\right)^n(n^2-9n+20)}{2d\,(n^3\,1\mathrm{i}-n^2\,12\mathrm{i}+n\,47\mathrm{i}-60\mathrm{i})} + \frac{(\cos(4c+4dx)+\sin(4c+4dx)\ 1\mathrm{i})\left(a+\frac{a\sin(c+dx)\,1\mathrm{i}}{\cos(c+dx)}\right)^n(n^2-11n+30)}{2d\,(n^3\,1\mathrm{i}-n^2\,12\mathrm{i}+n\,47\mathrm{i}-60\mathrm{i})}\right)$$

```
[In] int((e/cos(c + d*x))^(6 - 2*n)*(a + a*tan(c + d*x)*1i)^n,x)
[Out] (cos(6*c + 6*d*x) - sin(6*c + 6*d*x)*1i)*(e/cos(c + d*x))^(6 - 2*n)*((a + (
a*sin(c + d*x)*1i)/cos(c + d*x))^n/(d*(n*47i - n^2*12i + n^3*1i - 60i)) - (
(2*n - 12)*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i)*(a + (a*sin(c + d*x)*1i
)/cos(c + d*x))^n)/(2*d*(n*47i - n^2*12i + n^3*1i - 60i)) + ((cos(6*c + 6*d
*x) + sin(6*c + 6*d*x)*1i)*(a + (a*sin(c + d*x)*1i)/cos(c + d*x))^n*(n^2 -
9*n + 20))/(2*d*(n*47i - n^2*12i + n^3*1i - 60i)) + ((cos(4*c + 4*d*x) + si
n(4*c + 4*d*x)*1i)*(a + (a*sin(c + d*x)*1i)/cos(c + d*x))^n*(n^2 - 11*n + 3
0))/(2*d*(n*47i - n^2*12i + n^3*1i - 60i)))
```

3.492 $\int (e\sec(c+dx))^{5-2n}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 30, antiderivative size = 97

$$\int (e\sec(c+dx))^{5-2n}(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{\frac{5}{2}-n}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{2}(-3+2n),\frac{7}{2},\frac{1}{2}(1+i\tan(c+dx))\right)(e\sec(c+dx))^{5-2n}(1-i\tan(c+dx)}{5d}$$

```
[Out] -1/5*I*2^(5/2-n)*hypergeom([5/2, -3/2+n],[7/2],1/2+1/2*I*tan(d*x+c))*(e*sec
(d*x+c))^(5-2*n)*(1-I*tan(d*x+c))^(-5/2+n)*(a+I*a*tan(d*x+c))^n/d
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3586, 3604, 7, 72, 71}

$$\int (e\sec(c+dx))^{5-2n}(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{\frac{5}{2}-n}(1-i\tan(c+dx))^{n-\frac{5}{2}}(a+ia\tan(c+dx))^n(e\sec(c+dx))^{5-2n}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{2}(2n-3),\right.}{5d}$$

```
[In] Int[(e*Sec[c + d*x])^(5 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-1/5*I)*2^(5/2 - n)*Hypergeometric2F1[5/2, (-3 + 2*n)/2, 7/2, (1 + I*Tan[
c + d*x])/2]*(e*Sec[c + d*x])^(5 - 2*n)*(1 - I*Tan[c + d*x])^(-5/2 + n)*(a
+ I*a*Tan[c + d*x])^n)/d
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \left((e\sec(c+dx))^{5-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(-5+2n)}(a+ia\tan(c+dx))^{\frac{1}{2}(-5+2n)}\right)\int(a-ia\tan(c+dx))^{\frac{1}{2}(5-2n)}(a+ia\tan(c+dx))^{\frac{1}{2}(5-2n)+n}\,dx$$

$$=\frac{\left(a^2(e\sec(c+dx))^{5-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(-5+2n)}(a+ia\tan(c+dx))^{\frac{1}{2}(-5+2n)}\right)\text{Subst}\left(\int(a-iax\right.}{d}$$

$$=\frac{\left(a^2(e\sec(c+dx))^{5-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(-5+2n)}(a+ia\tan(c+dx))^{\frac{1}{2}(-5+2n)}\right)\text{Subst}\left(\int(a-iax\right.}{d}$$

$$=\frac{\left(2^{\frac{3}{2}-n}a^3(e\sec(c+dx))^{5-2n}(a-ia\tan(c+dx))^{\frac{1}{2}-n+\frac{1}{2}(-5+2n)}\left(\frac{a-ia\tan(c+dx)}{a}\right)^{-\frac{1}{2}+n}(a+ia\tan(c+\right.}{d}$$

$$= -\frac{i2^{\frac{5}{2}-n}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{2}(-3+2n),\frac{7}{2},\frac{1}{2}(1+i\tan(c+dx))\right)(e\sec(c+dx))^{5-2n}(1-i\tan(c+dx))}{5d}$$

Mathematica [A] (verified)

Time = 15.72 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.71

$$\int (e\sec(c+dx))^{5-2n}(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{5-n}e^{5i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-n}\left(1+e^{2i(c+dx)}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{5}{2},5-n,\frac{7}{2},-e^{2i(c+dx)}\right)\sec^{-5+n}}{5d}$$

```
[In] Integrate[(e*Sec[c + d*x])^(5 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-1/5*I)*2^(5 - n)*E^((5*I)*(c + d*x))*(E^(I*d*x))^n*Hypergeometric2F1[5/2
, 5 - n, 7/2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(-5 + n)*(e*Sec[c + d*x])^
(5 - 2*n)*(a + I*a*Tan[c + d*x])^n)/(d*(E^(I*(c + d*x))/(1 + E^((2*I)*(c +
d*x))))^n*(1 + E^((2*I)*(c + d*x)))^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F]

$$\int (e\sec(dx+c))^{5-2n}(a+ia\tan(dx+c))^n\,dx$$

```
[In] int((e*sec(d*x+c))^(5-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^(5-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int (e\sec(c+dx))^{5-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^{-2n+5}(ia\tan(dx+c)+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(5-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas"
)
```

```
[Out] integral((2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n + 5)*e^(I*d*
n*x + I*c*n + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1)) + n*log(
a/e)), x)
```

Sympy [F]

$$\int (e\sec(c+dx))^{5-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec{(c+dx)})^{5-2n}\,(ia(\tan{(c+dx)}-i))^n\,dx$$

```
[In] integrate((e*sec(d*x+c))**(5-2*n)*(a+I*a*tan(d*x+c))**n,x)
[Out] Integral((e*sec(c + d*x))**(5 - 2*n)*(I*a*(tan(c + d*x) - I))**n, x)
```

Maxima [F]

$$\int (e\sec(c+dx))^{5-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec{(dx+c)})^{-2\,n+5}\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(5-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima"
)
[Out] integrate((e*sec(d*x + c))^(-2*n + 5)*(I*a*tan(d*x + c) + a)^n, x)
```

Giac [F]

$$\int (e\sec(c+dx))^{5-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec{(dx+c)})^{-2\,n+5}\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(5-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(-2*n + 5)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{5-2n}(a+ia\tan(c+dx))^n\,dx = \int \left(\frac{e}{\cos{(c+d\,x)}}\right)^{5-2\,n}\,(a+a\tan(c+d\,x)\;1\mathrm{i})^n\,dx$$

```
[In] int((e/cos(c + d*x))^(5 - 2*n)*(a + a*tan(c + d*x)*1i)^n,x)
[Out] int((e/cos(c + d*x))^(5 - 2*n)*(a + a*tan(c + d*x)*1i)^n, x)
```

3.493 $\int (e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 30, antiderivative size = 98

$$\int (e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^n\,dx = \frac{2ia^2(e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^{-2+n}}{d(6-5n+n^2)} + \frac{ia(e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^{-1+n}}{d(3-n)}$$

```
[Out] 2*I*a^2*(e*sec(d*x+c))^(4-2*n)*(a+I*a*tan(d*x+c))^(-2+n)/d/(n^2-5*n+6)+I*a*
(e*sec(d*x+c))^(4-2*n)*(a+I*a*tan(d*x+c))^(-1+n)/d/(3-n)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3575, 3574}

$$\int (e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^n\,dx = \frac{2ia^2(a+ia\tan(c+dx))^{n-2}(e\sec(c+dx))^{4-2n}}{d(n^2-5n+6)} + \frac{ia(a+ia\tan(c+dx))^{n-1}(e\sec(c+dx))^{4-2n}}{d(3-n)}$$

```
[In] Int[(e*Sec[c + d*x])^(4 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((2*I)*a^2*(e*Sec[c + d*x])^(4 - 2*n)*(a + I*a*Tan[c + d*x])^(-2 + n))/(d*(
6 - 5*n + n^2)) + (I*a*(e*Sec[c + d*x])^(4 - 2*n)*(a + I*a*Tan[c + d*x])^(-
1 + n))/(d*(3 - n))
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{ia(e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^{-1+n}}{d(3-n)} \\
&\quad + \frac{(2a)\int(e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^{-1+n}\,dx}{3-n} \\
&= \frac{2ia^2(e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^{-2+n}}{d\left(6-5n+n^2\right)} \\
&\quad + \frac{ia(e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^{-1+n}}{d(3-n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.56 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.93

$$\int (e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^n\,dx$$
$$= \frac{e^4\sec^2(c+dx)(e\sec(c+dx))^{-2n}(\cos(2(c+dx))-i\sin(2(c+dx)))(a+ia\tan(c+dx))^n(-i(-4+n)+(}{d(-3+n)(-2+n)}$$

```
[In] Integrate[(e*Sec[c + d*x])^(4 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (e^4*Sec[c + d*x]^2*(Cos[2*(c + d*x)] - I*Sin[2*(c + d*x)])*(a + I*a*Tan[c
+ d*x])^n*((-I)*(-4 + n) + (-2 + n)*Tan[c + d*x]))/(d*(-3 + n)*(-2 + n)*(e*
Sec[c + d*x])^(2*n))
```

Maple [F]

$$\int (e\sec (dx+c))^{4-2n} (a+ia\tan (dx+c))^{n} dx$$

```
[In] int((e*sec(d*x+c))^(4-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^(4-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.37

$$\int (e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^n\,dx = \frac{\left((-i\,n+3i)e^{(4i\,dx+4i\,c)}+(-i\,n+4i)e^{(2i\,dx+2i\,c)}+i\right)\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{-2\,n+4} e^{\left(i\,dnx+i\,cn-4i\,dx+n\log\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)+n\right.}}{2\,(dn^2-5\,dn+6\,d)}$$

```
[In] integrate((e*sec(d*x+c))^(4-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas"
)
```

```
[Out] 1/2*((-I*n + 3*I)*e^(4*I*d*x + 4*I*c) + (-I*n + 4*I)*e^(2*I*d*x + 2*I*c) +
I)*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n + 4)*e^(I*d*n*x +
I*c*n - 4*I*d*x + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1)) + n*
log(a/e) - 4*I*c)/(d*n^2 - 5*d*n + 6*d)
```

Sympy [F]

$$\int (e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec (c+dx))^{4-2n} (ia(\tan (c+dx)-i))^{n}\,dx$$

```
[In] integrate((e*sec(d*x+c))**(4-2*n)*(a+I*a*tan(d*x+c))**n,x)
```

```
[Out] Integral((e*sec(c + d*x))**(4 - 2*n)*(I*a*(tan(c + d*x) - I))**n, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 595 vs. 2(88) = 176.

Time = 1.08 (sec) , antiderivative size = 595, normalized size of antiderivative = 6.07

$$\int (e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^n\,dx$$
$$= \frac{8\left(\left(\cos\left(2\,dx+2\,c\right)^2+\sin\left(2\,dx+2\,c\right)^2+2\,\cos\left(2\,dx+2\,c\right)+1\right)^{\frac{1}{2}\,n}a^n e^4\cos\left(n\arctan\left(\sin\left(2\,dx+2\,c\right),\cos\right.\right.\right.}{}$$

```
[In] integrate((e*sec(d*x+c))^(4-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima"
)
```

```
[Out] 8*((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/2*
n)*a^n*e^4*cos(n*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + I*(cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/2*n)*a^n*e
^4*sin(n*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - (a^n*e^4*n - 3*
a^n*e^4)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/2*n)*cos(2*d*x + n*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1) + 2*
c) + (-I*a^n*e^4*n + 3*I*a^n*e^4)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/2*n)*sin(2*d*x + n*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1) + 2*c))/(((-I*e^(2*n)*n^2 + 5*I*e^(2*n)*n - 6*I*e^(2*n
))*2^n*cos(6*d*x + 6*c) - 3*(I*e^(2*n)*n^2 - 5*I*e^(2*n)*n + 6*I*e^(2*n))*2
^n*cos(4*d*x + 4*c) - 3*(I*e^(2*n)*n^2 - 5*I*e^(2*n)*n + 6*I*e^(2*n))*2^n*c
os(2*d*x + 2*c) + (e^(2*n)*n^2 - 5*e^(2*n)*n + 6*e^(2*n))*2^n*sin(6*d*x + 6
*c) + 3*(e^(2*n)*n^2 - 5*e^(2*n)*n + 6*e^(2*n))*2^n*sin(4*d*x + 4*c) + 3*(e
^(2*n)*n^2 - 5*e^(2*n)*n + 6*e^(2*n))*2^n*sin(2*d*x + 2*c) + (-I*e^(2*n)*n^
2 + 5*I*e^(2*n)*n - 6*I*e^(2*n))*2^n)*d)
```

Giac [F]

$$\int (e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^n\,dx = \int \left(e\sec\left(dx+c\right)\right)^{-2\,n+4}\left(i\,a\tan\left(dx+c\right)+a\right)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(4-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((e*sec(d*x + c))^(-2*n + 4)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [B] (verification not implemented)

Time = 7.47 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.78

$$\int (e\sec(c+dx))^{4-2n}(a+ia\tan(c+dx))^n\,dx = \frac{4\,e^4\,\left(\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}\right)^n\,\left(4\,\sin\left(2\,c+2\,d\,x\right)+\cos\left(2\,c+2\,d\,x\right)\,4\mathrm{i}+\cos\left(4\,c+4\,d\,x\right)\,1\mathrm{i}-n\,1\mathrm{i}+\cdots\right.}{d\,\left(\frac{e}{\cos(c+d\,x)}\right)^{2\,n}\,\left(4\,\cos\left(2\,c+2\,d\,x\right)+\cos\left(4\,c+4\,d\,x\right)+3\right)\cdots}$$

```
[In] int((e/cos(c + d*x))^(4 - 2*n)*(a + a*tan(c + d*x)*1i)^n,x)
```

```
[Out] (4*e^4*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x)
+ 1))^n*(cos(2*c + 2*d*x)*4i - n*1i + cos(4*c + 4*d*x)*1i + 4*sin(2*c + 2*d
*x) + sin(4*c + 4*d*x) - n*cos(2*c + 2*d*x)*1i - n*sin(2*c + 2*d*x) + 3i))/
(d*(e/cos(c + d*x))^(2*n)*(4*cos(2*c + 2*d*x) + cos(4*c + 4*d*x) + 3)*(n^2
- 5*n + 6))
```

3.494 $\int (e\sec(c+dx))^{3-2n}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 30, antiderivative size = 97

$$\int (e\sec(c+dx))^{3-2n}(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{\frac{3}{2}-n}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}(-1+2n),\frac{5}{2},\frac{1}{2}(1+i\tan(c+dx))\right)(e\sec(c+dx))^{3-2n}(1-i\tan(c+dx)}{3d}$$

```
[Out] -1/3*I*2^(3/2-n)*hypergeom([3/2, -1/2+n],[5/2],1/2+1/2*I*tan(d*x+c))*(e*sec
(d*x+c))^(3-2*n)*(1-I*tan(d*x+c))^(-3/2+n)*(a+I*a*tan(d*x+c))^n/d
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3586, 3604, 7, 72, 71}

$$\int (e\sec(c+dx))^{3-2n}(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{\frac{3}{2}-n}(1-i\tan(c+dx))^{n-\frac{3}{2}}(a+ia\tan(c+dx))^n(e\sec(c+dx))^{3-2n}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}(2n-1)\right.}{3d}$$

```
[In] Int[(e*Sec[c + d*x])^(3 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-1/3*I)*2^(3/2 - n)*Hypergeometric2F1[3/2, (-1 + 2*n)/2, 5/2, (1 + I*Tan[
c + d*x])/2]*(e*Sec[c + d*x])^(3 - 2*n)*(1 - I*Tan[c + d*x])^(-3/2 + n)*(a
+ I*a*Tan[c + d*x])^n)/d
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \left((e\sec(c+dx))^{3-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(-3+2n)}(a+ia\tan(c+dx))^{\frac{1}{2}(-3+2n)}\right)\int (a-ia\tan(c+dx))^{\frac{1}{2}(3-2n)}(a+ia\tan(c+dx))^{\frac{1}{2}(3-2n)+n}\,dx$$

$$= \frac{\left(a^2(e\sec(c+dx))^{3-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(-3+2n)}(a+ia\tan(c+dx))^{\frac{1}{2}(-3+2n)}\right)\text{Subst}\left(\int (a-iax\right.}{d}$$

$$= \frac{\left(a^2(e\sec(c+dx))^{3-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(-3+2n)}(a+ia\tan(c+dx))^{\frac{1}{2}(-3+2n)}\right)\text{Subst}\left(\int (a-iax\right.}{d}$$

$$= \frac{\left(2^{\frac{1}{2}-n}a^2(e\sec(c+dx))^{3-2n}(a-ia\tan(c+dx))^{\frac{1}{2}-n+\frac{1}{2}(-3+2n)}\left(\frac{a-ia\tan(c+dx)}{a}\right)^{-\frac{1}{2}+n}(a+ia\tan(c+d\right.}{d}$$

$$= -\frac{i2^{\frac{3}{2}-n}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{2}(-1+2n),\frac{5}{2},\frac{1}{2}(1+i\tan(c+dx))\right)(e\sec(c+dx))^{3-2n}(1-i\tan}{3d}$$

Mathematica [A] (verified)

Time = 14.59 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.71

$$\int (e\sec(c+dx))^{3-2n}(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{3-n}e^{3i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-n}\left(1+e^{2i(c+dx)}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{3}{2},3-n,\frac{5}{2},-e^{2i(c+dx)}\right)\sec^{-3}}{3d}$$

```
[In] Integrate[(e*Sec[c + d*x])^(3 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-1/3*I)*2^(3 - n)*E^((3*I)*(c + d*x))*(E^(I*d*x))^n*Hypergeometric2F1[3/2
, 3 - n, 5/2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(-3 + n)*(e*Sec[c + d*x])^
(3 - 2*n)*(a + I*a*Tan[c + d*x])^n)/(d*(E^(I*(c + d*x))/(1 + E^((2*I)*(c +
d*x))))^n*(1 + E^((2*I)*(c + d*x)))^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F]

$$\int (e\sec(dx+c))^{3-2n}\,(a+ia\tan(dx+c))^n\,dx$$

```
[In] int((e*sec(d*x+c))^(3-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^(3-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int (e\sec(c+dx))^{3-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^{-2n+3}\,(i\,a\tan(dx+c)+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(3-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas"
)
```

```
[Out] integral((2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n + 3)*e^(I*d*
n*x + I*c*n + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1)) + n*log(
a/e)), x)
```

Sympy [F]

$$\int (e\sec(c+dx))^{3-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec{(c+dx)})^{3-2n}\,(ia(\tan{(c+dx)}-i))^n\,dx$$

```
[In] integrate((e*sec(d*x+c))**(3-2*n)*(a+I*a*tan(d*x+c))**n,x)
[Out] Integral((e*sec(c + d*x))**(3 - 2*n)*(I*a*(tan(c + d*x) - I))**n, x)
```

Maxima [F]

$$\int (e\sec(c+dx))^{3-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec{(dx+c)})^{-2\,n+3}\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(3-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima"
)
[Out] integrate((e*sec(d*x + c))^(-2*n + 3)*(I*a*tan(d*x + c) + a)^n, x)
```

Giac [F]

$$\int (e\sec(c+dx))^{3-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec{(dx+c)})^{-2\,n+3}\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(3-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(-2*n + 3)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{3-2n}(a+ia\tan(c+dx))^n\,dx = \int \left(\frac{e}{\cos{(c+d\,x)}}\right)^{3-2\,n}\,(a+a\tan(c+d\,x)\;1\mathrm{i})^n\,dx$$

```
[In] int((e/cos(c + d*x))^(3 - 2*n)*(a + a*tan(c + d*x)*1i)^n,x)
[Out] int((e/cos(c + d*x))^(3 - 2*n)*(a + a*tan(c + d*x)*1i)^n, x)
```

3.495 $\int (e\sec(c+dx))^{2-2n}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 30, antiderivative size = 46

$$\int (e\sec(c+dx))^{2-2n}(a+ia\tan(c+dx))^n\,dx=\frac{ia(e\sec(c+dx))^{2-2n}(a+ia\tan(c+dx))^{-1+n}}{d(1-n)}$$

```
[Out] I*a*(e*sec(d*x+c))^(2-2*n)*(a+I*a*tan(d*x+c))^(-1+n)/d/(1-n)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {3574}

$$\int (e\sec(c+dx))^{2-2n}(a+ia\tan(c+dx))^n\,dx=\frac{ia(a+ia\tan(c+dx))^{n-1}(e\sec(c+dx))^{2-2n}}{d(1-n)}$$

```
[In] Int[(e*Sec[c + d*x])^(2 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (I*a*(e*Sec[c + d*x])^(2 - 2*n)*(a + I*a*Tan[c + d*x])^(-1 + n))/(d*(1 - n)
)
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\text{integral}=\frac{ia(e\sec(c+dx))^{2-2n}(a+ia\tan(c+dx))^{-1+n}}{d(1-n)}$$

Mathematica [A] (verified)

Time = 1.97 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.28

$$\int (e\sec(c+dx))^{2-2n}(a+ia\tan(c+dx))^n\,dx = -\frac{e^2(e\sec(c+dx))^{-2n}(i+\sec(c)\sec(c+dx)\sin(dx)+\tan(c))(a+ia\tan(c+dx))^n}{d(-1+n)}$$

```
[In] Integrate[(e*Sec[c + d*x])^(2 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] -((e^2*(I + Sec[c]*Sec[c + d*x]*Sin[d*x] + Tan[c])*(a + I*a*Tan[c + d*x])^n
)/(d*(-1 + n)*(e*Sec[c + d*x])^(2*n)))
```

Maple [F]

$$\int (e\sec(dx+c))^{2-2n}(a+ia\tan(dx+c))^n\,dx$$

```
[In] int((e*sec(d*x+c))^(2-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^(2-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(40) = 80.

Time = 0.25 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.37

$$\int (e\sec(c+dx))^{2-2n}(a+ia\tan(c+dx))^n\,dx = \frac{\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)^{-2\,n+2}\left(-i\,e^{(2i\,dx+2i\,c)}-i\right)e^{\left(i\,dnx+i\,cn-2i\,dx+n\log\left(\frac{2\,ee^{(i\,dx+i\,c)}}{e^{(2i\,dx+2i\,c)}+1}\right)+n\log\left(\frac{a}{e}\right)-2i\,c\right)}}{2\,(dn-d)}$$

```
[In] integrate((e*sec(d*x+c))^(2-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas"
)
```

```
[Out] 1/2*(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n + 2)*(-I*e^(2*I*d
*x + 2*I*c) - I)*e^(I*d*n*x + I*c*n - 2*I*d*x + n*log(2*e*e^(I*d*x + I*c)/(
e^(2*I*d*x + 2*I*c) + 1)) + n*log(a/e) - 2*I*c)/(d*n - d)
```

Sympy [F]

$$\int (e\sec(c+dx))^{2-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec{(c+dx)})^{2-2n}\,(ia(\tan{(c+dx)}-i))^n\,dx$$

```
[In] integrate((e*sec(d*x+c))**(2-2*n)*(a+I*a*tan(d*x+c))**n,x)
[Out] Integral((e*sec(c + d*x))**(2 - 2*n)*(I*a*(tan(c + d*x) - I))**n, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(40) = 80.

Time = 0.36 (sec) , antiderivative size = 217, normalized size of antiderivative = 4.72

$$\int (e\sec(c+dx))^{2-2n}(a+ia\tan(c+dx))^n\,dx = \frac{\left(-i\,a^n e^2 - \frac{2\,a^n e^2 \sin(dx+c)}{\cos(dx+c)+1} + \frac{i\,a^n e^2 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)e^{\left(n\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)+n\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)+n\log\left(-\frac{2i\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)}{(\cos(dx+c)+\cdots}\right.\right.}}{\left(e^{2\,n}(n-1) - \frac{e^{2\,n}(n-1)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)d}$$

```
[In] integrate((e*sec(d*x+c))^(2-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima"
)
[Out] (-I*a^n*e^2 - 2*a^n*e^2*sin(d*x + c)/(cos(d*x + c) + 1) + I*a^n*e^2*sin(d*x
 + c)^2/(cos(d*x + c) + 1)^2)*e^(n*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)
 + n*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1) + n*log(-2*I*sin(d*x + c)/(co
s(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1) - 2*n*log(-sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 - 1))/((e^(2*n)*(n - 1) - e^(2*n)*(n - 1)*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2)*d)
```

Giac [F]

$$\int (e\sec(c+dx))^{2-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec{(dx+c)})^{-2\,n+2}\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(2-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(-2*n + 2)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [B] (verification not implemented)

Time = 5.63 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.30

$$\int (e\sec(c+dx))^{2-2n}(a+ia\tan(c+dx))^n\,dx = -\frac{e^2\,(\cos(2\,c+2\,d\,x)\,1\mathrm{i}+\sin(2\,c+2\,d\,x)+1\mathrm{i})\,\left(\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i})}{\cos(2\,c+2\,d\,x)+1}\right)^n}{d\,(\cos(2\,c+2\,d\,x)+1)\,\left(\frac{e}{\cos(c+d\,x)}\right)^{2\,n}\,(n-1)}$$

```
[In] int((e/cos(c + d*x))^(2 - 2*n)*(a + a*tan(c + d*x)*1i)^n,x)
```

```
[Out] -(e^2*(cos(2*c + 2*d*x)*1i + sin(2*c + 2*d*x) + 1i)*((a*(cos(2*c + 2*d*x) +
 sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^n)/(d*(cos(2*c + 2*d*x)
+ 1)*(e/cos(c + d*x))^(2*n)*(n - 1))
```

3.496 $\int (e\sec(c+dx))^{1-2n}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 30, antiderivative size = 95

$$\int (e\sec(c+dx))^{1-2n}(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{\frac{1}{2}-n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+2n),\frac{3}{2},\frac{1}{2}(1+i\tan(c+dx))\right)(e\sec(c+dx))^{1-2n}(1-i\tan(c+dx)}{d}$$

```
[Out] -I*2^(1/2-n)*hypergeom([1/2, 1/2+n],[3/2],1/2+1/2*I*tan(d*x+c))*(e*sec(d*x+
c))^(1-2*n)*(1-I*tan(d*x+c))^(-1/2+n)*(a+I*a*tan(d*x+c))^n/d
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3586, 3604, 7, 72, 71}

$$\int (e\sec(c+dx))^{1-2n}(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{\frac{1}{2}-n}(1-i\tan(c+dx))^{n-\frac{1}{2}}(a+ia\tan(c+dx))^n(e\sec(c+dx))^{1-2n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(2n+1)\right.}{d}$$

```
[In] Int[(e*Sec[c + d*x])^(1 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(1/2 - n)*Hypergeometric2F1[1/2, (1 + 2*n)/2, 3/2, (1 + I*Tan[c + d
*x])/2]*(e*Sec[c + d*x])^(1 - 2*n)*(1 - I*Tan[c + d*x])^(-1/2 + n)*(a + I*a
*Tan[c + d*x])^n)/d
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \left((e\sec(c+dx))^{1-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(-1+2n)}(a+ia\tan(c+dx))^{\frac{1}{2}(-1+2n)}\right)\int (a-ia\tan(c+dx))^{\frac{1}{2}(1-2n)}(a+ia\tan(c+dx))^{\frac{1}{2}(1-2n)+n}\,dx$$

$$= \frac{\left(a^2(e\sec(c+dx))^{1-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(-1+2n)}(a+ia\tan(c+dx))^{\frac{1}{2}(-1+2n)}\right)\text{Subst}\left(\int (a-iax)\right.}{d}$$

$$= \frac{\left(a^2(e\sec(c+dx))^{1-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(-1+2n)}(a+ia\tan(c+dx))^{\frac{1}{2}(-1+2n)}\right)\text{Subst}\left(\int \frac{(a-iax)^{-1}}{\sqrt{a+}}\right.}{d}$$

$$= \frac{\left(2^{-\frac{1}{2}-n}a(e\sec(c+dx))^{1-2n}(a-ia\tan(c+dx))^{\frac{1}{2}-n+\frac{1}{2}(-1+2n)}\left(\frac{a-ia\tan(c+dx)}{a}\right)^{-\frac{1}{2}+n}(a+ia\tan(c+d\right.}{d}$$

$$= -\frac{i2^{\frac{1}{2}-n}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+2n),\frac{3}{2},\frac{1}{2}(1+i\tan(c+dx))\right)(e\sec(c+dx))^{1-2n}(1-i\tan(}{d}$$

Mathematica [A] (verified)

Time = 10.34 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.62

$$\int (e\sec(c+dx))^{1-2n}(a+ia\tan(c+dx))^n\,dx =$$

$$-\frac{i2^{1-n}e(e^{idx})^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{1-n}\left(1+e^{2i(c+dx)}\right)^{1-n}\text{Hypergeometric2F1}\left(\frac{1}{2},1-n,\frac{3}{2},-e^{2i(c+dx)}\right)\sec^n(c+d}{d}$$

```
[In] Integrate[(e*Sec[c + d*x])^(1 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(1 - n)*e*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))
^(1 - n)*(1 + E^((2*I)*(c + d*x)))^(1 - n)*Hypergeometric2F1[1/2, 1 - n, 3/
2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^n*(a + I*a*Tan[c + d*x])^n)/(d*(e*Sec
[c + d*x])^(2*n)*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F]

$$\int (e\sec(dx+c))^{1-2n}(a+ia\tan(dx+c))^n\,dx$$

```
[In] int((e*sec(d*x+c))^(1-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^(1-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int (e\sec(c+dx))^{1-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^{-2n+1}(ia\tan(dx+c)+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(1-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas"
)
```

```
[Out] integral((2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n + 1)*e^(I*d*
n*x + I*c*n + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1)) + n*log(
a/e)), x)
```

Sympy [F]

$$\int (e\sec(c+dx))^{1-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec{(c+dx)})^{1-2n}\,(ia(\tan{(c+dx)}-i))^n\,dx$$

```
[In] integrate((e*sec(d*x+c))**(1-2*n)*(a+I*a*tan(d*x+c))**n,x)
[Out] Integral((e*sec(c + d*x))**(1 - 2*n)*(I*a*(tan(c + d*x) - I))**n, x)
```

Maxima [F]

$$\int (e\sec(c+dx))^{1-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec{(dx+c)})^{-2\,n+1}\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(1-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima"
)
[Out] integrate((e*sec(d*x + c))^(-2*n + 1)*(I*a*tan(d*x + c) + a)^n, x)
```

Giac [F]

$$\int (e\sec(c+dx))^{1-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec{(dx+c)})^{-2\,n+1}\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(1-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(-2*n + 1)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{1-2n}(a+ia\tan(c+dx))^n\,dx = \int \left(\frac{e}{\cos{(c+d\,x)}}\right)^{1-2\,n}\,(a+a\tan(c+d\,x)\ 1\mathrm{i})^n\,dx$$

```
[In] int((e/cos(c + d*x))^(1 - 2*n)*(a + a*tan(c + d*x)*1i)^n,x)
[Out] int((e/cos(c + d*x))^(1 - 2*n)*(a + a*tan(c + d*x)*1i)^n, x)
```

3.497 $\int (e\sec(c+dx))^{-2n}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 28, antiderivative size = 65

$$\int (e\sec(c+dx))^{-2n}(a+ia\tan(c+dx))^n\,dx = -\frac{i\,\text{Hypergeometric2F1}\left(1,-n,1-n,\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{-2n}(a+ia\tan(c+dx))^n}{2dn}$$

```
[Out] -1/2*I*hypergeom([1, -n],[1-n],1/2-1/2*I*tan(d*x+c))*(a+I*a*tan(d*x+c))^n/d
/n/((e*sec(d*x+c))^(2*n))
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3573, 3562, 70}

$$\int (e\sec(c+dx))^{-2n}(a+ia\tan(c+dx))^n\,dx = -\frac{i(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-2n}\,\text{Hypergeometric2F1}\left(1,-n,1-n,\frac{1}{2}(1-i\tan(c+dx))\right)}{2dn}$$

```
[In] Int[(a + I*a*Tan[c + d*x])^n/(e*Sec[c + d*x])^(2*n),x]
```

```
[Out] ((-1/2*I)*Hypergeometric2F1[1, -n, 1 - n, (1 - I*Tan[c + d*x])/2]*(a + I*a*
Tan[c + d*x])^n)/(d*n*(e*Sec[c + d*x])^(2*n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
```

```
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3573

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Dist[(a/d)^(2*IntPart[n])*(a + b*Tan[e + f*x])^Frac
Part[n]*((a - b*Tan[e + f*x])^FracPart[n]/(d*Sec[e + f*x])^(2*FracPart[n]))
, Int[1/(a - b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] &
& EqQ[a^2 + b^2, 0] && EqQ[Simplify[m/2 + n], 0]
```

Rubi steps

$$\text{integral} = \left((e\sec(c+dx))^{-2n}(a-ia\tan(c+dx))^n(a+ia\tan(c+dx))^n\right)\int(a-ia\tan(c+dx))^{-n}\,dx$$

$$= \frac{(ia(e\sec(c+dx))^{-2n}(a-ia\tan(c+dx))^n(a+ia\tan(c+dx))^n)\,\text{Subst}\left(\int\frac{(a+x)^{-1-n}}{a-x}\,dx, x, -ia\tan(\right.}{d}$$

$$= -\frac{i\,\text{Hypergeometric2F1}\left(1,-n,1-n,\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{-2n}(a+ia\tan(c+dx))}{2dn}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 146 vs. 2(65) = 130.

Time = 2.24 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.25

$$\int(e\sec(c+dx))^{-2n}(a+ia\tan(c+dx))^n\,dx$$

$$= \frac{i2^{-1-n}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-n}\left(1+e^{2i(c+dx)}\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,1+e^{2i(c+dx)}\right)\sec^n(c+dx}{d(1+n)}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])^n/(e*Sec[c + d*x])^(2*n),x]
```

```
[Out] (I*2^(-1 - n)*(E^(I*d*x))^n*(1 + E^((2*I)*(c + d*x)))*Hypergeometric2F1[1,
1 + n, 2 + n, 1 + E^((2*I)*(c + d*x))]*Sec[c + d*x]^n*(a + I*a*Tan[c + d*x]
)^n)/(d*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^n*(1 + n)*(e*Sec[c + d*
x])^(2*n)*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F]

$$\int (a + ia\tan(dx + c))^n (e\sec(dx + c))^{-2n} \, dx$$

```
[In] int((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^(2*n)),x)
[Out] int((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^(2*n)),x)
```

Fricas [F]

$$\int (e\sec(c + dx))^{-2n}(a + ia\tan(c + dx))^n \, dx = \int \frac{(i\,a\tan(dx + c) + a)^n}{(e\sec(dx + c))^{2\,n}} \, dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^(2*n)),x, algorithm="fricas"
)
[Out] integral(e^(I*d*n*x + I*c*n + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c
) + 1)) + n*log(a/e))/(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(2*n)
, x)
```

Sympy [F]

$$\int (e\sec(c+dx))^{-2n}(a+ia\tan(c+dx))^n \, dx = \int (e\sec(c + dx))^{-2n} \, (ia(\tan(c + dx) - i))^n \, dx$$

```
[In] integrate((a+I*a*tan(d*x+c))**n/((e*sec(d*x+c))**(2*n)),x)
[Out] Integral((I*a*(tan(c + d*x) - I))**n/(e*sec(c + d*x))**(2*n), x)
```

Maxima [F]

$$\int (e\sec(c + dx))^{-2n}(a + ia\tan(c + dx))^n \, dx = \int \frac{(i\,a\tan(dx + c) + a)^n}{(e\sec(dx + c))^{2\,n}} \, dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^(2*n)),x, algorithm="maxima"
)
[Out] integrate((I*a*tan(d*x + c) + a)^n/(e*sec(d*x + c))^(2*n), x)
```

Giac [F]

$$\int (e\sec(c+dx))^{-2n}(a+ia\tan(c+dx))^n\,dx = \int \frac{(i\,a\tan(dx+c)+a)^n}{(e\sec(dx+c))^{2\,n}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^n/((e*sec(d*x+c))^(2*n)),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^n/(e*sec(d*x + c))^(2*n), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{-2n}(a+ia\tan(c+dx))^n\,dx = \int \frac{(a+a\tan(c+d\,x)\;1\mathrm{i})^n}{\left(\frac{e}{\cos(c+d\,x)}\right)^{2\,n}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(2*n),x)
[Out] int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(2*n), x)
```

3.498 $\int (e\sec(c+dx))^{-1-2n}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 30, antiderivative size = 95

$$\int (e\sec(c+dx))^{-1-2n}(a+ia\tan(c+dx))^n\,dx = \frac{i2^{-\frac{1}{2}-n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(3+2n),\frac{1}{2},\frac{1}{2}(1+i\tan(c+dx))\right)(e\sec(c+dx))^{-1-2n}(1-i\tan(c+dx)}{d}$$

```
[Out] I*2^(-1/2-n)*hypergeom([-1/2, 3/2+n],[1/2],1/2+1/2*I*tan(d*x+c))*(e*sec(d*x
+c))^(-1-2*n)*(1-I*tan(d*x+c))^(1/2+n)*(a+I*a*tan(d*x+c))^n/d
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3586, 3604, 7, 72, 71}

$$\int (e\sec(c+dx))^{-1-2n}(a+ia\tan(c+dx))^n\,dx = \frac{i2^{-n-\frac{1}{2}}(1-i\tan(c+dx))^{n+\frac{1}{2}}(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-2n-1}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(2n+\right.}{d}$$

```
[In] Int[(e*Sec[c + d*x])^(-1 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (I*2^(-1/2 - n)*Hypergeometric2F1[-1/2, (3 + 2*n)/2, 1/2, (1 + I*Tan[c + d*
x])/2]*(e*Sec[c + d*x])^(-1 - 2*n)*(1 - I*Tan[c + d*x])^(1/2 + n)*(a + I*a*
Tan[c + d*x])^n)/d
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \left((e\sec(c+dx))^{-1-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(1+2n)}(a+ia\tan(c+dx))^{\frac{1}{2}(1+2n)}\right)\int(a-ia\tan(c+dx))^{\frac{1}{2}(-1-2n)}(a+ia\tan(c+dx))^{\frac{1}{2}(-1-2n)+n}\,dx$$

$$=\frac{\left(a^2(e\sec(c+dx))^{-1-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(1+2n)}(a+ia\tan(c+dx))^{\frac{1}{2}(1+2n)}\right)\text{Subst}\left(\int(a-iax)^{-}\right.}{d}$$

$$=\frac{\left(a^2(e\sec(c+dx))^{-1-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(1+2n)}(a+ia\tan(c+dx))^{\frac{1}{2}(1+2n)}\right)\text{Subst}\left(\int\frac{(a-iax)^{-1+}}{(a+iax}\right.}{d}$$

$$=\frac{\left(2^{-\frac{3}{2}-n}a(e\sec(c+dx))^{-1-2n}(a-ia\tan(c+dx))^{-\frac{1}{2}-n+\frac{1}{2}(1+2n)}\left(\frac{a-ia\tan(c+dx)}{a}\right)^{\frac{1}{2}+n}(a+ia\tan(c+\right.}{d}$$

$$= \frac{i2^{-\frac{1}{2}-n}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{2}(3+2n),\frac{1}{2},\frac{1}{2}(1+i\tan(c+dx))\right)(e\sec(c+dx))^{-1-2n}(1-i\tan}{d}$$

Mathematica [A] (verified)

Time = 14.20 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.65

$$\int (e\sec(c+dx))^{-1-2n}(a+ia\tan(c+dx))^n\,dx$$
$$= \frac{i2^{-1-n}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-1-n}\left(1+e^{2i(c+dx)}\right)^{-1-n}\text{Hypergeometric2F1}\left(-\frac{1}{2},-1-n,\frac{1}{2},-e^{2i(c+dx)}\right)\sec^{1+n}}{d}$$

```
[In] Integrate[(e*Sec[c + d*x])^(-1 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (I*2^(-1 - n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^(-1
 - n)*(1 + E^((2*I)*(c + d*x)))^(-1 - n)*Hypergeometric2F1[-1/2, -1 - n, 1/
2, -E^((2*I)*(c + d*x))]*Sec[c + d*x]^(1 + n)*(e*Sec[c + d*x])^(-1 - 2*n)*(
a + I*a*Tan[c + d*x])^n)/(d*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F]

$$\int (e\sec(dx+c))^{-1-2n}(a+ia\tan(dx+c))^n\,dx$$

```
[In] int((e*sec(d*x+c))^(-1-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^(-1-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int (e\sec(c+dx))^{-1-2n}(a+ia\tan(c+dx))^n\,dx$$
$$= \int (e\sec(dx+c))^{-2n-1}(ia\tan(dx+c)+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(-1-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas
")
```

```
[Out] integral((2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n - 1)*e^(I*d*
n*x + I*c*n + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1)) + n*log(
a/e)), x)
```

Sympy [F]

$$\int (e\sec(c+dx))^{-1-2n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int (e\sec{(c+dx)})^{-2n-1}\,(ia(\tan{(c+dx)}-i))^n\,\,dx$$

```
[In] integrate((e*sec(d*x+c))**(-1-2*n)*(a+I*a*tan(d*x+c))**n,x)
[Out] Integral((e*sec(c + d*x))**(-2*n - 1)*(I*a*(tan(c + d*x) - I))**n, x)
```

Maxima [F]

$$\int (e\sec(c+dx))^{-1-2n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int (e\sec{(dx+c)})^{-2\,n-1}\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(-1-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima
")
[Out] integrate((e*sec(d*x + c))^(-2*n - 1)*(I*a*tan(d*x + c) + a)^n, x)
```

Giac [F]

$$\int (e\sec(c+dx))^{-1-2n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int (e\sec{(dx+c)})^{-2\,n-1}\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(-1-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(-2*n - 1)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{-1-2n}(a+ia\tan(c+dx))^n\,dx = \int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^n}{\left(\frac{e}{\cos(c+d\,x)}\right)^{2\,n+1}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(2*n + 1),x)
[Out] int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(2*n + 1), x)
```

3.499 $\int (e\sec(c+dx))^{-2-2n}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 30, antiderivative size = 74

$$\int (e\sec(c+dx))^{-2-2n}(a+ia\tan(c+dx))^n\,dx = -\frac{i\,\text{Hypergeometric2F1}\left(2,-1-n,-n,\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{-2(1+n)}(a+ia\tan(c+dx))^{1+n}}{4ad(1+n)}$$

```
[Out] -1/4*I*hypergeom([2, -1-n],[-n],1/2-1/2*I*tan(d*x+c))*(a+I*a*tan(d*x+c))^(1
+n)/a/d/(1+n)/((e*sec(d*x+c))^(2+2*n))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3586, 3604, 7, 70}

$$\int (e\sec(c+dx))^{-2-2n}(a+ia\tan(c+dx))^n\,dx = -\frac{i(a+ia\tan(c+dx))^{n+1}(e\sec(c+dx))^{-2(n+1)}\,\text{Hypergeometric2F1}\left(2,-n-1,-n,\frac{1}{2}(1-i\tan(c+dx))\right)}{4ad(n+1)}$$

```
[In] Int[(e*Sec[c + d*x])^(-2 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-1/4*I)*Hypergeometric2F1[2, -1 - n, -n, (1 - I*Tan[c + d*x])/2]*(a + I*a
*Tan[c + d*x])^(1 + n))/(a*d*(1 + n)*(e*Sec[c + d*x])^(2*(1 + n)))
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \left((e\sec(c+dx))^{-2-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(2+2n)}(a+ia\tan(c+dx))^{\frac{1}{2}(2+2n)}\right)\int(a-ia\tan(c+dx))^{\frac{1}{2}(-2-2n)}(a+ia\tan(c+dx))^{\frac{1}{2}(-2-2n)+n}\,dx$$

$$=\frac{\left(a^2(e\sec(c+dx))^{-2-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(2+2n)}(a+ia\tan(c+dx))^{\frac{1}{2}(2+2n)}\right)\text{Subst}\left(\int(a-iax)\right.}{d}$$

$$=\frac{\left(a^2(e\sec(c+dx))^{-2-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(2+2n)}(a+ia\tan(c+dx))^{\frac{1}{2}(2+2n)}\right)\text{Subst}\left(\int\frac{(a-iax)^{-1}}{(a+}\right.}{d}$$

$$=-\frac{i\,\text{Hypergeometric2F1}\left(2,-1-n,-n,\frac{1}{2}(1-i\tan(c+dx))\right)(e\sec(c+dx))^{-2(1+n)}(a+ia\tan(c}{4ad(1+n)}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 151 vs. 2(74) = 148.

Time = 14.69 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.04

$$\int (e\sec(c+dx))^{-2-2n}(a+ia\tan(c+dx))^n\,dx = -\frac{i2^{-3-n}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-n}\left(1+e^{2i(c+dx)}\right)^3\text{Hypergeometric2F1}\left(2,3+n,4+n,1+e^{2i(c+dx)}\right)\sec^n(c+}{de^2(3+n)}$$

```
[In] Integrate[(e*Sec[c + d*x])^(-2 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(-3 - n)*(E^(I*d*x))^n*(1 + E^((2*I)*(c + d*x)))^3*Hypergeometric2F
1[2, 3 + n, 4 + n, 1 + E^((2*I)*(c + d*x))]*Sec[c + d*x]^n*(a + I*a*Tan[c +
 d*x])^n)/(d*e^2*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^n*(3 + n)*(e*S
ec[c + d*x])^(2*n)*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F]

$$\int (e\sec(dx+c))^{-2-2n}(a+ia\tan(dx+c))^n\,dx$$

```
[In] int((e*sec(d*x+c))^(-2-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*sec(d*x+c))^(-2-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int (e\sec(c+dx))^{-2-2n}(a+ia\tan(c+dx))^n\,dx = \int (e\sec(dx+c))^{-2n-2}(ia\tan(dx+c)+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(-2-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas
")
```

```
[Out] integral((2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n - 2)*e^(I*d*
n*x + I*c*n + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1)) + n*log(
a/e)), x)
```

Sympy [F]

$$\int (e\sec(c+dx))^{-2-2n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int (e\sec(c+dx))^{-2n-2}\,(ia(\tan(c+dx)-i))^n\,dx$$

```
[In] integrate((e*sec(d*x+c))**(-2-2*n)*(a+I*a*tan(d*x+c))**n,x)
[Out] Integral((e*sec(c + d*x))**(-2*n - 2)*(I*a*(tan(c + d*x) - I))**n, x)
```

Maxima [F]

$$\int (e\sec(c+dx))^{-2-2n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int (e\sec(dx+c))^{-2n-2}\,(ia\tan(dx+c)+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(-2-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima
")
[Out] integrate((e*sec(d*x + c))^(-2*n - 2)*(I*a*tan(d*x + c) + a)^n, x)
```

Giac [F]

$$\int (e\sec(c+dx))^{-2-2n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int (e\sec(dx+c))^{-2n-2}\,(ia\tan(dx+c)+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(-2-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(-2*n - 2)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{-2-2n}(a+ia\tan(c+dx))^n\,dx = \int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^n}{\left(\frac{e}{\cos(c+d\,x)}\right)^{2\,n+2}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(2*n + 2),x)
[Out] int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(2*n + 2), x)
```

3.500 $\int (e\sec(c+dx))^{-3-2n}(a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 30, antiderivative size = 97

$$\int (e\sec(c+dx))^{-3-2n}(a+ia\tan(c+dx))^n\,dx = \frac{i2^{-\frac{3}{2}-n}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}(5+2n),-\frac{1}{2},\frac{1}{2}(1+i\tan(c+dx))\right)(e\sec(c+dx))^{-3-2n}(1-i\tan(c+dx))^{\frac{3}{2}+n}(a+ia\tan(c+dx))^n}{3d}$$

```
[Out] 1/3*I*2^(-3/2-n)*hypergeom([-3/2, 5/2+n],[-1/2],1/2+1/2*I*tan(d*x+c))*(e*se
c(d*x+c))^(-3-2*n)*(1-I*tan(d*x+c))^(3/2+n)*(a+I*a*tan(d*x+c))^n/d
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3586, 3604, 7, 72, 71}

$$\int (e\sec(c+dx))^{-3-2n}(a+ia\tan(c+dx))^n\,dx = \frac{i2^{-n-\frac{3}{2}}(1-i\tan(c+dx))^{n+\frac{3}{2}}(a+ia\tan(c+dx))^n(e\sec(c+dx))^{-2n-3}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}(2n+5),-\frac{1}{2},\frac{1}{2}(1+i\tan(c+dx))\right)}{3d}$$

```
[In] Int[(e*Sec[c + d*x])^(-3 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((I/3)*2^(-3/2 - n)*Hypergeometric2F1[-3/2, (5 + 2*n)/2, -1/2, (1 + I*Tan[c
 + d*x])/2]*(e*Sec[c + d*x])^(-3 - 2*n)*(1 - I*Tan[c + d*x])^(3/2 + n)*(a +
 I*a*Tan[c + d*x])^n)/d
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 && !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \left((e\sec(c+dx))^{-3-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(3+2n)}(a+ia\tan(c+dx))^{\frac{1}{2}(3+2n)}\right)\int (a-ia\tan(c+dx))^{\frac{1}{2}(-3-2n)}(a+ia\tan(c+dx))^{\frac{1}{2}(-3-2n)+n}\,dx$$

$$= \frac{\left(a^2(e\sec(c+dx))^{-3-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(3+2n)}(a+ia\tan(c+dx))^{\frac{1}{2}(3+2n)}\right)\text{Subst}\left(\int (a-iax)^{-}\right.}{d}$$

$$= \frac{\left(a^2(e\sec(c+dx))^{-3-2n}(a-ia\tan(c+dx))^{\frac{1}{2}(3+2n)}(a+ia\tan(c+dx))^{\frac{1}{2}(3+2n)}\right)\text{Subst}\left(\int \frac{(a-iax)^{-1+}}{(a+iax}\right.}{d}$$

$$= \frac{\left(2^{-\frac{5}{2}-n}(e\sec(c+dx))^{-3-2n}(a-ia\tan(c+dx))^{-\frac{1}{2}-n+\frac{1}{2}(3+2n)}\left(\frac{a-ia\tan(c+dx)}{a}\right)^{\frac{1}{2}+n}(a+ia\tan(c+dx\right.}{d}$$

$$= \frac{i2^{-\frac{3}{2}-n}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{2}(5+2n),-\frac{1}{2},\frac{1}{2}(1+i\tan(c+dx))\right)(e\sec(c+dx))^{-3-2n}(1-i\tan(c+dx)) \dots}{3d}$$

Mathematica [A] (verified)

Time = 14.99 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.71

$$\int (e\sec(c+dx))^{-3-2n}(a+ia\tan(c+dx))^n\,dx$$
$$= \frac{i2^{-3-n}e^{-3i(c+dx)}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^{-n}\left(1+e^{2i(c+dx)}\right)^{-n}\text{Hypergeometric2F1}\left(-\frac{3}{2},-3-n,-\frac{1}{2},-e^{2i(c+dx)}\right) \dots}{3d}$$

```
[In] Integrate[(e*Sec[c + d*x])^(-3 - 2*n)*(a + I*a*Tan[c + d*x])^n,x]
[Out] ((I/3)*2^(-3 - n)*(E^(I*d*x))^n*Hypergeometric2F1[-3/2, -3 - n, -1/2, -E^((
2*I)*(c + d*x))]*Sec[c + d*x]^(3 + n)*(e*Sec[c + d*x])^(-3 - 2*n)*(a + I*a*
Tan[c + d*x])^n)/(d*E^((3*I)*(c + d*x))*(E^(I*(c + d*x))/(1 + E^((2*I)*(c +
 d*x))))^n*(1 + E^((2*I)*(c + d*x)))^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F]

$$\int (e\sec(dx+c))^{-3-2n}(a+ia\tan(dx+c))^n\,dx$$

```
[In] int((e*sec(d*x+c))^(-3-2*n)*(a+I*a*tan(d*x+c))^n,x)
[Out] int((e*sec(d*x+c))^(-3-2*n)*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int (e\sec(c+dx))^{-3-2n}(a+ia\tan(c+dx))^n\,dx$$
$$= \int (e\sec(dx+c))^{-2n-3}(i\,a\tan(dx+c)+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(-3-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas
")
[Out] integral((2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1))^(-2*n - 3)*e^(I*d*
n*x + I*c*n + n*log(2*e*e^(I*d*x + I*c)/(e^(2*I*d*x + 2*I*c) + 1)) + n*log(
a/e)), x)
```

Sympy [F]

$$\int (e\sec(c+dx))^{-3-2n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int (e\sec(c+dx))^{-2n-3}\,(ia(\tan(c+dx)-i))^n\,dx$$

```
[In] integrate((e*sec(d*x+c))**(-3-2*n)*(a+I*a*tan(d*x+c))**n,x)
[Out] Integral((e*sec(c + d*x))**(-2*n - 3)*(I*a*(tan(c + d*x) - I))**n, x)
```

Maxima [F]

$$\int (e\sec(c+dx))^{-3-2n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int (e\sec(dx+c))^{-2n-3}(ia\tan(dx+c)+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(-3-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima
")
[Out] integrate((e*sec(d*x + c))^(-2*n - 3)*(I*a*tan(d*x + c) + a)^n, x)
```

Giac [F]

$$\int (e\sec(c+dx))^{-3-2n}(a+ia\tan(c+dx))^n\,dx$$
$$=\int (e\sec(dx+c))^{-2n-3}(ia\tan(dx+c)+a)^n\,dx$$

```
[In] integrate((e*sec(d*x+c))^(-3-2*n)*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((e*sec(d*x + c))^(-2*n - 3)*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\sec(c+dx))^{-3-2n}(a+ia\tan(c+dx))^n\,dx = \int \frac{(a+a\tan(c+d\,x)\ 1\mathrm{i})^n}{\left(\frac{e}{\cos(c+d\,x)}\right)^{2\,n+3}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(2*n + 3),x)
[Out] int((a + a*tan(c + d*x)*1i)^n/(e/cos(c + d*x))^(2*n + 3), x)
```

3.501 $\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-2-n}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 66

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-2-n}\,dx = \frac{i\,\text{Hypergeometric2F1}\left(3,n,1+n,\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}}{8a^2fn}$$

```
[Out] 1/8*I*hypergeom([3, n],[1+n],1/2-1/2*I*tan(f*x+e))*(d*sec(f*x+e))^(2*n)/a^2
/f/n/((a+I*a*tan(f*x+e))^n)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3586, 3603, 3568, 70}

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-2-n}\,dx = \frac{i(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\,\text{Hypergeometric2F1}\left(3,n,n+1,\frac{1}{2}(1-i\tan(e+fx))\right)}{8a^2fn}$$

```
[In] Int[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(-2 - n),x]
```

```
[Out] ((I/8)*Hypergeometric2F1[3, n, 1 + n, (1 - I*Tan[e + f*x])/2]*(d*Sec[e + f*
x])^(2*n))/(a^2*f*n*(a + I*a*Tan[e + f*x])^n)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
```

```
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\text{integral} = \left((d\sec(e+fx))^{2n}(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}\right)\int\frac{(a-ia\tan(e+fx))^n}{(a+ia\tan(e+fx))^2}\,dx$$

$$=\frac{\left((d\sec(e+fx))^{2n}(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}\right)\int\cos^4(e+fx)(a-ia\tan(e+f}{a^4}$$

$$=\frac{\left(ia(d\sec(e+fx))^{2n}(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}\right)\text{Subst}\left(\int\frac{(a+x)^{-1+n}}{(a-x)^3}\,dx,x,-ia\,\text{t}\right.}{f}$$

$$=\frac{i\,\text{Hypergeometric2F1}\left(3,n,1+n,\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}}{8a^2fn}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 165 vs. 2(66) = 132.

Time = 13.41 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.50

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-2-n}\,dx = -\frac{i2^{-3+n}e^{2ie}\left(e^{ifx}\right)^{-n}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^n\left(1+e^{2i(e+fx)}\right)^3\text{Hypergeometric2F1}\left(3,3-n,4-n,1+e^{2i(e+fx)}\right)\sec^{2-}}{f(-3+n)}$$

```
[In] Integrate[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(-2 - n),x]
```

```
[Out] ((-I)*2^(-3 + n)*E^((2*I)*e)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^n*
(1 + E^((2*I)*(e + f*x)))^3*Hypergeometric2F1[3, 3 - n, 4 - n, 1 + E^((2*I)
*(e + f*x))]*Sec[e + f*x]^(2 - n)*(d*Sec[e + f*x])^(2*n)*(Cos[f*x] + I*Sin[
f*x])^(2 + n)*(a + I*a*Tan[e + f*x])^(-2 - n))/((E^(I*f*x))^n*f*(-3 + n))
```

Maple [F]

$$\int (d\sec(fx+e))^{2n}(a+ia\tan(fx+e))^{-n-2}\,dx$$

```
[In] int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-n-2),x)
```

```
[Out] int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-n-2),x)
```

Fricas [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-2-n}\,dx = \int (d\sec(fx+e))^{2n}(ia\tan(fx+e)+a)^{-n-2}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-2-n),x, algorithm="fric
as")
```

```
[Out] integral((2*d*e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e) + 1))^(2*n)*e^(-I*e*n +
(-I*f*n - 2*I*f)*x - (n + 2)*log(2*d*e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e) +
 1)) - (n + 2)*log(a/d) - 2*I*e), x)
```

Sympy [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-2-n}\,dx$$
$$=\int (d\sec{(e+fx)})^{2n}\,(ia(\tan{(e+fx)}-i))^{-n-2}\,dx$$

```
[In] integrate((d*sec(f*x+e))**(2*n)*(a+I*a*tan(f*x+e))**(-2-n),x)
[Out] Integral((d*sec(e + f*x))**(2*n)*(I*a*(tan(e + f*x) - I))**(-n - 2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-2-n}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-2-n),x, algorithm="maxi
ma")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-2-n}\,dx$$
$$=\int (d\sec{(fx+e)})^{2n}\,(i\,a\tan{(fx+e)}+a)^{-n-2}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-2-n),x, algorithm="giac
")
[Out] integrate((d*sec(f*x + e))^(2*n)*(I*a*tan(f*x + e) + a)^(-n - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-2-n}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{2\,n}}{(a+a\tan\left(e+f\,x\right)\,1\mathrm{i})^{n+2}}\,dx$$

```
[In] int((d/cos(e + f*x))^(2*n)/(a + a*tan(e + f*x)*1i)^(n + 2),x)
[Out] int((d/cos(e + f*x))^(2*n)/(a + a*tan(e + f*x)*1i)^(n + 2), x)
```

3.502 $\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-1-n}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 66

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-1-n}\,dx = \frac{i\,\text{Hypergeometric2F1}\left(2,n,1+n,\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}}{4afn}$$

```
[Out] 1/4*I*hypergeom([2, n],[1+n],1/2-1/2*I*tan(f*x+e))*(d*sec(f*x+e))^(2*n)/a/f
/n/((a+I*a*tan(f*x+e))^n)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3586, 3603, 3568, 70}

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-1-n}\,dx = \frac{i(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\,\text{Hypergeometric2F1}\left(2,n,n+1,\frac{1}{2}(1-i\tan(e+fx))\right)}{4afn}$$

```
[In] Int[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(-1 - n),x]
```

```
[Out] ((I/4)*Hypergeometric2F1[2, n, 1 + n, (1 - I*Tan[e + f*x])/2]*(d*Sec[e + f*
x])^(2*n))/(a*f*n*(a + I*a*Tan[e + f*x])^n)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
```

```
+ 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3603

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Sec[e + f*x]^(2*m)*(c +
d*Tan[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b
*c + a*d, 0] && EqQ[a^2 + b^2, 0] && IntegerQ[m] &&  !(IGtQ[n, 0] && (LtQ[m
, 0] || GtQ[m, n]))
```

Rubi steps

$$\text{integral} = \left((d\sec(e+fx))^{2n}(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}\right)\int \frac{(a-ia\tan(e+fx))^n}{a+ia\tan(e+fx)}\,dx$$

$$= \frac{\left((d\sec(e+fx))^{2n}(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}\right)\int \cos^2(e+fx)(a-ia\tan(e+fx$$

$$= \frac{\left(ia(d\sec(e+fx))^{2n}(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}\right)\text{Subst}\left(\int \frac{(a+x)^{-1+n}}{(a-x)^2}\,dx, x, -ia\tan$$

$$= \frac{i\,\text{Hypergeometric2F1}\left(2,n,1+n,\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}}{4afn}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 165 vs. 2(66) = 132.

Time = 12.19 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.50

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-1-n}\,dx = \frac{i2^{-2+n}e^{ie}\left(e^{ifx}\right)^{-n}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^{n}\left(1+e^{2i(e+fx)}\right)^{2}\text{Hypergeometric2F1}\left(2,2-n,3-n,1+e^{2i(e+fx)}\right)\sec^{1-n}}{f(-2+n)}$$

```
[In] Integrate[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(-1 - n),x]
```

```
[Out] (I*2^(-2 + n)*E^(I*e)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^n*(1 + E^
((2*I)*(e + f*x)))^2*Hypergeometric2F1[2, 2 - n, 3 - n, 1 + E^((2*I)*(e + f
*x))]*Sec[e + f*x]^(1 - n)*(d*Sec[e + f*x])^(2*n)*(Cos[f*x] + I*Sin[f*x])^(
1 + n)*(a + I*a*Tan[e + f*x])^(-1 - n))/((E^(I*f*x))^n*f*(-2 + n))
```

Maple [F]

$$\int (d\sec(fx+e))^{2n}\,(a+ia\tan(fx+e))^{-1-n}\,dx$$

```
[In] int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-1-n),x)
```

```
[Out] int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-1-n),x)
```

Fricas [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-1-n}\,dx = \int (d\sec(fx+e))^{2n}\,(ia\tan(fx+e)+a)^{-n-1}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-1-n),x, algorithm="fric
as")
```

```
[Out] integral((2*d*e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e) + 1))^(2*n)*e^(-I*e*n +
(-I*f*n - I*f)*x - (n + 1)*log(2*d*e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e) + 1
)) - (n + 1)*log(a/d) - I*e), x)
```

Sympy [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-1-n}\,dx$$
$$=\int (d\sec{(e+fx)})^{2n}\,(ia(\tan{(e+fx)}-i))^{-n-1}\,dx$$

```
[In] integrate((d*sec(f*x+e))**(2*n)*(a+I*a*tan(f*x+e))**(-1-n),x)
[Out] Integral((d*sec(e + f*x))**(2*n)*(I*a*(tan(e + f*x) - I))**(-n - 1), x)
```

Maxima [F(-2)]

Exception generated.

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-1-n}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-1-n),x, algorithm="maxi
ma")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-1-n}\,dx$$
$$=\int (d\sec{(fx+e)})^{2\,n}\,(i\,a\tan{(fx+e)}+a)^{-n-1}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(-1-n),x, algorithm="giac
")
[Out] integrate((d*sec(f*x + e))^(2*n)*(I*a*tan(f*x + e) + a)^(-n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-1-n}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{2\,n}}{(a+a\tan\left(e+f\,x\right)\,1\mathrm{i})^{n+1}}\,dx$$

```
[In] int((d/cos(e + f*x))^(2*n)/(a + a*tan(e + f*x)*1i)^(n + 1),x)
[Out] int((d/cos(e + f*x))^(2*n)/(a + a*tan(e + f*x)*1i)^(n + 1), x)
```

3.503 $\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 63

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}\,dx = \frac{i\,\text{Hypergeometric2F1}\left(1,n,1+n,\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}}{2fn}$$

```
[Out] 1/2*I*hypergeom([1, n],[1+n],1/2-1/2*I*tan(f*x+e))*(d*sec(f*x+e))^(2*n)/f/n
/((a+I*a*tan(f*x+e))^n)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3573, 3562, 70}

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}\,dx = \frac{i(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}\,\text{Hypergeometric2F1}\left(1,n,n+1,\frac{1}{2}(1-i\tan(e+fx))\right)}{2fn}$$

```
[In] Int[(d*Sec[e + f*x])^(2*n)/(a + I*a*Tan[e + f*x])^n,x]
```

```
[Out] ((I/2)*Hypergeometric2F1[1, n, 1 + n, (1 - I*Tan[e + f*x])/2]*(d*Sec[e + f*
x])^(2*n))/(f*n*(a + I*a*Tan[e + f*x])^n)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
```

```
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3562

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-b/d, Su
bst[Int[(a + x)^(n - 1)/(a - x), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b,
 c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3573

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Dist[(a/d)^(2*IntPart[n])*(a + b*Tan[e + f*x])^Frac
Part[n]*((a - b*Tan[e + f*x])^FracPart[n]/(d*Sec[e + f*x])^(2*FracPart[n]))
, Int[1/(a - b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] &
& EqQ[a^2 + b^2, 0] && EqQ[Simplify[m/2 + n], 0]
```

Rubi steps

$$\text{integral} = \left((d\sec(e+fx))^{2n}(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n}\right)\int(a-ia\tan(e+fx))^n\,dx$$

$$=\frac{(ia(d\sec(e+fx))^{2n}(a-ia\tan(e+fx))^{-n}(a+ia\tan(e+fx))^{-n})\,\text{Subst}\left(\int\frac{(a+x)^{-1+n}}{a-x}\,dx,x,-ia\,\text{t}\right.}{f}$$

$$=\frac{i\,\text{Hypergeometric2F1}\left(1,n,1+n,\frac{1}{2}(1-i\tan(e+fx))\right)(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}}{2fn}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 150 vs. 2(63) = 126.

Time = 2.49 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.38

$$\int(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}\,dx =$$

$$-\frac{i2^{-1+n}\left(e^{ifx}\right)^{-n}\left(\frac{e^{i(e+fx)}}{1+e^{2i(e+fx)}}\right)^n\left(1+e^{2i(e+fx)}\right)\text{Hypergeometric2F1}\left(1,1-n,2-n,1+e^{2i(e+fx)}\right)\sec^{-n}(e}{f(-1+n)}$$

```
[In] Integrate[(d*Sec[e + f*x])^(2*n)/(a + I*a*Tan[e + f*x])^n,x]
[Out] ((-I)*2^(-1 + n)*(E^(I*(e + f*x))/(1 + E^((2*I)*(e + f*x))))^n*(1 + E^((2*I
)*(e + f*x)))*Hypergeometric2F1[1, 1 - n, 2 - n, 1 + E^((2*I)*(e + f*x))]*(
d*Sec[e + f*x])^(2*n)*(Cos[f*x] + I*Sin[f*x])^n)/((E^(I*f*x))^n*f*(-1 + n)*
Sec[e + f*x]^n*(a + I*a*Tan[e + f*x])^n)
```

Maple [F]

$$\int (a(i\tan(fx+e)+1))^{-n} (d\sec(fx+e))^{2n}\, dx$$

```
[In] int((d*sec(f*x+e))^(2*n)/((a+I*a*tan(f*x+e))^n),x)
```

```
[Out] int((d*sec(f*x+e))^(2*n)/((a+I*a*tan(f*x+e))^n),x)
```

Fricas [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}\, dx = \int \frac{(d\sec(fx+e))^{2n}}{(i\,a\tan(fx+e)+a)^{n}}\, dx$$

```
[In] integrate((d*sec(f*x+e))^(2*n)/((a+I*a*tan(f*x+e))^n),x, algorithm="fricas"
)
```

```
[Out] integral((2*d*e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e) + 1))^(2*n)*e^(-I*f*n*x
- I*e*n - n*log(2*d*e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e) + 1)) - n*log(a/d)
), x)
```

Sympy [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}\, dx = \int (d\sec(e+fx))^{2n} (ia(\tan(e+fx)-i))^{-n}\, dx$$

```
[In] integrate((d*sec(f*x+e))**(2*n)/((a+I*a*tan(f*x+e))**n),x)
```

```
[Out] Integral((d*sec(e + f*x))**(2*n)/(I*a*(tan(e + f*x) - I))**n, x)
```

Maxima [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}\, dx = \int \frac{(d\sec(fx+e))^{2n}}{(i\,a\tan(fx+e)+a)^{n}}\, dx$$

```
[In] integrate((d*sec(f*x+e))^(2*n)/((a+I*a*tan(f*x+e))^n),x, algorithm="maxima"
)
```

```
[Out] integrate((d*sec(f*x + e))^(2*n)/(I*a*tan(f*x + e) + a)^n, x)
```

Giac [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}\,dx=\int \frac{(d\sec(fx+e))^{2n}}{(ia\tan(fx+e)+a)^n}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(2*n)/((a+I*a*tan(f*x+e))^n),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(2*n)/(I*a*tan(f*x + e) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}\,dx=\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{2\,n}}{(a+a\tan(e+f\,x)\,1\mathrm{i})^n}\,dx$$

```
[In] int((d/cos(e + f*x))^(2*n)/(a + a*tan(e + f*x)*1i)^n,x)
[Out] int((d/cos(e + f*x))^(2*n)/(a + a*tan(e + f*x)*1i)^n, x)
```

3.504 $\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 40

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}\,dx=\frac{ia(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}}{fn}$$

```
[Out] I*a*(d*sec(f*x+e))^(2*n)/f/n/((a+I*a*tan(f*x+e))^n)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}=0.031$, Rules used = {3574}

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}\,dx=\frac{ia(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}}{fn}$$

```
[In] Int[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(1 - n),x]
```

```
[Out] (I*a*(d*Sec[e + f*x])^(2*n))/(f*n*(a + I*a*Tan[e + f*x])^n)
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rubi steps

$$\text{integral}=\frac{ia(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}}{fn}$$

Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}\,dx=\frac{ia(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}}{fn}$$

```
[In] Integrate[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(1 - n),x]
[Out] (I*a*(d*Sec[e + f*x])^(2*n))/(f*n*(a + I*a*Tan[e + f*x])^n)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 8.25 (sec) , antiderivative size = 1261, normalized size of antiderivative = 31.52

method	result	size
risch	Expression too large to display	1261

```
[In] int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(1-n),x,method=_RETURNVERBOSE)
[Out] -I/((exp(2*I*(f*x+e))+1)^n)/f/n*2^n*(d^n)^2*a/(a^n)*exp(-1/2*I*Pi*(-n*csgn(
I/(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e)))*csgn(I*a/(exp(2*I*(f*x+e))+1)*exp(
2*I*(f*x+e)))*csgn(I*a)+2*n*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csg
n(I*d)*csgn(I*d*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))+2*n*csgn(I/(exp(2*I*(f
*x+e))+1))*csgn(I*exp(I*(f*x+e)))*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1
))-n*csgn(I*exp(I*(f*x+e)))^2*csgn(I*exp(2*I*(f*x+e)))+2*n*csgn(I*exp(I*(f*
x+e))/(exp(2*I*(f*x+e))+1))^3+2*n*csgn(I*d*exp(I*(f*x+e))/(exp(2*I*(f*x+e))
+1))^3-n*csgn(I/(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e)))^3-n*csgn(I/(exp(2*I*
(f*x+e))+1))*csgn(I*exp(2*I*(f*x+e)))*csgn(I/(exp(2*I*(f*x+e))+1)*exp(2*I*(
f*x+e)))+n*csgn(I*exp(2*I*(f*x+e)))*csgn(I/(exp(2*I*(f*x+e))+1)*exp(2*I*(f*
x+e)))^2+csgn(I/(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e)))*csgn(I*a/(exp(2*I*(f
*x+e))+1)*exp(2*I*(f*x+e)))*csgn(I*a)+csgn(I/(exp(2*I*(f*x+e))+1)*exp(2*I*(
f*x+e)))^3-2*n*csgn(I/(exp(2*I*(f*x+e))+1))*csgn(I*exp(I*(f*x+e))/(exp(2*I*
(f*x+e))+1))^2+csgn(I/(exp(2*I*(f*x+e))+1))*csgn(I*exp(2*I*(f*x+e)))*csgn(I
/(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e)))-csgn(I*exp(2*I*(f*x+e)))*csgn(I/(ex
p(2*I*(f*x+e))+1)*exp(2*I*(f*x+e)))^2-csgn(I*a/(exp(2*I*(f*x+e))+1)*exp(2*I
*(f*x+e)))^2*csgn(I*a)+n*csgn(I*a/(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e)))^2*
csgn(I*a)+n*csgn(I/(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e)))*csgn(I*a/(exp(2*I
*(f*x+e))+1)*exp(2*I*(f*x+e)))^2+csgn(I*exp(2*I*(f*x+e)))^3+csgn(I*a/(exp(2
*I*(f*x+e))+1)*exp(2*I*(f*x+e)))^3+n*csgn(I/(exp(2*I*(f*x+e))+1))*csgn(I/(e
xp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e)))^2+2*n*csgn(I*exp(I*(f*x+e)))*csgn(I*ex
p(2*I*(f*x+e)))^2-csgn(I/(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e)))*csgn(I*a/(e
xp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e)))^2-csgn(I/(exp(2*I*(f*x+e))+1))*csgn(I/
(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e)))^2-2*n*csgn(I*exp(I*(f*x+e)))*csgn(I*
```

```
exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2-2*n*csgn(I*exp(I*(f*x+e))/(exp(2*I*(
f*x+e))+1))*csgn(I*d*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2-2*n*csgn(I*d)*c
sgn(I*d*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2-n*csgn(I*exp(2*I*(f*x+e)))^3
-n*csgn(I*a/(exp(2*I*(f*x+e))+1)*exp(2*I*(f*x+e)))^3+csgn(I*exp(I*(f*x+e)))
^2*csgn(I*exp(2*I*(f*x+e)))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(36) = 72$.

Time = 0.25 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.88

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}\,dx = \frac{\left(\frac{2\,de^{(i\,fx+i\,e)}}{e^{(2i\,fx+2i\,e)}+1}\right)^{2\,n}\left(i\,e^{(2i\,fx+2i\,e)}+i\right)e^{\left(-i\,en+(-i\,fn+i\,f)x-2i\,fx-(n-1)\log\left(\frac{2\,de^{(i\,fx+i\,e)}}{e^{(2i\,fx+2i\,e)}+1}\right)-(n-1)\log\left(\frac{a}{d}\right)-i\,e\right)}}{2\,fn}$$

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(1-n),x, algorithm="frica
s")
```

```
[Out] 1/2*(2*d*e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e) + 1))^(2*n)*(I*e^(2*I*f*x + 2
*I*e) + I)*e^(-I*e*n + (-I*f*n + I*f)*x - 2*I*f*x - (n - 1)*log(2*d*e^(I*f*
x + I*e)/(e^(2*I*f*x + 2*I*e) + 1)) - (n - 1)*log(a/d) - I*e)/(f*n)
```

Sympy [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}\,dx = \int (d\sec{(e+fx)})^{2n}\,(ia(\tan{(e+fx)}-i))^{1-n}\,dx$$

```
[In] integrate((d*sec(f*x+e))**(2*n)*(a+I*a*tan(f*x+e))**(1-n),x)
```

```
[Out] Integral((d*sec(e + f*x))**(2*n)*(I*a*(tan(e + f*x) - I))**(1 - n), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(36) = 72.

Time = 0.34 (sec) , antiderivative size = 137, normalized size of antiderivative = 3.42

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}\,dx$$
$$= \frac{i\,a^{-n+1}d^{2\,n}e^{\left(-n\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)-n\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)-n\log\left(-\frac{2i\,\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right)+2\,n\log\left(-\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-1\right)\right)}}{fn}$$

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(1-n),x, algorithm="maxim
a")
```

```
[Out] I*a^(-n + 1)*d^(2*n)*e^(-n*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) - n*log
(sin(f*x + e)/(cos(f*x + e) + 1) - 1) - n*log(-2*I*sin(f*x + e)/(cos(f*x +
e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1) + 2*n*log(-sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 - 1))/(f*n)
```

Giac [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}\,dx$$
$$= \int (d\sec(fx+e))^{2\,n}(i\,a\tan(fx+e)+a)^{-n+1}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(1-n),x, algorithm="giac"
)
```

```
[Out] integrate((d*sec(f*x + e))^(2*n)*(I*a*tan(f*x + e) + a)^(-n + 1), x)
```

Mupad [B] (verification not implemented)

Time = 5.44 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.55

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}\,dx = \frac{a\left(\frac{d}{\cos(e+f\,x)}\right)^{2\,n}1\mathrm{i}}{f\,n\left(\frac{a\,(\cos(2\,e+2\,f\,x)+1+\sin(2\,e+2\,f\,x)\,1\mathrm{i})}{2\cos(e+f\,x)^2}\right)^n}$$

```
[In] int((d/cos(e + f*x))^(2*n)*(a + a*tan(e + f*x)*1i)^(1 - n),x)
```

```
[Out] (a*(d/cos(e + f*x))^(2*n)*1i)/(f*n*((a*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)
*1i + 1))/(2*cos(e + f*x)^2))^n)
```

3.505 $\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 92

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}\,dx = \frac{ia(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}}{f(1+n)} + \frac{2ia^2(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}}{fn(1+n)}$$

```
[Out] I*a*(d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(1-n)/f/(1+n)+2*I*a^2*(d*sec(f*
x+e))^(2*n)/f/n/(1+n)/((a+I*a*tan(f*x+e))^n)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3575, 3574}

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}\,dx = \frac{2ia^2(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}}{fn(n+1)} + \frac{ia(a+ia\tan(e+fx))^{1-n}(d\sec(e+fx))^{2n}}{f(n+1)}$$

```
[In] Int[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(2 - n),x]
```

```
[Out] (I*a*(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(1 - n))/(f*(1 + n)) + (
(2*I)*a^2*(d*Sec[e + f*x])^(2*n))/(f*n*(1 + n)*(a + I*a*Tan[e + f*x])^n)
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\text{integral} = \frac{ia(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}}{f(1+n)} + \frac{(2a)\int(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}\,dx}{1+n}$$

$$= \frac{ia(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}}{f(1+n)} + \frac{2ia^2(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}}{fn(1+n)}$$

Mathematica [A] (verified)

Time = 2.44 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.66

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}\,dx = -\frac{a^2(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}(-i(2+n)+n\tan(e+fx))}{fn(1+n)}$$

```
[In] Integrate[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(2 - n),x]
[Out] -((a^2*(d*Sec[e + f*x])^(2*n)*((-I)*(2 + n) + n*Tan[e + f*x]))/(f*n*(1 + n)
*(a + I*a*Tan[e + f*x])^n))
```

Maple [F]

$$\int (d\sec(fx+e))^{2n}(a+ia\tan(fx+e))^{2-n}\,dx$$

```
[In] int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(2-n),x)
```

```
[Out] int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(2-n),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.51

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}\,dx = \frac{\left((in+i)e^{(4ifx+4ie)}+(in+2i)e^{(2ifx+2ie)}+i\right)\left(\frac{2de^{(ifx+ie)}}{e^{(2ifx+2ie)}+1}\right)^{2n}e^{\left(-ien+(-ifn+2if)x-4ifx-(n-2)\log\left(\frac{2de^{(ifx+ie)}}{e^{(2ifx+2ie)}+1}\right)\right.}}{2(fn^2+fn)}$$

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(2-n),x, algorithm="frica
s")
```

```
[Out] 1/2*((I*n + I)*e^(4*I*f*x + 4*I*e) + (I*n + 2*I)*e^(2*I*f*x + 2*I*e) + I)*(
2*d*e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e) + 1))^(2*n)*e^(-I*e*n + (-I*f*n +
2*I*f)*x - 4*I*f*x - (n - 2)*log(2*d*e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e) +
 1)) - (n - 2)*log(a/d) - 2*I*e)/(f*n^2 + f*n)
```

Sympy [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}\,dx = \int (d\sec(e+fx))^{2n}\,(ia(\tan(e+fx)-i))^{2-n}\,dx$$

```
[In] integrate((d*sec(f*x+e))**(2*n)*(a+I*a*tan(f*x+e))**(2-n),x)
```

```
[Out] Integral((d*sec(e + f*x))**(2*n)*(I*a*(tan(e + f*x) - I))**(2 - n), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(84) = 168.

Time = 0.44 (sec) , antiderivative size = 304, normalized size of antiderivative = 3.30

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}\,dx$$
$$= \frac{2^{n+1}a^2d^{2\,n}\cos\left(n\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,fx+2\,e\right)+1\right)\right)-i\cdot 2^{n+1}a^2d^{2\,n}\sin\left(n\arctan\left(\sin\left(2\,fx+2\right.\right.\right.}{\left(-i\,a^nn^2-i\,a^nn+\left(-\right.\right.}$$

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(2-n),x, algorithm="maxim
a")
```

```
[Out] (2^(n + 1)*a^2*d^(2*n)*cos(n*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1
)) - I*2^(n + 1)*a^2*d^(2*n)*sin(n*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*
e) + 1)) + 2*(a^2*d^(2*n)*n + a^2*d^(2*n))*2^n*cos(-2*f*x + n*arctan2(sin(2
*f*x + 2*e), cos(2*f*x + 2*e) + 1) - 2*e) + 2*(-I*a^2*d^(2*n)*n - I*a^2*d^(
2*n))*2^n*sin(-2*f*x + n*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1) -
2*e))/((-I*a^n*n^2 - I*a^n*n + (-I*a^n*n^2 - I*a^n*n)*cos(2*f*x + 2*e) + (a
^n*n^2 + a^n*n)*sin(2*f*x + 2*e))*(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2
+ 2*cos(2*f*x + 2*e) + 1)^(1/2*n)*f)
```

Giac [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}\,dx = \int \left(d\sec\left(fx+e\right)\right)^{2\,n}\left(i\,a\tan\left(fx+e\right)+a\right)^{-n+2}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(2-n),x, algorithm="giac"
)
```

```
[Out] integrate((d*sec(f*x + e))^(2*n)*(I*a*tan(f*x + e) + a)^(-n + 2), x)
```

Mupad [B] (verification not implemented)

Time = 9.24 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.83

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}\,dx$$
$$= -e^{-e\,4i-f\,x\,4i}\left(\frac{d}{\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}}\right)^{2\,n}\left(\frac{\left(a-\frac{a\,\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)\,1i}{e^{e\,2i+f\,x\,2i}+1}\right)^{2-n}}{2\,f\,n\,\left(n\,1i+1i\right)}+\frac{e^{e\,2i+f\,x\,2i}\,\left(a-\frac{a\,\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)\,1i}{e^{e\,2i+f\,x\,2i}+1}\right)^{2-n}\,\left(n+2\right)}{2\,f\,n\,\left(n\,1i+1i\right)}+\frac{e^{e\,4i+f\,x\,4i}\,\left(a-\frac{a\,\left(e^{e\,2i+f\,x\,2i}\,1i-i\right)\,1i}{e^{e\,2i+f\,x\,2i}+1}\right)^{2-n}\,\left(n+1\right)}{2\,f\,n\,\left(n\,1i+1i\right)}\right)$$

```
[In] int((d/cos(e + f*x))^(2*n)*(a + a*tan(e + f*x)*1i)^(2 - n),x)
```

```
[Out] -exp(- e*4i - f*x*4i)*(d/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(
2*n)*((a - (a*(exp(e*2i + f*x*2i)*1i - 1i)*1i)/(exp(e*2i + f*x*2i) + 1))^(2
 - n)/(2*f*n*(n*1i + 1i)) + (exp(e*2i + f*x*2i)*(a - (a*(exp(e*2i + f*x*2i)
*1i - 1i)*1i)/(exp(e*2i + f*x*2i) + 1))^(2 - n)*(n + 2))/(2*f*n*(n*1i + 1i)
) + (exp(e*4i + f*x*4i)*(a - (a*(exp(e*2i + f*x*2i)*1i - 1i)*1i)/(exp(e*2i
+ f*x*2i) + 1))^(2 - n)*(n + 1))/(2*f*n*(n*1i + 1i)))
```

3.506 $\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{3-n}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 148

$$\begin{aligned}\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{3-n}\,dx &= \frac{4ia^2(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}}{f(2+3n+n^2)}\\ &\quad+\frac{ia(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}}{f(2+n)}\\ &\quad+\frac{8ia^3(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}}{fn(2+3n+n^2)}\end{aligned}$$

```
[Out] 4*I*a^2*(d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(1-n)/f/(n^2+3*n+2)+I*a*(d*
sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(2-n)/f/(2+n)+8*I*a^3*(d*sec(f*x+e))^(
2*n)/f/n/(n^2+3*n+2)/((a+I*a*tan(f*x+e))^n)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {3575, 3574}

$$\begin{aligned}\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{3-n}\,dx &= \frac{8ia^3(a+ia\tan(e+fx))^{-n}(d\sec(e+fx))^{2n}}{fn(n^2+3n+2)}\\ &\quad+\frac{4ia^2(a+ia\tan(e+fx))^{1-n}(d\sec(e+fx))^{2n}}{f(n^2+3n+2)}\\ &\quad+\frac{ia(a+ia\tan(e+fx))^{2-n}(d\sec(e+fx))^{2n}}{f(n+2)}\end{aligned}$$

```
[In] Int[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(3 - n),x]
```

```
[Out] ((4*I)*a^2*(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(1 - n))/(f*(2 + 3
*n + n^2)) + (I*a*(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(2 - n))/(f
*(2 + n)) + ((8*I)*a^3*(d*Sec[e + f*x])^(2*n))/(f*n*(2 + 3*n + n^2)*(a + I*
a*Tan[e + f*x])^n)
```

Rule 3574

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[2*b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^
(n - 1)/(f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0]
&& EqQ[Simplify[m/2 + n - 1], 0]
```

Rule 3575

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m, n}, x] && EqQ[a^2 + b^2, 0] && IGtQ[Simplify[m/2 + n - 1], 0] &&  !Integ
erQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{ia(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}}{f(2+n)} \\
&\quad + \frac{(4a)\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}\,dx}{2+n} \\
&= \frac{4ia^2(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}}{f\left(2+3n+n^2\right)} \\
&\quad + \frac{ia(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}}{f(2+n)} \\
&\quad + \frac{(8a^2)\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}\,dx}{2+3n+n^2} \\
&= \frac{4ia^2(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{1-n}}{f\left(2+3n+n^2\right)} \\
&\quad + \frac{ia(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{2-n}}{f(2+n)} \\
&\quad + \frac{8ia^3(d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{-n}}{fn\left(2+3n+n^2\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 3.30 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.87

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{3-n}\,dx$$
$$= \frac{ia^3\sec^2(e+fx)(d\sec(e+fx))^{2n}(\cos(3fx)+i\sin(3fx))\,(2(2+n)+(4+3n+n^2)\cos(2(e+fx))+in(3}{fn(1+n)(2+n)(\cos(fx)+i\sin(fx))^3}$$

```
[In] Integrate[(d*Sec[e + f*x])^(2*n)*(a + I*a*Tan[e + f*x])^(3 - n),x]
```

```
[Out] (I*a^3*Sec[e + f*x]^2*(d*Sec[e + f*x])^(2*n)*(Cos[3*f*x] + I*Sin[3*f*x])*(2
*(2 + n) + (4 + 3*n + n^2)*Cos[2*(e + f*x)] + I*n*(3 + n)*Sin[2*(e + f*x)])
)/(f*n*(1 + n)*(2 + n)*(Cos[f*x] + I*Sin[f*x])^3*(a + I*a*Tan[e + f*x])^n)
```

Maple [F]

$$\int (d\sec(fx+e))^{2n}\,(a+ia\tan(fx+e))^{3-n}\,dx$$

```
[In] int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(3-n),x)
```

```
[Out] int((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(3-n),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.16

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{3-n}\,dx$$
$$= \frac{((i\,n^2+3i\,n+2i)e^{(6i\,fx+6i\,e)}+(i\,n^2+5i\,n+6i)e^{(4i\,fx+4i\,e)}-2\,(-i\,n-3i)e^{(2i\,fx+2i\,e)}+2i)\left(\frac{2\,de^{(i\,fx+i\,e)}}{e^{(2i\,fx+2i\,e)}+1}\right)^{2}}{2\,(fn^3+3\,fn^2+2\,fn)}$$

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(3-n),x, algorithm="frica
s")
```

```
[Out] 1/2*((I*n^2 + 3*I*n + 2*I)*e^(6*I*f*x + 6*I*e) + (I*n^2 + 5*I*n + 6*I)*e^(4
*I*f*x + 4*I*e) - 2*(-I*n - 3*I)*e^(2*I*f*x + 2*I*e) + 2*I)*(2*d*e^(I*f*x +
 I*e)/(e^(2*I*f*x + 2*I*e) + 1))^(2*n)*e^(-I*e*n + (-I*f*n + 3*I*f)*x - 6*I
*f*x - (n - 3)*log(2*d*e^(I*f*x + I*e)/(e^(2*I*f*x + 2*I*e) + 1)) - (n - 3)
*log(a/d) - 3*I*e)/(f*n^3 + 3*f*n^2 + 2*f*n)
```

Sympy [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{3-n}\,dx = \int (d\sec{(e+fx)})^{2n}\,(ia(\tan{(e+fx)}-i))^{3-n}\,dx$$

```
[In] integrate((d*sec(f*x+e))**(2*n)*(a+I*a*tan(f*x+e))**(3-n),x)
```

```
[Out] Integral((d*sec(e + f*x))**(2*n)*(I*a*(tan(e + f*x) - I))**(3 - n), x)
```

Maxima [F(-2)]

Exception generated.

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{3-n}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(3-n),x, algorithm="maxim
a")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{3-n}\,dx = \int (d\sec{(fx+e)})^{2\,n}\,(i\,a\tan{(fx+e)}+a)^{-n+3}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(2*n)*(a+I*a*tan(f*x+e))^(3-n),x, algorithm="giac"
)
```

```
[Out] integrate((d*sec(f*x + e))^(2*n)*(I*a*tan(f*x + e) + a)^(-n + 3), x)
```

Mupad [B] (verification not implemented)

Time = 13.76 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.17

$$\int (d\sec(e+fx))^{2n}(a+ia\tan(e+fx))^{3-n}\,dx$$
$$= -(\cos(6e+6fx)-\sin(6e+6fx)\,1\mathrm{i})\left(\frac{d}{\cos(e+fx)}\right)^{2n}\left(\frac{\left(a+\frac{a\sin(e+fx)\,1\mathrm{i}}{\cos(e+fx)}\right)^{3-n}}{f\,n\,(n^2\,1\mathrm{i}+n\,3\mathrm{i}+2\mathrm{i})}\right.$$
$$+\frac{(\cos(4e+4fx)+\sin(4e+4fx)\,1\mathrm{i})\left(a+\frac{a\sin(e+fx)\,1\mathrm{i}}{\cos(e+fx)}\right)^{3-n}(n^2+5n+6)}{2\,f\,n\,(n^2\,1\mathrm{i}+n\,3\mathrm{i}+2\mathrm{i})}$$
$$+\frac{(\cos(6e+6fx)+\sin(6e+6fx)\,1\mathrm{i})\left(a+\frac{a\sin(e+fx)\,1\mathrm{i}}{\cos(e+fx)}\right)^{3-n}(n^2+3n+2)}{2\,f\,n\,(n^2\,1\mathrm{i}+n\,3\mathrm{i}+2\mathrm{i})}$$
$$\left.+\frac{(2n+6)\,(\cos(2e+2fx)+\sin(2e+2fx)\,1\mathrm{i})\left(a+\frac{a\sin(e+fx)\,1\mathrm{i}}{\cos(e+fx)}\right)^{3-n}}{2\,f\,n\,(n^2\,1\mathrm{i}+n\,3\mathrm{i}+2\mathrm{i})}\right)$$

```
[In] int((d/cos(e + f*x))^(2*n)*(a + a*tan(e + f*x)*1i)^(3 - n),x)
```

```
[Out] -(cos(6*e + 6*f*x) - sin(6*e + 6*f*x)*1i)*(d/cos(e + f*x))^(2*n)*((a + (a*s
in(e + f*x)*1i)/cos(e + f*x))^(3 - n)/(f*n*(n*3i + n^2*1i + 2i)) + ((cos(4*
e + 4*f*x) + sin(4*e + 4*f*x)*1i)*(a + (a*sin(e + f*x)*1i)/cos(e + f*x))^(3
 - n)*(5*n + n^2 + 6))/(2*f*n*(n*3i + n^2*1i + 2i)) + ((cos(6*e + 6*f*x) +
sin(6*e + 6*f*x)*1i)*(a + (a*sin(e + f*x)*1i)/cos(e + f*x))^(3 - n)*(3*n +
n^2 + 2))/(2*f*n*(n*3i + n^2*1i + 2i)) + ((2*n + 6)*(cos(2*e + 2*f*x) + sin
(2*e + 2*f*x)*1i)*(a + (a*sin(e + f*x)*1i)/cos(e + f*x))^(3 - n))/(2*f*n*(n
*3i + n^2*1i + 2i)))
```

3.507 $\int \sec^6(c+dx)(a+b\tan(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 60

$$\int \sec^6(c+dx)(a+b\tan(c+dx))\,dx=\frac{b\sec^6(c+dx)}{6d}+\frac{a\tan(c+dx)}{d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan^5(c+dx)}{5d}$$

```
[Out] 1/6*b*sec(d*x+c)^6/d+a*tan(d*x+c)/d+2/3*a*tan(d*x+c)^3/d+1/5*a*tan(d*x+c)^5
/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3567, 3852}

$$\int \sec^6(c+dx)(a+b\tan(c+dx))\,dx=\frac{a\tan^5(c+dx)}{5d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\sec^6(c+dx)}{6d}$$

```
[In] Int[Sec[c + d*x]^6*(a + b*Tan[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^6)/(6*d) + (a*Tan[c + d*x])/d + (2*a*Tan[c + d*x]^3)/(3*d)
 + (a*Tan[c + d*x]^5)/(5*d)
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
```

```
| NeQ[a^2 + b^2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{b\sec^6(c+dx)}{6d} + a\int \sec^6(c+dx)\,dx \\
&= \frac{b\sec^6(c+dx)}{6d} - \frac{a\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x, -\tan(c+dx)\right)}{d} \\
&= \frac{b\sec^6(c+dx)}{6d} + \frac{a\tan(c+dx)}{d} + \frac{2a\tan^3(c+dx)}{3d} + \frac{a\tan^5(c+dx)}{5d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

$$
\int \sec^6(c+dx)(a+b\tan(c+dx))\,dx = \frac{b\sec^6(c+dx)}{6d} + \frac{a\left(\tan(c+dx)+\frac{2}{3}\tan^3(c+dx)+\frac{1}{5}\tan^5(c+dx)\right)}{d}
$$

```
[In] Integrate[Sec[c + d*x]^6*(a + b*Tan[c + d*x]),x]
[Out] (b*Sec[c + d*x]^6)/(6*d) + (a*(Tan[c + d*x] + (2*Tan[c + d*x]^3)/3 + Tan[c
+ d*x]^5/5))/d
```

Maple [A] (verified)

Time = 14.75 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.15

method	result	size
derivativedivides	$\frac{\frac{b\left(\tan^6(dx+c)\right)}{6}+\frac{a\left(\tan^5(dx+c)\right)}{5}+\frac{\left(\tan^4(dx+c)\right)b}{2}+\frac{2a\left(\tan^3(dx+c)\right)}{3}+\frac{b\left(\tan^2(dx+c)\right)}{2}+a\tan(dx+c)}{d}$	69
default	$\frac{\frac{b\left(\tan^6(dx+c)\right)}{6}+\frac{a\left(\tan^5(dx+c)\right)}{5}+\frac{\left(\tan^4(dx+c)\right)b}{2}+\frac{2a\left(\tan^3(dx+c)\right)}{3}+\frac{b\left(\tan^2(dx+c)\right)}{2}+a\tan(dx+c)}{d}$	69
risch	$\frac{\frac{32ia\,e^{6i(dx+c)}}{3}+\frac{32b\,e^{6i(dx+c)}}{3}+16ia\,e^{4i(dx+c)}+\frac{32ia\,e^{2i(dx+c)}}{5}+\frac{16ia}{15}}{d\left(e^{2i(dx+c)}+1\right)^6}$	75

```
[In] int(sec(d*x+c)^6*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/6*b*tan(d*x+c)^6+1/5*a*tan(d*x+c)^5+1/2*tan(d*x+c)^4*b+2/3*a*tan(d*x
+c)^3+1/2*b*tan(d*x+c)^2+a*tan(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int \sec^6(c+dx)(a+b\tan(c+dx))\,dx = \frac{2\left(8\,a\cos\left(dx+c\right)^5+4\,a\cos\left(dx+c\right)^3+3\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)+5\,b}{30\,d\cos\left(dx+c\right)^6}$$

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/30*(2*(8*a*cos(d*x + c)^5 + 4*a*cos(d*x + c)^3 + 3*a*cos(d*x + c))*sin(d*
x + c) + 5*b)/(d*cos(d*x + c)^6)
```

Sympy [A] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \sec^6(c+dx)(a+b\tan(c+dx))\,dx = \begin{cases} \frac{a\left(\frac{\tan^5(c+dx)}{5}+\frac{2\tan^3(c+dx)}{3}+\tan(c+dx)\right)+\frac{b\sec^6(c+dx)}{6}}{d} & \text{for}\ d\neq 0 \\ x(a+b\tan(c))\sec^6(c) & \text{otherwise} \end{cases}$$

```
[In] integrate(sec(d*x+c)**6*(a+b*tan(d*x+c)),x)
```

```
[Out] Piecewise(((a*(tan(c + d*x)**5/5 + 2*tan(c + d*x)**3/3 + tan(c + d*x)) + b*
sec(c + d*x)**6/6)/d, Ne(d, 0)), (x*(a + b*tan(c))*sec(c)**6, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.17

$$\int \sec^6(c+dx)(a+b\tan(c+dx))\,dx = \frac{5\,b\tan\left(dx+c\right)^6+6\,a\tan\left(dx+c\right)^5+15\,b\tan\left(dx+c\right)^4+20\,a\tan\left(dx+c\right)^3+15\,b\tan\left(dx+c\right)^2+30\,a\tan}{30\,d}$$

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/30*(5*b*tan(d*x + c)^6 + 6*a*tan(d*x + c)^5 + 15*b*tan(d*x + c)^4 + 20*a*
tan(d*x + c)^3 + 15*b*tan(d*x + c)^2 + 30*a*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.17

$$\int \sec^6(c+dx)(a+b\tan(c+dx))\,dx = \frac{5\,b\tan\left(dx+c\right)^6+6\,a\tan\left(dx+c\right)^5+15\,b\tan\left(dx+c\right)^4+20\,a\tan\left(dx+c\right)^3+15\,b\tan\left(dx+c\right)^2+30\,a\tan}{30\,d}$$

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/30*(5*b*tan(d*x + c)^6 + 6*a*tan(d*x + c)^5 + 15*b*tan(d*x + c)^4 + 20*a*
tan(d*x + c)^3 + 15*b*tan(d*x + c)^2 + 30*a*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.13

$$\int \sec^6(c+dx)(a+b\tan(c+dx))\,dx = \frac{\frac{b\tan(c+d\,x)^6}{6}+\frac{a\tan(c+d\,x)^5}{5}+\frac{b\tan(c+d\,x)^4}{2}+\frac{2\,a\tan(c+d\,x)^3}{3}+\frac{b\tan(c+d\,x)^2}{2}+a\tan(c+d\,x)}{d}$$

```
[In] int((a + b*tan(c + d*x))/cos(c + d*x)^6,x)
```

```
[Out] (a*tan(c + d*x) + (2*a*tan(c + d*x)^3)/3 + (a*tan(c + d*x)^5)/5 + (b*tan(c
+ d*x)^2)/2 + (b*tan(c + d*x)^4)/2 + (b*tan(c + d*x)^6)/6)/d
```

3.508 $\int \sec^5(c+dx)(a+b\tan(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 74

$$\int \sec^5(c+dx)(a+b\tan(c+dx))\,dx = \frac{3a\text{arctanh}(\sin(c+dx))}{8d}+\frac{b\sec^5(c+dx)}{5d}+\frac{3a\sec(c+dx)\tan(c+dx)}{8d}+\frac{a\sec^3(c+dx)\tan(c+dx)}{4d}$$

```
[Out] 3/8*a*arctanh(sin(d*x+c))/d+1/5*b*sec(d*x+c)^5/d+3/8*a*sec(d*x+c)*tan(d*x+c
)/d+1/4*a*sec(d*x+c)^3*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3567, 3853, 3855}

$$\int \sec^5(c+dx)(a+b\tan(c+dx))\,dx = \frac{3a\text{arctanh}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a\tan(c+dx)\sec(c+dx)}{8d}+\frac{b\sec^5(c+dx)}{5d}$$

```
[In] Int[Sec[c + d*x]^5*(a + b*Tan[c + d*x]),x]
```

```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) + (b*Sec[c + d*x]^5)/(5*d) + (3*a*Sec[c +
 d*x]*Tan[c + d*x])/(8*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b\sec^5(c+dx)}{5d} + a\int \sec^5(c+dx)\,dx \\
&= \frac{b\sec^5(c+dx)}{5d} + \frac{a\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}(3a)\int \sec^3(c+dx)\,dx \\
&= \frac{b\sec^5(c+dx)}{5d} + \frac{3a\sec(c+dx)\tan(c+dx)}{8d} \\
&\quad + \frac{a\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{8}(3a)\int \sec(c+dx)\,dx \\
&= \frac{3a\text{arctanh}(\sin(c+dx))}{8d} + \frac{b\sec^5(c+dx)}{5d} \\
&\quad + \frac{3a\sec(c+dx)\tan(c+dx)}{8d} + \frac{a\sec^3(c+dx)\tan(c+dx)}{4d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\begin{aligned}
\int \sec^5(c+dx)(a+b\tan(c+dx))\,dx &= \frac{3a\text{arctanh}(\sin(c+dx))}{8d} + \frac{b\sec^5(c+dx)}{5d} \\
&\quad + \frac{3a\sec(c+dx)\tan(c+dx)}{8d} \\
&\quad + \frac{a\sec^3(c+dx)\tan(c+dx)}{4d}
\end{aligned}$$

```
[In] Integrate[Sec[c + d*x]^5*(a + b*Tan[c + d*x]),x]
```

[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) + (b*Sec[c + d*x]^5)/(5*d) + (3*a*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)

Maple [A] (verified)

Time = 8.66 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{a\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{b}{5\cos(dx+c)^5}}{d}$
default	$\frac{a\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{b}{5\cos(dx+c)^5}}{d}$
risch	$\frac{-15ia\,e^{9i(dx+c)}-70ia\,e^{7i(dx+c)}+128b\,e^{5i(dx+c)}+70ia\,e^{3i(dx+c)}+15ia\,e^{i(dx+c)}}{20d\left(e^{2i(dx+c)}+1\right)^5}-\frac{3a\ln\left(e^{i(dx+c)}-i\right)}{8d}+\frac{3a\ln\left(e^{i(dx+c)}+\right.}{8d}$

[In] int(sec(d*x+c)^5*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(a*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))+1/5*b/cos(d*x+c)^5)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.19

$$\int \sec^5(c+dx)(a+b\tan(c+dx))\,dx$$
$$=\frac{15\,a\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-15\,a\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right)+10\left(3\,a\cos\left(dx+c\right)^3\right.}{80\,d\cos\left(dx+c\right)^5}$$

[In] integrate(sec(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="fricas")

[Out] 1/80*(15*a*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*a*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 10*(3*a*cos(d*x + c)^3 + 2*a*cos(d*x + c))*sin(d*x + c) + 16*b)/(d*cos(d*x + c)^5)

Sympy [F]

$$\int \sec^5(c+dx)(a+b\tan(c+dx))\,dx = \int (a+b\tan(c+dx))\sec^5(c+dx)\,dx$$

```
[In] integrate(sec(d*x+c)**5*(a+b*tan(d*x+c)),x)
[Out] Integral((a + b*tan(c + d*x))*sec(c + d*x)**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.16

$$\int \sec^5(c+dx)(a+b\tan(c+dx))\,dx = -\frac{5\,a\left(\frac{2\,\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-\frac{16\,b}{\cos(dx+c)^5}}{80\,d}$$

```
[In] integrate(sec(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="maxima")
[Out] -1/80*(5*a*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d
*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 16*b/
cos(d*x + c)^5)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(66) = 132.

Time = 0.41 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.91

$$\int \sec^5(c+dx)(a+b\tan(c+dx))\,dx = \frac{15\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\,\left(25\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-40\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-1\right.}{}}{40\,d}$$

```
[In] integrate(sec(d*x+c)^5*(a+b*tan(d*x+c)),x, algorithm="giac")
[Out] 1/40*(15*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*a*log(abs(tan(1/2*d*x +
1/2*c) - 1)) + 2*(25*a*tan(1/2*d*x + 1/2*c)^9 - 40*b*tan(1/2*d*x + 1/2*c)^8
 - 10*a*tan(1/2*d*x + 1/2*c)^7 - 80*b*tan(1/2*d*x + 1/2*c)^4 + 10*a*tan(1/2
*d*x + 1/2*c)^3 - 25*a*tan(1/2*d*x + 1/2*c) - 8*b)/(tan(1/2*d*x + 1/2*c)^2
- 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 7.98 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.36

$$\int \sec^5(c+dx)(a+b\tan(c+dx))\,dx = \frac{3\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d} - \frac{-\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2}+4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}+\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{2\,b}{5}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

```
[In] int((a + b*tan(c + d*x))/cos(c + d*x)^5,x)
```

```
[Out] (3*a*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((2*b)/5 + (5*a*tan(c/2 + (d*x)/2))
/4 - (a*tan(c/2 + (d*x)/2)^3)/2 + (a*tan(c/2 + (d*x)/2)^7)/2 - (5*a*tan(c/2
 + (d*x)/2)^9)/4 + 4*b*tan(c/2 + (d*x)/2)^4 + 2*b*tan(c/2 + (d*x)/2)^8)/(d*
(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6
 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.509 $\int \sec^4(c+dx)(a+b\tan(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 44

$$\int \sec^4(c+dx)(a+b\tan(c+dx))\,dx=\frac{b\sec^4(c+dx)}{4d}+\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}$$

```
[Out] 1/4*b*sec(d*x+c)^4/d+a*tan(d*x+c)/d+1/3*a*tan(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3567, 3852}

$$\int \sec^4(c+dx)(a+b\tan(c+dx))\,dx=\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\sec^4(c+dx)}{4d}$$

```
[In] Int[Sec[c + d*x]^4*(a + b*Tan[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^4)/(4*d) + (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{b\sec^4(c+dx)}{4d} + a\int \sec^4(c+dx)\,dx\\ &= \frac{b\sec^4(c+dx)}{4d} - \frac{a\text{Subst}\left(\int (1+x^2)\ dx, x, -\tan(c+dx)\right)}{d}\\ &= \frac{b\sec^4(c+dx)}{4d} + \frac{a\tan(c+dx)}{d} + \frac{a\tan^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \sec^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{b\sec^4(c+dx)}{4d} + \frac{a\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d}$$

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Tan[c + d*x]),x]
[Out] (b*Sec[c + d*x]^4)/(4*d) + (a*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Maple [A] (verified)

Time = 3.54 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$\frac{\frac{\left(\tan^4(dx+c)\right)b}{4}+\frac{a\left(\tan^3(dx+c)\right)}{3}+\frac{b\left(\tan^2(dx+c)\right)}{2}+a\tan(dx+c)}{d}$	47
default	$\frac{\frac{\left(\tan^4(dx+c)\right)b}{4}+\frac{a\left(\tan^3(dx+c)\right)}{3}+\frac{b\left(\tan^2(dx+c)\right)}{2}+a\tan(dx+c)}{d}$	47
risch	$\frac{4ia\,\mathrm{e}^{4i(dx+c)}+4b\,\mathrm{e}^{4i(dx+c)}+\frac{16ia\,\mathrm{e}^{2i(dx+c)}}{3}+\frac{4ia}{3}}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^4}$	62

```
[In] int(sec(d*x+c)^4*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(1/4*tan(d*x+c)^4*b+1/3*a*tan(d*x+c)^3+1/2*b*tan(d*x+c)^2+a*tan(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

$$\int \sec^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{4\left(2\,a\cos\left(dx+c\right)^3+a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)+3\,b}{12\,d\cos\left(dx+c\right)^4}$$

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*(2*a*cos(d*x + c)^3 + a*cos(d*x + c))*sin(d*x + c) + 3*b)/(d*cos(d*
x + c)^4)
```

Sympy [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \sec^4(c+dx)(a+b\tan(c+dx))\,dx = \begin{cases} \frac{a\left(\frac{\tan^3(c+dx)}{3}+\tan(c+dx)\right)+\frac{b\sec^4(c+dx)}{4}}{d} & \text{for } d\neq 0 \\ x(a+b\tan(c))\sec^4(c) & \text{otherwise} \end{cases}$$

```
[In] integrate(sec(d*x+c)**4*(a+b*tan(d*x+c)),x)
```

```
[Out] Piecewise(((a*(tan(c + d*x)**3/3 + tan(c + d*x)) + b*sec(c + d*x)**4/4)/d,
Ne(d, 0)), (x*(a + b*tan(c))*sec(c)**4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

$$\int \sec^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{3\,b\tan\left(dx+c\right)^4+4\,a\tan\left(dx+c\right)^3+6\,b\tan\left(dx+c\right)^2+12\,a\tan\left(dx+c\right)}{12\,d}$$

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*b*tan(d*x + c)^4 + 4*a*tan(d*x + c)^3 + 6*b*tan(d*x + c)^2 + 12*a*t
an(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

$$\int \sec^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{3\,b\tan\left(dx+c\right)^4+4\,a\tan\left(dx+c\right)^3+6\,b\tan\left(dx+c\right)^2+12\,a\tan\left(dx+c\right)}{12\,d}$$

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(3*b*tan(d*x + c)^4 + 4*a*tan(d*x + c)^3 + 6*b*tan(d*x + c)^2 + 12*a*t
an(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 4.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \sec^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{\frac{b\tan(c+d\,x)^4}{4}+\frac{a\tan(c+d\,x)^3}{3}+\frac{b\tan(c+d\,x)^2}{2}+a\tan(c+d\,x)}{d}$$

```
[In] int((a + b*tan(c + d*x))/cos(c + d*x)^4,x)
```

```
[Out] (a*tan(c + d*x) + (a*tan(c + d*x)^3)/3 + (b*tan(c + d*x)^2)/2 + (b*tan(c +
d*x)^4)/4)/d
```

3.510 $\int \sec^3(c+dx)(a+b\tan(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \sec^3(c+dx)(a+b\tan(c+dx))\,dx = \frac{a\text{arctanh}(\sin(c+dx))}{2d} + \frac{b\sec^3(c+dx)}{3d} + \frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

```
[Out] 1/2*a*arctanh(sin(d*x+c))/d+1/3*b*sec(d*x+c)^3/d+1/2*a*sec(d*x+c)*tan(d*x+c
)/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3567, 3853, 3855}

$$\int \sec^3(c+dx)(a+b\tan(c+dx))\,dx = \frac{a\text{arctanh}(\sin(c+dx))}{2d} + \frac{a\tan(c+dx)\sec(c+dx)}{2d} + \frac{b\sec^3(c+dx)}{3d}$$

```
[In] Int[Sec[c + d*x]^3*(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Sec[c + d*x]^3)/(3*d) + (a*Sec[c + d*x
]*Tan[c + d*x])/(2*d)
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
```

```
| NeQ[a^2 + b^2, 0])
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{b\sec^3(c+dx)}{3d} + a\int \sec^3(c+dx)\,dx \\ &= \frac{b\sec^3(c+dx)}{3d} + \frac{a\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}a\int \sec(c+dx)\,dx \\ &= \frac{a\text{arctanh}(\sin(c+dx))}{2d} + \frac{b\sec^3(c+dx)}{3d} + \frac{a\sec(c+dx)\tan(c+dx)}{2d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \sec^3(c+dx)(a+b\tan(c+dx))\,dx = \frac{a\text{arctanh}(\sin(c+dx))}{2d} + \frac{b\sec^3(c+dx)}{3d} + \frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Tan[c + d*x]),x]
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Sec[c + d*x]^3)/(3*d) + (a*Sec[c + d*x
]*Tan[c + d*x])/(2*d)
```

Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{b}{3\cos(dx+c)^3}}{d}$	50
default	$\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{b}{3\cos(dx+c)^3}}{d}$	50
risch	$\frac{-3ia\,e^{5i(dx+c)}+8b\,e^{3i(dx+c)}+3ia\,e^{i(dx+c)}}{3d\left(e^{2i(dx+c)}+1\right)^3}+\frac{a\ln\left(e^{i(dx+c)}+i\right)}{2d}-\frac{a\ln\left(e^{i(dx+c)}-i\right)}{2d}$	97

```
[In] int(sec(d*x+c)^3*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+1/3*b/cos(
d*x+c)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.42

$$\int \sec^3(c+dx)(a+b\tan(c+dx))\,dx = \frac{3\,a\cos(dx+c)^3\log(\sin(dx+c)+1)-3\,a\cos(dx+c)^3\log(-\sin(dx+c)+1)+6\,a\cos(dx+c)\sin(dx}{12\,d\cos(dx+c)^3}$$

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="fricas")
[Out] 1/12*(3*a*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a*cos(d*x + c)^3*log(-si
n(d*x + c) + 1) + 6*a*cos(d*x + c)*sin(d*x + c) + 4*b)/(d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \sec^3(c+dx)(a+b\tan(c+dx))\,dx = \int (a+b\tan(c+dx))\sec^3(c+dx)\,dx$$

```
[In] integrate(sec(d*x+c)**3*(a+b*tan(d*x+c)),x)
[Out] Integral((a + b*tan(c + d*x))*sec(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.17

$$\int \sec^3(c+dx)(a+b\tan(c+dx))\,dx$$
$$= -\frac{3\,a\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx+c\right)+1\right) + \log\left(\sin\left(dx+c\right)-1\right)\right) - \frac{4\,b}{\cos(dx+c)^3}}{12\,d}$$

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(3*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + l
og(sin(d*x + c) - 1)) - 4*b/cos(d*x + c)^3)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(46) = 92.

Time = 0.38 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.90

$$\int \sec^3(c+dx)(a+b\tan(c+dx))\,dx$$
$$= \frac{3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 6\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 - 3\,a\tan\left(\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{6\,d}$$

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) + 2*(3*a*tan(1/2*d*x + 1/2*c)^5 - 6*b*tan(1/2*d*x + 1/2*c)^4 - 3*
a*tan(1/2*d*x + 1/2*c) - 2*b)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 6.55 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.02

$$\int \sec^3(c+dx)(a+b\tan(c+dx))\,dx$$
$$= \frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5 + 2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right) + \frac{2\,b}{3}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 - 3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + 3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 - 1\right)}$$

```
[In] int((a + b*tan(c + d*x))/cos(c + d*x)^3,x)
```

```
[Out] (a*atanh(tan(c/2 + (d*x)/2)))/d - ((2*b)/3 + a*tan(c/2 + (d*x)/2) - a*tan(c
/2 + (d*x)/2)^5 + 2*b*tan(c/2 + (d*x)/2)^4)/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*
tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.511 $\int \sec^2(c+dx)(a+b\tan(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \sec^2(c+dx)(a+b\tan(c+dx))\,dx=\frac{b\sec^2(c+dx)}{2d}+\frac{a\tan(c+dx)}{d}$$

```
[Out] 1/2*b*sec(d*x+c)^2/d+a*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3567, 3852, 8}

$$\int \sec^2(c+dx)(a+b\tan(c+dx))\,dx=\frac{a\tan(c+dx)}{d}+\frac{b\sec^2(c+dx)}{2d}$$

```
[In] Int[Sec[c + d*x]^2*(a + b*Tan[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^2)/(2*d) + (a*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{b\sec^2(c+dx)}{2d} + a\int \sec^2(c+dx)\,dx \\ &= \frac{b\sec^2(c+dx)}{2d} - \frac{a\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{d} \\ &= \frac{b\sec^2(c+dx)}{2d} + \frac{a\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \sec^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{b\sec^2(c+dx)}{2d} + \frac{a\tan(c+dx)}{d}$$

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Tan[c + d*x]),x]
[Out] (b*Sec[c + d*x]^2)/(2*d) + (a*Tan[c + d*x])/d
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\frac{b\left(\tan^2(dx+c)\right)}{2}+a\tan(dx+c)}{d}$	25
default	$\frac{\frac{b\left(\tan^2(dx+c)\right)}{2}+a\tan(dx+c)}{d}$	25
risch	$\frac{2ia\,\mathrm{e}^{2i(dx+c)}+2b\,\mathrm{e}^{2i(dx+c)}+2ia}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}$	48

```
[In] int(sec(d*x+c)^2*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(1/2*b*tan(d*x+c)^2+a*tan(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \sec^2(c+dx)(a+b\tan(c+dx))\,dx=\frac{2\,a\cos\left(dx+c\right)\sin\left(dx+c\right)+b}{2\,d\cos\left(dx+c\right)^2}$$

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="fricas")
[Out] 1/2*(2*a*cos(d*x + c)*sin(d*x + c) + b)/(d*cos(d*x + c)^2)
```

Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \sec^2(c+dx)(a+b\tan(c+dx))\,dx=\begin{cases}\frac{a\tan\left(c+dx\right)+\frac{b\tan^2\left(c+dx\right)}{2}}{d} & \text{for } d\neq 0\\ x(a+b\tan\left(c\right))\sec^2\left(c\right) & \text{otherwise}\end{cases}$$

```
[In] integrate(sec(d*x+c)**2*(a+b*tan(d*x+c)),x)
[Out] Piecewise(((a*tan(c + d*x) + b*tan(c + d*x)**2/2)/d, Ne(d, 0)), (x*(a + b*t
an(c))*sec(c)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

$$\int \sec^2(c+dx)(a+b\tan(c+dx))\,dx=\frac{\left(b\tan\left(dx+c\right)+a\right)^2}{2\,bd}$$

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="maxima")
[Out] 1/2*(b*tan(d*x + c) + a)^2/(b*d)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \sec^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{b\tan(dx+c)^2+2\,a\tan(dx+c)}{2\,d}$$

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="giac")
[Out] 1/2*(b*tan(d*x + c)^2 + 2*a*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 4.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \sec^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{\tan(c+d\,x)\,\left(2\,a+b\tan(c+d\,x)\right)}{2\,d}$$

```
[In] int((a + b*tan(c + d*x))/cos(c + d*x)^2,x)
[Out] (tan(c + d*x)*(2*a + b*tan(c + d*x)))/(2*d)
```

3.512 $\int \sec(c+dx)(a+b\tan(c+dx))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 24

$$\int \sec(c+dx)(a+b\tan(c+dx))\,dx = \frac{a\,\text{arctanh}(\sin(c+dx))}{d} + \frac{b\sec(c+dx)}{d}$$

```
[Out] a*arctanh(sin(d*x+c))/d+b*sec(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3567, 3855}

$$\int \sec(c+dx)(a+b\tan(c+dx))\,dx = \frac{a\,\text{arctanh}(\sin(c+dx))}{d} + \frac{b\sec(c+dx)}{d}$$

```
[In] Int[Sec[c + d*x]*(a + b*Tan[c + d*x]),x]
[Out] (a*ArcTanh[Sin[c + d*x]])/d + (b*Sec[c + d*x])/d
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{b\sec(c+dx)}{d} + a\int \sec(c+dx)\,dx$$
$$= \frac{a\text{arctanh}(\sin(c+dx))}{d} + \frac{b\sec(c+dx)}{d}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \sec(c+dx)(a+b\tan(c+dx))\,dx = \frac{a\text{arctanh}(\sin(c+dx))}{d} + \frac{b\sec(c+dx)}{d}$$

```
[In] Integrate[Sec[c + d*x]*(a + b*Tan[c + d*x]),x]
[Out] (a*ArcTanh[Sin[c + d*x]])/d + (b*Sec[c + d*x])/d
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	$\frac{\frac{b}{\cos(dx+c)}+a\ln(\sec(dx+c)+\tan(dx+c))}{d}$	32
default	$\frac{\frac{b}{\cos(dx+c)}+a\ln(\sec(dx+c)+\tan(dx+c))}{d}$	32
risch	$\frac{2b\,\mathrm{e}^{i(dx+c)}}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)}+\frac{a\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d}-\frac{a\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{d}$	67

```
[In] int(sec(d*x+c)*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(b/cos(d*x+c)+a*ln(sec(d*x+c)+tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(24) = 48.

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.25

$$\int \sec(c+dx)(a+b\tan(c+dx))\,dx$$
$$= \frac{a\cos(dx+c)\log(\sin(dx+c)+1) - a\cos(dx+c)\log(-\sin(dx+c)+1) + 2b}{2d\cos(dx+c)}$$

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*cos(d*x + c)*log(sin(d*x + c) + 1) - a*cos(d*x + c)*log(-sin(d*x + c
) + 1) + 2*b)/(d*cos(d*x + c))
```

Sympy [A] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \sec(c+dx)(a+b\tan(c+dx))\,dx = \begin{cases} \frac{a\log\left(\tan\left(c+dx\right)+\sec\left(c+dx\right)\right)+b\sec\left(c+dx\right)}{d} & \text{for } d\neq 0 \\ x(a+b\tan\left(c\right))\sec\left(c\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c)),x)
[Out] Piecewise(((a*log(tan(c + d*x) + sec(c + d*x)) + b*sec(c + d*x))/d, Ne(d, 0
)), (x*(a + b*tan(c))*sec(c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int \sec(c+dx)(a+b\tan(c+dx))\,dx = \frac{a\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+\frac{b}{\cos(dx+c)}}{d}$$

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="maxima")
[Out] (a*log(sec(d*x + c) + tan(d*x + c)) + b/cos(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(24) = 48$.

Time = 0.37 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.25

$$\int \sec(c+dx)(a+b\tan(c+dx))\,dx = \frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,b}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}}{d}$$

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="giac")
[Out] (a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c) - 1)
) - 2*b/(tan(1/2*d*x + 1/2*c)^2 - 1))/d
```

Mupad [B] (verification not implemented)

Time = 4.53 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int \sec(c+dx)(a+b\tan(c+dx))\,dx = \frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{2\,b}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

```
[In] int((a + b*tan(c + d*x))/cos(c + d*x),x)
[Out] (2*a*atanh(tan(c/2 + (d*x)/2)))/d - (2*b)/(d*(tan(c/2 + (d*x)/2)^2 - 1))
```

3.513 $\int \cos(c+dx)(a+b\tan(c+dx))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 24

$$\int \cos(c+dx)(a+b\tan(c+dx))\,dx=-\frac{b\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d}$$

```
[Out] -b*cos(d*x+c)/d+a*sin(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3567, 2717}

$$\int \cos(c+dx)(a+b\tan(c+dx))\,dx=\frac{a\sin(c+dx)}{d}-\frac{b\cos(c+dx)}{d}$$

```
[In] Int[Cos[c + d*x]*(a + b*Tan[c + d*x]),x]
[Out] -((b*Cos[c + d*x])/d) + (a*Sin[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rubi steps

$$\text{integral} = -\frac{b\cos(c+dx)}{d} + a\int\cos(c+dx)\,dx$$
$$= -\frac{b\cos(c+dx)}{d} + \frac{a\sin(c+dx)}{d}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.92

$$\int\cos(c+dx)(a+b\tan(c+dx))\,dx = -\frac{b\cos(c)\cos(dx)}{d} + \frac{a\cos(dx)\sin(c)}{d} + \frac{a\cos(c)\sin(dx)}{d} + \frac{b\sin(c)\sin(dx)}{d}$$

```
[In] Integrate[Cos[c + d*x]*(a + b*Tan[c + d*x]),x]
[Out] -((b*Cos[c]*Cos[d*x])/d) + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d +
(b*Sin[c]*Sin[d*x])/d
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{-b\cos(dx+c)+a\sin(dx+c)}{d}$	23
default	$\frac{-b\cos(dx+c)+a\sin(dx+c)}{d}$	23
risch	$-\frac{b\cos(dx+c)}{d}+\frac{a\sin(dx+c)}{d}$	25

```
[In] int(cos(d*x+c)*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-b*cos(d*x+c)+a*sin(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \cos(c+dx)(a+b\tan(c+dx))\,dx=-\frac{b\cos\left(dx+c\right)-a\sin\left(dx+c\right)}{d}$$

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="fricas")
[Out] -(b*cos(d*x + c) - a*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos(c+dx)(a+b\tan(c+dx))\,dx=\int \left(a+b\tan\left(c+dx\right)\right)\cos\left(c+dx\right)\,dx$$

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c)),x)
[Out] Integral((a + b*tan(c + d*x))*cos(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \cos(c+dx)(a+b\tan(c+dx))\,dx=-\frac{b\cos\left(dx+c\right)-a\sin\left(dx+c\right)}{d}$$

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="maxima")
[Out] -(b*cos(d*x + c) - a*sin(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(24) = 48.

Time = 0.36 (sec) , antiderivative size = 129, normalized size of antiderivative = 5.38

$$\int \cos(c+dx)(a+b\tan(c+dx))\,dx=$$
$$-\frac{b\tan\left(\frac{1}{2}\,dx\right)^2\tan\left(\frac{1}{2}\,c\right)^2+2\,a\tan\left(\frac{1}{2}\,dx\right)^2\tan\left(\frac{1}{2}\,c\right)+2\,a\tan\left(\frac{1}{2}\,dx\right)\tan\left(\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx\right)^2-4\,b\tan\left(\right.}{d\tan\left(\frac{1}{2}\,dx\right)^2\tan\left(\frac{1}{2}\,c\right)^2+d\tan\left(\frac{1}{2}\,dx\right)^2+d\tan\left(\right.}$$

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -(b*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*tan(1/2*d*x)^2*tan(1/2*c) + 2*a*tan(1
/2*d*x)*tan(1/2*c)^2 - b*tan(1/2*d*x)^2 - 4*b*tan(1/2*d*x)*tan(1/2*c) - b*t
an(1/2*c)^2 - 2*a*tan(1/2*d*x) - 2*a*tan(1/2*c) + b)/(d*tan(1/2*d*x)^2*tan(
1/2*c)^2 + d*tan(1/2*d*x)^2 + d*tan(1/2*c)^2 + d)
```

Mupad [B] (verification not implemented)

Time = 4.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int \cos(c+dx)(a+b\tan(c+dx))\,dx=-\frac{2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

```
[In] int(cos(c + d*x)*(a + b*tan(c + d*x)),x)
```

```
[Out] -(2*cos(c/2 + (d*x)/2)*(b*cos(c/2 + (d*x)/2) - a*sin(c/2 + (d*x)/2)))/d
```

3.514 $\int \cos^2(c+dx)(a+b\tan(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 43

$$\int \cos^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{ax}{2} - \frac{b\cos^2(c+dx)}{2d} + \frac{a\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] 1/2*a*x-1/2*b*cos(d*x+c)^2/d+1/2*a*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3567, 2715, 8}

$$\int \cos^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{a\sin(c+dx)\cos(c+dx)}{2d} + \frac{ax}{2} - \frac{b\cos^2(c+dx)}{2d}$$

```
[In] Int[Cos[c + d*x]^2*(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*x)/2 - (b*Cos[c + d*x]^2)/(2*d) + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{b\cos^2(c+dx)}{2d} + a\int \cos^2(c+dx)\,dx\\ &= -\frac{b\cos^2(c+dx)}{2d} + \frac{a\cos(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}a\int 1\,dx\\ &= \frac{ax}{2} - \frac{b\cos^2(c+dx)}{2d} + \frac{a\cos(c+dx)\sin(c+dx)}{2d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \cos^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{a(c+dx)}{2d} - \frac{b\cos^2(c+dx)}{2d} + \frac{a\sin(2(c+dx))}{4d}$$

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Tan[c + d*x]),x]
[Out] (a*(c + d*x))/(2*d) - (b*Cos[c + d*x]^2)/(2*d) + (a*Sin[2*(c + d*x)])/(4*d)
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
risch	$\frac{ax}{2}-\frac{b\cos(2dx+2c)}{4d}+\frac{a\sin(2dx+2c)}{4d}$	36
derivativedivides	$\frac{-\frac{b\left(\cos^2(dx+c)\right)}{2}+a\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	41
default	$\frac{-\frac{b\left(\cos^2(dx+c)\right)}{2}+a\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	41

```
[In] int(cos(d*x+c)^2*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/2*a*x-1/4*b/d*cos(2*d*x+2*c)+1/4*a/d*sin(2*d*x+2*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \cos^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{adx - b\cos\left(dx+c\right)^2 + a\cos\left(dx+c\right)\sin\left(dx+c\right)}{2\,d}$$

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="fricas")
[Out] 1/2*(a*d*x - b*cos(d*x + c)^2 + a*cos(d*x + c)*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^2(c+dx)(a+b\tan(c+dx))\,dx = \int \left(a+b\tan\left(c+dx\right)\right)\cos^2\left(c+dx\right)\,dx$$

```
[In] integrate(cos(d*x+c)**2*(a+b*tan(d*x+c)),x)
[Out] Integral((a + b*tan(c + d*x))*cos(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \cos^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{(dx+c)a + \frac{a\tan(dx+c)-b}{\tan(dx+c)^2+1}}{2\,d}$$

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="maxima")
[Out] 1/2*((d*x + c)*a + (a*tan(d*x + c) - b)/(tan(d*x + c)^2 + 1))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(37) = 74.

Time = 0.37 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.40

$$\int \cos^2(c+dx)(a+b\tan(c+dx))\,dx$$
$$= \frac{2\,adx\tan\left(dx\right)^2\tan\left(c\right)^2 + 2\,adx\tan\left(dx\right)^2 + 2\,adx\tan\left(c\right)^2 - b\tan\left(dx\right)^2\tan\left(c\right)^2 - 2\,a\tan\left(dx\right)^2\tan\left(c\right) - 2}{4\left(d\tan\left(dx\right)^2\tan\left(c\right)^2 + d\,t\right.}$$

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(2*a*d*x*tan(d*x)^2*tan(c)^2 + 2*a*d*x*tan(d*x)^2 + 2*a*d*x*tan(c)^2 -
b*tan(d*x)^2*tan(c)^2 - 2*a*tan(d*x)^2*tan(c) - 2*a*tan(d*x)*tan(c)^2 + 2*a
*d*x + b*tan(d*x)^2 + 4*b*tan(d*x)*tan(c) + b*tan(c)^2 + 2*a*tan(d*x) + 2*a
*tan(c) - b)/(d*tan(d*x)^2*tan(c)^2 + d*tan(d*x)^2 + d*tan(c)^2 + d)
```

Mupad [B] (verification not implemented)

Time = 4.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

$$\int \cos^2(c+dx)(a+b\tan(c+dx))\,dx = \frac{a\,x}{2} - \frac{\cos\left(c+d\,x\right)^2\,\left(\frac{b}{2}-\frac{a\tan(c+d\,x)}{2}\right)}{d}$$

```
[In] int(cos(c + d*x)^2*(a + b*tan(c + d*x)),x)
```

```
[Out] (a*x)/2 - (cos(c + d*x)^2*(b/2 - (a*tan(c + d*x))/2))/d
```

3.515 $\int \cos^3(c+dx)(a+b\tan(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 44

$$\int \cos^3(c+dx)(a+b\tan(c+dx))\,dx=-\frac{b\cos^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}$$

```
[Out] -1/3*b*cos(d*x+c)^3/d+a*sin(d*x+c)/d-1/3*a*sin(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3567, 2713}

$$\int \cos^3(c+dx)(a+b\tan(c+dx))\,dx=-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{b\cos^3(c+dx)}{3d}$$

```
[In] Int[Cos[c + d*x]^3*(a + b*Tan[c + d*x]),x]
```

```
[Out] -1/3*(b*Cos[c + d*x]^3)/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
```

```
| NeQ[a^2 + b^2, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{b\cos^3(c+dx)}{3d} + a\int \cos^3(c+dx)\,dx\\ &= -\frac{b\cos^3(c+dx)}{3d} - \frac{a\text{Subst}\left(\int (1-x^2)\ dx, x, -\sin(c+dx)\right)}{d}\\ &= -\frac{b\cos^3(c+dx)}{3d} + \frac{a\sin(c+dx)}{d} - \frac{a\sin^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \cos^3(c+dx)(a+b\tan(c+dx))\,dx = -\frac{b\cos^3(c+dx)}{3d} + \frac{a\sin(c+dx)}{d} - \frac{a\sin^3(c+dx)}{3d}$$

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Tan[c + d*x]),x]
[Out] -1/3*(b*Cos[c + d*x]^3)/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d)
```

Maple [A] (verified)

Time = 2.89 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{-\frac{b\left(\cos^3(dx+c)\right)}{3}+\frac{a\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$	36
default	$\frac{-\frac{b\left(\cos^3(dx+c)\right)}{3}+\frac{a\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$	36
risch	$-\frac{b\cos(dx+c)}{4d}+\frac{3a\sin(dx+c)}{4d}-\frac{b\cos(3dx+3c)}{12d}+\frac{a\sin(3dx+3c)}{12d}$	56

```
[In] int(cos(d*x+c)^3*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/3*b*cos(d*x+c)^3+1/3*a*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \cos^3(c+dx)(a+b\tan(c+dx))\,dx = -\frac{b\cos\left(dx+c\right)^3 - \left(a\cos\left(dx+c\right)^2 + 2\,a\right)\sin\left(dx+c\right)}{3\,d}$$

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="fricas")
[Out] -1/3*(b*cos(d*x + c)^3 - (a*cos(d*x + c)^2 + 2*a)*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^3(c+dx)(a+b\tan(c+dx))\,dx = \int \left(a+b\tan\left(c+dx\right)\right)\cos^3\left(c+dx\right)\,dx$$

```
[In] integrate(cos(d*x+c)**3*(a+b*tan(d*x+c)),x)
[Out] Integral((a + b*tan(c + d*x))*cos(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \cos^3(c+dx)(a+b\tan(c+dx))\,dx = -\frac{b\cos\left(dx+c\right)^3 + \left(\sin\left(dx+c\right)^3 - 3\,\sin\left(dx+c\right)\right)a}{3\,d}$$

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="maxima")
[Out] -1/3*(b*cos(d*x + c)^3 + (sin(d*x + c)^3 - 3*sin(d*x + c))*a)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11886 vs. $2(40) = 80$.

Time = 2.42 (sec) , antiderivative size = 11886, normalized size of antiderivative = 270.14

$$\int \cos^3(c+dx)(a+b\tan(c+dx))\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/96*(3*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c)
+ tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(
1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*
tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 3*pi*b*sgn(tan(1/2*d*x)^2*t
an(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 -
 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2
*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*
tan(1/2*c)^6 - 3*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(
1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)
^6*tan(1/2*c)^6 + 3*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*t
an(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d
*x)^6*tan(1/2*c)^6 - 6*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^
2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)
^6 + 9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) +
 tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1
/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*t
an(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 + 9*pi*b*sgn(tan(1/2*d*x)^2*ta
n(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 -
2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*
c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*t
an(1/2*c)^4 + 9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan
(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c
)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 + 9*pi*b*sgn(tan(1/2*
d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/
2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)
*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2
*d*x)^4*tan(1/2*c)^6 + 6*pi*b*tan(1/2*d*x)^6*tan(1/2*c)^6 - 6*b*arctan((tan
(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2
*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 - 6*b*arc
tan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)
*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 +
 6*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(
1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/
2*c)^6 + 6*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) -
1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^
6*tan(1/2*c)^6 - 9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*ta
n(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*
x)^6*tan(1/2*c)^4 + 9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)
*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2
*d*x)^6*tan(1/2*c)^4 - 18*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*
x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2
*c)^4 - 9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^
2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(
1/2*c)^6 + 9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*
```

c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*t
an(1/2*c)^6 - 18*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*
tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 - 3
2*b*tan(1/2*d*x)^6*tan(1/2*c)^6 + 9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 +
2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c)
- 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/
2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 +
 9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan
(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c
)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1
/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 + 27*pi*b*sgn(tan(1/2*d*x)^2*tan(1
/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*t
an(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^
2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(
1/2*c)^4 + 27*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1
/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^
2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^
2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 + 18*pi*b*tan(1/2*d*x)^
6*tan(1/2*c)^4 - 18*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(
1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(
1/2*d*x)^6*tan(1/2*c)^4 - 18*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*
x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) +
 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 + 18*b*arctan((tan(1/2*d*x)*tan(1/2*c) + t
an(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan
(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 + 18*b*arctan((tan(1/2*d*x)*tan(1
/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d
*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 - 192*a*tan(1/2*d*x)^6*t
an(1/2*c)^5 + 9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan
(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c
)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^6 + 9*pi*b*sgn(tan(1/2*
d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/
2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)
*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2
*d*x)^2*tan(1/2*c)^6 + 18*pi*b*tan(1/2*d*x)^4*tan(1/2*c)^6 - 18*b*arctan((t
an(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1
/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^6 - 18*b*
arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d
*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^
6 + 18*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(
tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4*ta
n(1/2*c)^6 + 18*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*
c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*
d*x)^4*tan(1/2*c)^6 - 192*a*tan(1/2*d*x)^5*tan(1/2*c)^6 - 9*pi*b*sgn(tan(1/
2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(

1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 + 9*pi*b*sgn(tan
(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + t
an(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 - 18*pi*b*sgn
(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) -
tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 - 27*pi*b*sgn(tan(1/2*d*x)^2
*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2
+ 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 + 27*pi*b*sgn(tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*
c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 - 54*pi*b*sgn(tan(1/
2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/
2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 + 96*b*tan(1/2*d*x)^6*tan(1/2*c)^4
+ 384*b*tan(1/2*d*x)^5*tan(1/2*c)^5 - 9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2
*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^6 + 9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*
c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(
1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^6 - 18*pi*b*sgn(tan(1/2*d*x)^2*tan(
1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*t
an(1/2*d*x)^2*tan(1/2*c)^6 + 96*b*tan(1/2*d*x)^4*tan(1/2*c)^6 + 3*pi*b*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2
- tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(
1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)
*tan(1/2*d*x)^6 + 3*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2
*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1
/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6 + 27*pi*b*sgn(tan(1/2*d*x)^2*t
an(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 +
2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2
*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*
tan(1/2*c)^2 + 27*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*t
an(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2
*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^2 + 18*pi*b*tan(1/2*d
*x)^6*tan(1/2*c)^2 - 18*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) -
tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*
tan(1/2*d*x)^6*tan(1/2*c)^2 - 18*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/
2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*
c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 + 18*b*arctan((tan(1/2*d*x)*tan(1/2*c)
+ tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) -
tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 + 18*b*arctan((tan(1/2*d*x)*t
an(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1
/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 - 128*a*tan(1/2*d*x)
^6*tan(1/2*c)^3 + 27*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^
2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/
2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(
1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^4 + 27*pi*b*sgn(ta

n(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^4 + 54*pi*b*tan(1/2*d*x)^4*tan(1/2*c)^4 - 54*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 - 54*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 + 54*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 + 54*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 + 576*a*tan(1/2*d*x)^5*tan(1/2*c)^4 + 576*a*tan(1/2*d*x)^4*tan(1/2*c)^5 + 3*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^6 + 3*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^6 + 18*pi*b*tan(1/2*d*x)^2*tan(1/2*c)^6 - 18*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 - 18*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 + 18*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 + 18*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 - 128*a*tan(1/2*d*x)^3*tan(1/2*c)^6 - 3*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6 + 3*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6 - 6*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6 - 27*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^2 + 27*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^2 - 54*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^2 - 96*b*tan(1/2*d*x)^6*tan(1/2*c)^2 - 768*b*tan(1/2*d*x)^5*tan(1/2*c)^3 - 27*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^4 + 27*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan

(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^4 - 54*pi*b*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - t
an(1/2*c)^2 + 1)*tan(1/2*d*x)^2*tan(1/2*c)^4 - 1824*b*tan(1/2*d*x)^4*tan(1/
2*c)^4 - 768*b*tan(1/2*d*x)^3*tan(1/2*c)^5 - 3*pi*b*sgn(tan(1/2*d*x)^2*tan(
1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*
tan(1/2*d*x) - 1)*tan(1/2*c)^6 + 3*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2
*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x)
 - 1)*tan(1/2*c)^6 - 6*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^
2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^6 - 96*b*tan(1
/2*d*x)^2*tan(1/2*c)^6 + 9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2
*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2
 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4 + 9*pi*b*sgn(tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*
c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*t
an(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d
*x)^4 + 6*pi*b*tan(1/2*d*x)^6 - 6*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1
/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2
*c) + 1))*tan(1/2*d*x)^6 - 6*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*
x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) +
 1))*tan(1/2*d*x)^6 + 6*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) +
tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*
tan(1/2*d*x)^6 + 6*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1
/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1
/2*d*x)^6 - 192*a*tan(1/2*d*x)^6*tan(1/2*c) + 27*pi*b*sgn(tan(1/2*d*x)^2*ta
n(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 +
2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*
c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*t
an(1/2*c)^2 + 27*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*ta
n(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*
c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 54*pi*b*tan(1/2*d*
x)^4*tan(1/2*c)^2 - 54*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - t
an(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*t
an(1/2*d*x)^4*tan(1/2*c)^2 - 54*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2
*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c
) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^2 + 54*b*arctan((tan(1/2*d*x)*tan(1/2*c)
 + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) -
tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^2 + 54*b*arctan((tan(1/2*d*x)*ta
n(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/
2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^2 - 576*a*tan(1/2*d*x)^
5*tan(1/2*c)^2 - 2688*a*tan(1/2*d*x)^4*tan(1/2*c)^3 + 9*pi*b*sgn(tan(1/2*d*
x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*
c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*t
an(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c

)^4 + 9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c)
+ tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(
1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*
tan(1/2*d*x) - 1)*tan(1/2*c)^4 + 54*pi*b*tan(1/2*d*x)^2*tan(1/2*c)^4 - 54*b
*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*
d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)
^4 - 54*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/
(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2*t
an(1/2*c)^4 + 54*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2
*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2
*d*x)^2*tan(1/2*c)^4 + 54*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x)
- tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)
)*tan(1/2*d*x)^2*tan(1/2*c)^4 - 2688*a*tan(1/2*d*x)^3*tan(1/2*c)^4 - 576*a*
tan(1/2*d*x)^2*tan(1/2*c)^5 + 6*pi*b*tan(1/2*c)^6 - 6*b*arctan((tan(1/2*d*x
)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - ta
n(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*c)^6 - 6*b*arctan((tan(1/2*d*x)*tan(1
/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d
*x) - tan(1/2*c) + 1))*tan(1/2*c)^6 + 6*b*arctan((tan(1/2*d*x)*tan(1/2*c) +
 tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - t
an(1/2*c) - 1))*tan(1/2*c)^6 + 6*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/
2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*
c) - 1))*tan(1/2*c)^6 - 192*a*tan(1/2*d*x)*tan(1/2*c)^6 - 9*pi*b*sgn(tan(1/
2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(
1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4 + 9*pi*b*sgn(tan(1/2*d*x)^2*t
an(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 -
 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4 - 18*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)
^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2
*d*x)^4 + 32*b*tan(1/2*d*x)^6 + 384*b*tan(1/2*d*x)^5*tan(1/2*c) - 27*pi*b*s
gn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)
^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 27*pi
*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*
d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 5
4*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*ta
n(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 1824*b*tan(1/2*d
*x)^4*tan(1/2*c)^2 + 3584*b*tan(1/2*d*x)^3*tan(1/2*c)^3 - 9*pi*b*sgn(tan(1/
2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(
1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^4 + 9*pi*b*sgn(tan(1/2*d*x)^2*tan
(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2
*tan(1/2*d*x) - 1)*tan(1/2*c)^4 - 18*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 -
 tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^
4 + 1824*b*tan(1/2*d*x)^2*tan(1/2*c)^4 + 384*b*tan(1/2*d*x)*tan(1/2*c)^5 +
32*b*tan(1/2*c)^6 + 9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)
^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1
/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan
(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2 + 9*pi*b*sgn(tan(1/2*d*x)^2*

tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2
- 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/
2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2
+ 18*pi*b*tan(1/2*d*x)^4 - 18*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*
d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c)
+ 1))*tan(1/2*d*x)^4 - 18*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x)
+ tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1
))*tan(1/2*d*x)^4 + 18*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + t
an(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*t
an(1/2*d*x)^4 + 18*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1
/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1
/2*d*x)^4 + 192*a*tan(1/2*d*x)^5 + 576*a*tan(1/2*d*x)^4*tan(1/2*c) + 9*pi*b
*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*
x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2
*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x)
- 1)*tan(1/2*c)^2 + 9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x
)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(
1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + ta
n(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^2 + 54*pi*b*tan(1/2*d*x)^2*tan(
1/2*c)^2 - 54*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c)
+ 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*
x)^2*tan(1/2*c)^2 - 54*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + t
an(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*t
an(1/2*d*x)^2*tan(1/2*c)^2 + 54*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2
*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c
) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 54*b*arctan((tan(1/2*d*x)*tan(1/2*c)
- tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) +
tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2688*a*tan(1/2*d*x)^3*tan(1/
2*c)^2 + 2688*a*tan(1/2*d*x)^2*tan(1/2*c)^3 + 18*pi*b*tan(1/2*c)^4 - 18*b*a
rctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*
x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*c)^4 - 18*b*arctan(
(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan
(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*c)^4 + 18*b*arctan((tan(1
/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c
) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*c)^4 + 18*b*arctan((tan(1/2*d*x
)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + ta
n(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^4 + 576*a*tan(1/2*d*x)*tan(1/2*c)^
4 + 192*a*tan(1/2*c)^5 - 9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2
*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*ta
n(1/2*d*x)^2 + 9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(
1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)
^2 - 18*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d
*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2 - 96*b*tan(1/2*d*x)^4 - 7
68*b*tan(1/2*d*x)^3*tan(1/2*c) - 9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2
*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x)

- 1)*tan(1/2*c)^2 + 9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x
)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/
2*c)^2 - 18*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1
/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^2 - 1824*b*tan(1/2*d*x)^2
*tan(1/2*c)^2 - 768*b*tan(1/2*d*x)*tan(1/2*c)^3 - 96*b*tan(1/2*c)^4 + 3*pi*
b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d
*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 +
2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x
) - 1) + 3*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*
c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*t
an(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 -
 2*tan(1/2*d*x) - 1) + 18*pi*b*tan(1/2*d*x)^2 - 18*b*arctan((tan(1/2*d*x)*t
an(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1
/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2 - 18*b*arctan((tan(1/2*d*x)*tan(1
/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d
*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2 + 18*b*arctan((tan(1/2*d*x)*tan(1/2*c
) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x)
- tan(1/2*c) - 1))*tan(1/2*d*x)^2 + 18*b*arctan((tan(1/2*d*x)*tan(1/2*c) -
tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + ta
n(1/2*c) - 1))*tan(1/2*d*x)^2 + 128*a*tan(1/2*d*x)^3 - 576*a*tan(1/2*d*x)^2
*tan(1/2*c) + 18*pi*b*tan(1/2*c)^2 - 18*b*arctan((tan(1/2*d*x)*tan(1/2*c) +
 tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + t
an(1/2*c) + 1))*tan(1/2*c)^2 - 18*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1
/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2
*c) + 1))*tan(1/2*c)^2 + 18*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x
) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) -
1))*tan(1/2*c)^2 + 18*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - ta
n(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*ta
n(1/2*c)^2 - 576*a*tan(1/2*d*x)*tan(1/2*c)^2 + 128*a*tan(1/2*c)^3 - 3*pi*b*
sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1) + 3*pi*b*sgn(tan(1/2*d*x)^2*tan(1/
2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*ta
n(1/2*d*x) - 1) - 6*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 -
 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1) + 96*b*tan(1/2*d*x)^2 + 384*
b*tan(1/2*d*x)*tan(1/2*c) + 96*b*tan(1/2*c)^2 + 6*pi*b - 6*b*arctan((tan(1/
2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c)
 - tan(1/2*d*x) + tan(1/2*c) + 1)) - 6*b*arctan((tan(1/2*d*x)*tan(1/2*c) -
tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - ta
n(1/2*c) + 1)) + 6*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1
/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)) + 6*b
*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*
d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)) + 192*a*tan(1/2*d*x) + 19
2*a*tan(1/2*c) - 32*b)/(d*tan(1/2*d*x)^6*tan(1/2*c)^6 + 3*d*tan(1/2*d*x)^6*
tan(1/2*c)^4 + 3*d*tan(1/2*d*x)^4*tan(1/2*c)^6 + 3*d*tan(1/2*d*x)^6*tan(1/2
*c)^2 + 9*d*tan(1/2*d*x)^4*tan(1/2*c)^4 + 3*d*tan(1/2*d*x)^2*tan(1/2*c)^6 +

```
 d*tan(1/2*d*x)^6 + 9*d*tan(1/2*d*x)^4*tan(1/2*c)^2 + 9*d*tan(1/2*d*x)^2*ta
n(1/2*c)^4 + d*tan(1/2*c)^6 + 3*d*tan(1/2*d*x)^4 + 9*d*tan(1/2*d*x)^2*tan(1
/2*c)^2 + 3*d*tan(1/2*c)^4 + 3*d*tan(1/2*d*x)^2 + 3*d*tan(1/2*c)^2 + d)
```

Mupad [B] (verification not implemented)

Time = 4.65 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07

$$\int \cos^3(c+dx)(a+b\tan(c+dx))\,dx = \frac{2\,a\,\sin\left(c+d\,x\right)}{3\,d}-\frac{b\cos\left(c+d\,x\right)^3}{3\,d}+\frac{a\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}$$

```
[In] int(cos(c + d*x)^3*(a + b*tan(c + d*x)),x)
[Out] (2*a*sin(c + d*x))/(3*d) - (b*cos(c + d*x)^3)/(3*d) + (a*cos(c + d*x)^2*sin
(c + d*x))/(3*d)
```

3.516 $\int \cos^4(c+dx)(a+b\tan(c+dx))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 65

$$\int \cos^4(c+dx)(a+b\tan(c+dx))\,dx=\frac{3ax}{8}-\frac{b\cos^4(c+dx)}{4d}+\frac{3a\cos(c+dx)\sin(c+dx)}{8d}+\frac{a\cos^3(c+dx)\sin(c+dx)}{4d}$$

```
[Out] 3/8*a*x-1/4*b*cos(d*x+c)^4/d+3/8*a*cos(d*x+c)*sin(d*x+c)/d+1/4*a*cos(d*x+c)
^3*sin(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3567, 2715, 8}

$$\int \cos^4(c+dx)(a+b\tan(c+dx))\,dx=\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}-\frac{b\cos^4(c+dx)}{4d}$$

```
[In] Int[Cos[c + d*x]^4*(a + b*Tan[c + d*x]),x]
```

```
[Out] (3*a*x)/8 - (b*Cos[c + d*x]^4)/(4*d) + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8*d
) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b\cos^4(c+dx)}{4d} + a\int \cos^4(c+dx)\,dx \\
&= -\frac{b\cos^4(c+dx)}{4d} + \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}(3a)\int \cos^2(c+dx)\,dx \\
&= -\frac{b\cos^4(c+dx)}{4d} + \frac{3a\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{8}(3a)\int 1\,dx \\
&= \frac{3ax}{8} - \frac{b\cos^4(c+dx)}{4d} + \frac{3a\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos^3(c+dx)\sin(c+dx)}{4d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

$$\int \cos^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{3a(c+dx)}{8d} - \frac{b\cos^4(c+dx)}{4d} + \frac{a\sin(2(c+dx))}{4d} + \frac{a\sin(4(c+dx))}{32d}$$

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Tan[c + d*x]),x]
[Out] (3*a*(c + d*x))/(8*d) - (b*Cos[c + d*x]^4)/(4*d) + (a*Sin[2*(c + d*x)])/(4*
d) + (a*Sin[4*(c + d*x)])/(32*d)
```

Maple [A] (verified)

Time = 4.86 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{-\frac{b\left(\cos^4(dx+c)\right)}{4}+a\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$	52
default	$\frac{-\frac{b\left(\cos^4(dx+c)\right)}{4}+a\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$	52
risch	$\frac{3ax}{8}-\frac{b\cos(4dx+4c)}{32d}+\frac{a\sin(4dx+4c)}{32d}-\frac{b\cos(2dx+2c)}{8d}+\frac{a\sin(2dx+2c)}{4d}$	66

```
[In] int(cos(d*x+c)^4*(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/4*b*cos(d*x+c)^4+a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/
8*d*x+3/8*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

$$\int \cos^4(c+dx)(a+b\tan(c+dx))\,dx = -\frac{2\,b\cos\left(dx+c\right)^4-3\,adx-\left(2\,a\cos\left(dx+c\right)^3+3\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8\,d}$$

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="fricas")
[Out] -1/8*(2*b*cos(d*x + c)^4 - 3*a*d*x - (2*a*cos(d*x + c)^3 + 3*a*cos(d*x + c)
)*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^4(c+dx)(a+b\tan(c+dx))\,dx = \int \left(a+b\tan\left(c+dx\right)\right)\cos^4\left(c+dx\right)\,dx$$

```
[In] integrate(cos(d*x+c)**4*(a+b*tan(d*x+c)),x)
[Out] Integral((a + b*tan(c + d*x))*cos(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \cos^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{3\,(dx+c)a+\frac{3\,a\tan(dx+c)^3+5\,a\tan(dx+c)-2\,b}{\tan(dx+c)^4+2\,\tan(dx+c)^2+1}}{8\,d}$$

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/8*(3*(d*x + c)*a + (3*a*tan(d*x + c)^3 + 5*a*tan(d*x + c) - 2*b)/(tan(d*x
 + c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 426 vs. 2(57) = 114.

Time = 0.52 (sec) , antiderivative size = 426, normalized size of antiderivative = 6.55

$$\int \cos^4(c+dx)(a+b\tan(c+dx))\,dx$$
$$= \frac{12\,adx\tan{(dx)}^4\tan{(c)}^4+24\,adx\tan{(dx)}^4\tan{(c)}^2+24\,adx\tan{(dx)}^2\tan{(c)}^4-5\,b\tan{(dx)}^4\tan{(c)}^4-20}{}$$

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*(12*a*d*x*tan(d*x)^4*tan(c)^4 + 24*a*d*x*tan(d*x)^4*tan(c)^2 + 24*a*d*
x*tan(d*x)^2*tan(c)^4 - 5*b*tan(d*x)^4*tan(c)^4 - 20*a*tan(d*x)^4*tan(c)^3
- 20*a*tan(d*x)^3*tan(c)^4 + 12*a*d*x*tan(d*x)^4 + 48*a*d*x*tan(d*x)^2*tan(
c)^2 + 6*b*tan(d*x)^4*tan(c)^2 + 32*b*tan(d*x)^3*tan(c)^3 + 12*a*d*x*tan(c)
^4 + 6*b*tan(d*x)^2*tan(c)^4 - 12*a*tan(d*x)^4*tan(c) + 24*a*tan(d*x)^3*tan
(c)^2 + 24*a*tan(d*x)^2*tan(c)^3 - 12*a*tan(d*x)*tan(c)^4 + 24*a*d*x*tan(d*
x)^2 + 3*b*tan(d*x)^4 + 24*a*d*x*tan(c)^2 - 36*b*tan(d*x)^2*tan(c)^2 + 3*b*
tan(c)^4 + 12*a*tan(d*x)^3 - 24*a*tan(d*x)^2*tan(c) - 24*a*tan(d*x)*tan(c)^
2 + 12*a*tan(c)^3 + 12*a*d*x + 6*b*tan(d*x)^2 + 32*b*tan(d*x)*tan(c) + 6*b*
tan(c)^2 + 20*a*tan(d*x) + 20*a*tan(c) - 5*b)/(d*tan(d*x)^4*tan(c)^4 + 2*d*
tan(d*x)^4*tan(c)^2 + 2*d*tan(d*x)^2*tan(c)^4 + d*tan(d*x)^4 + 4*d*tan(d*x)
^2*tan(c)^2 + d*tan(c)^4 + 2*d*tan(d*x)^2 + 2*d*tan(c)^2 + d)
```

Mupad [B] (verification not implemented)

Time = 4.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.63

$$\int \cos^4(c+dx)(a+b\tan(c+dx))\,dx = \frac{3\,a\,x}{8}+\frac{\cos\left(c+d\,x\right)^4\,\left(\frac{3\,a\,\mathrm{tan}\left(c+d\,x\right)^3}{8}+\frac{5\,a\,\mathrm{tan}\left(c+d\,x\right)}{8}-\frac{b}{4}\right)}{d}$$

```
[In] int(cos(c + d*x)^4*(a + b*tan(c + d*x)),x)
```

```
[Out] (3*a*x)/8 + (cos(c + d*x)^4*((5*a*tan(c + d*x))/8 - b/4 + (3*a*tan(c + d*x)
^3)/8))/d
```

3.517 $\int \sec^8(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 119

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{ab\sec^8(c+dx)}{4d}+\frac{a^2\tan(c+dx)}{d}+\frac{(3a^2+b^2)\tan^3(c+dx)}{3d}+\frac{3(a^2+b^2)\tan^5(c+dx)}{5d}+\frac{(a^2+3b^2)\tan^7(c+dx)}{7d}+\frac{b^2\tan^9(c+dx)}{9d}$$

```
[Out] 1/4*a*b*sec(d*x+c)^8/d+a^2*tan(d*x+c)/d+1/3*(3*a^2+b^2)*tan(d*x+c)^3/d+3/5*
(a^2+b^2)*tan(d*x+c)^5/d+1/7*(a^2+3*b^2)*tan(d*x+c)^7/d+1/9*b^2*tan(d*x+c)^
9/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3587, 710, 1824}

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{(a^2+3b^2)\tan^7(c+dx)}{7d}+\frac{3(a^2+b^2)\tan^5(c+dx)}{5d}+\frac{(3a^2+b^2)\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}+\frac{ab\sec^8(c+dx)}{4d}+\frac{b^2\tan^9(c+dx)}{9d}$$

```
[In] Int[Sec[c + d*x]^8*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (a*b*Sec[c + d*x]^8)/(4*d) + (a^2*Tan[c + d*x])/d + ((3*a^2 + b^2)*Tan[c +
d*x]^3)/(3*d) + (3*(a^2 + b^2)*Tan[c + d*x]^5)/(5*d) + ((a^2 + 3*b^2)*Tan[c
 + d*x]^7)/(7*d) + (b^2*Tan[c + d*x]^9)/(9*d)
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*m*d^(m - 1)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Int[((d + e*x)^m -
e*m*d^(m - 1)*x)*(a + c*x^2)^p, x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
 a*e^2, 0] && IGtQ[p, 1] && IGtQ[m, 0] && LeQ[m, p]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (a+x)^2\left(1+\frac{x^2}{b^2}\right)^3 dx, x, b\tan(c+dx)\right)}{bd}\\
&= \frac{ab\sec^8(c+dx)}{4d}+\frac{\text{Subst}\left(\int \left(1+\frac{x^2}{b^2}\right)^3\left(-2ax+(a+x)^2\right) dx, x, b\tan(c+dx)\right)}{bd}\\
&= \frac{ab\sec^8(c+dx)}{4d}\\
&\quad+\frac{\text{Subst}\left(\int \left(a^2+\frac{(3a^2+b^2)x^2}{b^2}+\frac{3(a^2+b^2)x^4}{b^4}+\frac{(a^2+3b^2)x^6}{b^6}+\frac{x^8}{b^6}\right) dx, x, b\tan(c+dx)\right)}{bd}\\
&= \frac{ab\sec^8(c+dx)}{4d}+\frac{a^2\tan(c+dx)}{d}+\frac{(3a^2+b^2)\tan^3(c+dx)}{3d}\\
&\quad+\frac{3(a^2+b^2)\tan^5(c+dx)}{5d}+\frac{(a^2+3b^2)\tan^7(c+dx)}{7d}+\frac{b^2\tan^9(c+dx)}{9d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.12

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{\tan(c+dx)\left(1260a^2+1260ab\tan(c+dx)+420(3a^2+b^2)\tan^2(c+dx)+1890ab\tan^3(c+dx)+756(a^2+\right.}{}$$

[In] Integrate[Sec[c + d*x]^8*(a + b*Tan[c + d*x])^2,x]

[Out] (Tan[c + d*x]*(1260*a^2 + 1260*a*b*Tan[c + d*x] + 420*(3*a^2 + b^2)*Tan[c + d*x]^2 + 1890*a*b*Tan[c + d*x]^3 + 756*(a^2 + b^2)*Tan[c + d*x]^4 + 1260*a*b*Tan[c + d*x]^5 + 180*(a^2 + 3*b^2)*Tan[c + d*x]^6 + 315*a*b*Tan[c + d*x]^7 + 140*b^2*Tan[c + d*x]^8))/(1260*d)

Maple [A] (verified)

Time = 113.34 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{-a^2\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx+c)\right)}{35}-\frac{8\left(\sec^2(dx+c)\right)}{35}\right)\tan(dx+c)+\frac{ab}{4\cos(dx+c)^8}+b^2\left(\frac{\sin^3(dx+c)}{9\cos(dx+c)^9}+\frac{2\left(\sin^3(dx+c)\right)}{21\cos(dx+c)^7}+\right.}{d}$
default	$\frac{-a^2\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx+c)\right)}{35}-\frac{8\left(\sec^2(dx+c)\right)}{35}\right)\tan(dx+c)+\frac{ab}{4\cos(dx+c)^8}+b^2\left(\frac{\sin^3(dx+c)}{9\cos(dx+c)^9}+\frac{2\left(\sin^3(dx+c)\right)}{21\cos(dx+c)^7}+\right.}{d}$
risch	$\frac{32i\left(-630iab\,e^{10i(dx+c)}+315a^2e^{10i(dx+c)}-315b^2e^{10i(dx+c)}-630iab\,e^{8i(dx+c)}+819a^2e^{8i(dx+c)}+189b^2e^{8i(dx+c)}+756a^2e^{6}\right.}{315d\left(e^{2i(dx+c)}+1\right)^9}$

[In] int(sec(d*x+c)^8*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(-a^2*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan(d*x+c)+1/4*a*b/cos(d*x+c)^8+b^2*(1/9*sin(d*x+c)^3/cos(d*x+c)^9+2/21*sin(d*x+c)^3/cos(d*x+c)^7+8/105*sin(d*x+c)^3/cos(d*x+c)^5+16/315*sin(d*x+c)^3/cos(d*x+c)^3))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{315\,ab\cos\left(dx+c\right)+4\left(16\left(9\,a^2-b^2\right)\cos\left(dx+c\right)^8+8\left(9\,a^2-b^2\right)\cos\left(dx+c\right)^6+6\left(9\,a^2-b^2\right)\cos\left(dx+c\right)\right.}{1260\,d\cos\left(dx+c\right)^9}$$

```
[In] integrate(sec(d*x+c)^8*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/1260*(315*a*b*cos(d*x + c) + 4*(16*(9*a^2 - b^2)*cos(d*x + c)^8 + 8*(9*a^
2 - b^2)*cos(d*x + c)^6 + 6*(9*a^2 - b^2)*cos(d*x + c)^4 + 5*(9*a^2 - b^2)*
cos(d*x + c)^2 + 35*b^2)*sin(d*x + c))/(d*cos(d*x + c)^9)
```

Sympy [F]

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^2\,dx = \int \left(a+b\tan\left(c+dx\right)\right)^2\sec^8\left(c+dx\right)\,dx$$

```
[In] integrate(sec(d*x+c)**8*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*sec(c + d*x)**8, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.66 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.12

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{140\,b^2\tan\left(dx+c\right)^9+315\,ab\tan\left(dx+c\right)^8+1260\,ab\tan\left(dx+c\right)^6+180\left(a^2+3\,b^2\right)\tan\left(dx+c\right)^7+1890\,}{}$$

```
[In] integrate(sec(d*x+c)^8*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/1260*(140*b^2*tan(d*x + c)^9 + 315*a*b*tan(d*x + c)^8 + 1260*a*b*tan(d*x
+ c)^6 + 180*(a^2 + 3*b^2)*tan(d*x + c)^7 + 1890*a*b*tan(d*x + c)^4 + 756*(
a^2 + b^2)*tan(d*x + c)^5 + 1260*a*b*tan(d*x + c)^2 + 420*(3*a^2 + b^2)*tan
(d*x + c)^3 + 1260*a^2*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.53 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.31

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{140\,b^2\tan\left(dx+c\right)^9 + 315\,ab\tan\left(dx+c\right)^8 + 180\,a^2\tan\left(dx+c\right)^7 + 540\,b^2\tan\left(dx+c\right)^7 + 1260\,ab\tan\left(dx+\right.}{}$$

```
[In] integrate(sec(d*x+c)^8*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1260*(140*b^2*tan(d*x + c)^9 + 315*a*b*tan(d*x + c)^8 + 180*a^2*tan(d*x +
 c)^7 + 540*b^2*tan(d*x + c)^7 + 1260*a*b*tan(d*x + c)^6 + 756*a^2*tan(d*x
+ c)^5 + 756*b^2*tan(d*x + c)^5 + 1890*a*b*tan(d*x + c)^4 + 1260*a^2*tan(d*
x + c)^3 + 420*b^2*tan(d*x + c)^3 + 1260*a*b*tan(d*x + c)^2 + 1260*a^2*tan(
d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 4.54 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.11

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{\tan(c+d\,x)^3\left(a^2+\frac{b^2}{3}\right)+a^2\tan(c+d\,x)+\tan(c+d\,x)^5\left(\frac{3\,a^2}{5}+\frac{3\,b^2}{5}\right)+\tan(c+d\,x)^7\left(\frac{a^2}{7}+\frac{3\,b^2}{7}\right)+\frac{b^2\tan(c}{9}}{d}$$

```
[In] int((a + b*tan(c + d*x))^2/cos(c + d*x)^8,x)
```

```
[Out] (tan(c + d*x)^3*(a^2 + b^2/3) + a^2*tan(c + d*x) + tan(c + d*x)^5*((3*a^2)/
5 + (3*b^2)/5) + tan(c + d*x)^7*(a^2/7 + (3*b^2)/7) + (b^2*tan(c + d*x)^9)/
9 + a*b*tan(c + d*x)^2 + (3*a*b*tan(c + d*x)^4)/2 + a*b*tan(c + d*x)^6 + (a
*b*tan(c + d*x)^8)/4)/d
```

3.518 $\int \sec^6(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 97

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{ab\sec^6(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}+\frac{(2a^2+b^2)\tan^3(c+dx)}{3d}+\frac{(a^2+2b^2)\tan^5(c+dx)}{5d}+\frac{b^2\tan^7(c+dx)}{7d}$$

```
[Out] 1/3*a*b*sec(d*x+c)^6/d+a^2*tan(d*x+c)/d+1/3*(2*a^2+b^2)*tan(d*x+c)^3/d+1/5*
(a^2+2*b^2)*tan(d*x+c)^5/d+1/7*b^2*tan(d*x+c)^7/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3587, 710, 1824}

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{(a^2+2b^2)\tan^5(c+dx)}{5d}+\frac{(2a^2+b^2)\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}+\frac{ab\sec^6(c+dx)}{3d}+\frac{b^2\tan^7(c+dx)}{7d}$$

```
[In] Int[Sec[c + d*x]^6*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (a*b*Sec[c + d*x]^6)/(3*d) + (a^2*Tan[c + d*x])/d + ((2*a^2 + b^2)*Tan[c +
d*x]^3)/(3*d) + ((a^2 + 2*b^2)*Tan[c + d*x]^5)/(5*d) + (b^2*Tan[c + d*x]^7)
/(7*d)
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*m*d^(m - 1)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Int[((d + e*x)^m -
e*m*d^(m - 1)*x)*(a + c*x^2)^p, x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
 a*e^2, 0] && IGtQ[p, 1] && IGtQ[m, 0] && LeQ[m, p]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int (a+x)^2\left(1+\frac{x^2}{b^2}\right)^2\,dx,x,b\tan(c+dx)\right)}{bd}\\
&= \frac{ab\sec^6(c+dx)}{3d}+\frac{\text{Subst}\left(\int\left(1+\frac{x^2}{b^2}\right)^2\left(-2ax+(a+x)^2\right)\,dx,x,b\tan(c+dx)\right)}{bd}\\
&= \frac{ab\sec^6(c+dx)}{3d}+\frac{\text{Subst}\left(\int\left(a^2+\frac{(2a^2+b^2)x^2}{b^2}+\frac{(a^2+2b^2)x^4}{b^4}+\frac{x^6}{b^4}\right)\,dx,x,b\tan(c+dx)\right)}{bd}\\
&= \frac{ab\sec^6(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}+\frac{(2a^2+b^2)\tan^3(c+dx)}{3d}\\
&\quad+\frac{(a^2+2b^2)\tan^5(c+dx)}{5d}+\frac{b^2\tan^7(c+dx)}{7d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.07

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{\tan(c+dx)\left(105a^2+105ab\tan(c+dx)+35(2a^2+b^2)\tan^2(c+dx)+105ab\tan^3(c+dx)+21(a^2+2b^2)\,\mathrm{t}\right.}{105d}$$

```
[In] Integrate[Sec[c + d*x]^6*(a + b*Tan[c + d*x])^2,x]
[Out] (Tan[c + d*x]*(105*a^2 + 105*a*b*Tan[c + d*x] + 35*(2*a^2 + b^2)*Tan[c + d*
x]^2 + 105*a*b*Tan[c + d*x]^3 + 21*(a^2 + 2*b^2)*Tan[c + d*x]^4 + 35*a*b*Ta
n[c + d*x]^5 + 15*b^2*Tan[c + d*x]^6))/(105*d)
```

Maple [A] (verified)

Time = 28.99 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{-a^2\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+\frac{ab}{3\cos(dx+c)^6}+b^2\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8\left(\sin^3(dx+c)\right)}{105\cos(dx+c)^3}\right.}{d}$
default	$\frac{-a^2\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+\frac{ab}{3\cos(dx+c)^6}+b^2\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8\left(\sin^3(dx+c)\right)}{105\cos(dx+c)^3}\right.}{d}$
risch	$\frac{16i\left(-140iab\,e^{8i(dx+c)}+70a^2e^{8i(dx+c)}-70b^2e^{8i(dx+c)}-140iab\,e^{6i(dx+c)}+175a^2e^{6i(dx+c)}+35b^2e^{6i(dx+c)}+147a^2e^{4i(dx+c)}\right.}{105d\left(e^{2i(dx+c)}+1\right)^7}$

```
[In] int(sec(d*x+c)^6*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+1/3*a*b/cos
(d*x+c)^6+b^2*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d*x+c)^5
+8/105*sin(d*x+c)^3/cos(d*x+c)^3))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.03

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{35\,ab\cos\left(dx+c\right)+\left(8\left(7\,a^2-b^2\right)\cos\left(dx+c\right)^6+4\left(7\,a^2-b^2\right)\cos\left(dx+c\right)^4+3\left(7\,a^2-b^2\right)\cos\left(dx+c\right)^2+\right.}{105\,d\cos\left(dx+c\right)^7}$$

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/105*(35*a*b*cos(d*x + c) + (8*(7*a^2 - b^2)*cos(d*x + c)^6 + 4*(7*a^2 - b
^2)*cos(d*x + c)^4 + 3*(7*a^2 - b^2)*cos(d*x + c)^2 + 15*b^2)*sin(d*x + c))
/(d*cos(d*x + c)^7)
```

Sympy [F]

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^2\,dx = \int \left(a+b\tan\left(c+dx\right)\right)^2\sec^6\left(c+dx\right)\,dx$$

```
[In] integrate(sec(d*x+c)**6*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*sec(c + d*x)**6, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.07

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{15\,b^2\tan\left(dx+c\right)^7+35\,ab\tan\left(dx+c\right)^6+105\,ab\tan\left(dx+c\right)^4+21\left(a^2+2\,b^2\right)\tan\left(dx+c\right)^5+105\,ab\tan\left(\cdots\right.}{105\,d}$$

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/105*(15*b^2*tan(d*x + c)^7 + 35*a*b*tan(d*x + c)^6 + 105*a*b*tan(d*x + c)
^4 + 21*(a^2 + 2*b^2)*tan(d*x + c)^5 + 105*a*b*tan(d*x + c)^2 + 35*(2*a^2 +
 b^2)*tan(d*x + c)^3 + 105*a^2*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.53 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.22

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{15\,b^2\tan\left(dx+c\right)^7+35\,ab\tan\left(dx+c\right)^6+21\,a^2\tan\left(dx+c\right)^5+42\,b^2\tan\left(dx+c\right)^5+105\,ab\tan\left(dx+c\right)^4+\cdots}{105\,d}$$

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/105*(15*b^2*tan(d*x + c)^7 + 35*a*b*tan(d*x + c)^6 + 21*a^2*tan(d*x + c)^
5 + 42*b^2*tan(d*x + c)^5 + 105*a*b*tan(d*x + c)^4 + 70*a^2*tan(d*x + c)^3
+ 35*b^2*tan(d*x + c)^3 + 105*a*b*tan(d*x + c)^2 + 105*a^2*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 4.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.05

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{a^2\tan(c+d\,x)+\tan(c+d\,x)^3\left(\frac{2\,a^2}{3}+\frac{b^2}{3}\right)+\tan(c+d\,x)^5\left(\frac{a^2}{5}+\frac{2\,b^2}{5}\right)+\frac{b^2\tan(c+d\,x)^7}{7}+a\,b\tan(c+d\,x)^2+\cdots}{d}$$

```
[In] int((a + b*tan(c + d*x))^2/cos(c + d*x)^6,x)
```

```
[Out] (a^2*tan(c + d*x) + tan(c + d*x)^3*((2*a^2)/3 + b^2/3) + tan(c + d*x)^5*(a^
2/5 + (2*b^2)/5) + (b^2*tan(c + d*x)^7)/7 + a*b*tan(c + d*x)^2 + a*b*tan(c
+ d*x)^4 + (a*b*tan(c + d*x)^6)/3)/d
```

3.519 $\int \sec^4(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 75

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{(a^2+b^2)\,(a+b\tan(c+dx))^3}{3b^3d} - \frac{a(a+b\tan(c+dx))^4}{2b^3d} + \frac{(a+b\tan(c+dx))^5}{5b^3d}$$

```
[Out] 1/3*(a^2+b^2)*(a+b*tan(d*x+c))^3/b^3/d-1/2*a*(a+b*tan(d*x+c))^4/b^3/d+1/5*(
a+b*tan(d*x+c))^5/b^3/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3587, 711}

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{(a^2+b^2)\,(a+b\tan(c+dx))^3}{3b^3d} + \frac{(a+b\tan(c+dx))^5}{5b^3d} - \frac{a(a+b\tan(c+dx))^4}{2b^3d}$$

```
[In] Int[Sec[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((a^2 + b^2)*(a + b*Tan[c + d*x])^3)/(3*b^3*d) - (a*(a + b*Tan[c + d*x])^4)
/(2*b^3*d) + (a + b*Tan[c + d*x])^5/(5*b^3*d)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
```

```
x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int (a+x)^2\left(1+\frac{x^2}{b^2}\right)\,dx, x, b\tan(c+dx)\right)}{bd}\\ &= \frac{\text{Subst}\left(\int \left(\frac{(a^2+b^2)(a+x)^2}{b^2}-\frac{2a(a+x)^3}{b^2}+\frac{(a+x)^4}{b^2}\right)\,dx, x, b\tan(c+dx)\right)}{bd}\\ &= \frac{(a^2+b^2)(a+b\tan(c+dx))^3}{3b^3d}-\frac{a(a+b\tan(c+dx))^4}{2b^3d}+\frac{(a+b\tan(c+dx))^5}{5b^3d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.72

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{(a+b\tan(c+dx))^3\left(a^2+10b^2-3ab\tan(c+dx)+6b^2\tan^2(c+dx)\right)}{30b^3d}$$

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
[Out] ((a + b*Tan[c + d*x])^3*(a^2 + 10*b^2 - 3*a*b*Tan[c + d*x] + 6*b^2*Tan[c +
d*x]^2))/(30*b^3*d)
```

Maple [A] (verified)

Time = 7.98 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{-a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{ab}{2\cos(dx+c)^4}+b^2\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)}{d}$
default	$\frac{-a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{ab}{2\cos(dx+c)^4}+b^2\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)}{d}$
risch	$\frac{4i(-30iab\,e^{6i(dx+c)}+15a^2e^{6i(dx+c)}-15b^2e^{6i(dx+c)}-30iab\,e^{4i(dx+c)}+35a^2e^{4i(dx+c)}+5b^2e^{4i(dx+c)}+25a^2e^{2i(dx+c)}-5b^2}{15d\left(e^{2i(dx+c)}+1\right)^5}$

```
[In] int(sec(d*x+c)^4*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+1/2*a*b/cos(d*x+c)^4+b^2*(1/5*
sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{15\,ab\cos(dx+c)+2\left(2\left(5\,a^2-b^2\right)\cos(dx+c)^4+\left(5\,a^2-b^2\right)\cos(dx+c)^2+3\,b^2\right)\sin(dx+c)}{30\,d\cos(dx+c)^5}$$

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/30*(15*a*b*cos(d*x + c) + 2*(2*(5*a^2 - b^2)*cos(d*x + c)^4 + (5*a^2 - b^
2)*cos(d*x + c)^2 + 3*b^2)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F]

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^2\,dx = \int (a+b\tan(c+dx))^2\sec^4(c+dx)\,dx$$

```
[In] integrate(sec(d*x+c)**4*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*sec(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{6\,b^2\tan\left(dx+c\right)^5+15\,ab\tan\left(dx+c\right)^4+30\,ab\tan\left(dx+c\right)^2+10\,\left(a^2+b^2\right)\tan\left(dx+c\right)^3+30\,a^2\tan\left(dx+\right.}{30\,d}$$

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
[Out] 1/30*(6*b^2*tan(d*x + c)^5 + 15*a*b*tan(d*x + c)^4 + 30*a*b*tan(d*x + c)^2
+ 10*(a^2 + b^2)*tan(d*x + c)^3 + 30*a^2*tan(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.51 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.07

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{6\,b^2\tan\left(dx+c\right)^5+15\,ab\tan\left(dx+c\right)^4+10\,a^2\tan\left(dx+c\right)^3+10\,b^2\tan\left(dx+c\right)^3+30\,ab\tan\left(dx+c\right)^2+3}{30\,d}$$

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="giac")
[Out] 1/30*(6*b^2*tan(d*x + c)^5 + 15*a*b*tan(d*x + c)^4 + 10*a^2*tan(d*x + c)^3
+ 10*b^2*tan(d*x + c)^3 + 30*a*b*tan(d*x + c)^2 + 30*a^2*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 4.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{a^2\tan(c+d\,x)+\tan(c+d\,x)^3\left(\frac{a^2}{3}+\frac{b^2}{3}\right)+\frac{b^2\tan(c+d\,x)^5}{5}+a\,b\tan(c+d\,x)^2+\frac{a\,b\tan(c+d\,x)^4}{2}}{d}$$

```
[In] int((a + b*tan(c + d*x))^2/cos(c + d*x)^4,x)
[Out] (a^2*tan(c + d*x) + tan(c + d*x)^3*(a^2/3 + b^2/3) + (b^2*tan(c + d*x)^5)/5
 + a*b*tan(c + d*x)^2 + (a*b*tan(c + d*x)^4)/2)/d
```

3.520 $\int \sec^2(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 22

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{(a+b\tan(c+dx))^3}{3bd}$$

```
[Out] 1/3*(a+b*tan(d*x+c))^3/b/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3587, 32}

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{(a+b\tan(c+dx))^3}{3bd}$$

```
[In] Int[Sec[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
[Out] (a + b*Tan[c + d*x])^3/(3*b*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int (a+x)^2\,dx, x, b\tan(c+dx)\right)}{bd}\\ &= \frac{(a+b\tan(c+dx))^3}{3bd}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 46 vs. 2(22) = 44.

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.09

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{a^2\tan(c+dx)}{d} + \frac{ab\tan^2(c+dx)}{d} + \frac{b^2\tan^3(c+dx)}{3d}$$

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
[Out] (a^2*Tan[c + d*x])/d + (a*b*Tan[c + d*x]^2)/d + (b^2*Tan[c + d*x]^3)/(3*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(20) = 40.

Time = 2.44 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

method	result	size
derivativedivides	$\frac{\frac{b^2\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+\frac{ab}{\cos(dx+c)^2}+a^2\tan(dx+c)}{d}$	48
default	$\frac{\frac{b^2\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+\frac{ab}{\cos(dx+c)^2}+a^2\tan(dx+c)}{d}$	48
risch	$-\frac{2i\left(6iab\,e^{4i(dx+c)}-3a^2 e^{4i(dx+c)}+3b^2 e^{4i(dx+c)}+6iab\,e^{2i(dx+c)}-6a^2 e^{2i(dx+c)}-3a^2+b^2\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}$	99

```
[In] int(sec(d*x+c)^2*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(1/3*b^2*sin(d*x+c)^3/cos(d*x+c)^3+a*b/cos(d*x+c)^2+a^2*tan(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(20) = 40$.

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.50

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{3\,ab\cos\left(dx+c\right)+\left(\left(3\,a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^3}$$

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
[Out] 1/3*(3*a*b*cos(d*x + c) + ((3*a^2 - b^2)*cos(d*x + c)^2 + b^2)*sin(d*x + c)
)/(d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^2\,dx = \int \left(a+b\tan\left(c+dx\right)\right)^2\sec^2\left(c+dx\right)\,dx$$

```
[In] integrate(sec(d*x+c)**2*(a+b*tan(d*x+c))**2,x)
[Out] Integral((a + b*tan(c + d*x))**2*sec(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{\left(b\tan\left(dx+c\right)+a\right)^3}{3\,bd}$$

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
[Out] 1/3*(b*tan(d*x + c) + a)^3/(b*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(20) = 40$.

Time = 0.45 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.86

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{b^2\tan\left(dx+c\right)^3 + 3\,ab\tan\left(dx+c\right)^2 + 3\,a^2\tan\left(dx+c\right)}{3\,d}$$

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="giac")
[Out] 1/3*(b^2*tan(d*x + c)^3 + 3*a*b*tan(d*x + c)^2 + 3*a^2*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 4.36 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{a^2\tan(c+d\,x) + a\,b\tan(c+d\,x)^2 + \frac{b^2\tan(c+d\,x)^3}{3}}{d}$$

```
[In] int((a + b*tan(c + d*x))^2/cos(c + d*x)^2,x)
[Out] (a^2*tan(c + d*x) + (b^2*tan(c + d*x)^3)/3 + a*b*tan(c + d*x)^2)/d
```

3.521 $\int \cos^2(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{1}{2}\left(a^2+b^2\right)x-\frac{\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{2d}$$

```
[Out] 1/2*(a^2+b^2)*x-1/2*cos(d*x+c)^2*(b-a*tan(d*x+c))*(a+b*tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3587, 737, 209}

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{1}{2}x\left(a^2+b^2\right)-\frac{\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{2d}$$

```
[In] Int[Cos[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((a^2 + b^2)*x)/2 - (Cos[c + d*x]^2*(b - a*Tan[c + d*x])*(a + b*Tan[c + d*x
]))/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 737

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[(2*p + 3)*((c*d^2 + a*e^2)/(2*a*c*(p + 1))), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+x)^2}{\left(1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\tan(c+dx)\right)}{bd} \\
&= -\frac{\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{2d} \\
&\quad + \frac{(a^2+b^2)\,\text{Subst}\left(\int \frac{1}{1+\frac{x^2}{b^2}}\,dx, x, b\tan(c+dx)\right)}{2bd} \\
&= \frac{1}{2}\left(a^2+b^2\right)x - \frac{\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{2(a^2+b^2)\,(c+dx) - 2ab\cos(2(c+dx)) + (a^2-b^2)\sin(2(c+dx))}{4d}$$

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Tan[c + d*x])^2,x]
[Out] (2*(a^2 + b^2)*(c + d*x) - 2*a*b*Cos[2*(c + d*x)] + (a^2 - b^2)*Sin[2*(c +
d*x)])/(4*d)
```

Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.31

method	result	size
risch	$\frac{a^{2}x}{2}+\frac{b^{2}x}{2}-\frac{ab\cos(2dx+2c)}{2d}+\frac{\sin(2dx+2c)a^{2}}{4d}-\frac{\sin(2dx+2c)b^{2}}{4d}$	64
derivativedivides	$\frac{b^{2}\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-ab\left(\cos^{2}(dx+c)\right)+a^{2}\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	70
default	$\frac{b^{2}\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-ab\left(\cos^{2}(dx+c)\right)+a^{2}\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	70

```
[In] int(cos(d*x+c)^2*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/2*a^2*x+1/2*b^2*x-1/2*a*b/d*cos(2*d*x+2*c)+1/4/d*sin(2*d*x+2*c)*a^2-1/4/d
*sin(2*d*x+2*c)*b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{2\,ab\cos\left(dx+c\right)^{2}-\left(a^{2}+b^{2}\right)dx-\left(a^{2}-b^{2}\right)\cos\left(dx+c\right)\sin\left(dx+c\right)}{2\,d}$$

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
[Out] -1/2*(2*a*b*cos(d*x + c)^2 - (a^2 + b^2)*d*x - (a^2 - b^2)*cos(d*x + c)*sin
(d*x + c))/d
```

Sympy [F]

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^2\,dx = \int \left(a+b\tan\left(c+dx\right)\right)^{2}\cos^{2}\left(c+dx\right)\,dx$$

```
[In] integrate(cos(d*x+c)**2*(a+b*tan(d*x+c))**2,x)
[Out] Integral((a + b*tan(c + d*x))**2*cos(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{(a^2+b^2)(dx+c)-\frac{2\,ab-(a^2-b^2)\tan(dx+c)}{\tan(dx+c)^2+1}}{2\,d}$$

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*((a^2 + b^2)*(d*x + c) - (2*a*b - (a^2 - b^2)*tan(d*x + c))/(tan(d*x +
c)^2 + 1))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(46) = 92.

Time = 0.50 (sec) , antiderivative size = 245, normalized size of antiderivative = 5.00

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$= \frac{a^2dx\tan(dx)^2\tan(c)^2+b^2dx\tan(dx)^2\tan(c)^2+a^2dx\tan(dx)^2+b^2dx\tan(dx)^2+a^2dx\tan(c)^2+b^2dx\,\mathrm{t}}{}$$

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^2*d*x*tan(d*x)^2*tan(c)^2 + b^2*d*x*tan(d*x)^2*tan(c)^2 + a^2*d*x*ta
n(d*x)^2 + b^2*d*x*tan(d*x)^2 + a^2*d*x*tan(c)^2 + b^2*d*x*tan(c)^2 - a*b*t
an(d*x)^2*tan(c)^2 - a^2*tan(d*x)^2*tan(c) + b^2*tan(d*x)^2*tan(c) - a^2*ta
n(d*x)*tan(c)^2 + b^2*tan(d*x)*tan(c)^2 + a^2*d*x + b^2*d*x + a*b*tan(d*x)^
2 + 4*a*b*tan(d*x)*tan(c) + a*b*tan(c)^2 + a^2*tan(d*x) - b^2*tan(d*x) + a^
2*tan(c) - b^2*tan(c) - a*b)/(d*tan(d*x)^2*tan(c)^2 + d*tan(d*x)^2 + d*tan(
c)^2 + d)
```

Mupad [B] (verification not implemented)

Time = 3.99 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^2\,dx = x\left(\frac{a^2}{2}+\frac{b^2}{2}\right) - \frac{\cos(c+d\,x)^2\left(a\,b-\tan(c+d\,x)\left(\frac{a^2}{2}-\frac{b^2}{2}\right)\right)}{d}$$

```
[In] int(cos(c + d*x)^2*(a + b*tan(c + d*x))^2,x)
```

```
[Out] x*(a^2/2 + b^2/2) - (cos(c + d*x)^2*(a*b - tan(c + d*x)*(a^2/2 - b^2/2)))/d
```

3.522 $\int \cos^4(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 88

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{1}{8}\left(3a^2+b^2\right)x - \frac{\cos^4(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{4d} - \frac{\cos^2(c+dx)\left(2ab-\left(3a^2+b^2\right)\tan(c+dx)\right)}{8d}$$

```
[Out] 1/8*(3*a^2+b^2)*x-1/4*cos(d*x+c)^4*(b-a*tan(d*x+c))*(a+b*tan(d*x+c))/d-1/8*
cos(d*x+c)^2*(2*a*b-(3*a^2+b^2)*tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3587, 753, 653, 209}

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{\cos^2(c+dx)\left(2ab-\left(3a^2+b^2\right)\tan(c+dx)\right)}{8d} + \frac{1}{8}x\left(3a^2+b^2\right) - \frac{\cos^4(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{4d}$$

```
[In] Int[Cos[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((3*a^2 + b^2)*x)/8 - (Cos[c + d*x]^4*(b - a*Tan[c + d*x])*(a + b*Tan[c + d
*x]))/(4*d) - (Cos[c + d*x]^2*(2*a*b - (3*a^2 + b^2)*Tan[c + d*x]))/(8*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 753

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+x)^2}{\left(1+\frac{x^2}{b^2}\right)^3}\,dx, x, b\tan(c+dx)\right)}{bd} \\
&= -\frac{\cos^4(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{4d} \\
&\quad + \frac{b\text{Subst}\left(\int \frac{1+\frac{3a^2}{b^2}+\frac{2ax}{b^2}}{\left(1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\tan(c+dx)\right)}{4d} \\
&= -\frac{\cos^4(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{4d} \\
&\quad - \frac{\cos^2(c+dx)\left(2ab-(3a^2+b^2)\tan(c+dx)\right)}{8d} \\
&\quad + \frac{\left(\left(1+\frac{3a^2}{b^2}\right)b\right)\text{Subst}\left(\int \frac{1}{1+\frac{x^2}{b^2}}\,dx, x, b\tan(c+dx)\right)}{8d}
\end{aligned}$$

$$= \frac{1}{8}\left(3a^2+b^2\right)x - \frac{\cos^4(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{4d} - \frac{\cos^2(c+dx)\left(2ab-\left(3a^2+b^2\right)\tan(c+dx)\right)}{8d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 197 vs. 2(88) = 176.

Time = 3.32 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.24

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$= \frac{\left(3a^2+b^2\right)\left(2ab\left(2a^2+b^2\right)-2ab\left(a^2+b^2\right)\cos(2(c+dx))+\frac{b\left(a^2+b^2\right)^2\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)}{\sqrt{-b^2}}-\frac{b\left(a^2+b^2\right)^2\log\left(\sqrt{-b^2}\right.}{\sqrt{-b^2}}\right.}{16\left(a^2+\right.}$$

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((3*a^2 + b^2)*(2*a*b*(2*a^2 + b^2) - 2*a*b*(a^2 + b^2)*Cos[2*(c + d*x)] +
(b*(a^2 + b^2)^2*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/Sqrt[-b^2] - (b*(a^2 + b
^2)^2*Log[Sqrt[-b^2] + b*Tan[c + d*x]])/Sqrt[-b^2] + (a^4 - b^4)*Sin[2*(c +
 d*x)]) + 4*(a^2 + b^2)*Cos[c + d*x]^4*(b + a*Tan[c + d*x])*(a + b*Tan[c +
d*x])^3)/(16*(a^2 + b^2)^2*d)
```

Maple [A] (verified)

Time = 8.99 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{b^2\left(-\frac{\left(\cos^3(dx+c)\right)\sin(dx+c)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{ab\left(\cos^4(dx+c)\right)}{2}+a^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\right.}{d}$
default	$\frac{b^2\left(-\frac{\left(\cos^3(dx+c)\right)\sin(dx+c)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{ab\left(\cos^4(dx+c)\right)}{2}+a^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\right.}{d}$
risch	$\frac{3a^2x}{8}+\frac{b^2x}{8}-\frac{ab\cos(4dx+4c)}{16d}+\frac{\sin(4dx+4c)a^2}{32d}-\frac{\sin(4dx+4c)b^2}{32d}-\frac{ab\cos(2dx+2c)}{4d}+\frac{\sin(2dx+2c)a^2}{4d}$

```
[In] int(cos(d*x+c)^4*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*sin(d*x+c)*cos(d*x+c)+1/8*d*x+1/
8*c)-1/2*a*b*cos(d*x+c)^4+a^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)
+3/8*d*x+3/8*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.85

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{4\,ab\cos\left(dx+c\right)^4-(3\,a^2+b^2)dx-\left(2\,(a^2-b^2)\cos\left(dx+c\right)^3+(3\,a^2+b^2)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8\,d}$$

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*a*b*cos(d*x + c)^4 - (3*a^2 + b^2)*d*x - (2*(a^2 - b^2)*cos(d*x + c
)^3 + (3*a^2 + b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^2\,dx = \int \left(a+b\tan\left(c+dx\right)\right)^2\cos^4\left(c+dx\right)\,dx$$

```
[In] integrate(cos(d*x+c)**4*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*cos(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.62 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{(3\,a^2+b^2)(dx+c)+\frac{(3\,a^2+b^2)\tan(dx+c)^3-4\,ab+(5\,a^2-b^2)\tan(dx+c)}{\tan(dx+c)^4+2\,\tan(dx+c)^2+1}}{8\,d}$$

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*((3*a^2 + b^2)*(d*x + c) + ((3*a^2 + b^2)*tan(d*x + c)^3 - 4*a*b + (5*a
^2 - b^2)*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2286 vs. 2(83) = 166.

Time = 2.60 (sec) , antiderivative size = 2286, normalized size of antiderivative = 25.98

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^2\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/64*(3*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*
tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^4 + 24*a^2*d*x
*tan(d*x)^4*tan(c)^4 + 8*b^2*d*x*tan(d*x)^4*tan(c)^4 + 3*pi*b^2*sgn(-2*tan(
d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan
(c)^4 + 6*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) +
2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^2 + 6*pi*b^2
*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c
)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^4 + 6*b^2*arctan((tan(d*x) +
 tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^4*tan(c)^4 - 6*b^2*arctan(-(tan(d*
x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^4 + 48*a^2*d*x*tan(d*
x)^4*tan(c)^2 + 16*b^2*d*x*tan(d*x)^4*tan(c)^2 + 6*pi*b^2*sgn(-2*tan(d*x)^2
*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^2
+ 48*a^2*d*x*tan(d*x)^2*tan(c)^4 + 16*b^2*d*x*tan(d*x)^2*tan(c)^4 + 6*pi*b^
2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*t
an(d*x)^2*tan(c)^4 - 20*a*b*tan(d*x)^4*tan(c)^4 + 3*pi*b^2*sgn(2*tan(d*x)^2
*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x)
- 2*tan(c))*tan(d*x)^4 + 12*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*ta
n(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*t
an(c)^2 + 12*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)
^4*tan(c)^2 - 12*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan
(d*x)^4*tan(c)^2 - 40*a^2*tan(d*x)^4*tan(c)^3 + 8*b^2*tan(d*x)^4*tan(c)^3 +
 3*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d
*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^4 + 12*b^2*arctan((tan(d*x) +
tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^2*tan(c)^4 - 12*b^2*arctan(-(tan(d*
x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^4 - 40*a^2*tan(d*x)^3
*tan(c)^4 + 8*b^2*tan(d*x)^3*tan(c)^4 + 24*a^2*d*x*tan(d*x)^4 + 8*b^2*d*x*t
an(d*x)^4 + 3*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan
(d*x) - 2*tan(c))*tan(d*x)^4 + 96*a^2*d*x*tan(d*x)^2*tan(c)^2 + 32*b^2*d*x*
tan(d*x)^2*tan(c)^2 + 12*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c
)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^2 + 24*a*b*tan(d*x)^4*tan(c)
^2 + 128*a*b*tan(d*x)^3*tan(c)^3 + 24*a^2*d*x*tan(c)^4 + 8*b^2*d*x*tan(c)^4
 + 3*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2
*tan(c))*tan(c)^4 + 24*a*b*tan(d*x)^2*tan(c)^4 + 6*pi*b^2*sgn(2*tan(d*x)^2*
tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) -
 2*tan(c))*tan(d*x)^2 + 6*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) -
```

```
1))*tan(d*x)^4 - 6*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*
tan(d*x)^4 - 24*a^2*tan(d*x)^4*tan(c) - 8*b^2*tan(d*x)^4*tan(c) + 6*pi*b^2*
sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)
^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^2 + 24*b^2*arctan((tan(d*x) + tan(c))/(t
an(d*x)*tan(c) - 1))*tan(d*x)^2*tan(c)^2 - 24*b^2*arctan(-(tan(d*x) - tan(c
))/(tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^2 + 48*a^2*tan(d*x)^3*tan(c)^2
- 48*b^2*tan(d*x)^3*tan(c)^2 + 48*a^2*tan(d*x)^2*tan(c)^3 - 48*b^2*tan(d*x)
^2*tan(c)^3 + 6*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(c
)^4 - 6*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(c)^4 - 2
4*a^2*tan(d*x)*tan(c)^4 - 8*b^2*tan(d*x)*tan(c)^4 + 48*a^2*d*x*tan(d*x)^2 +
 16*b^2*d*x*tan(d*x)^2 + 6*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan
(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2 + 12*a*b*tan(d*x)^4 + 48*a^2*d*x*
tan(c)^2 + 16*b^2*d*x*tan(c)^2 + 6*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(
d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^2 - 144*a*b*tan(d*x)^2*tan(c)
^2 + 12*a*b*tan(c)^4 + 3*pi*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d
*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c)) + 12*b^2*arctan
((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^2 - 12*b^2*arctan(-(ta
n(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^2 + 24*a^2*tan(d*x)^3 + 8*
b^2*tan(d*x)^3 - 48*a^2*tan(d*x)^2*tan(c) + 48*b^2*tan(d*x)^2*tan(c) + 12*b
^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(c)^2 - 12*b^2*arct
an(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(c)^2 - 48*a^2*tan(d*x)*t
an(c)^2 + 48*b^2*tan(d*x)*tan(c)^2 + 24*a^2*tan(c)^3 + 8*b^2*tan(c)^3 + 24*
a^2*d*x + 8*b^2*d*x + 3*pi*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)
^2 + 2*tan(d*x) - 2*tan(c)) + 24*a*b*tan(d*x)^2 + 128*a*b*tan(d*x)*tan(c) +
 24*a*b*tan(c)^2 + 6*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))
- 6*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1)) + 40*a^2*tan(d*x
) - 8*b^2*tan(d*x) + 40*a^2*tan(c) - 8*b^2*tan(c) - 20*a*b)/(d*tan(d*x)^4*t
an(c)^4 + 2*d*tan(d*x)^4*tan(c)^2 + 2*d*tan(d*x)^2*tan(c)^4 + d*tan(d*x)^4
+ 4*d*tan(d*x)^2*tan(c)^2 + d*tan(c)^4 + 2*d*tan(d*x)^2 + 2*d*tan(c)^2 + d)
```

Mupad [B] (verification not implemented)

Time = 4.79 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.94

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=x\left(\frac{3\,a^2}{8}+\frac{b^2}{8}\right)+\frac{\left(\frac{3\,a^2}{8}+\frac{b^2}{8}\right)\,\tan(c+d\,x)^3+\left(\frac{5\,a^2}{8}-\frac{b^2}{8}\right)\,\tan(c+d\,x)-\frac{a\,b}{2}}{d\,\left(\tan\left(c+d\,x\right)^4+2\tan\left(c+d\,x\right)^2+1\right)}$$

```
[In] int(cos(c + d*x)^4*(a + b*tan(c + d*x))^2,x)
```

```
[Out] x*((3*a^2)/8 + b^2/8) + (tan(c + d*x)*((5*a^2)/8 - b^2/8) - (a*b)/2 + tan(c
 + d*x)^3*((3*a^2)/8 + b^2/8))/(d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 + 1))
```

3.523 $\int \sec^7(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 163

$$\int \sec^7(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{5(8a^2-b^2)\operatorname{arctanh}(\sin(c+dx))}{128d} + \frac{9ab\sec^7(c+dx)}{56d} + \frac{5(8a^2-b^2)\sec(c+dx)\tan(c+dx)}{128d} + \frac{5(8a^2-b^2)\sec^3(c+dx)\tan(c+dx)}{192d} + \frac{(8a^2-b^2)\sec^5(c+dx)\tan(c+dx)}{48d} + \frac{b\sec^7(c+dx)(a+b\tan(c+dx))}{8d}$$

```
[Out] 5/128*(8*a^2-b^2)*arctanh(sin(d*x+c))/d+9/56*a*b*sec(d*x+c)^7/d+5/128*(8*a^
2-b^2)*sec(d*x+c)*tan(d*x+c)/d+5/192*(8*a^2-b^2)*sec(d*x+c)^3*tan(d*x+c)/d+
1/48*(8*a^2-b^2)*sec(d*x+c)^5*tan(d*x+c)/d+1/8*b*sec(d*x+c)^7*(a+b*tan(d*x+
c))/d
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used

$= \{3589, 3567, 3853, 3855\}$

$$\int \sec^7(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{5(8a^2-b^2)\,\mathrm{arctanh}(\sin(c+dx))}{128d} + \frac{(8a^2-b^2)\tan(c+dx)\sec^5(c+dx)}{48d} + \frac{5(8a^2-b^2)\tan(c+dx)\sec^3(c+dx)}{192d} + \frac{5(8a^2-b^2)\tan(c+dx)\sec(c+dx)}{128d} + \frac{9ab\sec^7(c+dx)}{56d} + \frac{b\sec^7(c+dx)(a+b\tan(c+dx))}{8d}$$

```
[In] Int[Sec[c + d*x]^7*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (5*(8*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(128*d) + (9*a*b*Sec[c + d*x]^7)/(5
6*d) + (5*(8*a^2 - b^2)*Sec[c + d*x]*Tan[c + d*x])/(128*d) + (5*(8*a^2 - b^
2)*Sec[c + d*x]^3*Tan[c + d*x])/(192*d) + ((8*a^2 - b^2)*Sec[c + d*x]^5*Tan
[c + d*x])/(48*d) + (b*Sec[c + d*x]^7*(a + b*Tan[c + d*x]))/(8*d)
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{b\sec^7(c+dx)(a+b\tan(c+dx))}{8d}+\frac{1}{8}\int\sec^7(c+dx)\left(8a^2-b^2+9ab\tan(c+dx)\right)\,dx\\
&= \frac{9ab\sec^7(c+dx)}{56d}+\frac{b\sec^7(c+dx)(a+b\tan(c+dx))}{8d}+\frac{1}{8}\left(8a^2-b^2\right)\int\sec^7(c+dx)\,dx\\
&= \frac{9ab\sec^7(c+dx)}{56d}+\frac{(8a^2-b^2)\sec^5(c+dx)\tan(c+dx)}{48d}\\
&\quad+\frac{b\sec^7(c+dx)(a+b\tan(c+dx))}{8d}+\frac{1}{48}\left(5\left(8a^2-b^2\right)\right)\int\sec^5(c+dx)\,dx\\
&= \frac{9ab\sec^7(c+dx)}{56d}+\frac{5(8a^2-b^2)\sec^3(c+dx)\tan(c+dx)}{192d}\\
&\quad+\frac{(8a^2-b^2)\sec^5(c+dx)\tan(c+dx)}{48d}\\
&\quad+\frac{b\sec^7(c+dx)(a+b\tan(c+dx))}{8d}+\frac{1}{64}\left(5\left(8a^2-b^2\right)\right)\int\sec^3(c+dx)\,dx\\
&= \frac{9ab\sec^7(c+dx)}{56d}+\frac{5(8a^2-b^2)\sec(c+dx)\tan(c+dx)}{128d}\\
&\quad+\frac{5(8a^2-b^2)\sec^3(c+dx)\tan(c+dx)}{192d}+\frac{(8a^2-b^2)\sec^5(c+dx)\tan(c+dx)}{48d}\\
&\quad+\frac{b\sec^7(c+dx)(a+b\tan(c+dx))}{8d}+\frac{1}{128}\left(5\left(8a^2-b^2\right)\right)\int\sec(c+dx)\,dx\\
&= \frac{5(8a^2-b^2)\operatorname{arctanh}(\sin(c+dx))}{128d}+\frac{9ab\sec^7(c+dx)}{56d}\\
&\quad+\frac{5(8a^2-b^2)\sec(c+dx)\tan(c+dx)}{128d}+\frac{5(8a^2-b^2)\sec^3(c+dx)\tan(c+dx)}{192d}\\
&\quad+\frac{(8a^2-b^2)\sec^5(c+dx)\tan(c+dx)}{48d}+\frac{b\sec^7(c+dx)(a+b\tan(c+dx))}{8d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.33

$$\int \sec^7(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{5a^2\text{arctanh}(\sin(c+dx))}{16d} - \frac{5b^2\text{arctanh}(\sin(c+dx))}{128d} + \frac{2ab\sec^7(c+dx)}{7d} + \frac{5a^2\sec(c+dx)\tan(c+dx)}{16d} - \frac{5b^2\sec(c+dx)\tan(c+dx)}{128d} + \frac{5a^2\sec^3(c+dx)\tan(c+dx)}{24d} - \frac{5b^2\sec^3(c+dx)\tan(c+dx)}{192d} + \frac{a^2\sec^5(c+dx)\tan(c+dx)}{6d} - \frac{b^2\sec^5(c+dx)\tan(c+dx)}{48d} + \frac{b^2\sec^7(c+dx)\tan(c+dx)}{8d}$$

```
[In] Integrate[Sec[c + d*x]^7*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (5*a^2*ArcTanh[Sin[c + d*x]])/(16*d) - (5*b^2*ArcTanh[Sin[c + d*x]])/(128*d
) + (2*a*b*Sec[c + d*x]^7)/(7*d) + (5*a^2*Sec[c + d*x]*Tan[c + d*x])/(16*d)
 - (5*b^2*Sec[c + d*x]*Tan[c + d*x])/(128*d) + (5*a^2*Sec[c + d*x]^3*Tan[c
+ d*x])/(24*d) - (5*b^2*Sec[c + d*x]^3*Tan[c + d*x])/(192*d) + (a^2*Sec[c +
 d*x]^5*Tan[c + d*x])/(6*d) - (b^2*Sec[c + d*x]^5*Tan[c + d*x])/(48*d) + (b
^2*Sec[c + d*x]^7*Tan[c + d*x])/(8*d)
```

Maple [A] (verified)

Time = 67.51 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{a^2\left(-\left(-\frac{\left(\sec^5(dx+c)\right)}{6}-\frac{5\left(\sec^3(dx+c)\right)}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+\frac{2ab}{7\cos(dx+c)^7}+b^2\left(\frac{\sin^3(dx}{8\cos(dx}\right.}{d}$
default	$\frac{a^2\left(-\left(-\frac{\left(\sec^5(dx+c)\right)}{6}-\frac{5\left(\sec^3(dx+c)\right)}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+\frac{2ab}{7\cos(dx+c)^7}+b^2\left(\frac{\sin^3(dx}{8\cos(dx}\right.}{d}$
risch	$-\frac{ie^{i(dx+c)}\left(840a^2e^{14i(dx+c)}-105b^2e^{14i(dx+c)}+6440a^2e^{12i(dx+c)}-805b^2e^{12i(dx+c)}+21448a^2e^{10i(dx+c)}-2681b^2e^{10i(dx+\right.}{}$

```
[In] int(sec(d*x+c)^7*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-(-1/6*sec(d*x+c)^5-5/24*sec(d*x+c)^3-5/16*sec(d*x+c))*tan(d*x+c)
+5/16*ln(sec(d*x+c)+tan(d*x+c)))+2/7*a*b/cos(d*x+c)^7+b^2*(1/8*sin(d*x+c)^3
/cos(d*x+c)^8+5/48*sin(d*x+c)^3/cos(d*x+c)^6+5/64*sin(d*x+c)^3/cos(d*x+c)^4
+5/128*sin(d*x+c)^3/cos(d*x+c)^2+5/128*sin(d*x+c)-5/128*ln(sec(d*x+c)+tan(d
*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00

$$\int \sec^7(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{105\,(8\,a^2-b^2)\cos\left(dx+c\right)^8\log\left(\sin\left(dx+c\right)+1\right)-105\,(8\,a^2-b^2)\cos\left(dx+c\right)^8\log\left(-\sin\left(dx+c\right)+1\right)+}{}$$

```
[In] integrate(sec(d*x+c)^7*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/5376*(105*(8*a^2 - b^2)*cos(d*x + c)^8*log(sin(d*x + c) + 1) - 105*(8*a^2
 - b^2)*cos(d*x + c)^8*log(-sin(d*x + c) + 1) + 1536*a*b*cos(d*x + c) + 14*
(15*(8*a^2 - b^2)*cos(d*x + c)^6 + 10*(8*a^2 - b^2)*cos(d*x + c)^4 + 8*(8*a
^2 - b^2)*cos(d*x + c)^2 + 48*b^2)*sin(d*x + c))/(d*cos(d*x + c)^8)
```

Sympy [F]

$$\int \sec^7(c+dx)(a+b\tan(c+dx))^2\,dx = \int \left(a+b\tan\left(c+dx\right)\right)^2\sec^7\left(c+dx\right)dx$$

```
[In] integrate(sec(d*x+c)**7*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*sec(c + d*x)**7, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.35

$$\int \sec^7(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{7\,b^2\left(\frac{2\left(15\,\sin(dx+c)^7-55\,\sin(dx+c)^5+73\,\sin(dx+c)^3+15\,\sin(dx+c)\right)}{\sin(dx+c)^8-4\,\sin(dx+c)^6+6\,\sin(dx+c)^4-4\,\sin(dx+c)^2+1}-15\,\log\left(\sin\left(dx+c\right)+1\right)+15\,\log\left(\sin\left(dx+c\right)-\right.\right.}{}$$

```
[In] integrate(sec(d*x+c)^7*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/5376*(7*b^2*(2*(15*sin(d*x + c)^7 - 55*sin(d*x + c)^5 + 73*sin(d*x + c)^3
 + 15*sin(d*x + c))/(sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x + c)^4 -
 4*sin(d*x + c)^2 + 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1
)) - 56*a^2*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(s
in(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x +
 c) + 1) + 15*log(sin(d*x + c) - 1)) + 1536*a*b/cos(d*x + c)^7)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(151) = 302$.

Time = 0.59 (sec) , antiderivative size = 437, normalized size of antiderivative = 2.68

$$\int \sec^7(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$= \frac{105\,(8\,a^2-b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-105\,(8\,a^2-b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(1848\,a^2\tan\left(\frac{1}{2}\,dx+\right.\right.}{}}{}$$

```
[In] integrate(sec(d*x+c)^7*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2688*(105*(8*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*(8*a^2 -
 b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(1848*a^2*tan(1/2*d*x + 1/2*c)
^15 + 105*b^2*tan(1/2*d*x + 1/2*c)^15 - 5376*a*b*tan(1/2*d*x + 1/2*c)^14 -
3416*a^2*tan(1/2*d*x + 1/2*c)^13 + 2779*b^2*tan(1/2*d*x + 1/2*c)^13 + 5376*
a*b*tan(1/2*d*x + 1/2*c)^12 + 6328*a^2*tan(1/2*d*x + 1/2*c)^11 + 6265*b^2*t
an(1/2*d*x + 1/2*c)^11 - 26880*a*b*tan(1/2*d*x + 1/2*c)^10 - 4760*a^2*tan(1
/2*d*x + 1/2*c)^9 + 12355*b^2*tan(1/2*d*x + 1/2*c)^9 + 26880*a*b*tan(1/2*d*
x + 1/2*c)^8 - 4760*a^2*tan(1/2*d*x + 1/2*c)^7 + 12355*b^2*tan(1/2*d*x + 1/
2*c)^7 - 16128*a*b*tan(1/2*d*x + 1/2*c)^6 + 6328*a^2*tan(1/2*d*x + 1/2*c)^5
 + 6265*b^2*tan(1/2*d*x + 1/2*c)^5 + 16128*a*b*tan(1/2*d*x + 1/2*c)^4 - 341
6*a^2*tan(1/2*d*x + 1/2*c)^3 + 2779*b^2*tan(1/2*d*x + 1/2*c)^3 - 768*a*b*ta
n(1/2*d*x + 1/2*c)^2 + 1848*a^2*tan(1/2*d*x + 1/2*c) + 105*b^2*tan(1/2*d*x
+ 1/2*c) + 768*a*b)/(tan(1/2*d*x + 1/2*c)^2 - 1)^8)/d
```

Mupad [B] (verification not implemented)

Time = 8.17 (sec) , antiderivative size = 432, normalized size of antiderivative = 2.65

$$\int \sec^7(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{5\,a^2}{8}-\frac{5\,b^2}{64}\right)}{d}$$

$$+\frac{\left(\frac{11\,a^2}{8}+\frac{5\,b^2}{64}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}-4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+\left(\frac{397\,b^2}{192}-\frac{61\,a^2}{24}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}+4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{}$$

```
[In] int((a + b*tan(c + d*x))^2/cos(c + d*x)^7,x)

[Out] (atanh(tan(c/2 + (d*x)/2))*((5*a^2)/8 - (5*b^2)/64))/d + ((4*a*b)/7 + tan(c
/2 + (d*x)/2)^15*((11*a^2)/8 + (5*b^2)/64) - tan(c/2 + (d*x)/2)^3*((61*a^2)
/24 - (397*b^2)/192) - tan(c/2 + (d*x)/2)^13*((61*a^2)/24 - (397*b^2)/192)
+ tan(c/2 + (d*x)/2)^5*((113*a^2)/24 + (895*b^2)/192) + tan(c/2 + (d*x)/2)^
11*((113*a^2)/24 + (895*b^2)/192) - tan(c/2 + (d*x)/2)^7*((85*a^2)/24 - (17
65*b^2)/192) - tan(c/2 + (d*x)/2)^9*((85*a^2)/24 - (1765*b^2)/192) + tan(c/
2 + (d*x)/2)*((11*a^2)/8 + (5*b^2)/64) - (4*a*b*tan(c/2 + (d*x)/2)^2)/7 + 1
2*a*b*tan(c/2 + (d*x)/2)^4 - 12*a*b*tan(c/2 + (d*x)/2)^6 + 20*a*b*tan(c/2 +
 (d*x)/2)^8 - 20*a*b*tan(c/2 + (d*x)/2)^10 + 4*a*b*tan(c/2 + (d*x)/2)^12 -
4*a*b*tan(c/2 + (d*x)/2)^14)/(d*(28*tan(c/2 + (d*x)/2)^4 - 8*tan(c/2 + (d*x
)/2)^2 - 56*tan(c/2 + (d*x)/2)^6 + 70*tan(c/2 + (d*x)/2)^8 - 56*tan(c/2 + (
d*x)/2)^10 + 28*tan(c/2 + (d*x)/2)^12 - 8*tan(c/2 + (d*x)/2)^14 + tan(c/2 +
 (d*x)/2)^16 + 1))
```

3.524 $\int \sec^5(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 131

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{(6a^2-b^2)\,\text{arctanh}(\sin(c+dx))}{16d} + \frac{7ab\sec^5(c+dx)}{30d} + \frac{(6a^2-b^2)\sec(c+dx)\tan(c+dx)}{16d} + \frac{(6a^2-b^2)\sec^3(c+dx)\tan(c+dx)}{24d} + \frac{b\sec^5(c+dx)(a+b\tan(c+dx))}{6d}$$

```
[Out] 1/16*(6*a^2-b^2)*arctanh(sin(d*x+c))/d+7/30*a*b*sec(d*x+c)^5/d+1/16*(6*a^2-
b^2)*sec(d*x+c)*tan(d*x+c)/d+1/24*(6*a^2-b^2)*sec(d*x+c)^3*tan(d*x+c)/d+1/6
*b*sec(d*x+c)^5*(a+b*tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3589, 3567, 3853, 3855}

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{(6a^2-b^2)\,\text{arctanh}(\sin(c+dx))}{16d} + \frac{(6a^2-b^2)\tan(c+dx)\sec^3(c+dx)}{24d} + \frac{(6a^2-b^2)\tan(c+dx)\sec(c+dx)}{16d} + \frac{7ab\sec^5(c+dx)}{30d} + \frac{b\sec^5(c+dx)(a+b\tan(c+dx))}{6d}$$

```
[In] Int[Sec[c + d*x]^5*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((6*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(16*d) + (7*a*b*Sec[c + d*x]^5)/(30*d
) + ((6*a^2 - b^2)*Sec[c + d*x]*Tan[c + d*x])/(16*d) + ((6*a^2 - b^2)*Sec[c
 + d*x]^3*Tan[c + d*x])/(24*d) + (b*Sec[c + d*x]^5*(a + b*Tan[c + d*x]))/(6
*d)
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b\sec^5(c+dx)(a+b\tan(c+dx))}{6d}+\frac{1}{6}\int \sec^5(c+dx)\left(6a^2-b^2+7ab\tan(c+dx)\right)\,dx\\
&= \frac{7ab\sec^5(c+dx)}{30d}+\frac{b\sec^5(c+dx)(a+b\tan(c+dx))}{6d}+\frac{1}{6}\left(6a^2-b^2\right)\int \sec^5(c+dx)\,dx\\
&= \frac{7ab\sec^5(c+dx)}{30d}+\frac{\left(6a^2-b^2\right)\sec^3(c+dx)\tan(c+dx)}{24d}\\
&\quad +\frac{b\sec^5(c+dx)(a+b\tan(c+dx))}{6d}+\frac{1}{8}\left(6a^2-b^2\right)\int \sec^3(c+dx)\,dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{7ab\sec^5(c+dx)}{30d} + \frac{(6a^2-b^2)\sec(c+dx)\tan(c+dx)}{16d} \\
&\quad + \frac{(6a^2-b^2)\sec^3(c+dx)\tan(c+dx)}{24d} \\
&\quad + \frac{b\sec^5(c+dx)(a+b\tan(c+dx))}{6d} + \frac{1}{16}(6a^2-b^2)\int \sec(c+dx)\,dx \\
&= \frac{(6a^2-b^2)\operatorname{arctanh}(\sin(c+dx))}{16d} + \frac{7ab\sec^5(c+dx)}{30d} \\
&\quad + \frac{(6a^2-b^2)\sec(c+dx)\tan(c+dx)}{16d} + \frac{(6a^2-b^2)\sec^3(c+dx)\tan(c+dx)}{24d} \\
&\quad + \frac{b\sec^5(c+dx)(a+b\tan(c+dx))}{6d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.28

$$
\begin{aligned}
\int \sec^5(c+dx)(a+b\tan(c+dx))^2\,dx &= \frac{3a^2\operatorname{arctanh}(\sin(c+dx))}{8d} - \frac{b^2\operatorname{arctanh}(\sin(c+dx))}{16d} \\
&\quad + \frac{2ab\sec^5(c+dx)}{5d} + \frac{3a^2\sec(c+dx)\tan(c+dx)}{8d} \\
&\quad - \frac{b^2\sec(c+dx)\tan(c+dx)}{16d} \\
&\quad + \frac{a^2\sec^3(c+dx)\tan(c+dx)}{4d} \\
&\quad - \frac{b^2\sec^3(c+dx)\tan(c+dx)}{24d} \\
&\quad + \frac{b^2\sec^5(c+dx)\tan(c+dx)}{6d}
\end{aligned}
$$

```
[In] Integrate[Sec[c + d*x]^5*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (3*a^2*ArcTanh[Sin[c + d*x]])/(8*d) - (b^2*ArcTanh[Sin[c + d*x]])/(16*d) +
(2*a*b*Sec[c + d*x]^5)/(5*d) + (3*a^2*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (b
^2*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (a^2*Sec[c + d*x]^3*Tan[c + d*x])/(4
*d) - (b^2*Sec[c + d*x]^3*Tan[c + d*x])/(24*d) + (b^2*Sec[c + d*x]^5*Tan[c
+ d*x])/(6*d)
```

Maple [A] (verified)

Time = 15.70 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{a^2\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{2ab}{5\cos(dx+c)^5}+b^2\left(\frac{\sin^3(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^3(dx}{8\cos(dx}\right.}{d}$
default	$\frac{a^2\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{2ab}{5\cos(dx+c)^5}+b^2\left(\frac{\sin^3(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^3(dx}{8\cos(dx}\right.}{d}$
risch	$-\frac{ie^{i(dx+c)}\left(90a^2e^{10i(dx+c)}-15b^2e^{10i(dx+c)}+510a^2e^{8i(dx+c)}-85b^2e^{8i(dx+c)}+420a^2e^{6i(dx+c)}+570b^2e^{6i(dx+c)}+1536ia\right.}{120d\left(e^{2i(dx+\right.}}$

[In] int(sec(d*x+c)^5*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^2*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))+2/5*a*b/cos(d*x+c)^5+b^2*(1/6*sin(d*x+c)^3/cos(d*x+c)^6+1/8*sin(d*x+c)^3/cos(d*x+c)^4+1/16*sin(d*x+c)^3/cos(d*x+c)^2+1/16*sin(d*x+c)-1/16*ln(sec(d*x+c)+tan(d*x+c))))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.08

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{15\,(6\,a^2-b^2)\cos\left(dx+c\right)^6\log\left(\sin\left(dx+c\right)+1\right)-15\,(6\,a^2-b^2)\cos\left(dx+c\right)^6\log\left(-\sin\left(dx+c\right)+1\right)+1}{480\,d\cos\left(dx\right.}$$

[In] integrate(sec(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="fricas")

[Out] 1/480*(15*(6*a^2 - b^2)*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 15*(6*a^2 - b^2)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 192*a*b*cos(d*x + c) + 10*(3*(6*a^2 - b^2)*cos(d*x + c)^4 + 2*(6*a^2 - b^2)*cos(d*x + c)^2 + 8*b^2)*sin(d*x + c))/(d*cos(d*x + c)^6)

Sympy [F]

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^2\,dx = \int (a+b\tan(c+dx))^2\sec^5(c+dx)\,dx$$

[In] integrate(sec(d*x+c)**5*(a+b*tan(d*x+c))**2,x)

[Out] Integral((a + b*tan(c + d*x))**2*sec(c + d*x)**5, x)

Maxima [A] (verification not implemented)

none

Time = 0.58 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.37

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{5\,b^2\left(\frac{2\left(3\,\sin(dx+c)^5-8\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-30\,a^2\left(\frac{2\left(3\right.}{\sin(}\right.}{480\,d}$$

[In] integrate(sec(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="maxima")

[Out] 1/480*(5*b^2*(2*(3*sin(d*x + c)^5 - 8*sin(d*x + c)^3 - 3*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 30*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) + 192*a*b/cos(d*x + c)^5)/d

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(121) = 242.

Time = 0.54 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.62

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{15\left(6\,a^2-b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\left(6\,a^2-b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(150\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}{}}{}$$

[In] integrate(sec(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="giac")

[Out] 1/240*(15*(6*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(6*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(150*a^2*tan(1/2*d*x + 1/2*c)^11

```
+ 15*b^2*tan(1/2*d*x + 1/2*c)^11 - 480*a*b*tan(1/2*d*x + 1/2*c)^10 - 210*a^
2*tan(1/2*d*x + 1/2*c)^9 + 235*b^2*tan(1/2*d*x + 1/2*c)^9 + 480*a*b*tan(1/2
*d*x + 1/2*c)^8 + 60*a^2*tan(1/2*d*x + 1/2*c)^7 + 390*b^2*tan(1/2*d*x + 1/2
*c)^7 - 960*a*b*tan(1/2*d*x + 1/2*c)^6 + 60*a^2*tan(1/2*d*x + 1/2*c)^5 + 39
0*b^2*tan(1/2*d*x + 1/2*c)^5 + 960*a*b*tan(1/2*d*x + 1/2*c)^4 - 210*a^2*tan
(1/2*d*x + 1/2*c)^3 + 235*b^2*tan(1/2*d*x + 1/2*c)^3 - 96*a*b*tan(1/2*d*x +
 1/2*c)^2 + 150*a^2*tan(1/2*d*x + 1/2*c) + 15*b^2*tan(1/2*d*x + 1/2*c) + 96
*a*b)/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

Mupad [B] (verification not implemented)

Time = 6.89 (sec) , antiderivative size = 328, normalized size of antiderivative = 2.50

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$= \frac{\left(\frac{5a^2}{4}+\frac{b^2}{8}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}-4ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+\left(\frac{47b^2}{24}-\frac{7a^2}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}+4ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}+\left(\frac{a}{2}\right.}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-6\tan\right.}$$

$$+\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(\frac{3a^2}{4}-\frac{b^2}{8}\right)}{d}$$

```
[In] int((a + b*tan(c + d*x))^2/cos(c + d*x)^5,x)
```

```
[Out] ((4*a*b)/5 + tan(c/2 + (d*x)/2)^5*(a^2/2 + (13*b^2)/4) + tan(c/2 + (d*x)/2)
^7*(a^2/2 + (13*b^2)/4) + tan(c/2 + (d*x)/2)^11*((5*a^2)/4 + b^2/8) - tan(c
/2 + (d*x)/2)^3*((7*a^2)/4 - (47*b^2)/24) - tan(c/2 + (d*x)/2)^9*((7*a^2)/4
 - (47*b^2)/24) + tan(c/2 + (d*x)/2)*((5*a^2)/4 + b^2/8) - (4*a*b*tan(c/2 +
 (d*x)/2)^2)/5 + 8*a*b*tan(c/2 + (d*x)/2)^4 - 8*a*b*tan(c/2 + (d*x)/2)^6 +
4*a*b*tan(c/2 + (d*x)/2)^8 - 4*a*b*tan(c/2 + (d*x)/2)^10)/(d*(15*tan(c/2 +
(d*x)/2)^4 - 6*tan(c/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2
+ (d*x)/2)^8 - 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1)) + (ata
nh(tan(c/2 + (d*x)/2))*((3*a^2)/4 - b^2/8))/d
```

3.525 $\int \sec^3(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 99

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{(4a^2-b^2)\operatorname{arctanh}(\sin(c+dx))}{8d} + \frac{5ab\sec^3(c+dx)}{12d} + \frac{(4a^2-b^2)\sec(c+dx)\tan(c+dx)}{8d} + \frac{b\sec^3(c+dx)(a+b\tan(c+dx))}{4d}$$

```
[Out] 1/8*(4*a^2-b^2)*arctanh(sin(d*x+c))/d+5/12*a*b*sec(d*x+c)^3/d+1/8*(4*a^2-b^
2)*sec(d*x+c)*tan(d*x+c)/d+1/4*b*sec(d*x+c)^3*(a+b*tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3589, 3567, 3853, 3855}

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{(4a^2-b^2)\operatorname{arctanh}(\sin(c+dx))}{8d} + \frac{(4a^2-b^2)\tan(c+dx)\sec(c+dx)}{8d} + \frac{5ab\sec^3(c+dx)}{12d} + \frac{b\sec^3(c+dx)(a+b\tan(c+dx))}{4d}$$

```
[In] Int[Sec[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((4*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(8*d) + (5*a*b*Sec[c + d*x]^3)/(12*d)
 + ((4*a^2 - b^2)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (b*Sec[c + d*x]^3*(a +
 b*Tan[c + d*x]))/(4*d)
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{b\sec^3(c+dx)(a+b\tan(c+dx))}{4d}+\frac{1}{4}\int \sec^3(c+dx)\left(4a^2-b^2+5ab\tan(c+dx)\right)\,dx\\
&= \frac{5ab\sec^3(c+dx)}{12d}+\frac{b\sec^3(c+dx)(a+b\tan(c+dx))}{4d}+\frac{1}{4}(4a^2-b^2)\int \sec^3(c+dx)\,dx\\
&= \frac{5ab\sec^3(c+dx)}{12d}+\frac{(4a^2-b^2)\sec(c+dx)\tan(c+dx)}{8d}\\
&\quad+\frac{b\sec^3(c+dx)(a+b\tan(c+dx))}{4d}+\frac{1}{8}(4a^2-b^2)\int \sec(c+dx)\,dx\\
&= \frac{(4a^2-b^2)\operatorname{arctanh}(\sin(c+dx))}{8d}+\frac{5ab\sec^3(c+dx)}{12d}\\
&\quad+\frac{(4a^2-b^2)\sec(c+dx)\tan(c+dx)}{8d}+\frac{b\sec^3(c+dx)(a+b\tan(c+dx))}{4d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.21

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{a^2\operatorname{arctanh}(\sin(c+dx))}{2d} - \frac{b^2\operatorname{arctanh}(\sin(c+dx))}{8d} + \frac{2ab\sec^3(c+dx)}{3d} + \frac{a^2\sec(c+dx)\tan(c+dx)}{2d} - \frac{b^2\sec(c+dx)\tan(c+dx)}{8d} + \frac{b^2\sec^3(c+dx)\tan(c+dx)}{4d}$$

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/(2*d) - (b^2*ArcTanh[Sin[c + d*x]])/(8*d) + (2*
a*b*Sec[c + d*x]^3)/(3*d) + (a^2*Sec[c + d*x]*Tan[c + d*x])/(2*d) - (b^2*Se
c[c + d*x]*Tan[c + d*x])/(8*d) + (b^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Maple [A] (verified)

Time = 4.72 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.19

method	result
derivativedivides	$\frac{a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{2ab}{3\cos(dx+c)^3}+b^2\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)}{}\right.}{d}$
default	$\frac{a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{2ab}{3\cos(dx+c)^3}+b^2\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)}{}\right.}{d}$
risch	$-\frac{i e^{i(dx+c)}\left(12a^2 e^{6i(dx+c)}-3b^2 e^{6i(dx+c)}+12a^2 e^{4i(dx+c)}+21b^2 e^{4i(dx+c)}+64iab\, e^{4i(dx+c)}-12a^2 e^{2i(dx+c)}-21b^2 e^{2i(dx+c)}\right.}{12d\left(e^{2i(dx+c)}+1\right)^4}$

```
[In] int(sec(d*x+c)^3*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+2/3*a*b/
cos(d*x+c)^3+b^2*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/cos(d*x+c)
^2+1/8*sin(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.21

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{3\,(4\,a^2-b^2)\cos{(dx+c)}^4\log{(\sin{(dx+c)}+1)}-3\,(4\,a^2-b^2)\cos{(dx+c)}^4\log{(-\sin{(dx+c)}+1)}+32\,c}{48\,d\cos{(dx+c)}^4}$$

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
[Out] 1/48*(3*(4*a^2 - b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(4*a^2 - b^2
)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 32*a*b*cos(d*x + c) + 6*((4*a^2 -
 b^2)*cos(d*x + c)^2 + 2*b^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^2\,dx=\int (a+b\tan{(c+dx)})^2\sec^3{(c+dx)}\,dx$$

```
[In] integrate(sec(d*x+c)**3*(a+b*tan(d*x+c))**2,x)
[Out] Integral((a + b*tan(c + d*x))**2*sec(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.30

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{3\,b^2\left(\frac{2\left(\sin(dx+c)^3+\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-\log{(\sin{(dx+c)}+1)}+\log{(\sin{(dx+c)}-1)}\right)-12\,a^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log{(s}\right.}{48\,d}$$

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
[Out] 1/48*(3*b^2*(2*(sin(d*x + c)^3 + sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x
+ c)^2 + 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 12*a^2*(2*si
n(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c)
- 1)) + 32*a*b/cos(d*x + c)^3)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(91) = 182$.

Time = 0.51 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.52

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{3\,(4\,a^2-b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(4\,a^2-b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+3\cdots\right.}{\cdots}}{\cdots}$$

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(3*(4*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*a^2 - b^2)*
log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(12*a^2*tan(1/2*d*x + 1/2*c)^7 + 3*b
^2*tan(1/2*d*x + 1/2*c)^7 - 48*a*b*tan(1/2*d*x + 1/2*c)^6 - 12*a^2*tan(1/2*
d*x + 1/2*c)^5 + 21*b^2*tan(1/2*d*x + 1/2*c)^5 + 48*a*b*tan(1/2*d*x + 1/2*c
)^4 - 12*a^2*tan(1/2*d*x + 1/2*c)^3 + 21*b^2*tan(1/2*d*x + 1/2*c)^3 - 16*a*
b*tan(1/2*d*x + 1/2*c)^2 + 12*a^2*tan(1/2*d*x + 1/2*c) + 3*b^2*tan(1/2*d*x
+ 1/2*c) + 16*a*b)/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 7.25 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.18

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{\left(a^2+\frac{b^2}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7-4\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+\left(\frac{7\,b^2}{4}-a^2\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+4\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+\left(\frac{7\,b^2}{4}-a\cdots\right.}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-4\,\mathrm{t}\cdots\right.}$$
$$+\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(a^2-\frac{b^2}{4}\right)}{d}$$

```
[In] int((a + b*tan(c + d*x))^2/cos(c + d*x)^3,x)
```

```
[Out] ((4*a*b)/3 + tan(c/2 + (d*x)/2)*(a^2 + b^2/4) + tan(c/2 + (d*x)/2)^7*(a^2 +
 b^2/4) - tan(c/2 + (d*x)/2)^3*(a^2 - (7*b^2)/4) - tan(c/2 + (d*x)/2)^5*(a^
2 - (7*b^2)/4) - (4*a*b*tan(c/2 + (d*x)/2)^2)/3 + 4*a*b*tan(c/2 + (d*x)/2)^
4 - 4*a*b*tan(c/2 + (d*x)/2)^6)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d
*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (atanh(tan
(c/2 + (d*x)/2))*(a^2 - b^2/4))/d
```

3.526 $\int \sec(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 65

$$\int \sec(c+dx)(a+b\tan(c+dx))^2\,dx=\frac{(2a^2-b^2)\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{3ab\sec(c+dx)}{2d}+\frac{b\sec(c+dx)(a+b\tan(c+dx))}{2d}$$

```
[Out] 1/2*(2*a^2-b^2)*arctanh(sin(d*x+c))/d+3/2*a*b*sec(d*x+c)/d+1/2*b*sec(d*x+c)
*(a+b*tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3589, 3567, 3855}

$$\int \sec(c+dx)(a+b\tan(c+dx))^2\,dx=\frac{(2a^2-b^2)\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{3ab\sec(c+dx)}{2d}+\frac{b\sec(c+dx)(a+b\tan(c+dx))}{2d}$$

```
[In] Int[Sec[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((2*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (3*a*b*Sec[c + d*x])/(2*d) +
(b*Sec[c + d*x]*(a + b*Tan[c + d*x]))/(2*d)
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
```

```
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b\sec(c+dx)(a+b\tan(c+dx))}{2d}+\frac{1}{2}\int\sec(c+dx)\left(2a^2-b^2+3ab\tan(c+dx)\right)\,dx\\
&= \frac{3ab\sec(c+dx)}{2d}+\frac{b\sec(c+dx)(a+b\tan(c+dx))}{2d}+\frac{1}{2}(2a^2-b^2)\int\sec(c+dx)\,dx\\
&= \frac{(2a^2-b^2)\,\text{arctanh}(\sin(c+dx))}{2d}+\frac{3ab\sec(c+dx)}{2d}+\frac{b\sec(c+dx)(a+b\tan(c+dx))}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03

$$\int\sec(c+dx)(a+b\tan(c+dx))^2\,dx=\frac{a^2\text{arctanh}(\sin(c+dx))}{d}-\frac{b^2\text{arctanh}(\sin(c+dx))}{2d}+\frac{2ab\sec(c+dx)}{d}+\frac{b^2\sec(c+dx)\tan(c+dx)}{2d}$$

```
[In] Integrate[Sec[c + d*x]*(a + b*Tan[c + d*x])^2,x]
[Out] (a^2*ArcTanh[Sin[c + d*x]])/d - (b^2*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a*b*
Sec[c + d*x])/d + (b^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.28

method	result
derivativedivides	$\frac{b^2\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{2ab}{\cos(dx+c)}+a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}$
default	$\frac{b^2\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{2ab}{\cos(dx+c)}+a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}$
risch	$\frac{b\,e^{i(dx+c)}\left(-ib\,e^{2i(dx+c)}+4a\,e^{2i(dx+c)}+ib+4a\right)}{d\left(e^{2i(dx+c)}+1\right)^2}+\frac{a^2\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{\ln\left(e^{i(dx+c)}+i\right)b^2}{2d}-\frac{a^2\ln\left(e^{i(dx+c)}-i\right)}{d}+$

```
[In] int(sec(d*x+c)*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(b^2*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+ta
n(d*x+c)))+2*a*b/cos(d*x+c)+a^2*ln(sec(d*x+c)+tan(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.48

$$\int \sec(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{(2a^2-b^2)\cos(dx+c)^2\log(\sin(dx+c)+1)-(2a^2-b^2)\cos(dx+c)^2\log(-\sin(dx+c)+1)+8ab\cos}{4d\cos(dx+c)^2}$$

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
[Out] 1/4*((2*a^2 - b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a^2 - b^2)*cos
(d*x + c)^2*log(-sin(d*x + c) + 1) + 8*a*b*cos(d*x + c) + 2*b^2*sin(d*x + c
))/(d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \sec(c+dx)(a+b\tan(c+dx))^2\,dx=\int (a+b\tan(c+dx))^2\sec(c+dx)\,dx$$

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c))**2,x)
[Out] Integral((a + b*tan(c + d*x))**2*sec(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.26

$$\int \sec(c+dx)(a+b\tan(c+dx))^2\,dx =$$

$$-\frac{b^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)-4\,a^2\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{4\,d}$$

[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="maxima")

[Out] -1/4*(b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - lo
g(sin(d*x + c) - 1)) - 4*a^2*log(sec(d*x + c) + tan(d*x + c)) - 8*a*b/cos(d
*x + c))/d

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(59) = 118.

Time = 0.51 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.88

$$\int \sec(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$=\frac{(2\,a^2-b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-(2\,a^2-b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,ab\tan(}{\left(\tan(\frac{1}{2}\,d}}{2\,d}$$

[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="giac")

[Out] 1/2*((2*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (2*a^2 - b^2)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(b^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*b*tan(1
/2*d*x + 1/2*c)^2 + b^2*tan(1/2*d*x + 1/2*c) + 4*a*b)/(tan(1/2*d*x + 1/2*c)
^2 - 1)^2)/d

Mupad [B] (verification not implemented)

Time = 4.65 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.63

$$\int \sec(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-4\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+4\,a\,b}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}$$
$$+\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,\left(2\,a^2-b^2\right)}{d}$$

```
[In] int((a + b*tan(c + d*x))^2/cos(c + d*x),x)
```

```
[Out] (4*a*b + b^2*tan(c/2 + (d*x)/2)^3 + b^2*tan(c/2 + (d*x)/2) - 4*a*b*tan(c/2
+ (d*x)/2)^2)/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1)) + (at
anh(tan(c/2 + (d*x)/2))*(2*a^2 - b^2))/d
```

3.527 $\int \cos(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \cos(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{b^2\text{arctanh}(\sin(c+dx))}{d} - \frac{2ab\cos(c+dx)}{d} + \frac{(a^2-b^2)\sin(c+dx)}{d}$$

```
[Out] b^2*arctanh(sin(d*x+c))/d-2*a*b*cos(d*x+c)/d+(a^2-b^2)*sin(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3588}

$$\int \cos(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{(a^2-b^2)\sin(c+dx)}{d} - \frac{2ab\cos(c+dx)}{d} + \frac{b^2\text{arctanh}(\sin(c+dx))}{d}$$

```
[In] Int[Cos[c + d*x]*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (b^2*ArcTanh[Sin[c + d*x]])/d - (2*a*b*Cos[c + d*x])/d + ((a^2 - b^2)*Sin[c
 + d*x])/d
```

Rule 3588

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2/sec[(e_.) + (f_.)*(x_)], x_Sym
bol] :> Simp[b^2*(ArcTanh[Sin[e + f*x]]/f), x] + (-Simp[2*a*b*(Cos[e + f*x]
/f), x] + Simp[(a^2 - b^2)*(Sin[e + f*x]/f), x]) /; FreeQ[{a, b, e, f}, x]
&& NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{b^2\text{arctanh}(\sin(c+dx))}{d} - \frac{2ab\cos(c+dx)}{d} + \frac{(a^2-b^2)\sin(c+dx)}{d}$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.79

$$\int \cos(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$= \frac{-2ab\cos(c+dx)+b^2\left(-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right.\right.}{d}$$

```
[In] Integrate[Cos[c + d*x]*(a + b*Tan[c + d*x])^2,x]
[Out] (-2*a*b*Cos[c + d*x] + b^2*(-Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (a^2 - b^2)*Sin[c + d*x])/d
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{b^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))-2ab\cos(dx+c)+a^2\sin(dx+c)}{d}$
default	$\frac{b^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))-2ab\cos(dx+c)+a^2\sin(dx+c)}{d}$
risch	$-\frac{e^{i(dx+c)}ab}{d}-\frac{ia^2e^{i(dx+c)}}{2d}+\frac{ie^{i(dx+c)}b^2}{2d}-\frac{e^{-i(dx+c)}ab}{d}+\frac{ie^{-i(dx+c)}a^2}{2d}-\frac{ie^{-i(dx+c)}b^2}{2d}+\frac{\ln(e^{i(dx+c)}+i)b^2}{d}$

```
[In] int(cos(d*x+c)*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(b^2*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))-2*a*b*cos(d*x+c)+a^2*sin(d
*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.32

$$\int \cos(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{4\,ab\cos\left(dx+c\right)-b^2\log\left(\sin\left(dx+c\right)+1\right)+b^2\log\left(-\sin\left(dx+c\right)+1\right)-2\left(a^2-b^2\right)\sin\left(dx+c\right)}{2\,d}$$

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
[Out] -1/2*(4*a*b*cos(d*x + c) - b^2*log(sin(d*x + c) + 1) + b^2*log(-sin(d*x + c
) + 1) - 2*(a^2 - b^2)*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos(c+dx)(a+b\tan(c+dx))^2\,dx = \int \left(a+b\tan\left(c+dx\right)\right)^2\cos\left(c+dx\right)\,dx$$

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c))**2,x)
[Out] Integral((a + b*tan(c + d*x))**2*cos(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.28

$$\int \cos(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{b^2(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)-2\,\sin\left(dx+c\right))-4\,ab\cos\left(dx+c\right)+2\,a^2\sin\left(dx+c\right)}{2\,d}$$

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
[Out] 1/2*(b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)) -
 4*a*b*cos(d*x + c) + 2*a^2*sin(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1100 vs. $2(47) = 94$.

Time = 0.70 (sec) , antiderivative size = 1100, normalized size of antiderivative = 23.40

$$\int \cos(c+dx)(a+b\tan(c+dx))^2\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c))^2,x, algorithm="giac")
[Out] -1/2*(b^2*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c)
+ 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d
*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan
(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 - b^2*log(2*(tan(1/2*d*x)^2*tan
(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan
(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*
x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(
1/2*c)^2 + 4*a*b*tan(1/2*d*x)^2*tan(1/2*c)^2 + b^2*log(2*(tan(1/2*d*x)^2*ta
n(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + ta
n(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d
*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2 - b
^2*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan
(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2
*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)
^2 + 1))*tan(1/2*d*x)^2 + 4*a^2*tan(1/2*d*x)^2*tan(1/2*c) - 4*b^2*tan(1/2*d
*x)^2*tan(1/2*c) + b^2*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^
2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2
- 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2
*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^2 - b^2*log(2*(tan(1/2*d*x)^2*tan(1
/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1
/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)
^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^2 + 4*a^2*
tan(1/2*d*x)*tan(1/2*c)^2 - 4*b^2*tan(1/2*d*x)*tan(1/2*c)^2 - 4*a*b*tan(1/2
*d*x)^2 - 16*a*b*tan(1/2*d*x)*tan(1/2*c) - 4*a*b*tan(1/2*c)^2 + b^2*log(2*(
tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*
tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c
) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1)) -
 b^2*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*t
an(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) +
 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*
c)^2 + 1)) - 4*a^2*tan(1/2*d*x) + 4*b^2*tan(1/2*d*x) - 4*a^2*tan(1/2*c) + 4
*b^2*tan(1/2*c) + 4*a*b)/(d*tan(1/2*d*x)^2*tan(1/2*c)^2 + d*tan(1/2*d*x)^2
+ d*tan(1/2*c)^2 + d)
```

Mupad [B] (verification not implemented)

Time = 4.97 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.40

$$\int \cos(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{2\,b^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{4\,a\,b-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^2-2\,b^2\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

```
[In] int(cos(c + d*x)*(a + b*tan(c + d*x))^2,x)
```

```
[Out] (2*b^2*atanh(tan(c/2 + (d*x)/2)))/d - (4*a*b - tan(c/2 + (d*x)/2)*(2*a^2 -
2*b^2))/(d*(tan(c/2 + (d*x)/2)^2 + 1))
```

3.528 $\int \cos^3(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 90

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{ab\cos^3(c+dx)}{6d}+\frac{(2a^2+b^2)\sin(c+dx)}{2d} - \frac{(2a^2+b^2)\sin^3(c+dx)}{6d} - \frac{b\cos^3(c+dx)(a+b\tan(c+dx))}{2d}$$

```
[Out] -1/6*a*b*cos(d*x+c)^3/d+1/2*(2*a^2+b^2)*sin(d*x+c)/d-1/6*(2*a^2+b^2)*sin(d*
x+c)^3/d-1/2*b*cos(d*x+c)^3*(a+b*tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3589, 3567, 2713}

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{(2a^2+b^2)\sin^3(c+dx)}{6d} + \frac{(2a^2+b^2)\sin(c+dx)}{2d} - \frac{ab\cos^3(c+dx)}{6d} - \frac{b\cos^3(c+dx)(a+b\tan(c+dx))}{2d}$$

```
[In] Int[Cos[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -1/6*(a*b*Cos[c + d*x]^3)/d + ((2*a^2 + b^2)*Sin[c + d*x])/(2*d) - ((2*a^2
+ b^2)*Sin[c + d*x]^3)/(6*d) - (b*Cos[c + d*x]^3*(a + b*Tan[c + d*x]))/(2*d
)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b\cos^3(c+dx)(a+b\tan(c+dx))}{2d} \\
&\quad -\frac{1}{2}\int \cos^3(c+dx)\left(-2a^2-b^2-ab\tan(c+dx)\right)\,dx \\
&= -\frac{ab\cos^3(c+dx)}{6d}-\frac{b\cos^3(c+dx)(a+b\tan(c+dx))}{2d}-\frac{1}{2}\left(-2a^2-b^2\right)\int \cos^3(c+dx)\,dx \\
&= -\frac{ab\cos^3(c+dx)}{6d}-\frac{b\cos^3(c+dx)(a+b\tan(c+dx))}{2d} \\
&\quad -\frac{(2a^2+b^2)\,\text{Subst}\left(\int (1-x^2)\,dx, x, -\sin(c+dx)\right)}{2d} \\
&= -\frac{ab\cos^3(c+dx)}{6d}+\frac{(2a^2+b^2)\sin(c+dx)}{2d} \\
&\quad -\frac{(2a^2+b^2)\sin^3(c+dx)}{6d}-\frac{b\cos^3(c+dx)(a+b\tan(c+dx))}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{2ab\cos^3(c+dx)}{3d} + \frac{a^2\sin(c+dx)}{d} - \frac{a^2\sin^3(c+dx)}{3d} + \frac{b^2\sin^3(c+dx)}{3d}$$

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Tan[c + d*x])^2,x]
[Out] (-2*a*b*Cos[c + d*x]^3)/(3*d) + (a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*x]^3)
/(3*d) + (b^2*Sin[c + d*x]^3)/(3*d)
```

Maple [A] (verified)

Time = 4.72 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.58

method	result	size
derivativedivides	$\frac{\frac{b^2\left(\sin^3(dx+c)\right)}{3}-\frac{2ab\left(\cos^3(dx+c)\right)}{3}+\frac{a^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$	52
default	$\frac{\frac{b^2\left(\sin^3(dx+c)\right)}{3}-\frac{2ab\left(\cos^3(dx+c)\right)}{3}+\frac{a^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$	52
risch	$-\frac{ab\cos(dx+c)}{2d}+\frac{3a^2\sin(dx+c)}{4d}+\frac{\sin(dx+c)b^2}{4d}-\frac{ab\cos(3dx+3c)}{6d}+\frac{\sin(3dx+3c)a^2}{12d}-\frac{\sin(3dx+3c)b^2}{12d}$	93

```
[In] int(cos(d*x+c)^3*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(1/3*b^2*sin(d*x+c)^3-2/3*a*b*cos(d*x+c)^3+1/3*a^2*(2+cos(d*x+c)^2)*sin
(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.59

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{2\,ab\cos\left(dx+c\right)^3 - \left(\left(a^2-b^2\right)\cos\left(dx+c\right)^2 + 2\,a^2 + b^2\right)\sin\left(dx+c\right)}{3\,d}$$

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
[Out] -1/3*(2*a*b*cos(d*x + c)^3 - ((a^2 - b^2)*cos(d*x + c)^2 + 2*a^2 + b^2)*sin
(d*x + c))/d
```

Sympy [F]

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^2\,dx = \int (a+b\tan(c+dx))^2\cos^3(c+dx)\,dx$$

```
[In] integrate(cos(d*x+c)**3*(a+b*tan(d*x+c))**2,x)
[Out] Integral((a + b*tan(c + d*x))**2*cos(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.58

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{2\,ab\cos(dx+c)^3 - b^2\sin(dx+c)^3 + \left(\sin(dx+c)^3 - 3\,\sin(dx+c)\right)a^2}{3\,d}$$

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
[Out] -1/3*(2*a*b*cos(d*x + c)^3 - b^2*sin(d*x + c)^3 + (sin(d*x + c)^3 - 3*sin(d
*x + c))*a^2)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11162 vs. 2(82) = 164.

Time = 16.59 (sec) , antiderivative size = 11162, normalized size of antiderivative = 124.02

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^2\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^2,x, algorithm="giac")
[Out] -1/48*(3*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*
c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*t
an(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 +
 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 3*pi*a*b*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c
)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*ta
n(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*
x)^6*tan(1/2*c)^6 + 3*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*
x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2
*d*x)^6*tan(1/2*c)^6 + 3*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2
```

*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(
1/2*d*x)^6*tan(1/2*c)^6 + 9*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(
1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*s
gn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)
^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 + 9*pi*
a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2
*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
- 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d
*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 + 9*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*
c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(
1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 -
 tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2
*c)^6 + 9*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2
*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*
tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2
- 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 - 6*a*b*arctan((tan(1/2*d
*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) -
tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 - 6*a*b*arctan(
(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan
(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 + 6*a
*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/
2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*
c)^6 + 6*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) -
1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^
6*tan(1/2*c)^6 + 9*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^
2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*
x)^6*tan(1/2*c)^4 + 9*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*
x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2
*d*x)^6*tan(1/2*c)^4 + 9*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2
*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(
1/2*d*x)^4*tan(1/2*c)^6 + 9*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(
1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*t
an(1/2*d*x)^4*tan(1/2*c)^6 + 32*a*b*tan(1/2*d*x)^6*tan(1/2*c)^6 + 9*pi*a*b*
sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x
)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*
tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x)
- 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 + 9*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*
c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan
(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^
2 + 27*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c)
 + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan
(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2
*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 + 27*pi*a*b*sgn(tan(1/2*d*x)
^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)

^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan
(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x
)^4*tan(1/2*c)^4 - 18*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) -
tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*
tan(1/2*d*x)^6*tan(1/2*c)^4 - 18*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(
1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/
2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 + 18*a*b*arctan((tan(1/2*d*x)*tan(1/
2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*
x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 + 18*a*b*arctan((tan(1/2*
d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) +
tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 + 96*a^2*tan(1
/2*d*x)^6*tan(1/2*c)^5 + 9*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1
/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sg
n(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^
2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^6 + 9*pi*a
*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*
d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 -
2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*
x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^6 - 18*a*b*arctan((tan(1/2*d*x)*tan(1/2*c
) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x)
+ tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^6 - 18*a*b*arctan((tan(1/2*d*x
)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + ta
n(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^6 + 18*a*b*arctan((
tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(
1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^6 + 18*a
*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/
2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*
c)^6 + 96*a^2*tan(1/2*d*x)^5*tan(1/2*c)^6 + 9*pi*a*b*sgn(tan(1/2*d*x)^2*tan
(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2
*tan(1/2*c) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 + 9*pi*a*b*sgn(tan(1/2*d*x)^2*
tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2
- 2*tan(1/2*c) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 + 27*pi*a*b*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c
)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 + 27*pi*a*b*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1
/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 - 96*a*b*tan(1/2*d*
x)^6*tan(1/2*c)^4 - 384*a*b*tan(1/2*d*x)^5*tan(1/2*c)^5 + 9*pi*a*b*sgn(tan(
1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - ta
n(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^6 + 9*pi*a*b*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^6 - 96*a*b*tan(
1/2*d*x)^4*tan(1/2*c)^6 + 3*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(
1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*s
gn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)
^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6 + 3*pi*a*b*sgn(tan(1

/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan (1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d *x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6 + 27*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*t an(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d *x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2 *c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^2 + 27*pi*a*b*sgn(tan (1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - t an(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2 *d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*ta n(1/2*d*x)^4*tan(1/2*c)^2 - 18*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/ 2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2* c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 - 18*a*b*arctan((tan(1/2*d*x)*tan(1/2* c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 + 18*a*b*arctan((tan(1/2*d* x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - t an(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 + 18*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan (1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 + 64* a^2*tan(1/2*d*x)^6*tan(1/2*c)^3 + 128*b^2*tan(1/2*d*x)^6*tan(1/2*c)^3 + 27* pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c) ^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/ 2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^4 + 27*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2* tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c) ^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan (1/2*c)^4 - 54*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2 *c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2 *d*x)^4*tan(1/2*c)^4 - 54*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 + 54*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - ta n(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 + 54*a*b*arctan((tan(1/2*d*x)*ta n(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/ 2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 - 288*a^2*tan(1/2*d*x)^5*tan(1/2*c)^4 + 384*b^2*tan(1/2*d*x)^5*tan(1/2*c)^4 - 288*a^2*tan(1/2*d* x)^4*tan(1/2*c)^5 + 384*b^2*tan(1/2*d*x)^4*tan(1/2*c)^5 + 3*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - ta n(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2* d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan (1/2*c)^6 + 3*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan (1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^6 - 18*a*b*arctan((tan(1/2*d*x)*tan(1/

2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*
x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 - 18*a*b*arctan((tan(1/2*
d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) +
 tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 + 18*a*b*arcta
n((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*t
an(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 + 1
8*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan
(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1
/2*c)^6 + 64*a^2*tan(1/2*d*x)^3*tan(1/2*c)^6 + 128*b^2*tan(1/2*d*x)^3*tan(1
/2*c)^6 + 3*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1
/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^6 +
3*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + ta
n(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^6 + 27*pi*a*b*
sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x
)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^2 + 27*pi*
a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2
*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^2 + 96
*a*b*tan(1/2*d*x)^6*tan(1/2*c)^2 + 768*a*b*tan(1/2*d*x)^5*tan(1/2*c)^3 + 27
*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan
(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^4
+ 27*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) +
 tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^2*tan(1/2*c
)^4 + 1824*a*b*tan(1/2*d*x)^4*tan(1/2*c)^4 + 768*a*b*tan(1/2*d*x)^3*tan(1/2
*c)^5 + 3*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2
*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*c)^6 + 3*pi
*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/
2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*c)^6 + 96*a*b*tan(1/2*d
*x)^2*tan(1/2*c)^6 + 9*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d
*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4 + 9*pi*a*b*sgn(tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*
c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*t
an(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d
*x)^4 - 6*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) +
 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)
^6 - 6*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)
/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6
+ 6*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(t
an(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6 + 6
*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(
1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6 + 96*a
^2*tan(1/2*d*x)^6*tan(1/2*c) + 27*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 +
2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c)
- 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/

2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 +
27*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) +
tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/
2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*ta
n(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 54*a*b*arctan((tan(1/2*d*x)*t
an(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1
/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^2 - 54*a*b*arctan((tan
(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2
*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^2 + 54*a*b*
arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d
*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^
2 + 54*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)
/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4*
tan(1/2*c)^2 + 288*a^2*tan(1/2*d*x)^5*tan(1/2*c)^2 - 384*b^2*tan(1/2*d*x)^5
*tan(1/2*c)^2 + 1344*a^2*tan(1/2*d*x)^4*tan(1/2*c)^3 - 1152*b^2*tan(1/2*d*x
)^4*tan(1/2*c)^3 + 9*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x
)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(
1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + ta
n(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^4 + 9*pi*a*b*sgn(tan(1/2*d*x)^2
*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2
- 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1
/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^4
- 54*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(
tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2*ta
n(1/2*c)^4 - 54*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/
2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/
2*d*x)^2*tan(1/2*c)^4 + 54*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*
x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) -
1))*tan(1/2*d*x)^2*tan(1/2*c)^4 + 54*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) -
tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + t
an(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^4 + 1344*a^2*tan(1/2*d*x)^3*tan(1
/2*c)^4 - 1152*b^2*tan(1/2*d*x)^3*tan(1/2*c)^4 + 288*a^2*tan(1/2*d*x)^2*tan
(1/2*c)^5 - 384*b^2*tan(1/2*d*x)^2*tan(1/2*c)^5 - 6*a*b*arctan((tan(1/2*d*x
)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - ta
n(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*c)^6 - 6*a*b*arctan((tan(1/2*d*x)*tan
(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2
*d*x) - tan(1/2*c) + 1))*tan(1/2*c)^6 + 6*a*b*arctan((tan(1/2*d*x)*tan(1/2*
c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x)
- tan(1/2*c) - 1))*tan(1/2*c)^6 + 6*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) -
tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + ta
n(1/2*c) - 1))*tan(1/2*c)^6 + 96*a^2*tan(1/2*d*x)*tan(1/2*c)^6 + 9*pi*a*b*s
gn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)
^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^4 + 9*pi*a*b*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1
/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^4 - 32*a*b*tan(1/2*d*x)^6 - 384*a*

b*tan(1/2*d*x)^5*tan(1/2*c) + 27*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2
*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) -
 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 27*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*
c) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 1824*a*b*tan(1/2*d*x)^4*tan(1/2*c)^2
- 3584*a*b*tan(1/2*d*x)^3*tan(1/2*c)^3 + 9*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/
2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*ta
n(1/2*c) - 1)*tan(1/2*c)^4 + 9*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*t
an(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1
)*tan(1/2*c)^4 - 1824*a*b*tan(1/2*d*x)^2*tan(1/2*c)^4 - 384*a*b*tan(1/2*d*x
)*tan(1/2*c)^5 - 32*a*b*tan(1/2*c)^6 + 9*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*
c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(
1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 -
 tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2 + 9*pi*
a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2
*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
- 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d
*x) - 1)*tan(1/2*d*x)^2 - 18*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*
d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c)
 + 1))*tan(1/2*d*x)^4 - 18*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*
x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) +
 1))*tan(1/2*d*x)^4 + 18*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x)
 + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1
))*tan(1/2*d*x)^4 + 18*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) -
 tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))
*tan(1/2*d*x)^4 - 96*a^2*tan(1/2*d*x)^5 - 288*a^2*tan(1/2*d*x)^4*tan(1/2*c)
 + 384*b^2*tan(1/2*d*x)^4*tan(1/2*c) + 9*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*
c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(
1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 -
 tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^2 + 9*pi*a*
b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d
*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 -
2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x
) - 1)*tan(1/2*c)^2 - 54*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x)
 - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1
))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 54*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - t
an(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan
(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 54*a*b*arctan((tan(1/2*d*x)*tan
(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2
*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 54*a*b*arctan((tan(1
/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c
) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 1344*a^2*
tan(1/2*d*x)^3*tan(1/2*c)^2 + 1152*b^2*tan(1/2*d*x)^3*tan(1/2*c)^2 - 1344*a
^2*tan(1/2*d*x)^2*tan(1/2*c)^3 + 1152*b^2*tan(1/2*d*x)^2*tan(1/2*c)^3 - 18*
a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1

/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*c)^4 - 18*a*b*
arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d
*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*c)^4 + 18*a*b*arct
an((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*
tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*c)^4 + 18*a*b*arctan((
tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(
1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^4 - 288*a^2*tan(1/2*d*x
)*tan(1/2*c)^4 + 384*b^2*tan(1/2*d*x)*tan(1/2*c)^4 - 96*a^2*tan(1/2*c)^5 +
9*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + ta
n(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^2 + 9*pi*a*b*s
gn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)
^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^2 + 96*a*b*tan(1/2*d*x)^
4 + 768*a*b*tan(1/2*d*x)^3*tan(1/2*c) + 9*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2
*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan
(1/2*c) - 1)*tan(1/2*c)^2 + 9*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*ta
n(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)
*tan(1/2*c)^2 + 1824*a*b*tan(1/2*d*x)^2*tan(1/2*c)^2 + 768*a*b*tan(1/2*d*x)
*tan(1/2*c)^3 + 96*a*b*tan(1/2*c)^4 + 3*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c
)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1
/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 -
tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1) + 3*pi*a*b*sgn(tan(1/2*
d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/
2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)
*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1) - 18*a*
b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2
*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2 - 18*a*b*
arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d
*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2 + 18*a*b*ar
ctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x
)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2 + 18*a*b*arct
an((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*
tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2 - 64*a^2*tan(1/
2*d*x)^3 - 128*b^2*tan(1/2*d*x)^3 + 288*a^2*tan(1/2*d*x)^2*tan(1/2*c) - 384
*b^2*tan(1/2*d*x)^2*tan(1/2*c) - 18*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + t
an(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan
(1/2*c) + 1))*tan(1/2*c)^2 - 18*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1
/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2
*c) + 1))*tan(1/2*c)^2 + 18*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d
*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c)
- 1))*tan(1/2*c)^2 + 18*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x)
- tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)
)*tan(1/2*c)^2 + 288*a^2*tan(1/2*d*x)*tan(1/2*c)^2 - 384*b^2*tan(1/2*d*x)*t
an(1/2*c)^2 - 64*a^2*tan(1/2*c)^3 - 128*b^2*tan(1/2*c)^3 + 3*pi*a*b*sgn(tan
(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - t
an(1/2*c)^2 + 2*tan(1/2*c) - 1) + 3*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2

```
- 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c
) - 1) - 96*a*b*tan(1/2*d*x)^2 - 384*a*b*tan(1/2*d*x)*tan(1/2*c) - 96*a*b*t
an(1/2*c)^2 - 6*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/
2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)) - 6*a*
b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2
*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)) + 6*a*b*arctan((tan(1/2*
d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) -
 tan(1/2*d*x) - tan(1/2*c) - 1)) + 6*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) -
tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + ta
n(1/2*c) - 1)) - 96*a^2*tan(1/2*d*x) - 96*a^2*tan(1/2*c) + 32*a*b)/(d*tan(1
/2*d*x)^6*tan(1/2*c)^6 + 3*d*tan(1/2*d*x)^6*tan(1/2*c)^4 + 3*d*tan(1/2*d*x)
^4*tan(1/2*c)^6 + 3*d*tan(1/2*d*x)^6*tan(1/2*c)^2 + 9*d*tan(1/2*d*x)^4*tan(
1/2*c)^4 + 3*d*tan(1/2*d*x)^2*tan(1/2*c)^6 + d*tan(1/2*d*x)^6 + 9*d*tan(1/2
*d*x)^4*tan(1/2*c)^2 + 9*d*tan(1/2*d*x)^2*tan(1/2*c)^4 + d*tan(1/2*c)^6 + 3
*d*tan(1/2*d*x)^4 + 9*d*tan(1/2*d*x)^2*tan(1/2*c)^2 + 3*d*tan(1/2*c)^4 + 3*
d*tan(1/2*d*x)^2 + 3*d*tan(1/2*c)^2 + d)
```

Mupad [B] (verification not implemented)

Time = 4.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.86

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{2\left(\frac{\sin(c+d\,x)\,a^2\cos(c+d\,x)^2}{2}+\sin\left(c+d\,x\right)\,a^2-a\,b\cos\left(c+d\,x\right)^3-\frac{\sin(c+d\,x)\,b^2\cos(c+d\,x)^2}{2}+\frac{\sin(c+d\,x)\,b^2}{2}\right)}{3\,d}$$

```
[In] int(cos(c + d*x)^3*(a + b*tan(c + d*x))^2,x)
```

```
[Out] (2*(a^2*sin(c + d*x) + (b^2*sin(c + d*x))/2 + (a^2*cos(c + d*x)^2*sin(c + d
*x))/2 - (b^2*cos(c + d*x)^2*sin(c + d*x))/2 - a*b*cos(c + d*x)^3))/(3*d)
```

3.529 $\int \cos^5(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 114

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{3ab\cos^5(c+dx)}{20d}+\frac{(4a^2+b^2)\sin(c+dx)}{4d}-\frac{(4a^2+b^2)\sin^3(c+dx)}{6d}+\frac{(4a^2+b^2)\sin^5(c+dx)}{20d}-\frac{b\cos^5(c+dx)(a+b\tan(c+dx))}{4d}$$

```
[Out] -3/20*a*b*cos(d*x+c)^5/d+1/4*(4*a^2+b^2)*sin(d*x+c)/d-1/6*(4*a^2+b^2)*sin(d
*x+c)^3/d+1/20*(4*a^2+b^2)*sin(d*x+c)^5/d-1/4*b*cos(d*x+c)^5*(a+b*tan(d*x+c
))/d
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3589, 3567, 2713}

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{(4a^2+b^2)\sin^5(c+dx)}{20d}-\frac{(4a^2+b^2)\sin^3(c+dx)}{6d}+\frac{(4a^2+b^2)\sin(c+dx)}{4d}-\frac{3ab\cos^5(c+dx)}{20d}-\frac{b\cos^5(c+dx)(a+b\tan(c+dx))}{4d}$$

```
[In] Int[Cos[c + d*x]^5*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-3*a*b*Cos[c + d*x]^5)/(20*d) + ((4*a^2 + b^2)*Sin[c + d*x])/(4*d) - ((4*a
^2 + b^2)*Sin[c + d*x]^3)/(6*d) + ((4*a^2 + b^2)*Sin[c + d*x]^5)/(20*d) - (
b*Cos[c + d*x]^5*(a + b*Tan[c + d*x]))/(4*d)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{b\cos^5(c+dx)(a+b\tan(c+dx))}{4d} \\
&\quad -\frac{1}{4}\int \cos^5(c+dx)\left(-4a^2-b^2-3ab\tan(c+dx)\right)\,dx \\
&= -\frac{3ab\cos^5(c+dx)}{20d}-\frac{b\cos^5(c+dx)(a+b\tan(c+dx))}{4d}-\frac{1}{4}\left(-4a^2-b^2\right)\int \cos^5(c+dx)\,dx \\
&= -\frac{3ab\cos^5(c+dx)}{20d}-\frac{b\cos^5(c+dx)(a+b\tan(c+dx))}{4d} \\
&\quad -\frac{(4a^2+b^2)\,\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,-\sin(c+dx)\right)}{4d} \\
&= -\frac{3ab\cos^5(c+dx)}{20d}+\frac{(4a^2+b^2)\sin(c+dx)}{4d}-\frac{(4a^2+b^2)\sin^3(c+dx)}{6d} \\
&\quad +\frac{(4a^2+b^2)\sin^5(c+dx)}{20d}-\frac{b\cos^5(c+dx)(a+b\tan(c+dx))}{4d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.61

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^2\,dx$$
$$=\frac{-6ab\cos^5(c+dx)+15a^2\sin(c+dx)+5(-2a^2+b^2)\sin^3(c+dx)+3(a^2-b^2)\sin^5(c+dx)}{15d}$$

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-6*a*b*Cos[c + d*x]^5 + 15*a^2*Sin[c + d*x] + 5*(-2*a^2 + b^2)*Sin[c + d*x
]^3 + 3*(a^2 - b^2)*Sin[c + d*x]^5)/(15*d)
```

Maple [A] (verified)

Time = 21.46 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{b^2\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)-\frac{2ab\left(\cos^5(dx+c)\right)}{5}+\frac{a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+}{5}}{d}$
default	$\frac{b^2\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)-\frac{2ab\left(\cos^5(dx+c)\right)}{5}+\frac{a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+}{5}}{d}$
risch	$-\frac{ab\cos(dx+c)}{4d}+\frac{5a^2\sin(dx+c)}{8d}+\frac{\sin(dx+c)b^2}{8d}-\frac{ab\cos(5dx+5c)}{40d}+\frac{\sin(5dx+5c)a^2}{80d}-\frac{\sin(5dx+5c)b^2}{80d}-\frac{ab\,c}{}$

```
[In] int(cos(d*x+c)^5*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-2/
5*a*b*cos(d*x+c)^5+1/5*a^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.65

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^2\,dx =$$
$$-\frac{6\,ab\cos(dx+c)^5-\left(3\,(a^2-b^2)\cos(dx+c)^4+(4\,a^2+b^2)\cos(dx+c)^2+8\,a^2+2\,b^2\right)\sin(dx+c)}{15\,d}$$

```
[In] integrate(cos(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/15*(6*a*b*cos(d*x + c)^5 - (3*(a^2 - b^2)*cos(d*x + c)^4 + (4*a^2 + b^2)
*cos(d*x + c)^2 + 8*a^2 + 2*b^2)*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^2\,dx = \int (a+b\tan(c+dx))^2\cos^5(c+dx)\,dx$$

```
[In] integrate(cos(d*x+c)**5*(a+b*tan(d*x+c))**2,x)
[Out] Integral((a + b*tan(c + d*x))**2*cos(c + d*x)**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.68

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{6\,ab\cos(dx+c)^5 - (3\sin(dx+c)^5 - 10\sin(dx+c)^3 + 15\sin(dx+c))a^2 + (3\sin(dx+c)^5 - 5\sin(a}{15\,d}$$

```
[In] integrate(cos(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
[Out] -1/15*(6*a*b*cos(d*x + c)^5 - (3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*si
n(d*x + c))*a^2 + (3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*b^2)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28204 vs. 2(104) = 208.

Time = 34.08 (sec) , antiderivative size = 28204, normalized size of antiderivative = 247.40

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^2\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^5*(a+b*tan(d*x+c))^2,x, algorithm="giac")
[Out] -1/960*(45*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/
2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2
*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2
 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 + 45*pi*a*b*sgn(tan(1/
2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(
1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*
```

x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1
/2*d*x)^10*tan(1/2*c)^10 + 45*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)
*tan(1/2*d*x)^10*tan(1/2*c)^10 + 45*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
- 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c
) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 + 60*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2
*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(
1/2*d*x)^10*tan(1/2*c)^10 + 225*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*
tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) -
1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*
d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^8 +
225*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) +
tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/
2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*ta
n(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^8 + 225*pi*a*b*sgn(tan(1/2*d*x)^
2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^
2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(
1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)
^8*tan(1/2*c)^10 + 225*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d
*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^10 - 90*a*b*ar
ctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x
)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^1
0 - 90*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)
/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^10
*tan(1/2*c)^10 + 90*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + ta
n(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*ta
n(1/2*d*x)^10*tan(1/2*c)^10 + 90*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(
1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/
2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^10 + 225*pi*a*b*sgn(tan(1/2*d*x)^2*ta
n(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 +
2*tan(1/2*c) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^8 + 225*pi*a*b*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c
)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^8 + 300*pi*a*b*sgn(tan(1
/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1
/2*c)^2 + 1)*tan(1/2*d*x)^10*tan(1/2*c)^8 + 225*pi*a*b*sgn(tan(1/2*d*x)^2*t
an(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 +
2*tan(1/2*c) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^10 + 225*pi*a*b*sgn(tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*
c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^10 + 300*pi*a*b*sgn(tan(
1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(
1/2*c)^2 + 1)*tan(1/2*d*x)^8*tan(1/2*c)^10 + 384*a*b*tan(1/2*d*x)^10*tan(1/
2*c)^10 + 450*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan
(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x

)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c
)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^6 + 450*pi*a*b*sgn(tan
(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - t
an(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2
*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*ta
n(1/2*d*x)^10*tan(1/2*c)^6 + 1125*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 +
2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c)
- 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/
2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^8 +
1125*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c)
+ tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(
1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*
tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^8 - 450*a*b*arctan((tan(1/2*d*x
)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - ta
n(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^8 - 450*a*b*arctan
((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*ta
n(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^8 + 4
50*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(ta
n(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan
(1/2*c)^8 + 450*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/
2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/
2*d*x)^10*tan(1/2*c)^8 + 1920*a^2*tan(1/2*d*x)^10*tan(1/2*c)^9 + 450*pi*a*b
*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*
x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2
*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x)
- 1)*tan(1/2*d*x)^6*tan(1/2*c)^10 + 450*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*
c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(
1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 -
tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2
*c)^10 - 450*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c
) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d
*x)^8*tan(1/2*c)^10 - 450*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x
) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) +
1))*tan(1/2*d*x)^8*tan(1/2*c)^10 + 450*a*b*arctan((tan(1/2*d*x)*tan(1/2*c)
+ tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) -
tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^10 + 450*a*b*arctan((tan(1/2*d*x
)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + ta
n(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^10 + 1920*a^2*tan(1
/2*d*x)^9*tan(1/2*c)^10 + 450*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)
*tan(1/2*d*x)^10*tan(1/2*c)^6 + 450*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
- 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c
) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^6 + 600*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2
*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(
1/2*d*x)^10*tan(1/2*c)^6 + 1125*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*

tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^8 + 1125*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^8 + 1500*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^8*tan(1/2*c)^8 - 1920*a*b*tan(1/2*d*x)^10*tan(1/2*c)^8 - 7680*a*b*tan(1/2*d*x)^9*tan(1/2*c)^9 + 450*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^10 + 450*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^10 + 600*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^10 - 1920*a*b*tan(1/2*d*x)^8*tan(1/2*c)^10 + 450*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^4 + 450*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^4 + 2250*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^6 + 2250*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^6 - 900*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^6 - 900*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^6 + 900*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^6 + 900*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^6 + 2560*a^2*tan(1/2*d*x)^10*tan(1/2*c)^7 + 2560*b^2*tan(1/2*d*x)^10*tan(1/2*c)^7 + 2250*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^8 + 2250*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^8 - 2250*a*b*arctan((tan(1/2*d*x)*tan(1/2*

c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 - 2250*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 + 2250*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 + 2250*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 - 9600*a^2*tan(1/2*d*x)^9*tan(1/2*c)^8 + 7680*b^2*tan(1/2*d*x)^9*tan(1/2*c)^8 - 9600*a^2*tan(1/2*d*x)^8*tan(1/2*c)^9 + 7680*b^2*tan(1/2*d*x)^8*tan(1/2*c)^9 + 450*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^10 + 450*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^10 - 900*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^10 - 900*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^10 + 900*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^10 + 900*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^10 + 2560*a^2*tan(1/2*d*x)^7*tan(1/2*c)^10 + 2560*b^2*tan(1/2*d*x)^7*tan(1/2*c)^10 + 450*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^4 + 450*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^4 + 600*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^10*tan(1/2*c)^4 + 2250*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^6 + 2250*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^6 + 3000*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^8*tan(1/2*c)^6 + 3840*a*b*tan(1/2*d*x)^10*tan(1/2*c)^6 + 30720*a*b*tan(1/2*d*x)^9*tan(1/2*c)^7 + 2250*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^8 + 2250*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^8 + 3000*pi*a*b*sgn(tan(1/2*d*x)^2*tan(

1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*t
an(1/2*d*x)^6*tan(1/2*c)^8 + 71040*a*b*tan(1/2*d*x)^8*tan(1/2*c)^8 + 30720*
a*b*tan(1/2*d*x)^7*tan(1/2*c)^9 + 450*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2
*c) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^10 + 450*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1
/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*t
an(1/2*c) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^10 + 600*pi*a*b*sgn(tan(1/2*d*x)^2
*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 +
1)*tan(1/2*d*x)^4*tan(1/2*c)^10 + 3840*a*b*tan(1/2*d*x)^6*tan(1/2*c)^10 +
225*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) +
tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/
2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*ta
n(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^2 + 225*pi*a*b*sgn(tan(1/2*d*x)^
2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^
2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(
1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)
^10*tan(1/2*c)^2 + 2250*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^4 + 2250*pi*a
*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*
d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 -
2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*
x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^4 - 900*a*b*arctan((tan(1/2*d*x)*tan(1/2*
c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x)
+ tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^4 - 900*a*b*arctan((tan(1/2*
d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) +
tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^4 + 900*a*b*arc
tan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)
*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^4
+ 900*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/
(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^10*
tan(1/2*c)^4 + 7424*a^2*tan(1/2*d*x)^10*tan(1/2*c)^5 - 1024*b^2*tan(1/2*d*x
)^10*tan(1/2*c)^5 + 4500*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2
*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2
+ tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 4500*pi*
a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2
*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
- 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d
*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 - 4500*a*b*arctan((tan(1/2*d*x)*tan(1/
2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*
x) + tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^6 - 4500*a*b*arctan((tan(1/
2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c)
+ tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^6 + 4500*a*b*a

rctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^6 + 4500*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^6 + 19200*a^2*tan(1/2*d*x)^9*tan(1/2*c)^6 - 23040*b^2*tan(1/2*d*x)^9*tan(1/2*c)^6 + 89600*a^2*tan(1/2*d*x)^8*tan(1/2*c)^7 - 64000*b^2*tan(1/2*d*x)^8*tan(1/2*c)^7 + 2250*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^8 + 2250*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^8 - 4500*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^8 - 4500*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^8 + 4500*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^8 + 4500*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^8 + 89600*a^2*tan(1/2*d*x)^7*tan(1/2*c)^8 - 64000*b^2*tan(1/2*d*x)^7*tan(1/2*c)^8 + 19200*a^2*tan(1/2*d*x)^6*tan(1/2*c)^9 - 23040*b^2*tan(1/2*d*x)^6*tan(1/2*c)^9 + 225*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^10 + 225*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^10 - 900*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^10 - 900*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^10 + 900*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^10 + 900*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^10 + 7424*a^2*tan(1/2*d*x)^5*tan(1/2*c)^10 - 1024*b^2*tan(1/2*d*x)^5*tan(1/2*c)^10 + 225*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^2 + 225*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*ta

n(1/2*c) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^2 + 300*pi*a*b*sgn(tan(1/2*d*x)^2*
tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 +
1)*tan(1/2*d*x)^10*tan(1/2*c)^2 + 2250*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)
^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/
2*c) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^4 + 2250*pi*a*b*sgn(tan(1/2*d*x)^2*tan(
1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*
tan(1/2*c) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^4 + 3000*pi*a*b*sgn(tan(1/2*d*x)^
2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2
+ 1)*tan(1/2*d*x)^8*tan(1/2*c)^4 - 3840*a*b*tan(1/2*d*x)^10*tan(1/2*c)^4 -
46080*a*b*tan(1/2*d*x)^9*tan(1/2*c)^5 + 4500*pi*a*b*sgn(tan(1/2*d*x)^2*tan(
1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*
tan(1/2*c) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 4500*pi*a*b*sgn(tan(1/2*d*x)^
2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^
2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 6000*pi*a*b*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2
*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 - 203520*a*b*tan(1/2*d*x)^8*tan(1/2*
c)^6 - 368640*a*b*tan(1/2*d*x)^7*tan(1/2*c)^7 + 2250*pi*a*b*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c
)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^8 + 2250*pi*a*b*sgn(tan(1
/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan
(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^8 + 3000*pi*a*b*sgn
(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) -
tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^8 - 203520*a*b*tan(1/2*d*x)^6*
tan(1/2*c)^8 - 46080*a*b*tan(1/2*d*x)^5*tan(1/2*c)^9 + 225*pi*a*b*sgn(tan(1
/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan
(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^10 + 225*pi*a*b*sgn
(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2
- tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^10 + 300*pi*a
*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/
2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2*tan(1/2*c)^10 - 3840*a*b*tan(1/2*d*
x)^4*tan(1/2*c)^10 + 45*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10 + 45*pi*a*b*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1
/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x
)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/
2*d*x)^10 + 1125*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*
tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*
d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/
2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^2 + 1125*pi*a*b*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2
- tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(
1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)
*tan(1/2*d*x)^8*tan(1/2*c)^2 - 450*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + ta

n(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(
1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^2 - 450*a*b*arctan((tan(1/2*d*x)*ta
n(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/
2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^2 + 450*a*b*arctan((ta
n(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/
2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^2 + 450*a
*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/
2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2
*c)^2 + 2560*a^2*tan(1/2*d*x)^10*tan(1/2*c)^3 + 2560*b^2*tan(1/2*d*x)^10*ta
n(1/2*c)^3 + 4500*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2
*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1
/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 + 4500*pi*a*b*sgn
(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2
 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan
(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1
)*tan(1/2*d*x)^6*tan(1/2*c)^4 - 4500*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) +
tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + ta
n(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^4 - 4500*a*b*arctan((tan(1/2*d*x)*
tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(
1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^4 + 4500*a*b*arctan((
tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(
1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^4 + 4500
*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(
1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/
2*c)^4 - 19200*a^2*tan(1/2*d*x)^9*tan(1/2*c)^4 + 23040*b^2*tan(1/2*d*x)^9*t
an(1/2*c)^4 - 78080*a^2*tan(1/2*d*x)^8*tan(1/2*c)^5 + 133120*b^2*tan(1/2*d*
x)^8*tan(1/2*c)^5 + 4500*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2
*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2
 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 + 4500*pi*
a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2
*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d
*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 - 9000*a*b*arctan((tan(1/2*d*x)*tan(1/
2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*
x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 - 9000*a*b*arctan((tan(1/
2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c)
 + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 + 9000*a*b*a
rctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*
x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^6
 + 9000*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1
)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6
*tan(1/2*c)^6 - 281600*a^2*tan(1/2*d*x)^7*tan(1/2*c)^6 + 302080*b^2*tan(1/2
*d*x)^7*tan(1/2*c)^6 - 281600*a^2*tan(1/2*d*x)^6*tan(1/2*c)^7 + 302080*b^2*

tan(1/2*d*x)^6*tan(1/2*c)^7 + 1125*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^8 + 1125*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^8 - 4500*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^8 - 4500*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^8 + 4500*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^8 + 4500*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^8 - 78080*a^2*tan(1/2*d*x)^5*tan(1/2*c)^8 + 133120*b^2*tan(1/2*d*x)^5*tan(1/2*c)^8 - 19200*a^2*tan(1/2*d*x)^4*tan(1/2*c)^9 + 23040*b^2*tan(1/2*d*x)^4*tan(1/2*c)^9 + 45*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^10 + 45*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^10 - 450*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^10 - 450*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^10 + 450*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^10 + 450*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^10 + 2560*a^2*tan(1/2*d*x)^3*tan(1/2*c)^10 + 2560*b^2*tan(1/2*d*x)^3*tan(1/2*c)^10 + 45*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^10 + 45*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^10 + 60*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^10 + 1125*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^2 + 1125*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^2 + 1500*pi*a*b*sgn(tan(1/2

```
*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2
*c)^2 + 1)*tan(1/2*d*x)^8*tan(1/2*c)^2 + 1920*a*b*tan(1/2*d*x)^10*tan(1/2*c
)^2 + 30720*a*b*tan(1/2*d*x)^9*tan(1/2*c)^3 + 4500*pi*a*b*sgn(tan(1/2*d*x)^
2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^
2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 + 4500*pi*a*b*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1
/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 + 6000*pi*a*b*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - t
an(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 + 203520*a*b*tan(1/2*d*x)^8*ta
n(1/2*c)^4 + 675840*a*b*tan(1/2*d*x)^7*tan(1/2*c)^5 + 4500*pi*a*b*sgn(tan(1
/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan
(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 + 4500*pi*a*b*sgn
(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2
 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 + 6000*pi*a
*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/
2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 + 1082880*a*b*tan(1/2*
d*x)^6*tan(1/2*c)^6 + 675840*a*b*tan(1/2*d*x)^5*tan(1/2*c)^7 + 1125*pi*a*b*
sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x
)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^8 + 1125*p
i*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1
/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^8 +
1500*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*
x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2*tan(1/2*c)^8 + 203520*a*b*
tan(1/2*d*x)^4*tan(1/2*c)^8 + 30720*a*b*tan(1/2*d*x)^3*tan(1/2*c)^9 + 45*pi
*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/
2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*c)^10 + 45*pi*a*b*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
 tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*c)^10 + 60*pi*a*b*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^
2 + 1)*tan(1/2*c)^10 + 1920*a*b*tan(1/2*d*x)^2*tan(1/2*c)^10 + 225*pi*a*b*s
gn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)
^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*t
an(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) -
 1)*tan(1/2*d*x)^8 + 225*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2
*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2
+ tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8 - 90*a*b*arctan((tan(1/
2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c)
 - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^10 - 90*a*b*arctan((tan(1/2
*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c)
+ tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^10 + 90*a*b*arctan((tan(1/2*
d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) -
 tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^10 + 90*a*b*arctan((tan(1/2*d
*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) +
tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^10 + 1920*a^2*tan(1/2*d*x)^10*
```

tan(1/2*c) + 2250*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2
*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1
/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 + 2250*pi*a*b*sgn
(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2
 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan
(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1
)*tan(1/2*d*x)^6*tan(1/2*c)^2 - 2250*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) +
tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + ta
n(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^2 - 2250*a*b*arctan((tan(1/2*d*x)*
tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(
1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^2 + 2250*a*b*arctan((
tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(
1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^2 + 2250
*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(
1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/
2*c)^2 + 9600*a^2*tan(1/2*d*x)^9*tan(1/2*c)^2 - 7680*b^2*tan(1/2*d*x)^9*tan
(1/2*c)^2 + 89600*a^2*tan(1/2*d*x)^8*tan(1/2*c)^3 - 64000*b^2*tan(1/2*d*x)^
8*tan(1/2*c)^3 + 4500*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*
x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan
(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + t
an(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 + 4500*pi*a*b
*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*
x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2
*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x)
 - 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 - 9000*a*b*arctan((tan(1/2*d*x)*tan(1/2*c
) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x)
+ tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 - 9000*a*b*arctan((tan(1/2*d
*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) +
tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 + 9000*a*b*arct
an((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*
tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 +
9000*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(
tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*ta
n(1/2*c)^4 + 281600*a^2*tan(1/2*d*x)^7*tan(1/2*c)^4 - 302080*b^2*tan(1/2*d*
x)^7*tan(1/2*c)^4 + 688640*a^2*tan(1/2*d*x)^6*tan(1/2*c)^5 - 640000*b^2*tan
(1/2*d*x)^6*tan(1/2*c)^5 + 2250*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*
tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) -
1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*
d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^6 + 2
250*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) +
tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/
2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*ta
n(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^6 - 9000*a*b*arctan((tan(1/2*d*x)
*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan

(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^6 - 9000*a*b*arctan(
(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan
(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^6 + 900
0*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan
(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1
/2*c)^6 + 9000*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2
*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2
*d*x)^4*tan(1/2*c)^6 + 688640*a^2*tan(1/2*d*x)^5*tan(1/2*c)^6 - 640000*b^2*
tan(1/2*d*x)^5*tan(1/2*c)^6 + 281600*a^2*tan(1/2*d*x)^4*tan(1/2*c)^7 - 3020
80*b^2*tan(1/2*d*x)^4*tan(1/2*c)^7 + 225*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*
c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(
1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 -
 tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^8 + 225*pi*
a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2
*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
- 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d
*x) - 1)*tan(1/2*c)^8 - 2250*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*
d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c)
 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^8 - 2250*a*b*arctan((tan(1/2*d*x)*tan(1/2*
c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x)
 - tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^8 + 2250*a*b*arctan((tan(1/2*
d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) -
 tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^8 + 2250*a*b*arc
tan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)
*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^8 +
 89600*a^2*tan(1/2*d*x)^3*tan(1/2*c)^8 - 64000*b^2*tan(1/2*d*x)^3*tan(1/2*c
)^8 + 9600*a^2*tan(1/2*d*x)^2*tan(1/2*c)^9 - 7680*b^2*tan(1/2*d*x)^2*tan(1/
2*c)^9 - 90*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c)
 + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*c)
^10 - 90*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) +
1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*c)^10
 + 90*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/
(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*c)^10 +
90*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(ta
n(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^10 + 192
0*a^2*tan(1/2*d*x)*tan(1/2*c)^10 + 225*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)
^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/
2*c) - 1)*tan(1/2*d*x)^8 + 225*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*t
an(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1
)*tan(1/2*d*x)^8 + 300*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x
)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^8 - 384*a*
b*tan(1/2*d*x)^10 - 7680*a*b*tan(1/2*d*x)^9*tan(1/2*c) + 2250*pi*a*b*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 + 2250*pi*a*b*
sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x

)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 + 3000*p
i*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan
(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 - 71040*a*b*tan(1/2
*d*x)^8*tan(1/2*c)^2 - 368640*a*b*tan(1/2*d*x)^7*tan(1/2*c)^3 + 4500*pi*a*b
*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*
x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 + 4500*
pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(
1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 +
 6000*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d
*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 - 1082880*a*
b*tan(1/2*d*x)^6*tan(1/2*c)^4 - 1652736*a*b*tan(1/2*d*x)^5*tan(1/2*c)^5 + 2
250*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) +
tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^2*tan(1/2*c)
^6 + 2250*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2
*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^2*tan(
1/2*c)^6 + 3000*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4
*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2*tan(1/2*c)^6 -
1082880*a*b*tan(1/2*d*x)^4*tan(1/2*c)^6 - 368640*a*b*tan(1/2*d*x)^3*tan(1/2
*c)^7 + 225*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1
/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*c)^8 + 22
5*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + ta
n(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*c)^8 + 300*pi*a*b*s
gn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c)
 - tan(1/2*c)^2 + 1)*tan(1/2*c)^8 - 71040*a*b*tan(1/2*d*x)^2*tan(1/2*c)^8 -
 7680*a*b*tan(1/2*d*x)*tan(1/2*c)^9 - 384*a*b*tan(1/2*c)^10 + 450*pi*a*b*sg
n(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^
2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) -
1)*tan(1/2*d*x)^6 + 450*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*
d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
 tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6 - 450*a*b*arctan((tan(1/
2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c)
 - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^8 - 450*a*b*arctan((tan(1/2
*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c)
+ tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8 + 450*a*b*arctan((tan(1/2*
d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) -
 tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^8 + 450*a*b*arctan((tan(1/2*d
*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) +
tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^8 - 1920*a^2*tan(1/2*d*x)^9 -
9600*a^2*tan(1/2*d*x)^8*tan(1/2*c) + 7680*b^2*tan(1/2*d*x)^8*tan(1/2*c) + 2
250*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) +
tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/
2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*ta
n(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^2 + 2250*pi*a*b*sgn(tan(1/2*d*x)^

2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^
2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(
1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)
^4*tan(1/2*c)^2 - 4500*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) -
tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))
*tan(1/2*d*x)^6*tan(1/2*c)^2 - 4500*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - t
an(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan
(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 + 4500*a*b*arctan((tan(1/2*d*x)*t
an(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1
/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 + 4500*a*b*arctan((t
an(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1
/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 - 89600
*a^2*tan(1/2*d*x)^7*tan(1/2*c)^2 + 64000*b^2*tan(1/2*d*x)^7*tan(1/2*c)^2 -
281600*a^2*tan(1/2*d*x)^6*tan(1/2*c)^3 + 302080*b^2*tan(1/2*d*x)^6*tan(1/2*
c)^3 + 2250*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1
/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^
2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^
2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^4 + 2250*pi*a*b*sgn(tan(1
/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan
(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d
*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(
1/2*d*x)^2*tan(1/2*c)^4 - 9000*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/
2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*
c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 - 9000*a*b*arctan((tan(1/2*d*x)*tan(1/
2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*
x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 + 9000*a*b*arctan((tan(1/
2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c)
- tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 + 9000*a*b*a
rctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*
x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^4
- 688640*a^2*tan(1/2*d*x)^5*tan(1/2*c)^4 + 640000*b^2*tan(1/2*d*x)^5*tan(1
/2*c)^4 - 688640*a^2*tan(1/2*d*x)^4*tan(1/2*c)^5 + 640000*b^2*tan(1/2*d*x)^
4*tan(1/2*c)^5 + 450*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x
)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(
1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + ta
n(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^6 + 450*pi*a*b*sgn(tan(1/2*d*x)
^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)
^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan
(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^
6 - 4500*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) +
1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^
2*tan(1/2*c)^6 - 4500*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) +
tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*
tan(1/2*d*x)^2*tan(1/2*c)^6 + 4500*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + ta
n(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(

1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 + 4500*a*b*arctan((tan(1/2*d*x)*ta
n(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/
2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 - 281600*a^2*tan(1/2*
d*x)^3*tan(1/2*c)^6 + 302080*b^2*tan(1/2*d*x)^3*tan(1/2*c)^6 - 89600*a^2*ta
n(1/2*d*x)^2*tan(1/2*c)^7 + 64000*b^2*tan(1/2*d*x)^2*tan(1/2*c)^7 - 450*a*b
*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*
d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*c)^8 - 450*a*b*ar
ctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x
)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*c)^8 + 450*a*b*arcta
n((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*t
an(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*c)^8 + 450*a*b*arctan((
tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(
1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^8 - 9600*a^2*tan(1/2*d*
x)*tan(1/2*c)^8 + 7680*b^2*tan(1/2*d*x)*tan(1/2*c)^8 - 1920*a^2*tan(1/2*c)^
9 + 450*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c
) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^6 + 450*
pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(
1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^6 + 600*pi*a*b*s
gn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c)
 - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6 + 1920*a*b*tan(1/2*d*x)^8 + 30720*a*b*t
an(1/2*d*x)^7*tan(1/2*c) + 2250*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*
tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) -
1)*tan(1/2*d*x)^4*tan(1/2*c)^2 + 2250*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2
*c) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^2 + 3000*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1
/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*ta
n(1/2*d*x)^4*tan(1/2*c)^2 + 203520*a*b*tan(1/2*d*x)^6*tan(1/2*c)^2 + 675840
*a*b*tan(1/2*d*x)^5*tan(1/2*c)^3 + 2250*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c
)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1
/2*c) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^4 + 2250*pi*a*b*sgn(tan(1/2*d*x)^2*tan
(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2
*tan(1/2*c) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^4 + 3000*pi*a*b*sgn(tan(1/2*d*x)
^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2
 + 1)*tan(1/2*d*x)^2*tan(1/2*c)^4 + 1082880*a*b*tan(1/2*d*x)^4*tan(1/2*c)^4
 + 675840*a*b*tan(1/2*d*x)^3*tan(1/2*c)^5 + 450*pi*a*b*sgn(tan(1/2*d*x)^2*t
an(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 +
 2*tan(1/2*c) - 1)*tan(1/2*c)^6 + 450*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2
*c) - 1)*tan(1/2*c)^6 + 600*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/
2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^6 + 203
520*a*b*tan(1/2*d*x)^2*tan(1/2*c)^6 + 30720*a*b*tan(1/2*d*x)*tan(1/2*c)^7 +
 1920*a*b*tan(1/2*c)^8 + 450*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan
(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*
sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4 + 450*pi*a*b*sgn(ta

n(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4 - 900*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6 - 900*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6 + 900*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6 + 900*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6 - 2560*a^2*tan(1/2*d*x)^7 - 2560*b^2*tan(1/2*d*x)^7 + 19200*a^2*tan(1/2*d*x)^6*tan(1/2*c) - 23040*b^2*tan(1/2*d*x)^6*tan(1/2*c) + 1125*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 1125*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4500*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^2 - 4500*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^2 + 4500*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^2 + 4500*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^2 + 78080*a^2*tan(1/2*d*x)^5*tan(1/2*c)^2 - 133120*b^2*tan(1/2*d*x)^5*tan(1/2*c)^2 + 281600*a^2*tan(1/2*d*x)^4*tan(1/2*c)^3 - 302080*b^2*tan(1/2*d*x)^4*tan(1/2*c)^3 + 450*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^4 + 450*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^4 - 4500*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^4 - 4500*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^4 + 4500*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^4 + 4500*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2

*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c
) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^4 + 281600*a^2*tan(1/2*d*x)^3*tan(1/2*c)^
4 - 302080*b^2*tan(1/2*d*x)^3*tan(1/2*c)^4 + 78080*a^2*tan(1/2*d*x)^2*tan(1
/2*c)^5 - 133120*b^2*tan(1/2*d*x)^2*tan(1/2*c)^5 - 900*a*b*arctan((tan(1/2*
d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) -
 tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*c)^6 - 900*a*b*arctan((tan(1/2*d*x
)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + ta
n(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*c)^6 + 900*a*b*arctan((tan(1/2*d*x)*t
an(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1
/2*d*x) - tan(1/2*c) - 1))*tan(1/2*c)^6 + 900*a*b*arctan((tan(1/2*d*x)*tan(
1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*
d*x) + tan(1/2*c) - 1))*tan(1/2*c)^6 + 19200*a^2*tan(1/2*d*x)*tan(1/2*c)^6
 - 23040*b^2*tan(1/2*d*x)*tan(1/2*c)^6 - 2560*a^2*tan(1/2*c)^7 - 2560*b^2*ta
n(1/2*c)^7 + 450*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*
tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)
^4 + 450*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*
c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^4 + 600
*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*t
an(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4 - 3840*a*b*tan(1/2*d*x)^6 - 46
080*a*b*tan(1/2*d*x)^5*tan(1/2*c) + 1125*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*
c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(
1/2*c) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 1125*pi*a*b*sgn(tan(1/2*d*x)^2*ta
n(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 -
2*tan(1/2*c) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 1500*pi*a*b*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^
2 + 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 203520*a*b*tan(1/2*d*x)^4*tan(1/2*c)^2
 - 368640*a*b*tan(1/2*d*x)^3*tan(1/2*c)^3 + 450*pi*a*b*sgn(tan(1/2*d*x)^2*t
an(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 +
 2*tan(1/2*c) - 1)*tan(1/2*c)^4 + 450*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2
*c) - 1)*tan(1/2*c)^4 + 600*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/
2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^4 - 203
520*a*b*tan(1/2*d*x)^2*tan(1/2*c)^4 - 46080*a*b*tan(1/2*d*x)*tan(1/2*c)^5 -
 3840*a*b*tan(1/2*c)^6 + 225*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan
(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*
sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2 + 225*pi*a*b*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/
2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*t
an(1/2*d*x)^2 - 900*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - ta
n(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*ta
n(1/2*d*x)^4 - 900*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan
(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan
(1/2*d*x)^4 + 900*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(

1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(
1/2*d*x)^4 + 900*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1
/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1
/2*d*x)^4 - 7424*a^2*tan(1/2*d*x)^5 + 1024*b^2*tan(1/2*d*x)^5 - 19200*a^2*t
an(1/2*d*x)^4*tan(1/2*c) + 23040*b^2*tan(1/2*d*x)^4*tan(1/2*c) + 225*pi*a*b
*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*
x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2
*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x)
 - 1)*tan(1/2*c)^2 + 225*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2
*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2
 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^2 - 2250*a*b*arctan((tan(1/
2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c)
 - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2250*a*b*a
rctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*
x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2
 + 2250*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1
)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2
*tan(1/2*c)^2 + 2250*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - t
an(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*t
an(1/2*d*x)^2*tan(1/2*c)^2 - 89600*a^2*tan(1/2*d*x)^3*tan(1/2*c)^2 + 64000*
b^2*tan(1/2*d*x)^3*tan(1/2*c)^2 - 89600*a^2*tan(1/2*d*x)^2*tan(1/2*c)^3 + 6
4000*b^2*tan(1/2*d*x)^2*tan(1/2*c)^3 - 900*a*b*arctan((tan(1/2*d*x)*tan(1/2
*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x
) + tan(1/2*c) + 1))*tan(1/2*c)^4 - 900*a*b*arctan((tan(1/2*d*x)*tan(1/2*c)
 - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) -
 tan(1/2*c) + 1))*tan(1/2*c)^4 + 900*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) +
tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - ta
n(1/2*c) - 1))*tan(1/2*c)^4 + 900*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan
(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1
/2*c) - 1))*tan(1/2*c)^4 - 19200*a^2*tan(1/2*d*x)*tan(1/2*c)^4 + 23040*b^2*
tan(1/2*d*x)*tan(1/2*c)^4 - 7424*a^2*tan(1/2*c)^5 + 1024*b^2*tan(1/2*c)^5 +
 225*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) +
 tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^2 + 225*pi*
a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2
*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^2 + 300*pi*a*b*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) -
tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2 + 3840*a*b*tan(1/2*d*x)^4 + 30720*a*b*tan(
1/2*d*x)^3*tan(1/2*c) + 225*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(
1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*t
an(1/2*c)^2 + 225*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2
*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*c)^
2 + 300*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2
*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^2 + 71040*a*b*tan(1/2*d*x)^
2*tan(1/2*c)^2 + 30720*a*b*tan(1/2*d*x)*tan(1/2*c)^3 + 3840*a*b*tan(1/2*c)^

4 + 45*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1) + 45*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1) - 450*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2 - 450*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2 + 450*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2 + 450*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2 - 2560*a^2*tan(1/2*d*x)^3 - 2560*b^2*tan(1/2*d*x)^3 + 9600*a^2*tan(1/2*d*x)^2*tan(1/2*c) - 7680*b^2*tan(1/2*d*x)^2*tan(1/2*c) - 450*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*c)^2 - 450*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*c)^2 + 450*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*c)^2 + 450*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^2 + 9600*a^2*tan(1/2*d*x)*tan(1/2*c)^2 - 7680*b^2*tan(1/2*d*x)*tan(1/2*c)^2 - 2560*a^2*tan(1/2*c)^3 - 2560*b^2*tan(1/2*c)^3 + 45*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1) + 45*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1) + 60*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1) - 1920*a*b*tan(1/2*d*x)^2 - 7680*a*b*tan(1/2*d*x)*tan(1/2*c) - 1920*a*b*tan(1/2*c)^2 - 90*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)) - 90*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)) + 90*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)) + 90*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)) - 1920*a^2*tan(1/2*d*x) - 1920*a^2*tan(1/2*c) + 384*a*b)/(d*tan(1/2*d*x)^10*tan(1/2*c)^10 + 5*d*tan(1/2*d*x)^10*tan(1/2*c)^8 + 5*d*tan(1/2*d*x)^8*tan(1/2*c)^10 + 10*d*tan(1/2*d*x)^10*tan(1/2*c)^6 + 25*d*tan(1/2*d*x)^8*tan(1/2*c)^8 + 10*d*tan(1/2*d*x)^6*tan(1/2*c)^10 + 10*d*tan(1/2*d*x)^10*tan(1/2*c)^4 + 50*d*tan(1/2*d*x)^8*tan(1/2*c)^6 + 50*d*tan(1/2*d*x)^6*tan(1/2*c)^8 + 10*d*tan(1/2*d*x)^4*tan(1/2*c)^10 + 5*d*tan(1/2*d*x)^10*tan(1/2*c)^2 + 50*d*tan(1/2*d*x)^8*tan(1/2*c)^4

```
+ 100*d*tan(1/2*d*x)^6*tan(1/2*c)^6 + 50*d*tan(1/2*d*x)^4*tan(1/2*c)^8 + 5*
d*tan(1/2*d*x)^2*tan(1/2*c)^10 + d*tan(1/2*d*x)^10 + 25*d*tan(1/2*d*x)^8*ta
n(1/2*c)^2 + 100*d*tan(1/2*d*x)^6*tan(1/2*c)^4 + 100*d*tan(1/2*d*x)^4*tan(1
/2*c)^6 + 25*d*tan(1/2*d*x)^2*tan(1/2*c)^8 + d*tan(1/2*c)^10 + 5*d*tan(1/2*
d*x)^8 + 50*d*tan(1/2*d*x)^6*tan(1/2*c)^2 + 100*d*tan(1/2*d*x)^4*tan(1/2*c)
^4 + 50*d*tan(1/2*d*x)^2*tan(1/2*c)^6 + 5*d*tan(1/2*c)^8 + 10*d*tan(1/2*d*x
)^6 + 50*d*tan(1/2*d*x)^4*tan(1/2*c)^2 + 50*d*tan(1/2*d*x)^2*tan(1/2*c)^4 +
 10*d*tan(1/2*c)^6 + 10*d*tan(1/2*d*x)^4 + 25*d*tan(1/2*d*x)^2*tan(1/2*c)^2
 + 10*d*tan(1/2*c)^4 + 5*d*tan(1/2*d*x)^2 + 5*d*tan(1/2*c)^2 + d)
```

Mupad [B] (verification not implemented)

Time = 4.57 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.01

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^2\,dx$$

$$= \frac{2\left(\frac{3\sin(c+dx)\,a^2\cos(c+dx)^4}{2} + 2\sin(c+dx)\,a^2\cos(c+dx)^2 + 4\sin(c+dx)\,a^2 - 3\,a\,b\cos(c+dx)^5 - \frac{3\sin(c-}{}\right.}{15\,d}$$

```
[In] int(cos(c + d*x)^5*(a + b*tan(c + d*x))^2,x)
```

```
[Out] (2*(4*a^2*sin(c + d*x) + b^2*sin(c + d*x) + 2*a^2*cos(c + d*x)^2*sin(c + d*
x) + (3*a^2*cos(c + d*x)^4*sin(c + d*x))/2 + (b^2*cos(c + d*x)^2*sin(c + d*
x))/2 - (3*b^2*cos(c + d*x)^4*sin(c + d*x))/2 - 3*a*b*cos(c + d*x)^5))/(15*
d)
```

3.530 $\int \cos^7(c+dx)(a+b\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 138

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{5ab\cos^7(c+dx)}{42d}+\frac{(6a^2+b^2)\sin(c+dx)}{6d}-\frac{(6a^2+b^2)\sin^3(c+dx)}{6d}+\frac{(6a^2+b^2)\sin^5(c+dx)}{10d}-\frac{(6a^2+b^2)\sin^7(c+dx)}{42d}-\frac{b\cos^7(c+dx)(a+b\tan(c+dx))}{6d}$$

```
[Out] -5/42*a*b*cos(d*x+c)^7/d+1/6*(6*a^2+b^2)*sin(d*x+c)/d-1/6*(6*a^2+b^2)*sin(d
*x+c)^3/d+1/10*(6*a^2+b^2)*sin(d*x+c)^5/d-1/42*(6*a^2+b^2)*sin(d*x+c)^7/d-1
/6*b*cos(d*x+c)^7*(a+b*tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3589, 3567, 2713}

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{(6a^2+b^2)\sin^7(c+dx)}{42d}+\frac{(6a^2+b^2)\sin^5(c+dx)}{10d}-\frac{(6a^2+b^2)\sin^3(c+dx)}{6d}+\frac{(6a^2+b^2)\sin(c+dx)}{6d}-\frac{5ab\cos^7(c+dx)}{42d}-\frac{b\cos^7(c+dx)(a+b\tan(c+dx))}{6d}$$

```
[In] Int[Cos[c + d*x]^7*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-5*a*b*Cos[c + d*x]^7)/(42*d) + ((6*a^2 + b^2)*Sin[c + d*x])/(6*d) - ((6*a
^2 + b^2)*Sin[c + d*x]^3)/(6*d) + ((6*a^2 + b^2)*Sin[c + d*x]^5)/(10*d) - (
(6*a^2 + b^2)*Sin[c + d*x]^7)/(42*d) - (b*Cos[c + d*x]^7*(a + b*Tan[c + d*x
]))/(6*d)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{b\cos^7(c+dx)(a+b\tan(c+dx))}{6d} \\
&\quad -\frac{1}{6}\int \cos^7(c+dx)\left(-6a^2-b^2-5ab\tan(c+dx)\right)\,dx \\
&= -\frac{5ab\cos^7(c+dx)}{42d}-\frac{b\cos^7(c+dx)(a+b\tan(c+dx))}{6d}-\frac{1}{6}\left(-6a^2-b^2\right)\int \cos^7(c+dx)\,dx \\
&= -\frac{5ab\cos^7(c+dx)}{42d}-\frac{b\cos^7(c+dx)(a+b\tan(c+dx))}{6d} \\
&\quad -\frac{(6a^2+b^2)\,\text{Subst}\left(\int \left(1-3x^2+3x^4-x^6\right)\,dx,x,-\sin(c+dx)\right)}{6d} \\
&= -\frac{5ab\cos^7(c+dx)}{42d}+\frac{(6a^2+b^2)\sin(c+dx)}{6d} \\
&\quad -\frac{(6a^2+b^2)\sin^3(c+dx)}{6d}+\frac{(6a^2+b^2)\sin^5(c+dx)}{10d} \\
&\quad -\frac{(6a^2+b^2)\sin^7(c+dx)}{42d}-\frac{b\cos^7(c+dx)(a+b\tan(c+dx))}{6d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.67

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^2\,dx = \frac{-30ab\cos^7(c+dx)+105a^2\sin(c+dx)-35(3a^2-b^2)\sin^3(c+dx)+21(3a^2-2b^2)\sin^5(c+dx)-15(a^2}{105d}$$

```
[In] Integrate[Cos[c + d*x]^7*(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-30*a*b*Cos[c + d*x]^7 + 105*a^2*Sin[c + d*x] - 35*(3*a^2 - b^2)*Sin[c + d
*x]^3 + 21*(3*a^2 - 2*b^2)*Sin[c + d*x]^5 - 15*(a^2 - b^2)*Sin[c + d*x]^7)/
(105*d)
```

Maple [A] (verified)

Time = 68.94 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{b^2\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)-\frac{2ab\left(\cos^7(dx+c)\right)}{7}+\frac{a^2\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6(\cos}{}\right.}{}}{d}$
default	$\frac{b^2\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)-\frac{2ab\left(\cos^7(dx+c)\right)}{7}+\frac{a^2\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6(\cos}{}\right.}{}}{d}$
risch	$-\frac{5ab\cos(dx+c)}{32d}+\frac{35a^2\sin(dx+c)}{64d}+\frac{5\sin(dx+c)b^2}{64d}-\frac{ab\cos(7dx+7c)}{224d}+\frac{\sin(7dx+7c)a^2}{448d}-\frac{\sin(7dx+7c)b^2}{448d}-$

```
[In] int(cos(d*x+c)^7*(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c
)^2)*sin(d*x+c))-2/7*a*b*cos(d*x+c)^7+1/7*a^2*(16/5+cos(d*x+c)^6+6/5*cos(d*
x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.68

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{30\,ab\cos\left(dx+c\right)^7 - \left(15\,(a^2-b^2)\cos\left(dx+c\right)^6 + 3\,(6\,a^2+b^2)\cos\left(dx+c\right)^4 + 4\,(6\,a^2+b^2)\cos\left(dx+c\right)^2\right.}{105\,d}$$

```
[In] integrate(cos(d*x+c)^7*(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/105*(30*a*b*cos(d*x + c)^7 - (15*(a^2 - b^2)*cos(d*x + c)^6 + 3*(6*a^2 +
 b^2)*cos(d*x + c)^4 + 4*(6*a^2 + b^2)*cos(d*x + c)^2 + 48*a^2 + 8*b^2)*sin
(d*x + c))/d
```

Sympy [F]

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^2\,dx = \int (a+b\tan{(c+dx)})^2\cos^7{(c+dx)}\,dx$$

```
[In] integrate(cos(d*x+c)**7*(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**2*cos(c + d*x)**7, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.71

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^2\,dx = -\frac{30\,ab\cos\left(dx+c\right)^7 + 3\,\left(5\,\sin\left(dx+c\right)^7 - 21\,\sin\left(dx+c\right)^5 + 35\,\sin\left(dx+c\right)^3 - 35\,\sin\left(dx+c\right)\right)a^2 - (15}{105\,d}$$

```
[In] integrate(cos(d*x+c)^7*(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/105*(30*a*b*cos(d*x + c)^7 + 3*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 3
5*sin(d*x + c)^3 - 35*sin(d*x + c))*a^2 - (15*sin(d*x + c)^7 - 42*sin(d*x +
 c)^5 + 35*sin(d*x + c)^3)*b^2)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52002 vs. $2(126) = 252$.

Time = 61.66 (sec) , antiderivative size = 52002, normalized size of antiderivative = 376.83

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^2\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^7*(a+b*tan(d*x+c))^2,x, algorithm="giac")
[Out] -1/26880*(945*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan
(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c
)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^14 + 945*pi*a*b*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/
2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*t
an(1/2*d*x)^14*tan(1/2*c)^14 + 945*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 +
 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c)
 - 1)*tan(1/2*d*x)^14*tan(1/2*c)^14 + 945*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2
*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan
(1/2*c) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^14 + 8400*pi*a*b*sgn(tan(1/2*d*x)^2
*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 +
 1)*tan(1/2*d*x)^14*tan(1/2*c)^14 + 6615*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*
c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(
1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 -
 tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/
2*c)^12 + 6615*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*ta
n(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*
c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^12 + 6615*pi*a*b*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2
- tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(
1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)
*tan(1/2*d*x)^12*tan(1/2*c)^14 + 6615*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2
*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - ta
n(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c
)^14 - 5880*pi*a*b*tan(1/2*d*x)^14*tan(1/2*c)^14 - 1890*a*b*arctan((tan(1/2
*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c)
- tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^14*tan(1/2*c)^14 - 1890*a*b*
arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d
*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^14*tan(1/2*c)
^14 - 9870*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c)
- 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x
```

)^14*tan(1/2*c)^14 - 9870*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x
) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) -
1))*tan(1/2*d*x)^14*tan(1/2*c)^14 + 6615*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*
c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(
1/2*c) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^12 + 6615*pi*a*b*sgn(tan(1/2*d*x)^2*
tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2
- 2*tan(1/2*c) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^12 + 58800*pi*a*b*sgn(tan(1/
2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/
2*c)^2 + 1)*tan(1/2*d*x)^14*tan(1/2*c)^12 + 6615*pi*a*b*sgn(tan(1/2*d*x)^2*
tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2
+ 2*tan(1/2*c) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^14 + 6615*pi*a*b*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1
/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^14 + 58800*pi*a*b*sg
n(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c)
- tan(1/2*c)^2 + 1)*tan(1/2*d*x)^12*tan(1/2*c)^14 + 7680*a*b*tan(1/2*d*x)^1
4*tan(1/2*c)^14 + 19845*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^10 + 19845*p
i*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1
/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2
*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^10 + 46305*pi*a*b*sgn(tan(1/2*d*x)^2*
tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2
+ 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/
2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^1
2*tan(1/2*c)^12 + 46305*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*
d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^12 - 41160*p
i*a*b*tan(1/2*d*x)^14*tan(1/2*c)^12 - 13230*a*b*arctan((tan(1/2*d*x)*tan(1/
2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*
x) + tan(1/2*c) + 1))*tan(1/2*d*x)^14*tan(1/2*c)^12 - 13230*a*b*arctan((tan
(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2
*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^14*tan(1/2*c)^12 - 69090
*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(
1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^14*tan(1
/2*c)^12 - 69090*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1
/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1
/2*d*x)^14*tan(1/2*c)^12 + 53760*a^2*tan(1/2*d*x)^14*tan(1/2*c)^13 + 19845*
pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(
1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)
^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/
2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^14 + 19845*pi*a*b*sgn(tan(1/2*d*x)^2
*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2

- 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^14 - 41160*pi*a*b*tan(1/2*d*x)^12*tan(1/2*c)^14 - 13230*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^12*tan(1/2*c)^14 - 13230*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^12*tan(1/2*c)^14 - 69090*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^12*tan(1/2*c)^14 - 69090*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^12*tan(1/2*c)^14 + 53760*a^2*tan(1/2*d*x)^13*tan(1/2*c)^14 + 19845*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^10 + 19845*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^10 + 176400*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^14*tan(1/2*c)^10 + 46305*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^12 + 46305*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^12 + 411600*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^12*tan(1/2*c)^12 - 53760*a*b*tan(1/2*d*x)^14*tan(1/2*c)^12 - 215040*a*b*tan(1/2*d*x)^13*tan(1/2*c)^13 + 19845*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^14 + 19845*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^14 + 176400*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^10*tan(1/2*c)^14 - 53760*a*b*tan(1/2*d*x)^12*tan(1/2*c)^14 + 33075*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^8 + 33075*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^8 + 138915*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^10 + 138915*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c

) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(
1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^
10 - 123480*pi*a*b*tan(1/2*d*x)^14*tan(1/2*c)^10 - 39690*a*b*arctan((tan(1/
2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c)
 - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^14*tan(1/2*c)^10 - 39690*a*
b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2
*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^14*tan(1/2*
c)^10 - 207270*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2
*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2
*d*x)^14*tan(1/2*c)^10 - 207270*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1
/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2
*c) - 1))*tan(1/2*d*x)^14*tan(1/2*c)^10 + 107520*a^2*tan(1/2*d*x)^14*tan(1/
2*c)^11 + 71680*b^2*tan(1/2*d*x)^14*tan(1/2*c)^11 + 138915*pi*a*b*sgn(tan(1
/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan
(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d
*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(
1/2*d*x)^10*tan(1/2*c)^12 + 138915*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 -
 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c)
 - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1
/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^1
2 - 288120*pi*a*b*tan(1/2*d*x)^12*tan(1/2*c)^12 - 92610*a*b*arctan((tan(1/2
*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c)
- tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^12*tan(1/2*c)^12 - 92610*a*b
*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*
d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^12*tan(1/2*c
)^12 - 483630*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*
c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*
d*x)^12*tan(1/2*c)^12 - 483630*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/
2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*
c) - 1))*tan(1/2*d*x)^12*tan(1/2*c)^12 - 376320*a^2*tan(1/2*d*x)^13*tan(1/2
*c)^12 + 215040*b^2*tan(1/2*d*x)^13*tan(1/2*c)^12 - 376320*a^2*tan(1/2*d*x)
^12*tan(1/2*c)^13 + 215040*b^2*tan(1/2*d*x)^12*tan(1/2*c)^13 + 33075*pi*a*b
*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*
x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2
*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x)
 - 1)*tan(1/2*d*x)^8*tan(1/2*c)^14 + 33075*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/
2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*ta
n(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2
 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1
/2*c)^14 - 123480*pi*a*b*tan(1/2*d*x)^10*tan(1/2*c)^14 - 39690*a*b*arctan((
tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(
1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^14 - 39
690*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(t
an(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^10*ta
n(1/2*c)^14 - 207270*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + t

an(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*t
an(1/2*d*x)^10*tan(1/2*c)^14 - 207270*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) -
tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + t
an(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^14 + 107520*a^2*tan(1/2*d*x)^11*
tan(1/2*c)^14 + 71680*b^2*tan(1/2*d*x)^11*tan(1/2*c)^14 + 33075*pi*a*b*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2
- tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^8 + 33075*pi*
a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2
*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^8 + 2
94000*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d
*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^14*tan(1/2*c)^8 + 138915*pi
*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/
2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^10 +
138915*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c
) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^12*tan(1
/2*c)^10 + 1234800*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2
- 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^12*tan(1/2*c)^
10 + 161280*a*b*tan(1/2*d*x)^14*tan(1/2*c)^10 + 1290240*a*b*tan(1/2*d*x)^13
*tan(1/2*c)^11 + 138915*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1
/2*d*x)^10*tan(1/2*c)^12 + 138915*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 -
2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c)
- 1)*tan(1/2*d*x)^10*tan(1/2*c)^12 + 1234800*pi*a*b*sgn(tan(1/2*d*x)^2*tan(
1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*t
an(1/2*d*x)^10*tan(1/2*c)^12 + 2956800*a*b*tan(1/2*d*x)^12*tan(1/2*c)^12 +
1290240*a*b*tan(1/2*d*x)^11*tan(1/2*c)^13 + 33075*pi*a*b*sgn(tan(1/2*d*x)^2
*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2
+ 2*tan(1/2*c) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^14 + 33075*pi*a*b*sgn(tan(1/
2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(
1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^14 + 294000*pi*a*b*s
gn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c)
- tan(1/2*c)^2 + 1)*tan(1/2*d*x)^8*tan(1/2*c)^14 + 161280*a*b*tan(1/2*d*x)
^10*tan(1/2*c)^14 + 33075*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/
2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn
(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2
+ tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^6 + 33075*
pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(
1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)
^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/
2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^6 + 231525*pi*a*b*sgn(tan(1/2*d*x)^2
*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2
+ 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1
/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^
12*tan(1/2*c)^8 + 231525*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2
*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(

tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^8 - 205800*pi*a*b*tan(1/2*d*x)^14*tan(1/2*c)^8 - 66150*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^14*tan(1/2*c)^8 - 66150*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^14*tan(1/2*c)^8 - 345450*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^14*tan(1/2*c)^8 - 345450*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^14*tan(1/2*c)^8 + 462336*a^2*tan(1/2*d*x)^14*tan(1/2*c)^9 - 57344*b^2*tan(1/2*d*x)^14*tan(1/2*c)^9 + 416745*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 + 416745*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 - 864360*pi*a*b*tan(1/2*d*x)^12*tan(1/2*c)^10 - 277830*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^12*tan(1/2*c)^10 - 277830*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^12*tan(1/2*c)^10 - 1450890*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^12*tan(1/2*c)^10 - 1450890*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^12*tan(1/2*c)^10 + 1128960*a^2*tan(1/2*d*x)^13*tan(1/2*c)^10 - 1075200*b^2*tan(1/2*d*x)^13*tan(1/2*c)^10 + 5268480*a^2*tan(1/2*d*x)^12*tan(1/2*c)^11 - 2938880*b^2*tan(1/2*d*x)^12*tan(1/2*c)^11 + 231525*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^12 + 231525*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^12 - 864360*pi*a*b*tan(1/2*d*x)^10*tan(1/2*c)^12 - 277830*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^12 - 277830*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^12 - 1450890*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2

*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2
*d*x)^10*tan(1/2*c)^12 - 1450890*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(
1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/
2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^12 + 5268480*a^2*tan(1/2*d*x)^11*tan(
1/2*c)^12 - 2938880*b^2*tan(1/2*d*x)^11*tan(1/2*c)^12 + 1128960*a^2*tan(1/2
*d*x)^10*tan(1/2*c)^13 - 1075200*b^2*tan(1/2*d*x)^10*tan(1/2*c)^13 + 33075*
pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(
1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)
^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/
2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^14 + 33075*pi*a*b*sgn(tan(1/2*d*x)^2*
tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2
- 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/
2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6
*tan(1/2*c)^14 - 205800*pi*a*b*tan(1/2*d*x)^8*tan(1/2*c)^14 - 66150*a*b*arc
tan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)
*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^14
- 66150*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1
)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8
*tan(1/2*c)^14 - 345450*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x)
+ tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)
)*tan(1/2*d*x)^8*tan(1/2*c)^14 - 345450*a*b*arctan((tan(1/2*d*x)*tan(1/2*c)
- tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) +
tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^14 + 462336*a^2*tan(1/2*d*x)^9*
tan(1/2*c)^14 - 57344*b^2*tan(1/2*d*x)^9*tan(1/2*c)^14 + 33075*pi*a*b*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^6 + 33075*pi*a
*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*
d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^6 + 29
4000*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*
x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^14*tan(1/2*c)^6 + 231525*pi*
a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2
*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^8 + 2
31525*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c)
+ tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^12*tan(1/2
*c)^8 + 2058000*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4
*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^12*tan(1/2*c)^8 -
268800*a*b*tan(1/2*d*x)^14*tan(1/2*c)^8 - 3225600*a*b*tan(1/2*d*x)^13*tan(
1/2*c)^9 + 416745*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2
*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x
)^10*tan(1/2*c)^10 + 416745*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(
1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*t
an(1/2*d*x)^10*tan(1/2*c)^10 + 3704400*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)
^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2
*d*x)^10*tan(1/2*c)^10 - 14031360*a*b*tan(1/2*d*x)^12*tan(1/2*c)^10 - 24944
640*a*b*tan(1/2*d*x)^11*tan(1/2*c)^11 + 231525*pi*a*b*sgn(tan(1/2*d*x)^2*ta

n(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^12 + 231525*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^12 + 2058000*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^8*tan(1/2*c)^12 - 14031360*a*b*tan(1/2*d*x)^10*tan(1/2*c)^12 - 3225600*a*b*tan(1/2*d*x)^9*tan(1/2*c)^13 + 33075*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^14 + 33075*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^14 + 294000*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^14 - 268800*a*b*tan(1/2*d*x)^8*tan(1/2*c)^14 + 19845*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^4 + 19845*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^4 + 231525*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^6 + 231525*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^6 - 205800*pi*a*b*tan(1/2*d*x)^14*tan(1/2*c)^6 - 66150*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^14*tan(1/2*c)^6 - 66150*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^14*tan(1/2*c)^6 - 345450*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^14*tan(1/2*c)^6 - 345450*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^14*tan(1/2*c)^6 + 325632*a^2*tan(1/2*d*x)^14*tan(1/2*c)^7 + 233472*b^2*tan(1/2*d*x)^14*tan(1/2*c)^7 + 694575*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^8 + 694575*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1

/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^8 - 1440600*pi*a*b*tan(1/2*d*x)^12*tan(1/2*c)^8 - 463050*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^12*tan(1/2*c)^8 - 463050*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^12*tan(1/2*c)^8 - 2418150*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^12*tan(1/2*c)^8 - 2418150*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^12*tan(1/2*c)^8 - 1881600*a^2*tan(1/2*d*x)^13*tan(1/2*c)^8 + 2150400*b^2*tan(1/2*d*x)^13*tan(1/2*c)^8 - 8053248*a^2*tan(1/2*d*x)^12*tan(1/2*c)^9 + 11425792*b^2*tan(1/2*d*x)^12*tan(1/2*c)^9 + 694575*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^10 + 694575*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^10 - 2593080*pi*a*b*tan(1/2*d*x)^10*tan(1/2*c)^10 - 833490*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^10 - 833490*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^10 - 4352670*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^10 - 4352670*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^10 - 27847680*a^2*tan(1/2*d*x)^11*tan(1/2*c)^10 + 25159680*b^2*tan(1/2*d*x)^11*tan(1/2*c)^10 - 27847680*a^2*tan(1/2*d*x)^10*tan(1/2*c)^11 + 25159680*b^2*tan(1/2*d*x)^10*tan(1/2*c)^11 + 231525*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^12 + 231525*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^12 - 1440600*pi*a*b*tan(1/2*d*x)^8*tan(1/2*c)^12 - 463050*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^12 - 463050*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^12 - 2418150*a*b*arctan((tan(1/2*d*x)*tan(

1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*
d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^12 - 2418150*a*b*arctan((
tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(
1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^12 - 805
3248*a^2*tan(1/2*d*x)^9*tan(1/2*c)^12 + 11425792*b^2*tan(1/2*d*x)^9*tan(1/2
*c)^12 - 1881600*a^2*tan(1/2*d*x)^8*tan(1/2*c)^13 + 2150400*b^2*tan(1/2*d*x
)^8*tan(1/2*c)^13 + 19845*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/
2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn
(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2
 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^14 + 19845*
pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(
1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)
^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/
2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^14 - 205800*pi*a*b*tan(1/2*d*x)^6*tan
(1/2*c)^14 - 66150*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan
(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan
(1/2*d*x)^6*tan(1/2*c)^14 - 66150*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan
(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1
/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^14 - 345450*a*b*arctan((tan(1/2*d*x)*
tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(
1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^14 - 345450*a*b*arcta
n((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*t
an(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^14 +
325632*a^2*tan(1/2*d*x)^7*tan(1/2*c)^14 + 233472*b^2*tan(1/2*d*x)^7*tan(1/2
*c)^14 + 19845*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*ta
n(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^1
4*tan(1/2*c)^4 + 19845*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d
*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/
2*d*x)^14*tan(1/2*c)^4 + 176400*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - ta
n(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^1
4*tan(1/2*c)^4 + 231525*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1
/2*d*x)^12*tan(1/2*c)^6 + 231525*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2
*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) -
 1)*tan(1/2*d*x)^12*tan(1/2*c)^6 + 2058000*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/
2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan
(1/2*d*x)^12*tan(1/2*c)^6 + 268800*a*b*tan(1/2*d*x)^14*tan(1/2*c)^6 + 43008
00*a*b*tan(1/2*d*x)^13*tan(1/2*c)^7 + 694575*pi*a*b*sgn(tan(1/2*d*x)^2*tan(
1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*
tan(1/2*c) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^8 + 694575*pi*a*b*sgn(tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*
c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^8 + 6174000*pi*a*b*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) -
tan(1/2*c)^2 + 1)*tan(1/2*d*x)^10*tan(1/2*c)^8 + 27686400*a*b*tan(1/2*d*x)^
12*tan(1/2*c)^8 + 88166400*a*b*tan(1/2*d*x)^11*tan(1/2*c)^9 + 694575*pi*a*b

*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*
x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^10 + 6945
75*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + t
an(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^
10 + 6174000*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*ta
n(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^8*tan(1/2*c)^10 + 13
6711680*a*b*tan(1/2*d*x)^10*tan(1/2*c)^10 + 88166400*a*b*tan(1/2*d*x)^9*tan
(1/2*c)^11 + 231525*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)
^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d
*x)^6*tan(1/2*c)^12 + 231525*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan
(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*
tan(1/2*d*x)^6*tan(1/2*c)^12 + 2058000*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)
^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2
*d*x)^6*tan(1/2*c)^12 + 27686400*a*b*tan(1/2*d*x)^8*tan(1/2*c)^12 + 4300800
*a*b*tan(1/2*d*x)^7*tan(1/2*c)^13 + 19845*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2
*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan
(1/2*c) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^14 + 19845*pi*a*b*sgn(tan(1/2*d*x)^2
*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2
 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^14 + 176400*pi*a*b*sgn(tan(1
/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1
/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^14 + 268800*a*b*tan(1/2*d*x)^6*tan(1
/2*c)^14 + 6615*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*t
an(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2
*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^2 + 6615*pi*a*b*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2
- tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(
1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)
*tan(1/2*d*x)^14*tan(1/2*c)^2 + 138915*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)
^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/
2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - t
an(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*
c)^4 + 138915*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan
(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c
)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^4 - 123480*pi*a*b*tan(
1/2*d*x)^14*tan(1/2*c)^4 - 39690*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(
1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/
2*c) + 1))*tan(1/2*d*x)^14*tan(1/2*c)^4 - 39690*a*b*arctan((tan(1/2*d*x)*ta
n(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/
2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^14*tan(1/2*c)^4 - 207270*a*b*arctan(
(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan
(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^14*tan(1/2*c)^4 - 20
7270*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(
tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^14*t

an(1/2*c)^4 + 462336*a^2*tan(1/2*d*x)^14*tan(1/2*c)^5 - 57344*b^2*tan(1/2*d*x)^14*tan(1/2*c)^5 + 694575*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^6 + 694575*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^6 - 1440600*pi*a*b*tan(1/2*d*x)^12*tan(1/2*c)^6 - 463050*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^12*tan(1/2*c)^6 - 463050*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^12*tan(1/2*c)^6 - 2418150*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^12*tan(1/2*c)^6 - 2418150*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^12*tan(1/2*c)^6 + 1881600*a^2*tan(1/2*d*x)^13*tan(1/2*c)^6 - 2150400*b^2*tan(1/2*d*x)^13*tan(1/2*c)^6 + 17332224*a^2*tan(1/2*d*x)^12*tan(1/2*c)^7 - 15568896*b^2*tan(1/2*d*x)^12*tan(1/2*c)^7 + 1157625*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^8 + 1157625*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^8 - 4321800*pi*a*b*tan(1/2*d*x)^10*tan(1/2*c)^8 - 1389150*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^8 - 1389150*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^8 - 7254450*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^8 - 7254450*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^8 + 56448000*a^2*tan(1/2*d*x)^11*tan(1/2*c)^8 - 61286400*b^2*tan(1/2*d*x)^11*tan(1/2*c)^8 + 130131456*a^2*tan(1/2*d*x)^10*tan(1/2*c)^9 - 120551424*b^2*tan(1/2*d*x)^10*tan(1/2*c)^9 + 694575*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^10 + 694575*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) -

1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^10 - 4321800*pi*a*b*tan(1/2*d*x)^8*tan(1/2*c)^10 - 1389150*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^10 - 1389150*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^10 - 7254450*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^10 - 7254450*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^10 + 130131456*a^2*tan(1/2*d*x)^9*tan(1/2*c)^10 - 120551424*b^2*tan(1/2*d*x)^9*tan(1/2*c)^10 + 56448000*a^2*tan(1/2*d*x)^8*tan(1/2*c)^11 - 61286400*b^2*tan(1/2*d*x)^8*tan(1/2*c)^11 + 138915*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^12 + 138915*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^12 - 1440600*pi*a*b*tan(1/2*d*x)^6*tan(1/2*c)^12 - 463050*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^12 - 463050*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^12 - 2418150*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^12 - 2418150*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^12 + 17332224*a^2*tan(1/2*d*x)^7*tan(1/2*c)^12 - 15568896*b^2*tan(1/2*d*x)^7*tan(1/2*c)^12 + 1881600*a^2*tan(1/2*d*x)^6*tan(1/2*c)^13 - 2150400*b^2*tan(1/2*d*x)^6*tan(1/2*c)^13 + 6615*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^14 + 6615*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^14 - 123480*pi*a*b*tan(1/2*d*x)^4*tan(1/2*c)^14 - 39690*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^14 - 39690*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2

```
*d*x)^4*tan(1/2*c)^14 - 207270*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/
2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*
c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^14 - 207270*a*b*arctan((tan(1/2*d*x)*tan
(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2
*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^14 + 462336*a^2*tan(1/2*
d*x)^5*tan(1/2*c)^14 - 57344*b^2*tan(1/2*d*x)^5*tan(1/2*c)^14 + 6615*pi*a*b
*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*
x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^2 + 6615
*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan
(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^2
 + 58800*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/
2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^14*tan(1/2*c)^2 + 138915
*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan
(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^4
 + 138915*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2
*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^12*tan
(1/2*c)^4 + 1234800*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2
 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^12*tan(1/2*c)
^4 - 161280*a*b*tan(1/2*d*x)^14*tan(1/2*c)^4 - 3225600*a*b*tan(1/2*d*x)^13*
tan(1/2*c)^5 + 694575*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*
x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2
*d*x)^10*tan(1/2*c)^6 + 694575*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*t
an(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1
)*tan(1/2*d*x)^10*tan(1/2*c)^6 + 6174000*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*
c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1
/2*d*x)^10*tan(1/2*c)^6 - 27686400*a*b*tan(1/2*d*x)^12*tan(1/2*c)^6 - 12902
4000*a*b*tan(1/2*d*x)^11*tan(1/2*c)^7 + 1157625*pi*a*b*sgn(tan(1/2*d*x)^2*t
an(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 +
 2*tan(1/2*c) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^8 + 1157625*pi*a*b*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1
/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^8 + 10290000*pi*a*b*s
gn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c)
 - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^8*tan(1/2*c)^8 - 341107200*a*b*tan(1/2*d*
x)^10*tan(1/2*c)^8 - 488785920*a*b*tan(1/2*d*x)^9*tan(1/2*c)^9 + 694575*pi*
a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2
*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^10 + 6
94575*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c)
+ tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^6*tan(1/2*
c)^10 + 6174000*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4
*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^10 -
 341107200*a*b*tan(1/2*d*x)^8*tan(1/2*c)^10 - 129024000*a*b*tan(1/2*d*x)^7*
tan(1/2*c)^11 + 138915*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d
*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/
2*d*x)^4*tan(1/2*c)^12 + 138915*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*
tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) -
```

1)*tan(1/2*d*x)^4*tan(1/2*c)^12 + 1234800*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^12 - 27686400*a*b*tan(1/2*d*x)^6*tan(1/2*c)^12 - 3225600*a*b*tan(1/2*d*x)^5*tan(1/2*c)^13 + 6615*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^14 + 6615*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^14 + 58800*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2*tan(1/2*c)^14 - 161280*a*b*tan(1/2*d*x)^4*tan(1/2*c)^14 + 945*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14 + 945*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14 + 46305*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^2 + 46305*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^2 - 41160*pi*a*b*tan(1/2*d*x)^14*tan(1/2*c)^2 - 13230*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^14*tan(1/2*c)^2 - 13230*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^14*tan(1/2*c)^2 - 69090*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^14*tan(1/2*c)^2 - 69090*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^14*tan(1/2*c)^2 + 107520*a^2*tan(1/2*d*x)^14*tan(1/2*c)^3 + 71680*b^2*tan(1/2*d*x)^14*tan(1/2*c)^3 + 416745*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^4 + 416745*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^4 - 864360*pi*a*b*tan(1/2*d*x)^12*tan(1/2*c)^4 - 277830*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^12*tan(1/2*c)^4 -

277830*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)
/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^12
*tan(1/2*c)^4 - 1450890*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x)
+ tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)
)*tan(1/2*d*x)^12*tan(1/2*c)^4 - 1450890*a*b*arctan((tan(1/2*d*x)*tan(1/2*c
) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x)
+ tan(1/2*c) - 1))*tan(1/2*d*x)^12*tan(1/2*c)^4 - 1128960*a^2*tan(1/2*d*x)^
13*tan(1/2*c)^4 + 1075200*b^2*tan(1/2*d*x)^13*tan(1/2*c)^4 - 8053248*a^2*ta
n(1/2*d*x)^12*tan(1/2*c)^5 + 11425792*b^2*tan(1/2*d*x)^12*tan(1/2*c)^5 + 11
57625*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c)
+ tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(
1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*
tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^6 + 1157625*pi*a*b*sgn(tan(1/2*
d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/
2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)
*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2
*d*x)^8*tan(1/2*c)^6 - 4321800*pi*a*b*tan(1/2*d*x)^10*tan(1/2*c)^6 - 138915
0*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan
(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(
1/2*c)^6 - 1389150*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan
(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan
(1/2*d*x)^10*tan(1/2*c)^6 - 7254450*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + t
an(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan
(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^6 - 7254450*a*b*arctan((tan(1/2*d*
x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + t
an(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^6 - 56448000*a^2*
tan(1/2*d*x)^11*tan(1/2*c)^6 + 61286400*b^2*tan(1/2*d*x)^11*tan(1/2*c)^6 -
173795328*a^2*tan(1/2*d*x)^10*tan(1/2*c)^7 + 198438912*b^2*tan(1/2*d*x)^10*
tan(1/2*c)^7 + 1157625*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d
*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^8 + 1157625*pi
*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/
2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*
d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^8 - 7203000*pi*a*b*tan(1/2*d*x)^8*tan(1
/2*c)^8 - 2315250*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(
1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(
1/2*d*x)^8*tan(1/2*c)^8 - 2315250*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan
(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1
/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 - 12090750*a*b*arctan((tan(1/2*d*x)
*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan
(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 - 12090750*a*b*arc
tan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)
*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 -

353364480*a^2*tan(1/2*d*x)^9*tan(1/2*c)^8 + 359976960*b^2*tan(1/2*d*x)^9*t
an(1/2*c)^8 - 353364480*a^2*tan(1/2*d*x)^8*tan(1/2*c)^9 + 359976960*b^2*tan
(1/2*d*x)^8*tan(1/2*c)^9 + 416745*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 +
2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c)
- 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/
2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^10
+ 416745*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*
c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*t
an(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 -
2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^10 - 4321800*pi*a*b*tan(1/2*
d*x)^6*tan(1/2*c)^10 - 1389150*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/
2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*
c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^10 - 1389150*a*b*arctan((tan(1/2*d*x)*ta
n(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/
2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^10 - 7254450*a*b*arctan
((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*ta
n(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^10 - 7
254450*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)
/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*
tan(1/2*c)^10 - 173795328*a^2*tan(1/2*d*x)^7*tan(1/2*c)^10 + 198438912*b^2*
tan(1/2*d*x)^7*tan(1/2*c)^10 - 56448000*a^2*tan(1/2*d*x)^6*tan(1/2*c)^11 +
61286400*b^2*tan(1/2*d*x)^6*tan(1/2*c)^11 + 46305*pi*a*b*sgn(tan(1/2*d*x)^2
*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2
+ 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1
/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^
2*tan(1/2*c)^12 + 46305*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*
d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^12 - 864360*p
i*a*b*tan(1/2*d*x)^4*tan(1/2*c)^12 - 277830*a*b*arctan((tan(1/2*d*x)*tan(1/
2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*
x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^12 - 277830*a*b*arctan((tan
(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2
*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^12 - 145089
0*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan
(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1
/2*c)^12 - 1450890*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan
(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan
(1/2*d*x)^4*tan(1/2*c)^12 - 8053248*a^2*tan(1/2*d*x)^5*tan(1/2*c)^12 + 1142
5792*b^2*tan(1/2*d*x)^5*tan(1/2*c)^12 - 1128960*a^2*tan(1/2*d*x)^4*tan(1/2*
c)^13 + 1075200*b^2*tan(1/2*d*x)^4*tan(1/2*c)^13 + 945*pi*a*b*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2
*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*
tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*
c)^14 + 945*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1

/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^
2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^
2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^14 - 41160*pi*a*b*tan(1/2*d*x)^2*tan(1/2
*c)^14 - 13230*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2
*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2
*d*x)^2*tan(1/2*c)^14 - 13230*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2
*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c
) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^14 - 69090*a*b*arctan((tan(1/2*d*x)*tan(1
/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d
*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^14 - 69090*a*b*arctan((tan
(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2
*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^14 + 107520
*a^2*tan(1/2*d*x)^3*tan(1/2*c)^14 + 71680*b^2*tan(1/2*d*x)^3*tan(1/2*c)^14
+ 945*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c)
+ tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^14 + 945*p
i*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1
/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^14 + 8400*pi*a*b*
sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c
) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^14 + 46305*pi*a*b*sgn(tan(1/2*d*x)^2*tan
(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2
*tan(1/2*c) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^2 + 46305*pi*a*b*sgn(tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*
c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^2 + 411600*pi*a*b*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - t
an(1/2*c)^2 + 1)*tan(1/2*d*x)^12*tan(1/2*c)^2 + 53760*a*b*tan(1/2*d*x)^14*t
an(1/2*c)^2 + 1290240*a*b*tan(1/2*d*x)^13*tan(1/2*c)^3 + 416745*pi*a*b*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2
- tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^4 + 416745*pi
*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/
2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^4 +
3704400*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2
*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^10*tan(1/2*c)^4 + 1403136
0*a*b*tan(1/2*d*x)^12*tan(1/2*c)^4 + 88166400*a*b*tan(1/2*d*x)^11*tan(1/2*c
)^5 + 1157625*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan
(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^8*
tan(1/2*c)^6 + 1157625*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d
*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/
2*d*x)^8*tan(1/2*c)^6 + 10290000*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - t
an(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^
8*tan(1/2*c)^6 + 341107200*a*b*tan(1/2*d*x)^10*tan(1/2*c)^6 + 807690240*a*b
*tan(1/2*d*x)^9*tan(1/2*c)^7 + 1157625*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)
^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/
2*c) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^8 + 1157625*pi*a*b*sgn(tan(1/2*d*x)^2*t
an(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 -
 2*tan(1/2*c) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^8 + 10290000*pi*a*b*sgn(tan(1/

2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/
2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^8 + 1106972160*a*b*tan(1/2*d*x)^8*tan
(1/2*c)^8 + 807690240*a*b*tan(1/2*d*x)^7*tan(1/2*c)^9 + 416745*pi*a*b*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
 tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^10 + 416745*pi*
a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2
*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^10 + 3
704400*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*
d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^10 + 34110720
0*a*b*tan(1/2*d*x)^6*tan(1/2*c)^10 + 88166400*a*b*tan(1/2*d*x)^5*tan(1/2*c)
^11 + 46305*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1
/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^2*ta
n(1/2*c)^12 + 46305*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)
^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d
*x)^2*tan(1/2*c)^12 + 411600*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1
/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2*ta
n(1/2*c)^12 + 14031360*a*b*tan(1/2*d*x)^4*tan(1/2*c)^12 + 1290240*a*b*tan(1
/2*d*x)^3*tan(1/2*c)^13 + 945*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)
*tan(1/2*c)^14 + 945*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x
)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*
c)^14 + 8400*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*ta
n(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^14 + 53760*a*b*tan(1/2
*d*x)^2*tan(1/2*c)^14 + 6615*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan
(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*
sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12 + 6615*pi*a*b*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2
 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(
1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)
*tan(1/2*d*x)^12 - 5880*pi*a*b*tan(1/2*d*x)^14 - 1890*a*b*arctan((tan(1/2*d
*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) -
tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^14 - 1890*a*b*arctan((tan(1/2*
d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) +
 tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^14 - 9870*a*b*arctan((tan(1/2
*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c)
 - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^14 - 9870*a*b*arctan((tan(1/
2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c)
 + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^14 + 53760*a^2*tan(1/2*d*x)
^14*tan(1/2*c) + 138915*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
 tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^2 + 138915*p
i*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1
/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^

2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2
*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^2 - 288120*pi*a*b*tan(1/2*d*x)^12*tan
(1/2*c)^2 - 92610*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(
1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(
1/2*d*x)^12*tan(1/2*c)^2 - 92610*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(
1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/
2*c) + 1))*tan(1/2*d*x)^12*tan(1/2*c)^2 - 483630*a*b*arctan((tan(1/2*d*x)*t
an(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1
/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^12*tan(1/2*c)^2 - 483630*a*b*arctan
((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*ta
n(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^12*tan(1/2*c)^2 + 3
76320*a^2*tan(1/2*d*x)^13*tan(1/2*c)^2 - 215040*b^2*tan(1/2*d*x)^13*tan(1/2
*c)^2 + 5268480*a^2*tan(1/2*d*x)^12*tan(1/2*c)^3 - 2938880*b^2*tan(1/2*d*x)
^12*tan(1/2*c)^3 + 694575*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/
2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn
(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2
 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^4 + 694575*
pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(
1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)
^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/
2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^4 - 2593080*pi*a*b*tan(1/2*d*x)^10*ta
n(1/2*c)^4 - 833490*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - ta
n(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*ta
n(1/2*d*x)^10*tan(1/2*c)^4 - 833490*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - t
an(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan
(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^4 - 4352670*a*b*arctan((tan(1/2*d*
x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - t
an(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^4 - 4352670*a*b*a
rctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*
x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^
4 + 27847680*a^2*tan(1/2*d*x)^11*tan(1/2*c)^4 - 25159680*b^2*tan(1/2*d*x)^1
1*tan(1/2*c)^4 + 130131456*a^2*tan(1/2*d*x)^10*tan(1/2*c)^5 - 120551424*b^2
*tan(1/2*d*x)^10*tan(1/2*c)^5 + 1157625*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c
)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1
/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 -
tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*
c)^6 + 1157625*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*ta
n(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*
c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 - 7203000*pi*a*b*tan
(1/2*d*x)^8*tan(1/2*c)^6 - 2315250*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + ta
n(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(
1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^6 - 2315250*a*b*arctan((tan(1/2*d*x)
*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan
(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^6 - 12090750*a*b*arc

tan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)
*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^6 -
12090750*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) -
1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)
^8*tan(1/2*c)^6 + 353364480*a^2*tan(1/2*d*x)^9*tan(1/2*c)^6 - 359976960*b^2
*tan(1/2*d*x)^9*tan(1/2*c)^6 + 652646400*a^2*tan(1/2*d*x)^8*tan(1/2*c)^7 -
629207040*b^2*tan(1/2*d*x)^8*tan(1/2*c)^7 + 694575*pi*a*b*sgn(tan(1/2*d*x)^
2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^
2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(
1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)
^4*tan(1/2*c)^8 + 694575*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2
*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2
+ tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^8 - 7203000*
pi*a*b*tan(1/2*d*x)^6*tan(1/2*c)^8 - 2315250*a*b*arctan((tan(1/2*d*x)*tan(1
/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d
*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^8 - 2315250*a*b*arctan((ta
n(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/
2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^8 - 120907
50*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(ta
n(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(
1/2*c)^8 - 12090750*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - ta
n(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*ta
n(1/2*d*x)^6*tan(1/2*c)^8 + 652646400*a^2*tan(1/2*d*x)^7*tan(1/2*c)^8 - 629
207040*b^2*tan(1/2*d*x)^7*tan(1/2*c)^8 + 353364480*a^2*tan(1/2*d*x)^6*tan(1
/2*c)^9 - 359976960*b^2*tan(1/2*d*x)^6*tan(1/2*c)^9 + 138915*pi*a*b*sgn(tan
(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - t
an(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2
*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*ta
n(1/2*d*x)^2*tan(1/2*c)^10 + 138915*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
- 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c
) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(
1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^1
0 - 2593080*pi*a*b*tan(1/2*d*x)^4*tan(1/2*c)^10 - 833490*a*b*arctan((tan(1/
2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c)
- tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^10 - 833490*a*
b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2
*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c
)^10 - 4352670*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2
*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2
*d*x)^4*tan(1/2*c)^10 - 4352670*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1
/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2
*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^10 + 130131456*a^2*tan(1/2*d*x)^5*tan(1
/2*c)^10 - 120551424*b^2*tan(1/2*d*x)^5*tan(1/2*c)^10 + 27847680*a^2*tan(1/
2*d*x)^4*tan(1/2*c)^11 - 25159680*b^2*tan(1/2*d*x)^4*tan(1/2*c)^11 + 6615*p

```
i*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1
/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2
*d*x) - 1)*tan(1/2*c)^12 + 6615*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*
tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) -
1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*
d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^12 - 288120*pi*a*b*t
an(1/2*d*x)^2*tan(1/2*c)^12 - 92610*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + t
an(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan
(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^12 - 92610*a*b*arctan((tan(1/2*d*x)
*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan
(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^12 - 483630*a*b*arct
an((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*
tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^12 -
 483630*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1
)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2
*tan(1/2*c)^12 + 5268480*a^2*tan(1/2*d*x)^3*tan(1/2*c)^12 - 2938880*b^2*tan
(1/2*d*x)^3*tan(1/2*c)^12 + 376320*a^2*tan(1/2*d*x)^2*tan(1/2*c)^13 - 21504
0*b^2*tan(1/2*d*x)^2*tan(1/2*c)^13 - 5880*pi*a*b*tan(1/2*c)^14 - 1890*a*b*a
rctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*
x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*c)^14 - 1890*a*b*ar
ctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x
)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*c)^14 - 9870*a*b*arc
tan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)
*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*c)^14 - 9870*a*b*arct
an((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*
tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^14 + 53760*a^2*tan(
1/2*d*x)*tan(1/2*c)^14 + 6615*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)
*tan(1/2*d*x)^12 + 6615*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*
d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1
/2*d*x)^12 + 58800*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2
- 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^12 - 7680*a*b*
tan(1/2*d*x)^14 - 215040*a*b*tan(1/2*d*x)^13*tan(1/2*c) + 138915*pi*a*b*sgn
(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2
 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^2 + 138915*p
i*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1
/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^2 +
 1234800*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/
2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^10*tan(1/2*c)^2 - 295680
0*a*b*tan(1/2*d*x)^12*tan(1/2*c)^2 - 24944640*a*b*tan(1/2*d*x)^11*tan(1/2*c
)^3 + 694575*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(
1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^8*t
an(1/2*c)^4 + 694575*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x
)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*
```

d*x)^8*tan(1/2*c)^4 + 6174000*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(
1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^8*t
an(1/2*c)^4 - 136711680*a*b*tan(1/2*d*x)^10*tan(1/2*c)^4 - 488785920*a*b*ta
n(1/2*d*x)^9*tan(1/2*c)^5 + 1157625*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
+ 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c
) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 1157625*pi*a*b*sgn(tan(1/2*d*x)^2*tan(
1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*
tan(1/2*c) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 10290000*pi*a*b*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c
)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 - 1106972160*a*b*tan(1/2*d*x)^8*tan(1/
2*c)^6 - 1491763200*a*b*tan(1/2*d*x)^7*tan(1/2*c)^7 + 694575*pi*a*b*sgn(tan
(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - t
an(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^8 + 694575*pi*a*b
*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*
x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^8 + 61740
00*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)
*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^8 - 1106972160*a*
b*tan(1/2*d*x)^6*tan(1/2*c)^8 - 488785920*a*b*tan(1/2*d*x)^5*tan(1/2*c)^9 +
 138915*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c
) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^2*tan(1/
2*c)^10 + 138915*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*
tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)
^2*tan(1/2*c)^10 + 1234800*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2
*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2*tan(
1/2*c)^10 - 136711680*a*b*tan(1/2*d*x)^4*tan(1/2*c)^10 - 24944640*a*b*tan(1
/2*d*x)^3*tan(1/2*c)^11 + 6615*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*t
an(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1
)*tan(1/2*c)^12 + 6615*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d
*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/
2*c)^12 + 58800*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4
*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^12 - 2956800*a*b*ta
n(1/2*d*x)^2*tan(1/2*c)^12 - 215040*a*b*tan(1/2*d*x)*tan(1/2*c)^13 - 7680*a
*b*tan(1/2*c)^14 + 19845*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2
*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2
+ tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10 + 19845*pi*a*b*sgn(tan
(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - t
an(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2
*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*ta
n(1/2*d*x)^10 - 41160*pi*a*b*tan(1/2*d*x)^12 - 13230*a*b*arctan((tan(1/2*d*
x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - t
an(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^12 - 13230*a*b*arctan((tan(1/2*
d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) +
 tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^12 - 69090*a*b*arctan((tan(1/
2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c)

- tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^12 - 69090*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^12 - 53760*a^2*tan(1/2*d*x)^13 - 376320*a^2*tan(1/2*d*x)^12*tan(1/2*c) + 215040*b^2*tan(1/2*d*x)^12*tan(1/2*c) + 231525*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^2 + 231525*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^2 - 864360*pi*a*b*tan(1/2*d*x)^10*tan(1/2*c)^2 - 277830*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^2 - 277830*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^2 - 1450890*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^2 - 1450890*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^2 - 5268480*a^2*tan(1/2*d*x)^11*tan(1/2*c)^2 + 2938880*b^2*tan(1/2*d*x)^11*tan(1/2*c)^2 - 27847680*a^2*tan(1/2*d*x)^10*tan(1/2*c)^3 + 25159680*b^2*tan(1/2*d*x)^10*tan(1/2*c)^3 + 694575*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 + 694575*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 - 4321800*pi*a*b*tan(1/2*d*x)^8*tan(1/2*c)^4 - 1389150*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^4 - 1389150*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^4 - 7254450*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^4 - 7254450*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^4 - 130131456*a^2*tan(1/2*d*x)^9*tan(1/2*c)^4 + 120551424*b^2*tan(1/2*d*x)^9*tan(1/2*c)^4 - 353364480*a^2*tan(1/2*d*x)^8*tan(1/2*c)^5 + 359976960*b^2*tan(1/2*d*x)^8*tan(1/2*c)^5 + 694575*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(

1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 + 694575*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 - 7203000*pi*a*b*tan(1/2*d*x)^6*tan(1/2*c)^6 - 2315250*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 - 2315250*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 - 12090750*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 - 12090750*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 - 652646400*a^2*tan(1/2*d*x)^7*tan(1/2*c)^6 + 629207040*b^2*tan(1/2*d*x)^7*tan(1/2*c)^6 - 652646400*a^2*tan(1/2*d*x)^6*tan(1/2*c)^7 + 629207040*b^2*tan(1/2*d*x)^6*tan(1/2*c)^7 + 231525*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^8 + 231525*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^8 - 4321800*pi*a*b*tan(1/2*d*x)^4*tan(1/2*c)^8 - 1389150*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^8 - 1389150*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^8 - 7254450*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^8 - 7254450*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^8 - 353364480*a^2*tan(1/2*d*x)^5*tan(1/2*c)^8 + 359976960*b^2*tan(1/2*d*x)^5*tan(1/2*c)^8 - 130131456*a^2*tan(1/2*d*x)^4*tan(1/2*c)^9 + 120551424*b^2*tan(1/2*d*x)^4*tan(1/2*c)^9 + 19845*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^10 + 19845*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^10 - 864360*pi*a*b*tan(1/2*d*x)^2*tan(1/2*c)^10 - 277830*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1

/2*d*x)^2*tan(1/2*c)^10 - 277830*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(
1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/
2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^10 - 1450890*a*b*arctan((tan(1/2*d*x)*
tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(
1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^10 - 1450890*a*b*arct
an((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*
tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^10 -
 27847680*a^2*tan(1/2*d*x)^3*tan(1/2*c)^10 + 25159680*b^2*tan(1/2*d*x)^3*ta
n(1/2*c)^10 - 5268480*a^2*tan(1/2*d*x)^2*tan(1/2*c)^11 + 2938880*b^2*tan(1/
2*d*x)^2*tan(1/2*c)^11 - 41160*pi*a*b*tan(1/2*c)^12 - 13230*a*b*arctan((tan
(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2
*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*c)^12 - 13230*a*b*arctan((tan
(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2
*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*c)^12 - 69090*a*b*arctan((tan
(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2
*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*c)^12 - 69090*a*b*arctan((tan
(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2
*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^12 - 376320*a^2*tan(1/2*d*
x)*tan(1/2*c)^12 + 215040*b^2*tan(1/2*d*x)*tan(1/2*c)^12 - 53760*a^2*tan(1/
2*c)^13 + 19845*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*t
an(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^
10 + 19845*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/
2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^10 +
176400*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*
d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^10 + 53760*a*b*tan(1/2*d*x
)^12 + 1290240*a*b*tan(1/2*d*x)^11*tan(1/2*c) + 231525*pi*a*b*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2
*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^2 + 231525*pi*a*b*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
 tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^2 + 2058000*pi*
a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1
/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^8*tan(1/2*c)^2 + 14031360*a*b*tan(1/
2*d*x)^10*tan(1/2*c)^2 + 88166400*a*b*tan(1/2*d*x)^9*tan(1/2*c)^3 + 694575*
pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(
1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 +
 694575*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c
) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^6*tan(1/
2*c)^4 + 6174000*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 -
4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 +
 341107200*a*b*tan(1/2*d*x)^8*tan(1/2*c)^4 + 807690240*a*b*tan(1/2*d*x)^7*t
an(1/2*c)^5 + 694575*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x
)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*
d*x)^4*tan(1/2*c)^6 + 694575*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan
(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*
tan(1/2*d*x)^4*tan(1/2*c)^6 + 6174000*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^

2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*
d*x)^4*tan(1/2*c)^6 + 1106972160*a*b*tan(1/2*d*x)^6*tan(1/2*c)^6 + 80769024
0*a*b*tan(1/2*d*x)^5*tan(1/2*c)^7 + 231525*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/
2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*ta
n(1/2*c) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^8 + 231525*pi*a*b*sgn(tan(1/2*d*x)^
2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^
2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^8 + 2058000*pi*a*b*sgn(tan(
1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(
1/2*c)^2 + 1)*tan(1/2*d*x)^2*tan(1/2*c)^8 + 341107200*a*b*tan(1/2*d*x)^4*ta
n(1/2*c)^8 + 88166400*a*b*tan(1/2*d*x)^3*tan(1/2*c)^9 + 19845*pi*a*b*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*c)^10 + 19845*pi*a*b*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2
*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*c)^10 + 176400*pi*a*b*sgn(tan(1/2*d*x)^2*
tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 +
1)*tan(1/2*c)^10 + 14031360*a*b*tan(1/2*d*x)^2*tan(1/2*c)^10 + 1290240*a*b*
tan(1/2*d*x)*tan(1/2*c)^11 + 53760*a*b*tan(1/2*c)^12 + 33075*pi*a*b*sgn(tan
(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - t
an(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2
*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*ta
n(1/2*d*x)^8 + 33075*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x
)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(
1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + ta
n(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8 - 123480*pi*a*b*tan(1/2*d*x
)^10 - 39690*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c
) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d
*x)^10 - 39690*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2
*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2
*d*x)^10 - 207270*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(
1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(
1/2*d*x)^10 - 207270*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - t
an(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*t
an(1/2*d*x)^10 - 107520*a^2*tan(1/2*d*x)^11 - 71680*b^2*tan(1/2*d*x)^11 + 1
128960*a^2*tan(1/2*d*x)^10*tan(1/2*c) - 1075200*b^2*tan(1/2*d*x)^10*tan(1/2
*c) + 231525*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(
1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)
^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)
^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 + 231525*pi*a*b*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/
2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*t
an(1/2*d*x)^6*tan(1/2*c)^2 - 1440600*pi*a*b*tan(1/2*d*x)^8*tan(1/2*c)^2 - 4
63050*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/
(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^8*t
an(1/2*c)^2 - 463050*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + t

an(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*t
an(1/2*d*x)^8*tan(1/2*c)^2 - 2418150*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) +
tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - ta
n(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^2 - 2418150*a*b*arctan((tan(1/2*d*
x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + t
an(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^2 + 8053248*a^2*ta
n(1/2*d*x)^9*tan(1/2*c)^2 - 11425792*b^2*tan(1/2*d*x)^9*tan(1/2*c)^2 + 5644
8000*a^2*tan(1/2*d*x)^8*tan(1/2*c)^3 - 61286400*b^2*tan(1/2*d*x)^8*tan(1/2*
c)^3 + 416745*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan
(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c
)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 + 416745*pi*a*b*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1
/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*
tan(1/2*d*x)^4*tan(1/2*c)^4 - 4321800*pi*a*b*tan(1/2*d*x)^6*tan(1/2*c)^4 -
1389150*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1
)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6
*tan(1/2*c)^4 - 1389150*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x)
+ tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)
)*tan(1/2*d*x)^6*tan(1/2*c)^4 - 7254450*a*b*arctan((tan(1/2*d*x)*tan(1/2*c)
+ tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) -
tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 - 7254450*a*b*arctan((tan(1/2
*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c)
+ tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 + 173795328*a
^2*tan(1/2*d*x)^7*tan(1/2*c)^4 - 198438912*b^2*tan(1/2*d*x)^7*tan(1/2*c)^4
+ 353364480*a^2*tan(1/2*d*x)^6*tan(1/2*c)^5 - 359976960*b^2*tan(1/2*d*x)^6*
tan(1/2*c)^5 + 231525*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*
x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan
(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + t
an(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^6 + 231525*pi*a
*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*
d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 -
2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*
x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^6 - 4321800*pi*a*b*tan(1/2*d*x)^4*tan(1/2
*c)^6 - 1389150*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/
2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/
2*d*x)^4*tan(1/2*c)^6 - 1389150*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1
/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2
*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^6 - 7254450*a*b*arctan((tan(1/2*d*x)*ta
n(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/
2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^6 - 7254450*a*b*arctan(
(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan
(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^6 + 353
364480*a^2*tan(1/2*d*x)^5*tan(1/2*c)^6 - 359976960*b^2*tan(1/2*d*x)^5*tan(1

/2*c)^6 + 173795328*a^2*tan(1/2*d*x)^4*tan(1/2*c)^7 - 198438912*b^2*tan(1/2*d*x)^4*tan(1/2*c)^7 + 33075*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^8 + 33075*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^8 - 1440600*pi*a*b*tan(1/2*d*x)^2*tan(1/2*c)^8 - 463050*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^8 - 463050*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^8 - 2418150*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^8 - 2418150*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^8 + 56448000*a^2*tan(1/2*d*x)^3*tan(1/2*c)^8 - 61286400*b^2*tan(1/2*d*x)^3*tan(1/2*c)^8 + 8053248*a^2*tan(1/2*d*x)^2*tan(1/2*c)^9 - 11425792*b^2*tan(1/2*d*x)^2*tan(1/2*c)^9 - 123480*pi*a*b*tan(1/2*c)^10 - 39690*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*c)^10 - 39690*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*c)^10 - 207270*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*c)^10 - 207270*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^10 + 1128960*a^2*tan(1/2*d*x)*tan(1/2*c)^10 - 1075200*b^2*tan(1/2*d*x)*tan(1/2*c)^10 - 107520*a^2*tan(1/2*c)^11 - 71680*b^2*tan(1/2*c)^11 + 33075*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^8 + 33075*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^8 + 294000*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^8 - 161280*a*b*tan(1/2*d*x)^10 - 3225600*a*b*tan(1/2*d*x)^9*tan(1/2*c) + 231525*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 + 231525*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 + 2058000*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 - 27686400*a*b*tan(1/2*d*x)^8*tan(1/2*c)^2 - 129024000*a*b*tan(1/2*d*x)^7*tan(1/2*c)^3 + 416745*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*

d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 + 416
745*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) +
tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^4*tan(1/2*c)
^4 + 3704400*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*ta
n(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 - 341
107200*a*b*tan(1/2*d*x)^6*tan(1/2*c)^4 - 488785920*a*b*tan(1/2*d*x)^5*tan(1
/2*c)^5 + 231525*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*
tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)
^2*tan(1/2*c)^6 + 231525*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2
*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(
1/2*d*x)^2*tan(1/2*c)^6 + 2058000*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 -
tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)
^2*tan(1/2*c)^6 - 341107200*a*b*tan(1/2*d*x)^4*tan(1/2*c)^6 - 129024000*a*b
*tan(1/2*d*x)^3*tan(1/2*c)^7 + 33075*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*
c) - 1)*tan(1/2*c)^8 + 33075*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan
(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*
tan(1/2*c)^8 + 294000*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)
^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^8 - 27686400*
a*b*tan(1/2*d*x)^2*tan(1/2*c)^8 - 3225600*a*b*tan(1/2*d*x)*tan(1/2*c)^9 - 1
61280*a*b*tan(1/2*c)^10 + 33075*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*
tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) -
1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*
d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6 + 33075*pi*a*b*s
gn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)
^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*t
an(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) -
 1)*tan(1/2*d*x)^6 - 205800*pi*a*b*tan(1/2*d*x)^8 - 66150*a*b*arctan((tan(1
/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c
) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^8 - 66150*a*b*arctan((tan(
1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*
c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8 - 345450*a*b*arctan((ta
n(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/
2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^8 - 345450*a*b*arctan((
tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(
1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^8 - 462336*a^2*tan(1/
2*d*x)^9 + 57344*b^2*tan(1/2*d*x)^9 - 1881600*a^2*tan(1/2*d*x)^8*tan(1/2*c)
 + 2150400*b^2*tan(1/2*d*x)^8*tan(1/2*c) + 138915*pi*a*b*sgn(tan(1/2*d*x)^2
*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2
 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1
/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^
4*tan(1/2*c)^2 + 138915*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*
d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
 tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^2 - 1440600*p

i*a*b*tan(1/2*d*x)^6*tan(1/2*c)^2 - 463050*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 - 463050*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 - 2418150*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 - 2418150*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 - 17332224*a^2*tan(1/2*d*x)^7*tan(1/2*c)^2 + 15568896*b^2*tan(1/2*d*x)^7*tan(1/2*c)^2 - 56448000*a^2*tan(1/2*d*x)^6*tan(1/2*c)^3 + 61286400*b^2*tan(1/2*d*x)^6*tan(1/2*c)^3 + 138915*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^4 + 138915*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^4 - 2593080*pi*a*b*tan(1/2*d*x)^4*tan(1/2*c)^4 - 833490*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 - 833490*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 - 4352670*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 - 4352670*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 - 130131456*a^2*tan(1/2*d*x)^5*tan(1/2*c)^4 + 120551424*b^2*tan(1/2*d*x)^5*tan(1/2*c)^4 - 130131456*a^2*tan(1/2*d*x)^4*tan(1/2*c)^5 + 120551424*b^2*tan(1/2*d*x)^4*tan(1/2*c)^5 + 33075*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^6 + 33075*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^6 - 1440600*pi*a*b*tan(1/2*d*x)^2*tan(1/2*c)^6 - 463050*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 - 463050*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 - 2418150*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 - 2418150*a*b*arcta

n((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*t
an(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 - 5
6448000*a^2*tan(1/2*d*x)^3*tan(1/2*c)^6 + 61286400*b^2*tan(1/2*d*x)^3*tan(1
/2*c)^6 - 17332224*a^2*tan(1/2*d*x)^2*tan(1/2*c)^7 + 15568896*b^2*tan(1/2*d
*x)^2*tan(1/2*c)^7 - 205800*pi*a*b*tan(1/2*c)^8 - 66150*a*b*arctan((tan(1/2
*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c)
- tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*c)^8 - 66150*a*b*arctan((tan(1/2*
d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) +
tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*c)^8 - 345450*a*b*arctan((tan(1/2*
d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) -
tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*c)^8 - 345450*a*b*arctan((tan(1/2*
d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) +
tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^8 - 1881600*a^2*tan(1/2*d*x)*ta
n(1/2*c)^8 + 2150400*b^2*tan(1/2*d*x)*tan(1/2*c)^8 - 462336*a^2*tan(1/2*c)^
9 + 57344*b^2*tan(1/2*c)^9 + 33075*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 +
2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c)
- 1)*tan(1/2*d*x)^6 + 33075*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan
(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*
tan(1/2*d*x)^6 + 294000*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*
x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6 + 26880
0*a*b*tan(1/2*d*x)^8 + 4300800*a*b*tan(1/2*d*x)^7*tan(1/2*c) + 138915*pi*a*
b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d
*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^2 + 1389
15*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + t
an(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^
2 + 1234800*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan
(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^2 + 2768
6400*a*b*tan(1/2*d*x)^6*tan(1/2*c)^2 + 88166400*a*b*tan(1/2*d*x)^5*tan(1/2*
c)^3 + 138915*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan
(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^2*
tan(1/2*c)^4 + 138915*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*
x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2
*d*x)^2*tan(1/2*c)^4 + 1234800*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan
(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2*
tan(1/2*c)^4 + 136711680*a*b*tan(1/2*d*x)^4*tan(1/2*c)^4 + 88166400*a*b*tan
(1/2*d*x)^3*tan(1/2*c)^5 + 33075*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2
*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) -
1)*tan(1/2*c)^6 + 33075*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2
*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(
1/2*c)^6 + 294000*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 -
4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^6 + 27686400*a*b*
tan(1/2*d*x)^2*tan(1/2*c)^6 + 4300800*a*b*tan(1/2*d*x)*tan(1/2*c)^7 + 26880
0*a*b*tan(1/2*c)^8 + 19845*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1
/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sg
n(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^

2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4 + 19845*pi*a*b*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/
2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*t
an(1/2*d*x)^4 - 205800*pi*a*b*tan(1/2*d*x)^6 - 66150*a*b*arctan((tan(1/2*d*
x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - t
an(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6 - 66150*a*b*arctan((tan(1/2*d
*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) +
tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6 - 345450*a*b*arctan((tan(1/2
*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c)
- tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6 - 345450*a*b*arctan((tan(1
/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c
) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6 - 325632*a^2*tan(1/2*d*x
)^7 - 233472*b^2*tan(1/2*d*x)^7 + 1881600*a^2*tan(1/2*d*x)^6*tan(1/2*c) - 2
150400*b^2*tan(1/2*d*x)^6*tan(1/2*c) + 46305*pi*a*b*sgn(tan(1/2*d*x)^2*tan(
1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*
tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)
^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan
(1/2*c)^2 + 46305*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2
*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1
/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 864360*pi*a*b*t
an(1/2*d*x)^4*tan(1/2*c)^2 - 277830*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + t
an(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan
(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^2 - 277830*a*b*arctan((tan(1/2*d*x)
*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan
(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^2 - 1450890*a*b*arct
an((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*
tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^2 -
1450890*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1
)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4
*tan(1/2*c)^2 + 8053248*a^2*tan(1/2*d*x)^5*tan(1/2*c)^2 - 11425792*b^2*tan(
1/2*d*x)^5*tan(1/2*c)^2 + 27847680*a^2*tan(1/2*d*x)^4*tan(1/2*c)^3 - 251596
80*b^2*tan(1/2*d*x)^4*tan(1/2*c)^3 + 19845*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/
2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*ta
n(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2
 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^4 + 19845
*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan
(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c
)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1
/2*d*x) - 1)*tan(1/2*c)^4 - 864360*pi*a*b*tan(1/2*d*x)^2*tan(1/2*c)^4 - 277
830*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(t
an(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan
(1/2*c)^4 - 277830*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan
(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan

(1/2*d*x)^2*tan(1/2*c)^4 - 1450890*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + ta
n(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(
1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^4 - 1450890*a*b*arctan((tan(1/2*d*x)
*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan
(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^4 + 27847680*a^2*tan
(1/2*d*x)^3*tan(1/2*c)^4 - 25159680*b^2*tan(1/2*d*x)^3*tan(1/2*c)^4 + 80532
48*a^2*tan(1/2*d*x)^2*tan(1/2*c)^5 - 11425792*b^2*tan(1/2*d*x)^2*tan(1/2*c)
^5 - 205800*pi*a*b*tan(1/2*c)^6 - 66150*a*b*arctan((tan(1/2*d*x)*tan(1/2*c)
 + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) +
 tan(1/2*c) + 1))*tan(1/2*c)^6 - 66150*a*b*arctan((tan(1/2*d*x)*tan(1/2*c)
 - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) -
tan(1/2*c) + 1))*tan(1/2*c)^6 - 345450*a*b*arctan((tan(1/2*d*x)*tan(1/2*c)
+ tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) -
tan(1/2*c) - 1))*tan(1/2*c)^6 - 345450*a*b*arctan((tan(1/2*d*x)*tan(1/2*c)
 - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) +
tan(1/2*c) - 1))*tan(1/2*c)^6 + 1881600*a^2*tan(1/2*d*x)*tan(1/2*c)^6 - 215
0400*b^2*tan(1/2*d*x)*tan(1/2*c)^6 - 325632*a^2*tan(1/2*c)^7 - 233472*b^2*t
an(1/2*c)^7 + 19845*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)
^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d
*x)^4 + 19845*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan
(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^4
+ 176400*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/
2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4 - 268800*a*b*tan(1/2*d
*x)^6 - 3225600*a*b*tan(1/2*d*x)^5*tan(1/2*c) + 46305*pi*a*b*sgn(tan(1/2*d*
x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*
c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 46305*pi*a*b*sgn(tan
(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - t
an(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 411600*pi*a*b
*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*
c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 14031360*a*b*tan(1/2*d
*x)^4*tan(1/2*c)^2 - 24944640*a*b*tan(1/2*d*x)^3*tan(1/2*c)^3 + 19845*pi*a*
b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d
*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*c)^4 + 19845*pi*a*b*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*c)^4 + 176400*pi*a*b*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c
)^2 + 1)*tan(1/2*c)^4 - 14031360*a*b*tan(1/2*d*x)^2*tan(1/2*c)^4 - 3225600*
a*b*tan(1/2*d*x)*tan(1/2*c)^5 - 268800*a*b*tan(1/2*c)^6 + 6615*pi*a*b*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
 tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1
/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*
tan(1/2*d*x)^2 + 6615*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*
x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan
(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + t
an(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2 - 123480*pi*a*b*tan(1/2*d*

x)^4 - 39690*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c
) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d
*x)^4 - 39690*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*
c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*
d*x)^4 - 207270*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/
2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/
2*d*x)^4 - 207270*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(
1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(
1/2*d*x)^4 - 462336*a^2*tan(1/2*d*x)^5 + 57344*b^2*tan(1/2*d*x)^5 - 1128960
*a^2*tan(1/2*d*x)^4*tan(1/2*c) + 1075200*b^2*tan(1/2*d*x)^4*tan(1/2*c) + 66
15*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + t
an(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2
*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan
(1/2*d*x) - 1)*tan(1/2*c)^2 + 6615*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 -
 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c)
 - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1
/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^2 - 288120*pi*a*b
*tan(1/2*d*x)^2*tan(1/2*c)^2 - 92610*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) +
tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + ta
n(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 92610*a*b*arctan((tan(1/2*d*x)
*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan
(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 483630*a*b*arcta
n((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*t
an(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4
83630*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/
(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2*t
an(1/2*c)^2 - 5268480*a^2*tan(1/2*d*x)^3*tan(1/2*c)^2 + 2938880*b^2*tan(1/2
*d*x)^3*tan(1/2*c)^2 - 5268480*a^2*tan(1/2*d*x)^2*tan(1/2*c)^3 + 2938880*b^
2*tan(1/2*d*x)^2*tan(1/2*c)^3 - 123480*pi*a*b*tan(1/2*c)^4 - 39690*a*b*arct
an((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*
tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*c)^4 - 39690*a*b*arcta
n((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*t
an(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*c)^4 - 207270*a*b*arcta
n((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*t
an(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*c)^4 - 207270*a*b*arcta
n((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*t
an(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^4 - 1128960*a^2*tan(
1/2*d*x)*tan(1/2*c)^4 + 1075200*b^2*tan(1/2*d*x)*tan(1/2*c)^4 - 462336*a^2*
tan(1/2*c)^5 + 57344*b^2*tan(1/2*c)^5 + 6615*pi*a*b*sgn(tan(1/2*d*x)^2*tan(
1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*
tan(1/2*c) - 1)*tan(1/2*d*x)^2 + 6615*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2
*c) - 1)*tan(1/2*d*x)^2 + 58800*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - ta
n(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2
 + 161280*a*b*tan(1/2*d*x)^4 + 1290240*a*b*tan(1/2*d*x)^3*tan(1/2*c) + 6615

*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan
(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*tan(1/2*c)^2 + 6615*pi*a*b*s
gn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)
^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*tan(1/2*c)^2 + 58800*pi*a*b*sgn(tan(1
/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1
/2*c)^2 + 1)*tan(1/2*c)^2 + 2956800*a*b*tan(1/2*d*x)^2*tan(1/2*c)^2 + 12902
40*a*b*tan(1/2*d*x)*tan(1/2*c)^3 + 161280*a*b*tan(1/2*c)^4 + 945*pi*a*b*sgn
(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2
 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan
(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1
) + 945*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c
) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*ta
n(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 -
2*tan(1/2*d*x) - 1) - 41160*pi*a*b*tan(1/2*d*x)^2 - 13230*a*b*arctan((tan(1
/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c
) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2 - 13230*a*b*arctan((tan(
1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*
c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2 - 69090*a*b*arctan((tan
(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2
*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2 - 69090*a*b*arctan((ta
n(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/
2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2 - 107520*a^2*tan(1/2*
d*x)^3 - 71680*b^2*tan(1/2*d*x)^3 + 376320*a^2*tan(1/2*d*x)^2*tan(1/2*c) -
215040*b^2*tan(1/2*d*x)^2*tan(1/2*c) - 41160*pi*a*b*tan(1/2*c)^2 - 13230*a*
b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2
*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*c)^2 - 13230*a*b
*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*
d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*c)^2 - 69090*a*b*
arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d
*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*c)^2 - 69090*a*b*a
rctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*
x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^2 + 376320*a^2*t
an(1/2*d*x)*tan(1/2*c)^2 - 215040*b^2*tan(1/2*d*x)*tan(1/2*c)^2 - 107520*a^
2*tan(1/2*c)^3 - 71680*b^2*tan(1/2*c)^3 + 945*pi*a*b*sgn(tan(1/2*d*x)^2*tan
(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2
*tan(1/2*c) - 1) + 945*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d
*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1) + 8400
*pi*a*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*t
an(1/2*c) - tan(1/2*c)^2 + 1) - 53760*a*b*tan(1/2*d*x)^2 - 215040*a*b*tan(1
/2*d*x)*tan(1/2*c) - 53760*a*b*tan(1/2*c)^2 - 5880*pi*a*b - 1890*a*b*arctan
((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*ta
n(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)) - 1890*a*b*arctan((tan(1/2*d*x)*
tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(
1/2*d*x) - tan(1/2*c) + 1)) - 9870*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) + ta
n(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(

```
1/2*c) - 1)) - 9870*a*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - ta
n(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)) -
53760*a^2*tan(1/2*d*x) - 53760*a^2*tan(1/2*c) + 7680*a*b)/(d*tan(1/2*d*x)^1
4*tan(1/2*c)^14 + 7*d*tan(1/2*d*x)^14*tan(1/2*c)^12 + 7*d*tan(1/2*d*x)^12*t
an(1/2*c)^14 + 21*d*tan(1/2*d*x)^14*tan(1/2*c)^10 + 49*d*tan(1/2*d*x)^12*ta
n(1/2*c)^12 + 21*d*tan(1/2*d*x)^10*tan(1/2*c)^14 + 35*d*tan(1/2*d*x)^14*tan
(1/2*c)^8 + 147*d*tan(1/2*d*x)^12*tan(1/2*c)^10 + 147*d*tan(1/2*d*x)^10*tan
(1/2*c)^12 + 35*d*tan(1/2*d*x)^8*tan(1/2*c)^14 + 35*d*tan(1/2*d*x)^14*tan(1
/2*c)^6 + 245*d*tan(1/2*d*x)^12*tan(1/2*c)^8 + 441*d*tan(1/2*d*x)^10*tan(1/
2*c)^10 + 245*d*tan(1/2*d*x)^8*tan(1/2*c)^12 + 35*d*tan(1/2*d*x)^6*tan(1/2*
c)^14 + 21*d*tan(1/2*d*x)^14*tan(1/2*c)^4 + 245*d*tan(1/2*d*x)^12*tan(1/2*c
)^6 + 735*d*tan(1/2*d*x)^10*tan(1/2*c)^8 + 735*d*tan(1/2*d*x)^8*tan(1/2*c)^
10 + 245*d*tan(1/2*d*x)^6*tan(1/2*c)^12 + 21*d*tan(1/2*d*x)^4*tan(1/2*c)^14
 + 7*d*tan(1/2*d*x)^14*tan(1/2*c)^2 + 147*d*tan(1/2*d*x)^12*tan(1/2*c)^4 +
735*d*tan(1/2*d*x)^10*tan(1/2*c)^6 + 1225*d*tan(1/2*d*x)^8*tan(1/2*c)^8 + 7
35*d*tan(1/2*d*x)^6*tan(1/2*c)^10 + 147*d*tan(1/2*d*x)^4*tan(1/2*c)^12 + 7*
d*tan(1/2*d*x)^2*tan(1/2*c)^14 + d*tan(1/2*d*x)^14 + 49*d*tan(1/2*d*x)^12*t
an(1/2*c)^2 + 441*d*tan(1/2*d*x)^10*tan(1/2*c)^4 + 1225*d*tan(1/2*d*x)^8*ta
n(1/2*c)^6 + 1225*d*tan(1/2*d*x)^6*tan(1/2*c)^8 + 441*d*tan(1/2*d*x)^4*tan(
1/2*c)^10 + 49*d*tan(1/2*d*x)^2*tan(1/2*c)^12 + d*tan(1/2*c)^14 + 7*d*tan(1
/2*d*x)^12 + 147*d*tan(1/2*d*x)^10*tan(1/2*c)^2 + 735*d*tan(1/2*d*x)^8*tan(
1/2*c)^4 + 1225*d*tan(1/2*d*x)^6*tan(1/2*c)^6 + 735*d*tan(1/2*d*x)^4*tan(1/
2*c)^8 + 147*d*tan(1/2*d*x)^2*tan(1/2*c)^10 + 7*d*tan(1/2*c)^12 + 21*d*tan(
1/2*d*x)^10 + 245*d*tan(1/2*d*x)^8*tan(1/2*c)^2 + 735*d*tan(1/2*d*x)^6*tan(
1/2*c)^4 + 735*d*tan(1/2*d*x)^4*tan(1/2*c)^6 + 245*d*tan(1/2*d*x)^2*tan(1/2
*c)^8 + 21*d*tan(1/2*c)^10 + 35*d*tan(1/2*d*x)^8 + 245*d*tan(1/2*d*x)^6*tan
(1/2*c)^2 + 441*d*tan(1/2*d*x)^4*tan(1/2*c)^4 + 245*d*tan(1/2*d*x)^2*tan(1/
2*c)^6 + 35*d*tan(1/2*c)^8 + 35*d*tan(1/2*d*x)^6 + 147*d*tan(1/2*d*x)^4*tan
(1/2*c)^2 + 147*d*tan(1/2*d*x)^2*tan(1/2*c)^4 + 35*d*tan(1/2*c)^6 + 21*d*ta
n(1/2*d*x)^4 + 49*d*tan(1/2*d*x)^2*tan(1/2*c)^2 + 21*d*tan(1/2*c)^4 + 7*d*t
an(1/2*d*x)^2 + 7*d*tan(1/2*c)^2 + d)
```

Mupad [B] (verification not implemented)

Time = 4.66 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.28

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^2\,dx=\frac{16\,a^2\,\sin\left(c+d\,x\right)}{35\,d}+\frac{8\,b^2\,\sin\left(c+d\,x\right)}{105\,d}+\frac{8\,a^2\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{35\,d}+\frac{6\,a^2\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)}{35\,d}+\frac{a^2\cos\left(c+d\,x\right)^6\,\sin\left(c+d\,x\right)}{7\,d}+\frac{4\,b^2\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{105\,d}+\frac{b^2\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)}{35\,d}-\frac{b^2\cos\left(c+d\,x\right)^6\,\sin\left(c+d\,x\right)}{7\,d}-\frac{2\,a\,b\cos\left(c+d\,x\right)^7}{7\,d}$$

```
[In] int(cos(c + d*x)^7*(a + b*tan(c + d*x))^2,x)
```

```
[Out] (16*a^2*sin(c + d*x))/(35*d) + (8*b^2*sin(c + d*x))/(105*d) + (8*a^2*cos(c
+ d*x)^2*sin(c + d*x))/(35*d) + (6*a^2*cos(c + d*x)^4*sin(c + d*x))/(35*d)
+ (a^2*cos(c + d*x)^6*sin(c + d*x))/(7*d) + (4*b^2*cos(c + d*x)^2*sin(c + d
*x))/(105*d) + (b^2*cos(c + d*x)^4*sin(c + d*x))/(35*d) - (b^2*cos(c + d*x)
^6*sin(c + d*x))/(7*d) - (2*a*b*cos(c + d*x)^7)/(7*d)
```

3.531 $\int \sec^8(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 194

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{3a^2b\sec^8(c+dx)}{8d} + \frac{a^3\tan(c+dx)}{d} + \frac{a(a^2+b^2)\tan^3(c+dx)}{d} + \frac{b^3\tan^4(c+dx)}{4d} + \frac{3a(a^2+3b^2)\tan^5(c+dx)}{5d} + \frac{b^3\tan^6(c+dx)}{2d} + \frac{a(a^2+9b^2)\tan^7(c+dx)}{7d} + \frac{3b^3\tan^8(c+dx)}{8d} + \frac{ab^2\tan^9(c+dx)}{3d} + \frac{b^3\tan^{10}(c+dx)}{10d}$$

```
[Out] 3/8*a^2*b*sec(d*x+c)^8/d+a^3*tan(d*x+c)/d+a*(a^2+b^2)*tan(d*x+c)^3/d+1/4*b^
3*tan(d*x+c)^4/d+3/5*a*(a^2+3*b^2)*tan(d*x+c)^5/d+1/2*b^3*tan(d*x+c)^6/d+1/
7*a*(a^2+9*b^2)*tan(d*x+c)^7/d+3/8*b^3*tan(d*x+c)^8/d+1/3*a*b^2*tan(d*x+c)^
9/d+1/10*b^3*tan(d*x+c)^10/d
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used

= {3587, 710, 1824}

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{a^3\tan(c+dx)}{d} + \frac{a(a^2+9b^2)\tan^7(c+dx)}{7d} + \frac{3a(a^2+3b^2)\tan^5(c+dx)}{5d} + \frac{a(a^2+b^2)\tan^3(c+dx)}{d} + \frac{3a^2b\sec^8(c+dx)}{8d} + \frac{ab^2\tan^9(c+dx)}{3d} + \frac{b^3\tan^{10}(c+dx)}{10d} + \frac{3b^3\tan^8(c+dx)}{8d} + \frac{b^3\tan^6(c+dx)}{2d} + \frac{b^3\tan^4(c+dx)}{4d}$$

```
[In] Int[Sec[c + d*x]^8*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a^2*b*Sec[c + d*x]^8)/(8*d) + (a^3*Tan[c + d*x])/d + (a*(a^2 + b^2)*Tan[
c + d*x]^3)/d + (b^3*Tan[c + d*x]^4)/(4*d) + (3*a*(a^2 + 3*b^2)*Tan[c + d*x
]^5)/(5*d) + (b^3*Tan[c + d*x]^6)/(2*d) + (a*(a^2 + 9*b^2)*Tan[c + d*x]^7)/
(7*d) + (3*b^3*Tan[c + d*x]^8)/(8*d) + (a*b^2*Tan[c + d*x]^9)/(3*d) + (b^3*
Tan[c + d*x]^10)/(10*d)
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*m*d^(m - 1)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Int[((d + e*x)^m -
e*m*d^(m - 1)*x)*(a + c*x^2)^p, x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
 a*e^2, 0] && IGtQ[p, 1] && IGtQ[m, 0] && LeQ[m, p]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (a+x)^3 \left(1+\frac{x^2}{b^2}\right)^3 dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{3a^2b\sec^8(c+dx)}{8d} + \frac{\text{Subst}\left(\int \left(1+\frac{x^2}{b^2}\right)^3 \left(-3a^2x+(a+x)^3\right) dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{3a^2b\sec^8(c+dx)}{8d} + \frac{\text{Subst}\left(\int \left(a^3+\frac{3a(a^2+b^2)x^2}{b^2}+x^3+\frac{3a(a^2+3b^2)x^4}{b^4}+\frac{3x^5}{b^2}+\frac{a(a^2+9b^2)x^6}{b^6}+\frac{3x^7}{b^4}+\frac{3ax^8}{b^6}+\frac{x^9}{b^6}\right) dx, x, b\tan\right.}{bd}$$

$$= \frac{3a^2b\sec^8(c+dx)}{8d} + \frac{a^3\tan(c+dx)}{d} + \frac{a(a^2+b^2)\tan^3(c+dx)}{d} + \frac{b^3\tan^4(c+dx)}{4d} + \frac{3a(a^2+3b^2)\tan^5(c+dx)}{5d} + \frac{b^3\tan^6(c+dx)}{2d} + \frac{a(a^2+9b^2)\tan^7(c+dx)}{7d} + \frac{3b^3\tan^8(c+dx)}{8d} + \frac{ab^2\tan^9(c+dx)}{3d} + \frac{b^3\tan^{10}(c+dx)}{10d}$$

Mathematica [A] (verified)

Time = 2.25 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.91

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{\frac{1}{4}(a^2+b^2)^3(a+b\tan(c+dx))^4 - \frac{6}{5}a(a^2+b^2)^2(a+b\tan(c+dx))^5 + \frac{1}{2}(a^2+b^2)(5a^2+b^2)(a+b\tan(c+\right.}{}$$

```
[In] Integrate[Sec[c + d*x]^8*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (((a^2 + b^2)^3*(a + b*Tan[c + d*x])^4)/4 - (6*a*(a^2 + b^2)^2*(a + b*Tan[c
 + d*x])^5)/5 + ((a^2 + b^2)*(5*a^2 + b^2)*(a + b*Tan[c + d*x])^6)/2 - (4*a
*(5*a^2 + 3*b^2)*(a + b*Tan[c + d*x])^7)/7 + (3*(5*a^2 + b^2)*(a + b*Tan[c
+ d*x])^8)/8 - (2*a*(a + b*Tan[c + d*x])^9)/3 + (a + b*Tan[c + d*x])^10/10)
/(b^7*d)
```

Maple [A] (verified)

Time = 191.80 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{-a^3\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx+c)\right)}{35}-\frac{8\left(\sec^2(dx+c)\right)}{35}\right)\tan(dx+c)+\frac{3a^2b}{8\cos(dx+c)^8}+3a\,b^2\left(\frac{\sin^3(dx+c)}{9\cos(dx+c)^9}+\frac{2\left(\sin^3(dx+c)\right)}{21\cos(dx+c)^7}\right.}{d}$
default	$\frac{-a^3\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx+c)\right)}{35}-\frac{8\left(\sec^2(dx+c)\right)}{35}\right)\tan(dx+c)+\frac{3a^2b}{8\cos(dx+c)^8}+3a\,b^2\left(\frac{\sin^3(dx+c)}{9\cos(dx+c)^9}+\frac{2\left(\sin^3(dx+c)\right)}{21\cos(dx+c)^7}\right.}{d}$
risch	$-\frac{32\left(-30ia^3e^{2i(dx+c)}-360ia^3e^{6i(dx+c)}-315a^2b\,e^{12i(dx+c)}+105b^3e^{12i(dx+c)}-3ia^3+ia\,b^2-630a^2b\,e^{10i(dx+c)}-126b^3e^{10i}\right.}{}$

[In] int(sec(d*x+c)^8*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(-a^3*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan(d*x+c)+3/8*a^2*b/cos(d*x+c)^8+3*a*b^2*(1/9*sin(d*x+c)^3/cos(d*x+c)^9+2/21*sin(d*x+c)^3/cos(d*x+c)^7+8/105*sin(d*x+c)^3/cos(d*x+c)^5+16/315*sin(d*x+c)^3/cos(d*x+c)^3)+b^3*(1/10*sin(d*x+c)^4/cos(d*x+c)^10+3/40*sin(d*x+c)^4/cos(d*x+c)^8+1/20*sin(d*x+c)^4/cos(d*x+c)^6+1/40*sin(d*x+c)^4/cos(d*x+c)^4))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.77

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{84\,b^3+105\,(3\,a^2b-b^3)\cos{(dx+c)}^2+8\,\left(16\,(3\,a^3-ab^2)\cos{(dx+c)}^9+8\,(3\,a^3-ab^2)\cos{(dx+c)}^7+6\,(3\,a^3\right.}{840\,d\cos{(dx+c)}^{10}}$$

[In] integrate(sec(d*x+c)^8*(a+b*tan(d*x+c))^3,x, algorithm="fricas")

[Out] 1/840*(84*b^3 + 105*(3*a^2*b - b^3)*cos(d*x + c)^2 + 8*(16*(3*a^3 - a*b^2)*cos(d*x + c)^9 + 8*(3*a^3 - a*b^2)*cos(d*x + c)^7 + 6*(3*a^3 - a*b^2)*cos(d*x + c)^5 + 35*a*b^2*cos(d*x + c) + 5*(3*a^3 - a*b^2)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^10)

Sympy [F]

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^3\,dx = \int (a+b\tan(c+dx))^3\sec^8(c+dx)\,dx$$

```
[In] integrate(sec(d*x+c)**8*(a+b*tan(d*x+c))**3,x)
[Out] Integral((a + b*tan(c + d*x))**3*sec(c + d*x)**8, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.91

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{84\,b^3\tan(dx+c)^{10} + 280\,ab^2\tan(dx+c)^9 + 315\,(a^2b+b^3)\tan(dx+c)^8 + 120\,(a^3+9\,ab^2)\tan(dx+c)^7}{}$$

```
[In] integrate(sec(d*x+c)^8*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
[Out] 1/840*(84*b^3*tan(d*x + c)^10 + 280*a*b^2*tan(d*x + c)^9 + 315*(a^2*b + b^3
)*tan(d*x + c)^8 + 120*(a^3 + 9*a*b^2)*tan(d*x + c)^7 + 420*(3*a^2*b + b^3)
*tan(d*x + c)^6 + 504*(a^3 + 3*a*b^2)*tan(d*x + c)^5 + 1260*a^2*b*tan(d*x +
 c)^2 + 210*(9*a^2*b + b^3)*tan(d*x + c)^4 + 840*a^3*tan(d*x + c) + 840*(a^
3 + a*b^2)*tan(d*x + c)^3)/d
```

Giac [A] (verification not implemented)

none

Time = 0.82 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.13

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{84\,b^3\tan(dx+c)^{10} + 280\,ab^2\tan(dx+c)^9 + 315\,a^2b\tan(dx+c)^8 + 315\,b^3\tan(dx+c)^8 + 120\,a^3\tan(dx}{}$$

```
[In] integrate(sec(d*x+c)^8*(a+b*tan(d*x+c))^3,x, algorithm="giac")
[Out] 1/840*(84*b^3*tan(d*x + c)^10 + 280*a*b^2*tan(d*x + c)^9 + 315*a^2*b*tan(d*
x + c)^8 + 315*b^3*tan(d*x + c)^8 + 120*a^3*tan(d*x + c)^7 + 1080*a*b^2*tan
(d*x + c)^7 + 1260*a^2*b*tan(d*x + c)^6 + 420*b^3*tan(d*x + c)^6 + 504*a^3*
tan(d*x + c)^5 + 1512*a*b^2*tan(d*x + c)^5 + 1890*a^2*b*tan(d*x + c)^4 + 21
0*b^3*tan(d*x + c)^4 + 840*a^3*tan(d*x + c)^3 + 840*a*b^2*tan(d*x + c)^3 +
1260*a^2*b*tan(d*x + c)^2 + 840*a^3*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 4.11 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.90

$$\int \sec^8(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$=\frac{\tan(c+d\,x)^5\left(\frac{3\,a^3}{5}+\frac{9\,a\,b^2}{5}\right)+\tan(c+d\,x)^7\left(\frac{a^3}{7}+\frac{9\,a\,b^2}{7}\right)+\tan(c+d\,x)^6\left(\frac{3\,a^2\,b}{2}+\frac{b^3}{2}\right)+\tan(c+d\,x)^4\left(\frac{9\,a^2\,b}{4}\right.}{}$$

```
[In] int((a + b*tan(c + d*x))^3/cos(c + d*x)^8,x)
```

```
[Out] (tan(c + d*x)^5*((9*a*b^2)/5 + (3*a^3)/5) + tan(c + d*x)^7*((9*a*b^2)/7 + a
^3/7) + tan(c + d*x)^6*((3*a^2*b)/2 + b^3/2) + tan(c + d*x)^4*((9*a^2*b)/4
+ b^3/4) + a^3*tan(c + d*x) + (b^3*tan(c + d*x)^10)/10 + (3*a^2*b*tan(c + d
*x)^2)/2 + (a*b^2*tan(c + d*x)^9)/3 + a*tan(c + d*x)^3*(a^2 + b^2) + (3*b*t
an(c + d*x)^8*(a^2 + b^2))/8)/d
```

3.532 $\int \sec^6(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 138

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{(a^2+b^2)^2\,(a+b\tan(c+dx))^4}{4b^5d} - \frac{4a(a^2+b^2)\,(a+b\tan(c+dx))^5}{5b^5d} + \frac{(3a^2+b^2)\,(a+b\tan(c+dx))^6}{3b^5d} - \frac{4a(a+b\tan(c+dx))^7}{7b^5d} + \frac{(a+b\tan(c+dx))^8}{8b^5d}$$

```
[Out] 1/4*(a^2+b^2)^2*(a+b*tan(d*x+c))^4/b^5/d-4/5*a*(a^2+b^2)*(a+b*tan(d*x+c))^5
/b^5/d+1/3*(3*a^2+b^2)*(a+b*tan(d*x+c))^6/b^5/d-4/7*a*(a+b*tan(d*x+c))^7/b^
5/d+1/8*(a+b*tan(d*x+c))^8/b^5/d
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3587, 711}

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{(3a^2+b^2)\,(a+b\tan(c+dx))^6}{3b^5d} - \frac{4a(a^2+b^2)\,(a+b\tan(c+dx))^5}{5b^5d} + \frac{(a^2+b^2)^2\,(a+b\tan(c+dx))^4}{4b^5d} + \frac{(a+b\tan(c+dx))^8}{8b^5d} - \frac{4a(a+b\tan(c+dx))^7}{7b^5d}$$

```
[In] Int[Sec[c + d*x]^6*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] ((a^2 + b^2)^2*(a + b*Tan[c + d*x])^4)/(4*b^5*d) - (4*a*(a^2 + b^2)*(a + b*
Tan[c + d*x])^5)/(5*b^5*d) + ((3*a^2 + b^2)*(a + b*Tan[c + d*x])^6)/(3*b^5*
d) - (4*a*(a + b*Tan[c + d*x])^7)/(7*b^5*d) + (a + b*Tan[c + d*x])^8/(8*b^5
*d)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (a+x)^3\left(1+\frac{x^2}{b^2}\right)^2 dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{(a^2+b^2)^2(a+x)^3}{b^4} - \frac{4a(a^2+b^2)(a+x)^4}{b^4} + \frac{2(3a^2+b^2)(a+x)^5}{b^4} - \frac{4a(a+x)^6}{b^4} + \frac{(a+x)^7}{b^4}\right) dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{(a^2+b^2)^2(a+b\tan(c+dx))^4}{4b^5d} - \frac{4a(a^2+b^2)(a+b\tan(c+dx))^5}{5b^5d} + \frac{(3a^2+b^2)(a+b\tan(c+dx))^6}{3b^5d} - \frac{4a(a+b\tan(c+dx))^7}{7b^5d} + \frac{(a+b\tan(c+dx))^8}{8b^5d}$$

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.83

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{\frac{1}{4}(a^2+b^2)^2(a+b\tan(c+dx))^4 - \frac{4}{5}a(a^2+b^2)(a+b\tan(c+dx))^5 + \frac{1}{3}(3a^2+b^2)(a+b\tan(c+dx))^6 - \frac{4}{7}}{b^5d}$$

```
[In] Integrate[Sec[c + d*x]^6*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (((a^2 + b^2)^2*(a + b*Tan[c + d*x])^4)/4 - (4*a*(a^2 + b^2)*(a + b*Tan[c +
 d*x])^5)/5 + ((3*a^2 + b^2)*(a + b*Tan[c + d*x])^6)/3 - (4*a*(a + b*Tan[c
+ d*x])^7)/7 + (a + b*Tan[c + d*x])^8/8)/(b^5*d)
```

Maple [A] (verified)

Time = 69.61 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{-a^3\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+\frac{a^2b}{2\cos(dx+c)^6}+3a\,b^2\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8\left(\sin^3(dx+c\right.}{105\cos(dx+c}\right.}{d}$
default	$\frac{-a^3\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+\frac{a^2b}{2\cos(dx+c)^6}+3a\,b^2\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8\left(\sin^3(dx+c\right.}{105\cos(dx+c}\right.}{d}$
risch	$-\frac{16\left(-56ia^3\mathrm{e}^{2i(dx+c)}+24ia\,b^2\mathrm{e}^{2i(dx+c)}-210a^2b\,\mathrm{e}^{10i(dx+c)}+70b^3\mathrm{e}^{10i(dx+c)}-322ia^3\mathrm{e}^{6i(dx+c)}-7ia^3-420a^2b\,\mathrm{e}^{8i(dx+c)}\right.}{}$

```
[In] int(sec(d*x+c)^6*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^3*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+1/2*a^2*b/c
os(d*x+c)^6+3*a*b^2*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d*
x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)+b^3*(1/8*sin(d*x+c)^4/cos(d*x+c)^8+
1/12*sin(d*x+c)^4/cos(d*x+c)^6+1/24*sin(d*x+c)^4/cos(d*x+c)^4))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.93

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$=\frac{105\,b^3+140\,(3\,a^2b-b^3)\cos{(dx+c)}^2+8\,\left(8\,(7\,a^3-3\,ab^2)\cos{(dx+c)}^7+4\,(7\,a^3-3\,ab^2)\cos{(dx+c)}^5+\right.}{840\,d\cos{(dx+c)}^8}$$

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/840*(105*b^3 + 140*(3*a^2*b - b^3)*cos(d*x + c)^2 + 8*(8*(7*a^3 - 3*a*b^2
)*cos(d*x + c)^7 + 4*(7*a^3 - 3*a*b^2)*cos(d*x + c)^5 + 45*a*b^2*cos(d*x +
c) + 3*(7*a^3 - 3*a*b^2)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^8)
```

Sympy [F]

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^3\,dx = \int (a+b\tan{(c+dx)})^3\sec^6{(c+dx)}\,dx$$

```
[In] integrate(sec(d*x+c)**6*(a+b*tan(d*x+c))**3,x)
[Out] Integral((a + b*tan(c + d*x))**3*sec(c + d*x)**6, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.03

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{105\,b^3\tan{(dx+c)}^8+360\,ab^2\tan{(dx+c)}^7+140\,(3\,a^2b+2\,b^3)\tan{(dx+c)}^6+168\,(a^3+6\,ab^2)\tan{(dx+c}$$

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
[Out] 1/840*(105*b^3*tan(d*x + c)^8 + 360*a*b^2*tan(d*x + c)^7 + 140*(3*a^2*b + 2
*b^3)*tan(d*x + c)^6 + 168*(a^3 + 6*a*b^2)*tan(d*x + c)^5 + 1260*a^2*b*tan(
d*x + c)^2 + 210*(6*a^2*b + b^3)*tan(d*x + c)^4 + 840*a^3*tan(d*x + c) + 28
0*(2*a^3 + 3*a*b^2)*tan(d*x + c)^3)/d
```

Giac [A] (verification not implemented)

none

Time = 0.79 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.20

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{105\,b^3\tan{(dx+c)}^8+360\,ab^2\tan{(dx+c)}^7+420\,a^2b\tan{(dx+c)}^6+280\,b^3\tan{(dx+c)}^6+168\,a^3\tan{(dx}$$

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^3,x, algorithm="giac")
[Out] 1/840*(105*b^3*tan(d*x + c)^8 + 360*a*b^2*tan(d*x + c)^7 + 420*a^2*b*tan(d*
x + c)^6 + 280*b^3*tan(d*x + c)^6 + 168*a^3*tan(d*x + c)^5 + 1008*a*b^2*tan
(d*x + c)^5 + 1260*a^2*b*tan(d*x + c)^4 + 210*b^3*tan(d*x + c)^4 + 560*a^3*
tan(d*x + c)^3 + 840*a*b^2*tan(d*x + c)^3 + 1260*a^2*b*tan(d*x + c)^2 + 840
*a^3*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 4.86 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.01

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{\tan(c+d\,x)^3\left(\frac{2\,a^3}{3}+a\,b^2\right)+\tan(c+d\,x)^5\left(\frac{a^3}{5}+\frac{6\,a\,b^2}{5}\right)+\tan(c+d\,x)^6\left(\frac{a^2\,b}{2}+\frac{b^3}{3}\right)+\tan(c+d\,x)^4\left(\frac{3\,a^2\,b}{2}\right.}{d}$$

```
[In] int((a + b*tan(c + d*x))^3/cos(c + d*x)^6,x)
[Out] (tan(c + d*x)^3*(a*b^2 + (2*a^3)/3) + tan(c + d*x)^5*((6*a*b^2)/5 + a^3/5)
+ tan(c + d*x)^6*((a^2*b)/2 + b^3/3) + tan(c + d*x)^4*((3*a^2*b)/2 + b^3/4)
 + a^3*tan(c + d*x) + (b^3*tan(c + d*x)^8)/8 + (3*a^2*b*tan(c + d*x)^2)/2 +
 (3*a*b^2*tan(c + d*x)^7)/7)/d
```

3.533 $\int \sec^4(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 75

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^3\,dx=\frac{(a^2+b^2)\,(a+b\tan(c+dx))^4}{4b^3d}-\frac{2a(a+b\tan(c+dx))^5}{5b^3d}+\frac{(a+b\tan(c+dx))^6}{6b^3d}$$

```
[Out] 1/4*(a^2+b^2)*(a+b*tan(d*x+c))^4/b^3/d-2/5*a*(a+b*tan(d*x+c))^5/b^3/d+1/6*(
a+b*tan(d*x+c))^6/b^3/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3587, 711}

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^3\,dx=\frac{(a^2+b^2)\,(a+b\tan(c+dx))^4}{4b^3d}+\frac{(a+b\tan(c+dx))^6}{6b^3d}-\frac{2a(a+b\tan(c+dx))^5}{5b^3d}$$

```
[In] Int[Sec[c + d*x]^4*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] ((a^2 + b^2)*(a + b*Tan[c + d*x])^4)/(4*b^3*d) - (2*a*(a + b*Tan[c + d*x])^
5)/(5*b^3*d) + (a + b*Tan[c + d*x])^6/(6*b^3*d)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
```

```
x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int (a+x)^3 \left(1+\frac{x^2}{b^2}\right) dx, x, b\tan(c+dx)\right)}{bd}\\ &= \frac{\text{Subst}\left(\int \left(\frac{(a^2+b^2)(a+x)^3}{b^2}-\frac{2a(a+x)^4}{b^2}+\frac{(a+x)^5}{b^2}\right) dx, x, b\tan(c+dx)\right)}{bd}\\ &= \frac{(a^2+b^2)(a+b\tan(c+dx))^4}{4b^3d}-\frac{2a(a+b\tan(c+dx))^5}{5b^3d}+\frac{(a+b\tan(c+dx))^6}{6b^3d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.72

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{(a+b\tan(c+dx))^4\left(a^2+15b^2-4ab\tan(c+dx)+10b^2\tan^2(c+dx)\right)}{60b^3d}$$

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Tan[c + d*x])^3,x]
[Out] ((a + b*Tan[c + d*x])^4*(a^2 + 15*b^2 - 4*a*b*Tan[c + d*x] + 10*b^2*Tan[c +
 d*x]^2))/(60*b^3*d)
```

Maple [A] (verified)

Time = 16.39 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.69

method	result
derivativedivides	$\frac{-a^3\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{3a^2b}{4\cos(dx+c)^4}+3a\,b^2\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)+b^3\left(\frac{\sin^4(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^4(dx+c}{12\cos(dx+c}\right.}{d}$
default	$\frac{-a^3\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{3a^2b}{4\cos(dx+c)^4}+3a\,b^2\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)+b^3\left(\frac{\sin^4(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^4(dx+c}{12\cos(dx+c}\right.}{d}$
risch	$-\frac{4(-15ia^3e^{8i(dx+c)}+45ia\,b^2e^{8i(dx+c)}-45a^2b\,e^{8i(dx+c)}+15b^3e^{8i(dx+c)}-50ia^3e^{6i(dx+c)}+30ia\,b^2e^{6i(dx+c)}-90a^2b\,e^{6i(dx}}{15d(e^{2i(d}}$

[In] int(sec(d*x+c)^4*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(-a^3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+3/4*a^2*b/cos(d*x+c)^4+3*a*b^2 *(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)+b^3*(1/6*si n(d*x+c)^4/cos(d*x+c)^6+1/12*sin(d*x+c)^4/cos(d*x+c)^4))

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.40

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$=\frac{10\,b^3+15\,(3\,a^2b-b^3)\cos{(dx+c)}^2+4\,(2\,(5\,a^3-3\,ab^2)\cos{(dx+c)}^5+9\,ab^2\cos{(dx+c)}+(5\,a^3-3\,ab^2)\,c}{60\,d\cos{(dx+c)}^6}$$

[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="fricas")

[Out] 1/60*(10*b^3 + 15*(3*a^2*b - b^3)*cos(d*x + c)^2 + 4*(2*(5*a^3 - 3*a*b^2)*c os(d*x + c)^5 + 9*a*b^2*cos(d*x + c) + (5*a^3 - 3*a*b^2)*cos(d*x + c)^3)*si n(d*x + c))/(d*cos(d*x + c)^6)

Sympy [F]

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^3\,dx=\int (a+b\tan{(c+dx)})^3\sec^4{(c+dx)}\,dx$$

[In] integrate(sec(d*x+c)**4*(a+b*tan(d*x+c))**3,x)

[Out] Integral((a + b*tan(c + d*x))**3*sec(c + d*x)**4, x)

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.31

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$=\frac{10\,b^3\tan\left(dx+c\right)^6+36\,ab^2\tan\left(dx+c\right)^5+90\,a^2b\tan\left(dx+c\right)^2+15\left(3\,a^2b+b^3\right)\tan\left(dx+c\right)^4+60\,a^3\tan}{60\,d}$$

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(10*b^3*tan(d*x + c)^6 + 36*a*b^2*tan(d*x + c)^5 + 90*a^2*b*tan(d*x +
c)^2 + 15*(3*a^2*b + b^3)*tan(d*x + c)^4 + 60*a^3*tan(d*x + c) + 20*(a^3 +
3*a*b^2)*tan(d*x + c)^3)/d
```

Giac [A] (verification not implemented)

none

Time = 0.75 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.49

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$=\frac{10\,b^3\tan\left(dx+c\right)^6+36\,ab^2\tan\left(dx+c\right)^5+45\,a^2b\tan\left(dx+c\right)^4+15\,b^3\tan\left(dx+c\right)^4+20\,a^3\tan\left(dx+c\right)^3}{60\,d}$$

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(10*b^3*tan(d*x + c)^6 + 36*a*b^2*tan(d*x + c)^5 + 45*a^2*b*tan(d*x +
c)^4 + 15*b^3*tan(d*x + c)^4 + 20*a^3*tan(d*x + c)^3 + 60*a*b^2*tan(d*x + c
)^3 + 90*a^2*b*tan(d*x + c)^2 + 60*a^3*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 4.00 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.29

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$=\frac{\tan(c+d\,x)^3\left(\frac{a^3}{3}+a\,b^2\right)+\tan(c+d\,x)^4\left(\frac{3\,a^2\,b}{4}+\frac{b^3}{4}\right)+a^3\tan(c+d\,x)+\frac{b^3\tan(c+d\,x)^6}{6}+\frac{3\,a^2\,b\tan(c+d\,x)^2}{2}+}{d}$$

```
[In] int((a + b*tan(c + d*x))^3/cos(c + d*x)^4,x)
```

```
[Out] (tan(c + d*x)^3*(a*b^2 + a^3/3) + tan(c + d*x)^4*((3*a^2*b)/4 + b^3/4) + a^
3*tan(c + d*x) + (b^3*tan(c + d*x)^6)/6 + (3*a^2*b*tan(c + d*x)^2)/2 + (3*a
*b^2*tan(c + d*x)^5)/5)/d
```

3.534 $\int \sec^2(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 22

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^3\,dx=\frac{(a+b\tan(c+dx))^4}{4bd}$$

```
[Out] 1/4*(a+b*tan(d*x+c))^4/b/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3587, 32}

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^3\,dx=\frac{(a+b\tan(c+dx))^4}{4bd}$$

```
[In] Int[Sec[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (a + b*Tan[c + d*x])^4/(4*b*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1), 
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (a+x)^3\,dx, x, b\tan(c+dx)\right)}{bd}$$
$$= \frac{(a+b\tan(c+dx))^4}{4bd}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 57 vs. $2(22) = 44$.

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.59

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{\tan(c+dx)\left(4a^3+6a^2b\tan(c+dx)+4ab^2\tan^2(c+dx)+b^3\tan^3(c+dx)\right)}{4d}$$

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (Tan[c + d*x]*(4*a^3 + 6*a^2*b*Tan[c + d*x] + 4*a*b^2*Tan[c + d*x]^2 + b^3*
Tan[c + d*x]^3))/(4*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(20) = 40$.

Time = 4.50 (sec) , antiderivative size = 72, normalized size of antiderivative = 3.27

method	result
derivativedivides	$\frac{\frac{b^3\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}+\frac{a\,b^2\left(\sin^3(dx+c)\right)}{\cos(dx+c)^3}+\frac{3a^2b}{2\cos(dx+c)^2}+a^3\tan(dx+c)}{d}$
default	$\frac{\frac{b^3\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}+\frac{a\,b^2\left(\sin^3(dx+c)\right)}{\cos(dx+c)^3}+\frac{3a^2b}{2\cos(dx+c)^2}+a^3\tan(dx+c)}{d}$
risch	$-\frac{2\left(-ia^3e^{6i(dx+c)}+3ia\,b^2e^{6i(dx+c)}-3a^2b\,e^{6i(dx+c)}+b^3e^{6i(dx+c)}-3ia^3e^{4i(dx+c)}+3ia\,b^2e^{4i(dx+c)}-6a^2b\,e^{4i(dx+c)}-3ia\right.}{d\left(e^{2i(dx+c)}+1\right)^4}$

```
[In] int(sec(d*x+c)^2*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*b^3*sin(d*x+c)^4/cos(d*x+c)^4+a*b^2*sin(d*x+c)^3/cos(d*x+c)^3+3/2*
a^2*b/cos(d*x+c)^2+a^3*tan(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(20) = 40.

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.55

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{b^3+2\,(3\,a^2b-b^3)\cos{(dx+c)}^2+4\,\left(ab^2\cos{(dx+c)}+(a^3-ab^2)\cos{(dx+c)}^3\right)\sin{(dx+c)}}{4\,d\cos{(dx+c)}^4}$$

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
[Out] 1/4*(b^3 + 2*(3*a^2*b - b^3)*cos(d*x + c)^2 + 4*(a*b^2*cos(d*x + c) + (a^3
- a*b^2)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^3\,dx = \int (a+b\tan{(c+dx)})^3\sec^2{(c+dx)}\,dx$$

```
[In] integrate(sec(d*x+c)**2*(a+b*tan(d*x+c))**3,x)
[Out] Integral((a + b*tan(c + d*x))**3*sec(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{(b\tan{(dx+c)}+a)^4}{4\,bd}$$

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
[Out] 1/4*(b*tan(d*x + c) + a)^4/(b*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(20) = 40$.

Time = 0.74 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.59

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{b^3\tan\left(dx+c\right)^4+4\,ab^2\tan\left(dx+c\right)^3+6\,a^2b\tan\left(dx+c\right)^2+4\,a^3\tan\left(dx+c\right)}{4\,d}$$

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="giac")
[Out] 1/4*(b^3*tan(d*x + c)^4 + 4*a*b^2*tan(d*x + c)^3 + 6*a^2*b*tan(d*x + c)^2 +
 4*a^3*tan(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 4.41 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.50

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{a^3\tan(c+d\,x)+\frac{3\,a^2\,b\tan(c+d\,x)^2}{2}+a\,b^2\tan(c+d\,x)^3+\frac{b^3\tan(c+d\,x)^4}{4}}{d}$$

```
[In] int((a + b*tan(c + d*x))^3/cos(c + d*x)^2,x)
[Out] (a^3*tan(c + d*x) + (b^3*tan(c + d*x)^4)/4 + (3*a^2*b*tan(c + d*x)^2)/2 + a
*b^2*tan(c + d*x)^3)/d
```

3.535 $\int \cos^2(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 86

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{1}{2}a\left(a^2+3b^2\right)x-\frac{b^3\log(\cos(c+dx))}{d}-\frac{ab^2\tan(c+dx)}{2d}-\frac{\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))^2}{2d}$$

```
[Out] 1/2*a*(a^2+3*b^2)*x-b^3*ln(cos(d*x+c))/d-1/2*a*b^2*tan(d*x+c)/d-1/2*cos(d*x
+c)^2*(b-a*tan(d*x+c))*(a+b*tan(d*x+c))^2/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3587, 753, 788, 649, 209, 266}

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{1}{2}ax\left(a^2+3b^2\right)-\frac{ab^2\tan(c+dx)}{2d}-\frac{\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))^2}{2d}-\frac{b^3\log(\cos(c+dx))}{d}$$

```
[In] Int[Cos[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (a*(a^2 + 3*b^2)*x)/2 - (b^3*Log[Cos[c + d*x]])/d - (a*b^2*Tan[c + d*x])/(2
*d) - (Cos[c + d*x]^2*(b - a*Tan[c + d*x])*(a + b*Tan[c + d*x])^2)/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 753

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 788

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+x)^3}{\left(1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= -\frac{\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))^2}{2d} + \frac{b\text{Subst}\left(\int \frac{(a+x)\left(2+\frac{a^2}{b^2}-\frac{ax}{b^2}\right)}{1+\frac{x^2}{b^2}}dx, x, b\tan(c+dx)\right)}{2d}$$

$$= -\frac{ab^2\tan(c+dx)}{2d} - \frac{\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))^2}{2d} + \frac{b^3\text{Subst}\left(\int \frac{\frac{a}{b^2}+\frac{a\left(2+\frac{a^2}{b^2}\right)}{b^2}+\frac{2x}{b^2}}{1+\frac{x^2}{b^2}}dx, x, b\tan(c+dx)\right)}{2d}$$

$$= -\frac{ab^2\tan(c+dx)}{2d} - \frac{\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))^2}{2d} + \frac{b\text{Subst}\left(\int \frac{x}{1+\frac{x^2}{b^2}}dx, x, b\tan(c+dx)\right)}{d} + \frac{(a(a^2+3b^2))\,\text{Subst}\left(\int \frac{1}{1+\frac{x^2}{b^2}}dx, x, b\tan(c+dx)\right)}{2bd}$$

$$= \frac{1}{2}a\left(a^2+3b^2\right)x - \frac{b^3\log(\cos(c+dx))}{d} - \frac{ab^2\tan(c+dx)}{2d} - \frac{\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))^2}{2d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 401 vs. $2(86) = 172$.

Time = 0.84 (sec) , antiderivative size = 401, normalized size of antiderivative = 4.66

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$= \frac{5a^4b^2+2a^2b^4-b^6+(-3a^4b^2-2a^2b^4+b^6)\cos(2(c+dx))+2a^2b^4\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)+2b^6\log\left(\sqrt{\right.}\right.}{}$$

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (5*a^4*b^2 + 2*a^2*b^4 - b^6 + (-3*a^4*b^2 - 2*a^2*b^4 + b^6)*Cos[2*(c + d*
x)] + 2*a^2*b^4*Log[Sqrt[-b^2] - b*Tan[c + d*x]] + 2*b^6*Log[Sqrt[-b^2] - b
*Tan[c + d*x]] - a^5*Sqrt[-b^2]*Log[Sqrt[-b^2] - b*Tan[c + d*x]] + 4*a^3*(-
b^2)^(3/2)*Log[Sqrt[-b^2] - b*Tan[c + d*x]] - 3*a*(-b^2)^(5/2)*Log[Sqrt[-b^
2] - b*Tan[c + d*x]] + 2*a^2*b^4*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + 2*b^6*L
og[Sqrt[-b^2] + b*Tan[c + d*x]] + a^5*Sqrt[-b^2]*Log[Sqrt[-b^2] + b*Tan[c +
 d*x]] + 3*a*b^4*Sqrt[-b^2]*Log[Sqrt[-b^2] + b*Tan[c + d*x]] - 4*a^3*(-b^2)
^(3/2)*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + a*b*(a^4 - 2*a^2*b^2 - 3*b^4)*Sin
[2*(c + d*x)])/(4*b*(a^2 + b^2)*d)
```

Maple [A] (verified)

Time = 4.70 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{b^3\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+3a\,b^2\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\frac{3a^2b\left(\cos^2(dx+c)\right)}{2}+a^3\left(\frac{\sin(dx+c)\cos(dx+c)}{2}\right.}{d}$
default	$\frac{b^3\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+3a\,b^2\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\frac{3a^2b\left(\cos^2(dx+c)\right)}{2}+a^3\left(\frac{\sin(dx+c)\cos(dx+c)}{2}\right.}{d}$
risch	$ix\,b^3+\frac{a^3x}{2}+\frac{3xa\,b^2}{2}-\frac{3\,\mathrm{e}^{2i(dx+c)}b\,a^2}{8d}+\frac{\mathrm{e}^{2i(dx+c)}b^3}{8d}-\frac{i\mathrm{e}^{2i(dx+c)}a^3}{8d}+\frac{3i\mathrm{e}^{2i(dx+c)}a\,b^2}{8d}-\frac{3\,\mathrm{e}^{-2i(dx+c)}b\,a^2}{8d}$

[In] int(cos(d*x+c)^2*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(b^3*(-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))+3*a*b^2*(-1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)-3/2*a^2*b*cos(d*x+c)^2+a^3*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.92

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^3\,dx =$$
$$-\frac{2\,b^3\log\left(-\cos\left(dx+c\right)\right)-\left(a^3+3\,ab^2\right)dx+\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^2-\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)\sin\left(dx\right.}{2\,d}$$

[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="fricas")

[Out] -1/2*(2*b^3*log(-cos(d*x + c)) - (a^3 + 3*a*b^2)*d*x + (3*a^2*b - b^3)*cos(d*x + c)^2 - (a^3 - 3*a*b^2)*cos(d*x + c)*sin(d*x + c))/d

Sympy [F]

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^3\,dx = \int \left(a+b\tan\left(c+dx\right)\right)^3\cos^2\left(c+dx\right)\,dx$$

[In] integrate(cos(d*x+c)**2*(a+b*tan(d*x+c))**3,x)

[Out] Integral((a + b*tan(c + d*x))**3*cos(c + d*x)**2, x)

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{b^3\log\left(\tan(dx+c)^2+1\right)+(a^3+3\,ab^2)(dx+c)-\frac{3\,a^2b-b^3-(a^3-3\,ab^2)\tan(dx+c)}{\tan(dx+c)^2+1}}{2\,d}$$

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
[Out] 1/2*(b^3*log(tan(d*x + c)^2 + 1) + (a^3 + 3*a*b^2)*(d*x + c) - (3*a^2*b - b
^3 - (a^3 - 3*a*b^2)*tan(d*x + c))/(tan(d*x + c)^2 + 1))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 561 vs. 2(81) = 162.

Time = 0.89 (sec) , antiderivative size = 561, normalized size of antiderivative = 6.52

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{2\,a^3dx\tan(dx)^2\tan(c)^2+6\,ab^2dx\tan(dx)^2\tan(c)^2-2\,b^3\log\left(\frac{4\left(\tan(dx)^2\tan(c)^2-2\tan(dx)\tan(c)+1\right)}{\tan(dx)^2\tan(c)^2+\tan(dx)^2+\tan(c)^2+1}\right)\tan(dx)^2}{}$$

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^3,x, algorithm="giac")
[Out] 1/4*(2*a^3*d*x*tan(d*x)^2*tan(c)^2 + 6*a*b^2*d*x*tan(d*x)^2*tan(c)^2 - 2*b^
3*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2
+ tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^2*tan(c)^2 + 2*a^3*d*x*tan(d*x)^2 +
6*a*b^2*d*x*tan(d*x)^2 + 2*a^3*d*x*tan(c)^2 + 6*a*b^2*d*x*tan(c)^2 - 3*a^2*
b*tan(d*x)^2*tan(c)^2 + b^3*tan(d*x)^2*tan(c)^2 - 2*b^3*log(4*(tan(d*x)^2*t
an(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)
^2 + 1))*tan(d*x)^2 - 2*a^3*tan(d*x)^2*tan(c) + 6*a*b^2*tan(d*x)^2*tan(c) -
 2*b^3*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(
c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(c)^2 - 2*a^3*tan(d*x)*tan(c)^2 + 6*a
*b^2*tan(d*x)*tan(c)^2 + 2*a^3*d*x + 6*a*b^2*d*x + 3*a^2*b*tan(d*x)^2 - b^3
*tan(d*x)^2 + 12*a^2*b*tan(d*x)*tan(c) - 4*b^3*tan(d*x)*tan(c) + 3*a^2*b*ta
n(c)^2 - b^3*tan(c)^2 - 2*b^3*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c
) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1)) + 2*a^3*tan(d*x)
- 6*a*b^2*tan(d*x) + 2*a^3*tan(c) - 6*a*b^2*tan(c) - 3*a^2*b + b^3)/(d*tan(
d*x)^2*tan(c)^2 + d*tan(d*x)^2 + d*tan(c)^2 + d)
```

Mupad [B] (verification not implemented)

Time = 4.42 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.64

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{b^3\,\ln\left(\frac{1}{\cos(c+d\,x)^2}\right)}{2\,d}+\frac{b^3\cos\left(c+d\,x\right)^2}{2\,d} +\frac{a^3\,\mathrm{atan}\left(\frac{\sin(c+d\,x)}{\cos(c+d\,x)}\right)}{2\,d}-\frac{3\,a^2\,b\cos\left(c+d\,x\right)^2}{2\,d} +\frac{3\,a\,b^2\,\mathrm{atan}\left(\frac{\sin(c+d\,x)}{\cos(c+d\,x)}\right)}{2\,d} +\frac{a^3\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d} -\frac{3\,a\,b^2\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d}$$

```
[In] int(cos(c + d*x)^2*(a + b*tan(c + d*x))^3,x)
```

```
[Out] (b^3*log(1/cos(c + d*x)^2))/(2*d) + (b^3*cos(c + d*x)^2)/(2*d) + (a^3*atan(
sin(c + d*x)/cos(c + d*x)))/(2*d) - (3*a^2*b*cos(c + d*x)^2)/(2*d) + (3*a*b
^2*atan(sin(c + d*x)/cos(c + d*x)))/(2*d) + (a^3*cos(c + d*x)*sin(c + d*x))
/(2*d) - (3*a*b^2*cos(c + d*x)*sin(c + d*x))/(2*d)
```

3.536 $\int \cos^4(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 84

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{3}{8}a(a^2+b^2)\,x - \frac{3a\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{8d} + \frac{\cos^3(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{4d}$$

```
[Out] 3/8*a*(a^2+b^2)*x-3/8*a*cos(d*x+c)^2*(b-a*tan(d*x+c))*(a+b*tan(d*x+c))/d+1/
4*cos(d*x+c)^3*sin(d*x+c)*(a+b*tan(d*x+c))^3/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3587, 743, 737, 209}

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{3}{8}ax(a^2+b^2) - \frac{3a\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{8d} + \frac{\sin(c+dx)\cos^3(c+dx)(a+b\tan(c+dx))^3}{4d}$$

```
[In] Int[Cos[c + d*x]^4*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a*(a^2 + b^2)*x)/8 - (3*a*Cos[c + d*x]^2*(b - a*Tan[c + d*x])*(a + b*Tan
[c + d*x]))/(8*d) + (Cos[c + d*x]^3*Sin[c + d*x]*(a + b*Tan[c + d*x])^3)/(4
*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 737

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[(2*p + 3)*((c*d^2 + a*e^2)/(2*a*c*(p + 1))), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^m)*(2*c*x)*((a + c*x^2)^(p + 1)/(4*a*c*(p + 1))), x] - Dist[m*(
(2*c*d)/(4*a*c*(p + 1))), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x]
 /; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p +
3, 0] && LtQ[p, -1]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+x)^3}{\left(1+\frac{x^2}{b^2}\right)^3}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\cos^3(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{4d} + \frac{(3a)\text{Subst}\left(\int \frac{(a+x)^2}{\left(1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\tan(c+dx)\right)}{4bd}$$

$$= -\frac{3a\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{8d} + \frac{\cos^3(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{4d} + \frac{(3a(a^2+b^2))\,\text{Subst}\left(\int \frac{1}{1+\frac{x^2}{b^2}}\,dx, x, b\tan(c+dx)\right)}{8bd}$$

$$= \frac{3}{8}a(a^2+b^2)\,x - \frac{3a\cos^2(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))}{8d} + \frac{\cos^3(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{4d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 488 vs. $2(84) = 168$.

Time = 1.22 (sec) , antiderivative size = 488, normalized size of antiderivative = 5.81

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^3\,dx$$

$$= \frac{39a^6b^2+41a^4b^4+21a^2b^6+3b^8-4b^2(a^2+b^2)^2\,(3a^2+b^2)\cos(2(c+dx))+b^2(-3a^2+b^2)\,(a^2+b^2)^2\cos(4(}{}$$

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (39*a^6*b^2 + 41*a^4*b^4 + 21*a^2*b^6 + 3*b^8 - 4*b^2*(a^2 + b^2)^2*(3*a^2
+ b^2)*Cos[2*(c + d*x)] + b^2*(-3*a^2 + b^2)*(a^2 + b^2)^2*Cos[4*(c + d*x)]
 - 6*a^7*Sqrt[-b^2]*Log[Sqrt[-b^2] - b*Tan[c + d*x]] + 18*a^5*(-b^2)^(3/2)*
Log[Sqrt[-b^2] - b*Tan[c + d*x]] + 6*a*b^4*(-b^2)^(3/2)*Log[Sqrt[-b^2] - b*
Tan[c + d*x]] - 18*a^3*(-b^2)^(5/2)*Log[Sqrt[-b^2] - b*Tan[c + d*x]] + 6*a^
7*Sqrt[-b^2]*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + 18*a^3*b^4*Sqrt[-b^2]*Log[S
qrt[-b^2] + b*Tan[c + d*x]] + 6*a*b^6*Sqrt[-b^2]*Log[Sqrt[-b^2] + b*Tan[c +
 d*x]] - 18*a^5*(-b^2)^(3/2)*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + 8*a^7*b*Sin
[2*(c + d*x)] + 16*a^5*b^3*Sin[2*(c + d*x)] + 8*a^3*b^5*Sin[2*(c + d*x)] +
a^7*b*Sin[4*(c + d*x)] - a^5*b^3*Sin[4*(c + d*x)] - 5*a^3*b^5*Sin[4*(c + d*
x)] - 3*a*b^7*Sin[4*(c + d*x)])/(32*b*(a^2 + b^2)^2*d)
```

Maple [A] (verified)

Time = 20.99 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.36

method	result
derivativedivides	$\frac{\frac{b^3(\sin^4(dx+c))}{4}+3a\,b^2\left(-\frac{(\cos^3(dx+c))\sin(dx+c)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{3a^2b(\cos^4(dx+c))}{4}+a^3\left(\frac{(\cos^3(dx+c)+\frac{3\cos}{}}{}\right.}{d}$
default	$\frac{\frac{b^3(\sin^4(dx+c))}{4}+3a\,b^2\left(-\frac{(\cos^3(dx+c))\sin(dx+c)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{3a^2b(\cos^4(dx+c))}{4}+a^3\left(\frac{(\cos^3(dx+c)+\frac{3\cos}{}}{}\right.}{d}$
risch	$\frac{3a^3x}{8}+\frac{3xa\,b^2}{8}-\frac{3b\cos(4dx+4c)a^2}{32d}+\frac{b^3\cos(4dx+4c)}{32d}+\frac{a^3\sin(4dx+4c)}{32d}-\frac{3a\sin(4dx+4c)b^2}{32d}-\frac{3b\cos(2dx+2c)}{8d}$

```
[In] int(cos(d*x+c)^4*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

[Out] 1/d*(1/4*b^3*sin(d*x+c)^4+3*a*b^2*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*sin(d*x+c)*cos(d*x+c)+1/8*d*x+1/8*c)-3/4*a^2*b*cos(d*x+c)^4+a^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.19

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{4\,b^3\cos\left(dx+c\right)^2+2\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^4-3\left(a^3+ab^2\right)dx-\left(2\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^3+3\left(a^3+\right.\right.}{8\,d}$$

[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="fricas")

[Out] -1/8*(4*b^3*cos(d*x + c)^2 + 2*(3*a^2*b - b^3)*cos(d*x + c)^4 - 3*(a^3 + a*b^2)*d*x - (2*(a^3 - 3*a*b^2)*cos(d*x + c)^3 + 3*(a^3 + a*b^2)*cos(d*x + c))*sin(d*x + c))/d

Sympy [F]

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^3\,dx = \int (a+b\tan(c+dx))^3\cos^4(c+dx)\,dx$$

[In] integrate(cos(d*x+c)**4*(a+b*tan(d*x+c))**3,x)

[Out] Integral((a + b*tan(c + d*x))**3*cos(c + d*x)**4, x)

Maxima [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.31

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{3\left(a^3+ab^2\right)(dx+c)-\frac{4\,b^3\tan(dx+c)^2-3\left(a^3+ab^2\right)\tan(dx+c)^3+6\,a^2b+2\,b^3-\left(5\,a^3-3\,ab^2\right)\tan(dx+c)}{\tan(dx+c)^4+2\tan(dx+c)^2+1}}{8\,d}$$

[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="maxima")

[Out] 1/8*(3*(a^3 + a*b^2)*(d*x + c) - (4*b^3*tan(d*x + c)^2 - 3*(a^3 + a*b^2)*tan(d*x + c)^3 + 6*a^2*b + 2*b^3 - (5*a^3 - 3*a*b^2)*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1))/d

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2496 vs. 2(79) = 158.

Time = 12.25 (sec) , antiderivative size = 2496, normalized size of antiderivative = 29.71

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^3\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/64*(9*pi*a*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) +
2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^4 + 24*a^3*d
*x*tan(d*x)^4*tan(c)^4 + 24*a*b^2*d*x*tan(d*x)^4*tan(c)^4 + 9*pi*a*b^2*sgn(
-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x
)^4*tan(c)^4 + 18*pi*a*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2
*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(c)^2
+ 18*pi*a*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*t
an(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^4 + 18*a*b^2*ar
ctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^4*tan(c)^4 - 18*a*
b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(c)^4
+ 48*a^3*d*x*tan(d*x)^4*tan(c)^2 + 48*a*b^2*d*x*tan(d*x)^4*tan(c)^2 + 18*pi
*a*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(
c))*tan(d*x)^4*tan(c)^2 + 48*a^3*d*x*tan(d*x)^2*tan(c)^4 + 48*a*b^2*d*x*tan
(d*x)^2*tan(c)^4 + 18*pi*a*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)
^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^4 - 30*a^2*b*tan(d*x)^4*tan(c
)^4 - 6*b^3*tan(d*x)^4*tan(c)^4 + 9*pi*a*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)
*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*ta
n(d*x)^4 + 36*pi*a*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan
(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^2 + 36
*a*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^4*tan(c)^
2 - 36*a*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^4*
tan(c)^2 - 40*a^3*tan(d*x)^4*tan(c)^3 + 24*a*b^2*tan(d*x)^4*tan(c)^3 + 9*pi
*a*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)
*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^4 + 36*a*b^2*arctan((tan(d*x) + t
an(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^2*tan(c)^4 - 36*a*b^2*arctan(-(tan(d
*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^4 - 40*a^3*tan(d*x)^
3*tan(c)^4 + 24*a*b^2*tan(d*x)^3*tan(c)^4 + 24*a^3*d*x*tan(d*x)^4 + 24*a*b^
2*d*x*tan(d*x)^4 + 9*pi*a*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^
2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4 + 96*a^3*d*x*tan(d*x)^2*tan(c)^2 + 96
*a*b^2*d*x*tan(d*x)^2*tan(c)^2 + 36*pi*a*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*t
an(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(c)^2 + 36*a^2*b*ta
n(d*x)^4*tan(c)^2 - 12*b^3*tan(d*x)^4*tan(c)^2 + 192*a^2*b*tan(d*x)^3*tan(c
)^3 + 24*a^3*d*x*tan(c)^4 + 24*a*b^2*d*x*tan(c)^4 + 9*pi*a*b^2*sgn(-2*tan(d
*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^4 + 36*a
^2*b*tan(d*x)^2*tan(c)^4 - 12*b^3*tan(d*x)^2*tan(c)^4 + 18*pi*a*b^2*sgn(2*t
```

an(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2 + 18*a*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^4 - 18*a*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^4 - 24*a^3*tan(d*x)^4*tan(c) - 24*a*b^2*tan(d*x)^4*tan(c) + 18*pi*a*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^2 + 72*a*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^2*tan(c)^2 - 72*a*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^2 + 48*a^3*tan(d*x)^3*tan(c)^2 - 144*a*b^2*tan(d*x)^3*tan(c)^2 + 48*a^3*tan(d*x)^2*tan(c)^3 - 144*a*b^2*tan(d*x)^2*tan(c)^3 + 18*a*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(c)^4 - 18*a*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(c)^4 - 24*a^3*tan(d*x)*tan(c)^4 - 24*a*b^2*tan(d*x)*tan(c)^4 + 48*a^3*d*x*tan(d*x)^2 + 48*a*b^2*d*x*tan(d*x)^2 + 18*pi*a*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2 + 18*a^2*b*tan(d*x)^4 + 10*b^3*tan(d*x)^4 + 64*b^3*tan(d*x)^3*tan(c) + 48*a^3*d*x*tan(c)^2 + 48*a*b^2*d*x*tan(c)^2 + 18*pi*a*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(c)^2 - 216*a^2*b*tan(d*x)^2*tan(c)^2 + 72*b^3*tan(d*x)^2*tan(c)^2 + 64*b^3*tan(d*x)*tan(c)^3 + 18*a^2*b*tan(c)^4 + 10*b^3*tan(c)^4 + 9*pi*a*b^2*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c)) + 36*a*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^2 - 36*a*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^2 + 24*a^3*tan(d*x)^3 + 24*a*b^2*tan(d*x)^3 - 48*a^3*tan(d*x)^2*tan(c) + 144*a*b^2*tan(d*x)^2*tan(c) + 36*a*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(c)^2 - 36*a*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(c)^2 - 48*a^3*tan(d*x)*tan(c)^2 + 144*a*b^2*tan(d*x)*tan(c)^2 + 24*a^3*tan(c)^3 + 24*a*b^2*tan(c)^3 + 24*a^3*d*x + 24*a*b^2*d*x + 9*pi*a*b^2*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c)) + 36*a^2*b*tan(d*x)^2 - 12*b^3*tan(d*x)^2 + 192*a^2*b*tan(d*x)*tan(c) + 36*a^2*b*tan(c)^2 - 12*b^3*tan(c)^2 + 18*a*b^2*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1)) - 18*a*b^2*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1)) + 40*a^3*tan(d*x) - 24*a*b^2*tan(d*x) + 40*a^3*tan(c) - 24*a*b^2*tan(c) - 30*a^2*b - 6*b^3)/(d*tan(d*x)^4*tan(c)^4 + 2*d*tan(d*x)^4*tan(c)^2 + 2*d*tan(d*x)^2*tan(c)^4 + d*tan(d*x)^4 + 4*d*tan(d*x)^2*tan(c)^2 + d*tan(c)^4 + 2*d*tan(d*x)^2 + 2*d*tan(c)^2 + d)

Mupad [B] (verification not implemented)

Time = 4.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.30

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{3\,a^3\,x}{8} - \frac{6\,a^2\,b-\tan(c+d\,x)^3\,\left(3\,a^3+3\,a\,b^2\right)+2\,b^3+\tan(c+d\,x)\,\left(3\,a\,b^2-5\,a^3\right)+4\,b^3\,\tan(c+d\,x)^2}{d\,\left(8\,\tan\left(c+d\,x\right)^4+16\,\tan\left(c+d\,x\right)^2+8\right)} + \frac{3\,a\,b^2\,x}{8}$$

```
[In] int(cos(c + d*x)^4*(a + b*tan(c + d*x))^3,x)
```

```
[Out] (3*a^3*x)/8 - (6*a^2*b - tan(c + d*x)^3*(3*a*b^2 + 3*a^3) + 2*b^3 + tan(c +
 d*x)*(3*a*b^2 - 5*a^3) + 4*b^3*tan(c + d*x)^2)/(d*(16*tan(c + d*x)^2 + 8*t
an(c + d*x)^4 + 8)) + (3*a*b^2*x)/8
```

3.537 $\int \sec^5(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 159

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{3a(2a^2-b^2)\operatorname{arctanh}(\sin(c+dx))}{16d} + \frac{3a(2a^2-b^2)\sec(c+dx)\tan(c+dx)}{16d} + \frac{a(2a^2-b^2)\sec^3(c+dx)\tan(c+dx)}{8d} + \frac{b\sec^5(c+dx)(a+b\tan(c+dx))^2}{7d} + \frac{b\sec^5(c+dx)\left(4(8a^2-b^2)+15ab\tan(c+dx)\right)}{70d}$$

```
[Out] 3/16*a*(2*a^2-b^2)*arctanh(sin(d*x+c))/d+3/16*a*(2*a^2-b^2)*sec(d*x+c)*tan(
d*x+c)/d+1/8*a*(2*a^2-b^2)*sec(d*x+c)^3*tan(d*x+c)/d+1/7*b*sec(d*x+c)^5*(a+
b*tan(d*x+c))^2/d+1/70*b*sec(d*x+c)^5*(32*a^2-4*b^2+15*a*b*tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used

$= \{3593, 757, 794, 201, 221\}$

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{3a(2a^2-b^2)\sec(c+dx)\text{arcsinh}(\tan(c+dx))}{16d\sqrt{\sec^2(c+dx)}} + \frac{b\sec^5(c+dx)\left(4(8a^2-b^2)+15ab\tan(c+dx)\right)}{70d} + \frac{a(2a^2-b^2)\tan(c+dx)\sec^3(c+dx)}{8d} + \frac{3a(2a^2-b^2)\tan(c+dx)\sec(c+dx)}{16d} + \frac{b\sec^5(c+dx)(a+b\tan(c+dx))^2}{7d}$$

```
[In] Int[Sec[c + d*x]^5*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a*(2*a^2 - b^2)*ArcSinh[Tan[c + d*x]]*Sec[c + d*x])/(16*d*Sqrt[Sec[c + d
*x]^2]) + (3*a*(2*a^2 - b^2)*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (a*(2*a^2
- b^2)*Sec[c + d*x]^3*Tan[c + d*x])/(8*d) + (b*Sec[c + d*x]^5*(a + b*Tan[c
+ d*x])^2)/(7*d) + (b*Sec[c + d*x]^5*(4*(8*a^2 - b^2) + 15*a*b*Tan[c + d*x]
))/(70*d)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
```

```
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sec(c+dx)\text{Subst}\left(\int (a+x)^3\left(1+\frac{x^2}{b^2}\right)^{3/2}\,dx,x,b\tan(c+dx)\right)}{bd\sqrt{\sec^2(c+dx)}}\\
&= \frac{b\sec^5(c+dx)(a+b\tan(c+dx))^2}{7d}\\
&\quad+\frac{(b\sec(c+dx))\text{Subst}\left(\int (a+x)\left(-2+\frac{7a^2}{b^2}+\frac{9ax}{b^2}\right)\left(1+\frac{x^2}{b^2}\right)^{3/2}\,dx,x,b\tan(c+dx)\right)}{7d\sqrt{\sec^2(c+dx)}}\\
&= \frac{b\sec^5(c+dx)(a+b\tan(c+dx))^2}{7d}+\frac{b\sec^5(c+dx)\left(4(8a^2-b^2)+15ab\tan(c+dx)\right)}{70d}\\
&\quad-\frac{\left(\left(\frac{9a}{b^2}-\frac{6a\left(-2+\frac{7a^2}{b^2}\right)}{b^2}\right)b^3\sec(c+dx)\right)\text{Subst}\left(\int\left(1+\frac{x^2}{b^2}\right)^{3/2}\,dx,x,b\tan(c+dx)\right)}{42d\sqrt{\sec^2(c+dx)}}\\
&= \frac{a(2a^2-b^2)\sec^3(c+dx)\tan(c+dx)}{8d}+\frac{b\sec^5(c+dx)(a+b\tan(c+dx))^2}{7d}\\
&\quad+\frac{b\sec^5(c+dx)\left(4(8a^2-b^2)+15ab\tan(c+dx)\right)}{70d}\\
&\quad-\frac{\left(\left(\frac{9a}{b^2}-\frac{6a\left(-2+\frac{7a^2}{b^2}\right)}{b^2}\right)b^3\sec(c+dx)\right)\text{Subst}\left(\int\sqrt{1+\frac{x^2}{b^2}}\,dx,x,b\tan(c+dx)\right)}{56d\sqrt{\sec^2(c+dx)}}\\
&= \frac{3a(2a^2-b^2)\sec(c+dx)\tan(c+dx)}{16d}+\frac{a(2a^2-b^2)\sec^3(c+dx)\tan(c+dx)}{8d}\\
&\quad+\frac{b\sec^5(c+dx)(a+b\tan(c+dx))^2}{7d}+\frac{b\sec^5(c+dx)\left(4(8a^2-b^2)+15ab\tan(c+dx)\right)}{70d}\\
&\quad-\frac{\left(\left(\frac{9a}{b^2}-\frac{6a\left(-2+\frac{7a^2}{b^2}\right)}{b^2}\right)b^3\sec(c+dx)\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{b^2}}}\,dx,x,b\tan(c+dx)\right)}{112d\sqrt{\sec^2(c+dx)}}
\end{aligned}
$$

$$= \frac{3a(2a^2-b^2)\operatorname{arcsinh}(\tan(c+dx))\sec(c+dx)}{16d\sqrt{\sec^2(c+dx)}} + \frac{3a(2a^2-b^2)\sec(c+dx)\tan(c+dx)}{16d} + \frac{a(2a^2-b^2)\sec^3(c+dx)\tan(c+dx)}{8d} + \frac{b\sec^5(c+dx)(a+b\tan(c+dx))^2}{7d} + \frac{b\sec^5(c+dx)\left(4(8a^2-b^2)+15ab\tan(c+dx)\right)}{70d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 637 vs. 2(159) = 318.

Time = 3.32 (sec) , antiderivative size = 637, normalized size of antiderivative = 4.01

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{\sec^7(c+dx)\left(10752a^2b+1536b^3+3584(3a^2b-b^3)\cos(2(c+dx))-4410a^3\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+\right.\right.\right.}{}$$

```
[In] Integrate[Sec[c + d*x]^5*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^7*(10752*a^2*b + 1536*b^3 + 3584*(3*a^2*b - b^3)*Cos[2*(c + d
*x)] - 4410*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] +
 2205*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 147
0*a^3*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 735*a*b^2
*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 210*a^3*Cos[7*
(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 105*a*b^2*Cos[7*(c +
d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 3675*a*(2*a^2 - b^2)*Cos[c
 + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]]) + 4410*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]] - 2205*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] + 1470*a^3*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]] - 735*a*b^2*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] +
210*a^3*Cos[7*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 105*a*b
^2*Cos[7*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 4340*a^3*Sin
[2*(c + d*x)] + 6790*a*b^2*Sin[2*(c + d*x)] + 2800*a^3*Sin[4*(c + d*x)] - 1
400*a*b^2*Sin[4*(c + d*x)] + 420*a^3*Sin[6*(c + d*x)] - 210*a*b^2*Sin[6*(c
+ d*x)]))/(35840*d)
```

Maple [A] (verified)

Time = 31.31 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.56

method	result
derivativedivides	$\frac{a^3\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{3a^2b}{5\cos(dx+c)^5}+3ab^2\left(\frac{\sin^3(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^3}{8\cos(}\right.}{}$
default	$\frac{a^3\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{3a^2b}{5\cos(dx+c)^5}+3ab^2\left(\frac{\sin^3(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^3}{8\cos(}\right.}{}$
risch	$-\frac{e^{i(dx+c)}\left(-1400ia^3e^{2i(dx+c)}+700iab^2e^{2i(dx+c)}+1400ia^3e^{10i(dx+c)}-105iab^2e^{12i(dx+c)}+210ia^3e^{12i(dx+c)}-210ia^3-\right.}{}$

```
[In] int(sec(d*x+c)^5*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+
tan(d*x+c)))+3/5*a^2*b/cos(d*x+c)^5+3*a*b^2*(1/6*sin(d*x+c)^3/cos(d*x+c)^6+
1/8*sin(d*x+c)^3/cos(d*x+c)^4+1/16*sin(d*x+c)^3/cos(d*x+c)^2+1/16*sin(d*x+c
)-1/16*ln(sec(d*x+c)+tan(d*x+c)))+b^3*(1/7*sin(d*x+c)^4/cos(d*x+c)^7+3/35*s
in(d*x+c)^4/cos(d*x+c)^5+1/35*sin(d*x+c)^4/cos(d*x+c)^3-1/35*sin(d*x+c)^4/c
os(d*x+c)-1/35*(2+sin(d*x+c)^2)*cos(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.07

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{105\,(2\,a^3-ab^2)\cos{(dx+c)}^7\log{(\sin{(dx+c)}+1)}-105\,(2\,a^3-ab^2)\cos{(dx+c)}^7\log{(-\sin{(dx+c)}+1}}{}$$

```
[In] integrate(sec(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/1120*(105*(2*a^3 - a*b^2)*cos(d*x + c)^7*log(sin(d*x + c) + 1) - 105*(2*a
^3 - a*b^2)*cos(d*x + c)^7*log(-sin(d*x + c) + 1) + 160*b^3 + 224*(3*a^2*b
- b^3)*cos(d*x + c)^2 + 70*(3*(2*a^3 - a*b^2)*cos(d*x + c)^5 + 8*a*b^2*cos(
d*x + c) + 2*(2*a^3 - a*b^2)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^
7)
```

Sympy [F]

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^3\,dx = \int (a+b\tan(c+dx))^3\sec^5(c+dx)\,dx$$

```
[In] integrate(sec(d*x+c)**5*(a+b*tan(d*x+c))**3,x)
[Out] Integral((a + b*tan(c + d*x))**3*sec(c + d*x)**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.31

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{35\,ab^2\left(\frac{2\left(3\,\sin(dx+c)^5-8\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-70\,a^3\left(\cdots\right.}{11\cdots}$$

```
[In] integrate(sec(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
[Out] 1/1120*(35*a*b^2*(2*(3*sin(d*x + c)^5 - 8*sin(d*x + c)^3 - 3*sin(d*x + c))/
(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(d*x
+ c) + 1) + 3*log(sin(d*x + c) - 1)) - 70*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(
d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1)
 + 3*log(sin(d*x + c) - 1)) + 672*a^2*b/cos(d*x + c)^5 - 32*(7*cos(d*x + c)
^2 - 5)*b^3/cos(d*x + c)^7)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. 2(146) = 292.

Time = 0.84 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.92

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{105\left(2\,a^3-ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-105\left(2\,a^3-ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(350\,a^3\tan\left(\frac{1}{2}\,dx\cdots\right.\right.}{\cdots}}{\cdots}$$

```
[In] integrate(sec(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="giac")
[Out] 1/560*(105*(2*a^3 - a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*(2*a^3
- a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(350*a^3*tan(1/2*d*x + 1/2*
```

```
c)^13 + 105*a*b^2*tan(1/2*d*x + 1/2*c)^13 - 1680*a^2*b*tan(1/2*d*x + 1/2*c)
^12 - 840*a^3*tan(1/2*d*x + 1/2*c)^11 + 1540*a*b^2*tan(1/2*d*x + 1/2*c)^11
+ 3360*a^2*b*tan(1/2*d*x + 1/2*c)^10 - 1120*b^3*tan(1/2*d*x + 1/2*c)^10 + 6
30*a^3*tan(1/2*d*x + 1/2*c)^9 + 1085*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 5040*a^
2*b*tan(1/2*d*x + 1/2*c)^8 - 1120*b^3*tan(1/2*d*x + 1/2*c)^8 + 6720*a^2*b*t
an(1/2*d*x + 1/2*c)^6 - 2240*b^3*tan(1/2*d*x + 1/2*c)^6 - 630*a^3*tan(1/2*d
*x + 1/2*c)^5 - 1085*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 3696*a^2*b*tan(1/2*d*x
+ 1/2*c)^4 - 448*b^3*tan(1/2*d*x + 1/2*c)^4 + 840*a^3*tan(1/2*d*x + 1/2*c)^
3 - 1540*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 672*a^2*b*tan(1/2*d*x + 1/2*c)^2 -
224*b^3*tan(1/2*d*x + 1/2*c)^2 - 350*a^3*tan(1/2*d*x + 1/2*c) - 105*a*b^2*t
an(1/2*d*x + 1/2*c) - 336*a^2*b + 32*b^3)/(tan(1/2*d*x + 1/2*c)^2 - 1)^7)/d
```

Mupad [B] (verification not implemented)

Time = 8.11 (sec) , antiderivative size = 423, normalized size of antiderivative = 2.66

$$\int \sec^5(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{3\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(2\,a^2-b^2\right)}{8\,d}$$

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{5\,a^3}{4}+\frac{3\,a\,b^2}{8}\right)+\frac{6\,a^2\,b}{5}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{11\,a\,b^2}{2}-3\,a^3\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\,\left(\frac{11\,a\,b^2}{2}-3\,a^3\right)-\mathrm{ta}}{}$$

```
[In] int((a + b*tan(c + d*x))^3/cos(c + d*x)^5,x)
```

```
[Out] (3*a*atanh(tan(c/2 + (d*x)/2))*(2*a^2 - b^2))/(8*d) - (tan(c/2 + (d*x)/2)*(
(3*a*b^2)/8 + (5*a^3)/4) + (6*a^2*b)/5 + tan(c/2 + (d*x)/2)^3*((11*a*b^2)/2
 - 3*a^3) - tan(c/2 + (d*x)/2)^11*((11*a*b^2)/2 - 3*a^3) - tan(c/2 + (d*x)/
2)^13*((3*a*b^2)/8 + (5*a^3)/4) + tan(c/2 + (d*x)/2)^5*((31*a*b^2)/8 + (9*a
^3)/4) - tan(c/2 + (d*x)/2)^9*((31*a*b^2)/8 + (9*a^3)/4) - tan(c/2 + (d*x)/
2)^10*(12*a^2*b - 4*b^3) - tan(c/2 + (d*x)/2)^2*((12*a^2*b)/5 - (4*b^3)/5)
+ tan(c/2 + (d*x)/2)^8*(18*a^2*b + 4*b^3) - tan(c/2 + (d*x)/2)^6*(24*a^2*b
- 8*b^3) + tan(c/2 + (d*x)/2)^4*((66*a^2*b)/5 + (8*b^3)/5) - (4*b^3)/35 + 6
*a^2*b*tan(c/2 + (d*x)/2)^12)/(d*(7*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*
x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6 - 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2 +
(d*x)/2)^10 - 7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 - 1))
```

3.538 $\int \sec^3(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 126

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{a(4a^2-3b^2)\operatorname{arctanh}(\sin(c+dx))}{8d} + \frac{a(4a^2-3b^2)\sec(c+dx)\tan(c+dx)}{8d} + \frac{b\sec^3(c+dx)(a+b\tan(c+dx))^2}{5d} + \frac{b\sec^3(c+dx)\left(8(6a^2-b^2)+21ab\tan(c+dx)\right)}{60d}$$

```
[Out] 1/8*a*(4*a^2-3*b^2)*arctanh(sin(d*x+c))/d+1/8*a*(4*a^2-3*b^2)*sec(d*x+c)*ta
n(d*x+c)/d+1/5*b*sec(d*x+c)^3*(a+b*tan(d*x+c))^2/d+1/60*b*sec(d*x+c)^3*(48*
a^2-8*b^2+21*a*b*tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3593, 757, 794, 201, 221}

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{a(4a^2-3b^2)\sec(c+dx)\operatorname{arcsinh}(\tan(c+dx))}{8d\sqrt{\sec^2(c+dx)}} + \frac{b\sec^3(c+dx)\left(8(6a^2-b^2)+21ab\tan(c+dx)\right)}{60d} + \frac{a(4a^2-3b^2)\tan(c+dx)\sec(c+dx)}{8d} + \frac{b\sec^3(c+dx)(a+b\tan(c+dx))^2}{5d}$$

[In] Int[Sec[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]

[Out] (a*(4*a^2 - 3*b^2)*ArcSinh[Tan[c + d*x]]*Sec[c + d*x])/(8*d*Sqrt[Sec[c + d*x]^2]) + (a*(4*a^2 - 3*b^2)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (b*Sec[c + d*x]^3*(a + b*Tan[c + d*x])^2)/(5*d) + (b*Sec[c + d*x]^3*(8*(6*a^2 - b^2) + 21*a*b*Tan[c + d*x]))/(60*d)

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\sec(c+dx)\text{Subst}\left(\int (a+x)^3\sqrt{1+\frac{x^2}{b^2}}\,dx, x, b\tan(c+dx)\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$= \frac{b\sec^3(c+dx)(a+b\tan(c+dx))^2}{5d} + \frac{(b\sec(c+dx))\text{Subst}\left(\int (a+x)\left(-2+\frac{5a^2}{b^2}+\frac{7ax}{b^2}\right)\sqrt{1+\frac{x^2}{b^2}}\,dx, x, b\tan(c+dx)\right)}{5d\sqrt{\sec^2(c+dx)}}$$

$$= \frac{b\sec^3(c+dx)(a+b\tan(c+dx))^2}{5d} + \frac{b\sec^3(c+dx)\left(8(6a^2-b^2)+21ab\tan(c+dx)\right)}{60d} - \frac{\left(a\left(3-\frac{4a^2}{b^2}\right)b\sec(c+dx)\right)\text{Subst}\left(\int \sqrt{1+\frac{x^2}{b^2}}\,dx, x, b\tan(c+dx)\right)}{4d\sqrt{\sec^2(c+dx)}}$$

$$= \frac{a(4a^2-3b^2)\sec(c+dx)\tan(c+dx)}{8d} + \frac{b\sec^3(c+dx)(a+b\tan(c+dx))^2}{5d} + \frac{b\sec^3(c+dx)\left(8(6a^2-b^2)+21ab\tan(c+dx)\right)}{60d} - \frac{\left(a\left(3-\frac{4a^2}{b^2}\right)b\sec(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+dx)\right)}{8d\sqrt{\sec^2(c+dx)}}$$

$$= \frac{a(4a^2-3b^2)\text{arcsinh}(\tan(c+dx))\sec(c+dx)}{8d\sqrt{\sec^2(c+dx)}} + \frac{a(4a^2-3b^2)\sec(c+dx)\tan(c+dx)}{8d} + \frac{b\sec^3(c+dx)(a+b\tan(c+dx))^2}{5d} + \frac{b\sec^3(c+dx)\left(8(6a^2-b^2)+21ab\tan(c+dx)\right)}{60d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 464 vs. 2(126) = 252.

Time = 2.46 (sec) , antiderivative size = 464, normalized size of antiderivative = 3.68

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{\sec^5(c+dx)\left(960a^2b+64b^3+320(3a^2b-b^3)\cos(2(c+dx))-300a^3\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\right.\right.}{}$$

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^5*(960*a^2*b + 64*b^3 + 320*(3*a^2*b - b^3)*Cos[2*(c + d*x)]
- 300*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 225*a
```

```
*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 60*a^3*Cos
[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 45*a*b^2*Cos[5*(c
+ d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 150*a*(4*a^2 - 3*b^2)*Co
s[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]]) + 300*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]] - 225*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]] + 60*a^3*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
] - 45*a*b^2*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 24
0*a^3*Sin[2*(c + d*x)] + 540*a*b^2*Sin[2*(c + d*x)] + 120*a^3*Sin[4*(c + d*
x)] - 90*a*b^2*Sin[4*(c + d*x)]))/(1920*d)
```

Maple [A] (verified)

Time = 9.24 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.57

method	result
derivativedivides	$\frac{a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{a^2 b}{\cos(dx+c)^3}+3a\,b^2\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx}{}\right.}{d}$
default	$\frac{a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{a^2 b}{\cos(dx+c)^3}+3a\,b^2\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx}{}\right.}{d}$
risch	$-\frac{e^{i(dx+c)}\left(60ia^3 e^{8i(dx+c)}-45ia\,b^2 e^{8i(dx+c)}+120ia^3 e^{6i(dx+c)}+270ia\,b^2 e^{6i(dx+c)}-480a^2 b\,e^{6i(dx+c)}+160b^3 e^{6i(dx+c)}-\right.}{60d\left(e^{2}\right.}$

```
[In] int(sec(d*x+c)^3*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+a^2*b/co
s(d*x+c)^3+3*a*b^2*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/cos(d*x+
c)^2+1/8*sin(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c)))+b^3*(1/5*sin(d*x+c)^4/co
s(d*x+c)^5+1/15*sin(d*x+c)^4/cos(d*x+c)^3-1/15*sin(d*x+c)^4/cos(d*x+c)-1/15
*(2+sin(d*x+c)^2)*cos(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.17

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{15\,(4\,a^3-3\,ab^2)\cos(dx+c)^5\log(\sin(dx+c)+1)-15\,(4\,a^3-3\,ab^2)\cos(dx+c)^5\log(-\sin(dx+c)+1}{240\,d\cos}$$

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/240*(15*(4*a^3 - 3*a*b^2)*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(4*a^
3 - 3*a*b^2)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 48*b^3 + 80*(3*a^2*b -
 b^3)*cos(d*x + c)^2 + 30*(6*a*b^2*cos(d*x + c) + (4*a^3 - 3*a*b^2)*cos(d*x
 + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F]

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^3\,dx = \int (a+b\tan(c+dx))^3\sec^3(c+dx)\,dx$$

```
[In] integrate(sec(d*x+c)**3*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*sec(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.25

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{45\,ab^2\left(\frac{2\left(\sin(dx+c)^3+\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1} - \log\left(\sin\left(dx+c\right)+1\right) + \log\left(\sin\left(dx+c\right)-1\right)\right) - 60\,a^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log\left(\right.\right.}{240\,d}$$

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/240*(45*a*b^2*(2*(sin(d*x + c)^3 + sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(
d*x + c)^2 + 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 60*a^3*(
2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
 c) - 1)) + 240*a^2*b/cos(d*x + c)^3 - 16*(5*cos(d*x + c)^2 - 3)*b^3/cos(d*
x + c)^5)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(115) = 230.

Time = 0.75 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.64

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{15\left(4\,a^3-3\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 15\left(4\,a^3-3\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\left(60\,a^3\tan\left(\frac{1}{2}\,dx\right.\right.}{}}{}$$

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/120*(15*(4*a^3 - 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(4*a^3
- 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(60*a^3*tan(1/2*d*x + 1/2
*c)^9 + 45*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 360*a^2*b*tan(1/2*d*x + 1/2*c)^8
- 120*a^3*tan(1/2*d*x + 1/2*c)^7 + 270*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 720*a
^2*b*tan(1/2*d*x + 1/2*c)^6 - 240*b^3*tan(1/2*d*x + 1/2*c)^6 - 480*a^2*b*ta
n(1/2*d*x + 1/2*c)^4 - 80*b^3*tan(1/2*d*x + 1/2*c)^4 + 120*a^3*tan(1/2*d*x
+ 1/2*c)^3 - 270*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 240*a^2*b*tan(1/2*d*x + 1/2
*c)^2 - 80*b^3*tan(1/2*d*x + 1/2*c)^2 - 60*a^3*tan(1/2*d*x + 1/2*c) - 45*a*
b^2*tan(1/2*d*x + 1/2*c) - 120*a^2*b + 16*b^3)/(tan(1/2*d*x + 1/2*c)^2 - 1)
^5)/d
```

Mupad [B] (verification not implemented)

Time = 8.18 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.33

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9\left(a^3+\frac{3\,a\,b^2}{4}\right)-2\,a^2\,b-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{9\,a\,b^2}{2}-2\,a^3\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\left(\frac{9\,a\,b^2}{2}-2\,a^3\right)+\tan\left(\frac{c}{2}\right.}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\right.\right.}$$
$$-\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(\frac{3\,a\,b^2}{4}-a^3\right)}{d}$$

```
[In] int((a + b*tan(c + d*x))^3/cos(c + d*x)^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^9*((3*a*b^2)/4 + a^3) - 2*a^2*b - tan(c/2 + (d*x)/2)^3*
((9*a*b^2)/2 - 2*a^3) + tan(c/2 + (d*x)/2)^7*((9*a*b^2)/2 - 2*a^3) + tan(c/
2 + (d*x)/2)^2*(4*a^2*b - (4*b^3)/3) - tan(c/2 + (d*x)/2)^4*(8*a^2*b + (4*b
^3)/3) + tan(c/2 + (d*x)/2)^6*(12*a^2*b - 4*b^3) + (4*b^3)/15 - tan(c/2 + (
d*x)/2)*((3*a*b^2)/4 + a^3) - 6*a^2*b*tan(c/2 + (d*x)/2)^8)/(d*(5*tan(c/2 +
 (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2
 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1)) - (atanh(tan(c/2 + (d*x)/2))*((
3*a*b^2)/4 - a^3))/d
```

3.539 $\int \sec(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 19, antiderivative size = 91

$$\int \sec(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{a(2a^2-3b^2)\,\text{arctanh}(\sin(c+dx))}{2d} + \frac{b\sec(c+dx)(a+b\tan(c+dx))^2}{3d} + \frac{b\sec(c+dx)\,(4(4a^2-b^2)+5ab\tan(c+dx))}{6d}$$

```
[Out] 1/2*a*(2*a^2-3*b^2)*arctanh(sin(d*x+c))/d+1/3*b*sec(d*x+c)*(a+b*tan(d*x+c))
^2/d+1/6*b*sec(d*x+c)*(16*a^2-4*b^2+5*a*b*tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3593, 757, 794, 221}

$$\int \sec(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{a(2a^2-3b^2)\sec(c+dx)\text{arcsinh}(\tan(c+dx))}{2d\sqrt{\sec^2(c+dx)}} + \frac{b\sec(c+dx)\,(4(4a^2-b^2)+5ab\tan(c+dx))}{6d} + \frac{b\sec(c+dx)(a+b\tan(c+dx))^2}{3d}$$

```
[In] Int[Sec[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (a*(2*a^2 - 3*b^2)*ArcSinh[Tan[c + d*x]]*Sec[c + d*x])/(2*d*Sqrt[Sec[c + d*
x]^2]) + (b*Sec[c + d*x]*(a + b*Tan[c + d*x])^2)/(3*d) + (b*Sec[c + d*x]*(4
*(4*a^2 - b^2) + 5*a*b*Tan[c + d*x]))/(6*d)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\sec(c+dx)\text{Subst}\left(\int \frac{(a+x)^3}{\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+dx)\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$= \frac{b\sec(c+dx)(a+b\tan(c+dx))^2}{3d} + \frac{(b\sec(c+dx))\text{Subst}\left(\int \frac{(a+x)\left(-2+\frac{3a^2}{b^2}+\frac{5ax}{b^2}\right)}{\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+dx)\right)}{3d\sqrt{\sec^2(c+dx)}}$$

$$= \frac{b\sec(c+dx)(a+b\tan(c+dx))^2}{3d} + \frac{b\sec(c+dx)\left(4(4a^2-b^2)+5ab\tan(c+dx)\right)}{6d} - \frac{\left(a\left(3-\frac{2a^2}{b^2}\right)b\sec(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+dx)\right)}{2d\sqrt{\sec^2(c+dx)}}$$

$$= \frac{a(2a^2-3b^2)\operatorname{arcsinh}(\tan(c+dx))\sec(c+dx)}{2d\sqrt{\sec^2(c+dx)}} + \frac{b\sec(c+dx)(a+b\tan(c+dx))^2}{3d} + \frac{b\sec(c+dx)\left(4(4a^2-b^2)+5ab\tan(c+dx)\right)}{6d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 293 vs. 2(91) = 182.

Time = 2.44 (sec) , antiderivative size = 293, normalized size of antiderivative = 3.22

$$\int \sec(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{36a^2b-10b^3-6a(2a^2-3b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+12a^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(\right.\right.}{}$$

```
[In] Integrate[Sec[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (36*a^2*b - 10*b^3 - 6*a*(2*a^2 - 3*b^2)*Log[Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] + 12*a^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 18*a*b^2*Log[Cos
[(c + d*x)/2] + Sin[(c + d*x)/2]] + (9*a*b^2)/(Cos[(c + d*x)/2] - Sin[(c +
d*x)/2])^2 + b^3/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + 2*b*(18*a^2 - b^
2 + 2*b^2*Cos[c + d*x] + (18*a^2 - 5*b^2)*Cos[2*(c + d*x)])*Sec[c + d*x]^3*
Sin[(c + d*x)/2]^2 - (9*a*b^2)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + b^
3/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(12*d)
```

Maple [A] (verified)

Time = 2.56 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.60

method	result
derivativedivides	$\frac{b^3\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right)+3a\,b^2\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{}{\text{co}}}{d}$
default	$\frac{b^3\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right)+3a\,b^2\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{}{\text{co}}}{d}$
risch	$-\frac{b\,e^{i(dx+c)}\left(9iab\,e^{4i(dx+c)}-18a^2e^{4i(dx+c)}+6b^2e^{4i(dx+c)}-36a^2e^{2i(dx+c)}+4b^2e^{2i(dx+c)}-9iab-18a^2+6b^2\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}-\frac{a^3\ln\left(e^{i(d}\right.}{d}$

```
[In] int(sec(d*x+c)*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^3*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+
sin(d*x+c)^2)*cos(d*x+c))+3*a*b^2*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*
x+c)-1/2*ln(sec(d*x+c)+tan(d*x+c)))+3*a^2*b/cos(d*x+c)+a^3*ln(sec(d*x+c)+ta
n(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.35

$$\int \sec(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$=\frac{3\left(2\,a^3-3\,ab^2\right)\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\left(2\,a^3-3\,ab^2\right)\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)}{12\,d\cos\left(dx+c\right)^3}$$

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*(2*a^3 - 3*a*b^2)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(2*a^3 -
 3*a*b^2)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 18*a*b^2*cos(d*x + c)*sin
(d*x + c) + 4*b^3 + 12*(3*a^2*b - b^3)*cos(d*x + c)^2)/(d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \sec(c+dx)(a+b\tan(c+dx))^3\,dx=\int\left(a+b\tan\left(c+dx\right)\right)^3\sec\left(c+dx\right)dx$$

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*sec(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.22

$$\int \sec(c+dx)(a+b\tan(c+dx))^3\,dx=$$
$$-\frac{9\,ab^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)-12\,a^3\log\left(\sec\left(dx+c\right)+\tan\left(dx+\right.\right.}{12\,d}$$

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/12*(9*a*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1)
 - log(sin(d*x + c) - 1)) - 12*a^3*log(sec(d*x + c) + tan(d*x + c)) - 36*a^
2*b/cos(d*x + c) + 4*(3*cos(d*x + c)^2 - 1)*b^3/cos(d*x + c)^3)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(82) = 164.

Time = 0.71 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.88

$$\int \sec(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{3\,(2\,a^3-3\,ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(2\,a^3-3\,ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(9\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{}}{6\,d}$$

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(3*(2*a^3 - 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(2*a^3 - 3*
a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(9*a*b^2*tan(1/2*d*x + 1/2*c)
^5 - 18*a^2*b*tan(1/2*d*x + 1/2*c)^4 + 36*a^2*b*tan(1/2*d*x + 1/2*c)^2 - 12
*b^3*tan(1/2*d*x + 1/2*c)^2 - 9*a*b^2*tan(1/2*d*x + 1/2*c) - 18*a^2*b + 4*b
^3)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 6.26 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.76

$$\int \sec(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(3\,a\,b^2-2\,a^3\right)}{d}$$
$$-\frac{6\,a^2\,b-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(12\,a^2\,b-4\,b^3\right)-\frac{4\,b^3}{3}+3\,a\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+6\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-3\,a\,b^2\tan\left(\frac{c}{2}+\frac{d}{}\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

```
[In] int((a + b*tan(c + d*x))^3/cos(c + d*x),x)
```

```
[Out] - (atanh(tan(c/2 + (d*x)/2))*(3*a*b^2 - 2*a^3))/d - (6*a^2*b - tan(c/2 + (d
*x)/2)^2*(12*a^2*b - 4*b^3) - (4*b^3)/3 + 3*a*b^2*tan(c/2 + (d*x)/2) + 6*a^
2*b*tan(c/2 + (d*x)/2)^4 - 3*a*b^2*tan(c/2 + (d*x)/2)^5)/(d*(3*tan(c/2 + (d
*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.540 $\int \cos(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 19, antiderivative size = 84

$$\int \cos(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{3ab^2\text{arctanh}(\sin(c+dx))}{d} - \frac{\cos(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))^2}{d} - \frac{b\sec(c+dx)\left(2(a^2-b^2)+ab\tan(c+dx)\right)}{d}$$

```
[Out] 3*a*b^2*arctanh(sin(d*x+c))/d-cos(d*x+c)*(b-a*tan(d*x+c))*(a+b*tan(d*x+c))^
2/d-b*sec(d*x+c)*(2*a^2-2*b^2+a*b*tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.21, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3593, 753, 794, 221}

$$\int \cos(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{b\sec(c+dx)\left(2(a^2-b^2)+ab\tan(c+dx)\right)}{d} + \frac{3ab^2\cos(c+dx)\sqrt{\sec^2(c+dx)}\text{arcsinh}(\tan(c+dx))}{d} - \frac{\cos(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))^2}{d}$$

```
[In] Int[Cos[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a*b^2*ArcSinh[Tan[c + d*x]]*Cos[c + d*x]*Sqrt[Sec[c + d*x]^2])/d - (Cos[
c + d*x]*(b - a*Tan[c + d*x])*(a + b*Tan[c + d*x])^2)/d - (b*Sec[c + d*x]*(
2*(a^2 - b^2) + a*b*Tan[c + d*x]))/d
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 753

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\left(\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{(a+x)^3}{\left(1+\frac{x^2}{b^2}\right)^{3/2}}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= -\frac{\cos(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))^2}{d} + \frac{\left(b\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{(a+x)\left(2-\frac{2ax}{b^2}\right)}{\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))^2}{d} - \frac{b\sec(c+dx)\left(2(a^2-b^2)+ab\tan(c+dx)\right)}{d} + \frac{\left(3ab\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{b^2}}}dx,x,b\tan(c+dx)\right)}{d}$$

$$= \frac{3ab^2\text{arcsinh}(\tan(c+dx))\cos(c+dx)\sqrt{\sec^2(c+dx)}}{d} - \frac{\cos(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))^2}{d} - \frac{b\sec(c+dx)\left(2(a^2-b^2)+ab\tan(c+dx)\right)}{d}$$

Mathematica [A] (verified)

Time = 1.82 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.56

$$\int\cos(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{\sec(c+dx)\left(-3a^2b+3b^3+(-3a^2b+b^3)\cos(2(c+dx))-6ab^2\cos(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\sin\left(\frac{1}{2}\right.\right.\right.}{2d}$$

```
[In] Integrate[Cos[c + d*x]*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]*(-3*a^2*b + 3*b^3 + (-3*a^2*b + b^3)*Cos[2*(c + d*x)] - 6*a*b
^2*Cos[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]]) + a^3*Sin[2*(c + d*x)] - 3*a*b^2*Sin[2*(c + d*x)
]))/(2*d)
```

Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{b^3\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+3a\,b^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))-3a^2b\cos(dx+c)+a^3\sin}{d}$
default	$\frac{b^3\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+3a\,b^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))-3a^2b\cos(dx+c)+a^3\sin}{d}$
risch	$-\frac{3\,e^{i(dx+c)}b\,a^2}{2d}+\frac{e^{i(dx+c)}b^3}{2d}-\frac{ie^{i(dx+c)}a^3}{2d}+\frac{3ie^{i(dx+c)}a\,b^2}{2d}-\frac{3\,e^{-i(dx+c)}b\,a^2}{2d}+\frac{e^{-i(dx+c)}b^3}{2d}+\frac{ie^{-i(dx+c)}a}{2d}$

```
[In] int(cos(d*x+c)*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^3*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+3*a*b^2*(-si
n(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))-3*a^2*b*cos(d*x+c)+a^3*sin(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.30

$$\int \cos(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{3\,ab^2\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-3\,ab^2\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\,b^3-2\left(3\,a^2b-b^3\right)\mathrm{c}}{2\,d\cos\left(dx+c\right)}$$

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(3*a*b^2*cos(d*x + c)*log(sin(d*x + c) + 1) - 3*a*b^2*cos(d*x + c)*log(
-sin(d*x + c) + 1) + 2*b^3 - 2*(3*a^2*b - b^3)*cos(d*x + c)^2 + 2*(a^3 - 3*
a*b^2)*cos(d*x + c)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]

$$\int \cos(c+dx)(a+b\tan(c+dx))^3\,dx = \int \left(a+b\tan\left(c+dx\right)\right)^3\cos\left(c+dx\right)dx$$

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*cos(c + d*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00

$$\int \cos(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{2\,b^3\left(\frac{1}{\cos\left(dx+c\right)}+\cos\left(dx+c\right)\right)+3\,ab^2\left(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)-2\,\sin\left(dx+c\right)\right)-6\,a^{2}}{2\,d}$$

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^3*(1/cos(d*x + c) + cos(d*x + c)) + 3*a*b^2*(log(sin(d*x + c) + 1)
 - log(sin(d*x + c) - 1) - 2*sin(d*x + c)) - 6*a^2*b*cos(d*x + c) + 2*a^3*s
in(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4309 vs. $2(83) = 166$.

Time = 2.62 (sec) , antiderivative size = 4309, normalized size of antiderivative = 51.30

$$\int \cos(c+dx)(a+b\tan(c+dx))^3\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c))^3,x, algorithm="giac")
[Out] -1/4*(3*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1
/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 - 3*pi*a
^2*b*tan(1/2*d*x)^4*tan(1/2*c)^4 - 6*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c)
+ tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) -
tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 - 6*a^2*b*arctan((tan(1/2*d*x)
*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan
(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 + 6*a*b^2*log(2*(t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*t
an(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c)
 + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*ta
n(1/2*d*x)^4*tan(1/2*c)^4 - 6*a*b^2*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*
tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c
)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 - 12*
pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*
tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^3*tan(1/2*c)^3 + 12*a^2*b*tan(1
/2*d*x)^4*tan(1/2*c)^4 - 8*b^3*tan(1/2*d*x)^4*tan(1/2*c)^4 + 12*pi*a^2*b*ta
n(1/2*d*x)^3*tan(1/2*c)^3 + 24*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(
1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/
2*c) - 1))*tan(1/2*d*x)^3*tan(1/2*c)^3 + 24*a^2*b*arctan((tan(1/2*d*x)*tan(
1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*
d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^3*tan(1/2*c)^3 - 24*a*b^2*log(2*(tan(1
/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1
/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1
)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/
2*d*x)^3*tan(1/2*c)^3 + 24*a*b^2*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan
(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan
(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2
 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c)^3 + 8*a^3*
tan(1/2*d*x)^4*tan(1/2*c)^3 - 24*a*b^2*tan(1/2*d*x)^4*tan(1/2*c)^3 + 8*a^3*
tan(1/2*d*x)^3*tan(1/2*c)^4 - 24*a*b^2*tan(1/2*d*x)^3*tan(1/2*c)^4 - 3*pi*a
^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(
1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4 - 12*pi*a^2*b*sgn(tan(1/2*d*x)^2*
tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 +
1)*tan(1/2*d*x)^3*tan(1/2*c) - 24*a^2*b*tan(1/2*d*x)^4*tan(1/2*c)^2 - 12*pi
```

*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*ta
n(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)*tan(1/2*c)^3 - 96*a^2*b*tan(1/2*d
*x)^3*tan(1/2*c)^3 + 32*b^3*tan(1/2*d*x)^3*tan(1/2*c)^3 - 3*pi*a^2*b*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - ta
n(1/2*c)^2 + 1)*tan(1/2*c)^4 - 24*a^2*b*tan(1/2*d*x)^2*tan(1/2*c)^4 + 3*pi*
a^2*b*tan(1/2*d*x)^4 + 6*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*
x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) -
 1))*tan(1/2*d*x)^4 + 6*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x
) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) -
1))*tan(1/2*d*x)^4 - 6*a*b^2*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2
*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2
*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + t
an(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^4 + 6*a*b^2*log(2*(tan(1/2*
d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*
c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(
tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d
*x)^4 + 12*pi*a^2*b*tan(1/2*d*x)^3*tan(1/2*c) + 24*a^2*b*arctan((tan(1/2*d*
x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - t
an(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^3*tan(1/2*c) + 24*a^2*b*arctan(
(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan
(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^3*tan(1/2*c) - 24*a*
b^2*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*ta
n(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) -
2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c
)^2 + 1))*tan(1/2*d*x)^3*tan(1/2*c) + 24*a*b^2*log(2*(tan(1/2*d*x)^2*tan(1/
2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/
2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^
2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^3*tan(1/2
*c) - 8*a^3*tan(1/2*d*x)^4*tan(1/2*c) + 24*a*b^2*tan(1/2*d*x)^4*tan(1/2*c)
- 48*a^3*tan(1/2*d*x)^3*tan(1/2*c)^2 + 144*a*b^2*tan(1/2*d*x)^3*tan(1/2*c)^
2 + 12*pi*a^2*b*tan(1/2*d*x)*tan(1/2*c)^3 + 24*a^2*b*arctan((tan(1/2*d*x)*t
an(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1
/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)*tan(1/2*c)^3 + 24*a^2*b*arctan((tan
(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2
*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)*tan(1/2*c)^3 - 24*a*b^2*
log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/
2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*ta
n(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2
+ 1))*tan(1/2*d*x)*tan(1/2*c)^3 + 24*a*b^2*log(2*(tan(1/2*d*x)^2*tan(1/2*c)
^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*
x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*ta
n(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c)^3
- 48*a^3*tan(1/2*d*x)^2*tan(1/2*c)^3 + 144*a*b^2*tan(1/2*d*x)^2*tan(1/2*c)^
3 + 3*pi*a^2*b*tan(1/2*c)^4 + 6*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan
(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1

/2*c) - 1))*tan(1/2*c)^4 + 6*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^4 - 6*a*b^2*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^4 + 6*a*b^2*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*c)^4 - 8*a^3*tan(1/2*d*x)*tan(1/2*c)^4 + 24*a*b^2*tan(1/2*d*x)*tan(1/2*c)^4 + 12*a^2*b*tan(1/2*d*x)^4 - 8*b^3*tan(1/2*d*x)^4 - 12*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)*tan(1/2*c) + 96*a^2*b*tan(1/2*d*x)^3*tan(1/2*c) - 32*b^3*tan(1/2*d*x)^3*tan(1/2*c) + 240*a^2*b*tan(1/2*d*x)^2*tan(1/2*c)^2 - 96*b^3*tan(1/2*d*x)^2*tan(1/2*c)^2 + 96*a^2*b*tan(1/2*d*x)*tan(1/2*c)^3 - 32*b^3*tan(1/2*d*x)*tan(1/2*c)^3 + 12*a^2*b*tan(1/2*c)^4 - 8*b^3*tan(1/2*c)^4 + 8*a^3*tan(1/2*d*x)^3 - 24*a*b^2*tan(1/2*d*x)^3 + 12*pi*a^2*b*tan(1/2*d*x)*tan(1/2*c) + 24*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)*tan(1/2*c) + 24*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)*tan(1/2*c) - 24*a*b^2*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c) + 24*a*b^2*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)*tan(1/2*c) + 48*a^3*tan(1/2*d*x)^2*tan(1/2*c) - 144*a*b^2*tan(1/2*d*x)^2*tan(1/2*c) + 48*a^3*tan(1/2*d*x)*tan(1/2*c)^2 - 144*a*b^2*tan(1/2*d*x)*tan(1/2*c)^2 + 8*a^3*tan(1/2*c)^3 - 24*a*b^2*tan(1/2*c)^3 + 3*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1) - 24*a^2*b*tan(1/2*d*x)^2 - 96*a^2*b*tan(1/2*d*x)*tan(1/2*c) + 32*b^3*tan(1/2*d*x)*tan(1/2*c) - 24*a^2*b*tan(1/2*c)^2 - 3*pi*a^2*b - 6*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)) - 6*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)) + 6*a*b^2*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1)) - 6*a*b^2*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1)) - 8*a^3*tan(1/2*d*x) + 24*a*b^2*tan(1/2*d*x) - 8*a^3*ta

```
n(1/2*c) + 24*a*b^2*tan(1/2*c) + 12*a^2*b - 8*b^3)/(d*tan(1/2*d*x)^4*tan(1/
2*c)^4 - 4*d*tan(1/2*d*x)^3*tan(1/2*c)^3 - d*tan(1/2*d*x)^4 - 4*d*tan(1/2*d
*x)^3*tan(1/2*c) - 4*d*tan(1/2*d*x)*tan(1/2*c)^3 - d*tan(1/2*c)^4 - 4*d*tan
(1/2*d*x)*tan(1/2*c) + d)
```

Mupad [B] (verification not implemented)

Time = 4.69 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.38

$$\int \cos(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{6\,a\,b^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(6\,a\,b^2-2\,a^3\right)-6\,a^2\,b-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(6\,a\,b^2-2\,a^3\right)+4\,b^3+6\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-1\right)}$$

```
[In] int(cos(c + d*x)*(a + b*tan(c + d*x))^3,x)
```

```
[Out] (6*a*b^2*atanh(tan(c/2 + (d*x)/2)))/d - (tan(c/2 + (d*x)/2)^3*(6*a*b^2 - 2*
a^3) - 6*a^2*b - tan(c/2 + (d*x)/2)*(6*a*b^2 - 2*a^3) + 4*b^3 + 6*a^2*b*tan
(c/2 + (d*x)/2)^2)/(d*(tan(c/2 + (d*x)/2)^4 - 1))
```

3.541 $\int \cos^3(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 70

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{2(a^2+b^2)\cos(c+dx)(b-a\tan(c+dx))}{3d} - \frac{\cos^3(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))^2}{3d}$$

```
[Out] -2/3*(a^2+b^2)*cos(d*x+c)*(b-a*tan(d*x+c))/d-1/3*cos(d*x+c)^3*(b-a*tan(d*x+
c))*(a+b*tan(d*x+c))^2/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3593, 737, 651}

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{2(a^2+b^2)\cos(c+dx)(b-a\tan(c+dx))}{3d} - \frac{\cos^3(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))^2}{3d}$$

```
[In] Int[Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-2*(a^2 + b^2)*Cos[c + d*x]*(b - a*Tan[c + d*x]))/(3*d) - (Cos[c + d*x]^3*
(b - a*Tan[c + d*x])*(a + b*Tan[c + d*x])^2)/(3*d)
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 737

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[(2*p + 3)*((c*d^2 + a*e^2)/(2*a*c*(p + 1))), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\left(\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{(a+x)^3}{\left(1+\frac{x^2}{b^2}\right)^{5/2}}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= -\frac{\cos^3(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))^2}{3d} + \frac{\left(2(a^2+b^2)\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{a+x}{\left(1+\frac{x^2}{b^2}\right)^{3/2}}\,dx, x, b\tan(c+dx)\right)}{3bd}$$

$$= -\frac{2(a^2+b^2)\cos(c+dx)(b-a\tan(c+dx))}{3d} - \frac{\cos^3(c+dx)(b-a\tan(c+dx))(a+b\tan(c+dx))^2}{3d}$$

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.16

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$= \frac{-9b(a^2+b^2)\cos(c+dx)+(-3a^2b+b^3)\cos(3(c+dx))+2a(5a^2+3b^2+(a^2-3b^2)\cos(2(c+dx)))\sin(c}{12d}$$

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-9*b*(a^2 + b^2)*Cos[c + d*x] + (-3*a^2*b + b^3)*Cos[3*(c + d*x)] + 2*a*(5
*a^2 + 3*b^2 + (a^2 - 3*b^2)*Cos[2*(c + d*x)])*Sin[c + d*x])/(12*d)
```

Maple [A] (verified)

Time = 10.37 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{-\frac{b^3(2+\sin^2(dx+c))\cos(dx+c)}{3}+a\,b^2(\sin^3(dx+c))-a^2b(\cos^3(dx+c))+\frac{a^3(2+\cos^2(dx+c))\sin(dx+c)}{3}}{d}$
default	$\frac{-\frac{b^3(2+\sin^2(dx+c))\cos(dx+c)}{3}+a\,b^2(\sin^3(dx+c))-a^2b(\cos^3(dx+c))+\frac{a^3(2+\cos^2(dx+c))\sin(dx+c)}{3}}{d}$
risch	$-\frac{3b\cos(dx+c)a^2}{4d}-\frac{3b^3\cos(dx+c)}{4d}+\frac{3a^3\sin(dx+c)}{4d}+\frac{3a\sin(dx+c)b^2}{4d}-\frac{b\cos(3dx+3c)a^2}{4d}+\frac{b^3\cos(3dx+3c)}{12d}+$

```
[In] int(cos(d*x+c)^3*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*b^3*(2+sin(d*x+c)^2)*cos(d*x+c)+a*b^2*sin(d*x+c)^3-a^2*b*cos(d*x+
c)^3+1/3*a^3*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.10

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{3\,b^3\cos(dx+c)+(3\,a^2b-b^3)\cos(dx+c)^3-\left(2\,a^3+3\,ab^2+(a^3-3\,ab^2)\cos(dx+c)^2\right)\sin(dx+c)}{3\,d}$$

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^3*cos(d*x + c) + (3*a^2*b - b^3)*cos(d*x + c)^3 - (2*a^3 + 3*a*b^
2 + (a^3 - 3*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^3\,dx = \int (a+b\tan(c+dx))^3\cos^3(c+dx)\,dx$$

```
[In] integrate(cos(d*x+c)**3*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*cos(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.10

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{3\,a^2b\cos(dx+c)^3 - 3\,ab^2\sin(dx+c)^3 + (\sin(dx+c)^3 - 3\,\sin(dx+c))a^3 - (\cos(dx+c)^3 - 3\,\cos(dx}{3\,d}$$

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/3*(3*a^2*b*cos(d*x + c)^3 - 3*a*b^2*sin(d*x + c)^3 + (sin(d*x + c)^3 - 3
*sin(d*x + c))*a^3 - (cos(d*x + c)^3 - 3*cos(d*x + c))*b^3)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24430 vs. 2(68) = 136.

Time = 142.57 (sec) , antiderivative size = 24430, normalized size of antiderivative = 349.00

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^3\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/768*(72*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1
/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^
2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^
2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 - 105*pi*b^3*sgn(tan(1/
2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(
1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*
x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1
/2*d*x)^6*tan(1/2*c)^6 + 72*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*ta
n(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)
*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*
```

x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 - 105
*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan
(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c
)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1
/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 - 72*pi*a^2*b*sgn(tan(1/2*d*x)^2*t
an(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 +
 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 105*pi*b^3*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2
*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 72*pi*a^2*b*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
 tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 - 105*pi*b^
3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d
*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 - 14
4*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x
)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 210*pi*b^3*s
gn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c)
 - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 216*pi*a^2*b*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1
/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x
)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/
2*d*x)^6*tan(1/2*c)^4 - 315*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(
1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*s
gn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)
^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 + 216*p
i*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan
(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c
)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1
/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 - 315*pi*b^3*sgn(tan(1/2*d*x)^2*ta
n(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 -
2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*
c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*t
an(1/2*c)^4 + 216*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)
^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1
/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan
(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 - 315*pi*b^3*sg
n(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^
2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) -
1)*tan(1/2*d*x)^4*tan(1/2*c)^6 + 216*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)
^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/
2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - t
an(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c
)^6 - 315*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2
*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*
tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2

- 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 + 144*pi*a^2*b*tan(1/2*d*x)^6*tan(1/2*c)^6 - 210*pi*b^3*tan(1/2*d*x)^6*tan(1/2*c)^6 - 144*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 + 210*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 - 144*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 + 210*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 + 144*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 - 210*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 + 144*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 - 210*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 - 216*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 + 315*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 + 216*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 - 315*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 - 432*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 + 630*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 - 216*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 + 315*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 + 216*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 - 315*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 - 432*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 + 630*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 - 768*a^2*b*tan(1/2*d*x)^6*tan(1/2*c)^6 - 512*b^3*tan(1/2*d*x)^6*tan(1/2*c)^6 + 216*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1

/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*t
an(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^
2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(
1/2*c)^2 - 315*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*ta
n(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*
x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*
c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 + 216*pi*a^2*b*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1
/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*
tan(1/2*d*x)^6*tan(1/2*c)^2 - 315*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 -
2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c)
- 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/
2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 +
648*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c)
+ tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan
(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2
*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 - 945*pi*b^3*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c
)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*ta
n(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*
x)^4*tan(1/2*c)^4 + 648*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/
2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn
(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2
+ tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 - 945*pi*
b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2
*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
- 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d
*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 + 432*pi*a^2*b*tan(1/2*d*x)^6*tan(1/2*
c)^4 - 630*pi*b^3*tan(1/2*d*x)^6*tan(1/2*c)^4 - 432*a^2*b*arctan((tan(1/2*d
*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) -
tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 + 630*b^3*arcta
n((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*t
an(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 - 4
32*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(
tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*ta
n(1/2*c)^4 + 630*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1
/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1
/2*d*x)^6*tan(1/2*c)^4 + 432*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/
2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*
c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 - 630*b^3*arctan((tan(1/2*d*x)*tan(1/2
*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x
) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 + 432*a^2*b*arctan((tan(1/
2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c)
+ tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 - 630*b^3*ar

ctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x
)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^4
- 1536*a^3*tan(1/2*d*x)^6*tan(1/2*c)^5 + 216*pi*a^2*b*sgn(tan(1/2*d*x)^2*ta
n(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 +
2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*
c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*t
an(1/2*c)^6 - 315*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2
*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1
/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^6 + 216*pi*a^2*b*sg
n(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^
2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*ta
n(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) -
1)*tan(1/2*d*x)^2*tan(1/2*c)^6 - 315*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*
c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan
(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^
6 + 432*pi*a^2*b*tan(1/2*d*x)^4*tan(1/2*c)^6 - 630*pi*b^3*tan(1/2*d*x)^4*ta
n(1/2*c)^6 - 432*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan
(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan
(1/2*d*x)^4*tan(1/2*c)^6 + 630*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/
2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*
c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^6 - 432*a^2*b*arctan((tan(1/2*d*x)*tan(1
/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d
*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^6 + 630*b^3*arctan((tan(1/
2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c)
 + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^6 + 432*a^2*b*
arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d
*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^
6 - 630*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1
)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4
*tan(1/2*c)^6 + 432*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) -
tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*
tan(1/2*d*x)^4*tan(1/2*c)^6 - 630*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan
(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1
/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^6 - 1536*a^3*tan(1/2*d*x)^5*tan(1/2*c
)^6 - 216*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2
*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*
tan(1/2*c)^2 + 315*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*
tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*
d*x)^6*tan(1/2*c)^2 + 216*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(
1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)
*tan(1/2*d*x)^6*tan(1/2*c)^2 - 315*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 -
 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*
x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 - 432*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1

/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*ta n(1/2*d*x)^6*tan(1/2*c)^2 + 630*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - ta n(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6 *tan(1/2*c)^2 - 648*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d* x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1 /2*d*x)^4*tan(1/2*c)^4 + 945*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan (1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 + 648*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^ 2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2 *d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 - 945*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2* tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 - 1296*pi*a^2*b*sgn(tan(1/2*d *x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 + 1890*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1 /2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*ta n(1/2*d*x)^4*tan(1/2*c)^4 + 2304*a^2*b*tan(1/2*d*x)^6*tan(1/2*c)^4 - 1536*b ^3*tan(1/2*d*x)^6*tan(1/2*c)^4 + 9216*a^2*b*tan(1/2*d*x)^5*tan(1/2*c)^5 - 2 16*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2 *c)^6 + 315*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2 *c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2* tan(1/2*c)^6 + 216*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/ 2*d*x)^2*tan(1/2*c)^6 - 315*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1) *tan(1/2*d*x)^2*tan(1/2*c)^6 - 432*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d *x)^2*tan(1/2*c)^6 + 630*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d *x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2*tan(1/ 2*c)^6 + 2304*a^2*b*tan(1/2*d*x)^4*tan(1/2*c)^6 - 1536*b^3*tan(1/2*d*x)^4*t an(1/2*c)^6 + 72*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^ 2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/ 2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6 - 105*pi*b^3*sgn(tan(1/2*d*x) ^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c) ^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan (1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6 + 72*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/ 2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2 *tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6 - 105*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1 /2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*t an(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^ 2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6 + 64 8*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) +

tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/
2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*ta
n(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^2 - 945*pi*b^3*sgn(tan(1/2*d*x)^2
*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2
 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1
/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^
4*tan(1/2*c)^2 + 648*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d
*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^2 - 945*pi*b^3
*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*
x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2
*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x)
 - 1)*tan(1/2*d*x)^4*tan(1/2*c)^2 + 432*pi*a^2*b*tan(1/2*d*x)^6*tan(1/2*c)^
2 - 630*pi*b^3*tan(1/2*d*x)^6*tan(1/2*c)^2 - 432*a^2*b*arctan((tan(1/2*d*x)
*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan
(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 + 630*b^3*arctan((
tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(
1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 - 432*
a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan
(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1
/2*c)^2 + 630*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*
c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*
d*x)^6*tan(1/2*c)^2 + 432*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d
*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c)
- 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 - 630*b^3*arctan((tan(1/2*d*x)*tan(1/2*c)
 + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) -
 tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 + 432*a^2*b*arctan((tan(1/2*d
*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) +
tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 - 630*b^3*arcta
n((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*t
an(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 - 1
024*a^3*tan(1/2*d*x)^6*tan(1/2*c)^3 - 6144*a*b^2*tan(1/2*d*x)^6*tan(1/2*c)^
3 + 648*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2
*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*
tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2
+ 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^4 - 945*pi*b^3*sgn(tan(1/2*
d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/
2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)
*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2
*d*x)^2*tan(1/2*c)^4 + 648*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan
(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*
sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x
)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^4 - 945*
pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(

1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)
^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/
2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^4 + 1296*pi*a^2*b*tan(1/2*d*x)^4*tan(
1/2*c)^4 - 1890*pi*b^3*tan(1/2*d*x)^4*tan(1/2*c)^4 - 1296*a^2*b*arctan((tan
(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2
*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 + 1890*b^
3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2
*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c
)^4 - 1296*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c
) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d
*x)^4*tan(1/2*c)^4 + 1890*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x
) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) +
1))*tan(1/2*d*x)^4*tan(1/2*c)^4 + 1296*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c
) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x)
- tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 - 1890*b^3*arctan((tan(1/2*d
*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) -
tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 + 1296*a^2*b*ar
ctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x
)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^4
- 1890*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)
/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4*
tan(1/2*c)^4 + 4608*a^3*tan(1/2*d*x)^5*tan(1/2*c)^4 - 18432*a*b^2*tan(1/2*d
*x)^5*tan(1/2*c)^4 + 4608*a^3*tan(1/2*d*x)^4*tan(1/2*c)^5 - 18432*a*b^2*tan
(1/2*d*x)^4*tan(1/2*c)^5 + 72*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*
tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) -
1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*
d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^6 - 105*pi*b^3*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
 tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1
/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*
tan(1/2*c)^6 + 72*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)
^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1
/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan
(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^6 - 105*pi*b^3*sgn(tan(1/2*d*x)^
2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^
2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(
1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^6
 + 432*pi*a^2*b*tan(1/2*d*x)^2*tan(1/2*c)^6 - 630*pi*b^3*tan(1/2*d*x)^2*tan
(1/2*c)^6 - 432*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(
1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(
1/2*d*x)^2*tan(1/2*c)^6 + 630*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2
*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c
) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 - 432*a^2*b*arctan((tan(1/2*d*x)*tan(1/
2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*
x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 + 630*b^3*arctan((tan(1/2

*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 + 432*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 - 630*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 + 432*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 - 630*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 - 1024*a^3*tan(1/2*d*x)^3*tan(1/2*c)^6 - 6144*a*b^2*tan(1/2*d*x)^3*tan(1/2*c)^6 - 72*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6 + 105*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6 + 72*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6 - 105*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6 - 144*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6 + 210*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6 - 648*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^2 + 945*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^2 + 648*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^2 - 945*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^2 - 1296*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^2 + 1890*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^2 - 2304*a^2*b*tan(1/2*d*x)^6*tan(1/2*c)^2 + 1536*b^3*tan(1/2*d*x)^6*tan(1/2*c)^2 - 18432*a^2*b*tan(1/2*d*x)^5*tan(1/2*c)^3 + 12288*b^3*tan(1/2*d*x)^5*tan(1/2*c)^3 - 648*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^4 + 945*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^4 + 648*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^4 - 945*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1

)*tan(1/2*d*x)^2*tan(1/2*c)^4 - 1296*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2*tan(1/2*c)^4 + 1890*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2*tan(1/2*c)^4 - 43776*a^2*b*tan(1/2*d*x)^4*tan(1/2*c)^4 + 13824*b^3*tan(1/2*d*x)^4*tan(1/2*c)^4 - 18432*a^2*b*tan(1/2*d*x)^3*tan(1/2*c)^5 + 12288*b^3*tan(1/2*d*x)^3*tan(1/2*c)^5 - 72*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^6 + 105*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^6 + 72*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^6 - 105*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^6 - 144*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^6 + 210*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^6 - 2304*a^2*b*tan(1/2*d*x)^2*tan(1/2*c)^6 + 1536*b^3*tan(1/2*d*x)^2*tan(1/2*c)^6 + 216*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4 - 315*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4 + 216*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4 - 315*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4 + 144*pi*a^2*b*tan(1/2*d*x)^6 - 210*pi*b^3*tan(1/2*d*x)^6 - 144*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6 + 210*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6 - 144*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6 + 210*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6 + 144*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6 - 210*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6 + 144*a^2 *b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6 - 210*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6 - 1536*a^3*tan(1/2*d*x)^6*tan(1/2*c) + 648*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 945*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 648*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 945*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 1296*pi*a^2*b*tan(1/2*d*x)^4*tan(1/2*c)^2 - 1890*pi*b^3*tan(1/2*d*x)^4*tan(1/2*c)^2 - 1296*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^2 + 1890*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^2 - 1296*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^2 + 1890*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^2 + 1296*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^2 - 1890*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^2 + 1296*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^2 - 1890*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^2 - 4608*a^3*tan(1/2*d*x)^5*tan(1/2*c)^2 + 18432*a*b^2*tan(1/2*d*x)^5*tan(1/2*c)^2 - 21504*a^3*tan(1/2*d*x)^4*tan(1/2*c)^3 + 55296*a*b^2*tan(1/2*d*x)^4*tan(1/2*c)^3 + 216*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^4 - 315*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d *x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^4 + 216*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^4 - 315*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^4 + 1296*pi*a^2*b*tan(1/2*d*x)^2*tan(1/2*c)^4 - 1890*pi*b^3*tan(1/2*d*x)^2*tan(1/2*c)^4 - 1296*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^4 + 1890*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^4 - 1296*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^4 + 1890*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^4 + 1296*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^4 - 1890*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^4 + 1296*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^4 - 1890*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^4 - 21504*a^3*tan(1/2*d*x)^3*tan(1/2*c)^4 + 55296*a*b^2*tan(1/2*d*x)^3*tan(1/2*c)^4 - 4608*a^3*tan(1/2*d*x)^2*tan(1/2*c)^5 + 18432*a*b^2*tan(1/2*d*x)^2*tan(1/2*c)^5 + 144*pi*a^2*b*tan(1/2*c)^6 - 210*pi*b^3*tan(1/2*c)^6 - 144*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*c)^6 + 210*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*c)^6 - 144*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*c)^6 + 210*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*c)^6 + 144*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*c)^6 - 210*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*c)^6 + 144*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^6 - 210*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(

1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^6 - 1536*a^3*tan(1/2*d*x)*tan(1/2*c)^6 - 216*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4 + 315*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4 + 216*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4 - 315*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4 - 432*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4 + 630*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4 + 768*a^2*b*tan(1/2*d*x)^6 + 512*b^3*tan(1/2*d*x)^6 + 9216*a^2*b*tan(1/2*d*x)^5*tan(1/2*c) - 648*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 945*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 648*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 945*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 1296*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 1890*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 43776*a^2*b*tan(1/2*d*x)^4*tan(1/2*c)^2 - 13824*b^3*tan(1/2*d*x)^4*tan(1/2*c)^2 + 86016*a^2*b*tan(1/2*d*x)^3*tan(1/2*c)^3 - 16384*b^3*tan(1/2*d*x)^3*tan(1/2*c)^3 - 216*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^4 + 315*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^4 + 216*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^4 - 315*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^4 - 432*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^4 + 630*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^4 + 43776*a^2*b*tan(1/2*d*x)^2*tan(1/2*c)^4 - 13824*b^3*tan(1/2*d*x)^2*tan(1/2*c)^4 + 9216*a^2*b*tan(1/2*d*x)*tan(1/2*c)^5 + 768*a^2*b*tan(1/2*c)^6 + 512*b^3*tan(1/2*c)^6 + 216*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2 - 315*pi*b^3*sgn(tan(1/2*d*x)^2*tan(

1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*
tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)
^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2 + 2
16*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) +
 tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1
/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*t
an(1/2*d*x) - 1)*tan(1/2*d*x)^2 - 315*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2
*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - ta
n(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2 + 432*pi*a
^2*b*tan(1/2*d*x)^4 - 630*pi*b^3*tan(1/2*d*x)^4 - 432*a^2*b*arctan((tan(1/2
*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c)
- tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4 + 630*b^3*arctan((tan(1/2*
d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) -
 tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4 - 432*a^2*b*arctan((tan(1/2
*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c)
+ tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4 + 630*b^3*arctan((tan(1/2*
d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) +
 tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4 + 432*a^2*b*arctan((tan(1/2
*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c)
- tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4 - 630*b^3*arctan((tan(1/2*
d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) -
 tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4 + 432*a^2*b*arctan((tan(1/2
*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c)
+ tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4 - 630*b^3*arctan((tan(1/2*
d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) +
 tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4 + 1536*a^3*tan(1/2*d*x)^5 +
 4608*a^3*tan(1/2*d*x)^4*tan(1/2*c) - 18432*a*b^2*tan(1/2*d*x)^4*tan(1/2*c)
 + 216*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*
c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*t
an(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 +
 2*tan(1/2*d*x) - 1)*tan(1/2*c)^2 - 315*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c
)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1
/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 -
tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^2 + 216*pi*a
^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/
2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*
d*x) - 1)*tan(1/2*c)^2 - 315*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan
(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*
sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x
)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^2 + 1296*pi*a^2*b*tan(1
/2*d*x)^2*tan(1/2*c)^2 - 1890*pi*b^3*tan(1/2*d*x)^2*tan(1/2*c)^2 - 1296*a^2
*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/
2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*

c)^2 + 1890*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 1296*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 1890*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 1296*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 1890*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 1296*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 1890*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 21504*a^3*tan(1/2*d*x)^3*tan(1/2*c)^2 - 55296*a*b^2*tan(1/2*d*x)^3*tan(1/2*c)^2 + 21504*a^3*tan(1/2*d*x)^2*tan(1/2*c)^3 - 55296*a*b^2*tan(1/2*d*x)^2*tan(1/2*c)^3 + 432*pi*a^2*b*tan(1/2*c)^4 - 630*pi*b^3*tan(1/2*c)^4 - 432*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*c)^4 + 630*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*c)^4 - 432*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*c)^4 + 630*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*c)^4 + 432*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*c)^4 - 630*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*c)^4 + 432*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^4 - 630*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^4 + 4608*a^3*tan(1/2*d*x)*tan(1/2*c)^4 - 18432*a*b^2*tan(1/2*d*x)*tan(1/2*c)^4 + 1536*a^3*tan(1/2*c)^5 - 216*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2 + 315*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2 + 216*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2 - 315*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2 - 432*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2 + 630*pi*b^3*

sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c
) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2 - 2304*a^2*b*tan(1/2*d*x)^4 + 1536*b^3
*tan(1/2*d*x)^4 - 18432*a^2*b*tan(1/2*d*x)^3*tan(1/2*c) + 12288*b^3*tan(1/2
*d*x)^3*tan(1/2*c) - 216*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1
/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*
tan(1/2*c)^2 + 315*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*
tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*
c)^2 + 216*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/
2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^2 -
315*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 -
tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^2 - 432*pi*
a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan
(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^2 + 630*pi*b^3*sgn(tan(1/2*d*x)^2*ta
n(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)
*tan(1/2*c)^2 - 43776*a^2*b*tan(1/2*d*x)^2*tan(1/2*c)^2 + 13824*b^3*tan(1/2
*d*x)^2*tan(1/2*c)^2 - 18432*a^2*b*tan(1/2*d*x)*tan(1/2*c)^3 + 12288*b^3*ta
n(1/2*d*x)*tan(1/2*c)^3 - 2304*a^2*b*tan(1/2*c)^4 + 1536*b^3*tan(1/2*c)^4 +
72*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c)
+ tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(
1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*
tan(1/2*d*x) - 1) - 105*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1) + 72*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/
2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*ta
n(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2
- tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1) - 105*pi*b^3*sgn(tan
(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - t
an(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2
*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1) +
432*pi*a^2*b*tan(1/2*d*x)^2 - 630*pi*b^3*tan(1/2*d*x)^2 - 432*a^2*b*arctan(
(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan
(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2 + 630*b^3*arctan((
tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(
1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2 - 432*a^2*b*arctan(
(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan
(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2 + 630*b^3*arctan((
tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(
1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2 + 432*a^2*b*arctan(
(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan
(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2 - 630*b^3*arctan((
tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(
1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2 + 432*a^2*b*arctan(
(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan
(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2 - 630*b^3*arctan((

tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(
1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2 + 1024*a^3*tan(1/2*
d*x)^3 + 6144*a*b^2*tan(1/2*d*x)^3 - 4608*a^3*tan(1/2*d*x)^2*tan(1/2*c) + 1
8432*a*b^2*tan(1/2*d*x)^2*tan(1/2*c) + 432*pi*a^2*b*tan(1/2*c)^2 - 630*pi*b
^3*tan(1/2*c)^2 - 432*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x)
- tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)
)*tan(1/2*c)^2 + 630*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - t
an(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*t
an(1/2*c)^2 - 432*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + ta
n(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*ta
n(1/2*c)^2 + 630*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1
/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1
/2*c)^2 + 432*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/
2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/
2*c)^2 - 630*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c
) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*c
)^2 + 432*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c)
 - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)
^2 - 630*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) -
1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^2
- 4608*a^3*tan(1/2*d*x)*tan(1/2*c)^2 + 18432*a*b^2*tan(1/2*d*x)*tan(1/2*c)^
2 + 1024*a^3*tan(1/2*c)^3 + 6144*a*b^2*tan(1/2*c)^3 - 72*pi*a^2*b*sgn(tan(1
/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan
(1/2*c)^2 + 2*tan(1/2*d*x) - 1) + 105*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2
*d*x) - 1) + 72*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*t
an(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1) - 105*pi*
b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2
*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1) - 144*pi*a^2*b*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^
2 + 1) + 210*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*ta
n(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1) + 2304*a^2*b*tan(1/2*d*x)^2 - 153
6*b^3*tan(1/2*d*x)^2 + 9216*a^2*b*tan(1/2*d*x)*tan(1/2*c) + 2304*a^2*b*tan(
1/2*c)^2 - 1536*b^3*tan(1/2*c)^2 + 144*pi*a^2*b - 210*pi*b^3 - 144*a^2*b*ar
ctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x
)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)) + 210*b^3*arctan((tan(1/2*d*
x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - t
an(1/2*d*x) + tan(1/2*c) + 1)) - 144*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c)
- tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) -
tan(1/2*c) + 1)) + 210*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) +
 tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))
 + 144*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) -
1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)) - 210*b^3*arc
tan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)
*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)) + 144*a^2*b*arctan((tan(1/2*d

```
*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) +
tan(1/2*d*x) + tan(1/2*c) - 1)) - 210*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) -
 tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + t
an(1/2*c) - 1)) + 1536*a^3*tan(1/2*d*x) + 1536*a^3*tan(1/2*c) - 768*a^2*b -
 512*b^3)/(d*tan(1/2*d*x)^6*tan(1/2*c)^6 + 3*d*tan(1/2*d*x)^6*tan(1/2*c)^4
+ 3*d*tan(1/2*d*x)^4*tan(1/2*c)^6 + 3*d*tan(1/2*d*x)^6*tan(1/2*c)^2 + 9*d*t
an(1/2*d*x)^4*tan(1/2*c)^4 + 3*d*tan(1/2*d*x)^2*tan(1/2*c)^6 + d*tan(1/2*d*
x)^6 + 9*d*tan(1/2*d*x)^4*tan(1/2*c)^2 + 9*d*tan(1/2*d*x)^2*tan(1/2*c)^4 +
d*tan(1/2*c)^6 + 3*d*tan(1/2*d*x)^4 + 9*d*tan(1/2*d*x)^2*tan(1/2*c)^2 + 3*d
*tan(1/2*c)^4 + 3*d*tan(1/2*d*x)^2 + 3*d*tan(1/2*c)^2 + d)
```

Mupad [B] (verification not implemented)

Time = 4.22 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.49

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$=\frac{\frac{\sin(c+d\,x)\,a^3\cos(c+d\,x)^2}{3}+\frac{2\,\sin(c+d\,x)\,a^3}{3}-a^2\,b\cos\left(c+d\,x\right)^3-\sin\left(c+d\,x\right)\,a\,b^2\cos\left(c+d\,x\right)^2+\sin\left(c+d\,x\right)\,a\,b}{d}$$

```
[In] int(cos(c + d*x)^3*(a + b*tan(c + d*x))^3,x)
```

```
[Out] ((2*a^3*sin(c + d*x))/3 - b^3*cos(c + d*x) + (b^3*cos(c + d*x)^3)/3 - a^2*b
*cos(c + d*x)^3 + (a^3*cos(c + d*x)^2*sin(c + d*x))/3 + a*b^2*sin(c + d*x)
- a*b^2*cos(c + d*x)^2*sin(c + d*x))/d
```

3.542 $\int \cos^5(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{2(4a^2+b^2)\cos(c+dx)(b-a\tan(c+dx))}{15d} - \frac{\cos^3(c+dx)(b-4a\tan(c+dx))(a+b\tan(c+dx))^2}{15d} + \frac{\cos^4(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{5d}$$

```
[Out] -2/15*(4*a^2+b^2)*cos(d*x+c)*(b-a*tan(d*x+c))/d-1/15*cos(d*x+c)^3*(b-4*a*ta
n(d*x+c))*(a+b*tan(d*x+c))^2/d+1/5*cos(d*x+c)^4*sin(d*x+c)*(a+b*tan(d*x+c))
^3/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3593, 751, 819, 651}

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{2(4a^2+b^2)\cos(c+dx)(b-a\tan(c+dx))}{15d} - \frac{\cos^3(c+dx)(b-4a\tan(c+dx))(a+b\tan(c+dx))^2}{15d} + \frac{\sin(c+dx)\cos^4(c+dx)(a+b\tan(c+dx))^3}{5d}$$

```
[In] Int[Cos[c + d*x]^5*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-2*(4*a^2 + b^2)*Cos[c + d*x]*(b - a*Tan[c + d*x]))/(15*d) - (Cos[c + d*x]
^3*(b - 4*a*Tan[c + d*x])*(a + b*Tan[c + d*x])^2)/(15*d) + (Cos[c + d*x]^4*
Sin[c + d*x]*(a + b*Tan[c + d*x])^3)/(5*d)
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 751

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-x)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*(p + 1))), x] + Dist[1/(2*a*(p +
 1)), Int[(d + e*x)^(m - 1)*(d*(2*p + 3) + e*(m + 2*p + 3)*x)*(a + c*x^2)^(
p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p,
 -1] && GtQ[m, 0] && (LtQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) &&
IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Dist[m*((c*d*f + a*e*g)/(2*a*c*(p + 1))), Int[(d + e*x)^(m
 - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\left(\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int\frac{(a+x)^3}{\left(1+\frac{x^2}{b^2}\right)^{7/2}}\,dx,x,b\tan(c+dx)\right)}{bd}$$

$$=\frac{\cos^4(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{5d}-\frac{\left(\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int\frac{(-4a-x)(a+x)^2}{\left(1+\frac{x^2}{b^2}\right)^{5/2}}\,dx,x,b\tan(c+dx)\right)}{5bd}$$

$$= -\frac{\cos^3(c+dx)(b-4a\tan(c+dx))(a+b\tan(c+dx))^2}{15d} + \frac{\cos^4(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{5d} + \frac{\left(2(4a^2+b^2)\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{a+x}{\left(1+\frac{x^2}{b^2}\right)^{3/2}}\,dx, x, b\tan(c+dx)\right)}{15bd}$$

$$= -\frac{2(4a^2+b^2)\cos(c+dx)(b-a\tan(c+dx))}{15d} - \frac{\cos^3(c+dx)(b-4a\tan(c+dx))(a+b\tan(c+dx))^2}{15d} + \frac{\cos^4(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{5d}$$

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.15

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{-9a^2b\cos^5(c+dx)+15a^3\sin(c+dx)-5a(2a^2-3b^2)\sin^3(c+dx)+3a(a^2-3b^2)\sin^5(c+dx)+b^3\cos(c}{15d}$$

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-9*a^2*b*Cos[c + d*x]^5 + 15*a^3*Sin[c + d*x] - 5*a*(2*a^2 - 3*b^2)*Sin[c
+ d*x]^3 + 3*a*(a^2 - 3*b^2)*Sin[c + d*x]^5 + b^3*Cos[c + d*x]*(-2 + 2/Sqrt
[Cos[c + d*x]^2] - Sin[c + d*x]^2 + 3*Sin[c + d*x]^4))/(15*d)
```

Maple [A] (verified)

Time = 40.22 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.19

method	result
derivativedivides	$\frac{b^3\left(-\frac{(\cos^3(dx+c))(\sin^2(dx+c))}{5}-\frac{2(\cos^3(dx+c))}{15}\right)+3a\,b^2\left(-\frac{\sin(dx+c)(\cos^4(dx+c))}{5}+\frac{(2+\cos^2(dx+c))\sin(dx+c)}{15}\right)-\frac{3a^2b(\cos^5}{5}}{d}$
default	$\frac{b^3\left(-\frac{(\cos^3(dx+c))(\sin^2(dx+c))}{5}-\frac{2(\cos^3(dx+c))}{15}\right)+3a\,b^2\left(-\frac{\sin(dx+c)(\cos^4(dx+c))}{5}+\frac{(2+\cos^2(dx+c))\sin(dx+c)}{15}\right)-\frac{3a^2b(\cos^5}{5}}{d}$
risch	$-\frac{3b\cos(dx+c)a^2}{8d}-\frac{b^3\cos(dx+c)}{8d}+\frac{5a^3\sin(dx+c)}{8d}+\frac{3a\sin(dx+c)b^2}{8d}-\frac{3b\cos(5dx+5c)a^2}{80d}+\frac{b^3\cos(5dx+5c)}{80d}+$

```
[In] int(cos(d*x+c)^5*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^3*(-1/5*cos(d*x+c)^3*sin(d*x+c)^2-2/15*cos(d*x+c)^3)+3*a*b^2*(-1/5*s
in(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-3/5*a^2*b*cos(d*x+
c)^5+1/5*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.97

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{5\,b^3\cos\left(dx+c\right)^3 + 3\left(3\,a^2b - b^3\right)\cos\left(dx+c\right)^5 - \left(3\left(a^3 - 3\,ab^2\right)\cos\left(dx+c\right)^4 + 8\,a^3 + 6\,ab^2 + \left(4\,a^3 + 3\right.\right.}{15\,d}$$

```
[In] integrate(cos(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(5*b^3*cos(d*x + c)^3 + 3*(3*a^2*b - b^3)*cos(d*x + c)^5 - (3*(a^3 -
3*a*b^2)*cos(d*x + c)^4 + 8*a^3 + 6*a*b^2 + (4*a^3 + 3*a*b^2)*cos(d*x + c)^
2)*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^3\,dx = \int \left(a + b\tan\left(c + dx\right)\right)^3\cos^5\left(c + dx\right)dx$$

```
[In] integrate(cos(d*x+c)**5*(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**3*cos(c + d*x)**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.02

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{9\,a^2b\cos\left(dx+c\right)^5 - \left(3\,\sin\left(dx+c\right)^5 - 10\,\sin\left(dx+c\right)^3 + 15\,\sin\left(dx+c\right)\right)a^3 + 3\left(3\,\sin\left(dx+c\right)^5 - 5\,\mathrm{s}\right.}{15\,d}$$

```
[In] integrate(cos(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/15*(9*a^2*b*cos(d*x + c)^5 - (3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*
sin(d*x + c))*a^3 + 3*(3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*a*b^2 - (3*cos(
d*x + c)^5 - 5*cos(d*x + c)^3)*b^3)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56572 vs. $2(102) = 204$.

Time = 274.60 (sec) , antiderivative size = 56572, normalized size of antiderivative = 538.78

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^3\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^5*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/15360*(1080*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*t
an(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2
*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 - 105*pi*b^3*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2
- tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(
1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)
*tan(1/2*d*x)^10*tan(1/2*c)^10 + 1080*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c
)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1
/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 -
tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2
*c)^10 - 105*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(
1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)
^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)
^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 - 1800*pi*a^2*b*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2
+ tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 + 1995*p
i*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1
/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^1
0 + 1800*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*
c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*
tan(1/2*c)^10 - 1995*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x
)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/
2*d*x)^10*tan(1/2*c)^10 - 5760*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - t
an(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^
10*tan(1/2*c)^10 + 210*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x
)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^10*tan(1/2
*c)^10 + 5400*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*t
an(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2
*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^8 - 525*pi*b^3*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
 tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1
/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*
tan(1/2*d*x)^10*tan(1/2*c)^8 + 5400*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
```

2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2
*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - ta
n(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c
)^8 - 525*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2
*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*
tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2
- 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^8 + 5400*pi*a^2*b*sgn(tan(
1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - ta
n(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*
d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan
(1/2*d*x)^8*tan(1/2*c)^10 - 525*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*
tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) -
1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*
d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^10 +
5400*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c)
 + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan
(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2
*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^10 - 525*pi*b^3*sgn(tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*
c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*t
an(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d
*x)^8*tan(1/2*c)^10 + 5760*pi*a^2*b*tan(1/2*d*x)^10*tan(1/2*c)^10 - 210*pi*
b^3*tan(1/2*d*x)^10*tan(1/2*c)^10 - 2160*a^2*b*arctan((tan(1/2*d*x)*tan(1/2
*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x
) + tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^10 + 210*b^3*arctan((tan(1/
2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c)
 - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^10 - 2160*a^2
*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/
2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2
*c)^10 + 210*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c
) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d
*x)^10*tan(1/2*c)^10 + 9360*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2
*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c
) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^10 - 210*b^3*arctan((tan(1/2*d*x)*tan(1/
2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*
x) - tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^10 + 9360*a^2*b*arctan((ta
n(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/
2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^10 - 210*
b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1
/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/
2*c)^10 - 9000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*ta
n(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*
x)^10*tan(1/2*c)^8 + 9975*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/
2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*t
an(1/2*d*x)^10*tan(1/2*c)^8 + 9000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2

- 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*
d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^8 - 9975*pi*b^3*sgn(tan(1/2*d*x)^2*tan
(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2
*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^8 - 28800*pi*a^2*b*sgn(tan(1/
2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/
2*c)^2 + 1)*tan(1/2*d*x)^10*tan(1/2*c)^8 + 1050*pi*b^3*sgn(tan(1/2*d*x)^2*t
an(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1
)*tan(1/2*d*x)^10*tan(1/2*c)^8 - 9000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c
)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1
/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^10 + 9975*pi*b^3*sgn(tan(1/2*d*x)^2*
tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2
+ 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^10 + 9000*pi*a^2*b*sgn(tan(
1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + ta
n(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^10 - 9975*pi*b^3
*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*
x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^10 - 28
800*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d
*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^8*tan(1/2*c)^10 + 1050*pi*b
^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/
2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^8*tan(1/2*c)^10 - 9216*a^2*b*tan(1/2*
d*x)^10*tan(1/2*c)^10 - 2048*b^3*tan(1/2*d*x)^10*tan(1/2*c)^10 + 10800*pi*a
^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/
2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
+ 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*
d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^6 - 1050*pi*b^3*sgn(tan(1/2*d*x)^2*tan
(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2
*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c
)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*t
an(1/2*c)^6 + 10800*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*
x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan
(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + t
an(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^6 - 1050*pi*b^
3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d
*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 -
2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x
) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^6 + 27000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan
(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2
*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c
)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*ta
n(1/2*c)^8 - 2625*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2
*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1
/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^8 + 27000*pi*a^2*b*
sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x
)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*

tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x)
- 1)*tan(1/2*d*x)^8*tan(1/2*c)^8 - 2625*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c
)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1
/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 -
tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*
c)^8 + 28800*pi*a^2*b*tan(1/2*d*x)^10*tan(1/2*c)^8 - 1050*pi*b^3*tan(1/2*d*
x)^10*tan(1/2*c)^8 - 10800*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*
d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c)
 + 1))*tan(1/2*d*x)^10*tan(1/2*c)^8 + 1050*b^3*arctan((tan(1/2*d*x)*tan(1/2
*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x
) + tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^8 - 10800*a^2*b*arctan((tan
(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2
*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^8 + 1050*b
^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/
2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2
*c)^8 + 46800*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/
2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/
2*d*x)^10*tan(1/2*c)^8 - 1050*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2
*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c
) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^8 + 46800*a^2*b*arctan((tan(1/2*d*x)*tan
(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2
*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^8 - 1050*b^3*arctan((ta
n(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/
2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^8 - 30720
*a^3*tan(1/2*d*x)^10*tan(1/2*c)^9 + 10800*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1
/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*t
an(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^
2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(
1/2*c)^10 - 1050*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*
tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*
d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/
2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^10 + 10800*pi*a^2*b*
sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x
)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*
tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x)
- 1)*tan(1/2*d*x)^6*tan(1/2*c)^10 - 1050*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*
c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(
1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 -
 tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2
*c)^10 + 28800*pi*a^2*b*tan(1/2*d*x)^8*tan(1/2*c)^10 - 1050*pi*b^3*tan(1/2*
d*x)^8*tan(1/2*c)^10 - 10800*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/
2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*
c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^10 + 1050*b^3*arctan((tan(1/2*d*x)*tan(1
/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d
*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^10 - 10800*a^2*b*arctan((t

an(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1
/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^10 + 1050
*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(
1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/
2*c)^10 + 46800*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(
1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(
1/2*d*x)^8*tan(1/2*c)^10 - 1050*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1
/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2
*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^10 + 46800*a^2*b*arctan((tan(1/2*d*x)*t
an(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1
/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^10 - 1050*b^3*arctan((
tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(
1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^10 - 307
20*a^3*tan(1/2*d*x)^9*tan(1/2*c)^10 - 18000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan
(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2
*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^6 + 19950*pi*b^3*sgn(tan(1/2*
d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/
2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^6 + 18000*pi*a^2*b*
sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x
)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^6 - 199
50*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - t
an(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*
c)^6 - 57600*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*
tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^10*tan(1/2*c)^6 +
2100*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*
x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^10*tan(1/2*c)^6 - 45000*pi*a
^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/
2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^8 +
49875*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2
- tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1
/2*c)^8 + 45000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*t
an(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d
*x)^8*tan(1/2*c)^8 - 49875*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1
/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*
tan(1/2*d*x)^8*tan(1/2*c)^8 - 144000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)
^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2
*d*x)^8*tan(1/2*c)^8 + 5250*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/
2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^8*tan
(1/2*c)^8 + 46080*a^2*b*tan(1/2*d*x)^10*tan(1/2*c)^8 - 10240*b^3*tan(1/2*d*
x)^10*tan(1/2*c)^8 + 184320*a^2*b*tan(1/2*d*x)^9*tan(1/2*c)^9 - 18000*pi*a^
2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2
*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^10 +
19950*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2
- tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1
/2*c)^10 + 18000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*

tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^10 - 19950*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^10 - 57600*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^10 + 2100*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^10 + 46080*a^2*b*tan(1/2*d*x)^8*tan(1/2*c)^10 - 10240*b^3*tan(1/2*d*x)^8*tan(1/2*c)^10 + 10800*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^4 - 1050*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^4 + 10800*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^4 - 1050*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^4 + 54000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^6 - 5250*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^6 + 54000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^6 - 5250*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^6 + 57600*pi*a^2*b*tan(1/2*d*x)^10*tan(1/2*c)^6 - 2100*pi*b^3*tan(1/2*d*x)^10*tan(1/2*c)^6 - 21600*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^6 + 2100*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^6 - 21600*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*ta n(1/2*d*x)^10*tan(1/2*c)^6 + 2100*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^6 + 93600*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^6 - 2100*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^6 + 93600*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^6 - 2100*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^6 - 40960*a^3*tan(1/2*d*x)^10*tan(1/2*c)^7 - 122880*a*b^2*tan(1/2*d*x)^10*tan(1/2*c)^7 + 54000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^8 - 5250*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^8 + 54000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^8 - 5250*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^8 + 144000*pi*a^2*b*tan(1/2*d*x)^8*tan(1/2*c)^8 - 5250*pi*b^3*tan(1/2*d*x)^8*tan(1/2*c)^8 - 54000*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 + 5250*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 - 54000*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 + 5250*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 + 234000*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 - 5250*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 + 234000*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 - 5250*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c)

+ tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 + 153600*a^3
*tan(1/2*d*x)^9*tan(1/2*c)^8 - 368640*a*b^2*tan(1/2*d*x)^9*tan(1/2*c)^8 + 1
53600*a^3*tan(1/2*d*x)^8*tan(1/2*c)^9 - 368640*a*b^2*tan(1/2*d*x)^8*tan(1/2
*c)^9 + 10800*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*t
an(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2
*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^10 - 1050*pi*b^3*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2
- tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(
1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)
*tan(1/2*d*x)^4*tan(1/2*c)^10 + 10800*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c
)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1
/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 -
tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*
c)^10 - 1050*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(
1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)
^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)
^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^10 + 57600*pi*a^2*b*tan(
1/2*d*x)^6*tan(1/2*c)^10 - 2100*pi*b^3*tan(1/2*d*x)^6*tan(1/2*c)^10 - 21600
*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(ta
n(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(
1/2*c)^10 + 2100*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1
/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1
/2*d*x)^6*tan(1/2*c)^10 - 21600*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan
(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1
/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^10 + 2100*b^3*arctan((tan(1/2*d*x)*ta
n(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/
2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^10 + 93600*a^2*b*arctan
((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*ta
n(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^10 - 2
100*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(t
an(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan
(1/2*c)^10 + 93600*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - t
an(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*t
an(1/2*d*x)^6*tan(1/2*c)^10 - 2100*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - ta
n(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(
1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^10 - 40960*a^3*tan(1/2*d*x)^7*tan(1/
2*c)^10 - 122880*a*b^2*tan(1/2*d*x)^7*tan(1/2*c)^10 - 18000*pi*a^2*b*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^4 + 19950*pi*
b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2
*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^4 +
18000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)
^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*ta
n(1/2*c)^4 - 19950*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*

tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*
d*x)^10*tan(1/2*c)^4 - 57600*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan
(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^10
*tan(1/2*c)^4 + 2100*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^
2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^10*tan(1/2*c
)^4 - 90000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1
/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^
8*tan(1/2*c)^6 + 99750*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d
*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(
1/2*d*x)^8*tan(1/2*c)^6 + 90000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 -
2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x
) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^6 - 99750*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/
2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*ta
n(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^6 - 288000*pi*a^2*b*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c
)^2 + 1)*tan(1/2*d*x)^8*tan(1/2*c)^6 + 10500*pi*b^3*sgn(tan(1/2*d*x)^2*tan(
1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*t
an(1/2*d*x)^8*tan(1/2*c)^6 - 92160*a^2*b*tan(1/2*d*x)^10*tan(1/2*c)^6 + 409
60*b^3*tan(1/2*d*x)^10*tan(1/2*c)^6 - 737280*a^2*b*tan(1/2*d*x)^9*tan(1/2*c
)^7 + 245760*b^3*tan(1/2*d*x)^9*tan(1/2*c)^7 - 90000*pi*a^2*b*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2
*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^8 + 99750*pi*b^3*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2
+ tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^8 + 90000*pi
*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(
1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^8
 - 99750*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)
^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan
(1/2*c)^8 - 288000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^
2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)
^8 + 10500*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(
1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^8 - 17049
60*a^2*b*tan(1/2*d*x)^8*tan(1/2*c)^8 + 317440*b^3*tan(1/2*d*x)^8*tan(1/2*c)
^8 - 737280*a^2*b*tan(1/2*d*x)^7*tan(1/2*c)^9 + 245760*b^3*tan(1/2*d*x)^7*t
an(1/2*c)^9 - 18000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*
x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1
/2*d*x)^4*tan(1/2*c)^10 + 19950*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*
tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x)
- 1)*tan(1/2*d*x)^4*tan(1/2*c)^10 + 18000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1
/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*t
an(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^10 - 19950*pi*b^3*sgn(tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*
c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^10 - 57600*pi*a^2*b*sg
n(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c)
- tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^10 + 2100*pi*b^3*sgn(tan(1/2*

d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*
c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^10 - 92160*a^2*b*tan(1/2*d*x)^6*tan(1/2
*c)^10 + 40960*b^3*tan(1/2*d*x)^6*tan(1/2*c)^10 + 5400*pi*a^2*b*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1
/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x
)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/
2*d*x)^10*tan(1/2*c)^2 - 525*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan
(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*
sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^2 + 540
0*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) +
tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/
2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*ta
n(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^2 - 525*pi*b^3*sgn(tan(1/2*d*x)^
2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^
2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(
1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)
^10*tan(1/2*c)^2 + 54000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1
/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sg
n(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^
2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^4 - 5250*p
i*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1
/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2
*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^4 + 54000*pi*a^2*b*sgn(tan(1/2*d*x)^2*
tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2
- 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/
2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8
*tan(1/2*c)^4 - 5250*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x
)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(
1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + ta
n(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^4 + 57600*pi*a^2
*b*tan(1/2*d*x)^10*tan(1/2*c)^4 - 2100*pi*b^3*tan(1/2*d*x)^10*tan(1/2*c)^4
- 21600*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) +
1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)
^10*tan(1/2*c)^4 + 2100*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x)
- tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)
)*tan(1/2*d*x)^10*tan(1/2*c)^4 - 21600*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c
) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x)
- tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^4 + 2100*b^3*arctan((tan(1/2*
d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) +
tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^4 + 93600*a^2*b
*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*
d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c
)^4 - 2100*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c)

- 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^4 + 93600*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d *x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^4 - 2100*b^3*arctan((tan(1/2*d*x)*tan(1/2* c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^4 - 118784*a^3*tan(1/2*d*x)^ 10*tan(1/2*c)^5 + 49152*a*b^2*tan(1/2*d*x)^10*tan(1/2*c)^5 + 108000*pi*a^2* b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d *x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 - 10500*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/ 2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*ta n(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1 /2*c)^6 + 108000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^ 2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/ 2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 - 10500*pi*b^3*s gn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x) ^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*t an(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 288000*pi*a^2*b*tan(1/2*d*x)^8*tan(1/2*c) ^6 - 10500*pi*b^3*tan(1/2*d*x)^8*tan(1/2*c)^6 - 108000*a^2*b*arctan((tan(1/ 2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^6 + 10500*b^3* arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d *x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^ 6 - 108000*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d *x)^8*tan(1/2*c)^6 + 10500*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d* x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^6 + 468000*a^2*b*arctan((tan(1/2*d*x)*tan(1/ 2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d* x) - tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^6 - 10500*b^3*arctan((tan(1 /2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^6 + 468000*a^ 2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1 /2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2 *c)^6 - 10500*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2* c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2* d*x)^8*tan(1/2*c)^6 - 307200*a^3*tan(1/2*d*x)^9*tan(1/2*c)^6 + 1105920*a*b^ 2*tan(1/2*d*x)^9*tan(1/2*c)^6 - 1433600*a^3*tan(1/2*d*x)^8*tan(1/2*c)^7 + 3 072000*a*b^2*tan(1/2*d*x)^8*tan(1/2*c)^7 + 54000*pi*a^2*b*sgn(tan(1/2*d*x)^ 2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^ 2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(

1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)
^4*tan(1/2*c)^8 - 5250*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d
*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^8 + 54000*pi*a
^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/
2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*
d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^8 - 5250*pi*b^3*sgn(tan(1/2*d*x)^2*tan(
1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*
tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)
^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan
(1/2*c)^8 + 288000*pi*a^2*b*tan(1/2*d*x)^6*tan(1/2*c)^8 - 10500*pi*b^3*tan(
1/2*d*x)^6*tan(1/2*c)^8 - 108000*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + ta
n(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(
1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^8 + 10500*b^3*arctan((tan(1/2*d*x)*t
an(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1
/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^8 - 108000*a^2*b*arcta
n((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*t
an(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^8 + 1
0500*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(
tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*ta
n(1/2*c)^8 + 468000*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) +
tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*
tan(1/2*d*x)^6*tan(1/2*c)^8 - 10500*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + t
an(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan
(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^8 + 468000*a^2*b*arctan((tan(1/2*d*
x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + t
an(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^8 - 10500*b^3*arct
an((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*
tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^8 -
1433600*a^3*tan(1/2*d*x)^7*tan(1/2*c)^8 + 3072000*a*b^2*tan(1/2*d*x)^7*tan(
1/2*c)^8 - 307200*a^3*tan(1/2*d*x)^6*tan(1/2*c)^9 + 1105920*a*b^2*tan(1/2*d
*x)^6*tan(1/2*c)^9 + 5400*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(
1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*s
gn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)
^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^10 - 525*
pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(
1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)
^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/
2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^10 + 5400*pi*a^2*b*sgn(tan(1/2*d*x)^2
*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2
 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1
/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^
2*tan(1/2*c)^10 - 525*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*

x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan
(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + t
an(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^10 + 57600*pi*a
^2*b*tan(1/2*d*x)^4*tan(1/2*c)^10 - 2100*pi*b^3*tan(1/2*d*x)^4*tan(1/2*c)^1
0 - 21600*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c)
 + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*
x)^4*tan(1/2*c)^10 + 2100*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x
) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) +
1))*tan(1/2*d*x)^4*tan(1/2*c)^10 - 21600*a^2*b*arctan((tan(1/2*d*x)*tan(1/2
*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x
) - tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^10 + 2100*b^3*arctan((tan(1/
2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c)
 + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^10 + 93600*a^2
*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/
2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*
c)^10 - 2100*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c
) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d
*x)^4*tan(1/2*c)^10 + 93600*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2
*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c
) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^10 - 2100*b^3*arctan((tan(1/2*d*x)*tan(1/
2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*
x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^10 - 118784*a^3*tan(1/2*d*x
)^5*tan(1/2*c)^10 + 49152*a*b^2*tan(1/2*d*x)^5*tan(1/2*c)^10 - 9000*pi*a^2*
b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d
*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^2 + 9
975*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 -
tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2
*c)^2 + 9000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(
1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)
^10*tan(1/2*c)^2 - 9975*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*
d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan
(1/2*d*x)^10*tan(1/2*c)^2 - 28800*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
- tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*
x)^10*tan(1/2*c)^2 + 1050*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*
d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^10*tan(
1/2*c)^2 - 90000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*
tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*
d*x)^8*tan(1/2*c)^4 + 99750*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(
1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)
*tan(1/2*d*x)^8*tan(1/2*c)^4 + 90000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)
^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/
2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^4 - 99750*pi*b^3*sgn(tan(1/2*d*x)^2*t
an(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 -
 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^4 - 288000*pi*a^2*b*sgn(tan(
1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(

1/2*c)^2 + 1)*tan(1/2*d*x)^8*tan(1/2*c)^4 + 10500*pi*b^3*sgn(tan(1/2*d*x)^2
*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 +
1)*tan(1/2*d*x)^8*tan(1/2*c)^4 + 92160*a^2*b*tan(1/2*d*x)^10*tan(1/2*c)^4
- 40960*b^3*tan(1/2*d*x)^10*tan(1/2*c)^4 + 1105920*a^2*b*tan(1/2*d*x)^9*tan
(1/2*c)^5 - 491520*b^3*tan(1/2*d*x)^9*tan(1/2*c)^5 - 180000*pi*a^2*b*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 199500*pi*
b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2
*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 +
180000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)
^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan
(1/2*c)^6 - 199500*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*
tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*
d*x)^6*tan(1/2*c)^6 - 576000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan
(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*
tan(1/2*c)^6 + 21000*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^
2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)
^6 + 4884480*a^2*b*tan(1/2*d*x)^8*tan(1/2*c)^6 - 1884160*b^3*tan(1/2*d*x)^8
*tan(1/2*c)^6 + 8847360*a^2*b*tan(1/2*d*x)^7*tan(1/2*c)^7 - 2621440*b^3*tan
(1/2*d*x)^7*tan(1/2*c)^7 - 90000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 +
2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*
x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^8 + 99750*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1
/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*t
an(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^8 + 90000*pi*a^2*b*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2
*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^8 - 99750*pi*b^3*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2
+ tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^8 - 288000*p
i*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*t
an(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^8 + 10500*pi*b^3*sg
n(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c)
- tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^8 + 4884480*a^2*b*tan(1/2*d*x
)^6*tan(1/2*c)^8 - 1884160*b^3*tan(1/2*d*x)^6*tan(1/2*c)^8 + 1105920*a^2*b*
tan(1/2*d*x)^5*tan(1/2*c)^9 - 491520*b^3*tan(1/2*d*x)^5*tan(1/2*c)^9 - 9000
*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - t
an(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c
)^10 + 9975*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2
*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*
tan(1/2*c)^10 + 9000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d
*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(
1/2*d*x)^2*tan(1/2*c)^10 - 9975*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*
tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x)
- 1)*tan(1/2*d*x)^2*tan(1/2*c)^10 - 28800*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1
/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*ta
n(1/2*d*x)^2*tan(1/2*c)^10 + 1050*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 -

tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)
^2*tan(1/2*c)^10 + 92160*a^2*b*tan(1/2*d*x)^4*tan(1/2*c)^10 - 40960*b^3*tan
(1/2*d*x)^4*tan(1/2*c)^10 + 1080*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 +
2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c)
- 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1
/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10 - 105*pi*b^3
*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*
x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2
*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x)
- 1)*tan(1/2*d*x)^10 + 1080*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*t
an(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1
)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d
*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10 - 105*pi*b^3*sgn
(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2
- tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan
(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1
)*tan(1/2*d*x)^10 + 27000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(
1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*s
gn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)
^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^2 - 2625*
pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(
1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)
^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/
2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^2 + 27000*pi*a^2*b*sgn(tan(1/2*d*x)^2
*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2
- 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1
/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^
8*tan(1/2*c)^2 - 2625*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*
x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan
(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + t
an(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^2 + 28800*pi*a^
2*b*tan(1/2*d*x)^10*tan(1/2*c)^2 - 1050*pi*b^3*tan(1/2*d*x)^10*tan(1/2*c)^2
- 10800*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c)
+ 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x
)^10*tan(1/2*c)^2 + 1050*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x)
- tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1
))*tan(1/2*d*x)^10*tan(1/2*c)^2 - 10800*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*
c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x)
- tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^2 + 1050*b^3*arctan((tan(1/2
*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c)
+ tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^2 + 46800*a^2*
b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2
*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*
c)^2 - 1050*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c)
- 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*

x)^10*tan(1/2*c)^2 + 46800*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^2 - 1050*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^2 - 40960*a^3*tan(1/2*d*x)^10*tan(1/2*c)^3 - 122880*a*b^2*tan(1/2*d*x)^10*tan(1/2*c)^3 + 108000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 - 10500*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 + 108000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 - 10500*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 + 288000*pi*a^2*b*tan(1/2*d*x)^8*tan(1/2*c)^4 - 10500*pi*b^3*tan(1/2*d*x)^8*tan(1/2*c)^4 - 108000*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^4 + 10500*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^4 - 108000*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^4 + 10500*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^4 + 468000*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^4 - 10500*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^4 + 468000*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^4 - 10500*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^4 + 307200*a^3*tan(1/2*d*x)^9*tan(1/2*c)^4 - 1105920*a*b^2*tan(1/2*d*x)^9*tan(1/2*c)^4 + 1249280*a^3*tan(1/2*d*x)^8*tan(1/2*c)^5 - 6389760*a*b^2*tan(1/2*d*x)^8*tan(1/2*c)^5 + 108000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d* x)^4*tan(1/2*c)^6 - 10500*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 + 108000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 - 10500*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 + 576000*pi*a^2*b*tan(1/2*d*x)^6*tan(1/2*c)^6 - 21000*pi*b^3*tan(1/2*d*x)^6*tan(1/2*c)^6 - 216000*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 + 21000*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 - 216000*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 + 21000*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 + 936000*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 - 21000*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 + 936000*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 - 21000*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 + 4505600*a^3*tan(1/2*d*x)^7*tan(1/2*c)^6 - 14499840*a*b^2*tan(1/2*d*x)^7*tan(1/2*c)^6 + 4505600*a^3*tan(1/2*d*x)^6*tan(1/2*c)^7 - 14499840*a*b^2*tan(1/2*d*x)^6*tan(1/2*c)^7 + 27000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^8 - 2625*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^8 + 27000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^8 - 2625*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1

)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d
*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^8 + 28
8000*pi*a^2*b*tan(1/2*d*x)^4*tan(1/2*c)^8 - 10500*pi*b^3*tan(1/2*d*x)^4*tan
(1/2*c)^8 - 108000*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - t
an(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*t
an(1/2*d*x)^4*tan(1/2*c)^8 + 10500*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + ta
n(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(
1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^8 - 108000*a^2*b*arctan((tan(1/2*d*x
)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + ta
n(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^8 + 10500*b^3*arcta
n((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*t
an(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^8 + 4
68000*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1
)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4
*tan(1/2*c)^8 - 10500*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) +
tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*
tan(1/2*d*x)^4*tan(1/2*c)^8 + 468000*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c)
- tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) +
tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^8 - 10500*b^3*arctan((tan(1/2*d*
x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + t
an(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^8 + 1249280*a^3*ta
n(1/2*d*x)^5*tan(1/2*c)^8 - 6389760*a*b^2*tan(1/2*d*x)^5*tan(1/2*c)^8 + 307
200*a^3*tan(1/2*d*x)^4*tan(1/2*c)^9 - 1105920*a*b^2*tan(1/2*d*x)^4*tan(1/2*
c)^9 + 1080*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan
(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c
)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^10 - 105*pi*b^3*sgn(tan(1/2*d*x)^2*tan
(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2
*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c
)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^10 + 1
080*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c)
+ tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(
1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*
tan(1/2*d*x) - 1)*tan(1/2*c)^10 - 105*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2
*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - ta
n(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^10 + 28800*pi*
a^2*b*tan(1/2*d*x)^2*tan(1/2*c)^10 - 1050*pi*b^3*tan(1/2*d*x)^2*tan(1/2*c)^
10 - 10800*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c
) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d
*x)^2*tan(1/2*c)^10 + 1050*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*
x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) +
 1))*tan(1/2*d*x)^2*tan(1/2*c)^10 - 10800*a^2*b*arctan((tan(1/2*d*x)*tan(1/
2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*
x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^10 + 1050*b^3*arctan((tan(1

/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c
) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^10 + 46800*a^
2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1
/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2
*c)^10 - 1050*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*
c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*
d*x)^2*tan(1/2*c)^10 + 46800*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/
2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*
c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^10 - 1050*b^3*arctan((tan(1/2*d*x)*tan(1
/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d
*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^10 - 40960*a^3*tan(1/2*d*x
)^3*tan(1/2*c)^10 - 122880*a*b^2*tan(1/2*d*x)^3*tan(1/2*c)^10 - 1800*pi*a^2
*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*
d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10 + 1995*pi*b^3*s
gn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)
^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10 + 1800*pi*a^2*b*sgn
(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2
 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10 - 1995*pi*b^3*sgn(tan
(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + t
an(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10 - 5760*pi*a^2*b*sgn(tan(1
/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1
/2*c)^2 + 1)*tan(1/2*d*x)^10 + 210*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 -
 tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x
)^10 - 45000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(
1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)
^8*tan(1/2*c)^2 + 49875*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*
d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan
(1/2*d*x)^8*tan(1/2*c)^2 + 45000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 -
 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*
x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^2 - 49875*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1
/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*t
an(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^2 - 144000*pi*a^2*b*sgn(tan(1/2*
d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*
c)^2 + 1)*tan(1/2*d*x)^8*tan(1/2*c)^2 + 5250*pi*b^3*sgn(tan(1/2*d*x)^2*tan(
1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*t
an(1/2*d*x)^8*tan(1/2*c)^2 - 46080*a^2*b*tan(1/2*d*x)^10*tan(1/2*c)^2 + 102
40*b^3*tan(1/2*d*x)^10*tan(1/2*c)^2 - 737280*a^2*b*tan(1/2*d*x)^9*tan(1/2*c
)^3 + 245760*b^3*tan(1/2*d*x)^9*tan(1/2*c)^3 - 180000*pi*a^2*b*sgn(tan(1/2*
d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/
2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 + 199500*pi*b^3*sg
n(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^
2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 + 180000
*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - t
an(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c
)^4 - 199500*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/

2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6
*tan(1/2*c)^4 - 576000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d
*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/
2*c)^4 + 21000*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*
tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 - 4
884480*a^2*b*tan(1/2*d*x)^8*tan(1/2*c)^4 + 1884160*b^3*tan(1/2*d*x)^8*tan(1
/2*c)^4 - 16220160*a^2*b*tan(1/2*d*x)^7*tan(1/2*c)^5 + 6062080*b^3*tan(1/2*
d*x)^7*tan(1/2*c)^5 - 180000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*t
an(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) -
1)*tan(1/2*d*x)^4*tan(1/2*c)^6 + 199500*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*
c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(
1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 + 180000*pi*a^2*b*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c
)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 - 199500*pi*b^3*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 - 576000*pi
*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*ta
n(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 + 21000*pi*b^3*sgn
(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) -
tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 - 25989120*a^2*b*tan(1/2*d*x
)^6*tan(1/2*c)^6 + 8396800*b^3*tan(1/2*d*x)^6*tan(1/2*c)^6 - 16220160*a^2*b
*tan(1/2*d*x)^5*tan(1/2*c)^7 + 6062080*b^3*tan(1/2*d*x)^5*tan(1/2*c)^7 - 45
000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2
- tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/
2*c)^8 + 49875*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(
1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)
^2*tan(1/2*c)^8 + 45000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/
2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*t
an(1/2*d*x)^2*tan(1/2*c)^8 - 49875*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 -
2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*
x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^8 - 144000*pi*a^2*b*sgn(tan(1/2*d*x)^2*ta
n(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)
*tan(1/2*d*x)^2*tan(1/2*c)^8 + 5250*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
- tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*
x)^2*tan(1/2*c)^8 - 4884480*a^2*b*tan(1/2*d*x)^4*tan(1/2*c)^8 + 1884160*b^3
*tan(1/2*d*x)^4*tan(1/2*c)^8 - 737280*a^2*b*tan(1/2*d*x)^3*tan(1/2*c)^9 + 2
45760*b^3*tan(1/2*d*x)^3*tan(1/2*c)^9 - 1800*pi*a^2*b*sgn(tan(1/2*d*x)^2*ta
n(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 +
2*tan(1/2*d*x) - 1)*tan(1/2*c)^10 + 1995*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*
c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(
1/2*d*x) - 1)*tan(1/2*c)^10 + 1800*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
- 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*
d*x) - 1)*tan(1/2*c)^10 - 1995*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*t
an(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) -
1)*tan(1/2*c)^10 - 5760*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2

*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^10 + 210*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^10 - 46080*a^2*b*tan(1/2*d*x)^2*tan(1/2*c)^10 + 10240*b^3*tan(1/2*d*x)^2*tan(1/2*c)^10 + 5400*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8 - 525*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8 + 5400*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8 - 525*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8 + 5760*pi*a^2*b*tan(1/2*d*x)^10 - 210*pi*b^3*tan(1/2*d*x)^10 - 2160*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^10 + 210*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^10 - 2160*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^10 + 210*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^10 + 9360*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^10 - 210*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^10 + 9360*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^10 - 210*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^10 - 30720*a^3*tan(1/2*d*x)^10*tan(1/2*c) + 54000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 - 5250*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 + 54000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1

/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^2
- 5250*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c)
+ tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan
(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2
*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 + 144000*pi*a^2*b*tan(1/2*d*
x)^8*tan(1/2*c)^2 - 5250*pi*b^3*tan(1/2*d*x)^8*tan(1/2*c)^2 - 54000*a^2*b*a
rctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*
x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^2
+ 5250*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1
)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^8
*tan(1/2*c)^2 - 54000*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x)
+ tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)
)*tan(1/2*d*x)^8*tan(1/2*c)^2 + 5250*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) -
tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - ta
n(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^2 + 234000*a^2*b*arctan((tan(1/2*d
*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) -
tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^2 - 5250*b^3*arct
an((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*
tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^2 +
234000*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) -
1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^
8*tan(1/2*c)^2 - 5250*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) -
tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*
tan(1/2*d*x)^8*tan(1/2*c)^2 - 153600*a^3*tan(1/2*d*x)^9*tan(1/2*c)^2 + 3686
40*a*b^2*tan(1/2*d*x)^9*tan(1/2*c)^2 - 1433600*a^3*tan(1/2*d*x)^8*tan(1/2*c
)^3 + 3072000*a*b^2*tan(1/2*d*x)^8*tan(1/2*c)^3 + 108000*pi*a^2*b*sgn(tan(1
/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan
(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d
*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(
1/2*d*x)^4*tan(1/2*c)^4 - 10500*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*
tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) -
1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*
d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 + 1
08000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c
) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*ta
n(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 -
2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 - 10500*pi*b^3*sgn(tan(1/2*
d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/
2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)
*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2
*d*x)^4*tan(1/2*c)^4 + 576000*pi*a^2*b*tan(1/2*d*x)^6*tan(1/2*c)^4 - 21000*
pi*b^3*tan(1/2*d*x)^6*tan(1/2*c)^4 - 216000*a^2*b*arctan((tan(1/2*d*x)*tan(
1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*
d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 + 21000*b^3*arctan((tan
(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2

*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 - 216000*
a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan
(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1
/2*c)^4 + 21000*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/
2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/
2*d*x)^6*tan(1/2*c)^4 + 936000*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(
1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/
2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 - 21000*b^3*arctan((tan(1/2*d*x)*tan
(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2
*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 + 936000*a^2*b*arctan(
(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan
(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 - 210
00*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(ta
n(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(
1/2*c)^4 - 4505600*a^3*tan(1/2*d*x)^7*tan(1/2*c)^4 + 14499840*a*b^2*tan(1/2
*d*x)^7*tan(1/2*c)^4 - 11018240*a^3*tan(1/2*d*x)^6*tan(1/2*c)^5 + 30720000*
a*b^2*tan(1/2*d*x)^6*tan(1/2*c)^5 + 54000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1
/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*t
an(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^
2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(
1/2*c)^6 - 5250*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*t
an(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2
*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^6 + 54000*pi*a^2*b*sg
n(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^
2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*ta
n(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) -
1)*tan(1/2*d*x)^2*tan(1/2*c)^6 - 5250*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2
*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - ta
n(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)
^6 + 576000*pi*a^2*b*tan(1/2*d*x)^4*tan(1/2*c)^6 - 21000*pi*b^3*tan(1/2*d*x
)^4*tan(1/2*c)^6 - 216000*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d
*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c)
+ 1))*tan(1/2*d*x)^4*tan(1/2*c)^6 + 21000*b^3*arctan((tan(1/2*d*x)*tan(1/2*
c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x)
 + tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^6 - 216000*a^2*b*arctan((tan(
1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*
c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^6 + 21000*b^
3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2
*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c
)^6 + 936000*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2
*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2
*d*x)^4*tan(1/2*c)^6 - 21000*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*
d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c)

- 1))*tan(1/2*d*x)^4*tan(1/2*c)^6 + 936000*a^2*b*arctan((tan(1/2*d*x)*tan(
1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*
d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^6 - 21000*b^3*arctan((tan
(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2
*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^6 - 1101824
0*a^3*tan(1/2*d*x)^5*tan(1/2*c)^6 + 30720000*a*b^2*tan(1/2*d*x)^5*tan(1/2*c
)^6 - 4505600*a^3*tan(1/2*d*x)^4*tan(1/2*c)^7 + 14499840*a*b^2*tan(1/2*d*x)
^4*tan(1/2*c)^7 + 5400*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2
*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2
+ tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^8 - 525*pi*b^3*sgn(tan(1/2*
d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/
2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)
*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2
*c)^8 + 5400*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*ta
n(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*
c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^8 - 525*pi*b^3*sgn(tan(1/2*d*x)^2*tan
(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2
*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c
)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^8 + 14
4000*pi*a^2*b*tan(1/2*d*x)^2*tan(1/2*c)^8 - 5250*pi*b^3*tan(1/2*d*x)^2*tan(
1/2*c)^8 - 54000*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan
(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan
(1/2*d*x)^2*tan(1/2*c)^8 + 5250*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1
/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2
*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^8 - 54000*a^2*b*arctan((tan(1/2*d*x)*ta
n(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/
2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^8 + 5250*b^3*arctan((ta
n(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/
2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^8 + 234000
*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(ta
n(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(
1/2*c)^8 - 5250*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/
2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/
2*d*x)^2*tan(1/2*c)^8 + 234000*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(
1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/
2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^8 - 5250*b^3*arctan((tan(1/2*d*x)*tan(
1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*
d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^8 - 1433600*a^3*tan(1/2*d
*x)^3*tan(1/2*c)^8 + 3072000*a*b^2*tan(1/2*d*x)^3*tan(1/2*c)^8 - 153600*a^3
*tan(1/2*d*x)^2*tan(1/2*c)^9 + 368640*a*b^2*tan(1/2*d*x)^2*tan(1/2*c)^9 + 5
760*pi*a^2*b*tan(1/2*c)^10 - 210*pi*b^3*tan(1/2*c)^10 - 2160*a^2*b*arctan((
tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(
1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*c)^10 + 210*b^3*arctan((ta

n(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/
2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*c)^10 - 2160*a^2*b*arctan((t
an(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1
/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*c)^10 + 210*b^3*arctan((tan
(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2
*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*c)^10 + 9360*a^2*b*arctan((ta
n(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/
2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*c)^10 - 210*b^3*arctan((tan(
1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*
c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*c)^10 + 9360*a^2*b*arctan((tan
(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2
*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^10 - 210*b^3*arctan((tan(1
/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c
) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^10 - 30720*a^3*tan(1/2*d*x)*
tan(1/2*c)^10 - 9000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d
*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(
1/2*d*x)^8 + 9975*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*t
an(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d
*x)^8 + 9000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(
1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)
^8 - 9975*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c
)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8 -
28800*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2
*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^8 + 1050*pi*b^3*sgn(tan(1
/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1
/2*c)^2 + 1)*tan(1/2*d*x)^8 + 9216*a^2*b*tan(1/2*d*x)^10 + 2048*b^3*tan(1/2
*d*x)^10 + 184320*a^2*b*tan(1/2*d*x)^9*tan(1/2*c) - 90000*pi*a^2*b*sgn(tan(
1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + ta
n(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 + 99750*pi*b^3
*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*
x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 + 900
00*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 -
 tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2
*c)^2 - 99750*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1
/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^
6*tan(1/2*c)^2 - 288000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*
d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1
/2*c)^2 + 10500*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4
*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 +
1704960*a^2*b*tan(1/2*d*x)^8*tan(1/2*c)^2 - 317440*b^3*tan(1/2*d*x)^8*tan(1
/2*c)^2 + 8847360*a^2*b*tan(1/2*d*x)^7*tan(1/2*c)^3 - 2621440*b^3*tan(1/2*d
*x)^7*tan(1/2*c)^3 - 180000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) -
1)*tan(1/2*d*x)^4*tan(1/2*c)^4 + 199500*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c
)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1

/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 + 180000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 - 199500*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 - 576000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 + 21000*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 + 25989120*a^2*b*tan(1/2*d*x)^6*tan(1/2*c)^4 - 8396800*b^3*tan(1/2*d*x)^6*tan(1/2*c)^4 + 39665664*a^2*b*tan(1/2*d*x)^5*tan(1/2*c)^5 - 11665408*b^3*tan(1/2*d*x)^5*tan(1/2*c)^5 - 90000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^6 + 99750*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^6 + 90000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^6 - 99750*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^6 - 288000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2*tan(1/2*c)^6 + 10500*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2*tan(1/2*c)^6 + 25989120*a^2*b*tan(1/2*d*x)^4*tan(1/2*c)^6 - 8396800*b^3*tan(1/2*d*x)^4*tan(1/2*c)^6 + 8847360*a^2*b*tan(1/2*d*x)^3*tan(1/2*c)^7 - 2621440*b^3*tan(1/2*d*x)^3*tan(1/2*c)^7 - 9000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^8 + 9975*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^8 + 9000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^8 - 9975*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^8 - 28800*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^8 + 1050*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^8 + 1704960*a^2*b*tan(1/2*d*x)^2*tan(1/2*c)^8 - 317440*b^3*tan(1/2*d*x)^2*tan(1/2*c)^8 + 184320*a^2*b*tan(1/2*d*x)*tan(1/2*c)^9 + 9216*a^2*b*tan(1/2*c)^10 + 2048*b^3*tan(1/2*c)^10 + 10800*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6 - 1050*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 -

tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6 + 10800
*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + t
an(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2
*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan
(1/2*d*x) - 1)*tan(1/2*d*x)^6 - 1050*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*
c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan
(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6 + 28800*pi*
a^2*b*tan(1/2*d*x)^8 - 1050*pi*b^3*tan(1/2*d*x)^8 - 10800*a^2*b*arctan((tan
(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2
*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^8 + 1050*b^3*arctan((tan
(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2
*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^8 - 10800*a^2*b*arctan((
tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(
1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8 + 1050*b^3*arctan((
tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(
1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8 + 46800*a^2*b*arcta
n((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*t
an(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^8 - 1050*b^3*arcta
n((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*t
an(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^8 + 46800*a^2*b*ar
ctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x
)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^8 - 1050*b^3*ar
ctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x
)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^8 + 30720*a^3*t
an(1/2*d*x)^9 + 153600*a^3*tan(1/2*d*x)^8*tan(1/2*c) - 368640*a*b^2*tan(1/2
*d*x)^8*tan(1/2*c) + 54000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan
(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*
sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^2 - 5250
*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan
(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c
)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1
/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^2 + 54000*pi*a^2*b*sgn(tan(1/2*d*x)^
2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^
2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(
1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)
^4*tan(1/2*c)^2 - 5250*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d
*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^2 + 288000*pi*
a^2*b*tan(1/2*d*x)^6*tan(1/2*c)^2 - 10500*pi*b^3*tan(1/2*d*x)^6*tan(1/2*c)^
2 - 108000*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c
) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d
*x)^6*tan(1/2*c)^2 + 10500*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*

x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) +
1))*tan(1/2*d*x)^6*tan(1/2*c)^2 - 108000*a^2*b*arctan((tan(1/2*d*x)*tan(1/
2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*
x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 + 10500*b^3*arctan((tan(1
/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c
) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 + 468000*a^
2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1
/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2
*c)^2 - 10500*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*
c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*
d*x)^6*tan(1/2*c)^2 + 468000*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/
2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*
c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 - 10500*b^3*arctan((tan(1/2*d*x)*tan(1
/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d
*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 + 1433600*a^3*tan(1/2*d*
x)^7*tan(1/2*c)^2 - 3072000*a*b^2*tan(1/2*d*x)^7*tan(1/2*c)^2 + 4505600*a^3
*tan(1/2*d*x)^6*tan(1/2*c)^3 - 14499840*a*b^2*tan(1/2*d*x)^6*tan(1/2*c)^3 +
54000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*
c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*t
an(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 +
2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^4 - 5250*pi*b^3*sgn(tan(1/2*
d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/
2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)
*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2
*d*x)^2*tan(1/2*c)^4 + 54000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*t
an(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1
)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d
*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^4 - 52
50*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + t
an(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2
*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan
(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^4 + 576000*pi*a^2*b*tan(1/2*d*x)^4
*tan(1/2*c)^4 - 21000*pi*b^3*tan(1/2*d*x)^4*tan(1/2*c)^4 - 216000*a^2*b*arc
tan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)
*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 +
21000*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)
/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4*
tan(1/2*c)^4 - 216000*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x)
+ tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)
)*tan(1/2*d*x)^4*tan(1/2*c)^4 + 21000*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) -
tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - t
an(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 + 936000*a^2*b*arctan((tan(1/2*
d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) -
tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 - 21000*b^3*ar
ctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x

)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^4
+ 936000*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c)
- 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x
)^4*tan(1/2*c)^4 - 21000*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x)
 - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1
))*tan(1/2*d*x)^4*tan(1/2*c)^4 + 11018240*a^3*tan(1/2*d*x)^5*tan(1/2*c)^4 -
 30720000*a*b^2*tan(1/2*d*x)^5*tan(1/2*c)^4 + 11018240*a^3*tan(1/2*d*x)^4*t
an(1/2*c)^5 - 30720000*a*b^2*tan(1/2*d*x)^4*tan(1/2*c)^5 + 10800*pi*a^2*b*s
gn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)
^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*t
an(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) -
 1)*tan(1/2*c)^6 - 1050*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
 tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^6 + 10800*pi*a^2*b*sgn(tan(1
/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan
(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d
*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(
1/2*c)^6 - 1050*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*t
an(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2
*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^6 + 288000*pi*a^2*b*tan(1/2*d*x)^2*t
an(1/2*c)^6 - 10500*pi*b^3*tan(1/2*d*x)^2*tan(1/2*c)^6 - 108000*a^2*b*arcta
n((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*t
an(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 + 1
0500*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(
tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2*ta
n(1/2*c)^6 - 108000*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) +
tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*
tan(1/2*d*x)^2*tan(1/2*c)^6 + 10500*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - t
an(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan
(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 + 468000*a^2*b*arctan((tan(1/2*d*
x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - t
an(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 - 10500*b^3*arct
an((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*
tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 +
468000*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) -
1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^
2*tan(1/2*c)^6 - 10500*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) -
 tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))
*tan(1/2*d*x)^2*tan(1/2*c)^6 + 4505600*a^3*tan(1/2*d*x)^3*tan(1/2*c)^6 - 14
499840*a*b^2*tan(1/2*d*x)^3*tan(1/2*c)^6 + 1433600*a^3*tan(1/2*d*x)^2*tan(1
/2*c)^7 - 3072000*a*b^2*tan(1/2*d*x)^2*tan(1/2*c)^7 + 28800*pi*a^2*b*tan(1/
2*c)^8 - 1050*pi*b^3*tan(1/2*c)^8 - 10800*a^2*b*arctan((tan(1/2*d*x)*tan(1/
2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*

x) + tan(1/2*c) + 1))*tan(1/2*c)^8 + 1050*b^3*arctan((tan(1/2*d*x)*tan(1/2*
c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x)
 + tan(1/2*c) + 1))*tan(1/2*c)^8 - 10800*a^2*b*arctan((tan(1/2*d*x)*tan(1/2
*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x
) - tan(1/2*c) + 1))*tan(1/2*c)^8 + 1050*b^3*arctan((tan(1/2*d*x)*tan(1/2*c
) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x)
- tan(1/2*c) + 1))*tan(1/2*c)^8 + 46800*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*
c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x)
 - tan(1/2*c) - 1))*tan(1/2*c)^8 - 1050*b^3*arctan((tan(1/2*d*x)*tan(1/2*c)
 + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) -
 tan(1/2*c) - 1))*tan(1/2*c)^8 + 46800*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c
) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x)
+ tan(1/2*c) - 1))*tan(1/2*c)^8 - 1050*b^3*arctan((tan(1/2*d*x)*tan(1/2*c)
- tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) +
tan(1/2*c) - 1))*tan(1/2*c)^8 + 153600*a^3*tan(1/2*d*x)*tan(1/2*c)^8 - 3686
40*a*b^2*tan(1/2*d*x)*tan(1/2*c)^8 + 30720*a^3*tan(1/2*c)^9 - 18000*pi*a^2*
b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d
*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6 + 19950*pi*b^3*sg
n(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^
2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6 + 18000*pi*a^2*b*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2
+ tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6 - 19950*pi*b^3*sgn(tan(
1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + ta
n(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6 - 57600*pi*a^2*b*sgn(tan(1/
2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/
2*c)^2 + 1)*tan(1/2*d*x)^6 + 2100*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 -
tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)
^6 - 46080*a^2*b*tan(1/2*d*x)^8 + 10240*b^3*tan(1/2*d*x)^8 - 737280*a^2*b*t
an(1/2*d*x)^7*tan(1/2*c) + 245760*b^3*tan(1/2*d*x)^7*tan(1/2*c) - 90000*pi*
a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1
/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^2
+ 99750*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^
2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(
1/2*c)^2 + 90000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*
tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*
d*x)^4*tan(1/2*c)^2 - 99750*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(
1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)
*tan(1/2*d*x)^4*tan(1/2*c)^2 - 288000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c
)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/
2*d*x)^4*tan(1/2*c)^2 + 10500*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(
1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*t
an(1/2*c)^2 - 4884480*a^2*b*tan(1/2*d*x)^6*tan(1/2*c)^2 + 1884160*b^3*tan(1
/2*d*x)^6*tan(1/2*c)^2 - 16220160*a^2*b*tan(1/2*d*x)^5*tan(1/2*c)^3 + 60620
80*b^3*tan(1/2*d*x)^5*tan(1/2*c)^3 - 90000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(
1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*

tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^4 + 99750*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^4 + 90000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^4 - 99750*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^4 - 288000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2*tan(1/2*c)^4 + 10500*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2*tan(1/2*c)^4 - 25989120*a^2*b*tan(1/2*d*x)^4*tan(1/2*c)^4 + 8396800*b^3*tan(1/2*d*x)^4*tan(1/2*c)^4 - 16220160*a^2*b*tan(1/2*d*x)^3*tan(1/2*c)^5 + 6062080*b^3*tan(1/2*d*x)^3*tan(1/2*c)^5 - 18000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^6 + 19950*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^6 + 18000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^6 - 19950*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^6 - 57600*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^6 + 2100*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^6 - 4884480*a^2*b*tan(1/2*d*x)^2*tan(1/2*c)^6 + 1884160*b^3*tan(1/2*d*x)^2*tan(1/2*c)^6 - 737280*a^2*b*tan(1/2*d*x)*tan(1/2*c)^7 + 245760*b^3*tan(1/2*d*x)*tan(1/2*c)^7 - 46080*a^2*b*tan(1/2*c)^8 + 10240*b^3*tan(1/2*c)^8 + 10800*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4 - 1050*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4 + 10800*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4 - 1050*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4 + 57600*pi*a^2*b*tan(1/2*d*x)^6 - 2100*pi*b^3*tan(1/2*d*x)^6 - 21600*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6 + 2100*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x

)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6 - 21600*a^2*b
*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*
d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6 + 2100*b^3
*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*
d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6 + 93600*a^
2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1
/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6 - 2100*
b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1
/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6 + 93600
*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(ta
n(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6 - 21
00*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(ta
n(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6 + 40
960*a^3*tan(1/2*d*x)^7 + 122880*a*b^2*tan(1/2*d*x)^7 - 307200*a^3*tan(1/2*d
*x)^6*tan(1/2*c) + 1105920*a*b^2*tan(1/2*d*x)^6*tan(1/2*c) + 27000*pi*a^2*b
*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*
x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2
*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x)
 - 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2625*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*
c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(
1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 -
 tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2
*c)^2 + 27000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*t
an(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2
*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2625*pi*b^3*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
 tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1
/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*
tan(1/2*d*x)^2*tan(1/2*c)^2 + 288000*pi*a^2*b*tan(1/2*d*x)^4*tan(1/2*c)^2 -
 10500*pi*b^3*tan(1/2*d*x)^4*tan(1/2*c)^2 - 108000*a^2*b*arctan((tan(1/2*d*
x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - t
an(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^2 + 10500*b^3*arct
an((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*
tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^2 -
108000*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) +
1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^
4*tan(1/2*c)^2 + 10500*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) +
 tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))
*tan(1/2*d*x)^4*tan(1/2*c)^2 + 468000*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c)
 + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) -
 tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^2 - 10500*b^3*arctan((tan(1/2*d
*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) -
tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^2 + 468000*a^2*b*
arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d

*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^
2 - 10500*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) -
 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)
^4*tan(1/2*c)^2 - 1249280*a^3*tan(1/2*d*x)^5*tan(1/2*c)^2 + 6389760*a*b^2*t
an(1/2*d*x)^5*tan(1/2*c)^2 - 4505600*a^3*tan(1/2*d*x)^4*tan(1/2*c)^3 + 1449
9840*a*b^2*tan(1/2*d*x)^4*tan(1/2*c)^3 + 10800*pi*a^2*b*sgn(tan(1/2*d*x)^2*
tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2
+ 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/
2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^4 -
 1050*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c)
+ tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(
1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*
tan(1/2*d*x) - 1)*tan(1/2*c)^4 + 10800*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*
c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(
1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 -
 tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^4 - 1050*pi
*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/
2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*
d*x) - 1)*tan(1/2*c)^4 + 288000*pi*a^2*b*tan(1/2*d*x)^2*tan(1/2*c)^4 - 1050
0*pi*b^3*tan(1/2*d*x)^2*tan(1/2*c)^4 - 108000*a^2*b*arctan((tan(1/2*d*x)*ta
n(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/
2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^4 + 10500*b^3*arctan((t
an(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1
/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^4 - 10800
0*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(t
an(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan
(1/2*c)^4 + 10500*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(
1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(
1/2*d*x)^2*tan(1/2*c)^4 + 468000*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + ta
n(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(
1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^4 - 10500*b^3*arctan((tan(1/2*d*x)*t
an(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1
/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^4 + 468000*a^2*b*arcta
n((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*t
an(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^4 - 1
0500*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(
tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2*ta
n(1/2*c)^4 - 4505600*a^3*tan(1/2*d*x)^3*tan(1/2*c)^4 + 14499840*a*b^2*tan(1
/2*d*x)^3*tan(1/2*c)^4 - 1249280*a^3*tan(1/2*d*x)^2*tan(1/2*c)^5 + 6389760*
a*b^2*tan(1/2*d*x)^2*tan(1/2*c)^5 + 57600*pi*a^2*b*tan(1/2*c)^6 - 2100*pi*b
^3*tan(1/2*c)^6 - 21600*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x
) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) +
1))*tan(1/2*c)^6 + 2100*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x)
- tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)

)*tan(1/2*c)^6 - 21600*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x)
 + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1
))*tan(1/2*c)^6 + 2100*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) +
 tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))
*tan(1/2*c)^6 + 93600*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x)
+ tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)
)*tan(1/2*c)^6 - 2100*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) +
tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*
tan(1/2*c)^6 + 93600*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) -
 tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))
*tan(1/2*c)^6 - 2100*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - t
an(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*t
an(1/2*c)^6 - 307200*a^3*tan(1/2*d*x)*tan(1/2*c)^6 + 1105920*a*b^2*tan(1/2*
d*x)*tan(1/2*c)^6 + 40960*a^3*tan(1/2*c)^7 + 122880*a*b^2*tan(1/2*c)^7 - 18
000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2
 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4 + 1995
0*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - ta
n(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4 + 18000*pi
*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(
1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4 - 19950*pi*b
^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*
d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4 - 57600*pi*a^2*b
*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*
c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4 + 2100*pi*b^3*sgn(tan(1/2*d*x)^2*tan(
1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*t
an(1/2*d*x)^4 + 92160*a^2*b*tan(1/2*d*x)^6 - 40960*b^3*tan(1/2*d*x)^6 + 110
5920*a^2*b*tan(1/2*d*x)^5*tan(1/2*c) - 491520*b^3*tan(1/2*d*x)^5*tan(1/2*c)
 - 45000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*
c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*t
an(1/2*c)^2 + 49875*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)
*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2
*d*x)^2*tan(1/2*c)^2 + 45000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*t
an(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) -
 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 49875*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c
)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1
/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 144000*pi*a^2*b*sgn(tan(1/2*d*x)
^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2
 + 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 5250*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*
c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1
/2*d*x)^2*tan(1/2*c)^2 + 4884480*a^2*b*tan(1/2*d*x)^4*tan(1/2*c)^2 - 188416
0*b^3*tan(1/2*d*x)^4*tan(1/2*c)^2 + 8847360*a^2*b*tan(1/2*d*x)^3*tan(1/2*c)
^3 - 2621440*b^3*tan(1/2*d*x)^3*tan(1/2*c)^3 - 18000*pi*a^2*b*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2
*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^4 + 19950*pi*b^3*sgn(tan(1/2*d*x)^2*
tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2

+ 2*tan(1/2*d*x) - 1)*tan(1/2*c)^4 + 18000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^4 - 19950*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^4 - 57600*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^4 + 2100*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^4 + 4884480*a^2*b*tan(1/2*d*x)^2*tan(1/2*c)^4 - 1884160*b^3*tan(1/2*d*x)^2*tan(1/2*c)^4 + 1105920*a^2*b*tan(1/2*d*x)*tan(1/2*c)^5 - 491520*b^3*tan(1/2*d*x)*tan(1/2*c)^5 + 92160*a^2*b*tan(1/2*c)^6 - 40960*b^3*tan(1/2*c)^6 + 5400*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2 - 525*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2 + 5400*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2 - 525*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2 + 57600*pi*a^2*b*tan(1/2*d*x)^4 - 2100*pi*b^3*tan(1/2*d*x)^4 - 21600*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4 + 2100*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4 - 21600*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4 + 2100*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4 + 93600*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4 - 2100*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4 + 93600*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4 - 2100*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4 + 118784*a^3*tan(1/2*d*x)^5 - 49152*a*b^2*tan(1/2*d*x)^5 + 307200*a^3*tan(1/2*d*x)^4*tan(1/2*c) - 1105920*a*b^2*tan(1/2*d*x)^4*tan(1/2*c) + 5400*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*s

gn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)
^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^2 - 525*pi*b^3*sgn(tan(1
/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan
(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d
*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(
1/2*c)^2 + 5400*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2
*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1
/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^2 - 525*pi*b^3*sgn(tan(1/2*d*x)^2*
tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2
- 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/
2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^2 +
144000*pi*a^2*b*tan(1/2*d*x)^2*tan(1/2*c)^2 - 5250*pi*b^3*tan(1/2*d*x)^2*t
an(1/2*c)^2 - 54000*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) -
tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*
tan(1/2*d*x)^2*tan(1/2*c)^2 + 5250*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + ta
n(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(
1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 54000*a^2*b*arctan((tan(1/2*d*x)
*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan
(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 5250*b^3*arctan(
(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan
(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 234
000*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/
(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*t
an(1/2*c)^2 - 5250*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan
(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan
(1/2*d*x)^2*tan(1/2*c)^2 + 234000*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - t
an(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan
(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 5250*b^3*arctan((tan(1/2*d*x)*t
an(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1
/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 1433600*a^3*tan(1/
2*d*x)^3*tan(1/2*c)^2 - 3072000*a*b^2*tan(1/2*d*x)^3*tan(1/2*c)^2 + 1433600
*a^3*tan(1/2*d*x)^2*tan(1/2*c)^3 - 3072000*a*b^2*tan(1/2*d*x)^2*tan(1/2*c)^
3 + 57600*pi*a^2*b*tan(1/2*c)^4 - 2100*pi*b^3*tan(1/2*c)^4 - 21600*a^2*b*ar
ctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x
)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*c)^4 + 2100*b^3*arct
an((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*
tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*c)^4 - 21600*a^2*b*arc
tan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)
*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*c)^4 + 2100*b^3*arcta
n((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*t
an(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*c)^4 + 93600*a^2*b*arct
an((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*
tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*c)^4 - 2100*b^3*arctan
((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*ta

n(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*c)^4 + 93600*a^2*b*arcta
n((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*t
an(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^4 - 2100*b^3*arctan(
(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan
(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^4 + 307200*a^3*tan(1/2
*d*x)*tan(1/2*c)^4 - 1105920*a*b^2*tan(1/2*d*x)*tan(1/2*c)^4 + 118784*a^3*t
an(1/2*c)^5 - 49152*a*b^2*tan(1/2*c)^5 - 9000*pi*a^2*b*sgn(tan(1/2*d*x)^2*t
an(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 +
 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2 + 9975*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/
2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*ta
n(1/2*d*x) - 1)*tan(1/2*d*x)^2 + 9000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c
)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1
/2*d*x) - 1)*tan(1/2*d*x)^2 - 9975*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 -
 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*
x) - 1)*tan(1/2*d*x)^2 - 28800*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - t
an(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^
2 + 1050*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/
2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2 - 92160*a^2*b*tan(1/2*
d*x)^4 + 40960*b^3*tan(1/2*d*x)^4 - 737280*a^2*b*tan(1/2*d*x)^3*tan(1/2*c)
+ 245760*b^3*tan(1/2*d*x)^3*tan(1/2*c) - 9000*pi*a^2*b*sgn(tan(1/2*d*x)^2*t
an(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 +
 2*tan(1/2*d*x) - 1)*tan(1/2*c)^2 + 9975*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*
c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(
1/2*d*x) - 1)*tan(1/2*c)^2 + 9000*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
- 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d
*x) - 1)*tan(1/2*c)^2 - 9975*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan
(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1
)*tan(1/2*c)^2 - 28800*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d
*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^2 + 1050*p
i*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan
(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^2 - 1704960*a^2*b*tan(1/2*d*x)^2*tan
(1/2*c)^2 + 317440*b^3*tan(1/2*d*x)^2*tan(1/2*c)^2 - 737280*a^2*b*tan(1/2*d
*x)*tan(1/2*c)^3 + 245760*b^3*tan(1/2*d*x)*tan(1/2*c)^3 - 92160*a^2*b*tan(1
/2*c)^4 + 40960*b^3*tan(1/2*c)^4 + 1080*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2
*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan
(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2
- tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1) - 105*pi*b^3*sgn(tan(
1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - ta
n(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*
d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1) + 1
080*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c)
+ tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(
1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*
tan(1/2*d*x) - 1) - 105*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*
d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(t

an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1) + 28800*pi*a^2*b*tan(1/2*d*x)^2 - 1050*pi*b^3*tan(1/2*d*x)^2 - 10800*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2 + 1050*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2 - 10800*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2 + 1050*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2 + 46800*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2 - 1050*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2 + 46800*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2 - 1050*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2 + 40960*a^3*tan(1/2*d*x)^3 + 122880*a*b^2*tan(1/2*d*x)^3 - 153600*a^3*tan(1/2*d*x)^2*tan(1/2*c) + 368640*a*b^2*tan(1/2*d*x)^2*tan(1/2*c) + 28800*pi*a^2*b*tan(1/2*c)^2 - 1050*pi*b^3*tan(1/2*c)^2 - 10800*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*c)^2 + 1050*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*c)^2 - 10800*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*c)^2 + 1050*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*c)^2 + 46800*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*c)^2 - 1050*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*c)^2 + 46800*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^2 - 1050*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^2 - 153600*a^3*tan(1/2*d*x)*tan(1/2*c)^2 + 368640*a*b^2*tan(1/2*d*x)*tan(1/2*c)^2 + 40960*a^3*tan(1/2*c)^3 + 122880*a*b^2*tan(1/2*c)^3 - 1800*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1) + 1995*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1) + 1800*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1) - 1995*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^

```
2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1) - 5760*pi*a^2*b*sgn
(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) -
 tan(1/2*c)^2 + 1) + 210*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d
*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1) + 46080*a^2*b*tan(1/2
*d*x)^2 - 10240*b^3*tan(1/2*d*x)^2 + 184320*a^2*b*tan(1/2*d*x)*tan(1/2*c) +
 46080*a^2*b*tan(1/2*c)^2 - 10240*b^3*tan(1/2*c)^2 + 5760*pi*a^2*b - 210*pi
*b^3 - 2160*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*
c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)) + 210*b^
3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2
*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)) - 2160*a^2*b*arctan((tan
(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2
*c) + tan(1/2*d*x) - tan(1/2*c) + 1)) + 210*b^3*arctan((tan(1/2*d*x)*tan(1/
2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*
x) - tan(1/2*c) + 1)) + 9360*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/
2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*
c) - 1)) - 210*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2
*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)) + 9360*
a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan
(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)) - 210*b^3*arctan((ta
n(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/
2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)) + 30720*a^3*tan(1/2*d*x) + 30720*a^3
*tan(1/2*c) - 9216*a^2*b - 2048*b^3)/(d*tan(1/2*d*x)^10*tan(1/2*c)^10 + 5*d
*tan(1/2*d*x)^10*tan(1/2*c)^8 + 5*d*tan(1/2*d*x)^8*tan(1/2*c)^10 + 10*d*tan
(1/2*d*x)^10*tan(1/2*c)^6 + 25*d*tan(1/2*d*x)^8*tan(1/2*c)^8 + 10*d*tan(1/2
*d*x)^6*tan(1/2*c)^10 + 10*d*tan(1/2*d*x)^10*tan(1/2*c)^4 + 50*d*tan(1/2*d*
x)^8*tan(1/2*c)^6 + 50*d*tan(1/2*d*x)^6*tan(1/2*c)^8 + 10*d*tan(1/2*d*x)^4*
tan(1/2*c)^10 + 5*d*tan(1/2*d*x)^10*tan(1/2*c)^2 + 50*d*tan(1/2*d*x)^8*tan(
1/2*c)^4 + 100*d*tan(1/2*d*x)^6*tan(1/2*c)^6 + 50*d*tan(1/2*d*x)^4*tan(1/2*
c)^8 + 5*d*tan(1/2*d*x)^2*tan(1/2*c)^10 + d*tan(1/2*d*x)^10 + 25*d*tan(1/2*
d*x)^8*tan(1/2*c)^2 + 100*d*tan(1/2*d*x)^6*tan(1/2*c)^4 + 100*d*tan(1/2*d*x
)^4*tan(1/2*c)^6 + 25*d*tan(1/2*d*x)^2*tan(1/2*c)^8 + d*tan(1/2*c)^10 + 5*d
*tan(1/2*d*x)^8 + 50*d*tan(1/2*d*x)^6*tan(1/2*c)^2 + 100*d*tan(1/2*d*x)^4*t
an(1/2*c)^4 + 50*d*tan(1/2*d*x)^2*tan(1/2*c)^6 + 5*d*tan(1/2*c)^8 + 10*d*ta
n(1/2*d*x)^6 + 50*d*tan(1/2*d*x)^4*tan(1/2*c)^2 + 50*d*tan(1/2*d*x)^2*tan(1
/2*c)^4 + 10*d*tan(1/2*c)^6 + 10*d*tan(1/2*d*x)^4 + 25*d*tan(1/2*d*x)^2*tan
(1/2*c)^2 + 10*d*tan(1/2*c)^4 + 5*d*tan(1/2*d*x)^2 + 5*d*tan(1/2*c)^2 + d)
```

Mupad [B] (verification not implemented)

Time = 4.53 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.40

$$\int \cos^5(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$=\frac{2\left(\frac{3\sin(c+d\,x)\,a^3\cos(c+d\,x)^4}{2}+2\sin\left(c+d\,x\right)\,a^3\cos\left(c+d\,x\right)^2+4\sin\left(c+d\,x\right)\,a^3-\frac{9\,a^2\,b\cos(c+d\,x)^5}{2}-\frac{9\sin(c+d\,x)}{}\right.}{15\,d}$$

```
[In] int(cos(c + d*x)^5*(a + b*tan(c + d*x))^3,x)
```

```
[Out] (2*(4*a^3*sin(c + d*x) - (5*b^3*cos(c + d*x)^3)/2 + (3*b^3*cos(c + d*x)^5)/
2 - (9*a^2*b*cos(c + d*x)^5)/2 + 2*a^3*cos(c + d*x)^2*sin(c + d*x) + (3*a^3
*cos(c + d*x)^4*sin(c + d*x))/2 + 3*a*b^2*sin(c + d*x) + (3*a*b^2*cos(c + d
*x)^2*sin(c + d*x))/2 - (9*a*b^2*cos(c + d*x)^4*sin(c + d*x))/2))/(15*d)
```

3.543 $\int \cos^7(c+dx)(a+b\tan(c+dx))^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 142

$$\begin{aligned}\int \cos^7(c+dx)(a+b\tan(c+dx))^3\,dx &= \frac{8a(2a^2+b^2)\sin(c+dx)}{35d} - \frac{3\cos^5(c+dx)(b-2a\tan(c+dx))(a+b\tan(c+dx))^2}{35d} \\ &+ \frac{\cos^6(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{7d} \\ &- \frac{2\cos^3(c+dx)\left(b(6a^2+b^2)-a(4a^2-b^2)\tan(c+dx)\right)}{35d}\end{aligned}$$

```
[Out] 8/35*a*(2*a^2+b^2)*sin(d*x+c)/d-3/35*cos(d*x+c)^5*(b-2*a*tan(d*x+c))*(a+b*t
an(d*x+c))^2/d+1/7*cos(d*x+c)^6*sin(d*x+c)*(a+b*tan(d*x+c))^3/d-2/35*cos(d*
x+c)^3*(b*(6*a^2+b^2)-a*(4*a^2-b^2)*tan(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3593, 751, 835, 792, 197}

$$\begin{aligned}\int \cos^7(c+dx)(a+b\tan(c+dx))^3\,dx &= \frac{8a(2a^2+b^2)\sin(c+dx)}{35d} - \frac{2\cos^3(c+dx)\left(b(6a^2+b^2)-a(4a^2-b^2)\tan(c+dx)\right)}{35d} \\ &- \frac{3\cos^5(c+dx)(b-2a\tan(c+dx))(a+b\tan(c+dx))^2}{35d} \\ &+ \frac{\sin(c+dx)\cos^6(c+dx)(a+b\tan(c+dx))^3}{7d}\end{aligned}$$

```
[In] Int[Cos[c + d*x]^7*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (8*a*(2*a^2 + b^2)*Sin[c + d*x])/(35*d) - (3*Cos[c + d*x]^5*(b - 2*a*Tan[c
+ d*x])*(a + b*Tan[c + d*x])^2)/(35*d) + (Cos[c + d*x]^6*Sin[c + d*x]*(a +
b*Tan[c + d*x])^3)/(7*d) - (2*Cos[c + d*x]^3*(b*(6*a^2 + b^2) - a*(4*a^2 -
b^2)*Tan[c + d*x]))/(35*d)
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 751

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-x)*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*a*(p + 1))), x] + Dist[1/(2*a*(p +
 1)), Int[(d + e*x)^(m - 1)*(d*(2*p + 3) + e*(m + 2*p + 3)*x)*(a + c*x^2)^(
p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p,
 -1] && GtQ[m, 0] && (LtQ[m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) &&
IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\left(\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{(a+x)^3}{\left(1+\frac{x^2}{b^2}\right)^{9/2}}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\cos^6(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{7d} - \frac{\left(\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{(-6a-3x)(a+x)^2}{\left(1+\frac{x^2}{b^2}\right)^{7/2}}\,dx, x, b\tan(c+dx)\right)}{7bd}$$

$$= -\frac{3\cos^5(c+dx)(b-2a\tan(c+dx))(a+b\tan(c+dx))^2}{35d} + \frac{\cos^6(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{7d} - \frac{\left(b\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{(a+x)\left(-6\left(1+\frac{4a^2}{b^2}\right)-\frac{12ax}{b^2}\right)}{\left(1+\frac{x^2}{b^2}\right)^{5/2}}\,dx, x, b\tan(c+dx)\right)}{35d}$$

$$= -\frac{3\cos^5(c+dx)(b-2a\tan(c+dx))(a+b\tan(c+dx))^2}{35d} + \frac{\cos^6(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{7d} - \frac{2\cos^3(c+dx)\left(b(6a^2+b^2)-a(4a^2-b^2)\tan(c+dx)\right)}{35d} + \frac{\left(8a\left(1+\frac{2a^2}{b^2}\right)b\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{x^2}{b^2}\right)^{3/2}}\,dx, x, b\tan(c+dx)\right)}{35d}$$

$$= \frac{8a(2a^2+b^2)\sin(c+dx)}{35d} - \frac{3\cos^5(c+dx)(b-2a\tan(c+dx))(a+b\tan(c+dx))^2}{35d} + \frac{\cos^6(c+dx)\sin(c+dx)(a+b\tan(c+dx))^3}{7d} - \frac{2\cos^3(c+dx)\left(b(6a^2+b^2)-a(4a^2-b^2)\tan(c+dx)\right)}{35d}$$

Mathematica [A] (verified)

Time = 3.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.99

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^3\,dx$$
$$=\frac{-30a^2b\cos^7(c+dx)+b^3\cos^5(c+dx)(-9+5\cos(2(c+dx)))+4b^3\sqrt{\cos^2(c+dx)}\sec(c+dx)+2a\sin(c}{70d}$$

```
[In] Integrate[Cos[c + d*x]^7*(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-30*a^2*b*Cos[c + d*x]^7 + b^3*Cos[c + d*x]^5*(-9 + 5*Cos[2*(c + d*x)]) +
4*b^3*Sqrt[Cos[c + d*x]^2]*Sec[c + d*x] + 2*a*Sin[c + d*x]*(35*a^2 - 35*(a^
2 - b^2)*Sin[c + d*x]^2 + 21*(a^2 - 2*b^2)*Sin[c + d*x]^4 - 5*(a^2 - 3*b^2)
*Sin[c + d*x]^6))/(70*d)
```

Maple [A] (verified)

Time = 119.54 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{b^3\left(-\frac{\left(\cos^5(dx+c)\right)\left(\sin^2(dx+c)\right)}{7}-\frac{2\left(\cos^5(dx+c)\right)}{35}\right)+3a\,b^2\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin}{35}\right.}{d}$
default	$\frac{b^3\left(-\frac{\left(\cos^5(dx+c)\right)\left(\sin^2(dx+c)\right)}{7}-\frac{2\left(\cos^5(dx+c)\right)}{35}\right)+3a\,b^2\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin}{35}\right.}{d}$
risch	$-\frac{15b\cos(dx+c)a^2}{64d}-\frac{3b^3\cos(dx+c)}{64d}+\frac{35a^3\sin(dx+c)}{64d}+\frac{15a\sin(dx+c)b^2}{64d}-\frac{3b\cos(7dx+7c)a^2}{448d}+\frac{b^3\cos(7dx+7}{448d}$

```
[In] int(cos(d*x+c)^7*(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^3*(-1/7*cos(d*x+c)^5*sin(d*x+c)^2-2/35*cos(d*x+c)^5)+3*a*b^2*(-1/7*s
in(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
-3/7*a^2*b*cos(d*x+c)^7+1/7*a^3*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos
(d*x+c)^2)*sin(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.87

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{7\,b^3\cos\left(dx+c\right)^5+5\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^7-\left(5\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^6+3\left(2\,a^3+ab^2\right)\cos\left(dx+c\right.}{35\,d}$$

```
[In] integrate(cos(d*x+c)^7*(a+b*tan(d*x+c))^3,x, algorithm="fricas")
[Out] -1/35*(7*b^3*cos(d*x + c)^5 + 5*(3*a^2*b - b^3)*cos(d*x + c)^7 - (5*(a^3 -
3*a*b^2)*cos(d*x + c)^6 + 3*(2*a^3 + a*b^2)*cos(d*x + c)^4 + 16*a^3 + 8*a*b
^2 + 4*(2*a^3 + a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/d
```

Sympy [F]

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^3\,dx = \int \left(a+b\tan\left(c+dx\right)\right)^3\cos^7\left(c+dx\right)dx$$

```
[In] integrate(cos(d*x+c)**7*(a+b*tan(d*x+c))**3,x)
[Out] Integral((a + b*tan(c + d*x))**3*cos(c + d*x)**7, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.89

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^3\,dx = -\frac{15\,a^2b\cos\left(dx+c\right)^7+\left(5\,\sin\left(dx+c\right)^7-21\,\sin\left(dx+c\right)^5+35\,\sin\left(dx+c\right)^3-35\,\sin\left(dx+c\right)\right)a^3-\left(15\,\mathrm{s}\right.}{35\,d}$$

```
[In] integrate(cos(d*x+c)^7*(a+b*tan(d*x+c))^3,x, algorithm="maxima")
[Out] -1/35*(15*a^2*b*cos(d*x + c)^7 + (5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35
*sin(d*x + c)^3 - 35*sin(d*x + c))*a^3 - (15*sin(d*x + c)^7 - 42*sin(d*x +
c)^5 + 35*sin(d*x + c)^3)*a*b^2 - (5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*b^3
)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101962 vs. 2(136) = 272.

Time = 84.49 (sec) , antiderivative size = 101962, normalized size of antiderivative = 718.04

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^3\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^7*(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/17920*(945*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*ta
n(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*
x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*
c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^14 + 210*pi*b^3*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
 tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1
/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*
tan(1/2*d*x)^14*tan(1/2*c)^14 + 945*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2
*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - ta
n(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c
)^14 + 210*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/
2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2
*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2
 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^14 - 2205*pi*a^2*b*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^14 + 1995*pi*
b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2
*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^14
+ 2205*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)
^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*ta
n(1/2*c)^14 - 1995*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*
tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*
d*x)^14*tan(1/2*c)^14 - 4830*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan
(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^14
*tan(1/2*c)^14 - 420*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^
2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^14*tan(1/2*c
)^14 + 6615*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan
(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c
)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^12 + 1470*pi*b^3*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
 tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1
/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*
tan(1/2*d*x)^14*tan(1/2*c)^12 + 6615*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)
```

^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^12 + 1470*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^12 + 6615*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^14 + 1470*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^14 + 6615*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^14 + 1470*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^14 + 4830*pi*a^2*b*tan(1/2*d*x)^14*tan(1/2*c)^14 + 420*pi*b^3*tan(1/2*d*x)^14*tan(1/2*c)^14 - 1890*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^14*tan(1/2*c)^14 - 420*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^14*tan(1/2*c)^14 - 1890*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^14*tan(1/2*c)^14 - 420*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^14*tan(1/2*c)^14 + 7770*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^14*tan(1/2*c)^14 + 420*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^14*tan(1/2*c)^14 + 7770*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^14*tan(1/2*c)^14 + 420*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^14*tan(1/2*c)^14 - 15435*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^12 + 13965*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^12 + 15435*pi*a^2*b*sgn(tan(1/2*d*x)

^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)
^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^12 - 13965*pi*b^3*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
 tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^12 - 33810*p
i*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*t
an(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^14*tan(1/2*c)^12 - 2940*pi*b^3*s
gn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c)
 - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^14*tan(1/2*c)^12 - 15435*pi*a^2*b*sgn(tan
(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + t
an(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^14 + 13965*pi*
b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2
*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^14
 + 15435*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c
)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*t
an(1/2*c)^14 - 13965*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x
)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/
2*d*x)^12*tan(1/2*c)^14 - 33810*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 -
tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)
^12*tan(1/2*c)^14 - 2940*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d
*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^12*tan(1
/2*c)^14 - 7680*a^2*b*tan(1/2*d*x)^14*tan(1/2*c)^14 - 1024*b^3*tan(1/2*d*x)
^14*tan(1/2*c)^14 + 19845*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(
1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*s
gn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)
^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^10 + 441
0*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + ta
n(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*
c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(
1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^10 + 19845*pi*a^2*b*sgn(tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*
c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*t
an(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d
*x)^14*tan(1/2*c)^10 + 4410*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(
1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*s
gn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)
^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^10 + 463
05*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) +
 tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1
/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*t
an(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^12 + 10290*pi*b^3*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2
*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*
tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*
d*x)^12*tan(1/2*c)^12 + 46305*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*
tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) -

1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^12 + 10290*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^12 + 33810*pi*a^2*b*tan(1/2*d*x)^14*tan(1/2*c)^12 + 2940*pi*b^3*tan(1/2*d*x)^14*tan(1/2*c)^12 - 13230*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^14*tan(1/2*c)^12 - 2940*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^14*tan(1/2*c)^12 - 13230*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^14*tan(1/2*c)^12 - 2940*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^14*tan(1/2*c)^12 + 54390*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^14*tan(1/2*c)^12 + 2940*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^14*tan(1/2*c)^12 + 54390*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^14*tan(1/2*c)^12 + 2940*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^14*tan(1/2*c)^12 - 35840*a^3*tan(1/2*d*x)^14*tan(1/2*c)^13 + 19845*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^14 + 4410*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^14 + 19845*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^14 + 4410*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^14 + 33810*pi*a^2*b*tan(1/2*d*x)^12*tan(1/2*c)^14 + 2940*pi*b^3*tan(1/2*d*x)^12*tan(1/2*c)^14 - 13230*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^12*tan(1/2*c)^14 - 2940*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2

*c) + 1))*tan(1/2*d*x)^12*tan(1/2*c)^14 - 13230*a^2*b*arctan((tan(1/2*d*x)*
tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(
1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^12*tan(1/2*c)^14 - 2940*b^3*arctan
((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*ta
n(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^12*tan(1/2*c)^14 +
54390*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1
)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^1
2*tan(1/2*c)^14 + 2940*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) +
tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))
*tan(1/2*d*x)^12*tan(1/2*c)^14 + 54390*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c
) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x)
+ tan(1/2*c) - 1))*tan(1/2*d*x)^12*tan(1/2*c)^14 + 2940*b^3*arctan((tan(1/2
*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c)
+ tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^12*tan(1/2*c)^14 - 35840*a^3
*tan(1/2*d*x)^13*tan(1/2*c)^14 - 46305*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*
c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(
1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^10 + 41895*pi*b^3*sgn(tan(1/2*d*x)
^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)
^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^10 + 46305*pi*a^2*b*sgn
(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2
+ tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^10 - 41895
*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan
(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)
^10 - 101430*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*
tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^14*tan(1/2*c)^10 -
8820*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d
*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^14*tan(1/2*c)^10 - 108045*p
i*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan
(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)
^12 + 97755*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2
*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12
*tan(1/2*c)^12 + 108045*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/
2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*t
an(1/2*d*x)^12*tan(1/2*c)^12 - 97755*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
- 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*
d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^12 - 236670*pi*a^2*b*sgn(tan(1/2*d*x)^
2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2
+ 1)*tan(1/2*d*x)^12*tan(1/2*c)^12 - 20580*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/
2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan
(1/2*d*x)^12*tan(1/2*c)^12 + 53760*a^2*b*tan(1/2*d*x)^14*tan(1/2*c)^12 - 71
68*b^3*tan(1/2*d*x)^14*tan(1/2*c)^12 + 215040*a^2*b*tan(1/2*d*x)^13*tan(1/2
*c)^13 - 46305*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*ta
n(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*
x)^10*tan(1/2*c)^14 + 41895*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(
1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)

*tan(1/2*d*x)^10*tan(1/2*c)^14 + 46305*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^14 - 41895*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^14 - 101430*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^10*tan(1/2*c)^14 - 8820*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^10*tan(1/2*c)^14 + 53760*a^2*b*tan(1/2*d*x)^12*tan(1/2*c)^14 - 7168*b^3*tan(1/2*d*x)^12*tan(1/2*c)^14 + 33075*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^8 + 7350*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^8 + 33075*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^8 + 7350*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^8 + 138915*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^10 + 30870*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^10 + 138915*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^10 + 30870*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^10 + 101430*pi*a^2*b*tan(1/2*d*x)^14*tan(1/2*c)^10 + 8820*pi*b^3*tan(1/2*d*x)^14*tan(1/2*c)^10 - 39690*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^14*tan(1/2*c)^10 - 8820*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^14*tan(1/2*c)^10 - 39690*a^2*b*arctan

((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*ta
n(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^14*tan(1/2*c)^10 -
8820*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(
tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^14*t
an(1/2*c)^10 + 163170*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x)
+ tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)
)*tan(1/2*d*x)^14*tan(1/2*c)^10 + 8820*b^3*arctan((tan(1/2*d*x)*tan(1/2*c)
+ tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) -
tan(1/2*c) - 1))*tan(1/2*d*x)^14*tan(1/2*c)^10 + 163170*a^2*b*arctan((tan(1
/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c
) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^14*tan(1/2*c)^10 + 8820*b^
3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2
*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^14*tan(1/2*
c)^10 - 71680*a^3*tan(1/2*d*x)^14*tan(1/2*c)^11 - 143360*a*b^2*tan(1/2*d*x)
^14*tan(1/2*c)^11 + 138915*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan
(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*
sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^12 + 30
870*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) +
tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/
2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*ta
n(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^12 + 138915*pi*a^2*b*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1
/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x
)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/
2*d*x)^10*tan(1/2*c)^12 + 30870*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*
tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) -
1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*
d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^12 +
236670*pi*a^2*b*tan(1/2*d*x)^12*tan(1/2*c)^12 + 20580*pi*b^3*tan(1/2*d*x)^
12*tan(1/2*c)^12 - 92610*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*
x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) +
1))*tan(1/2*d*x)^12*tan(1/2*c)^12 - 20580*b^3*arctan((tan(1/2*d*x)*tan(1/2
*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x
) + tan(1/2*c) + 1))*tan(1/2*d*x)^12*tan(1/2*c)^12 - 92610*a^2*b*arctan((ta
n(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/
2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^12*tan(1/2*c)^12 - 2058
0*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan
(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^12*tan(
1/2*c)^12 + 380730*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + t
an(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*t
an(1/2*d*x)^12*tan(1/2*c)^12 + 20580*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) +
tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - ta
n(1/2*c) - 1))*tan(1/2*d*x)^12*tan(1/2*c)^12 + 380730*a^2*b*arctan((tan(1/2
*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c)

+ tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^12*tan(1/2*c)^12 + 20580*b^3
*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*
d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^12*tan(1/2*c
)^12 + 250880*a^3*tan(1/2*d*x)^13*tan(1/2*c)^12 - 430080*a*b^2*tan(1/2*d*x)
^13*tan(1/2*c)^12 + 250880*a^3*tan(1/2*d*x)^12*tan(1/2*c)^13 - 430080*a*b^2
*tan(1/2*d*x)^12*tan(1/2*c)^13 + 33075*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*
c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(
1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 -
tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2
*c)^14 + 7350*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan
(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c
)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^14 + 33075*pi*a^2*b*sgn
(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2
- tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan
(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1
)*tan(1/2*d*x)^8*tan(1/2*c)^14 + 7350*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2
*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - ta
n(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)
^14 + 101430*pi*a^2*b*tan(1/2*d*x)^10*tan(1/2*c)^14 + 8820*pi*b^3*tan(1/2*d
*x)^10*tan(1/2*c)^14 - 39690*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/
2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*
c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^14 - 8820*b^3*arctan((tan(1/2*d*x)*tan(
1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*
d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^14 - 39690*a^2*b*arctan(
(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan
(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^14 - 8
820*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(t
an(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^10*ta
n(1/2*c)^14 + 163170*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) +
tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))
*tan(1/2*d*x)^10*tan(1/2*c)^14 + 8820*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) +
tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - t
an(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^14 + 163170*a^2*b*arctan((tan(1/
2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c)
+ tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^14 + 8820*b^3
*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*
d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c
)^14 - 71680*a^3*tan(1/2*d*x)^11*tan(1/2*c)^14 - 143360*a*b^2*tan(1/2*d*x)^
11*tan(1/2*c)^14 - 77175*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1
/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*
tan(1/2*d*x)^14*tan(1/2*c)^8 + 69825*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
+ 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*
d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^8 + 77175*pi*a^2*b*sgn(tan(1/2*d*x)^2*

tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2
- 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^8 - 69825*pi*b^3*sgn(tan(1
/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan
(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^8 - 169050*pi*a^
2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1
/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^14*tan(1/2*c)^8 - 14700*pi*b^3*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - t
an(1/2*c)^2 + 1)*tan(1/2*d*x)^14*tan(1/2*c)^8 - 324135*pi*a^2*b*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1
/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^10 + 293265*pi*b^3
*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*
x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^10 + 3
24135*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^
2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan
(1/2*c)^10 - 293265*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)
*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2
*d*x)^12*tan(1/2*c)^10 - 710010*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 -
tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)
^12*tan(1/2*c)^10 - 61740*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*
d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^12*tan(
1/2*c)^10 - 161280*a^2*b*tan(1/2*d*x)^14*tan(1/2*c)^10 + 50176*b^3*tan(1/2*
d*x)^14*tan(1/2*c)^10 - 1290240*a^2*b*tan(1/2*d*x)^13*tan(1/2*c)^11 + 28672
0*b^3*tan(1/2*d*x)^13*tan(1/2*c)^11 - 324135*pi*a^2*b*sgn(tan(1/2*d*x)^2*ta
n(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 +
2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^12 + 293265*pi*b^3*sgn(tan(1
/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan
(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^12 + 324135*pi*a
^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/
2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^12
 - 293265*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c
)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*t
an(1/2*c)^12 - 710010*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*
x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^10*tan(1/
2*c)^12 - 61740*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4
*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^10*tan(1/2*c)^12
- 2956800*a^2*b*tan(1/2*d*x)^12*tan(1/2*c)^12 + 379904*b^3*tan(1/2*d*x)^12*
tan(1/2*c)^12 - 1290240*a^2*b*tan(1/2*d*x)^11*tan(1/2*c)^13 + 286720*b^3*ta
n(1/2*d*x)^11*tan(1/2*c)^13 - 77175*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2
*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^14 + 69825*pi*b^3*sgn(tan(1/2*d*x)^2*t
an(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 +
 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^14 + 77175*pi*a^2*b*sgn(tan(
1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + ta
n(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^14 - 69825*pi*b^
3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d

*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^14 - 1
69050*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2
*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^8*tan(1/2*c)^14 - 14700*p
i*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan
(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^8*tan(1/2*c)^14 - 161280*a^2*b*tan
(1/2*d*x)^10*tan(1/2*c)^14 + 50176*b^3*tan(1/2*d*x)^10*tan(1/2*c)^14 + 3307
5*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) +
tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/
2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*ta
n(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^6 + 7350*pi*b^3*sgn(tan(1/2*d*x)
^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)
^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan
(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x
)^14*tan(1/2*c)^6 + 33075*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(
1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*s
gn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)
^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^6 + 7350
*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan
(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c
)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1
/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^6 + 231525*pi*a^2*b*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c
)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*ta
n(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*
x)^12*tan(1/2*c)^8 + 51450*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1
/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sg
n(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^
2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^8 + 23152
5*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) +
tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/
2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*ta
n(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^8 + 51450*pi*b^3*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c
)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*ta
n(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*
x)^12*tan(1/2*c)^8 + 169050*pi*a^2*b*tan(1/2*d*x)^14*tan(1/2*c)^8 + 14700*p
i*b^3*tan(1/2*d*x)^14*tan(1/2*c)^8 - 66150*a^2*b*arctan((tan(1/2*d*x)*tan(1
/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d
*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^14*tan(1/2*c)^8 - 14700*b^3*arctan((tan
(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2
*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^14*tan(1/2*c)^8 - 66150*
a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan
(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^14*tan(
1/2*c)^8 - 14700*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1
/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1

/2*d*x)^14*tan(1/2*c)^8 + 271950*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + ta
n(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(
1/2*c) - 1))*tan(1/2*d*x)^14*tan(1/2*c)^8 + 14700*b^3*arctan((tan(1/2*d*x)*
tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(
1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^14*tan(1/2*c)^8 + 271950*a^2*b*arc
tan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)
*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^14*tan(1/2*c)^8
 + 14700*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1
)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^1
4*tan(1/2*c)^8 - 308224*a^3*tan(1/2*d*x)^14*tan(1/2*c)^9 + 114688*a*b^2*tan
(1/2*d*x)^14*tan(1/2*c)^9 + 416745*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*
c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan
(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)
^10 + 92610*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1
/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^
2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^
2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 + 416745*pi*a^2*b*sgn
(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2
 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan
(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1
)*tan(1/2*d*x)^10*tan(1/2*c)^10 + 92610*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c
)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1
/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 -
tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2
*c)^10 + 710010*pi*a^2*b*tan(1/2*d*x)^12*tan(1/2*c)^10 + 61740*pi*b^3*tan(1
/2*d*x)^12*tan(1/2*c)^10 - 277830*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + t
an(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan
(1/2*c) + 1))*tan(1/2*d*x)^12*tan(1/2*c)^10 - 61740*b^3*arctan((tan(1/2*d*x
)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - ta
n(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^12*tan(1/2*c)^10 - 277830*a^2*b*
arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d
*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^12*tan(1/2*c)
^10 - 61740*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c)
 + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*
x)^12*tan(1/2*c)^10 + 1142190*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1
/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2
*c) - 1))*tan(1/2*d*x)^12*tan(1/2*c)^10 + 61740*b^3*arctan((tan(1/2*d*x)*ta
n(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/
2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^12*tan(1/2*c)^10 + 1142190*a^2*b*arc
tan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)
*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^12*tan(1/2*c)^10
 + 61740*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) -
1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^
12*tan(1/2*c)^10 - 752640*a^3*tan(1/2*d*x)^13*tan(1/2*c)^10 + 2150400*a*b^2

*tan(1/2*d*x)^13*tan(1/2*c)^10 - 3512320*a^3*tan(1/2*d*x)^12*tan(1/2*c)^11
+ 5877760*a*b^2*tan(1/2*d*x)^12*tan(1/2*c)^11 + 231525*pi*a^2*b*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1
/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x
)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/
2*d*x)^8*tan(1/2*c)^12 + 51450*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*t
an(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1
)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d
*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^12 + 2
31525*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c
) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*ta
n(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 -
2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^12 + 51450*pi*b^3*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1
/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x
)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/
2*d*x)^8*tan(1/2*c)^12 + 710010*pi*a^2*b*tan(1/2*d*x)^10*tan(1/2*c)^12 + 61
740*pi*b^3*tan(1/2*d*x)^10*tan(1/2*c)^12 - 277830*a^2*b*arctan((tan(1/2*d*x
)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - ta
n(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^12 - 61740*b^3*arc
tan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)
*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^12
 - 277830*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c)
 + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*
x)^10*tan(1/2*c)^12 - 61740*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d
*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c)
+ 1))*tan(1/2*d*x)^10*tan(1/2*c)^12 + 1142190*a^2*b*arctan((tan(1/2*d*x)*ta
n(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/
2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^12 + 61740*b^3*arctan(
(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan
(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^12 + 1
142190*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) -
1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^
10*tan(1/2*c)^12 + 61740*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x)
 - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1
))*tan(1/2*d*x)^10*tan(1/2*c)^12 - 3512320*a^3*tan(1/2*d*x)^11*tan(1/2*c)^1
2 + 5877760*a*b^2*tan(1/2*d*x)^11*tan(1/2*c)^12 - 752640*a^3*tan(1/2*d*x)^1
0*tan(1/2*c)^13 + 2150400*a*b^2*tan(1/2*d*x)^10*tan(1/2*c)^13 + 33075*pi*a^
2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2
*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
+ 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d
*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^14 + 7350*pi*b^3*sgn(tan(1/2*d*x)^2*tan(
1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*
tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)
^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan

(1/2*c)^14 + 33075*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^14 + 7350*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^14 + 169050*pi*a^2*b*tan(1/2*d*x)^8*tan(1/2*c)^14 + 14700*pi*b^3*tan(1/2*d*x)^8*tan(1/2*c)^14 - 66150*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^14 - 14700*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^14 - 66150*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^14 - 14700*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^14 + 271950*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^14 + 14700*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^14 + 271950*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^14 + 14700*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^14 - 308224*a^3*tan(1/2*d*x)^9*tan(1/2*c)^14 + 114688*a*b^2*tan(1/2*d*x)^9*tan(1/2*c)^14 - 77175*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^6 + 69825*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^6 + 77175*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^6 - 69825*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^6 - 169050*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^14*tan(1/2*c)^6 - 14700*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^14*tan(1/2*c)^6 - 540225*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^8 + 488775*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^8 + 540225*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*t

an(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) -
 1)*tan(1/2*d*x)^12*tan(1/2*c)^8 - 488775*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2
*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan
(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^8 - 1183350*pi*a^2*b*sgn(tan(1/2*
d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*
c)^2 + 1)*tan(1/2*d*x)^12*tan(1/2*c)^8 - 102900*pi*b^3*sgn(tan(1/2*d*x)^2*t
an(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1
)*tan(1/2*d*x)^12*tan(1/2*c)^8 + 268800*a^2*b*tan(1/2*d*x)^14*tan(1/2*c)^8
 - 107520*b^3*tan(1/2*d*x)^14*tan(1/2*c)^8 + 3225600*a^2*b*tan(1/2*d*x)^13*t
an(1/2*c)^9 - 1146880*b^3*tan(1/2*d*x)^13*tan(1/2*c)^9 - 972405*pi*a^2*b*sg
n(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^
2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^10 + 8797
95*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - t
an(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*
c)^10 + 972405*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*ta
n(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*
x)^10*tan(1/2*c)^10 - 879795*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan
(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1
)*tan(1/2*d*x)^10*tan(1/2*c)^10 - 2130030*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1
/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*ta
n(1/2*d*x)^10*tan(1/2*c)^10 - 185220*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d
*x)^10*tan(1/2*c)^10 + 14031360*a^2*b*tan(1/2*d*x)^12*tan(1/2*c)^10 - 40929
28*b^3*tan(1/2*d*x)^12*tan(1/2*c)^10 + 24944640*a^2*b*tan(1/2*d*x)^11*tan(1
/2*c)^11 - 5734400*b^3*tan(1/2*d*x)^11*tan(1/2*c)^11 - 540225*pi*a^2*b*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2
 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^12 + 488775*
pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(
1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^1
2 + 540225*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/
2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8
*tan(1/2*c)^12 - 488775*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*
d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan
(1/2*d*x)^8*tan(1/2*c)^12 - 1183350*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*
d*x)^8*tan(1/2*c)^12 - 102900*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(
1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^8*t
an(1/2*c)^12 + 14031360*a^2*b*tan(1/2*d*x)^10*tan(1/2*c)^12 - 4092928*b^3*t
an(1/2*d*x)^10*tan(1/2*c)^12 + 3225600*a^2*b*tan(1/2*d*x)^9*tan(1/2*c)^13 -
 1146880*b^3*tan(1/2*d*x)^9*tan(1/2*c)^13 - 77175*pi*a^2*b*sgn(tan(1/2*d*x)
^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)
^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^14 + 69825*pi*b^3*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^14 + 77175*pi*
a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1

/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^14 - 69825*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^14 - 169050*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^14 - 14700*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^14 + 268800*a^2*b*tan(1/2*d*x)^8*tan(1/2*c)^14 - 107520*b^3*tan(1/2*d*x)^8*tan(1/2*c)^14 + 19845*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^4 + 4410*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^4 + 19845*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^4 + 4410*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^4 + 231525*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^6 + 51450*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^6 + 231525*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^6 + 51450*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^6 + 169050*pi*a^2*b*tan(1/2*d*x)^14*tan(1/2*c)^6 + 14700*pi*b^3*tan(1/2*d*x)^14*tan(1/2*c)^6 - 66150*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^14*tan(1/2*c)^6 - 14700*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^14*tan(1/2*c)^6 - 66150*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2

*d*x)^14*tan(1/2*c)^6 - 14700*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2
*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c
) + 1))*tan(1/2*d*x)^14*tan(1/2*c)^6 + 271950*a^2*b*arctan((tan(1/2*d*x)*ta
n(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/
2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^14*tan(1/2*c)^6 + 14700*b^3*arctan((
tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(
1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^14*tan(1/2*c)^6 + 271
950*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/
(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^14*
tan(1/2*c)^6 + 14700*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - t
an(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*t
an(1/2*d*x)^14*tan(1/2*c)^6 - 217088*a^3*tan(1/2*d*x)^14*tan(1/2*c)^7 - 466
944*a*b^2*tan(1/2*d*x)^14*tan(1/2*c)^7 + 694575*pi*a^2*b*sgn(tan(1/2*d*x)^2
*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2
 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1
/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^
10*tan(1/2*c)^8 + 154350*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2
*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2
+ tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^8 + 694575*
pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + ta
n(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*
c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(
1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^8 + 154350*pi*b^3*sgn(tan(1/2*d*x)
^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)
^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan
(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x
)^10*tan(1/2*c)^8 + 1183350*pi*a^2*b*tan(1/2*d*x)^12*tan(1/2*c)^8 + 102900*
pi*b^3*tan(1/2*d*x)^12*tan(1/2*c)^8 - 463050*a^2*b*arctan((tan(1/2*d*x)*tan
(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2
*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^12*tan(1/2*c)^8 - 102900*b^3*arctan((
tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(
1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^12*tan(1/2*c)^8 - 463
050*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/
(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^12*
tan(1/2*c)^8 - 102900*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) +
tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*
tan(1/2*d*x)^12*tan(1/2*c)^8 + 1903650*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c
) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x)
- tan(1/2*c) - 1))*tan(1/2*d*x)^12*tan(1/2*c)^8 + 102900*b^3*arctan((tan(1/
2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c)
 - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^12*tan(1/2*c)^8 + 1903650*a
^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(
1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^12*tan(1
/2*c)^8 + 102900*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1

/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1
/2*d*x)^12*tan(1/2*c)^8 + 1254400*a^3*tan(1/2*d*x)^13*tan(1/2*c)^8 - 430080
0*a*b^2*tan(1/2*d*x)^13*tan(1/2*c)^8 + 5368832*a^3*tan(1/2*d*x)^12*tan(1/2*
c)^9 - 22851584*a*b^2*tan(1/2*d*x)^12*tan(1/2*c)^9 + 694575*pi*a^2*b*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/
2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*t
an(1/2*d*x)^8*tan(1/2*c)^10 + 154350*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*
c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan
(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^
10 + 694575*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan
(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c
)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^10 + 154350*pi*b^3*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2
 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(
1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)
*tan(1/2*d*x)^8*tan(1/2*c)^10 + 2130030*pi*a^2*b*tan(1/2*d*x)^10*tan(1/2*c)
^10 + 185220*pi*b^3*tan(1/2*d*x)^10*tan(1/2*c)^10 - 833490*a^2*b*arctan((ta
n(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/
2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^10 - 1852
20*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(ta
n(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan
(1/2*c)^10 - 833490*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) +
tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*
tan(1/2*d*x)^10*tan(1/2*c)^10 - 185220*b^3*arctan((tan(1/2*d*x)*tan(1/2*c)
 - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) -
tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^10 + 3426570*a^2*b*arctan((tan(
1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*
c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^10 + 185220
*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(
1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1
/2*c)^10 + 3426570*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - t
an(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*t
an(1/2*d*x)^10*tan(1/2*c)^10 + 185220*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) -
 tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + t
an(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^10 + 18565120*a^3*tan(1/2*d*x)^1
1*tan(1/2*c)^10 - 50319360*a*b^2*tan(1/2*d*x)^11*tan(1/2*c)^10 + 18565120*a
^3*tan(1/2*d*x)^10*tan(1/2*c)^11 - 50319360*a*b^2*tan(1/2*d*x)^10*tan(1/2*c
)^11 + 231525*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*t
an(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2
*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^12 + 51450*pi*b^3*sgn
(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2

- tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan
(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1
)*tan(1/2*d*x)^6*tan(1/2*c)^12 + 231525*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2
*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan
(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2
- tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/
2*c)^12 + 51450*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*t
an(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2
*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^12 + 1183350*pi*a^2*b
*tan(1/2*d*x)^8*tan(1/2*c)^12 + 102900*pi*b^3*tan(1/2*d*x)^8*tan(1/2*c)^12
- 463050*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c)
+ 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x
)^8*tan(1/2*c)^12 - 102900*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*
x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) +
1))*tan(1/2*d*x)^8*tan(1/2*c)^12 - 463050*a^2*b*arctan((tan(1/2*d*x)*tan(1
/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d
*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^12 - 102900*b^3*arctan((ta
n(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/
2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^12 + 19036
50*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(
tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^8*ta
n(1/2*c)^12 + 102900*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + t
an(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*t
an(1/2*d*x)^8*tan(1/2*c)^12 + 1903650*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c)
- tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) +
tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^12 + 102900*b^3*arctan((tan(1/2
*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c)
+ tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^12 + 5368832*a^
3*tan(1/2*d*x)^9*tan(1/2*c)^12 - 22851584*a*b^2*tan(1/2*d*x)^9*tan(1/2*c)^1
2 + 1254400*a^3*tan(1/2*d*x)^8*tan(1/2*c)^13 - 4300800*a*b^2*tan(1/2*d*x)^8
*tan(1/2*c)^13 + 19845*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2
*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2
+ tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^14 + 4410*pi
*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/
2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
+ 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*
d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^14 + 19845*pi*a^2*b*sgn(tan(1/2*d*x)^2*
tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2
- 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/
2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4
*tan(1/2*c)^14 + 4410*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*
x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan
(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + t

an(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^14 + 169050*pi*
a^2*b*tan(1/2*d*x)^6*tan(1/2*c)^14 + 14700*pi*b^3*tan(1/2*d*x)^6*tan(1/2*c)
^14 - 66150*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*
c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*
d*x)^6*tan(1/2*c)^14 - 14700*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*
d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c)
 + 1))*tan(1/2*d*x)^6*tan(1/2*c)^14 - 66150*a^2*b*arctan((tan(1/2*d*x)*tan(
1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*
d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^14 - 14700*b^3*arctan((ta
n(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/
2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^14 + 27195
0*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(t
an(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan
(1/2*c)^14 + 14700*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan
(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan
(1/2*d*x)^6*tan(1/2*c)^14 + 271950*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) -
tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + ta
n(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^14 + 14700*b^3*arctan((tan(1/2*d*x
)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + ta
n(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^14 - 217088*a^3*tan
(1/2*d*x)^7*tan(1/2*c)^14 - 466944*a*b^2*tan(1/2*d*x)^7*tan(1/2*c)^14 - 463
05*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 -
 tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/
2*c)^4 + 41895*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(
1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)
^14*tan(1/2*c)^4 + 46305*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1
/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*
tan(1/2*d*x)^14*tan(1/2*c)^4 - 41895*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*
d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^4 - 101430*pi*a^2*b*sgn(tan(1/2*d*x)^2
*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 +
 1)*tan(1/2*d*x)^14*tan(1/2*c)^4 - 8820*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c
)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/
2*d*x)^14*tan(1/2*c)^4 - 540225*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 +
2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x
) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^6 + 488775*pi*b^3*sgn(tan(1/2*d*x)^2*tan(
1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*
tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^6 + 540225*pi*a^2*b*sgn(tan(1/
2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(
1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^6 - 488775*pi*b^3
*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*
x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^6 - 11
83350*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2
*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^12*tan(1/2*c)^6 - 102900*
pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*ta

n(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^12*tan(1/2*c)^6 - 268800*a^2*b*ta
n(1/2*d*x)^14*tan(1/2*c)^6 + 107520*b^3*tan(1/2*d*x)^14*tan(1/2*c)^6 - 4300
800*a^2*b*tan(1/2*d*x)^13*tan(1/2*c)^7 + 1720320*b^3*tan(1/2*d*x)^13*tan(1/
2*c)^7 - 1620675*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*
tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*
d*x)^10*tan(1/2*c)^8 + 1466325*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*t
an(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) -
1)*tan(1/2*d*x)^10*tan(1/2*c)^8 + 1620675*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(
1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*
tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^8 - 1466325*pi*b^3*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1
/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^8 - 3550050*pi*a^2
*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/
2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^10*tan(1/2*c)^8 - 308700*pi*b^3*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - t
an(1/2*c)^2 + 1)*tan(1/2*d*x)^10*tan(1/2*c)^8 - 27686400*a^2*b*tan(1/2*d*x)
^12*tan(1/2*c)^8 + 10429440*b^3*tan(1/2*d*x)^12*tan(1/2*c)^8 - 88166400*a^2
*b*tan(1/2*d*x)^11*tan(1/2*c)^9 + 29532160*b^3*tan(1/2*d*x)^11*tan(1/2*c)^9
- 1620675*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/
2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8
*tan(1/2*c)^10 + 1466325*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2
*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*ta
n(1/2*d*x)^8*tan(1/2*c)^10 + 1620675*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)
^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/
2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^10 - 1466325*pi*b^3*sgn(tan(1/2*d*x)^
2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^
2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^10 - 3550050*pi*a^2*b*sgn
(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) -
tan(1/2*c)^2 + 1)*tan(1/2*d*x)^8*tan(1/2*c)^10 - 308700*pi*b^3*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2
*c)^2 + 1)*tan(1/2*d*x)^8*tan(1/2*c)^10 - 136711680*a^2*b*tan(1/2*d*x)^10*t
an(1/2*c)^10 + 40406016*b^3*tan(1/2*d*x)^10*tan(1/2*c)^10 - 88166400*a^2*b*
tan(1/2*d*x)^9*tan(1/2*c)^11 + 29532160*b^3*tan(1/2*d*x)^9*tan(1/2*c)^11 -
540225*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)
^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan
(1/2*c)^12 + 488775*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)
*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2
*d*x)^6*tan(1/2*c)^12 + 540225*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2
*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x)
- 1)*tan(1/2*d*x)^6*tan(1/2*c)^12 - 488775*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1
/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*t
an(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^12 - 1183350*pi*a^2*b*sgn(tan(1/
2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/
2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^12 - 102900*pi*b^3*sgn(tan(1/2*d*x)^2
*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 +

1)*tan(1/2*d*x)^6*tan(1/2*c)^12 - 27686400*a^2*b*tan(1/2*d*x)^8*tan(1/2*c)^12 + 10429440*b^3*tan(1/2*d*x)^8*tan(1/2*c)^12 - 4300800*a^2*b*tan(1/2*d*x)^7*tan(1/2*c)^13 + 1720320*b^3*tan(1/2*d*x)^7*tan(1/2*c)^13 - 46305*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^14 + 41895*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^14 + 46305*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^14 - 41895*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^14 - 101430*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^14 - 8820*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^14 - 268800*a^2*b*tan(1/2*d*x)^6*tan(1/2*c)^14 + 107520*b^3*tan(1/2*d*x)^6*tan(1/2*c)^14 + 6615*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^2 + 1470*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^2 + 6615*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^2 + 1470*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^2 + 138915*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^4 + 30870*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^4 + 138915*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^4 + 30870*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2

*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^4 + 101430*pi*a^2*b*tan(1/2*d
*x)^14*tan(1/2*c)^4 + 8820*pi*b^3*tan(1/2*d*x)^14*tan(1/2*c)^4 - 39690*a^2*
b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2
*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^14*tan(1/2*
c)^4 - 8820*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c)
 + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*
x)^14*tan(1/2*c)^4 - 39690*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*
d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c)
 + 1))*tan(1/2*d*x)^14*tan(1/2*c)^4 - 8820*b^3*arctan((tan(1/2*d*x)*tan(1/2
*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x
) - tan(1/2*c) + 1))*tan(1/2*d*x)^14*tan(1/2*c)^4 + 163170*a^2*b*arctan((ta
n(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/
2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^14*tan(1/2*c)^4 + 8820*
b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1
/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^14*tan(1/
2*c)^4 + 163170*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(
1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(
1/2*d*x)^14*tan(1/2*c)^4 + 8820*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1
/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2
*c) - 1))*tan(1/2*d*x)^14*tan(1/2*c)^4 - 308224*a^3*tan(1/2*d*x)^14*tan(1/2
*c)^5 + 114688*a*b^2*tan(1/2*d*x)^14*tan(1/2*c)^5 + 694575*pi*a^2*b*sgn(tan
(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - t
an(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2
*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*ta
n(1/2*d*x)^10*tan(1/2*c)^6 + 154350*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c
) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(
1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^
6 + 694575*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(
1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)
^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)
^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^6 + 154350*pi*b^3*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
 tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1
/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*
tan(1/2*d*x)^10*tan(1/2*c)^6 + 1183350*pi*a^2*b*tan(1/2*d*x)^12*tan(1/2*c)^
6 + 102900*pi*b^3*tan(1/2*d*x)^12*tan(1/2*c)^6 - 463050*a^2*b*arctan((tan(1
/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c
) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^12*tan(1/2*c)^6 - 102900*b
^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/
2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^12*tan(1/2
*c)^6 - 463050*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1
/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1
/2*d*x)^12*tan(1/2*c)^6 - 102900*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(
1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/

2*c) + 1))*tan(1/2*d*x)^12*tan(1/2*c)^6 + 1903650*a^2*b*arctan((tan(1/2*d*x
)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - ta
n(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^12*tan(1/2*c)^6 + 102900*b^3*arc
tan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)
*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^12*tan(1/2*c)^6
+ 1903650*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c)
 - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*
x)^12*tan(1/2*c)^6 + 102900*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d
*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c)
- 1))*tan(1/2*d*x)^12*tan(1/2*c)^6 - 1254400*a^3*tan(1/2*d*x)^13*tan(1/2*c)
^6 + 4300800*a*b^2*tan(1/2*d*x)^13*tan(1/2*c)^6 - 11554816*a^3*tan(1/2*d*x)
^12*tan(1/2*c)^7 + 31137792*a*b^2*tan(1/2*d*x)^12*tan(1/2*c)^7 + 1157625*pi
*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(
1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)
^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/
2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^8 + 257250*pi*b^3*sgn(tan(1/2*d*x)^2*
tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2
+ 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/
2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8
*tan(1/2*c)^8 + 1157625*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/
2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn
(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2
 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^8 + 257250*
pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(
1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)
^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/
2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^8 + 3550050*pi*a^2*b*tan(1/2*d*x)^10*
tan(1/2*c)^8 + 308700*pi*b^3*tan(1/2*d*x)^10*tan(1/2*c)^8 - 1389150*a^2*b*a
rctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*
x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^
8 - 308700*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c)
+ 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x
)^10*tan(1/2*c)^8 - 1389150*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2
*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c
) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^8 - 308700*b^3*arctan((tan(1/2*d*x)*tan(
1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*
d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^8 + 5710950*a^2*b*arctan
((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*ta
n(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^8 + 3
08700*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/
(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^10*
tan(1/2*c)^8 + 5710950*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x)
 - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1
))*tan(1/2*d*x)^10*tan(1/2*c)^8 + 308700*b^3*arctan((tan(1/2*d*x)*tan(1/2*c
) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x)

+ tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^8 - 37632000*a^3*tan(1/2*d*x)^11*tan(1/2*c)^8 + 122572800*a*b^2*tan(1/2*d*x)^11*tan(1/2*c)^8 - 86754304*a^3*tan(1/2*d*x)^10*tan(1/2*c)^9 + 241102848*a*b^2*tan(1/2*d*x)^10*tan(1/2*c)^9 + 694575*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^10 + 154350*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^10 + 694575*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^10 + 154350*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^10 + 3550050*pi*a^2*b*tan(1/2*d*x)^8*tan(1/2*c)^10 + 308700*pi*b^3*tan(1/2*d*x)^8*tan(1/2*c)^10 - 1389150*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^10 - 308700*b^3*arctan((tan(1/2*d*x)*tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^10 - 1389150*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^10 - 308700*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^10 + 5710950*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^10 + 308700*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^10 + 5710950*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^10 + 308700*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^10 - 86754304*a^3*tan(1/2*d*x)^9*tan(1/2*c)^10 + 241102848*a*b^2*tan(1/2*d*x)^9*tan(1/2*c)^10 - 37632000*a^3*tan(1/2*d*x)^8*tan(1/2*c)^11 + 122572800*a*b^2*tan(1/2*d*x)^8*tan(1/2*c)^11 + 138915*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^12 + 30870*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*t an(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^12 + 138915*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^12 + 30870*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^12 + 1183350*pi*a^2*b*tan(1/2*d*x)^6*tan(1/2*c)^12 + 102900*pi*b^3*tan(1/2*d*x)^6*tan(1/2*c)^12 - 463050*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^12 - 102900*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^12 - 463050*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^12 - 102900*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^12 + 1903650*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^12 + 102900*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^12 + 1903650*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^12 + 102900*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^12 - 11554816*a^3*tan(1/2*d*x)^7*tan(1/2*c)^12 + 31137792*a*b^2*tan(1/2*d*x)^7*tan(1/2*c)^12 - 1254400*a^3*tan(1/2*d*x)^6*tan(1/2*c)^13 + 4300800*a*b^2*tan(1/2*d*x)^6*tan(1/2*c)^13 + 6615*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^14 + 1470*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^14 + 6615*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^14 + 1470*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*

c)^14 + 101430*pi*a^2*b*tan(1/2*d*x)^4*tan(1/2*c)^14 + 8820*pi*b^3*tan(1/2*
d*x)^4*tan(1/2*c)^14 - 39690*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/
2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*
c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^14 - 8820*b^3*arctan((tan(1/2*d*x)*tan(1
/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d
*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^14 - 39690*a^2*b*arctan((t
an(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1
/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^14 - 8820
*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(
1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/
2*c)^14 + 163170*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan
(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan
(1/2*d*x)^4*tan(1/2*c)^14 + 8820*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(
1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/
2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^14 + 163170*a^2*b*arctan((tan(1/2*d*x)
*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan
(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^14 + 8820*b^3*arctan
((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*ta
n(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^14 - 3
08224*a^3*tan(1/2*d*x)^5*tan(1/2*c)^14 + 114688*a*b^2*tan(1/2*d*x)^5*tan(1/
2*c)^14 - 15435*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*t
an(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d
*x)^14*tan(1/2*c)^2 + 13965*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(
1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)
*tan(1/2*d*x)^14*tan(1/2*c)^2 + 15435*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c
)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1
/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^2 - 13965*pi*b^3*sgn(tan(1/2*d*x)^2
*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2
 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14*tan(1/2*c)^2 - 33810*pi*a^2*b*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - ta
n(1/2*c)^2 + 1)*tan(1/2*d*x)^14*tan(1/2*c)^2 - 2940*pi*b^3*sgn(tan(1/2*d*x)
^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2
 + 1)*tan(1/2*d*x)^14*tan(1/2*c)^2 - 324135*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan
(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2
*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^4 + 293265*pi*b^3*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1
/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^4 + 324135*pi*a^2*
b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d
*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^4 - 2
93265*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2
 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1
/2*c)^4 - 710010*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2
 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^12*tan(1/2*c)^
4 - 61740*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1
/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^12*tan(1/2*c)^4 + 16128

0*a^2*b*tan(1/2*d*x)^14*tan(1/2*c)^4 - 50176*b^3*tan(1/2*d*x)^14*tan(1/2*c)
^4 + 3225600*a^2*b*tan(1/2*d*x)^13*tan(1/2*c)^5 - 1146880*b^3*tan(1/2*d*x)^
13*tan(1/2*c)^5 - 1620675*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(
1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)
*tan(1/2*d*x)^10*tan(1/2*c)^6 + 1466325*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c
)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1
/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^6 + 1620675*pi*a^2*b*sgn(tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*
c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^6 - 1466325*pi*b^3*sg
n(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^
2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^6 - 35500
50*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*
x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^10*tan(1/2*c)^6 - 308700*pi*
b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1
/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^10*tan(1/2*c)^6 + 27686400*a^2*b*tan
(1/2*d*x)^12*tan(1/2*c)^6 - 10429440*b^3*tan(1/2*d*x)^12*tan(1/2*c)^6 + 129
024000*a^2*b*tan(1/2*d*x)^11*tan(1/2*c)^7 - 48168960*b^3*tan(1/2*d*x)^11*ta
n(1/2*c)^7 - 2701125*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d
*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(
1/2*d*x)^8*tan(1/2*c)^8 + 2443875*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 +
2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x
) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^8 + 2701125*pi*a^2*b*sgn(tan(1/2*d*x)^2*ta
n(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 -
2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^8 - 2443875*pi*b^3*sgn(tan(1/
2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(
1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^8 - 5916750*pi*a^2
*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/
2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^8*tan(1/2*c)^8 - 514500*pi*b^3*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - ta
n(1/2*c)^2 + 1)*tan(1/2*d*x)^8*tan(1/2*c)^8 + 341107200*a^2*b*tan(1/2*d*x)^
10*tan(1/2*c)^8 - 119454720*b^3*tan(1/2*d*x)^10*tan(1/2*c)^8 + 488785920*a^
2*b*tan(1/2*d*x)^9*tan(1/2*c)^9 - 159186944*b^3*tan(1/2*d*x)^9*tan(1/2*c)^9
 - 1620675*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/
2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6
*tan(1/2*c)^10 + 1466325*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2
*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*ta
n(1/2*d*x)^6*tan(1/2*c)^10 + 1620675*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)
^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/
2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^10 - 1466325*pi*b^3*sgn(tan(1/2*d*x)^
2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^
2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^10 - 3550050*pi*a^2*b*sgn
(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) -
 tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^10 - 308700*pi*b^3*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2
*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^10 + 341107200*a^2*b*tan(1/2*d*x)^8*ta

n(1/2*c)^10 - 119454720*b^3*tan(1/2*d*x)^8*tan(1/2*c)^10 + 129024000*a^2*b*
tan(1/2*d*x)^7*tan(1/2*c)^11 - 48168960*b^3*tan(1/2*d*x)^7*tan(1/2*c)^11 -
324135*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)
^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan
(1/2*c)^12 + 293265*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)
*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2
*d*x)^4*tan(1/2*c)^12 + 324135*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2
*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x)
 - 1)*tan(1/2*d*x)^4*tan(1/2*c)^12 - 293265*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1
/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*t
an(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^12 - 710010*pi*a^2*b*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2
*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^12 - 61740*pi*b^3*sgn(tan(1/2*d*x)^2*t
an(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1
)*tan(1/2*d*x)^4*tan(1/2*c)^12 + 27686400*a^2*b*tan(1/2*d*x)^6*tan(1/2*c)^1
2 - 10429440*b^3*tan(1/2*d*x)^6*tan(1/2*c)^12 + 3225600*a^2*b*tan(1/2*d*x)^
5*tan(1/2*c)^13 - 1146880*b^3*tan(1/2*d*x)^5*tan(1/2*c)^13 - 15435*pi*a^2*b
*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*
x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^14 + 13
965*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 -
tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*
c)^14 + 15435*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan
(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x
)^2*tan(1/2*c)^14 - 13965*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/
2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*t
an(1/2*d*x)^2*tan(1/2*c)^14 - 33810*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*
d*x)^2*tan(1/2*c)^14 - 2940*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/
2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2*tan
(1/2*c)^14 + 161280*a^2*b*tan(1/2*d*x)^4*tan(1/2*c)^14 - 50176*b^3*tan(1/2*
d*x)^4*tan(1/2*c)^14 + 945*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan
(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*
sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14 + 210*pi*b^3*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
 tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1
/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*
tan(1/2*d*x)^14 + 945*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*
d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
 tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14 + 210*pi*b^3*sgn(tan(1/
2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(
1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*
x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1
/2*d*x)^14 + 46305*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x

)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(
1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + ta
n(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^2 + 10290*pi*b^
3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d
*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 +
2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x
) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^2 + 46305*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan
(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2
*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c
)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*t
an(1/2*c)^2 + 10290*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)
^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1
/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan
(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^2 + 33810*pi*a^2
*b*tan(1/2*d*x)^14*tan(1/2*c)^2 + 2940*pi*b^3*tan(1/2*d*x)^14*tan(1/2*c)^2
- 13230*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) +
 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)
^14*tan(1/2*c)^2 - 2940*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x)
- tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)
)*tan(1/2*d*x)^14*tan(1/2*c)^2 - 13230*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c
) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x)
- tan(1/2*c) + 1))*tan(1/2*d*x)^14*tan(1/2*c)^2 - 2940*b^3*arctan((tan(1/2*
d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) +
 tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^14*tan(1/2*c)^2 + 54390*a^2*b
*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*
d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^14*tan(1/2*c
)^2 + 2940*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c)
- 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x
)^14*tan(1/2*c)^2 + 54390*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d
*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c)
- 1))*tan(1/2*d*x)^14*tan(1/2*c)^2 + 2940*b^3*arctan((tan(1/2*d*x)*tan(1/2*
c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x)
 + tan(1/2*c) - 1))*tan(1/2*d*x)^14*tan(1/2*c)^2 - 71680*a^3*tan(1/2*d*x)^1
4*tan(1/2*c)^3 - 143360*a*b^2*tan(1/2*d*x)^14*tan(1/2*c)^3 + 416745*pi*a^2*
b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d
*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 +
2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x
) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^4 + 92610*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1
/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*t
an(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^
2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan
(1/2*c)^4 + 416745*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x
)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(
1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + ta
n(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^4 + 92610*pi*b^

3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d
*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 -
2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x
) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^4 + 710010*pi*a^2*b*tan(1/2*d*x)^12*tan(1
/2*c)^4 + 61740*pi*b^3*tan(1/2*d*x)^12*tan(1/2*c)^4 - 277830*a^2*b*arctan((
tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(
1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^12*tan(1/2*c)^4 - 617
40*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(ta
n(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^12*tan
(1/2*c)^4 - 277830*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + t
an(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*t
an(1/2*d*x)^12*tan(1/2*c)^4 - 61740*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - t
an(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan
(1/2*c) + 1))*tan(1/2*d*x)^12*tan(1/2*c)^4 + 1142190*a^2*b*arctan((tan(1/2*
d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) -
tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^12*tan(1/2*c)^4 + 61740*b^3*a
rctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*
x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^12*tan(1/2*c)^
4 + 1142190*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*
c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*
d*x)^12*tan(1/2*c)^4 + 61740*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*
d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c)
- 1))*tan(1/2*d*x)^12*tan(1/2*c)^4 + 752640*a^3*tan(1/2*d*x)^13*tan(1/2*c)
^4 - 2150400*a*b^2*tan(1/2*d*x)^13*tan(1/2*c)^4 + 5368832*a^3*tan(1/2*d*x)^
12*tan(1/2*c)^5 - 22851584*a*b^2*tan(1/2*d*x)^12*tan(1/2*c)^5 + 1157625*pi*
a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1
/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2
*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^6 + 257250*pi*b^3*sgn(tan(1/2*d*x)^2*t
an(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 +
2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2
*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*
tan(1/2*c)^6 + 1157625*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2
*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2
+ tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^6 + 257250*p
i*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1
/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2
*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^6 + 3550050*pi*a^2*b*tan(1/2*d*x)^10*t
an(1/2*c)^6 + 308700*pi*b^3*tan(1/2*d*x)^10*tan(1/2*c)^6 - 1389150*a^2*b*ar
ctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x
)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^6
- 308700*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) +
1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)

^10*tan(1/2*c)^6 - 1389150*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*
d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c)
 + 1))*tan(1/2*d*x)^10*tan(1/2*c)^6 - 308700*b^3*arctan((tan(1/2*d*x)*tan(1
/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d
*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^6 + 5710950*a^2*b*arctan(
(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan
(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^6 + 30
8700*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(
tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^10*t
an(1/2*c)^6 + 5710950*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x)
- tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)
)*tan(1/2*d*x)^10*tan(1/2*c)^6 + 308700*b^3*arctan((tan(1/2*d*x)*tan(1/2*c)
 - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) +
 tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^6 + 37632000*a^3*tan(1/2*d*x)^
11*tan(1/2*c)^6 - 122572800*a*b^2*tan(1/2*d*x)^11*tan(1/2*c)^6 + 115863552*
a^3*tan(1/2*d*x)^10*tan(1/2*c)^7 - 396877824*a*b^2*tan(1/2*d*x)^10*tan(1/2*
c)^7 + 1157625*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*
tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*
d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/
2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^8 + 257250*pi*b^3*sg
n(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^
2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) -
1)*tan(1/2*d*x)^6*tan(1/2*c)^8 + 1157625*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/
2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*ta
n(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2
 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1
/2*c)^8 + 257250*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*
tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*
d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/
2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^8 + 5916750*pi*a^2*b
*tan(1/2*d*x)^8*tan(1/2*c)^8 + 514500*pi*b^3*tan(1/2*d*x)^8*tan(1/2*c)^8 -
2315250*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) +
 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)
^8*tan(1/2*c)^8 - 514500*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x)
 - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1
))*tan(1/2*d*x)^8*tan(1/2*c)^8 - 2315250*a^2*b*arctan((tan(1/2*d*x)*tan(1/2
*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x
) - tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 - 514500*b^3*arctan((tan(1
/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c
) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 + 9518250*a
^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(
1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/
2*c)^8 + 514500*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/
2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/

2*d*x)^8*tan(1/2*c)^8 + 9518250*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan
(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1
/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 + 514500*b^3*arctan((tan(1/2*d*x)*t
an(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1
/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^8 + 235576320*a^3*tan(
1/2*d*x)^9*tan(1/2*c)^8 - 719953920*a*b^2*tan(1/2*d*x)^9*tan(1/2*c)^8 + 235
576320*a^3*tan(1/2*d*x)^8*tan(1/2*c)^9 - 719953920*a*b^2*tan(1/2*d*x)^8*tan
(1/2*c)^9 + 416745*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x
)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(
1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + ta
n(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^10 + 92610*pi*b^
3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d
*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 +
2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x
) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^10 + 416745*pi*a^2*b*sgn(tan(1/2*d*x)^2*ta
n(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 -
2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*
c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*t
an(1/2*c)^10 + 92610*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x
)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(
1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + ta
n(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^10 + 3550050*pi*
a^2*b*tan(1/2*d*x)^6*tan(1/2*c)^10 + 308700*pi*b^3*tan(1/2*d*x)^6*tan(1/2*c
)^10 - 1389150*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1
/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1
/2*d*x)^6*tan(1/2*c)^10 - 308700*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(
1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/
2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^10 - 1389150*a^2*b*arctan((tan(1/2*d*x
)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + ta
n(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^10 - 308700*b^3*arc
tan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)
*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^10
+ 5710950*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c)
 - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*
x)^6*tan(1/2*c)^10 + 308700*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d
*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c)
- 1))*tan(1/2*d*x)^6*tan(1/2*c)^10 + 5710950*a^2*b*arctan((tan(1/2*d*x)*tan
(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2
*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^10 + 308700*b^3*arctan((
tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(
1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^10 + 115
863552*a^3*tan(1/2*d*x)^7*tan(1/2*c)^10 - 396877824*a*b^2*tan(1/2*d*x)^7*ta
n(1/2*c)^10 + 37632000*a^3*tan(1/2*d*x)^6*tan(1/2*c)^11 - 122572800*a*b^2*t
an(1/2*d*x)^6*tan(1/2*c)^11 + 46305*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2

*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - ta
n(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)
^12 + 10290*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1
/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^
2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^
2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^12 + 46305*pi*a^2*b*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1
/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*
tan(1/2*d*x)^2*tan(1/2*c)^12 + 10290*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
- 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*
c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan
(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^
12 + 710010*pi*a^2*b*tan(1/2*d*x)^4*tan(1/2*c)^12 + 61740*pi*b^3*tan(1/2*d*
x)^4*tan(1/2*c)^12 - 277830*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2
*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c
) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^12 - 61740*b^3*arctan((tan(1/2*d*x)*tan(1
/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d
*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^12 - 277830*a^2*b*arctan((
tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(
1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^12 - 617
40*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(ta
n(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(
1/2*c)^12 + 1142190*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) +
tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*
tan(1/2*d*x)^4*tan(1/2*c)^12 + 61740*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) +
tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - ta
n(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^12 + 1142190*a^2*b*arctan((tan(1/2
*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c)
+ tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^12 + 61740*b^3*
arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d
*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^
12 + 5368832*a^3*tan(1/2*d*x)^5*tan(1/2*c)^12 - 22851584*a*b^2*tan(1/2*d*x)
^5*tan(1/2*c)^12 + 752640*a^3*tan(1/2*d*x)^4*tan(1/2*c)^13 - 2150400*a*b^2*
tan(1/2*d*x)^4*tan(1/2*c)^13 + 945*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
+ 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*
c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan
(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^14 + 210*pi*b^3
*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*
x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2
*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x)
- 1)*tan(1/2*c)^14 + 945*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(
1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*s
gn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)
^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^14 + 210*pi*b^3*sgn(tan(

1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - ta
n(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*
d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan
(1/2*c)^14 + 33810*pi*a^2*b*tan(1/2*d*x)^2*tan(1/2*c)^14 + 2940*pi*b^3*tan(
1/2*d*x)^2*tan(1/2*c)^14 - 13230*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + ta
n(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(
1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^14 - 2940*b^3*arctan((tan(1/2*d*x)*t
an(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1
/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^14 - 13230*a^2*b*arcta
n((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*t
an(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^14 -
2940*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(
tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2*ta
n(1/2*c)^14 + 54390*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) +
tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*
tan(1/2*d*x)^2*tan(1/2*c)^14 + 2940*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + t
an(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan
(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^14 + 54390*a^2*b*arctan((tan(1/2*d*
x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + t
an(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^14 + 2940*b^3*arct
an((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*
tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^14 -
 71680*a^3*tan(1/2*d*x)^3*tan(1/2*c)^14 - 143360*a*b^2*tan(1/2*d*x)^3*tan(1
/2*c)^14 - 2205*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*t
an(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d
*x)^14 + 1995*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1
/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^
14 + 2205*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2
*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14
 - 1995*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^
2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^14 - 4
830*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d
*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^14 - 420*pi*b^3*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2
*c)^2 + 1)*tan(1/2*d*x)^14 - 108045*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2
*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^2 + 97755*pi*b^3*sgn(tan(1/2*d*x)^2*t
an(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 +
 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^2 + 108045*pi*a^2*b*sgn(tan
(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + t
an(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^2 - 97755*pi*b
^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*
d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12*tan(1/2*c)^2 -
236670*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/
2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^12*tan(1/2*c)^2 - 20580*

pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*ta
n(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^12*tan(1/2*c)^2 - 53760*a^2*b*tan
(1/2*d*x)^14*tan(1/2*c)^2 + 7168*b^3*tan(1/2*d*x)^14*tan(1/2*c)^2 - 1290240
*a^2*b*tan(1/2*d*x)^13*tan(1/2*c)^3 + 286720*b^3*tan(1/2*d*x)^13*tan(1/2*c)
^3 - 972405*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1
/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^
10*tan(1/2*c)^4 + 879795*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2
*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*ta
n(1/2*d*x)^10*tan(1/2*c)^4 + 972405*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2
*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^4 - 879795*pi*b^3*sgn(tan(1/2*d*x)^2*
tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2
- 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^4 - 2130030*pi*a^2*b*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - t
an(1/2*c)^2 + 1)*tan(1/2*d*x)^10*tan(1/2*c)^4 - 185220*pi*b^3*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c
)^2 + 1)*tan(1/2*d*x)^10*tan(1/2*c)^4 - 14031360*a^2*b*tan(1/2*d*x)^12*tan(
1/2*c)^4 + 4092928*b^3*tan(1/2*d*x)^12*tan(1/2*c)^4 - 88166400*a^2*b*tan(1/
2*d*x)^11*tan(1/2*c)^5 + 29532160*b^3*tan(1/2*d*x)^11*tan(1/2*c)^5 - 270112
5*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 -
tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*
c)^6 + 2443875*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(
1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)
^8*tan(1/2*c)^6 + 2701125*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(
1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)
*tan(1/2*d*x)^8*tan(1/2*c)^6 - 2443875*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)
^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/
2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^6 - 5916750*pi*a^2*b*sgn(tan(1/2*d*x)
^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2
 + 1)*tan(1/2*d*x)^8*tan(1/2*c)^6 - 514500*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/
2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan
(1/2*d*x)^8*tan(1/2*c)^6 - 341107200*a^2*b*tan(1/2*d*x)^10*tan(1/2*c)^6 + 1
19454720*b^3*tan(1/2*d*x)^10*tan(1/2*c)^6 - 807690240*a^2*b*tan(1/2*d*x)^9*
tan(1/2*c)^7 + 276283392*b^3*tan(1/2*d*x)^9*tan(1/2*c)^7 - 2701125*pi*a^2*b
*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*
x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^8 + 244
3875*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 -
 tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2
*c)^8 + 2701125*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*t
an(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d
*x)^6*tan(1/2*c)^8 - 2443875*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan
(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1
)*tan(1/2*d*x)^6*tan(1/2*c)^8 - 5916750*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2
*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(
1/2*d*x)^6*tan(1/2*c)^8 - 514500*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - t

an(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^8 - 1106972160*a^2*b*tan(1/2*d*x)^8*tan(1/2*c)^8 + 362019840*b^3*tan(1/2*d*x)^8*tan(1/2*c)^8 - 807690240*a^2*b*tan(1/2*d*x)^7*tan(1/2*c)^9 + 276283392*b^3*tan(1/2*d*x)^7*tan(1/2*c)^9 - 972405*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^10 + 879795*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^10 + 972405*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^10 - 879795*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^10 - 2130030*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^10 - 185220*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^10 - 341107200*a^2*b*tan(1/2*d*x)^6*tan(1/2*c)^10 + 119454720*b^3*tan(1/2*d*x)^6*tan(1/2*c)^10 - 88166400*a^2*b*tan(1/2*d*x)^5*tan(1/2*c)^11 + 29532160*b^3*tan(1/2*d*x)^5*tan(1/2*c)^11 - 108045*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^12 + 97755*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^12 + 108045*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^12 - 97755*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^12 - 236670*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2*tan(1/2*c)^12 - 20580*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2*tan(1/2*c)^12 - 14031360*a^2*b*tan(1/2*d*x)^4*tan(1/2*c)^12 + 4092928*b^3*tan(1/2*d*x)^4*tan(1/2*c)^12 - 1290240*a^2*b*tan(1/2*d*x)^3*tan(1/2*c)^13 + 286720*b^3*tan(1/2*d*x)^3*tan(1/2*c)^13 - 2205*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^14 + 1995*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^14 + 2205*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^14 - 1995*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^14 - 4830*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^14 - 420*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^14 - 53760*a^2*b*tan(1/2*d*x)^2*tan(1/2*c)^14 + 7168

*b^3*tan(1/2*d*x)^2*tan(1/2*c)^14 + 6615*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/
2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*ta
n(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2
 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12 + 14
70*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + t
an(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2
*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan
(1/2*d*x) - 1)*tan(1/2*d*x)^12 + 6615*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c
)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1
/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 -
tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12 + 1470*
pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(
1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)
^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/
2*d*x) - 1)*tan(1/2*d*x)^12 + 4830*pi*a^2*b*tan(1/2*d*x)^14 + 420*pi*b^3*ta
n(1/2*d*x)^14 - 1890*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) -
 tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))
*tan(1/2*d*x)^14 - 420*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) -
 tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))
*tan(1/2*d*x)^14 - 1890*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x
) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) +
1))*tan(1/2*d*x)^14 - 420*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x
) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) +
1))*tan(1/2*d*x)^14 + 7770*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*
d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c)
 - 1))*tan(1/2*d*x)^14 + 420*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*
d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c)
 - 1))*tan(1/2*d*x)^14 + 7770*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1
/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2
*c) - 1))*tan(1/2*d*x)^14 + 420*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1
/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2
*c) - 1))*tan(1/2*d*x)^14 - 35840*a^3*tan(1/2*d*x)^14*tan(1/2*c) + 138915*p
i*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan
(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c
)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1
/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^2 + 30870*pi*b^3*sgn(tan(1/2*d*x)^2
*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2
 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1
/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^
10*tan(1/2*c)^2 + 138915*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1
/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sg
n(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^
2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^2 + 30870
*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan
(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c

)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^2 + 236670*pi*a^2*b*tan(1/2*d*x)^12*tan(1/2*c)^2 + 20580*pi*b^3*tan(1/2*d*x)^12*tan(1/2*c)^2 - 92610*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^12*tan(1/2*c)^2 - 20580*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^12*tan(1/2*c)^2 - 92610*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^12*tan(1/2*c)^2 - 20580*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^12*tan(1/2*c)^2 + 380730*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^12*tan(1/2*c)^2 + 20580*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^12*tan(1/2*c)^2 + 380730*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^12*tan(1/2*c)^2 + 20580*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^12*tan(1/2*c)^2 - 250880*a^3*tan(1/2*d*x)^13*tan(1/2*c)^2 + 430080*a*b^2*tan(1/2*d*x)^13*tan(1/2*c)^2 - 3512320*a^3*tan(1/2*d*x)^12*tan(1/2*c)^3 + 5877760*a*b^2*tan(1/2*d*x)^12*tan(1/2*c)^3 + 694575*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^4 + 154350*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^4 + 694575*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^4 + 154350*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^4 + 2130030*pi*a^2*b*tan(1/2*d*x)^10*tan(1/2*c)^4 + 185220*pi*b^3*tan(1/2*d*x)^10*tan(1/2*c)^4 - 833490*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^4 - 185220*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^4 - 833490*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1

))*tan(1/2*d*x)^10*tan(1/2*c)^4 - 185220*b^3*arctan((tan(1/2*d*x)*tan(1/2*c
) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x)
- tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^4 + 3426570*a^2*b*arctan((tan
(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2
*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^4 + 185220
*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(
1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1
/2*c)^4 + 3426570*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - ta
n(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*ta
n(1/2*d*x)^10*tan(1/2*c)^4 + 185220*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - t
an(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan
(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^4 - 18565120*a^3*tan(1/2*d*x)^11*t
an(1/2*c)^4 + 50319360*a*b^2*tan(1/2*d*x)^11*tan(1/2*c)^4 - 86754304*a^3*ta
n(1/2*d*x)^10*tan(1/2*c)^5 + 241102848*a*b^2*tan(1/2*d*x)^10*tan(1/2*c)^5 +
 1157625*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/
2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2
*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2
 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 257250*pi*b^3*sgn(tan(
1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - ta
n(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*
d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan
(1/2*d*x)^6*tan(1/2*c)^6 + 1157625*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*
c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan
(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^
6 + 257250*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/
2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2
*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2
 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 5916750*pi*a^2*b*tan(1
/2*d*x)^8*tan(1/2*c)^6 + 514500*pi*b^3*tan(1/2*d*x)^8*tan(1/2*c)^6 - 231525
0*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(t
an(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan
(1/2*c)^6 - 514500*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan
(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan
(1/2*d*x)^8*tan(1/2*c)^6 - 2315250*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) -
tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - ta
n(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^6 - 514500*b^3*arctan((tan(1/2*d*x
)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + ta
n(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^6 + 9518250*a^2*b*a
rctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*
x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^6
 + 514500*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) -
 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)
^8*tan(1/2*c)^6 + 9518250*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d
*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c)

- 1))*tan(1/2*d*x)^8*tan(1/2*c)^6 + 514500*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^6 - 235576320*a^3*tan(1/2*d*x)^9*tan(1/2*c)^6 + 719953920*a*b^2*tan(1/2*d*x)^9*tan(1/2*c)^6 - 435097600*a^3*tan(1/2*d*x)^8*tan(1/2*c)^7 + 1258414080*a*b^2*tan(1/2*d*x)^8*tan(1/2*c)^7 + 694575*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^8 + 154350*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^8 + 694575*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^8 + 154350*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^8 + 5916750*pi*a^2*b*tan(1/2*d*x)^6*tan(1/2*c)^8 + 514500*pi*b^3*tan(1/2*d*x)^6*tan(1/2*c)^8 - 2315250*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^8 - 514500*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^8 - 2315250*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^8 - 514500*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^8 + 9518250*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^8 + 514500*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^8 + 9518250*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^8 + 514500*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^8 - 435097600*a^3*tan(1/2*d*x)^7*tan(1/2*c)^8 + 1258414080*a*b^2*tan(1/2*d*x)^7*tan(1/2*c)^8 - 235576320*a^3*tan(1/2*d*x)^6*tan(1/2*c)^9 + 719953920*a*b^2*tan(1/2*d*x)^6*tan(1/2*c)^9 + 138915*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^10 + 30870*pi*b^3 *sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^10 + 138915*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^10 + 30870*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^10 + 2130030*pi*a^2*b*tan(1/2*d*x)^4*tan(1/2*c)^10 + 185220*pi*b^3*tan(1/2*d*x)^4*tan(1/2*c)^10 - 833490*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^10 - 185220*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^10 - 833490*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^10 - 185220*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^10 + 3426570*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^10 + 185220*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^10 + 3426570*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^10 + 185220*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^10 - 86754304*a^3*tan(1/2*d*x)^5*tan(1/2*c)^10 + 241102848*a*b^2*tan(1/2*d*x)^5*tan(1/2*c)^10 - 18565120*a^3*tan(1/2*d*x)^4*tan(1/2*c)^11 + 50319360*a*b^2*tan(1/2*d*x)^4*tan(1/2*c)^11 + 6615*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^12 + 1470*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^12 + 6615*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^12 + 1470*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*

x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1
/2*c)^12 + 236670*pi*a^2*b*tan(1/2*d*x)^2*tan(1/2*c)^12 + 20580*pi*b^3*tan(
1/2*d*x)^2*tan(1/2*c)^12 - 92610*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + ta
n(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(
1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^12 - 20580*b^3*arctan((tan(1/2*d*x)*
tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(
1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^12 - 92610*a^2*b*arct
an((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*
tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^12 -
 20580*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)
/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2*
tan(1/2*c)^12 + 380730*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x)
 + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1
))*tan(1/2*d*x)^2*tan(1/2*c)^12 + 20580*b^3*arctan((tan(1/2*d*x)*tan(1/2*c)
 + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) -
 tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^12 + 380730*a^2*b*arctan((tan(1
/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c
) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^12 + 20580*b^
3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2
*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c
)^12 - 3512320*a^3*tan(1/2*d*x)^3*tan(1/2*c)^12 + 5877760*a*b^2*tan(1/2*d*x
)^3*tan(1/2*c)^12 - 250880*a^3*tan(1/2*d*x)^2*tan(1/2*c)^13 + 430080*a*b^2*
tan(1/2*d*x)^2*tan(1/2*c)^13 + 4830*pi*a^2*b*tan(1/2*c)^14 + 420*pi*b^3*tan
(1/2*c)^14 - 1890*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - ta
n(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*ta
n(1/2*c)^14 - 420*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(
1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(
1/2*c)^14 - 1890*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan
(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan
(1/2*c)^14 - 420*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1
/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1
/2*c)^14 + 7770*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(
1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(
1/2*c)^14 + 420*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/
2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/
2*c)^14 + 7770*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1
/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1
/2*c)^14 + 420*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2
*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2
*c)^14 - 35840*a^3*tan(1/2*d*x)*tan(1/2*c)^14 - 15435*pi*a^2*b*sgn(tan(1/2*
d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/
2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12 + 13965*pi*b^3*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c
)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12 + 15435*pi*a^2*b*sgn(tan(1/2*d*x)
^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)

^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12 - 13965*pi*b^3*sgn(tan(1/2*d*x)^2*
tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2
- 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^12 - 33810*pi*a^2*b*sgn(tan(1/2*d*x)^2*t
an(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1
)*tan(1/2*d*x)^12 - 2940*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d
*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^12 + 768
0*a^2*b*tan(1/2*d*x)^14 + 1024*b^3*tan(1/2*d*x)^14 + 215040*a^2*b*tan(1/2*d
*x)^13*tan(1/2*c) - 324135*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan
(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1
)*tan(1/2*d*x)^10*tan(1/2*c)^2 + 293265*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c
)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1
/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^2 + 324135*pi*a^2*b*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c
)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^2 - 293265*pi*b^3*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2
+ tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10*tan(1/2*c)^2 - 710010*
pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*
tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^10*tan(1/2*c)^2 - 61740*pi*b^3*
sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c
) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^10*tan(1/2*c)^2 + 2956800*a^2*b*tan(1/2*
d*x)^12*tan(1/2*c)^2 - 379904*b^3*tan(1/2*d*x)^12*tan(1/2*c)^2 + 24944640*a
^2*b*tan(1/2*d*x)^11*tan(1/2*c)^3 - 5734400*b^3*tan(1/2*d*x)^11*tan(1/2*c)^
3 - 1620675*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1
/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^
8*tan(1/2*c)^4 + 1466325*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2
*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*ta
n(1/2*d*x)^8*tan(1/2*c)^4 + 1620675*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2
*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^4 - 1466325*pi*b^3*sgn(tan(1/2*d*x)^2*
tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2
- 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^4 - 3550050*pi*a^2*b*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - ta
n(1/2*c)^2 + 1)*tan(1/2*d*x)^8*tan(1/2*c)^4 - 308700*pi*b^3*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^
2 + 1)*tan(1/2*d*x)^8*tan(1/2*c)^4 + 136711680*a^2*b*tan(1/2*d*x)^10*tan(1/
2*c)^4 - 40406016*b^3*tan(1/2*d*x)^10*tan(1/2*c)^4 + 488785920*a^2*b*tan(1/
2*d*x)^9*tan(1/2*c)^5 - 159186944*b^3*tan(1/2*d*x)^9*tan(1/2*c)^5 - 2701125
*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - t
an(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c
)^6 + 2443875*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1
/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^
6*tan(1/2*c)^6 + 2701125*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1
/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*
tan(1/2*d*x)^6*tan(1/2*c)^6 - 2443875*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2

*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 - 5916750*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 - 514500*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 1106972160*a^2*b*tan(1/2*d*x)^8*tan(1/2*c)^6 - 362019840*b^3*tan(1/2*d*x)^8*tan(1/2*c)^6 + 1491763200*a^2*b*tan(1/2*d*x)^7*tan(1/2*c)^7 - 472252416*b^3*tan(1/2*d*x)^7*tan(1/2*c)^7 - 1620675*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^8 + 1466325*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^8 + 1620675*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^8 - 1466325*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^8 - 3550050*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^8 - 308700*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^8 + 1106972160*a^2*b*tan(1/2*d*x)^6*tan(1/2*c)^8 - 362019840*b^3*tan(1/2*d*x)^6*tan(1/2*c)^8 + 488785920*a^2*b*tan(1/2*d*x)^5*tan(1/2*c)^9 - 159186944*b^3*tan(1/2*d*x)^5*tan(1/2*c)^9 - 324135*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^10 + 293265*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^10 + 324135*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^10 - 293265*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^10 - 710010*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2*tan(1/2*c)^10 - 61740*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2*tan(1/2*c)^10 + 136711680*a^2*b*tan(1/2*d*x)^4*tan(1/2*c)^10 - 40406016*b^3*tan(1/2*d*x)^4*tan(1/2*c)^10 + 24944640*a^2*b*tan(1/2*d*x)^3*tan(1/2*c)^11 - 5734400*b^3*tan(1/2*d*x)^3*tan(1/2*c)^11 - 15435*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^12 + 13965*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^12 + 15435*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^12 - 13965*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^12 - 33810*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2

*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^12 - 294
0*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*
tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^12 + 2956800*a^2*b*tan(1/2*d*x)^2
*tan(1/2*c)^12 - 379904*b^3*tan(1/2*d*x)^2*tan(1/2*c)^12 + 215040*a^2*b*tan
(1/2*d*x)*tan(1/2*c)^13 + 7680*a^2*b*tan(1/2*c)^14 + 1024*b^3*tan(1/2*c)^14
 + 19845*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/
2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2
*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2
 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10 + 4410*pi*b^3*sgn(tan(1/2*d*x)^2*tan
(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2
*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c
)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10 +
 19845*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*
c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*t
an(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 -
 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10 + 4410*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1
/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*t
an(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^
2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^10 + 3
3810*pi*a^2*b*tan(1/2*d*x)^12 + 2940*pi*b^3*tan(1/2*d*x)^12 - 13230*a^2*b*a
rctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*
x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^12 - 2940*b^3*
arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d
*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^12 - 13230*a^
2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1
/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^12 - 2940
*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(
1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^12 + 543
90*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(
tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^12 +
 2940*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/
(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^12
+ 54390*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) -
 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)
^12 + 2940*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c)
- 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x
)^12 + 35840*a^3*tan(1/2*d*x)^13 + 250880*a^3*tan(1/2*d*x)^12*tan(1/2*c) -
430080*a*b^2*tan(1/2*d*x)^12*tan(1/2*c) + 231525*pi*a^2*b*sgn(tan(1/2*d*x)^
2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^
2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(
1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)
^8*tan(1/2*c)^2 + 51450*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
 tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^2 + 231525*pi

*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^2 + 51450*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^2 + 710010*pi*a^2*b*tan(1/2*d*x)^10*tan(1/2*c)^2 + 61740*pi*b^3*tan(1/2*d*x)^10*tan(1/2*c)^2 - 277830*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^2 - 61740*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^2 - 277830*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^2 - 61740*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^10*tan(1/2*c)^2 + 1142190*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^2 + 61740*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^2 + 1142190*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^2 + 61740*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^10*tan(1/2*c)^2 + 3512320*a^3*tan(1/2*d*x)^11*tan(1/2*c)^2 - 5877760*a*b^2*tan(1/2*d*x)^11*tan(1/2*c)^2 + 18565120*a^3*tan(1/2*d*x)^10*tan(1/2*c)^3 - 50319360*a*b^2*tan(1/2*d*x)^10*tan(1/2*c)^3 + 694575*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 + 154350*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 + 694575*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 + 154350*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 + 3550050*pi*a^2*b*tan(1/2*d*x)^8*tan(1/2*c)^4 + 308700*pi*b^3*tan(1/2*d*x)^8*tan(1/2*c)^4 - 1389150*a^2*b*arctan((tan(1/2*d*x)*tan(1/

2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^4 - 308700*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^4 - 1389150*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^4 - 308700*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^4 + 5710950*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^4 + 308700*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^4 + 5710950*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^4 + 308700*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^4 + 86754304*a^3*tan(1/2*d*x)^9*tan(1/2*c)^4 - 241102848*a*b^2*tan(1/2*d*x)^9*tan(1/2*c)^4 + 235576320*a^3*tan(1/2*d*x)^8*tan(1/2*c)^5 - 719953920*a*b^2*tan(1/2*d*x)^8*tan(1/2*c)^5 + 694575*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 + 154350*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 + 694575*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 + 154350*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 + 5916750*pi*a^2*b*tan(1/2*d*x)^6*tan(1/2*c)^6 + 514500*pi*b^3*tan(1/2*d*x)^6*tan(1/2*c)^6 - 2315250*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 - 514500*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 - 2315250*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 - 514500*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 + 9518250*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) +

tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - ta
n(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 + 514500*b^3*arctan((tan(1/2*d*x
)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - ta
n(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 + 9518250*a^2*b*a
rctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*
x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^6
 + 514500*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) -
 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)
^6*tan(1/2*c)^6 + 435097600*a^3*tan(1/2*d*x)^7*tan(1/2*c)^6 - 1258414080*a*
b^2*tan(1/2*d*x)^7*tan(1/2*c)^6 + 435097600*a^3*tan(1/2*d*x)^6*tan(1/2*c)^7
 - 1258414080*a*b^2*tan(1/2*d*x)^6*tan(1/2*c)^7 + 231525*pi*a^2*b*sgn(tan(1
/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan
(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d
*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(
1/2*d*x)^2*tan(1/2*c)^8 + 51450*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*
tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) -
1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*
d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^8 + 2
31525*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c
) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*ta
n(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 -
2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^8 + 51450*pi*b^3*sgn(tan(1/2*
d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/
2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)
*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2
*d*x)^2*tan(1/2*c)^8 + 3550050*pi*a^2*b*tan(1/2*d*x)^4*tan(1/2*c)^8 + 30870
0*pi*b^3*tan(1/2*d*x)^4*tan(1/2*c)^8 - 1389150*a^2*b*arctan((tan(1/2*d*x)*t
an(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1
/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^8 - 308700*b^3*arctan(
(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan
(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^8 - 138
9150*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)
/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4*
tan(1/2*c)^8 - 308700*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) +
tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*
tan(1/2*d*x)^4*tan(1/2*c)^8 + 5710950*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c)
 + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) -
 tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^8 + 308700*b^3*arctan((tan(1/2*
d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) -
 tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^8 + 5710950*a^2*
b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2
*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c
)^8 + 308700*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c
) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d
*x)^4*tan(1/2*c)^8 + 235576320*a^3*tan(1/2*d*x)^5*tan(1/2*c)^8 - 719953920*

a*b^2*tan(1/2*d*x)^5*tan(1/2*c)^8 + 86754304*a^3*tan(1/2*d*x)^4*tan(1/2*c)^
9 - 241102848*a*b^2*tan(1/2*d*x)^4*tan(1/2*c)^9 + 19845*pi*a^2*b*sgn(tan(1/
2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(
1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*
x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1
/2*c)^10 + 4410*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*t
an(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2
*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^10 + 19845*pi*a^2*b*sgn(tan(1/2*d*x)
^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)
^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan
(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^
10 + 4410*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2
*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*
tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2
- 2*tan(1/2*d*x) - 1)*tan(1/2*c)^10 + 710010*pi*a^2*b*tan(1/2*d*x)^2*tan(1/
2*c)^10 + 61740*pi*b^3*tan(1/2*d*x)^2*tan(1/2*c)^10 - 277830*a^2*b*arctan((
tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(
1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^10 - 617
40*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(ta
n(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(
1/2*c)^10 - 277830*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + t
an(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*t
an(1/2*d*x)^2*tan(1/2*c)^10 - 61740*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - t
an(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan
(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^10 + 1142190*a^2*b*arctan((tan(1/2*
d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) -
tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^10 + 61740*b^3*a
rctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*
x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^1
0 + 1142190*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*
c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*
d*x)^2*tan(1/2*c)^10 + 61740*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*
d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c)
- 1))*tan(1/2*d*x)^2*tan(1/2*c)^10 + 18565120*a^3*tan(1/2*d*x)^3*tan(1/2*c
)^10 - 50319360*a*b^2*tan(1/2*d*x)^3*tan(1/2*c)^10 + 3512320*a^3*tan(1/2*d*
x)^2*tan(1/2*c)^11 - 5877760*a*b^2*tan(1/2*d*x)^2*tan(1/2*c)^11 + 33810*pi*
a^2*b*tan(1/2*c)^12 + 2940*pi*b^3*tan(1/2*c)^12 - 13230*a^2*b*arctan((tan(1
/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c
) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*c)^12 - 2940*b^3*arctan((tan(1/
2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c)
- tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*c)^12 - 13230*a^2*b*arctan((tan(
1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*
c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*c)^12 - 2940*b^3*arctan((tan(1
/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c

) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*c)^12 + 54390*a^2*b*arctan((tan
(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2
*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*c)^12 + 2940*b^3*arctan((tan(
1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*
c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*c)^12 + 54390*a^2*b*arctan((ta
n(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/
2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^12 + 2940*b^3*arctan((tan
(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2
*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^12 + 250880*a^3*tan(1/2*d*
x)*tan(1/2*c)^12 - 430080*a*b^2*tan(1/2*d*x)*tan(1/2*c)^12 + 35840*a^3*tan(
1/2*c)^13 - 46305*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)
*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2
*d*x)^10 + 41895*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*ta
n(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*
x)^10 + 46305*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan
(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x
)^10 - 41895*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/
2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^1
0 - 101430*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*ta
n(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^10 - 8820*pi*b^3*sgn
(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) -
 tan(1/2*c)^2 + 1)*tan(1/2*d*x)^10 - 53760*a^2*b*tan(1/2*d*x)^12 + 7168*b^3
*tan(1/2*d*x)^12 - 1290240*a^2*b*tan(1/2*d*x)^11*tan(1/2*c) + 286720*b^3*ta
n(1/2*d*x)^11*tan(1/2*c) - 540225*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
+ 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d
*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^2 + 488775*pi*b^3*sgn(tan(1/2*d*x)^2*tan
(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2
*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^2 + 540225*pi*a^2*b*sgn(tan(1/
2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(
1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^2 - 488775*pi*b^3*
sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x
)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8*tan(1/2*c)^2 - 1183
350*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d
*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^8*tan(1/2*c)^2 - 102900*pi*
b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1
/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^8*tan(1/2*c)^2 - 14031360*a^2*b*tan(
1/2*d*x)^10*tan(1/2*c)^2 + 4092928*b^3*tan(1/2*d*x)^10*tan(1/2*c)^2 - 88166
400*a^2*b*tan(1/2*d*x)^9*tan(1/2*c)^3 + 29532160*b^3*tan(1/2*d*x)^9*tan(1/2
*c)^3 - 1620675*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*t
an(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d
*x)^6*tan(1/2*c)^4 + 1466325*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan
(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1
)*tan(1/2*d*x)^6*tan(1/2*c)^4 + 1620675*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2
*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan
(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 - 1466325*pi*b^3*sgn(tan(1/2*d*x

)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c
)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 - 3550050*pi*a^2*b*sg
n(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c)
- tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 - 308700*pi*b^3*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2
*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 - 341107200*a^2*b*tan(1/2*d*x)^8*tan
(1/2*c)^4 + 119454720*b^3*tan(1/2*d*x)^8*tan(1/2*c)^4 - 807690240*a^2*b*tan
(1/2*d*x)^7*tan(1/2*c)^5 + 276283392*b^3*tan(1/2*d*x)^7*tan(1/2*c)^5 - 1620
675*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2
- tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/
2*c)^6 + 1466325*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*ta
n(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*
x)^4*tan(1/2*c)^6 + 1620675*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*ta
n(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) -
1)*tan(1/2*d*x)^4*tan(1/2*c)^6 - 1466325*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*
c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(
1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 - 3550050*pi*a^2*b*sgn(tan(1/2*d*
x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)
^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 - 308700*pi*b^3*sgn(tan(1/2*d*x)^2*tan(
1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*t
an(1/2*d*x)^4*tan(1/2*c)^6 - 1106972160*a^2*b*tan(1/2*d*x)^6*tan(1/2*c)^6 +
 362019840*b^3*tan(1/2*d*x)^6*tan(1/2*c)^6 - 807690240*a^2*b*tan(1/2*d*x)^5
*tan(1/2*c)^7 + 276283392*b^3*tan(1/2*d*x)^5*tan(1/2*c)^7 - 540225*pi*a^2*b
*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*
x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^8 + 488
775*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 -
tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*
c)^8 + 540225*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan
(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x
)^2*tan(1/2*c)^8 - 488775*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/
2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*t
an(1/2*d*x)^2*tan(1/2*c)^8 - 1183350*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)
^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2
*d*x)^2*tan(1/2*c)^8 - 102900*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(
1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2*t
an(1/2*c)^8 - 341107200*a^2*b*tan(1/2*d*x)^4*tan(1/2*c)^8 + 119454720*b^3*t
an(1/2*d*x)^4*tan(1/2*c)^8 - 88166400*a^2*b*tan(1/2*d*x)^3*tan(1/2*c)^9 + 2
9532160*b^3*tan(1/2*d*x)^3*tan(1/2*c)^9 - 46305*pi*a^2*b*sgn(tan(1/2*d*x)^2
*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2
 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^10 + 41895*pi*b^3*sgn(tan(1/2*d*x)^2*tan(
1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*
tan(1/2*d*x) - 1)*tan(1/2*c)^10 + 46305*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2
*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan
(1/2*d*x) - 1)*tan(1/2*c)^10 - 41895*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*

d*x) - 1)*tan(1/2*c)^10 - 101430*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^10 - 8820*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^10 - 14031360*a^2*b*tan(1/2*d*x)^2*tan(1/2*c)^10 + 4092928*b^3*tan(1/2*d*x)^2*tan(1/2*c)^10 - 1290240*a^2*b*tan(1/2*d*x)*tan(1/2*c)^11 + 286720*b^3*tan(1/2*d*x)*tan(1/2*c)^11 - 53760*a^2*b*tan(1/2*c)^12 + 7168*b^3*tan(1/2*c)^12 + 33075*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8 + 7350*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8 + 33075*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8 + 7350*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^8 + 101430*pi*a^2*b*tan(1/2*d*x)^10 + 8820*pi*b^3*tan(1/2*d*x)^10 - 39690*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^10 - 8820*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^10 - 39690*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^10 - 8820*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^10 + 163170*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^10 + 8820*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^10 + 163170*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^10 + 8820*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^10 + 71680*a^3*tan(1/2*d*x)^11 + 143360*a*b^2*tan(1/2*d*x)^11 - 752640*a^3*tan(1/2*d*x)^10*tan(1/2*c) + 2150400*a*b^2*tan(1/2*d*x)^10*tan(1/2*c) + 231525*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 + 51450*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) -

1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 + 231525*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 + 51450*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 + 1183350*pi*a^2*b*tan(1/2*d*x)^8*tan(1/2*c)^2 + 102900*pi*b^3*tan(1/2*d*x)^8*tan(1/2*c)^2 - 463050*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^2 - 102900*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^2 - 463050*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^2 - 102900*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8*tan(1/2*c)^2 + 1903650*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^2 + 102900*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^2 + 1903650*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^2 + 102900*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^8*tan(1/2*c)^2 - 5368832*a^3*tan(1/2*d*x)^9*tan(1/2*c)^2 + 22851584*a*b^2*tan(1/2*d*x)^9*tan(1/2*c)^2 - 37632000*a^3*tan(1/2*d*x)^8*tan(1/2*c)^3 + 122572800*a*b^2*tan(1/2*d*x)^8*tan(1/2*c)^3 + 416745*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 + 92610*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 + 416745*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 + 92610*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)

^4*tan(1/2*c)^4 + 3550050*pi*a^2*b*tan(1/2*d*x)^6*tan(1/2*c)^4 + 308700*pi*
b^3*tan(1/2*d*x)^6*tan(1/2*c)^4 - 1389150*a^2*b*arctan((tan(1/2*d*x)*tan(1/
2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*
x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 - 308700*b^3*arctan((tan(
1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*
c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 - 1389150*
a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan
(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1
/2*c)^4 - 308700*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1
/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1
/2*d*x)^6*tan(1/2*c)^4 + 5710950*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + ta
n(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(
1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 + 308700*b^3*arctan((tan(1/2*d*x)*
tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(
1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 + 5710950*a^2*b*arc
tan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)
*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^4 +
 308700*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1
)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6
*tan(1/2*c)^4 - 115863552*a^3*tan(1/2*d*x)^7*tan(1/2*c)^4 + 396877824*a*b^2
*tan(1/2*d*x)^7*tan(1/2*c)^4 - 235576320*a^3*tan(1/2*d*x)^6*tan(1/2*c)^5 +
719953920*a*b^2*tan(1/2*d*x)^6*tan(1/2*c)^5 + 231525*pi*a^2*b*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2
*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*
tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*
d*x)^2*tan(1/2*c)^6 + 51450*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(
1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*s
gn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)
^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^6 + 23152
5*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) +
tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/
2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*ta
n(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^6 + 51450*pi*b^3*sgn(tan(1/2*d*x)
^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)
^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan
(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x
)^2*tan(1/2*c)^6 + 3550050*pi*a^2*b*tan(1/2*d*x)^4*tan(1/2*c)^6 + 308700*pi
*b^3*tan(1/2*d*x)^4*tan(1/2*c)^6 - 1389150*a^2*b*arctan((tan(1/2*d*x)*tan(1
/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d
*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^6 - 308700*b^3*arctan((tan
(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2
*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^6 - 1389150
*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(ta
n(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(
1/2*c)^6 - 308700*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(

1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(
1/2*d*x)^4*tan(1/2*c)^6 + 5710950*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + t
an(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan
(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^6 + 308700*b^3*arctan((tan(1/2*d*x)
*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan
(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^6 + 5710950*a^2*b*ar
ctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x
)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^6
+ 308700*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) -
1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^
4*tan(1/2*c)^6 - 235576320*a^3*tan(1/2*d*x)^5*tan(1/2*c)^6 + 719953920*a*b^
2*tan(1/2*d*x)^5*tan(1/2*c)^6 - 115863552*a^3*tan(1/2*d*x)^4*tan(1/2*c)^7 +
396877824*a*b^2*tan(1/2*d*x)^4*tan(1/2*c)^7 + 33075*pi*a^2*b*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2
*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*
tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*
c)^8 + 7350*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1
/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^
2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^
2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^8 + 33075*pi*a^2*b*sgn(tan(1/2*d*x)^2*ta
n(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 -
2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*
c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^8 + 7
350*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) +
tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/
2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*ta
n(1/2*d*x) - 1)*tan(1/2*c)^8 + 1183350*pi*a^2*b*tan(1/2*d*x)^2*tan(1/2*c)^8
 + 102900*pi*b^3*tan(1/2*d*x)^2*tan(1/2*c)^8 - 463050*a^2*b*arctan((tan(1/2
*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c)
- tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^8 - 102900*b^3*
arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d
*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^
8 - 463050*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c
) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d
*x)^2*tan(1/2*c)^8 - 102900*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d
*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c)
+ 1))*tan(1/2*d*x)^2*tan(1/2*c)^8 + 1903650*a^2*b*arctan((tan(1/2*d*x)*tan(
1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*
d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^8 + 102900*b^3*arctan((ta
n(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/
2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^8 + 190365
0*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(t
an(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan
(1/2*c)^8 + 102900*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan
(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan

(1/2*d*x)^2*tan(1/2*c)^8 - 37632000*a^3*tan(1/2*d*x)^3*tan(1/2*c)^8 + 12257
2800*a*b^2*tan(1/2*d*x)^3*tan(1/2*c)^8 - 5368832*a^3*tan(1/2*d*x)^2*tan(1/2
*c)^9 + 22851584*a*b^2*tan(1/2*d*x)^2*tan(1/2*c)^9 + 101430*pi*a^2*b*tan(1/
2*c)^10 + 8820*pi*b^3*tan(1/2*c)^10 - 39690*a^2*b*arctan((tan(1/2*d*x)*tan(
1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*
d*x) + tan(1/2*c) + 1))*tan(1/2*c)^10 - 8820*b^3*arctan((tan(1/2*d*x)*tan(1
/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d
*x) + tan(1/2*c) + 1))*tan(1/2*c)^10 - 39690*a^2*b*arctan((tan(1/2*d*x)*tan
(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2
*d*x) - tan(1/2*c) + 1))*tan(1/2*c)^10 - 8820*b^3*arctan((tan(1/2*d*x)*tan(
1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*
d*x) - tan(1/2*c) + 1))*tan(1/2*c)^10 + 163170*a^2*b*arctan((tan(1/2*d*x)*t
an(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1
/2*d*x) - tan(1/2*c) - 1))*tan(1/2*c)^10 + 8820*b^3*arctan((tan(1/2*d*x)*ta
n(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/
2*d*x) - tan(1/2*c) - 1))*tan(1/2*c)^10 + 163170*a^2*b*arctan((tan(1/2*d*x)
*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan
(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^10 + 8820*b^3*arctan((tan(1/2*d*x)*
tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(
1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^10 - 752640*a^3*tan(1/2*d*x)*tan(1/2
*c)^10 + 2150400*a*b^2*tan(1/2*d*x)*tan(1/2*c)^10 + 71680*a^3*tan(1/2*c)^11
 + 143360*a*b^2*tan(1/2*c)^11 - 77175*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c
)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1
/2*d*x) - 1)*tan(1/2*d*x)^8 + 69825*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
+ 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d
*x) - 1)*tan(1/2*d*x)^8 + 77175*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 -
2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x
) - 1)*tan(1/2*d*x)^8 - 69825*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*ta
n(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) -
1)*tan(1/2*d*x)^8 - 169050*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1
/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^8 -
14700*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d
*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^8 + 161280*a^2*b*tan(1/2*d*
x)^10 - 50176*b^3*tan(1/2*d*x)^10 + 3225600*a^2*b*tan(1/2*d*x)^9*tan(1/2*c)
 - 1146880*b^3*tan(1/2*d*x)^9*tan(1/2*c) - 540225*pi*a^2*b*sgn(tan(1/2*d*x)
^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)
^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 + 488775*pi*b^3*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 + 540225*pi*
a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1
/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^2
- 488775*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)
^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan
(1/2*c)^2 - 1183350*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)
^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c

)^2 - 102900*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*ta
n(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 + 276
86400*a^2*b*tan(1/2*d*x)^8*tan(1/2*c)^2 - 10429440*b^3*tan(1/2*d*x)^8*tan(1
/2*c)^2 + 129024000*a^2*b*tan(1/2*d*x)^7*tan(1/2*c)^3 - 48168960*b^3*tan(1/
2*d*x)^7*tan(1/2*c)^3 - 972405*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2
*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x)
 - 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 + 879795*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/
2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*ta
n(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 + 972405*pi*a^2*b*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2
*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 - 879795*pi*b^3*sgn
(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2
 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 - 2130030
*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)
*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 - 185220*pi*b^3
*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*
c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 + 341107200*a^2*b*tan(1/
2*d*x)^6*tan(1/2*c)^4 - 119454720*b^3*tan(1/2*d*x)^6*tan(1/2*c)^4 + 4887859
20*a^2*b*tan(1/2*d*x)^5*tan(1/2*c)^5 - 159186944*b^3*tan(1/2*d*x)^5*tan(1/2
*c)^5 - 540225*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*ta
n(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*
x)^2*tan(1/2*c)^6 + 488775*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1
/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*
tan(1/2*d*x)^2*tan(1/2*c)^6 + 540225*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)
^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/
2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^6 - 488775*pi*b^3*sgn(tan(1/2*d*x)^2*
tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2
 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^6 - 1183350*pi*a^2*b*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - ta
n(1/2*c)^2 + 1)*tan(1/2*d*x)^2*tan(1/2*c)^6 - 102900*pi*b^3*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^
2 + 1)*tan(1/2*d*x)^2*tan(1/2*c)^6 + 341107200*a^2*b*tan(1/2*d*x)^4*tan(1/2
*c)^6 - 119454720*b^3*tan(1/2*d*x)^4*tan(1/2*c)^6 + 129024000*a^2*b*tan(1/2
*d*x)^3*tan(1/2*c)^7 - 48168960*b^3*tan(1/2*d*x)^3*tan(1/2*c)^7 - 77175*pi*
a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1
/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^8 + 69825*pi*b^3*
sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^8 + 77175*pi*a^2*b*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2
 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^8 - 69825*pi*b^3*sgn(tan(1/
2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(
1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^8 - 169050*pi*a^2*b*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c
)^2 + 1)*tan(1/2*c)^8 - 14700*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(
1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^8 + 2

7686400*a^2*b*tan(1/2*d*x)^2*tan(1/2*c)^8 - 10429440*b^3*tan(1/2*d*x)^2*tan
(1/2*c)^8 + 3225600*a^2*b*tan(1/2*d*x)*tan(1/2*c)^9 - 1146880*b^3*tan(1/2*d
*x)*tan(1/2*c)^9 + 161280*a^2*b*tan(1/2*c)^10 - 50176*b^3*tan(1/2*c)^10 + 3
3075*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c)
 + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan
(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2
*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6 + 7350*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*
c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(
1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 -
 tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6 + 33075
*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + t
an(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2
*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan
(1/2*d*x) - 1)*tan(1/2*d*x)^6 + 7350*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*
c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan
(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6 + 169050*pi
*a^2*b*tan(1/2*d*x)^8 + 14700*pi*b^3*tan(1/2*d*x)^8 - 66150*a^2*b*arctan((t
an(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1
/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^8 - 14700*b^3*arctan((
tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(
1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^8 - 66150*a^2*b*arcta
n((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*t
an(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8 - 14700*b^3*arct
an((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*
tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^8 + 271950*a^2*b*
arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d
*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^8 + 14700*b^3
*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*
d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^8 + 271950*a
^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(
1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^8 + 1470
0*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan
(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^8 + 308
224*a^3*tan(1/2*d*x)^9 - 114688*a*b^2*tan(1/2*d*x)^9 + 1254400*a^3*tan(1/2*
d*x)^8*tan(1/2*c) - 4300800*a*b^2*tan(1/2*d*x)^8*tan(1/2*c) + 138915*pi*a^2
*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*
d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 +
 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*
x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^2 + 30870*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1
/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*t
an(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^
2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(
1/2*c)^2 + 138915*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)
^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1

/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan
(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^2 + 30870*pi*b^3*
sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x
)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*
tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x)
- 1)*tan(1/2*d*x)^4*tan(1/2*c)^2 + 1183350*pi*a^2*b*tan(1/2*d*x)^6*tan(1/2*
c)^2 + 102900*pi*b^3*tan(1/2*d*x)^6*tan(1/2*c)^2 - 463050*a^2*b*arctan((tan
(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2
*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 - 102900*
b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1
/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2
*c)^2 - 463050*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1
/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1
/2*d*x)^6*tan(1/2*c)^2 - 102900*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1
/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2
*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 + 1903650*a^2*b*arctan((tan(1/2*d*x)*
tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(
1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 + 102900*b^3*arctan
((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*ta
n(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^2 + 19
03650*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1
)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6
*tan(1/2*c)^2 + 102900*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) -
 tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))
*tan(1/2*d*x)^6*tan(1/2*c)^2 + 11554816*a^3*tan(1/2*d*x)^7*tan(1/2*c)^2 - 3
1137792*a*b^2*tan(1/2*d*x)^7*tan(1/2*c)^2 + 37632000*a^3*tan(1/2*d*x)^6*tan
(1/2*c)^3 - 122572800*a*b^2*tan(1/2*d*x)^6*tan(1/2*c)^3 + 138915*pi*a^2*b*s
gn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)
^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*t
an(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) -
 1)*tan(1/2*d*x)^2*tan(1/2*c)^4 + 30870*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c
)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1
/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 -
tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*
c)^4 + 138915*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*t
an(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2
*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^4 + 30870*pi*b^3*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2
- tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(
1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)
*tan(1/2*d*x)^2*tan(1/2*c)^4 + 2130030*pi*a^2*b*tan(1/2*d*x)^4*tan(1/2*c)^4
 + 185220*pi*b^3*tan(1/2*d*x)^4*tan(1/2*c)^4 - 833490*a^2*b*arctan((tan(1/2
*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c)
- tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 - 185220*b^3*

arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d
*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^
4 - 833490*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c
) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d
*x)^4*tan(1/2*c)^4 - 185220*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d
*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c)
+ 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 + 3426570*a^2*b*arctan((tan(1/2*d*x)*tan(
1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*
d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 + 185220*b^3*arctan((ta
n(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/
2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^4 + 342657
0*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(t
an(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan
(1/2*c)^4 + 185220*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan
(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan
(1/2*d*x)^4*tan(1/2*c)^4 + 86754304*a^3*tan(1/2*d*x)^5*tan(1/2*c)^4 - 24110
2848*a*b^2*tan(1/2*d*x)^5*tan(1/2*c)^4 + 86754304*a^3*tan(1/2*d*x)^4*tan(1/
2*c)^5 - 241102848*a*b^2*tan(1/2*d*x)^4*tan(1/2*c)^5 + 33075*pi*a^2*b*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1
/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*
tan(1/2*c)^6 + 7350*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)
^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1
/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan
(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^6 + 33075*pi*a^2*b*sgn(tan(1/2*d
*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2
*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*
tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*
c)^6 + 7350*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1
/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^
2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^
2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^6 + 1183350*pi*a^2*b*tan(1/2*d*x)^2*tan(
1/2*c)^6 + 102900*pi*b^3*tan(1/2*d*x)^2*tan(1/2*c)^6 - 463050*a^2*b*arctan(
(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan
(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 - 102
900*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(t
an(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan
(1/2*c)^6 - 463050*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + t
an(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*t
an(1/2*d*x)^2*tan(1/2*c)^6 - 102900*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - t
an(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan
(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 + 1903650*a^2*b*arctan((tan(1/2*d
*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) -
tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 + 102900*b^3*ar
ctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x

)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^6
+ 1903650*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c)
 - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*
x)^2*tan(1/2*c)^6 + 102900*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*
x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) -
 1))*tan(1/2*d*x)^2*tan(1/2*c)^6 + 37632000*a^3*tan(1/2*d*x)^3*tan(1/2*c)^6
 - 122572800*a*b^2*tan(1/2*d*x)^3*tan(1/2*c)^6 + 11554816*a^3*tan(1/2*d*x)^
2*tan(1/2*c)^7 - 31137792*a*b^2*tan(1/2*d*x)^2*tan(1/2*c)^7 + 169050*pi*a^2
*b*tan(1/2*c)^8 + 14700*pi*b^3*tan(1/2*c)^8 - 66150*a^2*b*arctan((tan(1/2*d
*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) -
tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*c)^8 - 14700*b^3*arctan((tan(1/2*d*
x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - t
an(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*c)^8 - 66150*a^2*b*arctan((tan(1/2*d
*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) +
tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*c)^8 - 14700*b^3*arctan((tan(1/2*d*
x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + t
an(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*c)^8 + 271950*a^2*b*arctan((tan(1/2*
d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) -
 tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*c)^8 + 14700*b^3*arctan((tan(1/2*d
*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) -
tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*c)^8 + 271950*a^2*b*arctan((tan(1/2
*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c)
+ tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^8 + 14700*b^3*arctan((tan(1/2*
d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) +
 tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^8 + 1254400*a^3*tan(1/2*d*x)*ta
n(1/2*c)^8 - 4300800*a*b^2*tan(1/2*d*x)*tan(1/2*c)^8 + 308224*a^3*tan(1/2*c
)^9 - 114688*a*b^2*tan(1/2*c)^9 - 77175*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2
*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan
(1/2*d*x) - 1)*tan(1/2*d*x)^6 + 69825*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^
2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2
*d*x) - 1)*tan(1/2*d*x)^6 + 77175*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2
- 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d
*x) - 1)*tan(1/2*d*x)^6 - 69825*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*
tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x)
- 1)*tan(1/2*d*x)^6 - 169050*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan
(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6
- 14700*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2
*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6 - 268800*a^2*b*tan(1/2*
d*x)^8 + 107520*b^3*tan(1/2*d*x)^8 - 4300800*a^2*b*tan(1/2*d*x)^7*tan(1/2*c
) + 1720320*b^3*tan(1/2*d*x)^7*tan(1/2*c) - 324135*pi*a^2*b*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c
)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^2 + 293265*pi*b^3*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 +
 tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^2 + 324135*pi
*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(

1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^2 - 293265*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^2 - 710010*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^2 - 61740*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^2 - 27686400*a^2*b*tan(1/2*d*x)^6*tan(1/2*c)^2 + 10429440*b^3*tan(1/2*d*x)^6*tan(1/2*c)^2 - 88166400*a^2*b*tan(1/2*d*x)^5*tan(1/2*c)^3 + 29532160*b^3*tan(1/2*d*x)^5*tan(1/2*c)^3 - 324135*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^4 + 293265*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^4 + 324135*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^4 - 293265*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^4 - 710010*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2*tan(1/2*c)^4 - 61740*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2*tan(1/2*c)^4 - 136711680*a^2*b*tan(1/2*d*x)^4*tan(1/2*c)^4 + 40406016*b^3*tan(1/2*d*x)^4*tan(1/2*c)^4 - 88166400*a^2*b*tan(1/2*d*x)^3*tan(1/2*c)^5 + 29532160*b^3*tan(1/2*d*x)^3*tan(1/2*c)^5 - 77175*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^6 + 69825*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^6 + 77175*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^6 - 69825*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^6 - 169050*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^6 - 14700*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^6 - 27686400*a^2*b*tan(1/2*d*x)^2*tan(1/2*c)^6 + 10429440*b^3*tan(1/2*d*x)^2*tan(1/2*c)^6 - 4300800*a^2*b*tan(1/2*d*x)*tan(1/2*c)^7 + 1720320*b^3*tan(1/2*d*x)*tan(1/2*c)^7 - 268800*a^2*b*tan(1/2*c)^8 + 107520*b^3*tan(1/2*c)^8 + 19845*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4 + 4410*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^

4 + 19845*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4 + 4410*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4 + 169050*pi*a^2*b*tan(1/2*d*x)^6 + 14700*pi*b^3*tan(1/2*d*x)^6 - 66150*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6 - 14700*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6 - 66150*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6 - 14700*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6 + 271950*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6 + 14700*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6 + 271950*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6 + 14700*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6 + 217088*a^3*tan(1/2*d*x)^7 + 466944*a*b^2*tan(1/2*d*x)^7 - 1254400*a^3*tan(1/2*d*x)^6*tan(1/2*c) + 4300800*a*b^2*tan(1/2*d*x)^6*tan(1/2*c) + 46305*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 10290*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 46305*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 10290*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 710010*pi*a^2*b*tan(1/2*d*x)^4*tan(1/2*c)^2 + 61740*pi*b^3*tan(1/2*d*x)^4*tan(1/2*c)^2 - 277830*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^2 - 61740*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(t

an(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4*tan
(1/2*c)^2 - 277830*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + t
an(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*t
an(1/2*d*x)^4*tan(1/2*c)^2 - 61740*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - ta
n(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(
1/2*c) + 1))*tan(1/2*d*x)^4*tan(1/2*c)^2 + 1142190*a^2*b*arctan((tan(1/2*d*
x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - t
an(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^2 + 61740*b^3*arct
an((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*
tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4*tan(1/2*c)^2 +
1142190*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) -
 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)
^4*tan(1/2*c)^2 + 61740*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x)
 - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)
)*tan(1/2*d*x)^4*tan(1/2*c)^2 - 5368832*a^3*tan(1/2*d*x)^5*tan(1/2*c)^2 + 2
2851584*a*b^2*tan(1/2*d*x)^5*tan(1/2*c)^2 - 18565120*a^3*tan(1/2*d*x)^4*tan
(1/2*c)^3 + 50319360*a*b^2*tan(1/2*d*x)^4*tan(1/2*c)^3 + 19845*pi*a^2*b*sgn
(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2
 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan
(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1
)*tan(1/2*c)^4 + 4410*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*
x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan
(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + t
an(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^4 + 19845*pi*a^2*b*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1
/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x
)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/
2*c)^4 + 4410*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan
(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x
)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c
)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^4 + 710010*pi*a^2*b*tan(1/2*d*x)^2*tan
(1/2*c)^4 + 61740*pi*b^3*tan(1/2*d*x)^2*tan(1/2*c)^4 - 277830*a^2*b*arctan(
(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan
(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^4 - 617
40*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(ta
n(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(
1/2*c)^4 - 277830*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + ta
n(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*ta
n(1/2*d*x)^2*tan(1/2*c)^4 - 61740*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan
(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1
/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^4 + 1142190*a^2*b*arctan((tan(1/2*d*x
)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - ta
n(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^4 + 61740*b^3*arcta
n((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*t
an(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^4 + 1

142190*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^4 + 61740*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^4 - 18565120*a^3*tan(1/2*d*x)^3*tan(1/2*c)^4 + 50319360*a*b^2*tan(1/2*d*x)^3*tan(1/2*c)^4 - 5368832*a^3*tan(1/2*d*x)^2*tan(1/2*c)^5 + 22851584*a*b^2*tan(1/2*d*x)^2*tan(1/2*c)^5 + 169050*pi*a^2*b*tan(1/2*c)^6 + 14700*pi*b^3*tan(1/2*c)^6 - 66150*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*c)^6 - 14700*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*c)^6 - 66150*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*c)^6 - 14700*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*c)^6 + 271950*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*c)^6 + 14700*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*c)^6 + 271950*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^6 + 14700*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^6 - 1254400*a^3*tan(1/2*d*x)*tan(1/2*c)^6 + 4300800*a*b^2*tan(1/2*d*x)*tan(1/2*c)^6 + 217088*a^3*tan(1/2*c)^7 + 466944*a*b^2*tan(1/2*c)^7 - 46305*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4 + 41895*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4 + 46305*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4 - 41895*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4 - 101430*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4 - 8820*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4 + 268800*a^2*b*tan(1/2*d*x)^6 - 107520*b^3*tan(1/2*d*x)^6 + 3225600*a^2*b*tan(1/2*d*x)^5*tan(1/2*c) - 1146880*b^3*tan(1/2*d*x)^5*tan(1/2*c) - 108045*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 97755*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 108045*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 97755 *pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 236670*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 20580*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2*tan(1/2*c)^2 + 14031360*a^2*b*tan(1/2*d*x)^4*tan(1/2*c)^2 - 4092928*b^3*tan(1/2*d*x)^4*tan(1/2*c)^2 + 24944640*a^2*b*tan(1/2*d*x)^3*tan(1/2*c)^3 - 5734400*b^3*tan(1/2*d*x)^3*tan(1/2*c)^3 - 46305*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^4 + 41895*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^4 + 46305*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^4 - 41895*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^4 - 101430*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^4 - 8820*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^4 + 14031360*a^2*b*tan(1/2*d*x)^2*tan(1/2*c)^4 - 4092928*b^3*tan(1/2*d*x)^2*tan(1/2*c)^4 + 3225600*a^2*b*tan(1/2*d*x)*tan(1/2*c)^5 - 1146880*b^3*tan(1/2*d*x)*tan(1/2*c)^5 + 268800*a^2*b*tan(1/2*c)^6 - 107520*b^3*tan(1/2*c)^6 + 6615*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2 + 1470*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2 + 6615*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2 + 1470*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2 + 101430*pi*a^2*b*tan(1/2*d*x)^4 + 8820*pi*b^3*tan(1/2*d*x)^4 - 39690*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4 - 8820*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^4 - 39690*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4 - 8820*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^4 + 1

63170*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1
)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4
 + 8820*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1
)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^4
 + 163170*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c)
 - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*
x)^4 + 8820*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c)
 - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*
x)^4 + 308224*a^3*tan(1/2*d*x)^5 - 114688*a*b^2*tan(1/2*d*x)^5 + 752640*a^3
*tan(1/2*d*x)^4*tan(1/2*c) - 2150400*a*b^2*tan(1/2*d*x)^4*tan(1/2*c) + 6615
*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + t
an(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2
*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan
(1/2*d*x) - 1)*tan(1/2*c)^2 + 1470*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 +
 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c)
 - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1
/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^2 + 6615*pi*a^2*b
*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*
x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2
*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x)
 - 1)*tan(1/2*c)^2 + 1470*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/
2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn
(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2
 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^2 + 236670*pi*a^2*b*tan(1/
2*d*x)^2*tan(1/2*c)^2 + 20580*pi*b^3*tan(1/2*d*x)^2*tan(1/2*c)^2 - 92610*a^
2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1
/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2
*c)^2 - 20580*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*
c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*
d*x)^2*tan(1/2*c)^2 - 92610*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2
*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c
) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 20580*b^3*arctan((tan(1/2*d*x)*tan(1/
2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*
x) - tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 380730*a^2*b*arctan((ta
n(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/
2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 20580*
b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1
/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^2*tan(1/2
*c)^2 + 380730*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1
/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1
/2*d*x)^2*tan(1/2*c)^2 + 20580*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/
2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*
c) - 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 3512320*a^3*tan(1/2*d*x)^3*tan(1/2*c
)^2 - 5877760*a*b^2*tan(1/2*d*x)^3*tan(1/2*c)^2 + 3512320*a^3*tan(1/2*d*x)^
2*tan(1/2*c)^3 - 5877760*a*b^2*tan(1/2*d*x)^2*tan(1/2*c)^3 + 101430*pi*a^2*

b*tan(1/2*c)^4 + 8820*pi*b^3*tan(1/2*c)^4 - 39690*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*c)^4 - 8820*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*c)^4 - 39690*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*c)^4 - 8820*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*c)^4 + 163170*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*c)^4 + 8820*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*c)^4 + 163170*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^4 + 8820*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^4 + 752640*a^3*tan(1/2*d*x)*tan(1/2*c)^4 - 2150400*a*b^2*tan(1/2*d*x)*tan(1/2*c)^4 + 308224*a^3*tan(1/2*c)^5 - 114688*a*b^2*tan(1/2*c)^5 - 15435*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2 + 13965*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2 + 15435*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2 - 13965*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^2 - 33810*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2 - 2940*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^2 - 161280*a^2*b*tan(1/2*d*x)^4 + 50176*b^3*tan(1/2*d*x)^4 - 1290240*a^2*b*tan(1/2*d*x)^3*tan(1/2*c) + 286720*b^3*tan(1/2*d*x)^3*tan(1/2*c) - 15435*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^2 + 13965*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*c)^2 + 15435*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^2 - 13965*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*c)^2 - 33810*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^2 - 2940*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*c)^2 - 2956800*a^2*b*tan(1/2*d*x)^2*tan(1/2*c)^2 + 379904*b^3*tan(1/2*d*x)^2*tan(1/2*c)^2 - 1290240*a^2*b*tan(1/2*d*x)*tan(1/2*c)^3 + 286720*b^3*tan(1/2*d*x)*tan(1/2*c)^3 - 161280*a^2*b*tan(1/2

*c)^4 + 50176*b^3*tan(1/2*c)^4 + 945*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)
^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/
2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - t
an(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1) + 210*pi*b^3*sgn(tan(1/2
*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1
/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x
)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1) + 945*
pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + ta
n(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*
c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(
1/2*d*x) - 1) + 210*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)
^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1
/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan
(1/2*c)^2 - 2*tan(1/2*d*x) - 1) + 33810*pi*a^2*b*tan(1/2*d*x)^2 + 2940*pi*b
^3*tan(1/2*d*x)^2 - 13230*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d
*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c)
+ 1))*tan(1/2*d*x)^2 - 2940*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d
*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c)
+ 1))*tan(1/2*d*x)^2 - 13230*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/
2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*
c) + 1))*tan(1/2*d*x)^2 - 2940*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/
2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*
c) + 1))*tan(1/2*d*x)^2 + 54390*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan
(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1
/2*c) - 1))*tan(1/2*d*x)^2 + 2940*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan
(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1
/2*c) - 1))*tan(1/2*d*x)^2 + 54390*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) -
tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + ta
n(1/2*c) - 1))*tan(1/2*d*x)^2 + 2940*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) -
tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + ta
n(1/2*c) - 1))*tan(1/2*d*x)^2 + 71680*a^3*tan(1/2*d*x)^3 + 143360*a*b^2*tan
(1/2*d*x)^3 - 250880*a^3*tan(1/2*d*x)^2*tan(1/2*c) + 430080*a*b^2*tan(1/2*d
*x)^2*tan(1/2*c) + 33810*pi*a^2*b*tan(1/2*c)^2 + 2940*pi*b^3*tan(1/2*c)^2 -
13230*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) +
1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*c)^2
- 2940*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)
/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*c)^2 -
13230*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1
)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*c)^2 -
2940*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/
(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*c)^2 + 5
4390*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)
/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*c)^2 +
2940*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(
tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*c)^2 + 54

390*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^2 + 2940*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*c)^2 - 250880*a^3*tan(1/2*d*x)*tan(1/2*c)^2 + 430080*a*b^2*tan(1/2*d*x)*tan(1/2*c)^2 + 71680*a^3*tan(1/2*c)^3 + 143360*a*b^2*tan(1/2*c)^3 - 2205*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1) + 1995*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1) + 2205*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1) - 1995*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1) - 4830*pi*a^2*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1) - 420*pi*b^3*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1) + 53760*a^2*b*tan(1/2*d*x)^2 - 7168*b^3*tan(1/2*d*x)^2 + 215040*a^2*b*tan(1/2*d*x)*tan(1/2*c) + 53760*a^2*b*tan(1/2*c)^2 - 7168*b^3*tan(1/2*c)^2 + 4830*pi*a^2*b + 420*pi*b^3 - 1890*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)) - 420*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)) - 1890*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)) - 420*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)) + 7770*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)) + 420*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)) + 7770*a^2*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)) + 420*b^3*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)) + 35840*a^3*tan(1/2*d*x) + 35840*a^3*tan(1/2*c) - 7680*a^2*b - 1024*b^3)/(d*tan(1/2*d*x)^14*tan(1/2*c)^14 + 7*d*tan(1/2*d*x)^14*tan(1/2*c)^12 + 7*d*tan(1/2*d*x)^12*tan(1/2*c)^14 + 21*d*tan(1/2*d*x)^14*tan(1/2*c)^10 + 49*d*tan(1/2*d*x)^12*tan(1/2*c)^12 + 21*d*tan(1/2*d*x)^10*tan(1/2*c)^14 + 35*d*tan(1/2*d*x)^14*tan(1/2*c)^8 + 147*d*tan(1/2*d*x)^12*tan(1/2*c)^10 + 147*d*tan(1/2*d*x)^10*tan(1/2*c)^12 + 35*d*tan(1/2*d*x)^8*tan(1/2*c)^14 + 35*d*tan(1/2*d*x)^14*tan(1/2*c)^6 + 245*d*tan(1/2*d*x)^12*tan(1/2*c)^8 + 441*d*tan(1/2*d*x)^10*tan(1/2*c)^10 + 245*d*tan(1/2*d*x)^8*tan(1/2*c)^12 + 35*d*tan(1/2*d*x)^6*tan(1/2*c)^14 + 21*d*tan(1/2*d*x)^14*tan(1/2*c)^4 + 245*d*tan(1/2*d*x)^12*tan(1/2*c)^6 + 735*d*tan(1/2*d*x)^10*tan(1/2*c)^8 + 735*d*tan(1/2*d*x)^8*tan(1/2*c)^10 + 245*d*tan(1/2*d*x)^6*tan(1/2*c)^12 + 21*d*tan(1/2*d*x)^4*tan(1/2*c)^14 + 7*d*tan(1/2*d*x)^14*tan(1/2*c)^2 + 147*d*tan(1/2*d*x)^12*tan(1/2*c)^4 + 735*d*tan(1/2*d*x)^10*tan(1

```
/2*c)^6 + 1225*d*tan(1/2*d*x)^8*tan(1/2*c)^8 + 735*d*tan(1/2*d*x)^6*tan(1/2
*c)^10 + 147*d*tan(1/2*d*x)^4*tan(1/2*c)^12 + 7*d*tan(1/2*d*x)^2*tan(1/2*c)
^14 + d*tan(1/2*d*x)^14 + 49*d*tan(1/2*d*x)^12*tan(1/2*c)^2 + 441*d*tan(1/2
*d*x)^10*tan(1/2*c)^4 + 1225*d*tan(1/2*d*x)^8*tan(1/2*c)^6 + 1225*d*tan(1/2
*d*x)^6*tan(1/2*c)^8 + 441*d*tan(1/2*d*x)^4*tan(1/2*c)^10 + 49*d*tan(1/2*d*
x)^2*tan(1/2*c)^12 + d*tan(1/2*c)^14 + 7*d*tan(1/2*d*x)^12 + 147*d*tan(1/2*
d*x)^10*tan(1/2*c)^2 + 735*d*tan(1/2*d*x)^8*tan(1/2*c)^4 + 1225*d*tan(1/2*d
*x)^6*tan(1/2*c)^6 + 735*d*tan(1/2*d*x)^4*tan(1/2*c)^8 + 147*d*tan(1/2*d*x)
^2*tan(1/2*c)^10 + 7*d*tan(1/2*c)^12 + 21*d*tan(1/2*d*x)^10 + 245*d*tan(1/2
*d*x)^8*tan(1/2*c)^2 + 735*d*tan(1/2*d*x)^6*tan(1/2*c)^4 + 735*d*tan(1/2*d*
x)^4*tan(1/2*c)^6 + 245*d*tan(1/2*d*x)^2*tan(1/2*c)^8 + 21*d*tan(1/2*c)^10
+ 35*d*tan(1/2*d*x)^8 + 245*d*tan(1/2*d*x)^6*tan(1/2*c)^2 + 441*d*tan(1/2*d
*x)^4*tan(1/2*c)^4 + 245*d*tan(1/2*d*x)^2*tan(1/2*c)^6 + 35*d*tan(1/2*c)^8
+ 35*d*tan(1/2*d*x)^6 + 147*d*tan(1/2*d*x)^4*tan(1/2*c)^2 + 147*d*tan(1/2*d
*x)^2*tan(1/2*c)^4 + 35*d*tan(1/2*c)^6 + 21*d*tan(1/2*d*x)^4 + 49*d*tan(1/2
*d*x)^2*tan(1/2*c)^2 + 21*d*tan(1/2*c)^4 + 7*d*tan(1/2*d*x)^2 + 7*d*tan(1/2
*c)^2 + d)
```

Mupad [B] (verification not implemented)

Time = 4.49 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.51

$$\int \cos^7(c+dx)(a+b\tan(c+dx))^3\,dx = \frac{16\,a^3\,\sin\left(c+d\,x\right)}{35\,d}-\frac{b^3\cos\left(c+d\,x\right)^5}{5\,d}+\frac{b^3\cos\left(c+d\,x\right)^7}{7\,d}-\frac{3\,a^2\,b\cos\left(c+d\,x\right)^7}{7\,d}+\frac{8\,a^3\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{35\,d}+\frac{6\,a^3\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)}{35\,d}+\frac{a^3\cos\left(c+d\,x\right)^6\,\sin\left(c+d\,x\right)}{7\,d}+\frac{8\,a\,b^2\,\sin\left(c+d\,x\right)}{35\,d}+\frac{4\,a\,b^2\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{35\,d}+\frac{3\,a\,b^2\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)}{35\,d}-\frac{3\,a\,b^2\cos\left(c+d\,x\right)^6\,\sin\left(c+d\,x\right)}{7\,d}$$

```
[In] int(cos(c + d*x)^7*(a + b*tan(c + d*x))^3,x)
```

```
[Out] (16*a^3*sin(c + d*x))/(35*d) - (b^3*cos(c + d*x)^5)/(5*d) + (b^3*cos(c + d*
x)^7)/(7*d) - (3*a^2*b*cos(c + d*x)^7)/(7*d) + (8*a^3*cos(c + d*x)^2*sin(c
```

+ d*x))/(35*d) + (6*a^3*cos(c + d*x)^4*sin(c + d*x))/(35*d) + (a^3*cos(c + d*x)^6*sin(c + d*x))/(7*d) + (8*a*b^2*sin(c + d*x))/(35*d) + (4*a*b^2*cos(c + d*x)^2*sin(c + d*x))/(35*d) + (3*a*b^2*cos(c + d*x)^4*sin(c + d*x))/(35*d) - (3*a*b^2*cos(c + d*x)^6*sin(c + d*x))/(7*d)

3.544 $\int \frac{\sec^6(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 116

$$\int \frac{\sec^6(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{(a^2+b^2)^2\log(a+b\tan(c+dx))}{b^5d} - \frac{a(a^2+2b^2)\tan(c+dx)}{b^4d} + \frac{(a^2+2b^2)\tan^2(c+dx)}{2b^3d} - \frac{a\tan^3(c+dx)}{3b^2d} + \frac{\tan^4(c+dx)}{4bd}$$

```
[Out] (a^2+b^2)^2*ln(a+b*tan(d*x+c))/b^5/d-a*(a^2+2*b^2)*tan(d*x+c)/b^4/d+1/2*(a^
2+2*b^2)*tan(d*x+c)^2/b^3/d-1/3*a*tan(d*x+c)^3/b^2/d+1/4*tan(d*x+c)^4/b/d
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3587, 711}

$$\int \frac{\sec^6(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{(a^2+b^2)^2\log(a+b\tan(c+dx))}{b^5d} - \frac{a(a^2+2b^2)\tan(c+dx)}{b^4d} + \frac{(a^2+2b^2)\tan^2(c+dx)}{2b^3d} - \frac{a\tan^3(c+dx)}{3b^2d} + \frac{\tan^4(c+dx)}{4bd}$$

```
[In] Int[Sec[c + d*x]^6/(a + b*Tan[c + d*x]),x]
```

```
[Out] ((a^2 + b^2)^2*Log[a + b*Tan[c + d*x]])/(b^5*d) - (a*(a^2 + 2*b^2)*Tan[c +
d*x])/(b^4*d) + ((a^2 + 2*b^2)*Tan[c + d*x]^2)/(2*b^3*d) - (a*Tan[c + d*x]^
3)/(3*b^2*d) + Tan[c + d*x]^4/(4*b*d)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^2}{a+x}\,dx, x, b\tan(c+dx)\right)}{bd} \\
&= \frac{\text{Subst}\left(\int \left(\frac{a(-a^2-2b^2)}{b^4} + \frac{(a^2+2b^2)x}{b^4} - \frac{ax^2}{b^4} + \frac{x^3}{b^4} + \frac{(a^2+b^2)^2}{b^4(a+x)}\right)\,dx, x, b\tan(c+dx)\right)}{bd} \\
&= \frac{(a^2+b^2)^2\log(a+b\tan(c+dx))}{b^5 d} - \frac{a(a^2+2b^2)\tan(c+dx)}{b^4 d} \\
&\quad + \frac{(a^2+2b^2)\tan^2(c+dx)}{2b^3 d} - \frac{a\tan^3(c+dx)}{3b^2 d} + \frac{\tan^4(c+dx)}{4bd}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.85

$$\int \frac{\sec^6(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{12(a^2+b^2)^2\log(a+b\tan(c+dx)) + 3b^4\sec^4(c+dx) - 12ab(a^2+2b^2)\tan(c+dx) + 6b^2(a^2+b^2)\tan^2(c+}{12b^5 d}$$

```
[In] Integrate[Sec[c + d*x]^6/(a + b*Tan[c + d*x]),x]
```

```
[Out] (12*(a^2 + b^2)^2*Log[a + b*Tan[c + d*x]] + 3*b^4*Sec[c + d*x]^4 - 12*a*b*(
a^2 + 2*b^2)*Tan[c + d*x] + 6*b^2*(a^2 + b^2)*Tan[c + d*x]^2 - 4*a*b^3*Tan[
c + d*x]^3)/(12*b^5*d)
```

Maple [A] (verified)

Time = 32.68 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{-\frac{-\frac{\left(\tan^4(dx+c)\right)b^3}{4}+\frac{a\left(\tan^3(dx+c)\right)b^2}{3}-\frac{\left(a^2+2b^2\right)\left(\tan^2(dx+c)\right)b}{2}+a\left(a^2+2b^2\right)\tan(dx+c)}{b^4}+\frac{\left(a^4+2a^2b^2+b^4\right)\ln(a+b\tan(dx+c))}{b^5}}{d}$
default	$\frac{-\frac{-\frac{\left(\tan^4(dx+c)\right)b^3}{4}+\frac{a\left(\tan^3(dx+c)\right)b^2}{3}-\frac{\left(a^2+2b^2\right)\left(\tan^2(dx+c)\right)b}{2}+a\left(a^2+2b^2\right)\tan(dx+c)}{b^4}+\frac{\left(a^4+2a^2b^2+b^4\right)\ln(a+b\tan(dx+c))}{b^5}}{d}$
risch	$\frac{-2ia^3e^{6i(dx+c)}-2ia\,b^2e^{6i(dx+c)}+2a^2b\,e^{6i(dx+c)}+2b^3e^{6i(dx+c)}-6ia^3e^{4i(dx+c)}-10ia\,b^2e^{4i(dx+c)}+4a^2b\,e^{4i(dx+c)}+8b^3e}{b^4d\left(e^{2i(dx+c)}+1\right)^4}$

```
[In] int(sec(d*x+c)^6/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b^4*(-1/4*tan(d*x+c)^4*b^3+1/3*a*tan(d*x+c)^3*b^2-1/2*(a^2+2*b^2)*t
an(d*x+c)^2*b+a*(a^2+2*b^2)*tan(d*x+c))+(a^4+2*a^2*b^2+b^4)/b^5*ln(a+b*tan(
d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.58

$$\int \frac{\sec^6(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{6\left(a^4+2\,a^2b^2+b^4\right)\cos\left(dx+c\right)^4\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)-6\left(a^4\right.}{}$$

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(6*(a^4 + 2*a^2*b^2 + b^4)*cos(d*x + c)^4*log(2*a*b*cos(d*x + c)*sin(d
*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 6*(a^4 + 2*a^2*b^2 + b^4)*cos
(d*x + c)^4*log(cos(d*x + c)^2) + 3*b^4 + 6*(a^2*b^2 + b^4)*cos(d*x + c)^2
- 4*(a*b^3*cos(d*x + c) + (3*a^3*b + 5*a*b^3)*cos(d*x + c)^3)*sin(d*x + c))
/(b^5*d*cos(d*x + c)^4)
```

Sympy [F]

$$\int \frac{\sec^6(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\sec^6(c+dx)}{a+b\tan(c+dx)}\,dx$$

```
[In] integrate(sec(d*x+c)**6/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**6/(a + b*tan(c + d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.93

$$\int \frac{\sec^6(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{3\,b^3\tan(dx+c)^4-4\,ab^2\tan(dx+c)^3+6\,(a^2b+2\,b^3)\tan(dx+c)^2-12\,(a^3+2\,ab^2)\tan(dx+c)}{b^4}+\frac{12\,(a^4+2\,a^2b^2+b^4)\log(b\tan(dx+c)+a)}{b^5}}{12\,d}$$

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*((3*b^3*tan(d*x + c)^4 - 4*a*b^2*tan(d*x + c)^3 + 6*(a^2*b + 2*b^3)*ta
n(d*x + c)^2 - 12*(a^3 + 2*a*b^2)*tan(d*x + c))/b^4 + 12*(a^4 + 2*a^2*b^2 +
 b^4)*log(b*tan(d*x + c) + a)/b^5)/d
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.03

$$\int \frac{\sec^6(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{3\,b^3\tan(dx+c)^4-4\,ab^2\tan(dx+c)^3+6\,a^2b\tan(dx+c)^2+12\,b^3\tan(dx+c)^2-12\,a^3\tan(dx+c)-24\,ab^2\tan(dx+c)}{b^4}+\frac{12\,(a^4+2\,a^2b^2+b^4)\log(|b\tan(d}{b^5}}{12\,d}$$

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*((3*b^3*tan(d*x + c)^4 - 4*a*b^2*tan(d*x + c)^3 + 6*a^2*b*tan(d*x + c)
^2 + 12*b^3*tan(d*x + c)^2 - 12*a^3*tan(d*x + c) - 24*a*b^2*tan(d*x + c))/b
^4 + 12*(a^4 + 2*a^2*b^2 + b^4)*log(abs(b*tan(d*x + c) + a))/b^5)/d
```

Mupad [B] (verification not implemented)

Time = 4.37 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03

$$\int \frac{\sec^6(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\tan(c+d\,x)^4}{4\,b\,d}+\frac{\tan(c+d\,x)^2\,\left(\frac{1}{b}+\frac{a^2}{2\,b^3}\right)}{d} + \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}{b^5\,d} - \frac{a\tan(c+d\,x)^3}{3\,b^2\,d}-\frac{a\tan(c+d\,x)\,\left(\frac{2}{b}+\frac{a^2}{b^3}\right)}{b\,d}$$

```
[In] int(1/(cos(c + d*x)^6*(a + b*tan(c + d*x))),x)
```

```
[Out] tan(c + d*x)^4/(4*b*d) + (tan(c + d*x)^2*(1/b + a^2/(2*b^3)))/d + (log(a +
b*tan(c + d*x))*(a^4 + b^4 + 2*a^2*b^2))/(b^5*d) - (a*tan(c + d*x)^3)/(3*b^
2*d) - (a*tan(c + d*x)*(2/b + a^2/b^3))/(b*d)
```

3.545 $\int \frac{\sec^4(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{\sec^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{(a^2+b^2)\log(a+b\tan(c+dx))}{b^3d} - \frac{a\tan(c+dx)}{b^2d} + \frac{\tan^2(c+dx)}{2bd}$$

```
[Out] (a^2+b^2)*ln(a+b*tan(d*x+c))/b^3/d-a*tan(d*x+c)/b^2/d+1/2*tan(d*x+c)^2/b/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3587, 711}

$$\int \frac{\sec^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{(a^2+b^2)\log(a+b\tan(c+dx))}{b^3d} - \frac{a\tan(c+dx)}{b^2d} + \frac{\tan^2(c+dx)}{2bd}$$

```
[In] Int[Sec[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```

```
[Out] ((a^2 + b^2)*Log[a + b*Tan[c + d*x]])/(b^3*d) - (a*Tan[c + d*x])/(b^2*d) +
Tan[c + d*x]^2/(2*b*d)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
```

```
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1+\frac{x^2}{b^2}}{a+x}\,dx, x, b\tan(c+dx)\right)}{bd}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{a}{b^2}+\frac{x}{b^2}+\frac{a^2+b^2}{b^2(a+x)}\right)\,dx, x, b\tan(c+dx)\right)}{bd}\\ &= \frac{(a^2+b^2)\log(a+b\tan(c+dx))}{b^3d}-\frac{a\tan(c+dx)}{b^2d}+\frac{\tan^2(c+dx)}{2bd}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int \frac{\sec^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{(a^2+b^2)\log(a+b\tan(c+dx))-ab\tan(c+dx)+\frac{1}{2}b^2\tan^2(c+dx)}{b^3d}$$

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Tan[c + d*x]),x]
[Out] ((a^2 + b^2)*Log[a + b*Tan[c + d*x]] - a*b*Tan[c + d*x] + (b^2*Tan[c + d*x]
^2)/2)/(b^3*d)
```

Maple [A] (verified)

Time = 7.50 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{-\frac{-\frac{b\left(\tan^2(dx+c)\right)}{2}+a\tan(dx+c)}{b^2}+\frac{\left(a^2+b^2\right)\ln(a+b\tan(dx+c))}{b^3}}{d}$
default	$\frac{-\frac{-\frac{b\left(\tan^2(dx+c)\right)}{2}+a\tan(dx+c)}{b^2}+\frac{\left(a^2+b^2\right)\ln(a+b\tan(dx+c))}{b^3}}{d}$
risch	$\frac{-2ia\,e^{2i(dx+c)}+2b\,e^{2i(dx+c)}-2ia}{b^2d\left(e^{2i(dx+c)}+1\right)^2}-\frac{\ln\left(e^{2i(dx+c)}+1\right)a^2}{b^3d}-\frac{\ln\left(e^{2i(dx+c)}+1\right)}{bd}+\frac{\ln\left(e^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)a^2}{b^3d}+\frac{\ln\left(e^{2i(d\right.}}{}$

```
[In] int(sec(d*x+c)^4/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b^2*(-1/2*b*tan(d*x+c)^2+a*tan(d*x+c))+(a^2+b^2)/b^3*ln(a+b*tan(d*x
+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(57) = 114$.

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.98

$$\int \frac{\sec^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{(a^2+b^2)\cos(dx+c)^2\log\left(2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2\right)-(a^2+b^2)\cos(dx}{2\,b^3d\cos(dx+c)^2}$$

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*((a^2 + b^2)*cos(d*x + c)^2*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2
- b^2)*cos(d*x + c)^2 + b^2) - (a^2 + b^2)*cos(d*x + c)^2*log(cos(d*x + c)^
2) - 2*a*b*cos(d*x + c)*sin(d*x + c) + b^2)/(b^3*d*cos(d*x + c)^2)
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\sec^4(c+dx)}{a+b\tan(c+dx)}\,dx$$

```
[In] integrate(sec(d*x+c)**4/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*tan(c + d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.62 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{\sec^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{b\tan(dx+c)^2-2\,a\tan(dx+c)}{b^2}+\frac{2\,(a^2+b^2)\log(b\tan(dx+c)+a)}{b^3}}{2\,d}$$

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*((b*tan(d*x + c)^2 - 2*a*tan(d*x + c))/b^2 + 2*(a^2 + b^2)*log(b*tan(d*
x + c) + a)/b^3)/d
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \frac{\sec^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{b\tan(dx+c)^2-2\,a\tan(dx+c)}{b^2}+\frac{2\,(a^2+b^2)\,\log(|b\tan(dx+c)+a|)}{b^3}}{2\,d}$$

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*((b*tan(d*x + c)^2 - 2*a*tan(d*x + c))/b^2 + 2*(a^2 + b^2)*log(abs(b*ta
n(d*x + c) + a))/b^3)/d
```

Mupad [B] (verification not implemented)

Time = 4.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int \frac{\sec^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\tan(c+d\,x)^2}{2\,b\,d}+\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(a^2+b^2\right)}{b^3\,d}-\frac{a\tan(c+d\,x)}{b^2\,d}$$

```
[In] int(1/(cos(c + d*x)^4*(a + b*tan(c + d*x))),x)
```

```
[Out] tan(c + d*x)^2/(2*b*d) + (log(a + b*tan(c + d*x))*(a^2 + b^2))/(b^3*d) - (a
*tan(c + d*x))/(b^2*d)
```

3.546 $\int \frac{\sec^2(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 18

$$\int \frac{\sec^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\log(a+b\tan(c+dx))}{bd}$$

```
[Out] ln(a+b*tan(d*x+c))/b/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3587, 31}

$$\int \frac{\sec^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\log(a+b\tan(c+dx))}{bd}$$

```
[In] Int[Sec[c + d*x]^2/(a + b*Tan[c + d*x]),x]
```

```
[Out] Log[a + b*Tan[c + d*x]]/(b*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
```

```
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\tan(c+dx)\right)}{bd}\\ &= \frac{\log(a+b\tan(c+dx))}{bd}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\log(a+b\tan(c+dx))}{bd}$$

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Tan[c + d*x]),x]
[Out] Log[a + b*Tan[c + d*x]]/(b*d)
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{\ln(a+b\tan(dx+c))}{bd}$	19
default	$\frac{\ln(a+b\tan(dx+c))}{bd}$	19
risch	$-\frac{\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{bd}+\frac{\ln\left(\mathrm{e}^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)}{bd}$	58

```
[In] int(sec(d*x+c)^2/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] ln(a+b*tan(d*x+c))/b/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(18) = 36$.

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 3.28

$$\int \frac{\sec^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)-\log\left(\cos\left(dx+c\right)^2\right)}{2\,bd}$$

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="fricas")
[Out] 1/2*(log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2
) - log(cos(d*x + c)^2))/(b*d)
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\sec^2\left(c+dx\right)}{a+b\tan\left(c+dx\right)}\,dx$$

```
[In] integrate(sec(d*x+c)**2/(a+b*tan(d*x+c)),x)
[Out] Integral(sec(c + d*x)**2/(a + b*tan(c + d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\log\left(b\tan\left(dx+c\right)+a\right)}{bd}$$

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="maxima")
[Out] log(b*tan(d*x + c) + a)/(b*d)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\sec^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\log\left(|b\tan\left(dx+c\right)+a|\right)}{bd}$$

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="giac")
[Out] log(abs(b*tan(d*x + c) + a))/(b*d)
```

Mupad [B] (verification not implemented)

Time = 3.94 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\ln\left(a+b\tan(c+d\,x)\right)}{b\,d}$$

```
[In] int(1/(cos(c + d*x)^2*(a + b*tan(c + d*x))),x)
[Out] log(a + b*tan(c + d*x))/(b*d)
```

3.547 $\int \frac{\cos^2(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 93

$$\int \frac{\cos^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{a(a^2+3b^2)\,x}{2\,(a^2+b^2)^2} + \frac{b^3\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^2\,d} + \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\,(a^2+b^2)\,d}$$

```
[Out] 1/2*a*(a^2+3*b^2)*x/(a^2+b^2)^2+b^3*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)
^2/d+1/2*cos(d*x+c)^2*(b+a*tan(d*x+c))/(a^2+b^2)/d
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3587, 755, 815, 649, 209, 266}

$$\int \frac{\cos^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\cos^2(c+dx)(a\tan(c+dx)+b)}{2d\,(a^2+b^2)} + \frac{ax(a^2+3b^2)}{2\,(a^2+b^2)^2} + \frac{b^3\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^2}$$

```
[In] Int[Cos[c + d*x]^2/(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*(a^2 + 3*b^2)*x)/(2*(a^2 + b^2)^2) + (b^3*Log[a*Cos[c + d*x] + b*Sin[c +
 d*x]])/((a^2 + b^2)^2*d) + (Cos[c + d*x]^2*(b + a*Tan[c + d*x]))/(2*(a^2 +
 b^2)*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(a+x)\left(1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\,(a^2+b^2)\,d} - \frac{b\text{Subst}\left(\int \frac{-2-\frac{a^2}{b^2}-\frac{ax}{b^2}}{(a+x)\left(1+\frac{x^2}{b^2}\right)}\,dx, x, b\tan(c+dx)\right)}{2\,(a^2+b^2)\,d}$$

$$
\begin{aligned}
&= \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2(a^2+b^2)d} \\
&\quad - \frac{b\text{Subst}\left(\int \left(-\frac{2b^2}{(a^2+b^2)(a+x)}+\frac{-a^3-3ab^2+2b^2x}{(a^2+b^2)(b^2+x^2)}\right)dx, x, b\tan(c+dx)\right)}{2(a^2+b^2)d} \\
&= \frac{b^3\log(a+b\tan(c+dx))}{(a^2+b^2)^2 d} + \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2(a^2+b^2)d} \\
&\quad - \frac{b\text{Subst}\left(\int \frac{-a^3-3ab^2+2b^2x}{b^2+x^2}dx, x, b\tan(c+dx)\right)}{2(a^2+b^2)^2 d} \\
&= \frac{b^3\log(a+b\tan(c+dx))}{(a^2+b^2)^2 d} + \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2(a^2+b^2)d} \\
&\quad - \frac{b^3\text{Subst}\left(\int \frac{x}{b^2+x^2}dx, x, b\tan(c+dx)\right)}{(a^2+b^2)^2 d} \\
&\quad + \frac{(ab(a^2+3b^2))\,\text{Subst}\left(\int \frac{1}{b^2+x^2}dx, x, b\tan(c+dx)\right)}{2(a^2+b^2)^2 d} \\
&= \frac{a(a^2+3b^2)\,x}{2(a^2+b^2)^2} + \frac{b^3\log(\cos(c+dx))}{(a^2+b^2)^2 d} \\
&\quad + \frac{b^3\log(a+b\tan(c+dx))}{(a^2+b^2)^2 d} + \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2(a^2+b^2)d}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.54

$$
\int \frac{\cos^2(c+dx)}{a+b\tan(c+dx)}\,dx
= \frac{2a^3c+6ab^2c+4ib^3c+2a^3dx+6ab^2dx+4ib^3dx-4ib^3\arctan(\tan(c+dx))+b(a^2+b^2)\cos(2(c+dx))+}{4(a^2+b^2)^2 d}
$$

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Tan[c + d*x]),x]
```

```
[Out] (2*a^3*c + 6*a*b^2*c + (4*I)*b^3*c + 2*a^3*d*x + 6*a*b^2*d*x + (4*I)*b^3*d*
x - (4*I)*b^3*ArcTan[Tan[c + d*x]] + b*(a^2 + b^2)*Cos[2*(c + d*x)] + 2*b^3
*Log[(a*Cos[c + d*x] + b*Sin[c + d*x])^2] + a^3*Sin[2*(c + d*x)] + a*b^2*Si
n[2*(c + d*x)])/(4*(a^2 + b^2)^2*d)
```

Maple [A] (verified)

Time = 2.29 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{\frac{\frac{\left(\frac{1}{2}a^3+\frac{1}{2}a\,b^2\right)\tan(dx+c)+\frac{a^2 b}{2}+\frac{b^3}{2}}{1+\tan^2(dx+c)}-\frac{b^3\ln\left(1+\tan^2(dx+c)\right)}{2}+\frac{\left(a^3+3a\,b^2\right)\arctan(\tan(dx+c))}{2}}{\left(a^2+b^2\right)^2}+\frac{b^3\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^2}}{d}$
default	$\frac{\frac{\frac{\left(\frac{1}{2}a^3+\frac{1}{2}a\,b^2\right)\tan(dx+c)+\frac{a^2 b}{2}+\frac{b^3}{2}}{1+\tan^2(dx+c)}-\frac{b^3\ln\left(1+\tan^2(dx+c)\right)}{2}+\frac{\left(a^3+3a\,b^2\right)\arctan(\tan(dx+c))}{2}}{\left(a^2+b^2\right)^2}+\frac{b^3\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^2}}{d}$
risch	$\frac{2ixb}{4iab-2a^2+2b^2}-\frac{xa}{4iab-2a^2+2b^2}-\frac{i\mathrm{e}^{2i(dx+c)}}{8(-ib+a)d}+\frac{i\mathrm{e}^{-2i(dx+c)}}{8(ib+a)d}-\frac{2ib^3x}{a^4+2a^2b^2+b^4}-\frac{2ib^3c}{d(a^4+2a^2b^2+b^4)}+\frac{b^3\ln\left(\mathrm{e}^{\cdots}\right.}{d(a^4\cdots}$

```
[In] int(cos(d*x+c)^2/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a^2+b^2)^2*(((1/2*a^3+1/2*a*b^2)*tan(d*x+c)+1/2*a^2*b+1/2*b^3)/(1+t
an(d*x+c)^2)-1/2*b^3*ln(1+tan(d*x+c)^2)+1/2*(a^3+3*a*b^2)*arctan(tan(d*x+c)
))+b^3/(a^2+b^2)^2*ln(a+b*tan(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.28

$$\int \frac{\cos^2(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$=\frac{b^3\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)+\left(a^3+3\,ab^2\right)dx+\left(a^2b+b^3\right)\cos\left(dx+\cdots\right.}{2\left(a^4+2\,a^2b^2+b^4\right)d}$$

```
[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 +
 b^2) + (a^3 + 3*a*b^2)*d*x + (a^2*b + b^3)*cos(d*x + c)^2 + (a^3 + a*b^2)*
cos(d*x + c)*sin(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d)
```

Sympy [F]

$$\int \frac{\cos^2(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\cos^2(c+dx)}{a+b\tan(c+dx)}\,dx$$

```
[In] integrate(cos(d*x+c)**2/(a+b*tan(d*x+c)),x)
[Out] Integral(cos(c + d*x)**2/(a + b*tan(c + d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.52

$$\int \frac{\cos^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{2\,b^3\log(b\tan(dx+c)+a)}{a^4+2\,a^2b^2+b^4} - \frac{b^3\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4} + \frac{(a^3+3\,ab^2)(dx+c)}{a^4+2\,a^2b^2+b^4} + \frac{a\tan(dx+c)+b}{(a^2+b^2)\tan(dx+c)^2+a^2+b^2}}{2\,d}$$

```
[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="maxima")
[Out] 1/2*(2*b^3*log(b*tan(d*x + c) + a)/(a^4 + 2*a^2*b^2 + b^4) - b^3*log(tan(d*
x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^3 + 3*a*b^2)*(d*x + c)/(a^4 + 2*
a^2*b^2 + b^4) + (a*tan(d*x + c) + b)/((a^2 + b^2)*tan(d*x + c)^2 + a^2 + b
^2))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(89) = 178.

Time = 0.36 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.96

$$\int \frac{\cos^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{2\,b^4\log(|b\tan(dx+c)+a|)}{a^4b+2\,a^2b^3+b^5} - \frac{b^3\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4} + \frac{(a^3+3\,ab^2)(dx+c)}{a^4+2\,a^2b^2+b^4} + \frac{b^3\tan(dx+c)^2+a^3\tan(dx+c)+ab^2\tan(dx+c)+a^2b+2\,b^3}{(a^4+2\,a^2b^2+b^4)\left(\tan(dx+c)^2+1\right)}}{2\,d}$$

```
[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c)),x, algorithm="giac")
[Out] 1/2*(2*b^4*log(abs(b*tan(d*x + c) + a))/(a^4*b + 2*a^2*b^3 + b^5) - b^3*log
(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^3 + 3*a*b^2)*(d*x + c)/(a
^4 + 2*a^2*b^2 + b^4) + (b^3*tan(d*x + c)^2 + a^3*tan(d*x + c) + a*b^2*tan(
d*x + c) + a^2*b + 2*b^3)/((a^4 + 2*a^2*b^2 + b^4)*(tan(d*x + c)^2 + 1)))/d
```

Mupad [B] (verification not implemented)

Time = 4.83 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.68

$$\int \frac{\cos^2(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\cos\left(c+d\,x\right)^2\left(\frac{b}{2\,\left(a^2+b^2\right)}+\frac{a\tan(c+d\,x)}{2\,\left(a^2+b^2\right)}\right)}{d} - \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(2\,b+a\,1\mathrm{i}\right)}{4\,d\,\left(-a^2+a\,b\,2\mathrm{i}+b^2\right)} - \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(a+b\,2\mathrm{i}\right)}{4\,d\,\left(-a^2\,1\mathrm{i}+2\,a\,b+b^2\,1\mathrm{i}\right)} + \frac{b^3\,\ln\left(a+b\tan(c+d\,x)\right)}{d\,\left(a^2+b^2\right)^2}$$

```
[In] int(cos(c + d*x)^2/(a + b*tan(c + d*x)),x)
```

```
[Out] (cos(c + d*x)^2*(b/(2*(a^2 + b^2)) + (a*tan(c + d*x))/(2*(a^2 + b^2))))/d -
 (log(tan(c + d*x) + 1i)*(a*1i + 2*b))/(4*d*(a*b*2i - a^2 + b^2)) - (log(ta
n(c + d*x) - 1i)*(a + b*2i))/(4*d*(2*a*b - a^2*1i + b^2*1i)) + (b^3*log(a +
 b*tan(c + d*x)))/(d*(a^2 + b^2)^2)
```

3.548 $\int \frac{\cos^4(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 152

$$\int \frac{\cos^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{a(3a^4+10a^2b^2+15b^4)\,x}{8\,(a^2+b^2)^3}+\frac{b^5\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^3\,d}+\frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\,(a^2+b^2)\,d}+\frac{\cos^2(c+dx)\,(4b^3+a(3a^2+7b^2)\tan(c+dx))}{8\,(a^2+b^2)^2\,d}$$

```
[Out] 1/8*a*(3*a^4+10*a^2*b^2+15*b^4)*x/(a^2+b^2)^3+b^5*ln(a*cos(d*x+c)+b*sin(d*x
+c))/(a^2+b^2)^3/d+1/4*cos(d*x+c)^4*(b+a*tan(d*x+c))/(a^2+b^2)/d+1/8*cos(d*
x+c)^2*(4*b^3+a*(3*a^2+7*b^2)*tan(d*x+c))/(a^2+b^2)^2/d
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3587, 755, 837, 815, 649, 209, 266}

$$\int \frac{\cos^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\cos^4(c+dx)(a\tan(c+dx)+b)}{4d\,(a^2+b^2)}+\frac{b^5\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^3}+\frac{\cos^2(c+dx)\,(a(3a^2+7b^2)\tan(c+dx)+4b^3)}{8d\,(a^2+b^2)^2}+\frac{ax(3a^4+10a^2b^2+15b^4)}{8\,(a^2+b^2)^3}$$

```
[In] Int[Cos[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```

```
[Out] (a*(3*a^4 + 10*a^2*b^2 + 15*b^4)*x)/(8*(a^2 + b^2)^3) + (b^5*Log[a*Cos[c +
d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^3*d) + (Cos[c + d*x]^4*(b + a*Tan[c +
d*x]))/(4*(a^2 + b^2)*d) + (Cos[c + d*x]^2*(4*b^3 + a*(3*a^2 + 7*b^2)*Tan[c
 + d*x]))/(8*(a^2 + b^2)^2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
```

```
[2*m, 2*p])
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(a+x)\left(1+\frac{x^2}{b^2}\right)^3}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4(a^2+b^2)d} - \frac{b\text{Subst}\left(\int \frac{-4-\frac{3a^2}{b^2}-\frac{3ax}{b^2}}{(a+x)\left(1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\tan(c+dx)\right)}{4(a^2+b^2)d}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4(a^2+b^2)d} + \frac{\cos^2(c+dx)\left(4b^3+a(3a^2+7b^2)\tan(c+dx)\right)}{8(a^2+b^2)^2 d} + \frac{b^5\text{Subst}\left(\int \frac{\frac{3a^4+7a^2b^2+8b^4}{b^6}+\frac{a\left(3a^2+7b^2\right)x}{b^6}}{(a+x)\left(1+\frac{x^2}{b^2}\right)}\,dx, x, b\tan(c+dx)\right)}{8(a^2+b^2)^2 d}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4(a^2+b^2)d} + \frac{\cos^2(c+dx)\left(4b^3+a(3a^2+7b^2)\tan(c+dx)\right)}{8(a^2+b^2)^2 d} + \frac{b^5\text{Subst}\left(\int \left(\frac{8}{(a^2+b^2)(a+x)}+\frac{3a^5+10a^3b^2+15ab^4-8b^4x}{b^4(a^2+b^2)(b^2+x^2)}\right)dx, x, b\tan(c+dx)\right)}{8(a^2+b^2)^2 d}$$

$$= \frac{b^5\log(a+b\tan(c+dx))}{(a^2+b^2)^3 d} + \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4(a^2+b^2)d} + \frac{\cos^2(c+dx)\left(4b^3+a(3a^2+7b^2)\tan(c+dx)\right)}{8(a^2+b^2)^2 d} + \frac{b\text{Subst}\left(\int \frac{3a^5+10a^3b^2+15ab^4-8b^4x}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{8(a^2+b^2)^3 d}$$

$$= \frac{b^5 \log(a+b\tan(c+dx))}{(a^2+b^2)^3 d} + \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4(a^2+b^2) d} + \frac{\cos^2(c+dx)\left(4b^3+a(3a^2+7b^2)\tan(c+dx)\right)}{8(a^2+b^2)^2 d} - \frac{b^5 \text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{(a^2+b^2)^3 d} + \frac{(ab(3a^4+10a^2b^2+15b^4))\,\text{Subst}\left(\int \frac{1}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{8(a^2+b^2)^3 d}$$

$$= \frac{a(3a^4+10a^2b^2+15b^4)\,x}{8(a^2+b^2)^3} + \frac{b^5 \log(\cos(c+dx))}{(a^2+b^2)^3 d} + \frac{b^5 \log(a+b\tan(c+dx))}{(a^2+b^2)^3 d} + \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4(a^2+b^2) d} + \frac{\cos^2(c+dx)\left(4b^3+a(3a^2+7b^2)\tan(c+dx)\right)}{8(a^2+b^2)^2 d}$$

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.48

$$\int \frac{\cos^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{-\left(\left(8b^6+\sqrt{-b^2}(3a^5+10a^3b^2+15ab^4)\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)\right)+16b^6\log(a+b\tan(c+dx))-(8b$$

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Tan[c + d*x]),x]
```

```
[Out] (-((8*b^6 + Sqrt[-b^2]*(3*a^5 + 10*a^3*b^2 + 15*a*b^4))*Log[Sqrt[-b^2] - b*
Tan[c + d*x]]) + 16*b^6*Log[a + b*Tan[c + d*x]] - (8*b^6 - Sqrt[-b^2]*(3*a^
5 + 10*a^3*b^2 + 15*a*b^4))*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + 4*b*(a^2 + b
^2)^2*Cos[c + d*x]^4*(b + a*Tan[c + d*x]) + 2*(a^2 + b^2)*Cos[c + d*x]^2*(4
*b^4 + a*b*(3*a^2 + 7*b^2)*Tan[c + d*x]))/(16*b*(a^2 + b^2)^3*d)
```

Maple [A] (verified)

Time = 8.19 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.30

method	result
derivativedivides	$\frac{\frac{\left(\frac{3}{8}a^5+\frac{5}{4}a^3b^2+\frac{7}{8}ab^4\right)\left(\tan^3(dx+c)\right)+\left(\frac{1}{2}a^2b^3+\frac{1}{2}b^5\right)\left(\tan^2(dx+c)\right)+\left(\frac{7}{4}a^3b^2+\frac{9}{8}ab^4+\frac{5}{8}a^5\right)\tan(dx+c)+\frac{a^4b}{4}+a^2b^3+\frac{3b^5}{4}}{\left(1+\tan^2(dx+c)\right)^2}-\frac{b^5\ln\left(1+\mathrm{t}\right.}{}}{\left(a^2+b^2\right)^3}}{d}$
default	$\frac{\frac{\left(\frac{3}{8}a^5+\frac{5}{4}a^3b^2+\frac{7}{8}ab^4\right)\left(\tan^3(dx+c)\right)+\left(\frac{1}{2}a^2b^3+\frac{1}{2}b^5\right)\left(\tan^2(dx+c)\right)+\left(\frac{7}{4}a^3b^2+\frac{9}{8}ab^4+\frac{5}{8}a^5\right)\tan(dx+c)+\frac{a^4b}{4}+a^2b^3+\frac{3b^5}{4}}{\left(1+\tan^2(dx+c)\right)^2}-\frac{b^5\ln\left(1+\mathrm{t}\right.}{}}{\left(a^2+b^2\right)^3}}{d}$
risch	$\frac{9xab}{8ia^3-24iab^2+24a^2b-8b^3}+\frac{3ixa^2}{8ia^3-24iab^2+24a^2b-8b^3}-\frac{8ixb^2}{8ia^3-24iab^2+24a^2b-8b^3}-\frac{3\,\mathrm{e}^{2i(dx+c)}b}{16(-2iab+a^2-b^2)d}-\frac{i}{8(-2}$

[In] int(cos(d*x+c)^4/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)

```
[Out] 1/d*(1/(a^2+b^2)^3*(((3/8*a^5+5/4*a^3*b^2+7/8*a*b^4)*tan(d*x+c)^3+(1/2*a^2*
b^3+1/2*b^5)*tan(d*x+c)^2+(7/4*a^3*b^2+9/8*a*b^4+5/8*a^5)*tan(d*x+c)+1/4*a^
4*b+a^2*b^3+3/4*b^5)/(1+tan(d*x+c)^2)^2-1/2*b^5*ln(1+tan(d*x+c)^2)+1/8*(3*a
^5+10*a^3*b^2+15*a*b^4)*arctan(tan(d*x+c)))+b^5/(a^2+b^2)^3*ln(a+b*tan(d*x+
c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.37

$$\int \frac{\cos^4(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$=\frac{4\,b^5\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)+2\left(a^4b+2\,a^2b^3+b^5\right)\cos\left(dx+c\right)^4+}{}$$

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*b^5*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
 + b^2) + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^4 + (3*a^5 + 10*a^3*b^2
+ 15*a*b^4)*d*x + 4*(a^2*b^3 + b^5)*cos(d*x + c)^2 + (2*(a^5 + 2*a^3*b^2 +
a*b^4)*cos(d*x + c)^3 + (3*a^5 + 10*a^3*b^2 + 7*a*b^4)*cos(d*x + c))*sin(d*
x + c))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d)
```

Sympy [F]

$$\int \frac{\cos^4(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\cos^4(c+dx)}{a+b\tan(c+dx)}\,dx$$

```
[In] integrate(cos(d*x+c)**4/(a+b*tan(d*x+c)),x)
[Out] Integral(cos(c + d*x)**4/(a + b*tan(c + d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.78

$$\int \frac{\cos^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{8\,b^5\log(b\tan(dx+c)+a)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{4\,b^5\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{(3\,a^5+10\,a^3b^2+15\,ab^4)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{4\,b^3\tan(dx+c)^2+(3\,a^3+7\,ab^2)\tan(dx+c)^3+2\,a^2b+}{(a^4+2\,a^2b^2+b^4)\tan(dx+c)^4+a^4+2\,a^2b^2+b^4+2\,(}}{8\,d}$$

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="maxima")
[Out] 1/8*(8*b^5*log(b*tan(d*x + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 4*
b^5*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (3*a^5 +
10*a^3*b^2 + 15*a*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (4*b
^3*tan(d*x + c)^2 + (3*a^3 + 7*a*b^2)*tan(d*x + c)^3 + 2*a^2*b + 6*b^3 + (5
*a^3 + 9*a*b^2)*tan(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*tan(d*x + c)^4 + a^4
 + 2*a^2*b^2 + b^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*tan(d*x + c)^2))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(146) = 292$.

Time = 0.40 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.12

$$\int \frac{\cos^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{8\,b^6\log(|b\tan(dx+c)+a|)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7} - \frac{4\,b^5\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{(3\,a^5+10\,a^3b^2+15\,ab^4)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{6\,b^5\tan(dx+c)^4+3\,a^5\tan(dx+c)^3+10\,a^3b^2\tan(dx}{}}{8\,d}$$

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c)),x, algorithm="giac")
[Out] 1/8*(8*b^6*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^
7) - 4*b^5*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (3
```

```
*a^5 + 10*a^3*b^2 + 15*a*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
 + (6*b^5*tan(d*x + c)^4 + 3*a^5*tan(d*x + c)^3 + 10*a^3*b^2*tan(d*x + c)^3
 + 7*a*b^4*tan(d*x + c)^3 + 4*a^2*b^3*tan(d*x + c)^2 + 16*b^5*tan(d*x + c)^
2 + 5*a^5*tan(d*x + c) + 14*a^3*b^2*tan(d*x + c) + 9*a*b^4*tan(d*x + c) + 2
*a^4*b + 8*a^2*b^3 + 12*b^5)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(tan(d*x
+ c)^2 + 1)^2))/d
```

Mupad [B] (verification not implemented)

Time = 4.57 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.09

$$\int \frac{\cos^4(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{a^2\,b+3\,b^3}{4\,(a^4+2\,a^2\,b^2+b^4)}+\frac{b^3\,\tan(c+d\,x)^2}{2\,(a^4+2\,a^2\,b^2+b^4)}+\frac{\tan(c+d\,x)^3\,(3\,a^3+7\,a\,b^2)}{8\,(a^4+2\,a^2\,b^2+b^4)}+\frac{\tan(c+d\,x)\,(5\,a^3+9\,a\,b^2)}{8\,(a^4+2\,a^2\,b^2+b^4)}}{d\,\left(\tan\left(c+d\,x\right)^4+2\tan\left(c+d\,x\right)^2+1\right)} - \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-a^2\,3\mathrm{i}+9\,a\,b+b^2\,8\mathrm{i}\right)}{16\,d\,\left(-a^3-a^2\,b\,3\mathrm{i}+3\,a\,b^2+b^3\,1\mathrm{i}\right)} + \frac{b^5\,\ln\left(a+b\tan(c+d\,x)\right)}{d\,(a^2+b^2)^3} - \frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(-3\,a^2+a\,b\,9\mathrm{i}+8\,b^2\right)}{16\,d\,\left(-a^3\,1\mathrm{i}-3\,a^2\,b+a\,b^2\,3\mathrm{i}+b^3\right)}$$

```
[In] int(cos(c + d*x)^4/(a + b*tan(c + d*x)),x)
```

```
[Out] ((a^2*b + 3*b^3)/(4*(a^4 + b^4 + 2*a^2*b^2)) + (b^3*tan(c + d*x)^2)/(2*(a^4
 + b^4 + 2*a^2*b^2)) + (tan(c + d*x)^3*(7*a*b^2 + 3*a^3))/(8*(a^4 + b^4 + 2
*a^2*b^2)) + (tan(c + d*x)*(9*a*b^2 + 5*a^3))/(8*(a^4 + b^4 + 2*a^2*b^2)))/
(d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 + 1)) - (log(tan(c + d*x) - 1i)*(9*a*
b - a^2*3i + b^2*8i))/(16*d*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)) + (b^5*log
(a + b*tan(c + d*x)))/(d*(a^2 + b^2)^3) - (log(tan(c + d*x) + 1i)*(a*b*9i -
 3*a^2 + 8*b^2))/(16*d*(a*b^2*3i - 3*a^2*b - a^3*1i + b^3))
```

3.549 $\int \frac{\sec^5(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 140

$$\int \frac{\sec^5(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{a(2a^2+3b^2)\operatorname{arctanh}(\sin(c+dx))}{2b^4d} - \frac{(a^2+b^2)^{3/2}\operatorname{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{b^4d} + \frac{(a^2+b^2)\sec(c+dx)}{b^3d} + \frac{\sec^3(c+dx)}{3bd} - \frac{a\sec(c+dx)\tan(c+dx)}{2b^2d}$$

```
[Out] -1/2*a*(2*a^2+3*b^2)*arctanh(sin(d*x+c))/b^4/d-(a^2+b^2)^(3/2)*arctanh(cos(
d*x+c)*(b-a*tan(d*x+c))/(a^2+b^2)^(1/2))/b^4/d+(a^2+b^2)*sec(d*x+c)/b^3/d+1
/3*sec(d*x+c)^3/b/d-1/2*a*sec(d*x+c)*tan(d*x+c)/b^2/d
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3591, 3567, 3853, 3855, 3590, 212}

$$\int \frac{\sec^5(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{a(a^2+b^2)\operatorname{arctanh}(\sin(c+dx))}{b^4d} - \frac{(a^2+b^2)^{3/2}\operatorname{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{b^4d} + \frac{(a^2+b^2)\sec(c+dx)}{b^3d} - \frac{a\operatorname{arctanh}(\sin(c+dx))}{2b^2d} - \frac{a\tan(c+dx)\sec(c+dx)}{2b^2d} + \frac{\sec^3(c+dx)}{3bd}$$

```
[In] Int[Sec[c + d*x]^5/(a + b*Tan[c + d*x]),x]
```

```
[Out] -1/2*(a*ArcTanh[Sin[c + d*x]])/(b^2*d) - (a*(a^2 + b^2)*ArcTanh[Sin[c + d*x
]])/(b^4*d) - ((a^2 + b^2)^(3/2)*ArcTanh[(Cos[c + d*x]*(b - a*Tan[c + d*x])
)/Sqrt[a^2 + b^2]])/(b^4*d) + ((a^2 + b^2)*Sec[c + d*x])/(b^3*d) + Sec[c +
d*x]^3/(3*b*d) - (a*Sec[c + d*x]*Tan[c + d*x])/(2*b^2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3590

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> Dist[-f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3591

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[-d^2/b^2, Int[(d*Sec[e + f*x])^(m - 2)*(a - b*Tan[e
 + f*x]), x], x] + Dist[d^2*((a^2 + b^2)/b^2), Int[(d*Sec[e + f*x])^(m - 2)
/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 + b^2
, 0] && IGtQ[m, 1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = -\frac{\int \sec^3(c+dx)(a-b\tan(c+dx))\,dx}{b^2} + \frac{(a^2+b^2)\int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}\,dx}{b^2}$$
$$= \frac{\sec^3(c+dx)}{3bd} - \frac{a\int \sec^3(c+dx)\,dx}{b^2} - \frac{(a^2+b^2)\int \sec(c+dx)(a-b\tan(c+dx))\,dx}{b^4} + \frac{(a^2+b^2)^2\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx}{b^4}$$
$$= \frac{(a^2+b^2)\sec(c+dx)}{b^3d} + \frac{\sec^3(c+dx)}{3bd} - \frac{a\sec(c+dx)\tan(c+dx)}{2b^2d} - \frac{a\int \sec(c+dx)\,dx}{2b^2} - \frac{(a(a^2+b^2))\int \sec(c+dx)\,dx}{b^4} - \frac{(a^2+b^2)^2\,\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx, x, \cos(c+dx)(b-a\tan(c+dx))\right)}{b^4d}$$
$$= -\frac{a\text{arctanh}(\sin(c+dx))}{2b^2d} - \frac{a(a^2+b^2)\,\text{arctanh}(\sin(c+dx))}{b^4d} - \frac{(a^2+b^2)^{3/2}\,\text{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{b^4d} + \frac{(a^2+b^2)\sec(c+dx)}{b^3d} + \frac{\sec^3(c+dx)}{3bd} - \frac{a\sec(c+dx)\tan(c+dx)}{2b^2d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 321 vs. $2(140) = 280$.

Time = 2.59 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.29

$$\int \frac{\sec^5(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{48(a^2+b^2)^{3/2}\,\text{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right) + \sec^3(c+dx)\left(12a^2b+20b^3+12b(a^2+b^2)\cos(2(c+dx))+6a\right.}{}$$

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Tan[c + d*x]),x]
```

```
[Out] (48*(a^2 + b^2)^(3/2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] +
Sec[c + d*x]^3*(12*a^2*b + 20*b^3 + 12*b*(a^2 + b^2)*Cos[2*(c + d*x)] + 6*a
^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 9*a*b^2*Cos[
3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 9*a*(2*a^2 + 3*b^2)
*Cos[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)
/2] + Sin[(c + d*x)/2]]) - 6*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]] - 9*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] - 6*a*b^2*Sin[2*(c + d*x)]))/(24*b^4*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(130) = 260$.

Time = 16.29 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.92

method	result
derivativedivides	$\frac{-\frac{1}{3b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{a+b}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{2a^2+ab+3b^2}{2b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\left(2a^2+3b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2b^4}+\frac{1}{3b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}}{}$
default	$\frac{-\frac{1}{3b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{a+b}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{2a^2+ab+3b^2}{2b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\left(2a^2+3b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2b^4}+\frac{1}{3b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}}{}$
risch	$\frac{e^{i(dx+c)}\left(3iab\,e^{4i(dx+c)}+6a^2e^{4i(dx+c)}+6b^2e^{4i(dx+c)}+12a^2e^{2i(dx+c)}+20b^2e^{2i(dx+c)}-3iab+6a^2+6b^2\right)}{3d\,b^3\left(e^{2i(dx+c)}+1\right)^3}+\frac{a^3\ln\left(e^{i(dx+c)}-\right.}{d\,b^4}$

```
[In] int(sec(d*x+c)^5/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3/b/(tan(1/2*d*x+1/2*c)-1)^3-1/2*(a+b)/b^2/(tan(1/2*d*x+1/2*c)-1)^2
-1/2*(2*a^2+a*b+3*b^2)/b^3/(tan(1/2*d*x+1/2*c)-1)+1/2*a*(2*a^2+3*b^2)/b^4*l
n(tan(1/2*d*x+1/2*c)-1)+1/3/b/(tan(1/2*d*x+1/2*c)+1)^3-1/2*(-a+b)/b^2/(tan(
1/2*d*x+1/2*c)+1)^2-1/2*(-2*a^2+a*b-3*b^2)/b^3/(tan(1/2*d*x+1/2*c)+1)-1/2*a
*(2*a^2+3*b^2)/b^4*ln(tan(1/2*d*x+1/2*c)+1)-2/b^4*(-a^4-2*a^2*b^2-b^4)/(a^2
+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.85

$$\int \frac{\sec^5(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{6\left(a^2+b^2\right)^{\frac{3}{2}}\cos\left(dx+c\right)^3\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}\left(b\cos(dx+c)-a\sin(dx+c)\right)}{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2+b^2}\right)-}{}$$

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(6*(a^2 + b^2)^(3/2)*cos(d*x + c)^3*log(-(2*a*b*cos(d*x + c)*sin(d*x +
 c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d
*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*c
os(d*x + c)^2 + b^2)) - 3*(2*a^3 + 3*a*b^2)*cos(d*x + c)^3*log(sin(d*x + c)
 + 1) + 3*(2*a^3 + 3*a*b^2)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) - 6*a*b^2
*cos(d*x + c)*sin(d*x + c) + 4*b^3 + 12*(a^2*b + b^3)*cos(d*x + c)^2)/(b^4*
d*cos(d*x + c)^3)
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\sec^5{(c+dx)}}{a+b\tan{(c+dx)}}\,dx$$

```
[In] integrate(sec(d*x+c)**5/(a+b*tan(d*x+c)),x)
[Out] Integral(sec(c + d*x)**5/(a + b*tan(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(132) = 264$.

Time = 0.30 (sec) , antiderivative size = 361, normalized size of antiderivative = 2.58

$$\int \frac{\sec^5(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{\frac{2\left(6a^2+8b^2-\frac{3ab\sin(dx+c)}{\cos(dx+c)+1}+\frac{3ab\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{12(a^2+b^2)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6(a^2+2b^2)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)}{b^3-\frac{3b^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3b^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{b^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}} - \frac{3(2a^3+3ab^2)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^4} + \frac{3(2a\cdots}{\cdots}}{6d}$$

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c)),x, algorithm="maxima")
[Out] 1/6*(2*(6*a^2 + 8*b^2 - 3*a*b*sin(d*x + c)/(cos(d*x + c) + 1) + 3*a*b*sin(d
*x + c)^5/(cos(d*x + c) + 1)^5 - 12*(a^2 + b^2)*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 6*(a^2 + 2*b^2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/(b^3 - 3*b^
3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*b^3*sin(d*x + c)^4/(cos(d*x + c)
+ 1)^4 - b^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) - 3*(2*a^3 + 3*a*b^2)*log
(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b^4 + 3*(2*a^3 + 3*a*b^2)*log(sin(d*x
 + c)/(cos(d*x + c) + 1) - 1)/b^4 - 6*(a^4 + 2*a^2*b^2 + b^4)*log((b - a*si
n(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d
*x + c) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(132) = 264$.

Time = 0.41 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.99

$$\int \frac{\sec^5(c+dx)}{a+b\tan(c+dx)}\,dx =$$
$$-\frac{\frac{3(2a^3+3ab^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^4} - \frac{3(2a^3+3ab^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^4} + \frac{6(a^4+2a^2b^2+b^4)\log\left(\frac{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b-2\sqrt{a^2\cdots}\right|}{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b+2\sqrt{a^2\cdots}\right|}\cdots\right)}{\sqrt{a^2+b^2}\,b^4}\cdots}{\cdots}$$

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(3*(2*a^3 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 - 3*(2*a^3
 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 + 6*(a^4 + 2*a^2*b^2 + b
^4)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan
(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4) + 2*(3*
a*b*tan(1/2*d*x + 1/2*c)^5 + 6*a^2*tan(1/2*d*x + 1/2*c)^4 + 12*b^2*tan(1/2*
d*x + 1/2*c)^4 - 12*a^2*tan(1/2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2*d*x + 1/2*c
)^2 - 3*a*b*tan(1/2*d*x + 1/2*c) + 6*a^2 + 8*b^2)/((tan(1/2*d*x + 1/2*c)^2
- 1)^3*b^3))/d
```

Mupad [B] (verification not implemented)

Time = 6.54 (sec) , antiderivative size = 724, normalized size of antiderivative = 5.17

$$\int \frac{\sec^5(c+dx)}{a+b\tan(c+dx)}\,dx$$

$$= \frac{b^3\left(\cos\left(c+d\,x\right)+\frac{\cos(2\,c+2\,d\,x)}{2}+\frac{\cos(3\,c+3\,d\,x)}{3}+\frac{5}{6}\right)-b^2\left(\frac{a\,\sin(2\,c+2\,d\,x)}{4}+\frac{3\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\cos(3\,c+3\,d\,x)}{4}+\frac{9\,a\,c}{}\right.}{}$$

```
[In] int(1/(cos(c + d*x)^5*(a + b*tan(c + d*x))),x)
```

```
[Out] (b^3*(cos(c + d*x) + cos(2*c + 2*d*x)/2 + cos(3*c + 3*d*x)/3 + 5/6) - b^2*(
(a*sin(2*c + 2*d*x))/4 + (3*a*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*
cos(3*c + 3*d*x))/4 + (9*a*cos(c + d*x)*atanh(sin(c/2 + (d*x)/2)/cos(c/2 +
(d*x)/2)))/4) + b*((3*a^2*cos(c + d*x))/4 + a^2/2 + (a^2*cos(2*c + 2*d*x))/
2 + (a^2*cos(3*c + 3*d*x))/4) + (atanh((a^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 +
 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 2*b^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2
*b^4 + 3*a^4*b^2)^(1/2) + a*b*cos(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3
*a^4*b^2)^(1/2))/(a^5*cos(c/2 + (d*x)/2) + 2*b^5*sin(c/2 + (d*x)/2) + a*b^4
*cos(c/2 + (d*x)/2) + 2*a^4*b*sin(c/2 + (d*x)/2) + 2*a^3*b^2*cos(c/2 + (d*x
)/2) + 4*a^2*b^3*sin(c/2 + (d*x)/2)))*cos(3*c + 3*d*x)*((a^2 + b^2)^3)^(1/2
))/2 - (3*a^3*cos(c + d*x)*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2
- (a^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/2 + (
3*cos(c + d*x)*atanh((a^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4
*b^2)^(1/2) + 2*b^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^
(1/2) + a*b*cos(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(
a^5*cos(c/2 + (d*x)/2) + 2*b^5*sin(c/2 + (d*x)/2) + a*b^4*cos(c/2 + (d*x)/2
) + 2*a^4*b*sin(c/2 + (d*x)/2) + 2*a^3*b^2*cos(c/2 + (d*x)/2) + 4*a^2*b^3*s
in(c/2 + (d*x)/2)))*((a^2 + b^2)^3)^(1/2))/2)/(b^4*d*((3*cos(c + d*x))/4 +
cos(3*c + 3*d*x)/4))
```

3.550 $\int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 79

$$\int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{a\text{arctanh}(\sin(c+dx))}{b^2d} - \frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{b^2d} + \frac{\sec(c+dx)}{bd}$$

```
[Out] -a*arctanh(sin(d*x+c))/b^2/d+sec(d*x+c)/b/d-arctanh(cos(d*x+c)*(b-a*tan(d*x
+c))/(a^2+b^2)^(1/2))*(a^2+b^2)^(1/2)/b^2/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3591, 3567, 3855, 3590, 212}

$$\int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{b^2d} - \frac{a\text{arctanh}(\sin(c+dx))}{b^2d} + \frac{\sec(c+dx)}{bd}$$

```
[In] Int[Sec[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

```
[Out] -((a*ArcTanh[Sin[c + d*x]])/(b^2*d)) - (Sqrt[a^2 + b^2]*ArcTanh[(Cos[c + d*
x]*(b - a*Tan[c + d*x]))/Sqrt[a^2 + b^2]])/(b^2*d) + Sec[c + d*x]/(b*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3590

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> Dist[-f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3591

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[-d^2/b^2, Int[(d*Sec[e + f*x])^(m - 2)*(a - b*Tan[e
 + f*x]), x], x] + Dist[d^2*((a^2 + b^2)/b^2), Int[(d*Sec[e + f*x])^(m - 2)
/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 + b^2
, 0] && IGtQ[m, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\int \sec(c+dx)(a-b\tan(c+dx))\,dx}{b^2}+\frac{(a^2+b^2)\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx}{b^2}\\
&= \frac{\sec(c+dx)}{bd}-\frac{a\int \sec(c+dx)\,dx}{b^2}\\
&\quad -\frac{(a^2+b^2)\,\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx,x,\cos(c+dx)(b-a\tan(c+dx))\right)}{b^2d}\\
&= -\frac{a\text{arctanh}(\sin(c+dx))}{b^2d}-\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{b^2d}+\frac{\sec(c+dx)}{bd}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.38

$$\int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{2\sqrt{a^2+b^2}\text{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)+a\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right.\right.}{b^2d}$$

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

```
[Out] (2*Sqrt[a^2 + b^2]*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + a*(
Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]]) + b*Sec[c + d*x])/(b^2*d)
```

Maple [A] (verified)

Time = 4.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.63

method	result
derivativedivides	$\frac{-\frac{2\left(-a^2-b^2\right)\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{1}{b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}+\frac{1}{b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{b^2}}{d}$
default	$\frac{-\frac{2\left(-a^2-b^2\right)\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{1}{b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}+\frac{1}{b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{b^2}}{d}$
risch	$\frac{2\,\mathrm{e}^{i(dx+c)}}{db\left(\mathrm{e}^{2i(dx+c)}+1\right)}+\frac{a\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{d\,b^2}-\frac{a\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d\,b^2}+\frac{\sqrt{a^2+b^2}\,\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{d\,b^2}-\frac{\sqrt{a^2+b^2}\,\ln\left(\mathrm{e}^{i(dx\right.}}{d\,b}$

```
[In] int(sec(d*x+c)^3/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/b^2*(-a^2-b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-
2*b)/(a^2+b^2)^(1/2))-1/b/(tan(1/2*d*x+1/2*c)-1)+a/b^2*ln(tan(1/2*d*x+1/2*c
)-1)+1/b/(tan(1/2*d*x+1/2*c)+1)-a/b^2*ln(tan(1/2*d*x+1/2*c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(77) = 154$.

Time = 0.27 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.42

$$\int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{a\cos(dx+c)\log(\sin(dx+c)+1) - a\cos(dx+c)\log(-\sin(dx+c)+1) - \sqrt{a^2+b^2}\cos(dx+c)\log(}{2\,b^2 d\cos(dx+c)}$$

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(a*cos(d*x + c)*log(sin(d*x + c) + 1) - a*cos(d*x + c)*log(-sin(d*x +
c) + 1) - sqrt(a^2 + b^2)*cos(d*x + c)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c
) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x
 + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos
(d*x + c)^2 + b^2)) - 2*b)/(b^2*d*cos(d*x + c))
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}\,dx$$

```
[In] integrate(sec(d*x+c)**3/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*tan(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(77) = 154$.

Time = 0.47 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.06

$$\int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{\frac{a\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^2} - \frac{a\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b^2} + \frac{\sqrt{a^2+b^2}\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{b^2} - \frac{2}{b-\frac{b\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}}{d}$$

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(a*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b^2 - a*log(sin(d*x + c)/(cos(
d*x + c) + 1) - 1)/b^2 + sqrt(a^2 + b^2)*log((b - a*sin(d*x + c)/(cos(d*x +
 c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a
^2 + b^2)))/b^2 - 2/(b - b*sin(d*x + c)^2/(cos(d*x + c) + 1)^2))/d
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.72

$$\int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^2} - \frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^2} + \frac{\sqrt{a^2+b^2}\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{b^2} + \frac{2}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)b}}{d}$$

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -(a*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^2 - a*log(abs(tan(1/2*d*x + 1/2*c)
 - 1))/b^2 + sqrt(a^2 + b^2)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqr
t(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/b^2
+ 2/((tan(1/2*d*x + 1/2*c)^2 - 1)*b))/d
```

Mupad [B] (verification not implemented)

Time = 4.31 (sec) , antiderivative size = 310, normalized size of antiderivative = 3.92

$$\int \frac{\sec^3(c+dx)}{a+b\tan(c+dx)}\,dx$$

$$= \frac{2\,\mathrm{atanh}\left(\frac{64\,a^2\,\sqrt{a^2+b^2}}{64\,a^2\,b+\frac{64\,a^4}{b}+128\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+128\,a\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)} + \frac{128\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a^2+b^2}}{64\,a^2+\frac{64\,a^4}{b^2}+\frac{128\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{b}+128\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)} + \frac{\cdots}{64\,a^4+128\cdots}\right.}{b^2\,d}$$

$$-\frac{2\,a\,\mathrm{atanh}\left(\frac{64\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a^2+\frac{64\,a^4}{b^2}} + \frac{64\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a^4+64\,a^2\,b^2}\right)}{b^2\,d} - \frac{2}{b\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

```
[In] int(1/(cos(c + d*x)^3*(a + b*tan(c + d*x))),x)
```

```
[Out] (2*atanh((64*a^2*(a^2 + b^2)^(1/2))/(64*a^2*b + (64*a^4)/b + 128*a^3*tan(c/
2 + (d*x)/2) + 128*a*b^2*tan(c/2 + (d*x)/2)) + (128*a*tan(c/2 + (d*x)/2)*(a
^2 + b^2)^(1/2))/(64*a^2 + (64*a^4)/b^2 + (128*a^3*tan(c/2 + (d*x)/2))/b +
128*a*b*tan(c/2 + (d*x)/2)) + (64*a^3*tan(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2))
/(64*a^4 + 64*a^2*b^2 + 128*a*b^3*tan(c/2 + (d*x)/2) + 128*a^3*b*tan(c/2 +
(d*x)/2)))*(a^2 + b^2)^(1/2))/(b^2*d) - (2*a*atanh((64*a^2*tan(c/2 + (d*x)/
2))/(64*a^2 + (64*a^4)/b^2) + (64*a^4*tan(c/2 + (d*x)/2))/(64*a^4 + 64*a^2*
b^2)))/(b^2*d) - 2/(b*d*(tan(c/2 + (d*x)/2)^2 - 1))
```

3.551 $\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 46

$$\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}$$

```
[Out] -arctanh(cos(d*x+c)*(b-a*tan(d*x+c))/(a^2+b^2)^(1/2))/d/(a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3590, 212}

$$\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$$

```
[In] Int[Sec[c + d*x]/(a + b*Tan[c + d*x]),x]
```

```
[Out] -(ArcTanh[(Cos[c + d*x]*(b - a*Tan[c + d*x]))/Sqrt[a^2 + b^2]]/(Sqrt[a^2 + b^2]*d))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3590

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> Dist[-f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
```

```
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx, x, \cos(c+dx)(b-a\tan(c+dx))\right)}{d}$$
$$= -\frac{\text{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}d}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{2\text{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}d}$$

```
[In] Integrate[Sec[c + d*x]/(a + b*Tan[c + d*x]),x]
[Out] (2*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(Sqrt[a^2 + b^2]*d)
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{2\ \text{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$	43
default	$\frac{2\ \text{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$	43
risch	$\frac{\ln\left(e^{i(dx+c)}+\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}-\frac{\ln\left(e^{i(dx+c)}-\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}$	88

```
[In] int(sec(d*x+c)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 2/d/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(44) = 88$.

Time = 0.25 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.85

$$\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)}{2\,\sqrt{a^2+b^2}d}$$

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*
a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos
(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2))/(sqrt(a^2 + b^2
)*d)
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx$$

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*tan(c + d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.51 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.74

$$\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}d}$$

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d
*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*d)
```

Giac [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.61

$$\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}d}$$

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/
2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*d)
```

Mupad [B] (verification not implemented)

Time = 4.57 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{b-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{d\,\sqrt{a^2+b^2}}$$

```
[In] int(1/(cos(c + d*x)*(a + b*tan(c + d*x))),x)
```

```
[Out] -(2*atanh((b - a*tan(c/2 + (d*x)/2))/(a^2 + b^2)^(1/2)))/(d*(a^2 + b^2)^(1/
2))
```

3.552 $\int \frac{\cos(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 90

$$\int \frac{\cos(c+dx)}{a+b\tan(c+dx)}\,dx=-\frac{b^2\text{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d}+\frac{b\cos(c+dx)}{(a^2+b^2)\,d}+\frac{a\sin(c+dx)}{(a^2+b^2)\,d}$$

```
[Out] -b^2*arctanh(cos(d*x+c)*(b-a*tan(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/d
+b*cos(d*x+c)/(a^2+b^2)/d+a*sin(d*x+c)/(a^2+b^2)/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3592, 3567, 2717, 3590, 212}

$$\int \frac{\cos(c+dx)}{a+b\tan(c+dx)}\,dx=-\frac{b^2\text{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{d\,(a^2+b^2)^{3/2}}+\frac{a\sin(c+dx)}{d\,(a^2+b^2)}+\frac{b\cos(c+dx)}{d\,(a^2+b^2)}$$

```
[In] Int[Cos[c + d*x]/(a + b*Tan[c + d*x]),x]
```

```
[Out] -((b^2*ArcTanh[(Cos[c + d*x]*(b - a*Tan[c + d*x]))/Sqrt[a^2 + b^2]])/((a^2
+ b^2)^(3/2)*d)) + (b*Cos[c + d*x])/((a^2 + b^2)*d) + (a*Sin[c + d*x])/((a^
2 + b^2)*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3590

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> Dist[-f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3592

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(d*Sec[e + f*x])^m*(a - b*Tan[e
+ f*x]), x], x] + Dist[b^2/(d^2*(a^2 + b^2)), Int[(d*Sec[e + f*x])^(m + 2)/
(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 + b^2,
 0] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \cos(c+dx)(a-b\tan(c+dx))\,dx}{a^2+b^2}+\frac{b^2\int \frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx}{a^2+b^2}\\
&= \frac{b\cos(c+dx)}{(a^2+b^2)\,d}+\frac{a\int \cos(c+dx)\,dx}{a^2+b^2}\\
&\quad -\frac{b^2\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx,x,\cos(c+dx)(b-a\tan(c+dx))\right)}{(a^2+b^2)\,d}\\
&= -\frac{b^2\text{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d}+\frac{b\cos(c+dx)}{(a^2+b^2)\,d}+\frac{a\sin(c+dx)}{(a^2+b^2)\,d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.88

$$\int \frac{\cos(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{2b^2\text{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)+\sqrt{a^2+b^2}(b\cos(c+dx)+a\sin(c+dx))}{(a^2+b^2)^{3/2}\,d}$$

```
[In] Integrate[Cos[c + d*x]/(a + b*Tan[c + d*x]),x]
```

```
[Out] (2*b^2*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + Sqrt[a^2 + b^2]
*(b*Cos[c + d*x] + a*Sin[c + d*x]))/((a^2 + b^2)^(3/2)*d)
```

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\frac{2b^2\,\text{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\left(-a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-b\right)}{\left(a^2+b^2\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{d}$	90
default	$\frac{\frac{2b^2\,\text{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\left(-a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-b\right)}{\left(a^2+b^2\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{d}$	90
risch	$-\frac{i\,\mathrm{e}^{i(dx+c)}}{2(-ib+a)d}+\frac{i\,\mathrm{e}^{-i(dx+c)}}{2(ib+a)d}+\frac{b^2\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^3+ia\,b^2-a^2b-b^3}{\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}d}-\frac{b^2\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ia^3+ia\,b^2-a^2b-b^3}{\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}d}$	174

```
[In] int(cos(d*x+c)/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*b^2/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^
2)^(1/2))-2/(a^2+b^2)*(-a*tan(1/2*d*x+1/2*c)-b)/(1+tan(1/2*d*x+1/2*c)^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(88) = 176$.

Time = 0.27 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.08

$$\int \frac{\cos(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\sqrt{a^2+b^2}\,b^2\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)+2\,(a^2b+b^3)\,c}{2\,(a^4+2\,a^2b^2+b^4)d}$$

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2 + b^2)*b^2*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2
)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(
d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 +
b^2)) + 2*(a^2*b + b^3)*cos(d*x + c) + 2*(a^3 + a*b^2)*sin(d*x + c))/((a^4
+ 2*a^2*b^2 + b^4)*d)
```

Sympy [F]

$$\int \frac{\cos(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\cos\left(c+dx\right)}{a+b\tan\left(c+dx\right)}\,dx$$

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)/(a + b*tan(c + d*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.46 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.58

$$\int \frac{\cos(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{\frac{b^2\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}}-\frac{2\left(b+\frac{a\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2+b^2+\frac{(a^2+b^2)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}}{d}$$

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(b^2*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*
sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(
b + a*sin(d*x + c)/(cos(d*x + c) + 1))/(a^2 + b^2 + (a^2 + b^2)*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2))/d
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.31

$$\int \frac{\cos(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{\frac{b^2\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}}-\frac{2\,\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b\right)}{(a^2+b^2)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)}}{d}$$

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -(b^2*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*t
an(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(a*ta
n(1/2*d*x + 1/2*c) + b)/((a^2 + b^2)*(tan(1/2*d*x + 1/2*c)^2 + 1)))/d
```

Mupad [B] (verification not implemented)

Time = 4.45 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.22

$$\int \frac{\cos(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{\frac{2\,b}{a^2+b^2}+\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2+b^2}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}-\frac{2\,b^2\,\mathrm{atanh}\left(\frac{a^2\,b+b^3-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a^2+b^2)}{(a^2+b^2)^{3/2}}\right)}{d\,(a^2+b^2)^{3/2}}$$

```
[In] int(cos(c + d*x)/(a + b*tan(c + d*x)),x)
```

```
[Out] ((2*b)/(a^2 + b^2) + (2*a*tan(c/2 + (d*x)/2))/(a^2 + b^2))/(d*(tan(c/2 + (d
*x)/2)^2 + 1)) - (2*b^2*atanh((a^2*b + b^3 - a*tan(c/2 + (d*x)/2)*(a^2 + b^
2))/(a^2 + b^2)^(3/2)))/(d*(a^2 + b^2)^(3/2))
```

3.553 $\int \frac{\cos^3(c+dx)}{a+b\tan(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 165

$$\int \frac{\cos^3(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{b^4\text{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}d}+\frac{b^3\cos(c+dx)}{\left(a^2+b^2\right)^2d}+\frac{b\cos^3(c+dx)}{3\left(a^2+b^2\right)d}+\frac{ab^2\sin(c+dx)}{\left(a^2+b^2\right)^2d}+\frac{a\sin(c+dx)}{\left(a^2+b^2\right)d}-\frac{a\sin^3(c+dx)}{3\left(a^2+b^2\right)d}$$

```
[Out] -b^4*arctanh(cos(d*x+c)*(b-a*tan(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)/d
+b^3*cos(d*x+c)/(a^2+b^2)^2/d+1/3*b*cos(d*x+c)^3/(a^2+b^2)/d+a*b^2*sin(d*x+
c)/(a^2+b^2)^2/d+a*sin(d*x+c)/(a^2+b^2)/d-1/3*a*sin(d*x+c)^3/(a^2+b^2)/d
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3592, 3567, 2713, 2717, 3590, 212}

$$\int \frac{\cos^3(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{b^4\text{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{5/2}}-\frac{a\sin^3(c+dx)}{3d\left(a^2+b^2\right)}+\frac{ab^2\sin(c+dx)}{d\left(a^2+b^2\right)^2}+\frac{a\sin(c+dx)}{d\left(a^2+b^2\right)}+\frac{b\cos^3(c+dx)}{3d\left(a^2+b^2\right)}+\frac{b^3\cos(c+dx)}{d\left(a^2+b^2\right)^2}$$

```
[In] Int[Cos[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

```
[Out] -((b^4*ArcTanh[(Cos[c + d*x]*(b - a*Tan[c + d*x]))/Sqrt[a^2 + b^2]])/((a^2
+ b^2)^(5/2)*d)) + (b^3*Cos[c + d*x])/((a^2 + b^2)^2*d) + (b*Cos[c + d*x]^3
```

```
)/(3*(a^2 + b^2)*d) + (a*b^2*Sin[c + d*x])/((a^2 + b^2)^2*d) + (a*Sin[c + d
*x])/((a^2 + b^2)*d) - (a*Sin[c + d*x]^3)/(3*(a^2 + b^2)*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3590

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> Dist[-f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3592

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(d*Sec[e + f*x])^m*(a - b*Tan[e
+ f*x]), x], x] + Dist[b^2/(d^2*(a^2 + b^2)), Int[(d*Sec[e + f*x])^(m + 2)/
(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 + b^2,
 0] && ILtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \cos^3(c+dx)(a-b\tan(c+dx))\,dx}{a^2+b^2} + \frac{b^2\int \frac{\cos(c+dx)}{a+b\tan(c+dx)}\,dx}{a^2+b^2}$$

$$= \frac{b\cos^3(c+dx)}{3(a^2+b^2)d} + \frac{b^2\int\cos(c+dx)(a-b\tan(c+dx))\,dx}{(a^2+b^2)^2} + \frac{b^4\int\frac{\sec(c+dx)}{a+b\tan(c+dx)}\,dx}{(a^2+b^2)^2} + \frac{a\int\cos^3(c+dx)\,dx}{a^2+b^2}$$

$$= \frac{b^3\cos(c+dx)}{(a^2+b^2)^2 d} + \frac{b\cos^3(c+dx)}{3(a^2+b^2)d} + \frac{(ab^2)\int\cos(c+dx)\,dx}{(a^2+b^2)^2} - \frac{b^4\text{Subst}\left(\int\frac{1}{a^2+b^2-x^2}\,dx, x, \cos(c+dx)(b-a\tan(c+dx))\right)}{(a^2+b^2)^2 d} - \frac{a\text{Subst}\left(\int(1-x^2)\,dx, x, -\sin(c+dx)\right)}{(a^2+b^2)d}$$

$$= -\frac{b^4\text{arctanh}\left(\frac{\cos(c+dx)(b-a\tan(c+dx))}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}d} + \frac{b^3\cos(c+dx)}{(a^2+b^2)^2 d} + \frac{b\cos^3(c+dx)}{3(a^2+b^2)d} + \frac{ab^2\sin(c+dx)}{(a^2+b^2)^2 d} + \frac{a\sin(c+dx)}{(a^2+b^2)d} - \frac{a\sin^3(c+dx)}{3(a^2+b^2)d}$$

Mathematica [A] (verified)

Time = 1.71 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.83

$$\int\frac{\cos^3(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{24b^4\text{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right) + \sqrt{a^2+b^2}(3b(a^2+5b^2)\cos(c+dx) + b(a^2+b^2)\cos(3(c+dx)) + 2a(5a^2}{12(a^2+b^2)^{5/2}d}$$

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Tan[c + d*x]),x]
```

```
[Out] (24*b^4*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + Sqrt[a^2 + b^2
]*(3*b*(a^2 + 5*b^2)*Cos[c + d*x] + b*(a^2 + b^2)*Cos[3*(c + d*x)] + 2*a*(5
*a^2 + 11*b^2 + (a^2 + b^2)*Cos[2*(c + d*x)])*Sin[c + d*x]))/(12*(a^2 + b^2
)^(5/2)*d)
```

Maple [A] (verified)

Time = 4.15 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{\frac{2b^4 \operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{(a^4+2a^2b^2+b^4)\sqrt{a^2+b^2}}-\frac{2\left(\left(-a^3-2a\,b^2\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-a^2b-2b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{2}{3}a^3-\frac{8}{3}a\,b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{\left(a^4+2a^2b^2+b^4\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{d}$
default	$\frac{\frac{2b^4 \operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{(a^4+2a^2b^2+b^4)\sqrt{a^2+b^2}}-\frac{2\left(\left(-a^3-2a\,b^2\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-a^2b-2b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{2}{3}a^3-\frac{8}{3}a\,b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{\left(a^4+2a^2b^2+b^4\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{d}$
risch	$-\frac{5\,e^{i(dx+c)}b}{8(-2iab+a^2-b^2)d}-\frac{3ie^{i(dx+c)}a}{8(-2iab+a^2-b^2)d}-\frac{5\,e^{-i(dx+c)}b}{8(ib+a)^2d}+\frac{3ie^{-i(dx+c)}a}{8(ib+a)^2d}+\frac{b^4\ln\left(e^{i(dx+c)}+\frac{ia^5+2ia^3b^2+ia\,b^4-a^4}{(a^2+b^2)^{\frac{5}{2}}}\right.}{(a^2+b^2)^{\frac{5}{2}}d}$

[In] int(cos(d*x+c)^3/(a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(2*b^4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x
+1/2*c)-2*b)/(a^2+b^2)^(1/2))-2/(a^4+2*a^2*b^2+b^4)*((-a^3-2*a*b^2)*tan(1/2
*d*x+1/2*c)^5+(-a^2*b-2*b^3)*tan(1/2*d*x+1/2*c)^4+(-2/3*a^3-8/3*a*b^2)*tan(
1/2*d*x+1/2*c)^3-2*b^3*tan(1/2*d*x+1/2*c)^2+(-a^3-2*a*b^2)*tan(1/2*d*x+1/2*
c)-1/3*a^2*b-4/3*b^3)/(1+tan(1/2*d*x+1/2*c)^2)^3)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.59

$$\int \frac{\cos^3(c+dx)}{a+b\tan(c+dx)}\,dx = \frac{3\sqrt{a^2+b^2}b^4\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b^2+2\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)+2\left(a^4b+2\,a^2\right.}{6\left(a^6+\right.}$$

[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="fricas")

[Out] 1/6*(3*sqrt(a^2 + b^2)*b^4*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b
^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*si
n(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
+ b^2)) + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^3 + 6*(a^2*b^3 + b^5)*co
s(d*x + c) + 2*(2*a^5 + 7*a^3*b^2 + 5*a*b^4 + (a^5 + 2*a^3*b^2 + a*b^4)*cos
(d*x + c)^2)*sin(d*x + c))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d)

Sympy [F]

$$\int \frac{\cos^3(c+dx)}{a+b\tan(c+dx)}\,dx = \int \frac{\cos^3(c+dx)}{a+b\tan(c+dx)}\,dx$$

[In] integrate(cos(d*x+c)**3/(a+b*tan(d*x+c)),x)

[Out] Integral(cos(c + d*x)**3/(a + b*tan(c + d*x)), x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. 2(159) = 318.

Time = 0.32 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.30

$$\int \frac{\cos^3(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{\frac{3\,b^4\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} - \frac{2\left(a^2b+4\,b^3+\frac{6\,b^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\left(a^3+2\,ab^2\right)\sin(dx+c)}{\cos(dx+c)+1}+\frac{2\left(a^3+4\,ab^2\right)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\left(a^2b+2\,b^3\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\cdots\right)}{a^4+2\,a^2b^2+b^4+\frac{3\left(a^4+2\,a^2b^2+b^4\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\left(a^4+2\,a^2b^2+b^4\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\left(a^4+2\,a^2b^2\cdots\right.}{(\cos(\cdots}}}{3\,d}$$

[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="maxima")

[Out] -1/3*(3*b^4*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2*(a^2*b + 4*b^3 + 6*b^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*(a^3 + 2*a*b^2)*sin(d*x + c)/(cos(d*x + c) + 1) + 2*(a^3 + 4*a*b^2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*(a^2*b + 2*b^3)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 3*(a^3 + 2*a*b^2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^4 + 2*a^2*b^2 + b^4 + 3*(a^4 + 2*a^2*b^2 + b^4)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*(a^4 + 2*a^2*b^2 + b^4)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (a^4 + 2*a^2*b^2 + b^4)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6))/d

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.73

$$\int \frac{\cos^3(c+dx)}{a+b\tan(c+dx)}\,dx = -\frac{\frac{3\,b^4\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} - \frac{2\left(3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+6\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}\cdots\right.}{(a^4\cdots}}{3\,d}$$

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3*(3*b^4*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs
(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b
^4)*sqrt(a^2 + b^2)) - 2*(3*a^3*tan(1/2*d*x + 1/2*c)^5 + 6*a*b^2*tan(1/2*d*
x + 1/2*c)^5 + 3*a^2*b*tan(1/2*d*x + 1/2*c)^4 + 6*b^3*tan(1/2*d*x + 1/2*c)^
4 + 2*a^3*tan(1/2*d*x + 1/2*c)^3 + 8*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*b^3*t
an(1/2*d*x + 1/2*c)^2 + 3*a^3*tan(1/2*d*x + 1/2*c) + 6*a*b^2*tan(1/2*d*x +
1/2*c) + a^2*b + 4*b^3)/((a^4 + 2*a^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 +
1)^3))/d
```

Mupad [B] (verification not implemented)

Time = 7.59 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.07

$$\int \frac{\cos^3(c+dx)}{a+b\tan(c+dx)}\,dx$$
$$= \frac{\frac{\frac{2a^2b}{3}+\frac{8b^3}{3}}{a^4+2a^2b^2+b^4} + \frac{4b^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{a^4+2a^2b^2+b^4} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5(2a^3+4ab^2)}{a^4+2a^2b^2+b^4} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{4a^3}{3}+\frac{16ab^2}{3}\right)}{a^4+2a^2b^2+b^4} + \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^3+2ab^2)}{a^4+2a^2b^2+b^4} + \frac{2b\tan\cdots}{\cdots}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}$$
$$- \frac{2b^4\,\mathrm{atanh}\left(\frac{a^4b+b^5+2a^2b^3-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^4+2a^2b^2+b^4)}{(a^2+b^2)^{5/2}}\right)}{d\,(a^2+b^2)^{5/2}}$$

```
[In] int(cos(c + d*x)^3/(a + b*tan(c + d*x)),x)
```

```
[Out] (((2*a^2*b)/3 + (8*b^3)/3)/(a^4 + b^4 + 2*a^2*b^2) + (4*b^3*tan(c/2 + (d*x)
/2)^2)/(a^4 + b^4 + 2*a^2*b^2) + (tan(c/2 + (d*x)/2)^5*(4*a*b^2 + 2*a^3))/(
a^4 + b^4 + 2*a^2*b^2) + (tan(c/2 + (d*x)/2)^3*((16*a*b^2)/3 + (4*a^3)/3))/
(a^4 + b^4 + 2*a^2*b^2) + (2*tan(c/2 + (d*x)/2)*(2*a*b^2 + a^3))/(a^4 + b^4
 + 2*a^2*b^2) + (2*b*tan(c/2 + (d*x)/2)^4*(a^2 + 2*b^2))/(a^4 + b^4 + 2*a^2
*b^2))/(d*(3*tan(c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x
)/2)^6 + 1)) - (2*b^4*atanh((a^4*b + b^5 + 2*a^2*b^3 - a*tan(c/2 + (d*x)/2)
*(a^4 + b^4 + 2*a^2*b^2))/(a^2 + b^2)^(5/2)))/(d*(a^2 + b^2)^(5/2))
```

3.554 $\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 178

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{6a(a^2+b^2)^2\log(a+b\tan(c+dx))}{b^7d} + \frac{(5a^4+9a^2b^2+3b^4)\tan(c+dx)}{b^6d} - \frac{a(2a^2+3b^2)\tan^2(c+dx)}{b^5d} + \frac{(a^2+b^2)\tan^3(c+dx)}{b^4d} - \frac{a\tan^4(c+dx)}{2b^3d} + \frac{\tan^5(c+dx)}{5b^2d} - \frac{(a^2+b^2)^3}{b^7d(a+b\tan(c+dx))}$$

```
[Out] -6*a*(a^2+b^2)^2*ln(a+b*tan(d*x+c))/b^7/d+(5*a^4+9*a^2*b^2+3*b^4)*tan(d*x+c
)/b^6/d-a*(2*a^2+3*b^2)*tan(d*x+c)^2/b^5/d+(a^2+b^2)*tan(d*x+c)^3/b^4/d-1/2
*a*tan(d*x+c)^4/b^3/d+1/5*tan(d*x+c)^5/b^2/d-(a^2+b^2)^3/b^7/d/(a+b*tan(d*x
+c))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used

$= \{3587, 711\}$

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{(a^2+b^2)^3}{b^7d(a+b\tan(c+dx))} - \frac{6a(a^2+b^2)^2\log(a+b\tan(c+dx))}{b^7d} - \frac{a(2a^2+3b^2)\tan^2(c+dx)}{b^5d} + \frac{(a^2+b^2)\tan^3(c+dx)}{b^4d} + \frac{(5a^4+9a^2b^2+3b^4)\tan(c+dx)}{b^6d} - \frac{a\tan^4(c+dx)}{2b^3d} + \frac{\tan^5(c+dx)}{5b^2d}$$

```
[In] Int[Sec[c + d*x]^8/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-6*a*(a^2 + b^2)^2*Log[a + b*Tan[c + d*x]])/(b^7*d) + ((5*a^4 + 9*a^2*b^2
+ 3*b^4)*Tan[c + d*x])/(b^6*d) - (a*(2*a^2 + 3*b^2)*Tan[c + d*x]^2)/(b^5*d)
 + ((a^2 + b^2)*Tan[c + d*x]^3)/(b^4*d) - (a*Tan[c + d*x]^4)/(2*b^3*d) + Ta
n[c + d*x]^5/(5*b^2*d) - (a^2 + b^2)^3/(b^7*d*(a + b*Tan[c + d*x]))
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^3}{(a+x)^2}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{5a^4+9a^2b^2+3b^4}{b^6} - \frac{2a(2a^2+3b^2)x}{b^6} + \frac{3(a^2+b^2)x^2}{b^6} - \frac{2ax^3}{b^6} + \frac{x^4}{b^6} + \frac{(a^2+b^2)^3}{b^6(a+x)^2} - \frac{6a(a^2+b^2)^2}{b^6(a+x)}\right) dx, x, b\tan(c+dx)\right)}{bd}$$

$$= -\frac{6a(a^2+b^2)^2\log(a+b\tan(c+dx))}{b^7d} + \frac{(5a^4+9a^2b^2+3b^4)\tan(c+dx)}{b^6d} - \frac{a(2a^2+3b^2)\tan^2(c+dx)}{b^5d} + \frac{(a^2+b^2)\tan^3(c+dx)}{b^4d} - \frac{a\tan^4(c+dx)}{2b^3d} + \frac{\tan^5(c+dx)}{5b^2d} - \frac{(a^2+b^2)^3}{b^7d(a+b\tan(c+dx))}$$

Mathematica [A] (verified)

Time = 2.62 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.29

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{2b^6\sec^6(c+dx)+b^4\sec^4(c+dx)\left(a^2+4b^2-3ab\tan(c+dx)\right)-2\Big(8(a^2+b^2)^3+30a^2(a^2+b^2)^2\log(a+}{}$$

```
[In] Integrate[Sec[c + d*x]^8/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (2*b^6*Sec[c + d*x]^6 + b^4*Sec[c + d*x]^4*(a^2 + 4*b^2 - 3*a*b*Tan[c + d*x
]) - 2*(8*(a^2 + b^2)^3 + 30*a^2*(a^2 + b^2)^2*Log[a + b*Tan[c + d*x]] + 2*
a*b*(-11*a^4 - 18*a^2*b^2 - 4*b^4 + 15*(a^2 + b^2)^2*Log[a + b*Tan[c + d*x]
])*Tan[c + d*x] - b^2*(15*a^4 + 29*a^2*b^2 + 8*b^4)*Tan[c + d*x]^2 + a*b^3*
(5*a^2 + 7*b^2)*Tan[c + d*x]^3 - 2*a^2*b^4*Tan[c + d*x]^4))/(10*b^7*d*(a +
b*Tan[c + d*x]))
```

Maple [A] (verified)

Time = 265.50 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{\frac{\frac{\left(\tan^5(dx+c)\right)b^4}{5}-\frac{a\,b^3\left(\tan^4(dx+c)\right)}{2}+a^2b^2\left(\tan^3(dx+c)\right)+b^4\left(\tan^3(dx+c)\right)-2a^3b\left(\tan^2(dx+c)\right)-3a\,b^3\left(\tan^2(dx+c)\right)+5a^4\tan(dx+}{b^6}}{d}$
default	$\frac{\frac{\frac{\left(\tan^5(dx+c)\right)b^4}{5}-\frac{a\,b^3\left(\tan^4(dx+c)\right)}{2}+a^2b^2\left(\tan^3(dx+c)\right)+b^4\left(\tan^3(dx+c)\right)-2a^3b\left(\tan^2(dx+c)\right)-3a\,b^3\left(\tan^2(dx+c)\right)+5a^4\tan(dx+}{b^6}}{d}$
risch	$-\frac{4i\left(25a^2b^3+40b^5e^{4i(dx+c)}+32b^5e^{2i(dx+c)}-33ia\,b^4e^{2i(dx+c)}-30ia^3b^2e^{10i(dx+c)}-135ia^3b^2e^{8i(dx+c)}-15ia\,b^4e^{10i(dx+}{}$

```
[In] int(sec(d*x+c)^8/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^6*(1/5*tan(d*x+c)^5*b^4-1/2*a*b^3*tan(d*x+c)^4+a^2*b^2*tan(d*x+c)^
3+b^4*tan(d*x+c)^3-2*a^3*b*tan(d*x+c)^2-3*a*b^3*tan(d*x+c)^2+5*a^4*tan(d*x+
c)+9*a^2*b^2*tan(d*x+c)+3*b^4*tan(d*x+c))-6*a/b^7*(a^4+2*a^2*b^2+b^4)*ln(a+
b*tan(d*x+c))-1/b^7*(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(a+b*tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. 2(174) = 348.

Time = 0.29 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.17

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^2} dx = -\frac{4\,(15\,a^4b^2+25\,a^2b^4+8\,b^6)\cos{(dx+c)}^6-2\,b^6-2\,(15\,a^4b^2+25\,a^2b^4+8\,b^6)\cos{(dx+c)}^4-(5\,a^2b^4+4\,b}{}$$

```
[In] integrate(sec(d*x+c)^8/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/10*(4*(15*a^4*b^2 + 25*a^2*b^4 + 8*b^6)*cos(d*x + c)^6 - 2*b^6 - 2*(15*a
^4*b^2 + 25*a^2*b^4 + 8*b^6)*cos(d*x + c)^4 - (5*a^2*b^4 + 4*b^6)*cos(d*x +
 c)^2 + 30*((a^6 + 2*a^4*b^2 + a^2*b^4)*cos(d*x + c)^6 + (a^5*b + 2*a^3*b^3
 + a*b^5)*cos(d*x + c)^5*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c)
+ (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 30*((a^6 + 2*a^4*b^2 + a^2*b^4)*cos(d
*x + c)^6 + (a^5*b + 2*a^3*b^3 + a*b^5)*cos(d*x + c)^5*sin(d*x + c))*log(co
s(d*x + c)^2) + (3*a*b^5*cos(d*x + c) - 4*(15*a^5*b + 25*a^3*b^3 + 8*a*b^5)
*cos(d*x + c)^5 + 2*(5*a^3*b^3 + 7*a*b^5)*cos(d*x + c)^3)*sin(d*x + c))/(a*
b^7*d*cos(d*x + c)^6 + b^8*d*cos(d*x + c)^5*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^2} dx = \int \frac{\sec^8{(c+dx)}}{(a+b\tan{(c+dx)})^2} dx$$

```
[In] integrate(sec(d*x+c)**8/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**8/(a + b*tan(c + d*x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.04

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^2} dx = -\frac{\frac{10\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)}{b^8\tan(dx+c)+ab^7}-\frac{2\,b^4\tan(dx+c)^5-5\,ab^3\tan(dx+c)^4+10\,(a^2b^2+b^4)\tan(dx+c)^3-10\,(2\,a^3b+3\,ab^3)\tan(dx+c)^2+10\,(5\,a^4+9\,a^2b^2}{b^6}}{10\,d}$$

```
[In] integrate(sec(d*x+c)^8/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/10*(10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)/(b^8*tan(d*x + c) + a*b^7) -
(2*b^4*tan(d*x + c)^5 - 5*a*b^3*tan(d*x + c)^4 + 10*(a^2*b^2 + b^4)*tan(d*x
 + c)^3 - 10*(2*a^3*b + 3*a*b^3)*tan(d*x + c)^2 + 10*(5*a^4 + 9*a^2*b^2 + 3
*b^4)*tan(d*x + c))/b^6 + 60*(a^5 + 2*a^3*b^2 + a*b^4)*log(b*tan(d*x + c) +
 a)/b^7)/d
```

Giac [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.42

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{60\,(a^5+2\,a^3b^2+ab^4)\log(|b\tan(dx+c)+a|)}{b^7} - \frac{10\,(6\,a^5b\tan(dx+c)+12\,a^3b^3\tan(dx+c)+6\,ab^5\tan(dx+c)+5\,a^6+9\,a^4b^2+3\,a^2b^4-b^6)}{(b\tan(dx+c)+a)b^7} - \frac{2\,b^8\,\mathrm{t}\cdots}{\cdots}}{\cdots}$$

```
[In] integrate(sec(d*x+c)^8/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/10*(60*(a^5 + 2*a^3*b^2 + a*b^4)*log(abs(b*tan(d*x + c) + a))/b^7 - 10*(
6*a^5*b*tan(d*x + c) + 12*a^3*b^3*tan(d*x + c) + 6*a*b^5*tan(d*x + c) + 5*a
^6 + 9*a^4*b^2 + 3*a^2*b^4 - b^6)/((b*tan(d*x + c) + a)*b^7) - (2*b^8*tan(d
*x + c)^5 - 5*a*b^7*tan(d*x + c)^4 + 10*a^2*b^6*tan(d*x + c)^3 + 10*b^8*tan
(d*x + c)^3 - 20*a^3*b^5*tan(d*x + c)^2 - 30*a*b^7*tan(d*x + c)^2 + 50*a^4*
b^4*tan(d*x + c) + 90*a^2*b^6*tan(d*x + c) + 30*b^8*tan(d*x + c))/b^10)/d
```

Mupad [B] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.45

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\tan(c+d\,x)^2\,\left(\frac{a^3}{b^5}-\frac{a\,\left(\frac{3}{b^2}+\frac{3\,a^2}{b^4}\right)}{b}\right)}{d} + \frac{\tan(c+d\,x)^5}{5\,b^2\,d} + \frac{\tan(c+d\,x)^3\,\left(\frac{1}{b^2}+\frac{a^2}{b^4}\right)}{d} - \frac{\tan(c+d\,x)\,\left(\frac{a^2\,\left(\frac{3}{b^2}+\frac{3\,a^2}{b^4}\right)}{b^2}-\frac{3}{b^2}+\frac{2\,a\,\left(\frac{2\,a^3}{b^5}-\frac{2\,a\,\left(\frac{3}{b^2}+\frac{3\,a^2}{b^4}\right)}{b}\right)}{b}\right)}{d} - \frac{a\tan(c+d\,x)^4}{2\,b^3\,d} - \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(6\,a^5+12\,a^3\,b^2+6\,a\,b^4\right)}{b^7\,d} - \frac{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}{b\,d\,\left(\tan\left(c+d\,x\right)\,b^7+a\,b^6\right)}$$

```
[In] int(1/(cos(c + d*x)^8*(a + b*tan(c + d*x))^2),x)
```

```
[Out] (tan(c + d*x)^2*(a^3/b^5 - (a*(3/b^2 + (3*a^2)/b^4))/b))/d + tan(c + d*x)^5
/(5*b^2*d) + (tan(c + d*x)^3*(1/b^2 + a^2/b^4))/d - (tan(c + d*x)*((a^2*(3/
b^2 + (3*a^2)/b^4))/b^2 - 3/b^2 + (2*a*((2*a^3)/b^5 - (2*a*(3/b^2 + (3*a^2)
/b^4))/b))/b))/d - (a*tan(c + d*x)^4)/(2*b^3*d) - (log(a + b*tan(c + d*x))*
(6*a*b^4 + 6*a^5 + 12*a^3*b^2))/(b^7*d) - (a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^
2)/(b*d*(a*b^6 + b^7*tan(c + d*x)))
```

3.555 $\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 116

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{4a(a^2+b^2)\log(a+b\tan(c+dx))}{b^5d}+\frac{(3a^2+2b^2)\tan(c+dx)}{b^4d} -\frac{a\tan^2(c+dx)}{b^3d}+\frac{\tan^3(c+dx)}{3b^2d}-\frac{(a^2+b^2)^2}{b^5d(a+b\tan(c+dx))}$$

```
[Out] -4*a*(a^2+b^2)*ln(a+b*tan(d*x+c))/b^5/d+(3*a^2+2*b^2)*tan(d*x+c)/b^4/d-a*ta
n(d*x+c)^2/b^3/d+1/3*tan(d*x+c)^3/b^2/d-(a^2+b^2)^2/b^5/d/(a+b*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3587, 711}

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{(a^2+b^2)^2}{b^5d(a+b\tan(c+dx))}-\frac{4a(a^2+b^2)\log(a+b\tan(c+dx))}{b^5d} +\frac{(3a^2+2b^2)\tan(c+dx)}{b^4d}-\frac{a\tan^2(c+dx)}{b^3d}+\frac{\tan^3(c+dx)}{3b^2d}$$

```
[In] Int[Sec[c + d*x]^6/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-4*a*(a^2 + b^2)*Log[a + b*Tan[c + d*x]])/(b^5*d) + ((3*a^2 + 2*b^2)*Tan[c
 + d*x])/(b^4*d) - (a*Tan[c + d*x]^2)/(b^3*d) + Tan[c + d*x]^3/(3*b^2*d) -
(a^2 + b^2)^2/(b^5*d*(a + b*Tan[c + d*x]))
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^2}{(a+x)^2}\,dx, x, b\tan(c+dx)\right)}{bd} \\
&= \frac{\text{Subst}\left(\int \left(\frac{3a^2+2b^2}{b^4} - \frac{2ax}{b^4} + \frac{x^2}{b^4} + \frac{\left(a^2+b^2\right)^2}{b^4(a+x)^2} - \frac{4a\left(a^2+b^2\right)}{b^4(a+x)}\right)\,dx, x, b\tan(c+dx)\right)}{bd} \\
&= -\frac{4a(a^2+b^2)\log(a+b\tan(c+dx))}{b^5d} + \frac{(3a^2+2b^2)\tan(c+dx)}{b^4d} \\
&\quad - \frac{a\tan^2(c+dx)}{b^3d} + \frac{\tan^3(c+dx)}{3b^2d} - \frac{\left(a^2+b^2\right)^2}{b^5d(a+b\tan(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 5.39 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.05

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{4b(2a^2+b^2)\tan(c+dx) - 2ab^2\tan^2(c+dx) + \frac{b^4\sec^4(c+dx)-4(a^2+b^2)\left(a^2+b^2+3a^2\log(a+b\tan(c+dx))+3ab\log(a+b\tan(c+\cdots\right.}{a+b\tan(c+dx)}}{3b^5d}$$

```
[In] Integrate[Sec[c + d*x]^6/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (4*b*(2*a^2 + b^2)*Tan[c + d*x] - 2*a*b^2*Tan[c + d*x]^2 + (b^4*Sec[c + d*x
]^4 - 4*(a^2 + b^2)*(a^2 + b^2 + 3*a^2*Log[a + b*Tan[c + d*x]] + 3*a*b*Log[
a + b*Tan[c + d*x]]*Tan[c + d*x]))/(a + b*Tan[c + d*x]))/(3*b^5*d)
```

Maple [A] (verified)

Time = 58.77 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{\frac{\frac{b^2\left(\tan^3(dx+c)\right)}{3}-ab\left(\tan^2(dx+c)\right)+3a^2\tan(dx+c)+2b^2\tan(dx+c)}{b^4}-\frac{a^4+2a^2b^2+b^4}{b^5(a+b\tan(dx+c))}-\frac{4a\left(a^2+b^2\right)\ln(a+b\tan(dx+c))}{b^5}}{d}$
default	$\frac{\frac{\frac{b^2\left(\tan^3(dx+c)\right)}{3}-ab\left(\tan^2(dx+c)\right)+3a^2\tan(dx+c)+2b^2\tan(dx+c)}{b^4}-\frac{a^4+2a^2b^2+b^4}{b^5(a+b\tan(dx+c))}-\frac{4a\left(a^2+b^2\right)\ln(a+b\tan(dx+c))}{b^5}}{d}$
risch	$-\frac{8i\left(3a^2b\,e^{4i(dx+c)}+6a^2b\,e^{2i(dx+c)}-3ia^3e^{6i(dx+c)}-9ia^3e^{4i(dx+c)}-9ia^3e^{2i(dx+c)}+2b^3+3a^2b-2ia\,b^2-3ia^3+4b^3e^{2i(dx}\right.}{3\left(e^{2i(dx+c)}+1\right)^3\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)b^4d}$

```
[In] int(sec(d*x+c)^6/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^4*(1/3*b^2*tan(d*x+c)^3-a*b*tan(d*x+c)^2+3*a^2*tan(d*x+c)+2*b^2*ta
n(d*x+c))-1/b^5*(a^4+2*a^2*b^2+b^4)/(a+b*tan(d*x+c))-4*a/b^5*(a^2+b^2)*ln(a
+b*tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. 2(114) = 228.

Time = 0.27 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.42

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx =$$
$$-\frac{4\left(3\,a^2b^2+2\,b^4\right)\cos\left(dx+c\right)^4-b^4-2\left(3\,a^2b^2+2\,b^4\right)\cos\left(dx+c\right)^2+6\left(\left(a^4+a^2b^2\right)\cos\left(dx+c\right)^4+\left(a^3b\right.\right.}{}$$

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(4*(3*a^2*b^2 + 2*b^4)*cos(d*x + c)^4 - b^4 - 2*(3*a^2*b^2 + 2*b^4)*co
s(d*x + c)^2 + 6*((a^4 + a^2*b^2)*cos(d*x + c)^4 + (a^3*b + a*b^3)*cos(d*x
+ c)^3*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(
d*x + c)^2 + b^2) - 6*((a^4 + a^2*b^2)*cos(d*x + c)^4 + (a^3*b + a*b^3)*cos
(d*x + c)^3*sin(d*x + c))*log(cos(d*x + c)^2) + 2*(a*b^3*cos(d*x + c) - 2*(
3*a^3*b + 2*a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(a*b^5*d*cos(d*x + c)^4 +
b^6*d*cos(d*x + c)^3*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\sec^6{(c+dx)}}{(a+b\tan{(c+dx)})^2}\,dx$$

[In] integrate(sec(d*x+c)**6/(a+b*tan(d*x+c))**2,x)

[Out] Integral(sec(c + d*x)**6/(a + b*tan(c + d*x))**2, x)

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.99

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{3\,(a^4+2\,a^2b^2+b^4)}{b^6\tan(dx+c)+ab^5} - \frac{b^2\tan(dx+c)^3-3\,ab\tan(dx+c)^2+3\,(3\,a^2+2\,b^2)\tan(dx+c)}{b^4} + \frac{12\,(a^3+ab^2)\log(b\tan(dx+c)+a)}{b^5}}{3\,d}$$

[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="maxima")

[Out] -1/3*(3*(a^4 + 2*a^2*b^2 + b^4)/(b^6*tan(d*x + c) + a*b^5) - (b^2*tan(d*x + c)^3 - 3*a*b*tan(d*x + c)^2 + 3*(3*a^2 + 2*b^2)*tan(d*x + c))/b^4 + 12*(a^3 + a*b^2)*log(b*tan(d*x + c) + a)/b^5)/d

Giac [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.28

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{12\,(a^3+ab^2)\log(|b\tan(dx+c)+a|)}{b^5} - \frac{b^4\tan(dx+c)^3-3\,ab^3\tan(dx+c)^2+9\,a^2b^2\tan(dx+c)+6\,b^4\tan(dx+c)}{b^6} - \frac{3\,(4\,a^3b\tan(dx+c)+4\,ab^3\tan(}{(b\tan(dx+c)}}{3\,d}$$

[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c))^2,x, algorithm="giac")

[Out] -1/3*(12*(a^3 + a*b^2)*log(abs(b*tan(d*x + c) + a))/b^5 - (b^4*tan(d*x + c)^3 - 3*a*b^3*tan(d*x + c)^2 + 9*a^2*b^2*tan(d*x + c) + 6*b^4*tan(d*x + c))/b^6 - 3*(4*a^3*b*tan(d*x + c) + 4*a*b^3*tan(d*x + c) + 3*a^4 + 2*a^2*b^2 - b^4)/((b*tan(d*x + c) + a)*b^5))/d

Mupad [B] (verification not implemented)

Time = 4.29 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.12

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\tan(c+d\,x)^3}{3\,b^2\,d}+\frac{\tan(c+d\,x)\,\left(\frac{2}{b^2}+\frac{3\,a^2}{b^4}\right)}{d}-\frac{a\tan(c+d\,x)^2}{b^3\,d}-\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(4\,a^3+4\,a\,b^2\right)}{b^5\,d}-\frac{a^4+2\,a^2\,b^2+b^4}{b\,d\,\left(\tan\left(c+d\,x\right)\,b^5+a\,b^4\right)}$$

```
[In] int(1/(cos(c + d*x)^6*(a + b*tan(c + d*x))^2),x)
```

```
[Out] tan(c + d*x)^3/(3*b^2*d) + (tan(c + d*x)*(2/b^2 + (3*a^2)/b^4))/d - (a*tan(
c + d*x)^2)/(b^3*d) - (log(a + b*tan(c + d*x))*(4*a*b^2 + 4*a^3))/(b^5*d) -
 (a^4 + b^4 + 2*a^2*b^2)/(b*d*(a*b^4 + b^5*tan(c + d*x)))
```

3.556 $\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{2a\log(a+b\tan(c+dx))}{b^3d} + \frac{\tan(c+dx)}{b^2d} - \frac{a^2+b^2}{b^3d(a+b\tan(c+dx))}$$

```
[Out] -2*a*ln(a+b*tan(d*x+c))/b^3/d+tan(d*x+c)/b^2/d+(-a^2-b^2)/b^3/d/(a+b*tan(d*
x+c))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3587, 711}

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{a^2+b^2}{b^3d(a+b\tan(c+dx))} - \frac{2a\log(a+b\tan(c+dx))}{b^3d} + \frac{\tan(c+dx)}{b^2d}$$

```
[In] Int[Sec[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-2*a*Log[a + b*Tan[c + d*x]])/(b^3*d) + Tan[c + d*x]/(b^2*d) - (a^2 + b^2)
/(b^3*d*(a + b*Tan[c + d*x]))
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
```

```
x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1+\frac{x^2}{b^2}}{(a+x)^2}\,dx, x, b\tan(c+dx)\right)}{bd} \\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{b^2}+\frac{a^2+b^2}{b^2(a+x)^2}-\frac{2a}{b^2(a+x)}\right)\,dx, x, b\tan(c+dx)\right)}{bd} \\
&= -\frac{2a\log(a+b\tan(c+dx))}{b^3d}+\frac{\tan(c+dx)}{b^2d}-\frac{a^2+b^2}{b^3d(a+b\tan(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{-2a\log(a+b\tan(c+dx))+b\tan(c+dx)-\frac{a^2+b^2}{a+b\tan(c+dx)}}{b^3d}$$

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
[Out] (-2*a*Log[a + b*Tan[c + d*x]] + b*Tan[c + d*x] - (a^2 + b^2)/(a + b*Tan[c +
 d*x]))/(b^3*d)
```

Maple [A] (verified)

Time = 15.50 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\frac{\tan(dx+c)}{b^2}-\frac{2a\ln(a+b\tan(dx+c))}{b^3}-\frac{a^2+b^2}{b^3(a+b\tan(dx+c))}}{d}$	57
default	$\frac{\frac{\tan(dx+c)}{b^2}-\frac{2a\ln(a+b\tan(dx+c))}{b^3}-\frac{a^2+b^2}{b^3(a+b\tan(dx+c))}}{d}$	57
risch	$-\frac{4i\left(-ia\,e^{2i(dx+c)}+b-ia\right)}{\left(e^{2i(dx+c)}+1\right)\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)b^2d}+\frac{2a\ln\left(e^{2i(dx+c)}+1\right)}{b^3d}-\frac{2a\ln\left(e^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)}{b^3d}$	136

```
[In] int(sec(d*x+c)^4/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^2*tan(d*x+c)-2/b^3*a*ln(a+b*tan(d*x+c))-1/b^3*(a^2+b^2)/(a+b*tan(d
*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(61) = 122$.

Time = 0.26 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.92

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx =$$
$$-\frac{2\,b^2\cos\left(dx+c\right)^2-2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)-b^2+\left(a^2\cos\left(dx+c\right)^2+ab\cos\left(dx+c\right)\sin\left(dx+c\right)\right)\mathrm{l}}{ab^3d\cos\left(c\right.}$$

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(2*b^2*cos(d*x + c)^2 - 2*a*b*cos(d*x + c)*sin(d*x + c) - b^2 + (a^2*cos(d
*x + c)^2 + a*b*cos(d*x + c)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x +
 c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - (a^2*cos(d*x + c)^2 + a*b*cos(d*x
 + c)*sin(d*x + c))*log(cos(d*x + c)^2))/(a*b^3*d*cos(d*x + c)^2 + b^4*d*co
s(d*x + c)*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\sec^4\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^2}\,dx$$

```
[In] integrate(sec(d*x+c)**4/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*tan(c + d*x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{a^2+b^2}{b^4\tan(dx+c)+ab^3}+\frac{2\,a\log(b\tan(dx+c)+a)}{b^3}-\frac{\tan(dx+c)}{b^2}}{d}$$

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -((a^2 + b^2)/(b^4*tan(d*x + c) + a*b^3) + 2*a*log(b*tan(d*x + c) + a)/b^3
- tan(d*x + c)/b^2)/d
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.16

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{2\,a\log(|b\tan(dx+c)+a|)}{b^3} - \frac{\tan(dx+c)}{b^2} - \frac{2\,ab\tan(dx+c)+a^2-b^2}{(b\tan(dx+c)+a)b^3}}{d}$$

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*a*log(abs(b*tan(d*x + c) + a))/b^3 - tan(d*x + c)/b^2 - (2*a*b*tan(d*x
+ c) + a^2 - b^2)/((b*tan(d*x + c) + a)*b^3))/d
```

Mupad [B] (verification not implemented)

Time = 4.56 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.10

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\tan(c+d\,x)}{b^2\,d} - \frac{a^2+b^2}{b\,d\,\left(\tan\left(c+d\,x\right)\,b^3+a\,b^2\right)} - \frac{2\,a\,\ln\left(a+b\tan(c+d\,x)\right)}{b^3\,d}$$

```
[In] int(1/(cos(c + d*x)^4*(a + b*tan(c + d*x))^2),x)
```

```
[Out] tan(c + d*x)/(b^2*d) - (a^2 + b^2)/(b*d*(a*b^2 + b^3*tan(c + d*x))) - (2*a*
log(a + b*tan(c + d*x)))/(b^3*d)
```

3.557 $\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 20

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^2} dx = -\frac{1}{bd(a+b\tan(c+dx))}$$

```
[Out] -1/b/d/(a+b*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3587, 32}

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^2} dx = -\frac{1}{bd(a+b\tan(c+dx))}$$

```
[In] Int[Sec[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
[Out] -(1/(b*d*(a + b*Tan[c + d*x])))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
```

```
] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(a+x)^2}\,dx, x, b\tan(c+dx)\right)}{bd}$$
$$= -\frac{1}{bd(a+b\tan(c+dx))}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.60

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\sin(c+dx)}{ad(a\cos(c+dx)+b\sin(c+dx))}$$

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
[Out] Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$-\frac{1}{bd(a+b\tan(dx+c))}$	21
default	$-\frac{1}{bd(a+b\tan(dx+c))}$	21
risch	$\frac{2i}{d(-ib+a)\left(-ib\,\mathrm{e}^{2i(dx+c)}+a\,\mathrm{e}^{2i(dx+c)}+ib+a\right)}$	47

```
[In] int(sec(d*x+c)^2/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] -1/b/d/(a+b*tan(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(20) = 40$.

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.85

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{b\cos(dx+c) - a\sin(dx+c)}{(a^3+ab^2)d\cos(dx+c) + (a^2b+b^3)d\sin(dx+c)}$$

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
[Out] -(b*cos(d*x + c) - a*sin(d*x + c))/((a^3 + a*b^2)*d*cos(d*x + c) + (a^2*b +
 b^3)*d*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

```
[In] integrate(sec(d*x+c)**2/(a+b*tan(d*x+c))**2,x)
[Out] Integral(sec(c + d*x)**2/(a + b*tan(c + d*x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{1}{(b\tan(dx+c)+a)bd}$$

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
[Out] -1/((b*tan(d*x + c) + a)*b*d)
```

Giac [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{1}{(b\tan(dx+c)+a)bd}$$

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="giac")
[Out] -1/((b*tan(d*x + c) + a)*b*d)
```

Mupad [B] (verification not implemented)

Time = 4.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{1}{b\,d\,\left(a+b\tan\left(c+d\,x\right)\right)}$$

```
[In] int(1/(cos(c + d*x)^2*(a + b*tan(c + d*x))^2),x)
[Out] -1/(b*d*(a + b*tan(c + d*x)))
```

3.558 $\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 152

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{(a^4+6a^2b^2-3b^4)\,x}{2\,(a^2+b^2)^3} + \frac{4ab^3\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^3\,d} + \frac{b(a^2-3b^2)}{2\,(a^2+b^2)^2\,d(a+b\tan(c+dx))} + \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\,(a^2+b^2)\,d(a+b\tan(c+dx))}$$

```
[Out] 1/2*(a^4+6*a^2*b^2-3*b^4)*x/(a^2+b^2)^3+4*a*b^3*ln(a*cos(d*x+c)+b*sin(d*x+c
))/(a^2+b^2)^3/d+1/2*b*(a^2-3*b^2)/(a^2+b^2)^2/d/(a+b*tan(d*x+c))+1/2*cos(d
*x+c)^2*(b+a*tan(d*x+c))/(a^2+b^2)/d/(a+b*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3587, 755, 815, 649, 209, 266}

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{b(a^2-3b^2)}{2d\,(a^2+b^2)^2\,(a+b\tan(c+dx))} + \frac{\cos^2(c+dx)(a\tan(c+dx)+b)}{2d\,(a^2+b^2)\,(a+b\tan(c+dx))} + \frac{4ab^3\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^3} + \frac{x(a^4+6a^2b^2-3b^4)}{2\,(a^2+b^2)^3}$$

```
[In] Int[Cos[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((a^4 + 6*a^2*b^2 - 3*b^4)*x)/(2*(a^2 + b^2)^3) + (4*a*b^3*Log[a*Cos[c + d*
x] + b*Sin[c + d*x]])/((a^2 + b^2)^3*d) + (b*(a^2 - 3*b^2))/(2*(a^2 + b^2)^
2*d*(a + b*Tan[c + d*x])) + (Cos[c + d*x]^2*(b + a*Tan[c + d*x]))/(2*(a^2 +
 b^2)*d*(a + b*Tan[c + d*x]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2(a^2+b^2)\,d(a+b\tan(c+dx))} - \frac{b\text{Subst}\left(\int \frac{-3-\frac{a^2}{b^2}-\frac{2ax}{b^2}}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)}\,dx, x, b\tan(c+dx)\right)}{2(a^2+b^2)\,d}$$

$$= \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2(a^2+b^2)\,d(a+b\tan(c+dx))} - \frac{b\text{Subst}\left(\int \left(\frac{a^2-3b^2}{(a^2+b^2)(a+x)^2} - \frac{8ab^2}{(a^2+b^2)^2(a+x)} + \frac{-a^4-6a^2b^2+3b^4+8ab^2x}{(a^2+b^2)^2(b^2+x^2)}\right)dx, x, b\tan(c+dx)\right)}{2(a^2+b^2)\,d}$$

$$= \frac{4ab^3\log(a+b\tan(c+dx))}{(a^2+b^2)^3\,d} + \frac{b(a^2-3b^2)}{2(a^2+b^2)^2\,d(a+b\tan(c+dx))} + \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2(a^2+b^2)\,d(a+b\tan(c+dx))} - \frac{b\text{Subst}\left(\int \frac{-a^4-6a^2b^2+3b^4+8ab^2x}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{2(a^2+b^2)^3\,d}$$

$$= \frac{4ab^3\log(a+b\tan(c+dx))}{(a^2+b^2)^3\,d} + \frac{b(a^2-3b^2)}{2(a^2+b^2)^2\,d(a+b\tan(c+dx))} + \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2(a^2+b^2)\,d(a+b\tan(c+dx))} - \frac{(4ab^3)\,\text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{(a^2+b^2)^3\,d} + \frac{(b(a^4+6a^2b^2-3b^4))\,\text{Subst}\left(\int \frac{1}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{2(a^2+b^2)^3\,d}$$

$$= \frac{(a^4+6a^2b^2-3b^4)\,x}{2(a^2+b^2)^3} + \frac{4ab^3\log(\cos(c+dx))}{(a^2+b^2)^3\,d} + \frac{4ab^3\log(a+b\tan(c+dx))}{(a^2+b^2)^3\,d} + \frac{b(a^2-3b^2)}{2(a^2+b^2)^2\,d(a+b\tan(c+dx))} + \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2(a^2+b^2)\,d(a+b\tan(c+dx))}$$

Mathematica [A] (verified)

Time = 4.41 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.00

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

$$= \frac{-\frac{ab\left(\left(-a+\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)-2\sqrt{-b^2}\log(a+b\tan(c+dx))+\left(a+\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}+b\tan(c+dx)\right)\right)}{\sqrt{-b^2}(a^2+b^2)}+\frac{\cos^2(c+dx)(b+a\tan(c+}{a+b\tan(c+dx)}}{2\,(a^2}$$

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-((a*b*((-a + Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] - 2*Sqrt[-b^2]*
Log[a + b*Tan[c + d*x]] + (a + Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]]
))/(Sqrt[-b^2]*(a^2 + b^2))) + (Cos[c + d*x]^2*(b + a*Tan[c + d*x]))/(a + b
*Tan[c + d*x]) + (b*(a^2 - 3*b^2)*((2*a + (-a^2 + b^2)/Sqrt[-b^2])*Log[Sqrt
[-b^2] - b*Tan[c + d*x]] - 4*a*Log[a + b*Tan[c + d*x]] + (2*a + (a^2 - b^2)
/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + (2*(a^2 + b^2))/(a + b*Tan[
c + d*x])))/(2*(a^2 + b^2)^2))/(2*(a^2 + b^2)*d)
```

Maple [A] (verified)

Time = 4.17 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{-\frac{b^3}{\left(a^2+b^2\right)^2(a+b\tan(dx+c))}+\frac{4b^3 a\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^3}+\frac{\frac{\left(\frac{a^4}{2}-\frac{b^4}{2}\right)\tan(dx+c)+a^3b+a\,b^3}{1+\tan^2(dx+c)}-2a\,b^3\ln\left(1+\tan^2(dx+c)\right)+\frac{(a^4+6a^2b^2-}{}}{\left(a^2+b^2\right)^3}}{d}$
default	$\frac{-\frac{b^3}{\left(a^2+b^2\right)^2(a+b\tan(dx+c))}+\frac{4b^3 a\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^3}+\frac{\frac{\left(\frac{a^4}{2}-\frac{b^4}{2}\right)\tan(dx+c)+a^3b+a\,b^3}{1+\tan^2(dx+c)}-2a\,b^3\ln\left(1+\tan^2(dx+c)\right)+\frac{(a^4+6a^2b^2-}{}}{\left(a^2+b^2\right)^3}}{d}$
risch	$\frac{3ixb}{6ib\,a^2-2ib^3-2a^3+6a\,b^2}-\frac{xa}{6ib\,a^2-2ib^3-2a^3+6a\,b^2}-\frac{i\mathrm{e}^{2i(dx+c)}}{8(-2iab+a^2-b^2)d}+\frac{i\mathrm{e}^{-2i(dx+c)}}{8(2iab+a^2-b^2)d}-\frac{8ia\,b^3x}{a^6+3a^4b^2+3a^2b^4}$

```
[In] int(cos(d*x+c)^2/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-b^3/(a^2+b^2)^2/(a+b*tan(d*x+c))+4*b^3/(a^2+b^2)^3*a*ln(a+b*tan(d*x+c
))+1/(a^2+b^2)^3*(((1/2*a^4-1/2*b^4)*tan(d*x+c)+a^3*b+a*b^3)/(1+tan(d*x+c)^
2)-2*a*b^3*ln(1+tan(d*x+c)^2)+1/2*(a^4+6*a^2*b^2-3*b^4)*arctan(tan(d*x+c)))
)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.84

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{(a^4b+2\,a^2b^3+b^5)\cos{(dx+c)}^3-(a^2b^3+3\,b^5-(a^5+6\,a^3b^2-3\,ab^4)dx)\cos{(dx+c)}+4\,(a^2b^3\cos{(dx+c}}{2\,((a^7+3\,a}$$

[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="fricas")

[Out] 1/2*((a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^3 - (a^2*b^3 + 3*b^5 - (a^5 + 6*a^3*b^2 - 3*a*b^4)*d*x)*cos(d*x + c) + 4*(a^2*b^3*cos(d*x + c) + a*b^4*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - (a^3*b^2 - a*b^4 - (a^4*b + 6*a^2*b^3 - 3*b^5)*d*x - (a^5 + 2*a^3*b^2 + a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*d*cos(d*x + c) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*d*sin(d*x + c))

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \text{Timed out}$$

[In] integrate(cos(d*x+c)**2/(a+b*tan(d*x+c))**2,x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.86

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{8\,ab^3\log(b\tan(dx+c)+a)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{4\,ab^3\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{(a^4+6\,a^2b^2-3\,b^4)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{2\,a^2b-2\,b^3+(a^2b-3\,b^3)\tan(dx}{a^5+2\,a^3b^2+ab^4+(a^4b+2\,a^2b^3+b^5)\tan(dx+c)^3+(a^5+2\,a}}{2\,d}$$

[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="maxima")

[Out] 1/2*(8*a*b^3*log(b*tan(d*x + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 4*a*b^3*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (a^4

```
+ 6*a^2*b^2 - 3*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (2*a^2
*b - 2*b^3 + (a^2*b - 3*b^3)*tan(d*x + c)^2 + (a^3 + a*b^2)*tan(d*x + c))/(
a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*tan(d*x + c)^3 + (a^5 +
 2*a^3*b^2 + a*b^4)*tan(d*x + c)^2 + (a^4*b + 2*a^2*b^3 + b^5)*tan(d*x + c)
))/d
```

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.64

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$=\frac{\frac{8\,ab^4\log(|b\tan(dx+c)+a|)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7}-\frac{4\,ab^3\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{(a^4+6\,a^2b^2-3\,b^4)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{a^2b\tan(dx+c)^2-3\,b^3\tan(dx+c)^2+a^3\tan(dx+c)+ab}{(a^4+2\,a^2b^2+b^4)\left(b\tan(dx+c)^3+a\tan(dx+c)^2\right.}}{2\,d}$$

```
[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(8*a*b^4*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 +
b^7) - 4*a*b^3*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
+ (a^4 + 6*a^2*b^2 - 3*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) +
 (a^2*b*tan(d*x + c)^2 - 3*b^3*tan(d*x + c)^2 + a^3*tan(d*x + c) + a*b^2*ta
n(d*x + c) + 2*a^2*b - 2*b^3)/((a^4 + 2*a^2*b^2 + b^4)*(b*tan(d*x + c)^3 +
a*tan(d*x + c)^2 + b*tan(d*x + c) + a)))/d
```

Mupad [B] (verification not implemented)

Time = 5.13 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.62

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^2}\,dx=\frac{\frac{a^2\,b-b^3}{{\left(a^2+b^2\right)}^2}+\frac{\tan(c+d\,x)^2\,\left(a^2\,b-3\,b^3\right)}{2\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{a\,\tan(c+d\,x)}{2\,\left(a^2+b^2\right)}}{d\,\left(b\,\tan\left(c+d\,x\right)^3+a\,\tan\left(c+d\,x\right)^2+b\,\tan\left(c+d\,x\right)+a\right)}+\frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-3\,b+a\,1\mathrm{i}\right)}{4\,d\,\left(-a^3-a^2\,b\,3\mathrm{i}+3\,a\,b^2+b^3\,1\mathrm{i}\right)}+\frac{\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(a-b\,3\mathrm{i}\right)}{4\,d\,\left(-a^3\,1\mathrm{i}-3\,a^2\,b+a\,b^2\,3\mathrm{i}+b^3\right)}+\frac{4\,a\,b^3\,\ln\left(a+b\,\tan(c+d\,x)\right)}{d\,{\left(a^2+b^2\right)}^3}$$

```
[In] int(cos(c + d*x)^2/(a + b*tan(c + d*x))^2,x)
```

```
[Out] ((a^2*b - b^3)/(a^2 + b^2)^2 + (tan(c + d*x)^2*(a^2*b - 3*b^3))/(2*(a^4 + b
^4 + 2*a^2*b^2)) + (a*tan(c + d*x))/(2*(a^2 + b^2)))/(d*(a + b*tan(c + d*x)
```

```
 + a*tan(c + d*x)^2 + b*tan(c + d*x)^3)) + (log(tan(c + d*x) - 1i)*(a*1i -
3*b))/(4*d*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)) + (log(tan(c + d*x) + 1i)*(
a - b*3i))/(4*d*(a*b^2*3i - 3*a^2*b - a^3*1i + b^3)) + (4*a*b^3*log(a + b*t
an(c + d*x)))/(d*(a^2 + b^2)^3)
```

3.559 $\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 235

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx &= \frac{3(a^6+5a^4b^2+15a^2b^4-5b^6)\,x}{8\,(a^2+b^2)^4} \\ &\quad+\frac{6ab^5\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^4\,d} \\ &\quad+\frac{3b(a^2-b^2)\,(a^2+5b^2)}{8\,(a^2+b^2)^3\,d(a+b\tan(c+dx))} \\ &\quad+\frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\,(a^2+b^2)\,d(a+b\tan(c+dx))} \\ &\quad-\frac{\cos^2(c+dx)\,(b(a^2-5b^2)-3a(a^2+3b^2)\tan(c+dx))}{8\,(a^2+b^2)^2\,d(a+b\tan(c+dx))}\end{aligned}$$

```
[Out] 3/8*(a^6+5*a^4*b^2+15*a^2*b^4-5*b^6)*x/(a^2+b^2)^4+6*a*b^5*ln(a*cos(d*x+c)+
b*sin(d*x+c))/(a^2+b^2)^4/d+3/8*b*(a^2-b^2)*(a^2+5*b^2)/(a^2+b^2)^3/d/(a+b*
tan(d*x+c))+1/4*cos(d*x+c)^4*(b+a*tan(d*x+c))/(a^2+b^2)/d/(a+b*tan(d*x+c))-
1/8*cos(d*x+c)^2*(b*(a^2-5*b^2)-3*a*(a^2+3*b^2)*tan(d*x+c))/(a^2+b^2)^2/d/(
a+b*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3587, 755, 837, 815, 649, 209, 266}

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{3b(a^2-b^2)(a^2+5b^2)}{8d(a^2+b^2)^3(a+b\tan(c+dx))} + \frac{\cos^4(c+dx)(a\tan(c+dx)+b)}{4d(a^2+b^2)(a+b\tan(c+dx))} - \frac{\cos^2(c+dx)(b(a^2-5b^2)-3a(a^2+3b^2)\tan(c+dx))}{8d(a^2+b^2)^2(a+b\tan(c+dx))} + \frac{6ab^5\log(a\cos(c+dx)+b\sin(c+dx))}{d(a^2+b^2)^4} + \frac{3x(a^6+5a^4b^2+15a^2b^4-5b^6)}{8(a^2+b^2)^4}$$

```
[In] Int[Cos[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (3*(a^6 + 5*a^4*b^2 + 15*a^2*b^4 - 5*b^6)*x)/(8*(a^2 + b^2)^4) + (6*a*b^5*L
og[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^4*d) + (3*b*(a^2 - b^2)*(
a^2 + 5*b^2))/(8*(a^2 + b^2)^3*d*(a + b*Tan[c + d*x])) + (Cos[c + d*x]^4*(b
 + a*Tan[c + d*x]))/(4*(a^2 + b^2)*d*(a + b*Tan[c + d*x])) - (Cos[c + d*x]^
2*(b*(a^2 - 5*b^2) - 3*a*(a^2 + 3*b^2)*Tan[c + d*x]))/(8*(a^2 + b^2)^2*d*(a
 + b*Tan[c + d*x]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
```

```
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^3}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\left(a^2+b^2\right)d(a+b\tan(c+dx))} - \frac{b\text{Subst}\left(\int \frac{-5-\frac{3a^2}{b^2}-\frac{4ax}{b^2}}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\tan(c+dx)\right)}{4\left(a^2+b^2\right)d}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4(a^2+b^2)d(a+b\tan(c+dx))} - \frac{\cos^2(c+dx)\left(b(a^2-5b^2)-3a(a^2+3b^2)\tan(c+dx)\right)}{8(a^2+b^2)^2 d(a+b\tan(c+dx))} + \frac{b^5\text{Subst}\left(\int \frac{\frac{3\left(a^4+2a^2b^2+5b^4\right)}{b^6}+\frac{6a\left(a^2+3b^2\right)x}{b^6}}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)}dx, x, b\tan(c+dx)\right)}{8(a^2+b^2)^2 d}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4(a^2+b^2)d(a+b\tan(c+dx))} - \frac{\cos^2(c+dx)\left(b(a^2-5b^2)-3a(a^2+3b^2)\tan(c+dx)\right)}{8(a^2+b^2)^2 d(a+b\tan(c+dx))} + \frac{b^5\text{Subst}\left(\int \left(\frac{3(-a^4-4a^2b^2+5b^4)}{b^4(a^2+b^2)(a+x)^2}+\frac{48a}{(a^2+b^2)^2(a+x)}-\frac{3(-a^6-5a^4b^2-15a^2b^4+5b^6+16ab^4x)}{b^4(a^2+b^2)^2(b^2+x^2)}\right)dx, x, b\tan(c+dx)\right)}{8(a^2+b^2)^2 d}$$

$$= \frac{6ab^5\log(a+b\tan(c+dx))}{(a^2+b^2)^4 d} + \frac{3b(a^2-b^2)(a^2+5b^2)}{8(a^2+b^2)^3 d(a+b\tan(c+dx))} + \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4(a^2+b^2)d(a+b\tan(c+dx))} - \frac{\cos^2(c+dx)\left(b(a^2-5b^2)-3a(a^2+3b^2)\tan(c+dx)\right)}{8(a^2+b^2)^2 d(a+b\tan(c+dx))} - \frac{(3b)\text{Subst}\left(\int \frac{-a^6-5a^4b^2-15a^2b^4+5b^6+16ab^4x}{b^2+x^2}dx, x, b\tan(c+dx)\right)}{8(a^2+b^2)^4 d}$$

$$= \frac{6ab^5\log(a+b\tan(c+dx))}{(a^2+b^2)^4 d} + \frac{3b(a^2-b^2)(a^2+5b^2)}{8(a^2+b^2)^3 d(a+b\tan(c+dx))} + \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4(a^2+b^2)d(a+b\tan(c+dx))} - \frac{\cos^2(c+dx)\left(b(a^2-5b^2)-3a(a^2+3b^2)\tan(c+dx)\right)}{8(a^2+b^2)^2 d(a+b\tan(c+dx))} - \frac{(6ab^5)\text{Subst}\left(\int \frac{x}{b^2+x^2}dx, x, b\tan(c+dx)\right)}{(a^2+b^2)^4 d} + \frac{(3b(a^6+5a^4b^2+15a^2b^4-5b^6))\text{Subst}\left(\int \frac{1}{b^2+x^2}dx, x, b\tan(c+dx)\right)}{8(a^2+b^2)^4 d}$$

$$= \frac{3(a^6+5a^4b^2+15a^2b^4-5b^6)x}{8(a^2+b^2)^4} + \frac{6ab^5\log(\cos(c+dx))}{(a^2+b^2)^4 d} + \frac{6ab^5\log(a+b\tan(c+dx))}{(a^2+b^2)^4 d} + \frac{3b(a^2-b^2)(a^2+5b^2)}{8(a^2+b^2)^3 d(a+b\tan(c+dx))} + \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4(a^2+b^2)d(a+b\tan(c+dx))} - \frac{\cos^2(c+dx)\left(b(a^2-5b^2)-3a(a^2+3b^2)\tan(c+dx)\right)}{8(a^2+b^2)^2 d(a+b\tan(c+dx))}$$

Mathematica [A] (verified)

Time = 3.60 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.77

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{4b\cos^4(c+dx)(b+a\tan(c+dx)) + \frac{2b\cos^2(c+dx)(-a^2b+5b^3+3a(a^2+3b^2)\tan(c+dx))}{a^2+b^2} - \frac{\sqrt{-b^2}\left(6a(a^2+b^2)(a^2+3b^2)\left(\left(a-\sqrt{-\right.\right.\right.}{}}{}$$

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (4*b*Cos[c + d*x]^4*(b + a*Tan[c + d*x]) + (2*b*Cos[c + d*x]^2*(-(a^2*b) +
5*b^3 + 3*a*(a^2 + 3*b^2)*Tan[c + d*x]))/(a^2 + b^2) - (Sqrt[-b^2]*(6*a*(a^
2 + b^2)*(a^2 + 3*b^2)*((a - Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]] +
 2*Sqrt[-b^2]*Log[a + b*Tan[c + d*x]] - (a + Sqrt[-b^2])*Log[Sqrt[-b^2] + b
*Tan[c + d*x]])*(a + b*Tan[c + d*x]) + 3*(a^4 + 4*a^2*b^2 - 5*b^4)*(2*Sqrt[
-b^2]*(a^2 + b^2) + (-a^2 + b^2 + 2*a*Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c
+ d*x]]*(a + b*Tan[c + d*x]) - 4*a*Sqrt[-b^2]*Log[a + b*Tan[c + d*x]]*(a +
b*Tan[c + d*x]) + (a^2 - b^2 + 2*a*Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d
*x]]*(a + b*Tan[c + d*x]))))/(a^2 + b^2)^3)/(16*b*(a^2 + b^2)*d*(a + b*Tan[
c + d*x]))
```

Maple [A] (verified)

Time = 16.90 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{\frac{\left(\frac{3}{8}a^6+\frac{15}{8}a^4b^2+\frac{5}{8}a^2b^4-\frac{7}{8}b^6\right)\left(\tan^3(dx+c)\right)+\left(2a^3b^3+2a\,b^5\right)\left(\tan^2(dx+c)\right)+\left(\frac{17}{8}a^4b^2+\frac{3}{8}a^2b^4-\frac{9}{8}b^6+\frac{5}{8}a^6\right)\tan(dx+c)+\frac{a^5b}{2}+3a^3b}{\left(1+\tan^2(dx+c)\right)^2}}{\left(a^2+b^2\right)^4}$
default	$\frac{\frac{\left(\frac{3}{8}a^6+\frac{15}{8}a^4b^2+\frac{5}{8}a^2b^4-\frac{7}{8}b^6\right)\left(\tan^3(dx+c)\right)+\left(2a^3b^3+2a\,b^5\right)\left(\tan^2(dx+c)\right)+\left(\frac{17}{8}a^4b^2+\frac{3}{8}a^2b^4-\frac{9}{8}b^6+\frac{5}{8}a^6\right)\tan(dx+c)+\frac{a^5b}{2}+3a^3b}{\left(1+\tan^2(dx+c)\right)^2}}{\left(a^2+b^2\right)^4}$
risch	$\frac{12ixab}{32ia^3b-32ia\,b^3-8a^4+48a^2b^2-8b^4}-\frac{3x\,a^2}{32ia^3b-32ia\,b^3-8a^4+48a^2b^2-8b^4}+\frac{15x\,b^2}{32ia^3b-32ia\,b^3-8a^4+48a^2b^2-8b^4}-$

```
[In] int(cos(d*x+c)^4/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a^2+b^2)^4*(((3/8*a^6+15/8*a^4*b^2+5/8*a^2*b^4-7/8*b^6)*tan(d*x+c)^
3+(2*a^3*b^3+2*a*b^5)*tan(d*x+c)^2+(17/8*a^4*b^2+3/8*a^2*b^4-9/8*b^6+5/8*a^
6)*tan(d*x+c)+1/2*a^5*b+3*a^3*b^3+5/2*a*b^5)/(1+tan(d*x+c)^2)^2-3*a*b^5*ln(
1+tan(d*x+c)^2)+3/8*(a^6+5*a^4*b^2+15*a^2*b^4-5*b^6)*arctan(tan(d*x+c)))-b^
5/(a^2+b^2)^3/(a+b*tan(d*x+c))+6*b^5/(a^2+b^2)^4*a*ln(a+b*tan(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.80

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{4\left(a^6b+3\,a^4b^3+3\,a^2b^5+b^7\right)\cos\left(dx+c\right)^5-2\left(a^6b-3\,a^4b^3-9\,a^2b^5-5\,b^7\right)\cos\left(dx+c\right)^3+\left(3\,a^6b+8\,a^4b^3\right.}{}$$

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cos(d*x + c)^5 - 2*(a^6*b - 3
*a^4*b^3 - 9*a^2*b^5 - 5*b^7)*cos(d*x + c)^3 + (3*a^6*b + 8*a^4*b^3 - 9*a^2
*b^5 - 30*b^7 + 6*(a^7 + 5*a^5*b^2 + 15*a^3*b^4 - 5*a*b^6)*d*x)*cos(d*x + c
) + 48*(a^2*b^5*cos(d*x + c) + a*b^6*sin(d*x + c))*log(2*a*b*cos(d*x + c)*s
in(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - (3*a^5*b^2 + 22*a^3*b^4 +
 3*a*b^6 - 4*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(d*x + c)^4 - 6*(a^6*
b + 5*a^4*b^3 + 15*a^2*b^5 - 5*b^7)*d*x - 6*(a^7 + 5*a^5*b^2 + 7*a^3*b^4 +
3*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^
3*b^6 + a*b^8)*d*cos(d*x + c) + (a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7
+ b^9)*d*sin(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**4/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 502 vs. 2(228) = 456.

Time = 0.48 (sec) , antiderivative size = 502, normalized size of antiderivative = 2.14

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{48\,ab^5\log\left(b\tan\left(dx+c\right)+a\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}-\frac{24\,ab^5\log\left(\tan\left(dx+c\right)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}+\frac{3\left(a^6+5\,a^4b^2+15\,a^2b^4-5\,b^6\right)\left(dx+c\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}+\frac{4\,a^4b+}{a^7+3\,a^5b^2+3\,a^3b^4+ab^6+\left(a^6b+3\,a^4\right.}}{}$$

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*(48*a*b^5*log(b*tan(d*x + c) + a)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*
b^6 + b^8) - 24*a*b^5*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4
+ 4*a^2*b^6 + b^8) + 3*(a^6 + 5*a^4*b^2 + 15*a^2*b^4 - 5*b^6)*(d*x + c)/(a^
8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + (4*a^4*b + 20*a^2*b^3 - 8*b^
5 + 3*(a^4*b + 4*a^2*b^3 - 5*b^5)*tan(d*x + c)^4 + 3*(a^5 + 4*a^3*b^2 + 3*a
*b^4)*tan(d*x + c)^3 + (5*a^4*b + 28*a^2*b^3 - 25*b^5)*tan(d*x + c)^2 + (5*
a^5 + 16*a^3*b^2 + 11*a*b^4)*tan(d*x + c))/(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a
*b^6 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*tan(d*x + c)^5 + (a^7 + 3*a^5*
b^2 + 3*a^3*b^4 + a*b^6)*tan(d*x + c)^4 + 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5
+ b^7)*tan(d*x + c)^3 + 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*tan(d*x + c
)^2 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*tan(d*x + c)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(228) = 456$.

Time = 0.47 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.97

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{48\,ab^6\log(|b\tan(dx+c)+a|)}{a^8b+4\,a^6b^3+6\,a^4b^5+4\,a^2b^7+b^9} - \frac{24\,ab^5\log\left(\tan(dx+c)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} + \frac{3\,(a^6+5\,a^4b^2+15\,a^2b^4-5\,b^6)\,(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} - \frac{8\,(6\,ab^6\tan(dx+c)+7\,a^2}{(a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8)(}}{}$$

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(48*a*b^6*log(abs(b*tan(d*x + c) + a))/(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 +
 4*a^2*b^7 + b^9) - 24*a*b^5*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a
^4*b^4 + 4*a^2*b^6 + b^8) + 3*(a^6 + 5*a^4*b^2 + 15*a^2*b^4 - 5*b^6)*(d*x +
 c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 8*(6*a*b^6*tan(d*x +
c) + 7*a^2*b^5 + b^7)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(b*t
an(d*x + c) + a)) + (36*a*b^5*tan(d*x + c)^4 + 3*a^6*tan(d*x + c)^3 + 15*a^
4*b^2*tan(d*x + c)^3 + 5*a^2*b^4*tan(d*x + c)^3 - 7*b^6*tan(d*x + c)^3 + 16
*a^3*b^3*tan(d*x + c)^2 + 88*a*b^5*tan(d*x + c)^2 + 5*a^6*tan(d*x + c) + 17
*a^4*b^2*tan(d*x + c) + 3*a^2*b^4*tan(d*x + c) - 9*b^6*tan(d*x + c) + 4*a^5
*b + 24*a^3*b^3 + 56*a*b^5)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8
)*(tan(d*x + c)^2 + 1)^2))/d
```

Mupad [B] (verification not implemented)

Time = 5.33 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.97

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{\frac{3\tan(c+dx)^4\,(a^4\,b+4\,a^2\,b^3-5\,b^5)}{8\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)}+\frac{a^4\,b+5\,a^2\,b^3-2\,b^5}{2\,(a^2+b^2)\,(a^4+2\,a^2\,b^2+b^4)}+\frac{\tan(c+d\,x)\,(5\,a^3+11\,a\,b^2)}{8\,(a^4+2\,a^2\,b^2+b^4)}+\frac{3\tan(c+d\,x)^3\,(a^3+3\,a\,b^2)}{8\,(a^4+2\,a^2\,b^2+b^4)}+\frac{\tan(c+d\,x)^2\,(5}{8\,(a^2+b^2)}}{d\,\left(b\tan\left(c+d\,x\right)^5+a\tan\left(c+d\,x\right)^4+2\,b\tan\left(c+d\,x\right)^3+2\,a\tan\left(c+d\,x\right)^2+b\tan\left(c+d\,x\right)+\right.}$$
$$+\frac{3\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(-a^2+a\,b\,4\mathrm{i}+5\,b^2\right)}{16\,d\,\left(a^4\,1\mathrm{i}+4\,a^3\,b-a^2\,b^2\,6\mathrm{i}-4\,a\,b^3+b^4\,1\mathrm{i}\right)}$$
$$+\frac{3\,\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(a^2+a\,b\,4\mathrm{i}-5\,b^2\right)}{16\,d\,\left(a^4\,1\mathrm{i}-4\,a^3\,b-a^2\,b^2\,6\mathrm{i}+4\,a\,b^3+b^4\,1\mathrm{i}\right)}+\frac{6\,a\,b^5\,\ln\left(a+b\tan(c+d\,x)\right)}{d\,(a^2+b^2)^4}$$

```
[In] int(cos(c + d*x)^4/(a + b*tan(c + d*x))^2,x)
```

```
[Out] ((3*tan(c + d*x)^4*(a^4*b - 5*b^5 + 4*a^2*b^3))/(8*(a^6 + b^6 + 3*a^2*b^4 +
 3*a^4*b^2)) + (a^4*b - 2*b^5 + 5*a^2*b^3)/(2*(a^2 + b^2)*(a^4 + b^4 + 2*a^
2*b^2)) + (tan(c + d*x)*(11*a*b^2 + 5*a^3))/(8*(a^4 + b^4 + 2*a^2*b^2)) + (
3*tan(c + d*x)^3*(3*a*b^2 + a^3))/(8*(a^4 + b^4 + 2*a^2*b^2)) + (tan(c + d*
x)^2*(5*a^4*b - 25*b^5 + 28*a^2*b^3))/(8*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2
)))/(d*(a + b*tan(c + d*x) + 2*a*tan(c + d*x)^2 + a*tan(c + d*x)^4 + 2*b*ta
n(c + d*x)^3 + b*tan(c + d*x)^5)) + (3*log(tan(c + d*x) + 1i)*(a*b*4i - a^2
 + 5*b^2))/(16*d*(4*a^3*b - 4*a*b^3 + a^4*1i + b^4*1i - a^2*b^2*6i)) + (3*l
og(tan(c + d*x) - 1i)*(a*b*4i + a^2 - 5*b^2))/(16*d*(4*a*b^3 - 4*a^3*b + a^
4*1i + b^4*1i - a^2*b^2*6i)) + (6*a*b^5*log(a + b*tan(c + d*x)))/(d*(a^2 +
b^2)^4)
```

3.560 $\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 235

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{5(8a^4+12a^2b^2+3b^4)\operatorname{arcsinh}(\tan(c+dx))\sec(c+dx)}{8b^6d\sqrt{\sec^2(c+dx)}} + \frac{5a(a^2+b^2)^{3/2}\operatorname{arctanh}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\sec(c+dx)}{b^6d\sqrt{\sec^2(c+dx)}} - \frac{5\sec^3(c+dx)(4a-3b\tan(c+dx))}{12b^3d} - \frac{\sec^5(c+dx)}{bd(a+b\tan(c+dx))} - \frac{5\sec(c+dx)\,(8a(a^2+b^2)-b(4a^2+3b^2)\tan(c+dx))}{8b^5d}$$

```
[Out] 5/8*(8*a^4+12*a^2*b^2+3*b^4)*arcsinh(tan(d*x+c))*sec(d*x+c)/b^6/d/(sec(d*x+
c)^2)^(1/2)+5*a*(a^2+b^2)^(3/2)*arctanh((b-a*tan(d*x+c))/(a^2+b^2)^(1/2)/(s
ec(d*x+c)^2)^(1/2))*sec(d*x+c)/b^6/d/(sec(d*x+c)^2)^(1/2)-5/12*sec(d*x+c)^3
*(4*a-3*b*tan(d*x+c))/b^3/d-sec(d*x+c)^5/b/d/(a+b*tan(d*x+c))-5/8*sec(d*x+c
)*(8*a*(a^2+b^2)-b*(4*a^2+3*b^2)*tan(d*x+c))/b^5/d
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

$= \{3593, 747, 829, 858, 221, 739, 212\}$

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{5a(a^2+b^2)^{3/2}\sec(c+dx)\text{arctanh}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)}{b^6d\sqrt{\sec^2(c+dx)}} - \frac{5\sec(c+dx)\left(8a(a^2+b^2)-b(4a^2+3b^2)\tan(c+dx)\right)}{8b^5d} + \frac{5(8a^4+12a^2b^2+3b^4)\sec(c+dx)\text{arcsinh}(\tan(c+dx))}{8b^6d\sqrt{\sec^2(c+dx)}} - \frac{5\sec^3(c+dx)(4a-3b\tan(c+dx))}{12b^3d} - \frac{\sec^5(c+dx)}{bd(a+b\tan(c+dx))}$$

```
[In] Int[Sec[c + d*x]^7/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (5*(8*a^4 + 12*a^2*b^2 + 3*b^4)*ArcSinh[Tan[c + d*x]]*Sec[c + d*x])/(8*b^6*
d*Sqrt[Sec[c + d*x]^2]) + (5*a*(a^2 + b^2)^(3/2)*ArcTanh[(b - a*Tan[c + d*x
])/(Sqrt[a^2 + b^2]*Sqrt[Sec[c + d*x]^2])]*Sec[c + d*x])/(b^6*d*Sqrt[Sec[c
+ d*x]^2]) - (5*Sec[c + d*x]^3*(4*a - 3*b*Tan[c + d*x]))/(12*b^3*d) - Sec[c
 + d*x]^5/(b*d*(a + b*Tan[c + d*x])) - (5*Sec[c + d*x]*(8*a*(a^2 + b^2) - b
*(4*a^2 + 3*b^2)*Tan[c + d*x]))/(8*b^5*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 747

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 1))), x] - Dist[2*c*(p/(e*(m + 1))
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\sec(c+dx)\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^{5/2}}{(a+x)^2}\,dx, x, b\tan(c+dx)\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{\sec^5(c+dx)}{bd(a+b\tan(c+dx))} + \frac{(5\sec(c+dx))\text{Subst}\left(\int \frac{x\left(1+\frac{x^2}{b^2}\right)^{3/2}}{a+x}\,dx, x, b\tan(c+dx)\right)}{b^3d\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{5\sec^3(c+dx)(4a-3b\tan(c+dx))}{12b^3d} - \frac{\sec^5(c+dx)}{bd(a+b\tan(c+dx))} + \frac{(5\sec(c+dx))\text{Subst}\left(\int \frac{\left(-\frac{a}{b^2}+\frac{(4a^2+3b^2)x}{b^4}\right)\sqrt{1+\frac{x^2}{b^2}}}{a+x}\,dx, x, b\tan(c+dx)\right)}{4bd\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{5\sec^3(c+dx)(4a-3b\tan(c+dx))}{12b^3d} - \frac{\sec^5(c+dx)}{bd(a+b\tan(c+dx))} - \frac{5\sec(c+dx)\left(8a(a^2+b^2)-b(4a^2+3b^2)\tan(c+dx)\right)}{8b^5d} + \frac{(5b\sec(c+dx))\text{Subst}\left(\int \frac{-\frac{a(4a^2+5b^2)}{b^6}+\frac{(8a^4+12a^2b^2+3b^4)x}{b^8}}{(a+x)\sqrt{1+\frac{x^2}{b^2}}}dx, x, b\tan(c+dx)\right)}{8d\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{5\sec^3(c+dx)(4a-3b\tan(c+dx))}{12b^3d} - \frac{\sec^5(c+dx)}{bd(a+b\tan(c+dx))} - \frac{5\sec(c+dx)\left(8a(a^2+b^2)-b(4a^2+3b^2)\tan(c+dx)\right)}{8b^5d} - \frac{\left(5a(a^2+b^2)^2\sec(c+dx)\right)\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{1+\frac{x^2}{b^2}}}dx, x, b\tan(c+dx)\right)}{b^7d\sqrt{\sec^2(c+dx)}} + \frac{(5(8a^4+12a^2b^2+3b^4)\sec(c+dx))\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{b^2}}}dx, x, b\tan(c+dx)\right)}{8b^7d\sqrt{\sec^2(c+dx)}}$$

$$= \frac{5(8a^4+12a^2b^2+3b^4)\text{arcsinh}(\tan(c+dx))\sec(c+dx)}{8b^6d\sqrt{\sec^2(c+dx)}} - \frac{5\sec^3(c+dx)(4a-3b\tan(c+dx))}{12b^3d} - \frac{\sec^5(c+dx)}{bd(a+b\tan(c+dx))} - \frac{5\sec(c+dx)\left(8a(a^2+b^2)-b(4a^2+3b^2)\tan(c+dx)\right)}{8b^5d} + \frac{\left(5a(a^2+b^2)^2\sec(c+dx)\right)\text{Subst}\left(\int \frac{1}{1+\frac{a^2}{b^2}-x^2}dx, x, \frac{1-\frac{a\tan(c+dx)}{b}}{\sqrt{\sec^2(c+dx)}}\right)}{b^7d\sqrt{\sec^2(c+dx)}}$$

$$= \frac{5(8a^4+12a^2b^2+3b^4)\text{arcsinh}(\tan(c+dx))\sec(c+dx)}{8b^6d\sqrt{\sec^2(c+dx)}} + \frac{5a(a^2+b^2)^{3/2}\text{arctanh}\left(\frac{b\left(1-\frac{a\tan(c+dx)}{b}\right)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\sec(c+dx)}{b^6d\sqrt{\sec^2(c+dx)}} - \frac{5\sec^3(c+dx)(4a-3b\tan(c+dx))}{12b^3d} - \frac{\sec^5(c+dx)}{bd(a+b\tan(c+dx))} - \frac{5\sec(c+dx)\left(8a(a^2+b^2)-b(4a^2+3b^2)\tan(c+dx)\right)}{8b^5d}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.76 (sec) , antiderivative size = 1152, normalized size of antiderivative = 4.90

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{(a-ib)^2(a+ib)^2\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))}{b^5d(a+b\tan(c+dx))^2}$$
$$-\frac{a(12a^2+13b^2)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{3b^5d(a+b\tan(c+dx))^2}$$
$$+\frac{10ia(a+ib)(ia+b)\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{\sqrt{a^2+b^2}\left(-b\cos\left(\frac{1}{2}(c+dx)\right)+a\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^2\cos\left(\frac{1}{2}(c+dx)\right)+b^2\cos\left(\frac{1}{2}(c+dx)\right)}\right)\sec^2(c+dx)(a\cos(c+dx)+}{b^6d(a+b\tan(c+dx))^2}$$
$$-\frac{5(8a^4+12a^2b^2+3b^4)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx}{8b^6d(a+b\tan(c+dx))^2}$$
$$+\frac{5(8a^4+12a^2b^2+3b^4)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx}{8b^6d(a+b\tan(c+dx))^2}$$
$$+\frac{\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{16b^2d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^4(a+b\tan(c+dx))^2}$$
$$+\frac{(36a^2-8ab+21b^2)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{48b^4d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a+b\tan(c+dx))^2}$$
$$-\frac{a\sec^2(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a\cos(c+dx)+b\sin(c+dx))^2}{3b^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3(a+b\tan(c+dx))^2}$$
$$-\frac{\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{16b^2d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4(a+b\tan(c+dx))^2}$$
$$+\frac{a\sec^2(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a\cos(c+dx)+b\sin(c+dx))^2}{3b^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3(a+b\tan(c+dx))^2}$$
$$+\frac{(-36a^2-8ab-21b^2)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{48b^4d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a+b\tan(c+dx))^2}$$
$$+\frac{\sec^2(c+dx)\left(-12a^3\sin\left(\frac{1}{2}(c+dx)\right)-13ab^2\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^2}{3b^5d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^2}$$
$$+\frac{\sec^2(c+dx)\left(12a^3\sin\left(\frac{1}{2}(c+dx)\right)+13ab^2\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^2}{3b^5d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^2}$$

```
[In] Integrate[Sec[c + d*x]^7/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((a - I*b)^2*(a + I*b)^2*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])
)/(b^5*d*(a + b*Tan[c + d*x])^2)) - (a*(12*a^2 + 13*b^2)*Sec[c + d*x]^2*(a*
Cos[c + d*x] + b*Sin[c + d*x])^2)/(3*b^5*d*(a + b*Tan[c + d*x])^2) + ((10*I
)*a*(a + I*b)*(I*a + b)*Sqrt[a^2 + b^2]*ArcTanh[(Sqrt[a^2 + b^2]*(-(b*Cos[(
c + d*x)/2]) + a*Sin[(c + d*x)/2]))/(a^2*Cos[(c + d*x)/2] + b^2*Cos[(c + d*
```

```
x)/2])]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(b^6*d*(a + b*T
an[c + d*x])^2) - (5*(8*a^4 + 12*a^2*b^2 + 3*b^4)*Log[Cos[(c + d*x)/2] - Si
n[(c + d*x)/2]]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(8*b^6*
d*(a + b*Tan[c + d*x])^2) + (5*(8*a^4 + 12*a^2*b^2 + 3*b^4)*Log[Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^
2)/(8*b^6*d*(a + b*Tan[c + d*x])^2) + (Sec[c + d*x]^2*(a*Cos[c + d*x] + b*S
in[c + d*x])^2)/(16*b^2*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4*(a + b*Ta
n[c + d*x])^2) + ((36*a^2 - 8*a*b + 21*b^2)*Sec[c + d*x]^2*(a*Cos[c + d*x]
 + b*Sin[c + d*x])^2)/(48*b^4*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(a +
 b*Tan[c + d*x])^2) - (a*Sec[c + d*x]^2*Sin[(c + d*x)/2]*(a*Cos[c + d*x] +
b*Sin[c + d*x])^2)/(3*b^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(a + b*
Tan[c + d*x])^2) - (Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(16
*b^2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(a + b*Tan[c + d*x])^2) + (a
*Sec[c + d*x]^2*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(3*b^
3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(a + b*Tan[c + d*x])^2) + ((-36
*a^2 - 8*a*b - 21*b^2)*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/
(48*b^4*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a + b*Tan[c + d*x])^2) +
 (Sec[c + d*x]^2*(-12*a^3*Sin[(c + d*x)/2] - 13*a*b^2*Sin[(c + d*x)/2])*(a*
Cos[c + d*x] + b*Sin[c + d*x])^2)/(3*b^5*d*(Cos[(c + d*x)/2] - Sin[(c + d*x
)/2])*(a + b*Tan[c + d*x])^2) + (Sec[c + d*x]^2*(12*a^3*Sin[(c + d*x)/2] +
13*a*b^2*Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(3*b^5*d*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 453 vs. 2(219) = 438.

Time = 152.14 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.93

method	result
derivativedivides	$-\frac{1}{4b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{4a-3b}{6b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{12a^2-8ab+11b^2}{8b^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{\left(40a^4+60a^2b^2+15b^4\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{8b^6}-\frac{32a^3-12a}{8b^5(\tan}$
default	$-\frac{1}{4b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{4a-3b}{6b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{12a^2-8ab+11b^2}{8b^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{\left(40a^4+60a^2b^2+15b^4\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{8b^6}-\frac{32a^3-12a}{8b^5(\tan}$
risch	$-\frac{120ia^3b\,e^{3i(dx+c)}+60ia^3b\,e^{i(dx+c)}+180a^2b^2e^{9i(dx+c)}+640a^2b^2e^{7i(dx+c)}+920a^2b^2e^{5i(dx+c)}+640a^2b^2e^{3i(dx+c)}+180a^2}{}$

```
[In] int(sec(d*x+c)^7/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4/b^2/(tan(1/2*d*x+1/2*c)+1)^4-1/6*(4*a-3*b)/b^3/(tan(1/2*d*x+1/2*c
```

```
)+1)^3-1/8*(12*a^2-8*a*b+11*b^2)/b^4/(tan(1/2*d*x+1/2*c)+1)^2+1/8/b^6*(40*a
^4+60*a^2*b^2+15*b^4)*ln(tan(1/2*d*x+1/2*c)+1)-1/8*(32*a^3-12*a^2*b+40*a*b^
2-9*b^3)/b^5/(tan(1/2*d*x+1/2*c)+1)+1/4/b^2/(tan(1/2*d*x+1/2*c)-1)^4-1/6*(-
4*a-3*b)/b^3/(tan(1/2*d*x+1/2*c)-1)^3-1/8*(-12*a^2-8*a*b-11*b^2)/b^4/(tan(1
/2*d*x+1/2*c)-1)^2+1/8/b^6*(-40*a^4-60*a^2*b^2-15*b^4)*ln(tan(1/2*d*x+1/2*c
)-1)-1/8*(-32*a^3-12*a^2*b-40*a*b^2-9*b^3)/b^5/(tan(1/2*d*x+1/2*c)-1)+2/b^6
*(((a^4+2*a^2*b^2+b^4)*b^2/a*tan(1/2*d*x+1/2*c)+b*(a^4+2*a^2*b^2+b^4))/(tan
(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)-5*a*(a^4+2*a^2*b^2+b^4)/(a^2+
b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(220) = 440$.

Time = 0.42 (sec) , antiderivative size = 472, normalized size of antiderivative = 2.01

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{12\,b^5 - 30\,(8\,a^4b + 12\,a^2b^3 + 3\,b^5)\cos{(dx+c)}^4 + 10\,(4\,a^2b^3 + 3\,b^5)\cos{(dx+c)}^2 + 120\,\left((a^4 + a^2b^2)\cos{(d}\right.}{}$$

```
[In] integrate(sec(d*x+c)^7/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(12*b^5 - 30*(8*a^4*b + 12*a^2*b^3 + 3*b^5)*cos(d*x + c)^4 + 10*(4*a^2
*b^3 + 3*b^5)*cos(d*x + c)^2 + 120*((a^4 + a^2*b^2)*cos(d*x + c)^5 + (a^3*b
 + a*b^3)*cos(d*x + c)^4*sin(d*x + c))*sqrt(a^2 + b^2)*log((2*a*b*cos(d*x +
 c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 +
b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) +
(a^2 - b^2)*cos(d*x + c)^2 + b^2)) + 15*((8*a^5 + 12*a^3*b^2 + 3*a*b^4)*cos
(d*x + c)^5 + (8*a^4*b + 12*a^2*b^3 + 3*b^5)*cos(d*x + c)^4*sin(d*x + c))*l
og(sin(d*x + c) + 1) - 15*((8*a^5 + 12*a^3*b^2 + 3*a*b^4)*cos(d*x + c)^5 +
(8*a^4*b + 12*a^2*b^3 + 3*b^5)*cos(d*x + c)^4*sin(d*x + c))*log(-sin(d*x +
c) + 1) - 10*(2*a*b^4*cos(d*x + c) + 3*(4*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^3
)*sin(d*x + c))/(a*b^6*d*cos(d*x + c)^5 + b^7*d*cos(d*x + c)^4*sin(d*x + c)
)
```

Sympy [F]

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\sec^7{(c+dx)}}{(a+b\tan{(c+dx)})^2}\,dx$$

```
[In] integrate(sec(d*x+c)**7/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**7/(a + b*tan(c + d*x))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 827 vs. $2(220) = 440$.

Time = 0.32 (sec) , antiderivative size = 827, normalized size of antiderivative = 3.52

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(sec(d*x+c)^7/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/24*(2*(120*a^5 + 160*a^3*b^2 + 24*a*b^4 + (180*a^4*b + 245*a^2*b^3 + 24*
b^5)*sin(d*x + c)/(cos(d*x + c) + 1) - 10*(48*a^5 + 68*a^3*b^2 + 15*a*b^4)*
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(300*a^4*b + 385*a^2*b^3 + 48*b^5)*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 10*(72*a^5 + 100*a^3*b^2 + 15*a*b^4)*
sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 48*(15*a^4*b + 20*a^2*b^3 + 3*b^5)*si
n(d*x + c)^5/(cos(d*x + c) + 1)^5 - 30*(16*a^5 + 20*a^3*b^2 + 3*a*b^4)*sin(
d*x + c)^6/(cos(d*x + c) + 1)^6 - 6*(60*a^4*b + 85*a^2*b^3 + 16*b^5)*sin(d*
x + c)^7/(cos(d*x + c) + 1)^7 + 30*(4*a^5 + 4*a^3*b^2 - a*b^4)*sin(d*x + c)
^8/(cos(d*x + c) + 1)^8 + 3*(20*a^4*b + 25*a^2*b^3 + 8*b^5)*sin(d*x + c)^9/
(cos(d*x + c) + 1)^9)/(a^2*b^5 + 2*a*b^6*sin(d*x + c)/(cos(d*x + c) + 1) -
5*a^2*b^5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 8*a*b^6*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3 + 10*a^2*b^5*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 12*a*b^
6*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 10*a^2*b^5*sin(d*x + c)^6/(cos(d*x
+ c) + 1)^6 - 8*a*b^6*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 5*a^2*b^5*sin(d
*x + c)^8/(cos(d*x + c) + 1)^8 + 2*a*b^6*sin(d*x + c)^9/(cos(d*x + c) + 1)^
9 - a^2*b^5*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) - 120*(a^4 + 2*a^2*b^2 +
 b^4)*a*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b -
a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^6)
 - 15*(8*a^4 + 12*a^2*b^2 + 3*b^4)*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)
/b^6 + 15*(8*a^4 + 12*a^2*b^2 + 3*b^4)*log(sin(d*x + c)/(cos(d*x + c) + 1)
- 1)/b^6)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 530 vs. 2(220) = 440.

Time = 0.53 (sec) , antiderivative size = 530, normalized size of antiderivative = 2.26

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

$$= \frac{\frac{15\,(8\,a^4+12\,a^2b^2+3\,b^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^6} - \frac{15\,(8\,a^4+12\,a^2b^2+3\,b^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^6} + \frac{120\,(a^5+2\,a^3b^2+ab^4)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\right.\right.}{\left|2\,a\tan\left(\frac{1}{2}\right.\right.}\right.}{\sqrt{a^2+b^2}b^6}}{}$$

```
[In] integrate(sec(d*x+c)^7/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(15*(8*a^4 + 12*a^2*b^2 + 3*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^
6 - 15*(8*a^4 + 12*a^2*b^2 + 3*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^6
+ 120*(a^5 + 2*a^3*b^2 + a*b^4)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*
sqrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(
sqrt(a^2 + b^2)*b^6) + 48*(a^4*b*tan(1/2*d*x + 1/2*c) + 2*a^2*b^3*tan(1/2*d
*x + 1/2*c) + b^5*tan(1/2*d*x + 1/2*c) + a^5 + 2*a^3*b^2 + a*b^4)/((a*tan(1
/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)*a*b^5) + 2*(36*a^2*b*tan(
1/2*d*x + 1/2*c)^7 + 27*b^3*tan(1/2*d*x + 1/2*c)^7 + 96*a^3*tan(1/2*d*x + 1
/2*c)^6 + 144*a*b^2*tan(1/2*d*x + 1/2*c)^6 - 36*a^2*b*tan(1/2*d*x + 1/2*c)^
5 - 3*b^3*tan(1/2*d*x + 1/2*c)^5 - 288*a^3*tan(1/2*d*x + 1/2*c)^4 - 336*a*b
^2*tan(1/2*d*x + 1/2*c)^4 - 36*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 3*b^3*tan(1/2
*d*x + 1/2*c)^3 + 288*a^3*tan(1/2*d*x + 1/2*c)^2 + 304*a*b^2*tan(1/2*d*x +
1/2*c)^2 + 36*a^2*b*tan(1/2*d*x + 1/2*c) + 27*b^3*tan(1/2*d*x + 1/2*c) - 96
*a^3 - 112*a*b^2)/((tan(1/2*d*x + 1/2*c)^2 - 1)^4*b^5))/d
```

Mupad [B] (verification not implemented)

Time = 7.56 (sec) , antiderivative size = 2654, normalized size of antiderivative = 11.29

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] int(1/(cos(c + d*x)^7*(a + b*tan(c + d*x))^2),x)
```

```
[Out] -((9*a*b^5)/64 + (15*a^5*b)/8 + (b^6*sin(c + d*x))/8 + (115*a^3*b^3)/48 + (
3*b^6*sin(3*c + 3*d*x))/16 + (b^6*sin(5*c + 5*d*x))/16 + (a^6*cos(c + d*x)*
atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*25i)/4 + (5*a*b^5*cos(2*c
+ 2*d*x))/8 + (5*a^5*b*cos(2*c + 2*d*x))/2 + (5*a*b^5*cos(3*c + 3*d*x))/16
+ (25*a^5*b*cos(3*c + 3*d*x))/16 + (15*a*b^5*cos(4*c + 4*d*x))/64 + (5*a^5*
b*cos(4*c + 4*d*x))/8 + (a*b^5*cos(5*c + 5*d*x))/16 + (5*a^5*b*cos(5*c + 5*
d*x))/16 + (25*a^3*b^3*cos(c + d*x))/6 + (5*a^2*b^4*sin(c + d*x))/6 + (5*a^
```

4*b^2*sin(c + d*x))/8 + (a^6*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x)*25i)/8 + (a^6*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(5*c + 5*d*x)*5i)/8 + (10*a^3*b^3*cos(2*c + 2*d*x))/3 + (25*a^3*b^3*cos(3*c + 3*d*x))/12 + (15*a^3*b^3*cos(4*c + 4*d*x))/16 + (5*a^3*b^3*cos(5*c + 5*d*x))/12 + (95*a^2*b^4*sin(2*c + 2*d*x))/96 + (5*a^4*b^2*sin(2*c + 2*d*x))/8 + (5*a^2*b^4*sin(3*c + 3*d*x))/4 + (15*a^4*b^2*sin(3*c + 3*d*x))/16 + (25*a^2*b^4*sin(4*c + 4*d*x))/64 + (5*a^4*b^2*sin(4*c + 4*d*x))/16 + (5*a^2*b^4*sin(5*c + 5*d*x))/12 + (5*a^4*b^2*sin(5*c + 5*d*x))/16 + (5*a*b^5*cos(c + d*x))/8 + (25*a^5*b*cos(c + d*x))/8 + (a*b^5*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*sin(3*c + 3*d*x)*45i)/64 + (a^5*b*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*sin(3*c + 3*d*x)*15i)/8 + (a*b^5*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*sin(5*c + 5*d*x)*15i)/64 + (a^5*b*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*sin(5*c + 5*d*x)*5i)/8 + (a^3*b^3*sin(c + d*x)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*15i)/8 + (a^2*b^4*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x)*75i)/64 + (a^4*b^2*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x)*75i)/16 + (a^2*b^4*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(5*c + 5*d*x)*15i)/64 + (a^4*b^2*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(5*c + 5*d*x)*15i)/16 + (a^3*b^3*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*sin(3*c + 3*d*x)*45i)/16 + (a^3*b^3*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*sin(5*c + 5*d*x)*15i)/16 + (25*a^3*cos(c + d*x)*atanh((a^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 2*b^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + a*b*cos(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(a^5*cos(c/2 + (d*x)/2) + 2*b^5*sin(c/2 + (d*x)/2) + a*b^4*cos(c/2 + (d*x)/2) + 2*a^4*b*sin(c/2 + (d*x)/2) + 2*a^3*b^2*cos(c/2 + (d*x)/2) + 4*a^2*b^3*sin(c/2 + (d*x)/2)))*((a^2 + b^2)^3)^(1/2))/4 + (a*b^5*sin(c + d*x)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*15i)/32 + (a^5*b*sin(c + d*x)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*5i)/4 + (25*a^3*atanh((a^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 2*b^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + a*b*cos(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(a^5*cos(c/2 + (d*x)/2) + 2*b^5*sin(c/2 + (d*x)/2) + a*b^4*cos(c/2 + (d*x)/2) + 2*a^4*b*sin(c/2 + (d*x)/2) + 2*a^3*b^2*cos(c/2 + (d*x)/2) + 4*a^2*b^3*sin(c/2 + (d*x)/2)))*cos(3*c + 3*d*x)*((a^2 + b^2)^3)^(1/2))/8 + (5*a^3*atanh((a^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 2*b^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + a*b*cos(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(a^5*cos(c/2 + (d*x)/2) + 2*b^5*sin(c/2 + (d*x)/2) + a*b^4*cos(c/2 + (d*x)/2) + 2*a^4*b*sin(c/2 + (d*x)/2) + 2*a^3*b^2*cos(c/2 + (d*x)/2) + 4*a^2*b^3*sin(c/2 + (d*x)/2)))*cos(5*c + 5*d*x)*((a^2 + b^2)^3)^(1/2))/8 + (a^2*b^4*cos(c + d*x)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*75i)/32 + (a^4*b^2*cos(c + d*x)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*75i)/8 + (15*a^2*b*atanh((a^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 2*b^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + a*b*cos(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4

```
+ 3*a^4*b^2)^(1/2))/(a^5*cos(c/2 + (d*x)/2) + 2*b^5*sin(c/2 + (d*x)/2) + a*
b^4*cos(c/2 + (d*x)/2) + 2*a^4*b*sin(c/2 + (d*x)/2) + 2*a^3*b^2*cos(c/2 + (
d*x)/2) + 4*a^2*b^3*sin(c/2 + (d*x)/2)))*sin(3*c + 3*d*x)*((a^2 + b^2)^3)^(
1/2))/8 + (5*a^2*b*atanh((a^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3
*a^4*b^2)^(1/2) + 2*b^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b
^2)^(1/2) + a*b*cos(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^(1/2
))/(a^5*cos(c/2 + (d*x)/2) + 2*b^5*sin(c/2 + (d*x)/2) + a*b^4*cos(c/2 + (d*
x)/2) + 2*a^4*b*sin(c/2 + (d*x)/2) + 2*a^3*b^2*cos(c/2 + (d*x)/2) + 4*a^2*b
^3*sin(c/2 + (d*x)/2)))*sin(5*c + 5*d*x)*((a^2 + b^2)^3)^(1/2))/8 + (5*a^2*
b*atanh((a^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^(1/2) +
 2*b^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + a*b*c
os(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(a^5*cos(c/2 +
 (d*x)/2) + 2*b^5*sin(c/2 + (d*x)/2) + a*b^4*cos(c/2 + (d*x)/2) + 2*a^4*b*s
in(c/2 + (d*x)/2) + 2*a^3*b^2*cos(c/2 + (d*x)/2) + 4*a^2*b^3*sin(c/2 + (d*x
)/2)))*sin(c + d*x)*((a^2 + b^2)^3)^(1/2))/4)/(a*b^6*d*((5*a*cos(c + d*x))/
8 + (b*sin(c + d*x))/8 + (5*a*cos(3*c + 3*d*x))/16 + (a*cos(5*c + 5*d*x))/1
6 + (3*b*sin(3*c + 3*d*x))/16 + (b*sin(5*c + 5*d*x))/16))
```

3.561 $\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 176

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{3(2a^2+b^2)\operatorname{arcsinh}(\tan(c+dx))\sec(c+dx)}{2b^4d\sqrt{\sec^2(c+dx)}} + \frac{3a\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\sec(c+dx)}{b^4d\sqrt{\sec^2(c+dx)}} - \frac{3\sec(c+dx)(2a-b\tan(c+dx))}{2b^3d} - \frac{\sec^3(c+dx)}{bd(a+b\tan(c+dx))}$$

```
[Out] 3/2*(2*a^2+b^2)*arcsinh(tan(d*x+c))*sec(d*x+c)/b^4/d/(sec(d*x+c)^2)^(1/2)+3
*a*arctanh((b-a*tan(d*x+c))/(a^2+b^2)^(1/2)/(sec(d*x+c)^2)^(1/2))*sec(d*x+c
)*(a^2+b^2)^(1/2)/b^4/d/(sec(d*x+c)^2)^(1/2)-3/2*sec(d*x+c)*(2*a-b*tan(d*x+
c))/b^3/d-sec(d*x+c)^3/b/d/(a+b*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3593, 747, 829, 858, 221, 739, 212}

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{3(2a^2+b^2)\sec(c+dx)\operatorname{arcsinh}(\tan(c+dx))}{2b^4d\sqrt{\sec^2(c+dx)}} + \frac{3a\sqrt{a^2+b^2}\sec(c+dx)\operatorname{arctanh}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)}{b^4d\sqrt{\sec^2(c+dx)}} - \frac{3\sec(c+dx)(2a-b\tan(c+dx))}{2b^3d} - \frac{\sec^3(c+dx)}{bd(a+b\tan(c+dx))}$$

```
[In] Int[Sec[c + d*x]^5/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (3*(2*a^2 + b^2)*ArcSinh[Tan[c + d*x]]*Sec[c + d*x])/(2*b^4*d*Sqrt[Sec[c +
d*x]^2]) + (3*a*Sqrt[a^2 + b^2]*ArcTanh[(b - a*Tan[c + d*x])/(Sqrt[a^2 + b^
2]*Sqrt[Sec[c + d*x]^2])]*Sec[c + d*x])/(b^4*d*Sqrt[Sec[c + d*x]^2]) - (3*S
ec[c + d*x]*(2*a - b*Tan[c + d*x]))/(2*b^3*d) - Sec[c + d*x]^3/(b*d*(a + b*
Tan[c + d*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 747

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 1))), x] - Dist[2*c*(p/(e*(m + 1))
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\sec(c+dx)\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^{3/2}}{(a+x)^2}\,dx, x, b\tan(c+dx)\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{\sec^3(c+dx)}{bd(a+b\tan(c+dx))} + \frac{(3\sec(c+dx))\text{Subst}\left(\int \frac{x\sqrt{1+\frac{x^2}{b^2}}}{a+x}\,dx, x, b\tan(c+dx)\right)}{b^3d\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{3\sec(c+dx)(2a-b\tan(c+dx))}{2b^3d} - \frac{\sec^3(c+dx)}{bd(a+b\tan(c+dx))} + \frac{(3\sec(c+dx))\text{Subst}\left(\int \frac{-\frac{a}{b^2}+\frac{(2a^2+b^2)x}{b^4}}{(a+x)\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+dx)\right)}{2bd\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{3\sec(c+dx)(2a-b\tan(c+dx))}{2b^3d} - \frac{\sec^3(c+dx)}{bd(a+b\tan(c+dx))} - \frac{(3a(a^2+b^2)\sec(c+dx))\,\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+dx)\right)}{b^5d\sqrt{\sec^2(c+dx)}} + \frac{(3(2a^2+b^2)\sec(c+dx))\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+dx)\right)}{2b^5d\sqrt{\sec^2(c+dx)}}$$

$$= \frac{3(2a^2+b^2)\,\text{arcsinh}(\tan(c+dx))\sec(c+dx)}{2b^4d\sqrt{\sec^2(c+dx)}} - \frac{3\sec(c+dx)(2a-b\tan(c+dx))}{2b^3d} - \frac{\sec^3(c+dx)}{bd(a+b\tan(c+dx))} + \frac{(3a(a^2+b^2)\sec(c+dx))\,\text{Subst}\left(\int \frac{1}{1+\frac{a^2}{b^2}-x^2}\,dx, x, \frac{1-\frac{a\tan(c+dx)}{b}}{\sqrt{\sec^2(c+dx)}}\right)}{b^5d\sqrt{\sec^2(c+dx)}}$$

$$= \frac{3(2a^2+b^2)\operatorname{arcsinh}(\tan(c+dx))\sec(c+dx)}{2b^4d\sqrt{\sec^2(c+dx)}} + \frac{3a\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{b\left(1-\frac{a\tan(c+dx)}{b}\right)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\sec(c+dx)}{b^4d\sqrt{\sec^2(c+dx)}} - \frac{3\sec(c+dx)(2a-b\tan(c+dx))}{2b^3d} - \frac{\sec^3(c+dx)}{bd(a+b\tan(c+dx))}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.58 (sec) , antiderivative size = 709, normalized size of antiderivative = 4.03

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{(a-ib)(a+ib)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))}{b^3d(a+b\tan(c+dx))^2} - \frac{2a\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{b^3d(a+b\tan(c+dx))^2} - \frac{6a\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{\sqrt{a^2+b^2}\left(-b\cos\left(\frac{1}{2}(c+dx)\right)+a\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^2\cos\left(\frac{1}{2}(c+dx)\right)+b^2\cos\left(\frac{1}{2}(c+dx)\right)}\right)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{b^4d(a+b\tan(c+dx))^2} - \frac{3(2a^2+b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{2b^4d(a+b\tan(c+dx))^2} + \frac{3(2a^2+b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{2b^4d(a+b\tan(c+dx))^2} + \frac{\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{4b^2d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a+b\tan(c+dx))^2} - \frac{2a\sec^2(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a\cos(c+dx)+b\sin(c+dx))^2}{b^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^2} - \frac{\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{4b^2d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a+b\tan(c+dx))^2} + \frac{2a\sec^2(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a\cos(c+dx)+b\sin(c+dx))^2}{b^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^2}$$

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((a - I*b)*(a + I*b)*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x]))/(b
^3*d*(a + b*Tan[c + d*x])^2)) - (2*a*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin
[c + d*x])^2)/(b^3*d*(a + b*Tan[c + d*x])^2) - (6*a*Sqrt[a^2 + b^2]*ArcTanh
[(Sqrt[a^2 + b^2]*(-(b*Cos[(c + d*x)/2]) + a*Sin[(c + d*x)/2]))/(a^2*Cos[(c
 + d*x)/2] + b^2*Cos[(c + d*x)/2])]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[
c + d*x])^2)/(b^4*d*(a + b*Tan[c + d*x])^2) - (3*(2*a^2 + b^2)*Log[Cos[(c +
```

```
d*x)/2] - Sin[(c + d*x)/2]]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x
])^2)/(2*b^4*d*(a + b*Tan[c + d*x])^2) + (3*(2*a^2 + b^2)*Log[Cos[(c + d*x)
/2] + Sin[(c + d*x)/2]]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)
/(2*b^4*d*(a + b*Tan[c + d*x])^2) + (Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin
[c + d*x])^2)/(4*b^2*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(a + b*Tan[c
 + d*x])^2) - (2*a*Sec[c + d*x]^2*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin[
c + d*x])^2)/(b^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(a + b*Tan[c + d*
x])^2) - (Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(4*b^2*d*(Cos
[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a + b*Tan[c + d*x])^2) + (2*a*Sec[c +
d*x]^2*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(b^3*d*(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^2)
```

Maple [A] (verified)

Time = 32.08 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.47

method	result
derivativedivides	$\frac{\frac{1}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-4a-b}{2b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-6a^2-3b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2b^4}-\frac{1}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{4a-b}{2b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}}{d}$
default	$\frac{\frac{1}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-4a-b}{2b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-6a^2-3b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2b^4}-\frac{1}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{4a-b}{2b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}}{d}$
risch	$-\frac{-3iab\,e^{5i(dx+c)}+6a^2e^{5i(dx+c)}+3b^2e^{5i(dx+c)}+12a^2e^{3i(dx+c)}+2b^2e^{3i(dx+c)}+3iab\,e^{i(dx+c)}+6a^2e^{i(dx+c)}+3b^2e^{i(dx+c)}}{\left(e^{2i(dx+c)}+1\right)^2\left(-ib\,e^{2i(dx+c)}+a\,e^{2i(dx+c)}+ib+a\right)d\,b^3}$

```
[In] int(sec(d*x+c)^5/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/b^2/(tan(1/2*d*x+1/2*c)-1)^2-1/2*(-4*a-b)/b^3/(tan(1/2*d*x+1/2*c)-
1)+1/2/b^4*(-6*a^2-3*b^2)*ln(tan(1/2*d*x+1/2*c)-1)-1/2/b^2/(tan(1/2*d*x+1/2
*c)+1)^2-1/2*(4*a-b)/b^3/(tan(1/2*d*x+1/2*c)+1)+1/2/b^4*(6*a^2+3*b^2)*ln(ta
n(1/2*d*x+1/2*c)+1)+2/b^4*(((a^2+b^2)*b^2/a*tan(1/2*d*x+1/2*c)+b*(a^2+b^2))
/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)-3*a*(a^2+b^2)^(1/2)*arct
anh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(163) = 326$.

Time = 0.32 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.02

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{6\,ab^2\cos(dx+c)\sin(dx+c) - 2\,b^3 + 6\,(2\,a^2b+b^3)\cos(dx+c)^2 - 6\,\left(a^2\cos(dx+c)^3 + ab\cos(dx+c\right.}{}$$

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/4*(6*a*b^2*cos(d*x + c)*sin(d*x + c) - 2*b^3 + 6*(2*a^2*b + b^3)*cos(d*x
 + c)^2 - 6*(a^2*cos(d*x + c)^3 + a*b*cos(d*x + c)^2*sin(d*x + c))*sqrt(a^2
 + b^2)*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 -
 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*
cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - 3*((2*a^3
+ a*b^2)*cos(d*x + c)^3 + (2*a^2*b + b^3)*cos(d*x + c)^2*sin(d*x + c))*log(
sin(d*x + c) + 1) + 3*((2*a^3 + a*b^2)*cos(d*x + c)^3 + (2*a^2*b + b^3)*cos
(d*x + c)^2*sin(d*x + c))*log(-sin(d*x + c) + 1))/(a*b^4*d*cos(d*x + c)^3 +
 b^5*d*cos(d*x + c)^2*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

```
[In] integrate(sec(d*x+c)**5/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**5/(a + b*tan(c + d*x))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 471 vs. $2(163) = 326$.

Time = 0.33 (sec) , antiderivative size = 471, normalized size of antiderivative = 2.68

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{2\left(6\,a^3+2\,ab^2+\frac{6\,a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{(9\,a^2b+2\,b^3)\sin(dx+c)}{\cos(dx+c)+1}-\frac{6\,(2\,a^3+ab^2)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4\,(3\,a^2b+b^3)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{(3\,a^2b+2\,b^3)\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^2b^3+\frac{2\,ab^4\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\,a^2b^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4\,ab^4\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,a^2b^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{2\,ab^4\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{a^2b^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}} - \frac{6\sqrt{a^2}}{}}{2\,d}$$

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(6*a^3 + 2*a*b^2 + 6*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (9*a
^2*b + 2*b^3)*sin(d*x + c)/(cos(d*x + c) + 1) - 6*(2*a^3 + a*b^2)*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 - 4*(3*a^2*b + b^3)*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + (3*a^2*b + 2*b^3)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^2*b^3 +
 2*a*b^4*sin(d*x + c)/(cos(d*x + c) + 1) - 3*a^2*b^3*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 - 4*a*b^4*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*a^2*b^3*sin
(d*x + c)^4/(cos(d*x + c) + 1)^4 + 2*a*b^4*sin(d*x + c)^5/(cos(d*x + c) + 1
)^5 - a^2*b^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) - 6*sqrt(a^2 + b^2)*a*lo
g((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x
+ c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/b^4 - 3*(2*a^2 + b^2)*log(sin(d
*x + c)/(cos(d*x + c) + 1) + 1)/b^4 + 3*(2*a^2 + b^2)*log(sin(d*x + c)/(cos
(d*x + c) + 1) - 1)/b^4)/d
```

Giac [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.59

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{3\,(2\,a^2+b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^4} - \frac{3\,(2\,a^2+b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^4} + \frac{6\,(a^3+ab^2)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b^4} + \frac{2\,\left(b\tan\right.}{}}{2\,d}$$

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*(2*a^2 + b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 - 3*(2*a^2 + b^
2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 + 6*(a^3 + a*b^2)*log(abs(2*a*tan
(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) -
 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4) + 2*(b*tan(1/2*d*x + 1/2*c
)^3 + 4*a*tan(1/2*d*x + 1/2*c)^2 + b*tan(1/2*d*x + 1/2*c) - 4*a)/((tan(1/2*
d*x + 1/2*c)^2 - 1)^2*b^3) + 4*(a^2*b*tan(1/2*d*x + 1/2*c) + b^3*tan(1/2*d*
x + 1/2*c) + a^3 + a*b^2)/((a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/
2*c) - a)*a*b^3))/d
```

Mupad [B] (verification not implemented)

Time = 5.94 (sec) , antiderivative size = 585, normalized size of antiderivative = 3.32

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$
$$= \frac{\operatorname{atanh}\left(\frac{648\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{216\,a\,b^2+648\,a^3+\frac{432\,a^5}{b^2}}+\frac{432\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{432\,a^5+648\,a^3\,b^2+216\,a\,b^4}+\frac{216\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{216\,a+\frac{648\,a^3}{b^2}+\frac{432\,a^5}{b^4}}\right)(6\,a^2+3\,b^2)}{b^4\,d}$$
$$-\frac{\frac{2\,(3\,a^2+b^2)}{b^3}+\frac{6\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{b^3}-\frac{6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2(2\,a^2+b^2)}{b^3}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)(9\,a^2+2\,b^2)}{a\,b^2}-\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3(3\,a^2+b^2)}{a\,b^2}+\tan}{d\left(-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\right.}$$
$$-\frac{6\,a\operatorname{atanh}\left(\frac{432\,a^3\sqrt{a^2+b^2}}{432\,a^3\,b+\frac{432\,a^5}{b}+864\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+864\,a^2\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{864\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a^2+b^2}}{432\,a^3+\frac{432\,a^5}{b^2}+864\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{864\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{b}}-\right.}{b^4\,d}$$

```
[In] int(1/(cos(c + d*x)^5*(a + b*tan(c + d*x))^2),x)
```

```
[Out] (atanh((648*a^3*tan(c/2 + (d*x)/2))/(216*a*b^2 + 648*a^3 + (432*a^5)/b^2) +
 (432*a^5*tan(c/2 + (d*x)/2))/(216*a*b^4 + 432*a^5 + 648*a^3*b^2) + (216*a*
tan(c/2 + (d*x)/2))/(216*a + (648*a^3)/b^2 + (432*a^5)/b^4))*(6*a^2 + 3*b^2
))/(b^4*d) - ((2*(3*a^2 + b^2))/b^3 + (6*a^2*tan(c/2 + (d*x)/2)^4)/b^3 - (6
*tan(c/2 + (d*x)/2)^2*(2*a^2 + b^2))/b^3 + (tan(c/2 + (d*x)/2)*(9*a^2 + 2*b
^2))/(a*b^2) - (4*tan(c/2 + (d*x)/2)^3*(3*a^2 + b^2))/(a*b^2) + (tan(c/2 +
(d*x)/2)^5*(3*a^2 + 2*b^2))/(a*b^2))/(d*(a + 2*b*tan(c/2 + (d*x)/2) - 3*a*t
an(c/2 + (d*x)/2)^2 + 3*a*tan(c/2 + (d*x)/2)^4 - a*tan(c/2 + (d*x)/2)^6 - 4
*b*tan(c/2 + (d*x)/2)^3 + 2*b*tan(c/2 + (d*x)/2)^5)) - (6*a*atanh((432*a^3*
(a^2 + b^2)^(1/2))/(432*a^3*b + (432*a^5)/b + 864*a^4*tan(c/2 + (d*x)/2) +
864*a^2*b^2*tan(c/2 + (d*x)/2)) + (864*a^2*tan(c/2 + (d*x)/2)*(a^2 + b^2)^(
1/2))/(432*a^3 + (432*a^5)/b^2 + 864*a^2*b*tan(c/2 + (d*x)/2) + (864*a^4*ta
n(c/2 + (d*x)/2))/b) + (432*a^4*tan(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2))/(432*
a^5 + 432*a^3*b^2 + 864*a^4*b*tan(c/2 + (d*x)/2) + 864*a^2*b^3*tan(c/2 + (d
*x)/2)))*(a^2 + b^2)^(1/2))/(b^4*d)
```

3.562 $\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 91

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{b^2d} + \frac{a\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}\,d} - \frac{\sec(c+dx)}{bd(a+b\tan(c+dx))}$$

```
[Out] arctanh(sin(d*x+c))/b^2/d+a*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(
1/2))/b^2/d/(a^2+b^2)^(1/2)-sec(d*x+c)/b/d/(a+b*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.45, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3593, 747, 858, 221, 739, 212}

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{a\sec(c+dx)\operatorname{arctanh}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)}{b^2d\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}} - \frac{\sec(c+dx)}{bd(a+b\tan(c+dx))} + \frac{\sec(c+dx)\operatorname{arcsinh}(\tan(c+dx))}{b^2d\sqrt{\sec^2(c+dx)}}$$

```
[In] Int[Sec[c + d*x]^3/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (ArcSinh[Tan[c + d*x]]*Sec[c + d*x])/(b^2*d*Sqrt[Sec[c + d*x]^2]) + (a*ArcT
anh[(b - a*Tan[c + d*x])/(Sqrt[a^2 + b^2]*Sqrt[Sec[c + d*x]^2])]*Sec[c + d*
x])/(b^2*Sqrt[a^2 + b^2]*d*Sqrt[Sec[c + d*x]^2]) - Sec[c + d*x]/(b*d*(a + b
*Tan[c + d*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 747

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 1))), x] - Dist[2*c*(p/(e*(m + 1))
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\sec(c+dx)\text{Subst}\left(\int \frac{\sqrt{1+\frac{x^2}{b^2}}}{(a+x)^2}\,dx, x, b\tan(c+dx)\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{\sec(c+dx)}{bd(a+b\tan(c+dx))} + \frac{\sec(c+dx)\text{Subst}\left(\int \frac{x}{(a+x)\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+dx)\right)}{b^3d\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{\sec(c+dx)}{bd(a+b\tan(c+dx))} + \frac{\sec(c+dx)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+dx)\right)}{b^3d\sqrt{\sec^2(c+dx)}} - \frac{(a\sec(c+dx))\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+dx)\right)}{b^3d\sqrt{\sec^2(c+dx)}}$$

$$= \frac{\text{arcsinh}(\tan(c+dx))\sec(c+dx)}{b^2d\sqrt{\sec^2(c+dx)}} - \frac{\sec(c+dx)}{bd(a+b\tan(c+dx))} + \frac{(a\sec(c+dx))\text{Subst}\left(\int \frac{1}{1+\frac{a^2}{b^2}-x^2}\,dx, x, \frac{1-\frac{a\tan(c+dx)}{b}}{\sqrt{\sec^2(c+dx)}}\right)}{b^3d\sqrt{\sec^2(c+dx)}}$$

$$= \frac{\text{arcsinh}(\tan(c+dx))\sec(c+dx)}{b^2d\sqrt{\sec^2(c+dx)}} + \frac{a\text{arctanh}\left(\frac{b\left(1-\frac{a\tan(c+dx)}{b}\right)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\sec(c+dx)}{b^2\sqrt{a^2+b^2}d\sqrt{\sec^2(c+dx)}} - \frac{\sec(c+dx)}{bd(a+b\tan(c+dx))}$$

Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.32

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{2a\text{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} + \log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) - \log\left(\cos\left(\frac{1}{2}(c+dx)\right) + \sin\left(\frac{1}{2}(c-\right.\right.}{b^2d}$$

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -(((2*a*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2]
 + Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]] + (b*Sec[c + d*x])/(a + b*Tan[c + d*x]))/(b^2*d))
```

Maple [A] (verified)

Time = 7.86 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.48

method	result
derivativedivides	$\frac{\frac{2\left(\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}+b\right)}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}-\frac{2a\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}}{b^2}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}}{d}$
default	$\frac{\frac{2\left(\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}+b\right)}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}-\frac{2a\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}}{b^2}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}}{d}$
risch	$-\frac{2\,\mathrm{e}^{i(dx+c)}}{db\left(-ib\,\mathrm{e}^{2i(dx+c)}+a\,\mathrm{e}^{2i(dx+c)}+ib+a\right)}+\frac{a\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d\,b^2}-\frac{a\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d\,b^2}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right.}{d\,b^2}$

```
[In] int(sec(d*x+c)^3/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/b^2*((b^2/a*tan(1/2*d*x+1/2*c)+b)/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/
2*d*x+1/2*c)-a)-a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/
(a^2+b^2)^(1/2)))-1/b^2*ln(tan(1/2*d*x+1/2*c)-1)+1/b^2*ln(tan(1/2*d*x+1/2*c
)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(87) = 174.

Time = 0.31 (sec) , antiderivative size = 293, normalized size of antiderivative = 3.22

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx =$$

$$-\frac{2\,a^2b+2\,b^3-\left(a^2\cos\left(dx+c\right)+ab\sin\left(dx+c\right)\right)\sqrt{a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b}\right.}{}$$

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*b + 2*b^3 - (a^2*cos(d*x + c) + a*b*sin(d*x + c))*sqrt(a^2 + b^
2)*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^
2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d
*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - ((a^3 + a*b^2)*
cos(d*x + c) + (a^2*b + b^3)*sin(d*x + c))*log(sin(d*x + c) + 1) + ((a^3 +
a*b^2)*cos(d*x + c) + (a^2*b + b^3)*sin(d*x + c))*log(-sin(d*x + c) + 1))/(
(a^3*b^2 + a*b^4)*d*cos(d*x + c) + (a^2*b^3 + b^5)*d*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx$$

```
[In] integrate(sec(d*x+c)**3/(a+b*tan(d*x+c))**2,x)
[Out] Integral(sec(c + d*x)**3/(a + b*tan(c + d*x))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(87) = 174$.

Time = 0.30 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.33

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{2\left(a+\frac{b\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2b+\frac{2ab^2\sin(dx+c)}{\cos(dx+c)+1}-\frac{a^2b\sin(dx+c)^2}{(\cos(dx+c)+1)^2}} - \frac{a\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b^2} - \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^2} + \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b^2}}{d}$$

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
[Out] -(2*(a + b*sin(d*x + c)/(cos(d*x + c) + 1))/(a^2*b + 2*a*b^2*sin(d*x + c)/(
cos(d*x + c) + 1) - a^2*b*sin(d*x + c)^2/(cos(d*x + c) + 1)^2) - a*log((b -
 a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c)/(
cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^2) - log(sin(d*x +
 c)/(cos(d*x + c) + 1) + 1)/b^2 + log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/
b^2)/d
```

Giac [A] (verification not implemented)

none

Time = 0.48 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.82

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{a\log\left(\frac{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b-2\sqrt{a^2+b^2}\right|}{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b+2\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b^2} + \frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^2} - \frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^2} + \frac{2\left(b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\right.}}{d}$$

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (a*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(
1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^2) + log(ab
s(tan(1/2*d*x + 1/2*c) + 1))/b^2 - log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^2 +
 2*(b*tan(1/2*d*x + 1/2*c) + a)/((a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*
x + 1/2*c) - a)*a*b))/d
```

Mupad [B] (verification not implemented)

Time = 5.23 (sec) , antiderivative size = 383, normalized size of antiderivative = 4.21

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{b^2\,\sin\left(c+d\,x\right)-\frac{2\,\left(a^2\,\cos(c+d\,x)\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\sqrt{a^2+b^2}+a^3\,\mathrm{atan}\left(\frac{1\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+1\mathrm{i}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b+2\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^2}{a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2+b^2}+2\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2+b^2}}\right)\,\mathrm{co}}{\sqrt{a^2+b^2}}}{a\,b^2\,d}$$

```
[In] int(1/(cos(c + d*x)^3*(a + b*tan(c + d*x))^2),x)
```

```
[Out] -(b^2*sin(c + d*x) - (2*(a^3*atan((a^2*sin(c/2 + (d*x)/2)*1i + b^2*sin(c/2
+ (d*x)/2)*2i + a*b*cos(c/2 + (d*x)/2)*1i)/(a*cos(c/2 + (d*x)/2)*(a^2 + b^2
)^(1/2) + 2*b*sin(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2)))*cos(c + d*x)*1i + a^2*
cos(c + d*x)*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*(a^2 + b^2)^(1/2)
))/(a^2 + b^2)^(1/2) + (2*b*((a*(a^2 + b^2)^(1/2))/2 + (a*cos(c + d*x)*(a^2
 + b^2)^(1/2))/2 - a^2*atan((a^2*sin(c/2 + (d*x)/2)*1i + b^2*sin(c/2 + (d*x
)/2)*2i + a*b*cos(c/2 + (d*x)/2)*1i)/(a*cos(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2
) + 2*b*sin(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2)))*sin(c + d*x)*1i - a*sin(c +
d*x)*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*(a^2 + b^2)^(1/2)))/(a^2
+ b^2)^(1/2))/(a*b^2*d*(a*cos(c + d*x) + b*sin(c + d*x)))
```

3.563 $\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{a\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} - \frac{b\sec(c+dx)}{(a^2+b^2)\,d(a+b\tan(c+dx))}$$

```
[Out] -a*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/d-b
*sec(d*x+c)/(a^2+b^2)/d/(a+b*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.28, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3593, 745, 739, 212}

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{a\sec(c+dx)\text{arctanh}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)}{d\,(a^2+b^2)^{3/2}\sqrt{\sec^2(c+dx)}} - \frac{b\sec(c+dx)}{d\,(a^2+b^2)\,(a+b\tan(c+dx))}$$

```
[In] Int[Sec[c + d*x]/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] -((a*ArcTanh[(b - a*Tan[c + d*x])/(Sqrt[a^2 + b^2]*Sqrt[Sec[c + d*x]^2])]*S
ec[c + d*x])/((a^2 + b^2)^(3/2)*d*Sqrt[Sec[c + d*x]^2])) - (b*Sec[c + d*x])
/((a^2 + b^2)*d*(a + b*Tan[c + d*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\sec(c+dx)\text{Subst}\left(\int \frac{1}{(a+x)^2\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+dx)\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{b\sec(c+dx)}{(a^2+b^2)\,d(a+b\tan(c+dx))} + \frac{(a\sec(c+dx))\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+dx)\right)}{b\,(a^2+b^2)\,d\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{b\sec(c+dx)}{(a^2+b^2)\,d(a+b\tan(c+dx))} - \frac{(a\sec(c+dx))\text{Subst}\left(\int \frac{1}{1+\frac{a^2}{b^2}-x^2}\,dx, x, \frac{1-\frac{a\tan(c+dx)}{b}}{\sqrt{\sec^2(c+dx)}}\right)}{b\,(a^2+b^2)\,d\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{a\text{arctanh}\left(\frac{b\left(1-\frac{a\tan(c+dx)}{b}\right)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\sec(c+dx)}{(a^2+b^2)^{3/2}\,d\sqrt{\sec^2(c+dx)}} - \frac{b\sec(c+dx)}{(a^2+b^2)\,d(a+b\tan(c+dx))}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{2a\,\mathrm{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}} - \frac{b\sec(c+dx)}{(a^2+b^2)(a+b\tan(c+dx))}}{d}$$

```
[In] Integrate[Sec[c + d*x]/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] ((2*a*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2)
 - (b*Sec[c + d*x])/((a^2 + b^2)*(a + b*Tan[c + d*x])))/d
```

Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.44

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(a^2+b^2\right)}-\frac{b}{a^2+b^2}\right)}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}+\frac{2a\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}}{d}$
default	$\frac{-\frac{2\left(-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(a^2+b^2\right)}-\frac{b}{a^2+b^2}\right)}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}+\frac{2a\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}}{d}$
risch	$-\frac{2ib\,\mathrm{e}^{i(dx+c)}}{(-ia+b)d(ia+b)\left(b\,\mathrm{e}^{2i(dx+c)}+ia\,\mathrm{e}^{2i(dx+c)}-b+ia\right)}+\frac{a\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^3+ia\,b^2-a^2b-b^3}{\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}d}-\frac{a\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ia^3+ia\,b}{(a^2}\right.}{(a^2+b^2)^{\frac{3}{2}}d}$

```
[In] int(sec(d*x+c)/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-b^2/a/(a^2+b^2)*tan(1/2*d*x+1/2*c)-b/(a^2+b^2))/(tan(1/2*d*x+1/2*
c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)+2*a/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tan(1
/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(78) = 156$.

Time = 0.27 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.62

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{2\,a^2b+2\,b^3-\left(a^2\cos\left(dx+c\right)+ab\sin\left(dx+c\right)\right)\sqrt{a^2+b^2}\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2}\right.}{2\left((a^5+2\,a^3b^2+ab^4)d\cos\left(dx+c\right)+(a^4b+2\,a^2b^3+b^5)d\sin\left(dx+c\right)\right)}$$

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*b + 2*b^3 - (a^2*cos(d*x + c) + a*b*sin(d*x + c))*sqrt(a^2 + b^
2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a
^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(
d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)))/((a^5 + 2*a^3*b
^2 + a*b^4)*d*cos(d*x + c) + (a^4*b + 2*a^2*b^3 + b^5)*d*sin(d*x + c))
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\sec\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^2}\,dx$$

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*tan(c + d*x))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(78) = 156$.

Time = 0.36 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.22

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{a\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}}+\frac{2\left(ab+\frac{b^2\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4+a^2b^2+\frac{2\left(a^3b+ab^3\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{\left(a^4+a^2b^2\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}}{d}$$

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(a*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*si
n(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) + 2*(a*
b + b^2*sin(d*x + c)/(cos(d*x + c) + 1))/(a^4 + a^2*b^2 + 2*(a^3*b + a*b^3)
*sin(d*x + c)/(cos(d*x + c) + 1) - (a^4 + a^2*b^2)*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2))/d
```

Giac [A] (verification not implemented)

none

Time = 0.44 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.68

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{a\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}} - \frac{2\left(b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+ab\right)}{(a^3+ab^2)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)}}{d}$$

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(a*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan
(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(b^2*ta
n(1/2*d*x + 1/2*c) + a*b)/((a^3 + a*b^2)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b*ta
n(1/2*d*x + 1/2*c) - a)))/d
```

Mupad [B] (verification not implemented)

Time = 4.65 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.66

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{2\,b}{a^2+b^2}+\frac{2\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,(a^2+b^2)}}{d\,\left(-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)} + \frac{a\,\mathrm{atan}\left(\frac{a^2\,b\,1\mathrm{i}+b^3\,1\mathrm{i}-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a^2+b^2)\,1\mathrm{i}}{(a^2+b^2)^{3/2}}\right)\,2\mathrm{i}}{d\,(a^2+b^2)^{3/2}}$$

```
[In] int(1/(cos(c + d*x)*(a + b*tan(c + d*x))^2),x)
```

```
[Out] (a*atan((a^2*b*1i + b^3*1i - a*tan(c/2 + (d*x)/2)*(a^2 + b^2)*1i)/(a^2 + b^
2)^(3/2))*2i)/(d*(a^2 + b^2)^(3/2)) - ((2*b)/(a^2 + b^2) + (2*b^2*tan(c/2 +
 (d*x)/2))/(a*(a^2 + b^2)))/(d*(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d
*x)/2)^2))
```

3.564 $\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 157

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{3ab^2\text{arctanh}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\cos(c+dx)\sqrt{\sec^2(c+dx)}}{(a^2+b^2)^{5/2}\,d} + \frac{b(a^2-2b^2)\sec(c+dx)}{(a^2+b^2)^2\,d(a+b\tan(c+dx))} + \frac{\cos(c+dx)(b+a\tan(c+dx))}{(a^2+b^2)\,d(a+b\tan(c+dx))}$$

```
[Out] -3*a*b^2*arctanh((b-a*tan(d*x+c))/(a^2+b^2)^(1/2)/(sec(d*x+c)^2)^(1/2))*cos
(d*x+c)*(sec(d*x+c)^2)^(1/2)/(a^2+b^2)^(5/2)/d+b*(a^2-2*b^2)*sec(d*x+c)/(a^
2+b^2)^2/d/(a+b*tan(d*x+c))+cos(d*x+c)*(b+a*tan(d*x+c))/(a^2+b^2)/d/(a+b*ta
n(d*x+c))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3593, 755, 821, 739, 212}

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{3ab^2\cos(c+dx)\sqrt{\sec^2(c+dx)}\text{arctanh}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)}{d\,(a^2+b^2)^{5/2}} + \frac{\cos(c+dx)(a\tan(c+dx)+b)}{d\,(a^2+b^2)\,(a+b\tan(c+dx))} + \frac{b(a^2-2b^2)\sec(c+dx)}{d\,(a^2+b^2)^2\,(a+b\tan(c+dx))}$$

```
[In] Int[Cos[c + d*x]/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-3*a*b^2*ArcTanh[(b - a*Tan[c + d*x])/(Sqrt[a^2 + b^2]*Sqrt[Sec[c + d*x]^2
])]*Cos[c + d*x]*Sqrt[Sec[c + d*x]^2])/((a^2 + b^2)^(5/2)*d) + (b*(a^2 - 2*
b^2)*Sec[c + d*x])/((a^2 + b^2)^2*d*(a + b*Tan[c + d*x])) + (Cos[c + d*x]*(
b + a*Tan[c + d*x]))/((a^2 + b^2)*d*(a + b*Tan[c + d*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\left(\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{1}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^{3/2}}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\cos(c+dx)(b+a\tan(c+dx))}{(a^2+b^2)\,d(a+b\tan(c+dx))} - \frac{\left(b\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{-2-\frac{ax}{b^2}}{(a+x)^2\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+dx)\right)}{(a^2+b^2)\,d}$$

$$= \frac{b(a^2-2b^2)\sec(c+dx)}{(a^2+b^2)^2\,d(a+b\tan(c+dx))} + \frac{\cos(c+dx)(b+a\tan(c+dx))}{(a^2+b^2)\,d(a+b\tan(c+dx))} + \frac{\left(3ab\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+dx)\right)}{(a^2+b^2)^2\,d}$$

$$= \frac{b(a^2-2b^2)\sec(c+dx)}{(a^2+b^2)^2\,d(a+b\tan(c+dx))} + \frac{\cos(c+dx)(b+a\tan(c+dx))}{(a^2+b^2)\,d(a+b\tan(c+dx))} - \frac{\left(3ab\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{1}{1+\frac{a^2}{b^2}-x^2}\,dx, x, \frac{1-\frac{a\tan(c+dx)}{b}}{\sqrt{\sec^2(c+dx)}}\right)}{(a^2+b^2)^2\,d}$$

$$= -\frac{3ab^2\text{arctanh}\left(\frac{b\left(1-\frac{a\tan(c+dx)}{b}\right)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\cos(c+dx)\sqrt{\sec^2(c+dx)}}{(a^2+b^2)^{5/2}\,d} + \frac{b(a^2-2b^2)\sec(c+dx)}{(a^2+b^2)^2\,d(a+b\tan(c+dx))} + \frac{\cos(c+dx)(b+a\tan(c+dx))}{(a^2+b^2)\,d(a+b\tan(c+dx))}$$

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.97

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\sec(c+dx)\left(12ab^2\sqrt{a^2+b^2}\text{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)(a\cos(c+dx)+b\sin(c+dx))+(a^2+b^2)\left(3b(a^2-\right.\right.}{2\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))}$$

```
[In] Integrate[Cos[c + d*x]/(a + b*Tan[c + d*x])^2,x]
[Out] (Sec[c + d*x]*(12*a*b^2*Sqrt[a^2 + b^2]*ArcTanh[(-b + a*Tan[(c + d*x)/2])/S
qrt[a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d*x]) + (a^2 + b^2)*(3*b*(a^2 -
 b^2) + b*(a^2 + b^2)*Cos[2*(c + d*x)] + a*(a^2 + b^2)*Sin[2*(c + d*x)])))/
(2*(a^2 + b^2)^3*d*(a + b*Tan[c + d*x]))
```

Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{-\frac{2\left(\left(-a^{2}+b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2ab\right)}{\left(a^{4}+2a^{2}b^{2}+b^{4}\right)\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{2b^{2}\left(\frac{-\frac{b^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}-b}{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}-\frac{3a\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{2}+b^{2}\right)^{2}}}{d}$
default	$\frac{-\frac{2\left(\left(-a^{2}+b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2ab\right)}{\left(a^{4}+2a^{2}b^{2}+b^{4}\right)\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{2b^{2}\left(\frac{-\frac{b^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}-b}{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}-\frac{3a\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{2}+b^{2}\right)^{2}}}{d}$
risch	$-\frac{i\mathrm{e}^{i(dx+c)}}{2(-2iab+a^{2}-b^{2})d}+\frac{i\mathrm{e}^{-i(dx+c)}}{2(2iab+a^{2}-b^{2})d}-\frac{2ib^{3}\mathrm{e}^{i(dx+c)}}{(-ia+b)^{2}d(ia+b)^{2}\left(b\,\mathrm{e}^{2i(dx+c)}+ia\,\mathrm{e}^{2i(dx+c)}-b+ia\right)}+\frac{3b^{2}a\ln\left(\mathrm{e}^{i(dx+c)}\right.}{}$

[In] int(cos(d*x+c)/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(-2/(a^4+2*a^2*b^2+b^4)*((-a^2+b^2)*tan(1/2*d*x+1/2*c)-2*a*b)/(1+tan(1/2*d*x+1/2*c)^2)-2*b^2/(a^2+b^2)^2*((-b^2/a*tan(1/2*d*x+1/2*c)-b)/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)-3*a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))))

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.92

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{2\,a^4b-2\,a^2b^3-4\,b^5+2\,(a^4b+2\,a^2b^3+b^5)\cos{(dx+c)}^2+2\,(a^5+2\,a^3b^2+ab^4)\cos{(dx+c)}\sin{(dx+c)}+}{2\,((a^7+3\,a^5b^2+3\,a^3b^4+ab^6)d\cos}$$

[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c))^2,x, algorithm="fricas")

[Out] 1/2*(2*a^4*b - 2*a^2*b^3 - 4*b^5 + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(d*x + c)*sin(d*x + c) + 3*(a^2*b^2*cos(d*x + c) + a*b^3*sin(d*x + c))*sqrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)))/((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*d*cos(d*x + c) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*d*sin(d*x + c))

Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \int \frac{\cos{(c+dx)}}{(a+b\tan{(c+dx)})^2}\,dx$$

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)/(a + b*tan(c + d*x))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(151) = 302$.

Time = 0.42 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.22

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{3\,ab^2\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} - \frac{2\left(2\,a^3b-ab^3-\frac{3\,ab^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{(a^4+3\,a^2b^2-b^4)\sin(dx+c)}{\cos(dx+c)+1}-\frac{(a^4-a^2b^2+b^4)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^6+2\,a^4b^2+a^2b^4+\frac{2\,(a^5b+2\,a^3b^3+ab^5)\sin(dx+c)}{\cos(dx+c)+1}+\frac{2\,(a^5b+2\,a^3b^3+ab^5)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{(a^6+2\,a^4b^2+a^2b^4)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}}{d}$$

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(3*a*b^2*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b
- a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 +
 b^4)*sqrt(a^2 + b^2)) - 2*(2*a^3*b - a*b^3 - 3*a*b^3*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + (a^4 + 3*a^2*b^2 - b^4)*sin(d*x + c)/(cos(d*x + c) + 1) -
(a^4 - a^2*b^2 + b^4)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a^6 + 2*a^4*b^2
 + a^2*b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*sin(d*x + c)/(cos(d*x + c) + 1)
+ 2*(a^5*b + 2*a^3*b^3 + a*b^5)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - (a^6
+ 2*a^4*b^2 + a^2*b^4)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4))/d
```

Giac [A] (verification not implemented)

none

Time = 0.51 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.82

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{3\,ab^2\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} - \frac{2\left(a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\cdots\right.}{(a^5+2\,a^3b^2+ab^4)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-2\,b\tan\left(\frac{1}{2}\,d\cdots\right.\right.}}{d}$$

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(3*a*b^2*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2
*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4
)*sqrt(a^2 + b^2)) - 2*(a^4*tan(1/2*d*x + 1/2*c)^3 - a^2*b^2*tan(1/2*d*x +
1/2*c)^3 + b^4*tan(1/2*d*x + 1/2*c)^3 + 3*a*b^3*tan(1/2*d*x + 1/2*c)^2 - a^
4*tan(1/2*d*x + 1/2*c) - 3*a^2*b^2*tan(1/2*d*x + 1/2*c) + b^4*tan(1/2*d*x +
 1/2*c) - 2*a^3*b + a*b^3)/((a^5 + 2*a^3*b^2 + a*b^4)*(a*tan(1/2*d*x + 1/2*
c)^4 - 2*b*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c) - a)))/d
```

Mupad [B] (verification not implemented)

Time = 6.54 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.82

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{4\,a^2\,b-2\,b^3}{a^4+2\,a^2\,b^2+b^4}-\frac{6\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{a^4+2\,a^2\,b^2+b^4}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^4+3\,a^2\,b^2-b^4\right)}{a\,\left(a^4+2\,a^2\,b^2+b^4\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(2\,a^4-2\,a^2\,b^2+2\,b^4\right)}{a\,\left(a^4+2\,a^2\,b^2+b^4\right)}}{d\,\left(-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)} - \frac{6\,a\,b^2\,\mathrm{atanh}\left(\frac{a^4\,b+b^5+2\,a^2\,b^3-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}{{\left(a^2+b^2\right)}^{5/2}}\right)}{d\,{\left(a^2+b^2\right)}^{5/2}}$$

```
[In] int(cos(c + d*x)/(a + b*tan(c + d*x))^2,x)
```

```
[Out] ((4*a^2*b - 2*b^3)/(a^4 + b^4 + 2*a^2*b^2) - (6*b^3*tan(c/2 + (d*x)/2)^2)/(
a^4 + b^4 + 2*a^2*b^2) + (2*tan(c/2 + (d*x)/2)*(a^4 - b^4 + 3*a^2*b^2))/(a*
(a^4 + b^4 + 2*a^2*b^2)) - (tan(c/2 + (d*x)/2)^3*(2*a^4 + 2*b^4 - 2*a^2*b^2
))/(a*(a^4 + b^4 + 2*a^2*b^2)))/(d*(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2
+ (d*x)/2)^4 + 2*b*tan(c/2 + (d*x)/2)^3)) - (6*a*b^2*atanh((a^4*b + b^5 + 2
*a^2*b^3 - a*tan(c/2 + (d*x)/2)*(a^4 + b^4 + 2*a^2*b^2))/(a^2 + b^2)^(5/2))
)/(d*(a^2 + b^2)^(5/2))
```

3.565 $\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 241

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{5ab^4\text{arctanh}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\cos(c+dx)\sqrt{\sec^2(c+dx)}}{(a^2+b^2)^{7/2}\,d} + \frac{b(2a^4+9a^2b^2-8b^4)\sec(c+dx)}{3\,(a^2+b^2)^3\,d(a+b\tan(c+dx))} + \frac{\cos^3(c+dx)(b+a\tan(c+dx))}{3\,(a^2+b^2)\,d(a+b\tan(c+dx))} - \frac{\cos(c+dx)\,(b(a^2-4b^2)-a(2a^2+7b^2)\tan(c+dx))}{3\,(a^2+b^2)^2\,d(a+b\tan(c+dx))}$$

```
[Out] -5*a*b^4*arctanh((b-a*tan(d*x+c))/(a^2+b^2)^(1/2)/(sec(d*x+c)^2)^(1/2))*cos
(d*x+c)*(sec(d*x+c)^2)^(1/2)/(a^2+b^2)^(7/2)/d+1/3*b*(2*a^4+9*a^2*b^2-8*b^4
)*sec(d*x+c)/(a^2+b^2)^3/d/(a+b*tan(d*x+c))+1/3*cos(d*x+c)^3*(b+a*tan(d*x+c
))/(a^2+b^2)/d/(a+b*tan(d*x+c))-1/3*cos(d*x+c)*(b*(a^2-4*b^2)-a*(2*a^2+7*b^
2)*tan(d*x+c))/(a^2+b^2)^2/d/(a+b*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used

$= \{3593, 755, 837, 821, 739, 212\}$

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{5ab^4\cos(c+dx)\sqrt{\sec^2(c+dx)}\text{arctanh}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)}{d\left(a^2+b^2\right)^{7/2}} + \frac{\cos^3(c+dx)(a\tan(c+dx)+b)}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))} - \frac{\cos(c+dx)\left(b(a^2-4b^2)-a(2a^2+7b^2)\tan(c+dx)\right)}{3d\left(a^2+b^2\right)^2(a+b\tan(c+dx))} + \frac{b(2a^4+9a^2b^2-8b^4)\sec(c+dx)}{3d\left(a^2+b^2\right)^3(a+b\tan(c+dx))}$$

```
[In] Int[Cos[c + d*x]^3/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (-5*a*b^4*ArcTanh[(b - a*Tan[c + d*x])/(Sqrt[a^2 + b^2]*Sqrt[Sec[c + d*x]^2
])]*Cos[c + d*x]*Sqrt[Sec[c + d*x]^2])/((a^2 + b^2)^(7/2)*d) + (b*(2*a^4 +
9*a^2*b^2 - 8*b^4)*Sec[c + d*x])/(3*(a^2 + b^2)^3*d*(a + b*Tan[c + d*x])) +
 (Cos[c + d*x]^3*(b + a*Tan[c + d*x]))/(3*(a^2 + b^2)*d*(a + b*Tan[c + d*x]
)) - (Cos[c + d*x]*(b*(a^2 - 4*b^2) - a*(2*a^2 + 7*b^2)*Tan[c + d*x]))/(3*(
a^2 + b^2)^2*d*(a + b*Tan[c + d*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
```

```
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\left(\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{1}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^{5/2}}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\cos^3(c+dx)(b+a\tan(c+dx))}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))} - \frac{\left(b\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{-2\left(2+\frac{a^2}{b^2}\right)-\frac{3ax}{b^2}}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^{3/2}}\,dx, x, b\tan(c+dx)\right)}{3\left(a^2+b^2\right)d}$$

$$= \frac{\cos^3(c+dx)(b+a\tan(c+dx))}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))} - \frac{\cos(c+dx)\left(b\left(a^2-4b^2\right)-a\left(2a^2+7b^2\right)\tan(c+dx)\right)}{3\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))} + \frac{\left(b^5\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{-\frac{2\left(a^2-4b^2\right)}{b^4}+\frac{a\left(2a^2+7b^2\right)x}{b^6}}{(a+x)^2\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+dx)\right)}{3\left(a^2+b^2\right)^2 d}$$

$$= \frac{b(2a^4+9a^2b^2-8b^4)\sec(c+dx)}{3(a^2+b^2)^3 d(a+b\tan(c+dx))} + \frac{\cos^3(c+dx)(b+a\tan(c+dx))}{3(a^2+b^2)d(a+b\tan(c+dx))} - \frac{\cos(c+dx)\left(b(a^2-4b^2)-a(2a^2+7b^2)\tan(c+dx)\right)}{3(a^2+b^2)^2 d(a+b\tan(c+dx))} + \frac{\left(5ab^3\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+dx)\right)}{(a^2+b^2)^3 d}$$

$$= \frac{b(2a^4+9a^2b^2-8b^4)\sec(c+dx)}{3(a^2+b^2)^3 d(a+b\tan(c+dx))} + \frac{\cos^3(c+dx)(b+a\tan(c+dx))}{3(a^2+b^2)d(a+b\tan(c+dx))} - \frac{\cos(c+dx)\left(b(a^2-4b^2)-a(2a^2+7b^2)\tan(c+dx)\right)}{3(a^2+b^2)^2 d(a+b\tan(c+dx))} - \frac{\left(5ab^3\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{1}{1+\frac{a^2}{b^2}-x^2}\,dx, x, \frac{1-\frac{a\tan(c+dx)}{b}}{\sqrt{\sec^2(c+dx)}}\right)}{(a^2+b^2)^3 d}$$

$$= -\frac{5ab^4\text{arctanh}\left(\frac{b\left(1-\frac{a\tan(c+dx)}{b}\right)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\cos(c+dx)\sqrt{\sec^2(c+dx)}}{(a^2+b^2)^{7/2} d} + \frac{b(2a^4+9a^2b^2-8b^4)\sec(c+dx)}{3(a^2+b^2)^3 d(a+b\tan(c+dx))} + \frac{\cos^3(c+dx)(b+a\tan(c+dx))}{3(a^2+b^2)d(a+b\tan(c+dx))} - \frac{\cos(c+dx)\left(b(a^2-4b^2)-a(2a^2+7b^2)\tan(c+dx)\right)}{3(a^2+b^2)^2 d(a+b\tan(c+dx))}$$

Mathematica [A] (verified)

Time = 1.80 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.03

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\sec(c+dx)\left(240ab^4\sqrt{a^2+b^2}\text{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)(a\cos(c+dx)+b\sin(c+dx))+(a^2+b^2)\left(15a^4b\right.\right.}{}$$

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Tan[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]*(240*a*b^4*Sqrt[a^2 + b^2]*ArcTanh[(-b + a*Tan[(c + d*x)/2])/
Sqrt[a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d*x]) + (a^2 + b^2)*(15*a^4*b
+ 90*a^2*b^3 - 45*b^5 + 20*b^3*(a^2 + b^2)*Cos[2*(c + d*x)] + b*(a^2 + b^2)
^2*Cos[4*(c + d*x)] + 10*a^5*Sin[2*(c + d*x)] + 40*a^3*b^2*Sin[2*(c + d*x)]
 + 30*a*b^4*Sin[2*(c + d*x)] + a^5*Sin[4*(c + d*x)] + 2*a^3*b^2*Sin[4*(c +
d*x)] + a*b^4*Sin[4*(c + d*x)])))/(24*(a^2 + b^2)^4*d*(a + b*Tan[c + d*x]))
```

Maple [A] (verified)

Time = 8.25 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.33

method	result
derivativedivides	$-\frac{2\left(\left(-a^4-3a^2b^2+2b^4\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-2a^3b-6a\,b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{2}{3}a^4-6a^2b^2+\frac{8}{3}b^4\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8a\,b^3\left(\tan^2\left(\frac{dx}{2}\right.\right.\right.}{\left(a^2+b^2\right)\left(a^4+2a^2b^2+b^4\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$
default	$-\frac{2\left(\left(-a^4-3a^2b^2+2b^4\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-2a^3b-6a\,b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{2}{3}a^4-6a^2b^2+\frac{8}{3}b^4\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8a\,b^3\left(\tan^2\left(\frac{dx}{2}\right.\right.\right.}{\left(a^2+b^2\right)\left(a^4+2a^2b^2+b^4\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$
risch	$-\frac{ie^{3i(dx+c)}}{24(-2iab+a^2-b^2)d}-\frac{7\,\mathrm{e}^{i(dx+c)}b}{8(-3ib\,a^2+ib^3+a^3-3a\,b^2)d}-\frac{3ie^{i(dx+c)}a}{8(-3ib\,a^2+ib^3+a^3-3a\,b^2)d}-\frac{7\,\mathrm{e}^{-i(dx+c)}b}{8(ib+a)^3d}+\frac{3ie^{-i(dx+}}{8(ib+a)^3}$

[In] int(cos(d*x+c)^3/(a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(-2/(a^2+b^2)/(a^4+2*a^2*b^2+b^4)*((-a^4-3*a^2*b^2+2*b^4)*tan(1/2*d*x+1/2*c)^5+(-2*a^3*b-6*a*b^3)*tan(1/2*d*x+1/2*c)^4+(-2/3*a^4-6*a^2*b^2+8/3*b^4)*tan(1/2*d*x+1/2*c)^3-8*a*b^3*tan(1/2*d*x+1/2*c)^2+(-a^4-3*a^2*b^2+2*b^4)*tan(1/2*d*x+1/2*c)-2/3*a^3*b-14/3*a*b^3)/(1+tan(1/2*d*x+1/2*c)^2)^3-2*b^4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*((-b^2/a*tan(1/2*d*x+1/2*c)-b)/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)-5*a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))))

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.73

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{4\,a^6b+22\,a^4b^3+2\,a^2b^5-16\,b^7+2\left(a^6b+3\,a^4b^3+3\,a^2b^5+b^7\right)\cos\left(dx+c\right)^4-2\left(a^6b-2\,a^4b^3-7\,a^2b^5-\right.}{}$$

[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c))^2,x, algorithm="fricas")

[Out] 1/6*(4*a^6*b + 22*a^4*b^3 + 2*a^2*b^5 - 16*b^7 + 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cos(d*x + c)^4 - 2*(a^6*b - 2*a^4*b^3 - 7*a^2*b^5 - 4*b^7)*cos(d*x + c)^2 + 15*(a^2*b^4*cos(d*x + c) + a*b^5*sin(d*x + c))*sqrt(a^2 + b^

```
2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a
^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(
d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) + 2*((a^7 + 3*a^
5*b^2 + 3*a^3*b^4 + a*b^6)*cos(d*x + c)^3 + (2*a^7 + 11*a^5*b^2 + 16*a^3*b^
4 + 7*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*
a^3*b^6 + a*b^8)*d*cos(d*x + c) + (a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^
7 + b^9)*d*sin(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**3/(a+b*tan(d*x+c))**2,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 772 vs. $2(229) = 458$.

Time = 0.46 (sec) , antiderivative size = 772, normalized size of antiderivative = 3.20

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{15\,ab^4\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}} - \frac{2\left(2\,a^5b+14\,a^3b^3-3\,ab^5-\frac{15\,ab^5\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{\left(3\,a^6+13\,a^4b^2+22\,a^2b^4-3\,b^6\right)\sin(dx+c)}{\cos(dx+c)+1}+\cdots\right.}{a^8+3\,a^6b^2+3\,a^4b^4+a^2b^6+\frac{2\left(a^7b+3\,a^5b^3+3\,a^3b^5+ab^7\right)\sin(dx+c)}{\cos(dx+c)+1}+\frac{2\left(a^8+3\,a^6b^2+3\,a^4b^4+a^2b^6\right)\sin(dx+c)\cdots}{(\cos(dx+c)+1)^2}\cdots}}{\cdots}$$

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c))^2,x, algorithm="maxima")
[Out] -1/3*(15*a*b^4*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2)
)/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a^6 + 3*a^4*
b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 2*(2*a^5*b + 14*a^3*b^3 - 3*a*b^5
 - 15*a*b^5*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + (3*a^6 + 13*a^4*b^2 + 22*
a^2*b^4 - 3*b^6)*sin(d*x + c)/(cos(d*x + c) + 1) + (4*a^5*b + 28*a^3*b^3 -
21*a*b^5)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - (a^6 - 9*a^4*b^2 - 46*a^2*b
^4 + 9*b^6)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*(2*a^5*b + 6*a^3*b^3 -
5*a*b^5)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (a^6 + 3*a^4*b^2 + 38*a^2*b^
4 - 9*b^6)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*(a^6 + 3*a^4*b^2 - 2*a^2
*b^4 + b^6)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/(a^8 + 3*a^6*b^2 + 3*a^4*b
^4 + a^2*b^6 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*sin(d*x + c)/(cos(
d*x + c) + 1) + 2*(a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*sin(d*x + c)^2/(c
```

```
os(d*x + c) + 1)^2 + 6*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*sin(d*x + c)
^3/(cos(d*x + c) + 1)^3 + 6*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*sin(d*x
 + c)^5/(cos(d*x + c) + 1)^5 - 2*(a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^
7)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - (a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2
*b^6)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8))/d
```

Giac [A] (verification not implemented)

none

Time = 0.51 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.82

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx = -\frac{\frac{15\,ab^4\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}} - \frac{6\,(b^6\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)+ab^5)}{(a^7+3\,a^5b^2+3\,a^3b^4+ab^6)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)} - \frac{2\left(3\,a^4\tan\left(\frac{1}{2}\,dx\right.\right.}{}}{}$$

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(15*a*b^4*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/
abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 6*(b^6*tan(1/2*d*x + 1/2*c) + a*b^5)/
((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(
1/2*d*x + 1/2*c) - a)) - 2*(3*a^4*tan(1/2*d*x + 1/2*c)^5 + 9*a^2*b^2*tan(1/
2*d*x + 1/2*c)^5 - 6*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*a^3*b*tan(1/2*d*x + 1/2
*c)^4 + 18*a*b^3*tan(1/2*d*x + 1/2*c)^4 + 2*a^4*tan(1/2*d*x + 1/2*c)^3 + 18
*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 8*b^4*tan(1/2*d*x + 1/2*c)^3 + 24*a*b^3*t
an(1/2*d*x + 1/2*c)^2 + 3*a^4*tan(1/2*d*x + 1/2*c) + 9*a^2*b^2*tan(1/2*d*x
+ 1/2*c) - 6*b^4*tan(1/2*d*x + 1/2*c) + 2*a^3*b + 14*a*b^3)/((a^6 + 3*a^4*b
^2 + 3*a^2*b^4 + b^6)*(tan(1/2*d*x + 1/2*c)^2 + 1)^3))/d
```

Mupad [B] (verification not implemented)

Time = 8.30 (sec) , antiderivative size = 674, normalized size of antiderivative = 2.80

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^2}\,dx = \frac{\frac{2\,(2\,a^4\,b+14\,a^2\,b^3-3\,b^5)}{3\,(a^2+b^2)\,(a^4+2\,a^2\,b^2+b^4)} - \frac{10\,b^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6} + \frac{10\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4(2\,a^4+6\,a^2\,b^2-5\,b^4)}{3\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)} + \frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2(4\,a^4\,b+28\,a^2\,b^3-21\,b^5)}{3\,(a^2+b^2)\,(a^4+2\,a^2\,b^2+b^4)}}{d\,\left(-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-2\,a\tan\left(\frac{c}{2}+\right.\right.} - \frac{10\,a\,b^4\,\mathrm{atanh}\left(\frac{2\,a^6\,b+2\,b^7+6\,a^2\,b^5+6\,a^4\,b^3-2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)}{2\,(a^2+b^2)^{7/2}}\right)}{d\,(a^2+b^2)^{7/2}}$$

```
[In] int(cos(c + d*x)^3/(a + b*tan(c + d*x))^2,x)
```

```
[Out] ((2*(2*a^4*b - 3*b^5 + 14*a^2*b^3))/(3*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2))
 - (10*b^5*tan(c/2 + (d*x)/2)^6)/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (10*
b*tan(c/2 + (d*x)/2)^4*(2*a^4 - 5*b^4 + 6*a^2*b^2))/(3*(a^6 + b^6 + 3*a^2*b
^4 + 3*a^4*b^2)) + (2*tan(c/2 + (d*x)/2)^2*(4*a^4*b - 21*b^5 + 28*a^2*b^3))
/(3*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)) - (2*tan(c/2 + (d*x)/2)^7*(a^6 + b
^6 - 2*a^2*b^4 + 3*a^4*b^2))/(a*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (2*t
an(c/2 + (d*x)/2)*(3*a^6 - 3*b^6 + 22*a^2*b^4 + 13*a^4*b^2))/(3*a*(a^2 + b^
2)*(a^4 + b^4 + 2*a^2*b^2)) + (2*tan(c/2 + (d*x)/2)^5*(a^6 - 9*b^6 + 38*a^2
*b^4 + 3*a^4*b^2))/(3*a*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)) - (2*tan(c/2 +
 (d*x)/2)^3*(a^6 + 9*b^6 - 46*a^2*b^4 - 9*a^4*b^2))/(3*a*(a^2 + b^2)*(a^4 +
 b^4 + 2*a^2*b^2)))/(d*(a + 2*b*tan(c/2 + (d*x)/2) + 2*a*tan(c/2 + (d*x)/2)
^2 - 2*a*tan(c/2 + (d*x)/2)^6 - a*tan(c/2 + (d*x)/2)^8 + 6*b*tan(c/2 + (d*x
)/2)^3 + 6*b*tan(c/2 + (d*x)/2)^5 + 2*b*tan(c/2 + (d*x)/2)^7)) - (10*a*b^4*
atanh((2*a^6*b + 2*b^7 + 6*a^2*b^5 + 6*a^4*b^3 - 2*a*tan(c/2 + (d*x)/2)*(a^
6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))/(2*(a^2 + b^2)^(7/2))))/(d*(a^2 + b^2)^(7
/2))
```

3.566 $\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 185

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{3(a^2+b^2)(5a^2+b^2)\log(a+b\tan(c+dx))}{b^7d} - \frac{a(10a^2+9b^2)\tan(c+dx)}{b^6d} + \frac{3(2a^2+b^2)\tan^2(c+dx)}{2b^5d} - \frac{a\tan^3(c+dx)}{b^4d} + \frac{\tan^4(c+dx)}{4b^3d} - \frac{(a^2+b^2)^3}{2b^7d(a+b\tan(c+dx))^2} + \frac{6a(a^2+b^2)^2}{b^7d(a+b\tan(c+dx))}$$

```
[Out] 3*(a^2+b^2)*(5*a^2+b^2)*ln(a+b*tan(d*x+c))/b^7/d-a*(10*a^2+9*b^2)*tan(d*x+c
)/b^6/d+3/2*(2*a^2+b^2)*tan(d*x+c)^2/b^5/d-a*tan(d*x+c)^3/b^4/d+1/4*tan(d*x
+c)^4/b^3/d-1/2*(a^2+b^2)^3/b^7/d/(a+b*tan(d*x+c))^2+6*a*(a^2+b^2)^2/b^7/d/
(a+b*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used

$= \{3587, 711\}$

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{6a(a^2+b^2)^2}{b^7d(a+b\tan(c+dx))} - \frac{(a^2+b^2)^3}{2b^7d(a+b\tan(c+dx))^2} + \frac{3(a^2+b^2)(5a^2+b^2)\log(a+b\tan(c+dx))}{b^7d} - \frac{a(10a^2+9b^2)\tan(c+dx)}{b^6d} + \frac{3(2a^2+b^2)\tan^2(c+dx)}{2b^5d} - \frac{a\tan^3(c+dx)}{b^4d} + \frac{\tan^4(c+dx)}{4b^3d}$$

```
[In] Int[Sec[c + d*x]^8/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*(a^2 + b^2)*(5*a^2 + b^2)*Log[a + b*Tan[c + d*x]])/(b^7*d) - (a*(10*a^2
+ 9*b^2)*Tan[c + d*x])/(b^6*d) + (3*(2*a^2 + b^2)*Tan[c + d*x]^2)/(2*b^5*d)
 - (a*Tan[c + d*x]^3)/(b^4*d) + Tan[c + d*x]^4/(4*b^3*d) - (a^2 + b^2)^3/(2
*b^7*d*(a + b*Tan[c + d*x])^2) + (6*a*(a^2 + b^2)^2)/(b^7*d*(a + b*Tan[c +
d*x]))
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^3}{(a+x)^3}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{-10a^3-9ab^2}{b^6} + \frac{3(2a^2+b^2)x}{b^6} - \frac{3ax^2}{b^6} + \frac{x^3}{b^6} + \frac{(a^2+b^2)^3}{b^6(a+x)^3} - \frac{6a(a^2+b^2)^2}{b^6(a+x)^2} + \frac{3(5a^4+6a^2b^2+b^4)}{b^6(a+x)}\right)\,dx, x, b\tan(\right.}{bd}$$

$$= \frac{3(a^2+b^2)(5a^2+b^2)\log(a+b\tan(c+dx))}{b^7d} - \frac{a(10a^2+9b^2)\tan(c+dx)}{b^6d} + \frac{3(2a^2+b^2)\tan^2(c+dx)}{2b^5d} - \frac{a\tan^3(c+dx)}{b^4d} + \frac{\tan^4(c+dx)}{4b^3d} - \frac{(a^2+b^2)^3}{2b^7d(a+b\tan(c+dx))^2} + \frac{6a(a^2+b^2)^2}{b^7d(a+b\tan(c+dx))}$$

Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.47

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{2(a^2+b^2)\left(19a^4+16a^2b^2-3b^4+6a^2(5a^2+b^2)\log(a+b\tan(c+dx))\right)+b^6\sec^6(c+dx)+4ab(4a^4+17a$$

```
[In] Integrate[Sec[c + d*x]^8/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (2*(a^2 + b^2)*(19*a^4 + 16*a^2*b^2 - 3*b^4 + 6*a^2*(5*a^2 + b^2)*Log[a + b
*Tan[c + d*x]]) + b^6*Sec[c + d*x]^6 + 4*a*b*(4*a^4 + 17*a^2*b^2 + 11*b^4 +
 6*(5*a^4 + 6*a^2*b^2 + b^4)*Log[a + b*Tan[c + d*x]])*Tan[c + d*x] + 4*b^2*
(-13*a^4 - 10*a^2*b^2 + 3*(5*a^4 + 6*a^2*b^2 + b^4)*Log[a + b*Tan[c + d*x]]
)*Tan[c + d*x]^2 - 20*a*b^3*(a^2 + b^2)*Tan[c + d*x]^3 + 4*a^2*b^4*Tan[c +
d*x]^4 + b^4*Sec[c + d*x]^4*(a^2 + 3*b^2 - 2*a*b*Tan[c + d*x]))/(4*b^7*d*(a
 + b*Tan[c + d*x])^2)
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.05

$$\frac{\frac{\frac{(\tan^4(dx+c))b^3}{4}-a(\tan^3(dx+c))b^2+3a^2b(\tan^2(dx+c))+\frac{3b^3(\tan^2(dx+c))}{2}-10a^3\tan(dx+c)-9a\,b^2\tan(dx+c)}{b^6}+\frac{6a(a^4+2a^2b^2+b^4)}{b^7(a+b\tan(dx+c))}+\frac{(15a^4+18a}{}}{d}$$

```
[In] int(sec(d*x+c)^8/(a+b*tan(d*x+c))^3,x)
```

```
[Out] 1/d*(1/b^6*(1/4*tan(d*x+c)^4*b^3-a*tan(d*x+c)^3*b^2+3*a^2*b*tan(d*x+c)^2+3/
2*b^3*tan(d*x+c)^2-10*a^3*tan(d*x+c)-9*a*b^2*tan(d*x+c))+6*a/b^7*(a^4+2*a^2
*b^2+b^4)/(a+b*tan(d*x+c))+(15*a^4+18*a^2*b^2+3*b^4)/b^7*ln(a+b*tan(d*x+c))
-1/2/b^7*(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(a+b*tan(d*x+c))^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(179) = 358$.

Time = 0.31 (sec) , antiderivative size = 476, normalized size of antiderivative = 2.57

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{8\left(15\,a^4b^2+13\,a^2b^4\right)\cos\left(dx+c\right)^6+b^6-2\left(45\,a^4b^2+44\,a^2b^4+3\,b^6\right)\cos\left(dx+c\right)^4+\left(5\,a^2b^4+3\,b^6\right)\cos\left(dx\right.}{}$$

```
[In] integrate(sec(d*x+c)^8/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(8*(15*a^4*b^2 + 13*a^2*b^4)*cos(d*x + c)^6 + b^6 - 2*(45*a^4*b^2 + 44*
a^2*b^4 + 3*b^6)*cos(d*x + c)^4 + (5*a^2*b^4 + 3*b^6)*cos(d*x + c)^2 + 6*((
5*a^6 + a^4*b^2 - 5*a^2*b^4 - b^6)*cos(d*x + c)^6 + 2*(5*a^5*b + 6*a^3*b^3
+ a*b^5)*cos(d*x + c)^5*sin(d*x + c) + (5*a^4*b^2 + 6*a^2*b^4 + b^6)*cos(d*
x + c)^4)*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
+ b^2) - 6*((5*a^6 + a^4*b^2 - 5*a^2*b^4 - b^6)*cos(d*x + c)^6 + 2*(5*a^5*b
 + 6*a^3*b^3 + a*b^5)*cos(d*x + c)^5*sin(d*x + c) + (5*a^4*b^2 + 6*a^2*b^4
+ b^6)*cos(d*x + c)^4)*log(cos(d*x + c)^2) - 2*(a*b^5*cos(d*x + c) + 2*(15*
a^5*b - 2*a^3*b^3 - 13*a*b^5)*cos(d*x + c)^5 + 10*(a^3*b^3 + a*b^5)*cos(d*x
 + c)^3)*sin(d*x + c))/(2*a*b^8*d*cos(d*x + c)^5*sin(d*x + c) + b^9*d*cos(d
*x + c)^4 + (a^2*b^7 - b^9)*d*cos(d*x + c)^6)
```

Sympy [F]

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\sec^8{(c+dx)}}{(a+b\tan{(c+dx)})^3}\,dx$$

```
[In] integrate(sec(d*x+c)**8/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**8/(a + b*tan(c + d*x))**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.08

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{2\,(11\,a^6+21\,a^4b^2+9\,a^2b^4-b^6+12\,(a^5b+2\,a^3b^3+ab^5)\tan(dx+c))}{b^9\tan(dx+c)^2+2\,ab^8\tan(dx+c)+a^2b^7} + \frac{b^3\tan(dx+c)^4-4\,ab^2\tan(dx+c)^3+6\,(2\,a^2b+b^3)\tan(dx+c)^2-4\,(10\,a^3+9\,ab^2)}{b^6}}{4\,d}$$

```
[In] integrate(sec(d*x+c)^8/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(11*a^6 + 21*a^4*b^2 + 9*a^2*b^4 - b^6 + 12*(a^5*b + 2*a^3*b^3 + a*b
^5)*tan(d*x + c))/(b^9*tan(d*x + c)^2 + 2*a*b^8*tan(d*x + c) + a^2*b^7) + (
b^3*tan(d*x + c)^4 - 4*a*b^2*tan(d*x + c)^3 + 6*(2*a^2*b + b^3)*tan(d*x + c
)^2 - 4*(10*a^3 + 9*a*b^2)*tan(d*x + c))/b^6 + 12*(5*a^4 + 6*a^2*b^2 + b^4)
*log(b*tan(d*x + c) + a)/b^7)/d
```

Giac [A] (verification not implemented)

none

Time = 0.62 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.31

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{12\,(5\,a^4+6\,a^2b^2+b^4)\log(|b\tan(dx+c)+a|)}{b^7} - \frac{2\left(45\,a^4b^2\tan(dx+c)^2+54\,a^2b^4\tan(dx+c)^2+9\,b^6\tan(dx+c)^2+78\,a^5b\tan(dx+c)+84\,a^3b^3\tan(dx\cdots\right.}{(b\tan(dx+c)+a)^2b^7}}{}$$

```
[In] integrate(sec(d*x+c)^8/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4*(12*(5*a^4 + 6*a^2*b^2 + b^4)*log(abs(b*tan(d*x + c) + a))/b^7 - 2*(45*
a^4*b^2*tan(d*x + c)^2 + 54*a^2*b^4*tan(d*x + c)^2 + 9*b^6*tan(d*x + c)^2 +
 78*a^5*b*tan(d*x + c) + 84*a^3*b^3*tan(d*x + c) + 6*a*b^5*tan(d*x + c) + 3
4*a^6 + 33*a^4*b^2 + b^6)/((b*tan(d*x + c) + a)^2*b^7) + (b^9*tan(d*x + c)^
4 - 4*a*b^8*tan(d*x + c)^3 + 12*a^2*b^7*tan(d*x + c)^2 + 6*b^9*tan(d*x + c)
^2 - 40*a^3*b^6*tan(d*x + c) - 36*a*b^8*tan(d*x + c))/b^12)/d
```

Mupad [B] (verification not implemented)

Time = 4.49 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.26

$$\int \frac{\sec^8(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{11\,a^6+21\,a^4\,b^2+9\,a^2\,b^4-b^6}{2\,b}+\tan(c+d\,x)\,\left(6\,a^5+12\,a^3\,b^2+6\,a\,b^4\right)}{d\,\left(a^2\,b^6+2\,a\,b^7\tan\left(c+d\,x\right)+b^8\tan\left(c+d\,x\right)^2\right)} + \frac{\tan(c+d\,x)^2\,\left(\frac{3}{2\,b^3}+\frac{3\,a^2}{b^5}\right)}{d} + \frac{\tan(c+d\,x)^4}{4\,b^3\,d} + \frac{\tan(c+d\,x)\,\left(\frac{8\,a^3}{b^6}-\frac{3\,a\,\left(\frac{3}{b^3}+\frac{6\,a^2}{b^5}\right)}{b}\right)}{d} - \frac{a\tan(c+d\,x)^3}{b^4\,d} + \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(15\,a^4+18\,a^2\,b^2+3\,b^4\right)}{b^7\,d}$$

```
[In] int(1/(cos(c + d*x)^8*(a + b*tan(c + d*x))^3),x)
```

```
[Out] ((11*a^6 - b^6 + 9*a^2*b^4 + 21*a^4*b^2)/(2*b) + tan(c + d*x)*(6*a*b^4 + 6*
a^5 + 12*a^3*b^2))/(d*(a^2*b^6 + b^8*tan(c + d*x)^2 + 2*a*b^7*tan(c + d*x))
) + (tan(c + d*x)^2*(3/(2*b^3) + (3*a^2)/b^5))/d + tan(c + d*x)^4/(4*b^3*d)
 + (tan(c + d*x)*((8*a^3)/b^6 - (3*a*(3/b^3 + (6*a^2)/b^5))/b))/d - (a*tan(
c + d*x)^3)/(b^4*d) + (log(a + b*tan(c + d*x))*(15*a^4 + 3*b^4 + 18*a^2*b^2
))/(b^7*d)
```

3.567 $\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 121

$$\begin{aligned}\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx &= \frac{2(3a^2+b^2)\log(a+b\tan(c+dx))}{b^5d} \\ &\quad -\frac{3a\tan(c+dx)}{b^4d}+\frac{\tan^2(c+dx)}{2b^3d} \\ &\quad -\frac{(a^2+b^2)^2}{2b^5d(a+b\tan(c+dx))^2}+\frac{4a(a^2+b^2)}{b^5d(a+b\tan(c+dx))}\end{aligned}$$

```
[Out] 2*(3*a^2+b^2)*ln(a+b*tan(d*x+c))/b^5/d-3*a*tan(d*x+c)/b^4/d+1/2*tan(d*x+c)^
2/b^3/d-1/2*(a^2+b^2)^2/b^5/d/(a+b*tan(d*x+c))^2+4*a*(a^2+b^2)/b^5/d/(a+b*t
an(d*x+c))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3587, 711}

$$\begin{aligned}\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx &= -\frac{(a^2+b^2)^2}{2b^5d(a+b\tan(c+dx))^2}+\frac{4a(a^2+b^2)}{b^5d(a+b\tan(c+dx))} \\ &\quad +\frac{2(3a^2+b^2)\log(a+b\tan(c+dx))}{b^5d} \\ &\quad -\frac{3a\tan(c+dx)}{b^4d}+\frac{\tan^2(c+dx)}{2b^3d}\end{aligned}$$

```
[In] Int[Sec[c + d*x]^6/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (2*(3*a^2 + b^2)*Log[a + b*Tan[c + d*x]])/(b^5*d) - (3*a*Tan[c + d*x])/(b^4
*d) + Tan[c + d*x]^2/(2*b^3*d) - (a^2 + b^2)^2/(2*b^5*d*(a + b*Tan[c + d*x]
)^2) + (4*a*(a^2 + b^2))/(b^5*d*(a + b*Tan[c + d*x]))
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^2}{(a+x)^3}\,dx, x, b\tan(c+dx)\right)}{bd} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{3a}{b^4}+\frac{x}{b^4}+\frac{\left(a^2+b^2\right)^2}{b^4(a+x)^3}-\frac{4a\left(a^2+b^2\right)}{b^4(a+x)^2}+\frac{2\left(3a^2+b^2\right)}{b^4(a+x)}\right)\,dx, x, b\tan(c+dx)\right)}{bd} \\
&= \frac{2\left(3a^2+b^2\right)\log(a+b\tan(c+dx))}{b^5d}-\frac{3a\tan(c+dx)}{b^4d}+\frac{\tan^2(c+dx)}{2b^3d} \\
&\quad -\frac{\left(a^2+b^2\right)^2}{2b^5d(a+b\tan(c+dx))^2}+\frac{4a\left(a^2+b^2\right)}{b^5d(a+b\tan(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 4.39 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.16

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{b^4\sec^4(c+dx)}{2(a+b\tan(c+dx))^2}-2a\left(-2a\log(a+b\tan(c+dx))+b\tan(c+dx)-\frac{a^2+b^2}{a+b\tan(c+dx)}\right)+2\left(a^2+b^2\right)\left(\log(a+b\,\mathrm{t}\right.}{b^5d}$$

```
[In] Integrate[Sec[c + d*x]^6/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] ((b^4*Sec[c + d*x]^4)/(2*(a + b*Tan[c + d*x])^2) - 2*a*(-2*a*Log[a + b*Tan[
c + d*x]] + b*Tan[c + d*x] - (a^2 + b^2)/(a + b*Tan[c + d*x])) + 2*(a^2 + b
^2)*(Log[a + b*Tan[c + d*x]] + (3*a^2 - b^2 + 4*a*b*Tan[c + d*x])/(2*(a + b
*Tan[c + d*x])^2)))/(b^5*d)
```

Maple [A] (verified)

Time = 174.19 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{-\frac{-\frac{b\left(\tan^2(dx+c)\right)}{2}+3a\tan(dx+c)}{b^4}-\frac{a^4+2a^2b^2+b^4}{2b^5(a+b\tan(dx+c))^2}+\frac{\left(6a^2+2b^2\right)\ln(a+b\tan(dx+c))}{b^5}+\frac{4a\left(a^2+b^2\right)}{b^5(a+b\tan(dx+c))}}{d}$
default	$\frac{-\frac{-\frac{b\left(\tan^2(dx+c)\right)}{2}+3a\tan(dx+c)}{b^4}-\frac{a^4+2a^2b^2+b^4}{2b^5(a+b\tan(dx+c))^2}+\frac{\left(6a^2+2b^2\right)\ln(a+b\tan(dx+c))}{b^5}+\frac{4a\left(a^2+b^2\right)}{b^5(a+b\tan(dx+c))}}{d}$
risch	$\frac{-36a^2b\,e^{2i(dx+c)}+36ia^3e^{2i(dx+c)}+4ia\,b^2e^{6i(dx+c)}+12ia\,b^2e^{4i(dx+c)}+12a^2b\,e^{6i(dx+c)}+12ia^3e^{6i(dx+c)}+36ia^3e^{4i(dx+c)}-}{\left(e^{2i(dx+c)}+1\right)^2\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)^2b^4d}$

```
[In] int(sec(d*x+c)^6/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b^4*(-1/2*b*tan(d*x+c)^2+3*a*tan(d*x+c))-1/2/b^5*(a^4+2*a^2*b^2+b^4
)/(a+b*tan(d*x+c))^2+(6*a^2+2*b^2)/b^5*ln(a+b*tan(d*x+c))+4*a/b^5*(a^2+b^2)
/(a+b*tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. 2(117) = 234.

Time = 0.30 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.93

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{24\,a^2b^2\cos\left(dx+c\right)^4+b^4-2\left(9\,a^2b^2+b^4\right)\cos\left(dx+c\right)^2+2\left(\left(3\,a^4-2\,a^2b^2-b^4\right)\cos\left(dx+c\right)^4+2\left(3\,a^3b+\right.\right.}{}$$

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(24*a^2*b^2*cos(d*x + c)^4 + b^4 - 2*(9*a^2*b^2 + b^4)*cos(d*x + c)^2 +
 2*((3*a^4 - 2*a^2*b^2 - b^4)*cos(d*x + c)^4 + 2*(3*a^3*b + a*b^3)*cos(d*x
+ c)^3*sin(d*x + c) + (3*a^2*b^2 + b^4)*cos(d*x + c)^2)*log(2*a*b*cos(d*x +
 c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 2*((3*a^4 - 2*a^2*b^
2 - b^4)*cos(d*x + c)^4 + 2*(3*a^3*b + a*b^3)*cos(d*x + c)^3*sin(d*x + c) +
 (3*a^2*b^2 + b^4)*cos(d*x + c)^2)*log(cos(d*x + c)^2) - 4*(a*b^3*cos(d*x +
 c) + 3*(a^3*b - a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(2*a*b^6*d*cos(d*x +
c)^3*sin(d*x + c) + b^7*d*cos(d*x + c)^2 + (a^2*b^5 - b^7)*d*cos(d*x + c)^4
)
```

Sympy [F]

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

```
[In] integrate(sec(d*x+c)**6/(a+b*tan(d*x+c))**3,x)
[Out] Integral(sec(c + d*x)**6/(a + b*tan(c + d*x))**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.06

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{7a^4+6a^2b^2-b^4+8(a^3b+ab^3)\tan(dx+c)}{b^7\tan(dx+c)^2+2ab^6\tan(dx+c)+a^2b^5} + \frac{b\tan(dx+c)^2-6a\tan(dx+c)}{b^4} + \frac{4(3a^2+b^2)\log(b\tan(dx+c)+a)}{b^5}}{2d}$$

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
[Out] 1/2*((7*a^4 + 6*a^2*b^2 - b^4 + 8*(a^3*b + a*b^3)*tan(d*x + c))/(b^7*tan(d*
x + c)^2 + 2*a*b^6*tan(d*x + c) + a^2*b^5) + (b*tan(d*x + c)^2 - 6*a*tan(d*
x + c))/b^4 + 4*(3*a^2 + b^2)*log(b*tan(d*x + c) + a)/b^5)/d
```

Giac [A] (verification not implemented)

none

Time = 0.58 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.16

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{4(3a^2+b^2)\log(|b\tan(dx+c)+a|)}{b^5} + \frac{b^3\tan(dx+c)^2-6ab^2\tan(dx+c)}{b^6} - \frac{18a^2b^2\tan(dx+c)^2+6b^4\tan(dx+c)^2+28a^3b\tan(dx+c)+4ab^3\tan(dx}{(b\tan(dx+c)+a)^2b^5}}{2d}$$

```
[In] integrate(sec(d*x+c)^6/(a+b*tan(d*x+c))^3,x, algorithm="giac")
[Out] 1/2*(4*(3*a^2 + b^2)*log(abs(b*tan(d*x + c) + a))/b^5 + (b^3*tan(d*x + c)^2
 - 6*a*b^2*tan(d*x + c))/b^6 - (18*a^2*b^2*tan(d*x + c)^2 + 6*b^4*tan(d*x +
 c)^2 + 28*a^3*b*tan(d*x + c) + 4*a*b^3*tan(d*x + c) + 11*a^4 + b^4)/((b*ta
n(d*x + c) + a)^2*b^5))/d
```

Mupad [B] (verification not implemented)

Time = 4.73 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.18

$$\int \frac{\sec^6(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{7\,a^4+6\,a^2\,b^2-b^4}{2\,b}+\tan(c+d\,x)\,\left(4\,a^3+4\,a\,b^2\right)}{d\,\left(a^2\,b^4+2\,a\,b^5\tan\left(c+d\,x\right)+b^6\tan\left(c+d\,x\right)^2\right)} + \frac{\tan(c+d\,x)^2}{2\,b^3\,d} - \frac{3\,a\tan(c+d\,x)}{b^4\,d} + \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(6\,a^2+2\,b^2\right)}{b^5\,d}$$

```
[In] int(1/(cos(c + d*x)^6*(a + b*tan(c + d*x))^3),x)
```

```
[Out] ((7*a^4 - b^4 + 6*a^2*b^2)/(2*b) + tan(c + d*x)*(4*a*b^2 + 4*a^3))/(d*(a^2*
b^4 + b^6*tan(c + d*x)^2 + 2*a*b^5*tan(c + d*x))) + tan(c + d*x)^2/(2*b^3*d
) - (3*a*tan(c + d*x))/(b^4*d) + (log(a + b*tan(c + d*x))*(6*a^2 + 2*b^2))/
(b^5*d)
```

3.568 $\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 69

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^3} dx = \frac{\log(a+b\tan(c+dx))}{b^3d} - \frac{a^2+b^2}{2b^3d(a+b\tan(c+dx))^2} + \frac{2a}{b^3d(a+b\tan(c+dx))}$$

```
[Out] ln(a+b*tan(d*x+c))/b^3/d+1/2*(-a^2-b^2)/b^3/d/(a+b*tan(d*x+c))^2+2*a/b^3/d/
(a+b*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3587, 711}

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^3} dx = -\frac{a^2+b^2}{2b^3d(a+b\tan(c+dx))^2} + \frac{2a}{b^3d(a+b\tan(c+dx))} + \frac{\log(a+b\tan(c+dx))}{b^3d}$$

```
[In] Int[Sec[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] Log[a + b*Tan[c + d*x]]/(b^3*d) - (a^2 + b^2)/(2*b^3*d*(a + b*Tan[c + d*x])
^2) + (2*a)/(b^3*d*(a + b*Tan[c + d*x]))
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
```

```
x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1+\frac{x^2}{b^2}}{(a+x)^3}\,dx, x, b\tan(c+dx)\right)}{bd} \\
&= \frac{\text{Subst}\left(\int \left(\frac{a^2+b^2}{b^2(a+x)^3} - \frac{2a}{b^2(a+x)^2} + \frac{1}{b^2(a+x)}\right)\,dx, x, b\tan(c+dx)\right)}{bd} \\
&= \frac{\log(a+b\tan(c+dx))}{b^3d} - \frac{a^2+b^2}{2b^3d(a+b\tan(c+dx))^2} + \frac{2a}{b^3d(a+b\tan(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\log(a+b\tan(c+dx)) - \frac{a^2+b^2}{2(a+b\tan(c+dx))^2} + \frac{2a}{a+b\tan(c+dx)}}{b^3d}$$

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
[Out] (Log[a + b*Tan[c + d*x]] - (a^2 + b^2)/(2*(a + b*Tan[c + d*x])^2) + (2*a)/(
a + b*Tan[c + d*x]))/(b^3*d)
```

Maple [A] (verified)

Time = 37.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{\frac{\ln(a+b\tan(dx+c))}{b^3}-\frac{a^2+b^2}{2b^3(a+b\tan(dx+c))^2}+\frac{2a}{b^3(a+b\tan(dx+c))}}{d}$	63
default	$\frac{\frac{\ln(a+b\tan(dx+c))}{b^3}-\frac{a^2+b^2}{2b^3(a+b\tan(dx+c))^2}+\frac{2a}{b^3(a+b\tan(dx+c))}}{d}$	63
risch	$\frac{-2a^2e^{2i(dx+c)}+2b^2e^{2i(dx+c)}+4iab\,e^{2i(dx+c)}-2a^2-2iab}{b^2(ia+b)\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)^2d}+\frac{\ln\left(e^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)}{b^3d}-\frac{\ln\left(e^{2i(dx+c)}+1\right)}{b^3d}$	160

```
[In] int(sec(d*x+c)^4/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 1/d*(1/b^3*ln(a+b*tan(d*x+c))-1/2*(a^2+b^2)/b^3/(a+b*tan(d*x+c))^2+2/b^3*a/
(a+b*tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(67) = 134$.

Time = 0.27 (sec) , antiderivative size = 284, normalized size of antiderivative = 4.12

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{4\,a^2b^2\cos(dx+c)^2 - 3\,a^2b^2 - b^4 - 2\,(a^3b-ab^3)\cos(dx+c)\sin(dx+c) + \left(a^2b^2+b^4+(a^4-b^4)\cos(dx\right.}{}$$

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
[Out] 1/2*(4*a^2*b^2*cos(d*x + c)^2 - 3*a^2*b^2 - b^4 - 2*(a^3*b - a*b^3)*cos(d*x
 + c)*sin(d*x + c) + (a^2*b^2 + b^4 + (a^4 - b^4)*cos(d*x + c)^2 + 2*(a^3*b
 + a*b^3)*cos(d*x + c)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) +
(a^2 - b^2)*cos(d*x + c)^2 + b^2) - (a^2*b^2 + b^4 + (a^4 - b^4)*cos(d*x +
c)^2 + 2*(a^3*b + a*b^3)*cos(d*x + c)*sin(d*x + c))*log(cos(d*x + c)^2))/((
a^4*b^3 - b^7)*d*cos(d*x + c)^2 + 2*(a^3*b^4 + a*b^6)*d*cos(d*x + c)*sin(d*
x + c) + (a^2*b^5 + b^7)*d)
```

Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

```
[In] integrate(sec(d*x+c)**4/(a+b*tan(d*x+c))**3,x)
[Out] Integral(sec(c + d*x)**4/(a + b*tan(c + d*x))**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{4\,ab\tan(dx+c)+3\,a^2-b^2}{b^5\tan(dx+c)^2+2\,ab^4\tan(dx+c)+a^2b^3} + \frac{2\,\log(b\tan(dx+c)+a)}{b^3}}{2\,d}$$

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((4*a*b*tan(d*x + c) + 3*a^2 - b^2)/(b^5*tan(d*x + c)^2 + 2*a*b^4*tan(d
*x + c) + a^2*b^3) + 2*log(b*tan(d*x + c) + a)/b^3)/d
```

Giac [A] (verification not implemented)

none

Time = 0.56 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.90

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{2\,\log(|b\tan(dx+c)+a|)}{b^3} - \frac{3\,b\tan(dx+c)^2+2\,a\tan(dx+c)+b}{(b\tan(dx+c)+a)^2b^2}}{2\,d}$$

```
[In] integrate(sec(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*log(abs(b*tan(d*x + c) + a))/b^3 - (3*b*tan(d*x + c)^2 + 2*a*tan(d*x
 + c) + b)/((b*tan(d*x + c) + a)^2*b^2))/d
```

Mupad [B] (verification not implemented)

Time = 4.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.16

$$\int \frac{\sec^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{3\,a^2-b^2}{2\,b^3} + \frac{2\,a\tan(c+d\,x)}{b^2}}{d\,\left(a^2+2\,a\,b\tan\left(c+d\,x\right)+b^2\tan\left(c+d\,x\right)^2\right)} + \frac{\ln\left(a+b\tan(c+d\,x)\right)}{b^3\,d}$$

```
[In] int(1/(cos(c + d*x)^4*(a + b*tan(c + d*x))^3),x)
```

```
[Out] ((3*a^2 - b^2)/(2*b^3) + (2*a*tan(c + d*x))/b^2)/(d*(a^2 + b^2*tan(c + d*x)
^2 + 2*a*b*tan(c + d*x))) + log(a + b*tan(c + d*x))/(b^3*d)
```

3.569 $\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^3} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 22

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^3} \, dx = -\frac{1}{2bd(a+b\tan(c+dx))^2}$$

```
[Out] -1/2/b/d/(a+b*tan(d*x+c))^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3587, 32}

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^3} \, dx = -\frac{1}{2bd(a+b\tan(c+dx))^2}$$

```
[In] Int[Sec[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -1/2*1/(b*d*(a + b*Tan[c + d*x])^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
```

```
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(a+x)^3}\,dx, x, b\tan(c+dx)\right)}{bd}\\ &= -\frac{1}{2bd(a+b\tan(c+dx))^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{1}{2bd(a+b\tan(c+dx))^2}$$

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
[Out] -1/2*1/(b*d*(a + b*Tan[c + d*x])^2)
```

Maple [A] (verified)

Time = 7.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$-\frac{1}{2bd(a+b\tan(dx+c))^2}$	21
default	$-\frac{1}{2bd(a+b\tan(dx+c))^2}$	21
risch	$\frac{2ia\,e^{2i(dx+c)}+2b\,e^{2i(dx+c)}+2ia}{\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)^2 d(ia+b)^2}$	77

```
[In] int(sec(d*x+c)^2/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] -1/2/b/d/(a+b*tan(d*x+c))^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(20) = 40.

Time = 0.26 (sec) , antiderivative size = 142, normalized size of antiderivative = 6.45

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{4\,a^2b\cos\left(dx+c\right)^2 - a^2b + b^3 - 2\left(a^3 - ab^2\right)\cos\left(dx+c\right)\sin\left(dx+c\right)}{2\left(\left(a^6 + a^4b^2 - a^2b^4 - b^6\right)d\cos\left(dx+c\right)^2 + 2\left(a^5b + 2\,a^3b^3 + ab^5\right)d\cos\left(dx+c\right)\sin\left(dx+c\right) + \left(a^4b^2 + 2\right.\right.}$$

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^2*b*cos(d*x + c)^2 - a^2*b + b^3 - 2*(a^3 - a*b^2)*cos(d*x + c)*s
in(d*x + c))/((a^6 + a^4*b^2 - a^2*b^4 - b^6)*d*cos(d*x + c)^2 + 2*(a^5*b +
 2*a^3*b^3 + a*b^5)*d*cos(d*x + c)*sin(d*x + c) + (a^4*b^2 + 2*a^2*b^4 + b^
6)*d)
```

Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\sec^2\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^3}\,dx$$

```
[In] integrate(sec(d*x+c)**2/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*tan(c + d*x))**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{1}{2\left(b\tan\left(dx+c\right)+a\right)^2bd}$$

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2/((b*tan(d*x + c) + a)^2*b*d)
```

Giac [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{1}{2\,(b\tan\left(dx+c\right)+a)^2bd}$$

```
[In] integrate(sec(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="giac")
[Out] -1/2/((b*tan(d*x + c) + a)^2*b*d)
```

Mupad [B] (verification not implemented)

Time = 4.52 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{\sec^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{1}{d\,\left(2\,a^2\,b+4\,a\,b^2\tan\left(c+d\,x\right)+2\,b^3\tan\left(c+d\,x\right)^2\right)}$$

```
[In] int(1/(cos(c + d*x)^2*(a + b*tan(c + d*x))^3),x)
[Out] -1/(d*(2*a^2*b + 2*b^3*tan(c + d*x)^2 + 4*a*b^2*tan(c + d*x)))
```

3.570 $\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 202

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{a(a^4+10a^2b^2-15b^4)\,x}{2\,(a^2+b^2)^4} + \frac{2b^3(5a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^4\,d} + \frac{b(a^2-2b^2)}{2\,(a^2+b^2)^2\,d(a+b\tan(c+dx))^2} + \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\,(a^2+b^2)\,d(a+b\tan(c+dx))^2} + \frac{ab(a^2-11b^2)}{2\,(a^2+b^2)^3\,d(a+b\tan(c+dx))}$$

```
[Out] 1/2*a*(a^4+10*a^2*b^2-15*b^4)*x/(a^2+b^2)^4+2*b^3*(5*a^2-b^2)*ln(a*cos(d*x+
c)+b*sin(d*x+c))/(a^2+b^2)^4/d+1/2*b*(a^2-2*b^2)/(a^2+b^2)^2/d/(a+b*tan(d*x
+c))^2+1/2*cos(d*x+c)^2*(b+a*tan(d*x+c))/(a^2+b^2)/d/(a+b*tan(d*x+c))^2+1/2
*a*b*(a^2-11*b^2)/(a^2+b^2)^3/d/(a+b*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used

$= \{3587, 755, 815, 649, 209, 266\}$

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{ab(a^2-11b^2)}{2d\,(a^2+b^2)^3\,(a+b\tan(c+dx))} + \frac{b(a^2-2b^2)}{2d\,(a^2+b^2)^2\,(a+b\tan(c+dx))^2} + \frac{\cos^2(c+dx)(a\tan(c+dx)+b)}{2d\,(a^2+b^2)\,(a+b\tan(c+dx))^2} + \frac{2b^3(5a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^4} + \frac{ax(a^4+10a^2b^2-15b^4)}{2\,(a^2+b^2)^4}$$

```
[In] Int[Cos[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (a*(a^4 + 10*a^2*b^2 - 15*b^4)*x)/(2*(a^2 + b^2)^4) + (2*b^3*(5*a^2 - b^2)*
Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^4*d) + (b*(a^2 - 2*b^2))
/(2*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^2) + (Cos[c + d*x]^2*(b + a*Tan[c
+ d*x]))/(2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) + (a*b*(a^2 - 11*b^2))/(2
*(a^2 + b^2)^3*d*(a + b*Tan[c + d*x]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
```

```
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(a+x)^3\left(1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{b\text{Subst}\left(\int \frac{-4-\frac{a^2}{b^2}-\frac{3ax}{b^2}}{(a+x)^3\left(1+\frac{x^2}{b^2}\right)}\,dx, x, b\tan(c+dx)\right)}{2\left(a^2+b^2\right)d}$$

$$= \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{b\text{Subst}\left(\int \left(\frac{2(a^2-2b^2)}{(a^2+b^2)(a+x)^3} + \frac{a^3-11ab^2}{(a^2+b^2)^2(a+x)^2} + \frac{4b^2(-5a^2+b^2)}{(a^2+b^2)^3(a+x)} + \frac{-a(a^4+10a^2b^2-15b^4)+4b^2(5a^2-b^2)x}{(a^2+b^2)^3(b^2+x^2)}\right) dx, x, b\right.}{2\left(a^2+b^2\right)d}$$

$$= \frac{2b^3(5a^2-b^2)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^4 d} + \frac{b(a^2-2b^2)}{2\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^2} + \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} + \frac{ab(a^2-11b^2)}{2\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))} - \frac{b\text{Subst}\left(\int \frac{-a(a^4+10a^2b^2-15b^4)+4b^2(5a^2-b^2)x}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{2\left(a^2+b^2\right)^4 d}$$

$$= \frac{2b^3(5a^2-b^2)\log(a+b\tan(c+dx))}{(a^2+b^2)^4 d} + \frac{b(a^2-2b^2)}{2(a^2+b^2)^2 d(a+b\tan(c+dx))^2} + \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2(a^2+b^2) d(a+b\tan(c+dx))^2} + \frac{ab(a^2-11b^2)}{2(a^2+b^2)^3 d(a+b\tan(c+dx))} - \frac{(2b^3(5a^2-b^2))\operatorname{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{(a^2+b^2)^4 d} + \frac{(ab(a^4+10a^2b^2-15b^4))\operatorname{Subst}\left(\int \frac{1}{b^2+x^2}\,dx, x, b\tan(c+dx)\right)}{2(a^2+b^2)^4 d}$$

$$= \frac{a(a^4+10a^2b^2-15b^4)x}{2(a^2+b^2)^4} + \frac{2b^3(5a^2-b^2)\log(\cos(c+dx))}{(a^2+b^2)^4 d} + \frac{2b^3(5a^2-b^2)\log(a+b\tan(c+dx))}{(a^2+b^2)^4 d} + \frac{b(a^2-2b^2)}{2(a^2+b^2)^2 d(a+b\tan(c+dx))^2} + \frac{\cos^2(c+dx)(b+a\tan(c+dx))}{2(a^2+b^2) d(a+b\tan(c+dx))^2} + \frac{ab(a^2-11b^2)}{2(a^2+b^2)^3 d(a+b\tan(c+dx))}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 458 vs. 2(202) = 404.

Time = 6.37 (sec) , antiderivative size = 458, normalized size of antiderivative = 2.27

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

$$= \frac{b^3\left(\frac{\cos^2(c+dx)(b^2+ab\tan(c+dx))}{2b^4(a^2+b^2)(a+b\tan(c+dx))^2} - \frac{(2a^2-4b^2)\left(-\frac{\left(3a^2-b^2-\frac{a^3-3ab^2}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)}{2\left(a^2+b^2\right)^3} + \frac{\left(3a^2-b^2\right)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^3} - \frac{\left(3a^2-b^2+\frac{a^3-3\ldots}{\sqrt{-\ldots}}\right.}{\ldots}\right.}{\ldots}\right.}{\ldots}$$

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (b^3*((Cos[c + d*x]^2*(b^2 + a*b*Tan[c + d*x]))/(2*b^4*(a^2 + b^2)*(a + b*T
an[c + d*x])^2) - ((2*a^2 - 4*b^2)*(-1/2*((3*a^2 - b^2 - (a^3 - 3*a*b^2)/Sq
rt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/(a^2 + b^2)^3 + ((3*a^2 - b^2)*
Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^3 - ((3*a^2 - b^2 + (a^3 - 3*a*b^2)/Sq
rt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]])/(2*(a^2 + b^2)^3) - 1/(2*(a^2 +
 b^2)*(a + b*Tan[c + d*x])^2) - (2*a)/((a^2 + b^2)^2*(a + b*Tan[c + d*x])))
 - 3*a*(-1/2*((2*a - (a^2 - b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x
]])/(a^2 + b^2)^2 + (2*a*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^2 - ((2*a + (
a^2 - b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]])/(2*(a^2 + b^2)^2)
- 1/((a^2 + b^2)*(a + b*Tan[c + d*x]))))/(2*b^2*(a^2 + b^2))))/d
```

Maple [A] (verified)

Time = 10.26 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{\frac{\frac{\left(\frac{1}{2}a^5-a^3b^2-\frac{3}{2}a\,b^4\right)\tan(dx+c)+\frac{3a^4b}{2}+a^2b^3-\frac{b^5}{2}}{1+\tan^2(dx+c)}+\frac{\left(-20a^2b^3+4b^5\right)\ln\left(1+\tan^2(dx+c)\right)}{4}+\frac{\left(a^5+10a^3b^2-15a\,b^4\right)\arctan(\tan(dx+c))}{2}}{\left(a^2+b^2\right)^4}}{d}$
default	$\frac{\frac{\frac{\left(\frac{1}{2}a^5-a^3b^2-\frac{3}{2}a\,b^4\right)\tan(dx+c)+\frac{3a^4b}{2}+a^2b^3-\frac{b^5}{2}}{1+\tan^2(dx+c)}+\frac{\left(-20a^2b^3+4b^5\right)\ln\left(1+\tan^2(dx+c)\right)}{4}+\frac{\left(a^5+10a^3b^2-15a\,b^4\right)\arctan(\tan(dx+c))}{2}}{\left(a^2+b^2\right)^4}}{d}$
risch	$\frac{4ixb}{8ia^3b-8ia\,b^3-2a^4+12a^2b^2-2b^4}-\frac{xa}{8ia^3b-8ia\,b^3-2a^4+12a^2b^2-2b^4}-\frac{ie^{2i(dx+c)}}{8(-3ib\,a^2+ib^3+a^3-3a\,b^2)d}+\frac{ie^{-2i}}{8(3ib\,a^2-ib^3}$

```
[In] int(cos(d*x+c)^2/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(a^2+b^2)^4*(((1/2*a^5-a^3*b^2-3/2*a*b^4)*tan(d*x+c)+3/2*a^4*b+a^2*b
^3-1/2*b^5)/(1+tan(d*x+c)^2)+1/4*(-20*a^2*b^3+4*b^5)*ln(1+tan(d*x+c)^2)+1/2
*(a^5+10*a^3*b^2-15*a*b^4)*arctan(tan(d*x+c)))-1/2*b^3/(a^2+b^2)^2/(a+b*tan
(d*x+c))^2-4*b^3/(a^2+b^2)^3*a/(a+b*tan(d*x+c))+2*b^3*(5*a^2-b^2)/(a^2+b^2)
^4*ln(a+b*tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 503 vs. $2(194) = 388$.

Time = 0.31 (sec) , antiderivative size = 503, normalized size of antiderivative = 2.49

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{3\,a^4b^3-16\,a^2b^5+b^7-2\left(a^6b+3\,a^4b^3+3\,a^2b^5+b^7\right)\cos\left(dx+c\right)^4-2\left(a^5b^2+10\,a^3b^4-15\,ab^6\right)dx-(a}{}$$

```
[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(3*a^4*b^3 - 16*a^2*b^5 + b^7 - 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7
)*cos(d*x + c)^4 - 2*(a^5*b^2 + 10*a^3*b^4 - 15*a*b^6)*d*x - (a^6*b - a^4*b
^3 - 45*a^2*b^5 - 3*b^7 + 2*(a^7 + 9*a^5*b^2 - 25*a^3*b^4 + 15*a*b^6)*d*x)*
cos(d*x + c)^2 - 4*(5*a^2*b^5 - b^7 + (5*a^4*b^3 - 6*a^2*b^5 + b^7)*cos(d*x
 + c)^2 + 2*(5*a^3*b^4 - a*b^6)*cos(d*x + c)*sin(d*x + c))*log(2*a*b*cos(d*
x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 2*((a^7 + 3*a^5*b
^2 + 3*a^3*b^4 + a*b^6)*cos(d*x + c)^3 - 2*(a^5*b^2 - 3*a^3*b^4 + 6*a*b^6 -
 (a^6*b + 10*a^4*b^3 - 15*a^2*b^5)*d*x)*cos(d*x + c))*sin(d*x + c))/((a^10
+ 3*a^8*b^2 + 2*a^6*b^4 - 2*a^4*b^6 - 3*a^2*b^8 - b^10)*d*cos(d*x + c)^2 +
2*(a^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*
x + c) + (a^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)*d)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Exception raised: AttributeError}$$

```
[In] integrate(cos(d*x+c)**2/(a+b*tan(d*x+c))**3,x)
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(194) = 388$.

Time = 0.31 (sec) , antiderivative size = 458, normalized size of antiderivative = 2.27

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{(a^5+10\,a^3b^2-15\,ab^4)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} + \frac{4\,(5\,a^2b^3-b^5)\log(b\tan(dx+c)+a)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} - \frac{2\,(5\,a^2b^3-b^5)\log\left(\tan(dx+c)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} + \frac{\cdots}{a^8+3\,a^6b^2+3\,a^4b^4+a^2b^6+(a^6b^2+\cdots}}{\cdots}$$

```
[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
[Out] 1/2*((a^5 + 10*a^3*b^2 - 15*a*b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
 4*a^2*b^6 + b^8) + 4*(5*a^2*b^3 - b^5)*log(b*tan(d*x + c) + a)/(a^8 + 4*a^
6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 2*(5*a^2*b^3 - b^5)*log(tan(d*x + c)
^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + (3*a^4*b - 10*a^2
*b^3 - b^5 + (a^3*b^2 - 11*a*b^4)*tan(d*x + c)^3 + 2*(a^4*b - 6*a^2*b^3 - b
^5)*tan(d*x + c)^2 + (a^5 + 3*a^3*b^2 - 10*a*b^4)*tan(d*x + c))/(a^8 + 3*a^
6*b^2 + 3*a^4*b^4 + a^2*b^6 + (a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*tan(d
*x + c)^4 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*tan(d*x + c)^3 + (a^8
 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*tan(d*x + c)^2 + 2*(a^7*b + 3*a
^5*b^3 + 3*a^3*b^5 + a*b^7)*tan(d*x + c)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(194) = 388$.

Time = 0.64 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.17

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{(a^5+10\,a^3b^2-15\,ab^4)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} - \frac{2\,(5\,a^2b^3-b^5)\log\left(\tan(dx+c)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} + \frac{4\,(5\,a^2b^4-b^6)\log(|b\tan(dx+c)+a|)}{a^8b+4\,a^6b^3+6\,a^4b^5+4\,a^2b^7+b^9} + \frac{10\,a^2b^3\tan(dx+c)^2-2\,b^5\tan(d\cdots}{(\cdots}}{\cdots}$$

```
[In] integrate(cos(d*x+c)^2/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*((a^5 + 10*a^3*b^2 - 15*a*b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
 4*a^2*b^6 + b^8) - 2*(5*a^2*b^3 - b^5)*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^
6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 4*(5*a^2*b^4 - b^6)*log(abs(b*tan(d*
x + c) + a))/(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9) + (10*a^2*b^
3*tan(d*x + c)^2 - 2*b^5*tan(d*x + c)^2 + a^5*tan(d*x + c) - 2*a^3*b^2*tan(
d*x + c) - 3*a*b^4*tan(d*x + c) + 3*a^4*b + 12*a^2*b^3 - 3*b^5)/((a^8 + 4*a
^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(tan(d*x + c)^2 + 1)) - (30*a^2*b^5*t
an(d*x + c)^2 - 6*b^7*tan(d*x + c)^2 + 68*a^3*b^4*tan(d*x + c) - 4*a*b^6*ta
n(d*x + c) + 39*a^4*b^3 + 4*a^2*b^5 + b^7)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 +
4*a^2*b^6 + b^8)*(b*tan(d*x + c) + a)^2))/d
```

Mupad [B] (verification not implemented)

Time = 5.26 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.07

$$\int \frac{\cos^2(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{10\,b^3}{{\left(a^2+b^2\right)}^3}-\frac{12\,b^5}{{\left(a^2+b^2\right)}^4}\right)}{d} - \frac{\frac{-3\,a^4\,b+10\,a^2\,b^3+b^5}{2\,\left(a^2+b^2\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{{\tan(c+d\,x)}^3\,\left(11\,a\,b^4-a^3\,b^2\right)}{2\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{{\tan(c+d\,x)}^2\,\left(-a^4\,b+6\,a^2\,b^3+b^5\right)}{\left(a^2+b^2\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}-\frac{a\tan(c+d\,x)\,\left(a^4+3\,a^2\,b^2-10\,b^4\right)}{2\,\left(a^2+b^2\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}}{d\,\left({\tan\left(c+d\,x\right)}^2\,\left(a^2+b^2\right)+a^2+b^2\,{\tan\left(c+d\,x\right)}^4+2\,a\,b\tan\left(c+d\,x\right)+2\,a\,b\,{\tan\left(c+d\,x\right)}^3\right)} + \frac{\ln\left(\tan(c+d\,x)+1{}\mathrm{i}\right)\,\left(b+\frac{a\,1{}\mathrm{i}}{4}\right)}{d\,\left(a^4-a^3\,b\,4{}\mathrm{i}-6\,a^2\,b^2+a\,b^3\,4{}\mathrm{i}+b^4\right)} + \frac{\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(a+b\,4{}\mathrm{i}\right)}{4\,d\,\left(a^4\,1{}\mathrm{i}-4\,a^3\,b-a^2\,b^2\,6{}\mathrm{i}+4\,a\,b^3+b^4\,1{}\mathrm{i}\right)}$$

```
[In] int(cos(c + d*x)^2/(a + b*tan(c + d*x))^3,x)
```

```
[Out] (log(a + b*tan(c + d*x))*((10*b^3)/(a^2 + b^2)^3 - (12*b^5)/(a^2 + b^2)^4))
/d - ((b^5 - 3*a^4*b + 10*a^2*b^3)/(2*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2))
+ (tan(c + d*x)^3*(11*a*b^4 - a^3*b^2))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b
^2)) + (tan(c + d*x)^2*(b^5 - a^4*b + 6*a^2*b^3))/((a^2 + b^2)*(a^4 + b^4 +
 2*a^2*b^2)) - (a*tan(c + d*x)*(a^4 - 10*b^4 + 3*a^2*b^2))/(2*(a^2 + b^2)*(
a^4 + b^4 + 2*a^2*b^2)))/(d*(tan(c + d*x)^2*(a^2 + b^2) + a^2 + b^2*tan(c +
 d*x)^4 + 2*a*b*tan(c + d*x) + 2*a*b*tan(c + d*x)^3)) + (log(tan(c + d*x) +
 1i)*((a*1i)/4 + b))/(d*(a*b^3*4i - a^3*b*4i + a^4 + b^4 - 6*a^2*b^2)) + (l
og(tan(c + d*x) - 1i)*(a + b*4i))/(4*d*(4*a*b^3 - 4*a^3*b + a^4*1i + b^4*1i
 - a^2*b^2*6i))
```

3.571 $\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 295

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{3a(a^6+7a^4b^2+35a^2b^4-35b^6)\,x}{8\,(a^2+b^2)^5} + \frac{3b^5(7a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^5\,d} + \frac{3b(a^4+5a^2b^2-4b^4)}{8\,(a^2+b^2)^3\,d(a+b\tan(c+dx))^2} + \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\,(a^2+b^2)\,d(a+b\tan(c+dx))^2} + \frac{3ab(a^4+6a^2b^2-27b^4)}{8\,(a^2+b^2)^4\,d(a+b\tan(c+dx))} - \frac{\cos^2(c+dx)\,(2b(a^2-3b^2)-a(3a^2+11b^2)\tan(c+dx))}{8\,(a^2+b^2)^2\,d(a+b\tan(c+dx))^2}$$

```
[Out] 3/8*a*(a^6+7*a^4*b^2+35*a^2*b^4-35*b^6)*x/(a^2+b^2)^5+3*b^5*(7*a^2-b^2)*ln(
a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^5/d+3/8*b*(a^4+5*a^2*b^2-4*b^4)/(a^2+b
^2)^3/d/(a+b*tan(d*x+c))^2+1/4*cos(d*x+c)^4*(b+a*tan(d*x+c))/(a^2+b^2)/d/(a
+b*tan(d*x+c))^2+3/8*a*b*(a^4+6*a^2*b^2-27*b^4)/(a^2+b^2)^4/d/(a+b*tan(d*x+
c))-1/8*cos(d*x+c)^2*(2*b*(a^2-3*b^2)-a*(3*a^2+11*b^2)*tan(d*x+c))/(a^2+b^2
)^2/d/(a+b*tan(d*x+c))^2
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3587, 755, 837, 815, 649, 209, 266}

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = {} & \frac{\cos^4(c+dx)(a\tan(c+dx)+b)}{4d\,(a^2+b^2)\,(a+b\tan(c+dx))^2} \\ & - \frac{\cos^2(c+dx)\,(2b(a^2-3b^2)-a(3a^2+11b^2)\tan(c+dx))}{8d\,(a^2+b^2)^2\,(a+b\tan(c+dx))^2} \\ & + \frac{3b^5(7a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^5} \\ & + \frac{3ab(a^4+6a^2b^2-27b^4)}{8d\,(a^2+b^2)^4\,(a+b\tan(c+dx))} \\ & + \frac{3b(a^4+5a^2b^2-4b^4)}{8d\,(a^2+b^2)^3\,(a+b\tan(c+dx))^2} \\ & + \frac{3ax(a^6+7a^4b^2+35a^2b^4-35b^6)}{8\,(a^2+b^2)^5}\end{aligned}$$

```
[In] Int[Cos[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a*(a^6 + 7*a^4*b^2 + 35*a^2*b^4 - 35*b^6)*x)/(8*(a^2 + b^2)^5) + (3*b^5*
(7*a^2 - b^2)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^5*d) + (3*
b*(a^4 + 5*a^2*b^2 - 4*b^4))/(8*(a^2 + b^2)^3*d*(a + b*Tan[c + d*x])^2) + (
Cos[c + d*x]^4*(b + a*Tan[c + d*x]))/(4*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^
2) + (3*a*b*(a^4 + 6*a^2*b^2 - 27*b^4))/(8*(a^2 + b^2)^4*d*(a + b*Tan[c + d
*x])) - (Cos[c + d*x]^2*(2*b*(a^2 - 3*b^2) - a*(3*a^2 + 11*b^2)*Tan[c + d*x
]))/(8*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(a+x)^3\left(1+\frac{x^2}{b^2}\right)^3}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{b\text{Subst}\left(\int \frac{-3\left(2+\frac{a^2}{b^2}\right)-\frac{5ax}{b^2}}{(a+x)^3\left(1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\tan(c+dx)\right)}{4\left(a^2+b^2\right)d}$$

$$=\frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\left(a^2+b^2\right)d(a+b\tan(c+dx))^2}-\frac{\cos^2(c+dx)\left(2b(a^2-3b^2)-a(3a^2+11b^2)\tan(c+dx)\right)}{8\left(a^2+b^2\right)^2d(a+b\tan(c+dx))^2}+\frac{b^5\text{Subst}\left(\int\frac{\frac{3\left(a^4+a^2b^2+8b^4\right)}{b^6}+\frac{3a\left(3a^2+11b^2\right)x}{b^6}}{(a+x)^3\left(1+\frac{x^2}{b^2}\right)}dx,x,b\tan(c+dx)\right)}{8\left(a^2+b^2\right)^2d}$$

$$=\frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\left(a^2+b^2\right)d(a+b\tan(c+dx))^2}-\frac{\cos^2(c+dx)\left(2b(a^2-3b^2)-a(3a^2+11b^2)\tan(c+dx)\right)}{8\left(a^2+b^2\right)^2d(a+b\tan(c+dx))^2}+\frac{b^5\text{Subst}\left(\int\left(\frac{6\left(-a^4-5a^2b^2+4b^4\right)}{b^4(a^2+b^2)(a+x)^3}+\frac{3a\left(-a^4-6a^2b^2+27b^4\right)}{b^4(a^2+b^2)^2(a+x)^2}+\frac{24\left(7a^2-b^2\right)}{(a^2+b^2)^3(a+x)}+\frac{3\left(a\left(a^6+7a^4b^2+35a^2b^4-35b^6\right)-8b^4(7a^2\right.}{b^4(a^2+b^2)^3(b^2+x^2)}\right.}{8\left(a^2+b^2\right)^2d}$$

$$=\frac{3b^5(7a^2-b^2)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^5d}+\frac{3b(a^4+5a^2b^2-4b^4)}{8\left(a^2+b^2\right)^3d(a+b\tan(c+dx))^2}+\frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\left(a^2+b^2\right)d(a+b\tan(c+dx))^2}+\frac{3ab(a^4+6a^2b^2-27b^4)}{8\left(a^2+b^2\right)^4d(a+b\tan(c+dx))}-\frac{\cos^2(c+dx)\left(2b(a^2-3b^2)-a(3a^2+11b^2)\tan(c+dx)\right)}{8\left(a^2+b^2\right)^2d(a+b\tan(c+dx))^2}+\frac{(3b)\text{Subst}\left(\int\frac{a\left(a^6+7a^4b^2+35a^2b^4-35b^6\right)-8b^4\left(7a^2-b^2\right)x}{b^2+x^2}dx,x,b\tan(c+dx)\right)}{8\left(a^2+b^2\right)^5d}$$

$$=\frac{3b^5(7a^2-b^2)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^5d}+\frac{3b(a^4+5a^2b^2-4b^4)}{8\left(a^2+b^2\right)^3d(a+b\tan(c+dx))^2}+\frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\left(a^2+b^2\right)d(a+b\tan(c+dx))^2}+\frac{3ab(a^4+6a^2b^2-27b^4)}{8\left(a^2+b^2\right)^4d(a+b\tan(c+dx))}-\frac{\cos^2(c+dx)\left(2b(a^2-3b^2)-a(3a^2+11b^2)\tan(c+dx)\right)}{8\left(a^2+b^2\right)^2d(a+b\tan(c+dx))^2}-\frac{\left(3b^5(7a^2-b^2)\right)\text{Subst}\left(\int\frac{x}{b^2+x^2}dx,x,b\tan(c+dx)\right)}{\left(a^2+b^2\right)^5d}+\frac{\left(3ab(a^6+7a^4b^2+35a^2b^4-35b^6)\right)\text{Subst}\left(\int\frac{1}{b^2+x^2}dx,x,b\tan(c+dx)\right)}{8\left(a^2+b^2\right)^5d}$$

$$=\frac{3a(a^6+7a^4b^2+35a^2b^4-35b^6)x}{8\left(a^2+b^2\right)^5}+\frac{3b^5(7a^2-b^2)\log(\cos(c+dx))}{\left(a^2+b^2\right)^5d}+\frac{3b^5(7a^2-b^2)\log(a+b\tan(c+dx))}{\left(a^2+b^2\right)^5d}+\frac{3b(a^4+5a^2b^2-4b^4)}{8\left(a^2+b^2\right)^3d(a+b\tan(c+dx))^2}+\frac{\cos^4(c+dx)(b+a\tan(c+dx))}{4\left(a^2+b^2\right)d(a+b\tan(c+dx))^2}+\frac{3ab(a^4+6a^2b^2-27b^4)}{8\left(a^2+b^2\right)^4d(a+b\tan(c+dx))}-\frac{\cos^2(c+dx)\left(2b(a^2-3b^2)-a(3a^2+11b^2)\tan(c+dx)\right)}{8\left(a^2+b^2\right)^2d(a+b\tan(c+dx))^2}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 596 vs. $2(295) = 590$.

Time = 6.29 (sec) , antiderivative size = 596, normalized size of antiderivative = 2.02

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

$$= \frac{b^5\left(\frac{\cos^4(c+dx)\left(b^2+ab\tan(c+dx)\right)}{4b^6(a^2+b^2)(a+b\tan(c+dx))^2} - \frac{\frac{\cos^2(c+dx)\left(5a^2b^2-3b^2\left(a^2+2b^2\right)+b\left(-5ab^2-3a\left(a^2+2b^2\right)\right)\tan(c+dx)\right)}{2b^4\left(a^2+b^2\right)(a+b\tan(c+dx))^2} - \frac{\left(-3a^2\left(3a^2+11b^2\right)+3\left(a^4+a^2b^2+8b^4\right)\right)\left(\cdots\right.}{\cdots}}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (b^5*((Cos[c + d*x]^4*(b^2 + a*b*Tan[c + d*x]))/(4*b^6*(a^2 + b^2)*(a + b*T
an[c + d*x])^2) - ((Cos[c + d*x]^2*(5*a^2*b^2 - 3*b^2*(a^2 + 2*b^2) + b*(-5
*a*b^2 - 3*a*(a^2 + 2*b^2))*Tan[c + d*x]))/(2*b^4*(a^2 + b^2)*(a + b*Tan[c
+ d*x])^2) - ((-3*a^2*(3*a^2 + 11*b^2) + 3*(a^4 + a^2*b^2 + 8*b^4))*(-1/2*(
(3*a^2 - b^2 - (a^3 - 3*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]]
)/(a^2 + b^2)^3 + ((3*a^2 - b^2)*Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^3 - (
(3*a^2 - b^2 + (a^3 - 3*a*b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]]
)/(2*(a^2 + b^2)^3) - 1/(2*(a^2 + b^2)*(a + b*Tan[c + d*x])^2) - (2*a)/((a^
2 + b^2)^2*(a + b*Tan[c + d*x]))) + 3*a*(3*a^2 + 11*b^2)*(-1/2*((2*a - (a^2
 - b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/(a^2 + b^2)^2 + (2*a*
Log[a + b*Tan[c + d*x]])/(a^2 + b^2)^2 - ((2*a + (a^2 - b^2)/Sqrt[-b^2])*Lo
g[Sqrt[-b^2] + b*Tan[c + d*x]])/(2*(a^2 + b^2)^2) - 1/((a^2 + b^2)*(a + b*T
an[c + d*x]))))/(2*b^2*(a^2 + b^2)))/(4*b^2*(a^2 + b^2))))/d
```

Maple [A] (verified)

Time = 43.90 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.06

method	result
derivativedivides	$-\frac{b^5}{2(a^2+b^2)^3(a+b\tan(dx+c))^2}-\frac{6b^5a}{(a^2+b^2)^4(a+b\tan(dx+c))}+\frac{3b^5(7a^2-b^2)\ln(a+b\tan(dx+c))}{(a^2+b^2)^5}+\frac{\left(\frac{3}{8}a^7+\frac{21}{8}a^5b^2-\frac{15}{8}a^3b^4-\frac{33}{8}ab^6\right.}{}$
default	$-\frac{b^5}{2(a^2+b^2)^3(a+b\tan(dx+c))^2}-\frac{6b^5a}{(a^2+b^2)^4(a+b\tan(dx+c))}+\frac{3b^5(7a^2-b^2)\ln(a+b\tan(dx+c))}{(a^2+b^2)^5}+\frac{\left(\frac{3}{8}a^7+\frac{21}{8}a^5b^2-\frac{15}{8}a^3b^4-\frac{33}{8}ab^6\right.}{}$
risch	$\frac{15xab}{8ia^5-80ia^3b^2+40iab^4+40a^4b-80a^2b^3+8b^5}-\frac{24ixb^2}{8ia^5-80ia^3b^2+40iab^4+40a^4b-80a^2b^3+8b^5}-\frac{ie^{2i(dx+c}}{8(-4ia^3b+4iab^3+a^4}$

```
[In] int(cos(d*x+c)^4/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*b^5/(a^2+b^2)^3/(a+b*tan(d*x+c))^2-6*b^5/(a^2+b^2)^4*a/(a+b*tan(d
*x+c))+3*b^5*(7*a^2-b^2)/(a^2+b^2)^5*ln(a+b*tan(d*x+c))+1/(a^2+b^2)^5*(((3/
8*a^7+21/8*a^5*b^2-15/8*a^3*b^4-33/8*a*b^6)*tan(d*x+c)^3+(5*a^4*b^3+4*a^2*b
^5-b^7)*tan(d*x+c)^2+(19/8*a^5*b^2-39/8*a*b^6+5/8*a^7-25/8*a^3*b^4)*tan(d*x
+c)+3/4*a^6*b+25/4*a^4*b^3+17/4*b^5*a^2-5/4*b^7)/(1+tan(d*x+c)^2)^2+3/16*(-
56*a^2*b^5+8*b^7)*ln(1+tan(d*x+c)^2)+3/8*(a^7+7*a^5*b^2+35*a^3*b^4-35*a*b^6
)*arctan(tan(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 671 vs. $2(284) = 568$.

Time = 0.33 (sec) , antiderivative size = 671, normalized size of antiderivative = 2.27

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{9\,a^6b^3+95\,a^4b^5-141\,a^2b^7-3\,b^9-8\,(a^8b+4\,a^6b^3+6\,a^4b^5+4\,a^2b^7+b^9)\cos(dx+c)^6+8\,(a^8b-6\,a^4b}{}$$

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/32*(9*a^6*b^3 + 95*a^4*b^5 - 141*a^2*b^7 - 3*b^9 - 8*(a^8*b + 4*a^6*b^3
 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cos(d*x + c)^6 + 8*(a^8*b - 6*a^4*b^5 - 8*a^
2*b^7 - 3*b^9)*cos(d*x + c)^4 - 12*(a^7*b^2 + 7*a^5*b^4 + 35*a^3*b^6 - 35*a
*b^8)*d*x - (15*a^8*b + 82*a^6*b^3 + 68*a^4*b^5 - 498*a^2*b^7 - 51*b^9 + 12
*(a^9 + 6*a^7*b^2 + 28*a^5*b^4 - 70*a^3*b^6 + 35*a*b^8)*d*x)*cos(d*x + c)^2
 - 48*(7*a^2*b^7 - b^9 + (7*a^4*b^5 - 8*a^2*b^7 + b^9)*cos(d*x + c)^2 + 2*(
7*a^3*b^6 - a*b^8)*cos(d*x + c)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*
```

```
x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 2*(4*(a^9 + 4*a^7*b^2 + 6*a^5*
b^4 + 4*a^3*b^6 + a*b^8)*cos(d*x + c)^5 + 2*(3*a^9 + 20*a^7*b^2 + 42*a^5*b^
4 + 36*a^3*b^6 + 11*a*b^8)*cos(d*x + c)^3 - (3*a^7*b^2 + 53*a^5*b^4 - 15*a^
3*b^6 + 159*a*b^8 - 12*(a^8*b + 7*a^6*b^3 + 35*a^4*b^5 - 35*a^2*b^7)*d*x)*c
os(d*x + c))*sin(d*x + c))/((a^12 + 4*a^10*b^2 + 5*a^8*b^4 - 5*a^4*b^8 - 4*
a^2*b^10 - b^12)*d*cos(d*x + c)^2 + 2*(a^11*b + 5*a^9*b^3 + 10*a^7*b^5 + 10
*a^5*b^7 + 5*a^3*b^9 + a*b^11)*d*cos(d*x + c)*sin(d*x + c) + (a^10*b^2 + 5*
a^8*b^4 + 10*a^6*b^6 + 10*a^4*b^8 + 5*a^2*b^10 + b^12)*d)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Exception raised: AttributeError}$$

```
[In] integrate(cos(d*x+c)**4/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 738 vs. $2(284) = 568$.

Time = 0.53 (sec) , antiderivative size = 738, normalized size of antiderivative = 2.50

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{3\,(a^7+7\,a^5b^2+35\,a^3b^4-35\,ab^6)\,(dx+c)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}} + \frac{24\,(7\,a^2b^5-b^7)\,\log(b\tan(dx+c)+a)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}} - \frac{12\,(7\,a^2b^5-b^7)\,\log\left(\tan(dx+c)^2+1\right)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}} + \frac{}{a^{10}+}}{}$$

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(a^7 + 7*a^5*b^2 + 35*a^3*b^4 - 35*a*b^6)*(d*x + c)/(a^10 + 5*a^8*b^
2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) + 24*(7*a^2*b^5 - b^7)*log(
b*tan(d*x + c) + a)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8
 + b^10) - 12*(7*a^2*b^5 - b^7)*log(tan(d*x + c)^2 + 1)/(a^10 + 5*a^8*b^2 +
 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) + (6*a^6*b + 44*a^4*b^3 - 62*a
^2*b^5 - 4*b^7 + 3*(a^5*b^2 + 6*a^3*b^4 - 27*a*b^6)*tan(d*x + c)^5 + 6*(a^6
*b + 6*a^4*b^3 - 13*a^2*b^5 - 2*b^7)*tan(d*x + c)^4 + (3*a^7 + 23*a^5*b^2 +
 61*a^3*b^4 - 151*a*b^6)*tan(d*x + c)^3 + 2*(5*a^6*b + 37*a^4*b^3 - 73*a^2*
b^5 - 9*b^7)*tan(d*x + c)^2 + (5*a^7 + 26*a^5*b^2 + 49*a^3*b^4 - 68*a*b^6)*
tan(d*x + c))/(a^10 + 4*a^8*b^2 + 6*a^6*b^4 + 4*a^4*b^6 + a^2*b^8 + (a^8*b^
2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)*tan(d*x + c)^6 + 2*(a^9*b + 4
```

```
*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*tan(d*x + c)^5 + (a^10 + 6*a^8*b^
2 + 14*a^6*b^4 + 16*a^4*b^6 + 9*a^2*b^8 + 2*b^10)*tan(d*x + c)^4 + 4*(a^9*b
 + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*tan(d*x + c)^3 + (2*a^10 + 9*
a^8*b^2 + 16*a^6*b^4 + 14*a^4*b^6 + 6*a^2*b^8 + b^10)*tan(d*x + c)^2 + 2*(a
^9*b + 4*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*tan(d*x + c)))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 587 vs. $2(284) = 568$.

Time = 0.61 (sec) , antiderivative size = 587, normalized size of antiderivative = 1.99

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{3\,(a^7+7\,a^5b^2+35\,a^3b^4-35\,ab^6)(dx+c)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}} - \frac{12\,(7\,a^2b^5-b^7)\log\left(\tan(dx+c)^2+1\right)}{a^{10}+5\,a^8b^2+10\,a^6b^4+10\,a^4b^6+5\,a^2b^8+b^{10}} + \frac{24\,(7\,a^2b^6-b^8)\log(|b\tan(dx+c)+a|)}{a^{10}b+5\,a^8b^3+10\,a^6b^5+10\,a^4b^7+5\,a^2b^9+b^{11}} + \frac{3}{\cdots}}{\cdots}$$

```
[In] integrate(cos(d*x+c)^4/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a^7 + 7*a^5*b^2 + 35*a^3*b^4 - 35*a*b^6)*(d*x + c)/(a^10 + 5*a^8*b^
2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8 + b^10) - 12*(7*a^2*b^5 - b^7)*log(
tan(d*x + c)^2 + 1)/(a^10 + 5*a^8*b^2 + 10*a^6*b^4 + 10*a^4*b^6 + 5*a^2*b^8
 + b^10) + 24*(7*a^2*b^6 - b^8)*log(abs(b*tan(d*x + c) + a))/(a^10*b + 5*a^
8*b^3 + 10*a^6*b^5 + 10*a^4*b^7 + 5*a^2*b^9 + b^11) + (3*a^5*b^2*tan(d*x +
c)^5 + 18*a^3*b^4*tan(d*x + c)^5 - 81*a*b^6*tan(d*x + c)^5 + 6*a^6*b*tan(d*
x + c)^4 + 36*a^4*b^3*tan(d*x + c)^4 - 78*a^2*b^5*tan(d*x + c)^4 - 12*b^7*t
an(d*x + c)^4 + 3*a^7*tan(d*x + c)^3 + 23*a^5*b^2*tan(d*x + c)^3 + 61*a^3*b
^4*tan(d*x + c)^3 - 151*a*b^6*tan(d*x + c)^3 + 10*a^6*b*tan(d*x + c)^2 + 74
*a^4*b^3*tan(d*x + c)^2 - 146*a^2*b^5*tan(d*x + c)^2 - 18*b^7*tan(d*x + c)^
2 + 5*a^7*tan(d*x + c) + 26*a^5*b^2*tan(d*x + c) + 49*a^3*b^4*tan(d*x + c)
- 68*a*b^6*tan(d*x + c) + 6*a^6*b + 44*a^4*b^3 - 62*a^2*b^5 - 4*b^7)/((a^8
+ 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(b*tan(d*x + c)^3 + a*tan(d*x +
c)^2 + b*tan(d*x + c) + a)^2))/d
```

Mupad [B] (verification not implemented)

Time = 6.49 (sec) , antiderivative size = 715, normalized size of antiderivative = 2.42

$$\int \frac{\cos^4(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{3\,a^6\,b+22\,a^4\,b^3-31\,a^2\,b^5-2\,b^7}{4\,(a^8+4\,a^6\,b^2+6\,a^4\,b^4+4\,a^2\,b^6+b^8)}+\frac{\tan(c+d\,x)\,(5\,a^7+26\,a^5\,b^2+49\,a^3\,b^4-68\,a\,b^6)}{8\,(a^8+4\,a^6\,b^2+6\,a^4\,b^4+4\,a^2\,b^6+b^8)}+\frac{3\tan(c+d\,x)^5\,(a^5\,b^2+6\,a^3\,b^4-27\,a\,b^6)}{8\,(a^8+4\,a^6\,b^2+6\,a^4\,b^4+4\,a^2\,b^6+b^8)}+\frac{\tan(c+d\,x)^3\,(3}{8\,(a^8+4}}{d\,\left(\tan\left(c+d\,x\right)^2\,\left(2\,a^2+b^2\right)+\tan\left(c+d\,x\right)^4\,\left(a^2+2\,b^2\right)+a^2+b^2\tan\left(c+d\,x\right.\right.}$$
$$+\frac{\ln\left(a+b\tan(c+d\,x)\right)\,\left(\frac{21\,b^5}{(a^2+b^2)^4}-\frac{24\,b^7}{(a^2+b^2)^5}\right)}{d}$$
$$+\frac{3\,\ln\left(\tan(c+d\,x)-\mathrm{i}\right)\,\left(-a^2\,1\mathrm{i}+5\,a\,b+b^2\,8\mathrm{i}\right)}{16\,d\,\left(a^5+a^4\,b\,5\mathrm{i}-10\,a^3\,b^2-a^2\,b^3\,10\mathrm{i}+5\,a\,b^4+b^5\,1\mathrm{i}\right)}$$
$$+\frac{3\,\ln\left(\tan(c+d\,x)+1\mathrm{i}\right)\,\left(a^2\,1\mathrm{i}+5\,a\,b-b^2\,8\mathrm{i}\right)}{16\,d\,\left(a^5-a^4\,b\,5\mathrm{i}-10\,a^3\,b^2+a^2\,b^3\,10\mathrm{i}+5\,a\,b^4-b^5\,1\mathrm{i}\right)}$$

```
[In] int(cos(c + d*x)^4/(a + b*tan(c + d*x))^3,x)
```

```
[Out] ((3*a^6*b - 2*b^7 - 31*a^2*b^5 + 22*a^4*b^3)/(4*(a^8 + b^8 + 4*a^2*b^6 + 6*
a^4*b^4 + 4*a^6*b^2)) + (tan(c + d*x)*(5*a^7 - 68*a*b^6 + 49*a^3*b^4 + 26*a
^5*b^2))/(8*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (3*tan(c + d
*x)^5*(6*a^3*b^4 - 27*a*b^6 + a^5*b^2))/(8*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b
^4 + 4*a^6*b^2)) + (tan(c + d*x)^3*(3*a^7 - 151*a*b^6 + 61*a^3*b^4 + 23*a^5
*b^2))/(8*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (3*tan(c + d*x
)^4*(a^6*b - 2*b^7 - 13*a^2*b^5 + 6*a^4*b^3))/(4*(a^8 + b^8 + 4*a^2*b^6 + 6
*a^4*b^4 + 4*a^6*b^2)) + (tan(c + d*x)^2*(5*a^6*b - 9*b^7 - 73*a^2*b^5 + 37
*a^4*b^3))/(4*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)))/(d*(tan(c +
 d*x)^2*(2*a^2 + b^2) + tan(c + d*x)^4*(a^2 + 2*b^2) + a^2 + b^2*tan(c + d*
x)^6 + 2*a*b*tan(c + d*x) + 4*a*b*tan(c + d*x)^3 + 2*a*b*tan(c + d*x)^5)) +
 (log(a + b*tan(c + d*x))*((21*b^5)/(a^2 + b^2)^4 - (24*b^7)/(a^2 + b^2)^5)
)/d + (3*log(tan(c + d*x) - 1i)*(5*a*b - a^2*1i + b^2*8i))/(16*d*(5*a*b^4 +
 a^4*b*5i + a^5 + b^5*1i - a^2*b^3*10i - 10*a^3*b^2)) + (3*log(tan(c + d*x)
 + 1i)*(5*a*b + a^2*1i - b^2*8i))/(16*d*(5*a*b^4 - a^4*b*5i + a^5 - b^5*1i
+ a^2*b^3*10i - 10*a^3*b^2))
```

3.572 $\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 239

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{5a(4a^2+3b^2)\,\text{arcsinh}(\tan(c+dx))\sec(c+dx)}{2b^6d\sqrt{\sec^2(c+dx)}} - \frac{5\sqrt{a^2+b^2}(4a^2+b^2)\,\text{arctanh}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\sec(c+dx)}{2b^6d\sqrt{\sec^2(c+dx)}} - \frac{\sec^5(c+dx)}{2bd(a+b\tan(c+dx))^2} + \frac{5\sec^3(c+dx)(4a+b\tan(c+dx))}{6b^3d(a+b\tan(c+dx))} + \frac{5\sec(c+dx)\left(4a^2+b^2-2ab\tan(c+dx)\right)}{2b^5d}$$

```
[Out] -5/2*a*(4*a^2+3*b^2)*arcsinh(tan(d*x+c))*sec(d*x+c)/b^6/d/(sec(d*x+c)^2)^(1
/2)-5/2*(4*a^2+b^2)*arctanh((b-a*tan(d*x+c))/(a^2+b^2)^(1/2)/(sec(d*x+c)^2)
^(1/2))*sec(d*x+c)*(a^2+b^2)^(1/2)/b^6/d/(sec(d*x+c)^2)^(1/2)-1/2*sec(d*x+c
)^5/b/d/(a+b*tan(d*x+c))^2+5/6*sec(d*x+c)^3*(4*a+b*tan(d*x+c))/b^3/d/(a+b*t
an(d*x+c))+5/2*sec(d*x+c)*(4*a^2+b^2-2*a*b*tan(d*x+c))/b^5/d
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used

$= \{3593, 747, 827, 829, 858, 221, 739, 212\}$

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{5a(4a^2+3b^2)\sec(c+dx)\text{arcsinh}(\tan(c+dx))}{2b^6d\sqrt{\sec^2(c+dx)}} - \frac{5\sqrt{a^2+b^2}(4a^2+b^2)\sec(c+dx)\text{arctanh}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)}{2b^6d\sqrt{\sec^2(c+dx)}} + \frac{5\sec(c+dx)\left(4a^2-2ab\tan(c+dx)+b^2\right)}{2b^5d} + \frac{5\sec^3(c+dx)(4a+b\tan(c+dx))}{6b^3d(a+b\tan(c+dx))} - \frac{\sec^5(c+dx)}{2bd(a+b\tan(c+dx))^2}$$

```
[In] Int[Sec[c + d*x]^7/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-5*a*(4*a^2 + 3*b^2)*ArcSinh[Tan[c + d*x]]*Sec[c + d*x])/(2*b^6*d*Sqrt[Sec
[c + d*x]^2]) - (5*Sqrt[a^2 + b^2]*(4*a^2 + b^2)*ArcTanh[(b - a*Tan[c + d*x
])/(Sqrt[a^2 + b^2]*Sqrt[Sec[c + d*x]^2])]*Sec[c + d*x])/(2*b^6*d*Sqrt[Sec[
c + d*x]^2]) - Sec[c + d*x]^5/(2*b*d*(a + b*Tan[c + d*x])^2) + (5*Sec[c + d
*x]^3*(4*a + b*Tan[c + d*x]))/(6*b^3*d*(a + b*Tan[c + d*x])) + (5*Sec[c + d
*x]*(4*a^2 + b^2 - 2*a*b*Tan[c + d*x]))/(2*b^5*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 747

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 1))), x] - Dist[2*c*(p/(e*(m + 1))
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\sec(c+dx)\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^{5/2}}{(a+x)^3}\,dx, x, b\tan(c+dx)\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$=-\frac{\sec^5(c+dx)}{2bd(a+b\tan(c+dx))^2}+\frac{(5\sec(c+dx))\text{Subst}\left(\int\frac{x\left(1+\frac{x^2}{b^2}\right)^{3/2}}{(a+x)^2}\,dx,x,b\tan(c+dx)\right)}{2b^3d\sqrt{\sec^2(c+dx)}}$$

$$=-\frac{\sec^5(c+dx)}{2bd(a+b\tan(c+dx))^2}+\frac{5\sec^3(c+dx)(4a+b\tan(c+dx))}{6b^3d(a+b\tan(c+dx))}-\frac{(5\sec(c+dx))\text{Subst}\left(\int\frac{\left(-2+\frac{8ax}{b^2}\right)\sqrt{1+\frac{x^2}{b^2}}}{a+x}\,dx,x,b\tan(c+dx)\right)}{4b^3d\sqrt{\sec^2(c+dx)}}$$

$$=-\frac{\sec^5(c+dx)}{2bd(a+b\tan(c+dx))^2}+\frac{5\sec^3(c+dx)(4a+b\tan(c+dx))}{6b^3d(a+b\tan(c+dx))}+\frac{5\sec(c+dx)\left(4a^2+b^2-2ab\tan(c+dx)\right)}{2b^5d}-\frac{(5\sec(c+dx))\text{Subst}\left(\int\frac{-\frac{4\left(2a^2+b^2\right)}{b^4}+\frac{4a\left(4a^2+3b^2\right)x}{b^6}}{(a+x)\sqrt{1+\frac{x^2}{b^2}}}\,dx,x,b\tan(c+dx)\right)}{8bd\sqrt{\sec^2(c+dx)}}$$

$$=-\frac{\sec^5(c+dx)}{2bd(a+b\tan(c+dx))^2}+\frac{5\sec^3(c+dx)(4a+b\tan(c+dx))}{6b^3d(a+b\tan(c+dx))}+\frac{5\sec(c+dx)\left(4a^2+b^2-2ab\tan(c+dx)\right)}{2b^5d}+\frac{\left(5(a^2+b^2)\left(4a^2+b^2\right)\sec(c+dx)\right)\text{Subst}\left(\int\frac{1}{(a+x)\sqrt{1+\frac{x^2}{b^2}}}\,dx,x,b\tan(c+dx)\right)}{2b^7d\sqrt{\sec^2(c+dx)}}-\frac{\left(5a(4a^2+3b^2)\sec(c+dx)\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{b^2}}}\,dx,x,b\tan(c+dx)\right)}{2b^7d\sqrt{\sec^2(c+dx)}}$$

$$=-\frac{5a(4a^2+3b^2)\text{arcsinh}(\tan(c+dx))\sec(c+dx)}{2b^6d\sqrt{\sec^2(c+dx)}}-\frac{\sec^5(c+dx)}{2bd(a+b\tan(c+dx))^2}+\frac{5\sec^3(c+dx)(4a+b\tan(c+dx))}{6b^3d(a+b\tan(c+dx))}+\frac{5\sec(c+dx)\left(4a^2+b^2-2ab\tan(c+dx)\right)}{2b^5d}-\frac{\left(5(a^2+b^2)\left(4a^2+b^2\right)\sec(c+dx)\right)\text{Subst}\left(\int\frac{1}{1+\frac{a^2}{b^2}-x^2}\,dx,x,\frac{1-\frac{a\tan(c+dx)}{b}}{\sqrt{\sec^2(c+dx)}}\right)}{2b^7d\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{5a(4a^2+3b^2)\operatorname{arcsinh}(\tan(c+dx))\sec(c+dx)}{2b^6d\sqrt{\sec^2(c+dx)}} - \frac{5\sqrt{a^2+b^2}(4a^2+b^2)\operatorname{arctanh}\left(\frac{b\left(1-\frac{a\tan(c+dx)}{b}\right)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\sec(c+dx)}{2b^6d\sqrt{\sec^2(c+dx)}} - \frac{\sec^5(c+dx)}{2bd(a+b\tan(c+dx))^2} + \frac{5\sec^3(c+dx)(4a+b\tan(c+dx))}{6b^3d(a+b\tan(c+dx))} + \frac{5\sec(c+dx)\left(4a^2+b^2-2ab\tan(c+dx)\right)}{2b^5d}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.61 (sec) , antiderivative size = 688, normalized size of antiderivative = 2.88

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(\frac{6b^2(a^2+b^2)^2\sin(c+dx)}{a} + \frac{6(a-ib)(a+ib)b(8a^2-b^2)(a\cos(c+dx)+b\sin(c+dx))}{a}\right.}{}$$

```
[In] Integrate[Sec[c + d*x]^7/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])*((6*b^2*(a^2 + b^2)^2*Sin
[c + d*x])/a + (6*(a - I*b)*(a + I*b)*b*(8*a^2 - b^2)*(a*Cos[c + d*x] + b*S
in[c + d*x]))/a + 2*b*(36*a^2 + 13*b^2)*(a*Cos[c + d*x] + b*Sin[c + d*x])^2
 + 60*Sqrt[a^2 + b^2]*(4*a^2 + b^2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[
a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2 + 30*a*(4*a^2 + 3*b^2)*Log[
Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2 -
30*a*(4*a^2 + 3*b^2)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a*Cos[c + d*
x] + b*Sin[c + d*x])^2 + (b^2*(-9*a + b)*(a*Cos[c + d*x] + b*Sin[c + d*x])^
2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (2*b^3*Sin[(c + d*x)/2]*(a*Cos
[c + d*x] + b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 + (2
*b*(36*a^2 + 13*b^2)*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/
(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (2*b^3*Sin[(c + d*x)/2]*(a*Cos[c +
d*x] + b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + (b^2*(9
*a + b)*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2] + Sin[(c + d
*x)/2])^2 - (2*b*(36*a^2 + 13*b^2)*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin
[c + d*x])^2)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/(12*b^6*d*(a + b*Tan[
c + d*x])^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 443 vs. 2(219) = 438.

Time = 299.23 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.86

method	result
derivativedivides	$-\frac{2\left(\frac{\frac{b^2(7a^4+5a^2b^2-2b^4)(\tan^3(\frac{dx}{2}+\frac{c}{2}))}{2a}+\frac{b(8a^6-9a^4b^2-15a^2b^4+2b^6)(\tan^2(\frac{dx}{2}+\frac{c}{2}))}{2a^2}-\frac{b^2(25a^4+23a^2b^2-2b^4)\tan(\frac{dx}{2}+\frac{c}{2})}{2a}-4}{((\tan^2(\frac{dx}{2}+\frac{c}{2}))a-2b\tan(\frac{dx}{2}+\frac{c}{2})-a)^2}\right.}{b^6}$
default	$-\frac{2\left(\frac{\frac{b^2(7a^4+5a^2b^2-2b^4)(\tan^3(\frac{dx}{2}+\frac{c}{2}))}{2a}+\frac{b(8a^6-9a^4b^2-15a^2b^4+2b^6)(\tan^2(\frac{dx}{2}+\frac{c}{2}))}{2a^2}-\frac{b^2(25a^4+23a^2b^2-2b^4)\tan(\frac{dx}{2}+\frac{c}{2})}{2a}-4}{((\tan^2(\frac{dx}{2}+\frac{c}{2}))a-2b\tan(\frac{dx}{2}+\frac{c}{2})-a)^2}\right.}{b^6}$
risch	$\frac{15a^2b^2e^{9i(dx+c)}+140a^2b^2e^{7i(dx+c)}+250a^2b^2e^{5i(dx+c)}+140a^2b^2e^{3i(dx+c)}+15a^2b^2e^{i(dx+c)}+240a^4e^{3i(dx+c)}+60a^4e^{i(dx+c)}}{}$

```
[In] int(sec(d*x+c)^7/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/b^6*((1/2*b^2*(7*a^4+5*a^2*b^2-2*b^4)/a*tan(1/2*d*x+1/2*c)^3+1/2*b*
(8*a^6-9*a^4*b^2-15*a^2*b^4+2*b^6)/a^2*tan(1/2*d*x+1/2*c)^2-1/2*b^2*(25*a^4
+23*a^2*b^2-2*b^4)/a*tan(1/2*d*x+1/2*c)-4*a^4*b-7/2*a^2*b^3+1/2*b^5)/(tan(1
/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^2-5/2*(4*a^4+5*a^2*b^2+b^4)/(a^
2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))+1/3
/b^3/(tan(1/2*d*x+1/2*c)+1)^3-1/2*(-3*a+b)/b^4/(tan(1/2*d*x+1/2*c)+1)^2-1/2
*(-12*a^2+3*a*b-5*b^2)/b^5/(tan(1/2*d*x+1/2*c)+1)-5/2*a*(4*a^2+3*b^2)/b^6*l
n(tan(1/2*d*x+1/2*c)+1)-1/3/b^3/(tan(1/2*d*x+1/2*c)-1)^3-1/2*(3*a+b)/b^4/(t
an(1/2*d*x+1/2*c)-1)^2-1/2*(12*a^2+3*a*b+5*b^2)/b^5/(tan(1/2*d*x+1/2*c)-1)+
5/2*a*(4*a^2+3*b^2)/b^6*ln(tan(1/2*d*x+1/2*c)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 564 vs. 2(223) = 446.

Time = 0.35 (sec) , antiderivative size = 564, normalized size of antiderivative = 2.36

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{4\,b^5+30\,(4\,a^4b+a^2b^3-b^5)\cos{(dx+c)}^4+20\,(2\,a^2b^3+b^5)\cos{(dx+c)}^2+15\,((4\,a^4-3\,a^2b^2-b^4)\cos{(dx}}{}$$

```
[In] integrate(sec(d*x+c)^7/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*b^5 + 30*(4*a^4*b + a^2*b^3 - b^5)*cos(d*x + c)^4 + 20*(2*a^2*b^3 +
 b^5)*cos(d*x + c)^2 + 15*((4*a^4 - 3*a^2*b^2 - b^4)*cos(d*x + c)^5 + 2*(4*
a^3*b + a*b^3)*cos(d*x + c)^4*sin(d*x + c) + (4*a^2*b^2 + b^4)*cos(d*x + c)
^3)*sqrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos
(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x +
 c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2))
 - 15*((4*a^5 - a^3*b^2 - 3*a*b^4)*cos(d*x + c)^5 + 2*(4*a^4*b + 3*a^2*b^3)
*cos(d*x + c)^4*sin(d*x + c) + (4*a^3*b^2 + 3*a*b^4)*cos(d*x + c)^3)*log(si
n(d*x + c) + 1) + 15*((4*a^5 - a^3*b^2 - 3*a*b^4)*cos(d*x + c)^5 + 2*(4*a^4
*b + 3*a^2*b^3)*cos(d*x + c)^4*sin(d*x + c) + (4*a^3*b^2 + 3*a*b^4)*cos(d*x
 + c)^3)*log(-sin(d*x + c) + 1) - 10*(a*b^4*cos(d*x + c) - 6*(3*a^3*b^2 + 2
*a*b^4)*cos(d*x + c)^3)*sin(d*x + c))/(2*a*b^7*d*cos(d*x + c)^4*sin(d*x + c
) + b^8*d*cos(d*x + c)^3 + (a^2*b^6 - b^8)*d*cos(d*x + c)^5)
```

Sympy [F]

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^3}\,dx=\int \frac{\sec^7{(c+dx)}}{(a+b\tan{(c+dx)})^3}\,dx$$

```
[In] integrate(sec(d*x+c)**7/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**7/(a + b*tan(c + d*x))**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 902 vs. $2(223) = 446$.

Time = 0.61 (sec) , antiderivative size = 902, normalized size of antiderivative = 3.77

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^3}\,dx=\text{Too large to display}$$

```
[In] integrate(sec(d*x+c)^7/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(60*a^6 + 35*a^4*b^2 - 3*a^2*b^4 + (210*a^5*b + 125*a^3*b^3 - 6*a*b^
5)*sin(d*x + c)/(cos(d*x + c) + 1) - 2*(120*a^6 - 10*a^4*b^2 - 55*a^2*b^4 +
 3*b^6)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(330*a^5*b + 205*a^3*b^3 -
12*a*b^5)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*(180*a^6 - 95*a^4*b^2 - 1
20*a^2*b^4 + 9*b^6)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 12*(60*a^5*b + 35
*a^3*b^3 - 3*a*b^5)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 6*(40*a^6 - 30*a^
4*b^2 - 35*a^2*b^4 + 3*b^6)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 6*(50*a^5
*b + 25*a^3*b^3 - 4*a*b^5)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 3*(20*a^6
- 15*a^4*b^2 - 15*a^2*b^4 + 2*b^6)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 3*
(10*a^5*b + 5*a^3*b^3 - 2*a*b^5)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(a^4*
```

```
b^5 + 4*a^3*b^6*sin(d*x + c)/(cos(d*x + c) + 1) - 16*a^3*b^6*sin(d*x + c)^3
/(cos(d*x + c) + 1)^3 + 24*a^3*b^6*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 16
*a^3*b^6*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 4*a^3*b^6*sin(d*x + c)^9/(co
s(d*x + c) + 1)^9 - a^4*b^5*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - (5*a^4*
b^5 - 4*a^2*b^7)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 2*(5*a^4*b^5 - 6*a^2
*b^7)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 2*(5*a^4*b^5 - 6*a^2*b^7)*sin(d
*x + c)^6/(cos(d*x + c) + 1)^6 + (5*a^4*b^5 - 4*a^2*b^7)*sin(d*x + c)^8/(co
s(d*x + c) + 1)^8) - 15*(4*a^3 + 3*a*b^2)*log(sin(d*x + c)/(cos(d*x + c) +
1) + 1)/b^6 + 15*(4*a^3 + 3*a*b^2)*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)
/b^6 - 15*(4*a^4 + 5*a^2*b^2 + b^4)*log((b - a*sin(d*x + c)/(cos(d*x + c) +
 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 +
b^2)))/(sqrt(a^2 + b^2)*b^6))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 510 vs. $2(223) = 446$.

Time = 0.65 (sec) , antiderivative size = 510, normalized size of antiderivative = 2.13

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^3}\,dx =$$

$$-\frac{\frac{15\,(4\,a^3+3\,ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^6}-\frac{15\,(4\,a^3+3\,ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^6}+\frac{15\,(4\,a^4+5\,a^2b^2+b^4)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{}\right.}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{}\right.}\right.}{\sqrt{a^2+b^2}b^6}}{}$$

```
[In] integrate(sec(d*x+c)^7/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(15*(4*a^3 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^6 - 15*(4*a
^3 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^6 + 15*(4*a^4 + 5*a^2*b^
2 + b^4)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*
a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^6) +
2*(9*a*b*tan(1/2*d*x + 1/2*c)^5 + 36*a^2*tan(1/2*d*x + 1/2*c)^4 + 18*b^2*ta
n(1/2*d*x + 1/2*c)^4 - 72*a^2*tan(1/2*d*x + 1/2*c)^2 - 24*b^2*tan(1/2*d*x +
 1/2*c)^2 - 9*a*b*tan(1/2*d*x + 1/2*c) + 36*a^2 + 14*b^2)/((tan(1/2*d*x + 1
/2*c)^2 - 1)^3*b^5) + 6*(7*a^5*b*tan(1/2*d*x + 1/2*c)^3 + 5*a^3*b^3*tan(1/2
*d*x + 1/2*c)^3 - 2*a*b^5*tan(1/2*d*x + 1/2*c)^3 + 8*a^6*tan(1/2*d*x + 1/2*
c)^2 - 9*a^4*b^2*tan(1/2*d*x + 1/2*c)^2 - 15*a^2*b^4*tan(1/2*d*x + 1/2*c)^2
 + 2*b^6*tan(1/2*d*x + 1/2*c)^2 - 25*a^5*b*tan(1/2*d*x + 1/2*c) - 23*a^3*b^
3*tan(1/2*d*x + 1/2*c) + 2*a*b^5*tan(1/2*d*x + 1/2*c) - 8*a^6 - 7*a^4*b^2 +
 a^2*b^4)/((a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^2*a^2*
b^5))/d
```

Mupad [B] (verification not implemented)

Time = 7.83 (sec) , antiderivative size = 1203, normalized size of antiderivative = 5.03

$$\int \frac{\sec^7(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] int(1/(cos(c + d*x)^7*(a + b*tan(c + d*x))^3),x)
```

```
[Out] ((60*a^4 - 3*b^4 + 35*a^2*b^2)/(3*b^5) + (tan(c/2 + (d*x)/2)*(210*a^4 - 6*b
^4 + 125*a^2*b^2))/(3*a*b^4) + (tan(c/2 + (d*x)/2)^8*(20*a^6 + 2*b^6 - 15*a
^2*b^4 - 15*a^4*b^2))/(a^2*b^5) - (2*tan(c/2 + (d*x)/2)^6*(40*a^6 + 3*b^6 -
 35*a^2*b^4 - 30*a^4*b^2))/(a^2*b^5) - (2*tan(c/2 + (d*x)/2)^2*(120*a^6 + 3
*b^6 - 55*a^2*b^4 - 10*a^4*b^2))/(3*a^2*b^5) + (2*tan(c/2 + (d*x)/2)^4*(180
*a^6 + 9*b^6 - 120*a^2*b^4 - 95*a^4*b^2))/(3*a^2*b^5) + (tan(c/2 + (d*x)/2)
^9*(10*a^4 - 2*b^4 + 5*a^2*b^2))/(a*b^4) - (2*tan(c/2 + (d*x)/2)^7*(50*a^4
- 4*b^4 + 25*a^2*b^2))/(a*b^4) + (4*tan(c/2 + (d*x)/2)^5*(60*a^4 - 3*b^4 +
35*a^2*b^2))/(a*b^4) - (2*tan(c/2 + (d*x)/2)^3*(330*a^4 - 12*b^4 + 205*a^2*
b^2))/(3*a*b^4))/(d*(tan(c/2 + (d*x)/2)^8*(5*a^2 - 4*b^2) - tan(c/2 + (d*x)
/2)^2*(5*a^2 - 4*b^2) + tan(c/2 + (d*x)/2)^4*(10*a^2 - 12*b^2) - tan(c/2 +
(d*x)/2)^6*(10*a^2 - 12*b^2) - a^2*tan(c/2 + (d*x)/2)^10 + a^2 - 16*a*b*tan
(c/2 + (d*x)/2)^3 + 24*a*b*tan(c/2 + (d*x)/2)^5 - 16*a*b*tan(c/2 + (d*x)/2)
^7 + 4*a*b*tan(c/2 + (d*x)/2)^9 + 4*a*b*tan(c/2 + (d*x)/2))) - (atanh((3000
*a^2*tan(c/2 + (d*x)/2))/(3000*a^2 + (7000*a^4)/b^2 + (4000*a^6)/b^4) + (70
00*a^4*tan(c/2 + (d*x)/2))/(7000*a^4 + 3000*a^2*b^2 + (4000*a^6)/b^2) + (40
00*a^6*tan(c/2 + (d*x)/2))/(4000*a^6 + 3000*a^2*b^4 + 7000*a^4*b^2))*(15*a*
b^2 + 20*a^3))/(b^6*d) + (5*atanh((1000*a^2*(a^2 + b^2)^(1/2))/(1000*a^2*b
+ (5000*a^4)/b + (4000*a^6)/b^3 + 10000*a^3*tan(c/2 + (d*x)/2) + 2000*a*b^2
*tan(c/2 + (d*x)/2) + (8000*a^5*tan(c/2 + (d*x)/2))/b^2) + (4000*a^4*(a^2 +
 b^2)^(1/2))/(5000*a^4*b + 1000*a^2*b^3 + (4000*a^6)/b + 8000*a^5*tan(c/2 +
 (d*x)/2) + 2000*a*b^4*tan(c/2 + (d*x)/2) + 10000*a^3*b^2*tan(c/2 + (d*x)/2
)) + (9000*a^3*tan(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2))/(5000*a^4 + 1000*a^2*b
^2 + (4000*a^6)/b^2 + 2000*a*b^3*tan(c/2 + (d*x)/2) + 10000*a^3*b*tan(c/2 +
 (d*x)/2) + (8000*a^5*tan(c/2 + (d*x)/2))/b) + (4000*a^5*tan(c/2 + (d*x)/2)
*(a^2 + b^2)^(1/2))/(4000*a^6 + 1000*a^2*b^4 + 5000*a^4*b^2 + 2000*a*b^5*ta
n(c/2 + (d*x)/2) + 8000*a^5*b*tan(c/2 + (d*x)/2) + 10000*a^3*b^3*tan(c/2 +
(d*x)/2)) + (2000*a*tan(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2))/(1000*a^2 + (5000
*a^4)/b^2 + (4000*a^6)/b^4 + (10000*a^3*tan(c/2 + (d*x)/2))/b + (8000*a^5*t
an(c/2 + (d*x)/2))/b^3 + 2000*a*b*tan(c/2 + (d*x)/2)))*(4*a^2 + b^2)*(a^2 +
 b^2)^(1/2))/(b^6*d)
```

3.573 $\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 148

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{3a\text{arctanh}(\sin(c+dx))}{b^4d} - \frac{3(2a^2+b^2)\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2b^4\sqrt{a^2+b^2}d} - \frac{\sec^3(c+dx)}{2bd(a+b\tan(c+dx))^2} + \frac{3\sec(c+dx)(2a+b\tan(c+dx))}{2b^3d(a+b\tan(c+dx))}$$

```
[Out] -3*a*arctanh(sin(d*x+c))/b^4/d-3/2*(2*a^2+b^2)*arctanh((b*cos(d*x+c)-a*sin(
d*x+c))/(a^2+b^2)^(1/2))/b^4/d/(a^2+b^2)^(1/2)-1/2*sec(d*x+c)^3/b/d/(a+b*ta
n(d*x+c))^2+3/2*sec(d*x+c)*(2*a+b*tan(d*x+c))/b^3/d/(a+b*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3593, 747, 827, 858, 221, 739, 212}

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{3(2a^2+b^2)\sec(c+dx)\text{arctanh}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)}{2b^4d\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}} - \frac{3a\sec(c+dx)\text{arcsinh}(\tan(c+dx))}{b^4d\sqrt{\sec^2(c+dx)}} + \frac{3\sec(c+dx)(2a+b\tan(c+dx))}{2b^3d(a+b\tan(c+dx))} - \frac{\sec^3(c+dx)}{2bd(a+b\tan(c+dx))^2}$$

```
[In] Int[Sec[c + d*x]^5/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-3*a*ArcSinh[Tan[c + d*x]]*Sec[c + d*x])/(b^4*d*Sqrt[Sec[c + d*x]^2]) - (3
*(2*a^2 + b^2)*ArcTanh[(b - a*Tan[c + d*x])/(Sqrt[a^2 + b^2]*Sqrt[Sec[c + d
*x]^2])]*Sec[c + d*x])/(2*b^4*Sqrt[a^2 + b^2]*d*Sqrt[Sec[c + d*x]^2]) - Sec
[c + d*x]^3/(2*b*d*(a + b*Tan[c + d*x])^2) + (3*Sec[c + d*x]*(2*a + b*Tan[c
 + d*x]))/(2*b^3*d*(a + b*Tan[c + d*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 747

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 1))), x] - Dist[2*c*(p/(e*(m + 1))
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
```

```
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\sec(c+dx)\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^{3/2}}{(a+x)^3}\,dx, x, b\tan(c+dx)\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{\sec^3(c+dx)}{2bd(a+b\tan(c+dx))^2} + \frac{(3\sec(c+dx))\text{Subst}\left(\int \frac{x\sqrt{1+\frac{x^2}{b^2}}}{(a+x)^2}\,dx, x, b\tan(c+dx)\right)}{2b^3d\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{\sec^3(c+dx)}{2bd(a+b\tan(c+dx))^2} + \frac{3\sec(c+dx)(2a+b\tan(c+dx))}{2b^3d(a+b\tan(c+dx))} - \frac{(3\sec(c+dx))\text{Subst}\left(\int \frac{-2+\frac{4ax}{b^2}}{(a+x)\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+dx)\right)}{4b^3d\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{\sec^3(c+dx)}{2bd(a+b\tan(c+dx))^2} + \frac{3\sec(c+dx)(2a+b\tan(c+dx))}{2b^3d(a+b\tan(c+dx))} - \frac{(3a\sec(c+dx))\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+dx)\right)}{b^5d\sqrt{\sec^2(c+dx)}} + \frac{\left(3\left(1+\frac{2a^2}{b^2}\right)\sec(c+dx)\right)\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+dx)\right)}{2b^3d\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{3a\text{arcsinh}(\tan(c+dx))\sec(c+dx)}{b^4d\sqrt{\sec^2(c+dx)}} - \frac{\sec^3(c+dx)}{2bd(a+b\tan(c+dx))^2} + \frac{3\sec(c+dx)(2a+b\tan(c+dx))}{2b^3d(a+b\tan(c+dx))} - \frac{\left(3\left(1+\frac{2a^2}{b^2}\right)\sec(c+dx)\right)\text{Subst}\left(\int \frac{1}{1+\frac{a^2}{b^2}-x^2}\,dx, x, \frac{1-\frac{a\tan(c+dx)}{b}}{\sqrt{\sec^2(c+dx)}}\right)}{2b^3d\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{3a\operatorname{arcsinh}(\tan(c+dx))\sec(c+dx)}{b^4d\sqrt{\sec^2(c+dx)}} - \frac{3(2a^2+b^2)\operatorname{arctanh}\left(\frac{b\left(1-\frac{a\tan(c+dx)}{b}\right)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\sec(c+dx)}{2b^4\sqrt{a^2+b^2}d\sqrt{\sec^2(c+dx)}} - \frac{\sec^3(c+dx)}{2bd(a+b\tan(c+dx))^2} + \frac{3\sec(c+dx)(2a+b\tan(c+dx))}{2b^3d(a+b\tan(c+dx))}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 396 vs. $2(148) = 296$.

Time = 3.68 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.68

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

$$= \frac{\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(\frac{b^2(a^2+b^2)\sin(c+dx)}{a} + \frac{(2a-b)b(2a+b)(a\cos(c+dx)+b\sin(c+dx))}{a} + 2b(a\cos\right.}{}$$

```
[In] Integrate[Sec[c + d*x]^5/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])*((b^2*(a^2 + b^2)*Sin[c +
 d*x])/a + ((2*a - b)*b*(2*a + b)*(a*Cos[c + d*x] + b*Sin[c + d*x]))/a + 2*
b*(a*Cos[c + d*x] + b*Sin[c + d*x])^2 + (6*(2*a^2 + b^2)*ArcTanh[(-b + a*Ta
n[(c + d*x)/2])/Sqrt[a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/Sqrt[
a^2 + b^2] + 6*a*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a*Cos[c + d*x] +
 b*Sin[c + d*x])^2 - 6*a*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a*Cos[c
+ d*x] + b*Sin[c + d*x])^2 + (2*b*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin[
c + d*x])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (2*b*Sin[(c + d*x)/2]*
(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
)/(2*b^4*d*(a + b*Tan[c + d*x])^3)
```

Maple [A] (verified)

Time = 94.52 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.82

method	result
derivativedivides	$\frac{\frac{1}{b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{3a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^4}-\frac{2\left(\frac{\frac{b^2(3a^2-2b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{b(4a^4-9a^2b^2+2b^4)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2}-\frac{b^2(13a^2-}{}}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}\right.}{b^4}}{d}$
default	$\frac{\frac{1}{b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{3a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^4}-\frac{2\left(\frac{\frac{b^2(3a^2-2b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{b(4a^4-9a^2b^2+2b^4)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2}-\frac{b^2(13a^2-}{}}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}\right.}{b^4}}{d}$
risch	$\frac{-9iab\,e^{5i(dx+c)}+6a^2e^{5i(dx+c)}-3b^2e^{5i(dx+c)}+12a^2e^{3i(dx+c)}+2b^2e^{3i(dx+c)}+9iab\,e^{i(dx+c)}+6a^2e^{i(dx+c)}-3b^2e^{i(dx+c)}}{\left(e^{2i(dx+c)}+1\right)\left(-ib\,e^{2i(dx+c)}+a\,e^{2i(dx+c)}+ib+a\right)^2b^3d}-$

[In] int(sec(d*x+c)^5/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(1/b^3/(tan(1/2*d*x+1/2*c)+1)-3*a/b^4*ln(tan(1/2*d*x+1/2*c)+1)-2/b^4*((1/2*b^2*(3*a^2-2*b^2)/a*tan(1/2*d*x+1/2*c)^3+1/2*b*(4*a^4-9*a^2*b^2+2*b^4)/a^2*tan(1/2*d*x+1/2*c)^2-1/2*b^2*(13*a^2-2*b^2)/a*tan(1/2*d*x+1/2*c)-2*a^2*b+1/2*b^3)/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^2-3/2*(2*a^2+b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))-1/b^3/(tan(1/2*d*x+1/2*c)-1)+3*a/b^4*ln(tan(1/2*d*x+1/2*c)-1))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 513 vs. 2(138) = 276.

Time = 0.34 (sec) , antiderivative size = 513, normalized size of antiderivative = 3.47

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$=\frac{4\,a^2b^3+4\,b^5+6\,(2\,a^4b+a^2b^3-b^5)\cos{(dx+c)}^2+18\,(a^3b^2+ab^4)\cos{(dx+c)}\sin{(dx+c)}+3\,\left((2\,a^4-a^2b\right.}{}$$

[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c))^3,x, algorithm="fricas")

[Out] 1/4*(4*a^2*b^3 + 4*b^5 + 6*(2*a^4*b + a^2*b^3 - b^5)*cos(d*x + c)^2 + 18*(a^3*b^2 + a*b^4)*cos(d*x + c)*sin(d*x + c) + 3*((2*a^4 - a^2*b^2 - b^4)*cos(d*x + c)^3 + 2*(2*a^3*b + a*b^3)*cos(d*x + c)^2*sin(d*x + c) + (2*a^2*b^2 + b^4)*cos(d*x + c))*sqrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - 6*((a^5 - a*b^4)*cos(d*x + c)^3 + 2*(a^4*b + a^2*b^3)*cos(d*x + c)^2*sin(d*x + c) + (a^3*b^2 + a*b^4)*cos(d*x + c))*log(sin(d*x + c

```
) + 1) + 6*((a^5 - a*b^4)*cos(d*x + c)^3 + 2*(a^4*b + a^2*b^3)*cos(d*x + c)
^2*sin(d*x + c) + (a^3*b^2 + a*b^4)*cos(d*x + c))*log(-sin(d*x + c) + 1))/(
(a^4*b^4 - b^8)*d*cos(d*x + c)^3 + 2*(a^3*b^5 + a*b^7)*d*cos(d*x + c)^2*sin
(d*x + c) + (a^2*b^6 + b^8)*d*cos(d*x + c))
```

Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\sec^5\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^3}\,dx$$

```
[In] integrate(sec(d*x+c)**5/(a+b*tan(d*x+c))**3,x)
[Out] Integral(sec(c + d*x)**5/(a + b*tan(c + d*x))**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(138) = 276$.

Time = 0.34 (sec) , antiderivative size = 518, normalized size of antiderivative = 3.50

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{2\left(6a^4-a^2b^2+\frac{\left(21a^3b-2ab^3\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\left(6a^4-9a^2b^2+b^4\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4\left(6a^3b-ab^3\right)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\left(6a^4-9a^2b^2+2b^4\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\left(3a^3b-2ab^3\right)\sin\cdots}{(\cos(dx+c)\cdots}}{a^4b^3+\frac{4a^3b^4\sin(dx+c)}{\cos(dx+c)+1}-\frac{8a^3b^4\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{4a^3b^4\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{a^4b^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{\left(3a^4b^3-4a^2b^5\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\left(3a^4b^3-4a^2b^5\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\cdots}}{2d}$$

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
[Out] 1/2*(2*(6*a^4 - a^2*b^2 + (21*a^3*b - 2*a*b^3)*sin(d*x + c)/(cos(d*x + c) +
 1) - 2*(6*a^4 - 9*a^2*b^2 + b^4)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4*(
6*a^3*b - a*b^3)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + (6*a^4 - 9*a^2*b^2 +
 2*b^4)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (3*a^3*b - 2*a*b^3)*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5)/(a^4*b^3 + 4*a^3*b^4*sin(d*x + c)/(cos(d*x + c)
 + 1) - 8*a^3*b^4*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 4*a^3*b^4*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5 - a^4*b^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 -
(3*a^4*b^3 - 4*a^2*b^5)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + (3*a^4*b^3 -
4*a^2*b^5)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - 6*a*log(sin(d*x + c)/(cos
(d*x + c) + 1) + 1)/b^4 + 6*a*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/b^4
- 3*(2*a^2 + b^2)*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b
^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 +
 b^2)*b^4))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(138) = 276.

Time = 0.61 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.12

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{\frac{6\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^4}-\frac{6\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^4}+\frac{3\,(2\,a^2+b^2)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^4}+\frac{4}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right.}}{}$$

```
[In] integrate(sec(d*x+c)^5/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(6*a*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 - 6*a*log(abs(tan(1/2*d*x
+ 1/2*c) - 1))/b^4 + 3*(2*a^2 + b^2)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b
 - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2
)))/(sqrt(a^2 + b^2)*b^4) + 4/((tan(1/2*d*x + 1/2*c)^2 - 1)*b^3) + 2*(3*a^3
*b*tan(1/2*d*x + 1/2*c)^3 - 2*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 4*a^4*tan(1/2*
d*x + 1/2*c)^2 - 9*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 2*b^4*tan(1/2*d*x + 1/2
*c)^2 - 13*a^3*b*tan(1/2*d*x + 1/2*c) + 2*a*b^3*tan(1/2*d*x + 1/2*c) - 4*a^
4 + a^2*b^2)/((a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^2*a
^2*b^3))/d
```

Mupad [B] (verification not implemented)

Time = 6.48 (sec) , antiderivative size = 1311, normalized size of antiderivative = 8.86

$$\int \frac{\sec^5(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] int(1/(cos(c + d*x)^5*(a + b*tan(c + d*x))^3),x)
```

```
[Out] ((6*a^2 - b^2)/b^3 - (2*tan(c/2 + (d*x)/2)^2*(6*a^4 + b^4 - 9*a^2*b^2))/(a^
2*b^3) + (tan(c/2 + (d*x)/2)*(21*a^2 - 2*b^2))/(a*b^2) + (tan(c/2 + (d*x)/2
)^4*(6*a^4 + 2*b^4 - 9*a^2*b^2))/(a^2*b^3) - (4*tan(c/2 + (d*x)/2)^3*(6*a^2
 - b^2))/(a*b^2) + (tan(c/2 + (d*x)/2)^5*(3*a^2 - 2*b^2))/(a*b^2))/(d*(tan(
c/2 + (d*x)/2)^4*(3*a^2 - 4*b^2) - tan(c/2 + (d*x)/2)^2*(3*a^2 - 4*b^2) - a
^2*tan(c/2 + (d*x)/2)^6 + a^2 - 8*a*b*tan(c/2 + (d*x)/2)^3 + 4*a*b*tan(c/2
+ (d*x)/2)^5 + 4*a*b*tan(c/2 + (d*x)/2))) - (6*a*atanh(tan(c/2 + (d*x)/2)))
/(b^4*d) + (atan((((2*a^2 + b^2)*(a^2 + b^2)^(1/2)*((288*a^4)/b^5 + (8*tan(
c/2 + (d*x)/2)*(9*a*b^7 + 108*a^3*b^5 + 72*a^5*b^3))/b^9 - (3*(2*a^2 + b^2)
*(a^2 + b^2)^(1/2)*((8*tan(c/2 + (d*x)/2)*(12*a*b^10 + 24*a^3*b^8))/b^9 - 4
8*a^2 + (3*(2*a^2 + b^2)*(a^2 + b^2)^(1/2)*(32*a^2*b^3 + (8*tan(c/2 + (d*x)
/2)*(12*a*b^13 + 8*a^3*b^11))/b^9))/(2*(b^6 + a^2*b^4))))/(2*(b^6 + a^2*b^4
```

```
)))*3i)/(2*(b^6 + a^2*b^4)) + ((2*a^2 + b^2)*(a^2 + b^2)^(1/2)*((288*a^4)/b
^5 + (8*tan(c/2 + (d*x)/2)*(9*a*b^7 + 108*a^3*b^5 + 72*a^5*b^3))/b^9 - (3*(
2*a^2 + b^2)*(a^2 + b^2)^(1/2)*(48*a^2 - (8*tan(c/2 + (d*x)/2)*(12*a*b^10 +
 24*a^3*b^8))/b^9 + (3*(2*a^2 + b^2)*(a^2 + b^2)^(1/2)*(32*a^2*b^3 + (8*tan
(c/2 + (d*x)/2)*(12*a*b^13 + 8*a^3*b^11))/b^9))/(2*(b^6 + a^2*b^4))))/(2*(b
^6 + a^2*b^4)))*3i)/(2*(b^6 + a^2*b^4)))/((16*(54*a^4 + 27*a^2*b^2))/b^8 -
(16*tan(c/2 + (d*x)/2)*(216*a^5 + 108*a^3*b^2))/b^9 - (3*(2*a^2 + b^2)*(a^2
 + b^2)^(1/2)*((288*a^4)/b^5 + (8*tan(c/2 + (d*x)/2)*(9*a*b^7 + 108*a^3*b^5
 + 72*a^5*b^3))/b^9 - (3*(2*a^2 + b^2)*(a^2 + b^2)^(1/2)*((8*tan(c/2 + (d*x
)/2)*(12*a*b^10 + 24*a^3*b^8))/b^9 - 48*a^2 + (3*(2*a^2 + b^2)*(a^2 + b^2)^
(1/2)*(32*a^2*b^3 + (8*tan(c/2 + (d*x)/2)*(12*a*b^13 + 8*a^3*b^11))/b^9))/(
2*(b^6 + a^2*b^4))))/(2*(b^6 + a^2*b^4))))/(2*(b^6 + a^2*b^4)) + (3*(2*a^2
+ b^2)*(a^2 + b^2)^(1/2)*((288*a^4)/b^5 + (8*tan(c/2 + (d*x)/2)*(9*a*b^7 +
108*a^3*b^5 + 72*a^5*b^3))/b^9 - (3*(2*a^2 + b^2)*(a^2 + b^2)^(1/2)*(48*a^2
 - (8*tan(c/2 + (d*x)/2)*(12*a*b^10 + 24*a^3*b^8))/b^9 + (3*(2*a^2 + b^2)*(
a^2 + b^2)^(1/2)*(32*a^2*b^3 + (8*tan(c/2 + (d*x)/2)*(12*a*b^13 + 8*a^3*b^1
1))/b^9))/(2*(b^6 + a^2*b^4))))/(2*(b^6 + a^2*b^4))))/(2*(b^6 + a^2*b^4))))
*(2*a^2 + b^2)*(a^2 + b^2)^(1/2)*3i)/(d*(b^6 + a^2*b^4))
```

3.574 $\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}d} - \frac{\sec(c+dx)(b-a\tan(c+dx))}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2}$$

```
[Out] -1/2*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/d
-1/2*sec(d*x+c)*(b-a*tan(d*x+c))/(a^2+b^2)/d/(a+b*tan(d*x+c))^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.24, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3593, 735, 739, 212}

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{\sec(c+dx)\operatorname{arctanh}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)}{2d\left(a^2+b^2\right)^{3/2}\sqrt{\sec^2(c+dx)}} - \frac{\sec(c+dx)(b-a\tan(c+dx))}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

```
[In] Int[Sec[c + d*x]^3/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -1/2*(ArcTanh[(b - a*Tan[c + d*x])/(Sqrt[a^2 + b^2]*Sqrt[Sec[c + d*x]^2])]*
Sec[c + d*x])/((a^2 + b^2)^(3/2)*d*Sqrt[Sec[c + d*x]^2]) - (Sec[c + d*x]*(b
 - a*Tan[c + d*x]))/(2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 735

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(-2*a*e + (2*c*d)*x)*((a + c*x^2)^p/(2*(m + 1)*(c*d^2
+ a*e^2))), x] - Dist[4*a*c*(p/(2*(m + 1)*(c*d^2 + a*e^2))), Int[(d + e*x)^
(m + 2)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2
+ a*e^2, 0] && EqQ[m + 2*p + 2, 0] && GtQ[p, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\sec(c+dx)\text{Subst}\left(\int \frac{\sqrt{1+\frac{x^2}{b^2}}}{(a+x)^3}\,dx, x, b\tan(c+dx)\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{\sec(c+dx)(b-a\tan(c+dx))}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} + \frac{\sec(c+dx)\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+dx)\right)}{2b\left(a^2+b^2\right)d\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{\sec(c+dx)(b-a\tan(c+dx))}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{\sec(c+dx)\text{Subst}\left(\int \frac{1}{1+\frac{a^2}{b^2}-x^2}\,dx, x, \frac{1-\frac{a\tan(c+dx)}{b}}{\sqrt{\sec^2(c+dx)}}\right)}{2b\left(a^2+b^2\right)d\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{\operatorname{arctanh}\left(\frac{b\left(1-\frac{a\tan(c+dx)}{b}\right)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\sec(c+dx)}{2\left(a^2+b^2\right)^{3/2}d\sqrt{\sec^2(c+dx)}} - \frac{\sec(c+dx)(b-a\tan(c+dx))}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.39

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$=\frac{(a^2+b^2)\left(-b\cos(c+dx)+a\sin(c+dx)\right)+2\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)(a\cos(c+dx)+b\sin(c+}{2(a-ib)^2(a+ib)^2d(a\cos(c+dx)+b\sin(c+dx))^2}$$

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] ((a^2 + b^2)*(-(b*Cos[c + d*x]) + a*Sin[c + d*x]) + 2*Sqrt[a^2 + b^2]*ArcTa
nh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d
*x])^2)/(2*(a - I*b)^2*(a + I*b)^2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(87) = 174.

Time = 17.40 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.01

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{(a^2+2b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a(a^2+b^2)}-\frac{b(a^2-2b^2)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a^2+b^2)a^2}-\frac{(a^2-2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a^2+b^2)a}+\frac{b}{2a^2+2b^2}\right)}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}+\frac{\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2}{2\sqrt{a^2+b^2}}\right.}{(a^2+b^2)^{\frac{3}{2}}}}{d}$
default	$\frac{-\frac{2\left(-\frac{(a^2+2b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a(a^2+b^2)}-\frac{b(a^2-2b^2)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a^2+b^2)a^2}-\frac{(a^2-2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a^2+b^2)a}+\frac{b}{2a^2+2b^2}\right)}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}+\frac{\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2}{2\sqrt{a^2+b^2}}\right.}{(a^2+b^2)^{\frac{3}{2}}}}{d}$
risch	$\frac{e^{i(dx+c)}\left(ia\,e^{2i(dx+c)}+b\,e^{2i(dx+c)}-ia+b\right)}{\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)^2(-ia+b)d(ia+b)}+\frac{\ln\left(e^{i(dx+c)}+\frac{ia^3+ia\,b^2-a^2b-b^3}{(a^2+b^2)^{\frac{3}{2}}}\right)}{2(a^2+b^2)^{\frac{3}{2}}d}-\frac{\ln\left(e^{i(dx+c)}-\frac{ia^3+ia\,b^2-a^2}{(a^2+b^2)}\right.}{2(a^2+b^2)^{\frac{3}{2}}d}$

```
[In] int(sec(d*x+c)^3/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-1/2*(a^2+2*b^2)/a/(a^2+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*b*(a^2-2*b^2
)/(a^2+b^2)/a^2*tan(1/2*d*x+1/2*c)^2-1/2*(a^2-2*b^2)/(a^2+b^2)/a*tan(1/2*d*
x+1/2*c)+1/2*b/(a^2+b^2))/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)
^2+1/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/
2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(88) = 176.

Time = 0.27 (sec) , antiderivative size = 294, normalized size of antiderivative = 3.09

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{(2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2)\sqrt{a^2+b^2}\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos}{2\,ab\cos(dx+c)\sin}\right.}{4\left((a^6+a^4b^2-a^2b^4-b^6)d\cos(dx+c)^2+2\,(a^5b+2\,a^3b^3+a\right.}$$

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)*s
qrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x
+ c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))
/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - 2*
(a^2*b + b^3)*cos(d*x + c) + 2*(a^3 + a*b^2)*sin(d*x + c))/((a^6 + a^4*b^2
- a^2*b^4 - b^6)*d*cos(d*x + c)^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d*cos(d*x
 + c)*sin(d*x + c) + (a^4*b^2 + 2*a^2*b^4 + b^6)*d)
```

Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

```
[In] integrate(sec(d*x+c)**3/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*tan(c + d*x))**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(88) = 176.

Time = 0.31 (sec) , antiderivative size = 326, normalized size of antiderivative = 3.43

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx =$$
$$-\frac{\frac{2\left(a^2b-\frac{(a^3-2\,ab^2)\sin(dx+c)}{\cos(dx+c)+1}-\frac{(a^2b-2\,b^3)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{(a^3+2\,ab^2)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^6+a^4b^2+\frac{4\,(a^5b+a^3b^3)\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\,(a^6-a^4b^2-2\,a^2b^4)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4\,(a^5b+a^3b^3)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{(a^6+a^4b^2)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-}\right.}{(a^2+b^2)^{\frac{3}{2}}}}{2\,d}$$

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(a^2*b - (a^3 - 2*a*b^2)*sin(d*x + c)/(cos(d*x + c) + 1) - (a^2*b -
 2*b^3)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - (a^3 + 2*a*b^2)*sin(d*x + c)^
3/(cos(d*x + c) + 1)^3)/(a^6 + a^4*b^2 + 4*(a^5*b + a^3*b^3)*sin(d*x + c)/(
cos(d*x + c) + 1) - 2*(a^6 - a^4*b^2 - 2*a^2*b^4)*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 - 4*(a^5*b + a^3*b^3)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + (a^6
 + a^4*b^2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + log((b - a*sin(d*x + c)/
(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1
) - sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(88) = 176$.

Time = 0.61 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.33

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{\frac{\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}} - \frac{2\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a^3\tan\right)}{(a^4+a^2b^2)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)^2}}{2\,d}$$

```
[In] integrate(sec(d*x+c)^3/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*t
an(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(a^3*
tan(1/2*d*x + 1/2*c)^3 + 2*a*b^2*tan(1/2*d*x + 1/2*c)^3 + a^2*b*tan(1/2*d*x
 + 1/2*c)^2 - 2*b^3*tan(1/2*d*x + 1/2*c)^2 + a^3*tan(1/2*d*x + 1/2*c) - 2*a
*b^2*tan(1/2*d*x + 1/2*c) - a^2*b)/((a^4 + a^2*b^2)*(a*tan(1/2*d*x + 1/2*c)
^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^2))/d
```

Mupad [B] (verification not implemented)

Time = 6.93 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.74

$$\int \frac{\sec^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a^2-2\,b^2)}{a\,(a^2+b^2)} - \frac{b}{a^2+b^2} + \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(a^2+2\,b^2)}{a\,(a^2+b^2)} + \frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(a^2-2\,b^2)}{a^2\,(a^2+b^2)}}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(2\,a^2-4\,b^2)+a^2-4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)} + \frac{\mathrm{atanh}\left(\frac{\left(2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{2\,a^2\,b+2\,b^3}{a^2+b^2}\right)\,\left(\frac{a^2}{2}+\frac{b^2}{2}\right)}{(a^2+b^2)^{3/2}}\right)}{d\,(a^2+b^2)^{3/2}}$$

```
[In] int(1/(cos(c + d*x)^3*(a + b*tan(c + d*x))^3),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)*(a^2 - 2*b^2))/(a*(a^2 + b^2)) - b/(a^2 + b^2) + (tan(
c/2 + (d*x)/2)^3*(a^2 + 2*b^2))/(a*(a^2 + b^2)) + (b*tan(c/2 + (d*x)/2)^2*(
a^2 - 2*b^2))/(a^2*(a^2 + b^2)))/(d*(a^2*tan(c/2 + (d*x)/2)^4 - tan(c/2 + (
d*x)/2)^2*(2*a^2 - 4*b^2) + a^2 - 4*a*b*tan(c/2 + (d*x)/2)^3 + 4*a*b*tan(c/
2 + (d*x)/2))) + atanh(((2*a*tan(c/2 + (d*x)/2) - (2*a^2*b + 2*b^3)/(a^2 +
b^2))*(a^2/2 + b^2/2))/(a^2 + b^2)^(3/2))/(d*(a^2 + b^2)^(3/2))
```

3.575 $\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 155

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{(2a^2-b^2)\,\mathrm{arctanh}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\sec(c+dx)}{2\,(a^2+b^2)^{5/2}\,d\sqrt{\sec^2(c+dx)}} - \frac{b\sec(c+dx)}{2\,(a^2+b^2)\,d(a+b\tan(c+dx))^2} - \frac{3ab\sec(c+dx)}{2\,(a^2+b^2)^2\,d(a+b\tan(c+dx))}$$

```
[Out] -1/2*(2*a^2-b^2)*arctanh((b-a*tan(d*x+c))/(a^2+b^2)^(1/2)/(sec(d*x+c)^2)^(1
/2))*sec(d*x+c)/(a^2+b^2)^(5/2)/d/(sec(d*x+c)^2)^(1/2)-1/2*b*sec(d*x+c)/(a^
2+b^2)/d/(a+b*tan(d*x+c))^2-3/2*a*b*sec(d*x+c)/(a^2+b^2)^2/d/(a+b*tan(d*x+c
))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3593, 759, 821, 739, 212}

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{(2a^2-b^2)\sec(c+dx)\mathrm{arctanh}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)}{2d\,(a^2+b^2)^{5/2}\sqrt{\sec^2(c+dx)}} - \frac{3ab\sec(c+dx)}{2d\,(a^2+b^2)^2\,(a+b\tan(c+dx))} - \frac{b\sec(c+dx)}{2d\,(a^2+b^2)\,(a+b\tan(c+dx))^2}$$

```
[In] Int[Sec[c + d*x]/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] -1/2*((2*a^2 - b^2)*ArcTanh[(b - a*Tan[c + d*x])/(Sqrt[a^2 + b^2]*Sqrt[Sec[
c + d*x]^2])]*Sec[c + d*x])/((a^2 + b^2)^(5/2)*d*Sqrt[Sec[c + d*x]^2]) - (b
*Sec[c + d*x])/(2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) - (3*a*b*Sec[c + d*
x])/(2*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\sec(c+dx)\text{Subst}\left(\int \frac{1}{(a+x)^3\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+dx)\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{b\sec(c+dx)}{2(a^2+b^2)\,d(a+b\tan(c+dx))^2} - \frac{\sec(c+dx)\text{Subst}\left(\int \frac{-2a+x}{(a+x)^2\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+dx)\right)}{2b(a^2+b^2)\,d\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{b\sec(c+dx)}{2(a^2+b^2)\,d(a+b\tan(c+dx))^2} - \frac{3ab\sec(c+dx)}{2(a^2+b^2)^2\,d(a+b\tan(c+dx))} + \frac{((2a^2-b^2)\sec(c+dx))\,\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+dx)\right)}{2b(a^2+b^2)^2\,d\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{b\sec(c+dx)}{2(a^2+b^2)\,d(a+b\tan(c+dx))^2} - \frac{3ab\sec(c+dx)}{2(a^2+b^2)^2\,d(a+b\tan(c+dx))} - \frac{((2a^2-b^2)\sec(c+dx))\,\text{Subst}\left(\int \frac{1}{1+\frac{a^2}{b^2}-x^2}\,dx, x, \frac{1-\frac{a\tan(c+dx)}{b}}{\sqrt{\sec^2(c+dx)}}\right)}{2b(a^2+b^2)^2\,d\sqrt{\sec^2(c+dx)}}$$

$$= -\frac{(2a^2-b^2)\,\text{arctanh}\left(\frac{b\left(1-\frac{a\tan(c+dx)}{b}\right)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\sec(c+dx)}{2(a^2+b^2)^{5/2}\,d\sqrt{\sec^2(c+dx)}} - \frac{b\sec(c+dx)}{2(a^2+b^2)\,d(a+b\tan(c+dx))^2} - \frac{3ab\sec(c+dx)}{2(a^2+b^2)^2\,d(a+b\tan(c+dx))}$$

Mathematica [A] (verified)

Time = 1.66 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.71

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{\frac{2(2a^2-b^2)\text{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} - \frac{b\sec(c+dx)(4a^2+b^2+3ab\tan(c+dx))}{(a^2+b^2)^2(a+b\tan(c+dx))^2}}{2d}$$

```
[In] Integrate[Sec[c + d*x]/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] ((2*(2*a^2 - b^2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^2
+ b^2)^(5/2) - (b*Sec[c + d*x]*(4*a^2 + b^2 + 3*a*b*Tan[c + d*x]))/((a^2 +
b^2)^2*(a + b*Tan[c + d*x])^2))/(2*d)
```

Maple [A] (verified)

Time = 3.62 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.81

method	result
derivativedivides	$-\frac{\frac{2\left(-\frac{b^2\left(5a^2+2b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a\left(a^4+2a^2b^2+b^4\right)}-\frac{b\left(4a^4-7a^2b^2-2b^4\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2\left(a^4+2a^2b^2+b^4\right)a^2}+\frac{b^2\left(11a^2+2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^4+2a^2b^2+b^4\right)a}+\frac{b\left(4a^2+b^2\right)}{2a^4+4a^2b^2+2b^4}\right)}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}+\cdots}{d}$
default	$-\frac{\frac{2\left(-\frac{b^2\left(5a^2+2b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a\left(a^4+2a^2b^2+b^4\right)}-\frac{b\left(4a^4-7a^2b^2-2b^4\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2\left(a^4+2a^2b^2+b^4\right)a^2}+\frac{b^2\left(11a^2+2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^4+2a^2b^2+b^4\right)a}+\frac{b\left(4a^2+b^2\right)}{2a^4+4a^2b^2+2b^4}\right)}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}+\cdots}{d}$
risch	$-\frac{b\,e^{i(dx+c)}\left(-3iab\,e^{2i(dx+c)}+4a^2e^{2i(dx+c)}+b^2e^{2i(dx+c)}+3iab+4a^2+b^2\right)}{(ib+a)^2\left(-ib\,e^{2i(dx+c)}+a\,e^{2i(dx+c)}+ib+a\right)^2d(-ib+a)^2}+\frac{\ln\left(e^{i(dx+c)}+\frac{ia^5+2ia^3b^2+ia\,b^4-a^4b-2a^2\cdots}{\left(a^2+b^2\right)^{\frac{5}{2}}}\cdots\right)}{\left(a^2+b^2\right)^{\frac{5}{2}}d}\cdots$

```
[In] int(sec(d*x+c)/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-1/2*b^2*(5*a^2+2*b^2)/a/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)^3-
1/2*b*(4*a^4-7*a^2*b^2-2*b^4)/(a^4+2*a^2*b^2+b^4)/a^2*tan(1/2*d*x+1/2*c)^2+
1/2*b^2*(11*a^2+2*b^2)/(a^4+2*a^2*b^2+b^4)/a*tan(1/2*d*x+1/2*c)+1/2*b*(4*a^
2+b^2)/(a^4+2*a^2*b^2+b^4))/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-
a)^2+(2*a^2-b^2)/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1
/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(143) = 286.

Time = 0.29 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.27

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{\left(2\,a^2b^2-b^4+\left(2\,a^4-3\,a^2b^2+b^4\right)\cos\left(dx+c\right)^2+2\left(2\,a^3b-ab^3\right)\cos\left(dx+c\right)\sin\left(dx+c\right)\right)\sqrt{a^2+b^2}\log\cdots}{4\left(\left(a^8+2\,a^6b^2-2\,a^2b^6-b^8\right)d\cos\left(dx+c\right)^2+2\left(\cdots\right.\right.}$$

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*((2*a^2*b^2 - b^4 + (2*a^4 - 3*a^2*b^2 + b^4)*cos(d*x + c)^2 + 2*(2*a^
3*b - a*b^3)*cos(d*x + c)*sin(d*x + c))*sqrt(a^2 + b^2)*log((2*a*b*cos(d*x
+ c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 +
 b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) +
 (a^2 - b^2)*cos(d*x + c)^2 + b^2)) + 2*(4*a^4*b + 5*a^2*b^3 + b^5)*cos(d*x
 + c) + 6*(a^3*b^2 + a*b^4)*sin(d*x + c))/((a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b
^8)*d*cos(d*x + c)^2 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*cos(d*x
+ c)*sin(d*x + c) + (a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d)
```

Sympy [F]

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c))**3,x)
[Out] Integral(sec(c + d*x)/(a + b*tan(c + d*x))**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(143) = 286$.

Time = 0.30 (sec) , antiderivative size = 412, normalized size of antiderivative = 2.66

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{\frac{(2a^2-b^2)\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^4+2a^2b^2+b^4)\sqrt{a^2+b^2}} + \frac{2\left(4a^4b+a^2b^3+\frac{(11a^3b^2+2ab^4)\sin(dx+c)}{\cos(dx+c)+1}-\frac{(4a^4b-7a^2b^3-2b^5)\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \cdots\right.}{a^8+2a^6b^2+a^4b^4+\frac{4(a^7b+2a^5b^3+a^3b^5)\sin(dx+c)}{\cos(dx+c)+1}-\frac{2(a^8-3a^4b^4-2a^2b^6)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4(a^7b+2a^5b \cdots}{(\cos \cdots}}}{2d}$$

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
[Out] -1/2*((2*a^2 - b^2)*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 +
 b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2
*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2*(4*a^4*b + a^2*b^3 + (11*a^3*b^2 + 2*a
*b^4)*sin(d*x + c)/(cos(d*x + c) + 1) - (4*a^4*b - 7*a^2*b^3 - 2*b^5)*sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 - (5*a^3*b^2 + 2*a*b^4)*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3)/(a^8 + 2*a^6*b^2 + a^4*b^4 + 4*(a^7*b + 2*a^5*b^3 + a^3*b^
5)*sin(d*x + c)/(cos(d*x + c) + 1) - 2*(a^8 - 3*a^4*b^4 - 2*a^2*b^6)*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2 - 4*(a^7*b + 2*a^5*b^3 + a^3*b^5)*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 + (a^8 + 2*a^6*b^2 + a^4*b^4)*sin(d*x + c)^4/(cos(
d*x + c) + 1)^4))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(143) = 286$.

Time = 0.54 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.89

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{\frac{(2a^2-b^2)\log\left(\frac{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b-2\sqrt{a^2+b^2}\right|}{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b+2\sqrt{a^2+b^2}\right|}\right)}{(a^4+2a^2b^2+b^4)\sqrt{a^2+b^2}} - \frac{2\left(5a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2ab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+4a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-7a^2b^3\tan\left(\frac{1}{2} \cdots\right.}{(a^6+2a^4b^2+a^2b^4)\left(a\tan\left(\frac{1}{2} \cdots\right.}}{2d}$$

```
[In] integrate(sec(d*x+c)/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*((2*a^2 - b^2)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b
^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2
*b^2 + b^4)*sqrt(a^2 + b^2)) - 2*(5*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*a*b^
4*tan(1/2*d*x + 1/2*c)^3 + 4*a^4*b*tan(1/2*d*x + 1/2*c)^2 - 7*a^2*b^3*tan(1
/2*d*x + 1/2*c)^2 - 2*b^5*tan(1/2*d*x + 1/2*c)^2 - 11*a^3*b^2*tan(1/2*d*x +
 1/2*c) - 2*a*b^4*tan(1/2*d*x + 1/2*c) - 4*a^4*b - a^2*b^3)/((a^6 + 2*a^4*b
^2 + a^2*b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^2))
/d
```

Mupad [B] (verification not implemented)

Time = 5.88 (sec) , antiderivative size = 443, normalized size of antiderivative = 2.86

$$\int \frac{\sec(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= \frac{\ln\left(\left(a^2+b^2\right)^{5/2}-a^4\,b-b^5-2\,a^2\,b^3+a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\,b^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+2\,a^3\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a^2-\frac{b}{2}\right.}{d\,\left(a^2+b^2\right)^{5/2}}$$
$$-\frac{\ln\left(\left(a^2+b^2\right)^{5/2}+a^4\,b+b^5+2\,a^2\,b^3-a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a\,b^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\,a^3\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(2\,a^2\right.}{2\,d\,\left(a^2+b^2\right)^{5/2}}$$
$$-\frac{\frac{4\,a^2\,b+b^3}{a^4+2\,a^2\,b^2+b^4}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(a^2-2\,b^2\right)\,\left(4\,a^2\,b+b^3\right)}{a^2\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(11\,a^2\,b+2\,b^3\right)}{a\,\left(a^4+2\,a^2\,b^2+b^4\right)}-\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(5\,a^2\,b+2\,b^3\right)}{a\,\left(a^4+2\,a^2\,b^2+b^4\right)}}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(2\,a^2-4\,b^2\right)+a^2-4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$

```
[In] int(1/(cos(c + d*x)*(a + b*tan(c + d*x))^3),x)
```

```
[Out] (log((a^2 + b^2)^(5/2) - a^4*b - b^5 - 2*a^2*b^3 + a^5*tan(c/2 + (d*x)/2) +
 a*b^4*tan(c/2 + (d*x)/2) + 2*a^3*b^2*tan(c/2 + (d*x)/2))*(a^2 - b^2/2))/(d
*(a^2 + b^2)^(5/2)) - (log((a^2 + b^2)^(5/2) + a^4*b + b^5 + 2*a^2*b^3 - a^
5*tan(c/2 + (d*x)/2) - a*b^4*tan(c/2 + (d*x)/2) - 2*a^3*b^2*tan(c/2 + (d*x)
/2))*(2*a^2 - b^2))/(2*d*(a^2 + b^2)^(5/2)) - ((4*a^2*b + b^3)/(a^4 + b^4 +
 2*a^2*b^2) - (tan(c/2 + (d*x)/2)^2*(a^2 - 2*b^2)*(4*a^2*b + b^3))/(a^2*(a^
4 + b^4 + 2*a^2*b^2)) + (b*tan(c/2 + (d*x)/2)*(11*a^2*b + 2*b^3))/(a*(a^4 +
 b^4 + 2*a^2*b^2)) - (b*tan(c/2 + (d*x)/2)^3*(5*a^2*b + 2*b^3))/(a*(a^4 + b
^4 + 2*a^2*b^2)))/(d*(a^2*tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^2*(2*a^
2 - 4*b^2) + a^2 - 4*a*b*tan(c/2 + (d*x)/2)^3 + 4*a*b*tan(c/2 + (d*x)/2)))
```

3.576 $\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 221

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^3}\,dx &= -\frac{3b^2(4a^2-b^2)\,\text{arctanh}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\cos(c+dx)\sqrt{\sec^2(c+dx)}}{2\left(a^2+b^2\right)^{7/2}d}\\ &\quad+\frac{b(2a^2-3b^2)\sec(c+dx)}{2\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^2}\\ &\quad+\frac{\cos(c+dx)(b+a\tan(c+dx))}{(a^2+b^2)\,d(a+b\tan(c+dx))^2}+\frac{ab(2a^2-13b^2)\sec(c+dx)}{2\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))}\end{aligned}$$

```
[Out] -3/2*b^2*(4*a^2-b^2)*arctanh((b-a*tan(d*x+c))/(a^2+b^2)^(1/2)/(sec(d*x+c)^2
)^(1/2))*cos(d*x+c)*(sec(d*x+c)^2)^(1/2)/(a^2+b^2)^(7/2)/d+1/2*b*(2*a^2-3*b
^2)*sec(d*x+c)/(a^2+b^2)^2/d/(a+b*tan(d*x+c))^2+cos(d*x+c)*(b+a*tan(d*x+c))
/(a^2+b^2)/d/(a+b*tan(d*x+c))^2+1/2*a*b*(2*a^2-13*b^2)*sec(d*x+c)/(a^2+b^2)
^3/d/(a+b*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used

= {3593, 755, 849, 821, 739, 212}

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= -\frac{3b^2(4a^2-b^2)\cos(c+dx)\sqrt{\sec^2(c+dx)}\operatorname{arctanh}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)}{2d\,(a^2+b^2)^{7/2}}$$
$$+\frac{\cos(c+dx)(a\tan(c+dx)+b)}{d\,(a^2+b^2)\,(a+b\tan(c+dx))^2}+\frac{ab(2a^2-13b^2)\sec(c+dx)}{2d\,(a^2+b^2)^3\,(a+b\tan(c+dx))}$$
$$+\frac{b(2a^2-3b^2)\sec(c+dx)}{2d\,(a^2+b^2)^2\,(a+b\tan(c+dx))^2}$$

```
[In] Int[Cos[c + d*x]/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-3*b^2*(4*a^2 - b^2)*ArcTanh[(b - a*Tan[c + d*x])/(Sqrt[a^2 + b^2]*Sqrt[Se
c[c + d*x]^2])]*Cos[c + d*x]*Sqrt[Sec[c + d*x]^2])/(2*(a^2 + b^2)^(7/2)*d)
+ (b*(2*a^2 - 3*b^2)*Sec[c + d*x])/(2*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^
2) + (Cos[c + d*x]*(b + a*Tan[c + d*x]))/((a^2 + b^2)*d*(a + b*Tan[c + d*x]
)^2) + (a*b*(2*a^2 - 13*b^2)*Sec[c + d*x])/(2*(a^2 + b^2)^3*d*(a + b*Tan[c
+ d*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
```

```
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\left(\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{1}{(a+x)^3\left(1+\frac{x^2}{b^2}\right)^{3/2}}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\cos(c+dx)(b+a\tan(c+dx))}{(a^2+b^2)\,d(a+b\tan(c+dx))^2} - \frac{\left(b\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{-3-\frac{2ax}{b^2}}{(a+x)^3\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+dx)\right)}{(a^2+b^2)\,d}$$

$$= \frac{b(2a^2-3b^2)\sec(c+dx)}{2\,(a^2+b^2)^2\,d(a+b\tan(c+dx))^2} + \frac{\cos(c+dx)(b+a\tan(c+dx))}{(a^2+b^2)\,d(a+b\tan(c+dx))^2} + \frac{\left(b^3\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{\frac{10a}{b^2}+\frac{(2a^2-3b^2)x}{b^4}}{(a+x)^2\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+dx)\right)}{2\,(a^2+b^2)^2\,d}$$

$$
\begin{aligned}
&= \frac{b(2a^2-3b^2)\sec(c+dx)}{2\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^2} + \frac{\cos(c+dx)(b+a\tan(c+dx))}{(a^2+b^2)\,d(a+b\tan(c+dx))^2} \\
&\quad + \frac{ab(2a^2-13b^2)\sec(c+dx)}{2\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))} \\
&\quad + \frac{\left(3b(4a^2-b^2)\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+dx)\right)}{2\left(a^2+b^2\right)^3 d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{b(2a^2-3b^2)\sec(c+dx)}{2\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^2} + \frac{\cos(c+dx)(b+a\tan(c+dx))}{(a^2+b^2)\,d(a+b\tan(c+dx))^2} \\
&\quad + \frac{ab(2a^2-13b^2)\sec(c+dx)}{2\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))} \\
&\quad - \frac{\left(3b(4a^2-b^2)\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{1}{1+\frac{a^2}{b^2}-x^2}\,dx, x, \frac{1-\frac{a\tan(c+dx)}{b}}{\sqrt{\sec^2(c+dx)}}\right)}{2\left(a^2+b^2\right)^3 d}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{3b^2(4a^2-b^2)\,\text{arctanh}\left(\frac{b\left(1-\frac{a\tan(c+dx)}{b}\right)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\cos(c+dx)\sqrt{\sec^2(c+dx)}}{2\left(a^2+b^2\right)^{7/2} d} \\
&\quad + \frac{b(2a^2-3b^2)\sec(c+dx)}{2\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^2} \\
&\quad + \frac{\cos(c+dx)(b+a\tan(c+dx))}{(a^2+b^2)\,d(a+b\tan(c+dx))^2} + \frac{ab(2a^2-13b^2)\sec(c+dx)}{2\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.71 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.83

$$
\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^3}\,dx
$$

$$
= \frac{-\frac{12b^2(-4a^2+b^2)\text{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}} + \frac{\sec^2(c+dx)\left(b(11a^4-22a^2b^2-3b^4)\cos(c+dx)+b(a^2+b^2)^2\cos(3(c+dx))+2a\left(a^4+4a^2b^2\right.\right.}{(a^2+b^2)^3(a+b\tan(c+dx))^2}}{4d}
$$

```
[In] Integrate[Cos[c + d*x]/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] ((-12*b^2*(-4*a^2 + b^2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]]
)/(a^2 + b^2)^(7/2) + (Sec[c + d*x]^2*(b*(11*a^4 - 22*a^2*b^2 - 3*b^4)*Cos[
c + d*x] + b*(a^2 + b^2)^2*Cos[3*(c + d*x)] + 2*a*(a^4 + 4*a^2*b^2 - 12*b^4
 + (a^2 + b^2)^2*Cos[2*(c + d*x)])*Sin[c + d*x]))/((a^2 + b^2)^3*(a + b*Tan
[c + d*x])^2))/(4*d)
```

Maple [A] (verified)

Time = 4.99 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.28

method	result
derivativedivides	$\frac{-\frac{2\left(\left(-a^3+3a\,b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3a^2b+b^3\right)}{\left(a^6+3a^4b^2+3a^2b^4+b^6\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{2b^2\left(\frac{-\frac{b^2\left(9a^2+2b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{b\left(8a^4-15a^2b^2-2b^4\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2}+\frac{b^2\left(2\cdots\right.}{\cdots}}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}\cdots\right)}{\left(a^2+b\cdots\right.}}{d}$
default	$\frac{-\frac{2\left(\left(-a^3+3a\,b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3a^2b+b^3\right)}{\left(a^6+3a^4b^2+3a^2b^4+b^6\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{2b^2\left(\frac{-\frac{b^2\left(9a^2+2b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{b\left(8a^4-15a^2b^2-2b^4\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2}+\frac{b^2\left(2\cdots\right.}{\cdots}}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}\cdots\right)}{\left(a^2+b\cdots\right.}}{d}$
risch	$-\frac{i e^{i(dx+c)}}{2(-3ib\,a^2+ib^3+a^3-3a\,b^2)d}+\frac{i e^{-i(dx+c)}}{2(3ib\,a^2-ib^3+a^3-3a\,b^2)d}+\frac{b^3 e^{i(dx+c)}\left(-7iab\,e^{2i(dx+c)}+8a^2e^{2i(dx+c)}+b^2e^{2i(dx+c)}\cdots\right.}{(-ia+b)^3\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)^2d(\cdots}$

```
[In] int(cos(d*x+c)/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*((-a^3+3*a*b^2)*tan(1/2*d*x+1/2*c)-3*
a^2*b+b^3)/(1+tan(1/2*d*x+1/2*c)^2)-2*b^2/(a^2+b^2)^3*((-1/2*b^2*(9*a^2+2*b
^2)/a*tan(1/2*d*x+1/2*c)^3-1/2*b*(8*a^4-15*a^2*b^2-2*b^4)/a^2*tan(1/2*d*x+1
/2*c)^2+1/2*b^2*(23*a^2+2*b^2)/a*tan(1/2*d*x+1/2*c)+4*a^2*b+1/2*b^3)/(tan(1
/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^2-3/2*(4*a^2-b^2)/(a^2+b^2)^(1/
2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. 2(209) = 418.

Time = 0.31 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.17

$$\int\frac{\cos(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \frac{4\left(a^6b+3\,a^4b^3+3\,a^2b^5+b^7\right)\cos\left(dx+c\right)^3-3\left(4\,a^2b^4-b^6+\left(4\,a^4b^2-5\,a^2b^4+b^6\right)\cos\left(dx+c\right)^2+2\left(4\,a^3b^3\cdots\right.\right.}{4\left(\left(a^{10}+3\cdots\right.\right.}$$

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cos(d*x + c)^3 - 3*(4*a^2*b^4
- b^6 + (4*a^4*b^2 - 5*a^2*b^4 + b^6)*cos(d*x + c)^2 + 2*(4*a^3*b^3 - a*b^5
)*cos(d*x + c)*sin(d*x + c))*sqrt(a^2 + b^2)*log((2*a*b*cos(d*x + c)*sin(d*
x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*co
```

```
s(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2
)*cos(d*x + c)^2 + b^2)) + 2*(4*a^6*b - 10*a^4*b^3 - 17*a^2*b^5 - 3*b^7)*co
s(d*x + c) + 2*(2*a^5*b^2 - 11*a^3*b^4 - 13*a*b^6 + 2*(a^7 + 3*a^5*b^2 + 3*
a^3*b^4 + a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^10 + 3*a^8*b^2 + 2*a^6*b
^4 - 2*a^4*b^6 - 3*a^2*b^8 - b^10)*d*cos(d*x + c)^2 + 2*(a^9*b + 4*a^7*b^3
+ 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*x + c) + (a^8*b^2 + 4
*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)*d)
```

Sympy [F]

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \int \frac{\cos\left(c+dx\right)}{\left(a+b\tan\left(c+dx\right)\right)^3}\,dx$$

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)/(a + b*tan(c + d*x))**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(209) = 418$.

Time = 0.32 (sec) , antiderivative size = 658, normalized size of antiderivative = 2.98

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{\frac{3\,(4\,a^2b^2-b^4)\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}}-\frac{2\left(6\,a^6b-10\,a^4b^3-a^2b^5+\frac{(2\,a^7+18\,a^5b^2-31\,a^3b^4-2\,ab^6)\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\,(2\,a^6b-2\,a^4b^3+\cdots}{(\cos(\cdots}\right.}{a^{10}+3\,a^8b^2+3\,a^6b^4+a^4b^6+\frac{4\,(a^9b+3\,a^7b^3+3\,a^5b^5+a^3b^7)\sin(dx+c)}{\cos(dx+c)+1}-\frac{(a^{10}-a^8b^2-9\,a^6b^4-11\,a^{\cdots}}{(\cos(dx+\cdots}}}{\cdots}$$

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(3*(4*a^2*b^2 - b^4)*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt
(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a
^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 2*(6*a^6*b - 10*a^4*b^
3 - a^2*b^5 + (2*a^7 + 18*a^5*b^2 - 31*a^3*b^4 - 2*a*b^6)*sin(d*x + c)/(cos
(d*x + c) + 1) - 2*(2*a^6*b - 2*a^4*b^3 + 12*a^2*b^5 + b^7)*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 - 2*(2*a^7 + 2*a^5*b^2 + 15*a^3*b^4)*sin(d*x + c)^3/(c
os(d*x + c) + 1)^3 - (2*a^6*b - 30*a^4*b^3 + 15*a^2*b^5 + 2*b^7)*sin(d*x +
c)^4/(cos(d*x + c) + 1)^4 + (2*a^7 - 6*a^5*b^2 + 9*a^3*b^4 + 2*a*b^6)*sin(d
*x + c)^5/(cos(d*x + c) + 1)^5)/(a^10 + 3*a^8*b^2 + 3*a^6*b^4 + a^4*b^6 + 4
*(a^9*b + 3*a^7*b^3 + 3*a^5*b^5 + a^3*b^7)*sin(d*x + c)/(cos(d*x + c) + 1)
- (a^10 - a^8*b^2 - 9*a^6*b^4 - 11*a^4*b^6 - 4*a^2*b^8)*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 - (a^10 - a^8*b^2 - 9*a^6*b^4 - 11*a^4*b^6 - 4*a^2*b^8)*si
```

```
n(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*(a^9*b + 3*a^7*b^3 + 3*a^5*b^5 + a^3*
b^7)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + (a^10 + 3*a^8*b^2 + 3*a^6*b^4 +
a^4*b^6)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6))/d
```

Giac [A] (verification not implemented)

none

Time = 0.63 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.81

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{\frac{3\,(4\,a^2b^2-b^4)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}} - \frac{4\,(a^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)-3\,ab^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)+3\,a^2b-b^3)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\left(\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^2+1\right)} - \frac{2\left(9\,a^3b^4\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)\right.}{}}{}$$

```
[In] integrate(cos(d*x+c)/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(3*(4*a^2*b^2 - b^4)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a
^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^6 +
 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 4*(a^3*tan(1/2*d*x + 1/2*c
) - 3*a*b^2*tan(1/2*d*x + 1/2*c) + 3*a^2*b - b^3)/((a^6 + 3*a^4*b^2 + 3*a^2
*b^4 + b^6)*(tan(1/2*d*x + 1/2*c)^2 + 1)) - 2*(9*a^3*b^4*tan(1/2*d*x + 1/2*
c)^3 + 2*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 8*a^4*b^3*tan(1/2*d*x + 1/2*c)^2 -
15*a^2*b^5*tan(1/2*d*x + 1/2*c)^2 - 2*b^7*tan(1/2*d*x + 1/2*c)^2 - 23*a^3*b
^4*tan(1/2*d*x + 1/2*c) - 2*a*b^6*tan(1/2*d*x + 1/2*c) - 8*a^4*b^3 - a^2*b^
5)/((a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b
*tan(1/2*d*x + 1/2*c) - a)^2))/d
```

Mupad [B] (verification not implemented)

Time = 8.10 (sec) , antiderivative size = 610, normalized size of antiderivative = 2.76

$$\int \frac{\cos(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{\frac{-6\,a^4\,b+10\,a^2\,b^3+b^5}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6} + \frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(2\,a^5+2\,a^3\,b^2+15\,a\,b^4)}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6} + \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,(2\,a^6\,b-30\,a^4\,b^3+15\,a^2\,b^5+2\,b^7)}{a^2\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)(2\,a^6+}{a\,(a^6+3\,a^4}}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+a^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(a^2-4\,b^2)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right.} - \frac{\mathrm{atan}\left(\frac{-1\mathrm{i}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^7+a^6\,b\,1\mathrm{i}-3\mathrm{i}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^5\,b^2+a^4\,b^3\,3\mathrm{i}-3\mathrm{i}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^3\,b^4+a^2\,b^5\,3\mathrm{i}-1\mathrm{i}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a\,b^6+b^7\,1\mathrm{i}}{(a^2+b^2)^{7/2}}\right)(3\,b^4-12}{d\,(a^2+b^2)^{7/2}}$$

```
[In] int(cos(c + d*x)/(a + b*tan(c + d*x))^3,x)
```

```
[Out] - ((b^5 - 6*a^4*b + 10*a^2*b^3)/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (2*ta
n(c/2 + (d*x)/2)^3*(15*a*b^4 + 2*a^5 + 2*a^3*b^2))/(a^6 + b^6 + 3*a^2*b^4 +
 3*a^4*b^2) + (tan(c/2 + (d*x)/2)^4*(2*a^6*b + 2*b^7 + 15*a^2*b^5 - 30*a^4*
b^3))/(a^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (tan(c/2 + (d*x)/2)*(2*a^
6 - 2*b^6 - 31*a^2*b^4 + 18*a^4*b^2))/(a*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2
)) - (tan(c/2 + (d*x)/2)^5*(2*a^6 + 2*b^6 + 9*a^2*b^4 - 6*a^4*b^2))/(a*(a^6
 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (2*tan(c/2 + (d*x)/2)^2*(2*a^6*b + b^7 +
 12*a^2*b^5 - 2*a^4*b^3))/(a^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))/(d*(a^
2*tan(c/2 + (d*x)/2)^6 + a^2 - tan(c/2 + (d*x)/2)^2*(a^2 - 4*b^2) - tan(c/2
 + (d*x)/2)^4*(a^2 - 4*b^2) - 4*a*b*tan(c/2 + (d*x)/2)^5 + 4*a*b*tan(c/2 +
(d*x)/2))) - (atan((a^6*b*1i + b^7*1i + a^2*b^5*3i + a^4*b^3*3i - a^7*tan(c
/2 + (d*x)/2)*1i - a*b^6*tan(c/2 + (d*x)/2)*1i - a^3*b^4*tan(c/2 + (d*x)/2)
*3i - a^5*b^2*tan(c/2 + (d*x)/2)*3i)/(a^2 + b^2)^(7/2))*(3*b^4 - 12*a^2*b^2
)*1i)/(d*(a^2 + b^2)^(7/2))
```

3.577 $\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 310

$$
\begin{aligned}
&\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx \\
&= -\frac{5b^4(6a^2-b^2)\operatorname{arctanh}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\cos(c+dx)\sqrt{\sec^2(c+dx)}}{2\left(a^2+b^2\right)^{9/2}d} \\
&\quad+\frac{b(4a^4+24a^2b^2-15b^4)\sec(c+dx)}{6\left(a^2+b^2\right)^3 d(a+b\tan(c+dx))^2}+\frac{\cos^3(c+dx)(b+a\tan(c+dx))}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} \\
&\quad+\frac{ab(4a^4+28a^2b^2-81b^4)\sec(c+dx)}{6\left(a^2+b^2\right)^4 d(a+b\tan(c+dx))} \\
&\quad-\frac{\cos(c+dx)\left(b(2a^2-5b^2)-a(2a^2+9b^2)\tan(c+dx)\right)}{3\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^2}
\end{aligned}
$$

```
[Out] -5/2*b^4*(6*a^2-b^2)*arctanh((b-a*tan(d*x+c))/(a^2+b^2)^(1/2)/(sec(d*x+c)^2
)^(1/2))*cos(d*x+c)*(sec(d*x+c)^2)^(1/2)/(a^2+b^2)^(9/2)/d+1/6*b*(4*a^4+24*
a^2*b^2-15*b^4)*sec(d*x+c)/(a^2+b^2)^3/d/(a+b*tan(d*x+c))^2+1/3*cos(d*x+c)^
3*(b+a*tan(d*x+c))/(a^2+b^2)/d/(a+b*tan(d*x+c))^2+1/6*a*b*(4*a^4+28*a^2*b^2
-81*b^4)*sec(d*x+c)/(a^2+b^2)^4/d/(a+b*tan(d*x+c))-1/3*cos(d*x+c)*(b*(2*a^2
-5*b^2)-a*(2*a^2+9*b^2)*tan(d*x+c))/(a^2+b^2)^2/d/(a+b*tan(d*x+c))^2
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3593, 755, 837, 849, 821, 739, 212}

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$= -\frac{5b^4(6a^2-b^2)\cos(c+dx)\sqrt{\sec^2(c+dx)}\text{arctanh}\left(\frac{b-a\tan(c+dx)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)}{2d\,(a^2+b^2)^{9/2}}$$
$$+\frac{\cos^3(c+dx)(a\tan(c+dx)+b)}{3d\,(a^2+b^2)\,(a+b\tan(c+dx))^2}$$
$$-\frac{\cos(c+dx)\,(b(2a^2-5b^2)-a(2a^2+9b^2)\tan(c+dx))}{3d\,(a^2+b^2)^2\,(a+b\tan(c+dx))^2}$$
$$+\frac{ab(4a^4+28a^2b^2-81b^4)\sec(c+dx)}{6d\,(a^2+b^2)^4\,(a+b\tan(c+dx))}+\frac{b(4a^4+24a^2b^2-15b^4)\sec(c+dx)}{6d\,(a^2+b^2)^3\,(a+b\tan(c+dx))^2}$$

```
[In] Int[Cos[c + d*x]^3/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (-5*b^4*(6*a^2 - b^2)*ArcTanh[(b - a*Tan[c + d*x])/(Sqrt[a^2 + b^2]*Sqrt[Se
c[c + d*x]^2])]*Cos[c + d*x]*Sqrt[Sec[c + d*x]^2])/(2*(a^2 + b^2)^(9/2)*d)
+ (b*(4*a^4 + 24*a^2*b^2 - 15*b^4)*Sec[c + d*x])/(6*(a^2 + b^2)^3*d*(a + b*
Tan[c + d*x])^2) + (Cos[c + d*x]^3*(b + a*Tan[c + d*x]))/(3*(a^2 + b^2)*d*(
a + b*Tan[c + d*x])^2) + (a*b*(4*a^4 + 28*a^2*b^2 - 81*b^4)*Sec[c + d*x])/(
6*(a^2 + b^2)^4*d*(a + b*Tan[c + d*x])) - (Cos[c + d*x]*(b*(2*a^2 - 5*b^2)
- a*(2*a^2 + 9*b^2)*Tan[c + d*x]))/(3*(a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^
2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
```

```
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\left(\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int\frac{1}{(a+x)^3\left(1+\frac{x^2}{b^2}\right)^{5/2}}\,dx,x,b\tan(c+dx)\right)}{bd}$$

$$= \frac{\cos^3(c+dx)(b+a\tan(c+dx))}{3(a^2+b^2)d(a+b\tan(c+dx))^2} - \frac{\left(b\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{-5-\frac{2a^2}{b^2}-\frac{4ax}{b^2}}{(a+x)^3\left(1+\frac{x^2}{b^2}\right)^{3/2}}dx, x, b\tan(c+dx)\right)}{3(a^2+b^2)d}$$

$$= \frac{\cos^3(c+dx)(b+a\tan(c+dx))}{3(a^2+b^2)d(a+b\tan(c+dx))^2} - \frac{\cos(c+dx)\left(b(2a^2-5b^2)-a(2a^2+9b^2)\tan(c+dx)\right)}{3(a^2+b^2)^2d(a+b\tan(c+dx))^2} + \frac{\left(b^5\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{-\frac{3\left(2a^2-5b^2\right)}{b^4}+\frac{2a\left(2a^2+9b^2\right)x}{b^6}}{(a+x)^3\sqrt{1+\frac{x^2}{b^2}}}dx, x, b\tan(c+dx)\right)}{3(a^2+b^2)^2d}$$

$$= \frac{b(4a^4+24a^2b^2-15b^4)\sec(c+dx)}{6(a^2+b^2)^3d(a+b\tan(c+dx))^2} + \frac{\cos^3(c+dx)(b+a\tan(c+dx))}{3(a^2+b^2)d(a+b\tan(c+dx))^2} - \frac{\cos(c+dx)\left(b(2a^2-5b^2)-a(2a^2+9b^2)\tan(c+dx)\right)}{3(a^2+b^2)^2d(a+b\tan(c+dx))^2} - \frac{\left(b^7\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{\frac{2a\left(2a^2-33b^2\right)}{b^6}-\frac{\left(4a^4+24a^2b^2-15b^4\right)x}{b^8}}{(a+x)^2\sqrt{1+\frac{x^2}{b^2}}}dx, x, b\tan(c+dx)\right)}{6(a^2+b^2)^3d}$$

$$= \frac{b(4a^4+24a^2b^2-15b^4)\sec(c+dx)}{6(a^2+b^2)^3d(a+b\tan(c+dx))^2} + \frac{\cos^3(c+dx)(b+a\tan(c+dx))}{3(a^2+b^2)d(a+b\tan(c+dx))^2} + \frac{ab(4a^4+28a^2b^2-81b^4)\sec(c+dx)}{6(a^2+b^2)^4d(a+b\tan(c+dx))} - \frac{\cos(c+dx)\left(b(2a^2-5b^2)-a(2a^2+9b^2)\tan(c+dx)\right)}{3(a^2+b^2)^2d(a+b\tan(c+dx))^2} + \frac{\left(5b^3(6a^2-b^2)\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{1}{(a+x)\sqrt{1+\frac{x^2}{b^2}}}dx, x, b\tan(c+dx)\right)}{2(a^2+b^2)^4d}$$

$$= \frac{b(4a^4+24a^2b^2-15b^4)\sec(c+dx)}{6(a^2+b^2)^3d(a+b\tan(c+dx))^2} + \frac{\cos^3(c+dx)(b+a\tan(c+dx))}{3(a^2+b^2)d(a+b\tan(c+dx))^2} + \frac{ab(4a^4+28a^2b^2-81b^4)\sec(c+dx)}{6(a^2+b^2)^4d(a+b\tan(c+dx))} - \frac{\cos(c+dx)\left(b(2a^2-5b^2)-a(2a^2+9b^2)\tan(c+dx)\right)}{3(a^2+b^2)^2d(a+b\tan(c+dx))^2} - \frac{\left(5b^3(6a^2-b^2)\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{1}{1+\frac{a^2}{b^2}-x^2}dx, x, \frac{1-\frac{a\tan(c+dx)}{b}}{\sqrt{\sec^2(c+dx)}}\right)}{2(a^2+b^2)^4d}$$

$$= -\frac{5b^4(6a^2-b^2)\operatorname{arctanh}\left(\frac{b\left(1-\frac{a\tan(c+dx)}{b}\right)}{\sqrt{a^2+b^2}\sqrt{\sec^2(c+dx)}}\right)\cos(c+dx)\sqrt{\sec^2(c+dx)}}{2(a^2+b^2)^{9/2}d} + \frac{b(4a^4+24a^2b^2-15b^4)\sec(c+dx)}{6(a^2+b^2)^3 d(a+b\tan(c+dx))^2} + \frac{\cos^3(c+dx)(b+a\tan(c+dx))}{3(a^2+b^2)d(a+b\tan(c+dx))^2} + \frac{ab(4a^4+28a^2b^2-81b^4)\sec(c+dx)}{6(a^2+b^2)^4 d(a+b\tan(c+dx))} - \frac{\cos(c+dx)\left(b(2a^2-5b^2)-a(2a^2+9b^2)\tan(c+dx)\right)}{3(a^2+b^2)^2 d(a+b\tan(c+dx))^2}$$

Mathematica [A] (verified)

Time = 3.04 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.20

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$

$$= \frac{\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(\frac{9b(a^4+14a^2b^2-3b^4)(a\cos(c+dx)+b\sin(c+dx))^2}{(a^2+b^2)^4} + \frac{6b^6\tan(c+dx)}{a(a^2+b^2)^3} + \frac{9a(a^4+6a^2\right.}{}$$

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Tan[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])*((9*b*(a^4 + 14*a^2*b^2 -
 3*b^4)*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(a^2 + b^2)^4 + (6*b^6*Tan[c +
 d*x])/(a*(a^2 + b^2)^3) + (9*a*(a^4 + 6*a^2*b^2 - 11*b^4)*(a*Cos[c + d*x]
+ b*Sin[c + d*x])^2*Tan[c + d*x])/(a^2 + b^2)^4 - (6*b^5*(12*a^2 + b^2)*(a
+ b*Tan[c + d*x]))/(a*(a^2 + b^2)^4) - (60*b^4*(-6*a^2 + b^2)*ArcTanh[(-b +
 a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]]*Cos[c + d*x]*(a + b*Tan[c + d*x])^2)/
(a^2 + b^2)^(9/2) - (b*(-3*a^2 + b^2)*Cos[c + d*x]*Cos[3*(c + d*x)]*(a + b*
Tan[c + d*x])^2)/(a^2 + b^2)^3 + (a*(a^2 - 3*b^2)*Cos[c + d*x]*Sin[3*(c + d
*x)]*(a + b*Tan[c + d*x])^2)/(a^2 + b^2)^3))/(12*d*(a + b*Tan[c + d*x])^3)
```

Maple [A] (verified)

Time = 23.38 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.47

method	result
derivativedivides	$-\frac{2b^4\left(\frac{-\frac{b^2(13a^2+2b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{b(12a^4-23a^2b^2-2b^4)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2}+\frac{b^2(35a^2+2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+6a^2b+\frac{b^3}{2}}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}-\frac{5(6a}{}\right.}{(a^2+b^2)(a^6+3a^4b^2+3a^2b^4+b^6)}$
default	$-\frac{2b^4\left(\frac{-\frac{b^2(13a^2+2b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{b(12a^4-23a^2b^2-2b^4)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2}+\frac{b^2(35a^2+2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+6a^2b+\frac{b^3}{2}}{\left(\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}-\frac{5(6a}{}\right.}{(a^2+b^2)(a^6+3a^4b^2+3a^2b^4+b^6)}$
risch	$-\frac{ie^{3i(dx+c)}}{24(-3iba^2+ib^3+a^3-3ab^2)d}-\frac{9\,e^{i(dx+c)}b}{8(-4ia^3b+4iab^3+a^4-6a^2b^2+b^4)d}-\frac{3ie^{i(dx+c)}a}{8(-4ia^3b+4iab^3+a^4-6a^2b^2+b^4)d}-\frac{}{8(ib}$

```
[In] int(cos(d*x+c)^3/(a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*b^4/(a^2+b^2)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*((-1/2*b^2*(13*a^2+2*b^
2)/a*tan(1/2*d*x+1/2*c)^3-1/2*b*(12*a^4-23*a^2*b^2-2*b^4)/a^2*tan(1/2*d*x+1
/2*c)^2+1/2*b^2*(35*a^2+2*b^2)/a*tan(1/2*d*x+1/2*c)+6*a^2*b+1/2*b^3)/(tan(1
/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^2-5/2*(6*a^2-b^2)/(a^2+b^2)^(1/
2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))-2/(a^6+3*a^4*
b^2+3*a^2*b^4+b^6)/(a^2+b^2)*((-a^5-4*a^3*b^2+9*a*b^4)*tan(1/2*d*x+1/2*c)^5
+(-3*a^4*b-12*a^2*b^3+3*b^5)*tan(1/2*d*x+1/2*c)^4+(-2/3*a^5-32/3*a^3*b^2+14
*a*b^4)*tan(1/2*d*x+1/2*c)^3+(-20*a^2*b^3+4*b^5)*tan(1/2*d*x+1/2*c)^2+(-a^5
-4*a^3*b^2+9*a*b^4)*tan(1/2*d*x+1/2*c)-a^4*b-32/3*a^2*b^3+7/3*b^5)/(1+tan(1
/2*d*x+1/2*c)^2)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 619 vs. $2(294) = 588$.

Time = 0.33 (sec) , antiderivative size = 619, normalized size of antiderivative = 2.00

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx$$
$$=\frac{4\,(a^8b+4\,a^6b^3+6\,a^4b^5+4\,a^2b^7+b^9)\cos{(dx+c)}^5-4\,(2\,a^8b+a^6b^3-9\,a^4b^5-13\,a^2b^7-5\,b^9)\cos{(dx+c}}{}$$

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cos(d*x + c)^5 -
4*(2*a^8*b + a^6*b^3 - 9*a^4*b^5 - 13*a^2*b^7 - 5*b^9)*cos(d*x + c)^3 - 15*
(6*a^2*b^6 - b^8 + (6*a^4*b^4 - 7*a^2*b^6 + b^8)*cos(d*x + c)^2 + 2*(6*a^3*
```

```
b^5 - a*b^7)*cos(d*x + c)*sin(d*x + c))*sqrt(a^2 + b^2)*log((2*a*b*cos(d*x
+ c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 +
 b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) +
 (a^2 - b^2)*cos(d*x + c)^2 + b^2)) + 2*(8*a^8*b + 64*a^6*b^3 - 16*a^4*b^5
- 87*a^2*b^7 - 15*b^9)*cos(d*x + c) + 2*(4*a^7*b^2 + 32*a^5*b^4 - 53*a^3*b^
6 - 81*a*b^8 + 2*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cos(d*x
+ c)^4 + 2*(2*a^9 + 15*a^7*b^2 + 33*a^5*b^4 + 29*a^3*b^6 + 9*a*b^8)*cos(d*x
 + c)^2)*sin(d*x + c))/((a^12 + 4*a^10*b^2 + 5*a^8*b^4 - 5*a^4*b^8 - 4*a^2*
b^10 - b^12)*d*cos(d*x + c)^2 + 2*(a^11*b + 5*a^9*b^3 + 10*a^7*b^5 + 10*a^5
*b^7 + 5*a^3*b^9 + a*b^11)*d*cos(d*x + c)*sin(d*x + c) + (a^10*b^2 + 5*a^8*
b^4 + 10*a^6*b^6 + 10*a^4*b^8 + 5*a^2*b^10 + b^12)*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**3/(a+b*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1229 vs. $2(294) = 588$.

Time = 0.36 (sec) , antiderivative size = 1229, normalized size of antiderivative = 3.96

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/6*(15*(6*a^2*b^4 - b^6)*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqr
t(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((
a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*sqrt(a^2 + b^2)) - 2*(6*a^8*
b + 64*a^6*b^3 - 50*a^4*b^5 - 3*a^2*b^7 + (6*a^9 + 48*a^7*b^2 + 202*a^5*b^4
 - 161*a^3*b^6 - 6*a*b^8)*sin(d*x + c)/(cos(d*x + c) + 1) + 2*(6*a^8*b + 56
*a^6*b^3 - 14*a^4*b^5 - 67*a^2*b^7 - 3*b^9)*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 - 4*(2*a^9 - 4*a^7*b^2 - 86*a^5*b^4 + 133*a^3*b^6 + 3*a*b^8)*sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3 + 2*(8*a^8*b + 28*a^6*b^3 + 188*a^4*b^5 - 156*a^
2*b^7 - 9*b^9)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 2*(2*a^9 + 4*a^7*b^2 +
 62*a^5*b^4 - 255*a^3*b^6)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 2*(14*a^8*
b + 56*a^6*b^3 - 246*a^4*b^5 + 141*a^2*b^7 + 9*b^9)*sin(d*x + c)^6/(cos(d*x
 + c) + 1)^6 - 4*(2*a^9 + 8*a^7*b^2 + 42*a^5*b^4 + 33*a^3*b^6 - 3*a*b^8)*si
```

```
n(d*x + c)^7/(cos(d*x + c) + 1)^7 - 3*(2*a^8*b + 8*a^6*b^3 - 78*a^4*b^5 + 2
3*a^2*b^7 + 2*b^9)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 3*(2*a^9 + 8*a^7*b
^2 - 18*a^5*b^4 + 13*a^3*b^6 + 2*a*b^8)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9
)/(a^12 + 4*a^10*b^2 + 6*a^8*b^4 + 4*a^6*b^6 + a^4*b^8 + 4*(a^11*b + 4*a^9*
b^3 + 6*a^7*b^5 + 4*a^5*b^7 + a^3*b^9)*sin(d*x + c)/(cos(d*x + c) + 1) + (a
^12 + 8*a^10*b^2 + 22*a^8*b^4 + 28*a^6*b^6 + 17*a^4*b^8 + 4*a^2*b^10)*sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 + 8*(a^11*b + 4*a^9*b^3 + 6*a^7*b^5 + 4*a^5*
b^7 + a^3*b^9)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 2*(a^12 - 2*a^10*b^2 -
 18*a^8*b^4 - 32*a^6*b^6 - 23*a^4*b^8 - 6*a^2*b^10)*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 - 2*(a^12 - 2*a^10*b^2 - 18*a^8*b^4 - 32*a^6*b^6 - 23*a^4*b^8
- 6*a^2*b^10)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 8*(a^11*b + 4*a^9*b^3 +
 6*a^7*b^5 + 4*a^5*b^7 + a^3*b^9)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + (a^
12 + 8*a^10*b^2 + 22*a^8*b^4 + 28*a^6*b^6 + 17*a^4*b^8 + 4*a^2*b^10)*sin(d*
x + c)^8/(cos(d*x + c) + 1)^8 - 4*(a^11*b + 4*a^9*b^3 + 6*a^7*b^5 + 4*a^5*b
^7 + a^3*b^9)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + (a^12 + 4*a^10*b^2 + 6*
a^8*b^4 + 4*a^6*b^6 + a^4*b^8)*sin(d*x + c)^10/(cos(d*x + c) + 1)^10))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 640 vs. $2(294) = 588$.

Time = 0.63 (sec) , antiderivative size = 640, normalized size of antiderivative = 2.06

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx = -\frac{\frac{15\,(6\,a^2b^4-b^6)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8)\sqrt{a^2+b^2}} - \frac{6\left(13\,a^3b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,ab^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+12\,a^4b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-23\cdots\right.}{(a^{10}+4\,a^8b^2+6\,a^6b^4+4\,a^4b\cdots}}{}$$

```
[In] integrate(cos(d*x+c)^3/(a+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(15*(6*a^2*b^4 - b^6)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(
a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^8
+ 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*sqrt(a^2 + b^2)) - 6*(13*a^3*b^6
*tan(1/2*d*x + 1/2*c)^3 + 2*a*b^8*tan(1/2*d*x + 1/2*c)^3 + 12*a^4*b^5*tan(1
/2*d*x + 1/2*c)^2 - 23*a^2*b^7*tan(1/2*d*x + 1/2*c)^2 - 2*b^9*tan(1/2*d*x +
 1/2*c)^2 - 35*a^3*b^6*tan(1/2*d*x + 1/2*c) - 2*a*b^8*tan(1/2*d*x + 1/2*c)
- 12*a^4*b^5 - a^2*b^7)/((a^10 + 4*a^8*b^2 + 6*a^6*b^4 + 4*a^4*b^6 + a^2*b^
8)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^2) - 4*(3*a^5*
tan(1/2*d*x + 1/2*c)^5 + 12*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 27*a*b^4*tan(1
/2*d*x + 1/2*c)^5 + 9*a^4*b*tan(1/2*d*x + 1/2*c)^4 + 36*a^2*b^3*tan(1/2*d*x
 + 1/2*c)^4 - 9*b^5*tan(1/2*d*x + 1/2*c)^4 + 2*a^5*tan(1/2*d*x + 1/2*c)^3 +
 32*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 42*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 60*a
^2*b^3*tan(1/2*d*x + 1/2*c)^2 - 12*b^5*tan(1/2*d*x + 1/2*c)^2 + 3*a^5*tan(1
/2*d*x + 1/2*c) + 12*a^3*b^2*tan(1/2*d*x + 1/2*c) - 27*a*b^4*tan(1/2*d*x +
```

```
1/2*c) + 3*a^4*b + 32*a^2*b^3 - 7*b^5)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^
2*b^6 + b^8)*(tan(1/2*d*x + 1/2*c)^2 + 1)^3))/d
```

Mupad [B] (verification not implemented)

Time = 9.86 (sec) , antiderivative size = 1128, normalized size of antiderivative = 3.64

$$\int \frac{\cos^3(c+dx)}{(a+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] int(cos(c + d*x)^3/(a + b*tan(c + d*x))^3,x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)^5*(2*a^7 - 255*a*b^6 + 62*a^3*b^4 + 4*a^5*b^2))/(3*(
a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (6*a^6*b - 3*b^7 - 50*a^2
*b^5 + 64*a^4*b^3)/(3*(a^2 + b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (2
*tan(c/2 + (d*x)/2)^2*(6*a^6*b - 3*b^7 - 64*a^2*b^5 + 50*a^4*b^3))/(3*a^2*(
a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (tan(c/2 + (d*x)/2)^9*(2*a^8 + 2*b^8
+ 13*a^2*b^6 - 18*a^4*b^4 + 8*a^6*b^2))/(a*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b
^4 + 4*a^6*b^2)) - (4*tan(c/2 + (d*x)/2)^7*(2*a^6 - 3*b^6 + 36*a^2*b^4 + 6*
a^4*b^2))/(3*a*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (tan(c/2 + (d*x)/2)^8
*(2*a^8*b + 2*b^9 + 23*a^2*b^7 - 78*a^4*b^5 + 8*a^6*b^3))/(a^2*(a^8 + b^8 +
 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (2*tan(c/2 + (d*x)/2)^4*(8*a^8*b - 9
*b^9 - 156*a^2*b^7 + 188*a^4*b^5 + 28*a^6*b^3))/(3*a^2*(a^2 + b^2)*(a^6 + b
^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (2*tan(c/2 + (d*x)/2)^6*(14*a^8*b + 9*b^9 +
141*a^2*b^7 - 246*a^4*b^5 + 56*a^6*b^3))/(3*a^2*(a^2 + b^2)*(a^6 + b^6 + 3*
a^2*b^4 + 3*a^4*b^2)) + (tan(c/2 + (d*x)/2)*(6*a^8 - 6*b^8 - 161*a^2*b^6 +
202*a^4*b^4 + 48*a^6*b^2))/(3*a*(a^2 + b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*
b^2)) - (4*tan(c/2 + (d*x)/2)^3*(2*a^8 + 3*b^8 + 133*a^2*b^6 - 86*a^4*b^4 -
 4*a^6*b^2))/(3*a*(a^2 + b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))/(d*(a^2
*tan(c/2 + (d*x)/2)^10 - tan(c/2 + (d*x)/2)^6*(2*a^2 - 12*b^2) - tan(c/2 +
(d*x)/2)^4*(2*a^2 - 12*b^2) + a^2 + tan(c/2 + (d*x)/2)^2*(a^2 + 4*b^2) + ta
n(c/2 + (d*x)/2)^8*(a^2 + 4*b^2) + 8*a*b*tan(c/2 + (d*x)/2)^3 - 8*a*b*tan(c
/2 + (d*x)/2)^7 - 4*a*b*tan(c/2 + (d*x)/2)^9 + 4*a*b*tan(c/2 + (d*x)/2))) +
 (b^4*atan((a^8*b*1i + b^9*1i + a^2*b^7*4i + a^4*b^5*6i + a^6*b^3*4i - a^9*
tan(c/2 + (d*x)/2)*1i - a*b^8*tan(c/2 + (d*x)/2)*1i - a^3*b^6*tan(c/2 + (d*
x)/2)*4i - a^5*b^4*tan(c/2 + (d*x)/2)*6i - a^7*b^2*tan(c/2 + (d*x)/2)*4i)/(
a^2 + b^2)^(9/2))*(6*a^2 - b^2)*5i)/(d*(a^2 + b^2)^(9/2))
```

3.578 $\int (d\sec(e+fx))^{7/2}(a+b\tan(e+fx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 121

$$\int (d\sec(e+fx))^{7/2}(a+b\tan(e+fx))\,dx = -\frac{6ad^4E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2b(d\sec(e+fx))^{7/2}}{7f}+\frac{6ad^3\sqrt{d\sec(e+fx)}\sin(e+fx)}{5f}+\frac{2ad(d\sec(e+fx))^{5/2}\sin(e+fx)}{5f}$$

```
[Out] 2/7*b*(d*sec(f*x+e))^(7/2)/f+2/5*a*d*(d*sec(f*x+e))^(5/2)*sin(f*x+e)/f-6/5*
a*d^4*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x
+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(d*sec(f*x+e))^(1/2)+6/5*a*d^3*sin(f*x+
e)*(d*sec(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3567, 3853, 3856, 2719}

$$\int (d\sec(e+fx))^{7/2}(a+b\tan(e+fx))\,dx = -\frac{6ad^4E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{6ad^3\sin(e+fx)\sqrt{d\sec(e+fx)}}{5f}+\frac{2ad\sin(e+fx)(d\sec(e+fx))^{5/2}}{5f}+\frac{2b(d\sec(e+fx))^{7/2}}{7f}$$

```
[In] Int[(d*Sec[e + f*x])^(7/2)*(a + b*Tan[e + f*x]),x]
```

```
[Out] (-6*a*d^4*EllipticE[(e + f*x)/2, 2])/(5*f*Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e +
 f*x]]) + (2*b*(d*Sec[e + f*x])^(7/2))/(7*f) + (6*a*d^3*Sqrt[d*Sec[e + f*x]
]*Sin[e + f*x])/(5*f) + (2*a*d*(d*Sec[e + f*x])^(5/2)*Sin[e + f*x])/(5*f)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2b(d\sec(e+fx))^{7/2}}{7f} + a\int (d\sec(e+fx))^{7/2}\,dx \\
&= \frac{2b(d\sec(e+fx))^{7/2}}{7f} + \frac{2ad(d\sec(e+fx))^{5/2}\sin(e+fx)}{5f} + \frac{1}{5}\left(3ad^2\right)\int (d\sec(e+fx))^{3/2}\,dx \\
&= \frac{2b(d\sec(e+fx))^{7/2}}{7f} + \frac{6ad^3\sqrt{d\sec(e+fx)}\sin(e+fx)}{5f} \\
&\quad + \frac{2ad(d\sec(e+fx))^{5/2}\sin(e+fx)}{5f} - \frac{1}{5}\left(3ad^4\right)\int \frac{1}{\sqrt{d\sec(e+fx)}}\,dx \\
&= \frac{2b(d\sec(e+fx))^{7/2}}{7f} + \frac{6ad^3\sqrt{d\sec(e+fx)}\sin(e+fx)}{5f} \\
&\quad + \frac{2ad(d\sec(e+fx))^{5/2}\sin(e+fx)}{5f} - \frac{\left(3ad^4\right)\int \sqrt{\cos(e+fx)}\,dx}{5\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}
\end{aligned}$$

$$= -\frac{6ad^4E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}} + \frac{2b(d\sec(e+fx))^{7/2}}{7f} + \frac{6ad^3\sqrt{d\sec(e+fx)}\sin(e+fx)}{5f} + \frac{2ad(d\sec(e+fx))^{5/2}\sin(e+fx)}{5f}$$

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.57

$$\int (d\sec(e+fx))^{7/2}(a+b\tan(e+fx))\,dx = \frac{(d\sec(e+fx))^{7/2}\left(40b-168a\cos^{\frac{7}{2}}(e+fx)E\left(\frac{1}{2}(e+fx)\middle|2\right)+70a\sin(2(e+fx))+21\right.}{140f}$$

```
[In] Integrate[(d*Sec[e + f*x])^(7/2)*(a + b*Tan[e + f*x]),x]
[Out] ((d*Sec[e + f*x])^(7/2)*(40*b - 168*a*Cos[e + f*x]^(7/2)*EllipticE[(e + f*x
)/2, 2] + 70*a*Sin[2*(e + f*x)] + 21*a*Sin[4*(e + f*x)]))/(140*f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 26.10 (sec) , antiderivative size = 436, normalized size of antiderivative = 3.60

method	result
default	$-\frac{2a\sqrt{d\sec(fx+e)}\,d^3\left(3iE(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,(\cos^2(fx+e))-3iF(i(\csc(fx+e)-\cot(fx+e)\right.}{}$
parts	$-\frac{2a\sqrt{d\sec(fx+e)}\,d^3\left(3iE(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,(\cos^2(fx+e))-3iF(i(\csc(fx+e)-\cot(fx+e)\right.}{}$

```
[In] int((d*sec(f*x+e))^(7/2)*(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
[Out] -2/5*a/f*(d*sec(f*x+e))^(1/2)*d^3/(cos(f*x+e)+1)*(3*I*EllipticE(I*(csc(f*x+
e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)*cos(f*x+e)^2-3*I*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1)
)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2+6*I*cos(f*x+e)*Ellip
ticE(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)-6*I*cos(f*x+e)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/
(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+3*I*(1/(cos(f*x+e)+
1))^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)-3*I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f
*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)-3*sin(f*x+e)-tan(f*x+e)-sec(f*x+e)*tan(f
*x+e))+2/7*b*(d*sec(f*x+e))^(7/2)/f
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.20

$$\int (d\sec(e+fx))^{7/2}(a+b\tan(e+fx))\,dx = \frac{-21i\sqrt{2}ad^{\frac{7}{2}}\cos(fx+e)^3\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)$$

```
[In] integrate((d*sec(f*x+e))^(7/2)*(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/35*(-21*I*sqrt(2)*a*d^(7/2)*cos(f*x + e)^3*weierstrassZeta(-4, 0, weierst
rassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 21*I*sqrt(2)*a*d^(7/2
)*cos(f*x + e)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x
+ e) - I*sin(f*x + e))) + 2*(5*b*d^3 + 7*(3*a*d^3*cos(f*x + e)^3 + a*d^3*co
s(f*x + e))*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(f*cos(f*x + e)^3)
```

Sympy [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{7/2}(a+b\tan(e+fx))\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))**(7/2)*(a+b*tan(f*x+e)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (d\sec(e+fx))^{7/2}(a+b\tan(e+fx))\,dx = \int (d\sec(fx+e))^{\frac{7}{2}}(b\tan(fx+e)+a)\,dx$$

```
[In] integrate((d*sec(f*x+e))^(7/2)*(a+b*tan(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*sec(f*x + e))^(7/2)*(b*tan(f*x + e) + a), x)
```

Giac [F]

$$\int (d\sec(e+fx))^{7/2}(a+b\tan(e+fx))\,dx = \int (d\sec(fx+e))^{\frac{7}{2}}\,(b\tan(fx+e)+a)\,dx$$

```
[In] integrate((d*sec(f*x+e))^(7/2)*(a+b*tan(f*x+e)),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(7/2)*(b*tan(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{7/2}(a+b\tan(e+fx))\,dx = \int \left(\frac{d}{\cos(e+f\,x)}\right)^{7/2}\,(a+b\tan(e+f\,x))\,dx$$

```
[In] int((d/cos(e + f*x))^(7/2)*(a + b*tan(e + f*x)),x)
[Out] int((d/cos(e + f*x))^(7/2)*(a + b*tan(e + f*x)), x)
```

3.579 $\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 92

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))\,dx = \frac{2ad^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{3f} + \frac{2b(d\sec(e+fx))^{5/2}}{5f} + \frac{2ad(d\sec(e+fx))^{3/2}\sin(e+fx)}{3f}$$

```
[Out] 2/5*b*(d*sec(f*x+e))^(5/2)/f+2/3*a*d*(d*sec(f*x+e))^(3/2)*sin(f*x+e)/f+2/3*
a*d^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x
+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(d*sec(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3567, 3853, 3856, 2720}

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))\,dx = \frac{2ad^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{3f} + \frac{2ad\sin(e+fx)(d\sec(e+fx))^{3/2}}{3f} + \frac{2b(d\sec(e+fx))^{5/2}}{5f}$$

```
[In] Int[(d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x]),x]
```

```
[Out] (2*a*d^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]])
/(3*f) + (2*b*(d*Sec[e + f*x])^(5/2))/(5*f) + (2*a*d*(d*Sec[e + f*x])^(3/2)
*Sin[e + f*x])/(3*f)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2b(d\sec(e+fx))^{5/2}}{5f} + a\int (d\sec(e+fx))^{5/2}\,dx \\
&= \frac{2b(d\sec(e+fx))^{5/2}}{5f} + \frac{2ad(d\sec(e+fx))^{3/2}\sin(e+fx)}{3f} + \frac{1}{3}\left(ad^2\right)\int \sqrt{d\sec(e+fx)}\,dx \\
&= \frac{2b(d\sec(e+fx))^{5/2}}{5f} + \frac{2ad(d\sec(e+fx))^{3/2}\sin(e+fx)}{3f} \\
&\quad + \frac{1}{3}\left(ad^2\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\right)\int \frac{1}{\sqrt{\cos(e+fx)}}\,dx \\
&= \frac{2ad^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{3f} \\
&\quad + \frac{2b(d\sec(e+fx))^{5/2}}{5f} + \frac{2ad(d\sec(e+fx))^{3/2}\sin(e+fx)}{3f}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.63

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))\,dx = \frac{(d\sec(e+fx))^{5/2}\left(6b+10a\cos^{\frac{5}{2}}(e+fx)\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)+5a\sin(2(e+fx))\right)}{15f}$$

```
[In] Integrate[(d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x]),x]
[Out] ((d*Sec[e + f*x])^(5/2)*(6*b + 10*a*Cos[e + f*x]^(5/2)*EllipticF[(e + f*x)/
2, 2] + 5*a*Sin[2*(e + f*x)]))/(15*f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 24.30 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.76

method	result
default	$-\frac{2a\sqrt{d\sec(fx+e)}\,d^2\left(i\cos(fx+e)F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}+i\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)-\cot\right.}{3f}$
parts	$-\frac{2a\sqrt{d\sec(fx+e)}\,d^2\left(i\cos(fx+e)F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}+i\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)-\cot\right.}{3f}$

```
[In] int((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
[Out] -2/3*a/f*(d*sec(f*x+e))^(1/2)*d^2*(I*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)
*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)+I*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(
cos(f*x+e)+1))^(1/2)-tan(f*x+e))+2/5*b*(d*sec(f*x+e))^(5/2)/f
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.34

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))\,dx = \frac{-5i\sqrt{2}ad^{\frac{5}{2}}\cos(fx+e)^2\,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))+5i\sqrt{}}{}$$

```
[In] integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*a*d^(5/2)*cos(f*x + e)^2*weierstrassPInverse(-4, 0, cos(
f*x + e) + I*sin(f*x + e)) + 5*I*sqrt(2)*a*d^(5/2)*cos(f*x + e)^2*weierstra
ssPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) + 2*(5*a*d^2*cos(f*x + e)*
sin(f*x + e) + 3*b*d^2)*sqrt(d/cos(f*x + e)))/(f*cos(f*x + e)^2)
```

Sympy [F]

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))\,dx = \int (d\sec(e+fx))^{\frac{5}{2}}\,(a+b\tan(e+fx))\,dx$$

```
[In] integrate((d*sec(f*x+e))**(5/2)*(a+b*tan(f*x+e)),x)
[Out] Integral((d*sec(e + f*x))**(5/2)*(a + b*tan(e + f*x)), x)
```

Maxima [F]

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))\,dx = \int (d\sec(fx+e))^{\frac{5}{2}}\,(b\tan(fx+e)+a)\,dx$$

```
[In] integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e)),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a), x)
```

Giac [F]

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))\,dx = \int (d\sec(fx+e))^{\frac{5}{2}}\,(b\tan(fx+e)+a)\,dx$$

```
[In] integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e)),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))\,dx = \int \left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}(a+b\tan(e+f\,x))\,dx$$

```
[In] int((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x)),x)
[Out] int((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x)), x)
```

3.580 $\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 88

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))\,dx = -\frac{2ad^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}} + \frac{2b(d\sec(e+fx))^{3/2}}{3f} + \frac{2ad\sqrt{d\sec(e+fx)}\sin(e+fx)}{f}$$

```
[Out] 2/3*b*(d*sec(f*x+e))^(3/2)/f-2*a*d^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f
*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(d*sec(f
*x+e))^(1/2)+2*a*d*sin(f*x+e)*(d*sec(f*x+e))^(1/2)/f
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3567, 3853, 3856, 2719}

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))\,dx = -\frac{2ad^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}} + \frac{2ad\sin(e+fx)\sqrt{d\sec(e+fx)}}{f} + \frac{2b(d\sec(e+fx))^{3/2}}{3f}$$

```
[In] Int[(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x]),x]
```

```
[Out] (-2*a*d^2*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e + f
*x]]) + (2*b*(d*Sec[e + f*x])^(3/2))/(3*f) + (2*a*d*Sqrt[d*Sec[e + f*x]]*Si
n[e + f*x])/f
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2b(d\sec(e+fx))^{3/2}}{3f} + a\int (d\sec(e+fx))^{3/2}\,dx\\ &= \frac{2b(d\sec(e+fx))^{3/2}}{3f} + \frac{2ad\sqrt{d\sec(e+fx)}\sin(e+fx)}{f} - (ad^2)\int \frac{1}{\sqrt{d\sec(e+fx)}}\,dx\\ &= \frac{2b(d\sec(e+fx))^{3/2}}{3f} + \frac{2ad\sqrt{d\sec(e+fx)}\sin(e+fx)}{f} - \frac{(ad^2)\int \sqrt{\cos(e+fx)}\,dx}{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}\\ &= -\frac{2ad^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}} + \frac{2b(d\sec(e+fx))^{3/2}}{3f} + \frac{2ad\sqrt{d\sec(e+fx)}\sin(e+fx)}{f}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.66

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))\,dx=\frac{(d\sec(e+fx))^{3/2}\left(2b-6a\cos^{\frac{3}{2}}(e+fx)E\left(\frac{1}{2}(e+fx)\middle|2\right)+3a\sin(2(e+fx))\right)}{3f}$$

```
[In] Integrate[(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x]),x]
```

```
[Out] ((d*Sec[e + f*x])^(3/2)*(2*b - 6*a*Cos[e + f*x]^(3/2)*EllipticE[(e + f*x)/2
, 2] + 3*a*Sin[2*(e + f*x)]))/(3*f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.59 (sec) , antiderivative size = 412, normalized size of antiderivative = 4.68

method	result
default	$-\frac{2a\left(iE(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\left(\cos^2(fx+e)\right)-iF(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\cdots}\right.}{\cdots}$
parts	$-\frac{2a\left(iE(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\left(\cos^2(fx+e)\right)-iF(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\cdots}\right.}{\cdots}$

```
[In] int((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*a/f*(I*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2-I*EllipticF(I*(csc(f*x+e)-cot
(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(
f*x+e)^2+2*I*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2
)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)-2*I*EllipticF(I*(csc(f*x+e)-
cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*c
os(f*x+e)+I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elli
pticE(I*(csc(f*x+e)-cot(f*x+e)),I)-I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elli
pticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)-sin(f*x+e))*(d*
sec(f*x+e))^(1/2)*d/(cos(f*x+e)+1)+2/3*b*(d*sec(f*x+e))^(3/2)/f
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.36

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))\,dx = \frac{-3i\sqrt{2}ad^{\frac{3}{2}}\cos(fx+e)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)$$

[In] integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e)),x, algorithm="fricas")

[Out] 1/3*(-3*I*sqrt(2)*a*d^(3/2)*cos(f*x + e)*weierstrassZeta(-4, 0, weierstrass PInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*I*sqrt(2)*a*d^(3/2)*cos (f*x + e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) + 2*(3*a*d*cos(f*x + e)*sin(f*x + e) + b*d)*sqrt(d/cos(f*x + e)))/(f*cos(f*x + e))

Sympy [F]

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))\,dx = \int (d\sec(e+fx))^{\frac{3}{2}}(a+b\tan(e+fx))\,dx$$

[In] integrate((d*sec(f*x+e))**(3/2)*(a+b*tan(f*x+e)),x)

[Out] Integral((d*sec(e + f*x))**(3/2)*(a + b*tan(e + f*x)), x)

Maxima [F]

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))\,dx = \int (d\sec(fx+e))^{\frac{3}{2}}(b\tan(fx+e)+a)\,dx$$

[In] integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e)),x, algorithm="maxima")

[Out] integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a), x)

Giac [F]

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))\,dx = \int (d\sec(fx+e))^{\frac{3}{2}}\,(b\tan(fx+e)+a)\,dx$$

```
[In] integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e)),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))\,dx = \int \left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}(a+b\tan(e+f\,x))\,dx$$

```
[In] int((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x)),x)
[Out] int((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x)), x)
```

3.581 $\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 58

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))\,dx = \frac{2b\sqrt{d\sec(e+fx)}}{f}+\frac{2a\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{f}$$

```
[Out] 2*b*(d*sec(f*x+e))^(1/2)/f+2*a*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2
*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(d*sec(f*x+e))^(
1/2)/f
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3567, 3856, 2720}

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))\,dx = \frac{2a\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{f}+\frac{2b\sqrt{d\sec(e+fx)}}{f}$$

```
[In] Int[Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x]),x]
```

```
[Out] (2*b*Sqrt[d*Sec[e + f*x]])/f + (2*a*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/
2, 2]*Sqrt[d*Sec[e + f*x]])/f
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2b\sqrt{d\sec(e+fx)}}{f} + a\int\sqrt{d\sec(e+fx)}\,dx \\
&= \frac{2b\sqrt{d\sec(e+fx)}}{f} + \left(a\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\right)\int\frac{1}{\sqrt{\cos(e+fx)}}\,dx \\
&= \frac{2b\sqrt{d\sec(e+fx)}}{f} + \frac{2a\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.72

$$\int\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))\,dx = \frac{2\left(b+a\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\right)\sqrt{d\sec(e+fx)}}{f}$$

```
[In] Integrate[Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x]),x]
[Out] (2*(b + a*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])*Sqrt[d*Sec[e + f*x]
])/f
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 11.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.64

method	result
parts	$-\frac{2ia(\cos(fx+e)+1)F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{d\sec(fx+e)}\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}}{f}+\frac{2b\sqrt{d\sec(fx+e)}}{f}$
default	$-\frac{2\left(i\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{1}{\cos(fx+e)+1}}F(i(\csc(fx+e)-\cot(fx+e)),i)a\cos(fx+e)+i\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{1}{\cos(fx+e)+1}}F(i(\csc(fx+e)-\cot\right.}{f}$

[In] int((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)

[Out] -2*I*a/f*(cos(f*x+e)+1)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(d*sec(f*x+e))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+2*b*(d*sec(f*x+e))^(1/2)/f

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.28

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))\,dx = \frac{-i\sqrt{2}a\sqrt{d}\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))+i\sqrt{2}a\sqrt{d}\text{weierstrassPInverse}(-4,0}{f}$$

[In] integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e)),x, algorithm="fricas")

[Out] (-I*sqrt(2)*a*sqrt(d)*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e)) + I*sqrt(2)*a*sqrt(d)*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) + 2*b*sqrt(d/cos(f*x + e)))/f

Sympy [F]

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))\,dx = \int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))\,dx$$

[In] integrate((d*sec(f*x+e))**(1/2)*(a+b*tan(f*x+e)),x)

[Out] Integral(sqrt(d*sec(e + f*x))*(a + b*tan(e + f*x)), x)

Maxima [F]

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))\,dx = \int \sqrt{d\sec\left(fx+e\right)}(b\tan\left(fx+e\right)+a)\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e)),x, algorithm="maxima")
[Out] integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a), x)
```

Giac [F]

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))\,dx = \int \sqrt{d\sec\left(fx+e\right)}(b\tan\left(fx+e\right)+a)\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e)),x, algorithm="giac")
[Out] integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.67

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))\,dx = \frac{2\left(b+a\sqrt{\cos\left(e+f\,x\right)}\,\mathrm{F}\left(\frac{e}{2}+\frac{f\,x}{2}\middle|2\right)\right)\sqrt{\frac{d}{\cos(e+f\,x)}}}{f}$$

```
[In] int((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x)),x)
[Out] (2*(b + a*cos(e + f*x)^(1/2)*ellipticF(e/2 + (f*x)/2, 2))*(d/cos(e + f*x))^
(1/2))/f
```

3.582 $\int \frac{a+b\tan(e+fx)}{\sqrt{d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 58

$$\int \frac{a+b\tan(e+fx)}{\sqrt{d\sec(e+fx)}}\,dx = -\frac{2b}{f\sqrt{d\sec(e+fx)}} + \frac{2aE\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}$$

```
[Out] -2*b/f/(d*sec(f*x+e))^(1/2)+2*a*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/
2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(d*sec(f*x+e)
)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3567, 3856, 2719}

$$\int \frac{a+b\tan(e+fx)}{\sqrt{d\sec(e+fx)}}\,dx = \frac{2aE\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}} - \frac{2b}{f\sqrt{d\sec(e+fx)}}$$

```
[In] Int[(a + b*Tan[e + f*x])/Sqrt[d*Sec[e + f*x]],x]
```

```
[Out] (-2*b)/(f*Sqrt[d*Sec[e + f*x]]) + (2*a*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[C
os[e + f*x]]*Sqrt[d*Sec[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2b}{f\sqrt{d\sec(e+fx)}} + a\int \frac{1}{\sqrt{d\sec(e+fx)}}\,dx \\
&= -\frac{2b}{f\sqrt{d\sec(e+fx)}} + \frac{a\int \sqrt{\cos(e+fx)}\,dx}{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}} \\
&= -\frac{2b}{f\sqrt{d\sec(e+fx)}} + \frac{2aE\left(\frac{1}{2}(e+fx)\middle|\,2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int \frac{a+b\tan(e+fx)}{\sqrt{d\sec(e+fx)}}\,dx = \frac{-2b\sqrt{\cos(e+fx)} + 2aE\left(\frac{1}{2}(e+fx)\middle|\,2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}$$

```
[In] Integrate[(a + b*Tan[e + f*x])/Sqrt[d*Sec[e + f*x]],x]
[Out] (-2*b*Sqrt[Cos[e + f*x]] + 2*a*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[Cos[e + f
*x]]*Sqrt[d*Sec[e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 10.44 (sec) , antiderivative size = 306, normalized size of antiderivative = 5.28

method	result
risch	$-\frac{i(-ib+a)\sqrt{2}}{f\sqrt{\frac{d\,\mathrm{e}^{i(fx+e)}}{\mathrm{e}^{2i(fx+e)}+1}}}-\frac{ia\left(-\frac{2\left(\mathrm{e}^{2i(fx+e)}d+d\right)}{d\sqrt{\mathrm{e}^{i(fx+e)}\left(\mathrm{e}^{2i(fx+e)}d+d\right)}}+\frac{i\sqrt{-i\left(\mathrm{e}^{i(fx+e)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(\mathrm{e}^{i(fx+e)}-i\right)}\,\sqrt{i\mathrm{e}^{i(fx+e)}}\,\left(-2iE\left(\sqrt{-i\left(\mathrm{e}^{i(fx+e)}+i\right)}\right.\right.}{\sqrt{d\,\mathrm{e}^{3i(fx+e)}+d\,\mathrm{e}^{i(fx+e)}}}}{f\sqrt{\frac{d\,\mathrm{e}^{i(fx+e)}}{\mathrm{e}^{2i(fx+e)}+1}}\,\left(\mathrm{e}^{2i(fx+e)}+1\right)}$
parts	$\frac{2a\left(i\cos(fx+e)E(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}-i\cos(fx+e)F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e}}\right.}{}$
default	Expression too large to display

```
[In] int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*(a-I*b)/f*2^(1/2)/(d*exp(I*(f*x+e))/(exp(I*(f*x+e))^2+1))^(1/2)-I*a/f*(-
2*(exp(I*(f*x+e))^2*d+d)/d/(exp(I*(f*x+e))*(exp(I*(f*x+e))^2*d+d))^(1/2)+I*
(-I*(exp(I*(f*x+e))+I))^(1/2)*2^(1/2)*(I*(exp(I*(f*x+e))-I))^(1/2)*(I*exp(I
*(f*x+e)))^(1/2)/(d*exp(I*(f*x+e))^3+d*exp(I*(f*x+e)))^(1/2)*(-2*I*Elliptic
E((-I*(exp(I*(f*x+e))+I))^(1/2),1/2*2^(1/2))+I*EllipticF((-I*(exp(I*(f*x+e)
)+I))^(1/2),1/2*2^(1/2))))*2^(1/2)/(d*exp(I*(f*x+e))/(exp(I*(f*x+e))^2+1))^
(1/2)*(d*exp(I*(f*x+e))*(exp(I*(f*x+e))^2+1))^(1/2)/(exp(I*(f*x+e))^2+1)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.53

$$\int \frac{a+b\tan(e+fx)}{\sqrt{d\sec(e+fx)}}\,dx$$
$$=\frac{i\sqrt{2}a\sqrt{d}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos{(fx+e)}+i\sin{(fx+e)}))-i\sqrt{2}a\sqrt{d}\text{weie}}{d}$$

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] (I*sqrt(2)*a*sqrt(d)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(
f*x + e) + I*sin(f*x + e))) - I*sqrt(2)*a*sqrt(d)*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) - 2*b*sqrt(d/cos(f
*x + e))*cos(f*x + e))/(d*f)
```

Sympy [F]

$$\int \frac{a+b\tan(e+fx)}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{a+b\tan\left(e+fx\right)}{\sqrt{d\sec\left(e+fx\right)}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))**(1/2),x)
[Out] Integral((a + b*tan(e + f*x))/sqrt(d*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{a+b\tan(e+fx)}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{b\tan\left(fx+e\right)+a}{\sqrt{d\sec\left(fx+e\right)}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e) + a)/sqrt(d*sec(f*x + e)), x)
```

Giac [F]

$$\int \frac{a+b\tan(e+fx)}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{b\tan\left(fx+e\right)+a}{\sqrt{d\sec\left(fx+e\right)}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)/sqrt(d*sec(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\tan(e+fx)}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{a+b\tan(e+f\,x)}{\sqrt{\frac{d}{\cos(e+f\,x)}}}\,dx$$

```
[In] int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(1/2),x)
[Out] int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(1/2), x)
```

3.583 $\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 94

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{3/2}}\,dx=-\frac{2b}{3f(d\sec(e+fx))^{3/2}}+\frac{2a\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{3d^2f}+\frac{2a\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}$$

```
[Out] -2/3*b/f/(d*sec(f*x+e))^(3/2)+2/3*a*sin(f*x+e)/d/f/(d*sec(f*x+e))^(1/2)+2/3
*a*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/
2*e),2^(1/2))*cos(f*x+e)^(1/2)*(d*sec(f*x+e))^(1/2)/d^2/f
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3567, 3854, 3856, 2720}

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{3/2}}\,dx=\frac{2a\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{3d^2f}+\frac{2a\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}-\frac{2b}{3f(d\sec(e+fx))^{3/2}}$$

```
[In] Int[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-2*b)/(3*f*(d*Sec[e + f*x])^(3/2)) + (2*a*Sqrt[Cos[e + f*x]]*EllipticF[(e
+ f*x)/2, 2]*Sqrt[d*Sec[e + f*x]])/(3*d^2*f) + (2*a*Sin[e + f*x])/(3*d*f*Sq
rt[d*Sec[e + f*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2b}{3f(d\sec(e+fx))^{3/2}}+a\int\frac{1}{(d\sec(e+fx))^{3/2}}\,dx\\
&= -\frac{2b}{3f(d\sec(e+fx))^{3/2}}+\frac{2a\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}+\frac{a\int\sqrt{d\sec(e+fx)}\,dx}{3d^2}\\
&= -\frac{2b}{3f(d\sec(e+fx))^{3/2}}+\frac{2a\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}\\
&\quad+\frac{\left(a\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\right)\int\frac{1}{\sqrt{\cos(e+fx)}}\,dx}{3d^2}\\
&= -\frac{2b}{3f(d\sec(e+fx))^{3/2}}\\
&\quad+\frac{2a\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{3d^2f}+\frac{2a\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.73

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{3/2}}\,dx = -\frac{\sqrt{d\sec(e+fx)}\left(b+b\cos(2(e+fx))-2a\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)-a\sin(2(e+fx))\right)}{3d^2f}$$

```
[In] Integrate[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(3/2),x]
```

```
[Out] -1/3*(Sqrt[d*Sec[e + f*x]]*(b + b*Cos[2*(e + f*x)] - 2*a*Sqrt[Cos[e + f*x]]
*EllipticF[(e + f*x)/2, 2] - a*Sin[2*(e + f*x)]))/(d^2*f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 10.85 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.65

method	result
default	$\frac{\frac{2iF(i(\cot(fx+e)-\csc(fx+e)),i)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{1}{\cos(fx+e)+1}}\,a}{3}+\frac{2i\sec(fx+e)F(i(\cot(fx+e)-\csc(fx+e)),i)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{1}{\cos(fx+e)+1}}\,a}{3}+}{df\sqrt{d\sec(fx+e)}}$
parts	$-\frac{2a\left(i\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}+i\sec(fx+e)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{}{co}}\right.}{3f\sqrt{d\sec(fx+e)}\,d}$

```
[In] int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/d/f/(d*sec(f*x+e))^(1/2)*(I*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*a+I*sec(f*x+e)*Ellip
ticF(I*(cot(f*x+e)-csc(f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos
(f*x+e)+1))^(1/2)*a+sin(f*x+e)*a-b*cos(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.10

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{3/2}}\,dx = \frac{-i\sqrt{2}a\sqrt{d}\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))+i\sqrt{2}a\sqrt{d}\text{w}}{}$$

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*a*sqrt(d)*weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f
*x + e)) + I*sqrt(2)*a*sqrt(d)*weierstrassPInverse(-4, 0, cos(f*x + e) - I*
sin(f*x + e)) - 2*(b*cos(f*x + e)^2 - a*cos(f*x + e)*sin(f*x + e))*sqrt(d/c
os(f*x + e)))/(d^2*f)
```

Sympy [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{a+b\tan{(e+fx)}}{(d\sec{(e+fx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))**(3/2),x)
[Out] Integral((a + b*tan(e + f*x))/(d*sec(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{b\tan{(fx+e)}+a}{(d\sec{(fx+e)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{b\tan{(fx+e)}+a}{(d\sec{(fx+e)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{a+b\tan(e+f\,x)}{{\left(\frac{d}{\cos(e+f\,x)}\right)}^{3/2}}\,dx$$

```
[In] int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(3/2),x)
[Out] int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(3/2), x)
```

3.584 $\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 94

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/2}}\,dx = -\frac{2b}{5f(d\sec(e+fx))^{5/2}} + \frac{6aE\left(\frac{1}{2}(e+fx)\middle|2\right)}{5d^2f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}} + \frac{2a\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}$$

```
[Out] -2/5*b/f/(d*sec(f*x+e))^(5/2)+2/5*a*sin(f*x+e)/d/f/(d*sec(f*x+e))^(3/2)+6/5
*a*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/
2*e),2^(1/2))/d^2/f/cos(f*x+e)^(1/2)/(d*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3567, 3854, 3856, 2719}

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/2}}\,dx = \frac{6aE\left(\frac{1}{2}(e+fx)\middle|2\right)}{5d^2f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}} + \frac{2a\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}} - \frac{2b}{5f(d\sec(e+fx))^{5/2}}$$

```
[In] Int[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-2*b)/(5*f*(d*Sec[e + f*x])^(5/2)) + (6*a*EllipticE[(e + f*x)/2, 2])/(5*d^
2*f*Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e + f*x]]) + (2*a*Sin[e + f*x])/(5*d*f*(d
*Sec[e + f*x])^(3/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2b}{5f(d\sec(e+fx))^{5/2}}+a\int\frac{1}{(d\sec(e+fx))^{5/2}}\,dx\\
&= -\frac{2b}{5f(d\sec(e+fx))^{5/2}}+\frac{2a\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}+\frac{(3a)\int\frac{1}{\sqrt{d\sec(e+fx)}}\,dx}{5d^2}\\
&= -\frac{2b}{5f(d\sec(e+fx))^{5/2}}+\frac{2a\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}+\frac{(3a)\int\sqrt{\cos(e+fx)}\,dx}{5d^2\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}\\
&= -\frac{2b}{5f(d\sec(e+fx))^{5/2}}+\frac{6aE\left(\frac{1}{2}(e+fx)\middle|2\right)}{5d^2f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2a\sin(e+fx)}{5df(d\sec(e+fx))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.42 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.79

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/2}}\,dx = \frac{2\sqrt{d\sec(e+fx)}\left(3a\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)+\cos^2(e+fx)(-b\cos(e+f\right.}{5d^3f}$$

```
[In] Integrate[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*Sqrt[d*Sec[e + f*x]]*(3*a*Sqrt[Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2] +
 Cos[e + f*x]^2*(-(b*Cos[e + f*x]) + a*Sin[e + f*x])))/(5*d^3*f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.15 (sec) , antiderivative size = 438, normalized size of antiderivative = 4.66

method	result
default	$\frac{2a\left(3i\cos(fx+e)E(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}-3i\cos(fx+e)F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx}}\right.}{}$
parts	$\frac{2a\left(3i\cos(fx+e)E(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}-3i\cos(fx+e)F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx}}\right.}{}$

```
[In] int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*a/f/(cos(f*x+e)+1)/(d*sec(f*x+e))^(1/2)/d^2*(3*I*cos(f*x+e)*EllipticE(I
*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)-3*I*cos(f*x+e)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f
*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+6*I*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I
)-6*I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e))
,I)*(1/(cos(f*x+e)+1))^(1/2)+3*I*sec(f*x+e)*EllipticE(I*(csc(f*x+e)-cot(f*x
+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-3*I*sec(
f*x+e)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)+sin(f*x+e)*cos(f*x+e)^2+sin(f*x+e)*cos(f*x+e)
+3*sin(f*x+e))-2/5*b/f/(d*sec(f*x+e))^(5/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.18

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/2}}\,dx = \frac{3i\sqrt{2}a\sqrt{d}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(f}{}$$

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/5*(3*I*sqrt(2)*a*sqrt(d)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(f*x + e) + I*sin(f*x + e))) - 3*I*sqrt(2)*a*sqrt(d)*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) - 2*(b*cos
(f*x + e)^3 - a*cos(f*x + e)^2*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(d^3*f)
```

Sympy [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))/(d*sec(e + f*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{b\tan(fx+e)+a}{(d\sec(fx+e))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(5/2), x)
```

Giac [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{b\tan(fx+e)+a}{(d\sec(fx+e))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/2}}\,dx=\int \frac{a+b\tan(e+f\,x)}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

```
[In] int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(5/2),x)
[Out] int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(5/2), x)
```

3.585 $\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 123

$$\begin{aligned}\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{7/2}}\,dx = & -\frac{2b}{7f(d\sec(e+fx))^{7/2}} \\ & + \frac{10a\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{21d^4f} \\ & + \frac{2a\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}} + \frac{10a\sin(e+fx)}{21d^3f\sqrt{d\sec(e+fx)}}\end{aligned}$$

```
[Out] -2/7*b/f/(d*sec(f*x+e))^(7/2)+2/7*a*sin(f*x+e)/d/f/(d*sec(f*x+e))^(5/2)+10/
21*a*sin(f*x+e)/d^3/f/(d*sec(f*x+e))^(1/2)+10/21*a*(cos(1/2*f*x+1/2*e)^2)^(
1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1
/2)*(d*sec(f*x+e))^(1/2)/d^4/f
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3567, 3854, 3856, 2720}

$$\begin{aligned}\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{7/2}}\,dx = & \frac{10a\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{21d^4f} \\ & + \frac{10a\sin(e+fx)}{21d^3f\sqrt{d\sec(e+fx)}} + \frac{2a\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}} - \frac{2b}{7f(d\sec(e+fx))^{7/2}}\end{aligned}$$

```
[In] Int[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(7/2),x]
```

```
[Out] (-2*b)/(7*f*(d*Sec[e + f*x])^(7/2)) + (10*a*Sqrt[Cos[e + f*x]]*EllipticF[(e
 + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]])/(21*d^4*f) + (2*a*Sin[e + f*x])/(7*d*f*
(d*Sec[e + f*x])^(5/2)) + (10*a*Sin[e + f*x])/(21*d^3*f*Sqrt[d*Sec[e + f*x]
])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2b}{7f(d\sec(e+fx))^{7/2}} + a\int \frac{1}{(d\sec(e+fx))^{7/2}}\,dx \\
&= -\frac{2b}{7f(d\sec(e+fx))^{7/2}} + \frac{2a\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}} + \frac{(5a)\int \frac{1}{(d\sec(e+fx))^{3/2}}\,dx}{7d^2} \\
&= -\frac{2b}{7f(d\sec(e+fx))^{7/2}} + \frac{2a\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}} \\
&\quad + \frac{10a\sin(e+fx)}{21d^3f\sqrt{d\sec(e+fx)}} + \frac{(5a)\int \sqrt{d\sec(e+fx)}\,dx}{21d^4} \\
&= -\frac{2b}{7f(d\sec(e+fx))^{7/2}} + \frac{2a\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}} + \frac{10a\sin(e+fx)}{21d^3f\sqrt{d\sec(e+fx)}} \\
&\quad + \frac{\left(5a\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\right)\int \frac{1}{\sqrt{\cos(e+fx)}}\,dx}{21d^4}
\end{aligned}$$

$$= -\frac{2b}{7f(d\sec(e+fx))^{7/2}} + \frac{10a\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{21d^4f} + \frac{2a\sin(e+fx)}{7df(d\sec(e+fx))^{5/2}} + \frac{10a\sin(e+fx)}{21d^3f\sqrt{d\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.76

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{7/2}}\,dx = \frac{\sqrt{d\sec(e+fx)}\Big(-9b-12b\cos(2(e+fx))-3b\cos(4(e+fx))+40a\sqrt{\cos(e+fx)}}{84d^4f}$$

```
[In] Integrate[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(7/2),x]
```

```
[Out] (Sqrt[d*Sec[e + f*x]]*(-9*b - 12*b*Cos[2*(e + f*x)] - 3*b*Cos[4*(e + f*x)]
+ 40*a*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2] + 26*a*Sin[2*(e + f*x)]
 + 3*a*Sin[4*(e + f*x)]))/(84*d^4*f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.18 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.45

method	result
default	$-\frac{2a\left(5i\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}+5i\sec(fx+e)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{}{}}}{21f\sqrt{d\sec(fx+e)}\,d^3}$
parts	$-\frac{2a\left(5i\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}+5i\sec(fx+e)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{}{}}}{21f\sqrt{d\sec(fx+e)}\,d^3}$

```
[In] int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21*a/f/(d*sec(f*x+e))^(1/2)/d^3*(5*I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*E
llipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)+5*I*sec(f*x+
e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)
*(1/(cos(f*x+e)+1))^(1/2)-3*sin(f*x+e)*cos(f*x+e)^2-5*sin(f*x+e))-2/7*b/f/(
d*sec(f*x+e))^(7/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.96

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{7/2}}\,dx = \frac{-5i\sqrt{2}a\sqrt{d}\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e))+5i\sqrt{2}a\sqrt{}}{}$$

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/21*(-5*I*sqrt(2)*a*sqrt(d)*weierstrassPInverse(-4, 0, cos(f*x + e) + I*si
n(f*x + e)) + 5*I*sqrt(2)*a*sqrt(d)*weierstrassPInverse(-4, 0, cos(f*x + e)
 - I*sin(f*x + e)) - 2*(3*b*cos(f*x + e)^4 - (3*a*cos(f*x + e)^3 + 5*a*cos(
f*x + e))*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(d^4*f)
```

Sympy [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))**(7/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))/(d*sec(e + f*x))**(7/2), x)
```

Maxima [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{b\tan(fx+e)+a}{(d\sec(fx+e))^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(7/2), x)
```

Giac [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{b\tan(fx+e)+a}{(d\sec(fx+e))^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{a+b\tan(e+f\,x)}{\left(\frac{d}{\cos(e+f\,x)}\right)^{7/2}}\,dx$$

```
[In] int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(7/2),x)
[Out] int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(7/2), x)
```

3.586 $\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 143

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2\,dx = \frac{2(7a^2-2b^2)\,d^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{21f} + \frac{18ab(d\sec(e+fx))^{5/2}}{35f} + \frac{2(7a^2-2b^2)\,d(d\sec(e+fx))^{3/2}\sin(e+fx)}{21f} + \frac{2b(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}{7f}$$

```
[Out] 18/35*a*b*(d*sec(f*x+e))^(5/2)/f+2/21*(7*a^2-2*b^2)*d*(d*sec(f*x+e))^(3/2)*
sin(f*x+e)/f+2/21*(7*a^2-2*b^2)*d^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*
x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(d*sec(f*x+
e))^(1/2)/f+2/7*b*(d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))/f
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used

$= \{3589, 3567, 3853, 3856, 2720\}$

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2\,dx = \frac{2d^2(7a^2-2b^2)\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{21f} + \frac{2d(7a^2-2b^2)\sin(e+fx)(d\sec(e+fx))^{3/2}}{21f} + \frac{18ab(d\sec(e+fx))^{5/2}}{35f} + \frac{2b(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}{7f}$$

```
[In] Int[(d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (2*(7*a^2 - 2*b^2)*d^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[d*
Sec[e + f*x]])/(21*f) + (18*a*b*(d*Sec[e + f*x])^(5/2))/(35*f) + (2*(7*a^2
- 2*b^2)*d*(d*Sec[e + f*x])^(3/2)*Sin[e + f*x])/(21*f) + (2*b*(d*Sec[e + f*
x])^(5/2)*(a + b*Tan[e + f*x]))/(7*f)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2b(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}{7f} \\
&\quad + \frac{2}{7}\int (d\sec(e+fx))^{5/2}\left(\frac{7a^2}{2}-b^2+\frac{9}{2}ab\tan(e+fx)\right)dx \\
&= \frac{18ab(d\sec(e+fx))^{5/2}}{35f}+\frac{2b(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}{7f} \\
&\quad + \frac{1}{7}\left(7a^2-2b^2\right)\int (d\sec(e+fx))^{5/2}\,dx \\
&= \frac{18ab(d\sec(e+fx))^{5/2}}{35f}+\frac{2\left(7a^2-2b^2\right)d(d\sec(e+fx))^{3/2}\sin(e+fx)}{21f} \\
&\quad + \frac{2b(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}{7f} \\
&\quad + \frac{1}{21}\left(\left(7a^2-2b^2\right)d^2\right)\int \sqrt{d\sec(e+fx)}\,dx \\
&= \frac{18ab(d\sec(e+fx))^{5/2}}{35f}+\frac{2\left(7a^2-2b^2\right)d(d\sec(e+fx))^{3/2}\sin(e+fx)}{21f} \\
&\quad + \frac{2b(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}{7f} \\
&\quad + \frac{1}{21}\left(\left(7a^2-2b^2\right)d^2\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\right)\int \frac{1}{\sqrt{\cos(e+fx)}}\,dx \\
&= \frac{2\left(7a^2-2b^2\right)d^2\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{21f} \\
&\quad + \frac{18ab(d\sec(e+fx))^{5/2}}{35f}+\frac{2\left(7a^2-2b^2\right)d(d\sec(e+fx))^{3/2}\sin(e+fx)}{21f} \\
&\quad + \frac{2b(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}{7f}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.44 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.89

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2\,dx = \frac{2d^2\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2\left(5(7a^2-2b^2)\cos^{\frac{5}{2}}(e+fx)\,\text{EllipticF}\left(\frac{1}{2}(e+fx\right.\right.}{105f(a\cos(e+fx)+b\sin(e+}$$

```
[In] Integrate[(d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (2*d^2*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^2*(5*(7*a^2 - 2*b^2)*Cos[e
 + f*x]^(5/2)*EllipticF[(e + f*x)/2, 2] + (5*(7*a^2 - 2*b^2)*Sin[2*(e + f*x
)])/2 + 3*b*(14*a + 5*b*Tan[e + f*x])))/(105*f*(a*Cos[e + f*x] + b*Sin[e +
f*x])^2)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 146.29 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.22

method	result
default	$-\frac{2d^2\sqrt{d\sec(fx+e)}\left(35i\cos(fx+e)F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{1}{\cos(fx+e)+1}}\,a^2-10i\cos(fx+e)F(i(\csc(fx+e)-c\right.}{}$
parts	$-\frac{2a^2\sqrt{d\sec(fx+e)}\,d^2\left(i\cos(fx+e)F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}+i\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}F(i(\csc(fx+e)-co\right.}{3f}$

```
[In] int((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/105*d^2/f*(d*sec(f*x+e))^(1/2)*(35*I*cos(f*x+e)*EllipticF(I*(csc(f*x+e)-
cot(f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*a
^2-10*I*cos(f*x+e)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*b^2+35*I*(cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*a
^2-10*I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*Elliptic
F(I*(csc(f*x+e)-cot(f*x+e)),I)*b^2-35*tan(f*x+e)*a^2+10*tan(f*x+e)*b^2-42*s
ec(f*x+e)^2*a*b-15*tan(f*x+e)*sec(f*x+e)^2*b^2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.20

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2\,dx = \frac{-5i\sqrt{2}(7a^2-2b^2)d^{\frac{5}{2}}\cos(fx+e)^3\,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\,\sin(f$$

```
[In] integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
[Out] 1/105*(-5*I*sqrt(2)*(7*a^2 - 2*b^2)*d^(5/2)*cos(f*x + e)^3*weierstrassPInve
rse(-4, 0, cos(f*x + e) + I*sin(f*x + e)) + 5*I*sqrt(2)*(7*a^2 - 2*b^2)*d^(
5/2)*cos(f*x + e)^3*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e
)) + 2*(42*a*b*d^2*cos(f*x + e) + 5*((7*a^2 - 2*b^2)*d^2*cos(f*x + e)^2 + 3
*b^2*d^2)*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(f*cos(f*x + e)^3)
```

Sympy [F]

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2\,dx = \int (d\sec(e+fx))^{\frac{5}{2}}\,(a+b\tan(e+fx))^2\,dx$$

```
[In] integrate((d*sec(f*x+e))**(5/2)*(a+b*tan(f*x+e))**2,x)
[Out] Integral((d*sec(e + f*x))**(5/2)*(a + b*tan(e + f*x))**2, x)
```

Maxima [F]

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2\,dx = \int (d\sec(fx+e))^{\frac{5}{2}}\,(b\tan(fx+e)+a)^2\,dx$$

```
[In] integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a)^2, x)
```

Giac [F]

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2\,dx=\int (d\sec{(fx+e)})^{\frac{5}{2}}\,(b\tan{(fx+e)}+a)^2\,dx$$

```
[In] integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2\,dx=\int \left(\frac{d}{\cos{(e+f\,x)}}\right)^{5/2}\,(a+b\tan(e+f\,x))^2\,dx$$

```
[In] int((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x))^2,x)
[Out] int((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x))^2, x)
```

3.587 $\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 143

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2\,dx = -\frac{2(5a^2-2b^2)\,d^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{14ab(d\sec(e+fx))^{3/2}}{15f}+\frac{2(5a^2-2b^2)\,d\sqrt{d\sec(e+fx)}\sin(e+fx)}{5f}+\frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}{5f}$$

```
[Out] 14/15*a*b*(d*sec(f*x+e))^(3/2)/f-2/5*(5*a^2-2*b^2)*d^2*(cos(1/2*f*x+1/2*e)^
2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/f/cos(f*x
+e)^(1/2)/(d*sec(f*x+e))^(1/2)+2/5*(5*a^2-2*b^2)*d*sin(f*x+e)*(d*sec(f*x+e)
)^(1/2)/f+2/5*b*(d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))/f
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3589, 3567, 3853, 3856, 2719}

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2\,dx = -\frac{2d^2(5a^2-2b^2)\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2d(5a^2-2b^2)\sin(e+fx)\sqrt{d\sec(e+fx)}}{5f}+\frac{14ab(d\sec(e+fx))^{3/2}}{15f}+\frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}{5f}$$

```
[In] Int[(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (-2*(5*a^2 - 2*b^2)*d^2*EllipticE[(e + f*x)/2, 2])/(5*f*Sqrt[Cos[e + f*x]]*
Sqrt[d*Sec[e + f*x]]) + (14*a*b*(d*Sec[e + f*x])^(3/2))/(15*f) + (2*(5*a^2
- 2*b^2)*d*Sqrt[d*Sec[e + f*x]]*Sin[e + f*x])/(5*f) + (2*b*(d*Sec[e + f*x])
^(3/2)*(a + b*Tan[e + f*x]))/(5*f)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\text{integral} = \frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}{5f} + \frac{2}{5}\int (d\sec(e+fx))^{3/2}\left(\frac{5a^2}{2}-b^2+\frac{7}{2}ab\tan(e+fx)\right)dx$$

$$= \frac{14ab(d\sec(e+fx))^{3/2}}{15f} + \frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}{5f} + \frac{1}{5}(5a^2-2b^2)\int (d\sec(e+fx))^{3/2}\,dx$$

$$= \frac{14ab(d\sec(e+fx))^{3/2}}{15f} + \frac{2(5a^2-2b^2)\,d\sqrt{d\sec(e+fx)}\sin(e+fx)}{5f} + \frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}{5f} - \frac{1}{5}\left((5a^2-2b^2)\,d^2\right)\int \frac{1}{\sqrt{d\sec(e+fx)}}\,dx$$

$$= \frac{14ab(d\sec(e+fx))^{3/2}}{15f} + \frac{2(5a^2-2b^2)\,d\sqrt{d\sec(e+fx)}\sin(e+fx)}{5f} + \frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}{5f} - \frac{((5a^2-2b^2)\,d^2)\int \sqrt{\cos(e+fx)}\,dx}{5\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$= -\frac{2(5a^2-2b^2)\,d^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}} + \frac{14ab(d\sec(e+fx))^{3/2}}{15f} + \frac{2(5a^2-2b^2)\,d\sqrt{d\sec(e+fx)}\sin(e+fx)}{5f} + \frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}{5f}$$

Mathematica [A] (verified)

Time = 1.85 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.88

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2\,dx = -\frac{2d^2(a+b\tan(e+fx))^2\left(3(5a^2-2b^2)\cos^{\frac{3}{2}}(e+fx)E\left(\frac{1}{2}(e+fx)\middle|2\right)+\left(-\frac{15a^2}{2}+3b^2\right)\sin(2(e+fx))-b(\right.}{15f\sqrt{d\sec(e+fx)}(a\cos(e+fx)+b\sin(e+fx))^2}$$

```
[In] Integrate[(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (-2*d^2*(a + b*Tan[e + f*x])^2*(3*(5*a^2 - 2*b^2)*Cos[e + f*x]^(3/2)*Ellipt
icE[(e + f*x)/2, 2] + ((-15*a^2)/2 + 3*b^2)*Sin[2*(e + f*x)] - b*(10*a + 3*
b*Tan[e + f*x])))/(15*f*Sqrt[d*Sec[e + f*x]]*(a*Cos[e + f*x] + b*Sin[e + f*
x])^2)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 21.00 (sec) , antiderivative size = 833, normalized size of antiderivative = 5.83

method	result	size
parts	Expression too large to display	833
default	Expression too large to display	844

```
[In] int((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*a^2/f*(I*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2-I*EllipticF(I*(csc(f*x+e)-c
ot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*co
s(f*x+e)^2+2*I*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)-2*I*EllipticF(I*(csc(f*x+e
)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)
*cos(f*x+e)+I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*El
lipticE(I*(csc(f*x+e)-cot(f*x+e)),I)-I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*El
lipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)-sin(f*x+e))*(
d*sec(f*x+e))^(1/2)*d/(cos(f*x+e)+1)-2/5*b^2/f*(d*sec(f*x+e))^(1/2)*d/(cos(
f*x+e)+1)*(2*I*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2-2*I*EllipticE(I*(csc(f*x
+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)*cos(f*x+e)^2+4*I*cos(f*x+e)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(c
os(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-4*I*cos(f*x+e)*Ellipt
icE(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)+2*I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x
+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)-2*I*(1/(cos(f*x+e)+1))^(1/2)*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)+2*si
n(f*x+e)-tan(f*x+e)-sec(f*x+e)*tan(f*x+e))+4/3*a*b*(d*sec(f*x+e))^(3/2)/f
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.20

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2\,dx = \frac{-3i\sqrt{2}(5a^2-2b^2)d^{\frac{3}{2}}\cos(fx+e)^2\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,c$$

```
[In] integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/15*(-3*I*sqrt(2)*(5*a^2 - 2*b^2)*d^(3/2)*cos(f*x + e)^2*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*I*sqrt
(2)*(5*a^2 - 2*b^2)*d^(3/2)*cos(f*x + e)^2*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) + 2*(10*a*b*d*cos(f*x + e
) + 3*((5*a^2 - 2*b^2)*d*cos(f*x + e)^2 + b^2*d)*sin(f*x + e))*sqrt(d/cos(f
*x + e)))/(f*cos(f*x + e)^2)
```

Sympy [F]

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2\,dx=\int (d\sec{(e+fx)})^{\frac{3}{2}}\,(a+b\tan{(e+fx)})^2\,dx$$

```
[In] integrate((d*sec(f*x+e))**(3/2)*(a+b*tan(f*x+e))**2,x)
[Out] Integral((d*sec(e + f*x))**(3/2)*(a + b*tan(e + f*x))**2, x)
```

Maxima [F]

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2\,dx=\int (d\sec{(fx+e)})^{\frac{3}{2}}\,(b\tan{(fx+e)}+a)^2\,dx$$

```
[In] integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)^2, x)
```

Giac [F]

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2\,dx=\int (d\sec{(fx+e)})^{\frac{3}{2}}\,(b\tan{(fx+e)}+a)^2\,dx$$

```
[In] integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2\,dx = \int \left(\frac{d}{\cos\left(e+f\,x\right)}\right)^{3/2}\left(a+b\tan(e+f\,x)\right)^2\,dx$$

```
[In] int((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x))^2,x)
[Out] int((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x))^2, x)
```

3.588 $\int \sqrt{d \sec(e+fx)}(a+b\tan(e+fx))^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 103

$$\int \sqrt{d \sec(e+fx)}(a+b\tan(e+fx))^2\,dx = \frac{10ab\sqrt{d\sec(e+fx)}}{3f} + \frac{2(3a^2-2b^2)\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{3f} + \frac{2b\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}{3f}$$

```
[Out] 10/3*a*b*(d*sec(f*x+e))^(1/2)/f+2/3*(3*a^2-2*b^2)*(cos(1/2*f*x+1/2*e)^2)^(1
/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/
2)*(d*sec(f*x+e))^(1/2)/f+2/3*b*(d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))/f
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3589, 3567, 3856, 2720}

$$\int \sqrt{d \sec(e+fx)}(a+b\tan(e+fx))^2\,dx = \frac{2(3a^2-2b^2)\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{3f} + \frac{10ab\sqrt{d\sec(e+fx)}}{3f} + \frac{2b\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}{3f}$$

```
[In] Int[Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (10*a*b*Sqrt[d*Sec[e + f*x]])/(3*f) + (2*(3*a^2 - 2*b^2)*Sqrt[Cos[e + f*x]]
*EllipticF[(e + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]])/(3*f) + (2*b*Sqrt[d*Sec[e
+ f*x]]*(a + b*Tan[e + f*x]))/(3*f)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2b\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}{3f} \\
&\quad+\frac{2}{3}\int\sqrt{d\sec(e+fx)}\left(\frac{3a^2}{2}-b^2+\frac{5}{2}ab\tan(e+fx)\right)\,dx \\
&= \frac{10ab\sqrt{d\sec(e+fx)}}{3f}+\frac{2b\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}{3f} \\
&\quad+\frac{1}{3}\left(3a^2-2b^2\right)\int\sqrt{d\sec(e+fx)}\,dx \\
&= \frac{10ab\sqrt{d\sec(e+fx)}}{3f}+\frac{2b\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}{3f} \\
&\quad+\frac{1}{3}\left(\left(3a^2-2b^2\right)\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\right)\int\frac{1}{\sqrt{\cos(e+fx)}}\,dx
\end{aligned}
$$

$$= \frac{10ab\sqrt{d\sec(e+fx)}}{3f} + \frac{2(3a^2-2b^2)\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{3f} + \frac{2b\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}{3f}$$

Mathematica [A] (verified)

Time = 1.65 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx$$
$$= \frac{2\sec^2(e+fx)\sqrt{d\sec(e+fx)}\Big((3a^2-2b^2)\cos^{\frac{5}{2}}(e+fx)\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)+b\cos(e+fx)(6a\cos(e}{3f}$$

```
[In] Integrate[Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (2*Sec[e + f*x]^2*Sqrt[d*Sec[e + f*x]]*((3*a^2 - 2*b^2)*Cos[e + f*x]^(5/2)*
EllipticF[(e + f*x)/2, 2] + b*Cos[e + f*x]*(6*a*Cos[e + f*x] + b*Sin[e + f*
x])))/(3*f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 15.54 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.32

method	result
parts	$-\frac{2ia^2(\cos(fx+e)+1)F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{d\sec(fx+e)}\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}}{f}+\frac{2b^2\sqrt{d\sec(fx+e)}\left(2i\cos(fx\right.}{}$
default	$-\frac{2\sqrt{d\sec(fx+e)}\left(3i\cos(fx+e)F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{1}{\cos(fx+e)+1}}a^2-2i\cos(fx+e)F(i(\csc(fx+e)-\cot(\right.}{}$

```
[In] int((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*I*a^2/f*(cos(f*x+e)+1)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(d*sec(f*x
+e))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+2/3*b
^2/f*(d*sec(f*x+e))^(1/2)*(2*I*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(c
os(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)+2*I*(cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(
f*x+e)+1))^(1/2)+tan(f*x+e))+4*a*b*(d*sec(f*x+e))^(1/2)/f
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.29

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx$$
$$= \frac{\sqrt{2}(-3i\,a^2+2i\,b^2)\sqrt{d}\cos{(fx+e)}\,\text{weierstrassPInverse}(-4,0,\cos{(fx+e)}+i\,\sin{(fx+e)})+\sqrt{2}(3i\,a^2-}{}$$

```
[In] integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-3*I*a^2 + 2*I*b^2)*sqrt(d)*cos(f*x + e)*weierstrassPInverse(
-4, 0, cos(f*x + e) + I*sin(f*x + e)) + sqrt(2)*(3*I*a^2 - 2*I*b^2)*sqrt(d)
*cos(f*x + e)*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) + 2
*(6*a*b*cos(f*x + e) + b^2*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(f*cos(f*x +
 e))
```

Sympy [F]

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx = \int \sqrt{d\sec{(e+fx)}}(a+b\tan{(e+fx)})^2\,dx$$

```
[In] integrate((d*sec(f*x+e))**(1/2)*(a+b*tan(f*x+e))**2,x)
```

```
[Out] Integral(sqrt(d*sec(e + f*x))*(a + b*tan(e + f*x))**2, x)
```

Maxima [F]

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx = \int \sqrt{d\sec{(fx+e)}}(b\tan{(fx+e)}+a)^2\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)^2, x)
```

Giac [F]

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx = \int \sqrt{d\sec{(fx+e)}}(b\tan{(fx+e)}+a)^2\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2,x, algorithm="giac")
[Out] integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx = \int \sqrt{\frac{d}{\cos{(e+f\,x)}}}\,(a+b\tan(e+f\,x))^2\,dx$$

```
[In] int((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x))^2,x)
[Out] int((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x))^2, x)
```

3.589 $\int \frac{(a+b\tan(e+fx))^2}{\sqrt{d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt{d\sec(e+fx)}}\,dx=-\frac{6ab}{f\sqrt{d\sec(e+fx)}}+\frac{2(a^2-2b^2)\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2b(a+b\tan(e+fx))}{f\sqrt{d\sec(e+fx)}}$$

```
[Out] -6*a*b/f/(d*sec(f*x+e))^(1/2)+2*(a^2-2*b^2)*(cos(1/2*f*x+1/2*e)^2)^(1/2)/co
s(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(
d*sec(f*x+e))^(1/2)+2*b*(a+b*tan(f*x+e))/f/(d*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3589, 3567, 3856, 2719}

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt{d\sec(e+fx)}}\,dx=\frac{2(a^2-2b^2)\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}-\frac{6ab}{f\sqrt{d\sec(e+fx)}}+\frac{2b(a+b\tan(e+fx))}{f\sqrt{d\sec(e+fx)}}$$

```
[In] Int[(a + b*Tan[e + f*x])^2/Sqrt[d*Sec[e + f*x]],x]
```

```
[Out] (-6*a*b)/(f*Sqrt[d*Sec[e + f*x]]) + (2*(a^2 - 2*b^2)*EllipticE[(e + f*x)/2,
 2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e + f*x]]) + (2*b*(a + b*Tan[e + f*x]
))/(f*Sqrt[d*Sec[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2b(a+b\tan(e+fx))}{f\sqrt{d\sec(e+fx)}}+2\int\frac{\frac{a^2}{2}-b^2+\frac{3}{2}ab\tan(e+fx)}{\sqrt{d\sec(e+fx)}}\,dx\\
&= -\frac{6ab}{f\sqrt{d\sec(e+fx)}}+\frac{2b(a+b\tan(e+fx))}{f\sqrt{d\sec(e+fx)}}+\left(a^2-2b^2\right)\int\frac{1}{\sqrt{d\sec(e+fx)}}\,dx\\
&= -\frac{6ab}{f\sqrt{d\sec(e+fx)}}+\frac{2b(a+b\tan(e+fx))}{f\sqrt{d\sec(e+fx)}}+\frac{\left(a^2-2b^2\right)\int\sqrt{\cos(e+fx)}\,dx}{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}\\
&= -\frac{6ab}{f\sqrt{d\sec(e+fx)}}+\frac{2\left(a^2-2b^2\right)E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2b(a+b\tan(e+fx))}{f\sqrt{d\sec(e+fx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.65 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.67

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt{d\sec(e+fx)}}\,dx=\frac{\frac{2(a^2-2b^2)E\left(\frac{1}{2}(e+fx)|2\right)}{\sqrt{\cos(e+fx)}}+2b(-2a+b\tan(e+fx))}{f\sqrt{d\sec(e+fx)}}$$

```
[In] Integrate[(a + b*Tan[e + f*x])^2/Sqrt[d*Sec[e + f*x]],x]
```

```
[Out] ((2*(a^2 - 2*b^2)*EllipticE[(e + f*x)/2, 2])/Sqrt[Cos[e + f*x]] + 2*b*(-2*a
 + b*Tan[e + f*x]))/(f*Sqrt[d*Sec[e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 16.29 (sec) , antiderivative size = 805, normalized size of antiderivative = 8.47

method	result
parts	$\frac{2a^2\left(i\cos(fx+e)E(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}-i\cos(fx+e)F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)}}\right.}{}$
default	Expression too large to display

```
[In] int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a^2/f/(cos(f*x+e)+1)/(d*sec(f*x+e))^(1/2)*(I*EllipticE(I*(csc(f*x+e)-cot(
f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f
*x+e)-I*cos(f*x+e)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1)
)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+2*I*(1/(cos(f*x+e)+1))^(1/2)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)-2*I*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(
cos(f*x+e)+1))^(1/2)+I*sec(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)-I*sec(f*x+e)*(cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos
(f*x+e)+1))^(1/2)+sin(f*x+e))-2*b^2/f/(cos(f*x+e)+1)/(d*sec(f*x+e))^(1/2)*(
2*I*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)-2*I*EllipticF(I*(csc(f*x+e)-cot(f*x+e
)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)
+4*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I
*(csc(f*x+e)-cot(f*x+e)),I)-4*I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF
(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)+2*I*sec(f*x+e)*(1/(c
os(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(csc(f*x+
e)-cot(f*x+e)),I)-2*I*sec(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elliptic
F(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)+sin(f*x+e)-tan(f*x+
e))-4*a*b/f/(d*sec(f*x+e))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.28

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt{d\sec(e+fx)}}\,dx$$
$$= \frac{\sqrt{2}(i\,a^2-2i\,b^2)\sqrt{d}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos{(fx+e)}+i\,\sin{(fx+e)}))+\sqrt{2}}{}$$

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*(I*a^2 - 2*I*b^2)*sqrt(d)*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + sqrt(2)*(-I*a^2 + 2*I*b^2)*sqrt
(d)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(
f*x + e))) - 2*(2*a*b*cos(f*x + e) - b^2*sin(f*x + e))*sqrt(d/cos(f*x + e))
)/(d*f)
```

Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{(a+b\tan{(e+fx)})^2}{\sqrt{d\sec{(e+fx)}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))**2/(d*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**2/sqrt(d*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{(b\tan{(fx+e)}+a)^2}{\sqrt{d\sec{(fx+e)}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^2/sqrt(d*sec(f*x + e)), x)
```

Giac [F]

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{(b\tan(fx+e)+a)^2}{\sqrt{d\sec(fx+e)}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)^2/sqrt(d*sec(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{(a+b\tan(e+f\,x))^2}{\sqrt{\frac{d}{\cos(e+f\,x)}}}\,dx$$

```
[In] int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(1/2),x)
[Out] int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(1/2), x)
```

3.590 $\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 139

$$\begin{aligned}\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{3/2}}\,dx &= \frac{2ab}{3f(d\sec(e+fx))^{3/2}} \\ &+ \frac{2(a^2+2b^2)\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{3d^2f} \\ &+ \frac{2(a^2+2b^2)\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}} - \frac{2b(a+b\tan(e+fx))}{f(d\sec(e+fx))^{3/2}}\end{aligned}$$

```
[Out] 2/3*a*b/f/(d*sec(f*x+e))^(3/2)+2/3*(a^2+2*b^2)*sin(f*x+e)/d/f/(d*sec(f*x+e)
)^(1/2)+2/3*(a^2+2*b^2)*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Ell
ipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(d*sec(f*x+e))^(1/2)/d^
2/f-2*b*(a+b*tan(f*x+e))/f/(d*sec(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3589, 3567, 3854, 3856, 2720}

$$\begin{aligned}\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{3/2}}\,dx &= \frac{2(a^2+2b^2)\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{3d^2f} \\ &+ \frac{2(a^2+2b^2)\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}} + \frac{2ab}{3f(d\sec(e+fx))^{3/2}} - \frac{2b(a+b\tan(e+fx))}{f(d\sec(e+fx))^{3/2}}\end{aligned}$$

```
[In] Int[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*a*b)/(3*f*(d*Sec[e + f*x])^(3/2)) + (2*(a^2 + 2*b^2)*Sqrt[Cos[e + f*x]]*
EllipticF[(e + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]])/(3*d^2*f) + (2*(a^2 + 2*b^2
)*Sin[e + f*x])/(3*d*f*Sqrt[d*Sec[e + f*x]]) - (2*b*(a + b*Tan[e + f*x]))/(
f*(d*Sec[e + f*x])^(3/2))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2b(a+b\tan(e+fx))}{f(d\sec(e+fx))^{3/2}}-2\int \frac{-\frac{a^2}{2}-b^2+\frac{1}{2}ab\tan(e+fx)}{(d\sec(e+fx))^{3/2}}\,dx \\
&= \frac{2ab}{3f(d\sec(e+fx))^{3/2}}-\frac{2b(a+b\tan(e+fx))}{f(d\sec(e+fx))^{3/2}}-\left(-a^2-2b^2\right)\int \frac{1}{(d\sec(e+fx))^{3/2}}\,dx
\end{aligned}$$

$$= \frac{2ab}{3f(d\sec(e+fx))^{3/2}} + \frac{2(a^2+2b^2)\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}} - \frac{2b(a+b\tan(e+fx))}{f(d\sec(e+fx))^{3/2}} + \frac{(a^2+2b^2)\int\sqrt{d\sec(e+fx)}\,dx}{3d^2}$$

$$= \frac{2ab}{3f(d\sec(e+fx))^{3/2}} + \frac{2(a^2+2b^2)\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}} - \frac{2b(a+b\tan(e+fx))}{f(d\sec(e+fx))^{3/2}} + \frac{\left((a^2+2b^2)\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\right)\int\frac{1}{\sqrt{\cos(e+fx)}}\,dx}{3d^2}$$

$$= \frac{2ab}{3f(d\sec(e+fx))^{3/2}} + \frac{2(a^2+2b^2)\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{3d^2f} + \frac{2(a^2+2b^2)\sin(e+fx)}{3df\sqrt{d\sec(e+fx)}} - \frac{2b(a+b\tan(e+fx))}{f(d\sec(e+fx))^{3/2}}$$

Mathematica [A] (verified)

Time = 2.43 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.73

$$\int\frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{3/2}}\,dx = \frac{\sec^2(e+fx)\left(-2ab-2ab\cos(2(e+fx))+2(a^2+2b^2)\sqrt{\cos(e+fx)}\,\text{Elliptic}\right.}{3f(d\sec(e+fx))^{3/2}}$$

```
[In] Integrate[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (Sec[e + f*x]^2*(-2*a*b - 2*a*b*Cos[2*(e + f*x)] + 2*(a^2 + 2*b^2)*Sqrt[Cos
[e + f*x]]*EllipticF[(e + f*x)/2, 2] + a^2*Sin[2*(e + f*x)] - b^2*Sin[2*(e
+ f*x)]))/(3*f*(d*Sec[e + f*x])^(3/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 15.54 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.12

method	result
default	$\frac{2iF(i(\cot(fx+e)-\csc(fx+e)),i)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{1}{\cos(fx+e)+1}}\,a^2}{3} + \frac{4iF(i(\cot(fx+e)-\csc(fx+e)),i)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{1}{\cos(fx+e)+1}}\,b^2}{3} + \frac{2i\sec(f}{}$
parts	$-\frac{2a^2\left(i\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}+i\sec(fx+e)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{}{c}}\right.}{3f\sqrt{d\sec(fx+e)}\,d}$

```
[In] int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/d/f/(d*sec(f*x+e))^(1/2)*(I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*a^2+2*I*(1/(cos(f*x+
e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(
f*x+e)),I)*b^2+I*sec(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*a^2+2*I*sec(f*x+e)*(1/(c
os(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+
e)-csc(f*x+e)),I)*b^2-2*cos(f*x+e)*a*b+a^2*sin(f*x+e)-sin(f*x+e)*b^2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.94

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{2}(-i\,a^2-2i\,b^2)\sqrt{d}\text{weierstrassPInverse}(-4,0,\cos{(fx+e)}+i\,\sin{(fx+e)})+$$

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-I*a^2 - 2*I*b^2)*sqrt(d)*weierstrassPInverse(-4, 0, cos(f*x
+ e) + I*sin(f*x + e)) + sqrt(2)*(I*a^2 + 2*I*b^2)*sqrt(d)*weierstrassPInve
rse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) - 2*(2*a*b*cos(f*x + e)^2 - (a^2
- b^2)*cos(f*x + e)*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(d^2*f)
```

Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(a+b\tan{(e+fx)})^2}{(d\sec{(e+fx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))**2/(d*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**2/(d*sec(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(b\tan{(fx+e)}+a)^2}{(d\sec{(fx+e)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(b\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(a+b\tan(e+f\,x))^2}{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(3/2),x)
[Out] int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(3/2), x)
```

3.591 $\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 145

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/2}}\,dx=-\frac{2ab}{15f(d\sec(e+fx))^{5/2}}+\frac{2(3a^2+2b^2)\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5d^2f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(3a^2+2b^2)\sin(e+fx)}{15df(d\sec(e+fx))^{3/2}}-\frac{2b(a+b\tan(e+fx))}{3f(d\sec(e+fx))^{5/2}}$$

```
[Out] -2/15*a*b/f/(d*sec(f*x+e))^(5/2)+2/15*(3*a^2+2*b^2)*sin(f*x+e)/d/f/(d*sec(f
*x+e))^(3/2)+2/5*(3*a^2+2*b^2)*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2
*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/d^2/f/cos(f*x+e)^(1/2)/(d*sec(f*x
+e))^(1/2)-2/3*b*(a+b*tan(f*x+e))/f/(d*sec(f*x+e))^(5/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3589, 3567, 3854, 3856, 2719}

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/2}}\,dx=\frac{2(3a^2+2b^2)\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5d^2f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}+\frac{2(3a^2+2b^2)\sin(e+fx)}{15df(d\sec(e+fx))^{3/2}}-\frac{2ab}{15f(d\sec(e+fx))^{5/2}}-\frac{2b(a+b\tan(e+fx))}{3f(d\sec(e+fx))^{5/2}}$$

```
[In] Int[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-2*a*b)/(15*f*(d*Sec[e + f*x])^(5/2)) + (2*(3*a^2 + 2*b^2)*EllipticE[(e +
f*x)/2, 2])/(5*d^2*f*Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e + f*x]]) + (2*(3*a^2 +
```

```
2*b^2)*Sin[e + f*x])/(15*d*f*(d*Sec[e + f*x])^(3/2)) - (2*b*(a + b*Tan[e +
f*x]))/(3*f*(d*Sec[e + f*x])^(5/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2b(a+b\tan(e+fx))}{3f(d\sec(e+fx))^{5/2}}-\frac{2}{3}\int\frac{-\frac{3a^2}{2}-b^2-\frac{1}{2}ab\tan(e+fx)}{(d\sec(e+fx))^{5/2}}\,dx\\
&= -\frac{2ab}{15f(d\sec(e+fx))^{5/2}}-\frac{2b(a+b\tan(e+fx))}{3f(d\sec(e+fx))^{5/2}}-\frac{1}{3}\left(-3a^2-2b^2\right)\int\frac{1}{(d\sec(e+fx))^{5/2}}\,dx\\
&= -\frac{2ab}{15f(d\sec(e+fx))^{5/2}}+\frac{2(3a^2+2b^2)\sin(e+fx)}{15df(d\sec(e+fx))^{3/2}}\\
&\quad-\frac{2b(a+b\tan(e+fx))}{3f(d\sec(e+fx))^{5/2}}+\frac{(3a^2+2b^2)\int\frac{1}{\sqrt{d\sec(e+fx)}}\,dx}{5d^2}
\end{aligned}$$

$$= -\frac{2ab}{15f(d\sec(e+fx))^{5/2}} + \frac{2(3a^2+2b^2)\sin(e+fx)}{15df(d\sec(e+fx))^{3/2}} - \frac{2b(a+b\tan(e+fx))}{3f(d\sec(e+fx))^{5/2}} + \frac{(3a^2+2b^2)\int\sqrt{\cos(e+fx)}\,dx}{5d^2\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}}$$

$$= -\frac{2ab}{15f(d\sec(e+fx))^{5/2}} + \frac{2(3a^2+2b^2)\,E\left(\frac{1}{2}(e+fx)\middle|\,2\right)}{5d^2f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}} + \frac{2(3a^2+2b^2)\sin(e+fx)}{15df(d\sec(e+fx))^{3/2}} - \frac{2b(a+b\tan(e+fx))}{3f(d\sec(e+fx))^{5/2}}$$

Mathematica [A] (verified)

Time = 2.83 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.63

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/2}}\,dx = \frac{(6a^2+4b^2)\,E\left(\frac{1}{2}(e+fx)\middle|\,2\right)+2\cos^{\frac{3}{2}}(e+fx)\left(-2ab\cos(e+fx)+(a^2-b^2)\sin\right.}{5f\cos^{\frac{5}{2}}(e+fx)(d\sec(e+fx))^{5/2}}$$

```
[In] Integrate[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((6*a^2 + 4*b^2)*EllipticE[(e + f*x)/2, 2] + 2*Cos[e + f*x]^(3/2)*(-2*a*b*C
os[e + f*x] + (a^2 - b^2)*Sin[e + f*x]))/(5*f*Cos[e + f*x]^(5/2)*(d*Sec[e +
 f*x])^(5/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 23.41 (sec) , antiderivative size = 863, normalized size of antiderivative = 5.95

method	result	size
parts	Expression too large to display	863
default	Expression too large to display	890

```
[In] int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*a^2/f/(cos(f*x+e)+1)/(d*sec(f*x+e))^(1/2)/d^2*(3*I*cos(f*x+e)*EllipticE
(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)-3*I*cos(f*x+e)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos
(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+6*I*(1/(cos(f*x+e)+1))^
(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e))
,I)-6*I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e
)),I)*(1/(cos(f*x+e)+1))^(1/2)+3*I*sec(f*x+e)*EllipticE(I*(csc(f*x+e)-cot(f
*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-3*I*se
c(f*x+e)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(c
```

```
os(f*x+e)/(cos(f*x+e)+1))^(1/2)+sin(f*x+e)*cos(f*x+e)^2+sin(f*x+e)*cos(f*x+
e)+3*sin(f*x+e))-2/5*b^2/f/(cos(f*x+e)+1)/(d*sec(f*x+e))^(1/2)/d^2*(-2*I*co
s(f*x+e)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)+2*I*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*
(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)-4*I*E
llipticE(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)+4*I*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*
x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+sin(f*x+e)*cos(f*x+e)^2-2*
I*sec(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ell
ipticE(I*(csc(f*x+e)-cot(f*x+e)),I)+2*I*sec(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)+s
in(f*x+e)*cos(f*x+e)-2*sin(f*x+e))-4/5*a*b/f/(d*sec(f*x+e))^(5/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{2}(3i\,a^2+2i\,b^2)\sqrt{d}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos{(fx}$$

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/5*(sqrt(2)*(3*I*a^2 + 2*I*b^2)*sqrt(d)*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + sqrt(2)*(-3*I*a^2 - 2*I*b
^2)*sqrt(d)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e)
- I*sin(f*x + e))) - 2*(2*a*b*cos(f*x + e)^3 - (a^2 - b^2)*cos(f*x + e)^2*s
in(f*x + e))*sqrt(d/cos(f*x + e)))/(d^3*f)
```

Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{(a+b\tan{(e+fx)})^2}{(d\sec{(e+fx)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))**2/(d*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**2/(d*sec(e + f*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{(b\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(5/2), x)
```

Giac [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{(b\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{(a+b\tan(e+f\,x))^2}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

```
[In] int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(5/2),x)
[Out] int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(5/2), x)
```

3.592 $\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 184

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{7/2}}\,dx = -\frac{6ab}{35f(d\sec(e+fx))^{7/2}} + \frac{2(5a^2+2b^2)\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{21d^4f} + \frac{2(5a^2+2b^2)\sin(e+fx)}{35df(d\sec(e+fx))^{5/2}} + \frac{2(5a^2+2b^2)\sin(e+fx)}{21d^3f\sqrt{d\sec(e+fx)}} - \frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}}$$

```
[Out] -6/35*a*b/f/(d*sec(f*x+e))^(7/2)+2/35*(5*a^2+2*b^2)*sin(f*x+e)/d/f/(d*sec(f
*x+e))^(5/2)+2/21*(5*a^2+2*b^2)*sin(f*x+e)/d^3/f/(d*sec(f*x+e))^(1/2)+2/21*
(5*a^2+2*b^2)*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin
(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(d*sec(f*x+e))^(1/2)/d^4/f-2/5*b*
(a+b*tan(f*x+e))/f/(d*sec(f*x+e))^(7/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3589, 3567, 3854, 3856, 2720}

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{7/2}}\,dx = \frac{2(5a^2+2b^2)\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{21d^4f} + \frac{2(5a^2+2b^2)\sin(e+fx)}{21d^3f\sqrt{d\sec(e+fx)}} + \frac{2(5a^2+2b^2)\sin(e+fx)}{35df(d\sec(e+fx))^{5/2}} - \frac{6ab}{35f(d\sec(e+fx))^{7/2}} - \frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}}$$

```
[In] Int[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(7/2),x]
```

```
[Out] (-6*a*b)/(35*f*(d*Sec[e + f*x])^(7/2)) + (2*(5*a^2 + 2*b^2)*Sqrt[Cos[e + f*
x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[d*Sec[e + f*x]])/(21*d^4*f) + (2*(5*a^2
+ 2*b^2)*Sin[e + f*x])/(35*d*f*(d*Sec[e + f*x])^(5/2)) + (2*(5*a^2 + 2*b^2)
*Sin[e + f*x])/(21*d^3*f*Sqrt[d*Sec[e + f*x]]) - (2*b*(a + b*Tan[e + f*x]))
/(5*f*(d*Sec[e + f*x])^(7/2))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\text{integral} = -\frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}} - \frac{2}{5}\int \frac{-\frac{5a^2}{2} - b^2 - \frac{3}{2}ab\tan(e+fx)}{(d\sec(e+fx))^{7/2}}\,dx$$

$$= -\frac{6ab}{35f(d\sec(e+fx))^{7/2}} - \frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}} - \frac{1}{5}\left(-5a^2-2b^2\right)\int \frac{1}{(d\sec(e+fx))^{7/2}}\,dx$$

$$= -\frac{6ab}{35f(d\sec(e+fx))^{7/2}} + \frac{2(5a^2+2b^2)\sin(e+fx)}{35df(d\sec(e+fx))^{5/2}} - \frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}} + \frac{(5a^2+2b^2)\int\frac{1}{(d\sec(e+fx))^{3/2}}dx}{7d^2}$$

$$= -\frac{6ab}{35f(d\sec(e+fx))^{7/2}} + \frac{2(5a^2+2b^2)\sin(e+fx)}{35df(d\sec(e+fx))^{5/2}} + \frac{2(5a^2+2b^2)\sin(e+fx)}{21d^3f\sqrt{d\sec(e+fx)}} - \frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}} + \frac{(5a^2+2b^2)\int\sqrt{d\sec(e+fx)}dx}{21d^4}$$

$$= -\frac{6ab}{35f(d\sec(e+fx))^{7/2}} + \frac{2(5a^2+2b^2)\sin(e+fx)}{35df(d\sec(e+fx))^{5/2}} + \frac{2(5a^2+2b^2)\sin(e+fx)}{21d^3f\sqrt{d\sec(e+fx)}} - \frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}} + \frac{\left((5a^2+2b^2)\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\right)\int\frac{1}{\sqrt{\cos(e+fx)}}dx}{21d^4}$$

$$= -\frac{6ab}{35f(d\sec(e+fx))^{7/2}} + \frac{2(5a^2+2b^2)\sqrt{\cos(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\sqrt{d\sec(e+fx)}}{21d^4f} + \frac{2(5a^2+2b^2)\sin(e+fx)}{35df(d\sec(e+fx))^{5/2}} + \frac{2(5a^2+2b^2)\sin(e+fx)}{21d^3f\sqrt{d\sec(e+fx)}} - \frac{2b(a+b\tan(e+fx))}{5f(d\sec(e+fx))^{7/2}}$$

Mathematica [A] (verified)

Time = 4.66 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.69

$$\int\frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{7/2}}dx = \frac{-18ab\cos(e+fx) - 6ab\cos(3(e+fx)) + \frac{4(5a^2+2b^2)\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)}{\sqrt{\cos(e+fx)}} + 23a^2\,\text{si}}{42d^3f\sqrt{d\sec(e}}$$

```
[In] Integrate[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(7/2),x]
```

```
[Out] (-18*a*b*Cos[e + f*x] - 6*a*b*Cos[3*(e + f*x)] + (4*(5*a^2 + 2*b^2)*Ellipti
cF[(e + f*x)/2, 2])/Sqrt[Cos[e + f*x]] + 23*a^2*Sin[e + f*x] + 5*b^2*Sin[e
+ f*x] + 3*a^2*Sin[3*(e + f*x)] - 3*b^2*Sin[3*(e + f*x)])/(42*d^3*f*Sqrt[d*
Sec[e + f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 21.61 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.83

method	result
default	$\frac{10iF(i(\cot(fx+e)-\csc(fx+e)),i)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{1}{\cos(fx+e)+1}}\,a^2}{21}+\frac{4iF(i(\cot(fx+e)-\csc(fx+e)),i)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{1}{\cos(fx+e)+1}}\,b^2}{21}-\frac{4ab(\cos^{3}$
parts	$-\frac{2a^2\left(5i\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}+5i\sec(fx+e)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\cdots}\right.}{21f\sqrt{d\sec(fx+e)}\,d^3}$

```
[In] int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21/d^3/f/(d*sec(f*x+e))^(1/2)*(5*I*(1/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(
cot(f*x+e)-csc(f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a^2+2*I*(1/(cos
(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)
-csc(f*x+e)),I)*b^2-6*a*b*cos(f*x+e)^3+3*a^2*cos(f*x+e)^2*sin(f*x+e)-3*b^2*
cos(f*x+e)^2*sin(f*x+e)+5*I*sec(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*EllipticF(I
*(cot(f*x+e)-csc(f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a^2+2*I*sec(f
*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(
I*(cot(f*x+e)-csc(f*x+e)),I)*b^2+5*a^2*sin(f*x+e)+2*sin(f*x+e)*b^2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{7/2}}\,dx = \frac{\sqrt{2}(-5i\,a^2-2i\,b^2)\sqrt{d}\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\,\sin(fx+e))}{}$$

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/21*(sqrt(2)*(-5*I*a^2 - 2*I*b^2)*sqrt(d)*weierstrassPInverse(-4, 0, cos(f
*x + e) + I*sin(f*x + e)) + sqrt(2)*(5*I*a^2 + 2*I*b^2)*sqrt(d)*weierstrass
PInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) - 2*(6*a*b*cos(f*x + e)^4 -
(3*(a^2 - b^2)*cos(f*x + e)^3 + (5*a^2 + 2*b^2)*cos(f*x + e))*sin(f*x + e))
*sqrt(d/cos(f*x + e)))/(d^4*f)
```

Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))**2/(d*sec(f*x+e))**(7/2),x)
[Out] Integral((a + b*tan(e + f*x))**2/(d*sec(e + f*x))**(7/2), x)
```

Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{(b\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(7/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(7/2), x)
```

Giac [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{(b\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(7/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{(a+b\tan(e+f\,x))^2}{\left(\frac{d}{\cos(e+f\,x)}\right)^{7/2}}\,dx$$

```
[In] int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(7/2),x)
[Out] int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(7/2), x)
```

3.593 $\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{9/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 184

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{9/2}}\,dx = -\frac{10ab}{63f(d\sec(e+fx))^{9/2}} + \frac{2(7a^2+2b^2)\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{15d^4f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}} + \frac{2(7a^2+2b^2)\sin(e+fx)}{63df(d\sec(e+fx))^{7/2}} + \frac{2(7a^2+2b^2)\sin(e+fx)}{45d^3f(d\sec(e+fx))^{3/2}} - \frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}}$$

```
[Out] -10/63*a*b/f/(d*sec(f*x+e))^(9/2)+2/63*(7*a^2+2*b^2)*sin(f*x+e)/d/f/(d*sec(
f*x+e))^(7/2)+2/45*(7*a^2+2*b^2)*sin(f*x+e)/d^3/f/(d*sec(f*x+e))^(3/2)+2/15
*(7*a^2+2*b^2)*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(si
n(1/2*f*x+1/2*e),2^(1/2))/d^4/f/cos(f*x+e)^(1/2)/(d*sec(f*x+e))^(1/2)-2/7*b
*(a+b*tan(f*x+e))/f/(d*sec(f*x+e))^(9/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3589, 3567, 3854, 3856, 2719}

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{9/2}}\,dx = \frac{2(7a^2+2b^2)\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{15d^4f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}} + \frac{2(7a^2+2b^2)\sin(e+fx)}{45d^3f(d\sec(e+fx))^{3/2}} + \frac{2(7a^2+2b^2)\sin(e+fx)}{63df(d\sec(e+fx))^{7/2}} - \frac{10ab}{63f(d\sec(e+fx))^{9/2}} - \frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}}$$

```
[In] Int[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(9/2),x]
```

```
[Out] (-10*a*b)/(63*f*(d*Sec[e + f*x])^(9/2)) + (2*(7*a^2 + 2*b^2)*EllipticE[(e +
 f*x)/2, 2])/(15*d^4*f*Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e + f*x]]) + (2*(7*a^2
 + 2*b^2)*Sin[e + f*x])/(63*d*f*(d*Sec[e + f*x])^(7/2)) + (2*(7*a^2 + 2*b^2
)*Sin[e + f*x])/(45*d^3*f*(d*Sec[e + f*x])^(3/2)) - (2*b*(a + b*Tan[e + f*x
]))/(7*f*(d*Sec[e + f*x])^(9/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\text{integral} = -\frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}} - \frac{2}{7}\int \frac{-\frac{7a^2}{2} - b^2 - \frac{5}{2}ab\tan(e+fx)}{(d\sec(e+fx))^{9/2}}\,dx$$

$$= -\frac{10ab}{63f(d\sec(e+fx))^{9/2}} - \frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}} - \frac{1}{7}\left(-7a^2-2b^2\right)\int \frac{1}{(d\sec(e+fx))^{9/2}}\,dx$$

$$
\begin{aligned}
&= -\frac{10ab}{63f(d\sec(e+fx))^{9/2}} + \frac{2(7a^2+2b^2)\sin(e+fx)}{63df(d\sec(e+fx))^{7/2}} \\
&\quad - \frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}} + \frac{(7a^2+2b^2)\int \frac{1}{(d\sec(e+fx))^{5/2}}\,dx}{9d^2} \\
&= -\frac{10ab}{63f(d\sec(e+fx))^{9/2}} + \frac{2(7a^2+2b^2)\sin(e+fx)}{63df(d\sec(e+fx))^{7/2}} + \frac{2(7a^2+2b^2)\sin(e+fx)}{45d^3f(d\sec(e+fx))^{3/2}} \\
&\quad - \frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}} + \frac{(7a^2+2b^2)\int \frac{1}{\sqrt{d\sec(e+fx)}}\,dx}{15d^4} \\
&= -\frac{10ab}{63f(d\sec(e+fx))^{9/2}} + \frac{2(7a^2+2b^2)\sin(e+fx)}{63df(d\sec(e+fx))^{7/2}} + \frac{2(7a^2+2b^2)\sin(e+fx)}{45d^3f(d\sec(e+fx))^{3/2}} \\
&\quad - \frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}} + \frac{(7a^2+2b^2)\int \sqrt{\cos(e+fx)}\,dx}{15d^4\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}} \\
&= -\frac{10ab}{63f(d\sec(e+fx))^{9/2}} + \frac{2(7a^2+2b^2)\,E\left(\frac{1}{2}(e+fx)\middle|\,2\right)}{15d^4f\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}} \\
&\quad + \frac{2(7a^2+2b^2)\sin(e+fx)}{63df(d\sec(e+fx))^{7/2}} + \frac{2(7a^2+2b^2)\sin(e+fx)}{45d^3f(d\sec(e+fx))^{3/2}} - \frac{2b(a+b\tan(e+fx))}{7f(d\sec(e+fx))^{9/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 6.01 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.68

$$
\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{9/2}}\,dx = \frac{\frac{48(7a^2+2b^2)E\left(\frac{1}{2}(e+fx)\middle|2\right)}{\sqrt{\cos(e+fx)}} + 4\cos(e+fx)\left(-30ab\cos(e+fx) - 10ab\cos(3(e+fx)\right.}{360d^4f\sqrt{d\sec(e+fx)}}
$$

```
[In] Integrate[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(9/2),x]
```

```
[Out] ((48*(7*a^2 + 2*b^2)*EllipticE[(e + f*x)/2, 2])/Sqrt[Cos[e + f*x]] + 4*Cos[
e + f*x]*(-30*a*b*Cos[e + f*x] - 10*a*b*Cos[3*(e + f*x)] + 2*(19*a^2 - b^2
+ 5*(a^2 - b^2)*Cos[2*(e + f*x)])*Sin[e + f*x]))/(360*d^4*f*Sqrt[d*Sec[e +
f*x]])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 33.43 (sec) , antiderivative size = 931, normalized size of antiderivative = 5.06

method	result	size
parts	Expression too large to display	931
default	Expression too large to display	968

```
[In] int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/45*a^2/f/(cos(f*x+e)+1)/(d*sec(f*x+e))^(1/2)/d^4*(21*I*cos(f*x+e)*Ellipti
cE(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)-21*I*cos(f*x+e)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(
cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+5*cos(f*x+e)^4*sin(f
*x+e)+42*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellip
ticE(I*(csc(f*x+e)-cot(f*x+e)),I)-42*I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*El
lipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)+5*cos(f*x+e)^
3*sin(f*x+e)+21*I*sec(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+
e)+1))^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)-21*I*sec(f*x+e)*(cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f
*x+e)+1))^(1/2)+7*sin(f*x+e)*cos(f*x+e)^2+7*sin(f*x+e)*cos(f*x+e)+21*sin(f*
x+e))+2/45*b^2/f/(cos(f*x+e)+1)/(d*sec(f*x+e))^(1/2)/d^4*(6*I*cos(f*x+e)*El
lipticE(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(
cos(f*x+e)+1))^(1/2)-6*I*cos(f*x+e)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*
(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-5*cos(f*x+e)^4*s
in(f*x+e)+12*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*E
llipticE(I*(csc(f*x+e)-cot(f*x+e)),I)-12*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)-5*cos(f*x
+e)^3*sin(f*x+e)+6*I*sec(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)-6*I*sec(f*x+e)*(cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos
(f*x+e)+1))^(1/2)+2*sin(f*x+e)*cos(f*x+e)^2+2*sin(f*x+e)*cos(f*x+e)+6*sin(f
*x+e))-4/9*a*b/f/(d*sec(f*x+e))^(9/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.89

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{9/2}}\,dx = -\frac{3\sqrt{2}(-7i\,a^2-2i\,b^2)\sqrt{d}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\,\sin(fx+e)))}{}$$

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(9/2),x, algorithm="fricas")
```

```
[Out] -1/45*(3*sqrt(2)*(-7*I*a^2 - 2*I*b^2)*sqrt(d)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*sqrt(2)*(7*I*a^2 +
 2*I*b^2)*sqrt(d)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x
 + e) - I*sin(f*x + e))) + 2*(10*a*b*cos(f*x + e)^5 - (5*(a^2 - b^2)*cos(f*
x + e)^4 + (7*a^2 + 2*b^2)*cos(f*x + e)^2)*sin(f*x + e))*sqrt(d/cos(f*x + e
)))/(d^5*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*tan(f*x+e))**2/(d*sec(f*x+e))**(9/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{9/2}}\,dx = \int \frac{(b\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{9}{2}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(9/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(9/2), x)
```

Giac [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{9/2}}\,dx = \int \frac{(b\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{9}{2}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(9/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{9/2}}\,dx = \int \frac{(a+b\tan(e+f\,x))^2}{\left(\frac{d}{\cos(e+f\,x)}\right)^{9/2}}\,dx$$

```
[In] int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(9/2),x)
[Out] int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(9/2), x)
```

3.594 $\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3\,dx$

Optimal result

Integrand size = 25, antiderivative size = 198

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3\,dx = \frac{2a(7a^2-6b^2)\,d^2\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{21f\sqrt[4]{\sec^2(e+fx)}} + \frac{2a(7a^2-6b^2)\,d^2\sqrt{d\sec(e+fx)}\tan(e+fx)}{21f} + \frac{2bd^2\sec^2(e+fx)\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}{9f} + \frac{2bd^2\sec^2(e+fx)\sqrt{d\sec(e+fx)}(14(11a^2-2b^2)+65ab\tan(e+fx))}{315f}$$

```
[Out] 2/21*a*(7*a^2-6*b^2)*d^2*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arct
an(tan(f*x+e)))*EllipticF(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(d*sec(f*x+e
))^(1/2)/f/(sec(f*x+e)^2)^(1/4)+2/21*a*(7*a^2-6*b^2)*d^2*(d*sec(f*x+e))^(1/
2)*tan(f*x+e)/f+2/9*b*d^2*sec(f*x+e)^2*(d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e)
)^2/f+2/315*b*d^2*sec(f*x+e)^2*(d*sec(f*x+e))^(1/2)*(154*a^2-28*b^2+65*a*b*
tan(f*x+e))/f
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3593, 757, 794, 201, 237}

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3\,dx=\frac{2ad^2(7a^2-6b^2)\sqrt{d\sec(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)}{21f\sqrt[4]{\sec^2(e+fx)}}+\frac{2bd^2\sec^2(e+fx)\sqrt{d\sec(e+fx)}(14(11a^2-2b^2)+65ab\tan(e+fx))}{315f}+\frac{2ad^2(7a^2-6b^2)\tan(e+fx)\sqrt{d\sec(e+fx)}}{21f}+\frac{2bd^2\sec^2(e+fx)\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}{9f}$$

```
[In] Int[(d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (2*a*(7*a^2 - 6*b^2)*d^2*EllipticF[ArcTan[Tan[e + f*x]]/2, 2]*Sqrt[d*Sec[e
+ f*x]])/(21*f*(Sec[e + f*x]^2)^(1/4)) + (2*a*(7*a^2 - 6*b^2)*d^2*Sqrt[d*Se
c[e + f*x]]*Tan[e + f*x])/(21*f) + (2*b*d^2*Sec[e + f*x]^2*Sqrt[d*Sec[e + f
*x]]*(a + b*Tan[e + f*x])^2)/(9*f) + (2*b*d^2*Sec[e + f*x]^2*Sqrt[d*Sec[e +
 f*x]]*(14*(11*a^2 - 2*b^2) + 65*a*b*Tan[e + f*x]))/(315*f)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
```

```
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(d^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int(a+x)^3\sqrt[4]{1+\frac{x^2}{b^2}}\,dx,x,b\tan(e+fx)\right)}{bf\sqrt[4]{\sec^2(e+fx)}}\\
&= \frac{2bd^2\sec^2(e+fx)\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}{9f}\\
&\quad+\frac{\left(2bd^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int(a+x)\left(\frac{1}{2}\left(-4+\frac{9a^2}{b^2}\right)+\frac{13ax}{2b^2}\right)\sqrt[4]{1+\frac{x^2}{b^2}}\,dx,x,b\tan(e+fx)\right)}{9f\sqrt[4]{\sec^2(e+fx)}}\\
&= \frac{2bd^2\sec^2(e+fx)\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}{9f}\\
&\quad+\frac{2bd^2\sec^2(e+fx)\sqrt{d\sec(e+fx)}(14(11a^2-2b^2)+65ab\tan(e+fx))}{315f}\\
&\quad-\frac{\left(a\left(6-\frac{7a^2}{b^2}\right)bd^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int\sqrt[4]{1+\frac{x^2}{b^2}}\,dx,x,b\tan(e+fx)\right)}{7f\sqrt[4]{\sec^2(e+fx)}}
\end{aligned}$$

$$= \frac{2a(7a^2-6b^2)\,d^2\sqrt{d\sec(e+fx)}\tan(e+fx)}{21f} + \frac{2bd^2\sec^2(e+fx)\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}{9f} + \frac{2bd^2\sec^2(e+fx)\sqrt{d\sec(e+fx)}(14(11a^2-2b^2)+65ab\tan(e+fx))}{315f} - \frac{\left(a\left(6-\frac{7a^2}{b^2}\right)bd^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx,x,b\tan(e+fx)\right)}{21f\sqrt[4]{\sec^2(e+fx)}}$$

$$= \frac{2a(7a^2-6b^2)\,d^2\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{21f\sqrt[4]{\sec^2(e+fx)}} + \frac{2a(7a^2-6b^2)\,d^2\sqrt{d\sec(e+fx)}\tan(e+fx)}{21f} + \frac{2bd^2\sec^2(e+fx)\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}{9f} + \frac{2bd^2\sec^2(e+fx)\sqrt{d\sec(e+fx)}(14(11a^2-2b^2)+65ab\tan(e+fx))}{315f}$$

Mathematica [A] (verified)

Time = 4.21 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.79

$$\int(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3\,dx =$$

$$-\frac{2d(d\sec(e+fx))^{3/2}\left(63b(-3a^2+b^2)\cos^2(e+fx)-15a(7a^2-6b^2)\cos^{\frac{9}{2}}(e+fx)\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)\right.}{315f(a\cos(e+fx)+b}$$

```
[In] Integrate[(d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (-2*d*(d*Sec[e + f*x])^(3/2)*(63*b*(-3*a^2 + b^2)*Cos[e + f*x]^2 - 15*a*(7*
a^2 - 6*b^2)*Cos[e + f*x]^(9/2)*EllipticF[(e + f*x)/2, 2] - 15*a*(7*a^2 - 6
*b^2)*Cos[e + f*x]^3*Sin[e + f*x] - (5*b^2*(14*b + 27*a*Sin[2*(e + f*x)]))/
2)*(a + b*Tan[e + f*x])^3)/(315*f*(a*Cos[e + f*x] + b*Sin[e + f*x])^3)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 619.10 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.76

method	result
default	$-\frac{2d^2\sqrt{d\sec(fx+e)}\left(105i\cos(fx+e)\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)-\cot(fx+e)),i)a^3-90i\cos(fx+e)\sqrt{\frac{1}{\cos(fx+e)+1}}\right.}{}$
parts	$-\frac{2a^3\sqrt{d\sec(fx+e)}\,d^2\left(i\cos(fx+e)F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}\,\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}+i\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)-\right.}{3f}$

```
[In] int((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/315*d^2/f*(d*sec(f*x+e))^(1/2)*(105*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2
)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*
a^3-90*I*cos(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*a*b^2+105*I*(1/(cos(f*x+e)+1))^(
1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)*a^3-90*I*(1/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a*b^2-105*tan(f*x+e)*a^3+90*tan(f*x+e)*a
*b^2-189*sec(f*x+e)^2*a^2*b-135*tan(f*x+e)*sec(f*x+e)^2*a*b^2+63*b^3*sec(f*
x+e)^2-35*sec(f*x+e)^4*b^3)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.02

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3\,dx = \frac{-15i\sqrt{2}(7a^3-6ab^2)d^{\frac{5}{2}}\cos(fx+e)^4\,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin}{}$$

```
[In] integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/315*(-15*I*sqrt(2)*(7*a^3 - 6*a*b^2)*d^(5/2)*cos(f*x + e)^4*weierstrassPI
nverse(-4, 0, cos(f*x + e) + I*sin(f*x + e)) + 15*I*sqrt(2)*(7*a^3 - 6*a*b^
2)*d^(5/2)*cos(f*x + e)^4*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f
*x + e)) + 2*(35*b^3*d^2 + 63*(3*a^2*b - b^3)*d^2*cos(f*x + e)^2 + 15*(9*a*
b^2*d^2*cos(f*x + e) + (7*a^3 - 6*a*b^2)*d^2*cos(f*x + e)^3)*sin(f*x + e))*
sqrt(d/cos(f*x + e)))/(f*cos(f*x + e)^4)
```

Sympy [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3\,dx=\text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))**(5/2)*(a+b*tan(f*x+e))**3,x)
[Out] Timed out
```

Maxima [F]

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3\,dx=\int (d\sec(fx+e))^{\frac{5}{2}}(b\tan(fx+e)+a)^3\,dx$$

```
[In] integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^3,x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a)^3, x)
```

Giac [F]

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3\,dx=\int (d\sec(fx+e))^{\frac{5}{2}}(b\tan(fx+e)+a)^3\,dx$$

```
[In] integrate((d*sec(f*x+e))^(5/2)*(a+b*tan(f*x+e))^3,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3\,dx=\int \left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}(a+b\tan(e+f\,x))^3\,dx$$

```
[In] int((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x))^3,x)
[Out] int((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x))^3, x)
```

3.595 $\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3\,dx$

Optimal result

Integrand size = 25, antiderivative size = 176

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3\,dx = -\frac{2a(5a^2-6b^2)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{5f\sec^2(e+fx)^{3/4}} + \frac{2a(5a^2-6b^2)\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{5f} + \frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}{7f} + \frac{2b(d\sec(e+fx))^{3/2}\left(10(9a^2-2b^2)+33ab\tan(e+fx)\right)}{105f}$$

```
[Out] -2/5*a*(5*a^2-6*b^2)*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(t
an(f*x+e)))*EllipticE(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(d*sec(f*x+e))^(
3/2)/f/(sec(f*x+e)^2)^(3/4)+2/5*a*(5*a^2-6*b^2)*cos(f*x+e)*(d*sec(f*x+e))^(
3/2)*sin(f*x+e)/f+2/7*b*(d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^2/f+2/105*b*(
d*sec(f*x+e))^(3/2)*(90*a^2-20*b^2+33*a*b*tan(f*x+e))/f
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used

= {3593, 757, 794, 233, 202}

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3\,dx = -\frac{2a(5a^2-6b^2)\,(d\sec(e+fx))^{3/2}E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|\,2\right)}{5f\sec^2(e+fx)^{3/4}} + \frac{2b(d\sec(e+fx))^{3/2}\,(10(9a^2-2b^2)+33ab\tan(e+fx))}{105f} + \frac{2a(5a^2-6b^2)\sin(e+fx)\cos(e+fx)(d\sec(e+fx))^{3/2}}{5f} + \frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}{7f}$$

```
[In] Int[(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (-2*a*(5*a^2 - 6*b^2)*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(d*Sec[e + f*x])
^(3/2))/(5*f*(Sec[e + f*x]^2)^(3/4)) + (2*a*(5*a^2 - 6*b^2)*Cos[e + f*x]*(d
*Sec[e + f*x])^(3/2)*Sin[e + f*x])/(5*f) + (2*b*(d*Sec[e + f*x])^(3/2)*(a +
 b*Tan[e + f*x])^2)/(7*f) + (2*b*(d*Sec[e + f*x])^(3/2)*(10*(9*a^2 - 2*b^2)
 + 33*a*b*Tan[e + f*x]))/(105*f)
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
```

```
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] && !IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(d\sec(e+fx))^{3/2}\text{Subst}\left(\int \frac{(a+x)^3}{\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{3/4}} \\
&= \frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}{7f} \\
&\quad + \frac{\left(2b(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{(a+x)\left(\frac{1}{2}\left(-4+\frac{7a^2}{b^2}\right)+\frac{11ax}{2b^2}\right)}{\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{7f\sec^2(e+fx)^{3/4}} \\
&= \frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}{7f} \\
&\quad + \frac{2b(d\sec(e+fx))^{3/2}\left(10(9a^2-2b^2)+33ab\tan(e+fx)\right)}{105f} \\
&\quad - \frac{\left(a\left(6-\frac{5a^2}{b^2}\right)b(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{5f\sec^2(e+fx)^{3/4}}
\end{aligned}
$$

$$= \frac{2a(5a^2-6b^2)\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{5f} + \frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}{7f} + \frac{2b(d\sec(e+fx))^{3/2}\left(10(9a^2-2b^2)+33ab\tan(e+fx)\right)}{105f} + \frac{\left(a\left(6-\frac{5a^2}{b^2}\right)b(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int\frac{1}{\left(1+\frac{x^2}{b^2}\right)^{5/4}}dx,x,b\tan(e+fx)\right)}{5f\sec^2(e+fx)^{3/4}}$$

$$= -\frac{2a(5a^2-6b^2)E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{5f\sec^2(e+fx)^{3/4}} + \frac{2a(5a^2-6b^2)\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{5f} + \frac{2b(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}{7f} + \frac{2b(d\sec(e+fx))^{3/2}\left(10(9a^2-2b^2)+33ab\tan(e+fx)\right)}{105f}$$

Mathematica [A] (verified)

Time = 3.86 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.88

$$\int(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3\,dx =$$

$$-\frac{d\sqrt{d\sec(e+fx)}\left(70b(-3a^2+b^2)\cos^2(e+fx)+42a(5a^2-6b^2)\cos^{\frac{7}{2}}(e+fx)E\left(\frac{1}{2}(e+fx)\middle|2\right)-42a(5a^2\right.}{105f(a\cos(e+fx)+b\sin(e}$$

```
[In] Integrate[(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^3,x]
```

```
[Out] -1/105*(d*Sqrt[d*Sec[e + f*x]]*(70*b*(-3*a^2 + b^2)*Cos[e + f*x]^2 + 42*a*(
5*a^2 - 6*b^2)*Cos[e + f*x]^(7/2)*EllipticE[(e + f*x)/2, 2] - 42*a*(5*a^2 -
 6*b^2)*Cos[e + f*x]^3*Sin[e + f*x] - 3*b^2*(10*b + 21*a*Sin[2*(e + f*x)]))
*(a + b*Tan[e + f*x])^3)/(f*(a*Cos[e + f*x] + b*Sin[e + f*x])^3)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 22.07 (sec) , antiderivative size = 872, normalized size of antiderivative = 4.95

method	result	size
parts	Expression too large to display	872
default	Expression too large to display	899

```
[In] int((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
[Out] -2*a^3/f*(I*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2-I*EllipticF(I*(csc(f*x+e)-c
ot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*co
s(f*x+e)^2+2*I*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)-2*I*EllipticF(I*(csc(f*x+e
)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)
*cos(f*x+e)+I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*El
lipticE(I*(csc(f*x+e)-cot(f*x+e)),I)-I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*El
lipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)-sin(f*x+e))*(
d*sec(f*x+e))^(1/2)*d/(cos(f*x+e)+1)+2*b^3/f/d^2*(1/7*(d*sec(f*x+e))^(7/2)-
1/3*d^2*(d*sec(f*x+e))^(3/2))+2*a^2*b/f*(d*sec(f*x+e))^(3/2)+6/5*a*b^2/f*(d
*sec(f*x+e))^(1/2)*d/(cos(f*x+e)+1)*(2*I*EllipticE(I*(csc(f*x+e)-cot(f*x+e)
),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^
2-2*I*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2+4*I*cos(f*x+e)*EllipticE(I*(csc(f
*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)-4*I*cos(f*x+e)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1
))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+2*I*(1/(cos(f*x+e)+1))^(1/2)*(co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)-2*I*(
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/
(cos(f*x+e)+1))^(1/2)-2*sin(f*x+e)+tan(f*x+e)+sec(f*x+e)*tan(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.14

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3\,dx = \frac{-21i\sqrt{2}(5a^3-6ab^2)d^{\frac{3}{2}}\cos(fx+e)^3\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,}{}$$

```
[In] integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

[Out] 1/105*(-21*I*sqrt(2)*(5*a^3 - 6*a*b^2)*d^(3/2)*cos(f*x + e)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 21*I*sqrt(2)*(5*a^3 - 6*a*b^2)*d^(3/2)*cos(f*x + e)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) + 2*(15*b^3*d + 35*(3*a^2*b - b^3)*d*cos(f*x + e)^2 + 21*(3*a*b^2*d*cos(f*x + e) + (5*a^3 - 6*a*b^2)*d*cos(f*x + e)^3)*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(f*cos(f*x + e)^3)

Sympy [F]

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3\,dx = \int (d\sec(e+fx))^{\frac{3}{2}}\,(a+b\tan(e+fx))^3\,dx$$

[In] integrate((d*sec(f*x+e))**(3/2)*(a+b*tan(f*x+e))**3,x)

[Out] Integral((d*sec(e + f*x))**(3/2)*(a + b*tan(e + f*x))**3, x)

Maxima [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3\,dx = \text{Timed out}$$

[In] integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^3,x, algorithm="maxima")

[Out] Timed out

Giac [F]

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3\,dx = \int (d\sec(fx+e))^{\frac{3}{2}}\,(b\tan(fx+e)+a)^3\,dx$$

[In] integrate((d*sec(f*x+e))^(3/2)*(a+b*tan(f*x+e))^3,x, algorithm="giac")

[Out] integrate((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)^3, x)

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3\,dx = \int \left(\frac{d}{\cos(e+f\,x)}\right)^{3/2} (a+b\tan(e+f\,x))^3\,dx$$

```
[In] int((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x))^3,x)
[Out] int((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x))^3, x)
```

3.596 $\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3\,dx$

Optimal result

Integrand size = 25, antiderivative size = 129

$$\begin{aligned}\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3\,dx &= \frac{2a(a^2-2b^2)\,\mathrm{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{f\sqrt[4]{\sec^2(e+fx)}}\\ &\quad+\frac{2b\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}{5f}\\ &\quad+\frac{2b\sqrt{d\sec(e+fx)}(2(7a^2-2b^2)+3ab\tan(e+fx))}{5f}\end{aligned}$$

```
[Out] 2*a*(a^2-2*b^2)*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(f*
x+e)))*EllipticF(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(d*sec(f*x+e))^(1/2)/
f/(sec(f*x+e)^2)^(1/4)+2/5*b*(d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^2/f+2/5*
b*(d*sec(f*x+e))^(1/2)*(14*a^2-4*b^2+3*a*b*tan(f*x+e))/f
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used

$= \{3593, 757, 794, 237\}$

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3\,dx = \frac{2a(a^2-2b^2)\sqrt{d\sec(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)}{f\sqrt[4]{\sec^2(e+fx)}} + \frac{2b\sqrt{d\sec(e+fx)}(2(7a^2-2b^2)+3ab\tan(e+fx))}{5f} + \frac{2b\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}{5f}$$

```
[In] Int[Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (2*a*(a^2 - 2*b^2)*EllipticF[ArcTan[Tan[e + f*x]]/2, 2]*Sqrt[d*Sec[e + f*x]
])/(f*(Sec[e + f*x]^2)^(1/4)) + (2*b*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*
x])^2)/(5*f) + (2*b*Sqrt[d*Sec[e + f*x]]*(2*(7*a^2 - 2*b^2) + 3*a*b*Tan[e +
 f*x]))/(5*f)
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
```

```
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{d\sec(e+fx)}\text{Subst}\left(\int \frac{(a+x)^3}{\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$= \frac{2b\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}{5f} + \frac{\left(2b\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{(a+x)\left(\frac{1}{2}\left(-4+\frac{5a^2}{b^2}\right)+\frac{9ax}{2b^2}\right)}{\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{5f\sqrt[4]{\sec^2(e+fx)}}$$

$$= \frac{2b\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}{5f} + \frac{2b\sqrt{d\sec(e+fx)}(2(7a^2-2b^2)+3ab\tan(e+fx))}{5f} - \frac{\left(a\left(2-\frac{a^2}{b^2}\right)b\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{f\sqrt[4]{\sec^2(e+fx)}}$$

$$= \frac{2a(a^2-2b^2)\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{f\sqrt[4]{\sec^2(e+fx)}} + \frac{2b\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}{5f} + \frac{2b\sqrt{d\sec(e+fx)}(2(7a^2-2b^2)+3ab\tan(e+fx))}{5f}$$

Mathematica [A] (verified)

Time = 3.91 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.02

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3\,dx =$$

$$-\frac{2\sqrt{d\sec(e+fx)}\left(5b(-3a^2+b^2)\cos^3(e+fx) - 5a(a^2-2b^2)\cos^{\frac{7}{2}}(e+fx)\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right) - \frac{1}{2}b\right.}{5f(a\cos(e+fx)+b\sin(e+fx))^3}$$

```
[In] Integrate[Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (-2*Sqrt[d*Sec[e + f*x]]*(5*b*(-3*a^2 + b^2)*Cos[e + f*x]^3 - 5*a*(a^2 - 2*
b^2)*Cos[e + f*x]^(7/2)*EllipticF[(e + f*x)/2, 2] - (b^2*Cos[e + f*x]*(2*b
+ 5*a*Sin[2*(e + f*x)]))/2)*(a + b*Tan[e + f*x])^3)/(5*f*(a*Cos[e + f*x] +
b*Sin[e + f*x])^3)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 21.26 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.31

method	result
default	$\frac{2\sqrt{d\sec(fx+e)}\left(-5i\cos(fx+e)\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}F(i(\csc(fx+e)-\cot(fx+e)),i)a^3+10i\cos(fx+e)\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{c}{\cos}}\right.}{}$
parts	$-\frac{2ia^3(\cos(fx+e)+1)F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{d\sec(fx+e)}\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}}{f}-\frac{b^3\sqrt{d\sec(fx+e)}\left(20\cos(fx-\right.}{}$

```
[In] int((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/f*(d*sec(f*x+e))^(1/2)*(-5*I*cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)*(1/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*a^3+10*I
*cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*Elli
pticF(I*(csc(f*x+e)-cot(f*x+e)),I)*a*b^2-5*I*(cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)*(1/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*a^3+10*
I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(c
sc(f*x+e)-cot(f*x+e)),I)*a*b^2+5*tan(f*x+e)*a*b^2+15*a^2*b-5*b^3+b^3*sec(f*
x+e)^2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.27

$$\int\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3\,dx =$$

$$-\frac{5\sqrt{2}(i\,a^3-2i\,ab^2)\sqrt{d}\cos(fx+e)^2\,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\,\sin(fx+e))+5\sqrt{2}(-}{}$$

```
[In] integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/5*(5*sqrt(2)*(I*a^3 - 2*I*a*b^2)*sqrt(d)*cos(f*x + e)^2*weierstrassPInve
rse(-4, 0, cos(f*x + e) + I*sin(f*x + e)) + 5*sqrt(2)*(-I*a^3 + 2*I*a*b^2)*
```

```
sqrt(d)*cos(f*x + e)^2*weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x
+ e)) - 2*(5*a*b^2*cos(f*x + e)*sin(f*x + e) + b^3 + 5*(3*a^2*b - b^3)*cos(
f*x + e)^2)*sqrt(d/cos(f*x + e)))/(f*cos(f*x + e)^2)
```

Sympy [F]

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3\,dx = \int \sqrt{d\sec{(e+fx)}}(a+b\tan{(e+fx)})^3\,dx$$

```
[In] integrate((d*sec(f*x+e))**(1/2)*(a+b*tan(f*x+e))**3,x)
[Out] Integral(sqrt(d*sec(e + f*x))*(a + b*tan(e + f*x))**3, x)
```

Maxima [F]

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3\,dx = \int \sqrt{d\sec{(fx+e)}}(b\tan{(fx+e)}+a)^3\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^3,x, algorithm="maxima")
[Out] integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)^3, x)
```

Giac [F]

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3\,dx = \int \sqrt{d\sec{(fx+e)}}(b\tan{(fx+e)}+a)^3\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/2)*(a+b*tan(f*x+e))^3,x, algorithm="giac")
[Out] integrate(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3\,dx = \int \sqrt{\frac{d}{\cos{(e+f\,x)}}}\,(a+b\tan(e+f\,x))^3\,dx$$

```
[In] int((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x))^3,x)
[Out] int((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x))^3, x)
```

3.597 $\int \frac{(a+b\tan(e+fx))^3}{\sqrt{d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 178

$$\int \frac{(a+b\tan(e+fx))^3}{\sqrt{d\sec(e+fx)}}\,dx = \frac{2a(a^2-6b^2)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{f\sqrt{d\sec(e+fx)}} - \frac{2a(a^2-6b^2)\tan(e+fx)}{f\sqrt{d\sec(e+fx)}} - \frac{2(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{f\sqrt{d\sec(e+fx)}} - \frac{2b\sec^2(e+fx)\,(2(3a^2-2b^2)+3ab\tan(e+fx))}{3f\sqrt{d\sec(e+fx)}}$$

```
[Out] 2*a*(a^2-6*b^2)*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(f*
x+e)))*EllipticE(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(sec(f*x+e)^2)^(1/4)/
f/(d*sec(f*x+e))^(1/2)-2*a*(a^2-6*b^2)*tan(f*x+e)/f/(d*sec(f*x+e))^(1/2)-2*
(b-a*tan(f*x+e))*(a+b*tan(f*x+e))^2/f/(d*sec(f*x+e))^(1/2)-2/3*b*sec(f*x+e)
^2*(6*a^2-4*b^2+3*a*b*tan(f*x+e))/f/(d*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used

$= \{3593, 753, 794, 233, 202\}$

$$\int \frac{(a+b\tan(e+fx))^3}{\sqrt{d\sec(e+fx)}}\,dx = \frac{2a(a^2-6b^2)\sqrt[4]{\sec^2(e+fx)}E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)}{f\sqrt{d\sec(e+fx)}} - \frac{2b\sec^2(e+fx)\,(2(3a^2-2b^2)+3ab\tan(e+fx))}{3f\sqrt{d\sec(e+fx)}} - \frac{2a(a^2-6b^2)\tan(e+fx)}{f\sqrt{d\sec(e+fx)}} - \frac{2(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{f\sqrt{d\sec(e+fx)}}$$

```
[In] Int[(a + b*Tan[e + f*x])^3/Sqrt[d*Sec[e + f*x]],x]
```

```
[Out] (2*a*(a^2 - 6*b^2)*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^(1
/4))/(f*Sqrt[d*Sec[e + f*x]]) - (2*a*(a^2 - 6*b^2)*Tan[e + f*x])/(f*Sqrt[d*
Sec[e + f*x]]) - (2*(b - a*Tan[e + f*x])*(a + b*Tan[e + f*x])^2)/(f*Sqrt[d*
Sec[e + f*x]]) - (2*b*Sec[e + f*x]^2*(2*(3*a^2 - 2*b^2) + 3*a*b*Tan[e + f*x
]))/(3*f*Sqrt[d*Sec[e + f*x]])
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 753

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
```

```
Q[p, -1]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt[4]{\sec^2(e+fx)}\text{Subst}\left(\int \frac{(a+x)^3}{\left(1+\frac{x^2}{b^2}\right)^{5/4}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt{d\sec(e+fx)}}$$

$$= -\frac{2(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{f\sqrt{d\sec(e+fx)}} + \frac{\left(2b\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{(a+x)\left(\frac{1}{2}\left(4-\frac{a^2}{b^2}\right)-\frac{5ax}{2b^2}\right)}{\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{f\sqrt{d\sec(e+fx)}}$$

$$= -\frac{2(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{f\sqrt{d\sec(e+fx)}} - \frac{2b\sec^2(e+fx)\left(2(3a^2-2b^2)+3ab\tan(e+fx)\right)}{3f\sqrt{d\sec(e+fx)}} + \frac{\left(a\left(6-\frac{a^2}{b^2}\right)b\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{f\sqrt{d\sec(e+fx)}}$$

$$= -\frac{2a(a^2-6b^2)\tan(e+fx)}{f\sqrt{d\sec(e+fx)}} - \frac{2(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{f\sqrt{d\sec(e+fx)}} - \frac{2b\sec^2(e+fx)\left(2(3a^2-2b^2)+3ab\tan(e+fx)\right)}{3f\sqrt{d\sec(e+fx)}} - \frac{\left(a\left(6-\frac{a^2}{b^2}\right)b\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{x^2}{b^2}\right)^{5/4}}\,dx, x, b\tan(e+fx)\right)}{f\sqrt{d\sec(e+fx)}}$$

$$= \frac{2a(a^2-6b^2)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|\,2\right)\sqrt[4]{\sec^2(e+fx)}}{f\sqrt{d\sec(e+fx)}} - \frac{2a(a^2-6b^2)\tan(e+fx)}{f\sqrt{d\sec(e+fx)}} - \frac{2(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{f\sqrt{d\sec(e+fx)}} - \frac{2b\sec^2(e+fx)\left(2(3a^2-2b^2)+3ab\tan(e+fx)\right)}{3f\sqrt{d\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 4.90 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.73

$$\int \frac{(a+b\tan(e+fx))^3}{\sqrt{d\sec(e+fx)}}\,dx = \frac{d\Big(6a(a^2-6b^2)\cos^{\frac{3}{2}}(e+fx)E\left(\frac{1}{2}(e+fx)\middle|\,2\right)+b(-9a^2+5b^2+(-9a^2+3b^2)\cos(2(e+fx))+9ab\sin(2(e}{3f(d\sec(e+fx))^{3/2}(a\cos(e+fx)+b\sin(e+fx))^3}$$

```
[In] Integrate[(a + b*Tan[e + f*x])^3/Sqrt[d*Sec[e + f*x]],x]
```

```
[Out] (d*(6*a*(a^2 - 6*b^2)*Cos[e + f*x]^(3/2)*EllipticE[(e + f*x)/2, 2] + b*(-9*
a^2 + 5*b^2 + (-9*a^2 + 3*b^2)*Cos[2*(e + f*x)] + 9*a*b*Sin[2*(e + f*x)]))*
(a + b*Tan[e + f*x])^3)/(3*f*(d*Sec[e + f*x])^(3/2)*(a*Cos[e + f*x] + b*Sin
[e + f*x])^3)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 21.30 (sec) , antiderivative size = 1114, normalized size of antiderivative = 6.26

method	result	size
parts	Expression too large to display	1114
default	Expression too large to display	2626

```
[In] int((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a^3/f/(cos(f*x+e)+1)/(d*sec(f*x+e))^(1/2)*(I*EllipticE(I*(csc(f*x+e)-cot(
f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f
*x+e)-I*cos(f*x+e)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1)
)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+2*I*(1/(cos(f*x+e)+1))^(1/2)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)-2*I*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(
cos(f*x+e)+1))^(1/2)+I*sec(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)-I*sec(f*x+e)*(cos(
```

```
f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos
(f*x+e)+1))^(1/2)+sin(f*x+e))-1/6*b^3/f/(cos(f*x+e)+1)/(d*sec(f*x+e))^(1/2)
/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*(-12*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)+3*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))-3*ln(2*(2
*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))-12*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)-4*sec(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-4*sec(f*x+e)^2*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))-6*a^2*b/(d*sec(f*x+e))^(1/2)/f-6*a*b^2/
f/(cos(f*x+e)+1)/(d*sec(f*x+e))^(1/2)*(2*I*EllipticE(I*(csc(f*x+e)-cot(f*x+
e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e
)-2*I*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)+4*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)-4*I*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(co
s(f*x+e)+1))^(1/2)+2*I*sec(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)-2*I*sec(f*x+e)*(co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(c
os(f*x+e)+1))^(1/2)+sin(f*x+e)-tan(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.95

$$\int \frac{(a+b\tan(e+fx))^3}{\sqrt{d\sec(e+fx)}}\,dx =$$

$$-\frac{3\sqrt{2}(-i\,a^3+6i\,ab^2)\sqrt{d}\cos(fx+e)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i$$

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*sqrt(2)*(-I*a^3 + 6*I*a*b^2)*sqrt(d)*cos(f*x + e)*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*sqrt(2
)*(I*a^3 - 6*I*a*b^2)*sqrt(d)*cos(f*x + e)*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) - 2*(9*a*b^2*cos(f*x + e)
*sin(f*x + e) + b^3 - 3*(3*a^2*b - b^3)*cos(f*x + e)^2)*sqrt(d/cos(f*x + e)
))/(d*f*cos(f*x + e))
```

Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^3}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{(a+b\tan(e+fx))^3}{\sqrt{d\sec(e+fx)}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))**3/(d*sec(f*x+e))**(1/2),x)
[Out] Integral((a + b*tan(e + f*x))**3/sqrt(d*sec(e + f*x)), x)
```

Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^3}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{(b\tan(fx+e)+a)^3}{\sqrt{d\sec(fx+e)}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e) + a)^3/sqrt(d*sec(f*x + e)), x)
```

Giac [F]

$$\int \frac{(a+b\tan(e+fx))^3}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{(b\tan(fx+e)+a)^3}{\sqrt{d\sec(fx+e)}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)^3/sqrt(d*sec(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^3}{\sqrt{d\sec(e+fx)}}\,dx = \int \frac{(a+b\tan(e+f\,x))^3}{\sqrt{\frac{d}{\cos(e+f\,x)}}}\,dx$$

```
[In] int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(1/2),x)
[Out] int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(1/2), x)
```

3.598 $\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 146

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{3/2}}\,dx = \frac{2a(a^2+6b^2)\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sec^2(e+fx)^{3/4}}{3f(d\sec(e+fx))^{3/2}} - \frac{2(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{3f(d\sec(e+fx))^{3/2}} - \frac{2b\sec^2(e+fx)\left(2(a^2-2b^2)+ab\tan(e+fx)\right)}{3f(d\sec(e+fx))^{3/2}}$$

```
[Out] 2/3*a*(a^2+6*b^2)*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(
f*x+e)))*EllipticF(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(sec(f*x+e)^2)^(3/4
)/f/(d*sec(f*x+e))^(3/2)-2/3*(b-a*tan(f*x+e))*(a+b*tan(f*x+e))^2/f/(d*sec(f
*x+e))^(3/2)-2/3*b*sec(f*x+e)^2*(2*a^2-4*b^2+a*b*tan(f*x+e))/f/(d*sec(f*x+e
))^(3/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3593, 753, 794, 237}

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{3/2}}\,dx = \frac{2a(a^2+6b^2)\sec^2(e+fx)^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)}{3f(d\sec(e+fx))^{3/2}} - \frac{2b\sec^2(e+fx)\left(2(a^2-2b^2)+ab\tan(e+fx)\right)}{3f(d\sec(e+fx))^{3/2}} - \frac{2(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{3f(d\sec(e+fx))^{3/2}}$$

```
[In] Int[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*a*(a^2 + 6*b^2)*EllipticF[ArcTan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^(3
/4))/(3*f*(d*Sec[e + f*x])^(3/2)) - (2*(b - a*Tan[e + f*x])*(a + b*Tan[e +
```

```
f*x])^2)/(3*f*(d*Sec[e + f*x])^(3/2)) - (2*b*Sec[e + f*x]^2*(2*(a^2 - 2*b^2
) + a*b*Tan[e + f*x]))/(3*f*(d*Sec[e + f*x])^(3/2))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 753

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sec^2(e+fx)^{3/4}\text{Subst}\left(\int \frac{(a+x)^3}{\left(1+\frac{x^2}{b^2}\right)^{7/4}}\,dx, x, b\tan(e+fx)\right)}{bf(d\sec(e+fx))^{3/2}}\\
&= -\frac{2(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{3f(d\sec(e+fx))^{3/2}}\\
&\quad+\frac{\left(2b\sec^2(e+fx)^{3/4}\right)\text{Subst}\left(\int \frac{(a+x)\left(\frac{1}{2}\left(4+\frac{a^2}{b^2}\right)-\frac{3ax}{2b^2}\right)}{\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{3f(d\sec(e+fx))^{3/2}}
\end{aligned}$$

$$= -\frac{2(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{3f(d\sec(e+fx))^{3/2}} - \frac{2b\sec^2(e+fx)\left(2(a^2-2b^2)+ab\tan(e+fx)\right)}{3f(d\sec(e+fx))^{3/2}} + \frac{\left(a\left(6+\frac{a^2}{b^2}\right)b\sec^2(e+fx)^{3/4}\right)\text{Subst}\left(\int\frac{1}{\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx,x,b\tan(e+fx)\right)}{3f(d\sec(e+fx))^{3/2}}$$

$$= \frac{2a(a^2+6b^2)\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sec^2(e+fx)^{3/4}}{3f(d\sec(e+fx))^{3/2}} - \frac{2(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{3f(d\sec(e+fx))^{3/2}} - \frac{2b\sec^2(e+fx)\left(2(a^2-2b^2)+ab\tan(e+fx)\right)}{3f(d\sec(e+fx))^{3/2}}$$

Mathematica [A] (verified)

Time = 3.80 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.80

$$\int\frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{3/2}}\,dx = \frac{\sec^2(e+fx)\left(-3a^2b+7b^3+(-3a^2b+b^3)\cos(2(e+fx))+2a(a^2+6b^2)\sqrt{\text{co}}\right.}{3f(d\sec(e}$$

```
[In] Integrate[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(3/2),x]
```

```
[Out] (Sec[e + f*x]^2*(-3*a^2*b + 7*b^3 + (-3*a^2*b + b^3)*Cos[2*(e + f*x)] + 2*a
*(a^2 + 6*b^2)*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2] + a^3*Sin[2*(e
+ f*x)] - 3*a*b^2*Sin[2*(e + f*x)]))/(3*f*(d*Sec[e + f*x])^(3/2))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 23.96 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.21

method	result
default	$-\frac{2\left(i\sqrt{\frac{1}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,a^3+6i\sqrt{\frac{1}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+}}\right.}{}$
parts	Expression too large to display

```
[In] int((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d/f/(d*sec(f*x+e))^(1/2)*(I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*a^3+6*I*(1/(cos(f*x
```

```
+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot
(f*x+e)),I)*a*b^2+I*sec(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*a^3+6*I*sec(f*x+e)*(1
/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f
*x+e)-cot(f*x+e)),I)*a*b^2+3*cos(f*x+e)*a^2*b-cos(f*x+e)*b^3-a^3*sin(f*x+e)
+3*a*b^2*sin(f*x+e)-3*sec(f*x+e)*b^3)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.01

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{3/2}}\,dx = \frac{\sqrt{2}(-i\,a^3-6i\,ab^2)\sqrt{d}\text{weierstrassPInverse}(-4,0,\cos{(fx+e)}+i\,\sin{(fx+e)})}{}$$

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-I*a^3 - 6*I*a*b^2)*sqrt(d)*weierstrassPInverse(-4, 0, cos(f*
x + e) + I*sin(f*x + e)) + sqrt(2)*(I*a^3 + 6*I*a*b^2)*sqrt(d)*weierstrassP
Inverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) + 2*(3*b^3 - (3*a^2*b - b^3)*
cos(f*x + e)^2 + (a^3 - 3*a*b^2)*cos(f*x + e)*sin(f*x + e))*sqrt(d/cos(f*x
+ e)))/(d^2*f)
```

Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(a+b\tan{(e+fx)})^3}{(d\sec{(e+fx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))**3/(d*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**3/(d*sec(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(b\tan{(fx+e)}+a)^3}{(d\sec{(fx+e)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(b\tan(fx+e)+a)^3}{(d\sec(fx+e))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{3/2}}\,dx = \int \frac{(a+b\tan(e+f\,x))^3}{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

```
[In] int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(3/2),x)
[Out] int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(3/2), x)
```

3.599 $\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 204

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{5/2}}\,dx = \frac{6a(a^2+2b^2)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{5d^2f\sqrt{d\sec(e+fx)}} - \frac{6a(a^2+2b^2)\tan(e+fx)}{5d^2f\sqrt{d\sec(e+fx)}} - \frac{2\cos^2(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{5d^2f\sqrt{d\sec(e+fx)}} - \frac{2(2b(a^2+2b^2)-a(3a^2+5b^2)\tan(e+fx))}{5d^2f\sqrt{d\sec(e+fx)}}$$

```
[Out] 6/5*a*(a^2+2*b^2)*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(
f*x+e)))*EllipticE(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(sec(f*x+e)^2)^(1/4
)/d^2/f/(d*sec(f*x+e))^(1/2)-6/5*a*(a^2+2*b^2)*tan(f*x+e)/d^2/f/(d*sec(f*x+
e))^(1/2)-2/5*cos(f*x+e)^2*(b-a*tan(f*x+e))*(a+b*tan(f*x+e))^2/d^2/f/(d*sec
(f*x+e))^(1/2)-2/5*(2*b*(a^2+2*b^2)-a*(3*a^2+5*b^2)*tan(f*x+e))/d^2/f/(d*se
c(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3593, 753, 792, 233, 202}

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{5/2}}\,dx = \frac{6a(a^2+2b^2)\sqrt[4]{\sec^2(e+fx)}E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)}{5d^2f\sqrt{d\sec(e+fx)}} - \frac{6a(a^2+2b^2)\tan(e+fx)}{5d^2f\sqrt{d\sec(e+fx)}} - \frac{2(2b(a^2+2b^2)-a(3a^2+5b^2)\tan(e+fx))}{5d^2f\sqrt{d\sec(e+fx)}} - \frac{2\cos^2(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{5d^2f\sqrt{d\sec(e+fx)}}$$

```
[In] Int[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(5/2),x]
```

```
[Out] (6*a*(a^2 + 2*b^2)*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^(1
/4))/(5*d^2*f*Sqrt[d*Sec[e + f*x]]) - (6*a*(a^2 + 2*b^2)*Tan[e + f*x])/(5*d
^2*f*Sqrt[d*Sec[e + f*x]]) - (2*Cos[e + f*x]^2*(b - a*Tan[e + f*x])*(a + b*
Tan[e + f*x])^2)/(5*d^2*f*Sqrt[d*Sec[e + f*x]]) - (2*(2*b*(a^2 + 2*b^2) - a
*(3*a^2 + 5*b^2)*Tan[e + f*x]))/(5*d^2*f*Sqrt[d*Sec[e + f*x]])
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 753

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt[4]{\sec^2(e+fx)}\text{Subst}\left(\int \frac{(a+x)^3}{\left(1+\frac{x^2}{b^2}\right)^{9/4}}\,dx, x, b\tan(e+fx)\right)}{bd^2 f\sqrt{d\sec(e+fx)}}$$

$$= -\frac{2\cos^2(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{5d^2 f\sqrt{d\sec(e+fx)}} + \frac{\left(2b\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{(a+x)\left(\frac{1}{2}\left(4+\frac{3a^2}{b^2}\right)-\frac{ax}{2b^2}\right)}{\left(1+\frac{x^2}{b^2}\right)^{5/4}}\,dx, x, b\tan(e+fx)\right)}{5d^2 f\sqrt{d\sec(e+fx)}}$$

$$= -\frac{2\cos^2(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{5d^2 f\sqrt{d\sec(e+fx)}} - \frac{2(2b(a^2+2b^2)-a(3a^2+5b^2)\tan(e+fx))}{5d^2 f\sqrt{d\sec(e+fx)}} - \frac{\left(3a\left(2+\frac{a^2}{b^2}\right)b\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{5d^2 f\sqrt{d\sec(e+fx)}}$$

$$= -\frac{6a(a^2+2b^2)\tan(e+fx)}{5d^2 f\sqrt{d\sec(e+fx)}} - \frac{2\cos^2(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{5d^2 f\sqrt{d\sec(e+fx)}} - \frac{2(2b(a^2+2b^2)-a(3a^2+5b^2)\tan(e+fx))}{5d^2 f\sqrt{d\sec(e+fx)}} + \frac{\left(3a\left(2+\frac{a^2}{b^2}\right)b\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{x^2}{b^2}\right)^{5/4}}\,dx, x, b\tan(e+fx)\right)}{5d^2 f\sqrt{d\sec(e+fx)}}$$

$$= \frac{6a(a^2+2b^2)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{5d^2 f\sqrt{d\sec(e+fx)}} - \frac{6a(a^2+2b^2)\tan(e+fx)}{5d^2 f\sqrt{d\sec(e+fx)}} - \frac{2\cos^2(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{5d^2 f\sqrt{d\sec(e+fx)}} - \frac{2(2b(a^2+2b^2)-a(3a^2+5b^2)\tan(e+fx))}{5d^2 f\sqrt{d\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 3.94 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.74

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{5/2}}\,dx = \frac{\sqrt{d\sec(e+fx)}\Big(-b(9a^2+17b^2)\cos(e+fx)-3a^2b\cos(3(e+fx))+b^3\cos(3(e+fx))$$

[In] Integrate[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(5/2),x]

[Out] (Sqrt[d*Sec[e + f*x]]*(-(b*(9*a^2 + 17*b^2)*Cos[e + f*x]) - 3*a^2*b*Cos[3*(e + f*x)] + b^3*Cos[3*(e + f*x)] + 12*a*(a^2 + 2*b^2)*Sqrt[Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2] + a^3*Sin[e + f*x] - 3*a*b^2*Sin[e + f*x] + a^3*Sin[3*(e + f*x)] - 3*a*b^2*Sin[3*(e + f*x)]))/(10*d^3*f)

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 26.00 (sec) , antiderivative size = 1289, normalized size of antiderivative = 6.32

method	result	size
parts	Expression too large to display	1289
default	Expression too large to display	1602

[In] int((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(5/2),x,method=_RETURNVERBOSE)

[Out] 2/5*a^3/f/(cos(f*x+e)+1)/(d*sec(f*x+e))^(1/2)/d^2*(3*I*cos(f*x+e)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-3*I*cos(f*x+e)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+6*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)-6*I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)+3*I*sec(f*x+e)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-3*I*sec(f*x+e)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+sin(f*x+e)*cos(f*x+e)^2+sin(f*x+e)*cos(f*x+e)+3*sin(f*x+e))-1/10*b^3/f/(d*sec(f*x+e))^(1/2)/d^2*(5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))-5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))-4*cos(f*x+e)^2+5*sec(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))-5*sec(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2

```
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))+20)-6/5
*a*b^2/f/(cos(f*x+e)+1)/(d*sec(f*x+e))^(1/2)/d^2*(-2*I*cos(f*x+e)*EllipticE
(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)+2*I*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))
^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)-4*I*EllipticE(I*(csc(f*
x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)+4*I*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)+sin(f*x+e)*cos(f*x+e)^2-2*I*sec(f*x+e)*(1/(c
os(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(csc(f*x+
e)-cot(f*x+e)),I)+2*I*sec(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elliptic
F(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)+sin(f*x+e)*cos(f*x+
e)-2*sin(f*x+e))-6/5*a^2*b/f/(d*sec(f*x+e))^(5/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.80

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{5/2}}\,dx = -\frac{3\sqrt{2}(-i\,a^3-2i\,ab^2)\sqrt{d}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\,\sin(fx+e)))+}{}$$

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/5*(3*sqrt(2)*(-I*a^3 - 2*I*a*b^2)*sqrt(d)*weierstrassZeta(-4, 0, weierst
rassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*sqrt(2)*(I*a^3 + 2*
I*a*b^2)*sqrt(d)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x
+ e) - I*sin(f*x + e))) + 2*(5*b^3*cos(f*x + e) + (3*a^2*b - b^3)*cos(f*x +
 e)^3 - (a^3 - 3*a*b^2)*cos(f*x + e)^2*sin(f*x + e))*sqrt(d/cos(f*x + e)))/
(d^3*f)
```

Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))**3/(d*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral((a + b*tan(e + f*x))**3/(d*sec(e + f*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{(b\tan(fx+e)+a)^3}{(d\sec(fx+e))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(5/2), x)
```

Giac [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{(b\tan(fx+e)+a)^3}{(d\sec(fx+e))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(5/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{5/2}}\,dx = \int \frac{(a+b\tan(e+f\,x))^3}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}}\,dx$$

```
[In] int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(5/2),x)
[Out] int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(5/2), x)
```

3.600 $\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 170

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{7/2}}\,dx = \frac{2a(5a^2+6b^2)\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sec^2(e+fx)^{3/4}}{21d^2f(d\sec(e+fx))^{3/2}} - \frac{2\cos^2(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{7d^2f(d\sec(e+fx))^{3/2}} - \frac{2(2b(3a^2+2b^2)-a(5a^2+3b^2)\tan(e+fx))}{21d^2f(d\sec(e+fx))^{3/2}}$$

```
[Out] 2/21*a*(5*a^2+6*b^2)*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(t
an(f*x+e)))*EllipticF(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(sec(f*x+e)^2)^(
3/4)/d^2/f/(d*sec(f*x+e))^(3/2)-2/7*cos(f*x+e)^2*(b-a*tan(f*x+e))*(a+b*tan(
f*x+e))^2/d^2/f/(d*sec(f*x+e))^(3/2)-2/21*(2*b*(3*a^2+2*b^2)-a*(5*a^2+3*b^2
)*tan(f*x+e))/d^2/f/(d*sec(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3593, 753, 792, 237}

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{7/2}}\,dx = \frac{2a(5a^2+6b^2)\sec^2(e+fx)^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)}{21d^2f(d\sec(e+fx))^{3/2}} - \frac{2(2b(3a^2+2b^2)-a(5a^2+3b^2)\tan(e+fx))}{21d^2f(d\sec(e+fx))^{3/2}} - \frac{2\cos^2(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{7d^2f(d\sec(e+fx))^{3/2}}$$

```
[In] Int[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(7/2),x]
```

```
[Out] (2*a*(5*a^2 + 6*b^2)*EllipticF[ArcTan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^
(3/4))/(21*d^2*f*(d*Sec[e + f*x])^(3/2)) - (2*Cos[e + f*x]^2*(b - a*Tan[e +
 f*x])*(a + b*Tan[e + f*x])^2)/(7*d^2*f*(d*Sec[e + f*x])^(3/2)) - (2*(2*b*(
3*a^2 + 2*b^2) - a*(5*a^2 + 3*b^2)*Tan[e + f*x]))/(21*d^2*f*(d*Sec[e + f*x]
)^(3/2))
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 753

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\sec^2(e+fx)^{3/4}\text{Subst}\left(\int \frac{(a+x)^3}{\left(1+\frac{x^2}{b^2}\right)^{11/4}}\,dx, x, b\tan(e+fx)\right)}{bd^2f(d\sec(e+fx))^{3/2}}$$

$$= -\frac{2\cos^2(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{7d^2f(d\sec(e+fx))^{3/2}} + \frac{(2b\sec^2(e+fx)^{3/4})\operatorname{Subst}\left(\int \frac{(a+x)\left(\frac{1}{2}\left(4+\frac{5a^2}{b^2}\right)+\frac{ax}{2b^2}\right)}{\left(1+\frac{x^2}{b^2}\right)^{7/4}}dx, x, b\tan(e+fx)\right)}{7d^2f(d\sec(e+fx))^{3/2}}$$

$$= -\frac{2\cos^2(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{7d^2f(d\sec(e+fx))^{3/2}} - \frac{2(2b(3a^2+2b^2)-a(5a^2+3b^2)\tan(e+fx))}{21d^2f(d\sec(e+fx))^{3/2}} + \frac{\left(a\left(6+\frac{5a^2}{b^2}\right)b\sec^2(e+fx)^{3/4}\right)\operatorname{Subst}\left(\int \frac{1}{\left(1+\frac{x^2}{b^2}\right)^{3/4}}dx, x, b\tan(e+fx)\right)}{21d^2f(d\sec(e+fx))^{3/2}}$$

$$= \frac{2a(5a^2+6b^2)\operatorname{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sec^2(e+fx)^{3/4}}{21d^2f(d\sec(e+fx))^{3/2}} - \frac{2\cos^2(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{7d^2f(d\sec(e+fx))^{3/2}} - \frac{2(2b(3a^2+2b^2)-a(5a^2+3b^2)\tan(e+fx))}{21d^2f(d\sec(e+fx))^{3/2}}$$

Mathematica [A] (verified)

Time = 5.49 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{7/2}}\,dx = \frac{\sqrt{\cos(e+fx)}\sqrt{d\sec(e+fx)}\left(4(5a^3+6ab^2)\operatorname{EllipticF}\left(\frac{1}{2}(e+fx),2\right)+\sqrt{\cos(e\right.}$$

```
[In] Integrate[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(7/2),x]
```

```
[Out] (Sqrt[Cos[e + f*x]]*Sqrt[d*Sec[e + f*x]]*(4*(5*a^3 + 6*a*b^2)*EllipticF[(e
+ f*x)/2, 2] + Sqrt[Cos[e + f*x]]*(-(b*(27*a^2 + 19*b^2)*Cos[e + f*x]) + (-
9*a^2*b + 3*b^3)*Cos[3*(e + f*x)] + 2*a*(13*a^2 + 3*b^2 + 3*(a^2 - 3*b^2)*C
os[2*(e + f*x)])*Sin[e + f*x])))/(42*d^4*f)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 23.65 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.15

method	result
default	$\frac{10i\sqrt{\frac{1}{\cos(fx+e)+1}}\,F(i(\cot(fx+e)-\csc(fx+e)),i)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,a^3}{21}+\frac{4i\sqrt{\frac{1}{\cos(fx+e)+1}}\,F(i(\cot(fx+e)-\csc(fx+e)),i)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,a\,b^2}{7}-\frac{6\left(\mathrm{co}\right.}{}$
parts	$-\frac{2a^3\left(5i\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}+5i\sec(fx+e)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)-\cot(fx+e)),i)\right.}{21f\sqrt{d\sec(fx+e)}\,d^3}$

```
[In] int((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21/d^3/f/(d*sec(f*x+e))^(1/2)*(5*I*(1/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(
cot(f*x+e)-csc(f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a^3+6*I*(1/(cos
(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*(cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)*a*b^2-9*cos(f*x+e)^3*a^2*b+3*cos(f*x+e)^3*b^3+3*cos(f*x+e)^
2*sin(f*x+e)*a^3-9*cos(f*x+e)^2*sin(f*x+e)*a*b^2+5*I*sec(f*x+e)*(1/(cos(f*x
+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*a^3+6*I*sec(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*
x+e)-csc(f*x+e)),I)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*a*b^2-7*cos(f*x+e)*b^
3+5*a^3*sin(f*x+e)+6*a*b^2*sin(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{7/2}}\,dx = \frac{\sqrt{2}(-5i\,a^3-6i\,ab^2)\sqrt{d}\,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\,\sin(fx+e}{}$$

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/21*(sqrt(2)*(-5*I*a^3 - 6*I*a*b^2)*sqrt(d)*weierstrassPInverse(-4, 0, cos
(f*x + e) + I*sin(f*x + e)) + sqrt(2)*(5*I*a^3 + 6*I*a*b^2)*sqrt(d)*weierst
rassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) - 2*(7*b^3*cos(f*x + e)^
2 + 3*(3*a^2*b - b^3)*cos(f*x + e)^4 - (3*(a^3 - 3*a*b^2)*cos(f*x + e)^3 +
(5*a^3 + 6*a*b^2)*cos(f*x + e))*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(d^4*f)
```

Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{(a+b\tan{(e+fx)})^3}{(d\sec{(e+fx)})^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))**3/(d*sec(f*x+e))**(7/2),x)
[Out] Integral((a + b*tan(e + f*x))**3/(d*sec(e + f*x))**(7/2), x)
```

Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{(b\tan{(fx+e)}+a)^3}{(d\sec{(fx+e)})^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(7/2),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(7/2), x)
```

Giac [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{(b\tan{(fx+e)}+a)^3}{(d\sec{(fx+e)})^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(7/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{7/2}}\,dx = \int \frac{(a+b\tan(e+f\,x))^3}{\left(\frac{d}{\cos(e+f\,x)}\right)^{7/2}}\,dx$$

```
[In] int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(7/2),x)
[Out] int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(7/2), x)
```

3.601 $\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{9/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 176

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{9/2}}\,dx = \frac{2a(7a^2+6b^2)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{15d^4f\sqrt{d\sec(e+fx)}} - \frac{2\cos^4(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{9d^4f\sqrt{d\sec(e+fx)}} - \frac{2\cos^2(e+fx)\left(2b(5a^2+2b^2)-a(7a^2+b^2)\tan(e+fx)\right)}{45d^4f\sqrt{d\sec(e+fx)}}$$

```
[Out] 2/15*a*(7*a^2+6*b^2)*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(t
an(f*x+e)))*EllipticE(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(sec(f*x+e)^2)^(
1/4)/d^4/f/(d*sec(f*x+e))^(1/2)-2/9*cos(f*x+e)^4*(b-a*tan(f*x+e))*(a+b*tan(
f*x+e))^2/d^4/f/(d*sec(f*x+e))^(1/2)-2/45*cos(f*x+e)^2*(2*b*(5*a^2+2*b^2)-a
*(7*a^2+b^2)*tan(f*x+e))/d^4/f/(d*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3593, 753, 792, 202}

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{9/2}}\,dx = \frac{2a(7a^2+6b^2)\sqrt[4]{\sec^2(e+fx)}E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)}{15d^4f\sqrt{d\sec(e+fx)}} - \frac{2\cos^2(e+fx)\left(2b(5a^2+2b^2)-a(7a^2+b^2)\tan(e+fx)\right)}{45d^4f\sqrt{d\sec(e+fx)}} - \frac{2\cos^4(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{9d^4f\sqrt{d\sec(e+fx)}}$$

```
[In] Int[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(9/2),x]
```

```
[Out] (2*a*(7*a^2 + 6*b^2)*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^
(1/4))/(15*d^4*f*Sqrt[d*Sec[e + f*x]]) - (2*Cos[e + f*x]^4*(b - a*Tan[e + f
*x])*(a + b*Tan[e + f*x])^2)/(9*d^4*f*Sqrt[d*Sec[e + f*x]]) - (2*Cos[e + f*
x]^2*(2*b*(5*a^2 + 2*b^2) - a*(7*a^2 + b^2)*Tan[e + f*x]))/(45*d^4*f*Sqrt[d
*Sec[e + f*x]])
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 753

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt[4]{\sec^2(e+fx)}\,\text{Subst}\left(\int \frac{(a+x)^3}{\left(1+\frac{x^2}{b^2}\right)^{13/4}}\,dx, x, b\tan(e+fx)\right)}{bd^4f\sqrt{d\sec(e+fx)}}$$

$$= -\frac{2\cos^4(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{9d^4f\sqrt{d\sec(e+fx)}} + \frac{\left(2b\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{(a+x)\left(\frac{1}{2}\left(4+\frac{7a^2}{b^2}\right)+\frac{3ax}{2b^2}\right)}{\left(1+\frac{x^2}{b^2}\right)^{9/4}}dx,x,b\tan(e+fx)\right)}{9d^4f\sqrt{d\sec(e+fx)}}$$

$$= -\frac{2\cos^4(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{9d^4f\sqrt{d\sec(e+fx)}} - \frac{2\cos^2(e+fx)\left(2b(5a^2+2b^2)-a(7a^2+b^2)\tan(e+fx)\right)}{45d^4f\sqrt{d\sec(e+fx)}} + \frac{\left(a\left(6+\frac{7a^2}{b^2}\right)b\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\left(1+\frac{x^2}{b^2}\right)^{5/4}}dx,x,b\tan(e+fx)\right)}{15d^4f\sqrt{d\sec(e+fx)}}$$

$$= \frac{2a(7a^2+6b^2)E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{15d^4f\sqrt{d\sec(e+fx)}} - \frac{2\cos^4(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{9d^4f\sqrt{d\sec(e+fx)}} - \frac{2\cos^2(e+fx)\left(2b(5a^2+2b^2)-a(7a^2+b^2)\tan(e+fx)\right)}{45d^4f\sqrt{d\sec(e+fx)}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 372 vs. 2(176) = 352.

Time = 9.41 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.11

$$\int\frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{9/2}}dx = \frac{\sec^{\frac{3}{2}}(e+fx)\left(\frac{2(56a^3+48ab^2)E\left(\frac{1}{2}(e+fx)\middle|2\right)}{\sqrt{\cos(e+fx)}\sqrt{\sec(e+fx)}} - \frac{2(15a^2b+7b^3)\sin^2(e+fx)}{\sqrt{1-\cos^2(e+fx)}\sqrt{\sec(e+fx)}\sqrt{\cos^2(e+fx)(-1+\text{se}}}\right.}{120f(d\sec(e+fx))^{9/2}(a\cos(e+fx)+b\sin(e+f}$$
$$+\frac{\sec^2(e+fx)\left(-\frac{1}{90}b(15a^2+4b^2)\cos(e+fx)-\frac{1}{360}b(75a^2+11b^2)\cos(3(e+fx))-\frac{1}{72}b(3a^2-b^2)\cos(5(e+\right.}{f(d\sec(e+fx))}$$

```
[In] Integrate[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(9/2),x]
```

```
[Out] (Sec[e + f*x]^(3/2)*((2*(56*a^3 + 48*a*b^2)*EllipticE[(e + f*x)/2, 2])/(Sqr
t[Cos[e + f*x]]*Sqrt[Sec[e + f*x]]) - (2*(15*a^2*b + 7*b^3)*Sin[e + f*x]^2)
/(Sqrt[1 - Cos[e + f*x]^2]*Sqrt[Sec[e + f*x]]*Sqrt[Cos[e + f*x]^2*(-1 + Sec
[e + f*x]^2)]))*(a + b*Tan[e + f*x])^3)/(120*f*(d*Sec[e + f*x])^(9/2)*(a*Co
s[e + f*x] + b*Sin[e + f*x])^3) + (Sec[e + f*x]^2*(-1/90*(b*(15*a^2 + 4*b^2
)*Cos[e + f*x]) - (b*(75*a^2 + 11*b^2)*Cos[3*(e + f*x)])/360 - (b*(3*a^2 -
b^2)*Cos[5*(e + f*x)])/72 + (a*(19*a^2 - 3*b^2)*Sin[e + f*x])/180 + (a*(43*
a^2 - 21*b^2)*Sin[3*(e + f*x)])/360 + (a*(a^2 - 3*b^2)*Sin[5*(e + f*x)])/72
)*(a + b*Tan[e + f*x])^3)/(f*(d*Sec[e + f*x])^(9/2)*(a*Cos[e + f*x] + b*Sin
[e + f*x])^3)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 32.98 (sec) , antiderivative size = 976, normalized size of antiderivative = 5.55

method	result	size
parts	Expression too large to display	976
default	Expression too large to display	1035

```
[In] int((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/45*a^3/f/(cos(f*x+e)+1)/(d*sec(f*x+e))^(1/2)/d^4*(21*I*cos(f*x+e)*Ellipti
cE(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)-21*I*cos(f*x+e)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(
cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+5*cos(f*x+e)^4*sin(f
*x+e)+42*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellip
ticE(I*(csc(f*x+e)-cot(f*x+e)),I)-42*I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*El
lipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)+5*cos(f*x+e)^
3*sin(f*x+e)+21*I*sec(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+
e)+1))^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)-21*I*sec(f*x+e)*(cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f
*x+e)+1))^(1/2)+7*sin(f*x+e)*cos(f*x+e)^2+7*sin(f*x+e)*cos(f*x+e)+21*sin(f*
x+e))+2/45*b^3/f/(d*sec(f*x+e))^(1/2)/d^4*(5*cos(f*x+e)^4-9*cos(f*x+e)^2)+2
/15*a*b^2/f/(cos(f*x+e)+1)/(d*sec(f*x+e))^(1/2)/d^4*(6*I*cos(f*x+e)*Ellipti
cE(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)-6*I*cos(f*x+e)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(c
os(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-5*cos(f*x+e)^4*sin(f*
x+e)+12*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellipt
icE(I*(csc(f*x+e)-cot(f*x+e)),I)-12*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)-5*cos(f*x+e)^3
*sin(f*x+e)+6*I*sec(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*EllipticE(I*(csc(f*x+e)-cot(f*x+e)),I)-6*I*sec(f*x+e)*(cos(f*x+e
)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*(1/(cos(f*x+
e)+1))^(1/2)+2*sin(f*x+e)*cos(f*x+e)^2+2*sin(f*x+e)*cos(f*x+e)+6*sin(f*x+e)
)-2/3*a^2*b/f/(d*sec(f*x+e))^(9/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{9/2}}\,dx = -\frac{3\sqrt{2}(-7i\,a^3-6i\,ab^2)\sqrt{d}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))}{}$$

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(9/2),x, algorithm="fricas")
```

```
[Out] -1/45*(3*sqrt(2)*(-7*I*a^3 - 6*I*a*b^2)*sqrt(d)*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*sqrt(2)*(7*I*a^3
 + 6*I*a*b^2)*sqrt(d)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
(f*x + e) - I*sin(f*x + e))) + 2*(9*b^3*cos(f*x + e)^3 + 5*(3*a^2*b - b^3)*
cos(f*x + e)^5 - (5*(a^3 - 3*a*b^2)*cos(f*x + e)^4 + (7*a^3 + 6*a*b^2)*cos(
f*x + e)^2)*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(d^5*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*tan(f*x+e))**3/(d*sec(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{9/2}}\,dx = \int \frac{(b\tan(fx+e)+a)^3}{(d\sec(fx+e))^{\frac{9}{2}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(9/2), x)
```

Giac [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{9/2}}\,dx = \int \frac{\left(b\tan\left(fx+e\right)+a\right)^3}{\left(d\sec\left(fx+e\right)\right)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(9/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{9/2}}\,dx = \int \frac{\left(a+b\tan(e+f\,x)\right)^3}{\left(\frac{d}{\cos(e+f\,x)}\right)^{9/2}}\,dx$$

```
[In] int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(9/2),x)
[Out] int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(9/2), x)
```

3.602 $\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{11/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 218

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{11/2}}\,dx = \frac{10a(3a^2+2b^2)\operatorname{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sec^2(e+fx)^{3/4}}{77d^4f(d\sec(e+fx))^{3/2}} + \frac{10a(3a^2+2b^2)\tan(e+fx)}{77d^4f(d\sec(e+fx))^{3/2}} - \frac{2\cos^4(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{11d^4f(d\sec(e+fx))^{3/2}} - \frac{2\cos^2(e+fx)\left(2b(7a^2+2b^2)-a(9a^2-b^2)\tan(e+fx)\right)}{77d^4f(d\sec(e+fx))^{3/2}}$$

```
[Out] 10/77*a*(3*a^2+2*b^2)*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(
tan(f*x+e)))*EllipticF(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(sec(f*x+e)^2)^
(3/4)/d^4/f/(d*sec(f*x+e))^(3/2)+10/77*a*(3*a^2+2*b^2)*tan(f*x+e)/d^4/f/(d*
sec(f*x+e))^(3/2)-2/11*cos(f*x+e)^4*(b-a*tan(f*x+e))*(a+b*tan(f*x+e))^2/d^4
/f/(d*sec(f*x+e))^(3/2)-2/77*cos(f*x+e)^2*(2*b*(7*a^2+2*b^2)-a*(9*a^2-b^2)*
tan(f*x+e))/d^4/f/(d*sec(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3593, 753, 792, 205, 237}

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{11/2}}\,dx = \frac{10a(3a^2+2b^2)\sec^2(e+fx)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)}{77d^4f(d\sec(e+fx))^{3/2}} + \frac{10a(3a^2+2b^2)\tan(e+fx)}{77d^4f(d\sec(e+fx))^{3/2}} - \frac{2\cos^2(e+fx)\left(2b(7a^2+2b^2)-a(9a^2-b^2)\tan(e+fx)\right)}{77d^4f(d\sec(e+fx))^{3/2}} - \frac{2\cos^4(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{11d^4f(d\sec(e+fx))^{3/2}}$$

```
[In] Int[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(11/2),x]
```

```
[Out] (10*a*(3*a^2 + 2*b^2)*EllipticF[ArcTan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)
^(3/4))/(77*d^4*f*(d*Sec[e + f*x])^(3/2)) + (10*a*(3*a^2 + 2*b^2)*Tan[e + f
*x])/(77*d^4*f*(d*Sec[e + f*x])^(3/2)) - (2*Cos[e + f*x]^4*(b - a*Tan[e + f
*x])*(a + b*Tan[e + f*x])^2)/(11*d^4*f*(d*Sec[e + f*x])^(3/2)) - (2*Cos[e +
 f*x]^2*(2*b*(7*a^2 + 2*b^2) - a*(9*a^2 - b^2)*Tan[e + f*x]))/(77*d^4*f*(d*
Sec[e + f*x])^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 753

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\sec^2(e+fx)^{3/4}\text{Subst}\left(\int \frac{(a+x)^3}{\left(1+\frac{x^2}{b^2}\right)^{15/4}}\,dx, x, b\tan(e+fx)\right)}{bd^4f(d\sec(e+fx))^{3/2}}$$

$$= -\frac{2\cos^4(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{11d^4f(d\sec(e+fx))^{3/2}} + \frac{\left(2b\sec^2(e+fx)^{3/4}\right)\text{Subst}\left(\int \frac{(a+x)\left(\frac{1}{2}\left(4+\frac{9a^2}{b^2}\right)+\frac{5ax}{2b^2}\right)}{\left(1+\frac{x^2}{b^2}\right)^{11/4}}\,dx, x, b\tan(e+fx)\right)}{11d^4f(d\sec(e+fx))^{3/2}}$$

$$= -\frac{2\cos^4(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{11d^4f(d\sec(e+fx))^{3/2}} - \frac{2\cos^2(e+fx)\left(2b(7a^2+2b^2)-a(9a^2-b^2)\tan(e+fx)\right)}{77d^4f(d\sec(e+fx))^{3/2}} + \frac{\left(15a\left(2+\frac{3a^2}{b^2}\right)b\sec^2(e+fx)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{x^2}{b^2}\right)^{7/4}}\,dx, x, b\tan(e+fx)\right)}{77d^4f(d\sec(e+fx))^{3/2}}$$

$$= \frac{10a(3a^2+2b^2)\tan(e+fx)}{77d^4f(d\sec(e+fx))^{3/2}} - \frac{2\cos^4(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{11d^4f(d\sec(e+fx))^{3/2}} - \frac{2\cos^2(e+fx)\left(2b(7a^2+2b^2)-a(9a^2-b^2)\tan(e+fx)\right)}{77d^4f(d\sec(e+fx))^{3/2}} + \frac{\left(5a\left(2+\frac{3a^2}{b^2}\right)b\sec^2(e+fx)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{77d^4f(d\sec(e+fx))^{3/2}}$$

$$= \frac{10a(3a^2+2b^2)\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sec^2(e+fx)^{3/4}}{77d^4f(d\sec(e+fx))^{3/2}} + \frac{10a(3a^2+2b^2)\tan(e+fx)}{77d^4f(d\sec(e+fx))^{3/2}} - \frac{2\cos^4(e+fx)(b-a\tan(e+fx))(a+b\tan(e+fx))^2}{11d^4f(d\sec(e+fx))^{3/2}} - \frac{2\cos^2(e+fx)\left(2b(7a^2+2b^2)-a(9a^2-b^2)\tan(e+fx)\right)}{77d^4f(d\sec(e+fx))^{3/2}}$$

Mathematica [A] (verified)

Time = 10.23 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.36

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{11/2}}\,dx = \frac{10a(3a^2+2b^2)\,\text{EllipticF}\left(\frac{1}{2}(e+fx),2\right)(a+b\tan(e+fx))^3}{77f\cos^{\frac{5}{2}}(e+fx)(d\sec(e+fx))^{11/2}(a\cos(e+fx)+b\sin(e+fx))^3}$$
$$+\frac{\sec^3(e+fx)\left(-\frac{1}{616}b(105a^2+31b^2)-\frac{b(315a^2+71b^2)\cos(2(e+fx))}{1232}-\frac{1}{616}b(63a^2+b^2)\cos(4(e+fx))-\frac{1}{176}b(3a^2-\right.}{f(d\sec(e+}$$

```
[In] Integrate[(a + b*Tan[e + f*x])^3/(d*Sec[e + f*x])^(11/2),x]
```

```
[Out] (10*a*(3*a^2 + 2*b^2)*EllipticF[(e + f*x)/2, 2]*(a + b*Tan[e + f*x])^3)/(77
*f*Cos[e + f*x]^(5/2)*(d*Sec[e + f*x])^(11/2)*(a*Cos[e + f*x] + b*Sin[e + f
*x])^3) + (Sec[e + f*x]^3*(-1/616*(b*(105*a^2 + 31*b^2)) - (b*(315*a^2 + 71
*b^2)*Cos[2*(e + f*x)])/1232 - (b*(63*a^2 + b^2)*Cos[4*(e + f*x)])/616 - (b
*(3*a^2 - b^2)*Cos[6*(e + f*x)])/176 + (a*(347*a^2 + 103*b^2)*Sin[2*(e + f*
x)])/1232 + (a*(16*a^2 - 15*b^2)*Sin[4*(e + f*x)])/308 + (a*(a^2 - 3*b^2)*S
in[6*(e + f*x)])/176)*(a + b*Tan[e + f*x])^3)/(f*(d*Sec[e + f*x])^(11/2)*(a
*Cos[e + f*x] + b*Sin[e + f*x])^3)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 39.40 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.87

method	result
default	$\frac{-\frac{6a^2b\left(\cos^5(fx+e)\right)}{11}+\frac{2b^3\left(\cos^5(fx+e)\right)}{11}+\frac{2a^3\left(\cos^4(fx+e)\right)\sin(fx+e)}{11}-\frac{6ab^2\left(\cos^4(fx+e)\right)\sin(fx+e)}{11}+\frac{30i\sqrt{\frac{1}{\cos(fx+e)+1}}\,F(i(\cot(fx+e)-\csc(f}{77}}{}$
parts	$-\frac{2a^3\left(-7\left(\cos^4(fx+e)\right)\sin(fx+e)+15i\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F(i(\csc(fx+e)-\cot(fx+e)),i)\sqrt{\frac{1}{\cos(fx+e)+1}}+15i\sec(fx+e)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,F}{77f\sqrt{d\sec(fx+e)}\,d^5}$

```
[In] int((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/77/d^5/f/(d*sec(f*x+e))^(1/2)*(-21*a^2*b*cos(f*x+e)^5+7*b^3*cos(f*x+e)^5+
7*a^3*cos(f*x+e)^4*sin(f*x+e)-21*a*b^2*cos(f*x+e)^4*sin(f*x+e)+15*I*(cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)
-csc(f*x+e)),I)*a^3+10*I*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1
))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*a*b^2-11*cos(f*x+e)^3*b^3+9
*cos(f*x+e)^2*sin(f*x+e)*a^3+6*cos(f*x+e)^2*sin(f*x+e)*a*b^2+15*I*sec(f*x+e
)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(c
ot(f*x+e)-csc(f*x+e)),I)*a^3+10*I*sec(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)*(1/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*a*b^2+1
5*a^3*sin(f*x+e)+10*a*b^2*sin(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.94

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{11/2}}\,dx = -\frac{5\sqrt{2}(3i\,a^3+2i\,ab^2)\sqrt{d}\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\,\sin(fx+e))+5\sqrt{2}(-3i\,a^3-2i\,ab^2)\sqrt{}}{}$$

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(11/2),x, algorithm="fricas")
```

```
[Out] -1/77*(5*sqrt(2)*(3*I*a^3 + 2*I*a*b^2)*sqrt(d)*weierstrassPInverse(-4, 0, c
os(f*x + e) + I*sin(f*x + e)) + 5*sqrt(2)*(-3*I*a^3 - 2*I*a*b^2)*sqrt(d)*we
ierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e)) + 2*(11*b^3*cos(f*x
 + e)^4 + 7*(3*a^2*b - b^3)*cos(f*x + e)^6 - (7*(a^3 - 3*a*b^2)*cos(f*x + e
)^5 + 3*(3*a^3 + 2*a*b^2)*cos(f*x + e)^3 + 5*(3*a^3 + 2*a*b^2)*cos(f*x + e)
)*sin(f*x + e))*sqrt(d/cos(f*x + e)))/(d^6*f)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{11/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*tan(f*x+e))**3/(d*sec(f*x+e))**(11/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{11/2}}\,dx = \int \frac{(b\tan(fx+e)+a)^3}{(d\sec(fx+e))^{\frac{11}{2}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(11/2), x)
```

Giac [F]

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{11/2}}\,dx = \int \frac{(b\tan(fx+e)+a)^3}{(d\sec(fx+e))^{\frac{11}{2}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))^3/(d*sec(f*x+e))^(11/2),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)^3/(d*sec(f*x + e))^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^3}{(d\sec(e+fx))^{11/2}}\,dx = \int \frac{(a+b\tan(e+f\,x))^3}{\left(\frac{d}{\cos(e+f\,x)}\right)^{11/2}}\,dx$$

```
[In] int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(11/2),x)
[Out] int((a + b*tan(e + f*x))^3/(d/cos(e + f*x))^(11/2), x)
```

3.603 $\int \frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 456

$$\int \frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}\,dx = \frac{2d^2(d\sec(e+fx))^{3/2}}{3bf}$$
$$+\frac{(a^2+b^2)^{3/4}\,d^2\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{b^{5/2}f\sec^2(e+fx)^{3/4}}$$
$$-\frac{(a^2+b^2)^{3/4}\,d^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{b^{5/2}f\sec^2(e+fx)^{3/4}}$$
$$+\frac{2ad^2E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{b^2f\sec^2(e+fx)^{3/4}}$$
$$-\frac{2ad^2\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{b^2f}$$
$$-\frac{a\sqrt{a^2+b^2}d^2\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e}}{b^3f\sec^2(e+fx)^{3/4}}$$
$$+\frac{a\sqrt{a^2+b^2}d^2\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+}}{b^3f\sec^2(e+fx)^{3/4}}$$

```
[Out] 2/3*d^2*(d*sec(f*x+e))^(3/2)/b/f+(a^2+b^2)^(3/4)*d^2*arctan((sec(f*x+e)^2)^
(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(d*sec(f*x+e))^(3/2)/b^(5/2)/f/(sec(f*x+e)^2
)^(3/4)-(a^2+b^2)^(3/4)*d^2*arctanh((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^
(1/4))*(d*sec(f*x+e))^(3/2)/b^(5/2)/f/(sec(f*x+e)^2)^(3/4)+2*a*d^2*(cos(1/2
*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*EllipticE(sin(1/2
*arctan(tan(f*x+e))),2^(1/2))*(d*sec(f*x+e))^(3/2)/b^2/f/(sec(f*x+e)^2)^(3/
4)-2*a*d^2*cos(f*x+e)*(d*sec(f*x+e))^(3/2)*sin(f*x+e)/b^2/f-a*d^2*cot(f*x+e
```

```
)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(3/2
)*(a^2+b^2)^(1/2)*(-tan(f*x+e)^2)^(1/2)/b^3/f/(sec(f*x+e)^2)^(3/4)+a*d^2*co
t(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e)
)^(3/2)*(a^2+b^2)^(1/2)*(-tan(f*x+e)^2)^(1/2)/b^3/f/(sec(f*x+e)^2)^(3/4)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3593, 749, 858, 233, 202, 760, 408, 504, 1227, 551, 455, 65, 304, 211, 214}

$$\int \frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}\,dx =$$
$$-\frac{ad^2\sqrt{a^2+b^2}\sqrt{-\tan^2(e+fx)}\cot(e+fx)(d\sec(e+fx))^{3/2}\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),\right.}{b^3f\sec^2(e+fx)^{3/4}}$$
$$+\frac{ad^2\sqrt{a^2+b^2}\sqrt{-\tan^2(e+fx)}\cot(e+fx)(d\sec(e+fx))^{3/2}\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-\right.}{b^3f\sec^2(e+fx)^{3/4}}$$
$$+\frac{d^2(a^2+b^2)^{3/4}(d\sec(e+fx))^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{b^{5/2}f\sec^2(e+fx)^{3/4}}$$
$$-\frac{d^2(a^2+b^2)^{3/4}(d\sec(e+fx))^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{b^{5/2}f\sec^2(e+fx)^{3/4}}$$
$$+\frac{2ad^2(d\sec(e+fx))^{3/2}E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)}{b^2f\sec^2(e+fx)^{3/4}}$$
$$-\frac{2ad^2\sin(e+fx)\cos(e+fx)(d\sec(e+fx))^{3/2}}{b^2f}+\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf}$$

```
[In] Int[(d*Sec[e + f*x])^(7/2)/(a + b*Tan[e + f*x]),x]
```

```
[Out] (2*d^2*(d*Sec[e + f*x])^(3/2))/(3*b*f) + ((a^2 + b^2)^(3/4)*d^2*ArcTan[(Sqr
t[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(d*Sec[e + f*x])^(3/2))/(b^
(5/2)*f*(Sec[e + f*x]^2)^(3/4)) - ((a^2 + b^2)^(3/4)*d^2*ArcTanh[(Sqrt[b]*(
Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(d*Sec[e + f*x])^(3/2))/(b^(5/2)*
f*(Sec[e + f*x]^2)^(3/4)) + (2*a*d^2*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(
d*Sec[e + f*x])^(3/2))/(b^2*f*(Sec[e + f*x]^2)^(3/4)) - (2*a*d^2*Cos[e + f*
x]*(d*Sec[e + f*x])^(3/2)*Sin[e + f*x])/(b^2*f) - (a*Sqrt[a^2 + b^2]*d^2*Co
t[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)],
 -1]*(d*Sec[e + f*x])^(3/2)*Sqrt[-Tan[e + f*x]^2])/(b^3*f*(Sec[e + f*x]^2)^
(3/4)) + (a*Sqrt[a^2 + b^2]*d^2*Cot[e + f*x]*EllipticPi[b/Sqrt[a^2 + b^2],
ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(d*Sec[e + f*x])^(3/2)*Sqrt[-Tan[e + f*
x]^2])/(b^3*f*(Sec[e + f*x]^2)^(3/4))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 749

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] + Dist[2*(p/(e*(m +
 2*p + 1))), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 760

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
```

```
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\left(d^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int\frac{\left(1+\frac{x^2}{b^2}\right)^{3/4}}{a+x}\,dx,x,b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$=\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf}+\frac{\left(d^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int\frac{1-\frac{ax}{b^2}}{(a+x)\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx,x,b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$=\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf}-\frac{\left(ad^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int\frac{1}{\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx,x,b\tan(e+fx)\right)}{b^3f\sec^2(e+fx)^{3/4}}$$

$$+\frac{\left(\left(1+\frac{a^2}{b^2}\right)d^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int\frac{1}{(a+x)\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx,x,b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$=\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf}-\frac{2ad^2\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{b^2f}$$
$$+\frac{\left(ad^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int\frac{1}{\left(1+\frac{x^2}{b^2}\right)^{5/4}}dx,x,b\tan(e+fx)\right)}{b^3f\sec^2(e+fx)^{3/4}}$$
$$-\frac{\left(\left(1+\frac{a^2}{b^2}\right)d^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int\frac{x}{(a^2-x^2)\sqrt[4]{1+\frac{x^2}{b^2}}}dx,x,b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{3/4}}$$
$$+\frac{\left(a\left(1+\frac{a^2}{b^2}\right)d^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int\frac{1}{(a^2-x^2)\sqrt[4]{1+\frac{x^2}{b^2}}}dx,x,b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{3/4}}$$
$$=\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf}+\frac{2ad^2E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{b^2f\sec^2(e+fx)^{3/4}}$$
$$-\frac{2ad^2\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{b^2f}$$
$$-\frac{\left(\left(1+\frac{a^2}{b^2}\right)d^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int\frac{1}{(a^2-x)\sqrt[4]{1+\frac{x}{b^2}}}dx,x,b^2\tan^2(e+fx)\right)}{2bf\sec^2(e+fx)^{3/4}}$$
$$+\frac{\left(2a\left(1+\frac{a^2}{b^2}\right)d^2\cot(e+fx)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{1-x^4}\left(1+\frac{a^2}{b^2}-x^4\right)}dx,x,\right.}{b^2f\sec^2(e+fx)^{3/4}}$$
$$=\frac{2d^2(d\sec(e+fx))^{3/2}}{3bf}+\frac{2ad^2E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{b^2f\sec^2(e+fx)^{3/4}}$$
$$-\frac{2ad^2\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{b^2f}$$
$$-\frac{\left(2\left(1+\frac{a^2}{b^2}\right)bd^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int\frac{x^2}{a^2+b^2-b^2x^4}dx,x,\sqrt[4]{\sec^2(e+fx)}\right)}{f\sec^2(e+fx)^{3/4}}$$
$$+\frac{\left(a\left(1+\frac{a^2}{b^2}\right)d^2\cot(e+fx)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\left(\sqrt{a^2+b^2}-bx^2\right)\sqrt{1-x^4}}dx,x\right.}{bf\sec^2(e+fx)^{3/4}}$$
$$-\frac{\left(a\left(1+\frac{a^2}{b^2}\right)d^2\cot(e+fx)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\left(\sqrt{a^2+b^2}+bx^2\right)\sqrt{1-x^4}}dx,x\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$= \frac{2d^2(d\sec(e+fx))^{3/2}}{3bf} + \frac{2ad^2E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{b^2f\sec^2(e+fx)^{3/4}}$$
$$- \frac{2ad^2\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{b^2f}$$
$$- \frac{\left(\left(1+\frac{a^2}{b^2}\right)d^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{a^2+b^2}-bx^2}\,dx, x, \sqrt[4]{\sec^2(e+fx)}\right)}{f\sec^2(e+fx)^{3/4}}$$
$$+ \frac{\left(\left(1+\frac{a^2}{b^2}\right)d^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{a^2+b^2}+bx^2}\,dx, x, \sqrt[4]{\sec^2(e+fx)}\right)}{f\sec^2(e+fx)^{3/4}}$$
$$+ \frac{\left(a\left(1+\frac{a^2}{b^2}\right)d^2\cot(e+fx)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(\sqrt{a^2+b^2}-bx^2\right.}\right.}{bf\sec^2(e+fx)^{3/4}}$$
$$- \frac{\left(a\left(1+\frac{a^2}{b^2}\right)d^2\cot(e+fx)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(\sqrt{a^2+b^2}+bx^2\right.}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$= \frac{2d^2(d\sec(e+fx))^{3/2}}{3bf}$$
$$+ \frac{(a^2+b^2)^{3/4}d^2\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{b^{5/2}f\sec^2(e+fx)^{3/4}}$$
$$- \frac{(a^2+b^2)^{3/4}d^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{b^{5/2}f\sec^2(e+fx)^{3/4}}$$
$$+ \frac{2ad^2E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{b^2f\sec^2(e+fx)^{3/4}}$$
$$- \frac{2ad^2\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{b^2f}$$
$$- \frac{a\sqrt{a^2+b^2}d^2\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}}, \arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right), -1\right)(d\sec(e+fx))^{3/2}}{b^3f\sec^2(e+fx)^{3/4}}$$
$$+ \frac{a\sqrt{a^2+b^2}d^2\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}}, \arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right), -1\right)(d\sec(e+fx))^{3/2}\sqrt{}}{b^3f\sec^2(e+fx)^{3/4}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 11117 vs. 2(456) = 912.

Time = 90.48 (sec) , antiderivative size = 11117, normalized size of antiderivative = 24.38

$$\int \frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(d*Sec[e + f*x])^(7/2)/(a + b*Tan[e + f*x]),x]
[Out] Result too large to show
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 31612 vs. 2(421) = 842.

Time = 33.06 (sec) , antiderivative size = 31613, normalized size of antiderivative = 69.33

method	result	size
default	Expression too large to display	31613

```
[In] int((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))**(7/2)/(a+b*tan(f*x+e)),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}\,dx = \int \frac{(d\sec(fx+e))^{\frac{7}{2}}}{b\tan(fx+e)+a}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e)),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^(7/2)/(b*tan(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}\,dx = \int \frac{(d\sec(fx+e))^{\frac{7}{2}}}{b\tan(fx+e)+a}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e)),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(7/2)/(b*tan(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{a+b\tan(e+fx)}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{7/2}}{a+b\tan(e+f\,x)}\,dx$$

```
[In] int((d/cos(e + f*x))^(7/2)/(a + b*tan(e + f*x)),x)
[Out] int((d/cos(e + f*x))^(7/2)/(a + b*tan(e + f*x)), x)
```

3.604 $\int \frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 396

$$\int \frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}\,dx = \frac{2d^2\sqrt{d\sec(e+fx)}}{bf} - \frac{\sqrt[4]{a^2+b^2}d^2\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{b^{3/2}f\sqrt[4]{\sec^2(e+fx)}} - \frac{\sqrt[4]{a^2+b^2}d^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{b^{3/2}f\sqrt[4]{\sec^2(e+fx)}} - \frac{2ad^2\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{b^2f\sqrt[4]{\sec^2(e+fx)}} + \frac{ad^2\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}{b^2f\sqrt[4]{\sec^2(e+fx)}} + \frac{ad^2\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}{b^2f\sqrt[4]{\sec^2(e+fx)}}$$

```
[Out] 2*d^2*(d*sec(f*x+e))^(1/2)/b/f-(a^2+b^2)^(1/4)*d^2*arctan((sec(f*x+e)^2)^(1
/4)*b^(1/2)/(a^2+b^2)^(1/4))*(d*sec(f*x+e))^(1/2)/b^(3/2)/f/(sec(f*x+e)^2)^
(1/4)-(a^2+b^2)^(1/4)*d^2*arctanh((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1
/4))*(d*sec(f*x+e))^(1/2)/b^(3/2)/f/(sec(f*x+e)^2)^(1/4)-2*a*d^2*(cos(1/2*a
rctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*EllipticF(sin(1/2*a
rctan(tan(f*x+e))),2^(1/2))*(d*sec(f*x+e))^(1/2)/b^2/f/(sec(f*x+e)^2)^(1/4)
+a*d^2*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(d*
sec(f*x+e))^(1/2)*(-tan(f*x+e)^2)^(1/2)/b^2/f/(sec(f*x+e)^2)^(1/4)+a*d^2*co
```

```
t(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e)
)^(1/2)*(-tan(f*x+e)^2)^(1/2)/b^2/f/(sec(f*x+e)^2)^(1/4)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3593, 749, 858, 237, 761, 410, 109, 418, 1227, 551, 455, 65, 218, 214, 211}

$$\int \frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}\,dx = \frac{ad^2\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sqrt{d\sec(e+fx)}\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\right.\right.}{b^2f\sqrt[4]{\sec^2(e+fx)}}$$
$$+\frac{ad^2\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sqrt{d\sec(e+fx)}\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)}{b^2f\sqrt[4]{\sec^2(e+fx)}}$$
$$-\frac{d^2\sqrt[4]{a^2+b^2}\sqrt{d\sec(e+fx)}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{b^{3/2}f\sqrt[4]{\sec^2(e+fx)}}$$
$$-\frac{d^2\sqrt[4]{a^2+b^2}\sqrt{d\sec(e+fx)}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{b^{3/2}f\sqrt[4]{\sec^2(e+fx)}}$$
$$-\frac{2ad^2\sqrt{d\sec(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)}{b^2f\sqrt[4]{\sec^2(e+fx)}}+\frac{2d^2\sqrt{d\sec(e+fx)}}{bf}$$

```
[In] Int[(d*Sec[e + f*x])^(5/2)/(a + b*Tan[e + f*x]),x]
```

```
[Out] (2*d^2*Sqrt[d*Sec[e + f*x]])/(b*f) - ((a^2 + b^2)^(1/4)*d^2*ArcTan[(Sqrt[b]
*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*Sqrt[d*Sec[e + f*x]])/(b^(3/2)*
f*(Sec[e + f*x]^2)^(1/4)) - ((a^2 + b^2)^(1/4)*d^2*ArcTanh[(Sqrt[b]*(Sec[e
+ f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*Sqrt[d*Sec[e + f*x]])/(b^(3/2)*f*(Sec[e
 + f*x]^2)^(1/4)) - (2*a*d^2*EllipticF[ArcTan[Tan[e + f*x]]/2, 2]*Sqrt[d*Se
c[e + f*x]])/(b^2*f*(Sec[e + f*x]^2)^(1/4)) + (a*d^2*Cot[e + f*x]*EllipticP
i[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[d*Sec[e +
f*x]]*Sqrt[-Tan[e + f*x]^2])/(b^2*f*(Sec[e + f*x]^2)^(1/4)) + (a*d^2*Cot[e
+ f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sq
rt[d*Sec[e + f*x]]*Sqrt[-Tan[e + f*x]^2])/(b^2*f*(Sec[e + f*x]^2)^(1/4))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 109

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e
/f) + d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-f/(d*e - c*f), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[(-b)*(x^2/a)]/(2*x), Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 749

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] + Dist[2*(p/(e*(m +
 2*p + 1))), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 761

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(3/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(3/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(3/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
```

```
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral}=\frac{\left(d^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int\frac{\sqrt[4]{1+\frac{x^2}{b^2}}}{a+x}\,dx,x,b\tan(e+fx)\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$=\frac{2d^2\sqrt{d\sec(e+fx)}}{bf}+\frac{\left(d^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int\frac{1-\frac{ax}{b^2}}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx,x,b\tan(e+fx)\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$=\frac{2d^2\sqrt{d\sec(e+fx)}}{bf}-\frac{\left(ad^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx,x,b\tan(e+fx)\right)}{b^3f\sqrt[4]{\sec^2(e+fx)}}+\frac{\left(\left(1+\frac{a^2}{b^2}\right)d^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int\frac{1}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx,x,b\tan(e+fx)\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$=\frac{2d^2\sqrt{d\sec(e+fx)}}{bf}-\frac{2ad^2\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{b^2f\sqrt[4]{\sec^2(e+fx)}}-\frac{\left(\left(1+\frac{a^2}{b^2}\right)d^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int\frac{x}{(a^2-x^2)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx,x,b\tan(e+fx)\right)}{bf\sqrt[4]{\sec^2(e+fx)}}+\frac{\left(a\left(1+\frac{a^2}{b^2}\right)d^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int\frac{1}{(a^2-x^2)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx,x,b\tan(e+fx)\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$=\frac{2d^2\sqrt{d\sec(e+fx)}}{bf}-\frac{2ad^2\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{b^2f\sqrt[4]{\sec^2(e+fx)}}-\frac{\left(\left(1+\frac{a^2}{b^2}\right)d^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int\frac{1}{(a^2-x)\left(1+\frac{x}{b^2}\right)^{3/4}}\,dx,x,b^2\tan^2(e+fx)\right)}{2bf\sqrt[4]{\sec^2(e+fx)}}+\frac{\left(a\left(1+\frac{a^2}{b^2}\right)d^2\cot(e+fx)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{(a^2-x)\sqrt{-\frac{x}{b^2}}\left(1+\frac{x}{b^2}\right)^{3/4}}\,dx,x}{2b^2f\sqrt[4]{\sec^2(e+fx)}}$$

$$= \frac{2d^2\sqrt{d\sec(e+fx)}}{bf} - \frac{2ad^2 \operatorname{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{b^2 f\sqrt[4]{\sec^2(e+fx)}}$$
$$- \frac{\left(2\left(1+\frac{a^2}{b^2}\right)bd^2\sqrt{d\sec(e+fx)}\right)\operatorname{Subst}\left(\int \frac{1}{a^2+b^2-b^2x^4}\,dx, x, \sqrt[4]{\sec^2(e+fx)}\right)}{f\sqrt[4]{\sec^2(e+fx)}}$$
$$- \frac{\left(2a\left(1+\frac{a^2}{b^2}\right)d^2\cot(e+fx)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^4}\left(-1-\frac{a^2}{b^2}+x^4\right)}\,dx, x\right.}{b^2 f\sqrt[4]{\sec^2(e+fx)}}$$

$$= \frac{2d^2\sqrt{d\sec(e+fx)}}{bf} - \frac{2ad^2 \operatorname{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{b^2 f\sqrt[4]{\sec^2(e+fx)}}$$
$$- \frac{\left(\left(1+\frac{a^2}{b^2}\right)bd^2\sqrt{d\sec(e+fx)}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a^2+b^2}-bx^2}\,dx, x, \sqrt[4]{\sec^2(e+fx)}\right)}{\sqrt{a^2+b^2}f\sqrt[4]{\sec^2(e+fx)}}$$
$$- \frac{\left(\left(1+\frac{a^2}{b^2}\right)bd^2\sqrt{d\sec(e+fx)}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a^2+b^2}+bx^2}\,dx, x, \sqrt[4]{\sec^2(e+fx)}\right)}{\sqrt{a^2+b^2}f\sqrt[4]{\sec^2(e+fx)}}$$
$$+ \frac{\left(a\left(1+\frac{a^2}{b^2}\right)d^2\cot(e+fx)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\operatorname{Subst}\left(\int \frac{1}{\left(1-\frac{bx^2}{\sqrt{a^2+b^2}}\right)\sqrt{1-x^4}}\,dx, x,\right.}{(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}}$$
$$+ \frac{\left(a\left(1+\frac{a^2}{b^2}\right)d^2\cot(e+fx)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\operatorname{Subst}\left(\int \frac{1}{\left(1+\frac{bx^2}{\sqrt{a^2+b^2}}\right)\sqrt{1-x^4}}\,dx, x,\right.}{(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}}$$

$$= \frac{2d^2\sqrt{d\sec(e+fx)}}{bf} - \frac{\sqrt[4]{a^2+b^2}d^2\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{b^{3/2}f\sqrt[4]{\sec^2(e+fx)}}$$
$$- \frac{\sqrt[4]{a^2+b^2}d^2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{b^{3/2}f\sqrt[4]{\sec^2(e+fx)}}$$
$$- \frac{2ad^2 \operatorname{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{b^2 f\sqrt[4]{\sec^2(e+fx)}}$$
$$+ \frac{\left(a\left(1+\frac{a^2}{b^2}\right)d^2\cot(e+fx)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(1-\frac{bx^2}{\sqrt{a^2+b^2}}\right)}\,d\right.}{(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}}$$
$$+ \frac{\left(a\left(1+\frac{a^2}{b^2}\right)d^2\cot(e+fx)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(1+\frac{bx^2}{\sqrt{a^2+b^2}}\right)}\,d\right.}{(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}}$$

$$= \frac{2d^2\sqrt{d\sec(e+fx)}}{bf} - \frac{\sqrt[4]{a^2+b^2}d^2\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{b^{3/2}f\sqrt[4]{\sec^2(e+fx)}}$$
$$- \frac{\sqrt[4]{a^2+b^2}d^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{b^{3/2}f\sqrt[4]{\sec^2(e+fx)}}$$
$$- \frac{2ad^2\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{b^2f\sqrt[4]{\sec^2(e+fx)}}$$
$$+ \frac{ad^2\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+}}{b^2f\sqrt[4]{\sec^2(e+fx)}}$$
$$+ \frac{ad^2\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+f}}{b^2f\sqrt[4]{\sec^2(e+fx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 27.35 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.73

$$\int \frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}\,dx = \frac{d^2\cot(e+fx)\sqrt{d\sec(e+fx)}\left(-\sqrt{b}\sqrt[4]{a^2+b^2}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\tan(e+\right.}{}$$

```
[In] Integrate[(d*Sec[e + f*x])^(5/2)/(a + b*Tan[e + f*x]),x]
```

```
[Out] (d^2*Cot[e + f*x]*Sqrt[d*Sec[e + f*x]]*(-(Sqrt[b]*(a^2 + b^2)^(1/4)*ArcTan[
(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*Tan[e + f*x]) - Sqrt[b]
*(a^2 + b^2)^(1/4)*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/
4)]*Tan[e + f*x] + 2*b*(Sec[e + f*x]^2)^(1/4)*Tan[e + f*x] - a*Hypergeometr
ic2F1[1/2, 3/4, 3/2, -Tan[e + f*x]^2]*Tan[e + f*x]^2 + a*EllipticPi[-(b/Sqr
t[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[-Tan[e + f*x]^2] +
a*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[-T
an[e + f*x]^2]))/(b^2*f*(Sec[e + f*x]^2)^(1/4))
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 7346 vs. 2(369) = 738.

Time = 30.43 (sec) , antiderivative size = 7347, normalized size of antiderivative = 18.55

method	result	size
default	Expression too large to display	7347

```
[In] int((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{2}}}{b\tan(fx+e)+a}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")
[Out] integral(sqrt(d*sec(f*x + e))*d^2*sec(f*x + e)^2/(b*tan(f*x + e) + a), x)
```

Sympy [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}\,dx = \int \frac{(d\sec(e+fx))^{\frac{5}{2}}}{a+b\tan(e+fx)}\,dx$$

```
[In] integrate((d*sec(f*x+e))**(5/2)/(a+b*tan(f*x+e)),x)
[Out] Integral((d*sec(e + f*x))**(5/2)/(a + b*tan(e + f*x)), x)
```

Maxima [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{2}}}{b\tan(fx+e)+a}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}\,dx = \int \frac{(d\sec\left(fx+e\right))^{\frac{5}{2}}}{b\tan\left(fx+e\right)+a}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/2}}{a+b\tan(e+fx)}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}}{a+b\tan\left(e+f\,x\right)}\,dx$$

```
[In] int((d/cos(e + f*x))^(5/2)/(a + b*tan(e + f*x)),x)
[Out] int((d/cos(e + f*x))^(5/2)/(a + b*tan(e + f*x)), x)
```

3.605 $\int \frac{(d\sec(e+fx))^{3/2}}{a+b\tan(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 334

$$\int \frac{(d\sec(e+fx))^{3/2}}{a+b\tan(e+fx)}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{\sqrt{b}\sqrt[4]{a^2+b^2}f\sec^2(e+fx)^{3/4}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{\sqrt{b}\sqrt[4]{a^2+b^2}f\sec^2(e+fx)^{3/4}} - \frac{a\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}}{b\sqrt{a^2+b^2}f\sec^2(e+fx)^{3/4}} + \frac{a\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}}{b\sqrt{a^2+b^2}f\sec^2(e+fx)^{3/4}}$$

```
[Out] arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(d*sec(f*x+e))^(3/2)/(
a^2+b^2)^(1/4)/f/(sec(f*x+e)^2)^(3/4)/b^(1/2)-arctanh((sec(f*x+e)^2)^(1/4)*
b^(1/2)/(a^2+b^2)^(1/4))*(d*sec(f*x+e))^(3/2)/(a^2+b^2)^(1/4)/f/(sec(f*x+e)
^2)^(3/4)/b^(1/2)-a*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2)
^(1/2),I)*(d*sec(f*x+e))^(3/2)*(-tan(f*x+e)^2)^(1/2)/b/f/(sec(f*x+e)^2)^(3/
4)/(a^2+b^2)^(1/2)+a*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)
^(1/2),I)*(d*sec(f*x+e))^(3/2)*(-tan(f*x+e)^2)^(1/2)/b/f/(sec(f*x+e)^2)^(3/
4)/(a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3593, 760, 408, 504, 1227, 551, 455, 65, 304, 211, 214}

$$\int \frac{(d\sec(e+fx))^{3/2}}{a+b\tan(e+fx)}\,dx = -\frac{a\sqrt{-\tan^2(e+fx)}\cot(e+fx)(d\sec(e+fx))^{3/2}\operatorname{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)}{bf\sqrt{a^2+b^2}\sec^2(e+fx)^{3/4}}$$
$$+\frac{a\sqrt{-\tan^2(e+fx)}\cot(e+fx)(d\sec(e+fx))^{3/2}\operatorname{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)}{bf\sqrt{a^2+b^2}\sec^2(e+fx)^{3/4}}$$
$$+\frac{(d\sec(e+fx))^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{\sqrt{b}f\sqrt[4]{a^2+b^2}\sec^2(e+fx)^{3/4}}$$
$$-\frac{(d\sec(e+fx))^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{\sqrt{b}f\sqrt[4]{a^2+b^2}\sec^2(e+fx)^{3/4}}$$

```
[In] Int[(d*Sec[e + f*x])^(3/2)/(a + b*Tan[e + f*x]),x]
```

```
[Out] (ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(d*Sec[e + f*x]
)^(3/2))/(Sqrt[b]*(a^2 + b^2)^(1/4)*f*(Sec[e + f*x]^2)^(3/4)) - (ArcTanh[(S
qrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(d*Sec[e + f*x])^(3/2))/(
Sqrt[b]*(a^2 + b^2)^(1/4)*f*(Sec[e + f*x]^2)^(3/4)) - (a*Cot[e + f*x]*Ellip
ticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(d*Sec[e +
f*x])^(3/2)*Sqrt[-Tan[e + f*x]^2])/(b*Sqrt[a^2 + b^2]*f*(Sec[e + f*x]^2)^(3
/4)) + (a*Cot[e + f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2
)^(1/4)], -1]*(d*Sec[e + f*x])^(3/2)*Sqrt[-Tan[e + f*x]^2])/(b*Sqrt[a^2 + b
^2]*f*(Sec[e + f*x]^2)^(3/4))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 760

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
```

```
2 + a*e^2, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{(d\sec(e+fx))^{3/2}\text{Subst}\left(\int \frac{1}{(a+x)\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$= -\frac{(d\sec(e+fx))^{3/2}\text{Subst}\left(\int \frac{x}{(a^2-x^2)\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{3/4}} + \frac{\left(a(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{1}{(a^2-x^2)\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$= -\frac{(d\sec(e+fx))^{3/2}\text{Subst}\left(\int \frac{1}{(a^2-x)\sqrt[4]{1+\frac{x}{b^2}}}\,dx, x, b^2\tan^2(e+fx)\right)}{2bf\sec^2(e+fx)^{3/4}} + \frac{\left(2a\cot(e+fx)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-x^4}\left(1+\frac{a^2}{b^2}-x^4\right)}\,dx, x, \sqrt[4]{\sec^2(e+\right.}{b^2f\sec^2(e+fx)^{3/4}}$$

$$= -\frac{\left(2b(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{x^2}{a^2+b^2-b^2x^4}\,dx, x, \sqrt[4]{\sec^2(e+fx)}\right)}{f\sec^2(e+fx)^{3/4}}$$

$$+ \frac{\left(a\cot(e+fx)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{a^2+b^2}-bx^2\right)\sqrt{1-x^4}}\,dx, x, \sqrt[4]{\sec^2(e}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$- \frac{\left(a\cot(e+fx)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{a^2+b^2}+bx^2\right)\sqrt{1-x^4}}\,dx, x, \sqrt[4]{\sec^2(e}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$= -\frac{(d\sec(e+fx))^{3/2}\text{Subst}\left(\int \frac{1}{\sqrt{a^2+b^2}-bx^2}\,dx, x, \sqrt[4]{\sec^2(e+fx)}\right)}{f\sec^2(e+fx)^{3/4}}$$

$$+ \frac{(d\sec(e+fx))^{3/2}\text{Subst}\left(\int \frac{1}{\sqrt{a^2+b^2}+bx^2}\,dx, x, \sqrt[4]{\sec^2(e+fx)}\right)}{f\sec^2(e+fx)^{3/4}}$$

$$+ \frac{\left(a\cot(e+fx)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(\sqrt{a^2+b^2}-bx^2\right)}\,dx, x, \sqrt[4]{\text{se}}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$- \frac{\left(a\cot(e+fx)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(\sqrt{a^2+b^2}+bx^2\right)}\,dx, x, \sqrt[4]{\text{se}}\right.}{bf\sec^2(e+fx)^{3/4}}$$

$$= \frac{\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{\sqrt{b}\sqrt[4]{a^2+b^2}f\sec^2(e+fx)^{3/4}}$$

$$- \frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{\sqrt{b}\sqrt[4]{a^2+b^2}f\sec^2(e+fx)^{3/4}}$$

$$- \frac{a\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}}, \arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right), -1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e}}{b\sqrt{a^2+b^2}f\sec^2(e+fx)^{3/4}}$$

$$+ \frac{a\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}}, \arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right), -1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+}}{b\sqrt{a^2+b^2}f\sec^2(e+fx)^{3/4}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 3.43 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.83

$$\int \frac{(d\sec(e+fx))^{3/2}}{a+b\tan(e+fx)}\,dx = -\frac{12d^2\,\mathrm{AppellF1}\left(\frac{1}{2},\frac{1}{4},\frac{1}{4},\frac{3}{2},\frac{a-ib}{a+b\tan(e+fx)},\cdots\right.}{bf\sqrt{d\sec(e+fx)}\left((a+ib)\,\mathrm{AppellF1}\left(\frac{3}{2},\frac{1}{4},\frac{5}{4},\frac{5}{2},\frac{a-ib}{a+b\tan(e+fx)},\frac{a+ib}{a+b\tan(e+fx)}\right)+(a-ib)\,\mathrm{AppellF1}\left(\frac{3}{2},\frac{5}{4},\frac{1}{4},\frac{5}{2},\cdots\right.\right.}$$

```
[In] Integrate[(d*Sec[e + f*x])^(3/2)/(a + b*Tan[e + f*x]),x]
```

```
[Out] (-12*d^2*AppellF1[1/2, 1/4, 1/4, 3/2, (a - I*b)/(a + b*Tan[e + f*x]), (a +
I*b)/(a + b*Tan[e + f*x])]*(a + b*Tan[e + f*x]))/(b*f*Sqrt[d*Sec[e + f*x]]*
((a + I*b)*AppellF1[3/2, 1/4, 5/4, 5/2, (a - I*b)/(a + b*Tan[e + f*x]), (a
+ I*b)/(a + b*Tan[e + f*x])] + (a - I*b)*AppellF1[3/2, 5/4, 1/4, 5/2, (a -
I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])] + 6*AppellF1[1/2
, 1/4, 1/4, 3/2, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f
*x])]*(a + b*Tan[e + f*x])))
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3633 vs. 2(282) = 564.

Time = 9.43 (sec) , antiderivative size = 3634, normalized size of antiderivative = 10.88

method	result	size
default	Expression too large to display	3634

```
[In] int((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*d/f*(cos(f*x+e)+1)*(-I*(a^2+b^2)^(3/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a
^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+
b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*ln(2)*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)*a^2-I*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3
)/a^4)^(1/2)*arctanh(1/2*(cos(f*x+e)*(a^2+b^2)^(1/2)*b+a^2*cos(f*x+e)+b^2*c
os(f*x+e)-b*(a^2+b^2)^(1/2)-b^2)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)/(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)
^(1/2)/a^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*a^2*b^3-I*(b*((a^2+b^2)^(1
/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*arctanh(1/2*(cos(f*
x+e)*(a^2+b^2)^(1/2)*b-a^2*cos(f*x+e)-b^2*cos(f*x+e)-b*(a^2+b^2)^(1/2)+b^2)
/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(-b*((a^2+b^2)^(1/2)*a
^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)/a^2)*(-cos(f*x+e)/(cos(f
```

```
*x+e)+1)^2)^(1/2)*a^4*b-I*(a^2+b^2)^(1/2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^
2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*arctanh(1/2*(cos(f*x+e)*(a^2+b^2)^(1
/2)*b-a^2*cos(f*x+e)-b^2*cos(f*x+e)-b*(a^2+b^2)^(1/2)+b^2)/(cos(f*x+e)+1)/(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1
/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)/a^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
*b^4+I*(a^2+b^2)^(3/2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*
b+2*b^3)/a^4)^(1/2)*arctanh(1/2*(cos(f*x+e)*(a^2+b^2)^(1/2)*b-a^2*cos(f*x+e
)-b^2*cos(f*x+e)-b*(a^2+b^2)^(1/2)+b^2)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)/(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b
^3)/a^4)^(1/2)/a^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*b^2+I*(a^2+b^2)^(1
/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2
)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*l
n((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)*a^2*b^2-4*(a^2+b^2)^(3/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/
2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*
b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*EllipticF(I*(csc(f*x+e)-cot(f*x+e)),I)*a*b+4*(a^2+b^2)^(1/2)*
(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*(b
*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*(1/(c
os(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(csc(f*x+
e)-cot(f*x+e)),I)*a*b^3+4*b*a^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)*EllipticPi(I*(csc(f*x+e)-cot(f*x+e)),-1/(-b+(a^2+b^2)^(1/2)
)^2*a^2,I)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4
)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^
(1/2)*(a^2+b^2)^(1/2)+4*b*a^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*EllipticPi(I*(csc(f*x+e)-cot(f*x+e)),-1/(b+(a^2+b^2)^(1/2))^2
*a^2,I)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(
1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/
2)*(a^2+b^2)^(1/2)-I*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b
-2*b^3)/a^4)^(1/2)*arctanh(1/2*(cos(f*x+e)*(a^2+b^2)^(1/2)*b+a^2*cos(f*x+e)
+b^2*cos(f*x+e)-b*(a^2+b^2)^(1/2)-b^2)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)/(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3
)/a^4)^(1/2)/a^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*a^4*b-I*(b*((a^2+b^2
)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*arctanh(1/2*(co
s(f*x+e)*(a^2+b^2)^(1/2)*b-a^2*cos(f*x+e)-b^2*cos(f*x+e)-b*(a^2+b^2)^(1/2)+
b^2)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(-b*((a^2+b^2)^(1/
2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)/a^2)*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)*a^2*b^3+I*(a^2+b^2)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*
(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^
2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*ln(2)*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)*a^2*b^2-I*(a^2+b^2)^(3/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(
1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2
)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)
```

```
+1))*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*a^2-I*(a^2+b^2)^(3/2)*(-b*((a^2+b
^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*arctanh(1/2*(
cos(f*x+e)*(a^2+b^2)^(1/2)*b+a^2*cos(f*x+e)+b^2*cos(f*x+e)-b*(a^2+b^2)^(1/2
)-b^2)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(b*((a^2+b^2)^(1
/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)/a^2)*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)*b^2-I*(a^2+b^2)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^
2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b
^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(
cos(f*x+e)+1))*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*a^2*b^2+I*(a^2+b^2)^(1/
2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)
*arctanh(1/2*(cos(f*x+e)*(a^2+b^2)^(1/2)*b+a^2*cos(f*x+e)+b^2*cos(f*x+e)-b*
(a^2+b^2)^(1/2)-b^2)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(b
*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)/a^2)*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*b^4+I*(a^2+b^2)^(3/2)*(-b*((a^2+b^2)^(
1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*(b*((a^2+b^2)^(1/2
)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*ln(2*(2*cos(f*x+e)*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-c
os(f*x+e)+1)/(cos(f*x+e)+1))*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*a^2)*(d*s
ec(f*x+e))^(1/2)/b/(a^2+b^2)^(1/2)/(b+(a^2+b^2)^(1/2))/a^2/(b*((a^2+b^2)^(1
/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)/(-b+(a^2+b^2)^(1/2)
)/(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(d\sec(e+fx))^{3/2}}{a+b\tan(e+fx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{a+b\tan(e+fx)}\,dx = \int \frac{(d\sec{(e+fx)})^{\frac{3}{2}}}{a+b\tan{(e+fx)}}\,dx$$

```
[In] integrate((d*sec(f*x+e))**(3/2)/(a+b*tan(f*x+e)),x)
[Out] Integral((d*sec(e + f*x))**(3/2)/(a + b*tan(e + f*x)), x)
```

Maxima [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{a+b\tan(e+fx)}\,dx = \int \frac{(d\sec(fx+e))^{\frac{3}{2}}}{b\tan(fx+e)+a}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^(3/2)/(b*tan(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{a+b\tan(e+fx)}\,dx = \int \frac{(d\sec(fx+e))^{\frac{3}{2}}}{b\tan(fx+e)+a}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(3/2)/(b*tan(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{3/2}}{a+b\tan(e+fx)}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}{a+b\tan(e+f\,x)}\,dx$$

```
[In] int((d/cos(e + f*x))^(3/2)/(a + b*tan(e + f*x)),x)
[Out] int((d/cos(e + f*x))^(3/2)/(a + b*tan(e + f*x)), x)
```

3.606 $\int \frac{\sqrt{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 324

$$\begin{aligned}\int \frac{\sqrt{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx &= -\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{(a^2+b^2)^{3/4}f\sqrt[4]{\sec^2(e+fx)}}\\
&-\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{(a^2+b^2)^{3/4}f\sqrt[4]{\sec^2(e+fx)}}\\
&+\frac{a\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}{(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}}\\
&+\frac{a\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}{(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}}\end{aligned}$$

```
[Out] -arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*b^(1/2)*(d*sec(f*x+e)
)^(1/2)/(a^2+b^2)^(3/4)/f/(sec(f*x+e)^2)^(1/4)-arctanh((sec(f*x+e)^2)^(1/4)
*b^(1/2)/(a^2+b^2)^(1/4))*b^(1/2)*(d*sec(f*x+e))^(1/2)/(a^2+b^2)^(3/4)/f/(s
ec(f*x+e)^2)^(1/4)+a*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2
)^(1/2),I)*(d*sec(f*x+e))^(1/2)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)/f/(sec(f*x+
e)^2)^(1/4)+a*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),
I)*(d*sec(f*x+e))^(1/2)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)/f/(sec(f*x+e)^2)^(1
/4)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3593, 761, 410, 109, 418, 1227, 551, 455, 65, 218, 214, 211}

$$\begin{aligned}
&\int \frac{\sqrt{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx \\
&= \frac{a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sqrt{d\sec(e+fx)}\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)}{f\left(a^2+b^2\right)\sqrt[4]{\sec^2(e+fx)}} \\
&+\frac{a\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sqrt{d\sec(e+fx)}\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)}{f\left(a^2+b^2\right)\sqrt[4]{\sec^2(e+fx)}} \\
&-\frac{\sqrt{b}\sqrt{d\sec(e+fx)}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{f\left(a^2+b^2\right)^{3/4}\sqrt[4]{\sec^2(e+fx)}} \\
&-\frac{\sqrt{b}\sqrt{d\sec(e+fx)}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{f\left(a^2+b^2\right)^{3/4}\sqrt[4]{\sec^2(e+fx)}}
\end{aligned}$$

```
[In] Int[Sqrt[d*Sec[e + f*x]]/(a + b*Tan[e + f*x]),x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*Sqrt[
d*Sec[e + f*x]])/((a^2 + b^2)^(3/4)*f*(Sec[e + f*x]^2)^(1/4))) - (Sqrt[b]*A
rcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*Sqrt[d*Sec[e + f
*x]])/((a^2 + b^2)^(3/4)*f*(Sec[e + f*x]^2)^(1/4)) + (a*Cot[e + f*x]*Ellipt
icPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[d*Sec[e
 + f*x]]*Sqrt[-Tan[e + f*x]^2])/((a^2 + b^2)*f*(Sec[e + f*x]^2)^(1/4)) + (a
*Cot[e + f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)],
 -1]*Sqrt[d*Sec[e + f*x]]*Sqrt[-Tan[e + f*x]^2])/((a^2 + b^2)*f*(Sec[e + f*
x]^2)^(1/4))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 109

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e
/f) + d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
```

```
x] && GtQ[-f/(d*e - c*f), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[(-b)*(x^2/a)]/(2*x), Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 761

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(3/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(3/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(3/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{d\sec(e+fx)}\text{Subst}\left(\int \frac{1}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{\sqrt{d\sec(e+fx)}\text{Subst}\left(\int \frac{x}{(a^2-x^2)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[4]{\sec^2(e+fx)}} + \frac{\left(a\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{(a^2-x^2)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{\sqrt{d\sec(e+fx)}\text{Subst}\left(\int \frac{1}{(a^2-x)\left(1+\frac{x}{b^2}\right)^{3/4}}\,dx, x, b^2\tan^2(e+fx)\right)}{2bf\sqrt[4]{\sec^2(e+fx)}} + \frac{\left(a\cot(e+fx)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{(a^2-x)\sqrt{-\frac{x}{b^2}}\left(1+\frac{x}{b^2}\right)^{3/4}}\,dx, x, b^2\tan^2(e\right.}{2b^2f\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{\left(2b\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{a^2+b^2-b^2x^4}\,dx, x, \sqrt[4]{\sec^2(e+fx)}\right)}{f\sqrt[4]{\sec^2(e+fx)}}$$
$$-\frac{\left(2a\cot(e+fx)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^4}\left(-1-\frac{a^2}{b^2}+x^4\right)}\,dx, x, \sqrt[4]{\sec^2(e+f}\right.}{b^2f\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{\left(b\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a^2+b^2}-bx^2}\,dx, x, \sqrt[4]{\sec^2(e+fx)}\right)}{\sqrt{a^2+b^2}f\sqrt[4]{\sec^2(e+fx)}}$$
$$-\frac{\left(b\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a^2+b^2}+bx^2}\,dx, x, \sqrt[4]{\sec^2(e+fx)}\right)}{\sqrt{a^2+b^2}f\sqrt[4]{\sec^2(e+fx)}}$$
$$+\frac{\left(a\cot(e+fx)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{bx^2}{\sqrt{a^2+b^2}}\right)\sqrt{1-x^4}}\,dx, x, \sqrt[4]{\sec^2(e+fx}\right.}{(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}}$$
$$+\frac{\left(a\cot(e+fx)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{bx^2}{\sqrt{a^2+b^2}}\right)\sqrt{1-x^4}}\,dx, x, \sqrt[4]{\sec^2(e+fx}\right.}{(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{(a^2+b^2)^{3/4}f\sqrt[4]{\sec^2(e+fx)}}$$
$$-\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{(a^2+b^2)^{3/4}f\sqrt[4]{\sec^2(e+fx)}}$$
$$+\frac{\left(a\cot(e+fx)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(1-\frac{bx^2}{\sqrt{a^2+b^2}}\right)}\,dx, x, \sqrt[4]{\sec^2(e}\right.}{(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}}$$
$$+\frac{\left(a\cot(e+fx)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(1+\frac{bx^2}{\sqrt{a^2+b^2}}\right)}\,dx, x, \sqrt[4]{\sec^2(e}\right.}{(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{(a^2+b^2)^{3/4}f\sqrt[4]{\sec^2(e+fx)}}$$
$$-\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{(a^2+b^2)^{3/4}f\sqrt[4]{\sec^2(e+fx)}}$$
$$+\frac{a\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+}}{(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}}$$
$$+\frac{a\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+f}}{(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}}$$

Mathematica [A] (verified)

Time = 2.87 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx$$
$$= \frac{\sqrt{d\sec(e+fx)}\left(-\sqrt{b}\sqrt[4]{a^2+b^2}\left(\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)+\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\right)+a\cot(e\right.}{}$$

```
[In] Integrate[Sqrt[d*Sec[e + f*x]]/(a + b*Tan[e + f*x]),x]
```

```
[Out] (Sqrt[d*Sec[e + f*x]]*(-(Sqrt[b]*(a^2 + b^2)^(1/4)*(ArcTan[(Sqrt[b]*(Sec[e
 + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)] + ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/
4))/(a^2 + b^2)^(1/4)])) + a*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]),
ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[-Tan[e + f*x]^2] + a*Cot[e + f*x]*
EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[-Tan
[e + f*x]^2]))/((a^2 + b^2)*f*(Sec[e + f*x]^2)^(1/4))
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3037 vs. 2(276) = 552.

Time = 8.02 (sec) , antiderivative size = 3038, normalized size of antiderivative = 9.38

method	result	size
default	Expression too large to display	3038

```
[In] int((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(cos(f*x+e)+1)*(4*I*b*a^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*EllipticPi(I*(cot(f*x+e)-csc(f*x+e)),-1/(b+(a^2+b^2)^(1/2)
)^2*a^2,I)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4
)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^
(1/2)*(a^2+b^2)^(1/2)-4*I*b*a^3*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)*EllipticPi(I*(cot(f*x+e)-csc(f*x+e)),-1/(-b+(a^2+b^2)^(1/2)
)^2*a^2,I)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4
)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^
(1/2)*(a^2+b^2)^(1/2)-4*I*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*
a^2*b-2*b^3)/a^4)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1)
)^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*(b*((a^2+b^2)^(1/2)*a^2+2*(a
^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*a^5-4*I*(-b*((a^2+b^2)^(1/2)*a^
2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*(1/(cos(f*x+e)+1))^(1/2)*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(cot(f*x+e)-csc(f*x+e)),I)*(b
*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*a^3*b
^2-(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+
b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*a^4*b+(-b*((a^2+b^2)^(1/2)*a^2+2*(
a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)*ln(2*(2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*(b*((a^2+b^2)^(1/2
)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*a^4*b-(-b*((a^2+b^2)^
(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)*ln((2*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+2
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)+1)/(cos(f*x+e)+1))*(b*((a^
2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*a^4*b-(a^2
+b^2)^(3/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a
^4)^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*arctanh(1/2*(cos(f*x+e)*(a^2
+b^2)^(1/2)*b+a^2*cos(f*x+e)+b^2*cos(f*x+e)-b*(a^2+b^2)^(1/2)-b^2)/(cos(f*x
+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+
b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)/a^2)*b^2-(a^2+b^2)^(3/2)*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)*arctanh(1/2*(cos(f*x+e)*(a^2+b^2)^(1/2)*b-a^2*c
os(f*x+e)-b^2*cos(f*x+e)-b*(a^2+b^2)^(1/2)+b^2)/(cos(f*x+e)+1)/(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)/(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a
^2*b-2*b^3)/a^4)^(1/2)/a^2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2
*a^2*b+2*b^3)/a^4)^(1/2)*b^2+(a^2+b^2)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^
2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)*arctanh(1/2*(cos(f*x+e)*(a^2+b^2)^(1/2)*b+a^2*cos(f*x+e)+b^2*cos(f*x+e
)-b*(a^2+b^2)^(1/2)-b^2)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)/(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)/a
^2)*b^4+(a^2+b^2)^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*arctanh(1/2*(c
os(f*x+e)*(a^2+b^2)^(1/2)*b-a^2*cos(f*x+e)-b^2*cos(f*x+e)-b*(a^2+b^2)^(1/2)
+b^2)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(-b*((a^2+b^2)^(1
/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)/a^2)*(b*((a^2+b^2)^
(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*b^4-(-b*((a^2+b^2
```

```
)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)*arctanh(1/2*(cos(f*x+e)*(a^2+b^2)^(1/2)*b+a^2*cos(f*x
+e)+b^2*cos(f*x+e)-b*(a^2+b^2)^(1/2)-b^2)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)/(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*
b^3)/a^4)^(1/2)/a^2)*a^4*b-(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2
*a^2*b-2*b^3)/a^4)^(1/2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*arctanh(1/2*(
cos(f*x+e)*(a^2+b^2)^(1/2)*b+a^2*cos(f*x+e)+b^2*cos(f*x+e)-b*(a^2+b^2)^(1/2
)-b^2)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(b*((a^2+b^2)^(1
/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)/a^2)*a^2*b^3+(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*arctanh(1/2*(cos(f*x+e)*(a^2+b^2)^(1/2)*b-a^
2*cos(f*x+e)-b^2*cos(f*x+e)-b*(a^2+b^2)^(1/2)+b^2)/(cos(f*x+e)+1)/(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)/(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-
2*a^2*b-2*b^3)/a^4)^(1/2)/a^2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^
2+2*a^2*b+2*b^3)/a^4)^(1/2)*a^4*b+(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*arct
anh(1/2*(cos(f*x+e)*(a^2+b^2)^(1/2)*b-a^2*cos(f*x+e)-b^2*cos(f*x+e)-b*(a^2+
b^2)^(1/2)+b^2)/(cos(f*x+e)+1)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(-b*((a
^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)/a^2)*(b*(
(a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*a^2*b^3
)*(d*sec(f*x+e))^(1/2)/(a^2+b^2)/a^2/(-b+(a^2+b^2)^(1/2))/(b*((a^2+b^2)^(1/
2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)/(b+(a^2+b^2)^(1/2))/
(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx = \int \frac{\sqrt{d\sec{(e+fx)}}}{a+b\tan{(e+fx)}}\,dx$$

```
[In] integrate((d*sec(f*x+e))**(1/2)/(a+b*tan(f*x+e)),x)
[Out] Integral(sqrt(d*sec(e + f*x))/(a + b*tan(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx = \int \frac{\sqrt{d\sec\left(fx+e\right)}}{b\tan\left(fx+e\right)+a}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x, algorithm="maxima")
[Out] integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx = \int \frac{\sqrt{d\sec\left(fx+e\right)}}{b\tan\left(fx+e\right)+a}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x, algorithm="giac")
[Out] integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx = \int \frac{\sqrt{\frac{d}{\cos(e+f\,x)}}}{a+b\tan\left(e+f\,x\right)}\,dx$$

```
[In] int((d/cos(e + f*x))^(1/2)/(a + b*tan(e + f*x)),x)
[Out] int((d/cos(e + f*x))^(1/2)/(a + b*tan(e + f*x)), x)
```

3.607 $\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 451

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx = \frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{(a^2+b^2)^{5/4}f\sqrt{d\sec(e+fx)}} - \frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{(a^2+b^2)^{5/4}f\sqrt{d\sec(e+fx)}} + \frac{2aE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{(a^2+b^2)f\sqrt{d\sec(e+fx)}} - \frac{2a\tan(e+fx)}{(a^2+b^2)f\sqrt{d\sec(e+fx)}} - \frac{ab\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}}{(a^2+b^2)^{3/2}f\sqrt{d\sec(e+fx)}} + \frac{ab\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}}{(a^2+b^2)^{3/2}f\sqrt{d\sec(e+fx)}} + \frac{2(b+a\tan(e+fx))}{(a^2+b^2)f\sqrt{d\sec(e+fx)}}$$

```
[Out] b^(3/2)*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(sec(f*x+e)^2)
^(1/4)/(a^2+b^2)^(5/4)/f/(d*sec(f*x+e))^(1/2)-b^(3/2)*arctanh((sec(f*x+e)^2
)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(5/4)/f/(d*
sec(f*x+e))^(1/2)+2*a*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(
tan(f*x+e)))*EllipticE(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(sec(f*x+e)^2)^
(1/4)/(a^2+b^2)/f/(d*sec(f*x+e))^(1/2)-a*b*cot(f*x+e)*EllipticPi((sec(f*x+e
)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2)^(1/4)*(-tan(f*x+e)^2)^(1/2)
/(a^2+b^2)^(3/2)/f/(d*sec(f*x+e))^(1/2)+a*b*cot(f*x+e)*EllipticPi((sec(f*x+
```

```
e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2)^(1/4)*(-tan(f*x+e)^2)^(1/2)
/(a^2+b^2)^(3/2)/f/(d*sec(f*x+e))^(1/2)-2*a*tan(f*x+e)/(a^2+b^2)/f/(d*sec(f
*x+e))^(1/2)+2*(b+a*tan(f*x+e))/(a^2+b^2)/f/(d*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3593, 755, 858, 233, 202, 760, 408, 504, 1227, 551, 455, 65, 304, 211, 214}

$$\begin{aligned}
&\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx = \\
&-\frac{ab\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)}{f\left(a^2+b^2\right)^{3/2}\sqrt{d\sec(e+fx)}} \\
&+\frac{ab\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)}{f\left(a^2+b^2\right)^{3/2}\sqrt{d\sec(e+fx)}} \\
&+\frac{2a\sqrt[4]{\sec^2(e+fx)}E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)}{f\left(a^2+b^2\right)\sqrt{d\sec(e+fx)}} \\
&+\frac{b^{3/2}\sqrt[4]{\sec^2(e+fx)}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{f\left(a^2+b^2\right)^{5/4}\sqrt{d\sec(e+fx)}} \\
&-\frac{b^{3/2}\sqrt[4]{\sec^2(e+fx)}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{f\left(a^2+b^2\right)^{5/4}\sqrt{d\sec(e+fx)}} \\
&-\frac{2a\tan(e+fx)}{f\left(a^2+b^2\right)\sqrt{d\sec(e+fx)}}+\frac{2(a\tan(e+fx)+b)}{f\left(a^2+b^2\right)\sqrt{d\sec(e+fx)}}
\end{aligned}$$

```
[In] Int[1/(Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])),x]
```

```
[Out] (b^(3/2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Sec[e
+ f*x]^2)^(1/4))/((a^2 + b^2)^(5/4)*f*Sqrt[d*Sec[e + f*x]]) - (b^(3/2)*ArcT
anh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Sec[e + f*x]^2)^(1
/4))/((a^2 + b^2)^(5/4)*f*Sqrt[d*Sec[e + f*x]]) + (2*a*EllipticE[ArcTan[Tan
[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^(1/4))/((a^2 + b^2)*f*Sqrt[d*Sec[e + f*x]
]) - (2*a*Tan[e + f*x])/((a^2 + b^2)*f*Sqrt[d*Sec[e + f*x]]) - (a*b*Cot[e +
 f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*
(Sec[e + f*x]^2)^(1/4)*Sqrt[-Tan[e + f*x]^2])/((a^2 + b^2)^(3/2)*f*Sqrt[d*S
ec[e + f*x]]) + (a*b*Cot[e + f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec
[e + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2)^(1/4)*Sqrt[-Tan[e + f*x]^2])/((a^
2 + b^2)^(3/2)*f*Sqrt[d*Sec[e + f*x]]) + (2*(b + a*Tan[e + f*x]))/((a^2 + b
^2)*f*Sqrt[d*Sec[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 760

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
```

```
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt[4]{\sec^2(e+fx)}\text{Subst}\left(\int \frac{1}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{5/4}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt{d\sec(e+fx)}}$$

$$= \frac{2(b+a\tan(e+fx))}{(a^2+b^2)\,f\sqrt{d\sec(e+fx)}} - \frac{\left(2b\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\frac{1}{2}\left(-1+\frac{a^2}{b^2}\right)+\frac{ax}{2b^2}}{(a+x)\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{(a^2+b^2)\,f\sqrt{d\sec(e+fx)}}$$

$$= \frac{2(b+a\tan(e+fx))}{(a^2+b^2)\,f\sqrt{d\sec(e+fx)}} - \frac{\left(a\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{b\,(a^2+b^2)\,f\sqrt{d\sec(e+fx)}} + \frac{\left(b\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{(a+x)\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{(a^2+b^2)\,f\sqrt{d\sec(e+fx)}}$$

$$= -\frac{2a\tan(e+fx)}{(a^2+b^2) f\sqrt{d\sec(e+fx)}} + \frac{2(b+a\tan(e+fx))}{(a^2+b^2) f\sqrt{d\sec(e+fx)}} + \frac{\left(a\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{x^2}{b^2}\right)^{5/4}}\,dx, x, b\tan(e+fx)\right)}{b\,(a^2+b^2) f\sqrt{d\sec(e+fx)}} - \frac{\left(b\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{x}{(a^2-x^2)\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{(a^2+b^2) f\sqrt{d\sec(e+fx)}} + \frac{\left(ab\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{(a^2-x^2)\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{(a^2+b^2) f\sqrt{d\sec(e+fx)}}$$

$$= \frac{2aE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{(a^2+b^2) f\sqrt{d\sec(e+fx)}} - \frac{2a\tan(e+fx)}{(a^2+b^2) f\sqrt{d\sec(e+fx)}} + \frac{2(b+a\tan(e+fx))}{(a^2+b^2) f\sqrt{d\sec(e+fx)}} - \frac{\left(b\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{(a^2-x)\sqrt[4]{1+\frac{x}{b^2}}}\,dx, x, b^2\tan^2(e+fx)\right)}{2\,(a^2+b^2) f\sqrt{d\sec(e+fx)}} + \frac{\left(2a\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-x^4}\left(1+\frac{a^2}{b^2}-x^4\right)}\,dx, x, \sqrt[4]{\sec^2(e+fx)}\right.}{(a^2+b^2) f\sqrt{d\sec(e+fx)}}$$

$$= \frac{2aE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{(a^2+b^2)f\sqrt{d\sec(e+fx)}} - \frac{2a\tan(e+fx)}{(a^2+b^2)f\sqrt{d\sec(e+fx)}} + \frac{2(b+a\tan(e+fx))}{(a^2+b^2)f\sqrt{d\sec(e+fx)}} - \frac{\left(2b^3\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{x^2}{a^2+b^2-b^2x^4}\,dx,x,\sqrt[4]{\sec^2(e+fx)}\right)}{(a^2+b^2)f\sqrt{d\sec(e+fx)}} + \frac{\left(ab\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\left(\sqrt{a^2+b^2}-bx^2\right)\sqrt{1-x^4}}\,dx,x,\sqrt[4]{\sec^2(e+\right.}{(a^2+b^2)f\sqrt{d\sec(e+fx)}} - \frac{\left(ab\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\left(\sqrt{a^2+b^2}+bx^2\right)\sqrt{1-x^4}}\,dx,x,\sqrt[4]{\sec^2(e+\right.}{(a^2+b^2)f\sqrt{d\sec(e+fx)}}$$

$$= \frac{2aE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{(a^2+b^2)f\sqrt{d\sec(e+fx)}} - \frac{2a\tan(e+fx)}{(a^2+b^2)f\sqrt{d\sec(e+fx)}} + \frac{2(b+a\tan(e+fx))}{(a^2+b^2)f\sqrt{d\sec(e+fx)}} - \frac{\left(b^2\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{a^2+b^2}-bx^2}\,dx,x,\sqrt[4]{\sec^2(e+fx)}\right)}{(a^2+b^2)f\sqrt{d\sec(e+fx)}} + \frac{\left(b^2\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{a^2+b^2}+bx^2}\,dx,x,\sqrt[4]{\sec^2(e+fx)}\right)}{(a^2+b^2)f\sqrt{d\sec(e+fx)}} + \frac{\left(ab\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(\sqrt{a^2+b^2}-bx^2\right)}\,dx,x,\sqrt[4]{\text{sec}\right.}{(a^2+b^2)f\sqrt{d\sec(e+fx)}} - \frac{\left(ab\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(\sqrt{a^2+b^2}+bx^2\right)}\,dx,x,\sqrt[4]{\text{sec}\right.}{(a^2+b^2)f\sqrt{d\sec(e+fx)}}$$

$$= \frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{(a^2+b^2)^{5/4}f\sqrt{d\sec(e+fx)}} - \frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{(a^2+b^2)^{5/4}f\sqrt{d\sec(e+fx)}} + \frac{2aE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{(a^2+b^2)f\sqrt{d\sec(e+fx)}} - \frac{2a\tan(e+fx)}{(a^2+b^2)f\sqrt{d\sec(e+fx)}} - \frac{ab\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+}}{(a^2+b^2)^{3/2}f\sqrt{d\sec(e+fx)}} + \frac{ab\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx}}{(a^2+b^2)^{3/2}f\sqrt{d\sec(e+fx)}} + \frac{2(b+a\tan(e+fx))}{(a^2+b^2)f\sqrt{d\sec(e+fx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 4.48 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.63

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx = -\frac{28d\,\text{AppellF1}\left(\frac{5}{2},\frac{5}{4},\frac{5}{4},\frac{7}{2},\frac{a-ib}{a+b\tan(e+fx)},\frac{}{a-}\right.}{5bf(d\sec(e+fx))^{3/2}\left(5(a+ib)\,\text{AppellF1}\left(\frac{7}{2},\frac{5}{4},\frac{9}{4},\frac{9}{2},\frac{a-ib}{a+b\tan(e+fx)},\frac{a+ib}{a+b\tan(e+fx)}\right)+5(a-ib)\,\text{AppellF1}\left(\frac{7}{2}\right.\right.}$$

```
[In] Integrate[1/(Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])),x]
```

```
[Out] (-28*d*AppellF1[5/2, 5/4, 5/4, 7/2, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*
b)/(a + b*Tan[e + f*x])]*(a*Cos[e + f*x] + b*Sin[e + f*x]))/(5*b*f*(d*Sec[e
 + f*x])^(3/2)*(5*(a + I*b)*AppellF1[7/2, 5/4, 9/4, 9/2, (a - I*b)/(a + b*T
an[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])] + 5*(a - I*b)*AppellF1[7/2, 9
/4, 5/4, 9/2, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x]
)] + 14*AppellF1[5/2, 5/4, 5/4, 7/2, (a - I*b)/(a + b*Tan[e + f*x]), (a + I
*b)/(a + b*Tan[e + f*x])]*(a + b*Tan[e + f*x])))
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 6967 vs. 2(418) = 836.

Time = 9.99 (sec) , antiderivative size = 6968, normalized size of antiderivative = 15.45

method	result	size
default	Expression too large to display	6968

```
[In] int(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx = \text{Timed out}$$

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx = \int \frac{1}{\sqrt{d\sec{(e+fx)}}\,(a+b\tan{(e+fx)})}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))**(1/2)/(a+b*tan(f*x+e)),x)
[Out] Integral(1/(sqrt(d*sec(e + f*x))*(a + b*tan(e + f*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx = \int \frac{1}{\sqrt{d\sec{(fx+e)}}(b\tan{(fx+e)}+a)}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x, algorithm="maxima")
[Out] integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx=\int \frac{1}{\sqrt{d\sec\left(fx+e\right)}(b\tan\left(fx+e\right)+a)}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e)),x, algorithm="giac")
[Out] integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx=\int \frac{1}{\sqrt{\frac{d}{\cos(e+f\,x)}}\,\left(a+b\tan\left(e+f\,x\right)\right)}\,dx$$

```
[In] int(1/((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x))),x)
[Out] int(1/((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x))), x)
```

3.608 $\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 422

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}\,dx = -\frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{(a^2+b^2)^{7/4}f(d\sec(e+fx))^{3/2}} - \frac{b^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{(a^2+b^2)^{7/4}f(d\sec(e+fx))^{3/2}} + \frac{2a\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sec^2(e+fx)^{3/4}}{3(a^2+b^2)f(d\sec(e+fx))^{3/2}} + \frac{ab^2\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sec^2(e+fx)^{3/4}\sqrt{-\tan^2(e+fx)}}{(a^2+b^2)^2f(d\sec(e+fx))^{3/2}} + \frac{ab^2\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sec^2(e+fx)^{3/4}\sqrt{-\tan^2(e+fx)}}{(a^2+b^2)^2f(d\sec(e+fx))^{3/2}} + \frac{2(b+a\tan(e+fx))}{3(a^2+b^2)f(d\sec(e+fx))^{3/2}}$$

```
[Out] -b^(5/2)*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(sec(f*x+e)^2
)^(3/4)/(a^2+b^2)^(7/4)/f/(d*sec(f*x+e))^(3/2)-b^(5/2)*arctanh((sec(f*x+e)^
2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(sec(f*x+e)^2)^(3/4)/(a^2+b^2)^(7/4)/f/(d
*sec(f*x+e))^(3/2)+2/3*a*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arct
an(tan(f*x+e)))*EllipticF(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(sec(f*x+e)^
2)^(3/4)/(a^2+b^2)/f/(d*sec(f*x+e))^(3/2)+a*b^2*cot(f*x+e)*EllipticPi((sec(
f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2)^(3/4)*(-tan(f*x+e)^2)^
```

```
(1/2)/(a^2+b^2)^2/f/(d*sec(f*x+e))^(3/2)+a*b^2*cot(f*x+e)*EllipticPi((sec(f
*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2)^(3/4)*(-tan(f*x+e)^2)^(1
/2)/(a^2+b^2)^2/f/(d*sec(f*x+e))^(3/2)+2/3*(b+a*tan(f*x+e))/(a^2+b^2)/f/(d*
sec(f*x+e))^(3/2)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3593, 755, 858, 237, 761, 410, 109, 418, 1227, 551, 455, 65, 218, 214, 211}

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}\,dx = \frac{ab^2\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sec^2(e+fx)^{3/4}\,\text{EllipticPi}\left(-\right.}{f\left(a^2+b^2\right)^2(d\sec(e+fx))}$$
$$+\frac{ab^2\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sec^2(e+fx)^{3/4}\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)}{f\left(a^2+b^2\right)^2(d\sec(e+fx))^{3/2}}$$
$$+\frac{2a\sec^2(e+fx)^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)}{3f\left(a^2+b^2\right)(d\sec(e+fx))^{3/2}}$$
$$-\frac{b^{5/2}\sec^2(e+fx)^{3/4}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{f\left(a^2+b^2\right)^{7/4}(d\sec(e+fx))^{3/2}}$$
$$-\frac{b^{5/2}\sec^2(e+fx)^{3/4}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{f\left(a^2+b^2\right)^{7/4}(d\sec(e+fx))^{3/2}}+\frac{2(a\tan(e+fx)+b)}{3f\left(a^2+b^2\right)(d\sec(e+fx))^{3/2}}$$

```
[In] Int[1/((d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])),x]
```

```
[Out] -((b^(5/2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Sec[
e + f*x]^2)^(3/4))/((a^2 + b^2)^(7/4)*f*(d*Sec[e + f*x])^(3/2))) - (b^(5/2)
*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Sec[e + f*x]^
2)^(3/4))/((a^2 + b^2)^(7/4)*f*(d*Sec[e + f*x])^(3/2)) + (2*a*EllipticF[Arc
Tan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^(3/4))/(3*(a^2 + b^2)*f*(d*Sec[e +
 f*x])^(3/2)) + (a*b^2*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin
[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2)^(3/4)*Sqrt[-Tan[e + f*x]^2])
/((a^2 + b^2)^2*f*(d*Sec[e + f*x])^(3/2)) + (a*b^2*Cot[e + f*x]*EllipticPi[
b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2)^(3/
4)*Sqrt[-Tan[e + f*x]^2])/((a^2 + b^2)^2*f*(d*Sec[e + f*x])^(3/2)) + (2*(b
+ a*Tan[e + f*x]))/(3*(a^2 + b^2)*f*(d*Sec[e + f*x])^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 109

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e
/f) + d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-f/(d*e - c*f), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[(-b)*(x^2/a)]/(2*x), Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 761

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(3/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(3/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(3/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sec^2(e+fx)^{3/4}\text{Subst}\left(\int \frac{1}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{7/4}}\,dx, x, b\tan(e+fx)\right)}{bf(d\sec(e+fx))^{3/2}} \\
&= \frac{2(b+a\tan(e+fx))}{3\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}} \\
&\quad - \frac{\left(2b\sec^2(e+fx)^{3/4}\right)\text{Subst}\left(\int \frac{\frac{1}{2}\left(-3-\frac{a^2}{b^2}\right)-\frac{ax}{2b^2}}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{3\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}} \\
&= \frac{2(b+a\tan(e+fx))}{3\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}} \\
&\quad + \frac{\left(a\sec^2(e+fx)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{3b\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}} \\
&\quad + \frac{\left(b\sec^2(e+fx)^{3/4}\right)\text{Subst}\left(\int \frac{1}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}} \\
&= \frac{2a\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sec^2(e+fx)^{3/4}}{3\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}} \\
&\quad + \frac{2(b+a\tan(e+fx))}{3\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}} \\
&\quad - \frac{\left(b\sec^2(e+fx)^{3/4}\right)\text{Subst}\left(\int \frac{x}{\left(a^2-x^2\right)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}} \\
&\quad + \frac{\left(ab\sec^2(e+fx)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\left(a^2-x^2\right)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}}
\end{aligned}$$

$$= \frac{2a \operatorname{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sec^2(e+fx)^{3/4}}{3\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}} + \frac{2(b+a\tan(e+fx))}{3\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}} - \frac{\left(b\sec^2(e+fx)^{3/4}\right)\operatorname{Subst}\left(\int \frac{1}{(a^2-x)\left(1+\frac{x}{b^2}\right)^{3/4}}\,dx,x,b^2\tan^2(e+fx)\right)}{2\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}} + \frac{\left(a\cot(e+fx)\sec^2(e+fx)^{3/4}\sqrt{-\tan^2(e+fx)}\right)\operatorname{Subst}\left(\int \frac{1}{(a^2-x)\sqrt{-\frac{x}{b^2}}\left(1+\frac{x}{b^2}\right)^{3/4}}\,dx,x,b^2\tan^2(e+\right.}{2\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}}$$

$$= \frac{2a \operatorname{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sec^2(e+fx)^{3/4}}{3\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}} + \frac{2(b+a\tan(e+fx))}{3\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}} - \frac{\left(2b^3\sec^2(e+fx)^{3/4}\right)\operatorname{Subst}\left(\int \frac{1}{a^2+b^2-b^2x^4}\,dx,x,\sqrt[4]{\sec^2(e+fx)}\right)}{\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}} - \frac{\left(2a\cot(e+fx)\sec^2(e+fx)^{3/4}\sqrt{-\tan^2(e+fx)}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^4}\left(-1-\frac{a^2}{b^2}+x^4\right)}\,dx,x,\sqrt[4]{\sec^2(e+f}\right.}{\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}}$$

$$= \frac{2a \operatorname{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sec^2(e+fx)^{3/4}}{3\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}} + \frac{2(b+a\tan(e+fx))}{3\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}} - \frac{\left(b^3\sec^2(e+fx)^{3/4}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a^2+b^2}-bx^2}\,dx,x,\sqrt[4]{\sec^2(e+fx)}\right)}{\left(a^2+b^2\right)^{3/2}f(d\sec(e+fx))^{3/2}} - \frac{\left(b^3\sec^2(e+fx)^{3/4}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a^2+b^2}+bx^2}\,dx,x,\sqrt[4]{\sec^2(e+fx)}\right)}{\left(a^2+b^2\right)^{3/2}f(d\sec(e+fx))^{3/2}} + \frac{\left(ab^2\cot(e+fx)\sec^2(e+fx)^{3/4}\sqrt{-\tan^2(e+fx)}\right)\operatorname{Subst}\left(\int \frac{1}{\left(1-\frac{bx^2}{\sqrt{a^2+b^2}}\right)\sqrt{1-x^4}}\,dx,x,\sqrt[4]{\sec^2(e+}\right.}{\left(a^2+b^2\right)^2 f(d\sec(e+fx))^{3/2}} + \frac{\left(ab^2\cot(e+fx)\sec^2(e+fx)^{3/4}\sqrt{-\tan^2(e+fx)}\right)\operatorname{Subst}\left(\int \frac{1}{\left(1+\frac{bx^2}{\sqrt{a^2+b^2}}\right)\sqrt{1-x^4}}\,dx,x,\sqrt[4]{\sec^2(e+}\right.}{\left(a^2+b^2\right)^2 f(d\sec(e+fx))^{3/2}}$$

$$= -\frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{(a^2+b^2)^{7/4}f(d\sec(e+fx))^{3/2}} - \frac{b^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{(a^2+b^2)^{7/4}f(d\sec(e+fx))^{3/2}} + \frac{2a\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sec^2(e+fx)^{3/4}}{3(a^2+b^2)f(d\sec(e+fx))^{3/2}} + \frac{2(b+a\tan(e+fx))}{3(a^2+b^2)f(d\sec(e+fx))^{3/2}} + \frac{\left(ab^2\cot(e+fx)\sec^2(e+fx)^{3/4}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(1-\frac{bx^2}{\sqrt{a^2+b^2}}\right)}dx,x,\sqrt[4]{\text{se}}\right.}{(a^2+b^2)^2f(d\sec(e+fx))^{3/2}} + \frac{\left(ab^2\cot(e+fx)\sec^2(e+fx)^{3/4}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(1+\frac{bx^2}{\sqrt{a^2+b^2}}\right)}dx,x,\sqrt[4]{\text{se}}\right.}{(a^2+b^2)^2f(d\sec(e+fx))^{3/2}}$$

$$= -\frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{(a^2+b^2)^{7/4}f(d\sec(e+fx))^{3/2}} - \frac{b^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{(a^2+b^2)^{7/4}f(d\sec(e+fx))^{3/2}} + \frac{2a\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sec^2(e+fx)^{3/4}}{3(a^2+b^2)f(d\sec(e+fx))^{3/2}} + \frac{ab^2\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sec^2(e+fx)^{3/4}\sqrt{-\tan^2(e+}}{(a^2+b^2)^2f(d\sec(e+fx))^{3/2}} + \frac{ab^2\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sec^2(e+fx)^{3/4}\sqrt{-\tan^2(e+}}{(a^2+b^2)^2f(d\sec(e+fx))^{3/2}} + \frac{2(b+a\tan(e+fx))}{3(a^2+b^2)f(d\sec(e+fx))^{3/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.17 (sec) , antiderivative size = 418, normalized size of antiderivative = 0.99

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}\,dx = \frac{a^2b\sec^2(e+fx)+b^3\sec^2(e+fx)+a^2b\cos(2(e+fx))\sec^2(e\cdot}{}$$

```
[In] Integrate[1/((d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])),x]
```

```
[Out] (a^2*b*Sec[e + f*x]^2 + b^3*Sec[e + f*x]^2 + a^2*b*Cos[2*(e + f*x)]*Sec[e +
 f*x]^2 + b^3*Cos[2*(e + f*x)]*Sec[e + f*x]^2 - 3*b^(5/2)*(a^2 + b^2)^(1/4)
*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Sec[e + f*x]^2
)^(3/4) - 3*b^(5/2)*(a^2 + b^2)^(1/4)*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/
4))/(a^2 + b^2)^(1/4)]*(Sec[e + f*x]^2)^(3/4) + 2*a^3*Tan[e + f*x] + 2*a*b^
2*Tan[e + f*x] + a*(a^2 + b^2)*Hypergeometric2F1[1/2, 3/4, 3/2, -Tan[e + f*
x]^2]*(Sec[e + f*x]^2)^(3/4)*Tan[e + f*x] + 3*a*b^2*Cot[e + f*x]*EllipticPi
[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2)
^(3/4)*Sqrt[-Tan[e + f*x]^2] + 3*a*b^2*Cot[e + f*x]*EllipticPi[b/Sqrt[a^2 +
 b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2)^(3/4)*Sqrt[-Tan
[e + f*x]^2])/(3*(a^2 + b^2)^2*f*(d*Sec[e + f*x])^(3/2))
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 6051 vs. 2(391) = 782.

Time = 10.47 (sec) , antiderivative size = 6052, normalized size of antiderivative = 14.34

method	result	size
default	Expression too large to display	6052

```
[In] int(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}\,dx=\text{Timed out}$$

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}\,dx=\int \frac{1}{(d\sec{(e+fx)})^{\frac{3}{2}}\,(a+b\tan{(e+fx)})}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))**(3/2)/(a+b*tan(f*x+e)),x)
[Out] Integral(1/((d*sec(e + f*x))**(3/2)*(a + b*tan(e + f*x))), x)
```

Maxima [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}\,dx=\int \frac{1}{(d\sec{(fx+e)})^{\frac{3}{2}}\,(b\tan{(fx+e)}+a)}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x, algorithm="maxima")
[Out] integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)), x)
```

Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}\,dx=\int \frac{1}{(d\sec{(fx+e)})^{\frac{3}{2}}\,(b\tan{(fx+e)}+a)}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e)),x, algorithm="giac")
[Out] integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}\,dx=\int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}\,(a+b\tan\left(e+f\,x\right))}\,dx$$

```
[In] int(1/((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x))),x)
[Out] int(1/((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x))), x)
```

3.609 $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 568

$$\begin{aligned}\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}\,dx &= \frac{b^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{(a^2+b^2)^{9/4}\,d^2f\sqrt{d\sec(e+fx)}} \\
&- \frac{b^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{(a^2+b^2)^{9/4}\,d^2f\sqrt{d\sec(e+fx)}} \\
&+ \frac{2a(3a^2+8b^2)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{5\,(a^2+b^2)^2\,d^2f\sqrt{d\sec(e+fx)}} \\
&- \frac{2a(3a^2+8b^2)\tan(e+fx)}{5\,(a^2+b^2)^2\,d^2f\sqrt{d\sec(e+fx)}} \\
&- \frac{ab^3\cot(e+fx)\operatorname{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}}{(a^2+b^2)^{5/2}\,d^2f\sqrt{d\sec(e+fx)}} \\
&+ \frac{ab^3\cot(e+fx)\operatorname{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}}{(a^2+b^2)^{5/2}\,d^2f\sqrt{d\sec(e+fx)}} \\
&+ \frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5\,(a^2+b^2)\,d^2f\sqrt{d\sec(e+fx)}} + \frac{2(5b^3+a(3a^2+8b^2)\tan(e+fx))}{5\,(a^2+b^2)^2\,d^2f\sqrt{d\sec(e+fx)}}\end{aligned}$$

```
[Out] b^(7/2)*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(sec(f*x+e)^2)
^(1/4)/(a^2+b^2)^(9/4)/d^2/f/(d*sec(f*x+e))^(1/2)-b^(7/2)*arctanh((sec(f*x+
e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(9/4)/d
^2/f/(d*sec(f*x+e))^(1/2)+2/5*a*(3*a^2+8*b^2)*(cos(1/2*arctan(tan(f*x+e)))^
2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*EllipticE(sin(1/2*arctan(tan(f*x+e))),
2^(1/2))*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^2/d^2/f/(d*sec(f*x+e))^(1/2)-a*b^3*
```

```
cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(sec(f*x+e
)^2)^(1/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^(5/2)/d^2/f/(d*sec(f*x+e))^(1/2)
+a*b^3*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(sec
(f*x+e)^2)^(1/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^(5/2)/d^2/f/(d*sec(f*x+e))
^(1/2)-2/5*a*(3*a^2+8*b^2)*tan(f*x+e)/(a^2+b^2)^2/d^2/f/(d*sec(f*x+e))^(1/2
)+2/5*cos(f*x+e)^2*(b+a*tan(f*x+e))/(a^2+b^2)/d^2/f/(d*sec(f*x+e))^(1/2)+2/
5*(5*b^3+a*(3*a^2+8*b^2)*tan(f*x+e))/(a^2+b^2)^2/d^2/f/(d*sec(f*x+e))^(1/2)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3593, 755, 837, 858, 233, 202, 760, 408, 504, 1227, 551, 455, 65, 304, 211, 214}

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}\,dx = -\frac{ab^3\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)}{d^2f\left(a^2+b^2\right)^{5/2}\sqrt{d\sec(e+fx)}}$$
$$+\frac{ab^3\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)}{d^2f\left(a^2+b^2\right)^{5/2}\sqrt{d\sec(e+fx)}}$$
$$+\frac{2a(3a^2+8b^2)\sqrt[4]{\sec^2(e+fx)}E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)}{5d^2f\left(a^2+b^2\right)^2\sqrt{d\sec(e+fx)}}$$
$$+\frac{b^{7/2}\sqrt[4]{\sec^2(e+fx)}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{d^2f\left(a^2+b^2\right)^{9/4}\sqrt{d\sec(e+fx)}}$$
$$-\frac{b^{7/2}\sqrt[4]{\sec^2(e+fx)}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{d^2f\left(a^2+b^2\right)^{9/4}\sqrt{d\sec(e+fx)}}-\frac{2a(3a^2+8b^2)\tan(e+fx)}{5d^2f\left(a^2+b^2\right)^2\sqrt{d\sec(e+fx)}}$$
$$+\frac{2\cos^2(e+fx)(a\tan(e+fx)+b)}{5d^2f\left(a^2+b^2\right)\sqrt{d\sec(e+fx)}}+\frac{2(a(3a^2+8b^2)\tan(e+fx)+5b^3)}{5d^2f\left(a^2+b^2\right)^2\sqrt{d\sec(e+fx)}}$$

```
[In] Int[1/((d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])),x]
```

```
[Out] (b^(7/2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Sec[e
+ f*x]^2)^(1/4))/((a^2 + b^2)^(9/4)*d^2*f*Sqrt[d*Sec[e + f*x]]) - (b^(7/2)*
ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Sec[e + f*x]^2
)^(1/4))/((a^2 + b^2)^(9/4)*d^2*f*Sqrt[d*Sec[e + f*x]]) + (2*a*(3*a^2 + 8*b
^2)*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^(1/4))/(5*(a^2 +
b^2)^2*d^2*f*Sqrt[d*Sec[e + f*x]]) - (2*a*(3*a^2 + 8*b^2)*Tan[e + f*x])/(5*
(a^2 + b^2)^2*d^2*f*Sqrt[d*Sec[e + f*x]]) - (a*b^3*Cot[e + f*x]*EllipticPi[
-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2)^
```

```
(1/4)*Sqrt[-Tan[e + f*x]^2])/((a^2 + b^2)^(5/2)*d^2*f*Sqrt[d*Sec[e + f*x]])
 + (a*b^3*Cot[e + f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2
)^(1/4)], -1]*(Sec[e + f*x]^2)^(1/4)*Sqrt[-Tan[e + f*x]^2])/((a^2 + b^2)^(5
/2)*d^2*f*Sqrt[d*Sec[e + f*x]]) + (2*Cos[e + f*x]^2*(b + a*Tan[e + f*x]))/(
5*(a^2 + b^2)*d^2*f*Sqrt[d*Sec[e + f*x]]) + (2*(5*b^3 + a*(3*a^2 + 8*b^2)*T
an[e + f*x]))/(5*(a^2 + b^2)^2*d^2*f*Sqrt[d*Sec[e + f*x]])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
```

```
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 760

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
```

```
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt[4]{\sec^2(e+fx)}\text{Subst}\left(\int \frac{1}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{9/4}}\,dx, x, b\tan(e+fx)\right)}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$= \frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5\left(a^2+b^2\right)d^2f\sqrt{d\sec(e+fx)}} - \frac{\left(2b\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\frac{1}{2}\left(-5-\frac{3a^2}{b^2}\right)-\frac{3ax}{2b^2}}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{5/4}}\,dx, x, b\tan(e+fx)\right)}{5\left(a^2+b^2\right)d^2f\sqrt{d\sec(e+fx)}}$$

$$= \frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5(a^2+b^2)d^2f\sqrt{d\sec(e+fx)}} + \frac{2(5b^3+a(3a^2+8b^2)\tan(e+fx))}{5(a^2+b^2)^2d^2f\sqrt{d\sec(e+fx)}} + \frac{\left(4b^5\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{-\frac{3a^4+8a^2b^2-5b^4}{4b^6}-\frac{a(3a^2+8b^2)x}{4b^6}}{(a+x)\sqrt[4]{1+\frac{x^2}{b^2}}}dx,x,b\tan(e+fx)\right)}{5(a^2+b^2)^2d^2f\sqrt{d\sec(e+fx)}}$$

$$= \frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5(a^2+b^2)d^2f\sqrt{d\sec(e+fx)}} + \frac{2(5b^3+a(3a^2+8b^2)\tan(e+fx))}{5(a^2+b^2)^2d^2f\sqrt{d\sec(e+fx)}} + \frac{\left(b^3\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{(a+x)\sqrt[4]{1+\frac{x^2}{b^2}}}dx,x,b\tan(e+fx)\right)}{(a^2+b^2)^2d^2f\sqrt{d\sec(e+fx)}} - \frac{\left(a(3a^2+8b^2)\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt[4]{1+\frac{x^2}{b^2}}}dx,x,b\tan(e+fx)\right)}{5b(a^2+b^2)^2d^2f\sqrt{d\sec(e+fx)}}$$

$$= -\frac{2a(3a^2+8b^2)\tan(e+fx)}{5(a^2+b^2)^2d^2f\sqrt{d\sec(e+fx)}} + \frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5(a^2+b^2)d^2f\sqrt{d\sec(e+fx)}} + \frac{2(5b^3+a(3a^2+8b^2)\tan(e+fx))}{5(a^2+b^2)^2d^2f\sqrt{d\sec(e+fx)}} - \frac{\left(b^3\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{x}{(a^2-x^2)\sqrt[4]{1+\frac{x^2}{b^2}}}dx,x,b\tan(e+fx)\right)}{(a^2+b^2)^2d^2f\sqrt{d\sec(e+fx)}} + \frac{\left(ab^3\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{(a^2-x^2)\sqrt[4]{1+\frac{x^2}{b^2}}}dx,x,b\tan(e+fx)\right)}{(a^2+b^2)^2d^2f\sqrt{d\sec(e+fx)}} + \frac{\left(a(3a^2+8b^2)\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\left(1+\frac{x^2}{b^2}\right)^{5/4}}dx,x,b\tan(e+fx)\right)}{5b(a^2+b^2)^2d^2f\sqrt{d\sec(e+fx)}}$$

$$= \frac{2a(3a^2+8b^2)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{5\left(a^2+b^2\right)^2 d^2 f\sqrt{d\sec(e+fx)}}$$
$$-\frac{2a(3a^2+8b^2)\tan(e+fx)}{5\left(a^2+b^2\right)^2 d^2 f\sqrt{d\sec(e+fx)}}+\frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5\left(a^2+b^2\right) d^2 f\sqrt{d\sec(e+fx)}}$$
$$+\frac{2(5b^3+a(3a^2+8b^2)\tan(e+fx))}{5\left(a^2+b^2\right)^2 d^2 f\sqrt{d\sec(e+fx)}}$$
$$-\frac{\left(b^3\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{(a^2-x)\sqrt[4]{1+\frac{x}{b^2}}}\,dx,x,b^2\tan^2(e+fx)\right)}{2\left(a^2+b^2\right)^2 d^2 f\sqrt{d\sec(e+fx)}}$$
$$+\frac{\left(2ab^2\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{1-x^4}\left(1+\frac{a^2}{b^2}-x^4\right)}\,dx,x,\sqrt[4]{\sec^2(e+\right.}{\left(a^2+b^2\right)^2 d^2 f\sqrt{d\sec(e+fx)}}$$

$$= \frac{2a(3a^2+8b^2)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{5\left(a^2+b^2\right)^2 d^2 f\sqrt{d\sec(e+fx)}}$$
$$-\frac{2a(3a^2+8b^2)\tan(e+fx)}{5\left(a^2+b^2\right)^2 d^2 f\sqrt{d\sec(e+fx)}}+\frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5\left(a^2+b^2\right) d^2 f\sqrt{d\sec(e+fx)}}$$
$$+\frac{2(5b^3+a(3a^2+8b^2)\tan(e+fx))}{5\left(a^2+b^2\right)^2 d^2 f\sqrt{d\sec(e+fx)}}$$
$$-\frac{\left(2b^5\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{x^2}{a^2+b^2-b^2x^4}\,dx,x,\sqrt[4]{\sec^2(e+fx)}\right)}{\left(a^2+b^2\right)^2 d^2 f\sqrt{d\sec(e+fx)}}$$
$$+\frac{\left(ab^3\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\left(\sqrt{a^2+b^2}-bx^2\right)\sqrt{1-x^4}}\,dx,x,\sqrt[4]{\sec^2(e-\right.}{\left(a^2+b^2\right)^2 d^2 f\sqrt{d\sec(e+fx)}}$$
$$-\frac{\left(ab^3\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\left(\sqrt{a^2+b^2}+bx^2\right)\sqrt{1-x^4}}\,dx,x,\sqrt[4]{\sec^2(e-\right.}{\left(a^2+b^2\right)^2 d^2 f\sqrt{d\sec(e+fx)}}$$

$$= \frac{2a(3a^2+8b^2)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{5\left(a^2+b^2\right)^2 d^2 f\sqrt{d\sec(e+fx)}} - \frac{2a(3a^2+8b^2)\tan(e+fx)}{5\left(a^2+b^2\right)^2 d^2 f\sqrt{d\sec(e+fx)}} + \frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5\left(a^2+b^2\right) d^2 f\sqrt{d\sec(e+fx)}} + \frac{2(5b^3+a(3a^2+8b^2)\tan(e+fx))}{5\left(a^2+b^2\right)^2 d^2 f\sqrt{d\sec(e+fx)}} - \frac{\left(b^4\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{a^2+b^2}-bx^2}\,dx,x,\sqrt[4]{\sec^2(e+fx)}\right)}{\left(a^2+b^2\right)^2 d^2 f\sqrt{d\sec(e+fx)}} + \frac{\left(b^4\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{a^2+b^2}+bx^2}\,dx,x,\sqrt[4]{\sec^2(e+fx)}\right)}{\left(a^2+b^2\right)^2 d^2 f\sqrt{d\sec(e+fx)}} + \frac{\left(ab^3\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(\sqrt{a^2+b^2}-bx^2\right)}\,dx,x,\sqrt[4]{\sec^2(e+fx)}\right)}{\left(a^2+b^2\right)^2 d^2 f\sqrt{d\sec(e+fx)}} - \frac{\left(ab^3\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(\sqrt{a^2+b^2}+bx^2\right)}\,dx,x,\sqrt[4]{\sec^2(e+fx)}\right)}{\left(a^2+b^2\right)^2 d^2 f\sqrt{d\sec(e+fx)}}$$

$$= \frac{b^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{\left(a^2+b^2\right)^{9/4} d^2 f\sqrt{d\sec(e+fx)}} - \frac{b^{7/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{\left(a^2+b^2\right)^{9/4} d^2 f\sqrt{d\sec(e+fx)}} + \frac{2a(3a^2+8b^2)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{5\left(a^2+b^2\right)^2 d^2 f\sqrt{d\sec(e+fx)}} - \frac{2a(3a^2+8b^2)\tan(e+fx)}{5\left(a^2+b^2\right)^2 d^2 f\sqrt{d\sec(e+fx)}} - \frac{ab^3\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}}{\left(a^2+b^2\right)^{5/2} d^2 f\sqrt{d\sec(e+fx)}} + \frac{ab^3\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}}{\left(a^2+b^2\right)^{5/2} d^2 f\sqrt{d\sec(e+fx)}} + \frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5\left(a^2+b^2\right) d^2 f\sqrt{d\sec(e+fx)}} + \frac{2(5b^3+a(3a^2+8b^2)\tan(e+fx))}{5\left(a^2+b^2\right)^2 d^2 f\sqrt{d\sec(e+fx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 77.21 (sec) , antiderivative size = 2596, normalized size of antiderivative = 4.57

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}\,dx = \text{Result too large to show}$$

```
[In] Integrate[1/((d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])),x]
```

```
[Out] (3*a^2*Cos[2*(e + f*x)]*Sec[e + f*x]^(9/2)*(((1/4 + I/4)*(a^4 - b^4)*(2*Arc
Tan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sec[e + f*x]])/(a^2 + b^2)^(1/4)] - 2*ArcTan[
1 + ((1 + I)*Sqrt[b]*Sqrt[Sec[e + f*x]])/(a^2 + b^2)^(1/4)] - Log[Sqrt[a^2
+ b^2] - (1 + I)*Sqrt[b]*(a^2 + b^2)^(1/4)*Sqrt[Sec[e + f*x]] + I*b*Sec[e +
 f*x]] + Log[Sqrt[a^2 + b^2] + (1 + I)*Sqrt[b]*(a^2 + b^2)^(1/4)*Sqrt[Sec[e
 + f*x]] + I*b*Sec[e + f*x]]))/(Sqrt[b]*(a^2 + b^2)^(9/4)) + (4*(b + a*Sqrt
[1 - Cos[e + f*x]^2]*Sec[e + f*x]))/((a^2 + b^2)*Sqrt[Sec[e + f*x]]) + (2*a
*(3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, Sec[e + f*x]^2, (b^2*Sec[e + f*x]
^2)/(a^2 + b^2)]*Sqrt[1 - Cos[e + f*x]^2]*Sec[e + f*x]^(5/2))/(3*(a^2 + b^2
)^2*Sqrt[1 - Sec[e + f*x]^2]) - (4*a*b^2*AppellF1[7/4, 1/2, 1, 11/4, Sec[e
+ f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)]*Sqrt[1 - Cos[e + f*x]^2]*Sec[e
+ f*x]^(9/2))/(7*(a^2 + b^2)^2*Sqrt[1 - Sec[e + f*x]^2]))*(a + b*Sec[e + f*
x]*Sqrt[Cos[e + f*x]^2*(-1 + Sec[e + f*x]^2)])*Sin[e + f*x])/(10*(a - I*b)*
(a + I*b)*f*Sqrt[1 - Cos[e + f*x]^2]*(d*Sec[e + f*x])^(5/2)*(2 - Sec[e + f*
x]^2)*(a + b*Tan[e + f*x])) + (11*b^2*Cos[2*(e + f*x)]*Sec[e + f*x]^(9/2)*(
((1/4 + I/4)*(a^4 - b^4)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sec[e + f*x]])
/(a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sec[e + f*x]])/(a^
2 + b^2)^(1/4)] - Log[Sqrt[a^2 + b^2] - (1 + I)*Sqrt[b]*(a^2 + b^2)^(1/4)*S
qrt[Sec[e + f*x]] + I*b*Sec[e + f*x]] + Log[Sqrt[a^2 + b^2] + (1 + I)*Sqrt[
b]*(a^2 + b^2)^(1/4)*Sqrt[Sec[e + f*x]] + I*b*Sec[e + f*x]]))/(Sqrt[b]*(a^2
 + b^2)^(9/4)) + (4*(b + a*Sqrt[1 - Cos[e + f*x]^2]*Sec[e + f*x]))/((a^2 +
b^2)*Sqrt[Sec[e + f*x]]) + (2*a*(3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, Se
c[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)]*Sqrt[1 - Cos[e + f*x]^2]*Se
c[e + f*x]^(5/2))/(3*(a^2 + b^2)^2*Sqrt[1 - Sec[e + f*x]^2]) - (4*a*b^2*App
ellF1[7/4, 1/2, 1, 11/4, Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)]*
Sqrt[1 - Cos[e + f*x]^2]*Sec[e + f*x]^(9/2))/(7*(a^2 + b^2)^2*Sqrt[1 - Sec[
e + f*x]^2]))*(a + b*Sec[e + f*x]*Sqrt[Cos[e + f*x]^2*(-1 + Sec[e + f*x]^2)
])*Sin[e + f*x])/(20*(a - I*b)*(a + I*b)*f*Sqrt[1 - Cos[e + f*x]^2]*(d*Sec[
e + f*x])^(5/2)*(2 - Sec[e + f*x]^2)*(a + b*Tan[e + f*x])) + (a*b*Sec[e + f
*x]^(7/2)*(a + b*Sec[e + f*x]*Sqrt[Cos[e + f*x]^2*(-1 + Sec[e + f*x]^2)])*(
(4*(-a + b*Sqrt[1 - Cos[e + f*x]^2]*Sec[e + f*x]))/((a^2 + b^2)*Sqrt[Sec[e
+ f*x]]) + (56*b*(2*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, Sec[e + f*x]^2, (
b^2*Sec[e + f*x]^2)/(a^2 + b^2)]*Sqrt[1 - Cos[e + f*x]^2]*Sec[e + f*x]^(5/2
) - 24*b^3*AppellF1[7/4, 1/2, 1, 11/4, Sec[e + f*x]^2, (b^2*Sec[e + f*x]^2)
/(a^2 + b^2)]*Sqrt[1 - Cos[e + f*x]^2]*Sec[e + f*x]^(9/2) + (21 + 21*I)*a*S
```

```
qrt[b]*(a^2 + b^2)^(3/4)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Sec[e + f*x]])
/(a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Sec[e + f*x]])/(a^
2 + b^2)^(1/4)] - Log[Sqrt[a^2 + b^2] - (1 + I)*Sqrt[b]*(a^2 + b^2)^(1/4)*S
qrt[Sec[e + f*x]] + I*b*Sec[e + f*x]] + Log[Sqrt[a^2 + b^2] + (1 + I)*Sqrt[
b]*(a^2 + b^2)^(1/4)*Sqrt[Sec[e + f*x]] + I*b*Sec[e + f*x]])*Sqrt[1 - Sec[e
 + f*x]^2])/(42*(a^2 + b^2)^2*Sqrt[1 - Sec[e + f*x]^2]))*Sin[e + f*x]*Sin[2
*(e + f*x)])/(8*(a - I*b)*(a + I*b)*f*Sqrt[1 - Cos[e + f*x]^2]*(d*Sec[e + f
*x])^(5/2)*Sqrt[Cos[e + f*x]^2*(-1 + Sec[e + f*x]^2)]*(a + b*Tan[e + f*x]))
 + (Sec[e + f*x]^4*(a*Cos[e + f*x] + b*Sin[e + f*x])*((b*Cos[e + f*x])/(10*
(a - I*b)*(a + I*b)) + (b*Cos[3*(e + f*x)])/(10*(a - I*b)*(a + I*b)) + (a*S
in[e + f*x])/(10*(a - I*b)*(a + I*b)) + (a*Sin[3*(e + f*x)])/(10*(a - I*b)*
(a + I*b))))/(f*(d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])) - (18*a^2*Appe
llF1[1/2, 1/4, 1/4, 3/2, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*T
an[e + f*x])]*Sec[e + f*x]^3*(a*Cos[e + f*x] + b*Sin[e + f*x]))/(5*(a - I*b
)*(a + I*b)*b*f*(d*Sec[e + f*x])^(5/2)*((a + I*b)*AppellF1[3/2, 1/4, 5/4, 5
/2, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])] + (a -
I*b)*AppellF1[3/2, 5/4, 1/4, 5/2, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)
/(a + b*Tan[e + f*x])] + 6*AppellF1[1/2, 1/4, 1/4, 3/2, (a - I*b)/(a + b*Ta
n[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*(a + b*Tan[e + f*x]))) - (27*b
*AppellF1[1/2, 1/4, 1/4, 3/2, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a
+ b*Tan[e + f*x])]*Sec[e + f*x]^3*(a*Cos[e + f*x] + b*Sin[e + f*x]))/(5*(a
- I*b)*(a + I*b)*f*(d*Sec[e + f*x])^(5/2)*((a + I*b)*AppellF1[3/2, 1/4, 5/4
, 5/2, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])] + (a
 - I*b)*AppellF1[3/2, 5/4, 1/4, 5/2, (a - I*b)/(a + b*Tan[e + f*x]), (a + I
*b)/(a + b*Tan[e + f*x])] + 6*AppellF1[1/2, 1/4, 1/4, 3/2, (a - I*b)/(a + b
*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*(a + b*Tan[e + f*x])))
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 12840 vs. $2(525) = 1050$.

Time = 14.01 (sec) , antiderivative size = 12841, normalized size of antiderivative = 22.61

method	result	size
default	Expression too large to display	12841

```
[In] int(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}\,dx = \text{Timed out}$$

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}\,dx = \int \frac{1}{(d\sec{(e+fx)})^{\frac{5}{2}}\,(a+b\tan{(e+fx)})}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))**(5/2)/(a+b*tan(f*x+e)),x)
[Out] Integral(1/((d*sec(e + f*x))**(5/2)*(a + b*tan(e + f*x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}\,dx = \int \frac{1}{(d\sec{(fx+e)})^{\frac{5}{2}}\,(b\tan{(fx+e)}+a)}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e)),x, algorithm="giac")
[Out] integrate(1/((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))}\,dx = \int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}\,(a+b\tan\left(e+f\,x\right))}\,dx$$

```
[In] int(1/((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x))),x)
[Out] int(1/((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x))), x)
```

3.610 $\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 480

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^2}\,dx = -\frac{3ad^2\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{2b^{5/2}\sqrt[4]{a^2+b^2}\,f\sec^2(e+fx)^{3/4}}$$
$$+\frac{3ad^2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{2b^{5/2}\sqrt[4]{a^2+b^2}\,f\sec^2(e+fx)^{3/4}}$$
$$-\frac{3d^2E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{b^2f\sec^2(e+fx)^{3/4}}$$
$$+\frac{3d^2\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{b^2f}$$
$$+\frac{3a^2d^2\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}}{2b^3\sqrt{a^2+b^2}\,f\sec^2(e+fx)^{3/4}}$$
$$-\frac{3a^2d^2\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}}{2b^3\sqrt{a^2+b^2}\,f\sec^2(e+fx)^{3/4}}$$
$$-\frac{d^2(d\sec(e+fx))^{3/2}}{bf(a+b\tan(e+fx))}$$

```
[Out] -3/2*a*d^2*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(d*sec(f*x+
e))^(3/2)/b^(5/2)/(a^2+b^2)^(1/4)/f/(sec(f*x+e)^2)^(3/4)+3/2*a*d^2*arctanh(
(sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(d*sec(f*x+e))^(3/2)/b^(5/2)/
(a^2+b^2)^(1/4)/f/(sec(f*x+e)^2)^(3/4)-3*d^2*(cos(1/2*arctan(tan(f*x+e)))^2
)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*EllipticE(sin(1/2*arctan(tan(f*x+e))),2
^(1/2))*(d*sec(f*x+e))^(3/2)/b^2/f/(sec(f*x+e)^2)^(3/4)+3*d^2*cos(f*x+e)*(d
```

```
*sec(f*x+e))^(3/2)*sin(f*x+e)/b^2/f+3/2*a^2*d^2*cot(f*x+e)*EllipticPi((sec(
f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(3/2)*(-tan(f*x+e)^2)^
(1/2)/b^3/f/(sec(f*x+e)^2)^(3/4)/(a^2+b^2)^(1/2)-3/2*a^2*d^2*cot(f*x+e)*Ell
ipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(3/2)*(-ta
n(f*x+e)^2)^(1/2)/b^3/f/(sec(f*x+e)^2)^(3/4)/(a^2+b^2)^(1/2)-d^2*(d*sec(f*x
+e))^(3/2)/b/f/(a+b*tan(f*x+e))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3593, 747, 858, 233, 202, 760, 408, 504, 1227, 551, 455, 65, 304, 211, 214}

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^2}\,dx = \frac{3a^2d^2\sqrt{-\tan^2(e+fx)}\cot(e+fx)(d\sec(e+fx))^{3/2}\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\text{arcsi}\right.}{2b^3f\sqrt{a^2+b^2}\sec^2(e+fx)^{3/4}}$$
$$-\frac{3a^2d^2\sqrt{-\tan^2(e+fx)}\cot(e+fx)(d\sec(e+fx))^{3/2}\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)}{2b^3f\sqrt{a^2+b^2}\sec^2(e+fx)^{3/4}}$$
$$-\frac{3ad^2(d\sec(e+fx))^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2b^{5/2}f\sqrt[4]{a^2+b^2}\sec^2(e+fx)^{3/4}}$$
$$+\frac{3ad^2(d\sec(e+fx))^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2b^{5/2}f\sqrt[4]{a^2+b^2}\sec^2(e+fx)^{3/4}}-\frac{d^2(d\sec(e+fx))^{3/2}}{bf(a+b\tan(e+fx))}$$
$$-\frac{3d^2(d\sec(e+fx))^{3/2}E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)}{b^2f\sec^2(e+fx)^{3/4}}$$
$$+\frac{3d^2\sin(e+fx)\cos(e+fx)(d\sec(e+fx))^{3/2}}{b^2f}$$

```
[In] Int[(d*Sec[e + f*x])^(7/2)/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (-3*a*d^2*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(d*Sec
[e + f*x])^(3/2))/(2*b^(5/2)*(a^2 + b^2)^(1/4)*f*(Sec[e + f*x]^2)^(3/4)) +
(3*a*d^2*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(d*Sec
[e + f*x])^(3/2))/(2*b^(5/2)*(a^2 + b^2)^(1/4)*f*(Sec[e + f*x]^2)^(3/4)) -
(3*d^2*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(d*Sec[e + f*x])^(3/2))/(b^2*f*
(Sec[e + f*x]^2)^(3/4)) + (3*d^2*Cos[e + f*x]*(d*Sec[e + f*x])^(3/2)*Sin[e
+ f*x])/(b^2*f) + (3*a^2*d^2*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]),
ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(d*Sec[e + f*x])^(3/2)*Sqrt[-Tan[e + f*
x]^2])/(2*b^3*Sqrt[a^2 + b^2]*f*(Sec[e + f*x]^2)^(3/4)) - (3*a^2*d^2*Cot[e
+ f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(d
*Sec[e + f*x])^(3/2)*Sqrt[-Tan[e + f*x]^2])/(2*b^3*Sqrt[a^2 + b^2]*f*(Sec[e
 + f*x]^2)^(3/4)) - (d^2*(d*Sec[e + f*x])^(3/2))/(b*f*(a + b*Tan[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 747

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 1))), x] - Dist[2*c*(p/(e*(m + 1))
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 760

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
```

```
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\left(d^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int\frac{\left(1+\frac{x^2}{b^2}\right)^{3/4}}{(a+x)^2}\,dx,x,b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$=-\frac{d^2(d\sec(e+fx))^{3/2}}{bf(a+b\tan(e+fx))}+\frac{\left(3d^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int\frac{x}{(a+x)\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx,x,b\tan(e+fx)\right)}{2b^3f\sec^2(e+fx)^{3/4}}$$

$$=-\frac{d^2(d\sec(e+fx))^{3/2}}{bf(a+b\tan(e+fx))}+\frac{\left(3d^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int\frac{1}{\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx,x,b\tan(e+fx)\right)}{2b^3f\sec^2(e+fx)^{3/4}}-\frac{\left(3ad^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int\frac{1}{(a+x)\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx,x,b\tan(e+fx)\right)}{2b^3f\sec^2(e+fx)^{3/4}}$$

$$= \frac{3d^2\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{b^2f} - \frac{d^2(d\sec(e+fx))^{3/2}}{bf(a+b\tan(e+fx))}$$
$$- \frac{\left(3d^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{x^2}{b^2}\right)^{5/4}}\,dx, x, b\tan(e+fx)\right)}{2b^3f\sec^2(e+fx)^{3/4}}$$
$$+ \frac{\left(3ad^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{x}{(a^2-x^2)\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{2b^3f\sec^2(e+fx)^{3/4}}$$
$$- \frac{\left(3a^2d^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{1}{(a^2-x^2)\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{2b^3f\sec^2(e+fx)^{3/4}}$$

$$= -\frac{3d^2E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{b^2f\sec^2(e+fx)^{3/4}}$$
$$+ \frac{3d^2\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{b^2f} - \frac{d^2(d\sec(e+fx))^{3/2}}{bf(a+b\tan(e+fx))}$$
$$+ \frac{\left(3ad^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{1}{(a^2-x)\sqrt[4]{1+\frac{x}{b^2}}}\,dx, x, b^2\tan^2(e+fx)\right)}{4b^3f\sec^2(e+fx)^{3/4}}$$
$$- \frac{\left(3a^2d^2\cot(e+fx)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-x^4}\left(1+\frac{a^2}{b^2}-x^4\right)}\,dx, x, \sqrt[4]{\sec^2(e\right.}{b^4f\sec^2(e+fx)^{3/4}}$$

$$= -\frac{3d^2E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{b^2f\sec^2(e+fx)^{3/4}}$$
$$+ \frac{3d^2\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{b^2f} - \frac{d^2(d\sec(e+fx))^{3/2}}{bf(a+b\tan(e+fx))}$$
$$+ \frac{\left(3ad^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{x^2}{a^2+b^2-b^2x^4}\,dx, x, \sqrt[4]{\sec^2(e+fx)}\right)}{bf\sec^2(e+fx)^{3/4}}$$
$$- \frac{\left(3a^2d^2\cot(e+fx)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{a^2+b^2}-bx^2\right)\sqrt{1-x^4}}\,dx, x, \sqrt[4]{\sec^2\right.}{2b^3f\sec^2(e+fx)^{3/4}}$$
$$+ \frac{\left(3a^2d^2\cot(e+fx)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{a^2+b^2}+bx^2\right)\sqrt{1-x^4}}\,dx, x, \sqrt[4]{\sec^2\right.}{2b^3f\sec^2(e+fx)^{3/4}}$$

$$= -\frac{3d^2 E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{b^2 f \sec^2(e+fx)^{3/4}}$$

$$+ \frac{3d^2\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{b^2 f} - \frac{d^2(d\sec(e+fx))^{3/2}}{bf(a+b\tan(e+fx))}$$

$$+ \frac{\left(3ad^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{a^2+b^2}-bx^2}\,dx, x, \sqrt[4]{\sec^2(e+fx)}\right)}{2b^2 f \sec^2(e+fx)^{3/4}}$$

$$- \frac{\left(3ad^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{a^2+b^2}+bx^2}\,dx, x, \sqrt[4]{\sec^2(e+fx)}\right)}{2b^2 f \sec^2(e+fx)^{3/4}}$$

$$- \frac{\left(3a^2d^2\cot(e+fx)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(\sqrt{a^2+b^2}-bx^2\right)}\,dx, x,\right.}{2b^3 f \sec^2(e+fx)^{3/4}}$$

$$+ \frac{\left(3a^2d^2\cot(e+fx)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(\sqrt{a^2+b^2}+bx^2\right)}\,dx, x,\right.}{2b^3 f \sec^2(e+fx)^{3/4}}$$

$$= -\frac{3ad^2\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{2b^{5/2}\sqrt[4]{a^2+b^2}f\sec^2(e+fx)^{3/4}}$$

$$+ \frac{3ad^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{2b^{5/2}\sqrt[4]{a^2+b^2}f\sec^2(e+fx)^{3/4}}$$

$$- \frac{3d^2 E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{b^2 f \sec^2(e+fx)^{3/4}}$$

$$+ \frac{3d^2\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{b^2 f}$$

$$+ \frac{3a^2d^2\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}}, \arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right), -1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan}}{2b^3\sqrt{a^2+b^2}f\sec^2(e+fx)^{3/4}}$$

$$- \frac{3a^2d^2\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}}, \arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right), -1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2}}{2b^3\sqrt{a^2+b^2}f\sec^2(e+fx)^{3/4}}$$

$$- \frac{d^2(d\sec(e+fx))^{3/2}}{bf(a+b\tan(e+fx))}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 29.03 (sec) , antiderivative size = 1129, normalized size of antiderivative = 2.35

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^2}\,dx = \frac{\cos(e+fx)(d\sec(e+fx))^{7/2}(a\cos(e+fx)+b\sin(e+fx))^2\left(\frac{3\cos(e+fx)}{ab}+\frac{3\sin(\ldots}{\ldots}\right.}{f(a+b\tan(e+fx))^2}$$

$$+\frac{3(d\sec(e+fx))^{7/2}(a\cos(e+fx)+b\sin(e+fx))^2\left(-\frac{aE\left(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)\middle|-1\right)\sqrt{1+\tan^2\left(\frac{1}{2}(e+fx)\right)}}{\sqrt{1-\tan^2\left(\frac{1}{2}(e+fx)\right)}}+\frac{2a\,\mathrm{EllipticF}(\ldots}{\ldots}\right.}{\ldots}$$

```
[In] Integrate[(d*Sec[e + f*x])^(7/2)/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (Cos[e + f*x]*(d*Sec[e + f*x])^(7/2)*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*((
3*Cos[e + f*x])/(a*b) + (3*Sin[e + f*x])/b^2 - 1/(b*(a*Cos[e + f*x] + b*Sin
[e + f*x]))))/(f*(a + b*Tan[e + f*x])^2) + (3*(d*Sec[e + f*x])^(7/2)*(a*Cos
[e + f*x] + b*Sin[e + f*x])^2*(-((a*EllipticE[ArcSin[Tan[(e + f*x)/2]], -1]
*Sqrt[1 + Tan[(e + f*x)/2]^2])/Sqrt[1 - Tan[(e + f*x)/2]^2]) + (2*a*Ellipti
cF[ArcSin[Tan[(e + f*x)/2]], -1]*Sqrt[1 + Tan[(e + f*x)/2]^2])/Sqrt[1 - Tan
[(e + f*x)/2]^2] + (-2*Sqrt[2]*a*b*Sqrt[a^2 + b^2]*EllipticF[ArcSin[Sqrt[((
1 + I)*(1 + Tan[(e + f*x)/2]))/(I + Tan[(e + f*x)/2])]/Sqrt[2]], 2]*Sqrt[-(
(1 + I*Tan[(e + f*x)/2])/(I + Tan[(e + f*x)/2]))] + Sqrt[2]*a^2*Sqrt[a^2 +
b^2]*EllipticPi[((1 + I)*(a - I*(b + Sqrt[a^2 + b^2])))/(a + b + Sqrt[a^2 +
 b^2]), ArcSin[Sqrt[((1 + I)*(1 + Tan[(e + f*x)/2]))/(I + Tan[(e + f*x)/2])
]/Sqrt[2]], 2]*Sqrt[-((1 + I*Tan[(e + f*x)/2])/(I + Tan[(e + f*x)/2]))] + a
^2*(a + I*b + Sqrt[a^2 + b^2])*EllipticPi[((1 + I)*(a + I*(-b + Sqrt[a^2 +
b^2])))/(a + b - Sqrt[a^2 + b^2]), ArcSin[Sqrt[((1 + I)*(1 + Tan[(e + f*x)/
2]))/(I + Tan[(e + f*x)/2])]/Sqrt[2]], 2]*Sqrt[-((2 + (2*I)*Tan[(e + f*x)/2
])/(I + Tan[(e + f*x)/2]))] - a^3*EllipticPi[((1 + I)*(a - I*(b + Sqrt[a^2
+ b^2])))/(a + b + Sqrt[a^2 + b^2]), ArcSin[Sqrt[((1 + I)*(1 + Tan[(e + f*x
)/2]))/(I + Tan[(e + f*x)/2])]/Sqrt[2]], 2]*Sqrt[-((2 + (2*I)*Tan[(e + f*x)
/2])/(I + Tan[(e + f*x)/2]))] - I*a^2*b*EllipticPi[((1 + I)*(a - I*(b + Sqr
t[a^2 + b^2])))/(a + b + Sqrt[a^2 + b^2]), ArcSin[Sqrt[((1 + I)*(1 + Tan[(e
 + f*x)/2]))/(I + Tan[(e + f*x)/2])]/Sqrt[2]], 2]*Sqrt[-((2 + (2*I)*Tan[(e
+ f*x)/2])/(I + Tan[(e + f*x)/2]))] - 2*b^2*Sqrt[a^2 + b^2]*Sqrt[(-1 + Tan[
(e + f*x)/2]^2)/(I + Tan[(e + f*x)/2])^2] - 2*a*b*Sqrt[a^2 + b^2]*Tan[(e +
f*x)/2]*Sqrt[(-1 + Tan[(e + f*x)/2]^2)/(I + Tan[(e + f*x)/2])^2])/(2*b*Sqrt
[a^2 + b^2]*Sqrt[(-1 + Tan[(e + f*x)/2]^2)/(I + Tan[(e + f*x)/2])^2])))/(a*
```

```
b^2*f*Sec[e + f*x]^(3/2)*Sqrt[(1 + Tan[(e + f*x)/2]^2)/(1 - Tan[(e + f*x)/2
]^2)]*(a + b*Tan[e + f*x])^2)
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 33210 vs. 2(439) = 878.

Time = 132.90 (sec) , antiderivative size = 33211, normalized size of antiderivative = 69.19

method	result	size
default	Expression too large to display	33211

```
[In] int((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F]

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\sec(fx+e))^{\frac{7}{2}}}{(b\tan(fx+e)+a)^2}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(f*x + e))*d^3*sec(f*x + e)^3/(b^2*tan(f*x + e)^2 + 2*a*
b*tan(f*x + e) + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))**(7/2)/(a+b*tan(f*x+e))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\sec(fx+e))^{\frac{7}{2}}}{(b\tan(fx+e)+a)^2}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(7/2)/(b*tan(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{7/2}}{(a+b\tan(e+f\,x))^2}\,dx$$

```
[In] int((d/cos(e + f*x))^(7/2)/(a + b*tan(e + f*x))^2,x)
[Out] int((d/cos(e + f*x))^(7/2)/(a + b*tan(e + f*x))^2, x)
```

3.611 $\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 440

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx = \frac{ad^2\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{2b^{3/2}(a^2+b^2)^{3/4}f\sqrt[4]{\sec^2(e+fx)}}$$
$$+\frac{ad^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{2b^{3/2}(a^2+b^2)^{3/4}f\sqrt[4]{\sec^2(e+fx)}}$$
$$+\frac{d^2\operatorname{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{b^2f\sqrt[4]{\sec^2(e+fx)}}$$
$$-\frac{a^2d^2\cot(e+fx)\operatorname{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}{2b^2(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}}$$
$$-\frac{a^2d^2\cot(e+fx)\operatorname{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}{2b^2(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}}$$
$$-\frac{d^2\sqrt{d\sec(e+fx)}}{bf(a+b\tan(e+fx))}$$

```
[Out] 1/2*a*d^2*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(d*sec(f*x+e
))^(1/2)/b^(3/2)/(a^2+b^2)^(3/4)/f/(sec(f*x+e)^2)^(1/4)+1/2*a*d^2*arctanh((
sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(d*sec(f*x+e))^(1/2)/b^(3/2)/(
a^2+b^2)^(3/4)/f/(sec(f*x+e)^2)^(1/4)+d^2*(cos(1/2*arctan(tan(f*x+e)))^2)^(
1/2)/cos(1/2*arctan(tan(f*x+e)))*EllipticF(sin(1/2*arctan(tan(f*x+e))),2^(1
/2))*(d*sec(f*x+e))^(1/2)/b^2/f/(sec(f*x+e)^2)^(1/4)-1/2*a^2*d^2*cot(f*x+e)
*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(1/2)
*(-tan(f*x+e)^2)^(1/2)/b^2/(a^2+b^2)/f/(sec(f*x+e)^2)^(1/4)-1/2*a^2*d^2*cot
```

```
(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))
^(1/2)*(-tan(f*x+e)^2)^(1/2)/b^2/(a^2+b^2)/f/(sec(f*x+e)^2)^(1/4)-d^2*(d*se
c(f*x+e))^(1/2)/b/f/(a+b*tan(f*x+e))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3593, 747, 858, 237, 761, 410, 109, 418, 1227, 551, 455, 65, 218, 214, 211}

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx = -\frac{a^2d^2\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sqrt{d\sec(e+fx)}\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)}{2b^2f\left(a^2+b^2\right)\sqrt[4]{\sec^2(e+fx)}}$$
$$-\frac{a^2d^2\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sqrt{d\sec(e+fx)}\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)}{2b^2f\left(a^2+b^2\right)\sqrt[4]{\sec^2(e+fx)}}$$
$$+\frac{ad^2\sqrt{d\sec(e+fx)}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2b^{3/2}f\left(a^2+b^2\right)^{3/4}\sqrt[4]{\sec^2(e+fx)}}$$
$$+\frac{ad^2\sqrt{d\sec(e+fx)}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2b^{3/2}f\left(a^2+b^2\right)^{3/4}\sqrt[4]{\sec^2(e+fx)}}-\frac{d^2\sqrt{d\sec(e+fx)}}{bf(a+b\tan(e+fx))}$$
$$+\frac{d^2\sqrt{d\sec(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)}{b^2f\sqrt[4]{\sec^2(e+fx)}}$$

```
[In] Int[(d*Sec[e + f*x])^(5/2)/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (a*d^2*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*Sqrt[d*Se
c[e + f*x]])/(2*b^(3/2)*(a^2 + b^2)^(3/4)*f*(Sec[e + f*x]^2)^(1/4)) + (a*d^
2*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*Sqrt[d*Sec[e
+ f*x]])/(2*b^(3/2)*(a^2 + b^2)^(3/4)*f*(Sec[e + f*x]^2)^(1/4)) + (d^2*Elli
pticF[ArcTan[Tan[e + f*x]]/2, 2]*Sqrt[d*Sec[e + f*x]])/(b^2*f*(Sec[e + f*x]
^2)^(1/4)) - (a^2*d^2*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[
(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[d*Sec[e + f*x]]*Sqrt[-Tan[e + f*x]^2])/(2
*b^2*(a^2 + b^2)*f*(Sec[e + f*x]^2)^(1/4)) - (a^2*d^2*Cot[e + f*x]*Elliptic
Pi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[d*Sec[e + f*
x]]*Sqrt[-Tan[e + f*x]^2])/(2*b^2*(a^2 + b^2)*f*(Sec[e + f*x]^2)^(1/4)) - (
d^2*Sqrt[d*Sec[e + f*x]])/(b*f*(a + b*Tan[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 109

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e
/f) + d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-f/(d*e - c*f), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[(-b)*(x^2/a)]/(2*x), Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 747

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 1))), x] - Dist[2*c*(p/(e*(m + 1))
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 761

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(3/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(3/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(3/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\left(d^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt[4]{1+\frac{x^2}{b^2}}}{(a+x)^2}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{d^2\sqrt{d\sec(e+fx)}}{bf(a+b\tan(e+fx))} + \frac{\left(d^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{x}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{2b^3f\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{d^2\sqrt{d\sec(e+fx)}}{bf(a+b\tan(e+fx))} + \frac{\left(d^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{2b^3f\sqrt[4]{\sec^2(e+fx)}} - \frac{\left(ad^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{2b^3f\sqrt[4]{\sec^2(e+fx)}}$$

$$= \frac{d^2\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)), 2\right)\sqrt{d\sec(e+fx)}}{b^2f\sqrt[4]{\sec^2(e+fx)}} - \frac{d^2\sqrt{d\sec(e+fx)}}{bf(a+b\tan(e+fx))} + \frac{\left(ad^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{x}{(a^2-x^2)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{2b^3f\sqrt[4]{\sec^2(e+fx)}} - \frac{\left(a^2d^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{(a^2-x^2)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{2b^3f\sqrt[4]{\sec^2(e+fx)}}$$

$$= \frac{d^2 \text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{b^2 f\sqrt[4]{\sec^2(e+fx)}} - \frac{d^2\sqrt{d\sec(e+fx)}}{bf(a+b\tan(e+fx))}$$
$$+ \frac{\left(ad^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{(a^2-x)\left(1+\frac{x}{b^2}\right)^{3/4}}\,dx, x, b^2\tan^2(e+fx)\right)}{4b^3 f\sqrt[4]{\sec^2(e+fx)}}$$
$$- \frac{\left(a^2d^2\cot(e+fx)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{(a^2-x)\sqrt{-\frac{x}{b^2}}\left(1+\frac{x}{b^2}\right)^{3/4}}\,dx, x, b^2\tan^2(\right.}{4b^4 f\sqrt[4]{\sec^2(e+fx)}}$$

$$= \frac{d^2 \text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{b^2 f\sqrt[4]{\sec^2(e+fx)}} - \frac{d^2\sqrt{d\sec(e+fx)}}{bf(a+b\tan(e+fx))}$$
$$+ \frac{\left(ad^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{a^2+b^2-b^2x^4}\,dx, x, \sqrt[4]{\sec^2(e+fx)}\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$
$$+ \frac{\left(a^2d^2\cot(e+fx)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^4}\left(-1-\frac{a^2}{b^2}+x^4\right)}\,dx, x, \sqrt[4]{\sec^2(e+\right.}{b^4 f\sqrt[4]{\sec^2(e+fx)}}$$

$$= \frac{d^2 \text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{b^2 f\sqrt[4]{\sec^2(e+fx)}} - \frac{d^2\sqrt{d\sec(e+fx)}}{bf(a+b\tan(e+fx))}$$
$$+ \frac{\left(ad^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a^2+b^2}-bx^2}\,dx, x, \sqrt[4]{\sec^2(e+fx)}\right)}{2b\sqrt{a^2+b^2}f\sqrt[4]{\sec^2(e+fx)}}$$
$$+ \frac{\left(ad^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a^2+b^2}+bx^2}\,dx, x, \sqrt[4]{\sec^2(e+fx)}\right)}{2b\sqrt{a^2+b^2}f\sqrt[4]{\sec^2(e+fx)}}$$
$$- \frac{\left(a^2d^2\cot(e+fx)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{bx^2}{\sqrt{a^2+b^2}}\right)\sqrt{1-x^4}}\,dx, x, \sqrt[4]{\sec^2(e+\right.}{2b^2(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}}$$
$$- \frac{\left(a^2d^2\cot(e+fx)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{bx^2}{\sqrt{a^2+b^2}}\right)\sqrt{1-x^4}}\,dx, x, \sqrt[4]{\sec^2(e+\right.}{2b^2(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}}$$

$$= \frac{ad^2 \arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{2b^{3/2}\left(a^2+b^2\right)^{3/4} f\sqrt[4]{\sec^2(e+fx)}} + \frac{ad^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{2b^{3/2}\left(a^2+b^2\right)^{3/4} f\sqrt[4]{\sec^2(e+fx)}} + \frac{d^2 \operatorname{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{b^2 f\sqrt[4]{\sec^2(e+fx)}} - \frac{d^2\sqrt{d\sec(e+fx)}}{bf(a+b\tan(e+fx))} - \frac{\left(a^2d^2\cot(e+fx)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(1-\frac{bx^2}{\sqrt{a^2+b^2}}\right)}dx, x, \sqrt[4]{s\ldots}\right.}{2b^2\left(a^2+b^2\right)f\sqrt[4]{\sec^2(e+fx)}} - \frac{\left(a^2d^2\cot(e+fx)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(1+\frac{bx^2}{\sqrt{a^2+b^2}}\right)}dx, x, \sqrt[4]{s\ldots}\right.}{2b^2\left(a^2+b^2\right)f\sqrt[4]{\sec^2(e+fx)}}$$

$$= \frac{ad^2 \arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{2b^{3/2}\left(a^2+b^2\right)^{3/4} f\sqrt[4]{\sec^2(e+fx)}} + \frac{ad^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{2b^{3/2}\left(a^2+b^2\right)^{3/4} f\sqrt[4]{\sec^2(e+fx)}} + \frac{d^2 \operatorname{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{b^2 f\sqrt[4]{\sec^2(e+fx)}} - \frac{a^2d^2\cot(e+fx)\operatorname{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e\ldots}}{2b^2\left(a^2+b^2\right)f\sqrt[4]{\sec^2(e+fx)}} - \frac{a^2d^2\cot(e+fx)\operatorname{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e\ldots}}{2b^2\left(a^2+b^2\right)f\sqrt[4]{\sec^2(e+fx)}} - \frac{d^2\sqrt{d\sec(e+fx)}}{bf(a+b\tan(e+fx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 12.02 (sec) , antiderivative size = 378, normalized size of antiderivative = 0.86

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx = \frac{(d\sec(e+fx))^{5/2}(a\cos(e+fx)+b\sin(e+fx))^2\left(-\frac{1}{ab}+\frac{\sin(e+fx)}{a(a\cos(e+fx)+b\sin(e+fx))}\right.}{f(a+b\tan(e+fx))^2}$$

$$+\frac{\cos^2(e+fx)(d\sec(e+fx))^{5/2}\sec^2(e+fx)^{3/4}(a\cos(e+fx)+b\sin(e+fx))^2\left(\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\right.\right.}{}$$

```
[In] Integrate[(d*Sec[e + f*x])^(5/2)/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] ((d*Sec[e + f*x])^(5/2)*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(-(1/(a*b)) + S
in[e + f*x]/(a*(a*Cos[e + f*x] + b*Sin[e + f*x]))))/(f*(a + b*Tan[e + f*x])
^2) + (Cos[e + f*x]^2*(d*Sec[e + f*x])^(5/2)*(Sec[e + f*x]^2)^(3/4)*(a*Cos[
e + f*x] + b*Sin[e + f*x])^2*(Hypergeometric2F1[1/2, 3/4, 3/2, -Tan[e + f*x
]^2]*Tan[e + f*x] + (a*(a*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e +
f*x]^2)^(1/4)], -1]*Tan[e + f*x] + a*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(
Sec[e + f*x]^2)^(1/4)], -1]*Tan[e + f*x] + Sqrt[b]*(a^2 + b^2)^(1/4)*(ArcTa
n[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)] + ArcTanh[(Sqrt[b]*(S
ec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)])*Sqrt[-Tan[e + f*x]^2]))/((a^2 + b
^2)*Sqrt[-Tan[e + f*x]^2])))/(2*b^2*f*(a + b*Tan[e + f*x])^2)
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4811 vs. 2(405) = 810.

Time = 128.71 (sec) , antiderivative size = 4812, normalized size of antiderivative = 10.94

method	result	size
default	Expression too large to display	4812

```
[In] int((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*d^2/f*(-d*(csc(f*x+e)^2*(1-cos(f*x+e))^2+1)/(csc(f*x+e)^2*(1-cos(f*x+e
))^2-1))^(1/2)*(csc(f*x+e)^2*(1-cos(f*x+e))^2-1)*(8*I*(b*((a^2+b^2)^(1/2)*a
^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2
+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*(csc(f*x+e)^2*(1-cos(f*x+e
))^2+1)^(1/2)*(-csc(f*x+e)^2*(1-cos(f*x+e))^2+1)^(1/2)*EllipticPi(I*(csc(f*
x+e)-cot(f*x+e)),-1/(-b+(a^2+b^2)^(1/2))^2*a^2,I)*a*b*(csc(f*x+e)-cot(f*x+e
```

))+4*I*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1
/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2
)*(csc(f*x+e)^2*(1-cos(f*x+e))^2+1)^(1/2)*(-csc(f*x+e)^2*(1-cos(f*x+e))^2+1
)^(1/2)*EllipticPi(I*(csc(f*x+e)-cot(f*x+e)),-1/(-b+(a^2+b^2)^(1/2))^2*a^2,
I)*a^2-4*I*csc(f*x+e)^2*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2
*b+2*b^3)/a^4)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b
-2*b^3)/a^4)^(1/2)*(csc(f*x+e)^2*(1-cos(f*x+e))^2+1)^(1/2)*(-csc(f*x+e)^2*(
1-cos(f*x+e))^2+1)^(1/2)*EllipticPi(I*(csc(f*x+e)-cot(f*x+e)),-1/(-b+(a^2+b
^2)^(1/2))^2*a^2,I)*a^2*(1-cos(f*x+e))^2-4*I*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2
+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b
^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*(csc(f*x+e)^2*(1-cos(f*x+e))^2+1)^(
1/2)*(-csc(f*x+e)^2*(1-cos(f*x+e))^2+1)^(1/2)*EllipticPi(I*(csc(f*x+e)-cot(
f*x+e)),-1/(b+(a^2+b^2)^(1/2))^2*a^2,I)*a^2-8*I*(b*((a^2+b^2)^(1/2)*a^2+2*(
a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^
2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*(csc(f*x+e)^2*(1-cos(f*x+e))^2+1
)^(1/2)*(-csc(f*x+e)^2*(1-cos(f*x+e))^2+1)^(1/2)*EllipticPi(I*(csc(f*x+e)-c
ot(f*x+e)),-1/(b+(a^2+b^2)^(1/2))^2*a^2,I)*a*b*(csc(f*x+e)-cot(f*x+e))+4*I*
csc(f*x+e)^2*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a
^4)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4
)^(1/2)*(csc(f*x+e)^2*(1-cos(f*x+e))^2+1)^(1/2)*(-csc(f*x+e)^2*(1-cos(f*x+e
))^2+1)^(1/2)*EllipticPi(I*(csc(f*x+e)-cot(f*x+e)),-1/(b+(a^2+b^2)^(1/2))^2
*a^2,I)*a^2*(1-cos(f*x+e))^2+4*csc(f*x+e)^4*(a^2+b^2)^(1/2)*(b*((a^2+b^2)^(
1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*(-b*((a^2+b^2)^(1/
2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*a*(1-cos(f*x+e))^4+c
sc(f*x+e)^2*(a^2+b^2)^(1/2)*(csc(f*x+e)^4*(1-cos(f*x+e))^4-1)^(1/2)*(b*((a^
2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*arctanh(1/
2*(2*csc(f*x+e)^2*(1-cos(f*x+e))^2*b*(a^2+b^2)^(1/2)-csc(f*x+e)^2*a^2*(1-co
s(f*x+e))^2-2*csc(f*x+e)^2*b^2*(1-cos(f*x+e))^2+a^2)/(-b*((a^2+b^2)^(1/2)*a
^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)/(csc(f*x+e)^4*(1-cos(f*x
+e))^4-1)^(1/2)/a^2)*a*(1-cos(f*x+e))^2-csc(f*x+e)^2*(a^2+b^2)^(1/2)*(csc(f
x+e)^4(1-cos(f*x+e))^4-1)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2
)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*arctanh(1/2*(2*csc(f*x+e)^2*(1-cos(f*x+e))^
2*b*(a^2+b^2)^(1/2)+csc(f*x+e)^2*a^2*(1-cos(f*x+e))^2+2*csc(f*x+e)^2*b^2*(1
-cos(f*x+e))^2-a^2)/(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2
*b^3)/a^4)^(1/2)/(csc(f*x+e)^4*(1-cos(f*x+e))^4-1)^(1/2)/a^2)*a*(1-cos(f*x+
e))^2-csc(f*x+e)^2*(csc(f*x+e)^4*(1-cos(f*x+e))^4-1)^(1/2)*(b*((a^2+b^2)^(1
/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*arctanh(1/2*(2*csc(
f*x+e)^2*(1-cos(f*x+e))^2*b*(a^2+b^2)^(1/2)-csc(f*x+e)^2*a^2*(1-cos(f*x+e))
^2-2*csc(f*x+e)^2*b^2*(1-cos(f*x+e))^2+a^2)/(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2
+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)/(csc(f*x+e)^4*(1-cos(f*x+e))^4-1)
^(1/2)/a^2)*a*b*(1-cos(f*x+e))^2-csc(f*x+e)^2*(csc(f*x+e)^4*(1-cos(f*x+e))^
4-1)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^
4)^(1/2)*arctanh(1/2*(2*csc(f*x+e)^2*(1-cos(f*x+e))^2*b*(a^2+b^2)^(1/2)+csc
(f*x+e)^2*a^2*(1-cos(f*x+e))^2+2*csc(f*x+e)^2*b^2*(1-cos(f*x+e))^2-a^2)/(b*
((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)/(csc(f

```
*x+e)^4*(1-cos(f*x+e))^4-1)^(1/2)/a^2)*a*b*(1-cos(f*x+e))^2-2*(a^2+b^2)^(1/
2)*(csc(f*x+e)^4*(1-cos(f*x+e))^4-1)^(1/2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b
^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*arctanh(1/2*(2*csc(f*x+e)^2*(1-cos(
f*x+e))^2*b*(a^2+b^2)^(1/2)-csc(f*x+e)^2*a^2*(1-cos(f*x+e))^2-2*csc(f*x+e)^
2*b^2*(1-cos(f*x+e))^2+a^2)/(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-
2*a^2*b-2*b^3)/a^4)^(1/2)/(csc(f*x+e)^4*(1-cos(f*x+e))^4-1)^(1/2)/a^2)*b*(c
sc(f*x+e)-cot(f*x+e))+2*(a^2+b^2)^(1/2)*(csc(f*x+e)^4*(1-cos(f*x+e))^4-1)^(
1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/
2)*arctanh(1/2*(2*csc(f*x+e)^2*(1-cos(f*x+e))^2*b*(a^2+b^2)^(1/2)+csc(f*x+e
)^2*a^2*(1-cos(f*x+e))^2+2*csc(f*x+e)^2*b^2*(1-cos(f*x+e))^2-a^2)/(b*((a^2+
b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)/(csc(f*x+e)^
4*(1-cos(f*x+e))^4-1)^(1/2)/a^2)*b*(csc(f*x+e)-cot(f*x+e))+2*(csc(f*x+e)^4*
(1-cos(f*x+e))^4-1)^(1/2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a
^2*b+2*b^3)/a^4)^(1/2)*arctanh(1/2*(2*csc(f*x+e)^2*(1-cos(f*x+e))^2*b*(a^2+
b^2)^(1/2)-csc(f*x+e)^2*a^2*(1-cos(f*x+e))^2-2*csc(f*x+e)^2*b^2*(1-cos(f*x+
e))^2+a^2)/(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^
4)^(1/2)/(csc(f*x+e)^4*(1-cos(f*x+e))^4-1)^(1/2)/a^2)*b^2*(csc(f*x+e)-cot(f
*x+e))+2*(csc(f*x+e)^4*(1-cos(f*x+e))^4-1)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2
*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*arctanh(1/2*(2*csc(f*x+e)^2*
(1-cos(f*x+e))^2*b*(a^2+b^2)^(1/2)+csc(f*x+e)^2*a^2*(1-cos(f*x+e))^2+2*csc(
f*x+e)^2*b^2*(1-cos(f*x+e))^2-a^2)/(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2
)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)/(csc(f*x+e)^4*(1-cos(f*x+e))^4-1)^(1/2)/a^2
)*b^2*(csc(f*x+e)-cot(f*x+e))-(a^2+b^2)^(1/2)*(csc(f*x+e)^4*(1-cos(f*x+e))^
4-1)^(1/2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4
)^(1/2)*arctanh(1/2*(2*csc(f*x+e)^2*(1-cos(f*x+e))^2*b*(a^2+b^2)^(1/2)-csc(
f*x+e)^2*a^2*(1-cos(f*x+e))^2-2*csc(f*x+e)^2*b^2*(1-cos(f*x+e))^2+a^2)/(-b*
((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)/(csc(f
*x+e)^4*(1-cos(f*x+e))^4-1)^(1/2)/a^2)*a+(a^2+b^2)^(1/2)*(csc(f*x+e)^4*(1-c
os(f*x+e))^4-1)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*
b-2*b^3)/a^4)^(1/2)*arctanh(1/2*(2*csc(f*x+e)^2*(1-cos(f*x+e))^2*b*(a^2+b^2
)^(1/2)+csc(f*x+e)^2*a^2*(1-cos(f*x+e))^2+2*csc(f*x+e)^2*b^2*(1-cos(f*x+e))
^2-a^2)/(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(
1/2)/(csc(f*x+e)^4*(1-cos(f*x+e))^4-1)^(1/2)/a^2)*a+(csc(f*x+e)^4*(1-cos(f*
x+e))^4-1)^(1/2)*(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^
3)/a^4)^(1/2)*arctanh(1/2*(2*csc(f*x+e)^2*(1-cos(f*x+e))^2*b*(a^2+b^2)^(1/2
)-csc(f*x+e)^2*a^2*(1-cos(f*x+e))^2-2*csc(f*x+e)^2*b^2*(1-cos(f*x+e))^2+a^2
)/(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)/
(csc(f*x+e)^4*(1-cos(f*x+e))^4-1)^(1/2)/a^2)*a*b+(csc(f*x+e)^4*(1-cos(f*x+e
))^4-1)^(1/2)*(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)
/a^4)^(1/2)*arctanh(1/2*(2*csc(f*x+e)^2*(1-cos(f*x+e))^2*b*(a^2+b^2)^(1/2)+
csc(f*x+e)^2*a^2*(1-cos(f*x+e))^2+2*csc(f*x+e)^2*b^2*(1-cos(f*x+e))^2-a^2)/
(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)/(cs
c(f*x+e)^4*(1-cos(f*x+e))^4-1)^(1/2)/a^2)*a*b-4*(a^2+b^2)^(1/2)*(b*((a^2+b^
2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*b^3)/a^4)^(1/2)*(-b*((a^2+b^2)
^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b-2*b^3)/a^4)^(1/2)*a)/(csc(f*x+e)^2
```

```
*a*(1-cos(f*x+e))^2-2*b*(csc(f*x+e)-cot(f*x+e))-a)/(csc(f*x+e)^4*(1-cos(f*x
+e))^4-1)^(1/2)/((csc(f*x+e)^2*(1-cos(f*x+e))^2+1)*(csc(f*x+e)^2*(1-cos(f*x
+e))^2-1))^(1/2)/a/b/(-b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2-2*a^2*b
-2*b^3)/a^4)^(1/2)/(b*((a^2+b^2)^(1/2)*a^2+2*(a^2+b^2)^(1/2)*b^2+2*a^2*b+2*
b^3)/a^4)^(1/2)/(a^2+b^2)^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))**(5/2)/(a+b*tan(f*x+e))**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{2}}}{(b\tan(fx+e)+a)^2}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e) + a)^2, x)
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{2}}}{(b\tan(fx+e)+a)^2}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}}{(a+b\tan(e+f\,x))^2}\,dx$$

```
[In] int((d/cos(e + f*x))^(5/2)/(a + b*tan(e + f*x))^2,x)
[Out] int((d/cos(e + f*x))^(5/2)/(a + b*tan(e + f*x))^2, x)
```

3.612 $\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 477

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}\,dx = \frac{a\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{2\sqrt{b}\,(a^2+b^2)^{5/4}\,f\sec^2(e+fx)^{3/4}} - \frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{2\sqrt{b}\,(a^2+b^2)^{5/4}\,f\sec^2(e+fx)^{3/4}} - \frac{E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{(a^2+b^2)\,f\sec^2(e+fx)^{3/4}} + \frac{\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{(a^2+b^2)\,f} - \frac{a^2\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}}{2b\,(a^2+b^2)^{3/2}\,f\sec^2(e+fx)^{3/4}} + \frac{a^2\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}}{2b\,(a^2+b^2)^{3/2}\,f\sec^2(e+fx)^{3/4}} - \frac{b(d\sec(e+fx))^{3/2}}{(a^2+b^2)\,f(a+b\tan(e+fx))}$$

```
[Out] -(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*Elliptic
E(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(d*sec(f*x+e))^(3/2)/(a^2+b^2)/f/(se
c(f*x+e)^2)^(3/4)+cos(f*x+e)*(d*sec(f*x+e))^(3/2)*sin(f*x+e)/(a^2+b^2)/f+1/
2*a*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(d*sec(f*x+e))^(3/
2)/(a^2+b^2)^(5/4)/f/(sec(f*x+e)^2)^(3/4)/b^(1/2)-1/2*a*arctanh((sec(f*x+e)
^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(d*sec(f*x+e))^(3/2)/(a^2+b^2)^(5/4)/f/(
```

```
sec(f*x+e)^2)^(3/4)/b^(1/2)-1/2*a^2*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1
/4),-b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(3/2)*(-tan(f*x+e)^2)^(1/2)/b/(a^2
+b^2)^(3/2)/f/(sec(f*x+e)^2)^(3/4)+1/2*a^2*cot(f*x+e)*EllipticPi((sec(f*x+e
)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(3/2)*(-tan(f*x+e)^2)^(1/2)/
b/(a^2+b^2)^(3/2)/f/(sec(f*x+e)^2)^(3/4)-b*(d*sec(f*x+e))^(3/2)/(a^2+b^2)/f
/(a+b*tan(f*x+e))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3593, 759, 858, 233, 202, 760, 408, 504, 1227, 551, 455, 65, 304, 211, 214}

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}\,dx =$$

$$-\frac{a^2\sqrt{-\tan^2(e+fx)}\cot(e+fx)(d\sec(e+fx))^{3/2}\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)}{2bf\left(a^2+b^2\right)^{3/2}\sec^2(e+fx)^{3/4}}$$
$$+\frac{a^2\sqrt{-\tan^2(e+fx)}\cot(e+fx)(d\sec(e+fx))^{3/2}\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)}{2bf\left(a^2+b^2\right)^{3/2}\sec^2(e+fx)^{3/4}}$$
$$+\frac{a(d\sec(e+fx))^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2\sqrt{b}f\left(a^2+b^2\right)^{5/4}\sec^2(e+fx)^{3/4}}$$
$$-\frac{(d\sec(e+fx))^{3/2}E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)}{f\left(a^2+b^2\right)\sec^2(e+fx)^{3/4}}$$
$$-\frac{a(d\sec(e+fx))^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2\sqrt{b}f\left(a^2+b^2\right)^{5/4}\sec^2(e+fx)^{3/4}}$$
$$-\frac{b(d\sec(e+fx))^{3/2}}{f\left(a^2+b^2\right)(a+b\tan(e+fx))}+\frac{\sin(e+fx)\cos(e+fx)(d\sec(e+fx))^{3/2}}{f\left(a^2+b^2\right)}$$

```
[In] Int[(d*Sec[e + f*x])^(3/2)/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (a*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(d*Sec[e + f*
x])^(3/2))/(2*Sqrt[b]*(a^2 + b^2)^(5/4)*f*(Sec[e + f*x]^2)^(3/4)) - (a*ArcT
anh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(d*Sec[e + f*x])^(3
/2))/(2*Sqrt[b]*(a^2 + b^2)^(5/4)*f*(Sec[e + f*x]^2)^(3/4)) - (EllipticE[Ar
cTan[Tan[e + f*x]]/2, 2]*(d*Sec[e + f*x])^(3/2))/((a^2 + b^2)*f*(Sec[e + f*
x]^2)^(3/4)) + (Cos[e + f*x]*(d*Sec[e + f*x])^(3/2)*Sin[e + f*x])/((a^2 + b
^2)*f) - (a^2*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e +
 f*x]^2)^(1/4)], -1]*(d*Sec[e + f*x])^(3/2)*Sqrt[-Tan[e + f*x]^2])/(2*b*(a^
2 + b^2)^(3/2)*f*(Sec[e + f*x]^2)^(3/4)) + (a^2*Cot[e + f*x]*EllipticPi[b/S
```

qrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(d*Sec[e + f*x])^(3/2)*
Sqrt[-Tan[e + f*x]^2])/(2*b*(a^2 + b^2)^(3/2)*f*(Sec[e + f*x]^2)^(3/4)) - (
b*(d*Sec[e + f*x])^(3/2))/((a^2 + b^2)*f*(a + b*Tan[e + f*x]))

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 760

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{(d\sec(e+fx))^{3/2}\text{Subst}\left(\int \frac{1}{(a+x)^2\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$= -\frac{b(d\sec(e+fx))^{3/2}}{(a^2+b^2)\,f(a+b\tan(e+fx))} - \frac{(d\sec(e+fx))^{3/2}\text{Subst}\left(\int \frac{-a-\frac{x}{2}}{(a+x)\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{b\,(a^2+b^2)\,f\sec^2(e+fx)^{3/4}}$$

$$= -\frac{b(d\sec(e+fx))^{3/2}}{(a^2+b^2)\,f(a+b\tan(e+fx))} + \frac{(d\sec(e+fx))^{3/2}\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{2b\,(a^2+b^2)\,f\sec^2(e+fx)^{3/4}} + \frac{\left(a(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{1}{(a+x)\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{2b\,(a^2+b^2)\,f\sec^2(e+fx)^{3/4}}$$

$$= \frac{\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{(a^2+b^2)f} - \frac{b(d\sec(e+fx))^{3/2}}{(a^2+b^2)f(a+b\tan(e+fx))} - \frac{(d\sec(e+fx))^{3/2}\text{Subst}\left(\int \frac{1}{\left(1+\frac{x^2}{b^2}\right)^{5/4}}\,dx, x, b\tan(e+fx)\right)}{2b(a^2+b^2)f\sec^2(e+fx)^{3/4}} - \frac{\left(a(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{x}{(a^2-x^2)\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{2b(a^2+b^2)f\sec^2(e+fx)^{3/4}} + \frac{\left(a^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{1}{(a^2-x^2)\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{2b(a^2+b^2)f\sec^2(e+fx)^{3/4}}$$

$$= -\frac{E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{(a^2+b^2)f\sec^2(e+fx)^{3/4}} + \frac{\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{(a^2+b^2)f} - \frac{b(d\sec(e+fx))^{3/2}}{(a^2+b^2)f(a+b\tan(e+fx))} - \frac{\left(a(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{1}{(a^2-x)\sqrt[4]{1+\frac{x}{b^2}}}\,dx, x, b^2\tan^2(e+fx)\right)}{4b(a^2+b^2)f\sec^2(e+fx)^{3/4}} + \frac{\left(a^2\cot(e+fx)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-x^4}\left(1+\frac{a^2}{b^2}-x^4\right)}\,dx, x, \sqrt[4]{\sec^2(e+fx)}\right)}{b^2(a^2+b^2)f\sec^2(e+fx)^{3/4}}$$

$$= -\frac{E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{(a^2+b^2)f\sec^2(e+fx)^{3/4}} + \frac{\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{(a^2+b^2)f} - \frac{b(d\sec(e+fx))^{3/2}}{(a^2+b^2)f(a+b\tan(e+fx))} - \frac{\left(ab(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{x^2}{a^2+b^2-b^2x^4}\,dx, x, \sqrt[4]{\sec^2(e+fx)}\right)}{(a^2+b^2)f\sec^2(e+fx)^{3/4}} + \frac{\left(a^2\cot(e+fx)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{a^2+b^2}-bx^2\right)\sqrt{1-x^4}}\,dx, x, \sqrt[4]{\sec^2(e+fx)}\right)}{2b(a^2+b^2)f\sec^2(e+fx)^{3/4}} - \frac{\left(a^2\cot(e+fx)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{a^2+b^2}+bx^2\right)\sqrt{1-x^4}}\,dx, x, \sqrt[4]{\sec^2(e+fx)}\right)}{2b(a^2+b^2)f\sec^2(e+fx)^{3/4}}$$

$$= -\frac{E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{(a^2+b^2)f\sec^2(e+fx)^{3/4}} + \frac{\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{(a^2+b^2)f} - \frac{b(d\sec(e+fx))^{3/2}}{(a^2+b^2)f(a+b\tan(e+fx))} - \frac{\left(a(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{a^2+b^2-bx^2}}dx,x,\sqrt[4]{\sec^2(e+fx)}\right)}{2(a^2+b^2)f\sec^2(e+fx)^{3/4}} + \frac{\left(a(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{a^2+b^2+bx^2}}dx,x,\sqrt[4]{\sec^2(e+fx)}\right)}{2(a^2+b^2)f\sec^2(e+fx)^{3/4}} + \frac{\left(a^2\cot(e+fx)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(\sqrt{a^2+b^2}-bx^2\right)}dx,x,\sqrt[4]{s\ldots}\right.}{2b(a^2+b^2)f\sec^2(e+fx)^{3/4}} - \frac{\left(a^2\cot(e+fx)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(\sqrt{a^2+b^2}+bx^2\right)}dx,x,\sqrt[4]{\ldots}\right.}{2b(a^2+b^2)f\sec^2(e+fx)^{3/4}}$$

$$= \frac{a\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{2\sqrt{b}(a^2+b^2)^{5/4}f\sec^2(e+fx)^{3/4}} - \frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{2\sqrt{b}(a^2+b^2)^{5/4}f\sec^2(e+fx)^{3/4}} - \frac{E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{(a^2+b^2)f\sec^2(e+fx)^{3/4}} + \frac{\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{(a^2+b^2)f} - \frac{a^2\cot(e+fx)\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e\ldots}}{2b(a^2+b^2)^{3/2}f\sec^2(e+fx)^{3/4}} + \frac{a^2\cot(e+fx)\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e-\ldots}}{2b(a^2+b^2)^{3/2}f\sec^2(e+fx)^{3/4}} - \frac{b(d\sec(e+fx))^{3/2}}{(a^2+b^2)f(a+b\tan(e+fx))}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 27.97 (sec) , antiderivative size = 1125, normalized size of antiderivative = 2.36

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}\,dx = \frac{\sec(e+fx)(d\sec(e+fx))^{3/2}(a\cos(e+fx)+b\sin(e+fx))^2\left(\frac{b\cos(e+fx)}{a(a-ib)(a+ib)}+\frac{\mathrm{s}}{(a\cdots}\right.}{f(a+b\tan(e+fx))^2}$$

$$+\frac{\sqrt{\sec(e+fx)}(d\sec(e+fx))^{3/2}(a\cos(e+fx)+b\sin(e+fx))^2\left(-\frac{aE\left(\arcsin\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)\middle|-1\right)\sqrt{1+\tan^2\left(\frac{1}{2}(e+fx)\right)}}{\sqrt{1-\tan^2\left(\frac{1}{2}(e+fx)\right)}}\cdots\right.}{\cdots}$$

```
[In] Integrate[(d*Sec[e + f*x])^(3/2)/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (Sec[e + f*x]*(d*Sec[e + f*x])^(3/2)*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*((
b*Cos[e + f*x])/(a*(a - I*b)*(a + I*b)) + Sin[e + f*x]/((a - I*b)*(a + I*b)
) - b/((a - I*b)*(a + I*b)*(a*Cos[e + f*x] + b*Sin[e + f*x]))))/(f*(a + b*T
an[e + f*x])^2) + (Sqrt[Sec[e + f*x]]*(d*Sec[e + f*x])^(3/2)*(a*Cos[e + f*x
] + b*Sin[e + f*x])^2*(-((a*EllipticE[ArcSin[Tan[(e + f*x)/2]], -1]*Sqrt[1
+ Tan[(e + f*x)/2]^2])/Sqrt[1 - Tan[(e + f*x)/2]^2]) - (-2*Sqrt[2]*a*b*Sqrt
[a^2 + b^2]*EllipticF[ArcSin[Sqrt[((1 + I)*(1 + Tan[(e + f*x)/2]))/(I + Tan
[(e + f*x)/2])]/Sqrt[2]], 2]*Sqrt[-((1 + I*Tan[(e + f*x)/2])/(I + Tan[(e +
f*x)/2]))] + Sqrt[2]*a^2*Sqrt[a^2 + b^2]*EllipticPi[((1 + I)*(a - I*(b + Sq
rt[a^2 + b^2])))/(a + b + Sqrt[a^2 + b^2]), ArcSin[Sqrt[((1 + I)*(1 + Tan[(
e + f*x)/2]))/(I + Tan[(e + f*x)/2])]/Sqrt[2]], 2]*Sqrt[-((1 + I*Tan[(e + f
*x)/2])/(I + Tan[(e + f*x)/2]))] + a^2*(a + I*b + Sqrt[a^2 + b^2])*Elliptic
Pi[((1 + I)*(a + I*(-b + Sqrt[a^2 + b^2])))/(a + b - Sqrt[a^2 + b^2]), ArcS
in[Sqrt[((1 + I)*(1 + Tan[(e + f*x)/2]))/(I + Tan[(e + f*x)/2])]/Sqrt[2]],
2]*Sqrt[-((2 + (2*I)*Tan[(e + f*x)/2])/(I + Tan[(e + f*x)/2]))] - a^3*Ellip
ticPi[((1 + I)*(a - I*(b + Sqrt[a^2 + b^2])))/(a + b + Sqrt[a^2 + b^2]), Ar
cSin[Sqrt[((1 + I)*(1 + Tan[(e + f*x)/2]))/(I + Tan[(e + f*x)/2])]/Sqrt[2]]
, 2]*Sqrt[-((2 + (2*I)*Tan[(e + f*x)/2])/(I + Tan[(e + f*x)/2]))] - I*a^2*b
*EllipticPi[((1 + I)*(a - I*(b + Sqrt[a^2 + b^2])))/(a + b + Sqrt[a^2 + b^2
]), ArcSin[Sqrt[((1 + I)*(1 + Tan[(e + f*x)/2]))/(I + Tan[(e + f*x)/2])]/Sq
rt[2]], 2]*Sqrt[-((2 + (2*I)*Tan[(e + f*x)/2])/(I + Tan[(e + f*x)/2]))] + 2
*b^2*Sqrt[a^2 + b^2]*Sqrt[(-1 + Tan[(e + f*x)/2]^2)/(I + Tan[(e + f*x)/2])^
2] + 2*a*b*Sqrt[a^2 + b^2]*Tan[(e + f*x)/2]*Sqrt[(-1 + Tan[(e + f*x)/2]^2)/
(I + Tan[(e + f*x)/2])^2])/(2*b*Sqrt[a^2 + b^2]*Sqrt[(-1 + Tan[(e + f*x)/2]
^2)/(I + Tan[(e + f*x)/2])^2])))/(a*(a^2 + b^2)*f*Sqrt[(1 + Tan[(e + f*x)/2
]^2)/(1 - Tan[(e + f*x)/2]^2)]*(a + b*Tan[e + f*x])^2)
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 16453 vs. 2(436) = 872.

Time = 8.87 (sec) , antiderivative size = 16454, normalized size of antiderivative = 34.49

method	result	size
default	Expression too large to display	16454

```
[In] int((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\sec(e+fx))^{\frac{3}{2}}}{(a+b\tan(e+fx))^2}\,dx$$

```
[In] integrate((d*sec(f*x+e))**(3/2)/(a+b*tan(f*x+e))**2,x)
[Out] Integral((d*sec(e + f*x))**(3/2)/(a + b*tan(e + f*x))**2, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}\,dx=\int \frac{(d\sec(fx+e))^{\frac{3}{2}}}{(b\tan(fx+e)+a)^2}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(3/2)/(b*tan(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^2}\,dx=\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}{(a+b\tan(e+f\,x))^2}\,dx$$

```
[In] int((d/cos(e + f*x))^(3/2)/(a + b*tan(e + f*x))^2,x)
[Out] int((d/cos(e + f*x))^(3/2)/(a + b*tan(e + f*x))^2, x)
```

3.613 $\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 430

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx = -\frac{3a\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{2(a^2+b^2)^{7/4}f\sqrt[4]{\sec^2(e+fx)}}$$
$$-\frac{3a\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{2(a^2+b^2)^{7/4}f\sqrt[4]{\sec^2(e+fx)}}$$
$$-\frac{\operatorname{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}}$$
$$+\frac{3a^2\cot(e+fx)\operatorname{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}{2(a^2+b^2)^2f\sqrt[4]{\sec^2(e+fx)}}$$
$$+\frac{3a^2\cot(e+fx)\operatorname{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}{2(a^2+b^2)^2f\sqrt[4]{\sec^2(e+fx)}}$$
$$-\frac{b\sqrt{d\sec(e+fx)}}{(a^2+b^2)f(a+b\tan(e+fx))}$$

```
[Out] -(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*Elliptic
F(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(d*sec(f*x+e))^(1/2)/(a^2+b^2)/f/(se
c(f*x+e)^2)^(1/4)-3/2*a*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4)
)*b^(1/2)*(d*sec(f*x+e))^(1/2)/(a^2+b^2)^(7/4)/f/(sec(f*x+e)^2)^(1/4)-3/2*a
*arctanh((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*b^(1/2)*(d*sec(f*x+e
))^(1/2)/(a^2+b^2)^(7/4)/f/(sec(f*x+e)^2)^(1/4)+3/2*a^2*cot(f*x+e)*Elliptic
Pi((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(1/2)*(-tan(f*
x+e)^2)^(1/2)/(a^2+b^2)^2/f/(sec(f*x+e)^2)^(1/4)+3/2*a^2*cot(f*x+e)*Ellipti
```

```
cPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(1/2)*(-tan(f*
x+e)^2)^(1/2)/(a^2+b^2)^2/f/(sec(f*x+e)^2)^(1/4)-b*(d*sec(f*x+e))^(1/2)/(a^
2+b^2)/f/(a+b*tan(f*x+e))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3593, 759, 858, 237, 761, 410, 109, 418, 1227, 551, 455, 65, 218, 214, 211}

$$\begin{aligned}&\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx\\&=\frac{3a^2\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sqrt{d\sec(e+fx)}\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)}{2f\left(a^2+b^2\right)^2\sqrt[4]{\sec^2(e+fx)}}\\&+\frac{3a^2\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sqrt{d\sec(e+fx)}\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)}{2f\left(a^2+b^2\right)^2\sqrt[4]{\sec^2(e+fx)}}\\&-\frac{3a\sqrt{b}\sqrt{d\sec(e+fx)}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2f\left(a^2+b^2\right)^{7/4}\sqrt[4]{\sec^2(e+fx)}}\\&-\frac{\sqrt{d\sec(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)}{f\left(a^2+b^2\right)\sqrt[4]{\sec^2(e+fx)}}\\&-\frac{3a\sqrt{b}\sqrt{d\sec(e+fx)}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2f\left(a^2+b^2\right)^{7/4}\sqrt[4]{\sec^2(e+fx)}}-\frac{b\sqrt{d\sec(e+fx)}}{f\left(a^2+b^2\right)(a+b\tan(e+fx))}\end{aligned}$$

```
[In] Int[Sqrt[d*Sec[e + f*x]]/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (-3*a*Sqrt[b]*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*Sq
rt[d*Sec[e + f*x]])/(2*(a^2 + b^2)^(7/4)*f*(Sec[e + f*x]^2)^(1/4)) - (3*a*S
qrt[b]*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*Sqrt[d*S
ec[e + f*x]])/(2*(a^2 + b^2)^(7/4)*f*(Sec[e + f*x]^2)^(1/4)) - (EllipticF[A
rcTan[Tan[e + f*x]]/2, 2]*Sqrt[d*Sec[e + f*x]])/((a^2 + b^2)*f*(Sec[e + f*x
]^2)^(1/4)) + (3*a^2*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(
Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[d*Sec[e + f*x]]*Sqrt[-Tan[e + f*x]^2])/(2*
(a^2 + b^2)^2*f*(Sec[e + f*x]^2)^(1/4)) + (3*a^2*Cot[e + f*x]*EllipticPi[b/
Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[d*Sec[e + f*x]]*S
qrt[-Tan[e + f*x]^2])/(2*(a^2 + b^2)^2*f*(Sec[e + f*x]^2)^(1/4)) - (b*Sqrt[
d*Sec[e + f*x]])/((a^2 + b^2)*f*(a + b*Tan[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 109

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e
/f) + d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-f/(d*e - c*f), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[(-b)*(x^2/a)]/(2*x), Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 761

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(3/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(3/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(3/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{d\sec(e+fx)}\text{Subst}\left(\int \frac{1}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{b\sqrt{d\sec(e+fx)}}{(a^2+b^2)\,f(a+b\tan(e+fx))} - \frac{\sqrt{d\sec(e+fx)}\text{Subst}\left(\int \frac{-a+\frac{x}{2}}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{b\,(a^2+b^2)\,f\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{b\sqrt{d\sec(e+fx)}}{(a^2+b^2)\,f(a+b\tan(e+fx))} - \frac{\sqrt{d\sec(e+fx)}\text{Subst}\left(\int \frac{1}{\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{2b\,(a^2+b^2)\,f\sqrt[4]{\sec^2(e+fx)}} + \frac{\left(3a\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{2b\,(a^2+b^2)\,f\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{(a^2+b^2)\,f\sqrt[4]{\sec^2(e+fx)}} - \frac{b\sqrt{d\sec(e+fx)}}{(a^2+b^2)\,f(a+b\tan(e+fx))} - \frac{\left(3a\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{x}{(a^2-x^2)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+\right.}{2b\,(a^2+b^2)\,f\sqrt[4]{\sec^2(e+fx)}} + \frac{\left(3a^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{(a^2-x^2)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{2b\,(a^2+b^2)\,f\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}} - \frac{b\sqrt{d\sec(e+fx)}}{(a^2+b^2)f(a+b\tan(e+fx))} - \frac{\left(3a\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int\frac{1}{(a^2-x)\left(1+\frac{x}{b^2}\right)^{3/4}}dx,x,b^2\tan^2(e+\right.}{4b(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}} + \frac{\left(3a^2\cot(e+fx)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{(a^2-x)\sqrt{-\frac{x}{b^2}}\left(1+\frac{x}{b^2}\right)^{3/4}}dx,x,b^2\tan^2(e\right.}{4b^2(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}} - \frac{b\sqrt{d\sec(e+fx)}}{(a^2+b^2)f(a+b\tan(e+fx))} - \frac{\left(3ab\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int\frac{1}{a^2+b^2-b^2x^4}dx,x,\sqrt[4]{\sec^2(e+fx)}\right)}{(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}} - \frac{\left(3a^2\cot(e+fx)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^4}\left(-1-\frac{a^2}{b^2}+x^4\right)}dx,x,\sqrt[4]{\sec^2(e+\right.}{b^2(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}} - \frac{b\sqrt{d\sec(e+fx)}}{(a^2+b^2)f(a+b\tan(e+fx))} - \frac{\left(3ab\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{a^2+b^2}-bx^2}dx,x,\sqrt[4]{\sec^2(e+fx)}\right)}{2(a^2+b^2)^{3/2}f\sqrt[4]{\sec^2(e+fx)}} - \frac{\left(3ab\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{a^2+b^2}+bx^2}dx,x,\sqrt[4]{\sec^2(e+fx)}\right)}{2(a^2+b^2)^{3/2}f\sqrt[4]{\sec^2(e+fx)}} + \frac{\left(3a^2\cot(e+fx)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\left(1-\frac{bx^2}{\sqrt{a^2+b^2}}\right)\sqrt{1-x^4}}dx,x,\sqrt[4]{\sec^2(e+\right.}{2(a^2+b^2)^2f\sqrt[4]{\sec^2(e+fx)}} + \frac{\left(3a^2\cot(e+fx)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\left(1+\frac{bx^2}{\sqrt{a^2+b^2}}\right)\sqrt{1-x^4}}dx,x,\sqrt[4]{\sec^2(e+\right.}{2(a^2+b^2)^2f\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{3a\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{2(a^2+b^2)^{7/4}f\sqrt[4]{\sec^2(e+fx)}}$$
$$-\frac{3a\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{2(a^2+b^2)^{7/4}f\sqrt[4]{\sec^2(e+fx)}}$$
$$-\frac{\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}}$$
$$-\frac{b\sqrt{d\sec(e+fx)}}{(a^2+b^2)f(a+b\tan(e+fx))}$$
$$+\frac{\left(3a^2\cot(e+fx)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(1-\frac{bx^2}{\sqrt{a^2+b^2}}\right)}dx,x,\sqrt[4]{\text{se}\right.}{2(a^2+b^2)^2f\sqrt[4]{\sec^2(e+fx)}}$$
$$+\frac{\left(3a^2\cot(e+fx)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(1+\frac{bx^2}{\sqrt{a^2+b^2}}\right)}dx,x,\sqrt[4]{\text{se}\right.}{2(a^2+b^2)^2f\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{3a\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{2(a^2+b^2)^{7/4}f\sqrt[4]{\sec^2(e+fx)}}$$
$$-\frac{3a\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{2(a^2+b^2)^{7/4}f\sqrt[4]{\sec^2(e+fx)}}$$
$$-\frac{\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}}$$
$$+\frac{3a^2\cot(e+fx)\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e}}{2(a^2+b^2)^2f\sqrt[4]{\sec^2(e+fx)}}$$
$$+\frac{3a^2\cot(e+fx)\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+}}{2(a^2+b^2)^2f\sqrt[4]{\sec^2(e+fx)}}$$
$$-\frac{b\sqrt{d\sec(e+fx)}}{(a^2+b^2)f(a+b\tan(e+fx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.76 (sec) , antiderivative size = 422, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx$$
$$= \frac{\sec^2(e+fx)\sqrt{d\sec(e+fx)}(a\cos(e+fx)+b\sin(e+fx))^2\left(-\frac{b}{a(a-ib)(a+ib)}+\frac{b^2\sin(e+fx)}{a(a-ib)(a+ib)(a\cos(e+fx)+b\sin(e+f\right.}}{f(a+b\tan(e+fx))^2}$$
$$+\frac{\sqrt{d\sec(e+fx)}\sec^2(e+fx)^{3/4}(a\cos(e+fx)+b\sin(e+fx))^2\left(-\left((a^2+b^2)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\right.\right.\right.}{}$$

```
[In] Integrate[Sqrt[d*Sec[e + f*x]]/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (Sec[e + f*x]^2*Sqrt[d*Sec[e + f*x]]*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(-
(b/(a*(a - I*b)*(a + I*b))) + (b^2*Sin[e + f*x])/(a*(a - I*b)*(a + I*b)*(a*
Cos[e + f*x] + b*Sin[e + f*x]))))/(f*(a + b*Tan[e + f*x])^2) + (Sqrt[d*Sec[
e + f*x]]*(Sec[e + f*x]^2)^(3/4)*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*(-((a^
2 + b^2)*Hypergeometric2F1[1/2, 3/4, 3/2, -Tan[e + f*x]^2]*Tan[e + f*x]) +
3*a*(-(Sqrt[b]*(a^2 + b^2)^(1/4)*(ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(
a^2 + b^2)^(1/4)] + ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1
/4)])) + a*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*
x]^2)^(1/4)], -1]*Sqrt[-Tan[e + f*x]^2] + a*Cot[e + f*x]*EllipticPi[b/Sqrt[
a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[-Tan[e + f*x]^2])))/(2
*(a^2 + b^2)^2*f*(a + b*Tan[e + f*x])^2)
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 12205 vs. 2(395) = 790.

Time = 11.21 (sec) , antiderivative size = 12206, normalized size of antiderivative = 28.39

method	result	size
default	Expression too large to display	12206

```
[In] int((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{\sqrt{d\sec{(e+fx)}}}{(a+b\tan{(e+fx)})^2}\,dx$$

```
[In] integrate((d*sec(f*x+e))**(1/2)/(a+b*tan(f*x+e))**2,x)
[Out] Integral(sqrt(d*sec(e + f*x))/(a + b*tan(e + f*x))**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{\sqrt{d\sec(fx+e)}}{(b\tan(fx+e)+a)^2}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
[Out] integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e) + a)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{\sqrt{d\sec(fx+e)}}{(b\tan(fx+e)+a)^2}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
[Out] integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{\sqrt{\frac{d}{\cos(e+f\,x)}}}{\left(a+b\tan\left(e+f\,x\right)\right)^2}\,dx$$

```
[In] int((d/cos(e + f*x))^(1/2)/(a + b*tan(e + f*x))^2,x)
[Out] int((d/cos(e + f*x))^(1/2)/(a + b*tan(e + f*x))^2, x)
```

3.614 $\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 555

$$
\begin{aligned}
&\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx \\
&= \frac{5ab^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{2\left(a^2+b^2\right)^{9/4}f\sqrt{d\sec(e+fx)}} \\
&\quad - \frac{5ab^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{2\left(a^2+b^2\right)^{9/4}f\sqrt{d\sec(e+fx)}} \\
&\quad + \frac{(2a^2-3b^2)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{\left(a^2+b^2\right)^2f\sqrt{d\sec(e+fx)}} - \frac{(2a^2-3b^2)\tan(e+fx)}{\left(a^2+b^2\right)^2f\sqrt{d\sec(e+fx)}} \\
&\quad - \frac{5a^2b\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}}{2\left(a^2+b^2\right)^{5/2}f\sqrt{d\sec(e+fx)}} \\
&\quad + \frac{5a^2b\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}}{2\left(a^2+b^2\right)^{5/2}f\sqrt{d\sec(e+fx)}} \\
&\quad + \frac{b(2a^2-3b^2)\sec^2(e+fx)}{\left(a^2+b^2\right)^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} \\
&\quad + \frac{2(b+a\tan(e+fx))}{\left(a^2+b^2\right)f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}
\end{aligned}
$$

```
[Out] 5/2*a*b^(3/2)*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(sec(f*x
+e)^2)^(1/4)/(a^2+b^2)^(9/4)/f/(d*sec(f*x+e))^(1/2)-5/2*a*b^(3/2)*arctanh((
sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)
```

```
^(9/4)/f/(d*sec(f*x+e))^(1/2)+(2*a^2-3*b^2)*(cos(1/2*arctan(tan(f*x+e)))^2)
^(1/2)/cos(1/2*arctan(tan(f*x+e)))*EllipticE(sin(1/2*arctan(tan(f*x+e))),2^
(1/2))*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^2/f/(d*sec(f*x+e))^(1/2)-5/2*a^2*b*co
t(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^
2)^(1/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^(5/2)/f/(d*sec(f*x+e))^(1/2)+5/2*a
^2*b*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(sec(f
*x+e)^2)^(1/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^(5/2)/f/(d*sec(f*x+e))^(1/2)
-(2*a^2-3*b^2)*tan(f*x+e)/(a^2+b^2)^2/f/(d*sec(f*x+e))^(1/2)+b*(2*a^2-3*b^2
)*sec(f*x+e)^2/(a^2+b^2)^2/f/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))+2*(b+a*t
an(f*x+e))/(a^2+b^2)/f/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3593, 755, 849, 858, 233, 202, 760, 408, 504, 1227, 551, 455, 65, 304, 211, 214}

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx = -\frac{5a^2b\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)}{2f\left(a^2+b^2\right)^{5/2}\sqrt{d\sec(e+fx)}}$$
$$+\frac{5a^2b\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)}{2f\left(a^2+b^2\right)^{5/2}\sqrt{d\sec(e+fx)}}$$
$$+\frac{(2a^2-3b^2)\sqrt[4]{\sec^2(e+fx)}E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)}{f\left(a^2+b^2\right)^2\sqrt{d\sec(e+fx)}}$$
$$+\frac{5ab^{3/2}\sqrt[4]{\sec^2(e+fx)}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2f\left(a^2+b^2\right)^{9/4}\sqrt{d\sec(e+fx)}}$$
$$-\frac{5ab^{3/2}\sqrt[4]{\sec^2(e+fx)}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2f\left(a^2+b^2\right)^{9/4}\sqrt{d\sec(e+fx)}}$$
$$+\frac{b(2a^2-3b^2)\sec^2(e+fx)}{f\left(a^2+b^2\right)^2\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}$$
$$-\frac{(2a^2-3b^2)\tan(e+fx)}{f\left(a^2+b^2\right)^2\sqrt{d\sec(e+fx)}}+\frac{2(a\tan(e+fx)+b)}{f\left(a^2+b^2\right)\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}$$

```
[In] Int[1/(Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^2),x]
```

```
[Out] (5*a*b^(3/2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Se
c[e + f*x]^2)^(1/4))/(2*(a^2 + b^2)^(9/4)*f*Sqrt[d*Sec[e + f*x]]) - (5*a*b^
```

```
(3/2)*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Sec[e +
f*x]^2)^(1/4))/(2*(a^2 + b^2)^(9/4)*f*Sqrt[d*Sec[e + f*x]]) + ((2*a^2 - 3*b
^2)*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^(1/4))/((a^2 + b^
2)^2*f*Sqrt[d*Sec[e + f*x]]) - ((2*a^2 - 3*b^2)*Tan[e + f*x])/((a^2 + b^2)^
2*f*Sqrt[d*Sec[e + f*x]]) - (5*a^2*b*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 +
 b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2)^(1/4)*Sqrt[-Ta
n[e + f*x]^2])/(2*(a^2 + b^2)^(5/2)*f*Sqrt[d*Sec[e + f*x]]) + (5*a^2*b*Cot[
e + f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*
(Sec[e + f*x]^2)^(1/4)*Sqrt[-Tan[e + f*x]^2])/(2*(a^2 + b^2)^(5/2)*f*Sqrt[d
*Sec[e + f*x]]) + (b*(2*a^2 - 3*b^2)*Sec[e + f*x]^2)/((a^2 + b^2)^2*f*Sqrt[
d*Sec[e + f*x]]*(a + b*Tan[e + f*x])) + (2*(b + a*Tan[e + f*x]))/((a^2 + b^
2)*f*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
```

```
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 760

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
```

```
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt[4]{\sec^2(e+fx)}\text{Subst}\left(\int \frac{1}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^{5/4}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt{d\sec(e+fx)}}$$

$$= \frac{2(b+a\tan(e+fx))}{(a^2+b^2)f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} - \frac{\left(2b\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\frac{1}{2}\left(-3+\frac{a^2}{b^2}\right)-\frac{ax}{2b^2}}{(a+x)^2\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{(a^2+b^2)f\sqrt{d\sec(e+fx)}}$$

$$= \frac{b(2a^2-3b^2)\sec^2(e+fx)}{(a^2+b^2)^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} + \frac{2(b+a\tan(e+fx))}{(a^2+b^2)f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} + \frac{\left(2b^3\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{-\frac{a(a^2-4b^2)}{2b^4}-\frac{(2a^2-3b^2)x}{4b^4}}{(a+x)\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{(a^2+b^2)^2 f\sqrt{d\sec(e+fx)}}$$

$$= \frac{b(2a^2-3b^2)\sec^2(e+fx)}{(a^2+b^2)^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} + \frac{2(b+a\tan(e+fx))}{(a^2+b^2)f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} + \frac{\left(5ab\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{(a+x)\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{2(a^2+b^2)^2 f\sqrt{d\sec(e+fx)}} - \frac{\left((2a^2-3b^2)\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{2b(a^2+b^2)^2 f\sqrt{d\sec(e+fx)}}$$

$$= -\frac{(2a^2-3b^2)\tan(e+fx)}{(a^2+b^2)^2 f\sqrt{d\sec(e+fx)}} + \frac{b(2a^2-3b^2)\sec^2(e+fx)}{(a^2+b^2)^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}$$
$$+ \frac{2(b+a\tan(e+fx))}{(a^2+b^2) f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}$$
$$- \frac{\left(5ab\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{x}{(a^2-x^2)\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{2(a^2+b^2)^2 f\sqrt{d\sec(e+fx)}}$$
$$+ \frac{\left(5a^2b\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{(a^2-x^2)\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{2(a^2+b^2)^2 f\sqrt{d\sec(e+fx)}}$$
$$+ \frac{\left((2a^2-3b^2)\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{x^2}{b^2}\right)^{5/4}}\,dx, x, b\tan(e+fx)\right)}{2b(a^2+b^2)^2 f\sqrt{d\sec(e+fx)}}$$

$$= \frac{(2a^2-3b^2)E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{(a^2+b^2)^2 f\sqrt{d\sec(e+fx)}}$$
$$- \frac{(2a^2-3b^2)\tan(e+fx)}{(a^2+b^2)^2 f\sqrt{d\sec(e+fx)}} + \frac{b(2a^2-3b^2)\sec^2(e+fx)}{(a^2+b^2)^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}$$
$$+ \frac{2(b+a\tan(e+fx))}{(a^2+b^2) f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}$$
$$- \frac{\left(5ab\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{(a^2-x)\sqrt[4]{1+\frac{x}{b^2}}}\,dx, x, b^2\tan^2(e+fx)\right)}{4(a^2+b^2)^2 f\sqrt{d\sec(e+fx)}}$$
$$+ \frac{\left(5a^2\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-x^4}\left(1+\frac{a^2}{b^2}-x^4\right)}\,dx, x, \sqrt[4]{\sec^2(e+\right.}{(a^2+b^2)^2 f\sqrt{d\sec(e+fx)}}$$

$$= \frac{(2a^2-3b^2)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{(a^2+b^2)^2 f\sqrt{d\sec(e+fx)}} - \frac{(2a^2-3b^2)\tan(e+fx)}{(a^2+b^2)^2 f\sqrt{d\sec(e+fx)}} + \frac{b(2a^2-3b^2)\sec^2(e+fx)}{(a^2+b^2)^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} + \frac{2(b+a\tan(e+fx))}{(a^2+b^2) f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} - \frac{\left(5ab^3\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{x^2}{a^2+b^2-b^2x^4}\,dx, x, \sqrt[4]{\sec^2(e+fx)}\right)}{(a^2+b^2)^2 f\sqrt{d\sec(e+fx)}} + \frac{\left(5a^2b\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\left(\sqrt{a^2+b^2}-bx^2\right)\sqrt{1-x^4}}\,dx, x, \sqrt[4]{\sec^2(e+}\right.}{2(a^2+b^2)^2 f\sqrt{d\sec(e+fx)}} - \frac{\left(5a^2b\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\left(\sqrt{a^2+b^2}+bx^2\right)\sqrt{1-x^4}}\,dx, x, \sqrt[4]{\sec^2(e+}\right.}{2(a^2+b^2)^2 f\sqrt{d\sec(e+fx)}}$$

$$= \frac{(2a^2-3b^2)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{(a^2+b^2)^2 f\sqrt{d\sec(e+fx)}} - \frac{(2a^2-3b^2)\tan(e+fx)}{(a^2+b^2)^2 f\sqrt{d\sec(e+fx)}} + \frac{b(2a^2-3b^2)\sec^2(e+fx)}{(a^2+b^2)^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} + \frac{2(b+a\tan(e+fx))}{(a^2+b^2) f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} - \frac{\left(5ab^2\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{a^2+b^2}-bx^2}\,dx, x, \sqrt[4]{\sec^2(e+fx)}\right)}{2(a^2+b^2)^2 f\sqrt{d\sec(e+fx)}} + \frac{\left(5ab^2\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{a^2+b^2}+bx^2}\,dx, x, \sqrt[4]{\sec^2(e+fx)}\right)}{2(a^2+b^2)^2 f\sqrt{d\sec(e+fx)}} + \frac{\left(5a^2b\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(\sqrt{a^2+b^2}-bx^2\right)}\,dx, x, \sqrt[4]{\text{se}}\right.}{2(a^2+b^2)^2 f\sqrt{d\sec(e+fx)}} - \frac{\left(5a^2b\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(\sqrt{a^2+b^2}+bx^2\right)}\,dx, x, \sqrt[4]{\text{se}}\right.}{2(a^2+b^2)^2 f\sqrt{d\sec(e+fx)}}$$

$$= \frac{5ab^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{2(a^2+b^2)^{9/4}f\sqrt{d\sec(e+fx)}}$$
$$- \frac{5ab^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{2(a^2+b^2)^{9/4}f\sqrt{d\sec(e+fx)}}$$
$$+ \frac{(2a^2-3b^2)E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{(a^2+b^2)^2f\sqrt{d\sec(e+fx)}}$$
$$- \frac{(2a^2-3b^2)\tan(e+fx)}{(a^2+b^2)^2f\sqrt{d\sec(e+fx)}}$$
$$- \frac{5a^2b\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e}{2(a^2+b^2)^{5/2}f\sqrt{d\sec(e+fx)}}$$
$$+ \frac{5a^2b\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+}{2(a^2+b^2)^{5/2}f\sqrt{d\sec(e+fx)}}$$
$$+ \frac{b(2a^2-3b^2)\sec^2(e+fx)}{(a^2+b^2)^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}$$
$$+ \frac{2(b+a\tan(e+fx))}{(a^2+b^2)f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 35.31 (sec) , antiderivative size = 8379, normalized size of antiderivative = 15.10

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[1/(Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^2),x]
[Out] Result too large to show
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 29177 vs. $2(512) = 1024$.

Time = 11.81 (sec) , antiderivative size = 29178, normalized size of antiderivative = 52.57

method	result	size
default	Expression too large to display	29178

```
[In] int(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{\sqrt{d\sec(e+fx)}\,(a+b\tan(e+fx))^2}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))**(1/2)/(a+b*tan(f*x+e))**2,x)
[Out] Integral(1/(sqrt(d*sec(e + f*x))*(a + b*tan(e + f*x))**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{\sqrt{d\sec(fx+e)}(b\tan(fx+e)+a)^2}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
[Out] integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)^2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{\sqrt{d\sec\left(fx+e\right)}\left(b\tan\left(fx+e\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
[Out] integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{\sqrt{\frac{d}{\cos(e+f\,x)}}\,\left(a+b\tan\left(e+f\,x\right)\right)^2}\,dx$$

```
[In] int(1/((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x))^2),x)
[Out] int(1/((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x))^2), x)
```

3.615 $\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 520

$$\begin{aligned}
&\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}\,dx = \\
&-\frac{7ab^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{2\left(a^2+b^2\right)^{11/4}f(d\sec(e+fx))^{3/2}} \\
&-\frac{7ab^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{2\left(a^2+b^2\right)^{11/4}f(d\sec(e+fx))^{3/2}} \\
&+\frac{(2a^2-5b^2)\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sec^2(e+fx)^{3/4}}{3\left(a^2+b^2\right)^2f(d\sec(e+fx))^{3/2}} \\
&+\frac{7a^2b^2\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sec^2(e+fx)^{3/4}\sqrt{-\tan^2(e+fx)}}{2\left(a^2+b^2\right)^3f(d\sec(e+fx))^{3/2}} \\
&+\frac{7a^2b^2\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sec^2(e+fx)^{3/4}\sqrt{-\tan^2(e+fx)}}{2\left(a^2+b^2\right)^3f(d\sec(e+fx))^{3/2}} \\
&+\frac{b(2a^2-5b^2)\sec^2(e+fx)}{3\left(a^2+b^2\right)^2f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))} \\
&+\frac{2(b+a\tan(e+fx))}{3\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}
\end{aligned}$$

```
[Out] -7/2*a*b^(5/2)*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(sec(f*
x+e)^2)^(3/4)/(a^2+b^2)^(11/4)/f/(d*sec(f*x+e))^(3/2)-7/2*a*b^(5/2)*arctanh
((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(sec(f*x+e)^2)^(3/4)/(a^2+b^
2)^(11/4)/f/(d*sec(f*x+e))^(3/2)+1/3*(2*a^2-5*b^2)*(cos(1/2*arctan(tan(f*x+
```

```
e)))^2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*EllipticF(sin(1/2*arctan(tan(f*x+
e))),2^(1/2))*(sec(f*x+e)^2)^(3/4)/(a^2+b^2)^2/f/(d*sec(f*x+e))^(3/2)+7/2*a
^2*b^2*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(se
c(f*x+e)^2)^(3/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^3/f/(d*sec(f*x+e))^(3/2)+
7/2*a^2*b^2*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)
*(sec(f*x+e)^2)^(3/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^3/f/(d*sec(f*x+e))^(3
/2)+1/3*b*(2*a^2-5*b^2)*sec(f*x+e)^2/(a^2+b^2)^2/f/(d*sec(f*x+e))^(3/2)/(a+
b*tan(f*x+e))+2/3*(b+a*tan(f*x+e))/(a^2+b^2)/f/(d*sec(f*x+e))^(3/2)/(a+b*ta
n(f*x+e))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3593, 755, 849, 858, 237, 761, 410, 109, 418, 1227, 551, 455, 65, 218, 214, 211}

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}\,dx = \frac{7a^2b^2\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sec^2(e+fx)^{3/4}\,\text{EllipticP}}{2f\left(a^2+b^2\right)^3(d\sec(e+}$$
$$+\frac{7a^2b^2\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sec^2(e+fx)^{3/4}\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)}{2f\left(a^2+b^2\right)^3(d\sec(e+fx))^{3/2}}$$
$$+\frac{(2a^2-5b^2)\sec^2(e+fx)^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)}{3f\left(a^2+b^2\right)^2(d\sec(e+fx))^{3/2}}$$
$$-\frac{7ab^{5/2}\sec^2(e+fx)^{3/4}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2f\left(a^2+b^2\right)^{11/4}(d\sec(e+fx))^{3/2}}$$
$$-\frac{7ab^{5/2}\sec^2(e+fx)^{3/4}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2f\left(a^2+b^2\right)^{11/4}(d\sec(e+fx))^{3/2}}$$
$$+\frac{b(2a^2-5b^2)\sec^2(e+fx)}{3f\left(a^2+b^2\right)^2(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}$$
$$+\frac{2(a\tan(e+fx)+b)}{3f\left(a^2+b^2\right)(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}$$

```
[In] Int[1/((d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^2),x]
```

```
[Out] (-7*a*b^(5/2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(S
ec[e + f*x]^2)^(3/4))/(2*(a^2 + b^2)^(11/4)*f*(d*Sec[e + f*x])^(3/2)) - (7*
a*b^(5/2)*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Sec[
e + f*x]^2)^(3/4))/(2*(a^2 + b^2)^(11/4)*f*(d*Sec[e + f*x])^(3/2)) + ((2*a^
2 - 5*b^2)*EllipticF[ArcTan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^(3/4))/(3*
(a^2 + b^2)^2*f*(d*Sec[e + f*x])^(3/2)) + (7*a^2*b^2*Cot[e + f*x]*EllipticP
```

```
i[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2
)^(3/4)*Sqrt[-Tan[e + f*x]^2])/(2*(a^2 + b^2)^3*f*(d*Sec[e + f*x])^(3/2)) +
 (7*a^2*b^2*Cot[e + f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]
^2)^(1/4)], -1]*(Sec[e + f*x]^2)^(3/4)*Sqrt[-Tan[e + f*x]^2])/(2*(a^2 + b^2
)^3*f*(d*Sec[e + f*x])^(3/2)) + (b*(2*a^2 - 5*b^2)*Sec[e + f*x]^2)/(3*(a^2
+ b^2)^2*f*(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])) + (2*(b + a*Tan[e +
 f*x]))/(3*(a^2 + b^2)*f*(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 109

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e
/f) + d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-f/(d*e - c*f), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[(-b)*(x^2/a)]/(2*x), Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 761

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(3/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(3/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(3/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
```

```
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\sec^2(e+fx)^{3/4}\text{Subst}\left(\int \frac{1}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^{7/4}}\,dx, x, b\tan(e+fx)\right)}{bf(d\sec(e+fx))^{3/2}}$$

$$= \frac{2(b+a\tan(e+fx))}{3\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))} - \frac{\left(2b\sec^2(e+fx)^{3/4}\right)\text{Subst}\left(\int \frac{\frac{1}{2}\left(-5-\frac{a^2}{b^2}\right)-\frac{3ax}{2b^2}}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{3\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}}$$

$$= \frac{b(2a^2-5b^2)\sec^2(e+fx)}{3(a^2+b^2)^2 f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))} + \frac{2(b+a\tan(e+fx))}{3(a^2+b^2) f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))} + \frac{\left(2b^3\sec^2(e+fx)^{3/4}\right)\text{Subst}\left(\int \frac{\frac{a(a^2+8b^2)}{2b^4}+\frac{(2a^2-5b^2)x}{4b^4}}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{3(a^2+b^2)^2 f(d\sec(e+fx))^{3/2}}$$

$$= \frac{b(2a^2-5b^2)\sec^2(e+fx)}{3(a^2+b^2)^2 f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))} + \frac{2(b+a\tan(e+fx))}{3(a^2+b^2) f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))} + \frac{\left(7ab\sec^2(e+fx)^{3/4}\right)\text{Subst}\left(\int \frac{1}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{2(a^2+b^2)^2 f(d\sec(e+fx))^{3/2}} + \frac{\left((2a^2-5b^2)\sec^2(e+fx)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{6b(a^2+b^2)^2 f(d\sec(e+fx))^{3/2}}$$

$$= \frac{(2a^2-5b^2)\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sec^2(e+fx)^{3/4}}{3(a^2+b^2)^2 f(d\sec(e+fx))^{3/2}} + \frac{b(2a^2-5b^2)\sec^2(e+fx)}{3(a^2+b^2)^2 f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))} + \frac{2(b+a\tan(e+fx))}{3(a^2+b^2) f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))} - \frac{\left(7ab\sec^2(e+fx)^{3/4}\right)\text{Subst}\left(\int \frac{x}{(a^2-x^2)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{2(a^2+b^2)^2 f(d\sec(e+fx))^{3/2}} + \frac{\left(7a^2b\sec^2(e+fx)^{3/4}\right)\text{Subst}\left(\int \frac{1}{(a^2-x^2)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{2(a^2+b^2)^2 f(d\sec(e+fx))^{3/2}}$$

$$= \frac{(2a^2-5b^2)\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sec^2(e+fx)^{3/4}}{3\left(a^2+b^2\right)^2 f(d\sec(e+fx))^{3/2}} + \frac{b(2a^2-5b^2)\sec^2(e+fx)}{3\left(a^2+b^2\right)^2 f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))} + \frac{2(b+a\tan(e+fx))}{3\left(a^2+b^2\right) f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))} - \frac{\left(7ab\sec^2(e+fx)^{3/4}\right)\text{Subst}\left(\int \frac{1}{(a^2-x)\left(1+\frac{x}{b^2}\right)^{3/4}}\,dx,x,b^2\tan^2(e+fx)\right)}{4\left(a^2+b^2\right)^2 f(d\sec(e+fx))^{3/2}} + \frac{\left(7a^2\cot(e+fx)\sec^2(e+fx)^{3/4}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{(a^2-x)\sqrt{-\frac{x}{b^2}}\left(1+\frac{x}{b^2}\right)^{3/4}}\,dx,x,b^2\tan^2(e\right.}{4\left(a^2+b^2\right)^2 f(d\sec(e+fx))^{3/2}}$$

$$= \frac{(2a^2-5b^2)\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sec^2(e+fx)^{3/4}}{3\left(a^2+b^2\right)^2 f(d\sec(e+fx))^{3/2}} + \frac{b(2a^2-5b^2)\sec^2(e+fx)}{3\left(a^2+b^2\right)^2 f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))} + \frac{2(b+a\tan(e+fx))}{3\left(a^2+b^2\right) f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))} - \frac{\left(7ab^3\sec^2(e+fx)^{3/4}\right)\text{Subst}\left(\int \frac{1}{a^2+b^2-b^2x^4}\,dx,x,\sqrt[4]{\sec^2(e+fx)}\right)}{\left(a^2+b^2\right)^2 f(d\sec(e+fx))^{3/2}} - \frac{\left(7a^2\cot(e+fx)\sec^2(e+fx)^{3/4}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^4}\left(-1-\frac{a^2}{b^2}+x^4\right)}\,dx,x,\sqrt[4]{\sec^2(e+\right.}{\left(a^2+b^2\right)^2 f(d\sec(e+fx))^{3/2}}$$

$$= \frac{(2a^2-5b^2)\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sec^2(e+fx)^{3/4}}{3(a^2+b^2)^2 f(d\sec(e+fx))^{3/2}} + \frac{b(2a^2-5b^2)\sec^2(e+fx)}{3(a^2+b^2)^2 f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))} + \frac{2(b+a\tan(e+fx))}{3(a^2+b^2) f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))} - \frac{(7ab^3\sec^2(e+fx)^{3/4})\,\text{Subst}\left(\int \frac{1}{\sqrt{a^2+b^2}-bx^2}dx,x,\sqrt[4]{\sec^2(e+fx)}\right)}{2(a^2+b^2)^{5/2} f(d\sec(e+fx))^{3/2}} - \frac{(7ab^3\sec^2(e+fx)^{3/4})\,\text{Subst}\left(\int \frac{1}{\sqrt{a^2+b^2}+bx^2}dx,x,\sqrt[4]{\sec^2(e+fx)}\right)}{2(a^2+b^2)^{5/2} f(d\sec(e+fx))^{3/2}} + \frac{\left(7a^2b^2\cot(e+fx)\sec^2(e+fx)^{3/4}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{bx^2}{\sqrt{a^2+b^2}}\right)\sqrt{1-x^4}}dx,x,\sqrt[4]{\sec^2(e}\right.}{2(a^2+b^2)^3 f(d\sec(e+fx))^{3/2}} + \frac{\left(7a^2b^2\cot(e+fx)\sec^2(e+fx)^{3/4}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{bx^2}{\sqrt{a^2+b^2}}\right)\sqrt{1-x^4}}dx,x,\sqrt[4]{\sec^2(e}\right.}{2(a^2+b^2)^3 f(d\sec(e+fx))^{3/2}}$$

$$= -\frac{7ab^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{2(a^2+b^2)^{11/4} f(d\sec(e+fx))^{3/2}} - \frac{7ab^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{2(a^2+b^2)^{11/4} f(d\sec(e+fx))^{3/2}} + \frac{(2a^2-5b^2)\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sec^2(e+fx)^{3/4}}{3(a^2+b^2)^2 f(d\sec(e+fx))^{3/2}} + \frac{b(2a^2-5b^2)\sec^2(e+fx)}{3(a^2+b^2)^2 f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))} + \frac{2(b+a\tan(e+fx))}{3(a^2+b^2) f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))} + \frac{\left(7a^2b^2\cot(e+fx)\sec^2(e+fx)^{3/4}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(1-\frac{bx^2}{\sqrt{a^2+b^2}}\right)}dx,x,\sqrt[4]{}\right.}{2(a^2+b^2)^3 f(d\sec(e+fx))^{3/2}} + \frac{\left(7a^2b^2\cot(e+fx)\sec^2(e+fx)^{3/4}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(1+\frac{bx^2}{\sqrt{a^2+b^2}}\right)}dx,x,\sqrt[4]{}\right.}{2(a^2+b^2)^3 f(d\sec(e+fx))^{3/2}}$$

$$= -\frac{7ab^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{2(a^2+b^2)^{11/4}f(d\sec(e+fx))^{3/2}}$$
$$-\frac{7ab^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{2(a^2+b^2)^{11/4}f(d\sec(e+fx))^{3/2}}$$
$$+\frac{(2a^2-5b^2)\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sec^2(e+fx)^{3/4}}{3(a^2+b^2)^2 f(d\sec(e+fx))^{3/2}}$$
$$+\frac{7a^2b^2\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sec^2(e+fx)^{3/4}\sqrt{-\tan^2(e}}{2(a^2+b^2)^3 f(d\sec(e+fx))^{3/2}}$$
$$+\frac{7a^2b^2\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sec^2(e+fx)^{3/4}\sqrt{-\tan^2(e+}}{2(a^2+b^2)^3 f(d\sec(e+fx))^{3/2}}$$
$$+\frac{b(2a^2-5b^2)\sec^2(e+fx)}{3(a^2+b^2)^2 f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}$$
$$+\frac{2(b+a\tan(e+fx))}{3(a^2+b^2)f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 12.23 (sec) , antiderivative size = 528, normalized size of antiderivative = 1.02

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}\,dx = \frac{\sec^4(e+fx)(a\cos(e+fx)+b\sin(e+fx))^2\left(\frac{b(2a^2-3b^2)}{3a(a-ib)^2(a+ib)^2}+\right.}{f(d\sec(e+fx}$$
$$+\frac{\sec^2(e+fx)\sec^2(e+fx)^{3/4}(a\cos(e+fx)+b\sin(e+fx))^2\left((2a^4-3a^2b^2-5b^4)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4}\right.\right.}{}$$

```
[In] Integrate[1/((d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^2),x]
```

```
[Out] (Sec[e + f*x]^4*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*((b*(2*a^2 - 3*b^2))/(3
*a*(a - I*b)^2*(a + I*b)^2) + (2*a*b*Cos[2*(e + f*x)])/(3*(a - I*b)^2*(a +
I*b)^2) + (b^4*Sin[e + f*x])/(a*(a - I*b)^2*(a + I*b)^2*(a*Cos[e + f*x] + b
*Sin[e + f*x])) + ((a^2 - b^2)*Sin[2*(e + f*x)])/(3*(a - I*b)^2*(a + I*b)^2
)))/(f*(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^2) + (Sec[e + f*x]^2*(Se
c[e + f*x]^2)^(3/4)*(a*Cos[e + f*x] + b*Sin[e + f*x])^2*((2*a^4 - 3*a^2*b^2
 - 5*b^4)*Hypergeometric2F1[1/2, 3/4, 3/2, -Tan[e + f*x]^2]*Tan[e + f*x] +
21*a*b^2*(-(Sqrt[b]*(a^2 + b^2)^(1/4)*(ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/
4))/(a^2 + b^2)^(1/4)] + ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^
2)^(1/4)])) + a*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e
```

```
 + f*x]^2)^(1/4)], -1]*Sqrt[-Tan[e + f*x]^2] + a*Cot[e + f*x]*EllipticPi[b/
Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[-Tan[e + f*x]^2])
))/(6*(a^2 + b^2)^3*f*(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^2)
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 14916 vs. $2(477) = 954$.

Time = 13.29 (sec) , antiderivative size = 14917, normalized size of antiderivative = 28.69

method	result	size
default	Expression too large to display	14917

```
[In] int(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{(d\sec{(e+fx)})^{\frac{3}{2}}\,(a+b\tan{(e+fx)})^2}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))**(3/2)/(a+b*tan(f*x+e))**2,x)
[Out] Integral(1/((d*sec(e + f*x))**(3/2)*(a + b*tan(e + f*x))**2), x)
```

Maxima [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{3}{2}}(b\tan(fx+e)+a)^2}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
[Out] integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)^2), x)
```

Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{3}{2}}(b\tan(fx+e)+a)^2}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
[Out] integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}(a+b\tan(e+f\,x))^2}\,dx$$

```
[In] int(1/((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x))^2),x)
[Out] int(1/((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x))^2), x)
```

3.616 $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 700

$$\begin{aligned}
\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx &= \frac{9ab^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{2\left(a^2+b^2\right)^{13/4}d^2f\sqrt{d\sec(e+fx)}} \\
&- \frac{9ab^{7/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{2\left(a^2+b^2\right)^{13/4}d^2f\sqrt{d\sec(e+fx)}} \\
&+ \frac{3(2a^4+10a^2b^2-7b^4)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{5\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}} \\
&- \frac{3(2a^4+10a^2b^2-7b^4)\tan(e+fx)}{5\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}} \\
&- \frac{9a^2b^3\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}}{2\left(a^2+b^2\right)^{7/2}d^2f\sqrt{d\sec(e+fx)}} \\
&+ \frac{9a^2b^3\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}}{2\left(a^2+b^2\right)^{7/2}d^2f\sqrt{d\sec(e+fx)}} \\
&+ \frac{3b(2a^4+10a^2b^2-7b^4)\sec^2(e+fx)}{5\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} \\
&+ \frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5\left(a^2+b^2\right)d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} \\
&- \frac{2(b(2a^2-7b^2)-3a(a^2+4b^2)\tan(e+fx))}{5\left(a^2+b^2\right)^2d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}
\end{aligned}$$

```
[Out] 9/2*a*b^(7/2)*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(sec(f*x
+e)^2)^(1/4)/(a^2+b^2)^(13/4)/d^2/f/(d*sec(f*x+e))^(1/2)-9/2*a*b^(7/2)*arct
```

```
anh((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(sec(f*x+e)^2)^(1/4)/(a^2
+b^2)^(13/4)/d^2/f/(d*sec(f*x+e))^(1/2)+3/5*(2*a^4+10*a^2*b^2-7*b^4)*(cos(1
/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*EllipticE(sin(1
/2*arctan(tan(f*x+e))),2^(1/2))*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^3/d^2/f/(d*s
ec(f*x+e))^(1/2)-9/2*a^2*b^3*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/
(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2)^(1/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^(7/
2)/d^2/f/(d*sec(f*x+e))^(1/2)+9/2*a^2*b^3*cot(f*x+e)*EllipticPi((sec(f*x+e)
^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2)^(1/4)*(-tan(f*x+e)^2)^(1/2)/(
a^2+b^2)^(7/2)/d^2/f/(d*sec(f*x+e))^(1/2)-3/5*(2*a^4+10*a^2*b^2-7*b^4)*tan(
f*x+e)/(a^2+b^2)^3/d^2/f/(d*sec(f*x+e))^(1/2)+3/5*b*(2*a^4+10*a^2*b^2-7*b^4
)*sec(f*x+e)^2/(a^2+b^2)^3/d^2/f/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))+2/5*
cos(f*x+e)^2*(b+a*tan(f*x+e))/(a^2+b^2)/d^2/f/(d*sec(f*x+e))^(1/2)/(a+b*tan
(f*x+e))-2/5*(b*(2*a^2-7*b^2)-3*a*(a^2+4*b^2)*tan(f*x+e))/(a^2+b^2)^2/d^2/f
/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 700, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules

used = {3593, 755, 837, 849, 858, 233, 202, 760, 408, 504, 1227, 551, 455, 65, 304, 211, 214}

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx = -\frac{9a^2b^3\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\,\mathrm{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)}{2d^2f\left(a^2+b^2\right)^{7/2}\sqrt{d\sec(e+fx)}}$$
$$+\frac{9a^2b^3\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\,\mathrm{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)}{2d^2f\left(a^2+b^2\right)^{7/2}\sqrt{d\sec(e+fx)}}$$
$$+\frac{9ab^{7/2}\sqrt[4]{\sec^2(e+fx)}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2d^2f\left(a^2+b^2\right)^{13/4}\sqrt{d\sec(e+fx)}}$$
$$-\frac{9ab^{7/2}\sqrt[4]{\sec^2(e+fx)}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{2d^2f\left(a^2+b^2\right)^{13/4}\sqrt{d\sec(e+fx)}}$$
$$-\frac{2(b(2a^2-7b^2)-3a(a^2+4b^2)\tan(e+fx))}{5d^2f\left(a^2+b^2\right)^2\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}$$
$$+\frac{2\cos^2(e+fx)(a\tan(e+fx)+b)}{5d^2f\left(a^2+b^2\right)\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}$$
$$+\frac{3(2a^4+10a^2b^2-7b^4)\sqrt[4]{\sec^2(e+fx)}E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)}{5d^2f\left(a^2+b^2\right)^3\sqrt{d\sec(e+fx)}}$$
$$+\frac{3b(2a^4+10a^2b^2-7b^4)\sec^2(e+fx)}{5d^2f\left(a^2+b^2\right)^3\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}-\frac{3(2a^4+10a^2b^2-7b^4)\tan(e+fx)}{5d^2f\left(a^2+b^2\right)^3\sqrt{d\sec(e+fx)}}$$

```
[In] Int[1/((d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])^2),x]
```

```
[Out] (9*a*b^(7/2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Se
c[e + f*x]^2)^(1/4))/(2*(a^2 + b^2)^(13/4)*d^2*f*Sqrt[d*Sec[e + f*x]]) - (9
*a*b^(7/2)*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]*(Sec
[e + f*x]^2)^(1/4))/(2*(a^2 + b^2)^(13/4)*d^2*f*Sqrt[d*Sec[e + f*x]]) + (3*
(2*a^4 + 10*a^2*b^2 - 7*b^4)*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(Sec[e +
f*x]^2)^(1/4))/(5*(a^2 + b^2)^3*d^2*f*Sqrt[d*Sec[e + f*x]]) - (3*(2*a^4 + 1
0*a^2*b^2 - 7*b^4)*Tan[e + f*x])/(5*(a^2 + b^2)^3*d^2*f*Sqrt[d*Sec[e + f*x]
]) - (9*a^2*b^3*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e
 + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2)^(1/4)*Sqrt[-Tan[e + f*x]^2])/(2*(a^
2 + b^2)^(7/2)*d^2*f*Sqrt[d*Sec[e + f*x]]) + (9*a^2*b^3*Cot[e + f*x]*Ellipt
icPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2
)^(1/4)*Sqrt[-Tan[e + f*x]^2])/(2*(a^2 + b^2)^(7/2)*d^2*f*Sqrt[d*Sec[e + f*
x]]) + (3*b*(2*a^4 + 10*a^2*b^2 - 7*b^4)*Sec[e + f*x]^2)/(5*(a^2 + b^2)^3*d
^2*f*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])) + (2*Cos[e + f*x]^2*(b + a*
Tan[e + f*x]))/(5*(a^2 + b^2)*d^2*f*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x
```

```
])) - (2*(b*(2*a^2 - 7*b^2) - 3*a*(a^2 + 4*b^2)*Tan[e + f*x]))/(5*(a^2 + b^
2)^2*d^2*f*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 760

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
```

```
a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt[4]{\sec^2(e+fx)}\,\text{Subst}\left(\int \frac{1}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^{9/4}}\,dx, x, b\tan(e+fx)\right)}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$= \frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5\left(a^2+b^2\right)d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} - \frac{\left(2b\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{\frac{1}{2}\left(-7-\frac{3a^2}{b^2}\right)-\frac{5ax}{2b^2}}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^{5/4}}dx,x,b\tan(e+fx)\right)}{5\left(a^2+b^2\right)d^2f\sqrt{d\sec(e+fx)}}$$

$$= \frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5\left(a^2+b^2\right)d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} - \frac{2(b(2a^2-7b^2)-3a(a^2+4b^2)\tan(e+fx))}{5\left(a^2+b^2\right)^2d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} + \frac{\left(4b^5\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{-\frac{3\left(a^4+6a^2b^2-7b^4\right)}{4b^6}+\frac{3a\left(a^2+4b^2\right)x}{4b^6}}{(a+x)^2\sqrt[4]{1+\frac{x^2}{b^2}}}dx,x,b\tan(e+fx)\right)}{5\left(a^2+b^2\right)^2d^2f\sqrt{d\sec(e+fx)}}$$

$$= \frac{3b(2a^4+10a^2b^2-7b^4)\sec^2(e+fx)}{5\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} + \frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5\left(a^2+b^2\right)d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} - \frac{2(b(2a^2-7b^2)-3a(a^2+4b^2)\tan(e+fx))}{5\left(a^2+b^2\right)^2d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} - \frac{\left(4b^7\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{\frac{3a\left(a^4+5a^2b^2-11b^4\right)}{4b^8}+\frac{3\left(2a^4+10a^2b^2-7b^4\right)x}{8b^8}}{(a+x)\sqrt[4]{1+\frac{x^2}{b^2}}}dx,x,b\tan(e+fx)\right)}{5\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}}$$

$$= \frac{3b(2a^4+10a^2b^2-7b^4)\sec^2(e+fx)}{5(a^2+b^2)^3 d^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} + \frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5(a^2+b^2)d^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} - \frac{2(b(2a^2-7b^2)-3a(a^2+4b^2)\tan(e+fx))}{5(a^2+b^2)^2 d^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} + \frac{\left(9ab^3\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{(a+x)\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{2(a^2+b^2)^3 d^2 f\sqrt{d\sec(e+fx)}} - \frac{\left(3(2a^4+10a^2b^2-7b^4)\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{10b(a^2+b^2)^3 d^2 f\sqrt{d\sec(e+fx)}}$$

$$= -\frac{3(2a^4+10a^2b^2-7b^4)\tan(e+fx)}{5(a^2+b^2)^3 d^2 f\sqrt{d\sec(e+fx)}} + \frac{3b(2a^4+10a^2b^2-7b^4)\sec^2(e+fx)}{5(a^2+b^2)^3 d^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} + \frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5(a^2+b^2)d^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} - \frac{2(b(2a^2-7b^2)-3a(a^2+4b^2)\tan(e+fx))}{5(a^2+b^2)^2 d^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} - \frac{\left(9ab^3\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{x}{(a^2-x^2)\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{2(a^2+b^2)^3 d^2 f\sqrt{d\sec(e+fx)}} + \frac{\left(9a^2b^3\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{(a^2-x^2)\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{2(a^2+b^2)^3 d^2 f\sqrt{d\sec(e+fx)}} + \frac{\left(3(2a^4+10a^2b^2-7b^4)\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{x^2}{b^2}\right)^{5/4}}\,dx, x, b\tan(e+fx)\right)}{10b(a^2+b^2)^3 d^2 f\sqrt{d\sec(e+fx)}}$$

$$= \frac{3(2a^4+10a^2b^2-7b^4)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{5\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}} - \frac{3(2a^4+10a^2b^2-7b^4)\tan(e+fx)}{5\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}} + \frac{3b(2a^4+10a^2b^2-7b^4)\sec^2(e+fx)}{5\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} + \frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5\left(a^2+b^2\right)d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} - \frac{2(b(2a^2-7b^2)-3a(a^2+4b^2)\tan(e+fx))}{5\left(a^2+b^2\right)^2d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} - \frac{\left(9ab^3\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{(a^2-x)\sqrt[4]{1+\frac{x}{b^2}}}\,dx,x,b^2\tan^2(e+fx)\right)}{4\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}} + \frac{\left(9a^2b^2\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{1-x^4}\left(1+\frac{a^2}{b^2}-x^4\right)}\,dx,x,\sqrt[4]{\sec^2(e+\right.}{\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}}$$

$$= \frac{3(2a^4+10a^2b^2-7b^4)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{5\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}} - \frac{3(2a^4+10a^2b^2-7b^4)\tan(e+fx)}{5\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}} + \frac{3b(2a^4+10a^2b^2-7b^4)\sec^2(e+fx)}{5\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} + \frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5\left(a^2+b^2\right)d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} - \frac{2(b(2a^2-7b^2)-3a(a^2+4b^2)\tan(e+fx))}{5\left(a^2+b^2\right)^2d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} - \frac{\left(9ab^5\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{x^2}{a^2+b^2-b^2x^4}\,dx,x,\sqrt[4]{\sec^2(e+fx)}\right)}{\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}} + \frac{\left(9a^2b^3\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\left(\sqrt{a^2+b^2}-bx^2\right)\sqrt{1-x^4}}\,dx,x,\sqrt[4]{\sec^2(\right.}{2\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}} - \frac{\left(9a^2b^3\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\left(\sqrt{a^2+b^2}+bx^2\right)\sqrt{1-x^4}}\,dx,x,\sqrt[4]{\sec^2(\right.}{2\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}}$$

$$= \frac{3(2a^4+10a^2b^2-7b^4)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{5\left(a^2+b^2\right)^3 d^2 f\sqrt{d\sec(e+fx)}}$$
$$-\frac{3(2a^4+10a^2b^2-7b^4)\tan(e+fx)}{5\left(a^2+b^2\right)^3 d^2 f\sqrt{d\sec(e+fx)}}$$
$$+\frac{3b(2a^4+10a^2b^2-7b^4)\sec^2(e+fx)}{5\left(a^2+b^2\right)^3 d^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}$$
$$+\frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5\left(a^2+b^2\right) d^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}$$
$$-\frac{2(b(2a^2-7b^2)-3a(a^2+4b^2)\tan(e+fx))}{5\left(a^2+b^2\right)^2 d^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}$$
$$-\frac{\left(9ab^4\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{a^2+b^2-bx^2}}\,dx,x,\sqrt[4]{\sec^2(e+fx)}\right)}{2\left(a^2+b^2\right)^3 d^2 f\sqrt{d\sec(e+fx)}}$$
$$+\frac{\left(9ab^4\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{a^2+b^2+bx^2}}\,dx,x,\sqrt[4]{\sec^2(e+fx)}\right)}{2\left(a^2+b^2\right)^3 d^2 f\sqrt{d\sec(e+fx)}}$$
$$+\frac{\left(9a^2b^3\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(\sqrt{a^2+b^2}-bx^2\right)}\,dx,x,\sqrt[4]{\text{se}}\right.}{2\left(a^2+b^2\right)^3 d^2 f\sqrt{d\sec(e+fx)}}$$
$$-\frac{\left(9a^2b^3\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(\sqrt{a^2+b^2}+bx^2\right)}\,dx,x,\sqrt[4]{\text{se}}\right.}{2\left(a^2+b^2\right)^3 d^2 f\sqrt{d\sec(e+fx)}}$$

$$= \frac{9ab^{7/2}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{2\left(a^2+b^2\right)^{13/4}d^2f\sqrt{d\sec(e+fx)}} - \frac{9ab^{7/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{2\left(a^2+b^2\right)^{13/4}d^2f\sqrt{d\sec(e+fx)}} + \frac{3(2a^4+10a^2b^2-7b^4)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{5\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}} - \frac{3(2a^4+10a^2b^2-7b^4)\tan(e+fx)}{5\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}} - \frac{9a^2b^3\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(}}{2\left(a^2+b^2\right)^{7/2}d^2f\sqrt{d\sec(e+fx)}} + \frac{9a^2b^3\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e}}{2\left(a^2+b^2\right)^{7/2}d^2f\sqrt{d\sec(e+fx)}} + \frac{3b(2a^4+10a^2b^2-7b^4)\sec^2(e+fx)}{5\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} + \frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5\left(a^2+b^2\right)d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} - \frac{2(b(2a^2-7b^2)-3a(a^2+4b^2)\tan(e+fx))}{5\left(a^2+b^2\right)^2d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 32.27 (sec) , antiderivative size = 9161, normalized size of antiderivative = 13.09

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[1/((d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])^2),x]
[Out] Result too large to show
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 37752 vs. $2(645) = 1290$.

Time = 17.40 (sec) , antiderivative size = 37753, normalized size of antiderivative = 53.93

method	result	size
default	Expression too large to display	37753

```
[In] int(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{(d\sec(e+fx))^{\frac{5}{2}}(a+b\tan(e+fx))^2}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))**(5/2)/(a+b*tan(f*x+e))**2,x)
[Out] Integral(1/((d*sec(e + f*x))**(5/2)*(a + b*tan(e + f*x))**2), x)
```

Maxima [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{5}{2}}(b\tan(fx+e)+a)^2}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
[Out] integrate(1/((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a)^2), x)
```

Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{5}{2}}(b\tan(fx+e)+a)^2}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
[Out] integrate(1/((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}(a+b\tan(e+f\,x))^2}\,dx$$

```
[In] int(1/((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x))^2),x)
[Out] int(1/((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x))^2), x)
```

3.617 $\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 583

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx = \frac{3(a^2+2b^2)\,d^2\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{8b^{5/2}\left(a^2+b^2\right)^{5/4}f\sec^2(e+fx)^{3/4}}$$
$$-\frac{3(a^2+2b^2)\,d^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{8b^{5/2}\left(a^2+b^2\right)^{5/4}f\sec^2(e+fx)^{3/4}}$$
$$+\frac{3ad^2E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{4b^2\left(a^2+b^2\right)f\sec^2(e+fx)^{3/4}}$$
$$-\frac{3ad^2\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{4b^2\left(a^2+b^2\right)f}$$
$$-\frac{3a(a^2+2b^2)\,d^2\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(}}{8b^3\left(a^2+b^2\right)^{3/2}f\sec^2(e+fx)^{3/4}}$$
$$+\frac{3a(a^2+2b^2)\,d^2\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e}}{8b^3\left(a^2+b^2\right)^{3/2}f\sec^2(e+fx)^{3/4}}$$
$$-\frac{d^2(d\sec(e+fx))^{3/2}}{2bf(a+b\tan(e+fx))^2}+\frac{3ad^2(d\sec(e+fx))^{3/2}}{4b\left(a^2+b^2\right)f(a+b\tan(e+fx))}$$

```
[Out] 3/8*(a^2+2*b^2)*d^2*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(d
*sec(f*x+e))^(3/2)/b^(5/2)/(a^2+b^2)^(5/4)/f/(sec(f*x+e)^2)^(3/4)-3/8*(a^2+
2*b^2)*d^2*arctanh((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(d*sec(f*x
+e))^(3/2)/b^(5/2)/(a^2+b^2)^(5/4)/f/(sec(f*x+e)^2)^(3/4)+3/4*a*d^2*(cos(1/
2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*EllipticE(sin(1/
2*arctan(tan(f*x+e))),2^(1/2))*(d*sec(f*x+e))^(3/2)/b^2/(a^2+b^2)/f/(sec(f*
```

```
x+e)^2)^(3/4)-3/4*a*d^2*cos(f*x+e)*(d*sec(f*x+e))^(3/2)*sin(f*x+e)/b^2/(a^2
+b^2)/f-3/8*a*(a^2+2*b^2)*d^2*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b
/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(3/2)*(-tan(f*x+e)^2)^(1/2)/b^3/(a^2+b^2
)^(3/2)/f/(sec(f*x+e)^2)^(3/4)+3/8*a*(a^2+2*b^2)*d^2*cot(f*x+e)*EllipticPi(
(sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(3/2)*(-tan(f*x+e)
^2)^(1/2)/b^3/(a^2+b^2)^(3/2)/f/(sec(f*x+e)^2)^(3/4)-1/2*d^2*(d*sec(f*x+e))
^(3/2)/b/f/(a+b*tan(f*x+e))^2+3/4*a*d^2*(d*sec(f*x+e))^(3/2)/b/(a^2+b^2)/f/
(a+b*tan(f*x+e))
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 583, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3593, 747, 849, 858, 233, 202, 760, 408, 504, 1227, 551, 455, 65, 304, 211, 214}

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx =$$

$$-\frac{3ad^2(a^2+2b^2)\sqrt{-\tan^2(e+fx)}\cot(e+fx)(d\sec(e+fx))^{3/2}\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+f\cdots}\right.\right.}{8b^3f\left(a^2+b^2\right)^{3/2}\sec^2(e+fx)^{3/4}}$$

$$+\frac{3ad^2(a^2+2b^2)\sqrt{-\tan^2(e+fx)}\cot(e+fx)(d\sec(e+fx))^{3/2}\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right)\cdots\right.}{8b^3f\left(a^2+b^2\right)^{3/2}\sec^2(e+fx)^{3/4}}$$

$$+\frac{3ad^2(d\sec(e+fx))^{3/2}E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)}{4b^2f\left(a^2+b^2\right)\sec^2(e+fx)^{3/4}}$$

$$+\frac{3d^2(a^2+2b^2)\,(d\sec(e+fx))^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8b^{5/2}f\left(a^2+b^2\right)^{5/4}\sec^2(e+fx)^{3/4}}$$

$$-\frac{3d^2(a^2+2b^2)\,(d\sec(e+fx))^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8b^{5/2}f\left(a^2+b^2\right)^{5/4}\sec^2(e+fx)^{3/4}}$$

$$+\frac{3ad^2(d\sec(e+fx))^{3/2}}{4bf\left(a^2+b^2\right)(a+b\tan(e+fx))}$$

$$-\frac{3ad^2\sin(e+fx)\cos(e+fx)(d\sec(e+fx))^{3/2}}{4b^2f\left(a^2+b^2\right)}-\frac{d^2(d\sec(e+fx))^{3/2}}{2bf(a+b\tan(e+fx))^2}$$

```
[In] Int[(d*Sec[e + f*x])^(7/2)/(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (3*(a^2 + 2*b^2)*d^2*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1
/4)]*(d*Sec[e + f*x])^(3/2))/(8*b^(5/2)*(a^2 + b^2)^(5/4)*f*(Sec[e + f*x]^2
)^(3/4)) - (3*(a^2 + 2*b^2)*d^2*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a
^2 + b^2)^(1/4)]*(d*Sec[e + f*x])^(3/2))/(8*b^(5/2)*(a^2 + b^2)^(5/4)*f*(Se
c[e + f*x]^2)^(3/4)) + (3*a*d^2*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(d*Sec
```

```
[e + f*x])^(3/2))/(4*b^2*(a^2 + b^2)*f*(Sec[e + f*x]^2)^(3/4)) - (3*a*d^2*C
os[e + f*x]*(d*Sec[e + f*x])^(3/2)*Sin[e + f*x])/(4*b^2*(a^2 + b^2)*f) - (3
*a*(a^2 + 2*b^2)*d^2*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(
Sec[e + f*x]^2)^(1/4)], -1]*(d*Sec[e + f*x])^(3/2)*Sqrt[-Tan[e + f*x]^2])/(
8*b^3*(a^2 + b^2)^(3/2)*f*(Sec[e + f*x]^2)^(3/4)) + (3*a*(a^2 + 2*b^2)*d^2*
Cot[e + f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)],
-1]*(d*Sec[e + f*x])^(3/2)*Sqrt[-Tan[e + f*x]^2])/(8*b^3*(a^2 + b^2)^(3/2)*
f*(Sec[e + f*x]^2)^(3/4)) - (d^2*(d*Sec[e + f*x])^(3/2))/(2*b*f*(a + b*Tan[
e + f*x])^2) + (3*a*d^2*(d*Sec[e + f*x])^(3/2))/(4*b*(a^2 + b^2)*f*(a + b*T
an[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 747

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 1))), x] - Dist[2*c*(p/(e*(m + 1))
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 760

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\left(d^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^{3/4}}{(a+x)^3}\,dx, x, b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$= -\frac{d^2(d\sec(e+fx))^{3/2}}{2bf(a+b\tan(e+fx))^2} + \frac{\left(3d^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{x}{(a+x)^2\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{4b^3f\sec^2(e+fx)^{3/4}}$$

$$= -\frac{d^2(d\sec(e+fx))^{3/2}}{2bf(a+b\tan(e+fx))^2} + \frac{3ad^2(d\sec(e+fx))^{3/2}}{4b(a^2+b^2)f(a+b\tan(e+fx))} - \frac{\left(3d^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{-1+\frac{ax}{2b^2}}{(a+x)\sqrt[4]{1+\frac{x^2}{b^2}}}dx, x, b\tan(e+fx)\right)}{4b(a^2+b^2)f\sec^2(e+fx)^{3/4}}$$

$$= -\frac{d^2(d\sec(e+fx))^{3/2}}{2bf(a+b\tan(e+fx))^2} + \frac{3ad^2(d\sec(e+fx))^{3/2}}{4b(a^2+b^2)f(a+b\tan(e+fx))} - \frac{\left(3ad^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+\frac{x^2}{b^2}}}dx, x, b\tan(e+fx)\right)}{8b^3(a^2+b^2)f\sec^2(e+fx)^{3/4}} - \frac{\left(3\left(-2-\frac{a^2}{b^2}\right)d^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{1}{(a+x)\sqrt[4]{1+\frac{x^2}{b^2}}}dx, x, b\tan(e+fx)\right)}{8b(a^2+b^2)f\sec^2(e+fx)^{3/4}}$$

$$= -\frac{3ad^2\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{4b^2(a^2+b^2)f} - \frac{d^2(d\sec(e+fx))^{3/2}}{2bf(a+b\tan(e+fx))^2} + \frac{3ad^2(d\sec(e+fx))^{3/2}}{4b(a^2+b^2)f(a+b\tan(e+fx))} + \frac{\left(3ad^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{x^2}{b^2}\right)^{5/4}}dx, x, b\tan(e+fx)\right)}{8b^3(a^2+b^2)f\sec^2(e+fx)^{3/4}} + \frac{\left(3\left(-2-\frac{a^2}{b^2}\right)d^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{x}{(a^2-x^2)\sqrt[4]{1+\frac{x^2}{b^2}}}dx, x, b\tan(e+fx)\right)}{8b(a^2+b^2)f\sec^2(e+fx)^{3/4}} - \frac{\left(3a\left(-2-\frac{a^2}{b^2}\right)d^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{1}{(a^2-x^2)\sqrt[4]{1+\frac{x^2}{b^2}}}dx, x, b\tan(e+fx)\right)}{8b(a^2+b^2)f\sec^2(e+fx)^{3/4}}$$

$$=\frac{3ad^2E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{4b^2(a^2+b^2)f\sec^2(e+fx)^{3/4}}-\frac{3ad^2\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{4b^2(a^2+b^2)f}-\frac{d^2(d\sec(e+fx))^{3/2}}{2bf(a+b\tan(e+fx))^2}+\frac{3ad^2(d\sec(e+fx))^{3/2}}{4b(a^2+b^2)f(a+b\tan(e+fx))}+\frac{\left(3\left(-2-\frac{a^2}{b^2}\right)d^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int\frac{1}{(a^2-x)\sqrt[4]{1+\frac{x}{b^2}}}dx,x,b^2\tan^2(e+fx)\right)}{16b(a^2+b^2)f\sec^2(e+fx)^{3/4}}-\frac{\left(3a\left(-2-\frac{a^2}{b^2}\right)d^2\cot(e+fx)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{1-x^4}\left(1+\frac{a^2}{b^2}-x^4\right)}dx,x\right.}{4b^2(a^2+b^2)f\sec^2(e+fx)^{3/4}}$$

$$=\frac{3ad^2E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{4b^2(a^2+b^2)f\sec^2(e+fx)^{3/4}}-\frac{3ad^2\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{4b^2(a^2+b^2)f}-\frac{d^2(d\sec(e+fx))^{3/2}}{2bf(a+b\tan(e+fx))^2}+\frac{3ad^2(d\sec(e+fx))^{3/2}}{4b(a^2+b^2)f(a+b\tan(e+fx))}+\frac{\left(3\left(-2-\frac{a^2}{b^2}\right)bd^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int\frac{x^2}{a^2+b^2-b^2x^4}dx,x,\sqrt[4]{\sec^2(e+fx)}\right)}{4(a^2+b^2)f\sec^2(e+fx)^{3/4}}-\frac{\left(3a\left(-2-\frac{a^2}{b^2}\right)d^2\cot(e+fx)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\left(\sqrt{a^2+b^2}-bx^2\right)\sqrt{1-x^4}}d\right.}{8b(a^2+b^2)f\sec^2(e+fx)^{3/4}}+\frac{\left(3a\left(-2-\frac{a^2}{b^2}\right)d^2\cot(e+fx)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\left(\sqrt{a^2+b^2}+bx^2\right)\sqrt{1-x^4}}d\right.}{8b(a^2+b^2)f\sec^2(e+fx)^{3/4}}$$

$$= \frac{3ad^2E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{4b^2(a^2+b^2)f\sec^2(e+fx)^{3/4}} - \frac{3ad^2\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{4b^2(a^2+b^2)f} - \frac{d^2(d\sec(e+fx))^{3/2}}{2bf(a+b\tan(e+fx))^2} + \frac{3ad^2(d\sec(e+fx))^{3/2}}{4b(a^2+b^2)f(a+b\tan(e+fx))} + \frac{\left(3\left(-2-\frac{a^2}{b^2}\right)d^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{a^2+b^2-bx^2}}dx,x,\sqrt[4]{\sec^2(e+fx)}\right)}{8(a^2+b^2)f\sec^2(e+fx)^{3/4}} - \frac{\left(3\left(-2-\frac{a^2}{b^2}\right)d^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{a^2+b^2+bx^2}}dx,x,\sqrt[4]{\sec^2(e+fx)}\right)}{8(a^2+b^2)f\sec^2(e+fx)^{3/4}} - \frac{\left(3a\left(-2-\frac{a^2}{b^2}\right)d^2\cot(e+fx)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(\sqrt{a^2+b^2}\right.}\right.}{8b(a^2+b^2)f\sec^2(e+fx)^{3/4}} + \frac{\left(3a\left(-2-\frac{a^2}{b^2}\right)d^2\cot(e+fx)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(\sqrt{a^2+b^2}\right.}\right.}{8b(a^2+b^2)f\sec^2(e+fx)^{3/4}}$$

$$= \frac{3(a^2+2b^2)d^2\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{8b^{5/2}(a^2+b^2)^{5/4}f\sec^2(e+fx)^{3/4}} - \frac{3(a^2+2b^2)d^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{8b^{5/2}(a^2+b^2)^{5/4}f\sec^2(e+fx)^{3/4}} + \frac{3ad^2E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{4b^2(a^2+b^2)f\sec^2(e+fx)^{3/4}} - \frac{3ad^2\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{4b^2(a^2+b^2)f} - \frac{3a(a^2+2b^2)d^2\cot(e+fx)\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3}}{8b^3(a^2+b^2)^{3/2}f\sec^2(e+fx)^{3/4}} + \frac{3a(a^2+2b^2)d^2\cot(e+fx)\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}}{8b^3(a^2+b^2)^{3/2}f\sec^2(e+fx)^{3/4}} - \frac{d^2(d\sec(e+fx))^{3/2}}{2bf(a+b\tan(e+fx))^2} + \frac{3ad^2(d\sec(e+fx))^{3/2}}{4b(a^2+b^2)f(a+b\tan(e+fx))}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 35.71 (sec) , antiderivative size = 14225, normalized size of antiderivative = 24.40

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(d*Sec[e + f*x])^(7/2)/(a + b*Tan[e + f*x])^3,x]
[Out] Result too large to show
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 72184 vs. 2(532) = 1064.

Time = 1486.38 (sec) , antiderivative size = 72185, normalized size of antiderivative = 123.82

method	result	size
default	Expression too large to display	72185

```
[In] int((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx=\text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))**(7/2)/(a+b*tan(f*x+e))**3,x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx=\text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx=\int \frac{(d\sec(fx+e))^{\frac{7}{2}}}{(b\tan(fx+e)+a)^3}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(7/2)/(a+b*tan(f*x+e))^3,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(7/2)/(b*tan(f*x + e) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{7/2}}{(a+b\tan(e+fx))^3}\,dx=\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{7/2}}{(a+b\tan(e+f\,x))^3}\,dx$$

```
[In] int((d/cos(e + f*x))^(7/2)/(a + b*tan(e + f*x))^3,x)
[Out] int((d/cos(e + f*x))^(7/2)/(a + b*tan(e + f*x))^3, x)
```

3.618 $\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 532

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx = \frac{(a^2-2b^2)\,d^2\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{8b^{3/2}\left(a^2+b^2\right)^{7/4}f\sqrt[4]{\sec^2(e+fx)}} + \frac{(a^2-2b^2)\,d^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{8b^{3/2}\left(a^2+b^2\right)^{7/4}f\sqrt[4]{\sec^2(e+fx)}} + \frac{ad^2\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{4b^2\left(a^2+b^2\right)f\sqrt[4]{\sec^2(e+fx)}} - \frac{a(a^2-2b^2)\,d^2\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+}}{8b^2\left(a^2+b^2\right)^2f\sqrt[4]{\sec^2(e+fx)}} - \frac{a(a^2-2b^2)\,d^2\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+}}{8b^2\left(a^2+b^2\right)^2f\sqrt[4]{\sec^2(e+fx)}} - \frac{d^2\sqrt{d\sec(e+fx)}}{2bf(a+b\tan(e+fx))^2} + \frac{ad^2\sqrt{d\sec(e+fx)}}{4b\left(a^2+b^2\right)f(a+b\tan(e+fx))}$$

```
[Out] 1/8*(a^2-2*b^2)*d^2*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(d
*sec(f*x+e))^(1/2)/b^(3/2)/(a^2+b^2)^(7/4)/f/(sec(f*x+e)^2)^(1/4)+1/8*(a^2-
2*b^2)*d^2*arctanh((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(d*sec(f*x
+e))^(1/2)/b^(3/2)/(a^2+b^2)^(7/4)/f/(sec(f*x+e)^2)^(1/4)+1/4*a*d^2*(cos(1/
2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*EllipticF(sin(1/
2*arctan(tan(f*x+e))),2^(1/2))*(d*sec(f*x+e))^(1/2)/b^2/(a^2+b^2)/f/(sec(f*
x+e)^2)^(1/4)-1/8*a*(a^2-2*b^2)*d^2*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1
/4),-b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(1/2)*(-tan(f*x+e)^2)^(1/2)/b^2/(a
```

```
^2+b^2)^2/f/(sec(f*x+e)^2)^(1/4)-1/8*a*(a^2-2*b^2)*d^2*cot(f*x+e)*EllipticP
i((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(1/2)*(-tan(f*x+
e)^2)^(1/2)/b^2/(a^2+b^2)^2/f/(sec(f*x+e)^2)^(1/4)-1/2*d^2*(d*sec(f*x+e))^(
1/2)/b/f/(a+b*tan(f*x+e))^2+1/4*a*d^2*(d*sec(f*x+e))^(1/2)/b/(a^2+b^2)/f/(a
+b*tan(f*x+e))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3593, 747, 849, 858, 237, 761, 410, 109, 418, 1227, 551, 455, 65, 218, 214, 211}

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx = -\frac{ad^2(a^2-2b^2)\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sqrt{d\sec(e+fx)}\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right)\right.}{8b^2f(a^2+b^2)^2\sqrt[4]{\sec^2(e+fx)}} - \frac{ad^2(a^2-2b^2)\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sqrt{d\sec(e+fx)}\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),\right.}{8b^2f(a^2+b^2)^2\sqrt[4]{\sec^2(e+fx)}} + \frac{ad^2\sqrt{d\sec(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)}{4b^2f(a^2+b^2)\sqrt[4]{\sec^2(e+fx)}} + \frac{d^2(a^2-2b^2)\sqrt{d\sec(e+fx)}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8b^{3/2}f(a^2+b^2)^{7/4}\sqrt[4]{\sec^2(e+fx)}} + \frac{d^2(a^2-2b^2)\sqrt{d\sec(e+fx)}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8b^{3/2}f(a^2+b^2)^{7/4}\sqrt[4]{\sec^2(e+fx)}} + \frac{ad^2\sqrt{d\sec(e+fx)}}{4bf(a^2+b^2)(a+b\tan(e+fx))} - \frac{d^2\sqrt{d\sec(e+fx)}}{2bf(a+b\tan(e+fx))^2}$$

```
[In] Int[(d*Sec[e + f*x])^(5/2)/(a + b*Tan[e + f*x])^3,x]
```

```
[Out] ((a^2 - 2*b^2)*d^2*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4
)]*Sqrt[d*Sec[e + f*x]])/(8*b^(3/2)*(a^2 + b^2)^(7/4)*f*(Sec[e + f*x]^2)^(1
/4)) + ((a^2 - 2*b^2)*d^2*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b
^2)^(1/4)]*Sqrt[d*Sec[e + f*x]])/(8*b^(3/2)*(a^2 + b^2)^(7/4)*f*(Sec[e + f*
x]^2)^(1/4)) + (a*d^2*EllipticF[ArcTan[Tan[e + f*x]]/2, 2]*Sqrt[d*Sec[e + f
*x]])/(4*b^2*(a^2 + b^2)*f*(Sec[e + f*x]^2)^(1/4)) - (a*(a^2 - 2*b^2)*d^2*C
ot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)]
, -1]*Sqrt[d*Sec[e + f*x]]*Sqrt[-Tan[e + f*x]^2])/(8*b^2*(a^2 + b^2)^2*f*(S
ec[e + f*x]^2)^(1/4)) - (a*(a^2 - 2*b^2)*d^2*Cot[e + f*x]*EllipticPi[b/Sqrt
[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[d*Sec[e + f*x]]*Sqrt[
```

```
-Tan[e + f*x]^2])/(8*b^2*(a^2 + b^2)^2*f*(Sec[e + f*x]^2)^(1/4)) - (d^2*Sqr
t[d*Sec[e + f*x]])/(2*b*f*(a + b*Tan[e + f*x])^2) + (a*d^2*Sqrt[d*Sec[e + f
*x]])/(4*b*(a^2 + b^2)*f*(a + b*Tan[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 109

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e
/f) + d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-f/(d*e - c*f), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[(-b)*(x^2/a)]/(2*x), Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 747

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 1))), x] - Dist[2*c*(p/(e*(m + 1))
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 761

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(3/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(3/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(3/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
```

```
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\left(d^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int\frac{\sqrt[4]{1+\frac{x^2}{b^2}}}{(a+x)^3}\,dx,x,b\tan(e+fx)\right)}{bf\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{d^2\sqrt{d\sec(e+fx)}}{2bf(a+b\tan(e+fx))^2} + \frac{\left(d^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int\frac{x}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx,x,b\tan(e+fx)\right)}{4b^3f\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{d^2\sqrt{d\sec(e+fx)}}{2bf(a+b\tan(e+fx))^2} + \frac{ad^2\sqrt{d\sec(e+fx)}}{4b\left(a^2+b^2\right)f(a+b\tan(e+fx))} - \frac{\left(d^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int\frac{-1-\frac{ax}{2b^2}}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx,x,b\tan(e+fx)\right)}{4b\left(a^2+b^2\right)f\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{d^2\sqrt{d\sec(e+fx)}}{2bf(a+b\tan(e+fx))^2} + \frac{ad^2\sqrt{d\sec(e+fx)}}{4b(a^2+b^2)f(a+b\tan(e+fx))} + \frac{\left(ad^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\left(1+\frac{x^2}{b^2}\right)^{3/4}}dx, x, b\tan(e+fx)\right)}{8b^3(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}} - \frac{\left(\left(-2+\frac{a^2}{b^2}\right)d^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int\frac{1}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{3/4}}dx, x, b\tan(e+fx)\right)}{8b(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}}$$

$$= \frac{ad^2\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)), 2\right)\sqrt{d\sec(e+fx)}}{4b^2(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}} - \frac{d^2\sqrt{d\sec(e+fx)}}{2bf(a+b\tan(e+fx))^2} + \frac{ad^2\sqrt{d\sec(e+fx)}}{4b(a^2+b^2)f(a+b\tan(e+fx))} + \frac{\left(\left(-2+\frac{a^2}{b^2}\right)d^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int\frac{x}{(a^2-x^2)\left(1+\frac{x^2}{b^2}\right)^{3/4}}dx, x, b\tan(e+fx)\right)}{8b(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}} - \frac{\left(a\left(-2+\frac{a^2}{b^2}\right)d^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int\frac{1}{(a^2-x^2)\left(1+\frac{x^2}{b^2}\right)^{3/4}}dx, x, b\tan(e+fx)\right)}{8b(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}}$$

$$= \frac{ad^2\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)), 2\right)\sqrt{d\sec(e+fx)}}{4b^2(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}} - \frac{d^2\sqrt{d\sec(e+fx)}}{2bf(a+b\tan(e+fx))^2} + \frac{ad^2\sqrt{d\sec(e+fx)}}{4b(a^2+b^2)f(a+b\tan(e+fx))} + \frac{\left(\left(-2+\frac{a^2}{b^2}\right)d^2\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int\frac{1}{(a^2-x)\left(1+\frac{x}{b^2}\right)^{3/4}}dx, x, b^2\tan^2(e+fx)\right)}{16b(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}} - \frac{\left(a\left(-2+\frac{a^2}{b^2}\right)d^2\cot(e+fx)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{(a^2-x)\sqrt{-\frac{x}{b^2}}\left(1+\frac{x}{b^2}\right)^{3/4}}dx\right.}{16b^2(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}}$$

$$= \frac{ad^2 \operatorname{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{4b^2(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}} - \frac{d^2\sqrt{d\sec(e+fx)}}{2bf(a+b\tan(e+fx))^2} + \frac{ad^2\sqrt{d\sec(e+fx)}}{4b(a^2+b^2)f(a+b\tan(e+fx))} + \frac{\left(\left(-2+\frac{a^2}{b^2}\right)bd^2\sqrt{d\sec(e+fx)}\right)\operatorname{Subst}\left(\int \frac{1}{a^2+b^2-b^2x^4}\,dx, x, \sqrt[4]{\sec^2(e+fx)}\right)}{4(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}} + \frac{\left(a\left(-2+\frac{a^2}{b^2}\right)d^2\cot(e+fx)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^4}\left(-1-\frac{a^2}{b^2}+x^4\right)}\,dx, x,\right.}{4b^2(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}}$$

$$= \frac{ad^2 \operatorname{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{4b^2(a^2+b^2)f\sqrt[4]{\sec^2(e+fx)}} - \frac{d^2\sqrt{d\sec(e+fx)}}{2bf(a+b\tan(e+fx))^2} + \frac{ad^2\sqrt{d\sec(e+fx)}}{4b(a^2+b^2)f(a+b\tan(e+fx))} + \frac{\left(\left(-2+\frac{a^2}{b^2}\right)bd^2\sqrt{d\sec(e+fx)}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a^2+b^2}-bx^2}\,dx, x, \sqrt[4]{\sec^2(e+fx)}\right)}{8(a^2+b^2)^{3/2}f\sqrt[4]{\sec^2(e+fx)}} + \frac{\left(\left(-2+\frac{a^2}{b^2}\right)bd^2\sqrt{d\sec(e+fx)}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a^2+b^2}+bx^2}\,dx, x, \sqrt[4]{\sec^2(e+fx)}\right)}{8(a^2+b^2)^{3/2}f\sqrt[4]{\sec^2(e+fx)}} - \frac{\left(a\left(-2+\frac{a^2}{b^2}\right)d^2\cot(e+fx)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\operatorname{Subst}\left(\int \frac{1}{\left(1-\frac{bx^2}{\sqrt{a^2+b^2}}\right)\sqrt{1-x^4}}\,dx, x,\right.}{8(a^2+b^2)^2f\sqrt[4]{\sec^2(e+fx)}} - \frac{\left(a\left(-2+\frac{a^2}{b^2}\right)d^2\cot(e+fx)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\operatorname{Subst}\left(\int \frac{1}{\left(1+\frac{bx^2}{\sqrt{a^2+b^2}}\right)\sqrt{1-x^4}}\,dx, x,\right.}{8(a^2+b^2)^2f\sqrt[4]{\sec^2(e+fx)}}$$

$$= \frac{(a^2-2b^2)\,d^2\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{8b^{3/2}\left(a^2+b^2\right)^{7/4}f\sqrt[4]{\sec^2(e+fx)}} + \frac{(a^2-2b^2)\,d^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{8b^{3/2}\left(a^2+b^2\right)^{7/4}f\sqrt[4]{\sec^2(e+fx)}} + \frac{ad^2\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{4b^2\left(a^2+b^2\right)f\sqrt[4]{\sec^2(e+fx)}} - \frac{d^2\sqrt{d\sec(e+fx)}}{2bf(a+b\tan(e+fx))^2} + \frac{ad^2\sqrt{d\sec(e+fx)}}{4b\left(a^2+b^2\right)f(a+b\tan(e+fx))} - \frac{\left(a\left(-2+\frac{a^2}{b^2}\right)d^2\cot(e+fx)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(1-\frac{bx^2}{\sqrt{a^2+b^2}}\right)}\right.}{8\left(a^2+b^2\right)^2f\sqrt[4]{\sec^2(e+fx)}} - \frac{\left(a\left(-2+\frac{a^2}{b^2}\right)d^2\cot(e+fx)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(1+\frac{bx^2}{\sqrt{a^2+b^2}}\right)}\right.}{8\left(a^2+b^2\right)^2f\sqrt[4]{\sec^2(e+fx)}}$$

$$= \frac{(a^2-2b^2)\,d^2\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{8b^{3/2}\left(a^2+b^2\right)^{7/4}f\sqrt[4]{\sec^2(e+fx)}} + \frac{(a^2-2b^2)\,d^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{8b^{3/2}\left(a^2+b^2\right)^{7/4}f\sqrt[4]{\sec^2(e+fx)}} + \frac{ad^2\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{4b^2\left(a^2+b^2\right)f\sqrt[4]{\sec^2(e+fx)}} + \frac{a\left(2-\frac{a^2}{b^2}\right)d^2\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{}}{8\left(a^2+b^2\right)^2f\sqrt[4]{\sec^2(e+fx)}} + \frac{a\left(2-\frac{a^2}{b^2}\right)d^2\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-}}{8\left(a^2+b^2\right)^2f\sqrt[4]{\sec^2(e+fx)}} - \frac{d^2\sqrt{d\sec(e+fx)}}{2bf(a+b\tan(e+fx))^2} + \frac{ad^2\sqrt{d\sec(e+fx)}}{4b\left(a^2+b^2\right)f(a+b\tan(e+fx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.26 (sec) , antiderivative size = 352, normalized size of antiderivative = 0.66

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx = \frac{d(d\sec(e+fx))^{3/2}(a\cos(e+fx)+b\sin(e+fx))^3\left(-\frac{2b(a^2+b^2)\sec^2(e+fx)(a^2+2b^2-}{(a+b\tan(e+fx))^2}\right.}{}$$

```
[In] Integrate[(d*Sec[e + f*x])^(5/2)/(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (d*(d*Sec[e + f*x])^(3/2)*(a*Cos[e + f*x] + b*Sin[e + f*x])^3*((-2*b*(a^2 +
 b^2)*Sec[e + f*x]^2*(a^2 + 2*b^2 - a*b*Tan[e + f*x]))/(a + b*Tan[e + f*x])
^2 + (Sec[e + f*x]^2)^(3/4)*(a*(a^2 + b^2)*Hypergeometric2F1[1/2, 3/4, 3/2,
 -Tan[e + f*x]^2]*Tan[e + f*x] + ((a^2 - 2*b^2)*(a*EllipticPi[-(b/Sqrt[a^2
+ b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Tan[e + f*x] + a*EllipticPi[b/
Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Tan[e + f*x] + Sqrt[b]
*(a^2 + b^2)^(1/4)*(ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/
4)] + ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)])*Sqrt[-Ta
n[e + f*x]^2]))/Sqrt[-Tan[e + f*x]^2])))/(8*b^2*(a^2 + b^2)^2*f*(a + b*Tan[
e + f*x])^3)
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 36013 vs. 2(489) = 978.

Time = 1496.34 (sec) , antiderivative size = 36014, normalized size of antiderivative = 67.70

method	result	size
default	Expression too large to display	36014

```
[In] int((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))**(5/2)/(a+b*tan(f*x+e))**3,x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{2}}}{(b\tan(fx+e)+a)^3}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(5/2)/(b*tan(f*x + e) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/2}}{(a+b\tan(e+fx))^3}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}}{\left(a+b\tan\left(e+f\,x\right)\right)^3}\,dx$$

```
[In] int((d/cos(e + f*x))^(5/2)/(a + b*tan(e + f*x))^3,x)
[Out] int((d/cos(e + f*x))^(5/2)/(a + b*tan(e + f*x))^3, x)
```

3.619 $\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 566

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx = \frac{(3a^2-2b^2)\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{8\sqrt{b}\,(a^2+b^2)^{9/4}\,f\sec^2(e+fx)^{3/4}}$$

$$-\frac{(3a^2-2b^2)\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{8\sqrt{b}\,(a^2+b^2)^{9/4}\,f\sec^2(e+fx)^{3/4}}$$

$$-\frac{5aE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{4\,(a^2+b^2)^2\,f\sec^2(e+fx)^{3/4}}$$

$$+\frac{5a\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{4\,(a^2+b^2)^2\,f}$$

$$-\frac{a(3a^2-2b^2)\cot(e+fx)\,\mathrm{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e}}{8b\,(a^2+b^2)^{5/2}\,f\sec^2(e+fx)^{3/4}}$$

$$+\frac{a(3a^2-2b^2)\cot(e+fx)\,\mathrm{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+}}{8b\,(a^2+b^2)^{5/2}\,f\sec^2(e+fx)^{3/4}}$$

$$-\frac{b(d\sec(e+fx))^{3/2}}{2\,(a^2+b^2)\,f(a+b\tan(e+fx))^2}-\frac{5ab(d\sec(e+fx))^{3/2}}{4\,(a^2+b^2)^2\,f(a+b\tan(e+fx))}$$

```
[Out] -5/4*a*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*El
lipticE(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(d*sec(f*x+e))^(3/2)/(a^2+b^2)
^2/f/(sec(f*x+e)^2)^(3/4)+5/4*a*cos(f*x+e)*(d*sec(f*x+e))^(3/2)*sin(f*x+e)/
(a^2+b^2)^2/f+1/8*(3*a^2-2*b^2)*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^
2)^(1/4))*(d*sec(f*x+e))^(3/2)/(a^2+b^2)^(9/4)/f/(sec(f*x+e)^2)^(3/4)/b^(1/
2)-1/8*(3*a^2-2*b^2)*arctanh((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*
```

```
(d*sec(f*x+e))^(3/2)/(a^2+b^2)^(9/4)/f/(sec(f*x+e)^2)^(3/4)/b^(1/2)-1/8*a*(
3*a^2-2*b^2)*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),
I)*(d*sec(f*x+e))^(3/2)*(-tan(f*x+e)^2)^(1/2)/b/(a^2+b^2)^(5/2)/f/(sec(f*x+
e)^2)^(3/4)+1/8*a*(3*a^2-2*b^2)*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),
b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(3/2)*(-tan(f*x+e)^2)^(1/2)/b/(a^2+b^2)
^(5/2)/f/(sec(f*x+e)^2)^(3/4)-1/2*b*(d*sec(f*x+e))^(3/2)/(a^2+b^2)/f/(a+b*t
an(f*x+e))^2-5/4*a*b*(d*sec(f*x+e))^(3/2)/(a^2+b^2)^2/f/(a+b*tan(f*x+e))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 566, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3593, 759, 849, 858, 233, 202, 760, 408, 504, 1227, 551, 455, 65, 304, 211, 214}

$$
\begin{aligned}
\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx = \\
&-\frac{a(3a^2-2b^2)\sqrt{-\tan^2(e+fx)}\cot(e+fx)(d\sec(e+fx))^{3/2}\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right)\right.}{8bf\left(a^2+b^2\right)^{5/2}\sec^2(e+fx)^{3/4}} \\
&+\frac{a(3a^2-2b^2)\sqrt{-\tan^2(e+fx)}\cot(e+fx)(d\sec(e+fx))^{3/2}\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-\right.}{8bf\left(a^2+b^2\right)^{5/2}\sec^2(e+fx)^{3/4}} \\
&+\frac{(3a^2-2b^2)(d\sec(e+fx))^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8\sqrt{b}f\left(a^2+b^2\right)^{9/4}\sec^2(e+fx)^{3/4}} \\
&-\frac{5a(d\sec(e+fx))^{3/2}E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)}{4f\left(a^2+b^2\right)^2\sec^2(e+fx)^{3/4}} \\
&-\frac{(3a^2-2b^2)(d\sec(e+fx))^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8\sqrt{b}f\left(a^2+b^2\right)^{9/4}\sec^2(e+fx)^{3/4}} \\
&-\frac{5ab(d\sec(e+fx))^{3/2}}{4f\left(a^2+b^2\right)^2(a+b\tan(e+fx))}-\frac{b(d\sec(e+fx))^{3/2}}{2f\left(a^2+b^2\right)(a+b\tan(e+fx))^2} \\
&+\frac{5a\sin(e+fx)\cos(e+fx)(d\sec(e+fx))^{3/2}}{4f\left(a^2+b^2\right)^2}
\end{aligned}
$$

```
[In] Int[(d*Sec[e + f*x])^(3/2)/(a + b*Tan[e + f*x])^3,x]
```

```
[Out] ((3*a^2 - 2*b^2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)]
*(d*Sec[e + f*x])^(3/2))/(8*Sqrt[b]*(a^2 + b^2)^(9/4)*f*(Sec[e + f*x]^2)^(3
/4)) - ((3*a^2 - 2*b^2)*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2
)^(1/4)]*(d*Sec[e + f*x])^(3/2))/(8*Sqrt[b]*(a^2 + b^2)^(9/4)*f*(Sec[e + f*
x]^2)^(3/4)) - (5*a*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(d*Sec[e + f*x])^(
3/2))/(4*(a^2 + b^2)^2*f*(Sec[e + f*x]^2)^(3/4)) + (5*a*Cos[e + f*x]*(d*Sec
```

```
[e + f*x])^(3/2)*Sin[e + f*x])/(4*(a^2 + b^2)^2*f) - (a*(3*a^2 - 2*b^2)*Cot
[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)],
-1]*(d*Sec[e + f*x])^(3/2)*Sqrt[-Tan[e + f*x]^2])/(8*b*(a^2 + b^2)^(5/2)*f*
(Sec[e + f*x]^2)^(3/4)) + (a*(3*a^2 - 2*b^2)*Cot[e + f*x]*EllipticPi[b/Sqrt
[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(d*Sec[e + f*x])^(3/2)*Sqr
t[-Tan[e + f*x]^2])/(8*b*(a^2 + b^2)^(5/2)*f*(Sec[e + f*x]^2)^(3/4)) - (b*(
d*Sec[e + f*x])^(3/2))/(2*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^2) - (5*a*b*(d
*Sec[e + f*x])^(3/2))/(4*(a^2 + b^2)^2*f*(a + b*Tan[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 760

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{(d\sec(e+fx))^{3/2}\text{Subst}\left(\int \frac{1}{(a+x)^3\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{3/4}}$$

$$= -\frac{b(d\sec(e+fx))^{3/2}}{2\left(a^2+b^2\right)f(a+b\tan(e+fx))^2} - \frac{(d\sec(e+fx))^{3/2}\text{Subst}\left(\int \frac{-2a+\frac{x}{2}}{(a+x)^2\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{2b\left(a^2+b^2\right)f\sec^2(e+fx)^{3/4}}$$

$$= -\frac{b(d\sec(e+fx))^{3/2}}{2(a^2+b^2)f(a+b\tan(e+fx))^2} - \frac{5ab(d\sec(e+fx))^{3/2}}{4(a^2+b^2)^2 f(a+b\tan(e+fx))} + \frac{\left(b(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{\frac{1}{2}\left(-1+\frac{4a^2}{b^2}\right)+\frac{5ax}{4b^2}}{(a+x)\sqrt[4]{1+\frac{x^2}{b^2}}}dx, x, b\tan(e+fx)\right)}{2(a^2+b^2)^2 f\sec^2(e+fx)^{3/4}}$$

$$= -\frac{b(d\sec(e+fx))^{3/2}}{2(a^2+b^2)f(a+b\tan(e+fx))^2} - \frac{5ab(d\sec(e+fx))^{3/2}}{4(a^2+b^2)^2 f(a+b\tan(e+fx))} + \frac{\left(5a(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+\frac{x^2}{b^2}}}dx, x, b\tan(e+fx)\right)}{8b(a^2+b^2)^2 f\sec^2(e+fx)^{3/4}} + \frac{\left(\left(-2+\frac{3a^2}{b^2}\right)b(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{1}{(a+x)\sqrt[4]{1+\frac{x^2}{b^2}}}dx, x, b\tan(e+fx)\right)}{8(a^2+b^2)^2 f\sec^2(e+fx)^{3/4}}$$

$$= \frac{5a\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{4(a^2+b^2)^2 f} - \frac{b(d\sec(e+fx))^{3/2}}{2(a^2+b^2)f(a+b\tan(e+fx))^2} - \frac{5ab(d\sec(e+fx))^{3/2}}{4(a^2+b^2)^2 f(a+b\tan(e+fx))} - \frac{\left(5a(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{x^2}{b^2}\right)^{5/4}}dx, x, b\tan(e+fx)\right)}{8b(a^2+b^2)^2 f\sec^2(e+fx)^{3/4}} - \frac{\left(\left(-2+\frac{3a^2}{b^2}\right)b(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{x}{(a^2-x^2)\sqrt[4]{1+\frac{x^2}{b^2}}}dx, x, b\tan(e+fx)\right)}{8(a^2+b^2)^2 f\sec^2(e+fx)^{3/4}} + \frac{\left(a\left(-2+\frac{3a^2}{b^2}\right)b(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int \frac{1}{(a^2-x^2)\sqrt[4]{1+\frac{x^2}{b^2}}}dx, x, b\tan(e+fx)\right)}{8(a^2+b^2)^2 f\sec^2(e+fx)^{3/4}}$$

$$= -\frac{5aE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{4\left(a^2+b^2\right)^2 f\sec^2(e+fx)^{3/4}} + \frac{5a\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{4\left(a^2+b^2\right)^2 f} - \frac{b(d\sec(e+fx))^{3/2}}{2\left(a^2+b^2\right)f(a+b\tan(e+fx))^2} - \frac{5ab(d\sec(e+fx))^{3/2}}{4\left(a^2+b^2\right)^2 f(a+b\tan(e+fx))} - \frac{\left(\left(-2+\frac{3a^2}{b^2}\right)b(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int\frac{1}{(a^2-x)\sqrt[4]{1+\frac{x}{b^2}}}\,dx,x,b^2\tan^2(e+fx)\right)}{16\left(a^2+b^2\right)^2 f\sec^2(e+fx)^{3/4}} + \frac{\left(a\left(-2+\frac{3a^2}{b^2}\right)\cot(e+fx)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{1-x^4}\left(1+\frac{a^2}{b^2}-x^4\right)}\,dx,x,\right.}{4\left(a^2+b^2\right)^2 f\sec^2(e+fx)^{3/4}}$$

$$= -\frac{5aE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{4\left(a^2+b^2\right)^2 f\sec^2(e+fx)^{3/4}} + \frac{5a\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{4\left(a^2+b^2\right)^2 f} - \frac{b(d\sec(e+fx))^{3/2}}{2\left(a^2+b^2\right)f(a+b\tan(e+fx))^2} - \frac{5ab(d\sec(e+fx))^{3/2}}{4\left(a^2+b^2\right)^2 f(a+b\tan(e+fx))} - \frac{\left(\left(-2+\frac{3a^2}{b^2}\right)b^3(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int\frac{x^2}{a^2+b^2-b^2x^4}\,dx,x,\sqrt[4]{\sec^2(e+fx)}\right)}{4\left(a^2+b^2\right)^2 f\sec^2(e+fx)^{3/4}} + \frac{\left(a\left(-2+\frac{3a^2}{b^2}\right)b\cot(e+fx)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\left(\sqrt{a^2+b^2}-bx^2\right)\sqrt{1-x^4}}\,dx\right.}{8\left(a^2+b^2\right)^2 f\sec^2(e+fx)^{3/4}} - \frac{\left(a\left(-2+\frac{3a^2}{b^2}\right)b\cot(e+fx)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\left(\sqrt{a^2+b^2}+bx^2\right)\sqrt{1-x^4}}\,dx\right.}{8\left(a^2+b^2\right)^2 f\sec^2(e+fx)^{3/4}}$$

$$= -\frac{5aE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{4(a^2+b^2)^2 f\sec^2(e+fx)^{3/4}} + \frac{5a\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{4(a^2+b^2)^2 f} - \frac{b(d\sec(e+fx))^{3/2}}{2(a^2+b^2)f(a+b\tan(e+fx))^2} - \frac{5ab(d\sec(e+fx))^{3/2}}{4(a^2+b^2)^2 f(a+b\tan(e+fx))} - \frac{\left(\left(-2+\frac{3a^2}{b^2}\right)b^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{a^2+b^2}-bx^2}dx,x,\sqrt[4]{\sec^2(e+fx)}\right)}{8(a^2+b^2)^2 f\sec^2(e+fx)^{3/4}} + \frac{\left(\left(-2+\frac{3a^2}{b^2}\right)b^2(d\sec(e+fx))^{3/2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{a^2+b^2}+bx^2}dx,x,\sqrt[4]{\sec^2(e+fx)}\right)}{8(a^2+b^2)^2 f\sec^2(e+fx)^{3/4}} + \frac{\left(a\left(-2+\frac{3a^2}{b^2}\right)b\cot(e+fx)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(\sqrt{a^2+b^2}-bx^2\right)}\right.}{8(a^2+b^2)^2 f\sec^2(e+fx)^{3/4}} - \frac{\left(a\left(-2+\frac{3a^2}{b^2}\right)b\cot(e+fx)(d\sec(e+fx))^{3/2}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(\sqrt{a^2+b^2}+bx^2\right)}\right.}{8(a^2+b^2)^2 f\sec^2(e+fx)^{3/4}}$$

$$= -\frac{\left(2-\frac{3a^2}{b^2}\right)b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{8(a^2+b^2)^{9/4}f\sec^2(e+fx)^{3/4}} + \frac{\left(2-\frac{3a^2}{b^2}\right)b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)(d\sec(e+fx))^{3/2}}{8(a^2+b^2)^{9/4}f\sec^2(e+fx)^{3/4}} - \frac{5aE\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)(d\sec(e+fx))^{3/2}}{4(a^2+b^2)^2 f\sec^2(e+fx)^{3/4}} + \frac{5a\cos(e+fx)(d\sec(e+fx))^{3/2}\sin(e+fx)}{4(a^2+b^2)^2 f} + \frac{a\left(2-\frac{3a^2}{b^2}\right)b\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}\sqrt{}}{8(a^2+b^2)^{5/2}f\sec^2(e+fx)^{3/4}} - \frac{a\left(2-\frac{3a^2}{b^2}\right)b\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)(d\sec(e+fx))^{3/2}\sqrt{-}}{8(a^2+b^2)^{5/2}f\sec^2(e+fx)^{3/4}} - \frac{b(d\sec(e+fx))^{3/2}}{2(a^2+b^2)f(a+b\tan(e+fx))^2} - \frac{5ab(d\sec(e+fx))^{3/2}}{4(a^2+b^2)^2 f(a+b\tan(e+fx))}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 35.27 (sec) , antiderivative size = 14364, normalized size of antiderivative = 25.38

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(d*Sec[e + f*x])^(3/2)/(a + b*Tan[e + f*x])^3,x]
[Out] Result too large to show
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 60918 vs. 2(515) = 1030.

Time = 13.11 (sec) , antiderivative size = 60919, normalized size of antiderivative = 107.63

method	result	size
default	Expression too large to display	60919

```
[In] int((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx = \int \frac{(d\sec{(e+fx)})^{\frac{3}{2}}}{(a+b\tan{(e+fx)})^3}\,dx$$

```
[In] integrate((d*sec(f*x+e))**(3/2)/(a+b*tan(f*x+e))**3,x)
[Out] Integral((d*sec(e + f*x))**(3/2)/(a + b*tan(e + f*x))**3, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx = \int \frac{(d\sec{(fx+e)})^{\frac{3}{2}}}{(b\tan{(fx+e)}+a)^3}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(3/2)/(b*tan(f*x + e) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{3/2}}{(a+b\tan(e+fx))^3}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}}{(a+b\tan{(e+f\,x)})^3}\,dx$$

```
[In] int((d/cos(e + f*x))^(3/2)/(a + b*tan(e + f*x))^3,x)
[Out] int((d/cos(e + f*x))^(3/2)/(a + b*tan(e + f*x))^3, x)
```

3.620 $\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 515

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}\,dx = -\frac{3\sqrt{b}(5a^2-2b^2)\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{8\left(a^2+b^2\right)^{11/4}f\sqrt[4]{\sec^2(e+fx)}}$$
$$-\frac{3\sqrt{b}(5a^2-2b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{8\left(a^2+b^2\right)^{11/4}f\sqrt[4]{\sec^2(e+fx)}}$$
$$-\frac{7a\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{4\left(a^2+b^2\right)^2f\sqrt[4]{\sec^2(e+fx)}}$$
$$+\frac{3a(5a^2-2b^2)\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2}}{8\left(a^2+b^2\right)^3f\sqrt[4]{\sec^2(e+fx)}}$$
$$+\frac{3a(5a^2-2b^2)\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e}}{8\left(a^2+b^2\right)^3f\sqrt[4]{\sec^2(e+fx)}}$$
$$-\frac{b\sqrt{d\sec(e+fx)}}{2\left(a^2+b^2\right)f(a+b\tan(e+fx))^2}-\frac{7ab\sqrt{d\sec(e+fx)}}{4\left(a^2+b^2\right)^2f(a+b\tan(e+fx))}$$

```
[Out] -7/4*a*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*El
lipticF(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(d*sec(f*x+e))^(1/2)/(a^2+b^2)
^2/f/(sec(f*x+e)^2)^(1/4)-3/8*(5*a^2-2*b^2)*arctan((sec(f*x+e)^2)^(1/4)*b^(
1/2)/(a^2+b^2)^(1/4))*b^(1/2)*(d*sec(f*x+e))^(1/2)/(a^2+b^2)^(11/4)/f/(sec(
f*x+e)^2)^(1/4)-3/8*(5*a^2-2*b^2)*arctanh((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2
+b^2)^(1/4))*b^(1/2)*(d*sec(f*x+e))^(1/2)/(a^2+b^2)^(11/4)/f/(sec(f*x+e)^2)
^(1/4)+3/8*a*(5*a^2-2*b^2)*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a
^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(1/2)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^3/f/(
```

```
sec(f*x+e)^2)^(1/4)+3/8*a*(5*a^2-2*b^2)*cot(f*x+e)*EllipticPi((sec(f*x+e)^2
)^(1/4),b/(a^2+b^2)^(1/2),I)*(d*sec(f*x+e))^(1/2)*(-tan(f*x+e)^2)^(1/2)/(a^
2+b^2)^3/f/(sec(f*x+e)^2)^(1/4)-1/2*b*(d*sec(f*x+e))^(1/2)/(a^2+b^2)/f/(a+b
*tan(f*x+e))^2-7/4*a*b*(d*sec(f*x+e))^(1/2)/(a^2+b^2)^2/f/(a+b*tan(f*x+e))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3593, 759, 849, 858, 237, 761, 410, 109, 418, 1227, 551, 455, 65, 218, 214, 211}

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}\,dx$$
$$= \frac{3a(5a^2-2b^2)\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sqrt{d\sec(e+fx)}\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),\right.}{8f(a^2+b^2)^3\sqrt[4]{\sec^2(e+fx)}}$$
$$+\frac{3a(5a^2-2b^2)\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sqrt{d\sec(e+fx)}\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right)\right.}{8f(a^2+b^2)^3\sqrt[4]{\sec^2(e+fx)}}$$
$$-\frac{3\sqrt{b}(5a^2-2b^2)\sqrt{d\sec(e+fx)}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8f(a^2+b^2)^{11/4}\sqrt[4]{\sec^2(e+fx)}}$$
$$-\frac{7a\sqrt{d\sec(e+fx)}\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)}{4f(a^2+b^2)^2\sqrt[4]{\sec^2(e+fx)}}$$
$$-\frac{3\sqrt{b}(5a^2-2b^2)\sqrt{d\sec(e+fx)}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8f(a^2+b^2)^{11/4}\sqrt[4]{\sec^2(e+fx)}}$$
$$-\frac{7ab\sqrt{d\sec(e+fx)}}{4f(a^2+b^2)^2(a+b\tan(e+fx))}-\frac{b\sqrt{d\sec(e+fx)}}{2f(a^2+b^2)(a+b\tan(e+fx))^2}$$

```
[In] Int[Sqrt[d*Sec[e + f*x]]/(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (-3*Sqrt[b]*(5*a^2 - 2*b^2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 +
b^2)^(1/4)]*Sqrt[d*Sec[e + f*x]])/(8*(a^2 + b^2)^(11/4)*f*(Sec[e + f*x]^2)^
(1/4)) - (3*Sqrt[b]*(5*a^2 - 2*b^2)*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4)
)/(a^2 + b^2)^(1/4)]*Sqrt[d*Sec[e + f*x]])/(8*(a^2 + b^2)^(11/4)*f*(Sec[e +
 f*x]^2)^(1/4)) - (7*a*EllipticF[ArcTan[Tan[e + f*x]]/2, 2]*Sqrt[d*Sec[e +
f*x]])/(4*(a^2 + b^2)^2*f*(Sec[e + f*x]^2)^(1/4)) + (3*a*(5*a^2 - 2*b^2)*Co
t[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)],
 -1]*Sqrt[d*Sec[e + f*x]]*Sqrt[-Tan[e + f*x]^2])/(8*(a^2 + b^2)^3*f*(Sec[e
+ f*x]^2)^(1/4)) + (3*a*(5*a^2 - 2*b^2)*Cot[e + f*x]*EllipticPi[b/Sqrt[a^2
+ b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[d*Sec[e + f*x]]*Sqrt[-Tan[
e + f*x]^2])/(8*(a^2 + b^2)^3*f*(Sec[e + f*x]^2)^(1/4)) - (b*Sqrt[d*Sec[e +
```

```
 f*x]])/(2*(a^2 + b^2)*f*(a + b*Tan[e + f*x])^2) - (7*a*b*Sqrt[d*Sec[e + f*
x]])/(4*(a^2 + b^2)^2*f*(a + b*Tan[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 109

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e
/f) + d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-f/(d*e - c*f), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[(-b)*(x^2/a)]/(2*x), Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 761

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(3/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(3/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(3/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
```

```
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{d\sec(e+fx)}\text{Subst}\left(\int \frac{1}{(a+x)^3\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[4]{\sec^2(e+fx)}} \\
&= -\frac{b\sqrt{d\sec(e+fx)}}{2\left(a^2+b^2\right)f(a+b\tan(e+fx))^2} \\
&\quad - \frac{\sqrt{d\sec(e+fx)}\text{Subst}\left(\int \frac{-2a+\frac{3x}{2}}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{2b\left(a^2+b^2\right)f\sqrt[4]{\sec^2(e+fx)}} \\
&= -\frac{b\sqrt{d\sec(e+fx)}}{2\left(a^2+b^2\right)f(a+b\tan(e+fx))^2} - \frac{7ab\sqrt{d\sec(e+fx)}}{4\left(a^2+b^2\right)^2f(a+b\tan(e+fx))} \\
&\quad + \frac{\left(b\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{\frac{1}{2}\left(-3+\frac{4a^2}{b^2}\right)-\frac{7ax}{4b^2}}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{2\left(a^2+b^2\right)^2f\sqrt[4]{\sec^2(e+fx)}}
\end{aligned}$$

$$= -\frac{b\sqrt{d\sec(e+fx)}}{2(a^2+b^2)f(a+b\tan(e+fx))^2} - \frac{7ab\sqrt{d\sec(e+fx)}}{4(a^2+b^2)^2 f(a+b\tan(e+fx))} - \frac{\left(7a\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{8b(a^2+b^2)^2 f\sqrt[4]{\sec^2(e+fx)}} - \frac{\left(3\left(2-\frac{5a^2}{b^2}\right)b\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{8(a^2+b^2)^2 f\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{7a\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{4(a^2+b^2)^2 f\sqrt[4]{\sec^2(e+fx)}} - \frac{b\sqrt{d\sec(e+fx)}}{2(a^2+b^2)f(a+b\tan(e+fx))^2} - \frac{7ab\sqrt{d\sec(e+fx)}}{4(a^2+b^2)^2 f(a+b\tan(e+fx))} + \frac{\left(3\left(2-\frac{5a^2}{b^2}\right)b\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{x}{(a^2-x^2)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{8(a^2+b^2)^2 f\sqrt[4]{\sec^2(e+fx)}} - \frac{\left(3a\left(2-\frac{5a^2}{b^2}\right)b\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{(a^2-x^2)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{8(a^2+b^2)^2 f\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{7a\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{4(a^2+b^2)^2 f\sqrt[4]{\sec^2(e+fx)}} - \frac{b\sqrt{d\sec(e+fx)}}{2(a^2+b^2)f(a+b\tan(e+fx))^2} - \frac{7ab\sqrt{d\sec(e+fx)}}{4(a^2+b^2)^2 f(a+b\tan(e+fx))} + \frac{\left(3\left(2-\frac{5a^2}{b^2}\right)b\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{(a^2-x)\left(1+\frac{x}{b^2}\right)^{3/4}}\,dx, x, b^2\tan^2(e+fx)\right)}{16(a^2+b^2)^2 f\sqrt[4]{\sec^2(e+fx)}} - \frac{\left(3a\left(2-\frac{5a^2}{b^2}\right)\cot(e+fx)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{(a^2-x)\sqrt{-\frac{x}{b^2}}\left(1+\frac{x}{b^2}\right)^{3/4}}\,dx, x,\right.}{16(a^2+b^2)^2 f\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{7a\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{4\left(a^2+b^2\right)^2 f\sqrt[4]{\sec^2(e+fx)}} - \frac{b\sqrt{d\sec(e+fx)}}{2\left(a^2+b^2\right)f(a+b\tan(e+fx))^2} - \frac{7ab\sqrt{d\sec(e+fx)}}{4\left(a^2+b^2\right)^2 f(a+b\tan(e+fx))} + \frac{\left(3\left(2-\frac{5a^2}{b^2}\right)b^3\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{a^2+b^2-b^2x^4}\,dx, x, \sqrt[4]{\sec^2(e+fx)}\right)}{4\left(a^2+b^2\right)^2 f\sqrt[4]{\sec^2(e+fx)}} + \frac{\left(3a\left(2-\frac{5a^2}{b^2}\right)\cot(e+fx)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^4}\left(-1-\frac{a^2}{b^2}+x^4\right)}\,dx, x, \sqrt[4]{\cdots}\right.}{4\left(a^2+b^2\right)^2 f\sqrt[4]{\sec^2(e+fx)}}$$

$$= -\frac{7a\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{4\left(a^2+b^2\right)^2 f\sqrt[4]{\sec^2(e+fx)}} - \frac{b\sqrt{d\sec(e+fx)}}{2\left(a^2+b^2\right)f(a+b\tan(e+fx))^2} - \frac{7ab\sqrt{d\sec(e+fx)}}{4\left(a^2+b^2\right)^2 f(a+b\tan(e+fx))} + \frac{\left(3\left(2-\frac{5a^2}{b^2}\right)b^3\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a^2+b^2}-bx^2}\,dx, x, \sqrt[4]{\sec^2(e+fx)}\right)}{8\left(a^2+b^2\right)^{5/2} f\sqrt[4]{\sec^2(e+fx)}} + \frac{\left(3\left(2-\frac{5a^2}{b^2}\right)b^3\sqrt{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a^2+b^2}+bx^2}\,dx, x, \sqrt[4]{\sec^2(e+fx)}\right)}{8\left(a^2+b^2\right)^{5/2} f\sqrt[4]{\sec^2(e+fx)}} - \frac{\left(3a\left(2-\frac{5a^2}{b^2}\right)b^2\cot(e+fx)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{bx^2}{\sqrt{a^2+b^2}}\right)\sqrt{1-x^4}}\,dx, x\cdots\right.}{8\left(a^2+b^2\right)^3 f\sqrt[4]{\sec^2(e+fx)}} - \frac{\left(3a\left(2-\frac{5a^2}{b^2}\right)b^2\cot(e+fx)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{bx^2}{\sqrt{a^2+b^2}}\right)\sqrt{1-x^4}}\,dx, x\cdots\right.}{8\left(a^2+b^2\right)^3 f\sqrt[4]{\sec^2(e+fx)}}$$

$$= \frac{3\left(2-\frac{5a^2}{b^2}\right)b^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{8\left(a^2+b^2\right)^{11/4}f\sqrt[4]{\sec^2(e+fx)}} + \frac{3\left(2-\frac{5a^2}{b^2}\right)b^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{8\left(a^2+b^2\right)^{11/4}f\sqrt[4]{\sec^2(e+fx)}} - \frac{7a\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{4\left(a^2+b^2\right)^2f\sqrt[4]{\sec^2(e+fx)}} - \frac{b\sqrt{d\sec(e+fx)}}{2\left(a^2+b^2\right)f(a+b\tan(e+fx))^2} - \frac{7ab\sqrt{d\sec(e+fx)}}{4\left(a^2+b^2\right)^2f(a+b\tan(e+fx))} - \frac{\left(3a\left(2-\frac{5a^2}{b^2}\right)b^2\cot(e+fx)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(1-\frac{bx^2}{\sqrt{a^2+b^2}}\right)}d\right.}{8\left(a^2+b^2\right)^3f\sqrt[4]{\sec^2(e+fx)}} - \frac{\left(3a\left(2-\frac{5a^2}{b^2}\right)b^2\cot(e+fx)\sqrt{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(1+\frac{bx^2}{\sqrt{a^2+b^2}}\right)}d\right.}{8\left(a^2+b^2\right)^3f\sqrt[4]{\sec^2(e+fx)}}$$

$$= \frac{3\left(2-\frac{5a^2}{b^2}\right)b^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{8\left(a^2+b^2\right)^{11/4}f\sqrt[4]{\sec^2(e+fx)}} + \frac{3\left(2-\frac{5a^2}{b^2}\right)b^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt{d\sec(e+fx)}}{8\left(a^2+b^2\right)^{11/4}f\sqrt[4]{\sec^2(e+fx)}} - \frac{7a\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sqrt{d\sec(e+fx)}}{4\left(a^2+b^2\right)^2f\sqrt[4]{\sec^2(e+fx)}} + \frac{3a(5a^2-2b^2)\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{}}{8\left(a^2+b^2\right)^3f\sqrt[4]{\sec^2(e+fx)}} + \frac{3a(5a^2-2b^2)\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt{d\sec(e+fx)}\sqrt{-}}{8\left(a^2+b^2\right)^3f\sqrt[4]{\sec^2(e+fx)}} - \frac{b\sqrt{d\sec(e+fx)}}{2\left(a^2+b^2\right)f(a+b\tan(e+fx))^2} - \frac{7ab\sqrt{d\sec(e+fx)}}{4\left(a^2+b^2\right)^2f(a+b\tan(e+fx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.70 (sec) , antiderivative size = 369, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}\,dx$$

$$= \frac{(d\sec(e+fx))^{3/2}(a\cos(e+fx)+b\sin(e+fx))^3\left(-\frac{2b\sec^2(e+fx)(9a^2+2b^2+7ab\tan(e+fx))}{(a-ib)^2(a+ib)^2(a+b\tan(e+fx))^2}+\frac{\sec^2(e+fx)^{3/4}\left(-7a(a\right.}{}\right.}{}$$

```
[In] Integrate[Sqrt[d*Sec[e + f*x]]/(a + b*Tan[e + f*x])^3,x]
```

```
[Out] ((d*Sec[e + f*x])^(3/2)*(a*Cos[e + f*x] + b*Sin[e + f*x])^3*((-2*b*Sec[e +
f*x]^2*(9*a^2 + 2*b^2 + 7*a*b*Tan[e + f*x]))/((a - I*b)^2*(a + I*b)^2*(a +
b*Tan[e + f*x])^2) + ((Sec[e + f*x]^2)^(3/4)*(-7*a*(a^2 + b^2)*Hypergeometr
ic2F1[1/2, 3/4, 3/2, -Tan[e + f*x]^2]*Tan[e + f*x] + 3*(5*a^2 - 2*b^2)*(-(S
qrt[b]*(a^2 + b^2)^(1/4)*(ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^
2)^(1/4)] + ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b^2)^(1/4)])) +
 a*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1
/4)], -1]*Sqrt[-Tan[e + f*x]^2] + a*Cot[e + f*x]*EllipticPi[b/Sqrt[a^2 + b^
2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Sqrt[-Tan[e + f*x]^2])))/(a^2 + b^2
)^3))/(8*d*f*(a + b*Tan[e + f*x])^3)
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75606 vs. 2(472) = 944.

Time = 14.53 (sec) , antiderivative size = 75607, normalized size of antiderivative = 146.81

method	result	size
default	Expression too large to display	75607

```
[In] int((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}\,dx = \int \frac{\sqrt{d\sec{(e+fx)}}}{(a+b\tan{(e+fx)})^3}\,dx$$

```
[In] integrate((d*sec(f*x+e))**(1/2)/(a+b*tan(f*x+e))**3,x)
[Out] Integral(sqrt(d*sec(e + f*x))/(a + b*tan(e + f*x))**3, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{\sqrt{d\sec(e+fx)}}{(a+b\tan(e+fx))^3}\,dx = \int \frac{\sqrt{d\sec{(fx+e)}}}{(b\tan{(fx+e)}+a)^3}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x, algorithm="giac")
[Out] integrate(sqrt(d*sec(f*x + e))/(b*tan(f*x + e) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d \sec(e+fx)}}{(a+b\tan(e+fx))^3}\,dx = \int \frac{\sqrt{\frac{d}{\cos(e+f\,x)}}}{(a+b\tan\left(e+f\,x\right))^3}\,dx$$

```
[In] int((d/cos(e + f*x))^(1/2)/(a + b*tan(e + f*x))^3,x)
[Out] int((d/cos(e + f*x))^(1/2)/(a + b*tan(e + f*x))^3, x)
```

3.621 $\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 664

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}\,dx$$
$$= \frac{5b^{3/2}(7a^2-2b^2)\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{8(a^2+b^2)^{13/4}f\sqrt{d\sec(e+fx)}}$$
$$- \frac{5b^{3/2}(7a^2-2b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{8(a^2+b^2)^{13/4}f\sqrt{d\sec(e+fx)}}$$
$$+ \frac{a(8a^2-37b^2)E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{4(a^2+b^2)^3f\sqrt{d\sec(e+fx)}}$$
$$- \frac{a(8a^2-37b^2)\tan(e+fx)}{4(a^2+b^2)^3f\sqrt{d\sec(e+fx)}}$$
$$- \frac{5ab(7a^2-2b^2)\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(\ldots}}{8(a^2+b^2)^{7/2}f\sqrt{d\sec(e+fx)}}$$
$$+ \frac{5ab(7a^2-2b^2)\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e\ldots}}{8(a^2+b^2)^{7/2}f\sqrt{d\sec(e+fx)}}$$
$$+ \frac{b(4a^2-5b^2)\sec^2(e+fx)}{2(a^2+b^2)^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}$$
$$+ \frac{2(b+a\tan(e+fx))}{(a^2+b^2)f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}$$
$$+ \frac{ab(8a^2-37b^2)\sec^2(e+fx)}{4(a^2+b^2)^3f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}$$

```
[Out] 5/8*b^(3/2)*(7*a^2-2*b^2)*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/
4))*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(13/4)/f/(d*sec(f*x+e))^(1/2)-5/8*b^(3/2
)*(7*a^2-2*b^2)*arctanh((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(sec(
f*x+e)^2)^(1/4)/(a^2+b^2)^(13/4)/f/(d*sec(f*x+e))^(1/2)+1/4*a*(8*a^2-37*b^2
)*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*Ellipti
cE(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^3/f/
(d*sec(f*x+e))^(1/2)-5/8*a*b*(7*a^2-2*b^2)*cot(f*x+e)*EllipticPi((sec(f*x+e
)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2)^(1/4)*(-tan(f*x+e)^2)^(1/2)
/(a^2+b^2)^(7/2)/f/(d*sec(f*x+e))^(1/2)+5/8*a*b*(7*a^2-2*b^2)*cot(f*x+e)*El
lipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2)^(1/4)*(-t
an(f*x+e)^2)^(1/2)/(a^2+b^2)^(7/2)/f/(d*sec(f*x+e))^(1/2)-1/4*a*(8*a^2-37*b
^2)*tan(f*x+e)/(a^2+b^2)^3/f/(d*sec(f*x+e))^(1/2)+1/2*b*(4*a^2-5*b^2)*sec(f
*x+e)^2/(a^2+b^2)^2/f/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2+2*(b+a*tan(f*
x+e))/(a^2+b^2)/f/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2+1/4*a*b*(8*a^2-37
*b^2)*sec(f*x+e)^2/(a^2+b^2)^3/f/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 664, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules

used = {3593, 755, 849, 858, 233, 202, 760, 408, 504, 1227, 551, 455, 65, 304, 211, 214}

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}\,dx = -\frac{5ab(7a^2-2b^2)\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right.\right.}{8f(a^2+b^2)^{7/2}\sqrt{d\sec(e+fx)}}$$
$$+\frac{5ab(7a^2-2b^2)\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right)\right.}{8f(a^2+b^2)^{7/2}\sqrt{d\sec(e+fx)}}$$
$$+\frac{a(8a^2-37b^2)\sqrt[4]{\sec^2(e+fx)}E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)}{4f(a^2+b^2)^3\sqrt{d\sec(e+fx)}}$$
$$+\frac{5b^{3/2}(7a^2-2b^2)\sqrt[4]{\sec^2(e+fx)}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8f(a^2+b^2)^{13/4}\sqrt{d\sec(e+fx)}}$$
$$-\frac{5b^{3/2}(7a^2-2b^2)\sqrt[4]{\sec^2(e+fx)}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8f(a^2+b^2)^{13/4}\sqrt{d\sec(e+fx)}}$$
$$+\frac{ab(8a^2-37b^2)\sec^2(e+fx)}{4f(a^2+b^2)^3\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}$$
$$+\frac{b(4a^2-5b^2)\sec^2(e+fx)}{2f(a^2+b^2)^2\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}$$
$$-\frac{a(8a^2-37b^2)\tan(e+fx)}{4f(a^2+b^2)^3\sqrt{d\sec(e+fx)}}+\frac{2(a\tan(e+fx)+b)}{f(a^2+b^2)\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}$$

```
[In] Int[1/(Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^3),x]
```

```
[Out] (5*b^(3/2)*(7*a^2 - 2*b^2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 + b
^2)^(1/4)]*(Sec[e + f*x]^2)^(1/4))/(8*(a^2 + b^2)^(13/4)*f*Sqrt[d*Sec[e + f
*x]]) - (5*b^(3/2)*(7*a^2 - 2*b^2)*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))
/(a^2 + b^2)^(1/4)]*(Sec[e + f*x]^2)^(1/4))/(8*(a^2 + b^2)^(13/4)*f*Sqrt[d*
Sec[e + f*x]]) + (a*(8*a^2 - 37*b^2)*EllipticE[ArcTan[Tan[e + f*x]]/2, 2]*(
Sec[e + f*x]^2)^(1/4))/(4*(a^2 + b^2)^3*f*Sqrt[d*Sec[e + f*x]]) - (a*(8*a^2
 - 37*b^2)*Tan[e + f*x])/(4*(a^2 + b^2)^3*f*Sqrt[d*Sec[e + f*x]]) - (5*a*b*
(7*a^2 - 2*b^2)*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e
 + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2)^(1/4)*Sqrt[-Tan[e + f*x]^2])/(8*(a^
2 + b^2)^(7/2)*f*Sqrt[d*Sec[e + f*x]]) + (5*a*b*(7*a^2 - 2*b^2)*Cot[e + f*x
]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e
+ f*x]^2)^(1/4)*Sqrt[-Tan[e + f*x]^2])/(8*(a^2 + b^2)^(7/2)*f*Sqrt[d*Sec[e
+ f*x]]) + (b*(4*a^2 - 5*b^2)*Sec[e + f*x]^2)/(2*(a^2 + b^2)^2*f*Sqrt[d*Sec
[e + f*x]]*(a + b*Tan[e + f*x])^2) + (2*(b + a*Tan[e + f*x]))/((a^2 + b^2)*
f*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^2) + (a*b*(8*a^2 - 37*b^2)*Sec[
e + f*x]^2)/(4*(a^2 + b^2)^3*f*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 760

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt[4]{\sec^2(e+fx)}\text{Subst}\left(\int \frac{1}{(a+x)^3\left(1+\frac{x^2}{b^2}\right)^{5/4}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt{d\sec(e+fx)}} \\
&= \frac{2(b+a\tan(e+fx))}{(a^2+b^2)\,f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2} \\
&\quad - \frac{\left(2b\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\frac{1}{2}\left(-5+\frac{a^2}{b^2}\right)-\frac{3ax}{2b^2}}{(a+x)^3\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{(a^2+b^2)\,f\sqrt{d\sec(e+fx)}} \\
&= \frac{b(4a^2-5b^2)\sec^2(e+fx)}{2\,(a^2+b^2)^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2} \\
&\quad + \frac{2(b+a\tan(e+fx))}{(a^2+b^2)\,f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2} \\
&\quad + \frac{\left(b^3\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{-\frac{a\left(a^2-8b^2\right)}{b^4}+\frac{\left(4a^2-5b^2\right)x}{4b^4}}{(a+x)^2\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{(a^2+b^2)^2 f\sqrt{d\sec(e+fx)}}
\end{aligned}$$

$$= \frac{b(4a^2-5b^2)\sec^2(e+fx)}{2\left(a^2+b^2\right)^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2} + \frac{2(b+a\tan(e+fx))}{(a^2+b^2)\, f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2} + \frac{ab(8a^2-37b^2)\sec^2(e+fx)}{4\left(a^2+b^2\right)^3 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} - \frac{\left(b^5\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\frac{1}{4}\left(\frac{4a^4}{b^6}-\frac{36a^2}{b^4}+\frac{5}{b^2}\right)+\frac{a\left(8a^2-37b^2\right)x}{8b^6}}{(a+x)\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{\left(a^2+b^2\right)^3 f\sqrt{d\sec(e+fx)}}$$

$$= \frac{b(4a^2-5b^2)\sec^2(e+fx)}{2\left(a^2+b^2\right)^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2} + \frac{2(b+a\tan(e+fx))}{(a^2+b^2)\, f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2} + \frac{ab(8a^2-37b^2)\sec^2(e+fx)}{4\left(a^2+b^2\right)^3 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} - \frac{\left(a(8a^2-37b^2)\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{8b\left(a^2+b^2\right)^3 f\sqrt{d\sec(e+fx)}} + \frac{\left(5b(7a^2-2b^2)\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{(a+x)\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{8\left(a^2+b^2\right)^3 f\sqrt{d\sec(e+fx)}}$$

$$
\begin{aligned}
&= -\frac{a(8a^2-37b^2)\tan(e+fx)}{4\left(a^2+b^2\right)^3 f\sqrt{d\sec(e+fx)}} + \frac{b(4a^2-5b^2)\sec^2(e+fx)}{2\left(a^2+b^2\right)^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2} \\
&+ \frac{2(b+a\tan(e+fx))}{(a^2+b^2) f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2} \\
&+ \frac{ab(8a^2-37b^2)\sec^2(e+fx)}{4\left(a^2+b^2\right)^3 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} \\
&+ \frac{\left(a(8a^2-37b^2)\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\left(1+\frac{x^2}{b^2}\right)^{5/4}}\,dx,x,b\tan(e+fx)\right)}{8b\left(a^2+b^2\right)^3 f\sqrt{d\sec(e+fx)}} \\
&- \frac{\left(5b(7a^2-2b^2)\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{x}{(a^2-x^2)\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx,x,b\tan(e+fx)\right)}{8\left(a^2+b^2\right)^3 f\sqrt{d\sec(e+fx)}} \\
&+ \frac{\left(5ab(7a^2-2b^2)\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{(a^2-x^2)\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx,x,b\tan(e+fx)\right)}{8\left(a^2+b^2\right)^3 f\sqrt{d\sec(e+fx)}} \\
&= \frac{a(8a^2-37b^2)E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{4\left(a^2+b^2\right)^3 f\sqrt{d\sec(e+fx)}} \\
&- \frac{a(8a^2-37b^2)\tan(e+fx)}{4\left(a^2+b^2\right)^3 f\sqrt{d\sec(e+fx)}} + \frac{b(4a^2-5b^2)\sec^2(e+fx)}{2\left(a^2+b^2\right)^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2} \\
&+ \frac{2(b+a\tan(e+fx))}{(a^2+b^2) f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2} \\
&+ \frac{ab(8a^2-37b^2)\sec^2(e+fx)}{4\left(a^2+b^2\right)^3 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} \\
&- \frac{\left(5b(7a^2-2b^2)\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{(a^2-x)\sqrt[4]{1+\frac{x}{b^2}}}\,dx,x,b^2\tan^2(e+fx)\right)}{16\left(a^2+b^2\right)^3 f\sqrt{d\sec(e+fx)}} \\
&+ \frac{\left(5a(7a^2-2b^2)\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{1-x^4}\left(1+\frac{a^2}{b^2}-x^4\right)}\,dx,x,\sqrt[4]{}\right.}{4\left(a^2+b^2\right)^3 f\sqrt{d\sec(e+fx)}}
\end{aligned}
$$

$$= \frac{a(8a^2-37b^2)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{4\left(a^2+b^2\right)^3 f\sqrt{d\sec(e+fx)}}$$
$$-\frac{a(8a^2-37b^2)\tan(e+fx)}{4\left(a^2+b^2\right)^3 f\sqrt{d\sec(e+fx)}}+\frac{b(4a^2-5b^2)\sec^2(e+fx)}{2\left(a^2+b^2\right)^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}$$
$$+\frac{2(b+a\tan(e+fx))}{(a^2+b^2)\,f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}$$
$$+\frac{ab(8a^2-37b^2)\sec^2(e+fx)}{4\left(a^2+b^2\right)^3 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}$$
$$-\frac{\left(5b^3(7a^2-2b^2)\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{x^2}{a^2+b^2-b^2x^4}\,dx,x,\sqrt[4]{\sec^2(e+fx)}\right)}{4\left(a^2+b^2\right)^3 f\sqrt{d\sec(e+fx)}}$$
$$+\frac{\left(5ab(7a^2-2b^2)\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\left(\sqrt{a^2+b^2}-bx^2\right)\sqrt{1-x^4}}\,dx,x,\right.}{8\left(a^2+b^2\right)^3 f\sqrt{d\sec(e+fx)}}$$
$$-\frac{\left(5ab(7a^2-2b^2)\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\left(\sqrt{a^2+b^2}+bx^2\right)\sqrt{1-x^4}}\,dx,x,\right.}{8\left(a^2+b^2\right)^3 f\sqrt{d\sec(e+fx)}}$$

$$= \frac{a(8a^2-37b^2)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{4\left(a^2+b^2\right)^3 f\sqrt{d\sec(e+fx)}}$$
$$-\frac{a(8a^2-37b^2)\tan(e+fx)}{4\left(a^2+b^2\right)^3 f\sqrt{d\sec(e+fx)}}+\frac{b(4a^2-5b^2)\sec^2(e+fx)}{2\left(a^2+b^2\right)^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}$$
$$+\frac{2(b+a\tan(e+fx))}{(a^2+b^2)\,f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}$$
$$+\frac{ab(8a^2-37b^2)\sec^2(e+fx)}{4\left(a^2+b^2\right)^3 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}$$
$$-\frac{\left(5b^2(7a^2-2b^2)\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{a^2+b^2}-bx^2}\,dx,x,\sqrt[4]{\sec^2(e+fx)}\right)}{8\left(a^2+b^2\right)^3 f\sqrt{d\sec(e+fx)}}$$
$$+\frac{\left(5b^2(7a^2-2b^2)\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{a^2+b^2}+bx^2}\,dx,x,\sqrt[4]{\sec^2(e+fx)}\right)}{8\left(a^2+b^2\right)^3 f\sqrt{d\sec(e+fx)}}$$
$$+\frac{\left(5ab(7a^2-2b^2)\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(\sqrt{a^2+b^2}-bx^2\right)}\right.}{8\left(a^2+b^2\right)^3 f\sqrt{d\sec(e+fx)}}$$
$$-\frac{\left(5ab(7a^2-2b^2)\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(\sqrt{a^2+b^2}+bx^2\right)}\right.}{8\left(a^2+b^2\right)^3 f\sqrt{d\sec(e+fx)}}$$

$$= \frac{5b^{3/2}(7a^2-2b^2)\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{8\left(a^2+b^2\right)^{13/4}f\sqrt{d\sec(e+fx)}}$$
$$- \frac{5b^{3/2}(7a^2-2b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{8\left(a^2+b^2\right)^{13/4}f\sqrt{d\sec(e+fx)}}$$
$$+ \frac{a(8a^2-37b^2)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{4\left(a^2+b^2\right)^3f\sqrt{d\sec(e+fx)}}$$
$$- \frac{a(8a^2-37b^2)\tan(e+fx)}{4\left(a^2+b^2\right)^3f\sqrt{d\sec(e+fx)}}$$
$$- \frac{5ab(7a^2-2b^2)\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}}{8\left(a^2+b^2\right)^{7/2}f\sqrt{d\sec(e+fx)}}$$
$$+ \frac{5ab(7a^2-2b^2)\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{}}{8\left(a^2+b^2\right)^{7/2}f\sqrt{d\sec(e+fx)}}$$
$$+ \frac{b(4a^2-5b^2)\sec^2(e+fx)}{2\left(a^2+b^2\right)^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}$$
$$+ \frac{2(b+a\tan(e+fx))}{(a^2+b^2)f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}$$
$$+ \frac{ab(8a^2-37b^2)\sec^2(e+fx)}{4\left(a^2+b^2\right)^3f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 39.23 (sec) , antiderivative size = 14652, normalized size of antiderivative = 22.07

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}\,dx = \text{Result too large to show}$$

```
[In] Integrate[1/(Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^3),x]
[Out] Result too large to show
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 78531 vs. 2(611) = 1222.

Time = 15.80 (sec) , antiderivative size = 78532, normalized size of antiderivative = 118.27

method	result	size
default	Expression too large to display	78532

```
[In] int(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}\,dx = \int \frac{1}{\sqrt{d\sec{(e+fx)}}\,(a+b\tan{(e+fx)})^3}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))**(1/2)/(a+b*tan(f*x+e))**3,x)
[Out] Integral(1/(sqrt(d*sec(e + f*x))*(a + b*tan(e + f*x))**3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}\,dx = \int \frac{1}{\sqrt{d\sec{(fx+e)}}(b\tan{(fx+e)}+a)^3}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
[Out] integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)^3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}\,dx = \int \frac{1}{\sqrt{d\sec\left(fx+e\right)}{\left(b\tan\left(fx+e\right)+a\right)}^3}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^3,x, algorithm="giac")
[Out] integrate(1/(sqrt(d*sec(f*x + e))*(b*tan(f*x + e) + a)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^3}\,dx = \int \frac{1}{\sqrt{\frac{d}{\cos(e+f\,x)}}\,{\left(a+b\tan\left(e+f\,x\right)\right)}^3}\,dx$$

```
[In] int(1/((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x))^3),x)
[Out] int(1/((d/cos(e + f*x))^(1/2)*(a + b*tan(e + f*x))^3), x)
```

3.622 $\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 620

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx = -\frac{7b^{5/2}(9a^2-2b^2)\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{8\left(a^2+b^2\right)^{15/4}f(d\sec(e+fx))^{3/2}} - \frac{7b^{5/2}(9a^2-2b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{8\left(a^2+b^2\right)^{15/4}f(d\sec(e+fx))^{3/2}} + \frac{a(8a^2-69b^2)\operatorname{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sec^2(e+fx)^{3/4}}{12\left(a^2+b^2\right)^3 f(d\sec(e+fx))^{3/2}} + \frac{7ab^2(9a^2-2b^2)\cot(e+fx)\operatorname{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sec^2(e+fx)^{3/4}\sqrt{-\tan^2(e}}{8\left(a^2+b^2\right)^4 f(d\sec(e+fx))^{3/2}} + \frac{7ab^2(9a^2-2b^2)\cot(e+fx)\operatorname{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sec^2(e+fx)^{3/4}\sqrt{-\tan^2(e+}}{8\left(a^2+b^2\right)^4 f(d\sec(e+fx))^{3/2}} + \frac{b(4a^2-7b^2)\sec^2(e+fx)}{6\left(a^2+b^2\right)^2 f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2} + \frac{2(b+a\tan(e+fx))}{3\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2} + \frac{ab(8a^2-69b^2)\sec^2(e+fx)}{12\left(a^2+b^2\right)^3 f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}$$

```
[Out] -7/8*b^(5/2)*(9*a^2-2*b^2)*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1
/4))*(sec(f*x+e)^2)^(3/4)/(a^2+b^2)^(15/4)/f/(d*sec(f*x+e))^(3/2)-7/8*b^(5/
```

```
2)*(9*a^2-2*b^2)*arctanh((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))*(sec
(f*x+e)^2)^(3/4)/(a^2+b^2)^(15/4)/f/(d*sec(f*x+e))^(3/2)+1/12*a*(8*a^2-69*b
^2)*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arctan(tan(f*x+e)))*Ellip
ticF(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(sec(f*x+e)^2)^(3/4)/(a^2+b^2)^3/
f/(d*sec(f*x+e))^(3/2)+7/8*a*b^2*(9*a^2-2*b^2)*cot(f*x+e)*EllipticPi((sec(f
*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2)^(3/4)*(-tan(f*x+e)^2)^(
1/2)/(a^2+b^2)^4/f/(d*sec(f*x+e))^(3/2)+7/8*a*b^2*(9*a^2-2*b^2)*cot(f*x+e)*
EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^2)^(3/4)*(
-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^4/f/(d*sec(f*x+e))^(3/2)+1/6*b*(4*a^2-7*b^2)
*sec(f*x+e)^2/(a^2+b^2)^2/f/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2+2/3*(b+
a*tan(f*x+e))/(a^2+b^2)/f/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^2+1/12*a*b*
(8*a^2-69*b^2)*sec(f*x+e)^2/(a^2+b^2)^3/f/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x
+e))
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 620, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3593, 755, 849, 858, 237, 761, 410, 109, 418, 1227, 551, 455, 65, 218, 214, 211}

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx = \frac{7ab^2(9a^2-2b^2)\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sec^2(e+fx)^{3/\cdots}}{8f\left(a^2+b^2\right)^4(d\sec\cdots}$$
$$+\frac{7ab^2(9a^2-2b^2)\sqrt{-\tan^2(e+fx)}\cot(e+fx)\sec^2(e+fx)^{3/4}\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right)\cdots\right.}{8f\left(a^2+b^2\right)^4(d\sec(e+fx))^{3/2}}$$
$$+\frac{a(8a^2-69b^2)\sec^2(e+fx)^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)}{12f\left(a^2+b^2\right)^3(d\sec(e+fx))^{3/2}}$$
$$-\frac{7b^{5/2}(9a^2-2b^2)\sec^2(e+fx)^{3/4}\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8f\left(a^2+b^2\right)^{15/4}(d\sec(e+fx))^{3/2}}$$
$$-\frac{7b^{5/2}(9a^2-2b^2)\sec^2(e+fx)^{3/4}\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)}{8f\left(a^2+b^2\right)^{15/4}(d\sec(e+fx))^{3/2}}$$
$$+\frac{ab(8a^2-69b^2)\sec^2(e+fx)}{12f\left(a^2+b^2\right)^3(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}$$
$$+\frac{b(4a^2-7b^2)\sec^2(e+fx)}{6f\left(a^2+b^2\right)^2(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}$$
$$+\frac{2(a\tan(e+fx)+b)}{3f\left(a^2+b^2\right)(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}$$

```
[In] Int[1/((d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^3),x]
```

```
[Out] (-7*b^(5/2)*(9*a^2 - 2*b^2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 +
b^2)^(1/4)]*(Sec[e + f*x]^2)^(3/4))/(8*(a^2 + b^2)^(15/4)*f*(d*Sec[e + f*x]
)^(3/2)) - (7*b^(5/2)*(9*a^2 - 2*b^2)*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/
4))/(a^2 + b^2)^(1/4)]*(Sec[e + f*x]^2)^(3/4))/(8*(a^2 + b^2)^(15/4)*f*(d*S
ec[e + f*x])^(3/2)) + (a*(8*a^2 - 69*b^2)*EllipticF[ArcTan[Tan[e + f*x]]/2,
 2]*(Sec[e + f*x]^2)^(3/4))/(12*(a^2 + b^2)^3*f*(d*Sec[e + f*x])^(3/2)) + (
7*a*b^2*(9*a^2 - 2*b^2)*Cot[e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSi
n[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2)^(3/4)*Sqrt[-Tan[e + f*x]^2]
)/(8*(a^2 + b^2)^4*f*(d*Sec[e + f*x])^(3/2)) + (7*a*b^2*(9*a^2 - 2*b^2)*Cot
[e + f*x]*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]
*(Sec[e + f*x]^2)^(3/4)*Sqrt[-Tan[e + f*x]^2])/(8*(a^2 + b^2)^4*f*(d*Sec[e
+ f*x])^(3/2)) + (b*(4*a^2 - 7*b^2)*Sec[e + f*x]^2)/(6*(a^2 + b^2)^2*f*(d*S
ec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^2) + (2*(b + a*Tan[e + f*x]))/(3*(a
^2 + b^2)*f*(d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^2) + (a*b*(8*a^2 -
69*b^2)*Sec[e + f*x]^2)/(12*(a^2 + b^2)^3*f*(d*Sec[e + f*x])^(3/2)*(a + b*T
an[e + f*x]))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 109

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e
/f) + d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-f/(d*e - c*f), 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
```

```
, 0]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[(-b)*(x^2/a)]/(2*x), Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 761

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(3/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(3/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(3/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\sec^2(e+fx)^{3/4}\text{Subst}\left(\int \frac{1}{(a+x)^3\left(1+\frac{x^2}{b^2}\right)^{7/4}}\,dx, x, b\tan(e+fx)\right)}{bf(d\sec(e+fx))^{3/2}}$$

$$= \frac{2(b+a\tan(e+fx))}{3\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2} - \frac{\left(2b\sec^2(e+fx)^{3/4}\right)\text{Subst}\left(\int \frac{\frac{1}{2}\left(-7-\frac{a^2}{b^2}\right)-\frac{5ax}{2b^2}}{(a+x)^3\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{3\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}}$$

$$= \frac{b(4a^2-7b^2)\sec^2(e+fx)}{6\left(a^2+b^2\right)^2f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2} + \frac{2(b+a\tan(e+fx))}{3\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2} + \frac{\left(b^3\sec^2(e+fx)^{3/4}\right)\text{Subst}\left(\int \frac{\frac{a\left(a^2+12b^2\right)}{b^4}+\frac{3\left(4a^2-7b^2\right)x}{4b^4}}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{3\left(a^2+b^2\right)^2f(d\sec(e+fx))^{3/2}}$$

$$= \frac{b(4a^2-7b^2)\sec^2(e+fx)}{6\left(a^2+b^2\right)^2f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2} + \frac{2(b+a\tan(e+fx))}{3\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2} + \frac{ab(8a^2-69b^2)\sec^2(e+fx)}{12\left(a^2+b^2\right)^3f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))} - \frac{\left(b^5\sec^2(e+fx)^{3/4}\right)\text{Subst}\left(\int \frac{-\frac{4a^4+60a^2b^2-21b^4}{4b^6}-\frac{a\left(8a^2-69b^2\right)x}{8b^6}}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{3\left(a^2+b^2\right)^3f(d\sec(e+fx))^{3/2}}$$

$$= \frac{b(4a^2-7b^2)\sec^2(e+fx)}{6\left(a^2+b^2\right)^2f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2} + \frac{2(b+a\tan(e+fx))}{3\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2} + \frac{ab(8a^2-69b^2)\sec^2(e+fx)}{12\left(a^2+b^2\right)^3f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))} + \frac{\left(a(8a^2-69b^2)\sec^2(e+fx)^{3/4}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{24b\left(a^2+b^2\right)^3f(d\sec(e+fx))^{3/2}} + \frac{\left(7b(9a^2-2b^2)\sec^2(e+fx)^{3/4}\right)\text{Subst}\left(\int \frac{1}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx, x, b\tan(e+fx)\right)}{8\left(a^2+b^2\right)^3f(d\sec(e+fx))^{3/2}}$$

$$= \frac{a(8a^2-69b^2)\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sec^2(e+fx)^{3/4}}{12\,(a^2+b^2)^3\,f(d\sec(e+fx))^{3/2}} + \frac{b(4a^2-7b^2)\sec^2(e+fx)}{6\,(a^2+b^2)^2\,f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2} + \frac{2(b+a\tan(e+fx))}{3\,(a^2+b^2)\,f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2} + \frac{ab(8a^2-69b^2)\sec^2(e+fx)}{12\,(a^2+b^2)^3\,f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))} - \frac{\left(7b(9a^2-2b^2)\sec^2(e+fx)^{3/4}\right)\text{Subst}\left(\int\frac{x}{(a^2-x^2)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx,x,b\tan(e+fx)\right)}{8\,(a^2+b^2)^3\,f(d\sec(e+fx))^{3/2}} + \frac{\left(7ab(9a^2-2b^2)\sec^2(e+fx)^{3/4}\right)\text{Subst}\left(\int\frac{1}{(a^2-x^2)\left(1+\frac{x^2}{b^2}\right)^{3/4}}\,dx,x,b\tan(e+fx)\right)}{8\,(a^2+b^2)^3\,f(d\sec(e+fx))^{3/2}}$$

$$= \frac{a(8a^2-69b^2)\,\text{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sec^2(e+fx)^{3/4}}{12\,(a^2+b^2)^3\,f(d\sec(e+fx))^{3/2}} + \frac{b(4a^2-7b^2)\sec^2(e+fx)}{6\,(a^2+b^2)^2\,f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2} + \frac{2(b+a\tan(e+fx))}{3\,(a^2+b^2)\,f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2} + \frac{ab(8a^2-69b^2)\sec^2(e+fx)}{12\,(a^2+b^2)^3\,f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))} - \frac{\left(7b(9a^2-2b^2)\sec^2(e+fx)^{3/4}\right)\text{Subst}\left(\int\frac{1}{(a^2-x)\left(1+\frac{x}{b^2}\right)^{3/4}}\,dx,x,b^2\tan^2(e+fx)\right)}{16\,(a^2+b^2)^3\,f(d\sec(e+fx))^{3/2}} + \frac{\left(7a(9a^2-2b^2)\cot(e+fx)\sec^2(e+fx)^{3/4}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{(a^2-x)\sqrt{-\frac{x}{b^2}}\left(1+\frac{x}{b^2}\right)^{3/4}}\,dx,x\right.}{16\,(a^2+b^2)^3\,f(d\sec(e+fx))^{3/2}}$$

$$= \frac{a(8a^2-69b^2)\operatorname{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sec^2(e+fx)^{3/4}}{12\left(a^2+b^2\right)^3 f(d\sec(e+fx))^{3/2}}$$
$$+ \frac{b(4a^2-7b^2)\sec^2(e+fx)}{6\left(a^2+b^2\right)^2 f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}$$
$$+ \frac{2(b+a\tan(e+fx))}{3\left(a^2+b^2\right) f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}$$
$$+ \frac{ab(8a^2-69b^2)\sec^2(e+fx)}{12\left(a^2+b^2\right)^3 f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}$$
$$- \frac{\left(7b^3(9a^2-2b^2)\sec^2(e+fx)^{3/4}\right)\operatorname{Subst}\left(\int \frac{1}{a^2+b^2-b^2x^4}\,dx,x,\sqrt[4]{\sec^2(e+fx)}\right)}{4\left(a^2+b^2\right)^3 f(d\sec(e+fx))^{3/2}}$$
$$- \frac{\left(7a(9a^2-2b^2)\cot(e+fx)\sec^2(e+fx)^{3/4}\sqrt{-\tan^2(e+fx)}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^4}\left(-1-\frac{a^2}{b^2}+x^4\right)}\,dx,x,\right.}{4\left(a^2+b^2\right)^3 f(d\sec(e+fx))^{3/2}}$$

$$= \frac{a(8a^2-69b^2)\operatorname{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sec^2(e+fx)^{3/4}}{12\left(a^2+b^2\right)^3 f(d\sec(e+fx))^{3/2}}$$
$$+ \frac{b(4a^2-7b^2)\sec^2(e+fx)}{6\left(a^2+b^2\right)^2 f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}$$
$$+ \frac{2(b+a\tan(e+fx))}{3\left(a^2+b^2\right) f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}$$
$$+ \frac{ab(8a^2-69b^2)\sec^2(e+fx)}{12\left(a^2+b^2\right)^3 f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}$$
$$- \frac{\left(7b^3(9a^2-2b^2)\sec^2(e+fx)^{3/4}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a^2+b^2}-bx^2}\,dx,x,\sqrt[4]{\sec^2(e+fx)}\right)}{8\left(a^2+b^2\right)^{7/2} f(d\sec(e+fx))^{3/2}}$$
$$- \frac{\left(7b^3(9a^2-2b^2)\sec^2(e+fx)^{3/4}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a^2+b^2}+bx^2}\,dx,x,\sqrt[4]{\sec^2(e+fx)}\right)}{8\left(a^2+b^2\right)^{7/2} f(d\sec(e+fx))^{3/2}}$$
$$+ \frac{\left(7ab^2(9a^2-2b^2)\cot(e+fx)\sec^2(e+fx)^{3/4}\sqrt{-\tan^2(e+fx)}\right)\operatorname{Subst}\left(\int \frac{1}{\left(1-\frac{bx^2}{\sqrt{a^2+b^2}}\right)\sqrt{1-x^4}}\,dx,\right.}{8\left(a^2+b^2\right)^4 f(d\sec(e+fx))^{3/2}}$$
$$+ \frac{\left(7ab^2(9a^2-2b^2)\cot(e+fx)\sec^2(e+fx)^{3/4}\sqrt{-\tan^2(e+fx)}\right)\operatorname{Subst}\left(\int \frac{1}{\left(1+\frac{bx^2}{\sqrt{a^2+b^2}}\right)\sqrt{1-x^4}}\,dx,\right.}{8\left(a^2+b^2\right)^4 f(d\sec(e+fx))^{3/2}}$$

$$
\begin{aligned}
&= -\frac{7b^{5/2}(9a^2-2b^2)\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{8\left(a^2+b^2\right)^{15/4}f(d\sec(e+fx))^{3/2}} \\
&\quad - \frac{7b^{5/2}(9a^2-2b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{8\left(a^2+b^2\right)^{15/4}f(d\sec(e+fx))^{3/2}} \\
&\quad + \frac{a(8a^2-69b^2)\operatorname{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sec^2(e+fx)^{3/4}}{12\left(a^2+b^2\right)^3 f(d\sec(e+fx))^{3/2}} \\
&\quad + \frac{b(4a^2-7b^2)\sec^2(e+fx)}{6\left(a^2+b^2\right)^2 f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2} \\
&\quad + \frac{2(b+a\tan(e+fx))}{3\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2} \\
&\quad + \frac{ab(8a^2-69b^2)\sec^2(e+fx)}{12\left(a^2+b^2\right)^3 f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))} \\
&\quad + \frac{\left(7ab^2(9a^2-2b^2)\cot(e+fx)\sec^2(e+fx)^{3/4}\sqrt{-\tan^2(e+fx)}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(1-\frac{bx^2}{\sqrt{a^2+b^2}}\right)}\right.}{8\left(a^2+b^2\right)^4 f(d\sec(e+fx))^{3/2}} \\
&\quad + \frac{\left(7ab^2(9a^2-2b^2)\cot(e+fx)\sec^2(e+fx)^{3/4}\sqrt{-\tan^2(e+fx)}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(1+\frac{bx^2}{\sqrt{a^2+b^2}}\right)}\right.}{8\left(a^2+b^2\right)^4 f(d\sec(e+fx))^{3/2}}
\end{aligned}
$$

$$= -\frac{7b^{5/2}(9a^2-2b^2)\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{8\left(a^2+b^2\right)^{15/4}f(d\sec(e+fx))^{3/2}}$$

$$-\frac{7b^{5/2}(9a^2-2b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sec^2(e+fx)^{3/4}}{8\left(a^2+b^2\right)^{15/4}f(d\sec(e+fx))^{3/2}}$$

$$+\frac{a(8a^2-69b^2)\operatorname{EllipticF}\left(\frac{1}{2}\arctan(\tan(e+fx)),2\right)\sec^2(e+fx)^{3/4}}{12\left(a^2+b^2\right)^3f(d\sec(e+fx))^{3/2}}$$

$$+\frac{7ab^2(9a^2-2b^2)\cot(e+fx)\operatorname{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sec^2(e+fx)^{3/4}}{8\left(a^2+b^2\right)^4f(d\sec(e+fx))^{3/2}}$$

$$+\frac{7ab^2(9a^2-2b^2)\cot(e+fx)\operatorname{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sec^2(e+fx)^{3/4}}{8\left(a^2+b^2\right)^4f(d\sec(e+fx))^{3/2}}$$

$$+\frac{b(4a^2-7b^2)\sec^2(e+fx)}{6\left(a^2+b^2\right)^2f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}$$

$$+\frac{2(b+a\tan(e+fx))}{3\left(a^2+b^2\right)f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^2}$$

$$+\frac{ab(8a^2-69b^2)\sec^2(e+fx)}{12\left(a^2+b^2\right)^3f(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 12.85 (sec) , antiderivative size = 409, normalized size of antiderivative = 0.66

$$\int\frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx=\frac{\frac{2(a^2+b^2)\sec^3(e+fx)\left(b(22a^4-63a^2b^2-8b^4)\cos(e+fx)+2b(a^2+b^2)^2\cos(3(e+fx)\right.}{(a+b\tan(e+fx))^2}}{}$$

```
[In] Integrate[1/((d*Sec[e + f*x])^(3/2)*(a + b*Tan[e + f*x])^3),x]
```

```
[Out] ((2*(a^2 + b^2)*Sec[e + f*x]^3*(b*(22*a^4 - 63*a^2*b^2 - 8*b^4)*Cos[e + f*x
] + 2*b*(a^2 + b^2)^2*Cos[3*(e + f*x)] + a*(4*a^4 + 16*a^2*b^2 - 65*b^4 + 4
*(a^2 + b^2)^2*Cos[2*(e + f*x)])*Sin[e + f*x]))/(a + b*Tan[e + f*x])^2 + (S
ec[e + f*x]^2)^(3/4)*(a*(8*a^4 - 61*a^2*b^2 - 69*b^4)*Hypergeometric2F1[1/2
, 3/4, 3/2, -Tan[e + f*x]^2]*Tan[e + f*x] + (21*b^2*(-9*a^2 + 2*b^2)*(a*Ell
ipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*Tan[e + f
*x] + a*EllipticPi[b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*T
an[e + f*x] + Sqrt[b]*(a^2 + b^2)^(1/4)*(ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(
```

```
1/4))/(a^2 + b^2)^(1/4)] + ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 +
b^2)^(1/4)])*Sqrt[-Tan[e + f*x]^2]))/Sqrt[-Tan[e + f*x]^2]))/(24*(a^2 + b^2
)^4*f*(d*Sec[e + f*x])^(3/2))
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 76276 vs. 2(573) = 1146.

Time = 17.74 (sec) , antiderivative size = 76277, normalized size of antiderivative = 123.03

method	result	size
default	Expression too large to display	76277

```
[In] int(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx = \int \frac{1}{(d\sec{(e+fx)})^{\frac{3}{2}}\,(a+b\tan{(e+fx)})^3}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))**(3/2)/(a+b*tan(f*x+e))**3,x)
[Out] Integral(1/((d*sec(e + f*x))**(3/2)*(a + b*tan(e + f*x))**3), x)
```

Maxima [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{3}{2}}(b\tan(fx+e)+a)^3}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
[Out] integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)^3), x)
```

Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{3}{2}}(b\tan(fx+e)+a)^3}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(3/2)/(a+b*tan(f*x+e))^3,x, algorithm="giac")
[Out] integrate(1/((d*sec(f*x + e))^(3/2)*(b*tan(f*x + e) + a)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{3/2}(a+b\tan(e+fx))^3}\,dx = \int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{3/2}(a+b\tan(e+f\,x))^3}\,dx$$

```
[In] int(1/((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x))^3),x)
[Out] int(1/((d/cos(e + f*x))^(3/2)*(a + b*tan(e + f*x))^3), x)
```

3.623 $\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 814

$$
\begin{aligned}
\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx &= \frac{9b^{7/2}(11a^2-2b^2)\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{8\left(a^2+b^2\right)^{17/4}d^2f\sqrt{d\sec(e+fx)}} \\
&- \frac{9b^{7/2}(11a^2-2b^2)\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{8\left(a^2+b^2\right)^{17/4}d^2f\sqrt{d\sec(e+fx)}} \\
&+ \frac{3a(8a^4+64a^2b^2-139b^4)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{20\left(a^2+b^2\right)^4d^2f\sqrt{d\sec(e+fx)}} \\
&- \frac{3a(8a^4+64a^2b^2-139b^4)\tan(e+fx)}{20\left(a^2+b^2\right)^4d^2f\sqrt{d\sec(e+fx)}} \\
&- \frac{9ab^3(11a^2-2b^2)\cot(e+fx)\,\mathrm{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e}}{8\left(a^2+b^2\right)^{9/2}d^2f\sqrt{d\sec(e+fx)}} \\
&+ \frac{9ab^3(11a^2-2b^2)\cot(e+fx)\,\mathrm{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+}}{8\left(a^2+b^2\right)^{9/2}d^2f\sqrt{d\sec(e+fx)}} \\
&+ \frac{3b(4a^4+28a^2b^2-15b^4)\sec^2(e+fx)}{10\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2} \\
&+ \frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5\left(a^2+b^2\right)d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2} \\
&+ \frac{3ab(8a^4+64a^2b^2-139b^4)\sec^2(e+fx)}{20\left(a^2+b^2\right)^4d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} \\
&- \frac{2(b(4a^2-9b^2)-a(3a^2+16b^2)\tan(e+fx))}{5\left(a^2+b^2\right)^2d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}
\end{aligned}
$$

```
[Out] 9/8*b^(7/2)*(11*a^2-2*b^2)*arctan((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1
/4))*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(17/4)/d^2/f/(d*sec(f*x+e))^(1/2)-9/8*b
^(7/2)*(11*a^2-2*b^2)*arctanh((sec(f*x+e)^2)^(1/4)*b^(1/2)/(a^2+b^2)^(1/4))
*(sec(f*x+e)^2)^(1/4)/(a^2+b^2)^(17/4)/d^2/f/(d*sec(f*x+e))^(1/2)+3/20*a*(8
*a^4+64*a^2*b^2-139*b^4)*(cos(1/2*arctan(tan(f*x+e)))^2)^(1/2)/cos(1/2*arct
an(tan(f*x+e)))*EllipticE(sin(1/2*arctan(tan(f*x+e))),2^(1/2))*(sec(f*x+e)^
2)^(1/4)/(a^2+b^2)^4/d^2/f/(d*sec(f*x+e))^(1/2)-9/8*a*b^3*(11*a^2-2*b^2)*co
t(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),-b/(a^2+b^2)^(1/2),I)*(sec(f*x+e)^
2)^(1/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^(9/2)/d^2/f/(d*sec(f*x+e))^(1/2)+9
/8*a*b^3*(11*a^2-2*b^2)*cot(f*x+e)*EllipticPi((sec(f*x+e)^2)^(1/4),b/(a^2+b
^2)^(1/2),I)*(sec(f*x+e)^2)^(1/4)*(-tan(f*x+e)^2)^(1/2)/(a^2+b^2)^(9/2)/d^2
/f/(d*sec(f*x+e))^(1/2)-3/20*a*(8*a^4+64*a^2*b^2-139*b^4)*tan(f*x+e)/(a^2+b
^2)^4/d^2/f/(d*sec(f*x+e))^(1/2)+3/10*b*(4*a^4+28*a^2*b^2-15*b^4)*sec(f*x+e
)^2/(a^2+b^2)^3/d^2/f/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2+2/5*cos(f*x+e
)^2*(b+a*tan(f*x+e))/(a^2+b^2)/d^2/f/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^
2+3/20*a*b*(8*a^4+64*a^2*b^2-139*b^4)*sec(f*x+e)^2/(a^2+b^2)^4/d^2/f/(d*sec
(f*x+e))^(1/2)/(a+b*tan(f*x+e))-2/5*(b*(4*a^2-9*b^2)-a*(3*a^2+16*b^2)*tan(f
*x+e))/(a^2+b^2)^2/d^2/f/(d*sec(f*x+e))^(1/2)/(a+b*tan(f*x+e))^2
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 814, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.680$, Rules

used = {3593, 755, 837, 849, 858, 233, 202, 760, 408, 504, 1227, 551, 455, 65, 304, 211, 214}

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx = \frac{9(11a^2-2b^2)\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}b^{7/2}}{8\left(a^2+b^2\right)^{17/4}d^2f\sqrt{d\sec(e+fx)}}$$
$$-\frac{9(11a^2-2b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}b^{7/2}}{8\left(a^2+b^2\right)^{17/4}d^2f\sqrt{d\sec(e+fx)}}$$
$$-\frac{9a(11a^2-2b^2)\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+}}{8\left(a^2+b^2\right)^{9/2}d^2f\sqrt{d\sec(e+fx)}}$$
$$+\frac{9a(11a^2-2b^2)\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+}}{8\left(a^2+b^2\right)^{9/2}d^2f\sqrt{d\sec(e+fx)}}$$
$$+\frac{3a(8a^4+64b^2a^2-139b^4)\sec^2(e+fx)b}{20\left(a^2+b^2\right)^4d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}$$
$$+\frac{3(4a^4+28b^2a^2-15b^4)\sec^2(e+fx)b}{10\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}$$
$$-\frac{3a(8a^4+64b^2a^2-139b^4)\tan(e+fx)}{20\left(a^2+b^2\right)^4d^2f\sqrt{d\sec(e+fx)}}$$
$$-\frac{2(b(4a^2-9b^2)-a(3a^2+16b^2)\tan(e+fx))}{5\left(a^2+b^2\right)^2d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}$$
$$+\frac{3a(8a^4+64b^2a^2-139b^4)E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{20\left(a^2+b^2\right)^4d^2f\sqrt{d\sec(e+fx)}}$$
$$+\frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5\left(a^2+b^2\right)d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}$$

```
[In] Int[1/((d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])^3),x]
```

```
[Out] (9*b^(7/2)*(11*a^2 - 2*b^2)*ArcTan[(Sqrt[b]*(Sec[e + f*x]^2)^(1/4))/(a^2 +
b^2)^(1/4)]*(Sec[e + f*x]^2)^(1/4))/(8*(a^2 + b^2)^(17/4)*d^2*f*Sqrt[d*Sec[
e + f*x]]) - (9*b^(7/2)*(11*a^2 - 2*b^2)*ArcTanh[(Sqrt[b]*(Sec[e + f*x]^2)^
(1/4))/(a^2 + b^2)^(1/4)]*(Sec[e + f*x]^2)^(1/4))/(8*(a^2 + b^2)^(17/4)*d^2
*f*Sqrt[d*Sec[e + f*x]]) + (3*a*(8*a^4 + 64*a^2*b^2 - 139*b^4)*EllipticE[Ar
cTan[Tan[e + f*x]]/2, 2]*(Sec[e + f*x]^2)^(1/4))/(20*(a^2 + b^2)^4*d^2*f*Sq
rt[d*Sec[e + f*x]]) - (3*a*(8*a^4 + 64*a^2*b^2 - 139*b^4)*Tan[e + f*x])/(20
*(a^2 + b^2)^4*d^2*f*Sqrt[d*Sec[e + f*x]]) - (9*a*b^3*(11*a^2 - 2*b^2)*Cot[
e + f*x]*EllipticPi[-(b/Sqrt[a^2 + b^2]), ArcSin[(Sec[e + f*x]^2)^(1/4)], -
1]*(Sec[e + f*x]^2)^(1/4)*Sqrt[-Tan[e + f*x]^2])/(8*(a^2 + b^2)^(9/2)*d^2*f
*Sqrt[d*Sec[e + f*x]]) + (9*a*b^3*(11*a^2 - 2*b^2)*Cot[e + f*x]*EllipticPi[
b/Sqrt[a^2 + b^2], ArcSin[(Sec[e + f*x]^2)^(1/4)], -1]*(Sec[e + f*x]^2)^(1/
4)*Sqrt[-Tan[e + f*x]^2])/(8*(a^2 + b^2)^(9/2)*d^2*f*Sqrt[d*Sec[e + f*x]])
```

```
+ (3*b*(4*a^4 + 28*a^2*b^2 - 15*b^4)*Sec[e + f*x]^2)/(10*(a^2 + b^2)^3*d^2*
f*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])^2) + (2*Cos[e + f*x]^2*(b + a*T
an[e + f*x]))/(5*(a^2 + b^2)*d^2*f*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x]
)^2) + (3*a*b*(8*a^4 + 64*a^2*b^2 - 139*b^4)*Sec[e + f*x]^2)/(20*(a^2 + b^2
)^4*d^2*f*Sqrt[d*Sec[e + f*x]]*(a + b*Tan[e + f*x])) - (2*(b*(4*a^2 - 9*b^2
) - a*(3*a^2 + 16*b^2)*Tan[e + f*x]))/(5*(a^2 + b^2)^2*d^2*f*Sqrt[d*Sec[e +
 f*x]]*(a + b*Tan[e + f*x])^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 760

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(1/4)), x_Symbol] :> Dist[
d, Int[1/((d^2 - e^2*x^2)*(a + c*x^2)^(1/4)), x], x] - Dist[e, Int[x/((d^2
- e^2*x^2)*(a + c*x^2)^(1/4)), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^
2 + a*e^2, 0]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt[4]{\sec^2(e+fx)}\text{Subst}\left(\int \frac{1}{(a+x)^3\left(1+\frac{x^2}{b^2}\right)^{9/4}}\,dx, x, b\tan(e+fx)\right)}{bd^2f\sqrt{d\sec(e+fx)}}$$

$$=\frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5(a^2+b^2)d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}-\frac{\left(2b\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{-\frac{3}{2}\left(3+\frac{a^2}{b^2}\right)-\frac{7ax}{2b^2}}{(a+x)^3\left(1+\frac{x^2}{b^2}\right)^{5/4}}dx,x,b\tan(e+fx)\right)}{5(a^2+b^2)d^2f\sqrt{d\sec(e+fx)}}$$

$$=\frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5(a^2+b^2)d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}-\frac{2(b(4a^2-9b^2)-a(3a^2+16b^2)\tan(e+fx))}{5(a^2+b^2)^2d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}+\frac{\left(4b^5\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{-\frac{3(a^4+12a^2b^2-15b^4)}{4b^6}+\frac{3a(3a^2+16b^2)x}{4b^6}}{(a+x)^3\sqrt[4]{1+\frac{x^2}{b^2}}}dx,x,b\tan(e+fx)\right)}{5(a^2+b^2)^2d^2f\sqrt{d\sec(e+fx)}}$$

$$=\frac{3b(4a^4+28a^2b^2-15b^4)\sec^2(e+fx)}{10(a^2+b^2)^3d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}+\frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5(a^2+b^2)d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}-\frac{2(b(4a^2-9b^2)-a(3a^2+16b^2)\tan(e+fx))}{5(a^2+b^2)^2d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}-\frac{\left(2b^7\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{\frac{3a(a^4+9a^2b^2-31b^4)}{2b^8}-\frac{3(4a^4+28a^2b^2-15b^4)x}{8b^8}}{(a+x)^2\sqrt[4]{1+\frac{x^2}{b^2}}}dx,x,b\tan(e+fx)\right)}{5(a^2+b^2)^3d^2f\sqrt{d\sec(e+fx)}}$$

$$=\frac{3b(4a^4+28a^2b^2-15b^4)\sec^2(e+fx)}{10(a^2+b^2)^3d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}+\frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5(a^2+b^2)d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}+\frac{3ab(8a^4+64a^2b^2-139b^4)\sec^2(e+fx)}{20(a^2+b^2)^4d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}-\frac{2(b(4a^2-9b^2)-a(3a^2+16b^2)\tan(e+fx))}{5(a^2+b^2)^2d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}+\frac{\left(2b^9\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{-\frac{3(4a^6+32a^4b^2-152a^2b^4+15b^6)}{8b^{10}}-\frac{3a(8a^4+64a^2b^2-139b^4)x}{16b^{10}}}{(a+x)\sqrt[4]{1+\frac{x^2}{b^2}}}dx,x,b\tan(e+fx)\right)}{5(a^2+b^2)^4d^2f\sqrt{d\sec(e+fx)}}$$

$$= \frac{3b(4a^4+28a^2b^2-15b^4)\sec^2(e+fx)}{10\left(a^2+b^2\right)^3 d^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}$$
$$+\frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5\left(a^2+b^2\right) d^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}$$
$$+\frac{3ab(8a^4+64a^2b^2-139b^4)\sec^2(e+fx)}{20\left(a^2+b^2\right)^4 d^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}$$
$$-\frac{2(b(4a^2-9b^2)-a(3a^2+16b^2)\tan(e+fx))}{5\left(a^2+b^2\right)^2 d^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}$$
$$+\frac{\left(9b^3(11a^2-2b^2)\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{(a+x)\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx,x,b\tan(e+fx)\right)}{8\left(a^2+b^2\right)^4 d^2 f\sqrt{d\sec(e+fx)}}$$
$$-\frac{\left(3a(8a^4+64a^2b^2-139b^4)\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx,x,b\tan(e+fx)\right)}{40b\left(a^2+b^2\right)^4 d^2 f\sqrt{d\sec(e+fx)}}$$

$$
\begin{aligned}
&= -\frac{3a(8a^4+64a^2b^2-139b^4)\tan(e+fx)}{20\,(a^2+b^2)^4\,d^2f\sqrt{d\sec(e+fx)}} \\
&\quad + \frac{3b(4a^4+28a^2b^2-15b^4)\sec^2(e+fx)}{10\,(a^2+b^2)^3\,d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2} \\
&\quad + \frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5\,(a^2+b^2)\,d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2} \\
&\quad + \frac{3ab(8a^4+64a^2b^2-139b^4)\sec^2(e+fx)}{20\,(a^2+b^2)^4\,d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} \\
&\quad - \frac{2(b(4a^2-9b^2)-a(3a^2+16b^2)\tan(e+fx))}{5\,(a^2+b^2)^2\,d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2} \\
&\quad - \frac{\left(9b^3(11a^2-2b^2)\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{x}{(a^2-x^2)\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx,x,b\tan(e+fx)\right)}{8\,(a^2+b^2)^4\,d^2f\sqrt{d\sec(e+fx)}} \\
&\quad + \frac{\left(9ab^3(11a^2-2b^2)\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{(a^2-x^2)\sqrt[4]{1+\frac{x^2}{b^2}}}\,dx,x,b\tan(e+fx)\right)}{8\,(a^2+b^2)^4\,d^2f\sqrt{d\sec(e+fx)}} \\
&\quad + \frac{\left(3a(8a^4+64a^2b^2-139b^4)\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\left(1+\frac{x^2}{b^2}\right)^{5/4}}\,dx,x,b\tan(e+fx)\right)}{40b\,(a^2+b^2)^4\,d^2f\sqrt{d\sec(e+fx)}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3a(8a^4+64a^2b^2-139b^4)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{20\left(a^2+b^2\right)^4 d^2 f\sqrt{d\sec(e+fx)}} \\
&\quad - \frac{3a(8a^4+64a^2b^2-139b^4)\tan(e+fx)}{20\left(a^2+b^2\right)^4 d^2 f\sqrt{d\sec(e+fx)}} \\
&\quad + \frac{3b(4a^4+28a^2b^2-15b^4)\sec^2(e+fx)}{10\left(a^2+b^2\right)^3 d^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2} \\
&\quad + \frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5\left(a^2+b^2\right) d^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2} \\
&\quad + \frac{3ab(8a^4+64a^2b^2-139b^4)\sec^2(e+fx)}{20\left(a^2+b^2\right)^4 d^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} \\
&\quad - \frac{2(b(4a^2-9b^2)-a(3a^2+16b^2)\tan(e+fx))}{5\left(a^2+b^2\right)^2 d^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2} \\
&\quad - \frac{\left(9b^3(11a^2-2b^2)\sqrt[4]{\sec^2(e+fx)}\right)\operatorname{Subst}\left(\int\frac{1}{(a^2-x)\sqrt[4]{1+\frac{x}{b^2}}}\,dx,x,b^2\tan^2(e+fx)\right)}{16\left(a^2+b^2\right)^4 d^2 f\sqrt{d\sec(e+fx)}} \\
&\quad + \frac{\left(9ab^2(11a^2-2b^2)\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\operatorname{Subst}\left(\int\frac{x^2}{\sqrt{1-x^4}\left(1+\frac{a^2}{b^2}-x^4\right)}\,dx,x\right.}{4\left(a^2+b^2\right)^4 d^2 f\sqrt{d\sec(e+fx)}}
\end{aligned}
$$

$$= \frac{3a(8a^4+64a^2b^2-139b^4)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{20\,(a^2+b^2)^4\,d^2f\sqrt{d\sec(e+fx)}}$$

$$- \frac{3a(8a^4+64a^2b^2-139b^4)\tan(e+fx)}{20\,(a^2+b^2)^4\,d^2f\sqrt{d\sec(e+fx)}}$$

$$+ \frac{3b(4a^4+28a^2b^2-15b^4)\sec^2(e+fx)}{10\,(a^2+b^2)^3\,d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}$$

$$+ \frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5\,(a^2+b^2)\,d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}$$

$$+ \frac{3ab(8a^4+64a^2b^2-139b^4)\sec^2(e+fx)}{20\,(a^2+b^2)^4\,d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}$$

$$- \frac{2(b(4a^2-9b^2)-a(3a^2+16b^2)\tan(e+fx))}{5\,(a^2+b^2)^2\,d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}$$

$$- \frac{\left(9b^5(11a^2-2b^2)\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{x^2}{a^2+b^2-b^2x^4}\,dx,x,\sqrt[4]{\sec^2(e+fx)}\right)}{4\,(a^2+b^2)^4\,d^2f\sqrt{d\sec(e+fx)}}$$

$$+ \frac{\left(9ab^3(11a^2-2b^2)\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\left(\sqrt{a^2+b^2}-bx^2\right)\sqrt{1-x^4}}\,dx,\right.}{8\,(a^2+b^2)^4\,d^2f\sqrt{d\sec(e+fx)}}$$

$$- \frac{\left(9ab^3(11a^2-2b^2)\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\left(\sqrt{a^2+b^2}+bx^2\right)\sqrt{1-x^4}}\,dx,\right.}{8\,(a^2+b^2)^4\,d^2f\sqrt{d\sec(e+fx)}}$$

$$= \frac{3a(8a^4+64a^2b^2-139b^4)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{20\left(a^2+b^2\right)^4 d^2 f\sqrt{d\sec(e+fx)}}$$
$$-\frac{3a(8a^4+64a^2b^2-139b^4)\tan(e+fx)}{20\left(a^2+b^2\right)^4 d^2 f\sqrt{d\sec(e+fx)}}$$
$$+\frac{3b(4a^4+28a^2b^2-15b^4)\sec^2(e+fx)}{10\left(a^2+b^2\right)^3 d^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}$$
$$+\frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5\left(a^2+b^2\right) d^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}$$
$$+\frac{3ab(8a^4+64a^2b^2-139b^4)\sec^2(e+fx)}{20\left(a^2+b^2\right)^4 d^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))}$$
$$-\frac{2(b(4a^2-9b^2)-a(3a^2+16b^2)\tan(e+fx))}{5\left(a^2+b^2\right)^2 d^2 f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}$$
$$-\frac{\left(9b^4(11a^2-2b^2)\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{a^2+b^2-bx^2}}\,dx,x,\sqrt[4]{\sec^2(e+fx)}\right)}{8\left(a^2+b^2\right)^4 d^2 f\sqrt{d\sec(e+fx)}}$$
$$+\frac{\left(9b^4(11a^2-2b^2)\sqrt[4]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{a^2+b^2+bx^2}}\,dx,x,\sqrt[4]{\sec^2(e+fx)}\right)}{8\left(a^2+b^2\right)^4 d^2 f\sqrt{d\sec(e+fx)}}$$
$$+\frac{\left(9ab^3(11a^2-2b^2)\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(\sqrt{a^2+b^2}-b\right.}\right.}{8\left(a^2+b^2\right)^4 d^2 f\sqrt{d\sec(e+fx)}}$$
$$-\frac{\left(9ab^3(11a^2-2b^2)\cot(e+fx)\sqrt[4]{\sec^2(e+fx)}\sqrt{-\tan^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}\left(\sqrt{a^2+b^2}+b\right.}\right.}{8\left(a^2+b^2\right)^4 d^2 f\sqrt{d\sec(e+fx)}}$$

$$
\begin{aligned}
&= \frac{9b^{7/2}(11a^2-2b^2)\arctan\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{8\left(a^2+b^2\right)^{17/4}d^2f\sqrt{d\sec(e+fx)}} \\
&\quad - \frac{9b^{7/2}(11a^2-2b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt[4]{\sec^2(e+fx)}}{\sqrt[4]{a^2+b^2}}\right)\sqrt[4]{\sec^2(e+fx)}}{8\left(a^2+b^2\right)^{17/4}d^2f\sqrt{d\sec(e+fx)}} \\
&\quad + \frac{3a(8a^4+64a^2b^2-139b^4)\,E\left(\frac{1}{2}\arctan(\tan(e+fx))\middle|2\right)\sqrt[4]{\sec^2(e+fx)}}{20\left(a^2+b^2\right)^4d^2f\sqrt{d\sec(e+fx)}} \\
&\quad - \frac{3a(8a^4+64a^2b^2-139b^4)\tan(e+fx)}{20\left(a^2+b^2\right)^4d^2f\sqrt{d\sec(e+fx)}} \\
&\quad - \frac{9ab^3(11a^2-2b^2)\cot(e+fx)\,\text{EllipticPi}\left(-\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}}{8\left(a^2+b^2\right)^{9/2}d^2f\sqrt{d\sec(e+fx)}} \\
&\quad + \frac{9ab^3(11a^2-2b^2)\cot(e+fx)\,\text{EllipticPi}\left(\frac{b}{\sqrt{a^2+b^2}},\arcsin\left(\sqrt[4]{\sec^2(e+fx)}\right),-1\right)\sqrt[4]{\sec^2(e+fx)}\sqrt{}}{8\left(a^2+b^2\right)^{9/2}d^2f\sqrt{d\sec(e+fx)}} \\
&\quad + \frac{3b(4a^4+28a^2b^2-15b^4)\sec^2(e+fx)}{10\left(a^2+b^2\right)^3d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2} \\
&\quad + \frac{2\cos^2(e+fx)(b+a\tan(e+fx))}{5\left(a^2+b^2\right)d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2} \\
&\quad + \frac{3ab(8a^4+64a^2b^2-139b^4)\sec^2(e+fx)}{20\left(a^2+b^2\right)^4d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))} \\
&\quad - \frac{2(b(4a^2-9b^2)-a(3a^2+16b^2)\tan(e+fx))}{5\left(a^2+b^2\right)^2d^2f\sqrt{d\sec(e+fx)}(a+b\tan(e+fx))^2}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 33.97 (sec) , antiderivative size = 15481, normalized size of antiderivative = 19.02

$$
\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx = \text{Result too large to show}
$$

```
[In] Integrate[1/((d*Sec[e + f*x])^(5/2)*(a + b*Tan[e + f*x])^3),x]
[Out] Result too large to show
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 89814 vs. $2(755) = 1510$.

Time = 22.57 (sec) , antiderivative size = 89815, normalized size of antiderivative = 110.34

method	result	size
default	Expression too large to display	89815

```
[In] int(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx = \int \frac{1}{(d\sec(e+fx))^{\frac{5}{2}}\,(a+b\tan(e+fx))^3}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))**(5/2)/(a+b*tan(f*x+e))**3,x)
[Out] Integral(1/((d*sec(e + f*x))**(5/2)*(a + b*tan(e + f*x))**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{5}{2}}\,(b\tan(fx+e)+a)^3}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(5/2)/(a+b*tan(f*x+e))^3,x, algorithm="giac")
[Out] integrate(1/((d*sec(f*x + e))^(5/2)*(b*tan(f*x + e) + a)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/2}(a+b\tan(e+fx))^3}\,dx = \int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/2}\,(a+b\tan(e+f\,x))^3}\,dx$$

```
[In] int(1/((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x))^3),x)
[Out] int(1/((d/cos(e + f*x))^(5/2)*(a + b*tan(e + f*x))^3), x)
```

3.624 $\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 78

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))\,dx = \frac{3b(d\sec(e+fx))^{5/3}}{5f} + \frac{3ad\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(e+fx)\right)(d\sec(e+fx))^{2/3}\sin(e+fx)}{2f\sqrt{\sin^2(e+fx)}}$$

```
[Out] 3/5*b*(d*sec(f*x+e))^(5/3)/f+3/2*a*d*hypergeom([-1/3, 1/2],[2/3],cos(f*x+e)
^2)*(d*sec(f*x+e))^(2/3)*sin(f*x+e)/f/(sin(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3567, 3857, 2722}

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))\,dx = \frac{3ad\sin(e+fx)(d\sec(e+fx))^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(e+fx)\right)}{2f\sqrt{\sin^2(e+fx)}} + \frac{3b(d\sec(e+fx))^{5/3}}{5f}$$

```
[In] Int[(d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x]),x]
```

```
[Out] (3*b*(d*Sec[e + f*x])^(5/3))/(5*f) + (3*a*d*Hypergeometric2F1[-1/3, 1/2, 2/
3, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(2/3)*Sin[e + f*x])/(2*f*Sqrt[Sin[e + f
*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3b(d\sec(e+fx))^{5/3}}{5f} + a\int (d\sec(e+fx))^{5/3}\,dx \\
&= \frac{3b(d\sec(e+fx))^{5/3}}{5f} + \left(a\left(\frac{\cos(e+fx)}{d}\right)^{2/3}(d\sec(e+fx))^{2/3}\right)\int \frac{1}{\left(\frac{\cos(e+fx)}{d}\right)^{5/3}}\,dx \\
&= \frac{3b(d\sec(e+fx))^{5/3}}{5f} \\
&\quad + \frac{3ad\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(e+fx)\right)(d\sec(e+fx))^{2/3}\sin(e+fx)}{2f\sqrt{\sin^2(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))\,dx = \frac{3d(d\sec(e+fx))^{2/3}\left(b\sec(e+fx)+a\csc(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\sec^2(e+\ldots\right.\right.}{5f}$$

```
[In] Integrate[(d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x]),x]
```

```
[Out] (3*d*(d*Sec[e + f*x])^(2/3)*(b*Sec[e + f*x] + a*Csc[e + f*x]*Hypergeometric
2F1[1/2, 5/6, 11/6, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2]))/(5*f)
```

Maple [F]

$$\int (d\sec (fx+e))^{\frac{5}{3}} (a+b\tan (fx+e))\, dx$$

```
[In] int((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e)),x)
[Out] int((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e)),x)
```

Fricas [F]

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))\,dx = \int (d\sec (fx+e))^{\frac{5}{3}} (b\tan (fx+e)+a)\, dx$$

```
[In] integrate((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e)),x, algorithm="fricas")
[Out] integral((b*d*sec(f*x + e)*tan(f*x + e) + a*d*sec(f*x + e))*(d*sec(f*x + e)
)^(2/3), x)
```

Sympy [F]

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))\,dx = \int (d\sec (e+fx))^{\frac{5}{3}} (a+b\tan (e+fx))\, dx$$

```
[In] integrate((d*sec(f*x+e))**(5/3)*(a+b*tan(f*x+e)),x)
[Out] Integral((d*sec(e + f*x))**(5/3)*(a + b*tan(e + f*x)), x)
```

Maxima [F]

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))\,dx = \int (d\sec (fx+e))^{\frac{5}{3}} (b\tan (fx+e)+a)\, dx$$

```
[In] integrate((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e)),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^(5/3)*(b*tan(f*x + e) + a), x)
```

Giac [F]

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))\,dx = \int (d\sec(fx+e))^{\frac{5}{3}}(b\tan(fx+e)+a)\,dx$$

```
[In] integrate((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e)),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(5/3)*(b*tan(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))\,dx = \int \left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}(a+b\tan(e+f\,x))\,dx$$

```
[In] int((d/cos(e + f*x))^(5/3)*(a + b*tan(e + f*x)),x)
[Out] int((d/cos(e + f*x))^(5/3)*(a + b*tan(e + f*x)), x)
```

3.625 $\int \sqrt[3]{d \sec(e+fx)}(a+b\tan(e+fx))\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \sqrt[3]{d \sec(e+fx)}(a+b\tan(e+fx))\,dx = \frac{3b\sqrt[3]{d\sec(e+fx)}}{f} - \frac{3ad\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(e+fx)\right)\sin(e+fx)}{2f(d\sec(e+fx))^{2/3}\sqrt{\sin^2(e+fx)}}$$

```
[Out] 3*b*(d*sec(f*x+e))^(1/3)/f-3/2*a*d*hypergeom([1/3, 1/2],[4/3],cos(f*x+e)^2)
*sin(f*x+e)/f/(d*sec(f*x+e))^(2/3)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3567, 3857, 2722}

$$\int \sqrt[3]{d \sec(e+fx)}(a+b\tan(e+fx))\,dx = \frac{3b\sqrt[3]{d\sec(e+fx)}}{f} - \frac{3ad\sin(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(e+fx)\right)}{2f\sqrt{\sin^2(e+fx)}(d\sec(e+fx))^{2/3}}$$

```
[In] Int[(d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x]),x]
```

```
[Out] (3*b*(d*Sec[e + f*x])^(1/3))/f - (3*a*d*Hypergeometric2F1[1/3, 1/2, 4/3, Co
s[e + f*x]^2]*Sin[e + f*x])/(2*f*(d*Sec[e + f*x])^(2/3)*Sqrt[Sin[e + f*x]^2
])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{3b\sqrt[3]{d\sec(e+fx)}}{f} + a\int \sqrt[3]{d\sec(e+fx)}\,dx\\ &= \frac{3b\sqrt[3]{d\sec(e+fx)}}{f} + \left(a\sqrt[3]{\frac{\cos(e+fx)}{d}}\sqrt[3]{d\sec(e+fx)}\right)\int \frac{1}{\sqrt[3]{\frac{\cos(e+fx)}{d}}}\,dx\\ &= \frac{3b\sqrt[3]{d\sec(e+fx)}}{f}\\ &\quad - \frac{3a\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\sin(e+fx)}{2f\sqrt{\sin^2(e+fx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.78

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))\,dx = \frac{3\sqrt[3]{d\sec(e+fx)}\left(b+a\cot(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\sec^2(e+fx)\right)\sqrt{-\tan^2(e+fx)}\right)}{f}$$

```
[In] Integrate[(d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x]),x]
[Out] (3*(d*Sec[e + f*x])^(1/3)*(b + a*Cot[e + f*x]*Hypergeometric2F1[1/6, 1/2, 7
/6, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2]))/f
```

Maple [F]

$$\int (d\sec(fx+e))^{\frac{1}{3}} (a+b\tan(fx+e))\, dx$$

[In] int((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e)),x)

[Out] int((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e)),x)

Fricas [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))\, dx = \int (d\sec(fx+e))^{\frac{1}{3}} (b\tan(fx+e)+a)\, dx$$

[In] integrate((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e)),x, algorithm="fricas")

[Out] integral((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a), x)

Sympy [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))\, dx = \int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))\, dx$$

[In] integrate((d*sec(f*x+e))**(1/3)*(a+b*tan(f*x+e)),x)

[Out] Integral((d*sec(e + f*x))**(1/3)*(a + b*tan(e + f*x)), x)

Maxima [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))\, dx = \int (d\sec(fx+e))^{\frac{1}{3}} (b\tan(fx+e)+a)\, dx$$

[In] integrate((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e)),x, algorithm="maxima")

[Out] integrate((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a), x)

Giac [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))\,dx = \int (d\sec(fx+e))^{\frac{1}{3}}\,(b\tan(fx+e)+a)\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e)),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))\,dx = \int \left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}\,(a+b\tan(e+f\,x))\,dx$$

```
[In] int((d/cos(e + f*x))^(1/3)*(a + b*tan(e + f*x)),x)
[Out] int((d/cos(e + f*x))^(1/3)*(a + b*tan(e + f*x)), x)
```

3.626 $\int \frac{a+b\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \frac{a+b\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = -\frac{3b}{f\sqrt[3]{d\sec(e+fx)}} - \frac{3ad\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(e+fx)\right)\sin(e+fx)}{4f(d\sec(e+fx))^{4/3}\sqrt{\sin^2(e+fx)}}$$

```
[Out] -3*b/f/(d*sec(f*x+e))^(1/3)-3/4*a*d*hypergeom([1/2, 2/3],[5/3],cos(f*x+e)^2
)*sin(f*x+e)/f/(d*sec(f*x+e))^(4/3)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3567, 3857, 2722}

$$\int \frac{a+b\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = -\frac{3ad\sin(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(e+fx)\right)}{4f\sqrt{\sin^2(e+fx)}(d\sec(e+fx))^{4/3}} - \frac{3b}{f\sqrt[3]{d\sec(e+fx)}}$$

```
[In] Int[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(1/3),x]
```

```
[Out] (-3*b)/(f*(d*Sec[e + f*x])^(1/3)) - (3*a*d*Hypergeometric2F1[1/2, 2/3, 5/3,
 Cos[e + f*x]^2]*Sin[e + f*x])/(4*f*(d*Sec[e + f*x])^(4/3)*Sqrt[Sin[e + f*x
]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{3b}{f\sqrt[3]{d\sec(e+fx)}}+a\int\frac{1}{\sqrt[3]{d\sec(e+fx)}}\,dx \\
&= -\frac{3b}{f\sqrt[3]{d\sec(e+fx)}}+\left(a\left(\frac{\cos(e+fx)}{d}\right)^{2/3}(d\sec(e+fx))^{2/3}\right)\int\sqrt[3]{\frac{\cos(e+fx)}{d}}\,dx \\
&= -\frac{3b}{f\sqrt[3]{d\sec(e+fx)}} \\
&\quad -\frac{3a\cos^2(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(e+fx)\right)(d\sec(e+fx))^{2/3}\sin(e+fx)}{4df\sqrt{\sin^2(e+fx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.78

$$
\begin{aligned}
&\int\frac{a+b\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx \\
&= -\frac{3\left(b+a\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\sec^2(e+fx)\right)\sqrt{-\tan^2(e+fx)}\right)}{f\sqrt[3]{d\sec(e+fx)}}
\end{aligned}
$$

```
[In] Integrate[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(1/3),x]
[Out] (-3*(b + a*Cot[e + f*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[e + f*x]^2]*S
qrt[-Tan[e + f*x]^2]))/(f*(d*Sec[e + f*x])^(1/3))
```

Maple [F]

$$\int \frac{a+b\tan(fx+e)}{(d\sec(fx+e))^{\frac{1}{3}}}dx$$

```
[In] int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x)
[Out] int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int \frac{a+b\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{b\tan(fx+e)+a}{(d\sec(fx+e))^{\frac{1}{3}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x, algorithm="fricas")
[Out] integral((d*sec(f*x + e))^(2/3)*(b*tan(f*x + e) + a)/(d*sec(f*x + e)), x)
```

Sympy [F]

$$\int \frac{a+b\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{a+b\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))**(1/3),x)
[Out] Integral((a + b*tan(e + f*x))/(d*sec(e + f*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{a+b\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{b\tan(fx+e)+a}{(d\sec(fx+e))^{\frac{1}{3}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(1/3), x)
```

Giac [F]

$$\int \frac{a+b\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx=\int \frac{b\tan\left(fx+e\right)+a}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx=\int \frac{a+b\tan(e+f\,x)}{{\left(\frac{d}{\cos(e+f\,x)}\right)}^{1/3}}\,dx$$

```
[In] int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(1/3),x)
[Out] int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(1/3), x)
```

3.627 $\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/3}} dx$

Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/3}} dx = -\frac{3b}{5f(d\sec(e+fx))^{5/3}} - \frac{3ad\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(e+fx)\right)\sin(e+fx)}{8f(d\sec(e+fx))^{8/3}\sqrt{\sin^2(e+fx)}}$$

```
[Out] -3/5*b/f/(d*sec(f*x+e))^(5/3)-3/8*a*d*hypergeom([1/2, 4/3],[7/3],cos(f*x+e)
^2)*sin(f*x+e)/f/(d*sec(f*x+e))^(8/3)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3567, 3857, 2722}

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/3}} dx = -\frac{3ad\sin(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(e+fx)\right)}{8f\sqrt{\sin^2(e+fx)}(d\sec(e+fx))^{8/3}} - \frac{3b}{5f(d\sec(e+fx))^{5/3}}$$

```
[In] Int[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(5/3),x]
```

```
[Out] (-3*b)/(5*f*(d*Sec[e + f*x])^(5/3)) - (3*a*d*Hypergeometric2F1[1/2, 4/3, 7/
3, Cos[e + f*x]^2]*Sin[e + f*x])/(8*f*(d*Sec[e + f*x])^(8/3)*Sqrt[Sin[e + f
*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{3b}{5f(d\sec(e+fx))^{5/3}} + a\int \frac{1}{(d\sec(e+fx))^{5/3}}\,dx \\
&= -\frac{3b}{5f(d\sec(e+fx))^{5/3}} + \left(a\sqrt[3]{\frac{\cos(e+fx)}{d}}\sqrt[3]{d\sec(e+fx)}\right)\int \left(\frac{\cos(e+fx)}{d}\right)^{5/3}dx \\
&= -\frac{3b}{5f(d\sec(e+fx))^{5/3}} \\
&\quad -\frac{3a\cos^3(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\sin(e+fx)}{8d^2f\sqrt{\sin^2(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.78

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx = -\frac{3\left(b+a\cot(e+fx)\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},\sec^2(e+fx)\right)\sqrt{-\tan^2(e+fx)}\right)}{5f(d\sec(e+fx))^{5/3}}$$

```
[In] Integrate[(a + b*Tan[e + f*x])/(d*Sec[e + f*x])^(5/3),x]
[Out] (-3*(b + a*Cot[e + f*x]*Hypergeometric2F1[-5/6, 1/2, 1/6, Sec[e + f*x]^2]*S
qrt[-Tan[e + f*x]^2]))/(5*f*(d*Sec[e + f*x])^(5/3))
```

Maple [F]

$$\int \frac{a+b\tan(fx+e)}{(d\sec(fx+e))^{\frac{5}{3}}}dx$$

```
[In] int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x)
[Out] int((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x)
```

Fricas [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx = \int \frac{b\tan(fx+e)+a}{(d\sec(fx+e))^{\frac{5}{3}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x, algorithm="fricas")
[Out] integral((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a)/(d^2*sec(f*x + e)^2),
x)
```

Sympy [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx = \int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{\frac{5}{3}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))**(5/3),x)
[Out] Integral((a + b*tan(e + f*x))/(d*sec(e + f*x))**(5/3), x)
```

Maxima [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx = \int \frac{b\tan(fx+e)+a}{(d\sec(fx+e))^{\frac{5}{3}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(5/3), x)
```

Giac [F]

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx = \int \frac{b\tan\left(fx+e\right)+a}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))/(d*sec(f*x+e))^(5/3),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)/(d*sec(f*x + e))^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx = \int \frac{a+b\tan(e+f\,x)}{{\left(\frac{d}{\cos(e+f\,x)}\right)}^{5/3}}\,dx$$

```
[In] int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(5/3),x)
[Out] int((a + b*tan(e + f*x))/(d/cos(e + f*x))^(5/3), x)
```

3.628 $\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 119

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2\,dx = \frac{33ab(d\sec(e+fx))^{5/3}}{40f} + \frac{3(8a^2-3b^2)\,d\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(e+fx)\right)(d\sec(e+fx))^{2/3}\sin(e+fx)}{16f\sqrt{\sin^2(e+fx)}} + \frac{3b(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}{8f}$$

```
[Out] 33/40*a*b*(d*sec(f*x+e))^(5/3)/f+3/16*(8*a^2-3*b^2)*d*hypergeom([-1/3, 1/2]
,[2/3],cos(f*x+e)^2)*(d*sec(f*x+e))^(2/3)*sin(f*x+e)/f/(sin(f*x+e)^2)^(1/2)
+3/8*b*(d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e))/f
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3589, 3567, 3857, 2722}

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2\,dx = \frac{3d(8a^2-3b^2)\sin(e+fx)(d\sec(e+fx))^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(e+fx)\right)}{16f\sqrt{\sin^2(e+fx)}} + \frac{33ab(d\sec(e+fx))^{5/3}}{40f} + \frac{3b(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}{8f}$$

```
[In] Int[(d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (33*a*b*(d*Sec[e + f*x])^(5/3))/(40*f) + (3*(8*a^2 - 3*b^2)*d*Hypergeometri
c2F1[-1/3, 1/2, 2/3, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(2/3)*Sin[e + f*x])/(
16*f*Sqrt[Sin[e + f*x]^2]) + (3*b*(d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x
]))/(8*f)
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3b(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}{8f} \\
&\quad + \frac{3}{8}\int (d\sec(e+fx))^{5/3}\left(\frac{8a^2}{3}-b^2+\frac{11}{3}ab\tan(e+fx)\right)\,dx \\
&= \frac{33ab(d\sec(e+fx))^{5/3}}{40f}+\frac{3b(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}{8f} \\
&\quad + \frac{1}{8}\left(8a^2-3b^2\right)\int (d\sec(e+fx))^{5/3}\,dx
\end{aligned}$$

$$= \frac{33ab(d\sec(e+fx))^{5/3}}{40f} + \frac{3b(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}{8f} + \frac{1}{8}\left((8a^2-3b^2)\left(\frac{\cos(e+fx)}{d}\right)^{2/3}(d\sec(e+fx))^{2/3}\right)\int \frac{1}{\left(\frac{\cos(e+fx)}{d}\right)^{5/3}}\,dx$$

$$= \frac{33ab(d\sec(e+fx))^{5/3}}{40f} + \frac{3(8a^2-3b^2)\,d\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(e+fx)\right)(d\sec(e+fx))^{2/3}\sin(e+fx)}{16f\sqrt{\sin^2(e+fx)}} + \frac{3b(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}{8f}$$

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.92

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2\,dx = \frac{3(d\sec(e+fx))^{5/3}\left(b^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{5}{6},\frac{11}{6},\sec^2(e+fx)\right)\tan(e+fx)+a(\right.}{5f\sqrt{-\tan}}$$

```
[In] Integrate[(d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (3*(d*Sec[e + f*x])^(5/3)*(b^2*Hypergeometric2F1[-1/2, 5/6, 11/6, Sec[e + f
*x]^2]*Tan[e + f*x] + a*(-(a*Hypergeometric2F1[1/2, 5/6, 11/6, Sec[e + f*x]
^2]*Tan[e + f*x]) + 2*b*Sqrt[-Tan[e + f*x]^2])))/(5*f*Sqrt[-Tan[e + f*x]^2]
)
```

Maple [F]

$$\int (d\sec(fx+e))^{\frac{5}{3}}(a+b\tan(fx+e))^2\,dx$$

```
[In] int((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e))^2,x)
```

```
[Out] int((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e))^2,x)
```

Fricas [F]

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2\,dx = \int (d\sec(fx+e))^{\frac{5}{3}}(b\tan(fx+e)+a)^2\,dx$$

```
[In] integrate((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
[Out] integral((b^2*d*sec(f*x + e)*tan(f*x + e)^2 + 2*a*b*d*sec(f*x + e)*tan(f*x
+ e) + a^2*d*sec(f*x + e))*(d*sec(f*x + e))^(2/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))**(5/3)*(a+b*tan(f*x+e))**2,x)
[Out] Timed out
```

Maxima [F]

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2\,dx = \int (d\sec(fx+e))^{\frac{5}{3}}(b\tan(fx+e)+a)^2\,dx$$

```
[In] integrate((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^(5/3)*(b*tan(f*x + e) + a)^2, x)
```

Giac [F]

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2\,dx = \int (d\sec(fx+e))^{\frac{5}{3}}(b\tan(fx+e)+a)^2\,dx$$

```
[In] integrate((d*sec(f*x+e))^(5/3)*(a+b*tan(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(5/3)*(b*tan(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2\,dx=\int \left(\frac{d}{\cos{(e+f\,x)}}\right)^{5/3}(a+b\tan(e+f\,x))^2\,dx$$

```
[In] int((d/cos(e + f*x))^(5/3)*(a + b*tan(e + f*x))^2,x)
[Out] int((d/cos(e + f*x))^(5/3)*(a + b*tan(e + f*x))^2, x)
```

3.629 $\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 119

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx = \frac{21ab\sqrt[3]{d\sec(e+fx)}}{4f} - \frac{3(4a^2-3b^2)\,d\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(e+fx)\right)\sin(e+fx)}{8f(d\sec(e+fx))^{2/3}\sqrt{\sin^2(e+fx)}} + \frac{3b\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}{4f}$$

```
[Out] 21/4*a*b*(d*sec(f*x+e))^(1/3)/f-3/8*(4*a^2-3*b^2)*d*hypergeom([1/3, 1/2],[4
/3],cos(f*x+e)^2)*sin(f*x+e)/f/(d*sec(f*x+e))^(2/3)/(sin(f*x+e)^2)^(1/2)+3/
4*b*(d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e))/f
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3589, 3567, 3857, 2722}

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx = -\frac{3d(4a^2-3b^2)\sin(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(e+fx)\right)}{8f\sqrt{\sin^2(e+fx)}(d\sec(e+fx))^{2/3}} + \frac{21ab\sqrt[3]{d\sec(e+fx)}}{4f} + \frac{3b\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}{4f}$$

```
[In] Int[(d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (21*a*b*(d*Sec[e + f*x])^(1/3))/(4*f) - (3*(4*a^2 - 3*b^2)*d*Hypergeometric
2F1[1/3, 1/2, 4/3, Cos[e + f*x]^2]*Sin[e + f*x])/(8*f*(d*Sec[e + f*x])^(2/3
)*Sqrt[Sin[e + f*x]^2]) + (3*b*(d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x]))
/(4*f)
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{3b\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}{4f} \\
&\quad + \frac{3}{4}\int \sqrt[3]{d\sec(e+fx)}\left(\frac{4a^2}{3}-b^2+\frac{7}{3}ab\tan(e+fx)\right)\,dx \\
&= \frac{21ab\sqrt[3]{d\sec(e+fx)}}{4f}+\frac{3b\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}{4f} \\
&\quad + \frac{1}{4}\left(4a^2-3b^2\right)\int \sqrt[3]{d\sec(e+fx)}\,dx
\end{aligned}
$$

$$= \frac{21ab\sqrt[3]{d\sec(e+fx)}}{4f} + \frac{3b\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}{4f} + \frac{1}{4}\left((4a^2-3b^2)\sqrt[3]{\frac{\cos(e+fx)}{d}}\sqrt[3]{d\sec(e+fx)}\right)\int \frac{1}{\sqrt[3]{\frac{\cos(e+fx)}{d}}}dx$$

$$= \frac{21ab\sqrt[3]{d\sec(e+fx)}}{4f} - \frac{3(4a^2-3b^2)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\sin(e+fx)}{8f\sqrt{\sin^2(e+fx)}} + \frac{3b\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}{4f}$$

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.89

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx$$
$$= \frac{3\sqrt[3]{d\sec(e+fx)}\left(\frac{b^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{6},\frac{7}{6},\sec^2(e+fx)\right)\tan(e+fx)}{\sqrt{-\tan^2(e+fx)}} + a\left(2b + a\cot(e+fx)\,\text{Hypergeometric2F1}\,(\right.\right.}{f}$$

```
[In] Integrate[(d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (3*(d*Sec[e + f*x])^(1/3)*((b^2*Hypergeometric2F1[-1/2, 1/6, 7/6, Sec[e + f
*x]^2]*Tan[e + f*x])/Sqrt[-Tan[e + f*x]^2] + a*(2*b + a*Cot[e + f*x]*Hyperg
eometric2F1[1/6, 1/2, 7/6, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2])))/f
```

Maple [F]

$$\int (d\sec(fx+e))^{\frac{1}{3}}(a+b\tan(fx+e))^2\,dx$$

```
[In] int((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e))^2,x)
```

```
[Out] int((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e))^2,x)
```

Fricas [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx = \int (d\sec(fx+e))^{\frac{1}{3}}(b\tan(fx+e)+a)^2\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
[Out] integral((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)*(d*sec(f*x + e))^(
1/3), x)
```

Sympy [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx = \int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx$$

```
[In] integrate((d*sec(f*x+e))**(1/3)*(a+b*tan(f*x+e))**2,x)
[Out] Integral((d*sec(e + f*x))**(1/3)*(a + b*tan(e + f*x))**2, x)
```

Maxima [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx = \int (d\sec(fx+e))^{\frac{1}{3}}(b\tan(fx+e)+a)^2\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a)^2, x)
```

Giac [F]

$$\int \sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2\,dx = \int (d\sec(fx+e))^{\frac{1}{3}}(b\tan(fx+e)+a)^2\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/3)*(a+b*tan(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{d \sec(e+fx)}(a+b\tan(e+fx))^2\,dx = \int \left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}(a+b\tan(e+f\,x))^2\,dx$$

```
[In] int((d/cos(e + f*x))^(1/3)*(a + b*tan(e + f*x))^2,x)
[Out] int((d/cos(e + f*x))^(1/3)*(a + b*tan(e + f*x))^2, x)
```

3.630 $\int \frac{(a+b\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 119

$$\begin{aligned}\int \frac{(a+b\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx &= -\frac{15ab}{2f\sqrt[3]{d\sec(e+fx)}} \\ &\quad - \frac{3(2a^2-3b^2)\,d\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(e+fx)\right)\sin(e+fx)}{8f(d\sec(e+fx))^{4/3}\sqrt{\sin^2(e+fx)}} \\ &\quad + \frac{3b(a+b\tan(e+fx))}{2f\sqrt[3]{d\sec(e+fx)}}\end{aligned}$$

```
[Out] -15/2*a*b/f/(d*sec(f*x+e))^(1/3)-3/8*(2*a^2-3*b^2)*d*hypergeom([1/2, 2/3],[
5/3],cos(f*x+e)^2)*sin(f*x+e)/f/(d*sec(f*x+e))^(4/3)/(sin(f*x+e)^2)^(1/2)+3
/2*b*(a+b*tan(f*x+e))/f/(d*sec(f*x+e))^(1/3)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3589, 3567, 3857, 2722}

$$\begin{aligned}\int \frac{(a+b\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx &= -\frac{3d(2a^2-3b^2)\sin(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(e+fx)\right)}{8f\sqrt{\sin^2(e+fx)}(d\sec(e+fx))^{4/3}} \\ &\quad - \frac{15ab}{2f\sqrt[3]{d\sec(e+fx)}} + \frac{3b(a+b\tan(e+fx))}{2f\sqrt[3]{d\sec(e+fx)}}\end{aligned}$$

```
[In] Int[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(1/3),x]
```

```
[Out] (-15*a*b)/(2*f*(d*Sec[e + f*x])^(1/3)) - (3*(2*a^2 - 3*b^2)*d*Hypergeometri
c2F1[1/2, 2/3, 5/3, Cos[e + f*x]^2]*Sin[e + f*x])/(8*f*(d*Sec[e + f*x])^(4/
3)*Sqrt[Sin[e + f*x]^2]) + (3*b*(a + b*Tan[e + f*x]))/(2*f*(d*Sec[e + f*x])
^(1/3))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{3b(a+b\tan(e+fx))}{2f\sqrt[3]{d\sec(e+fx)}}+\frac{3}{2}\int \frac{\frac{2a^2}{3}-b^2+\frac{5}{3}ab\tan(e+fx)}{\sqrt[3]{d\sec(e+fx)}}\,dx\\
&= -\frac{15ab}{2f\sqrt[3]{d\sec(e+fx)}}+\frac{3b(a+b\tan(e+fx))}{2f\sqrt[3]{d\sec(e+fx)}}+\frac{1}{2}\left(2a^2-3b^2\right)\int \frac{1}{\sqrt[3]{d\sec(e+fx)}}\,dx\\
&= -\frac{15ab}{2f\sqrt[3]{d\sec(e+fx)}}+\frac{3b(a+b\tan(e+fx))}{2f\sqrt[3]{d\sec(e+fx)}}\\
&\quad+\frac{1}{2}\left(\left(2a^2-3b^2\right)\left(\frac{\cos(e+fx)}{d}\right)^{2/3}(d\sec(e+fx))^{2/3}\right)\int \sqrt[3]{\frac{\cos(e+fx)}{d}}\,dx
\end{aligned}
$$

$$= -\frac{15ab}{2f\sqrt[3]{d\sec(e+fx)}} - \frac{3(2a^2-3b^2)\cos^2(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(e+fx)\right)(d\sec(e+fx))^{2/3}\sin(e+}{8df\sqrt{\sin^2(e+fx)}} + \frac{3b(a+b\tan(e+fx))}{2f\sqrt[3]{d\sec(e+fx)}}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.90

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = -\frac{3\Big(b^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{6},\frac{5}{6},\sec^2(e+fx)\right)\tan(e+fx)+a\Big(-a\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6}\right.}{f\sqrt[3]{d\sec(e+fx)}\sqrt{-\tan^2(e+fx)}}$$

```
[In] Integrate[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(1/3),x]
[Out] (-3*(b^2*Hypergeometric2F1[-1/2, -1/6, 5/6, Sec[e + f*x]^2]*Tan[e + f*x] +
a*(-(a*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[e + f*x]^2]*Tan[e + f*x]) + 2*
b*Sqrt[-Tan[e + f*x]^2])))/(f*(d*Sec[e + f*x])^(1/3)*Sqrt[-Tan[e + f*x]^2])
```

Maple [F]

$$\int \frac{(a+b\tan(fx+e))^2}{(d\sec(fx+e))^{\frac{1}{3}}}dx$$

```
[In] int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x)
[Out] int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x)
```

Fricas [F]

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{(b\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{1}{3}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x, algorithm="fricas")
[Out] integral((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)*(d*sec(f*x + e))^(
2/3)/(d*sec(f*x + e)), x)
```

Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{(a+b\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))**2/(d*sec(f*x+e))**(1/3),x)
[Out] Integral((a + b*tan(e + f*x))**2/(d*sec(e + f*x))**(1/3), x)
```

Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{(b\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{1}{3}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(1/3), x)
```

Giac [F]

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{(b\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{1}{3}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(1/3),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^2}{\sqrt[3]{d\sec(e+fx)}}\,dx = \int \frac{(a+b\tan(e+f\,x))^2}{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}}\,dx$$

```
[In] int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(1/3),x)
[Out] int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(1/3), x)
```

3.631 $\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 119

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx = \frac{3ab}{10f(d\sec(e+fx))^{5/3}} - \frac{3(2a^2+3b^2)\,d\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(e+fx)\right)\sin(e+fx)}{16f(d\sec(e+fx))^{8/3}\sqrt{\sin^2(e+fx)}} - \frac{3b(a+b\tan(e+fx))}{2f(d\sec(e+fx))^{5/3}}$$

```
[Out] 3/10*a*b/f/(d*sec(f*x+e))^(5/3)-3/16*(2*a^2+3*b^2)*d*hypergeom([1/2, 4/3],[
7/3],cos(f*x+e)^2)*sin(f*x+e)/f/(d*sec(f*x+e))^(8/3)/(sin(f*x+e)^2)^(1/2)-3
/2*b*(a+b*tan(f*x+e))/f/(d*sec(f*x+e))^(5/3)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3589, 3567, 3857, 2722}

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx = -\frac{3d(2a^2+3b^2)\sin(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(e+fx)\right)}{16f\sqrt{\sin^2(e+fx)}(d\sec(e+fx))^{8/3}} + \frac{3ab}{10f(d\sec(e+fx))^{5/3}} - \frac{3b(a+b\tan(e+fx))}{2f(d\sec(e+fx))^{5/3}}$$

```
[In] Int[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(5/3),x]
```

```
[Out] (3*a*b)/(10*f*(d*Sec[e + f*x])^(5/3)) - (3*(2*a^2 + 3*b^2)*d*Hypergeometric
2F1[1/2, 4/3, 7/3, Cos[e + f*x]^2]*Sin[e + f*x])/(16*f*(d*Sec[e + f*x])^(8/
3)*Sqrt[Sin[e + f*x]^2]) - (3*b*(a + b*Tan[e + f*x]))/(2*f*(d*Sec[e + f*x])
^(5/3))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{3b(a+b\tan(e+fx))}{2f(d\sec(e+fx))^{5/3}} - \frac{3}{2}\int \frac{-\frac{2a^2}{3} - b^2 + \frac{1}{3}ab\tan(e+fx)}{(d\sec(e+fx))^{5/3}}\,dx \\
&= \frac{3ab}{10f(d\sec(e+fx))^{5/3}} - \frac{3b(a+b\tan(e+fx))}{2f(d\sec(e+fx))^{5/3}} - \frac{1}{2}\left(-2a^2-3b^2\right)\int \frac{1}{(d\sec(e+fx))^{5/3}}\,dx \\
&= \frac{3ab}{10f(d\sec(e+fx))^{5/3}} - \frac{3b(a+b\tan(e+fx))}{2f(d\sec(e+fx))^{5/3}} \\
&\quad - \frac{1}{2}\left(\left(-2a^2-3b^2\right)\sqrt[3]{\frac{\cos(e+fx)}{d}}\sqrt[3]{d\sec(e+fx)}\right)\int \left(\frac{\cos(e+fx)}{d}\right)^{5/3}\,dx
\end{aligned}$$

$$= \frac{3ab}{10f(d\sec(e+fx))^{5/3}} - \frac{3(2a^2+3b^2)\cos^3(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\sin(e+f}{16d^2f\sqrt{\sin^2(e+fx)}} - \frac{3b(a+b\tan(e+fx))}{2f(d\sec(e+fx))^{5/3}}$$

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.99

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx = -\frac{3\left(b^2\,\text{Hypergeometric2F1}\left(-\frac{5}{6},-\frac{1}{2},\frac{1}{6},\sec^2(e+fx)\right)\sin(e+fx)+a\left(-a\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},\sec\right.\right.\right.}{5df(d\sec(e+fx))^{2/3}\sqrt{-\tan^2(e+fx)}}$$

```
[In] Integrate[(a + b*Tan[e + f*x])^2/(d*Sec[e + f*x])^(5/3),x]
[Out] (-3*(b^2*Hypergeometric2F1[-5/6, -1/2, 1/6, Sec[e + f*x]^2]*Sin[e + f*x] +
a*(-(a*Hypergeometric2F1[-5/6, 1/2, 1/6, Sec[e + f*x]^2]*Sin[e + f*x]) + 2*
b*Cos[e + f*x]*Sqrt[-Tan[e + f*x]^2])))/(5*d*f*(d*Sec[e + f*x])^(2/3)*Sqrt[
-Tan[e + f*x]^2])
```

Maple [F]

$$\int \frac{(a+b\tan(fx+e))^2}{(d\sec(fx+e))^{\frac{5}{3}}}dx$$

```
[In] int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x)
[Out] int((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x)
```

Fricas [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx = \int \frac{(b\tan(fx+e)+a)^2}{(d\sec(fx+e))^{\frac{5}{3}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x, algorithm="fricas")
[Out] integral((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)*(d*sec(f*x + e))^(
1/3)/(d^2*sec(f*x + e)^2), x)
```

Sympy [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx = \int \frac{(a+b\tan{(e+fx)})^2}{(d\sec{(e+fx)})^{\frac{5}{3}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))**2/(d*sec(f*x+e))**(5/3),x)
[Out] Integral((a + b*tan(e + f*x))**2/(d*sec(e + f*x))**(5/3), x)
```

Maxima [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx = \int \frac{(b\tan{(fx+e)}+a)^2}{(d\sec{(fx+e)})^{\frac{5}{3}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(5/3), x)
```

Giac [F]

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx = \int \frac{(b\tan{(fx+e)}+a)^2}{(d\sec{(fx+e)})^{\frac{5}{3}}}\,dx$$

```
[In] integrate((a+b*tan(f*x+e))^2/(d*sec(f*x+e))^(5/3),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)^2/(d*sec(f*x + e))^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\tan(e+fx))^2}{(d\sec(e+fx))^{5/3}}\,dx = \int \frac{(a+b\tan(e+f\,x))^2}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}}\,dx$$

```
[In] int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(5/3),x)
[Out] int((a + b*tan(e + f*x))^2/(d/cos(e + f*x))^(5/3), x)
```

3.632 $\int \frac{(d\sec(e+fx))^{5/3}}{a+b\tan(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 552

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+b\tan(e+fx)}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)(d\sec(e+fx))^{5/3}}{2b^{2/3}\sqrt[6]{a^2+b^2}f\sec^2(e+fx)^{5/6}}$$
$$+\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)(d\sec(e+fx))^{5/3}}{2b^{2/3}\sqrt[6]{a^2+b^2}f\sec^2(e+fx)^{5/6}}$$
$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)(d\sec(e+fx))^{5/3}}{b^{2/3}\sqrt[6]{a^2+b^2}f\sec^2(e+fx)^{5/6}}$$
$$+\frac{\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)(d\sec(e+fx))^{5/3}}{4b^{2/3}\sqrt[6]{a^2+b^2}f\sec^2(e+fx)^{5/6}}$$
$$-\frac{\log\left(\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)(d\sec(e+fx))^{5/3}}{4b^{2/3}\sqrt[6]{a^2+b^2}f\sec^2(e+fx)^{5/6}}$$
$$+\frac{\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan(e+fx)}{af\sec^2(e+fx)^{5/6}}$$

```
[Out] -arctanh(b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6))*(d*sec(f*x+e))^(5/3)
/b^(2/3)/(a^2+b^2)^(1/6)/f/(sec(f*x+e)^2)^(5/6)+1/4*ln((a^2+b^2)^(1/3)-b^(1
/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)^(1/3))*(d*s
ec(f*x+e))^(5/3)/b^(2/3)/(a^2+b^2)^(1/6)/f/(sec(f*x+e)^2)^(5/6)-1/4*ln((a^2
+b^2)^(1/3)+b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e
)^2)^(1/3))*(d*sec(f*x+e))^(5/3)/b^(2/3)/(a^2+b^2)^(1/6)/f/(sec(f*x+e)^2)^(
5/6)+1/2*arctan(-1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/
```

```
6)*3^(1/2))*(d*sec(f*x+e))^(5/3)*3^(1/2)/b^(2/3)/(a^2+b^2)^(1/6)/f/(sec(f*x
+e)^2)^(5/6)+1/2*arctan(1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b
^2)^(1/6)*3^(1/2))*(d*sec(f*x+e))^(5/3)*3^(1/2)/b^(2/3)/(a^2+b^2)^(1/6)/f/(
sec(f*x+e)^2)^(5/6)+AppellF1(1/2,1,1/6,3/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+e)
^2)*(d*sec(f*x+e))^(5/3)*tan(f*x+e)/a/f/(sec(f*x+e)^2)^(5/6)
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3593, 771, 440, 455, 65, 302, 648, 632, 210, 642, 214}

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+b\tan(e+fx)}\,dx = \frac{\tan(e+fx)(d\sec(e+fx))^{5/3}\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{af\sec^2(e+fx)^{5/6}}$$
$$-\frac{\sqrt{3}(d\sec(e+fx))^{5/3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)}{2b^{2/3}f\sqrt[6]{a^2+b^2}\sec^2(e+fx)^{5/6}}$$
$$+\frac{\sqrt{3}(d\sec(e+fx))^{5/3}\arctan\left(\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}+\frac{1}{\sqrt{3}}\right)}{2b^{2/3}f\sqrt[6]{a^2+b^2}\sec^2(e+fx)^{5/6}}$$
$$-\frac{(d\sec(e+fx))^{5/3}\text{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)}{b^{2/3}f\sqrt[6]{a^2+b^2}\sec^2(e+fx)^{5/6}}$$
$$+\frac{(d\sec(e+fx))^{5/3}\log\left(-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+\sqrt[3]{a^2+b^2}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)}{4b^{2/3}f\sqrt[6]{a^2+b^2}\sec^2(e+fx)^{5/6}}$$
$$-\frac{(d\sec(e+fx))^{5/3}\log\left(\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+\sqrt[3]{a^2+b^2}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)}{4b^{2/3}f\sqrt[6]{a^2+b^2}\sec^2(e+fx)^{5/6}}$$

```
[In] Int[(d*Sec[e + f*x])^(5/3)/(a + b*Tan[e + f*x]),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[1/Sqrt[3] - (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqrt[3
]*(a^2 + b^2)^(1/6))]*(d*Sec[e + f*x])^(5/3))/(b^(2/3)*(a^2 + b^2)^(1/6)*f*
(Sec[e + f*x]^2)^(5/6)) + (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(Sec[e + f
*x]^2)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))]*(d*Sec[e + f*x])^(5/3))/(2*b^(2/
3)*(a^2 + b^2)^(1/6)*f*(Sec[e + f*x]^2)^(5/6)) - (ArcTanh[(b^(1/3)*(Sec[e +
 f*x]^2)^(1/6))/(a^2 + b^2)^(1/6)]*(d*Sec[e + f*x])^(5/3))/(b^(2/3)*(a^2 +
b^2)^(1/6)*f*(Sec[e + f*x]^2)^(5/6)) + (Log[(a^2 + b^2)^(1/3) - b^(1/3)*(a^
2 + b^2)^(1/6)*(Sec[e + f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)]*(d*
Sec[e + f*x])^(5/3))/(4*b^(2/3)*(a^2 + b^2)^(1/6)*f*(Sec[e + f*x]^2)^(5/6))
 - (Log[(a^2 + b^2)^(1/3) + b^(1/3)*(a^2 + b^2)^(1/6)*(Sec[e + f*x]^2)^(1/6
) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)]*(d*Sec[e + f*x])^(5/3))/(4*b^(2/3)*(a^2
```

```
 + b^2)^(1/6)*f*(Sec[e + f*x]^2)^(5/6)) + (AppellF1[1/2, 1, 1/6, 3/2, (b^2*
Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Sec[e + f*x])^(5/3)*Tan[e + f*x])/
(a*f*(Sec[e + f*x]^2)^(5/6))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{(d\sec(e+fx))^{5/3}\text{Subst}\left(\int \frac{1}{(a+x)\sqrt[6]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{5/6}}$$

$$= \frac{(d\sec(e+fx))^{5/3}\text{Subst}\left(\int \left(\frac{a}{(a^2-x^2)\sqrt[6]{1+\frac{x^2}{b^2}}} + \frac{x}{(-a^2+x^2)\sqrt[6]{1+\frac{x^2}{b^2}}}\right)dx, x, b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{5/6}}$$

$$
\begin{aligned}
&= \frac{(d\sec(e+fx))^{5/3}\text{Subst}\left(\int \frac{x}{(-a^2+x^2)\sqrt[6]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{5/6}} \\
&\quad + \frac{\left(a(d\sec(e+fx))^{5/3}\right)\text{Subst}\left(\int \frac{1}{(a^2-x^2)\sqrt[6]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{5/6}} \\
&= \frac{\text{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan(e+fx)}{af\sec^2(e+fx)^{5/6}} \\
&\quad + \frac{(d\sec(e+fx))^{5/3}\text{Subst}\left(\int \frac{1}{(-a^2+x)\sqrt[6]{1+\frac{x}{b^2}}}\,dx, x, b^2\tan^2(e+fx)\right)}{2bf\sec^2(e+fx)^{5/6}} \\
&= \frac{\text{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan(e+fx)}{af\sec^2(e+fx)^{5/6}} \\
&\quad + \frac{\left(3b(d\sec(e+fx))^{5/3}\right)\text{Subst}\left(\int \frac{x^4}{-a^2-b^2+b^2x^6}\,dx, x, \sqrt[6]{\sec^2(e+fx)}\right)}{f\sec^2(e+fx)^{5/6}} \\
&= \frac{\text{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan(e+fx)}{af\sec^2(e+fx)^{5/6}} \\
&\quad - \frac{(d\sec(e+fx))^{5/3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{a^2+b^2}-b^{2/3}x^2}\,dx, x, \sqrt[6]{\sec^2(e+fx)}\right)}{\sqrt[3]{b}f\sec^2(e+fx)^{5/6}} \\
&\quad - \frac{(d\sec(e+fx))^{5/3}\text{Subst}\left(\int \frac{-\frac{1}{2}\sqrt[6]{a^2+b^2}-\frac{\sqrt[3]{b}x}{2}}{\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}\,dx, x, \sqrt[6]{\sec^2(e+fx)}\right)}{\sqrt[3]{b}\sqrt[6]{a^2+b^2}f\sec^2(e+fx)^{5/6}} \\
&\quad - \frac{(d\sec(e+fx))^{5/3}\text{Subst}\left(\int \frac{-\frac{1}{2}\sqrt[6]{a^2+b^2}+\frac{\sqrt[3]{b}x}{2}}{\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}\,dx, x, \sqrt[6]{\sec^2(e+fx)}\right)}{\sqrt[3]{b}\sqrt[6]{a^2+b^2}f\sec^2(e+fx)^{5/6}}
\end{aligned}
$$

$$=-\frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)(d\sec(e+fx))^{5/3}}{b^{2/3}\sqrt[6]{a^2+b^2}f\sec^2(e+fx)^{5/6}}+\frac{\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan(e+fx)}{af\sec^2(e+fx)^{5/6}}+\frac{\left(3(d\sec(e+fx))^{5/3}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{4\sqrt[3]{b}f\sec^2(e+fx)^{5/6}}+\frac{\left(3(d\sec(e+fx))^{5/3}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{4\sqrt[3]{b}f\sec^2(e+fx)^{5/6}}+\frac{(d\sec(e+fx))^{5/3}\operatorname{Subst}\left(\int\frac{-\sqrt[3]{b}\sqrt[6]{a^2+b^2}+2b^{2/3}x}{\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{4b^{2/3}\sqrt[6]{a^2+b^2}f\sec^2(e+fx)^{5/6}}-\frac{(d\sec(e+fx))^{5/3}\operatorname{Subst}\left(\int\frac{\sqrt[3]{b}\sqrt[6]{a^2+b^2}+2b^{2/3}x}{\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{4b^{2/3}\sqrt[6]{a^2+b^2}f\sec^2(e+fx)^{5/6}}$$

$$=-\frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)(d\sec(e+fx))^{5/3}}{b^{2/3}\sqrt[6]{a^2+b^2}f\sec^2(e+fx)^{5/6}}+\frac{\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)(d\sec(e+fx))^{5/3}}{4b^{2/3}\sqrt[6]{a^2+b^2}f\sec^2(e+fx)^{5/6}}-\frac{\log\left(\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)(d\sec(e+fx))^{5/3}}{4b^{2/3}\sqrt[6]{a^2+b^2}f\sec^2(e+fx)^{5/6}}+\frac{\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan(e+fx)}{af\sec^2(e+fx)^{5/6}}+\frac{\left(3(d\sec(e+fx))^{5/3}\right)\operatorname{Subst}\left(\int\frac{1}{-3-x^2}dx,x,1-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)}{2b^{2/3}\sqrt[6]{a^2+b^2}f\sec^2(e+fx)^{5/6}}-\frac{\left(3(d\sec(e+fx))^{5/3}\right)\operatorname{Subst}\left(\int\frac{1}{-3-x^2}dx,x,1+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)}{2b^{2/3}\sqrt[6]{a^2+b^2}f\sec^2(e+fx)^{5/6}}$$

$$= -\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}}{\sqrt{3}}\right)(d\sec(e+fx))^{5/3}}{2b^{2/3}\sqrt[6]{a^2+b^2}f\sec^2(e+fx)^{5/6}}$$

$$+\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}}{\sqrt{3}}\right)(d\sec(e+fx))^{5/3}}{2b^{2/3}\sqrt[6]{a^2+b^2}f\sec^2(e+fx)^{5/6}}$$

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)(d\sec(e+fx))^{5/3}}{b^{2/3}\sqrt[6]{a^2+b^2}f\sec^2(e+fx)^{5/6}}$$

$$+\frac{\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)(d\sec(e+fx))^{5/3}}{4b^{2/3}\sqrt[6]{a^2+b^2}f\sec^2(e+fx)^{5/6}}$$

$$-\frac{\log\left(\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)(d\sec(e+fx))^{5/3}}{4b^{2/3}\sqrt[6]{a^2+b^2}f\sec^2(e+fx)^{5/6}}$$

$$+\frac{\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan(e+fx)}{af\sec^2(e+fx)^{5/6}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 17.91 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.50

$$\int\frac{(d\sec(e+fx))^{5/3}}{a+b\tan(e+fx)}\,dx =$$

$$-\frac{24d^2\operatorname{AppellF1}\left(\frac{1}{3},\frac{1}{6},\frac{1}{6},\frac{4}{3},\frac{a-ib}{a+b\tan(e+fx)},\right.}{bf\sqrt[3]{d\sec(e+fx)}\left((a+ib)\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{6},\frac{7}{6},\frac{7}{3},\frac{a-ib}{a+b\tan(e+fx)},\frac{a+ib}{a+b\tan(e+fx)}\right)+(a-ib)\operatorname{AppellF1}\left(\frac{4}{3},\frac{7}{6},\frac{1}{6},\frac{7}{3}\right.\right.}$$

```
[In] Integrate[(d*Sec[e + f*x])^(5/3)/(a + b*Tan[e + f*x]),x]
```

```
[Out] (-24*d^2*AppellF1[1/3, 1/6, 1/6, 4/3, (a - I*b)/(a + b*Tan[e + f*x]), (a +
I*b)/(a + b*Tan[e + f*x])]*(a + b*Tan[e + f*x]))/(b*f*(d*Sec[e + f*x])^(1/3
)*((a + I*b)*AppellF1[4/3, 1/6, 7/6, 7/3, (a - I*b)/(a + b*Tan[e + f*x]), (
a + I*b)/(a + b*Tan[e + f*x])] + (a - I*b)*AppellF1[4/3, 7/6, 1/6, 7/3, (a
- I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])] + 8*AppellF1[1
/3, 1/6, 1/6, 4/3, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e +
 f*x])]*(a + b*Tan[e + f*x])))
```

Maple [F]

$$\int \frac{(d\sec(fx+e))^{\frac{5}{3}}}{a+b\tan(fx+e)}dx$$

```
[In] int((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x)
[Out] int((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+b\tan(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+b\tan(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))**(5/3)/(a+b*tan(f*x+e)),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+b\tan(e+fx)}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{3}}}{b\tan(fx+e)+a}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^(5/3)/(b*tan(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+b\tan(e+fx)}\,dx = \int \frac{(d\sec{(fx+e)})^{\frac{5}{3}}}{b\tan{(fx+e)}+a}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(5/3)/(b*tan(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/3}}{a+b\tan(e+fx)}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}}{a+b\tan{(e+f\,x)}}\,dx$$

```
[In] int((d/cos(e + f*x))^(5/3)/(a + b*tan(e + f*x)),x)
[Out] int((d/cos(e + f*x))^(5/3)/(a + b*tan(e + f*x)), x)
```

3.633 $\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 552

$$
\begin{aligned}
\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx &= \frac{\sqrt{3}b^{2/3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)\sqrt[3]{d\sec(e+fx)}}{2(a^2+b^2)^{5/6}f\sqrt[6]{\sec^2(e+fx)}} \\
&- \frac{\sqrt{3}b^{2/3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)\sqrt[3]{d\sec(e+fx)}}{2(a^2+b^2)^{5/6}f\sqrt[6]{\sec^2(e+fx)}} \\
&- \frac{b^{2/3}\operatorname{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sqrt[3]{d\sec(e+fx)}}{(a^2+b^2)^{5/6}f\sqrt[6]{\sec^2(e+fx)}} \\
&+ \frac{b^{2/3}\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sqrt[3]{d\sec(e+fx)}}{4(a^2+b^2)^{5/6}f\sqrt[6]{\sec^2(e+fx)}} \\
&- \frac{b^{2/3}\log\left(\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sqrt[3]{d\sec(e+fx)}}{4(a^2+b^2)^{5/6}f\sqrt[6]{\sec^2(e+fx)}} \\
&+ \frac{\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan(e+fx)}{af\sqrt[6]{\sec^2(e+fx)}}
\end{aligned}
$$

```
[Out] -b^(2/3)*arctanh(b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6))*(d*sec(f*x+e
))^(1/3)/(a^2+b^2)^(5/6)/f/(sec(f*x+e)^2)^(1/6)+1/4*b^(2/3)*ln((a^2+b^2)^(1
/3)-b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)^(1/
3))*(d*sec(f*x+e))^(1/3)/(a^2+b^2)^(5/6)/f/(sec(f*x+e)^2)^(1/6)-1/4*b^(2/3)
*ln((a^2+b^2)^(1/3)+b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(s
ec(f*x+e)^2)^(1/3))*(d*sec(f*x+e))^(1/3)/(a^2+b^2)^(5/6)/f/(sec(f*x+e)^2)^(
```

```
1/6)-1/2*b^(2/3)*arctan(-1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+
b^2)^(1/6)*3^(1/2))*(d*sec(f*x+e))^(1/3)*3^(1/2)/(a^2+b^2)^(5/6)/f/(sec(f*x
+e)^2)^(1/6)-1/2*b^(2/3)*arctan(1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6
)/(a^2+b^2)^(1/6)*3^(1/2))*(d*sec(f*x+e))^(1/3)*3^(1/2)/(a^2+b^2)^(5/6)/f/(
sec(f*x+e)^2)^(1/6)+AppellF1(1/2,1,5/6,3/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+e)
^2)*(d*sec(f*x+e))^(1/3)*tan(f*x+e)/a/f/(sec(f*x+e)^2)^(1/6)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3593, 771, 440, 455, 65, 216, 648, 632, 210, 642, 214}

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx = \frac{\tan(e+fx)\sqrt[3]{d\sec(e+fx)}\,\text{AppellF1}\left(\frac{1}{2},1,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{af\sqrt[6]{\sec^2(e+fx)}} + \frac{\sqrt{3}b^{2/3}\sqrt[3]{d\sec(e+fx)}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)}{2f\left(a^2+b^2\right)^{5/6}\sqrt[6]{\sec^2(e+fx)}} - \frac{\sqrt{3}b^{2/3}\sqrt[3]{d\sec(e+fx)}\arctan\left(\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}+\frac{1}{\sqrt{3}}\right)}{2f\left(a^2+b^2\right)^{5/6}\sqrt[6]{\sec^2(e+fx)}} - \frac{b^{2/3}\sqrt[3]{d\sec(e+fx)}\text{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)}{f\left(a^2+b^2\right)^{5/6}\sqrt[6]{\sec^2(e+fx)}} + \frac{b^{2/3}\sqrt[3]{d\sec(e+fx)}\log\left(-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+\sqrt[3]{a^2+b^2}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)}{4f\left(a^2+b^2\right)^{5/6}\sqrt[6]{\sec^2(e+fx)}} - \frac{b^{2/3}\sqrt[3]{d\sec(e+fx)}\log\left(\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+\sqrt[3]{a^2+b^2}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)}{4f\left(a^2+b^2\right)^{5/6}\sqrt[6]{\sec^2(e+fx)}}$$

```
[In] Int[(d*Sec[e + f*x])^(1/3)/(a + b*Tan[e + f*x]),x]
```

```
[Out] (Sqrt[3]*b^(2/3)*ArcTan[1/Sqrt[3] - (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqr
t[3]*(a^2 + b^2)^(1/6))]*(d*Sec[e + f*x])^(1/3))/(2*(a^2 + b^2)^(5/6)*f*(Se
c[e + f*x]^2)^(1/6)) - (Sqrt[3]*b^(2/3)*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(Sec[
e + f*x]^2)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))]*(d*Sec[e + f*x])^(1/3))/(2*
(a^2 + b^2)^(5/6)*f*(Sec[e + f*x]^2)^(1/6)) - (b^(2/3)*ArcTanh[(b^(1/3)*(Se
c[e + f*x]^2)^(1/6))/(a^2 + b^2)^(1/6)]*(d*Sec[e + f*x])^(1/3))/((a^2 + b^2
)^(5/6)*f*(Sec[e + f*x]^2)^(1/6)) + (b^(2/3)*Log[(a^2 + b^2)^(1/3) - b^(1/3
```

```
)*(a^2 + b^2)^(1/6)*(Sec[e + f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)
]*(d*Sec[e + f*x])^(1/3))/(4*(a^2 + b^2)^(5/6)*f*(Sec[e + f*x]^2)^(1/6)) -
(b^(2/3)*Log[(a^2 + b^2)^(1/3) + b^(1/3)*(a^2 + b^2)^(1/6)*(Sec[e + f*x]^2)
^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)]*(d*Sec[e + f*x])^(1/3))/(4*(a^2 +
b^2)^(5/6)*f*(Sec[e + f*x]^2)^(1/6)) + (AppellF1[1/2, 1, 5/6, 3/2, (b^2*Tan
[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Sec[e + f*x])^(1/3)*Tan[e + f*x])/(a*
f*(Sec[e + f*x]^2)^(1/6))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt[3]{d\sec(e+fx)}\text{Subst}\left(\int \frac{1}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{5/6}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[6]{\sec^2(e+fx)}}$$

$$= \frac{\sqrt[3]{d\sec(e+fx)}\text{Subst}\left(\int \left(\frac{a}{(a^2-x^2)\left(1+\frac{x^2}{b^2}\right)^{5/6}} + \frac{x}{(-a^2+x^2)\left(1+\frac{x^2}{b^2}\right)^{5/6}}\right)\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[6]{\sec^2(e+fx)}}$$

$$
= \frac{\sqrt[3]{d\sec(e+fx)}\text{Subst}\left(\int \frac{x}{(-a^2+x^2)\left(1+\frac{x^2}{b^2}\right)^{5/6}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[6]{\sec^2(e+fx)}} + \frac{\left(a\sqrt[3]{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{(a^2-x^2)\left(1+\frac{x^2}{b^2}\right)^{5/6}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[6]{\sec^2(e+fx)}}
$$

$$
= \frac{\text{AppellF1}\left(\frac{1}{2}, 1, \frac{5}{6}, \frac{3}{2}, \frac{b^2\tan^2(e+fx)}{a^2}, -\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan(e+fx)}{af\sqrt[6]{\sec^2(e+fx)}} + \frac{\sqrt[3]{d\sec(e+fx)}\text{Subst}\left(\int \frac{1}{(-a^2+x)\left(1+\frac{x}{b^2}\right)^{5/6}}\,dx, x, b^2\tan^2(e+fx)\right)}{2bf\sqrt[6]{\sec^2(e+fx)}}
$$

$$
= \frac{\text{AppellF1}\left(\frac{1}{2}, 1, \frac{5}{6}, \frac{3}{2}, \frac{b^2\tan^2(e+fx)}{a^2}, -\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan(e+fx)}{af\sqrt[6]{\sec^2(e+fx)}} + \frac{\left(3b\sqrt[3]{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{-a^2-b^2+b^2x^6}\,dx, x, \sqrt[6]{\sec^2(e+fx)}\right)}{f\sqrt[6]{\sec^2(e+fx)}}
$$

$$
= \frac{\text{AppellF1}\left(\frac{1}{2}, 1, \frac{5}{6}, \frac{3}{2}, \frac{b^2\tan^2(e+fx)}{a^2}, -\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan(e+fx)}{af\sqrt[6]{\sec^2(e+fx)}} - \frac{\left(b\sqrt[3]{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt[6]{a^2+b^2}-\frac{\sqrt[3]{b}x}{2}}{\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}\,dx, x, \sqrt[6]{\sec^2(e+fx)}\right)}{(a^2+b^2)^{5/6}f\sqrt[6]{\sec^2(e+fx)}} - \frac{\left(b\sqrt[3]{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt[6]{a^2+b^2}+\frac{\sqrt[3]{b}x}{2}}{\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}\,dx, x, \sqrt[6]{\sec^2(e+fx)}\right)}{(a^2+b^2)^{5/6}f\sqrt[6]{\sec^2(e+fx)}} - \frac{\left(b\sqrt[3]{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a^2+b^2}-b^{2/3}x^2}\,dx, x, \sqrt[6]{\sec^2(e+fx)}\right)}{(a^2+b^2)^{2/3}f\sqrt[6]{\sec^2(e+fx)}}
$$

$$= -\frac{b^{2/3}\operatorname{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sqrt[3]{d\sec(e+fx)}}{(a^2+b^2)^{5/6} f\sqrt[6]{\sec^2(e+fx)}}$$
$$+\frac{\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan(e+fx)}{af\sqrt[6]{\sec^2(e+fx)}}$$
$$+\frac{\left(b^{2/3}\sqrt[3]{d\sec(e+fx)}\right)\operatorname{Subst}\left(\int\frac{-\sqrt[3]{b}\sqrt[6]{a^2+b^2}+2b^{2/3}x}{\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{4(a^2+b^2)^{5/6} f\sqrt[6]{\sec^2(e+fx)}}$$
$$-\frac{\left(b^{2/3}\sqrt[3]{d\sec(e+fx)}\right)\operatorname{Subst}\left(\int\frac{\sqrt[3]{b}\sqrt[6]{a^2+b^2}+2b^{2/3}x}{\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{4(a^2+b^2)^{5/6} f\sqrt[6]{\sec^2(e+fx)}}$$
$$-\frac{\left(3b\sqrt[3]{d\sec(e+fx)}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{4(a^2+b^2)^{2/3} f\sqrt[6]{\sec^2(e+fx)}}$$
$$-\frac{\left(3b\sqrt[3]{d\sec(e+fx)}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{4(a^2+b^2)^{2/3} f\sqrt[6]{\sec^2(e+fx)}}$$

$$= -\frac{b^{2/3}\operatorname{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sqrt[3]{d\sec(e+fx)}}{(a^2+b^2)^{5/6} f\sqrt[6]{\sec^2(e+fx)}}$$
$$+\frac{b^{2/3}\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sqrt[3]{d\sec(e+fx)}}{4(a^2+b^2)^{5/6} f\sqrt[6]{\sec^2(e+fx)}}$$
$$-\frac{b^{2/3}\log\left(\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sqrt[3]{d\sec(e+fx)}}{4(a^2+b^2)^{5/6} f\sqrt[6]{\sec^2(e+fx)}}$$
$$+\frac{\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan(e+fx)}{af\sqrt[6]{\sec^2(e+fx)}}$$
$$-\frac{\left(3b^{2/3}\sqrt[3]{d\sec(e+fx)}\right)\operatorname{Subst}\left(\int\frac{1}{-3-x^2}dx,x,1-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)}{2(a^2+b^2)^{5/6} f\sqrt[6]{\sec^2(e+fx)}}$$
$$+\frac{\left(3b^{2/3}\sqrt[3]{d\sec(e+fx)}\right)\operatorname{Subst}\left(\int\frac{1}{-3-x^2}dx,x,1+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)}{2(a^2+b^2)^{5/6} f\sqrt[6]{\sec^2(e+fx)}}$$

$$= \frac{\sqrt{3}b^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}}{\sqrt{3}}\right)\sqrt[3]{d\sec(e+fx)}}{2(a^2+b^2)^{5/6}f\sqrt[6]{\sec^2(e+fx)}} - \frac{\sqrt{3}b^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}}{\sqrt{3}}\right)\sqrt[3]{d\sec(e+fx)}}{2(a^2+b^2)^{5/6}f\sqrt[6]{\sec^2(e+fx)}} - \frac{b^{2/3}\text{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sqrt[3]{d\sec(e+fx)}}{(a^2+b^2)^{5/6}f\sqrt[6]{\sec^2(e+fx)}} + \frac{b^{2/3}\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sqrt[3]{d\sec(e+fx)}}{4(a^2+b^2)^{5/6}f\sqrt[6]{\sec^2(e+fx)}} - \frac{b^{2/3}\log\left(\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sqrt[3]{d\sec(e+fx)}}{4(a^2+b^2)^{5/6}f\sqrt[6]{\sec^2(e+fx)}} + \frac{\text{AppellF1}\left(\frac{1}{2},1,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan(e+fx)}{af\sqrt[6]{\sec^2(e+fx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 14.56 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx = -\frac{48d^2\,\text{AppellF1}\left(\frac{5}{3},\frac{5}{6},\frac{5}{6},\frac{8}{3},\frac{a-}{a+b\tan(}\right.}{5bf(d\sec(e+fx))^{5/3}\left(5(a+ib)\,\text{AppellF1}\left(\frac{8}{3},\frac{5}{6},\frac{11}{6},\frac{11}{3},\frac{a-ib}{a+b\tan(e+fx)},\frac{a+ib}{a+b\tan(e+fx)}\right)+5(a-ib)\,\text{AppellF1}\left(\right.\right.}$$

```
[In] Integrate[(d*Sec[e + f*x])^(1/3)/(a + b*Tan[e + f*x]),x]
```

```
[Out] (-48*d^2*AppellF1[5/3, 5/6, 5/6, 8/3, (a - I*b)/(a + b*Tan[e + f*x]), (a +
I*b)/(a + b*Tan[e + f*x])]*(a + b*Tan[e + f*x]))/(5*b*f*(d*Sec[e + f*x])^(5
/3)*(5*(a + I*b)*AppellF1[8/3, 5/6, 11/6, 11/3, (a - I*b)/(a + b*Tan[e + f*
x]), (a + I*b)/(a + b*Tan[e + f*x])] + 5*(a - I*b)*AppellF1[8/3, 11/6, 5/6,
 11/3, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])] + 16
*AppellF1[5/3, 5/6, 5/6, 8/3, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a
+ b*Tan[e + f*x])]*(a + b*Tan[e + f*x])))
```

Maple [F]

$$\int \frac{(d\sec(fx+e))^{\frac{1}{3}}}{a+b\tan(fx+e)}dx$$

```
[In] int((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x)
[Out] int((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx = \int \frac{\sqrt[3]{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx$$

```
[In] integrate((d*sec(f*x+e))**(1/3)/(a+b*tan(f*x+e)),x)
[Out] Integral((d*sec(e + f*x))**(1/3)/(a + b*tan(e + f*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx = \int \frac{(d\sec(fx+e))^{\frac{1}{3}}}{b\tan(fx+e)+a}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^(1/3)/(b*tan(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx=\int \frac{(d\sec(fx+e))^{\frac{1}{3}}}{b\tan(fx+e)+a}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(1/3)/(b*tan(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{a+b\tan(e+fx)}\,dx=\int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}}{a+b\tan(e+f\,x)}\,dx$$

```
[In] int((d/cos(e + f*x))^(1/3)/(a + b*tan(e + f*x)),x)
[Out] int((d/cos(e + f*x))^(1/3)/(a + b*tan(e + f*x)), x)
```

3.634 $\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 579

$$\begin{aligned}
&\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx \\
&= \frac{3b}{(a^2+b^2)\,f\sqrt[3]{d\sec(e+fx)}} - \frac{\sqrt{3}b^{4/3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)\sqrt[6]{\sec^2(e+fx)}}{2\,(a^2+b^2)^{7/6}\,f\sqrt[3]{d\sec(e+fx)}} \\
&+ \frac{\sqrt{3}b^{4/3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)\sqrt[6]{\sec^2(e+fx)}}{2\,(a^2+b^2)^{7/6}\,f\sqrt[3]{d\sec(e+fx)}} \\
&- \frac{b^{4/3}\operatorname{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sqrt[6]{\sec^2(e+fx)}}{(a^2+b^2)^{7/6}\,f\sqrt[3]{d\sec(e+fx)}} \\
&+ \frac{b^{4/3}\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sqrt[6]{\sec^2(e+fx)}}{4\,(a^2+b^2)^{7/6}\,f\sqrt[3]{d\sec(e+fx)}} \\
&- \frac{b^{4/3}\log\left(\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sqrt[6]{\sec^2(e+fx)}}{4\,(a^2+b^2)^{7/6}\,f\sqrt[3]{d\sec(e+fx)}} \\
&+ \frac{\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{7}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[6]{\sec^2(e+fx)}\tan(e+fx)}{af\sqrt[3]{d\sec(e+fx)}}
\end{aligned}$$

```
[Out] 3*b/(a^2+b^2)/f/(d*sec(f*x+e))^(1/3)-b^(4/3)*arctanh(b^(1/3)*(sec(f*x+e)^2)
^(1/6)/(a^2+b^2)^(1/6))*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(7/6)/f/(d*sec(f*x+e
))^(1/3)+1/4*b^(4/3)*ln((a^2+b^2)^(1/3)-b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)
^2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)^(1/3))*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(7/6
```

```
)/f/(d*sec(f*x+e))^(1/3)-1/4*b^(4/3)*ln((a^2+b^2)^(1/3)+b^(1/3)*(a^2+b^2)^(
1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)^(1/3))*(sec(f*x+e)^2)^(1/6
)/(a^2+b^2)^(7/6)/f/(d*sec(f*x+e))^(1/3)+1/2*b^(4/3)*arctan(-1/3*3^(1/2)+2/
3*b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6)*3^(1/2))*(sec(f*x+e)^2)^(1/6
)*3^(1/2)/(a^2+b^2)^(7/6)/f/(d*sec(f*x+e))^(1/3)+1/2*b^(4/3)*arctan(1/3*3^(
1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6)*3^(1/2))*(sec(f*x+e)^
2)^(1/6)*3^(1/2)/(a^2+b^2)^(7/6)/f/(d*sec(f*x+e))^(1/3)+AppellF1(1/2,1,7/6,
3/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+e)^2)*(sec(f*x+e)^2)^(1/6)*tan(f*x+e)/a/f
/(d*sec(f*x+e))^(1/3)
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3593, 771, 440, 455, 53, 65, 302, 648, 632, 210, 642, 214}

$$
\begin{aligned}
&\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}dx \\
&= \frac{\tan(e+fx)\sqrt[6]{\sec^2(e+fx)}\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{7}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{af\sqrt[3]{d\sec(e+fx)}} \\
&\quad -\frac{\sqrt{3}b^{4/3}\sqrt[6]{\sec^2(e+fx)}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)}{2f(a^2+b^2)^{7/6}\sqrt[3]{d\sec(e+fx)}} \\
&\quad +\frac{\sqrt{3}b^{4/3}\sqrt[6]{\sec^2(e+fx)}\arctan\left(\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}+\frac{1}{\sqrt{3}}\right)}{2f(a^2+b^2)^{7/6}\sqrt[3]{d\sec(e+fx)}} \\
&\quad -\frac{b^{4/3}\sqrt[6]{\sec^2(e+fx)}\operatorname{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)}{f(a^2+b^2)^{7/6}\sqrt[3]{d\sec(e+fx)}}+\frac{3b}{f(a^2+b^2)\sqrt[3]{d\sec(e+fx)}} \\
&\quad +\frac{b^{4/3}\sqrt[6]{\sec^2(e+fx)}\log\left(-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+\sqrt[3]{a^2+b^2}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)}{4f(a^2+b^2)^{7/6}\sqrt[3]{d\sec(e+fx)}} \\
&\quad -\frac{b^{4/3}\sqrt[6]{\sec^2(e+fx)}\log\left(\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+\sqrt[3]{a^2+b^2}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)}{4f(a^2+b^2)^{7/6}\sqrt[3]{d\sec(e+fx)}}
\end{aligned}
$$

```
[In] Int[1/((d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x])),x]
```

```
[Out] (3*b)/((a^2 + b^2)*f*(d*Sec[e + f*x])^(1/3)) - (Sqrt[3]*b^(4/3)*ArcTan[1/Sq
rt[3] - (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))]*(Se
c[e + f*x]^2)^(1/6))/(2*(a^2 + b^2)^(7/6)*f*(d*Sec[e + f*x])^(1/3)) + (Sqrt
[3]*b^(4/3)*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqrt[3]*
```

```
(a^2 + b^2)^(1/6))]*(Sec[e + f*x]^2)^(1/6))/(2*(a^2 + b^2)^(7/6)*f*(d*Sec[e
 + f*x])^(1/3)) - (b^(4/3)*ArcTanh[(b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(a^2 +
b^2)^(1/6)]*(Sec[e + f*x]^2)^(1/6))/((a^2 + b^2)^(7/6)*f*(d*Sec[e + f*x])^(
1/3)) + (b^(4/3)*Log[(a^2 + b^2)^(1/3) - b^(1/3)*(a^2 + b^2)^(1/6)*(Sec[e +
 f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)]*(Sec[e + f*x]^2)^(1/6))/(4
*(a^2 + b^2)^(7/6)*f*(d*Sec[e + f*x])^(1/3)) - (b^(4/3)*Log[(a^2 + b^2)^(1/
3) + b^(1/3)*(a^2 + b^2)^(1/6)*(Sec[e + f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*
x]^2)^(1/3)]*(Sec[e + f*x]^2)^(1/6))/(4*(a^2 + b^2)^(7/6)*f*(d*Sec[e + f*x]
)^(1/3)) + (AppellF1[1/2, 1, 7/6, 3/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f
*x]^2]*(Sec[e + f*x]^2)^(1/6)*Tan[e + f*x])/(a*f*(d*Sec[e + f*x])^(1/3))
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
```

```
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
```

```
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt[6]{\sec^2(e+fx)}\text{Subst}\left(\int \frac{1}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{7/6}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{\sqrt[6]{\sec^2(e+fx)}\text{Subst}\left(\int \left(\frac{a}{(a^2-x^2)\left(1+\frac{x^2}{b^2}\right)^{7/6}} + \frac{x}{(-a^2+x^2)\left(1+\frac{x^2}{b^2}\right)^{7/6}}\right)\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{\sqrt[6]{\sec^2(e+fx)}\text{Subst}\left(\int \frac{x}{(-a^2+x^2)\left(1+\frac{x^2}{b^2}\right)^{7/6}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[3]{d\sec(e+fx)}} + \frac{\left(a\sqrt[6]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{(a^2-x^2)\left(1+\frac{x^2}{b^2}\right)^{7/6}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{\text{AppellF1}\left(\frac{1}{2}, 1, \frac{7}{6}, \frac{3}{2}, \frac{b^2\tan^2(e+fx)}{a^2}, -\tan^2(e+fx)\right)\sqrt[6]{\sec^2(e+fx)}\tan(e+fx)}{af\sqrt[3]{d\sec(e+fx)}} + \frac{\sqrt[6]{\sec^2(e+fx)}\text{Subst}\left(\int \frac{1}{(-a^2+x)\left(1+\frac{x}{b^2}\right)^{7/6}}\,dx, x, b^2\tan^2(e+fx)\right)}{2bf\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{3b}{(a^2+b^2)\,f\sqrt[3]{d\sec(e+fx)}} + \frac{\text{AppellF1}\left(\frac{1}{2}, 1, \frac{7}{6}, \frac{3}{2}, \frac{b^2\tan^2(e+fx)}{a^2}, -\tan^2(e+fx)\right)\sqrt[6]{\sec^2(e+fx)}\tan(e+fx)}{af\sqrt[3]{d\sec(e+fx)}} + \frac{\left(b\sqrt[6]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{(-a^2+x)\sqrt[6]{1+\frac{x}{b^2}}}\,dx, x, b^2\tan^2(e+fx)\right)}{2(a^2+b^2)\,f\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{3b}{(a^2+b^2) f \sqrt[3]{d\sec(e+fx)}} + \frac{\text{AppellF1}\left(\frac{1}{2},1,\frac{7}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[6]{\sec^2(e+fx)}\tan(e+fx)}{af\sqrt[3]{d\sec(e+fx)}} + \frac{\left(3b^3\sqrt[6]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{x^4}{-a^2-b^2+b^2x^6}\,dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{(a^2+b^2) f\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{3b}{(a^2+b^2) f \sqrt[3]{d\sec(e+fx)}} + \frac{\text{AppellF1}\left(\frac{1}{2},1,\frac{7}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[6]{\sec^2(e+fx)}\tan(e+fx)}{af\sqrt[3]{d\sec(e+fx)}} - \frac{\left(b^{5/3}\sqrt[6]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{-\frac{1}{2}\sqrt[6]{a^2+b^2}-\frac{\sqrt[3]{b}x}{2}}{\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}\,dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{(a^2+b^2)^{7/6} f\sqrt[3]{d\sec(e+fx)}} - \frac{\left(b^{5/3}\sqrt[6]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{-\frac{1}{2}\sqrt[6]{a^2+b^2}+\frac{\sqrt[3]{b}x}{2}}{\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}\,dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{(a^2+b^2)^{7/6} f\sqrt[3]{d\sec(e+fx)}} - \frac{\left(b^{5/3}\sqrt[6]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt[3]{a^2+b^2}-b^{2/3}x^2}\,dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{(a^2+b^2) f\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{3b}{(a^2+b^2) f \sqrt[3]{d\sec(e+fx)}} - \frac{b^{4/3}\text{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sqrt[6]{\sec^2(e+fx)}}{(a^2+b^2)^{7/6} f \sqrt[3]{d\sec(e+fx)}}$$
$$+ \frac{\text{AppellF1}\left(\frac{1}{2},1,\frac{7}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[6]{\sec^2(e+fx)}\tan(e+fx)}{af\sqrt[3]{d\sec(e+fx)}}$$
$$+ \frac{\left(b^{4/3}\sqrt[6]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{-\sqrt[3]{b}\sqrt[6]{a^2+b^2}+2b^{2/3}x}{\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}dx, x, \sqrt[6]{\sec^2(e+fx)}\right)}{4(a^2+b^2)^{7/6} f \sqrt[3]{d\sec(e+fx)}}$$
$$- \frac{\left(b^{4/3}\sqrt[6]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt[3]{b}\sqrt[6]{a^2+b^2}+2b^{2/3}x}{\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}dx, x, \sqrt[6]{\sec^2(e+fx)}\right)}{4(a^2+b^2)^{7/6} f \sqrt[3]{d\sec(e+fx)}}$$
$$+ \frac{\left(3b^{5/3}\sqrt[6]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}dx, x, \sqrt[6]{\sec^2(e+fx)}\right)}{4(a^2+b^2) f \sqrt[3]{d\sec(e+fx)}}$$
$$+ \frac{\left(3b^{5/3}\sqrt[6]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}dx, x, \sqrt[6]{\sec^2(e+fx)}\right)}{4(a^2+b^2) f \sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{3b}{(a^2+b^2) f \sqrt[3]{d\sec(e+fx)}} - \frac{b^{4/3}\text{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sqrt[6]{\sec^2(e+fx)}}{(a^2+b^2)^{7/6} f \sqrt[3]{d\sec(e+fx)}}$$
$$+ \frac{b^{4/3}\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sqrt[6]{\sec^2(e+fx)}}{4(a^2+b^2)^{7/6} f \sqrt[3]{d\sec(e+fx)}}$$
$$- \frac{b^{4/3}\log\left(\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sqrt[6]{\sec^2(e+fx)}}{4(a^2+b^2)^{7/6} f \sqrt[3]{d\sec(e+fx)}}$$
$$+ \frac{\text{AppellF1}\left(\frac{1}{2},1,\frac{7}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[6]{\sec^2(e+fx)}\tan(e+fx)}{af\sqrt[3]{d\sec(e+fx)}}$$
$$+ \frac{\left(3b^{4/3}\sqrt[6]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}dx, x, 1-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)}{2(a^2+b^2)^{7/6} f \sqrt[3]{d\sec(e+fx)}}$$
$$- \frac{\left(3b^{4/3}\sqrt[6]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}dx, x, 1+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)}{2(a^2+b^2)^{7/6} f \sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{3b}{(a^2+b^2) f \sqrt[3]{d \sec(e+fx)}} - \frac{\sqrt{3} b^{4/3} \arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}}{\sqrt{3}}\right) \sqrt[6]{\sec^2(e+fx)}}{2(a^2+b^2)^{7/6} f \sqrt[3]{d \sec(e+fx)}} + \frac{\sqrt{3} b^{4/3} \arctan\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}}{\sqrt{3}}\right) \sqrt[6]{\sec^2(e+fx)}}{2(a^2+b^2)^{7/6} f \sqrt[3]{d \sec(e+fx)}} - \frac{b^{4/3} \operatorname{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right) \sqrt[6]{\sec^2(e+fx)}}{(a^2+b^2)^{7/6} f \sqrt[3]{d \sec(e+fx)}} + \frac{b^{4/3} \log\left(\sqrt[3]{a^2+b^2} - \sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)} + b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right) \sqrt[6]{\sec^2(e+fx)}}{4(a^2+b^2)^{7/6} f \sqrt[3]{d \sec(e+fx)}} - \frac{b^{4/3} \log\left(\sqrt[3]{a^2+b^2} + \sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)} + b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right) \sqrt[6]{\sec^2(e+fx)}}{4(a^2+b^2)^{7/6} f \sqrt[3]{d \sec(e+fx)}} + \frac{\operatorname{AppellF1}\left(\frac{1}{2}, 1, \frac{7}{6}, \frac{3}{2}, \frac{b^2 \tan^2(e+fx)}{a^2}, -\tan^2(e+fx)\right) \sqrt[6]{\sec^2(e+fx)} \tan(e+fx)}{af \sqrt[3]{d \sec(e+fx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 52.39 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.49

$$\int \frac{1}{\sqrt[3]{d \sec(e+fx)}(a+b\tan(e+fx))} \, dx = -\frac{60d \operatorname{AppellF1}\left(\frac{7}{3}, \frac{7}{6}, \frac{7}{6}, \frac{10}{3}, \frac{a-ib}{a+b\tan(e+fx)}\right.}{7bf(d\sec(e+fx))^{4/3}\left(7(a+ib)\operatorname{AppellF1}\left(\frac{10}{3}, \frac{7}{6}, \frac{13}{6}, \frac{13}{3}, \frac{a-ib}{a+b\tan(e+fx)}, \frac{a+ib}{a+b\tan(e+fx)}\right) + 7(a-ib)\operatorname{AppellF1}\right.}$$

```
[In] Integrate[1/((d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x])),x]
```

```
[Out] (-60*d*AppellF1[7/3, 7/6, 7/6, 10/3, (a - I*b)/(a + b*Tan[e + f*x]), (a + I
*b)/(a + b*Tan[e + f*x])]*(a*Cos[e + f*x] + b*Sin[e + f*x]))/(7*b*f*(d*Sec[
e + f*x])^(4/3)*(7*(a + I*b)*AppellF1[10/3, 7/6, 13/6, 13/3, (a - I*b)/(a +
 b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])] + 7*(a - I*b)*AppellF1[10
/3, 13/6, 7/6, 13/3, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e
```

```
 + f*x])] + 20*AppellF1[7/3, 7/6, 7/6, 10/3, (a - I*b)/(a + b*Tan[e + f*x])
, (a + I*b)/(a + b*Tan[e + f*x])]*(a + b*Tan[e + f*x])))
```

Maple [F]

$$\int \frac{1}{\left(d \sec \left(f x+e\right)\right)^{\frac{1}{3}} \left(a+b \tan \left(f x+e\right)\right)}dx$$

[In] int(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x)

[Out] int(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x)

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{d \sec(e+fx)}(a+b\tan(e+fx))}\,dx = \text{Timed out}$$

[In] integrate(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{1}{\sqrt[3]{d \sec(e+fx)}(a+b\tan(e+fx))}\,dx = \int \frac{1}{\sqrt[3]{d \sec\left(e+fx\right)}\left(a+b\tan\left(e+fx\right)\right)}\,dx$$

[In] integrate(1/(d*sec(f*x+e))**(1/3)/(a+b*tan(f*x+e)),x)

[Out] Integral(1/((d*sec(e + f*x))**(1/3)*(a + b*tan(e + f*x))), x)

Maxima [F]

$$\int \frac{1}{\sqrt[3]{d \sec(e+fx)}(a+b\tan(e+fx))}\,dx = \int \frac{1}{\left(d \sec\left(fx+e\right)\right)^{\frac{1}{3}}\left(b\tan\left(fx+e\right)+a\right)}\,dx$$

[In] integrate(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x, algorithm="maxima")

[Out] integrate(1/((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a)), x)

Giac [F]

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{1}{3}}(b\tan(fx+e)+a)}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e)),x, algorithm="giac")
[Out] integrate(1/((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))}\,dx = \int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}\,(a+b\tan(e+f\,x))}\,dx$$

```
[In] int(1/((d/cos(e + f*x))^(1/3)*(a + b*tan(e + f*x))),x)
[Out] int(1/((d/cos(e + f*x))^(1/3)*(a + b*tan(e + f*x))), x)
```

3.635 $\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 581

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}\,dx = \frac{3b}{5(a^2+b^2)f(d\sec(e+fx))^{5/3}} + \frac{\sqrt{3}b^{8/3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)\sec^2(e+fx)^{5/6}}{2(a^2+b^2)^{11/6}f(d\sec(e+fx))^{5/3}} - \frac{\sqrt{3}b^{8/3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)\sec^2(e+fx)^{5/6}}{2(a^2+b^2)^{11/6}f(d\sec(e+fx))^{5/3}} - \frac{b^{8/3}\text{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sec^2(e+fx)^{5/6}}{(a^2+b^2)^{11/6}f(d\sec(e+fx))^{5/3}} + \frac{b^{8/3}\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sec^2(e+fx)^{5/6}}{4(a^2+b^2)^{11/6}f(d\sec(e+fx))^{5/3}} - \frac{b^{8/3}\log\left(\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sec^2(e+fx)^{5/6}}{4(a^2+b^2)^{11/6}f(d\sec(e+fx))^{5/3}} + \frac{\text{AppellF1}\left(\frac{1}{2},1,\frac{11}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{5/6}\tan(e+fx)}{af(d\sec(e+fx))^{5/3}}$$

```
[Out] 3/5*b/(a^2+b^2)/f/(d*sec(f*x+e))^(5/3)-b^(8/3)*arctanh(b^(1/3)*(sec(f*x+e)^
2)^(1/6)/(a^2+b^2)^(1/6))*(sec(f*x+e)^2)^(5/6)/(a^2+b^2)^(11/6)/f/(d*sec(f*
x+e))^(5/3)+1/4*b^(8/3)*ln((a^2+b^2)^(1/3)-b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x
+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)^(1/3))*(sec(f*x+e)^2)^(5/6)/(a^2+b^2)^(
11/6)/f/(d*sec(f*x+e))^(5/3)-1/4*b^(8/3)*ln((a^2+b^2)^(1/3)+b^(1/3)*(a^2+b^
```

```
2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)^(1/3))*(sec(f*x+e)^2)^
(5/6)/(a^2+b^2)^(11/6)/f/(d*sec(f*x+e))^(5/3)-1/2*b^(8/3)*arctan(-1/3*3^(1/
2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6)*3^(1/2))*(sec(f*x+e)^2)
^(5/6)*3^(1/2)/(a^2+b^2)^(11/6)/f/(d*sec(f*x+e))^(5/3)-1/2*b^(8/3)*arctan(1
/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6)*3^(1/2))*(sec(f
*x+e)^2)^(5/6)*3^(1/2)/(a^2+b^2)^(11/6)/f/(d*sec(f*x+e))^(5/3)+AppellF1(1/2
,1,11/6,3/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+e)^2)*(sec(f*x+e)^2)^(5/6)*tan(f*
x+e)/a/f/(d*sec(f*x+e))^(5/3)
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 581, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3593, 771, 440, 455, 53, 65, 216, 648, 632, 210, 642, 214}

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}dx = \frac{\tan(e+fx)\sec^2(e+fx)^{5/6}\operatorname{AppellF1}\left(\frac{1}{2},1,\frac{11}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\right.}{af(d\sec(e+fx))^{5/3}}$$
$$+\frac{\sqrt{3}b^{8/3}\sec^2(e+fx)^{5/6}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)}{2f\left(a^2+b^2\right)^{11/6}(d\sec(e+fx))^{5/3}}$$
$$-\frac{\sqrt{3}b^{8/3}\sec^2(e+fx)^{5/6}\arctan\left(\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}+\frac{1}{\sqrt{3}}\right)}{2f\left(a^2+b^2\right)^{11/6}(d\sec(e+fx))^{5/3}}$$
$$-\frac{b^{8/3}\sec^2(e+fx)^{5/6}\operatorname{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)}{f\left(a^2+b^2\right)^{11/6}(d\sec(e+fx))^{5/3}}+\frac{3b}{5f\left(a^2+b^2\right)(d\sec(e+fx))^{5/3}}$$
$$+\frac{b^{8/3}\sec^2(e+fx)^{5/6}\log\left(-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+\sqrt[3]{a^2+b^2}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)}{4f\left(a^2+b^2\right)^{11/6}(d\sec(e+fx))^{5/3}}$$
$$-\frac{b^{8/3}\sec^2(e+fx)^{5/6}\log\left(\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+\sqrt[3]{a^2+b^2}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)}{4f\left(a^2+b^2\right)^{11/6}(d\sec(e+fx))^{5/3}}$$

```
[In] Int[1/((d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x])),x]
```

```
[Out] (3*b)/(5*(a^2 + b^2)*f*(d*Sec[e + f*x])^(5/3)) + (Sqrt[3]*b^(8/3)*ArcTan[1/
Sqrt[3] - (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))]*(
Sec[e + f*x]^2)^(5/6))/(2*(a^2 + b^2)^(11/6)*f*(d*Sec[e + f*x])^(5/3)) - (S
qrt[3]*b^(8/3)*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqrt[
3]*(a^2 + b^2)^(1/6))]*(Sec[e + f*x]^2)^(5/6))/(2*(a^2 + b^2)^(11/6)*f*(d*S
ec[e + f*x])^(5/3)) - (b^(8/3)*ArcTanh[(b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(a^
2 + b^2)^(1/6)]*(Sec[e + f*x]^2)^(5/6))/((a^2 + b^2)^(11/6)*f*(d*Sec[e + f*
x])^(5/3)) + (b^(8/3)*Log[(a^2 + b^2)^(1/3) - b^(1/3)*(a^2 + b^2)^(1/6)*(Se
```

```
c[e + f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)]*(Sec[e + f*x]^2)^(5/6
))/(4*(a^2 + b^2)^(11/6)*f*(d*Sec[e + f*x])^(5/3)) - (b^(8/3)*Log[(a^2 + b^
2)^(1/3) + b^(1/3)*(a^2 + b^2)^(1/6)*(Sec[e + f*x]^2)^(1/6) + b^(2/3)*(Sec[
e + f*x]^2)^(1/3)]*(Sec[e + f*x]^2)^(5/6))/(4*(a^2 + b^2)^(11/6)*f*(d*Sec[e
 + f*x])^(5/3)) + (AppellF1[1/2, 1, 11/6, 3/2, (b^2*Tan[e + f*x]^2)/a^2, -T
an[e + f*x]^2]*(Sec[e + f*x]^2)^(5/6)*Tan[e + f*x])/(a*f*(d*Sec[e + f*x])^(
5/3))
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\sec^2(e+fx)^{5/6}\text{Subst}\left(\int \frac{1}{(a+x)\left(1+\frac{x^2}{b^2}\right)^{11/6}}\,dx, x, b\tan(e+fx)\right)}{bf(d\sec(e+fx))^{5/3}}$$

$$= \frac{\sec^2(e+fx)^{5/6}\text{Subst}\left(\int \left(\frac{a}{(a^2-x^2)\left(1+\frac{x^2}{b^2}\right)^{11/6}} + \frac{x}{(-a^2+x^2)\left(1+\frac{x^2}{b^2}\right)^{11/6}}\right)\,dx, x, b\tan(e+fx)\right)}{bf(d\sec(e+fx))^{5/3}}$$

$$= \frac{\sec^2(e+fx)^{5/6}\text{Subst}\left(\int \frac{x}{(-a^2+x^2)\left(1+\frac{x^2}{b^2}\right)^{11/6}}\,dx, x, b\tan(e+fx)\right)}{bf(d\sec(e+fx))^{5/3}} + \frac{\left(a\sec^2(e+fx)^{5/6}\right)\text{Subst}\left(\int \frac{1}{(a^2-x^2)\left(1+\frac{x^2}{b^2}\right)^{11/6}}\,dx, x, b\tan(e+fx)\right)}{bf(d\sec(e+fx))^{5/3}}$$

$$= \frac{\text{AppellF1}\left(\frac{1}{2}, 1, \frac{11}{6}, \frac{3}{2}, \frac{b^2\tan^2(e+fx)}{a^2}, -\tan^2(e+fx)\right)\sec^2(e+fx)^{5/6}\tan(e+fx)}{af(d\sec(e+fx))^{5/3}} + \frac{\sec^2(e+fx)^{5/6}\text{Subst}\left(\int \frac{1}{(-a^2+x)\left(1+\frac{x}{b^2}\right)^{11/6}}\,dx, x, b^2\tan^2(e+fx)\right)}{2bf(d\sec(e+fx))^{5/3}}$$

$$= \frac{3b}{5\left(a^2+b^2\right)f(d\sec(e+fx))^{5/3}} + \frac{\text{AppellF1}\left(\frac{1}{2}, 1, \frac{11}{6}, \frac{3}{2}, \frac{b^2\tan^2(e+fx)}{a^2}, -\tan^2(e+fx)\right)\sec^2(e+fx)^{5/6}\tan(e+fx)}{af(d\sec(e+fx))^{5/3}} + \frac{\left(b\sec^2(e+fx)^{5/6}\right)\text{Subst}\left(\int \frac{1}{(-a^2+x)\left(1+\frac{x}{b^2}\right)^{5/6}}\,dx, x, b^2\tan^2(e+fx)\right)}{2\left(a^2+b^2\right)f(d\sec(e+fx))^{5/3}}$$

$$= \frac{3b}{5\left(a^2+b^2\right)f(d\sec(e+fx))^{5/3}} + \frac{\text{AppellF1}\left(\frac{1}{2}, 1, \frac{11}{6}, \frac{3}{2}, \frac{b^2\tan^2(e+fx)}{a^2}, -\tan^2(e+fx)\right)\sec^2(e+fx)^{5/6}\tan(e+fx)}{af(d\sec(e+fx))^{5/3}} + \frac{\left(3b^3\sec^2(e+fx)^{5/6}\right)\text{Subst}\left(\int \frac{1}{-a^2-b^2+b^2x^6}\,dx, x, \sqrt[6]{\sec^2(e+fx)}\right)}{\left(a^2+b^2\right)f(d\sec(e+fx))^{5/3}}$$

$$=\frac{3b}{5(a^2+b^2)f(d\sec(e+fx))^{5/3}}+\frac{\text{AppellF1}\left(\frac{1}{2},1,\frac{11}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{5/6}\tan(e+fx)}{af(d\sec(e+fx))^{5/3}}-\frac{\left(b^3\sec^2(e+fx)^{5/6}\right)\text{Subst}\left(\int\frac{\sqrt[6]{a^2+b^2}-\frac{\sqrt[3]{b}x}{2}}{\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{(a^2+b^2)^{11/6}f(d\sec(e+fx))^{5/3}}-\frac{\left(b^3\sec^2(e+fx)^{5/6}\right)\text{Subst}\left(\int\frac{\sqrt[6]{a^2+b^2}+\frac{\sqrt[3]{b}x}{2}}{\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{(a^2+b^2)^{11/6}f(d\sec(e+fx))^{5/3}}-\frac{\left(b^3\sec^2(e+fx)^{5/6}\right)\text{Subst}\left(\int\frac{1}{\sqrt[3]{a^2+b^2}-b^{2/3}x^2}dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{(a^2+b^2)^{5/3}f(d\sec(e+fx))^{5/3}}$$

$$=\frac{3b}{5(a^2+b^2)f(d\sec(e+fx))^{5/3}}-\frac{b^{8/3}\text{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sec^2(e+fx)^{5/6}}{(a^2+b^2)^{11/6}f(d\sec(e+fx))^{5/3}}+\frac{\text{AppellF1}\left(\frac{1}{2},1,\frac{11}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{5/6}\tan(e+fx)}{af(d\sec(e+fx))^{5/3}}+\frac{\left(b^{8/3}\sec^2(e+fx)^{5/6}\right)\text{Subst}\left(\int\frac{-\sqrt[3]{b}\sqrt[6]{a^2+b^2}+2b^{2/3}x}{\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{4(a^2+b^2)^{11/6}f(d\sec(e+fx))^{5/3}}-\frac{\left(b^{8/3}\sec^2(e+fx)^{5/6}\right)\text{Subst}\left(\int\frac{\sqrt[3]{b}\sqrt[6]{a^2+b^2}+2b^{2/3}x}{\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{4(a^2+b^2)^{11/6}f(d\sec(e+fx))^{5/3}}-\frac{\left(3b^3\sec^2(e+fx)^{5/6}\right)\text{Subst}\left(\int\frac{1}{\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{4(a^2+b^2)^{5/3}f(d\sec(e+fx))^{5/3}}-\frac{\left(3b^3\sec^2(e+fx)^{5/6}\right)\text{Subst}\left(\int\frac{1}{\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{4(a^2+b^2)^{5/3}f(d\sec(e+fx))^{5/3}}$$

$$= \frac{3b}{5(a^2+b^2)f(d\sec(e+fx))^{5/3}} - \frac{b^{8/3}\text{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sec^2(e+fx)^{5/6}}{(a^2+b^2)^{11/6}f(d\sec(e+fx))^{5/3}}$$
$$+ \frac{b^{8/3}\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sec^2(e+fx)^{5/6}}{4(a^2+b^2)^{11/6}f(d\sec(e+fx))^{5/3}}$$
$$- \frac{b^{8/3}\log\left(\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sec^2(e+fx)^{5/6}}{4(a^2+b^2)^{11/6}f(d\sec(e+fx))^{5/3}}$$
$$+ \frac{\text{AppellF1}\left(\frac{1}{2},1,\frac{11}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{5/6}\tan(e+fx)}{af(d\sec(e+fx))^{5/3}}$$
$$- \frac{\left(3b^{8/3}\sec^2(e+fx)^{5/6}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)}{2(a^2+b^2)^{11/6}f(d\sec(e+fx))^{5/3}}$$
$$+ \frac{\left(3b^{8/3}\sec^2(e+fx)^{5/6}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)}{2(a^2+b^2)^{11/6}f(d\sec(e+fx))^{5/3}}$$

$$= \frac{3b}{5(a^2+b^2)f(d\sec(e+fx))^{5/3}}$$
$$+ \frac{\sqrt{3}b^{8/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}}{\sqrt{3}}\right)\sec^2(e+fx)^{5/6}}{2(a^2+b^2)^{11/6}f(d\sec(e+fx))^{5/3}}$$
$$- \frac{\sqrt{3}b^{8/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}}{\sqrt{3}}\right)\sec^2(e+fx)^{5/6}}{2(a^2+b^2)^{11/6}f(d\sec(e+fx))^{5/3}}$$
$$- \frac{b^{8/3}\text{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sec^2(e+fx)^{5/6}}{(a^2+b^2)^{11/6}f(d\sec(e+fx))^{5/3}}$$
$$+ \frac{b^{8/3}\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sec^2(e+fx)^{5/6}}{4(a^2+b^2)^{11/6}f(d\sec(e+fx))^{5/3}}$$
$$- \frac{b^{8/3}\log\left(\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sec^2(e+fx)^{5/6}}{4(a^2+b^2)^{11/6}f(d\sec(e+fx))^{5/3}}$$
$$+ \frac{\text{AppellF1}\left(\frac{1}{2},1,\frac{11}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{5/6}\tan(e+fx)}{af(d\sec(e+fx))^{5/3}}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 6862 vs. $2(581) = 1162$.

Time = 111.46 (sec) , antiderivative size = 6862, normalized size of antiderivative = 11.81

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}\,dx = \text{Result too large to show}$$

```
[In] Integrate[1/((d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x])),x]
[Out] Result too large to show
```

Maple [F]

$$\int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}\left(a+b\tan\left(fx+e\right)\right)}dx$$

```
[In] int(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x)
[Out] int(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}\,dx = \text{Timed out}$$

```
[In] integrate(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}\,dx = \int \frac{1}{\left(d\sec\left(e+fx\right)\right)^{\frac{5}{3}}\left(a+b\tan\left(e+fx\right)\right)}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))**(5/3)/(a+b*tan(f*x+e)),x)
[Out] Integral(1/((d*sec(e + f*x))**(5/3)*(a + b*tan(e + f*x))), x)
```

Maxima [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}\,dx = \int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}\left(b\tan\left(fx+e\right)+a\right)}\,dx$$

[In] integrate(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x, algorithm="maxima")

[Out] integrate(1/((d*sec(f*x + e))^(5/3)*(b*tan(f*x + e) + a)), x)

Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}\,dx = \int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{5}{3}}\left(b\tan\left(fx+e\right)+a\right)}\,dx$$

[In] integrate(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e)),x, algorithm="giac")

[Out] integrate(1/((d*sec(f*x + e))^(5/3)*(b*tan(f*x + e) + a)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))}\,dx = \int \frac{1}{{\left(\frac{d}{\cos\left(e+f\,x\right)}\right)}^{5/3}\,\left(a+b\,\tan\left(e+f\,x\right)\right)}\,dx$$

[In] int(1/((d/cos(e + f*x))^(5/3)*(a + b*tan(e + f*x))),x)

[Out] int(1/((d/cos(e + f*x))^(5/3)*(a + b*tan(e + f*x))), x)

3.636 $\int \frac{(d\sec(e+fx))^{5/3}}{(a+b\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 687

$$\begin{aligned}\int \frac{(d\sec(e+fx))^{5/3}}{(a+b\tan(e+fx))^2}\,dx &= -\frac{a\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)(d\sec(e+fx))^{5/3}}{2\sqrt{3}b^{2/3}(a^2+b^2)^{7/6}f\sec^2(e+fx)^{5/6}} \\
&+\frac{a\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)(d\sec(e+fx))^{5/3}}{2\sqrt{3}b^{2/3}(a^2+b^2)^{7/6}f\sec^2(e+fx)^{5/6}} \\
&-\frac{a\text{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)(d\sec(e+fx))^{5/3}}{3b^{2/3}(a^2+b^2)^{7/6}f\sec^2(e+fx)^{5/6}} \\
&+\frac{a\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)(d\sec(e+fx))^{5/3}}{12b^{2/3}(a^2+b^2)^{7/6}f\sec^2(e+fx)^{5/6}} \\
&-\frac{a\log\left(\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)(d\sec(e+fx))^{5/3}}{12b^{2/3}(a^2+b^2)^{7/6}f\sec^2(e+fx)^{5/6}} \\
&+\frac{\text{AppellF1}\left(\frac{1}{2},2,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan(e+fx)}{a^2f\sec^2(e+fx)^{5/6}} \\
&+\frac{b^2\,\text{AppellF1}\left(\frac{3}{2},2,\frac{1}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan^3(e+fx)}{3a^4f\sec^2(e+fx)^{5/6}} \\
&-\frac{ab(d\sec(e+fx))^{5/3}}{(a^2+b^2)f(a^2-b^2\tan^2(e+fx))}\end{aligned}$$

```
[Out] -1/3*a*arctanh(b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6))*(d*sec(f*x+e))
^(5/3)/b^(2/3)/(a^2+b^2)^(7/6)/f/(sec(f*x+e)^2)^(5/6)+1/12*a*ln((a^2+b^2)^(
```

```
1/3)-b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)^(1
/3))*(d*sec(f*x+e))^(5/3)/b^(2/3)/(a^2+b^2)^(7/6)/f/(sec(f*x+e)^2)^(5/6)-1/
12*a*ln((a^2+b^2)^(1/3)+b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3
)*(sec(f*x+e)^2)^(1/3))*(d*sec(f*x+e))^(5/3)/b^(2/3)/(a^2+b^2)^(7/6)/f/(sec
(f*x+e)^2)^(5/6)+1/6*a*arctan(-1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)
/(a^2+b^2)^(1/6)*3^(1/2))*(d*sec(f*x+e))^(5/3)/b^(2/3)/(a^2+b^2)^(7/6)/f/(s
ec(f*x+e)^2)^(5/6)*3^(1/2)+1/6*a*arctan(1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)
^2)^(1/6)/(a^2+b^2)^(1/6)*3^(1/2))*(d*sec(f*x+e))^(5/3)/b^(2/3)/(a^2+b^2)^(
7/6)/f/(sec(f*x+e)^2)^(5/6)*3^(1/2)+AppellF1(1/2,2,1/6,3/2,b^2*tan(f*x+e)^2
/a^2,-tan(f*x+e)^2)*(d*sec(f*x+e))^(5/3)*tan(f*x+e)/a^2/f/(sec(f*x+e)^2)^(5
/6)+1/3*b^2*AppellF1(3/2,2,1/6,5/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+e)^2)*(d*s
ec(f*x+e))^(5/3)*tan(f*x+e)^3/a^4/f/(sec(f*x+e)^2)^(5/6)-a*b*(d*sec(f*x+e))
^(5/3)/(a^2+b^2)/f/(a^2-b^2*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 687, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3593, 771, 440, 455, 44, 65, 302, 648, 632, 210, 642, 214, 524}

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+b\tan(e+fx))^2}\,dx = \frac{\tan(e+fx)(d\sec(e+fx))^{5/3}\operatorname{AppellF1}\left(\frac{1}{2},2,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a^2 f\sec^2(e+fx)^{5/6}}$$
$$-\frac{a(d\sec(e+fx))^{5/3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)}{2\sqrt{3}b^{2/3}f\left(a^2+b^2\right)^{7/6}\sec^2(e+fx)^{5/6}}$$
$$+\frac{a(d\sec(e+fx))^{5/3}\arctan\left(\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}b^{2/3}f\left(a^2+b^2\right)^{7/6}\sec^2(e+fx)^{5/6}}$$
$$-\frac{a(d\sec(e+fx))^{5/3}\text{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)}{3b^{2/3}f\left(a^2+b^2\right)^{7/6}\sec^2(e+fx)^{5/6}}-\frac{ab(d\sec(e+fx))^{5/3}}{f\left(a^2+b^2\right)\left(a^2-b^2\tan^2(e+fx)\right)}$$
$$+\frac{a(d\sec(e+fx))^{5/3}\log\left(-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+\sqrt[3]{a^2+b^2}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)}{12b^{2/3}f\left(a^2+b^2\right)^{7/6}\sec^2(e+fx)^{5/6}}$$
$$-\frac{a(d\sec(e+fx))^{5/3}\log\left(\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+\sqrt[3]{a^2+b^2}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)}{12b^{2/3}f\left(a^2+b^2\right)^{7/6}\sec^2(e+fx)^{5/6}}$$
$$+\frac{b^2\tan^3(e+fx)(d\sec(e+fx))^{5/3}\operatorname{AppellF1}\left(\frac{3}{2},2,\frac{1}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{3a^4 f\sec^2(e+fx)^{5/6}}$$

```
[In] Int[(d*Sec[e + f*x])^(5/3)/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] -1/2*(a*ArcTan[1/Sqrt[3] - (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqrt[3]*(a^2
 + b^2)^(1/6))]*(d*Sec[e + f*x])^(5/3))/(Sqrt[3]*b^(2/3)*(a^2 + b^2)^(7/6)*
f*(Sec[e + f*x]^2)^(5/6)) + (a*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(Sec[e + f*x]^
2)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))]*(d*Sec[e + f*x])^(5/3))/(2*Sqrt[3]*b
^(2/3)*(a^2 + b^2)^(7/6)*f*(Sec[e + f*x]^2)^(5/6)) - (a*ArcTanh[(b^(1/3)*(S
ec[e + f*x]^2)^(1/6))/(a^2 + b^2)^(1/6)]*(d*Sec[e + f*x])^(5/3))/(3*b^(2/3)
*(a^2 + b^2)^(7/6)*f*(Sec[e + f*x]^2)^(5/6)) + (a*Log[(a^2 + b^2)^(1/3) - b
^(1/3)*(a^2 + b^2)^(1/6)*(Sec[e + f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^
(1/3)]*(d*Sec[e + f*x])^(5/3))/(12*b^(2/3)*(a^2 + b^2)^(7/6)*f*(Sec[e + f*x
]^2)^(5/6)) - (a*Log[(a^2 + b^2)^(1/3) + b^(1/3)*(a^2 + b^2)^(1/6)*(Sec[e +
 f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)]*(d*Sec[e + f*x])^(5/3))/(1
2*b^(2/3)*(a^2 + b^2)^(7/6)*f*(Sec[e + f*x]^2)^(5/6)) + (AppellF1[1/2, 2, 1
/6, 3/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Sec[e + f*x])^(5/3)*
Tan[e + f*x])/(a^2*f*(Sec[e + f*x]^2)^(5/6)) + (b^2*AppellF1[3/2, 2, 1/6, 5
/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Sec[e + f*x])^(5/3)*Tan[e
 + f*x]^3)/(3*a^4*f*(Sec[e + f*x]^2)^(5/6)) - (a*b*(d*Sec[e + f*x])^(5/3))/
((a^2 + b^2)*f*(a^2 - b^2*Tan[e + f*x]^2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{(d\sec(e+fx))^{5/3}\text{Subst}\left(\int \frac{1}{(a+x)^2\sqrt[6]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{5/6}}$$

$$= \frac{(d\sec(e+fx))^{5/3}\text{Subst}\left(\int \left(\frac{a^2}{(a^2-x^2)^2\sqrt[6]{1+\frac{x^2}{b^2}}} - \frac{2ax}{(a^2-x^2)^2\sqrt[6]{1+\frac{x^2}{b^2}}} + \frac{x^2}{(-a^2+x^2)^2\sqrt[6]{1+\frac{x^2}{b^2}}}\right) dx, x, b\right.}{bf\sec^2(e+fx)^{5/6}}$$

$$= \frac{(d\sec(e+fx))^{5/3}\text{Subst}\left(\int \frac{x^2}{(-a^2+x^2)^2\sqrt[6]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{5/6}}$$
$$- \frac{\left(2a(d\sec(e+fx))^{5/3}\right)\text{Subst}\left(\int \frac{x}{(a^2-x^2)^2\sqrt[6]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{5/6}}$$
$$+ \frac{\left(a^2(d\sec(e+fx))^{5/3}\right)\text{Subst}\left(\int \frac{1}{(a^2-x^2)^2\sqrt[6]{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(e+fx)\right)}{bf\sec^2(e+fx)^{5/6}}$$

$$= \frac{\text{AppellF1}\left(\frac{1}{2},2,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan(e+fx)}{a^2f\sec^2(e+fx)^{5/6}}$$
$$+ \frac{b^2\,\text{AppellF1}\left(\frac{3}{2},2,\frac{1}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan^3(e+fx)}{3a^4f\sec^2(e+fx)^{5/6}}$$
$$- \frac{\left(a(d\sec(e+fx))^{5/3}\right)\text{Subst}\left(\int \frac{1}{(a^2-x)^2\sqrt[6]{1+\frac{x}{b^2}}}\,dx, x, b^2\tan^2(e+fx)\right)}{bf\sec^2(e+fx)^{5/6}}$$

$$= \frac{\text{AppellF1}\left(\frac{1}{2},2,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan(e+fx)}{a^2f\sec^2(e+fx)^{5/6}}$$
$$+ \frac{b^2\,\text{AppellF1}\left(\frac{3}{2},2,\frac{1}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan^3(e+fx)}{3a^4f\sec^2(e+fx)^{5/6}}$$
$$- \frac{ab(d\sec(e+fx))^{5/3}}{(a^2+b^2)\,f\,(a^2-b^2\tan^2(e+fx))}$$
$$- \frac{\left(a(d\sec(e+fx))^{5/3}\right)\text{Subst}\left(\int \frac{1}{(a^2-x)\sqrt[6]{1+\frac{x}{b^2}}}\,dx, x, b^2\tan^2(e+fx)\right)}{6b\,(a^2+b^2)\,f\sec^2(e+fx)^{5/6}}$$

$$= \frac{\text{AppellF1}\left(\frac{1}{2},2,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan(e+fx)}{a^2f\sec^2(e+fx)^{5/6}}$$
$$+\frac{b^2\,\text{AppellF1}\left(\frac{3}{2},2,\frac{1}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan^3(e+fx)}{3a^4f\sec^2(e+fx)^{5/6}}$$
$$-\frac{ab(d\sec(e+fx))^{5/3}}{(a^2+b^2)f(a^2-b^2\tan^2(e+fx))}$$
$$-\frac{\left(ab(d\sec(e+fx))^{5/3}\right)\text{Subst}\left(\int\frac{x^4}{a^2+b^2-b^2x^6}\,dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{(a^2+b^2)f\sec^2(e+fx)^{5/6}}$$

$$= \frac{\text{AppellF1}\left(\frac{1}{2},2,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan(e+fx)}{a^2f\sec^2(e+fx)^{5/6}}$$
$$+\frac{b^2\,\text{AppellF1}\left(\frac{3}{2},2,\frac{1}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan^3(e+fx)}{3a^4f\sec^2(e+fx)^{5/6}}$$
$$-\frac{ab(d\sec(e+fx))^{5/3}}{(a^2+b^2)f(a^2-b^2\tan^2(e+fx))}$$
$$-\frac{\left(a(d\sec(e+fx))^{5/3}\right)\text{Subst}\left(\int\frac{-\frac{1}{2}\sqrt[6]{a^2+b^2}-\frac{\sqrt[3]{b}x}{2}}{\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}\,dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{3\sqrt[3]{b}\,(a^2+b^2)^{7/6}f\sec^2(e+fx)^{5/6}}$$
$$-\frac{\left(a(d\sec(e+fx))^{5/3}\right)\text{Subst}\left(\int\frac{-\frac{1}{2}\sqrt[6]{a^2+b^2}+\frac{\sqrt[3]{b}x}{2}}{\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}\,dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{3\sqrt[3]{b}\,(a^2+b^2)^{7/6}f\sec^2(e+fx)^{5/6}}$$
$$-\frac{\left(a(d\sec(e+fx))^{5/3}\right)\text{Subst}\left(\int\frac{1}{\sqrt[3]{a^2+b^2}-b^{2/3}x^2}\,dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{3\sqrt[3]{b}\,(a^2+b^2)f\sec^2(e+fx)^{5/6}}$$

$$= -\frac{a\text{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)(d\sec(e+fx))^{5/3}}{3b^{2/3}(a^2+b^2)^{7/6}f\sec^2(e+fx)^{5/6}}$$
$$+\frac{\text{AppellF1}\left(\frac{1}{2},2,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan(e+fx)}{a^2f\sec^2(e+fx)^{5/6}}$$
$$+\frac{b^2\,\text{AppellF1}\left(\frac{3}{2},2,\frac{1}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan^3(e+fx)}{3a^4f\sec^2(e+fx)^{5/6}}$$
$$-\frac{ab(d\sec(e+fx))^{5/3}}{(a^2+b^2)f(a^2-b^2\tan^2(e+fx))}$$
$$+\frac{\left(a(d\sec(e+fx))^{5/3}\right)\text{Subst}\left(\int\frac{-\sqrt[3]{b}\sqrt[6]{a^2+b^2}+2b^{2/3}x}{\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{12b^{2/3}(a^2+b^2)^{7/6}f\sec^2(e+fx)^{5/6}}$$
$$-\frac{\left(a(d\sec(e+fx))^{5/3}\right)\text{Subst}\left(\int\frac{\sqrt[3]{b}\sqrt[6]{a^2+b^2}+2b^{2/3}x}{\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{12b^{2/3}(a^2+b^2)^{7/6}f\sec^2(e+fx)^{5/6}}$$
$$+\frac{\left(a(d\sec(e+fx))^{5/3}\right)\text{Subst}\left(\int\frac{1}{\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{4\sqrt[3]{b}(a^2+b^2)f\sec^2(e+fx)^{5/6}}$$
$$+\frac{\left(a(d\sec(e+fx))^{5/3}\right)\text{Subst}\left(\int\frac{1}{\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{4\sqrt[3]{b}(a^2+b^2)f\sec^2(e+fx)^{5/6}}$$

$$= -\frac{a\operatorname{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)(d\sec(e+fx))^{5/3}}{3b^{2/3}(a^2+b^2)^{7/6} f\sec^2(e+fx)^{5/6}}$$

$$+\frac{a\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)(d\sec(e+fx))^{5/3}}{12b^{2/3}(a^2+b^2)^{7/6} f\sec^2(e+fx)^{5/6}}$$

$$-\frac{a\log\left(\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)(d\sec(e+fx))^{5/3}}{12b^{2/3}(a^2+b^2)^{7/6} f\sec^2(e+fx)^{5/6}}$$

$$+\frac{\operatorname{AppellF1}\left(\frac{1}{2},2,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan(e+fx)}{a^2 f\sec^2(e+fx)^{5/6}}$$

$$+\frac{b^2\operatorname{AppellF1}\left(\frac{3}{2},2,\frac{1}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan^3(e+fx)}{3a^4 f\sec^2(e+fx)^{5/6}}$$

$$-\frac{ab(d\sec(e+fx))^{5/3}}{(a^2+b^2) f(a^2-b^2\tan^2(e+fx))}$$

$$+\frac{\left(a(d\sec(e+fx))^{5/3}\right)\operatorname{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)}{2b^{2/3}(a^2+b^2)^{7/6} f\sec^2(e+fx)^{5/6}}$$

$$-\frac{\left(a(d\sec(e+fx))^{5/3}\right)\operatorname{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)}{2b^{2/3}(a^2+b^2)^{7/6} f\sec^2(e+fx)^{5/6}}$$

$$= -\frac{a\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}}{\sqrt{3}}\right)(d\sec(e+fx))^{5/3}}{2\sqrt{3}b^{2/3}(a^2+b^2)^{7/6}f\sec^2(e+fx)^{5/6}} + \frac{a\arctan\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}}{\sqrt{3}}\right)(d\sec(e+fx))^{5/3}}{2\sqrt{3}b^{2/3}(a^2+b^2)^{7/6}f\sec^2(e+fx)^{5/6}} - \frac{a\text{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)(d\sec(e+fx))^{5/3}}{3b^{2/3}(a^2+b^2)^{7/6}f\sec^2(e+fx)^{5/6}} + \frac{a\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)(d\sec(e+fx))^{5/3}}{12b^{2/3}(a^2+b^2)^{7/6}f\sec^2(e+fx)^{5/6}} - \frac{a\log\left(\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)(d\sec(e+fx))^{5/3}}{12b^{2/3}(a^2+b^2)^{7/6}f\sec^2(e+fx)^{5/6}} + \frac{\text{AppellF1}\left(\frac{1}{2},2,\frac{1}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan(e+fx)}{a^2f\sec^2(e+fx)^{5/6}} + \frac{b^2\text{AppellF1}\left(\frac{3}{2},2,\frac{1}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^{5/3}\tan^3(e+fx)}{3a^4f\sec^2(e+fx)^{5/6}} - \frac{ab(d\sec(e+fx))^{5/3}}{(a^2+b^2)f(a^2-b^2\tan^2(e+fx))}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 129.13 (sec) , antiderivative size = 8003, normalized size of antiderivative = 11.65

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+b\tan(e+fx))^2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(d*Sec[e + f*x])^(5/3)/(a + b*Tan[e + f*x])^2,x]
[Out] Result too large to show
```

Maple [F]

$$\int \frac{(d \sec (fx + e))^{\frac{5}{3}}}{(a + b \tan (fx + e))^2} dx$$

```
[In] int((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x)
[Out] int((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d \sec(e + fx))^{5/3}}{(a + b \tan(e + fx))^2} dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d \sec(e + fx))^{5/3}}{(a + b \tan(e + fx))^2} dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))**(5/3)/(a+b*tan(f*x+e))**2,x)
[Out] Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(d \sec(e + fx))^{5/3}}{(a + b \tan(e + fx))^2} dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
[Out] Timed out
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\sec(fx+e))^{\frac{5}{3}}}{(b\tan(fx+e)+a)^2}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(5/3)/(b*tan(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^{5/3}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}}{(a+b\tan(e+f\,x))^2}\,dx$$

```
[In] int((d/cos(e + f*x))^(5/3)/(a + b*tan(e + f*x))^2,x)
[Out] int((d/cos(e + f*x))^(5/3)/(a + b*tan(e + f*x))^2, x)
```

3.637 $\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 687

$$\begin{aligned}
\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx &= \frac{5ab^{2/3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)\sqrt[3]{d\sec(e+fx)}}{2\sqrt{3}\,(a^2+b^2)^{11/6}f\sqrt[6]{\sec^2(e+fx)}} \\
&\quad -\frac{5ab^{2/3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)\sqrt[3]{d\sec(e+fx)}}{2\sqrt{3}\,(a^2+b^2)^{11/6}f\sqrt[6]{\sec^2(e+fx)}} \\
&\quad -\frac{5ab^{2/3}\text{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sqrt[3]{d\sec(e+fx)}}{3\,(a^2+b^2)^{11/6}f\sqrt[6]{\sec^2(e+fx)}} \\
&\quad +\frac{5ab^{2/3}\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sqrt[3]{d\sec(e+fx)}}{12\,(a^2+b^2)^{11/6}f\sqrt[6]{\sec^2(e+fx)}} \\
&\quad -\frac{5ab^{2/3}\log\left(\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sqrt[3]{d\sec(e+fx)}}{12\,(a^2+b^2)^{11/6}f\sqrt[6]{\sec^2(e+fx)}} \\
&\quad +\frac{\text{AppellF1}\left(\frac{1}{2},2,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan(e+fx)}{a^2f\sqrt[6]{\sec^2(e+fx)}} \\
&\quad +\frac{b^2\,\text{AppellF1}\left(\frac{3}{2},2,\frac{5}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan^3(e+fx)}{3a^4f\sqrt[6]{\sec^2(e+fx)}} \\
&\quad -\frac{ab\sqrt[3]{d\sec(e+fx)}}{(a^2+b^2)f\,(a^2-b^2\tan^2(e+fx))}
\end{aligned}$$

```
[Out] -5/3*a*b^(2/3)*arctanh(b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6))*(d*sec
(f*x+e))^(1/3)/(a^2+b^2)^(11/6)/f/(sec(f*x+e)^2)^(1/6)+5/12*a*b^(2/3)*ln((a
^2+b^2)^(1/3)-b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(f*x
+e)^2)^(1/3))*(d*sec(f*x+e))^(1/3)/(a^2+b^2)^(11/6)/f/(sec(f*x+e)^2)^(1/6)-
5/12*a*b^(2/3)*ln((a^2+b^2)^(1/3)+b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)^2)^(1
/6)+b^(2/3)*(sec(f*x+e)^2)^(1/3))*(d*sec(f*x+e))^(1/3)/(a^2+b^2)^(11/6)/f/(
sec(f*x+e)^2)^(1/6)-5/6*a*b^(2/3)*arctan(-1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+
e)^2)^(1/6)/(a^2+b^2)^(1/6)*3^(1/2))*(d*sec(f*x+e))^(1/3)/(a^2+b^2)^(11/6)/
f/(sec(f*x+e)^2)^(1/6)*3^(1/2)-5/6*a*b^(2/3)*arctan(1/3*3^(1/2)+2/3*b^(1/3)
*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6)*3^(1/2))*(d*sec(f*x+e))^(1/3)/(a^2+b^
2)^(11/6)/f/(sec(f*x+e)^2)^(1/6)*3^(1/2)+AppellF1(1/2,2,5/6,3/2,b^2*tan(f*x
+e)^2/a^2,-tan(f*x+e)^2)*(d*sec(f*x+e))^(1/3)*tan(f*x+e)/a^2/f/(sec(f*x+e)^
2)^(1/6)+1/3*b^2*AppellF1(3/2,2,5/6,5/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+e)^2)
*(d*sec(f*x+e))^(1/3)*tan(f*x+e)^3/a^4/f/(sec(f*x+e)^2)^(1/6)-a*b*(d*sec(f*
x+e))^(1/3)/(a^2+b^2)/f/(a^2-b^2*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 687, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules

used = {3593, 771, 440, 455, 44, 65, 216, 648, 632, 210, 642, 214, 524}

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx$$
$$= \frac{\tan(e+fx)\sqrt[3]{d\sec(e+fx)}\,\text{AppellF1}\left(\frac{1}{2},2,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a^2f\sqrt[6]{\sec^2(e+fx)}}$$
$$+\frac{5ab^{2/3}\sqrt[3]{d\sec(e+fx)}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)}{2\sqrt{3}f(a^2+b^2)^{11/6}\sqrt[6]{\sec^2(e+fx)}}$$
$$-\frac{5ab^{2/3}\sqrt[3]{d\sec(e+fx)}\arctan\left(\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}f(a^2+b^2)^{11/6}\sqrt[6]{\sec^2(e+fx)}}$$
$$-\frac{5ab^{2/3}\sqrt[3]{d\sec(e+fx)}\,\text{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)}{3f(a^2+b^2)^{11/6}\sqrt[6]{\sec^2(e+fx)}}$$
$$-\frac{ab\sqrt[3]{d\sec(e+fx)}}{f(a^2+b^2)(a^2-b^2\tan^2(e+fx))}$$
$$+\frac{5ab^{2/3}\sqrt[3]{d\sec(e+fx)}\log\left(-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+\sqrt[3]{a^2+b^2}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)}{12f(a^2+b^2)^{11/6}\sqrt[6]{\sec^2(e+fx)}}$$
$$-\frac{5ab^{2/3}\sqrt[3]{d\sec(e+fx)}\log\left(\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+\sqrt[3]{a^2+b^2}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)}{12f(a^2+b^2)^{11/6}\sqrt[6]{\sec^2(e+fx)}}$$
$$+\frac{b^2\tan^3(e+fx)\sqrt[3]{d\sec(e+fx)}\,\text{AppellF1}\left(\frac{3}{2},2,\frac{5}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{3a^4f\sqrt[6]{\sec^2(e+fx)}}$$

```
[In] Int[(d*Sec[e + f*x])^(1/3)/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (5*a*b^(2/3)*ArcTan[1/Sqrt[3] - (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqrt[3]
*(a^2 + b^2)^(1/6))]*(d*Sec[e + f*x])^(1/3))/(2*Sqrt[3]*(a^2 + b^2)^(11/6)*
f*(Sec[e + f*x]^2)^(1/6)) - (5*a*b^(2/3)*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(Sec
[e + f*x]^2)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))]*(d*Sec[e + f*x])^(1/3))/(2
*Sqrt[3]*(a^2 + b^2)^(11/6)*f*(Sec[e + f*x]^2)^(1/6)) - (5*a*b^(2/3)*ArcTan
h[(b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(a^2 + b^2)^(1/6)]*(d*Sec[e + f*x])^(1/3
))/(3*(a^2 + b^2)^(11/6)*f*(Sec[e + f*x]^2)^(1/6)) + (5*a*b^(2/3)*Log[(a^2
+ b^2)^(1/3) - b^(1/3)*(a^2 + b^2)^(1/6)*(Sec[e + f*x]^2)^(1/6) + b^(2/3)*(
Sec[e + f*x]^2)^(1/3)]*(d*Sec[e + f*x])^(1/3))/(12*(a^2 + b^2)^(11/6)*f*(Se
c[e + f*x]^2)^(1/6)) - (5*a*b^(2/3)*Log[(a^2 + b^2)^(1/3) + b^(1/3)*(a^2 +
b^2)^(1/6)*(Sec[e + f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)]*(d*Sec[
e + f*x])^(1/3))/(12*(a^2 + b^2)^(11/6)*f*(Sec[e + f*x]^2)^(1/6)) + (Appell
F1[1/2, 2, 5/6, 3/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Sec[e +
```

```
f*x])^(1/3)*Tan[e + f*x])/(a^2*f*(Sec[e + f*x]^2)^(1/6)) + (b^2*AppellF1[3/
2, 2, 5/6, 5/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Sec[e + f*x])
^(1/3)*Tan[e + f*x]^3)/(3*a^4*f*(Sec[e + f*x]^2)^(1/6)) - (a*b*(d*Sec[e + f
*x])^(1/3))/((a^2 + b^2)*f*(a^2 - b^2*Tan[e + f*x]^2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
```

```
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt[3]{d\sec(e+fx)}\text{Subst}\left(\int \frac{1}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^{5/6}}\,dx,x,b\tan(e+fx)\right)}{bf\sqrt[6]{\sec^2(e+fx)}}$$

$$= \frac{\sqrt[3]{d\sec(e+fx)}\text{Subst}\left(\int \left(\frac{a^2}{(a^2-x^2)^2\left(1+\frac{x^2}{b^2}\right)^{5/6}} - \frac{2ax}{(a^2-x^2)^2\left(1+\frac{x^2}{b^2}\right)^{5/6}} + \frac{x^2}{(-a^2+x^2)^2\left(1+\frac{x^2}{b^2}\right)^{5/6}}\right)\,dx,x,b\tan(e+fx)\right)}{bf\sqrt[6]{\sec^2(e+fx)}}$$

$$= \frac{\sqrt[3]{d\sec(e+fx)}\text{Subst}\left(\int \frac{x^2}{(-a^2+x^2)^2\left(1+\frac{x^2}{b^2}\right)^{5/6}}\,dx,x,b\tan(e+fx)\right)}{bf\sqrt[6]{\sec^2(e+fx)}} - \frac{\left(2a\sqrt[3]{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{x}{(a^2-x^2)^2\left(1+\frac{x^2}{b^2}\right)^{5/6}}\,dx,x,b\tan(e+fx)\right)}{bf\sqrt[6]{\sec^2(e+fx)}} + \frac{\left(a^2\sqrt[3]{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{(a^2-x^2)^2\left(1+\frac{x^2}{b^2}\right)^{5/6}}\,dx,x,b\tan(e+fx)\right)}{bf\sqrt[6]{\sec^2(e+fx)}}$$

$$= \frac{\text{AppellF1}\left(\frac{1}{2},2,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan(e+fx)}{a^2f\sqrt[6]{\sec^2(e+fx)}} + \frac{b^2\,\text{AppellF1}\left(\frac{3}{2},2,\frac{5}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan^3(e+fx)}{3a^4f\sqrt[6]{\sec^2(e+fx)}} - \frac{\left(a\sqrt[3]{d\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{(a^2-x)^2\left(1+\frac{x}{b^2}\right)^{5/6}}\,dx,x,b^2\tan^2(e+fx)\right)}{bf\sqrt[6]{\sec^2(e+fx)}}$$

$$= \frac{\text{AppellF1}\left(\frac{1}{2},2,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan(e+fx)}{a^2f\sqrt[6]{\sec^2(e+fx)}} + \frac{b^2\,\text{AppellF1}\left(\frac{3}{2},2,\frac{5}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan^3(e+fx)}{3a^4f\sqrt[6]{\sec^2(e+fx)}} - \frac{ab\sqrt[3]{d\sec(e+fx)}}{(a^2+b^2)\,f\,(a^2-b^2\tan^2(e+fx))} - \frac{\left(5a\sqrt[3]{d\sec(e+fx)}\right)\text{Subst}\left(\int\frac{1}{(a^2-x)\left(1+\frac{x}{b^2}\right)^{5/6}}\,dx,x,b^2\tan^2(e+fx)\right)}{6b\,(a^2+b^2)\,f\sqrt[6]{\sec^2(e+fx)}}$$

$$= \frac{\text{AppellF1}\left(\frac{1}{2},2,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan(e+fx)}{a^2f\sqrt[6]{\sec^2(e+fx)}} + \frac{b^2\,\text{AppellF1}\left(\frac{3}{2},2,\frac{5}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan^3(e+fx)}{3a^4f\sqrt[6]{\sec^2(e+fx)}} - \frac{ab\sqrt[3]{d\sec(e+fx)}}{(a^2+b^2)\,f\,(a^2-b^2\tan^2(e+fx))} - \frac{\left(5ab\sqrt[3]{d\sec(e+fx)}\right)\text{Subst}\left(\int\frac{1}{a^2+b^2-b^2x^6}\,dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{(a^2+b^2)\,f\sqrt[6]{\sec^2(e+fx)}}$$

$$= \frac{\text{AppellF1}\left(\frac{1}{2},2,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan(e+fx)}{a^2f\sqrt[6]{\sec^2(e+fx)}} + \frac{b^2\,\text{AppellF1}\left(\frac{3}{2},2,\frac{5}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan^3(e+fx)}{3a^4f\sqrt[6]{\sec^2(e+fx)}} - \frac{ab\sqrt[3]{d\sec(e+fx)}}{(a^2+b^2)\,f\,(a^2-b^2\tan^2(e+fx))} - \frac{\left(5ab\sqrt[3]{d\sec(e+fx)}\right)\text{Subst}\left(\int\frac{\sqrt[6]{a^2+b^2}-\frac{\sqrt[3]{b}x}{2}}{\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}\,dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{3\,(a^2+b^2)^{11/6}\,f\sqrt[6]{\sec^2(e+fx)}} - \frac{\left(5ab\sqrt[3]{d\sec(e+fx)}\right)\text{Subst}\left(\int\frac{\sqrt[6]{a^2+b^2}+\frac{\sqrt[3]{b}x}{2}}{\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}\,dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{3\,(a^2+b^2)^{11/6}\,f\sqrt[6]{\sec^2(e+fx)}} - \frac{\left(5ab\sqrt[3]{d\sec(e+fx)}\right)\text{Subst}\left(\int\frac{1}{\sqrt[3]{a^2+b^2}-b^{2/3}x^2}\,dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{3\,(a^2+b^2)^{5/3}\,f\sqrt[6]{\sec^2(e+fx)}}$$

$$= -\frac{5ab^{2/3}\operatorname{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sqrt[3]{d\sec(e+fx)}}{3\left(a^2+b^2\right)^{11/6}f\sqrt[6]{\sec^2(e+fx)}}$$

$$+\frac{\operatorname{AppellF1}\left(\frac{1}{2},2,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan(e+fx)}{a^2f\sqrt[6]{\sec^2(e+fx)}}$$

$$+\frac{b^2\operatorname{AppellF1}\left(\frac{3}{2},2,\frac{5}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan^3(e+fx)}{3a^4f\sqrt[6]{\sec^2(e+fx)}}$$

$$-\frac{ab\sqrt[3]{d\sec(e+fx)}}{\left(a^2+b^2\right)f\left(a^2-b^2\tan^2(e+fx)\right)}$$

$$+\frac{\left(5ab^{2/3}\sqrt[3]{d\sec(e+fx)}\right)\operatorname{Subst}\left(\int\frac{-\sqrt[3]{b}\sqrt[6]{a^2+b^2}+2b^{2/3}x}{\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}\,dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{12\left(a^2+b^2\right)^{11/6}f\sqrt[6]{\sec^2(e+fx)}}$$

$$-\frac{\left(5ab^{2/3}\sqrt[3]{d\sec(e+fx)}\right)\operatorname{Subst}\left(\int\frac{\sqrt[3]{b}\sqrt[6]{a^2+b^2}+2b^{2/3}x}{\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}\,dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{12\left(a^2+b^2\right)^{11/6}f\sqrt[6]{\sec^2(e+fx)}}$$

$$-\frac{\left(5ab\sqrt[3]{d\sec(e+fx)}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}\,dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{4\left(a^2+b^2\right)^{5/3}f\sqrt[6]{\sec^2(e+fx)}}$$

$$-\frac{\left(5ab\sqrt[3]{d\sec(e+fx)}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}\,dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{4\left(a^2+b^2\right)^{5/3}f\sqrt[6]{\sec^2(e+fx)}}$$

$$
\begin{aligned}
&= -\frac{5ab^{2/3}\operatorname{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sqrt[3]{d\sec(e+fx)}}{3\left(a^2+b^2\right)^{11/6} f\sqrt[6]{\sec^2(e+fx)}} \\
&+ \frac{5ab^{2/3}\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sqrt[3]{d\sec(e+fx)}}{12\left(a^2+b^2\right)^{11/6} f\sqrt[6]{\sec^2(e+fx)}} \\
&- \frac{5ab^{2/3}\log\left(\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sqrt[3]{d\sec(e+fx)}}{12\left(a^2+b^2\right)^{11/6} f\sqrt[6]{\sec^2(e+fx)}} \\
&+ \frac{\operatorname{AppellF1}\left(\frac{1}{2},2,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan(e+fx)}{a^2 f\sqrt[6]{\sec^2(e+fx)}} \\
&+ \frac{b^2\operatorname{AppellF1}\left(\frac{3}{2},2,\frac{5}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan^3(e+fx)}{3a^4 f\sqrt[6]{\sec^2(e+fx)}} \\
&- \frac{ab\sqrt[3]{d\sec(e+fx)}}{\left(a^2+b^2\right) f\left(a^2-b^2\tan^2(e+fx)\right)} \\
&- \frac{\left(5ab^{2/3}\sqrt[3]{d\sec(e+fx)}\right)\operatorname{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{11/6} f\sqrt[6]{\sec^2(e+fx)}} \\
&+ \frac{\left(5ab^{2/3}\sqrt[3]{d\sec(e+fx)}\right)\operatorname{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{11/6} f\sqrt[6]{\sec^2(e+fx)}}
\end{aligned}
$$

$$= \frac{5ab^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}}{\sqrt{3}}\right)\sqrt[3]{d\sec(e+fx)}}{2\sqrt{3}\,(a^2+b^2)^{11/6}\,f\sqrt[6]{\sec^2(e+fx)}} - \frac{5ab^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}}{\sqrt{3}}\right)\sqrt[3]{d\sec(e+fx)}}{2\sqrt{3}\,(a^2+b^2)^{11/6}\,f\sqrt[6]{\sec^2(e+fx)}} - \frac{5ab^{2/3}\text{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sqrt[3]{d\sec(e+fx)}}{3\,(a^2+b^2)^{11/6}\,f\sqrt[6]{\sec^2(e+fx)}} + \frac{5ab^{2/3}\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sqrt[3]{d\sec(e+fx)}}{12\,(a^2+b^2)^{11/6}\,f\sqrt[6]{\sec^2(e+fx)}} - \frac{5ab^{2/3}\log\left(\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sqrt[3]{d\sec(e+fx)}}{12\,(a^2+b^2)^{11/6}\,f\sqrt[6]{\sec^2(e+fx)}} + \frac{\text{AppellF1}\left(\frac{1}{2},2,\frac{5}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan(e+fx)}{a^2f\sqrt[6]{\sec^2(e+fx)}} + \frac{b^2\,\text{AppellF1}\left(\frac{3}{2},2,\frac{5}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[3]{d\sec(e+fx)}\tan^3(e+fx)}{3a^4f\sqrt[6]{\sec^2(e+fx)}} - \frac{ab\sqrt[3]{d\sec(e+fx)}}{(a^2+b^2)\,f\,(a^2-b^2\tan^2(e+fx))}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7801 vs. 2(687) = 1374.

Time = 78.55 (sec) , antiderivative size = 7801, normalized size of antiderivative = 11.36

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(d*Sec[e + f*x])^(1/3)/(a + b*Tan[e + f*x])^2,x]
[Out] Result too large to show
```

Maple [F]

$$\int \frac{(d\sec(fx+e))^{\frac{1}{3}}}{(a+b\tan(fx+e))^{2}}dx$$

```
[In] int((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x)
[Out] int((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+b\tan(e+fx))^{2}}\,dx$$

```
[In] integrate((d*sec(f*x+e))**(1/3)/(a+b*tan(f*x+e))**2,x)
[Out] Integral((d*sec(e + f*x))**(1/3)/(a + b*tan(e + f*x))**2, x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\sec(fx+e))^{\frac{1}{3}}}{(b\tan(fx+e)+a)^{2}}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^(1/3)/(b*tan(f*x + e) + a)^2, x)
```

Giac [F]

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\sec(fx+e))^{\frac{1}{3}}}{(b\tan(fx+e)+a)^2}\,dx$$

```
[In] integrate((d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^(1/3)/(b*tan(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{d\sec(e+fx)}}{(a+b\tan(e+fx))^2}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}}{(a+b\tan(e+f\,x))^2}\,dx$$

```
[In] int((d/cos(e + f*x))^(1/3)/(a + b*tan(e + f*x))^2,x)
[Out] int((d/cos(e + f*x))^(1/3)/(a + b*tan(e + f*x))^2, x)
```

3.638 $\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 715

$$\begin{aligned}
&\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx \\
&= \frac{7ab}{(a^2+b^2)^2 f\sqrt[3]{d\sec(e+fx)}} - \frac{7ab^{4/3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)\sqrt[6]{\sec^2(e+fx)}}{2\sqrt{3}\,(a^2+b^2)^{13/6} f\sqrt[3]{d\sec(e+fx)}} \\
&+ \frac{7ab^{4/3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)\sqrt[6]{\sec^2(e+fx)}}{2\sqrt{3}\,(a^2+b^2)^{13/6} f\sqrt[3]{d\sec(e+fx)}} \\
&- \frac{7ab^{4/3}\text{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sqrt[6]{\sec^2(e+fx)}}{3\,(a^2+b^2)^{13/6} f\sqrt[3]{d\sec(e+fx)}} \\
&+ \frac{7ab^{4/3}\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sqrt[6]{\sec^2(e+fx)}}{12\,(a^2+b^2)^{13/6} f\sqrt[3]{d\sec(e+fx)}} \\
&- \frac{7ab^{4/3}\log\left(\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sqrt[6]{\sec^2(e+fx)}}{12\,(a^2+b^2)^{13/6} f\sqrt[3]{d\sec(e+fx)}} \\
&+ \frac{\text{AppellF1}\left(\frac{1}{2},2,\frac{7}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[6]{\sec^2(e+fx)}\tan(e+fx)}{a^2 f\sqrt[3]{d\sec(e+fx)}} \\
&+ \frac{b^2\,\text{AppellF1}\left(\frac{3}{2},2,\frac{7}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[6]{\sec^2(e+fx)}\tan^3(e+fx)}{3a^4 f\sqrt[3]{d\sec(e+fx)}} \\
&- \frac{ab}{(a^2+b^2) f\sqrt[3]{d\sec(e+fx)}\,(a^2-b^2\tan^2(e+fx))}
\end{aligned}$$

```
[Out] 7*a*b/(a^2+b^2)^2/f/(d*sec(f*x+e))^(1/3)-7/3*a*b^(4/3)*arctanh(b^(1/3)*(sec
(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6))*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(13/6)/f/(
d*sec(f*x+e))^(1/3)+7/12*a*b^(4/3)*ln((a^2+b^2)^(1/3)-b^(1/3)*(a^2+b^2)^(1/
6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)^(1/3))*(sec(f*x+e)^2)^(1/6)/
(a^2+b^2)^(13/6)/f/(d*sec(f*x+e))^(1/3)-7/12*a*b^(4/3)*ln((a^2+b^2)^(1/3)+b
^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)^(1/3))*(
sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(13/6)/f/(d*sec(f*x+e))^(1/3)+7/6*a*b^(4/3)*a
rctan(-1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6)*3^(1/2)
)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(13/6)/f/(d*sec(f*x+e))^(1/3)*3^(1/2)+7/6*
a*b^(4/3)*arctan(1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/
6)*3^(1/2))*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(13/6)/f/(d*sec(f*x+e))^(1/3)*3^
(1/2)+AppellF1(1/2,2,7/6,3/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+e)^2)*(sec(f*x+e
)^2)^(1/6)*tan(f*x+e)/a^2/f/(d*sec(f*x+e))^(1/3)+1/3*b^2*AppellF1(3/2,2,7/6
,5/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+e)^2)*(sec(f*x+e)^2)^(1/6)*tan(f*x+e)^3/
a^4/f/(d*sec(f*x+e))^(1/3)-a*b/(a^2+b^2)/f/(d*sec(f*x+e))^(1/3)/(a^2-b^2*ta
n(f*x+e)^2)
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 715, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules

used = {3593, 771, 440, 455, 44, 53, 65, 302, 648, 632, 210, 642, 214, 524}

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx$$
$$= \frac{\tan(e+fx)\sqrt[6]{\sec^2(e+fx)}\,\text{AppellF1}\left(\frac{1}{2},2,\frac{7}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a^2f\sqrt[3]{d\sec(e+fx)}}$$
$$-\frac{7ab^{4/3}\sqrt[6]{\sec^2(e+fx)}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)}{2\sqrt{3}f\left(a^2+b^2\right)^{13/6}\sqrt[3]{d\sec(e+fx)}}$$
$$+\frac{7ab^{4/3}\sqrt[6]{\sec^2(e+fx)}\arctan\left(\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}f\left(a^2+b^2\right)^{13/6}\sqrt[3]{d\sec(e+fx)}}$$
$$-\frac{7ab^{4/3}\sqrt[6]{\sec^2(e+fx)}\text{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)}{3f\left(a^2+b^2\right)^{13/6}\sqrt[3]{d\sec(e+fx)}}$$
$$+\frac{7ab}{f\left(a^2+b^2\right)^2\sqrt[3]{d\sec(e+fx)}}-\frac{ab}{f\left(a^2+b^2\right)\sqrt[3]{d\sec(e+fx)}\left(a^2-b^2\tan^2(e+fx)\right)}$$
$$+\frac{7ab^{4/3}\sqrt[6]{\sec^2(e+fx)}\log\left(-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+\sqrt[3]{a^2+b^2}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)}{12f\left(a^2+b^2\right)^{13/6}\sqrt[3]{d\sec(e+fx)}}$$
$$-\frac{7ab^{4/3}\sqrt[6]{\sec^2(e+fx)}\log\left(\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+\sqrt[3]{a^2+b^2}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)}{12f\left(a^2+b^2\right)^{13/6}\sqrt[3]{d\sec(e+fx)}}$$
$$+\frac{b^2\tan^3(e+fx)\sqrt[6]{\sec^2(e+fx)}\,\text{AppellF1}\left(\frac{3}{2},2,\frac{7}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{3a^4f\sqrt[3]{d\sec(e+fx)}}$$

```
[In] Int[1/((d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x])^2),x]
```

```
[Out] (7*a*b)/((a^2 + b^2)^2*f*(d*Sec[e + f*x])^(1/3)) - (7*a*b^(4/3)*ArcTan[1/Sq
rt[3] - (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))]*(Se
c[e + f*x]^2)^(1/6))/(2*Sqrt[3]*(a^2 + b^2)^(13/6)*f*(d*Sec[e + f*x])^(1/3)
) + (7*a*b^(4/3)*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqr
t[3]*(a^2 + b^2)^(1/6))]*(Sec[e + f*x]^2)^(1/6))/(2*Sqrt[3]*(a^2 + b^2)^(13
/6)*f*(d*Sec[e + f*x])^(1/3)) - (7*a*b^(4/3)*ArcTanh[(b^(1/3)*(Sec[e + f*x]
^2)^(1/6))/(a^2 + b^2)^(1/6)]*(Sec[e + f*x]^2)^(1/6))/(3*(a^2 + b^2)^(13/6)
*f*(d*Sec[e + f*x])^(1/3)) + (7*a*b^(4/3)*Log[(a^2 + b^2)^(1/3) - b^(1/3)*(
a^2 + b^2)^(1/6)*(Sec[e + f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)]*(
Sec[e + f*x]^2)^(1/6))/(12*(a^2 + b^2)^(13/6)*f*(d*Sec[e + f*x])^(1/3)) - (
7*a*b^(4/3)*Log[(a^2 + b^2)^(1/3) + b^(1/3)*(a^2 + b^2)^(1/6)*(Sec[e + f*x]
^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)]*(Sec[e + f*x]^2)^(1/6))/(12*(a^
2 + b^2)^(13/6)*f*(d*Sec[e + f*x])^(1/3)) + (AppellF1[1/2, 2, 7/6, 3/2, (b^
```

```
2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(1/6)*Tan[e + f*x]
)/(a^2*f*(d*Sec[e + f*x])^(1/3)) + (b^2*AppellF1[3/2, 2, 7/6, 5/2, (b^2*Tan
[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(1/6)*Tan[e + f*x]^3)/(
3*a^4*f*(d*Sec[e + f*x])^(1/3)) - (a*b)/((a^2 + b^2)*f*(d*Sec[e + f*x])^(1/
3)*(a^2 - b^2*Tan[e + f*x]^2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
```

```
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt[6]{\sec^2(e+fx)}\text{Subst}\left(\int \frac{1}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^{7/6}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{\sqrt[6]{\sec^2(e+fx)}\text{Subst}\left(\int \left(\frac{a^2}{(a^2-x^2)^2\left(1+\frac{x^2}{b^2}\right)^{7/6}} - \frac{2ax}{(a^2-x^2)^2\left(1+\frac{x^2}{b^2}\right)^{7/6}} + \frac{x^2}{(-a^2+x^2)^2\left(1+\frac{x^2}{b^2}\right)^{7/6}}\right) dx, x, b\tan(\ldots}{bf\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{\sqrt[6]{\sec^2(e+fx)}\text{Subst}\left(\int \frac{x^2}{(-a^2+x^2)^2\left(1+\frac{x^2}{b^2}\right)^{7/6}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[3]{d\sec(e+fx)}}$$
$$- \frac{\left(2a\sqrt[6]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{x}{(a^2-x^2)^2\left(1+\frac{x^2}{b^2}\right)^{7/6}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[3]{d\sec(e+fx)}}$$
$$+ \frac{\left(a^2\sqrt[6]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{(a^2-x^2)^2\left(1+\frac{x^2}{b^2}\right)^{7/6}}\,dx, x, b\tan(e+fx)\right)}{bf\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{\text{AppellF1}\left(\frac{1}{2},2,\frac{7}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[6]{\sec^2(e+fx)}\tan(e+fx)}{a^2f\sqrt[3]{d\sec(e+fx)}}$$
$$+\frac{b^2\,\text{AppellF1}\left(\frac{3}{2},2,\frac{7}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[6]{\sec^2(e+fx)}\tan^3(e+fx)}{3a^4f\sqrt[3]{d\sec(e+fx)}}$$
$$-\frac{\left(a\sqrt[6]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{(a^2-x)^2\left(1+\frac{x}{b^2}\right)^{7/6}}\,dx,x,b^2\tan^2(e+fx)\right)}{bf\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{\text{AppellF1}\left(\frac{1}{2},2,\frac{7}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[6]{\sec^2(e+fx)}\tan(e+fx)}{a^2f\sqrt[3]{d\sec(e+fx)}}$$
$$+\frac{b^2\,\text{AppellF1}\left(\frac{3}{2},2,\frac{7}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[6]{\sec^2(e+fx)}\tan^3(e+fx)}{3a^4f\sqrt[3]{d\sec(e+fx)}}$$
$$-\frac{ab}{(a^2+b^2)f\sqrt[3]{d\sec(e+fx)}\,(a^2-b^2\tan^2(e+fx))}$$
$$-\frac{\left(7a\sqrt[6]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{(a^2-x)\left(1+\frac{x}{b^2}\right)^{7/6}}\,dx,x,b^2\tan^2(e+fx)\right)}{6b\,(a^2+b^2)\,f\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{7ab}{(a^2+b^2)^2f\sqrt[3]{d\sec(e+fx)}}$$
$$+\frac{\text{AppellF1}\left(\frac{1}{2},2,\frac{7}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[6]{\sec^2(e+fx)}\tan(e+fx)}{a^2f\sqrt[3]{d\sec(e+fx)}}$$
$$+\frac{b^2\,\text{AppellF1}\left(\frac{3}{2},2,\frac{7}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[6]{\sec^2(e+fx)}\tan^3(e+fx)}{3a^4f\sqrt[3]{d\sec(e+fx)}}$$
$$-\frac{ab}{(a^2+b^2)f\sqrt[3]{d\sec(e+fx)}\,(a^2-b^2\tan^2(e+fx))}$$
$$-\frac{\left(7ab\sqrt[6]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{(a^2-x)\sqrt[6]{1+\frac{x}{b^2}}}\,dx,x,b^2\tan^2(e+fx)\right)}{6\,(a^2+b^2)^2f\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{7ab}{(a^2+b^2)^2 f \sqrt[3]{d\sec(e+fx)}} + \frac{\text{AppellF1}\left(\frac{1}{2},2,\frac{7}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[6]{\sec^2(e+fx)}\tan(e+fx)}{a^2 f\sqrt[3]{d\sec(e+fx)}} + \frac{b^2\,\text{AppellF1}\left(\frac{3}{2},2,\frac{7}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[6]{\sec^2(e+fx)}\tan^3(e+fx)}{3a^4 f\sqrt[3]{d\sec(e+fx)}} - \frac{ab}{(a^2+b^2) f\sqrt[3]{d\sec(e+fx)}\,(a^2-b^2\tan^2(e+fx))} - \frac{\left(7ab^3\sqrt[6]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{x^4}{a^2+b^2-b^2x^6}\,dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{(a^2+b^2)^2 f\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{7ab}{(a^2+b^2)^2 f \sqrt[3]{d\sec(e+fx)}} + \frac{\text{AppellF1}\left(\frac{1}{2},2,\frac{7}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[6]{\sec^2(e+fx)}\tan(e+fx)}{a^2 f\sqrt[3]{d\sec(e+fx)}} + \frac{b^2\,\text{AppellF1}\left(\frac{3}{2},2,\frac{7}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[6]{\sec^2(e+fx)}\tan^3(e+fx)}{3a^4 f\sqrt[3]{d\sec(e+fx)}} - \frac{ab}{(a^2+b^2) f\sqrt[3]{d\sec(e+fx)}\,(a^2-b^2\tan^2(e+fx))} - \frac{\left(7ab^{5/3}\sqrt[6]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{-\frac{1}{2}\sqrt[6]{a^2+b^2}-\frac{\sqrt[3]{b}x}{2}}{\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}\,dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{3(a^2+b^2)^{13/6} f\sqrt[3]{d\sec(e+fx)}} - \frac{\left(7ab^{5/3}\sqrt[6]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{-\frac{1}{2}\sqrt[6]{a^2+b^2}+\frac{\sqrt[3]{b}x}{2}}{\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}\,dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{3(a^2+b^2)^{13/6} f\sqrt[3]{d\sec(e+fx)}} - \frac{\left(7ab^{5/3}\sqrt[6]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a^2+b^2}-b^{2/3}x^2}\,dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{3(a^2+b^2)^2 f\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{7ab}{(a^2+b^2)^2 f \sqrt[3]{d\sec(e+fx)}} - \frac{7ab^{4/3}\text{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sqrt[6]{\sec^2(e+fx)}}{3(a^2+b^2)^{13/6} f \sqrt[3]{d\sec(e+fx)}}$$

$$+ \frac{\text{AppellF1}\left(\frac{1}{2},2,\frac{7}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[6]{\sec^2(e+fx)}\tan(e+fx)}{a^2 f\sqrt[3]{d\sec(e+fx)}}$$

$$+ \frac{b^2\,\text{AppellF1}\left(\frac{3}{2},2,\frac{7}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[6]{\sec^2(e+fx)}\tan^3(e+fx)}{3a^4 f\sqrt[3]{d\sec(e+fx)}}$$

$$- \frac{ab}{(a^2+b^2) f\sqrt[3]{d\sec(e+fx)}\,(a^2-b^2\tan^2(e+fx))}$$

$$+ \frac{\left(7ab^{4/3}\sqrt[6]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{-\sqrt[3]{b}\sqrt[6]{a^2+b^2}+2b^{2/3}x}{\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}\,dx, x, \sqrt[6]{\sec^2(e+fx)}\right)}{12(a^2+b^2)^{13/6} f\sqrt[3]{d\sec(e+fx)}}$$

$$- \frac{\left(7ab^{4/3}\sqrt[6]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt[3]{b}\sqrt[6]{a^2+b^2}+2b^{2/3}x}{\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}\,dx, x, \sqrt[6]{\sec^2(e+fx)}\right)}{12(a^2+b^2)^{13/6} f\sqrt[3]{d\sec(e+fx)}}$$

$$+ \frac{\left(7ab^{5/3}\sqrt[6]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}\,dx, x, \sqrt[6]{\sec^2(e+fx)}\right)}{4(a^2+b^2)^2 f\sqrt[3]{d\sec(e+fx)}}$$

$$+ \frac{\left(7ab^{5/3}\sqrt[6]{\sec^2(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}\,dx, x, \sqrt[6]{\sec^2(e+fx)}\right)}{4(a^2+b^2)^2 f\sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{7ab}{(a^2+b^2)^2 f \sqrt[3]{d\sec(e+fx)}} - \frac{7ab^{4/3}\text{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sqrt[6]{\sec^2(e+fx)}}{3(a^2+b^2)^{13/6} f \sqrt[3]{d\sec(e+fx)}}$$

$$+ \frac{7ab^{4/3}\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sqrt[6]{\sec^2(e+fx)}}{12(a^2+b^2)^{13/6} f \sqrt[3]{d\sec(e+fx)}}$$

$$- \frac{7ab^{4/3}\log\left(\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sqrt[6]{\sec^2(e+fx)}}{12(a^2+b^2)^{13/6} f \sqrt[3]{d\sec(e+fx)}}$$

$$+ \frac{\text{AppellF1}\left(\frac{1}{2},2,\frac{7}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[6]{\sec^2(e+fx)}\tan(e+fx)}{a^2 f \sqrt[3]{d\sec(e+fx)}}$$

$$+ \frac{b^2\,\text{AppellF1}\left(\frac{3}{2},2,\frac{7}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[6]{\sec^2(e+fx)}\tan^3(e+fx)}{3a^4 f \sqrt[3]{d\sec(e+fx)}}$$

$$- \frac{ab}{(a^2+b^2) f \sqrt[3]{d\sec(e+fx)}\,(a^2-b^2\tan^2(e+fx))}$$

$$+ \frac{\left(7ab^{4/3}\sqrt[6]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)}{2(a^2+b^2)^{13/6} f \sqrt[3]{d\sec(e+fx)}}$$

$$- \frac{\left(7ab^{4/3}\sqrt[6]{\sec^2(e+fx)}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)}{2(a^2+b^2)^{13/6} f \sqrt[3]{d\sec(e+fx)}}$$

$$= \frac{7ab}{(a^2+b^2)^2 f\sqrt[3]{d\sec(e+fx)}} - \frac{7ab^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}}{\sqrt{3}}\right)\sqrt[6]{\sec^2(e+fx)}}{2\sqrt{3}\,(a^2+b^2)^{13/6} f\sqrt[3]{d\sec(e+fx)}} + \frac{7ab^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}}{\sqrt{3}}\right)\sqrt[6]{\sec^2(e+fx)}}{2\sqrt{3}\,(a^2+b^2)^{13/6} f\sqrt[3]{d\sec(e+fx)}} - \frac{7ab^{4/3}\text{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sqrt[6]{\sec^2(e+fx)}}{3\,(a^2+b^2)^{13/6} f\sqrt[3]{d\sec(e+fx)}} + \frac{7ab^{4/3}\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sqrt[6]{\sec^2(e+fx)}}{12\,(a^2+b^2)^{13/6} f\sqrt[3]{d\sec(e+fx)}} - \frac{7ab^{4/3}\log\left(\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sqrt[6]{\sec^2(e+fx)}}{12\,(a^2+b^2)^{13/6} f\sqrt[3]{d\sec(e+fx)}} + \frac{\text{AppellF1}\left(\frac{1}{2},2,\frac{7}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[6]{\sec^2(e+fx)}\tan(e+fx)}{a^2 f\sqrt[3]{d\sec(e+fx)}} + \frac{b^2\,\text{AppellF1}\left(\frac{3}{2},2,\frac{7}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sqrt[6]{\sec^2(e+fx)}\tan^3(e+fx)}{3a^4 f\sqrt[3]{d\sec(e+fx)}} - \frac{ab}{(a^2+b^2)\,f\sqrt[3]{d\sec(e+fx)}\,(a^2-b^2\tan^2(e+fx))}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 137.80 (sec) , antiderivative size = 18832, normalized size of antiderivative = 26.34

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[1/((d*Sec[e + f*x])^(1/3)*(a + b*Tan[e + f*x])^2),x]
[Out] Result too large to show
```

Maple [F]

$$\int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}\left(a+b\tan\left(fx+e\right)\right)^{2}}dx$$

```
[In] int(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x)
[Out] int(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{\sqrt[3]{d\sec\left(e+fx\right)}\left(a+b\tan\left(e+fx\right)\right)^{2}}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))**(1/3)/(a+b*tan(f*x+e))**2,x)
[Out] Integral(1/((d*sec(e + f*x))**(1/3)*(a + b*tan(e + f*x))**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{\left(d\sec\left(fx+e\right)\right)^{\frac{1}{3}}\left(b\tan\left(fx+e\right)+a\right)^{2}}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
[Out] integrate(1/((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a)^2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx=\int \frac{1}{(d\sec{(fx+e)})^{\frac{1}{3}}\,(b\tan{(fx+e)}+a)^2}\,dx$$

[In] integrate(1/(d*sec(f*x+e))^(1/3)/(a+b*tan(f*x+e))^2,x, algorithm="giac")

[Out] integrate(1/((d*sec(f*x + e))^(1/3)*(b*tan(f*x + e) + a)^2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{d\sec(e+fx)}(a+b\tan(e+fx))^2}\,dx=\int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{1/3}\,(a+b\tan{(e+f\,x)})^2}\,dx$$

[In] int(1/((d/cos(e + f*x))^(1/3)*(a + b*tan(e + f*x))^2),x)

[Out] int(1/((d/cos(e + f*x))^(1/3)*(a + b*tan(e + f*x))^2), x)

3.639 $\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 717

$$\begin{aligned}
\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2}\,dx &= \frac{11ab}{5(a^2+b^2)^2 f(d\sec(e+fx))^{5/3}} \\
&+ \frac{11ab^{8/3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)\sec^2(e+fx)^{5/6}}{2\sqrt{3}(a^2+b^2)^{17/6} f(d\sec(e+fx))^{5/3}} \\
&- \frac{11ab^{8/3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)\sec^2(e+fx)^{5/6}}{2\sqrt{3}(a^2+b^2)^{17/6} f(d\sec(e+fx))^{5/3}} \\
&- \frac{11ab^{8/3}\text{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sec^2(e+fx)^{5/6}}{3(a^2+b^2)^{17/6} f(d\sec(e+fx))^{5/3}} \\
&+ \frac{11ab^{8/3}\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sec^2(e+fx)^{5/6}}{12(a^2+b^2)^{17/6} f(d\sec(e+fx))^{5/3}} \\
&- \frac{11ab^{8/3}\log\left(\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sec^2(e+fx)^{5/6}}{12(a^2+b^2)^{17/6} f(d\sec(e+fx))^{5/3}} \\
&+ \frac{\text{AppellF1}\left(\frac{1}{2},2,\frac{11}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{5/6}\tan(e+fx)}{a^2 f(d\sec(e+fx))^{5/3}} \\
&+ \frac{b^2\,\text{AppellF1}\left(\frac{3}{2},2,\frac{11}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{5/6}\tan^3(e+fx)}{3a^4 f(d\sec(e+fx))^{5/3}} \\
&- \frac{ab}{(a^2+b^2) f(d\sec(e+fx))^{5/3}(a^2-b^2\tan^2(e+fx))}
\end{aligned}$$

```
[Out] 11/5*a*b/(a^2+b^2)^2/f/(d*sec(f*x+e))^(5/3)-11/3*a*b^(8/3)*arctanh(b^(1/3)*
(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6))*(sec(f*x+e)^2)^(5/6)/(a^2+b^2)^(17/6)
/f/(d*sec(f*x+e))^(5/3)+11/12*a*b^(8/3)*ln((a^2+b^2)^(1/3)-b^(1/3)*(a^2+b^2
)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)^(1/3))*(sec(f*x+e)^2)^(
5/6)/(a^2+b^2)^(17/6)/f/(d*sec(f*x+e))^(5/3)-11/12*a*b^(8/3)*ln((a^2+b^2)^(
1/3)+b^(1/3)*(a^2+b^2)^(1/6)*(sec(f*x+e)^2)^(1/6)+b^(2/3)*(sec(f*x+e)^2)^(1
/3))*(sec(f*x+e)^2)^(5/6)/(a^2+b^2)^(17/6)/f/(d*sec(f*x+e))^(5/3)-11/6*a*b^
(8/3)*arctan(-1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+b^2)^(1/6)*
3^(1/2))*(sec(f*x+e)^2)^(5/6)/(a^2+b^2)^(17/6)/f/(d*sec(f*x+e))^(5/3)*3^(1/
2)-11/6*a*b^(8/3)*arctan(1/3*3^(1/2)+2/3*b^(1/3)*(sec(f*x+e)^2)^(1/6)/(a^2+
b^2)^(1/6)*3^(1/2))*(sec(f*x+e)^2)^(5/6)/(a^2+b^2)^(17/6)/f/(d*sec(f*x+e))^
(5/3)*3^(1/2)+AppellF1(1/2,2,11/6,3/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+e)^2)*(
sec(f*x+e)^2)^(5/6)*tan(f*x+e)/a^2/f/(d*sec(f*x+e))^(5/3)+1/3*b^2*AppellF1(
3/2,2,11/6,5/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+e)^2)*(sec(f*x+e)^2)^(5/6)*tan
(f*x+e)^3/a^4/f/(d*sec(f*x+e))^(5/3)-a*b/(a^2+b^2)/f/(d*sec(f*x+e))^(5/3)/(
a^2-b^2*tan(f*x+e)^2)
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 717, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.560, Rules

used = {3593, 771, 440, 455, 44, 53, 65, 216, 648, 632, 210, 642, 214, 524}

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2}\,dx = \frac{\tan(e+fx)\sec^2(e+fx)^{5/6}\operatorname{AppellF1}\left(\frac{1}{2},2,\frac{11}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2}\right.}{a^2f(d\sec(e+fx))^{5/3}}$$

$$+\frac{11ab^{8/3}\sec^2(e+fx)^{5/6}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}\right)}{2\sqrt{3}f\left(a^2+b^2\right)^{17/6}(d\sec(e+fx))^{5/3}}$$

$$-\frac{11ab^{8/3}\sec^2(e+fx)^{5/6}\arctan\left(\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt{3}\sqrt[6]{a^2+b^2}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}f\left(a^2+b^2\right)^{17/6}(d\sec(e+fx))^{5/3}}$$

$$-\frac{11ab^{8/3}\sec^2(e+fx)^{5/6}\operatorname{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)}{3f\left(a^2+b^2\right)^{17/6}(d\sec(e+fx))^{5/3}}$$

$$+\frac{11ab}{5f\left(a^2+b^2\right)^2(d\sec(e+fx))^{5/3}}-\frac{ab}{f\left(a^2+b^2\right)(d\sec(e+fx))^{5/3}\left(a^2-b^2\tan^2(e+fx)\right)}$$

$$+\frac{11ab^{8/3}\sec^2(e+fx)^{5/6}\log\left(-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+\sqrt[3]{a^2+b^2}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)}{12f\left(a^2+b^2\right)^{17/6}(d\sec(e+fx))^{5/3}}$$

$$-\frac{11ab^{8/3}\sec^2(e+fx)^{5/6}\log\left(\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+\sqrt[3]{a^2+b^2}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)}{12f\left(a^2+b^2\right)^{17/6}(d\sec(e+fx))^{5/3}}$$

$$+\frac{b^2\tan^3(e+fx)\sec^2(e+fx)^{5/6}\operatorname{AppellF1}\left(\frac{3}{2},2,\frac{11}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{3a^4f(d\sec(e+fx))^{5/3}}$$

```
[In] Int[1/((d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x])^2),x]
```

```
[Out] (11*a*b)/(5*(a^2 + b^2)^2*f*(d*Sec[e + f*x])^(5/3)) + (11*a*b^(8/3)*ArcTan[
1/Sqrt[3] - (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))/(Sqrt[3]*(a^2 + b^2)^(1/6))]
*(Sec[e + f*x]^2)^(5/6))/(2*Sqrt[3]*(a^2 + b^2)^(17/6)*f*(d*Sec[e + f*x])^(
5/3)) - (11*a*b^(8/3)*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(Sec[e + f*x]^2)^(1/6))
/(Sqrt[3]*(a^2 + b^2)^(1/6))]*(Sec[e + f*x]^2)^(5/6))/(2*Sqrt[3]*(a^2 + b^2
)^(17/6)*f*(d*Sec[e + f*x])^(5/3)) - (11*a*b^(8/3)*ArcTanh[(b^(1/3)*(Sec[e
+ f*x]^2)^(1/6))/(a^2 + b^2)^(1/6)]*(Sec[e + f*x]^2)^(5/6))/(3*(a^2 + b^2)^
(17/6)*f*(d*Sec[e + f*x])^(5/3)) + (11*a*b^(8/3)*Log[(a^2 + b^2)^(1/3) - b^
(1/3)*(a^2 + b^2)^(1/6)*(Sec[e + f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(
1/3)]*(Sec[e + f*x]^2)^(5/6))/(12*(a^2 + b^2)^(17/6)*f*(d*Sec[e + f*x])^(5/
3)) - (11*a*b^(8/3)*Log[(a^2 + b^2)^(1/3) + b^(1/3)*(a^2 + b^2)^(1/6)*(Sec[
e + f*x]^2)^(1/6) + b^(2/3)*(Sec[e + f*x]^2)^(1/3)]*(Sec[e + f*x]^2)^(5/6))
/(12*(a^2 + b^2)^(17/6)*f*(d*Sec[e + f*x])^(5/3)) + (AppellF1[1/2, 2, 11/6,
 3/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(5/6)*Tan
[e + f*x])/(a^2*f*(d*Sec[e + f*x])^(5/3)) + (b^2*AppellF1[3/2, 2, 11/6, 5/2
, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(5/6)*Tan[e +
```

```
f*x]^3)/(3*a^4*f*(d*Sec[e + f*x])^(5/3)) - (a*b)/((a^2 + b^2)*f*(d*Sec[e +
f*x])^(5/3)*(a^2 - b^2*Tan[e + f*x]^2))
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
```

```
gerQ[p] && ILtQ[m, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] && !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\sec^2(e+fx)^{5/6}\text{Subst}\left(\int \frac{1}{(a+x)^2\left(1+\frac{x^2}{b^2}\right)^{11/6}}\,dx, x, b\tan(e+fx)\right)}{bf(d\sec(e+fx))^{5/3}}$$

$$= \frac{\sec^2(e+fx)^{5/6}\text{Subst}\left(\int \left(\frac{a^2}{(a^2-x^2)^2\left(1+\frac{x^2}{b^2}\right)^{11/6}} - \frac{2ax}{(a^2-x^2)^2\left(1+\frac{x^2}{b^2}\right)^{11/6}} + \frac{x^2}{(-a^2+x^2)^2\left(1+\frac{x^2}{b^2}\right)^{11/6}}\right) dx, x, b\tan}{bf(d\sec(e+fx))^{5/3}}$$

$$= \frac{\sec^2(e+fx)^{5/6}\text{Subst}\left(\int \frac{x^2}{(-a^2+x^2)^2\left(1+\frac{x^2}{b^2}\right)^{11/6}}\,dx, x, b\tan(e+fx)\right)}{bf(d\sec(e+fx))^{5/3}}$$
$$- \frac{\left(2a\sec^2(e+fx)^{5/6}\right)\text{Subst}\left(\int \frac{x}{(a^2-x^2)^2\left(1+\frac{x^2}{b^2}\right)^{11/6}}\,dx, x, b\tan(e+fx)\right)}{bf(d\sec(e+fx))^{5/3}}$$
$$+ \frac{\left(a^2\sec^2(e+fx)^{5/6}\right)\text{Subst}\left(\int \frac{1}{(a^2-x^2)^2\left(1+\frac{x^2}{b^2}\right)^{11/6}}\,dx, x, b\tan(e+fx)\right)}{bf(d\sec(e+fx))^{5/3}}$$

$$= \frac{\text{AppellF1}\left(\frac{1}{2}, 2, \frac{11}{6}, \frac{3}{2}, \frac{b^2\tan^2(e+fx)}{a^2}, -\tan^2(e+fx)\right)\sec^2(e+fx)^{5/6}\tan(e+fx)}{a^2f(d\sec(e+fx))^{5/3}}$$
$$+ \frac{b^2\,\text{AppellF1}\left(\frac{3}{2}, 2, \frac{11}{6}, \frac{5}{2}, \frac{b^2\tan^2(e+fx)}{a^2}, -\tan^2(e+fx)\right)\sec^2(e+fx)^{5/6}\tan^3(e+fx)}{3a^4f(d\sec(e+fx))^{5/3}}$$
$$- \frac{\left(a\sec^2(e+fx)^{5/6}\right)\text{Subst}\left(\int \frac{1}{(a^2-x)^2\left(1+\frac{x}{b^2}\right)^{11/6}}\,dx, x, b^2\tan^2(e+fx)\right)}{bf(d\sec(e+fx))^{5/3}}$$

$$= \frac{\text{AppellF1}\left(\frac{1}{2},2,\frac{11}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{5/6}\tan(e+fx)}{a^2f(d\sec(e+fx))^{5/3}} + \frac{b^2\,\text{AppellF1}\left(\frac{3}{2},2,\frac{11}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{5/6}\tan^3(e+fx)}{3a^4f(d\sec(e+fx))^{5/3}} - \frac{ab}{(a^2+b^2)\,f(d\sec(e+fx))^{5/3}\,(a^2-b^2\tan^2(e+fx))} - \frac{(11a\sec^2(e+fx)^{5/6})\,\text{Subst}\left(\int\frac{1}{(a^2-x)\left(1+\frac{x}{b^2}\right)^{11/6}}\,dx,x,b^2\tan^2(e+fx)\right)}{6b\,(a^2+b^2)\,f(d\sec(e+fx))^{5/3}}$$

$$= \frac{11ab}{5\,(a^2+b^2)^2\,f(d\sec(e+fx))^{5/3}} + \frac{\text{AppellF1}\left(\frac{1}{2},2,\frac{11}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{5/6}\tan(e+fx)}{a^2f(d\sec(e+fx))^{5/3}} + \frac{b^2\,\text{AppellF1}\left(\frac{3}{2},2,\frac{11}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{5/6}\tan^3(e+fx)}{3a^4f(d\sec(e+fx))^{5/3}} - \frac{ab}{(a^2+b^2)\,f(d\sec(e+fx))^{5/3}\,(a^2-b^2\tan^2(e+fx))} - \frac{(11ab\sec^2(e+fx)^{5/6})\,\text{Subst}\left(\int\frac{1}{(a^2-x)\left(1+\frac{x}{b^2}\right)^{5/6}}\,dx,x,b^2\tan^2(e+fx)\right)}{6\,(a^2+b^2)^2\,f(d\sec(e+fx))^{5/3}}$$

$$= \frac{11ab}{5\,(a^2+b^2)^2\,f(d\sec(e+fx))^{5/3}} + \frac{\text{AppellF1}\left(\frac{1}{2},2,\frac{11}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{5/6}\tan(e+fx)}{a^2f(d\sec(e+fx))^{5/3}} + \frac{b^2\,\text{AppellF1}\left(\frac{3}{2},2,\frac{11}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{5/6}\tan^3(e+fx)}{3a^4f(d\sec(e+fx))^{5/3}} - \frac{ab}{(a^2+b^2)\,f(d\sec(e+fx))^{5/3}\,(a^2-b^2\tan^2(e+fx))} - \frac{(11ab^3\sec^2(e+fx)^{5/6})\,\text{Subst}\left(\int\frac{1}{a^2+b^2-b^2x^6}\,dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{(a^2+b^2)^2\,f(d\sec(e+fx))^{5/3}}$$

$$= \frac{11ab}{5\left(a^2+b^2\right)^2 f(d\sec(e+fx))^{5/3}}$$
$$+ \frac{\text{AppellF1}\left(\frac{1}{2},2,\frac{11}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{5/6}\tan(e+fx)}{a^2 f(d\sec(e+fx))^{5/3}}$$
$$+ \frac{b^2\,\text{AppellF1}\left(\frac{3}{2},2,\frac{11}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{5/6}\tan^3(e+fx)}{3a^4 f(d\sec(e+fx))^{5/3}}$$
$$- \frac{ab}{\left(a^2+b^2\right) f(d\sec(e+fx))^{5/3}\left(a^2-b^2\tan^2(e+fx)\right)}$$
$$- \frac{\left(11ab^3\sec^2(e+fx)^{5/6}\right)\text{Subst}\left(\int \frac{\sqrt[6]{a^2+b^2}-\frac{\sqrt[3]{b}x}{2}}{\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}\,dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{3\left(a^2+b^2\right)^{17/6} f(d\sec(e+fx))^{5/3}}$$
$$- \frac{\left(11ab^3\sec^2(e+fx)^{5/6}\right)\text{Subst}\left(\int \frac{\sqrt[6]{a^2+b^2}+\frac{\sqrt[3]{b}x}{2}}{\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}\,dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{3\left(a^2+b^2\right)^{17/6} f(d\sec(e+fx))^{5/3}}$$
$$- \frac{\left(11ab^3\sec^2(e+fx)^{5/6}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a^2+b^2}-b^{2/3}x^2}\,dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{3\left(a^2+b^2\right)^{8/3} f(d\sec(e+fx))^{5/3}}$$

$$= \frac{11ab}{5\left(a^2+b^2\right)^2 f(d\sec(e+fx))^{5/3}}$$
$$- \frac{11ab^{8/3}\operatorname{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sec^2(e+fx)^{5/6}}{3\left(a^2+b^2\right)^{17/6} f(d\sec(e+fx))^{5/3}}$$
$$+ \frac{\operatorname{AppellF1}\left(\frac{1}{2},2,\frac{11}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{5/6}\tan(e+fx)}{a^2 f(d\sec(e+fx))^{5/3}}$$
$$+ \frac{b^2\operatorname{AppellF1}\left(\frac{3}{2},2,\frac{11}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{5/6}\tan^3(e+fx)}{3a^4 f(d\sec(e+fx))^{5/3}}$$
$$- \frac{ab}{\left(a^2+b^2\right) f(d\sec(e+fx))^{5/3}\left(a^2-b^2\tan^2(e+fx)\right)}$$
$$+ \frac{\left(11ab^{8/3}\sec^2(e+fx)^{5/6}\right)\operatorname{Subst}\left(\int \frac{-\sqrt[3]{b}\sqrt[6]{a^2+b^2}+2b^{2/3}x}{\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}\,dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{12\left(a^2+b^2\right)^{17/6} f(d\sec(e+fx))^{5/3}}$$
$$- \frac{\left(11ab^{8/3}\sec^2(e+fx)^{5/6}\right)\operatorname{Subst}\left(\int \frac{\sqrt[3]{b}\sqrt[6]{a^2+b^2}+2b^{2/3}x}{\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}\,dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{12\left(a^2+b^2\right)^{17/6} f(d\sec(e+fx))^{5/3}}$$
$$- \frac{\left(11ab^3\sec^2(e+fx)^{5/6}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}\,dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{4\left(a^2+b^2\right)^{8/3} f(d\sec(e+fx))^{5/3}}$$
$$- \frac{\left(11ab^3\sec^2(e+fx)^{5/6}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}x+b^{2/3}x^2}\,dx,x,\sqrt[6]{\sec^2(e+fx)}\right)}{4\left(a^2+b^2\right)^{8/3} f(d\sec(e+fx))^{5/3}}$$

$$
\begin{aligned}
&= \frac{11ab}{5\left(a^2+b^2\right)^2 f(d\sec(e+fx))^{5/3}} \\
&\quad - \frac{11ab^{8/3}\operatorname{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sec^2(e+fx)^{5/6}}{3\left(a^2+b^2\right)^{17/6} f(d\sec(e+fx))^{5/3}} \\
&\quad + \frac{11ab^{8/3}\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sec^2(e+fx)^{5/6}}{12\left(a^2+b^2\right)^{17/6} f(d\sec(e+fx))^{5/3}} \\
&\quad - \frac{11ab^{8/3}\log\left(\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sec^2(e+fx)^{5/6}}{12\left(a^2+b^2\right)^{17/6} f(d\sec(e+fx))^{5/3}} \\
&\quad + \frac{\operatorname{AppellF1}\left(\frac{1}{2},2,\frac{11}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{5/6}\tan(e+fx)}{a^2 f(d\sec(e+fx))^{5/3}} \\
&\quad + \frac{b^2\operatorname{AppellF1}\left(\frac{3}{2},2,\frac{11}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{5/6}\tan^3(e+fx)}{3a^4 f(d\sec(e+fx))^{5/3}} \\
&\quad - \frac{ab}{\left(a^2+b^2\right) f(d\sec(e+fx))^{5/3}\left(a^2-b^2\tan^2(e+fx)\right)} \\
&\quad - \frac{\left(11ab^{8/3}\sec^2(e+fx)^{5/6}\right)\operatorname{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{17/6} f(d\sec(e+fx))^{5/3}} \\
&\quad + \frac{\left(11ab^{8/3}\sec^2(e+fx)^{5/6}\right)\operatorname{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{17/6} f(d\sec(e+fx))^{5/3}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{11ab}{5\left(a^2+b^2\right)^2 f(d\sec(e+fx))^{5/3}} \\
&+ \frac{11ab^{8/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}}{\sqrt{3}}\right)\sec^2(e+fx)^{5/6}}{2\sqrt{3}\left(a^2+b^2\right)^{17/6} f(d\sec(e+fx))^{5/3}} \\
&- \frac{11ab^{8/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}}{\sqrt{3}}\right)\sec^2(e+fx)^{5/6}}{2\sqrt{3}\left(a^2+b^2\right)^{17/6} f(d\sec(e+fx))^{5/3}} \\
&- \frac{11ab^{8/3}\text{arctanh}\left(\frac{\sqrt[3]{b}\sqrt[6]{\sec^2(e+fx)}}{\sqrt[6]{a^2+b^2}}\right)\sec^2(e+fx)^{5/6}}{3\left(a^2+b^2\right)^{17/6} f(d\sec(e+fx))^{5/3}} \\
&+ \frac{11ab^{8/3}\log\left(\sqrt[3]{a^2+b^2}-\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sec^2(e+fx)^{5/6}}{12\left(a^2+b^2\right)^{17/6} f(d\sec(e+fx))^{5/3}} \\
&- \frac{11ab^{8/3}\log\left(\sqrt[3]{a^2+b^2}+\sqrt[3]{b}\sqrt[6]{a^2+b^2}\sqrt[6]{\sec^2(e+fx)}+b^{2/3}\sqrt[3]{\sec^2(e+fx)}\right)\sec^2(e+fx)^{5/6}}{12\left(a^2+b^2\right)^{17/6} f(d\sec(e+fx))^{5/3}} \\
&+ \frac{\text{AppellF1}\left(\frac{1}{2},2,\frac{11}{6},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{5/6}\tan(e+fx)}{a^2 f(d\sec(e+fx))^{5/3}} \\
&+ \frac{b^2\,\text{AppellF1}\left(\frac{3}{2},2,\frac{11}{6},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{5/6}\tan^3(e+fx)}{3a^4 f(d\sec(e+fx))^{5/3}} \\
&- \frac{ab}{(a^2+b^2)\, f(d\sec(e+fx))^{5/3}\,(a^2-b^2\tan^2(e+fx))}
\end{aligned}
$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 11783 vs. $2(717) = 1434$.

Time = 81.40 (sec) , antiderivative size = 11783, normalized size of antiderivative = 16.43

$$
\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2}\,dx = \text{Result too large to show}
$$

```
[In] Integrate[1/((d*Sec[e + f*x])^(5/3)*(a + b*Tan[e + f*x])^2),x]
[Out] Result too large to show
```

Maple [F]

$$\int \frac{1}{\left(d \sec \left(f x+e\right)\right)^{\frac{5}{3}} \left(a+b \tan \left(f x+e\right)\right)^{2}}d x$$

```
[In] int(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x)
[Out] int(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{\left(d \sec \left(e+f x\right)\right)^{\frac{5}{3}} \left(a+b \tan \left(e+f x\right)\right)^{2}}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))**(5/3)/(a+b*tan(f*x+e))**2,x)
[Out] Integral(1/((d*sec(e + f*x))**(5/3)*(a + b*tan(e + f*x))**2), x)
```

Maxima [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{\left(d \sec \left(f x+e\right)\right)^{\frac{5}{3}} \left(b \tan \left(f x+e\right)+a\right)^{2}}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
[Out] integrate(1/((d*sec(f*x + e))^(5/3)*(b*tan(f*x + e) + a)^2), x)
```

Giac [F]

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{(d\sec(fx+e))^{\frac{5}{3}}(b\tan(fx+e)+a)^2}\,dx$$

```
[In] integrate(1/(d*sec(f*x+e))^(5/3)/(a+b*tan(f*x+e))^2,x, algorithm="giac")
[Out] integrate(1/((d*sec(f*x + e))^(5/3)*(b*tan(f*x + e) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d\sec(e+fx))^{5/3}(a+b\tan(e+fx))^2}\,dx = \int \frac{1}{\left(\frac{d}{\cos(e+f\,x)}\right)^{5/3}(a+b\tan(e+f\,x))^2}\,dx$$

```
[In] int(1/((d/cos(e + f*x))^(5/3)*(a + b*tan(e + f*x))^2),x)
[Out] int(1/((d/cos(e + f*x))^(5/3)*(a + b*tan(e + f*x))^2), x)
```

3.640 $\int (d\sec(e+fx))^m(a+b\tan(e+fx))^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 173

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^3\,dx = -\frac{a(3b^2-a^2(1+m))\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-\frac{m}{2},\frac{3}{2},-\tan^2(e+fx)\right)(d\sec(e+fx))^m\sec^2(e+fx)^{-m/2}}{f(1+m)} + \frac{b(d\sec(e+fx))^m(a+b\tan(e+fx))^2}{f(2+m)} - \frac{b(d\sec(e+fx))^m\,(2(1+m)\,(b^2-a^2(3+m))-abm(4+m)\tan(e+fx))}{fm\,(2+3m+m^2)}$$

```
[Out] -a*(3*b^2-a^2*(1+m))*hypergeom([1/2, 1-1/2*m],[3/2],-tan(f*x+e)^2)*(d*sec(f
*x+e))^m*tan(f*x+e)/f/(1+m)/((sec(f*x+e)^2)^(1/2*m))+b*(d*sec(f*x+e))^m*(a+
b*tan(f*x+e))^2/f/(2+m)-b*(d*sec(f*x+e))^m*(2*(1+m)*(b^2-a^2*(3+m))-a*b*m*(
4+m)*tan(f*x+e))/f/m/(m^2+3*m+2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used

$= \{3593, 757, 794, 251\}$

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^3\,dx$$

$$= \frac{a\left(a^2-\frac{3b^2}{m+1}\right)\tan(e+fx)\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},1-\frac{m}{2},\frac{3}{2},-\tan^2(e+\right.}{f}$$
$$-\frac{b(d\sec(e+fx))^m\,(2(m+1)\,(b^2-a^2(m+3))-abm(m+4)\tan(e+fx))}{fm\,(m^2+3m+2)}$$
$$+\frac{b(a+b\tan(e+fx))^2(d\sec(e+fx))^m}{f(m+2)}$$

```
[In] Int[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (a*(a^2 - (3*b^2)/(1 + m))*Hypergeometric2F1[1/2, 1 - m/2, 3/2, -Tan[e + f*
x]^2]*(d*Sec[e + f*x])^m*Tan[e + f*x])/(f*(Sec[e + f*x]^2)^(m/2)) + (b*(d*S
ec[e + f*x])^m*(a + b*Tan[e + f*x])^2)/(f*(2 + m)) - (b*(d*Sec[e + f*x])^m*
(2*(1 + m)*(b^2 - a^2*(3 + m)) - a*b*m*(4 + m)*Tan[e + f*x]))/(f*m*(2 + 3*m
 + m^2))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
```

```
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

integral

$$= \frac{\left((d\sec(e+fx))^m \sec^2(e+fx)^{-m/2}\right) \text{Subst}\left(\int (a+x)^3 \left(1+\frac{x^2}{b^2}\right)^{-1+\frac{m}{2}} dx, x, b\tan(e+fx)\right)}{bf}$$

$$= \frac{b(d\sec(e+fx))^m(a+b\tan(e+fx))^2}{f(2+m)} + \frac{\left(b(d\sec(e+fx))^m \sec^2(e+fx)^{-m/2}\right) \text{Subst}\left(\int (a+x) \left(-2+\frac{a^2(2+m)}{b^2}+\frac{a(4+m)x}{b^2}\right) \left(1+\frac{x^2}{b^2}\right)^{-1+\frac{m}{2}}\right.}{f(2+m)}$$

$$= \frac{b(d\sec(e+fx))^m(a+b\tan(e+fx))^2}{f(2+m)} - \frac{b(d\sec(e+fx))^m \left(2(1+m)\left(b^2-a^2(3+m)\right)-abm(4+m)\tan(e+fx)\right)}{fm\left(2+3m+m^2\right)} - \frac{\left(a\left(3b^2-a^2(1+m)\right)\left(d\sec(e+fx)\right)^m \sec^2(e+fx)^{-m/2}\right) \text{Subst}\left(\int \left(1+\frac{x^2}{b^2}\right)^{-1+\frac{m}{2}} dx, x, b\tan(e+\right.}{bf(1+m)}$$

$$= -\frac{a\left(3b^2-a^2(1+m)\right) \text{Hypergeometric2F1}\left(\frac{1}{2},1-\frac{m}{2},\frac{3}{2},-\tan^2(e+fx)\right)\left(d\sec(e+fx)\right)^m \sec^2(e+}{f(1+m)} + \frac{b(d\sec(e+fx))^m(a+b\tan(e+fx))^2}{f(2+m)} - \frac{b(d\sec(e+fx))^m \left(2(1+m)\left(b^2-a^2(3+m)\right)-abm(4+m)\tan(e+fx)\right)}{fm\left(2+3m+m^2\right)}$$

Mathematica [A] (verified)

Time = 2.15 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.92

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^3\,dx$$

$$= \frac{(d\sec(e+fx))^m \left(3ab^2(2+m)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m}{2},\frac{2+m}{2},\sec^2(e+fx)\right)\tan(e+fx)-a^3(2+m)\,\text{H}\right.}{fm(2+}$$

```
[In] Integrate[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^3,x]
```

```
[Out] ((d*Sec[e + f*x])^m*(3*a*b^2*(2 + m)*Hypergeometric2F1[-1/2, m/2, (2 + m)/2
, Sec[e + f*x]^2]*Tan[e + f*x] - a^3*(2 + m)*Hypergeometric2F1[1/2, m/2, (2
 + m)/2, Sec[e + f*x]^2]*Tan[e + f*x] + b*((3*a^2 - b^2)*(2 + m) + b^2*m*Se
c[e + f*x]^2)*Sqrt[-Tan[e + f*x]^2]))/(f*m*(2 + m)*Sqrt[-Tan[e + f*x]^2])
```

Maple [F]

$$\int (d\sec(fx+e))^m (a+b\tan(fx+e))^3 \, dx$$

```
[In] int((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^3,x)
```

```
[Out] int((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^3,x)
```

Fricas [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^3 \, dx = \int (b\tan(fx+e)+a)^3(d\sec(fx+e))^m \, dx$$

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*tan(f*x + e)^3 + 3*a*b^2*tan(f*x + e)^2 + 3*a^2*b*tan(f*x + e
) + a^3)*(d*sec(f*x + e))^m, x)
```

Sympy [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^3 \, dx = \int (d\sec(e+fx))^m (a+b\tan(e+fx))^3 \, dx$$

```
[In] integrate((d*sec(f*x+e))**m*(a+b*tan(f*x+e))**3,x)
```

```
[Out] Integral((d*sec(e + f*x))**m*(a + b*tan(e + f*x))**3, x)
```

Maxima [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^3 \, dx = \int (b\tan(fx+e)+a)^3(d\sec(fx+e))^m \, dx$$

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*tan(f*x + e) + a)^3*(d*sec(f*x + e))^m, x)
```

Giac [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^3\,dx = \int (b\tan(fx+e)+a)^3(d\sec(fx+e))^m\,dx$$

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^3,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)^3*(d*sec(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^3\,dx = \int \left(\frac{d}{\cos(e+f\,x)}\right)^m(a+b\tan(e+f\,x))^3\,dx$$

```
[In] int((d/cos(e + f*x))^m*(a + b*tan(e + f*x))^3,x)
[Out] int((d/cos(e + f*x))^m*(a + b*tan(e + f*x))^3, x)
```

3.641 $\int (d\sec(e+fx))^m(a+b\tan(e+fx))^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 147

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^2\,dx = \frac{ab(2+m)(d\sec(e+fx))^m}{fm(1+m)} + \frac{d(b^2-a^2(1+m))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-m}{2},\frac{3-m}{2},\cos^2(e+fx)\right)(d\sec(e+fx))^{-1+m}\sin(e+fx)}{f(1-m)(1+m)\sqrt{\sin^2(e+fx)}} + \frac{b(d\sec(e+fx))^m(a+b\tan(e+fx))}{f(1+m)}$$

```
[Out] a*b*(2+m)*(d*sec(f*x+e))^m/f/m/(1+m)+d*(b^2-a^2*(1+m))*hypergeom([1/2, 1/2-
1/2*m],[3/2-1/2*m],cos(f*x+e)^2)*(d*sec(f*x+e))^(-1+m)*sin(f*x+e)/f/(-m^2+1
)/(sin(f*x+e)^2)^(1/2)+b*(d*sec(f*x+e))^m*(a+b*tan(f*x+e))/f/(1+m)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3589, 3567, 3857, 2722}

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^2\,dx = \frac{d(b^2-a^2(m+1))\sin(e+fx)(d\sec(e+fx))^{m-1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-m}{2},\frac{3-m}{2},\cos^2(e+fx)\right)}{f(1-m)(m+1)\sqrt{\sin^2(e+fx)}} + \frac{ab(m+2)(d\sec(e+fx))^m}{fm(m+1)} + \frac{b(a+b\tan(e+fx))(d\sec(e+fx))^m}{f(m+1)}$$

```
[In] Int[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (a*b*(2 + m)*(d*Sec[e + f*x])^m)/(f*m*(1 + m)) + (d*(b^2 - a^2*(1 + m))*Hyp
ergeometric2F1[1/2, (1 - m)/2, (3 - m)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^
(-1 + m)*Sin[e + f*x])/(f*(1 - m)*(1 + m)*Sqrt[Sin[e + f*x]^2]) + (b*(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x]))/(f*(1 + m))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b(d\sec(e+fx))^m(a+b\tan(e+fx))}{f(1+m)} \\
&\quad + \frac{\int (d\sec(e+fx))^m\left(-b^2+a^2(1+m)+ab(2+m)\tan(e+fx)\right)\,dx}{1+m} \\
&= \frac{ab(2+m)(d\sec(e+fx))^m}{fm(1+m)}+\frac{b(d\sec(e+fx))^m(a+b\tan(e+fx))}{f(1+m)} \\
&\quad + \left(a^2-\frac{b^2}{1+m}\right)\int (d\sec(e+fx))^m\,dx
\end{aligned}$$

$$= \frac{ab(2+m)(d\sec(e+fx))^m}{fm(1+m)} + \frac{b(d\sec(e+fx))^m(a+b\tan(e+fx))}{f(1+m)} + \left(\left(a^2-\frac{b^2}{1+m}\right)\left(\frac{\cos(e+fx)}{d}\right)^m(d\sec(e+fx))^m\right)\int\left(\frac{\cos(e+fx)}{d}\right)^{-m}dx$$

$$= \frac{ab(2+m)(d\sec(e+fx))^m}{fm(1+m)} - \frac{\left(a^2-\frac{b^2}{1+m}\right)\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-m}{2},\frac{3-m}{2},\cos^2(e+fx)\right)(d\sec(e+fx))^m\sin(}{f(1-m)\sqrt{\sin^2(e+fx)}} + \frac{b(d\sec(e+fx))^m(a+b\tan(e+fx))}{f(1+m)}$$

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.81

$$\int(d\sec(e+fx))^m(a+b\tan(e+fx))^2\,dx = \frac{(d\sec(e+fx))^m\left(b^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m}{2},\frac{2+m}{2},\sec^2(e+fx)\right)\tan(e+fx)+a\Big(-a\,\text{Hypergeometr}}{fm\sqrt{-\tan^2(e+fx)}}$$

```
[In] Integrate[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] ((d*Sec[e + f*x])^m*(b^2*Hypergeometric2F1[-1/2, m/2, (2 + m)/2, Sec[e + f*
x]^2]*Tan[e + f*x] + a*(-(a*Hypergeometric2F1[1/2, m/2, (2 + m)/2, Sec[e +
f*x]^2]*Tan[e + f*x]) + 2*b*Sqrt[-Tan[e + f*x]^2])))/(f*m*Sqrt[-Tan[e + f*x
]^2])
```

Maple [F]

$$\int(d\sec(fx+e))^m(a+b\tan(fx+e))^2\,dx$$

```
[In] int((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^2,x)
```

```
[Out] int((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^2,x)
```

Fricas [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^2\,dx=\int (b\tan(fx+e)+a)^2(d\sec(fx+e))^m\,dx$$

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
[Out] integral((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)*(d*sec(f*x + e))^m
, x)
```

Sympy [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^2\,dx=\int (d\sec(e+fx))^m\,(a+b\tan(e+fx))^2\,dx$$

```
[In] integrate((d*sec(f*x+e))**m*(a+b*tan(f*x+e))**2,x)
[Out] Integral((d*sec(e + f*x))**m*(a + b*tan(e + f*x))**2, x)
```

Maxima [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^2\,dx=\int (b\tan(fx+e)+a)^2(d\sec(fx+e))^m\,dx$$

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e) + a)^2*(d*sec(f*x + e))^m, x)
```

Giac [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^2\,dx=\int (b\tan(fx+e)+a)^2(d\sec(fx+e))^m\,dx$$

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^2,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)^2*(d*sec(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^2\,dx = \int \left(\frac{d}{\cos\left(e+f\,x\right)}\right)^m\,\left(a+b\tan(e+f\,x)\right)^2\,dx$$

```
[In] int((d/cos(e + f*x))^m*(a + b*tan(e + f*x))^2,x)
[Out] int((d/cos(e + f*x))^m*(a + b*tan(e + f*x))^2, x)
```

3.642 $\int (d\sec(e+fx))^m(a+b\tan(e+fx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 93

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))\,dx = \frac{b(d\sec(e+fx))^m}{fm} - \frac{ad\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-m}{2},\frac{3-m}{2},\cos^2(e+fx)\right)(d\sec(e+fx))^{-1+m}\sin(e+fx)}{f(1-m)\sqrt{\sin^2(e+fx)}}$$

```
[Out] b*(d*sec(f*x+e))^m/f/m-a*d*hypergeom([1/2, 1/2-1/2*m],[3/2-1/2*m],cos(f*x+e
)^2)*(d*sec(f*x+e))^(-1+m)*sin(f*x+e)/f/(1-m)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3567, 3857, 2722}

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))\,dx = \frac{b(d\sec(e+fx))^m}{fm} - \frac{ad\sin(e+fx)(d\sec(e+fx))^{m-1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-m}{2},\frac{3-m}{2},\cos^2(e+fx)\right)}{f(1-m)\sqrt{\sin^2(e+fx)}}$$

```
[In] Int[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x]),x]
```

```
[Out] (b*(d*Sec[e + f*x])^m)/(f*m) - (a*d*Hypergeometric2F1[1/2, (1 - m)/2, (3 -
m)/2, Cos[e + f*x]^2]*(d*Sec[e + f*x])^(-1 + m)*Sin[e + f*x])/(f*(1 - m)*Sq
rt[Sin[e + f*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b(d\sec(e+fx))^m}{fm} + a\int (d\sec(e+fx))^m\,dx \\
&= \frac{b(d\sec(e+fx))^m}{fm} + \left(a\left(\frac{\cos(e+fx)}{d}\right)^m (d\sec(e+fx))^m\right)\int \left(\frac{\cos(e+fx)}{d}\right)^{-m} dx \\
&= \frac{b(d\sec(e+fx))^m}{fm} \\
&\quad - \frac{a\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-m}{2},\frac{3-m}{2},\cos^2(e+fx)\right)(d\sec(e+fx))^m\sin(e+fx)}{f(1-m)\sqrt{\sin^2(e+fx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.70

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))\,dx$$

$$= \frac{(d\sec(e+fx))^m\left(b+a\cot(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m}{2},\frac{2+m}{2},\sec^2(e+fx)\right)\sqrt{-\tan^2(e+fx)}\right)}{fm}$$

```
[In] Integrate[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x]),x]
[Out] ((d*Sec[e + f*x])^m*(b + a*Cot[e + f*x]*Hypergeometric2F1[1/2, m/2, (2 + m)
/2, Sec[e + f*x]^2]*Sqrt[-Tan[e + f*x]^2]))/(f*m)
```

Maple [F]

$$\int (d\sec(fx+e))^m (a+b\tan(fx+e))\,dx$$

```
[In] int((d*sec(f*x+e))^m*(a+b*tan(f*x+e)),x)
[Out] int((d*sec(f*x+e))^m*(a+b*tan(f*x+e)),x)
```

Fricas [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))\,dx = \int (b\tan(fx+e)+a)(d\sec(fx+e))^m\,dx$$

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e)),x, algorithm="fricas")
[Out] integral((b*tan(f*x + e) + a)*(d*sec(f*x + e))^m, x)
```

Sympy [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))\,dx = \int (d\sec(e+fx))^m (a+b\tan(e+fx))\,dx$$

```
[In] integrate((d*sec(f*x+e))**m*(a+b*tan(f*x+e)),x)
[Out] Integral((d*sec(e + f*x))**m*(a + b*tan(e + f*x)), x)
```

Maxima [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))\,dx = \int (b\tan(fx+e)+a)(d\sec(fx+e))^m\,dx$$

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e)),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e) + a)*(d*sec(f*x + e))^m, x)
```

Giac [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))\,dx = \int (b\tan(fx+e)+a)(d\sec(fx+e))^m\,dx$$

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e)),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)*(d*sec(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))\,dx = \int \left(\frac{d}{\cos(e+f\,x)}\right)^m (a+b\tan(e+f\,x))\,dx$$

```
[In] int((d/cos(e + f*x))^m*(a + b*tan(e + f*x)),x)
[Out] int((d/cos(e + f*x))^m*(a + b*tan(e + f*x)), x)
```

3.643 $\int \frac{(d\sec(e+fx))^m}{a+b\tan(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 141

$$\int \frac{(d\sec(e+fx))^m}{a+b\tan(e+fx)}\,dx=-\frac{b\,\text{Hypergeometric2F1}\left(1,\frac{m}{2},\frac{2+m}{2},\frac{b^2\sec^2(e+fx)}{a^2+b^2}\right)(d\sec(e+fx))^m}{(a^2+b^2)\,fm}+\frac{\text{AppellF1}\left(\frac{1}{2},1,1-\frac{m}{2},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^m\sec^2(e+fx)^{-m/2}\tan(e+fx)}{af}$$

```
[Out] -b*hypergeom([1, 1/2*m],[1+1/2*m],b^2*sec(f*x+e)^2/(a^2+b^2))*(d*sec(f*x+e)
)^m/(a^2+b^2)/f/m+AppellF1(1/2,1,1-1/2*m,3/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+
e)^2)*(d*sec(f*x+e))^m*tan(f*x+e)/a/f/((sec(f*x+e)^2)^(1/2*m))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3593, 771, 440, 455, 70}

$$\begin{gathered}\int \frac{(d\sec(e+fx))^m}{a+b\tan(e+fx)}\,dx\\ =\frac{\tan(e+fx)\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m\,\text{AppellF1}\left(\frac{1}{2},1,1-\frac{m}{2},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{af}\\ -\frac{b(d\sec(e+fx))^m\,\text{Hypergeometric2F1}\left(1,\frac{m}{2},\frac{m+2}{2},\frac{b^2\sec^2(e+fx)}{a^2+b^2}\right)}{fm\,(a^2+b^2)}\end{gathered}$$

```
[In] Int[(d*Sec[e + f*x])^m/(a + b*Tan[e + f*x]),x]
```

```
[Out] -((b*Hypergeometric2F1[1, m/2, (2 + m)/2, (b^2*Sec[e + f*x]^2)/(a^2 + b^2)]
*(d*Sec[e + f*x])^m)/((a^2 + b^2)*f*m)) + (AppellF1[1/2, 1, 1 - m/2, 3/2, (
b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Sec[e + f*x])^m*Tan[e + f*x])/
(a*f*(Sec[e + f*x]^2)^(m/2))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\left((d\sec(e+fx))^m \sec^2(e+fx)^{-m/2}\right) \text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^{-1+\frac{m}{2}}}{a+x}\,dx, x, b\tan(e+fx)\right)}{bf}$$

$$= \frac{\left((d\sec(e+fx))^m \sec^2(e+fx)^{-m/2}\right) \text{Subst}\left(\int \left(\frac{a\left(1+\frac{x^2}{b^2}\right)^{-1+\frac{m}{2}}}{a^2-x^2} + \frac{x\left(1+\frac{x^2}{b^2}\right)^{-1+\frac{m}{2}}}{-a^2+x^2}\right) dx, x, b\tan(e+fx)\right)}{bf}$$

$$= \frac{\left((d\sec(e+fx))^m \sec^2(e+fx)^{-m/2}\right) \text{Subst}\left(\int \frac{x\left(1+\frac{x^2}{b^2}\right)^{-1+\frac{m}{2}}}{-a^2+x^2} dx, x, b\tan(e+fx)\right)}{bf} + \frac{\left(a(d\sec(e+fx))^m \sec^2(e+fx)^{-m/2}\right) \text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^{-1+\frac{m}{2}}}{a^2-x^2} dx, x, b\tan(e+fx)\right)}{bf}$$

$$= \frac{\text{AppellF1}\left(\frac{1}{2}, 1, 1-\frac{m}{2}, \frac{3}{2}, \frac{b^2\tan^2(e+fx)}{a^2}, -\tan^2(e+fx)\right) (d\sec(e+fx))^m \sec^2(e+fx)^{-m/2} \tan(e+fx)}{af} + \frac{\left((d\sec(e+fx))^m \sec^2(e+fx)^{-m/2}\right) \text{Subst}\left(\int \frac{\left(1+\frac{x}{b^2}\right)^{-1+\frac{m}{2}}}{-a^2+x} dx, x, b^2\tan^2(e+fx)\right)}{2bf}$$

$$= -\frac{b\,\text{Hypergeometric2F1}\left(1, \frac{m}{2}, \frac{2+m}{2}, \frac{b^2\sec^2(e+fx)}{a^2+b^2}\right) (d\sec(e+fx))^m}{(a^2+b^2)\, fm} + \frac{\text{AppellF1}\left(\frac{1}{2}, 1, 1-\frac{m}{2}, \frac{3}{2}, \frac{b^2\tan^2(e+fx)}{a^2}, -\tan^2(e+fx)\right) (d\sec(e+fx))^m \sec^2(e+fx)^{-m/2} \tan(e+fx)}{af}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 15.16 (sec) , antiderivative size = 1158, normalized size of antiderivative = 8.21

$$\int \frac{(d\sec(e+fx))^m}{a+b\tan(e+fx)} dx$$

$$= \frac{}{f(a+b\tan(e+fx)) \left(am\,\text{Hypergeometric2F1}\left(\frac{1}{2}, 1-\frac{m}{2}, \frac{3}{2}, -\tan^2(e+fx)\right) \sec^2(e+fx) - bm\sec^2(e+f\right.}$$

```
[In] Integrate[(d*Sec[e + f*x])^m/(a + b*Tan[e + f*x]),x]
```

```
[Out] ((d*Sec[e + f*x])^m*(b - b*(Sec[e + f*x]^2)^(m/2) + a*m*Hypergeometric2F1[1
/2, 1 - m/2, 3/2, -Tan[e + f*x]^2]*Tan[e + f*x] + (b*AppellF1[-m, -1/2*m, -
1/2*m, 1 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x]
)]*(Sec[e + f*x]^2)^(m/2))/(((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^
```

```
(m/2)*((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2))))/(f*(a + b*Tan[
e + f*x])*(a*m*Hypergeometric2F1[1/2, 1 - m/2, 3/2, -Tan[e + f*x]^2]*Sec[e
+ f*x]^2 - b*m*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x] + (b*m*AppellF1[-m, -1/2
*m, -1/2*m, 1 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e +
 f*x])]*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x])/(((b*(-I + Tan[e + f*x]))/(a +
 b*Tan[e + f*x]))^(m/2)*((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)
) + (b*(Sec[e + f*x]^2)^(m/2)*(-1/2*((a - I*b)*b*m^2*AppellF1[1 - m, 1 - m/
2, -1/2*m, 2 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e +
f*x])]*Sec[e + f*x]^2)/((1 - m)*(a + b*Tan[e + f*x])^2) - ((a + I*b)*b*m^2*
AppellF1[1 - m, -1/2*m, 1 - m/2, 2 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a
+ I*b)/(a + b*Tan[e + f*x])]*Sec[e + f*x]^2)/(2*(1 - m)*(a + b*Tan[e + f*x]
)^2)))/(((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I + Tan[e
 + f*x]))/(a + b*Tan[e + f*x]))^(m/2)) - (b*m*AppellF1[-m, -1/2*m, -1/2*m,
1 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*(Sec
[e + f*x]^2)^(m/2)*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(-1 - m/2
)*(-((b^2*Sec[e + f*x]^2*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x])^2) + (b*
Sec[e + f*x]^2)/(a + b*Tan[e + f*x])))/(2*((b*(I + Tan[e + f*x]))/(a + b*Ta
n[e + f*x]))^(m/2)) - (b*m*AppellF1[-m, -1/2*m, -1/2*m, 1 - m, (a - I*b)/(a
 + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*(Sec[e + f*x]^2)^(m/2)*
((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(-1 - m/2)*(-((b^2*Sec[e + f*
x]^2*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x])^2) + (b*Sec[e + f*x]^2)/(a +
b*Tan[e + f*x])))/(2*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2))
+ a*m*Sec[e + f*x]^2*(-Hypergeometric2F1[1/2, 1 - m/2, 3/2, -Tan[e + f*x]^2
] + (1 + Tan[e + f*x]^2)^(-1 + m/2))))
```

Maple [F]

$$\int \frac{(d\sec(fx+e))^m}{a+b\tan(fx+e)}dx$$

```
[In] int((d*sec(f*x+e))^m/(a+b*tan(f*x+e)),x)
[Out] int((d*sec(f*x+e))^m/(a+b*tan(f*x+e)),x)
```

Fricas [F]

$$\int \frac{(d\sec(e+fx))^m}{a+b\tan(e+fx)}\,dx = \int \frac{(d\sec(fx+e))^m}{b\tan(fx+e)+a}\,dx$$

```
[In] integrate((d*sec(f*x+e))^m/(a+b*tan(f*x+e)),x, algorithm="fricas")
[Out] integral((d*sec(f*x + e))^m/(b*tan(f*x + e) + a), x)
```

Sympy [F]

$$\int \frac{(d\sec(e+fx))^m}{a+b\tan(e+fx)}\,dx = \int \frac{(d\sec{(e+fx)})^m}{a+b\tan{(e+fx)}}\,dx$$

```
[In] integrate((d*sec(f*x+e))**m/(a+b*tan(f*x+e)),x)
[Out] Integral((d*sec(e + f*x))**m/(a + b*tan(e + f*x)), x)
```

Maxima [F]

$$\int \frac{(d\sec(e+fx))^m}{a+b\tan(e+fx)}\,dx = \int \frac{(d\sec{(fx+e)})^m}{b\tan{(fx+e)}+a}\,dx$$

```
[In] integrate((d*sec(f*x+e))^m/(a+b*tan(f*x+e)),x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^m/(b*tan(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{(d\sec(e+fx))^m}{a+b\tan(e+fx)}\,dx = \int \frac{(d\sec{(fx+e)})^m}{b\tan{(fx+e)}+a}\,dx$$

```
[In] integrate((d*sec(f*x+e))^m/(a+b*tan(f*x+e)),x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^m/(b*tan(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^m}{a+b\tan(e+fx)}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^m}{a+b\tan{(e+f\,x)}}\,dx$$

```
[In] int((d/cos(e + f*x))^m/(a + b*tan(e + f*x)),x)
[Out] int((d/cos(e + f*x))^m/(a + b*tan(e + f*x)), x)
```

3.644 $\int \frac{(d\sec(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 227

$$\int \frac{(d\sec(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx$$

$$= -\frac{2ab\,\text{Hypergeometric2F1}\left(2,\frac{m}{2},\frac{2+m}{2},\frac{b^2\sec^2(e+fx)}{a^2+b^2}\right)(d\sec(e+fx))^m}{(a^2+b^2)^2\,fm}$$

$$+\frac{\text{AppellF1}\left(\frac{1}{2},2,1-\frac{m}{2},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^m\sec^2(e+fx)^{-m/2}\tan(e+fx)}{a^2f}$$

$$+\frac{b^2\,\text{AppellF1}\left(\frac{3}{2},2,1-\frac{m}{2},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^m\sec^2(e+fx)^{-m/2}\tan^3(e+f}{3a^4f}$$

```
[Out] -2*a*b*hypergeom([2, 1/2*m],[1+1/2*m],b^2*sec(f*x+e)^2/(a^2+b^2))*(d*sec(f*
x+e))^m/(a^2+b^2)^2/f/m+AppellF1(1/2,2,1-1/2*m,3/2,b^2*tan(f*x+e)^2/a^2,-ta
n(f*x+e)^2)*(d*sec(f*x+e))^m*tan(f*x+e)/a^2/f/((sec(f*x+e)^2)^(1/2*m))+1/3*
b^2*AppellF1(3/2,2,1-1/2*m,5/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+e)^2)*(d*sec(f
*x+e))^m*tan(f*x+e)^3/a^4/f/((sec(f*x+e)^2)^(1/2*m))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used

$= \{3593, 771, 440, 455, 70, 524\}$

$$\int \frac{(d\sec(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx$$
$$= \frac{\tan(e+fx)\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m \operatorname{AppellF1}\left(\frac{1}{2},2,1-\frac{m}{2},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a^2 f}$$
$$- \frac{2ab(d\sec(e+fx))^m \operatorname{Hypergeometric2F1}\left(2,\frac{m}{2},\frac{m+2}{2},\frac{b^2\sec^2(e+fx)}{a^2+b^2}\right)}{fm\left(a^2+b^2\right)^2}$$
$$+ \frac{b^2\tan^3(e+fx)\sec^2(e+fx)^{-m/2}(d\sec(e+fx))^m \operatorname{AppellF1}\left(\frac{3}{2},2,1-\frac{m}{2},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{3a^4 f}$$

```
[In] Int[(d*Sec[e + f*x])^m/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (-2*a*b*Hypergeometric2F1[2, m/2, (2 + m)/2, (b^2*Sec[e + f*x]^2)/(a^2 + b^
2)]*(d*Sec[e + f*x])^m)/((a^2 + b^2)^2*f*m) + (AppellF1[1/2, 2, 1 - m/2, 3/
2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Sec[e + f*x])^m*Tan[e + f*
x])/(a^2*f*(Sec[e + f*x]^2)^(m/2)) + (b^2*AppellF1[3/2, 2, 1 - m/2, 5/2, (b
^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Sec[e + f*x])^m*Tan[e + f*x]^3)
/(3*a^4*f*(Sec[e + f*x]^2)^(m/2))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
```

```
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] && !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\left((d\sec(e+fx))^m\sec^2(e+fx)^{-m/2}\right)\text{Subst}\left(\int\frac{\left(1+\frac{x^2}{b^2}\right)^{-1+\frac{m}{2}}}{(a+x)^2}\,dx,x,b\tan(e+fx)\right)}{bf}$$

$$= \frac{\left((d\sec(e+fx))^m\sec^2(e+fx)^{-m/2}\right)\text{Subst}\left(\int\left(\frac{a^2\left(1+\frac{x^2}{b^2}\right)^{-1+\frac{m}{2}}}{(a^2-x^2)^2}-\frac{2ax\left(1+\frac{x^2}{b^2}\right)^{-1+\frac{m}{2}}}{(a^2-x^2)^2}+\frac{x^2\left(1+\frac{x^2}{b^2}\right)^{-1+\frac{m}{2}}}{(-a^2+x^2)^2}\right.\right.}{bf}$$

$$= \frac{\left((d\sec(e+fx))^m\sec^2(e+fx)^{-m/2}\right)\text{Subst}\left(\int\frac{x^2\left(1+\frac{x^2}{b^2}\right)^{-1+\frac{m}{2}}}{(-a^2+x^2)^2}\,dx,x,b\tan(e+fx)\right)}{bf}$$
$$-\frac{\left(2a(d\sec(e+fx))^m\sec^2(e+fx)^{-m/2}\right)\text{Subst}\left(\int\frac{x\left(1+\frac{x^2}{b^2}\right)^{-1+\frac{m}{2}}}{(a^2-x^2)^2}\,dx,x,b\tan(e+fx)\right)}{bf}$$
$$+\frac{\left(a^2(d\sec(e+fx))^m\sec^2(e+fx)^{-m/2}\right)\text{Subst}\left(\int\frac{\left(1+\frac{x^2}{b^2}\right)^{-1+\frac{m}{2}}}{(a^2-x^2)^2}\,dx,x,b\tan(e+fx)\right)}{bf}$$

$$= \frac{\text{AppellF1}\left(\frac{1}{2},2,1-\frac{m}{2},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^m\sec^2(e+fx)^{-m/2}\tan(e+f}{a^2f}$$

$$+\frac{b^2\,\text{AppellF1}\left(\frac{3}{2},2,1-\frac{m}{2},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^m\sec^2(e+fx)^{-m/2}\tan^3}{3a^4f}$$

$$-\frac{\left(a(d\sec(e+fx))^m\sec^2(e+fx)^{-m/2}\right)\text{Subst}\left(\int\frac{\left(1+\frac{x}{b^2}\right)^{-1+\frac{m}{2}}}{(a^2-x)^2}\,dx,x,b^2\tan^2(e+fx)\right)}{bf}$$

$$=-\frac{2ab\,\text{Hypergeometric2F1}\left(2,\frac{m}{2},\frac{2+m}{2},\frac{b^2\sec^2(e+fx)}{a^2+b^2}\right)(d\sec(e+fx))^m}{(a^2+b^2)^2\,fm}$$

$$+\frac{\text{AppellF1}\left(\frac{1}{2},2,1-\frac{m}{2},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^m\sec^2(e+fx)^{-m/2}\tan(e-}{a^2f}$$

$$+\frac{b^2\,\text{AppellF1}\left(\frac{3}{2},2,1-\frac{m}{2},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\sec(e+fx))^m\sec^2(e+fx)^{-m/2}\tan^3}{3a^4f}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 18.79 (sec) , antiderivative size = 2453, normalized size of antiderivative = 10.81

$$\int\frac{(d\sec(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx=\text{Result too large to show}$$

```
[In] Integrate[(d*Sec[e + f*x])^m/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] ((d*Sec[e + f*x])^m*((-2*a*b*(-1 + (Sec[e + f*x]^2)^(m/2)))/m + (a^2 - b^2)
*Hypergeometric2F1[1/2, 1 - m/2, 3/2, -Tan[e + f*x]^2]*Tan[e + f*x] + (2*a*
b*AppellF1[-m, -1/2*m, -1/2*m, 1 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a +
I*b)/(a + b*Tan[e + f*x])]*(Sec[e + f*x]^2)^(m/2))/(m*((b*(-I + Tan[e + f*x
]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]
))^(m/2)) + (b*(a^2 + b^2)*AppellF1[1 - m, -1/2*m, -1/2*m, 2 - m, (a - I*b)
/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*(Sec[e + f*x]^2)^(m/
2))/((-1 + m)*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I +
 Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*(a + b*Tan[e + f*x]))))/(f*(a +
 b*Tan[e + f*x])^2*((a^2 - b^2)*Hypergeometric2F1[1/2, 1 - m/2, 3/2, -Tan[e
 + f*x]^2]*Sec[e + f*x]^2 - 2*a*b*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x] + (2*
a*b*AppellF1[-m, -1/2*m, -1/2*m, 1 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a
+ I*b)/(a + b*Tan[e + f*x])]*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x])/(((b*(-I
+ Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I + Tan[e + f*x]))/(a + b
*Tan[e + f*x]))^(m/2)) - (b^2*(a^2 + b^2)*AppellF1[1 - m, -1/2*m, -1/2*m, 2
```

- m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*(Sec[
e + f*x]^2)^(1 + m/2))/((-1 + m)*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*
x]))^(m/2)*((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*(a + b*Tan[e
 + f*x])^2) + (b*(a^2 + b^2)*m*AppellF1[1 - m, -1/2*m, -1/2*m, 2 - m, (a -
I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*(Sec[e + f*x]^2)
^(m/2)*Tan[e + f*x])/((-1 + m)*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]
))^(m/2)*((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*(a + b*Tan[e +
 f*x])) + (2*a*b*(Sec[e + f*x]^2)^(m/2)*(-1/2*((a - I*b)*b*m^2*AppellF1[1 -
 m, 1 - m/2, -1/2*m, 2 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a +
b*Tan[e + f*x])]*Sec[e + f*x]^2)/((1 - m)*(a + b*Tan[e + f*x])^2) - ((a + I
*b)*b*m^2*AppellF1[1 - m, -1/2*m, 1 - m/2, 2 - m, (a - I*b)/(a + b*Tan[e +
f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*Sec[e + f*x]^2)/(2*(1 - m)*(a + b*Ta
n[e + f*x])^2)))/(m*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((
b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)) + (b*(a^2 + b^2)*(Sec[e
+ f*x]^2)^(m/2)*(((a - I*b)*b*(1 - m)*m*AppellF1[2 - m, 1 - m/2, -1/2*m, 3
- m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*Sec[e
+ f*x]^2)/(2*(2 - m)*(a + b*Tan[e + f*x])^2) + ((a + I*b)*b*(1 - m)*m*Appel
lF1[2 - m, -1/2*m, 1 - m/2, 3 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b
)/(a + b*Tan[e + f*x])]*Sec[e + f*x]^2)/(2*(2 - m)*(a + b*Tan[e + f*x])^2))
)/((-1 + m)*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I + T
an[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*(a + b*Tan[e + f*x])) - (a*b*Appe
llF1[-m, -1/2*m, -1/2*m, 1 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(
a + b*Tan[e + f*x])]*(Sec[e + f*x]^2)^(m/2)*((b*(-I + Tan[e + f*x]))/(a + b
*Tan[e + f*x]))^(-1 - m/2)*(-((b^2*Sec[e + f*x]^2*(-I + Tan[e + f*x]))/(a +
 b*Tan[e + f*x])^2) + (b*Sec[e + f*x]^2)/(a + b*Tan[e + f*x])))/((b*(I + Ta
n[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2) - (b*(a^2 + b^2)*m*AppellF1[1 - m,
 -1/2*m, -1/2*m, 2 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Ta
n[e + f*x])]*(Sec[e + f*x]^2)^(m/2)*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e +
 f*x]))^(-1 - m/2)*(-((b^2*Sec[e + f*x]^2*(-I + Tan[e + f*x]))/(a + b*Tan[e
 + f*x])^2) + (b*Sec[e + f*x]^2)/(a + b*Tan[e + f*x])))/(2*(-1 + m)*((b*(I
+ Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*(a + b*Tan[e + f*x])) - (a*b*A
ppellF1[-m, -1/2*m, -1/2*m, 1 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b
)/(a + b*Tan[e + f*x])]*(Sec[e + f*x]^2)^(m/2)*((b*(I + Tan[e + f*x]))/(a +
 b*Tan[e + f*x]))^(-1 - m/2)*(-((b^2*Sec[e + f*x]^2*(I + Tan[e + f*x]))/(a
+ b*Tan[e + f*x])^2) + (b*Sec[e + f*x]^2)/(a + b*Tan[e + f*x])))/((b*(-I +
Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2) - (b*(a^2 + b^2)*m*AppellF1[1 -
m, -1/2*m, -1/2*m, 2 - m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*
Tan[e + f*x])]*(Sec[e + f*x]^2)^(m/2)*((b*(I + Tan[e + f*x]))/(a + b*Tan[e
+ f*x]))^(-1 - m/2)*(-((b^2*Sec[e + f*x]^2*(I + Tan[e + f*x]))/(a + b*Tan[e
 + f*x])^2) + (b*Sec[e + f*x]^2)/(a + b*Tan[e + f*x])))/(2*(-1 + m)*((b*(-I
 + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*(a + b*Tan[e + f*x])) + (a^2
- b^2)*Sec[e + f*x]^2*(-Hypergeometric2F1[1/2, 1 - m/2, 3/2, -Tan[e + f*x]^
2] + (1 + Tan[e + f*x]^2)^(-1 + m/2))))

Maple [F]

$$\int \frac{(d\sec (fx+e))^m}{(a+b\tan (fx+e))^2}dx$$

[In] `int((d*sec(f*x+e))^m/(a+b*tan(f*x+e))^2,x)`

[Out] `int((d*sec(f*x+e))^m/(a+b*tan(f*x+e))^2,x)`

Fricas [F]

$$\int \frac{(d\sec(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\sec (fx+e))^m}{(b\tan (fx+e)+a)^2}\,dx$$

[In] `integrate((d*sec(f*x+e))^m/(a+b*tan(f*x+e))^2,x, algorithm="fricas")`

[Out] `integral((d*sec(f*x + e))^m/(b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)
, x)`

Sympy [F]

$$\int \frac{(d\sec(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\sec (e+fx))^m}{(a+b\tan (e+fx))^2}\,dx$$

[In] `integrate((d*sec(f*x+e))**m/(a+b*tan(f*x+e))**2,x)`

[Out] `Integral((d*sec(e + f*x))**m/(a + b*tan(e + f*x))**2, x)`

Maxima [F]

$$\int \frac{(d\sec(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\sec (fx+e))^m}{(b\tan (fx+e)+a)^2}\,dx$$

[In] `integrate((d*sec(f*x+e))^m/(a+b*tan(f*x+e))^2,x, algorithm="maxima")`

[Out] `integrate((d*sec(f*x + e))^m/(b*tan(f*x + e) + a)^2, x)`

Giac [F]

$$\int \frac{(d\sec(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\sec(fx+e))^m}{(b\tan(fx+e)+a)^2}\,dx$$

```
[In] integrate((d*sec(f*x+e))^m/(a+b*tan(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^m/(b*tan(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\sec(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx = \int \frac{\left(\frac{d}{\cos(e+f\,x)}\right)^m}{(a+b\tan(e+f\,x))^2}\,dx$$

```
[In] int((d/cos(e + f*x))^m/(a + b*tan(e + f*x))^2,x)
[Out] int((d/cos(e + f*x))^m/(a + b*tan(e + f*x))^2, x)
```

3.645 $\int (d\sec(e+fx))^m(a+b\tan(e+fx))^n\,dx$

Optimal result

Integrand size = 23, antiderivative size = 181

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^n\,dx = \frac{b\,\text{AppellF1}\left(1+n,1-\frac{m}{2},1-\frac{m}{2},2+n,\frac{a+b\tan(e+fx)}{a+\sqrt{-b^2}},\frac{a+b\tan(e+fx)}{a-\sqrt{-b^2}}\right)(d\sec(e+fx))^m(a+b\tan(e+fx))^{1+n}}{(a^2+b^2)\,f(1+n)}$$

```
[Out] b*AppellF1(1+n,1-1/2*m,1-1/2*m,2+n,(a+b*tan(f*x+e))/(a-(-b^2)^(1/2)),(a+b*t
an(f*x+e))/(a+(-b^2)^(1/2)))*(d*sec(f*x+e))^m*(a+b*tan(f*x+e))^(1+n)/(a^2+b
^2)/f/(1+n)/((1+(a+b*tan(f*x+e))/(-a+(-b^2)^(1/2)))^(1/2*m))/((1+(-a-b*tan(
f*x+e))/(a+(-b^2)^(1/2)))^(1/2*m))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3593, 774, 138}

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^n\,dx = \frac{\cos^2(e+fx)(d\sec(e+fx))^m\left(1-\frac{a+b\tan(e+fx)}{a-\sqrt{-b^2}}\right)^{1-\frac{m}{2}}\left(1-\frac{a+b\tan(e+fx)}{a+\sqrt{-b^2}}\right)^{1-\frac{m}{2}}(a+b\tan(e+fx))^{n+1}\,\text{AppellF1}\ldots}{bf(n+1)}$$

```
[In] Int[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n,x]
```

```
[Out] (AppellF1[1 + n, 1 - m/2, 1 - m/2, 2 + n, (a + b*Tan[e + f*x])/(a - Sqrt[-b
^2]), (a + b*Tan[e + f*x])/(a + Sqrt[-b^2])]*Cos[e + f*x]^2*(d*Sec[e + f*x]
)^m*(a + b*Tan[e + f*x])^(1 + n)*(1 - (a + b*Tan[e + f*x])/(a - Sqrt[-b^2])
```

```
)^(1 - m/2)*(1 - (a + b*Tan[e + f*x])/(a + Sqrt[-b^2]))^(1 - m/2))/(b*f*(1
+ n))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 774

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + e*(q/c)))^p*
(1 - (d + e*x)/(d - e*(q/c)))^p), Subst[Int[x^m*Simp[1 - x/(d + e*(q/c)), x
]^p*Simp[1 - x/(d - e*(q/c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

integral

$$= \frac{\left((d\sec(e+fx))^m \sec^2(e+fx)^{-m/2}\right) \operatorname{Subst}\left(\int (a+x)^n \left(1+\frac{x^2}{b^2}\right)^{-1+\frac{m}{2}} dx, x, b\tan(e+fx)\right)}{bf}$$

$$= \frac{\left(\cos^2(e+fx)(d\sec(e+fx))^m \left(1-\frac{a+b\tan(e+fx)}{a-\frac{b^2}{\sqrt{-b^2}}}\right)^{1-\frac{m}{2}} \left(1-\frac{a+b\tan(e+fx)}{a+\frac{b^2}{\sqrt{-b^2}}}\right)^{1-\frac{m}{2}}\right) \operatorname{Subst}\left(\int x^n \left(1-\right.\right.}{bf}$$

$$= \frac{\operatorname{AppellF1}\left(1+n, 1-\frac{m}{2}, 1-\frac{m}{2}, 2+n, \frac{a+b\tan(e+fx)}{a-\sqrt{-b^2}}, \frac{a+b\tan(e+fx)}{a+\sqrt{-b^2}}\right) \cos^2(e+fx)(d\sec(e+fx))^m (a}{bf(1+n)}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.00 (sec) , antiderivative size = 699, normalized size of antiderivative = 3.86

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^n\,dx$$

$$= \frac{}{f\left(2b\,\text{AppellF1}\left(1+n,1-\frac{m}{2},1-\frac{m}{2},2+n,\frac{a+b\tan(e+fx)}{a-ib},\frac{a+b\tan(e+fx)}{a+ib}\right)\sec^2(e+fx)+2n\,\text{AppellF1}\left(1+n\right.\right.}$$

```
[In] Integrate[(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^n,x]
```

```
[Out] (2*AppellF1[1 + n, 1 - m/2, 1 - m/2, 2 + n, (a + b*Tan[e + f*x])/(a - I*b),
 (a + b*Tan[e + f*x])/(a + I*b)]*(d*Sec[e + f*x])^m*(a + b*Tan[e + f*x])^(1
 + n))/(f*(2*b*AppellF1[1 + n, 1 - m/2, 1 - m/2, 2 + n, (a + b*Tan[e + f*x]
)/(a - I*b), (a + b*Tan[e + f*x])/(a + I*b)]*Sec[e + f*x]^2 + 2*n*AppellF1[
1 + n, 1 - m/2, 1 - m/2, 2 + n, (a + b*Tan[e + f*x])/(a - I*b), (a + b*Tan[
e + f*x])/(a + I*b)]*(b - a*Tan[e + f*x]) - (b*(-2 + m)*(1 + n)*((a - I*b)*
AppellF1[2 + n, 1 - m/2, 2 - m/2, 3 + n, (a + b*Tan[e + f*x])/(a - I*b), (a
 + b*Tan[e + f*x])/(a + I*b)] + (a + I*b)*AppellF1[2 + n, 2 - m/2, 1 - m/2,
 3 + n, (a + b*Tan[e + f*x])/(a - I*b), (a + b*Tan[e + f*x])/(a + I*b)])*Se
c[e + f*x]^2*(a + b*Tan[e + f*x]))/((a - I*b)*(a + I*b)*(2 + n)) + 2*(m + n
)*AppellF1[1 + n, 1 - m/2, 1 - m/2, 2 + n, (a + b*Tan[e + f*x])/(a - I*b),
(a + b*Tan[e + f*x])/(a + I*b)]*Tan[e + f*x]*(a + b*Tan[e + f*x]) - (m*Appe
llF1[1 + n, 1 - m/2, 1 - m/2, 2 + n, (a + b*Tan[e + f*x])/(a - I*b), (a + b
*Tan[e + f*x])/(a + I*b)]*Sec[e + f*x]^2*(a + b*Tan[e + f*x]))/(-I + Tan[e
+ f*x]) - (m*AppellF1[1 + n, 1 - m/2, 1 - m/2, 2 + n, (a + b*Tan[e + f*x])/
(a - I*b), (a + b*Tan[e + f*x])/(a + I*b)]*Sec[e + f*x]^2*(a + b*Tan[e + f*
x]))/(I + Tan[e + f*x])))
```

Maple [F]

$$\int (d\sec(fx+e))^m\,(a+b\tan(fx+e))^n\,dx$$

```
[In] int((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^n,x)
```

```
[Out] int((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^n,x)
```

Fricas [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^n\,dx=\int (d\sec(fx+e))^m(b\tan(fx+e)+a)^n\,dx$$

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^n,x, algorithm="fricas")
[Out] integral((d*sec(f*x + e))^m*(b*tan(f*x + e) + a)^n, x)
```

Sympy [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^n\,dx=\int (d\sec(e+fx))^m(a+b\tan(e+fx))^n\,dx$$

```
[In] integrate((d*sec(f*x+e))**m*(a+b*tan(f*x+e))**n,x)
[Out] Integral((d*sec(e + f*x))**m*(a + b*tan(e + f*x))**n, x)
```

Maxima [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^n\,dx=\int (d\sec(fx+e))^m(b\tan(fx+e)+a)^n\,dx$$

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((d*sec(f*x + e))^m*(b*tan(f*x + e) + a)^n, x)
```

Giac [F]

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^n\,dx=\int (d\sec(fx+e))^m(b\tan(fx+e)+a)^n\,dx$$

```
[In] integrate((d*sec(f*x+e))^m*(a+b*tan(f*x+e))^n,x, algorithm="giac")
[Out] integrate((d*sec(f*x + e))^m*(b*tan(f*x + e) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\sec(e+fx))^m(a+b\tan(e+fx))^n\,dx=\int \left(\frac{d}{\cos\left(e+f\,x\right)}\right)^m\,{\left(a+b\tan(e+f\,x)\right)}^n\,dx$$

```
[In] int((d/cos(e + f*x))^m*(a + b*tan(e + f*x))^n,x)
[Out] int((d/cos(e + f*x))^m*(a + b*tan(e + f*x))^n, x)
```

3.646 $\int \sec^6(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 161

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{(a^2+b^2)^2(a+b\tan(c+dx))^{1+n}}{b^5d(1+n)} - \frac{4a(a^2+b^2)(a+b\tan(c+dx))^{2+n}}{b^5d(2+n)} + \frac{2(3a^2+b^2)(a+b\tan(c+dx))^{3+n}}{b^5d(3+n)} - \frac{4a(a+b\tan(c+dx))^{4+n}}{b^5d(4+n)} + \frac{(a+b\tan(c+dx))^{5+n}}{b^5d(5+n)}$$

```
[Out] (a^2+b^2)^2*(a+b*tan(d*x+c))^(1+n)/b^5/d/(1+n)-4*a*(a^2+b^2)*(a+b*tan(d*x+c
))^(2+n)/b^5/d/(2+n)+2*(3*a^2+b^2)*(a+b*tan(d*x+c))^(3+n)/b^5/d/(3+n)-4*a*(
a+b*tan(d*x+c))^(4+n)/b^5/d/(4+n)+(a+b*tan(d*x+c))^(5+n)/b^5/d/(5+n)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used

$= \{3587, 711\}$

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{(a^2+b^2)^2(a+b\tan(c+dx))^{n+1}}{b^5d(n+1)} - \frac{4a(a^2+b^2)(a+b\tan(c+dx))^{n+2}}{b^5d(n+2)} + \frac{2(3a^2+b^2)(a+b\tan(c+dx))^{n+3}}{b^5d(n+3)} - \frac{4a(a+b\tan(c+dx))^{n+4}}{b^5d(n+4)} + \frac{(a+b\tan(c+dx))^{n+5}}{b^5d(n+5)}$$

```
[In] Int[Sec[c + d*x]^6*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] ((a^2 + b^2)^2*(a + b*Tan[c + d*x])^(1 + n))/(b^5*d*(1 + n)) - (4*a*(a^2 +
b^2)*(a + b*Tan[c + d*x])^(2 + n))/(b^5*d*(2 + n)) + (2*(3*a^2 + b^2)*(a +
b*Tan[c + d*x])^(3 + n))/(b^5*d*(3 + n)) - (4*a*(a + b*Tan[c + d*x])^(4 + n
))/(b^5*d*(4 + n)) + (a + b*Tan[c + d*x])^(5 + n)/(b^5*d*(5 + n))
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int (a+x)^n\left(1+\frac{x^2}{b^2}\right)^2\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{(a^2+b^2)^2(a+x)^n}{b^4} - \frac{4a(a^2+b^2)(a+x)^{1+n}}{b^4} + \frac{2(3a^2+b^2)(a+x)^{2+n}}{b^4} - \frac{4a(a+x)^{3+n}}{b^4} + \frac{(a+x)^{4+n}}{b^4}\right)\,dx, x, b\tan(\right.}{bd}$$

$$= \frac{(a^2+b^2)^2 (a+b\tan(c+dx))^{1+n}}{b^5 d(1+n)} - \frac{4a(a^2+b^2)(a+b\tan(c+dx))^{2+n}}{b^5 d(2+n)} + \frac{2(3a^2+b^2)(a+b\tan(c+dx))^{3+n}}{b^5 d(3+n)} - \frac{4a(a+b\tan(c+dx))^{4+n}}{b^5 d(4+n)} + \frac{(a+b\tan(c+dx))^{5+n}}{b^5 d(5+n)}$$

Mathematica [A] (verified)

Time = 3.41 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^n\,dx$$
$$= \frac{(a+b\tan(c+dx))^{1+n}\left(b^4\sec^4(c+dx)+4(a^2+b^2)\left(\frac{a^2+b^2}{1+n}-\frac{2a(a+b\tan(c+dx))}{2+n}+\frac{(a+b\tan(c+dx))^2}{3+n}\right)-4a(a+\right.}{b^5 d(5+n)}$$

```
[In] Integrate[Sec[c + d*x]^6*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] ((a + b*Tan[c + d*x])^(1 + n)*(b^4*Sec[c + d*x]^4 + 4*(a^2 + b^2)*((a^2 + b
^2)/(1 + n) - (2*a*(a + b*Tan[c + d*x]))/(2 + n) + (a + b*Tan[c + d*x])^2/(
3 + n)) - 4*a*(a + b*Tan[c + d*x])*((a^2 + b^2)/(2 + n) - (2*a*(a + b*Tan[c
 + d*x]))/(3 + n) + (a + b*Tan[c + d*x])^2/(4 + n))))/(b^5*d*(5 + n))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. 2(161) = 322.

Time = 0.12 (sec) , antiderivative size = 462, normalized size of antiderivative = 2.87

$$\frac{(\tan^5(dx+c))\,e^{n\ln(a+b\tan(dx+c))}}{d(5+n)} + \frac{a(b^4n^4+14b^4n^3+4a^2b^2n^2+71b^4n^2+36a^2b^2n+154b^4n+24a^4+80a^2b^2}{b^5 d(n^5+15n^4+85n^3+225n^2+274n+120)}$$

```
[In] int(sec(d*x+c)^6*(a+b*tan(d*x+c))^n,x)
```

```
[Out] 1/d/(5+n)*tan(d*x+c)^5*exp(n*ln(a+b*tan(d*x+c)))+a*(b^4*n^4+14*b^4*n^3+4*a^
2*b^2*n^2+71*b^4*n^2+36*a^2*b^2*n+154*b^4*n+24*a^4+80*a^2*b^2+120*b^4)/b^5/
d/(n^5+15*n^4+85*n^3+225*n^2+274*n+120)*exp(n*ln(a+b*tan(d*x+c)))+a*n/b/d/(
n^2+9*n+20)*tan(d*x+c)^4*exp(n*ln(a+b*tan(d*x+c)))-2*(-b^2*n^2+2*a^2*n-9*b^
2*n-20*b^2)/b^2/d/(n^3+12*n^2+47*n+60)*tan(d*x+c)^3*exp(n*ln(a+b*tan(d*x+c)
))-(-b^4*n^4+4*a^2*b^2*n^3-14*b^4*n^3+36*a^2*b^2*n^2-71*b^4*n^2+24*a^4*n+80
*a^2*b^2*n-154*b^4*n-120*b^4)/b^4/(n^5+15*n^4+85*n^3+225*n^2+274*n+120)/d*t
an(d*x+c)*exp(n*ln(a+b*tan(d*x+c)))+2*(b^2*n^2+9*b^2*n+6*a^2+20*b^2)*a/b^3/
d*n/(n^4+14*n^3+71*n^2+154*n+120)*tan(d*x+c)^2*exp(n*ln(a+b*tan(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 420 vs. 2(161) = 322.

Time = 0.31 (sec) , antiderivative size = 420, normalized size of antiderivative = 2.61

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{(8\,(3\,a^5+10\,a^3b^2+15\,ab^4-(a^3b^2-3\,ab^4)n^2+3\,(a^3b^2+5\,ab^4)n)\cos{(dx+c)}^5+4\,(2\,ab^4n^3+3\,(a^3b^2+3\,a}{}$$

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] (8*(3*a^5 + 10*a^3*b^2 + 15*a*b^4 - (a^3*b^2 - 3*a*b^4)*n^2 + 3*(a^3*b^2 +
5*a*b^4)*n)*cos(d*x + c)^5 + 4*(2*a*b^4*n^3 + 3*(a^3*b^2 + 3*a*b^4)*n^2 + (
3*a^3*b^2 + 7*a*b^4)*n)*cos(d*x + c)^3 + (a*b^4*n^4 + 6*a*b^4*n^3 + 11*a*b^
4*n^2 + 6*a*b^4*n)*cos(d*x + c) + (b^5*n^4 + 10*b^5*n^3 + 35*b^5*n^2 + 50*b
^5*n + 24*b^5 + 8*(8*b^5 - (3*a^2*b^3 - b^5)*n^2 - 3*(a^4*b + 3*a^2*b^3 - 2
*b^5)*n)*cos(d*x + c)^4 + 4*(8*b^5 - (a^2*b^3 - b^5)*n^3 - (3*a^2*b^3 - 7*b
^5)*n^2 - 2*(a^2*b^3 - 7*b^5)*n)*cos(d*x + c)^2)*sin(d*x + c))*((a*cos(d*x
+ c) + b*sin(d*x + c))/cos(d*x + c))^n/((b^5*d*n^5 + 15*b^5*d*n^4 + 85*b^5*
d*n^3 + 225*b^5*d*n^2 + 274*b^5*d*n + 120*b^5*d)*cos(d*x + c)^5)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^n\,dx = \text{Timed out}$$

```
[In] integrate(sec(d*x+c)**6*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.78

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{\frac{(b\tan(dx+c)+a)^{n+1}}{b(n+1)} + \frac{2\left((n^2+3\,n+2)b^3\tan(dx+c)^3+(n^2+n)ab^2\tan(dx+c)^2-2\,a^2bn\tan(dx+c)+2\,a^3\right)(b\tan(dx+c)+a)^n}{(n^3+6\,n^2+11\,n+6)b^3} + \frac{\left((n^4+10\,n^3+35\right.}{}}{}$$

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] ((b*tan(d*x + c) + a)^(n + 1)/(b*(n + 1)) + 2*((n^2 + 3*n + 2)*b^3*tan(d*x
+ c)^3 + (n^2 + n)*a*b^2*tan(d*x + c)^2 - 2*a^2*b*n*tan(d*x + c) + 2*a^3)*(
b*tan(d*x + c) + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3) + ((n^4 + 10*n^3 + 35*
n^2 + 50*n + 24)*b^5*tan(d*x + c)^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*a*b^4*ta
n(d*x + c)^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*tan(d*x + c)^3 + 12*(n^2 + n)*
a^3*b^2*tan(d*x + c)^2 - 24*a^4*b*n*tan(d*x + c) + 24*a^5)*(b*tan(d*x + c)
+ a)^n/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^5))/d
```

Giac [F(-2)]

Exception generated.

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^n\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(sec(d*x+c)^6*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[0,1,4,0,0,0]%%%}+%%%{2,[0,1,2,2,0,0]%%%}+%%%{-4,[0,1,2,1,1
,0]%%%
```

Mupad [F(-1)]

Timed out.

$$\int \sec^6(c+dx)(a+b\tan(c+dx))^n\,dx = \int \frac{(a+b\tan(c+d\,x))^n}{\cos\left(c+d\,x\right)^6}\,dx$$

```
[In] int((a + b*tan(c + d*x))^n/cos(c + d*x)^6,x)
```

```
[Out] int((a + b*tan(c + d*x))^n/cos(c + d*x)^6, x)
```

3.647 $\int \sec^4(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 88

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{(a^2+b^2)\,(a+b\tan(c+dx))^{1+n}}{b^3d(1+n)} - \frac{2a(a+b\tan(c+dx))^{2+n}}{b^3d(2+n)} + \frac{(a+b\tan(c+dx))^{3+n}}{b^3d(3+n)}$$

```
[Out] (a^2+b^2)*(a+b*tan(d*x+c))^(1+n)/b^3/d/(1+n)-2*a*(a+b*tan(d*x+c))^(2+n)/b^3
/d/(2+n)+(a+b*tan(d*x+c))^(3+n)/b^3/d/(3+n)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3587, 711}

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{(a^2+b^2)\,(a+b\tan(c+dx))^{n+1}}{b^3d(n+1)} - \frac{2a(a+b\tan(c+dx))^{n+2}}{b^3d(n+2)} + \frac{(a+b\tan(c+dx))^{n+3}}{b^3d(n+3)}$$

```
[In] Int[Sec[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] ((a^2 + b^2)*(a + b*Tan[c + d*x])^(1 + n))/(b^3*d*(1 + n)) - (2*a*(a + b*Ta
n[c + d*x])^(2 + n))/(b^3*d*(2 + n)) + (a + b*Tan[c + d*x])^(3 + n)/(b^3*d*
(3 + n))
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int (a+x)^n\left(1+\frac{x^2}{b^2}\right)\,dx, x, b\tan(c+dx)\right)}{bd}\\ &= \frac{\text{Subst}\left(\int \left(\frac{(a^2+b^2)(a+x)^n}{b^2}-\frac{2a(a+x)^{1+n}}{b^2}+\frac{(a+x)^{2+n}}{b^2}\right)\,dx, x, b\tan(c+dx)\right)}{bd}\\ &= \frac{(a^2+b^2)\,(a+b\tan(c+dx))^{1+n}}{b^3d(1+n)}-\frac{2a(a+b\tan(c+dx))^{2+n}}{b^3d(2+n)}+\frac{(a+b\tan(c+dx))^{3+n}}{b^3d(3+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.81

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{(a+b\tan(c+dx))^{1+n}\left(\frac{a^2+b^2}{1+n}-\frac{2a(a+b\tan(c+dx))}{2+n}+\frac{(a+b\tan(c+dx))^2}{3+n}\right)}{b^3d}$$

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
[Out] ((a + b*Tan[c + d*x])^(1 + n)*((a^2 + b^2)/(1 + n) - (2*a*(a + b*Tan[c + d*
x]))/(2 + n) + (a + b*Tan[c + d*x])^2/(3 + n)))/(b^3*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(88) = 176.

Time = 109.94 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.32

method	result
derivativedivides	$\frac{(\tan^3(dx+c))e^{n\ln(a+b\tan(dx+c))}}{d(3+n)}+\frac{a(b^2n^2+5b^2n+2a^2+6b^2)e^{n\ln(a+b\tan(dx+c))}}{b^3d(n^3+6n^2+11n+6)}+\frac{an(\tan^2(dx+c))e^{n\ln(a+b\tan(dx+c}}{bd(n^2+5n+6)}$
default	$\frac{(\tan^3(dx+c))e^{n\ln(a+b\tan(dx+c))}}{d(3+n)}+\frac{a(b^2n^2+5b^2n+2a^2+6b^2)e^{n\ln(a+b\tan(dx+c))}}{b^3d(n^3+6n^2+11n+6)}+\frac{an(\tan^2(dx+c))e^{n\ln(a+b\tan(dx+c}}{bd(n^2+5n+6)}$

```
[In] int(sec(d*x+c)^4*(a+b*tan(d*x+c))^n,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/(3+n)*tan(d*x+c)^3*exp(n*ln(a+b*tan(d*x+c)))+a*(b^2*n^2+5*b^2*n+2*a^2+6
*b^2)/b^3/d/(n^3+6*n^2+11*n+6)*exp(n*ln(a+b*tan(d*x+c)))+a*n/b/d/(n^2+5*n+6
)*tan(d*x+c)^2*exp(n*ln(a+b*tan(d*x+c)))-(-b^2*n^2+2*a^2*n-5*b^2*n-6*b^2)/b
^2/(n^3+6*n^2+11*n+6)/d*tan(d*x+c)*exp(n*ln(a+b*tan(d*x+c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.00

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{(2(2ab^2n+a^3+3ab^2)\cos(dx+c)^3+(ab^2n^2+ab^2n)\cos(dx+c)+(b^3n^2+3b^3n+2b^3+2(2b^3-(a^2b}{(b^3dn^3+6b^3dn^2+11b^3dn+6b^3d)\cos(dx+c)^3}$$

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] (2*(2*a*b^2*n + a^3 + 3*a*b^2)*cos(d*x + c)^3 + (a*b^2*n^2 + a*b^2*n)*cos(d
*x + c) + (b^3*n^2 + 3*b^3*n + 2*b^3 + 2*(2*b^3 - (a^2*b - b^3)*n)*cos(d*x
+ c)^2)*sin(d*x + c))*((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*x + c))^n/((
b^3*d*n^3 + 6*b^3*d*n^2 + 11*b^3*d*n + 6*b^3*d)*cos(d*x + c)^3)
```

Sympy [F]

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^n\,dx = \int (a+b\tan(c+dx))^n\sec^4(c+dx)\,dx$$

```
[In] integrate(sec(d*x+c)**4*(a+b*tan(d*x+c))**n,x)
```

```
[Out] Integral((a + b*tan(c + d*x))**n*sec(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.56 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.32

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{\frac{(b\tan(dx+c)+a)^{n+1}}{b(n+1)} + \frac{\left((n^2+3n+2)b^3\tan(dx+c)^3+(n^2+n)ab^2\tan(dx+c)^2-2a^2bn\tan(dx+c)+2a^3\right)(b\tan(dx+c)+a)^n}{(n^3+6n^2+11n+6)b^3}}{d}$$

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] ((b*tan(d*x + c) + a)^(n + 1)/(b*(n + 1)) + ((n^2 + 3*n + 2)*b^3*tan(d*x +
c)^3 + (n^2 + n)*a*b^2*tan(d*x + c)^2 - 2*a^2*b*n*tan(d*x + c) + 2*a^3)*(b*
tan(d*x + c) + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3))/d
```

Giac [F(-2)]

Exception generated.

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^n\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(sec(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[0,1,2,0,0,0]%%%}+%%%{1,[0,1,0,2,0,0]%%%}+%%%{-2,[0,1,0,1,1
,0]%%%
```

Mupad [F(-1)]

Timed out.

$$\int \sec^4(c+dx)(a+b\tan(c+dx))^n\,dx = \int \frac{(a+b\tan(c+d\,x))^n}{\cos(c+d\,x)^4}\,dx$$

```
[In] int((a + b*tan(c + d*x))^n/cos(c + d*x)^4,x)
```

```
[Out] int((a + b*tan(c + d*x))^n/cos(c + d*x)^4, x)
```

3.648 $\int \sec^2(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 26

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^n\,dx=\frac{(a+b\tan(c+dx))^{1+n}}{bd(1+n)}$$

```
[Out] (a+b*tan(d*x+c))^(1+n)/b/d/(1+n)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3587, 32}

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^n\,dx=\frac{(a+b\tan(c+dx))^{n+1}}{bd(n+1)}$$

```
[In] Int[Sec[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
[Out] (a + b*Tan[c + d*x])^(1 + n)/(b*d*(1 + n))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int (a+x)^n\,dx, x, b\tan(c+dx)\right)}{bd}\\ &= \frac{(a+b\tan(c+dx))^{1+n}}{bd(1+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{(a+b\tan(c+dx))^{1+n}}{bd(1+n)}$$

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
[Out] (a + b*Tan[c + d*x])^(1 + n)/(b*d*(1 + n))
```

Maple [A] (verified)

Time = 8.99 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{(a+b\tan(dx+c))^{1+n}}{bd(1+n)}$	27
default	$\frac{(a+b\tan(dx+c))^{1+n}}{bd(1+n)}$	27

```
[In] int(sec(d*x+c)^2*(a+b*tan(d*x+c))^n,x,method=_RETURNVERBOSE)
[Out] (a+b*tan(d*x+c))^(1+n)/b/d/(1+n)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(26) = 52.

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.46

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{(a\cos(dx+c)+b\sin(dx+c))\left(\frac{a\cos(dx+c)+b\sin(dx+c)}{\cos(dx+c)}\right)^n}{(bdn+bd)\cos(dx+c)}$$

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
[Out] (a*cos(d*x + c) + b*sin(d*x + c))*((a*cos(d*x + c) + b*sin(d*x + c))/cos(d*
x + c))^n/((b*d*n + b*d)*cos(d*x + c))
```

Sympy [F]

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^n\,dx = \int (a+b\tan(c+dx))^n \sec^2(c+dx)\,dx$$

```
[In] integrate(sec(d*x+c)**2*(a+b*tan(d*x+c))**n,x)
[Out] Integral((a + b*tan(c + d*x))**n*sec(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{(b\tan(dx+c)+a)^{n+1}}{bd(n+1)}$$

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
[Out] (b*tan(d*x + c) + a)^(n + 1)/(b*d*(n + 1))
```

Giac [F(-2)]

Exception generated.

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^n\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(sec(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[0,1,0,0]%%%} / %%%{1,[0,0,1,1]%%%} Error: Bad Argument Val
ue
```

Mupad [B] (verification not implemented)

Time = 5.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.96

$$\int \sec^2(c+dx)(a+b\tan(c+dx))^n\,dx = \begin{cases} \frac{\ln(a+b\tan(c+dx))}{bd} & \text{if } n=-1 \\ \frac{(a+b\tan(c+dx))^{n+1}}{bd(n+1)} & \text{if } n\neq -1 \end{cases}$$

```
[In] int((a + b*tan(c + d*x))^n/cos(c + d*x)^2,x)
[Out] piecewise(n == -1, log(a + b*tan(c + d*x))/(b*d), n ~= -1, (a + b*tan(c + d
*x))^(n + 1)/(b*d*(n + 1)))
```

3.649 $\int \cos^2(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 272

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^n\,dx = -\frac{\left(\sqrt{-b^2}\left(1+\frac{a^2}{b^2}-n\right)-an\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)(a+b\tan(c+dx))^{1+n}}{4\left(1+\frac{a^2}{b^2}\right)b\left(a-\sqrt{-b^2}\right)d(1+n)} + \frac{b\left(\sqrt{-b^2}\left(1+\frac{a^2}{b^2}-n\right)+an\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)(a+b\tan(c+dx))^{1+n}}{4\left(a^2+b^2\right)\left(a+\sqrt{-b^2}\right)d(1+n)} + \frac{\cos^2(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{2\left(a^2+b^2\right)d}$$

```
[Out] -1/4*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a-(-b^2)^(1/2)))*(-a*n+(1+a
^2/b^2-n)*(-b^2)^(1/2))*(a+b*tan(d*x+c))^(1+n)/(1+a^2/b^2)/b/d/(1+n)/(a-(-b
^2)^(1/2))+1/4*b*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a+(-b^2)^(1/2))
)*(a*n+(1+a^2/b^2-n)*(-b^2)^(1/2))*(a+b*tan(d*x+c))^(1+n)/(a^2+b^2)/d/(1+n)
/(a+(-b^2)^(1/2))+1/2*cos(d*x+c)^2*(b+a*tan(d*x+c))*(a+b*tan(d*x+c))^(1+n)/
(a^2+b^2)/d
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used

= {3587, 755, 845, 70}

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^n\,dx = -\frac{\left(\sqrt{-b^2}\left(\frac{a^2}{b^2}-n+1\right)-an\right)(a+b\tan(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)}{4bd(n+1)\left(\frac{a^2}{b^2}+1\right)\left(a-\sqrt{-b^2}\right)} + \frac{b\left(\sqrt{-b^2}\left(\frac{a^2}{b^2}-n+1\right)+an\right)(a+b\tan(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)}{4d(n+1)\left(a^2+b^2\right)\left(a+\sqrt{-b^2}\right)} + \frac{\cos^2(c+dx)(a\tan(c+dx)+b)(a+b\tan(c+dx))^{n+1}}{2d\left(a^2+b^2\right)}$$

```
[In] Int[Cos[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] -1/4*((Sqrt[-b^2]*(1 + a^2/b^2 - n) - a*n)*Hypergeometric2F1[1, 1 + n, 2 +
n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/((1
 + a^2/b^2)*b*(a - Sqrt[-b^2])*d*(1 + n)) + (b*(Sqrt[-b^2]*(1 + a^2/b^2 - n
) + a*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt[
-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(4*(a^2 + b^2)*(a + Sqrt[-b^2])*d*(1
+ n)) + (Cos[c + d*x]^2*(b + a*Tan[c + d*x])*(a + b*Tan[c + d*x])^(1 + n))/
(2*(a^2 + b^2)*d)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(a+x)^n}{\left(1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\cos^2(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{2(a^2+b^2)d} - \frac{b\text{Subst}\left(\int \frac{(a+x)^n\left(-1-\frac{a^2}{b^2}+n+\frac{anx}{b^2}\right)}{1+\frac{x^2}{b^2}}\,dx, x, b\tan(c+dx)\right)}{2(a^2+b^2)d}$$

$$= \frac{\cos^2(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{2(a^2+b^2)d} - \frac{b\text{Subst}\left(\int \left(\frac{\left(-an+\sqrt{-b^2}\left(-1-\frac{a^2}{b^2}+n\right)\right)(a+x)^n}{2\left(\sqrt{-b^2}-x\right)} + \frac{\left(an+\sqrt{-b^2}\left(-1-\frac{a^2}{b^2}+n\right)\right)(a+x)^n}{2\left(\sqrt{-b^2}+x\right)}\right)dx, x, b\tan(c+dx)\right)}{2(a^2+b^2)d}$$

$$= \frac{\cos^2(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{2(a^2+b^2)d} + \frac{\left(b\left(\sqrt{-b^2}\left(1+\frac{a^2}{b^2}-n\right)-an\right)\right)\text{Subst}\left(\int \frac{(a+x)^n}{\sqrt{-b^2}+x}\,dx, x, b\tan(c+dx)\right)}{4(a^2+b^2)d} + \frac{\left(b\left(\sqrt{-b^2}\left(1+\frac{a^2}{b^2}-n\right)+an\right)\right)\text{Subst}\left(\int \frac{(a+x)^n}{\sqrt{-b^2}-x}\,dx, x, b\tan(c+dx)\right)}{4(a^2+b^2)d}$$

$$= -\frac{b\left(\sqrt{-b^2}\left(1+\frac{a^2}{b^2}-n\right)-an\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)(a+b\tan(c+}{4(a^2+b^2)\left(a-\sqrt{-b^2}\right)d(1+n)} + \frac{b\left(\sqrt{-b^2}\left(1+\frac{a^2}{b^2}-n\right)+an\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)(a+b\tan(c+}{4(a^2+b^2)\left(a+\sqrt{-b^2}\right)d(1+n)} + \frac{\cos^2(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{2(a^2+b^2)d}$$

Mathematica [A] (verified)

Time = 1.42 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.83

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^n\,dx$$
$$=\frac{(a+b\tan(c+dx))^{1+n}\left(-\frac{\left(\sqrt{-b^2}(a^2-b^2(-1+n))-ab^2n\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)}{\left(a-\sqrt{-b^2}\right)(1+n)}+\frac{\left(a^2\sqrt{-b^2}+(-b^2)^{3/2}(-\right.}{}\right.}{4b\left(a^2+b^2\right)d}$$

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] ((a + b*Tan[c + d*x])^(1 + n)*(-(((Sqrt[-b^2]*(a^2 - b^2*(-1 + n)) - a*b^2*
n)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2])
])/((a - Sqrt[-b^2])*(1 + n))) + ((a^2*Sqrt[-b^2] + (-b^2)^(3/2)*(-1 + n) +
 a*b^2*n)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt
[-b^2])])/((a + Sqrt[-b^2])*(1 + n)) + 2*b*Cos[c + d*x]^2*(b + a*Tan[c + d*
x])))/(4*b*(a^2 + b^2)*d)
```

Maple [F]

$$\int \left(\cos^2(dx+c)\right)(a+b\tan(dx+c))^n\,dx$$

```
[In] int(cos(d*x+c)^2*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(cos(d*x+c)^2*(a+b*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^n\,dx=\int (b\tan(dx+c)+a)^n\cos(dx+c)^2\,dx$$

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c) + a)^n*cos(d*x + c)^2, x)
```

Sympy [F]

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^n\,dx = \int (a+b\tan(c+dx))^n\cos^2(c+dx)\,dx$$

```
[In] integrate(cos(d*x+c)**2*(a+b*tan(d*x+c))**n,x)
[Out] Integral((a + b*tan(c + d*x))**n*cos(c + d*x)**2, x)
```

Maxima [F]

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n\cos(dx+c)^2\,dx$$

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^n*cos(d*x + c)^2, x)
```

Giac [F]

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n\cos(dx+c)^2\,dx$$

```
[In] integrate(cos(d*x+c)^2*(a+b*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^n*cos(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(c+dx)(a+b\tan(c+dx))^n\,dx = \int \cos(c+d\,x)^2\,(a+b\tan(c+d\,x))^n\,dx$$

```
[In] int(cos(c + d*x)^2*(a + b*tan(c + d*x))^n,x)
[Out] int(cos(c + d*x)^2*(a + b*tan(c + d*x))^n, x)
```

3.650 $\int \cos^4(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 434

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{b\left(\frac{a\left(5+\frac{3a^2}{b^2}-2n\right)n}{b^2}-\frac{\sqrt{-b^2}(3a^4+a^2b^2(6-2n-n^2)+b^4(3-4n+n^2))}{b^6}\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)(}{16\left(1+\frac{a^2}{b^2}\right)^2\left(a-\sqrt{-b^2}\right)d(1+n)}$$
$$+\frac{b\left(\frac{a\left(5+\frac{3a^2}{b^2}-2n\right)n}{b^2}+\frac{\sqrt{-b^2}(3a^4+a^2b^2(6-2n-n^2)+b^4(3-4n+n^2))}{b^6}\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)}{16\left(1+\frac{a^2}{b^2}\right)^2\left(a+\sqrt{-b^2}\right)d(1+n)}$$
$$+\frac{\cos^4(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{4(a^2+b^2)\,d}$$
$$+\frac{b\cos^2(c+dx)(a+b\tan(c+dx))^{1+n}\left(b^2(3-n)+a^2(1+n)+ab\left(5+\frac{3a^2}{b^2}-2n\right)\tan(c+dx)\right)}{8(a^2+b^2)^2\,d}$$

```
[Out] 1/16*b*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a-(-b^2)^(1/2)))*(a*(5+3*
a^2/b^2-2*n)*n/b^2-(3*a^4+a^2*b^2*(-n^2-2*n+6)+b^4*(n^2-4*n+3))*(-b^2)^(1/2
)/b^6)*(a+b*tan(d*x+c))^(1+n)/(1+a^2/b^2)^2/d/(1+n)/(a-(-b^2)^(1/2))+1/16*b
*hypergeom([1, 1+n],[2+n],(a+b*tan(d*x+c))/(a+(-b^2)^(1/2)))*(a*(5+3*a^2/b^
2-2*n)*n/b^2+(3*a^4+a^2*b^2*(-n^2-2*n+6)+b^4*(n^2-4*n+3))*(-b^2)^(1/2)/b^6)
*(a+b*tan(d*x+c))^(1+n)/(1+a^2/b^2)^2/d/(1+n)/(a+(-b^2)^(1/2))+1/4*cos(d*x+
c)^4*(b+a*tan(d*x+c))*(a+b*tan(d*x+c))^(1+n)/(a^2+b^2)/d+1/8*b*cos(d*x+c)^2
*(a+b*tan(d*x+c))^(1+n)*(b^2*(3-n)+a^2*(1+n)+a*b*(5+3*a^2/b^2-2*n)*tan(d*x+
c))/(a^2+b^2)^2/d
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3587, 755, 837, 845, 70}

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{\cos^4(c+dx)(a\tan(c+dx)+b)(a+b\tan(c+dx))^{n+1}}{4d(a^2+b^2)} + \frac{b\cos^2(c+dx)\left(ab\left(\frac{3a^2}{b^2}-2n+5\right)\tan(c+dx)+a^2(n+1)+b^2(3-n)\right)(a+b\tan(c+dx))^{n+1}}{8d(a^2+b^2)^2} + \frac{b\left(\frac{an\left(\frac{3a^2}{b^2}-2n+5\right)}{b^2}-\frac{\sqrt{-b^2}(3a^4+a^2b^2(-n^2-2n+6)+b^4(n^2-4n+3))}{b^6}\right)(a+b\tan(c+dx))^{n+1}\text{Hypergeometric2F1}\left(1,\right.}{16d(n+1)\left(\frac{a^2}{b^2}+1\right)^2\left(a-\sqrt{-b^2}\right)} + \frac{b\left(\frac{an\left(\frac{3a^2}{b^2}-2n+5\right)}{b^2}+\frac{\sqrt{-b^2}(3a^4+a^2b^2(-n^2-2n+6)+b^4(n^2-4n+3))}{b^6}\right)(a+b\tan(c+dx))^{n+1}\text{Hypergeometric2F1}\left(1,\right.}{16d(n+1)\left(\frac{a^2}{b^2}+1\right)^2\left(a+\sqrt{-b^2}\right)}$$

```
[In] Int[Cos[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (b*((a*(5 + (3*a^2)/b^2 - 2*n)*n)/b^2 - (Sqrt[-b^2]*(3*a^4 + a^2*b^2*(6 - 2
*n - n^2) + b^4*(3 - 4*n + n^2)))/b^6)*Hypergeometric2F1[1, 1 + n, 2 + n, (
a + b*Tan[c + d*x])/(a - Sqrt[-b^2])]*(a + b*Tan[c + d*x])^(1 + n))/(16*(1
+ a^2/b^2)^2*(a - Sqrt[-b^2])*d*(1 + n)) + (b*((a*(5 + (3*a^2)/b^2 - 2*n)*n
)/b^2 + (Sqrt[-b^2]*(3*a^4 + a^2*b^2*(6 - 2*n - n^2) + b^4*(3 - 4*n + n^2))
)/b^6)*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a + Sqrt[-b
^2])]*(a + b*Tan[c + d*x])^(1 + n))/(16*(1 + a^2/b^2)^2*(a + Sqrt[-b^2])*d*
(1 + n)) + (Cos[c + d*x]^4*(b + a*Tan[c + d*x])*(a + b*Tan[c + d*x])^(1 + n
))/(4*(a^2 + b^2)*d) + (b*Cos[c + d*x]^2*(a + b*Tan[c + d*x])^(1 + n)*(b^2*
(3 - n) + a^2*(1 + n) + a*b*(5 + (3*a^2)/b^2 - 2*n)*Tan[c + d*x]))/(8*(a^2
+ b^2)^2*d)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
```

```
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a+x)^n}{\left(1+\frac{x^2}{b^2}\right)^3}\,dx, x, b\tan(c+dx)\right)}{bd}\\ &= \frac{\cos^4(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{4\left(a^2+b^2\right)d}\\ &\quad - \frac{b\text{Subst}\left(\int \frac{(a+x)^n\left(-3-\frac{3a^2}{b^2}+n-\frac{a(2-n)x}{b^2}\right)}{\left(1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\tan(c+dx)\right)}{4\left(a^2+b^2\right)d}\end{aligned}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{4(a^2+b^2)d} + \frac{b\cos^2(c+dx)(a+b\tan(c+dx))^{1+n}\left(b^2(3-n)+a^2(1+n)+ab\left(5+\frac{3a^2}{b^2}-2n\right)\tan(c+dx)\right)}{8(a^2+b^2)^2 d} + \frac{b^5\text{Subst}\left(\int \frac{(a+x)^n\left(\frac{3a^4+a^2b^2(6-2n-n^2)+b^4(3-4n+n^2)}{b^6}-\frac{a\left(5+\frac{3a^2}{b^2}-2n\right)nx}{b^4}\right)}{1+\frac{x^2}{b^2}}\,dx, x, b\tan(c+dx)\right)}{8(a^2+b^2)^2 d}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{4(a^2+b^2)d} + \frac{b\cos^2(c+dx)(a+b\tan(c+dx))^{1+n}\left(b^2(3-n)+a^2(1+n)+ab\left(5+\frac{3a^2}{b^2}-2n\right)\tan(c+dx)\right)}{8(a^2+b^2)^2 d} + \frac{b^5\text{Subst}\left(\int\left(\frac{\left(\frac{a\left(5+\frac{3a^2}{b^2}-2n\right)n}{b^2}+\frac{\sqrt{-b^2}\left(3a^4+a^2b^2(6-2n-n^2)+b^4(3-4n+n^2)\right)}{b^6}\right)(a+x)^n}{2\left(\sqrt{-b^2}-x\right)}+\frac{\left(-\frac{a\left(5+\frac{3a^2}{b^2}-2n\right)n}{b^2}+\frac{\sqrt{-b^2}\left(3a^4\ldots\right.}{}\right.}{2(\ldots}\right.\right.}{8(a^2+b^2)^2 d}$$

$$= \frac{\cos^4(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{4(a^2+b^2)d} + \frac{b\cos^2(c+dx)(a+b\tan(c+dx))^{1+n}\left(b^2(3-n)+a^2(1+n)+ab\left(5+\frac{3a^2}{b^2}-2n\right)\tan(c+dx)\right)}{8(a^2+b^2)^2 d} - \frac{\left(b^5\left(\frac{a\left(5+\frac{3a^2}{b^2}-2n\right)n}{b^2}-\frac{\sqrt{-b^2}\left(3a^4+a^2b^2(6-2n-n^2)+b^4(3-4n+n^2)\right)}{b^6}\right)\right)\text{Subst}\left(\int\frac{(a+x)^n}{\sqrt{-b^2}+x}\,dx, x, b\tan(c+dx\right.}{16(a^2+b^2)^2 d} + \frac{\left(b^5\left(\frac{a\left(5+\frac{3a^2}{b^2}-2n\right)n}{b^2}+\frac{\sqrt{-b^2}\left(3a^4+a^2b^2(6-2n-n^2)+b^4(3-4n+n^2)\right)}{b^6}\right)\right)\text{Subst}\left(\int\frac{(a+x)^n}{\sqrt{-b^2}-x}\,dx, x, b\tan(c+dx\right.}{16(a^2+b^2)^2 d}$$

$$= \frac{b^5\left(\frac{a\left(5+\frac{3a^2}{b^2}-2n\right)n}{b^2} - \frac{\sqrt{-b^2}(3a^4+a^2b^2(6-2n-n^2)+b^4(3-4n+n^2))}{b^6}\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan}{a-\sqrt{}}\right.}{16\left(a^2+b^2\right)^2\left(a-\sqrt{-b^2}\right)d(1+n)}$$

$$+ \frac{b^5\left(\frac{a\left(5+\frac{3a^2}{b^2}-2n\right)n}{b^2} + \frac{\sqrt{-b^2}(3a^4+a^2b^2(6-2n-n^2)+b^4(3-4n+n^2))}{b^6}\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b}{a}\right.}{16\left(a^2+b^2\right)^2\left(a+\sqrt{-b^2}\right)d(1+n)}$$

$$+ \frac{\cos^4(c+dx)(b+a\tan(c+dx))(a+b\tan(c+dx))^{1+n}}{4\left(a^2+b^2\right)d}$$

$$+ \frac{b\cos^2(c+dx)(a+b\tan(c+dx))^{1+n}\left(b^2(3-n)+a^2(1+n)+ab\left(5+\frac{3a^2}{b^2}-2n\right)\tan(c+dx)\right)}{8\left(a^2+b^2\right)^2d}$$

Mathematica [A] (verified)

Time = 4.71 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.83

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^n\,dx$$

$$= \frac{(a+b\tan(c+dx))^{1+n}\left(\frac{\frac{\left(ab^2\left(3a^2+b^2(5-2n)\right)n+\sqrt{-b^2}\left(-3a^4-b^4\left(3-4n+n^2\right)+a^2b^2\left(-6+2n+n^2\right)\right)\right)\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+b\tan(c+d}{a-\sqrt{-b^2}}\right.}{a-\sqrt{-b^2}}}{(a^2}\right.}{}$$

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] ((a + b*Tan[c + d*x])^(1 + n)*((((a*b^2*(3*a^2 + b^2*(5 - 2*n))*n + Sqrt[-b
^2]*(-3*a^4 - b^4*(3 - 4*n + n^2) + a^2*b^2*(-6 + 2*n + n^2)))*Hypergeometr
ic2F1[1, 1 + n, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2])])/(a - Sqrt[-b
^2]) + ((a*b^2*(3*a^2 + b^2*(5 - 2*n))*n + Sqrt[-b^2]*(3*a^4 + b^4*(3 - 4*n
 + n^2) - a^2*b^2*(-6 + 2*n + n^2)))*Hypergeometric2F1[1, 1 + n, 2 + n, (a
+ b*Tan[c + d*x])/(a + Sqrt[-b^2])])/(a + Sqrt[-b^2]))/((a^2 + b^2)*(1 + n)
) + 4*b*Cos[c + d*x]^4*(b + a*Tan[c + d*x]) - (2*b*Cos[c + d*x]^2*(b^3*(-3
+ n) - a^2*b*(1 + n) - a*(3*a^2 + b^2*(5 - 2*n))*Tan[c + d*x]))/(a^2 + b^2)
))/(16*b*(a^2 + b^2)*d)
```

Maple [F]

$$\int \left(\cos^{4}\left(dx+c\right)\right)\left(a+b\tan\left(dx+c\right)\right)^{n}dx$$

```
[In] int(cos(d*x+c)^4*(a+b*tan(d*x+c))^n,x)
[Out] int(cos(d*x+c)^4*(a+b*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^n\,dx=\int \left(b\tan\left(dx+c\right)+a\right)^{n}\cos\left(dx+c\right)^{4}\,dx$$

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*tan(d*x + c) + a)^n*cos(d*x + c)^4, x)
```

Sympy [F]

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^n\,dx=\int \left(a+b\tan\left(c+dx\right)\right)^{n}\cos^{4}\left(c+dx\right)dx$$

```
[In] integrate(cos(d*x+c)**4*(a+b*tan(d*x+c))**n,x)
[Out] Integral((a + b*tan(c + d*x))**n*cos(c + d*x)**4, x)
```

Maxima [F]

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^n\,dx=\int \left(b\tan\left(dx+c\right)+a\right)^{n}\cos\left(dx+c\right)^{4}\,dx$$

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^n*cos(d*x + c)^4, x)
```

Giac [F]

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n\cos(dx+c)^4\,dx$$

```
[In] integrate(cos(d*x+c)^4*(a+b*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^n*cos(d*x + c)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^4(c+dx)(a+b\tan(c+dx))^n\,dx = \int \cos(c+d\,x)^4\,(a+b\tan(c+d\,x))^n\,dx$$

```
[In] int(cos(c + d*x)^4*(a + b*tan(c + d*x))^n,x)
[Out] int(cos(c + d*x)^4*(a + b*tan(c + d*x))^n, x)
```

3.651 $\int \sec^3(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 159

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{\operatorname{AppellF1}\left(1+n,-\frac{1}{2},-\frac{1}{2},2+n,\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)\sec(c+dx)(a+b\tan(c+dx))^{1+n}}{bd(1+n)\sqrt{1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}}\sqrt{1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}}}$$

```
[Out] AppellF1(1+n,-1/2,-1/2,2+n,(a+b*tan(d*x+c))/(a-(-b^2)^(1/2)),(a+b*tan(d*x+c
))/(a+(-b^2)^(1/2)))*sec(d*x+c)*(a+b*tan(d*x+c))^(1+n)/b/d/(1+n)/(1+(-a-b*t
an(d*x+c))/(a-(-b^2)^(1/2)))^(1/2)/(1+(-a-b*tan(d*x+c))/(a+(-b^2)^(1/2)))^(
1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3593, 774, 138}

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{\sec(c+dx)(a+b\tan(c+dx))^{n+1}\operatorname{AppellF1}\left(n+1,-\frac{1}{2},-\frac{1}{2},n+2,\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)}{bd(n+1)\sqrt{1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}}\sqrt{1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}}}$$

```
[In] Int[Sec[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (AppellF1[1 + n, -1/2, -1/2, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]),
(a + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*Sec[c + d*x]*(a + b*Tan[c + d*x])^(1
```

```
 + n))/(b*d*(1 + n)*Sqrt[1 - (a + b*Tan[c + d*x])/(a - Sqrt[-b^2])]*Sqrt[1
- (a + b*Tan[c + d*x])/(a + Sqrt[-b^2])])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 774

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + e*(q/c)))^p*
(1 - (d + e*x)/(d - e*(q/c)))^p), Subst[Int[x^m*Simp[1 - x/(d + e*(q/c)), x
]^p*Simp[1 - x/(d - e*(q/c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\sec(c+dx)\text{Subst}\left(\int (a+x)^n\sqrt{1+\frac{x^2}{b^2}}\,dx,x,b\tan(c+dx)\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$= \frac{\sec(c+dx)\text{Subst}\left(\int x^n\sqrt{1-\frac{x}{a-\sqrt{-b^2}}}\sqrt{1-\frac{x}{a+\sqrt{-b^2}}}\,dx,x,a+b\tan(c+dx)\right)}{bd\sqrt{1-\frac{a+b\tan(c+dx)}{a-\frac{b^2}{\sqrt{-b^2}}}}\sqrt{1-\frac{a+b\tan(c+dx)}{a+\frac{b^2}{\sqrt{-b^2}}}}}$$

$$= \frac{\text{AppellF1}\left(1+n,-\frac{1}{2},-\frac{1}{2},2+n,\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)\sec(c+dx)(a+b\tan(c+dx))^{1+n}}{bd(1+n)\sqrt{1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}}\sqrt{1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}}}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 14.54 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.92

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^n\,dx$$
$$=\frac{2(a-ib)(a+ib)(2+n)\operatorname{AppellF1}\left(1+\right.}{bd(1+n)\left(2\left(a^2+b^2\right)(2+n)\operatorname{AppellF1}\left(1+n,-\frac{1}{2},-\frac{1}{2},2+n,\frac{a+b\tan(c+dx)}{a-ib},\frac{a+b\tan(c+dx)}{a+ib}\right)-\left((a-ib)\operatorname{App}\right.\right.}$$

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (2*(a - I*b)*(a + I*b)*(2 + n)*AppellF1[1 + n, -1/2, -1/2, 2 + n, (a + b*Ta
n[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)]*Sec[c + d*x]*(a + b*
Tan[c + d*x])^(1 + n))/(b*d*(1 + n)*(2*(a^2 + b^2)*(2 + n)*AppellF1[1 + n,
-1/2, -1/2, 2 + n, (a + b*Tan[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a
+ I*b)] - ((a - I*b)*AppellF1[2 + n, -1/2, 1/2, 3 + n, (a + b*Tan[c + d*x])
/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)] + (a + I*b)*AppellF1[2 + n, 1/2
, -1/2, 3 + n, (a + b*Tan[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*
b)])*(a + b*Tan[c + d*x])))
```

Maple [F]

$$\int \left(\sec^3\left(dx+c\right)\right)\left(a+b\tan\left(dx+c\right)\right)^n dx$$

```
[In] int(sec(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(sec(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^n\,dx=\int \left(b\tan\left(dx+c\right)+a\right)^n\sec\left(dx+c\right)^3\,dx$$

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c) + a)^n*sec(d*x + c)^3, x)
```

Sympy [F]

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^n\,dx = \int (a+b\tan(c+dx))^n \sec^3(c+dx)\,dx$$

```
[In] integrate(sec(d*x+c)**3*(a+b*tan(d*x+c))**n,x)
[Out] Integral((a + b*tan(c + d*x))**n*sec(c + d*x)**3, x)
```

Maxima [F]

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n \sec(dx+c)^3\,dx$$

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^n*sec(d*x + c)^3, x)
```

Giac [F]

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n \sec(dx+c)^3\,dx$$

```
[In] integrate(sec(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^n*sec(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(c+dx)(a+b\tan(c+dx))^n\,dx = \int \frac{(a+b\tan(c+d\,x))^n}{\cos(c+d\,x)^3}\,dx$$

```
[In] int((a + b*tan(c + d*x))^n/cos(c + d*x)^3,x)
[Out] int((a + b*tan(c + d*x))^n/cos(c + d*x)^3, x)
```

3.652 $\int \sec(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 159

$$\int \sec(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{\text{AppellF1}\left(1+n,\frac{1}{2},\frac{1}{2},2+n,\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)\cos(c+dx)(a+b\tan(c+dx))^{1+n}\sqrt{1-\frac{a+b\tan(c+d}{a-\sqrt{-b^2}}}}{bd(1+n)}$$

```
[Out] AppellF1(1+n,1/2,1/2,2+n,(a+b*tan(d*x+c))/(a-(-b^2)^(1/2)),(a+b*tan(d*x+c))
/(a+(-b^2)^(1/2)))*cos(d*x+c)*(1+(-a-b*tan(d*x+c))/(a-(-b^2)^(1/2)))^(1/2)*
(1+(-a-b*tan(d*x+c))/(a+(-b^2)^(1/2)))^(1/2)*(a+b*tan(d*x+c))^(1+n)/b/d/(1+
n)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3593, 774, 138}

$$\int \sec(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{\cos(c+dx)\sqrt{1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}}\sqrt{1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}}(a+b\tan(c+dx))^{n+1}\,\text{AppellF1}\left(n+1,\frac{1}{2},\frac{1}{2},n+2,\frac{a+b\tan}{a-\sqrt{}}\right.}{bd(n+1)}$$

```
[In] Int[Sec[c + d*x]*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (AppellF1[1 + n, 1/2, 1/2, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]), (a
 + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*Cos[c + d*x]*(a + b*Tan[c + d*x])^(1 +
```

```
n)*Sqrt[1 - (a + b*Tan[c + d*x])/(a - Sqrt[-b^2])]*Sqrt[1 - (a + b*Tan[c +
d*x])/(a + Sqrt[-b^2])])/(b*d*(1 + n))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 774

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + e*(q/c)))^p*
(1 - (d + e*x)/(d - e*(q/c)))^p), Subst[Int[x^m*Simp[1 - x/(d + e*(q/c)), x
]^p*Simp[1 - x/(d - e*(q/c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\sec(c+dx)\text{Subst}\left(\int \frac{(a+x)^n}{\sqrt{1+\frac{x^2}{b^2}}}\,dx, x, b\tan(c+dx)\right)}{bd\sqrt{\sec^2(c+dx)}}$$

$$= \frac{\left(\cos(c+dx)\sqrt{1-\frac{a+b\tan(c+dx)}{a-\frac{b^2}{\sqrt{-b^2}}}}\sqrt{1-\frac{a+b\tan(c+dx)}{a+\frac{b^2}{\sqrt{-b^2}}}}\right)\text{Subst}\left(\int \frac{x^n}{\sqrt{1-\frac{x}{a-\sqrt{-b^2}}}\sqrt{1-\frac{x}{a+\sqrt{-b^2}}}}\,dx, x, a+b\tan(\right.}{bd}$$

$$= \frac{\text{AppellF1}\left(1+n,\frac{1}{2},\frac{1}{2},2+n,\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)\cos(c+dx)(a+b\tan(c+dx))^{1+n}\sqrt{1-\frac{a+b}{a}}}{bd(1+n)}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 5.18 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.14

$$\int \sec(c+dx)(a+b\tan(c+dx))^n\,dx$$

$$= \frac{2(a^2+b^2)^2\,(2+n)\,\text{AppellF1}\left(1+n,\frac{1}{2},\frac{1}{2},2+n,\frac{a\cdots}{\cdots}\right.}{(a-ib)(a+ib)bd(1+n)\left(2\,(a^2+b^2)\,(2+n)\,\text{AppellF1}\left(1+n,\frac{1}{2},\frac{1}{2},2+n,\frac{a+b\tan(c+dx)}{a-ib},\frac{a+b\tan(c+dx)}{a+ib}\right)+\left(\cdots\right.\right.}$$

```
[In] Integrate[Sec[c + d*x]*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (2*(a^2 + b^2)^2*(2 + n)*AppellF1[1 + n, 1/2, 1/2, 2 + n, (a + b*Tan[c + d*
x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)]*Cos[c + d*x]^3*(-I + Tan[c +
 d*x])*(I + Tan[c + d*x])*(a + b*Tan[c + d*x])^(1 + n))/((a - I*b)*(a + I*b
)*b*d*(1 + n)*(2*(a^2 + b^2)*(2 + n)*AppellF1[1 + n, 1/2, 1/2, 2 + n, (a +
b*Tan[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)] + ((a - I*b)*App
ellF1[2 + n, 1/2, 3/2, 3 + n, (a + b*Tan[c + d*x])/(a - I*b), (a + b*Tan[c
+ d*x])/(a + I*b)] + (a + I*b)*AppellF1[2 + n, 3/2, 1/2, 3 + n, (a + b*Tan[
c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)])*(a + b*Tan[c + d*x]))
)
```

Maple [F]

$$\int \sec\left(dx+c\right)\left(a+b\tan\left(dx+c\right)\right)^{n}dx$$

```
[In] int(sec(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(sec(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int \sec(c+dx)(a+b\tan(c+dx))^n\,dx = \int \left(b\tan\left(dx+c\right)+a\right)^{n}\sec\left(dx+c\right)\,dx$$

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c) + a)^n*sec(d*x + c), x)
```

Sympy [F]

$$\int \sec(c+dx)(a+b\tan(c+dx))^n\,dx = \int (a+b\tan{(c+dx)})^n\sec{(c+dx)}\,dx$$

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c))**n,x)
[Out] Integral((a + b*tan(c + d*x))**n*sec(c + d*x), x)
```

Maxima [F]

$$\int \sec(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan{(dx+c)}+a)^n\sec{(dx+c)}\,dx$$

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^n*sec(d*x + c), x)
```

Giac [F]

$$\int \sec(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan{(dx+c)}+a)^n\sec{(dx+c)}\,dx$$

```
[In] integrate(sec(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^n*sec(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(c+dx)(a+b\tan(c+dx))^n\,dx = \int \frac{(a+b\tan(c+d\,x))^n}{\cos{(c+d\,x)}}\,dx$$

```
[In] int((a + b*tan(c + d*x))^n/cos(c + d*x),x)
[Out] int((a + b*tan(c + d*x))^n/cos(c + d*x), x)
```

3.653 $\int \cos(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 19, antiderivative size = 161

$$\int \cos(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{\text{AppellF1}\left(1+n,\frac{3}{2},\frac{3}{2},2+n,\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)\cos^3(c+dx)(a+b\tan(c+dx))^{1+n}\left(1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)\ldots}{bd(1+n)}$$

```
[Out] AppellF1(1+n,3/2,3/2,2+n,(a+b*tan(d*x+c))/(a-(-b^2)^(1/2)),(a+b*tan(d*x+c))
/(a+(-b^2)^(1/2)))*cos(d*x+c)^3*(a+b*tan(d*x+c))^(1+n)*(1+(-a-b*tan(d*x+c))
/(a-(-b^2)^(1/2)))^(3/2)*(1+(-a-b*tan(d*x+c))/(a+(-b^2)^(1/2)))^(3/2)/b/d/(
1+n)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3593, 774, 138}

$$\int \cos(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{\cos^3(c+dx)\left(1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)^{3/2}\left(1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)^{3/2}(a+b\tan(c+dx))^{n+1}\text{AppellF1}\left(n+1,\frac{3}{2},\frac{3}{2},n+\ldots\right.}{bd(n+1)}$$

```
[In] Int[Cos[c + d*x]*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (AppellF1[1 + n, 3/2, 3/2, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]), (a
 + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*Cos[c + d*x]^3*(a + b*Tan[c + d*x])^(1
```

```
+ n)*(1 - (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]))^(3/2)*(1 - (a + b*Tan[c +
d*x])/(a + Sqrt[-b^2]))^(3/2))/(b*d*(1 + n))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 774

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + e*(q/c)))^p*
(1 - (d + e*x)/(d - e*(q/c)))^p), Subst[Int[x^m*Simp[1 - x/(d + e*(q/c)), x
]^p*Simp[1 - x/(d - e*(q/c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\left(\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{(a+x)^n}{\left(1+\frac{x^2}{b^2}\right)^{3/2}}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\left(\cos^3(c+dx)\left(1-\frac{a+b\tan(c+dx)}{a-\frac{b^2}{\sqrt{-b^2}}}\right)^{3/2}\left(1-\frac{a+b\tan(c+dx)}{a+\frac{b^2}{\sqrt{-b^2}}}\right)^{3/2}\right)\text{Subst}\left(\int \frac{x^n}{\left(1-\frac{x}{a-\sqrt{-b^2}}\right)^{3/2}\left(1-\frac{x}{a+\sqrt{-b^2}}\right)^{3/2}}\right.}{bd}$$

$$= \frac{\text{AppellF1}\left(1+n,\frac{3}{2},\frac{3}{2},2+n,\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)\cos^3(c+dx)(a+b\tan(c+dx))^{1+n}\left(1-\frac{a+b}{a}\right.}{bd(1+n)}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.57 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.12

$$\int \cos(c+dx)(a+b\tan(c+dx))^n\,dx$$

$$=\frac{2(a^2+b^2)^2(2+n)\,\text{AppellF1}\left(1+n,\tfrac{3}{2},\tfrac{3}{2},2+n,\tfrac{a}{}\right.}{(a-ib)(a+ib)bd(1+n)\left(2(a^2+b^2)(2+n)\,\text{AppellF1}\left(1+n,\tfrac{3}{2},\tfrac{3}{2},2+n,\tfrac{a+b\tan(c+dx)}{a-ib},\tfrac{a+b\tan(c+dx)}{a+ib}\right)+3\right.}$$

```
[In] Integrate[Cos[c + d*x]*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (2*(a^2 + b^2)^2*(2 + n)*AppellF1[1 + n, 3/2, 3/2, 2 + n, (a + b*Tan[c + d*
x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)]*Cos[c + d*x]^5*(-I + Tan[c +
 d*x])*(I + Tan[c + d*x])*(a + b*Tan[c + d*x])^(1 + n))/((a - I*b)*(a + I*b
)*b*d*(1 + n)*(2*(a^2 + b^2)*(2 + n)*AppellF1[1 + n, 3/2, 3/2, 2 + n, (a +
b*Tan[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)] + 3*((a - I*b)*A
ppellF1[2 + n, 3/2, 5/2, 3 + n, (a + b*Tan[c + d*x])/(a - I*b), (a + b*Tan[
c + d*x])/(a + I*b)] + (a + I*b)*AppellF1[2 + n, 5/2, 3/2, 3 + n, (a + b*Ta
n[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)])*(a + b*Tan[c + d*x]
)))
```

Maple [F]

$$\int \cos(dx+c)\,(a+b\tan(dx+c))^n\,dx$$

```
[In] int(cos(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(cos(d*x+c)*(a+b*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int \cos(c+dx)(a+b\tan(c+dx))^n\,dx=\int (b\tan(dx+c)+a)^n\cos(dx+c)\,dx$$

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c) + a)^n*cos(d*x + c), x)
```

Sympy [F]

$$\int \cos(c+dx)(a+b\tan(c+dx))^n\,dx = \int (a+b\tan(c+dx))^n\cos(c+dx)\,dx$$

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c))**n,x)
[Out] Integral((a + b*tan(c + d*x))**n*cos(c + d*x), x)
```

Maxima [F]

$$\int \cos(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n\cos(dx+c)\;dx$$

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^n*cos(d*x + c), x)
```

Giac [F]

$$\int \cos(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n\cos(dx+c)\;dx$$

```
[In] integrate(cos(d*x+c)*(a+b*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^n*cos(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(c+dx)(a+b\tan(c+dx))^n\,dx = \int \cos(c+d\,x)\;(a+b\tan(c+d\,x))^n\,dx$$

```
[In] int(cos(c + d*x)*(a + b*tan(c + d*x))^n,x)
[Out] int(cos(c + d*x)*(a + b*tan(c + d*x))^n, x)
```

3.654 $\int \cos^3(c+dx)(a+b\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 21, antiderivative size = 161

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{\text{AppellF1}\left(1+n,\frac{5}{2},\frac{5}{2},2+n,\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}},\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)\cos^5(c+dx)(a+b\tan(c+dx))^{1+n}\left(1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right.}{bd(1+n)}$$

```
[Out] AppellF1(1+n,5/2,5/2,2+n,(a+b*tan(d*x+c))/(a-(-b^2)^(1/2)),(a+b*tan(d*x+c))
/(a+(-b^2)^(1/2)))*cos(d*x+c)^5*(a+b*tan(d*x+c))^(1+n)*(1+(-a-b*tan(d*x+c))
/(a-(-b^2)^(1/2)))^(5/2)*(1+(-a-b*tan(d*x+c))/(a+(-b^2)^(1/2)))^(5/2)/b/d/(
1+n)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3593, 774, 138}

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^n\,dx = \frac{\cos^5(c+dx)\left(1-\frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}\right)^{5/2}\left(1-\frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)^{5/2}(a+b\tan(c+dx))^{n+1}\text{AppellF1}\left(n+1,\frac{5}{2},\frac{5}{2},n+\right.}{bd(n+1)}$$

```
[In] Int[Cos[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (AppellF1[1 + n, 5/2, 5/2, 2 + n, (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]), (a
 + b*Tan[c + d*x])/(a + Sqrt[-b^2])]*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^(1
```

```
 + n)*(1 - (a + b*Tan[c + d*x])/(a - Sqrt[-b^2]))^(5/2)*(1 - (a + b*Tan[c +
 d*x])/(a + Sqrt[-b^2]))^(5/2))/(b*d*(1 + n))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 774

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + e*(q/c)))^p*
(1 - (d + e*x)/(d - e*(q/c)))^p), Subst[Int[x^m*Simp[1 - x/(d + e*(q/c)), x
]^p*Simp[1 - x/(d - e*(q/c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\left(\cos(c+dx)\sqrt{\sec^2(c+dx)}\right)\text{Subst}\left(\int \frac{(a+x)^n}{\left(1+\frac{x^2}{b^2}\right)^{5/2}}\,dx, x, b\tan(c+dx)\right)}{bd}$$

$$= \frac{\left(\cos^5(c+dx)\left(1-\frac{a+b\tan(c+dx)}{a-\frac{b^2}{\sqrt{-b^2}}}\right)^{5/2}\left(1-\frac{a+b\tan(c+dx)}{a+\frac{b^2}{\sqrt{-b^2}}}\right)^{5/2}\right)\text{Subst}\left(\int \frac{x^n}{\left(1-\frac{x}{a-\sqrt{-b^2}}\right)^{5/2}\left(1-\frac{x}{a+\sqrt{-b^2}}\right)^{5/2}}\right.}{bd}$$

$$= \frac{\text{AppellF1}\left(1+n, \frac{5}{2}, \frac{5}{2}, 2+n, \frac{a+b\tan(c+dx)}{a-\sqrt{-b^2}}, \frac{a+b\tan(c+dx)}{a+\sqrt{-b^2}}\right)\cos^5(c+dx)(a+b\tan(c+dx))^{1+n}\left(1-\frac{a+}{}\right.}{bd(1+n)}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.43 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.12

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^n\,dx$$
$$=\frac{2(a^2+b^2)^2(2+n)\operatorname{AppellF1}\left(1+n,\frac{5}{2},\frac{5}{2},2+n,\right.}{(a-ib)(a+ib)bd(1+n)\left(2\left(a^2+b^2\right)(2+n)\operatorname{AppellF1}\left(1+n,\frac{5}{2},\frac{5}{2},2+n,\frac{a+b\tan(c+dx)}{a-ib},\frac{a+b\tan(c+dx)}{a+ib}\right)+5\right.}$$

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Tan[c + d*x])^n,x]
```

```
[Out] (2*(a^2 + b^2)^2*(2 + n)*AppellF1[1 + n, 5/2, 5/2, 2 + n, (a + b*Tan[c + d*
x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)]*Cos[c + d*x]^7*(-I + Tan[c +
 d*x])*(I + Tan[c + d*x])*(a + b*Tan[c + d*x])^(1 + n))/((a - I*b)*(a + I*b
)*b*d*(1 + n)*(2*(a^2 + b^2)*(2 + n)*AppellF1[1 + n, 5/2, 5/2, 2 + n, (a +
b*Tan[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)] + 5*((a - I*b)*A
ppellF1[2 + n, 5/2, 7/2, 3 + n, (a + b*Tan[c + d*x])/(a - I*b), (a + b*Tan[
c + d*x])/(a + I*b)] + (a + I*b)*AppellF1[2 + n, 7/2, 5/2, 3 + n, (a + b*Ta
n[c + d*x])/(a - I*b), (a + b*Tan[c + d*x])/(a + I*b)])*(a + b*Tan[c + d*x]
)))
```

Maple [F]

$$\int \left(\cos^3\left(dx+c\right)\right)\left(a+b\tan\left(dx+c\right)\right)^n dx$$

```
[In] int(cos(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

```
[Out] int(cos(d*x+c)^3*(a+b*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^n\,dx=\int \left(b\tan\left(dx+c\right)+a\right)^n\cos\left(dx+c\right)^3\,dx$$

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*tan(d*x + c) + a)^n*cos(d*x + c)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^n\,dx = \text{Timed out}$$

```
[In] integrate(cos(d*x+c)**3*(a+b*tan(d*x+c))**n,x)
[Out] Timed out
```

Maxima [F]

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n\cos(dx+c)^3\,dx$$

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*tan(d*x + c) + a)^n*cos(d*x + c)^3, x)
```

Giac [F]

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^n\,dx = \int (b\tan(dx+c)+a)^n\cos(dx+c)^3\,dx$$

```
[In] integrate(cos(d*x+c)^3*(a+b*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*tan(d*x + c) + a)^n*cos(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a+b\tan(c+dx))^n\,dx = \int \cos(c+d\,x)^3\,(a+b\tan(c+d\,x))^n\,dx$$

```
[In] int(cos(c + d*x)^3*(a + b*tan(c + d*x))^n,x)
[Out] int(cos(c + d*x)^3*(a + b*tan(c + d*x))^n, x)
```

3.655 $\int (e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 124

$$\int (e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx = -\frac{2ia(e\cos(c+dx))^{7/2}}{7d} + \frac{10a(e\cos(c+dx))^{7/2}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\cos^{\frac{7}{2}}(c+dx)} + \frac{2a(e\cos(c+dx))^{7/2}\tan(c+dx)}{7d} + \frac{10a(e\cos(c+dx))^{7/2}\sec^2(c+dx)\tan(c+dx)}{21d}$$

```
[Out] -2/7*I*a*(e*cos(d*x+c))^(7/2)/d+10/21*a*(e*cos(d*x+c))^(7/2)*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d/c
os(d*x+c)^(7/2)+2/7*a*(e*cos(d*x+c))^(7/2)*tan(d*x+c)/d+10/21*a*(e*cos(d*x+
c))^(7/2)*sec(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3596, 3567, 3854, 3856, 2720}

$$\int (e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx = -\frac{2ia(e\cos(c+dx))^{7/2}}{7d} + \frac{10a\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(e\cos(c+dx))^{7/2}}{21d\cos^{\frac{7}{2}}(c+dx)} + \frac{2a\tan(c+dx)(e\cos(c+dx))^{7/2}}{7d} + \frac{10a\tan(c+dx)\sec^2(c+dx)(e\cos(c+dx))^{7/2}}{21d}$$

```
[In] Int[(e*Cos[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (((-2*I)/7)*a*(e*Cos[c + d*x])^(7/2))/d + (10*a*(e*Cos[c + d*x])^(7/2)*Elli
pticF[(c + d*x)/2, 2])/(21*d*Cos[c + d*x]^(7/2)) + (2*a*(e*Cos[c + d*x])^(7
/2)*Tan[c + d*x])/(7*d) + (10*a*(e*Cos[c + d*x])^(7/2)*Sec[c + d*x]^2*Tan[c
 + d*x])/(21*d)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\text{integral} = \left((e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\right)\int\frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{7/2}}\,dx$$

$$= -\frac{2ia(e\cos(c+dx))^{7/2}}{7d} + \left(a(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\right)\int\frac{1}{(e\sec(c+dx))^{7/2}}\,dx$$

$$= -\frac{2ia(e\cos(c+dx))^{7/2}}{7d} + \frac{2a(e\cos(c+dx))^{7/2}\tan(c+dx)}{7d} + \frac{\left(5a(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\right)\int\frac{1}{(e\sec(c+dx))^{3/2}}\,dx}{7e^2}$$

$$= -\frac{2ia(e\cos(c+dx))^{7/2}}{7d} + \frac{2a(e\cos(c+dx))^{7/2}\tan(c+dx)}{7d} + \frac{10a(e\cos(c+dx))^{7/2}\sec^2(c+dx)\tan(c+dx)}{21d} + \frac{\left(5a(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\right)\int\sqrt{e\sec(c+dx)}\,dx}{21e^4}$$

$$= -\frac{2ia(e\cos(c+dx))^{7/2}}{7d} + \frac{2a(e\cos(c+dx))^{7/2}\tan(c+dx)}{7d} + \frac{10a(e\cos(c+dx))^{7/2}\sec^2(c+dx)\tan(c+dx)}{21d} + \frac{\left(5a(e\cos(c+dx))^{7/2}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{21\cos^{\frac{7}{2}}(c+dx)}$$

$$= -\frac{2ia(e\cos(c+dx))^{7/2}}{7d} + \frac{10a(e\cos(c+dx))^{7/2}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{21d\cos^{\frac{7}{2}}(c+dx)} + \frac{2a(e\cos(c+dx))^{7/2}\tan(c+dx)}{7d} + \frac{10a(e\cos(c+dx))^{7/2}\sec^2(c+dx)\tan(c+dx)}{21d}$$

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.07

$$\int(e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx = \frac{ae^3\sqrt{e\cos(c+dx)}(\cos(dx)-i\sin(dx))\left(10\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(\cos(c+dx)-i\sin\right.}{}$$

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (a*e^3*Sqrt[e*Cos[c + d*x]]*(Cos[d*x] - I*Sin[d*x])*(10*EllipticF[(c + d*x)
/2, 2]*(Cos[c + d*x] - I*Sin[c + d*x]) + Sqrt[Cos[c + d*x]]*(-8*I + (2*I)*C
os[2*(c + d*x)] + 5*Sin[2*(c + d*x)]))*(Cos[c + 2*d*x] + I*Sin[c + 2*d*x]))
/(21*d*Sqrt[Cos[c + d*x]])
```

Maple [A] (verified)

Time = 9.94 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.85

method	result
parts	$-\frac{2a\sqrt{e(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)(\sin^2(\frac{dx}{2}+\frac{c}{2}))}\, e^4 (48(\cos^9(\frac{dx}{2}+\frac{c}{2}))-120(\cos^7(\frac{dx}{2}+\frac{c}{2}))+128(\cos^5(\frac{dx}{2}+\frac{c}{2}))-72(\cos^3(\frac{dx}{2}+\frac{c}{2})\cdots}{21\sqrt{-e(2(\sin^4(\frac{dx}{2}+\frac{c}{2}))-(\sin^2(\frac{dx}{2}+\frac{c}{2})))}\,\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{e(2(c\cdots}}$
default	$-\frac{2a\, e^4 (48i(\sin^9(\frac{dx}{2}+\frac{c}{2}))+48\cos(\frac{dx}{2}+\frac{c}{2})(\sin^8(\frac{dx}{2}+\frac{c}{2}))-96i(\sin^7(\frac{dx}{2}+\frac{c}{2}))-72(\sin^6(\frac{dx}{2}+\frac{c}{2}))\cos(\frac{dx}{2}+\frac{c}{2})+72i(\sin^5(\frac{dx}{2}+\frac{c}{2}\cdots}{2\cdots}$

```
[In] int((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21*a*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*e^4*(48*c
os(1/2*d*x+1/2*c)^9-120*cos(1/2*d*x+1/2*c)^7+128*cos(1/2*d*x+1/2*c)^5-72*co
s(1/2*d*x+1/2*c)^3+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+16*cos(1/2*d*x+1/2*c))/(-e*(
2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(e*(
2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d-2/7*I*a*(e*cos(d*x+c))^(7/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.89

$$\int (e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx = \frac{(-20i\sqrt{2}ae^{\frac{7}{2}}e^{(i\,dx+i\,c)}\text{weierstrassPInverse}(-4,0,e^{(i\,dx+i\,c)})+\sqrt{\frac{1}{2}}(-3i\,ae^3e^{(4i\,dx+4i\,c)}-1\cdots}{42\,d}$$

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/42*(-20*I*sqrt(2)*a*e^(7/2)*e^(I*d*x + I*c)*weierstrassPInverse(-4, 0, e^
(I*d*x + I*c)) + sqrt(1/2)*(-3*I*a*e^3*e^(4*I*d*x + 4*I*c) - 16*I*a*e^3*e^(
2*I*d*x + 2*I*c) + 7*I*a*e^3)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*e^(-1/2*I*d*x
 - 1/2*I*c))*e^(-I*d*x - I*c)/d
```

Sympy [F(-1)]

Timed out.

$$\int (e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx = \text{Timed out}$$

```
[In] integrate((e*cos(d*x+c))**(7/2)*(a+I*a*tan(d*x+c)),x)
[Out] Timed out
```

Maxima [F]

$$\int (e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx = \int (e\cos(dx+c))^{\frac{7}{2}}\,(i\,a\tan(dx+c)+a)\,dx$$

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*cos(d*x + c))^(7/2)*(I*a*tan(d*x + c) + a), x)
```

Giac [F]

$$\int (e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx = \int (e\cos(dx+c))^{\frac{7}{2}}\,(i\,a\tan(dx+c)+a)\,dx$$

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] integrate((e*cos(d*x + c))^(7/2)*(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx = \int (e\,\cos(c+d\,x))^{7/2}\,(a+a\tan(c+d\,x)\,1\mathrm{i})\,dx$$

```
[In] int((e*cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i),x)
[Out] int((e*cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i), x)
```

3.656 $\int (e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 90

$$\int (e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx=-\frac{2ia(e\cos(c+dx))^{5/2}}{5d}+\frac{6a(e\cos(c+dx))^{5/2}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a(e\cos(c+dx))^{5/2}\tan(c+dx)}{5d}$$

```
[Out] -2/5*I*a*(e*cos(d*x+c))^(5/2)/d+6/5*a*(e*cos(d*x+c))^(5/2)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos
(d*x+c)^(5/2)+2/5*a*(e*cos(d*x+c))^(5/2)*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3596, 3567, 3854, 3856, 2719}

$$\int (e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx=-\frac{2ia(e\cos(c+dx))^{5/2}}{5d}+\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)(e\cos(c+dx))^{5/2}}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a\tan(c+dx)(e\cos(c+dx))^{5/2}}{5d}$$

```
[In] Int[(e*Cos[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (((-2*I)/5)*a*(e*Cos[c + d*x])^(5/2))/d + (6*a*(e*Cos[c + d*x])^(5/2)*Ellip
ticE[(c + d*x)/2, 2])/(5*d*Cos[c + d*x]^(5/2)) + (2*a*(e*Cos[c + d*x])^(5/2
)*Tan[c + d*x])/(5*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left((e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\right)\int\frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{5/2}}\,dx\\
&= -\frac{2ia(e\cos(c+dx))^{5/2}}{5d}+\left(a(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\right)\int\frac{1}{(e\sec(c+dx))^{5/2}}\,dx\\
&= -\frac{2ia(e\cos(c+dx))^{5/2}}{5d}+\frac{2a(e\cos(c+dx))^{5/2}\tan(c+dx)}{5d}\\
&\quad+\frac{\left(3a(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\right)\int\frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{5e^2}\\
&= -\frac{2ia(e\cos(c+dx))^{5/2}}{5d}+\frac{2a(e\cos(c+dx))^{5/2}\tan(c+dx)}{5d}\\
&\quad+\frac{\left(3a(e\cos(c+dx))^{5/2}\right)\int\sqrt{\cos(c+dx)}\,dx}{5\cos^{\frac{5}{2}}(c+dx)}
\end{aligned}$$

$$= -\frac{2ia(e\cos(c+dx))^{5/2}}{5d} + \frac{6a(e\cos(c+dx))^{5/2}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a(e\cos(c+dx))^{5/2}\tan(c+dx)}{5d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.26 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.78

$$\int (e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx = \frac{e^2\sqrt{e\cos(c+dx)}(\cos(dx)-i\sin(dx))\left(\left(9\cos(c-dx-\arctan(\tan(c)))(\csc(c)-i\sec(c\right.\right.}{}$$

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (e^2*Sqrt[e*Cos[c + d*x]]*(Cos[d*x] - I*Sin[d*x])*((9*Cos[c - d*x - ArcTan[
Tan[c]]]*(Csc[c] - I*Sec[c]) + 3*Cos[c + d*x + ArcTan[Tan[c]]]*(Csc[c] - I*
Sec[c]) - Csc[c]*Sqrt[Sec[c]^2]*(Cos[d*x] + I*Sin[d*x])*(6*Cos[c] + 3*Cos[c
 + 2*d*x] + 3*Cos[3*c + 2*d*x] + (2*I)*Sin[c] - (4*I)*Sin[c + 2*d*x] - (2*I
)*Sin[3*c + 2*d*x]))*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2] + (6*I)*Hypergeometr
icPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Ta
n[c]]]*(I + Tan[c]))*(a + I*a*Tan[c + d*x]))/(10*d*Sqrt[Sec[c]^2]*Sqrt[Sin[
d*x + ArcTan[Tan[c]]]^2])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(101) = 202$.

Time = 7.90 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.28

method	result
default	$\frac{2ae^3\left(8i\left(\sin^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-12i\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+6i\left(\sin^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\cos\right.}{5\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}$
parts	$-\frac{2a\sqrt{e\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,e^3\left(-8\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\right.\right.}{5\sqrt{-e\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{e\left(2\left(\cos^2\right.\right.}}$
risch	$-\frac{i\left(e^{2i(dx+c)}+7\right)\sqrt{2}\,e^2a\sqrt{e\left(e^{2i(dx+c)}+1\right)e^{-i(dx+c)}}}{10d} - \frac{3i\left(-\frac{2\left(e\,e^{2i(dx+c)}+e\right)}{e\sqrt{e^{i(dx+c)}\left(e\,e^{2i(dx+c)}+e\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(dx+c)}-i\right)}\sqrt{ie^{i(\right.}}{}\right.}{}$

```
[In] int((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a*e^3*(8*I*sin(1
/2*d*x+1/2*c)^7+8*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*I*sin(1/2*d*x+
1/2*c)^5-8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+6*I*sin(1/2*d*x+1/2*c)^3
+2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+3*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-I*sin
(1/2*d*x+1/2*c))/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.89

$$\int (e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx = \frac{6i\sqrt{2}ae^{\frac{5}{2}}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)+\sqrt{\frac{1}{2}}\left(-i\,ae^2e^{(\cdots}\right.}{5\,d}$$

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/5*(6*I*sqrt(2)*a*e^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, e^(I*d*x + I*c))) + sqrt(1/2)*(-I*a*e^2*e^(2*I*d*x + 2*I*c) + 5*I*a*e^2)*
sqrt(e*e^(2*I*d*x + 2*I*c) + e)*e^(-1/2*I*d*x - 1/2*I*c))/d
```

Sympy [F(-1)]

Timed out.

$$\int (e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx = \text{Timed out}$$

```
[In] integrate((e*cos(d*x+c))**(5/2)*(a+I*a*tan(d*x+c)),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx = \int (e\cos(dx+c))^{\frac{5}{2}}(i\,a\tan(dx+c)+a)\,dx$$

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a), x)
```

Giac [F]

$$\int (e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx = \int (e\cos(dx+c))^{\frac{5}{2}}\,(ia\tan(dx+c)+a)\,dx$$

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] integrate((e*cos(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx = \int (e\,\cos(c+d\,x))^{5/2}\,(a+a\tan(c+d\,x)\ 1\mathrm{i})\,dx$$

```
[In] int((e*cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i),x)
[Out] int((e*cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i), x)
```

3.657 $\int (e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 90

$$\int (e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx = -\frac{2ia(e\cos(c+dx))^{3/2}}{3d} + \frac{2a(e\cos(c+dx))^{3/2}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a(e\cos(c+dx))^{3/2}\tan(c+dx)}{3d}$$

```
[Out] -2/3*I*a*(e*cos(d*x+c))^(3/2)/d+2/3*a*(e*cos(d*x+c))^(3/2)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos
(d*x+c)^(3/2)+2/3*a*(e*cos(d*x+c))^(3/2)*tan(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3596, 3567, 3854, 3856, 2720}

$$\int (e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx = -\frac{2ia(e\cos(c+dx))^{3/2}}{3d} + \frac{2a\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(e\cos(c+dx))^{3/2}}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a\tan(c+dx)(e\cos(c+dx))^{3/2}}{3d}$$

```
[In] Int[(e*Cos[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (((-2*I)/3)*a*(e*Cos[c + d*x])^(3/2))/d + (2*a*(e*Cos[c + d*x])^(3/2)*Ellip
ticF[(c + d*x)/2, 2])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*(e*Cos[c + d*x])^(3/2
)*Tan[c + d*x])/(3*d)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left((e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\right)\int \frac{a+ia\tan(c+dx)}{(e\sec(c+dx))^{3/2}}\,dx \\
&= -\frac{2ia(e\cos(c+dx))^{3/2}}{3d}+\left(a(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\right)\int \frac{1}{(e\sec(c+dx))^{3/2}}\,dx \\
&= -\frac{2ia(e\cos(c+dx))^{3/2}}{3d}+\frac{2a(e\cos(c+dx))^{3/2}\tan(c+dx)}{3d} \\
&\quad +\frac{\left(a(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\right)\int \sqrt{e\sec(c+dx)}\,dx}{3e^2} \\
&= -\frac{2ia(e\cos(c+dx))^{3/2}}{3d}+\frac{2a(e\cos(c+dx))^{3/2}\tan(c+dx)}{3d} \\
&\quad +\frac{\left(a(e\cos(c+dx))^{3/2}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\cos^{\frac{3}{2}}(c+dx)}
\end{aligned}$$

$$= -\frac{2ia(e\cos(c+dx))^{3/2}}{3d} + \frac{2a(e\cos(c+dx))^{3/2}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a(e\cos(c+dx))^{3/2}\tan(c+dx)}{3d}$$

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.11

$$\int (e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx = \frac{2ae\sqrt{\cos(c+dx)}\sqrt{e\cos(c+dx)}\left(\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(i\cos(c)+\sin(c))+\sqrt{\cos(c\right.}$$
$$3d$$

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (2*a*e*Sqrt[Cos[c + d*x]]*Sqrt[e*Cos[c + d*x]]*(EllipticF[(c + d*x)/2, 2]*(
I*Cos[c] + Sin[c]) + Sqrt[Cos[c + d*x]]*(Cos[d*x] + I*Sin[d*x]))*(Cos[d*x]
- I*Sin[d*x])*(-I + Tan[c + d*x]))/(3*d)
```

Maple [A] (verified)

Time = 5.50 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.87

method	result
default	$-\frac{2a\,e^2\left(4i\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-4i\left(\sin^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\right.}{3\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\,d}$
parts	$-\frac{2a\sqrt{e\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,e^2\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\right.\right.}\right.}{3\sqrt{-e\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{e\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\,e}}$
risch	$-\frac{ie^{i(dx+c)}\sqrt{2}\,ea\sqrt{e\left(e^{2i(dx+c)}+1\right)e^{-i(dx+c)}}}{3d}+\frac{2\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{i\left(e^{i(dx+c)}-i\right)}\sqrt{ie^{i(dx+c)}}\,F\left(\sqrt{-i\left(e^{i(dx+c)}+i\right)},\frac{\sqrt{2}}{2}\right)ea\sqrt{e(}}{3d\sqrt{e\,e^{3i(dx+c)}+e\,e^{i(dx+c)}}\left(e^{2i(dx+c}\right.}$

```
[In] int((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a*e^2*(4*I*sin(
1/2*d*x+1/2*c)^5+4*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-4*I*sin(1/2*d*x+
1/2*c)^3-2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)
+I*sin(1/2*d*x+1/2*c))/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.64

$$\int (e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx = -\frac{2\left(i\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}aee^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+i\sqrt{2}ae^{\frac{3}{2}}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)}{3\,d}$$

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
[Out] -2/3*(I*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*a*e*e^(1/2*I*d*x + 1/2*I*
c) + I*sqrt(2)*a*e^(3/2)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))/d
```

Sympy [F(-1)]

Timed out.

$$\int (e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx = \text{Timed out}$$

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+I*a*tan(d*x+c)),x)
[Out] Timed out
```

Maxima [F]

$$\int (e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx = \int (e\cos(dx+c))^{\frac{3}{2}}(i\,a\tan(dx+c)+a)\,dx$$

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*cos(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a), x)
```

Giac [F]

$$\int (e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx = \int (e\cos(dx+c))^{\frac{3}{2}}(i\,a\tan(dx+c)+a)\,dx$$

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] integrate((e*cos(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx = \int (e\,\cos(c+d\,x))^{3/2}\,(a+a\tan(c+d\,x)\,1\mathrm{i})\,dx$$

```
[In] int((e*cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i),x)
[Out] int((e*cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i), x)
```

3.658 $\int \sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 26, antiderivative size = 60

$$\int \sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))\,dx = -\frac{2ia\sqrt{e\cos(c+dx)}}{d} + \frac{2a\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] -2*I*a*(e*cos(d*x+c))^(1/2)/d+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/d/cos(d*x
+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3596, 3567, 3856, 2719}

$$\int \sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))\,dx = \frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} - \frac{2ia\sqrt{e\cos(c+dx)}}{d}$$

```
[In] Int[Sqrt[e*Cos[c + d*x]]*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((-2*I)*a*Sqrt[e*Cos[c + d*x]])/d + (2*a*Sqrt[e*Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int\frac{a+ia\tan(c+dx)}{\sqrt{e\sec(c+dx)}}\,dx\\
&= -\frac{2ia\sqrt{e\cos(c+dx)}}{d}+\left(a\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int\frac{1}{\sqrt{e\sec(c+dx)}}\,dx\\
&= -\frac{2ia\sqrt{e\cos(c+dx)}}{d}+\frac{\left(a\sqrt{e\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}}\\
&= -\frac{2ia\sqrt{e\cos(c+dx)}}{d}+\frac{2a\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.68 (sec) , antiderivative size = 192, normalized size of antiderivative = 3.20

$$\int \sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))\,dx$$
$$= \frac{a\cos(c)\sqrt{e\cos(c+dx)}\sin(c)(\cos(dx)-i\sin(dx))\left({}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\arctan(\tan(c)))\right)(-i\csc(c)\right.}{}$$

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] (a*Cos[c]*Sqrt[e*Cos[c + d*x]]*Sin[c]*(Cos[d*x] - I*Sin[d*x])*(Hypergeometr
icPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*((-I)*Csc[c] - Sec[
c])*Sec[c]*Sin[d*x + ArcTan[Tan[c]]] + Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]*(2
*Cos[d*x + ArcTan[Tan[c]]]*Csc[c]*(I*Csc[c] + Sec[c]) + Sec[c]*((-2*I)*Cos[
c + d*x]*Csc[c]^2*Sqrt[Sec[c]^2] + (I*Csc[c] + Sec[c])*Sin[d*x + ArcTan[Tan
[c]]])))*(-I + Tan[c + d*x]))/(d*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c
]]]^2])
```

Maple [A] (verified)

Time = 4.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.80

method	result
default	$\frac{2ae\left(2i\left(\sin^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}-i\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\,d}$
parts	$\frac{2a\sqrt{e\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,e\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)}{\sqrt{-e\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{e\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}\,d}-\frac{2ia\sqrt{e\cos(dx+c)}}{d}$
risch	$-\frac{2i\sqrt{2}\,a\sqrt{e\left(\mathrm{e}^{2i(dx+c)}+1\right)\mathrm{e}^{-i(dx+c)}}}{d}-\frac{i\left(-\frac{2\left(e\,\mathrm{e}^{2i(dx+c)}+e\right)}{e\sqrt{\mathrm{e}^{i(dx+c)}\left(e\,\mathrm{e}^{2i(dx+c)}+e\right)}}+\frac{i\sqrt{-i\left(\mathrm{e}^{i(dx+c)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(\mathrm{e}^{i(dx+c)}-i\right)}\,\sqrt{i\mathrm{e}^{i(dx+c)}}\,\left(-2iE\left(\sqrt{-}\right.\right.}{\sqrt{e\,\mathrm{e}^{3i(dx+c)}+e\,\mathrm{e}^{i}}}\right.}{}$

```
[In] int((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*a*e*(2*I*sin(1/2*d
*x+1/2*c)^3+EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-I*sin(1/2*d*x+1/2*c))/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.45

$$\int \sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))\,dx = \frac{2i\sqrt{2}a\sqrt{e}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)\right)}{d}$$

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
[Out] 2*I*sqrt(2)*a*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(
I*d*x + I*c)))/d
```

Sympy [F]

$$\int \sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))\,dx = ia\left(\int \left(-i\sqrt{e\cos(c+dx)}\right)\,dx + \int \sqrt{e\cos(c+dx)}\tan(c+dx)\,dx\right)$$

```
[In] integrate((e*cos(d*x+c))**(1/2)*(a+I*a*tan(d*x+c)),x)
[Out] I*a*(Integral(-I*sqrt(e*cos(c + d*x)), x) + Integral(sqrt(e*cos(c + d*x))*t
an(c + d*x), x))
```

Maxima [F]

$$\int \sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))\,dx = \int \sqrt{e\cos(dx+c)}(i\,a\tan(dx+c)+a)\,dx$$

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
[Out] integrate(sqrt(e*cos(d*x + c))*(I*a*tan(d*x + c) + a), x)
```

Giac [F]

$$\int \sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))\,dx = \int \sqrt{e\cos(dx+c)}(i\,a\tan(dx+c)+a)\,dx$$

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] integrate(sqrt(e*cos(d*x + c))*(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))\,dx = \int \sqrt{e\,\cos(c+d\,x)}\,\left(a+a\tan(c+d\,x)\,1\mathrm{i}\right)\,dx$$

```
[In] int((e*cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i),x)
[Out] int((e*cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i), x)
```

3.659 $\int \frac{a+ia\tan(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 60

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx=\frac{2ia}{d\sqrt{e\cos(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{e\cos(c+dx)}}$$

```
[Out] 2*I*a/d/(e*cos(d*x+c))^(1/2)+2*a*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(e*cos(d*x+c
))^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3596, 3567, 3856, 2720}

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx=\frac{2a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{e\cos(c+dx)}}+\frac{2ia}{d\sqrt{e\cos(c+dx)}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] ((2*I)*a)/(d*Sqrt[e*Cos[c + d*x]]) + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2])/(d*Sqrt[e*Cos[c + d*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))\,dx}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}} \\
&= \frac{2ia}{d\sqrt{e\cos(c+dx)}} + \frac{a\int \sqrt{e\sec(c+dx)}\,dx}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}} \\
&= \frac{2ia}{d\sqrt{e\cos(c+dx)}} + \frac{\left(a\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{\sqrt{e\cos(c+dx)}} \\
&= \frac{2ia}{d\sqrt{e\cos(c+dx)}} + \frac{2a\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{d\sqrt{e\cos(c+dx)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.24 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.38

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx = -\frac{\sqrt{2}a\sqrt{e\cos(c+dx)}(-i+\cot(c))\left(\sqrt{2}\sqrt{\csc^2(c)}+i\cos(c+dx)\sqrt{1+\cos(2dx-2\arctan(\cot(c)))}\csc(c)\right.}{de}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])/Sqrt[e*Cos[c + d*x]],x]
```

[Out] -((Sqrt[2]*a*Sqrt[e*Cos[c + d*x]]*(-I + Cot[c])*(Sqrt[2]*Sqrt[Csc[c]^2] + I*Cos[c + d*x]*Sqrt[1 + Cos[2*d*x - 2*ArcTan[Cot[c]]]]*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]])*Sin[c]*(Cos[d*x] - I*Sin[d*x])*(-I + Tan[c + d*x]))/(d*e*Sqrt[Csc[c]^2]))

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.22

method	result	size
parts	$\frac{2a\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\operatorname{am}^{-1}\left(\frac{dx}{2}+\frac{c}{2}\middle|\sqrt{2}\right)}{d\sqrt{e\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}}+\frac{2ia}{d\sqrt{e\cos(dx+c)}}$	73
default	$-\frac{2\left(\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}-i\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}{\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)d}$	93

[In] int((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] 2*a/d/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)*(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))+2*I*a/d/(e*cos(d*x+c))^(1/2)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.48

$$\int \frac{a+ia\tan(c+dx)}{\sqrt{e\cos(c+dx)}}\,dx = -\frac{2\left(-2i\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\,ae^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}+\left(i\sqrt{2}ae^{(2i\,dx+2i\,c)}+i\sqrt{2}a\right)\sqrt{e}\mathrm{weierstrassPInverse}\left(-4,0,e^{(i\right.}\right.}{dee^{(2i\,dx+2i\,c)}+de}$$

[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] -2*(-2*I*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*a*e^(1/2*I*d*x + 1/2*I*c) + (I*sqrt(2)*a*e^(2*I*d*x + 2*I*c) + I*sqrt(2)*a)*sqrt(e)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))/(d*e*e^(2*I*d*x + 2*I*c) + d*e)

Sympy [F]

$$\int \frac{a + ia\tan(c + dx)}{\sqrt{e\cos(c + dx)}}\,dx = ia\left(\int \left(-\frac{i}{\sqrt{e\cos{(c + dx)}}}\right)dx + \int \frac{\tan{(c + dx)}}{\sqrt{e\cos{(c + dx)}}}\,dx\right)$$

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))**(1/2),x)
[Out] I*a*(Integral(-I/sqrt(e*cos(c + d*x)), x) + Integral(tan(c + d*x)/sqrt(e*co
s(c + d*x)), x))
```

Maxima [F]

$$\int \frac{a + ia\tan(c + dx)}{\sqrt{e\cos(c + dx)}}\,dx = \int \frac{i\,a\tan{(dx + c)} + a}{\sqrt{e\cos{(dx + c)}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)/sqrt(e*cos(d*x + c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a + ia\tan(c + dx)}{\sqrt{e\cos(c + dx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%{[%%%{-4,[1]%%%},0]:[1,0,%%%{1,[1]%%%}]%%},[2,1]%%%}+%%%{%%
%{8,[2
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.23

$$\int \frac{a + ia\tan(c + dx)}{\sqrt{e\cos(c + dx)}}\,dx = \frac{2\,a\,\sqrt{\cos{(c + d\,x)}}\,\mathrm{F}\left(\frac{c}{2} + \frac{d\,x}{2}\middle|2\right)}{d\,\sqrt{e\,\cos{(c + d\,x)}}} + \frac{a\,\cos{(c + d\,x)}\,\sqrt{e\,\cos{(c + d\,x)}}\,4\mathrm{i}}{d\,e\,\left(\cos{(2\,c + 2\,d\,x)} + 1\right)}$$

```
[In] int((a + a*tan(c + d*x)*1i)/(e*cos(c + d*x))^(1/2),x)
```

```
[Out] (2*a*cos(c + d*x)^(1/2)*ellipticF(c/2 + (d*x)/2, 2))/(d*(e*cos(c + d*x))^(1
/2)) + (a*cos(c + d*x)*(e*cos(c + d*x))^(1/2)*4i)/(d*e*(cos(2*c + 2*d*x) +
1))
```

3.660 $\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 89

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx = \frac{2ia}{3d(e\cos(c+dx))^{3/2}} - \frac{2a\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d(e\cos(c+dx))^{3/2}} + \frac{2a\sin(c+dx)}{de\sqrt{e\cos(c+dx)}}$$

```
[Out] 2/3*I*a/d/(e*cos(d*x+c))^(3/2)-2*a*cos(d*x+c)^(3/2)*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/(e*cos(d*x
+c))^(3/2)+2*a*sin(d*x+c)/d/e/(e*cos(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3596, 3567, 3853, 3856, 2719}

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx = \frac{2ia}{3d(e\cos(c+dx))^{3/2}} - \frac{2a\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d(e\cos(c+dx))^{3/2}} + \frac{2a\sin(c+dx)\cos(c+dx)}{d(e\cos(c+dx))^{3/2}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (((2*I)/3)*a)/(d*(e*Cos[c + d*x])^(3/2)) - (2*a*Cos[c + d*x]^(3/2)*Elliptic
E[(c + d*x)/2, 2])/(d*(e*Cos[c + d*x])^(3/2)) + (2*a*Cos[c + d*x]*Sin[c + d
*x])/(d*(e*Cos[c + d*x])^(3/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))\,dx}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}} \\
&= \frac{2ia}{3d(e\cos(c+dx))^{3/2}} + \frac{a\int (e\sec(c+dx))^{3/2}\,dx}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}} \\
&= \frac{2ia}{3d(e\cos(c+dx))^{3/2}} + \frac{2a\cos(c+dx)\sin(c+dx)}{d(e\cos(c+dx))^{3/2}} - \frac{(ae^2)\int \frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}} \\
&= \frac{2ia}{3d(e\cos(c+dx))^{3/2}} + \frac{2a\cos(c+dx)\sin(c+dx)}{d(e\cos(c+dx))^{3/2}} - \frac{\left(a\cos^{\frac{3}{2}}(c+dx)\right)\int \sqrt{\cos(c+dx)}\,dx}{(e\cos(c+dx))^{3/2}} \\
&= \frac{2ia}{3d(e\cos(c+dx))^{3/2}} - \frac{2a\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d(e\cos(c+dx))^{3/2}} + \frac{2a\cos(c+dx)\sin(c+dx)}{d(e\cos(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.02 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.13

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx = \frac{(\cos(dx)-i\sin(dx))\left(\frac{6\cos(c+dx)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\arctan(\tan(c)))\right)\sin(dx+\arctan(\tan(c)))(1\cdots}{\sqrt{\sin^2(dx+\arctan(\tan(c)))}}\cdots\right.}{\cdots}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[d*x] - I*Sin[d*x])*((6*Cos[c + d*x]*HypergeometricPFQ[{-1/2, -1/4}, {
3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*(1 - I*Tan[c])
)/Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2] + (Csc[c] - I*Sec[c])*(-3*Cos[c + d*x]*
(3*Cos[c - d*x - ArcTan[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]]) + (4*(I +
 3*Cos[d*x]*Cos[c + d*x]*Csc[c])*Tan[c])/Sqrt[Sec[c]^2]))*(a + I*a*Tan[c +
d*x]))/(6*d*e*Sqrt[e*Cos[c + d*x]]*Sqrt[Sec[c]^2])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 213 vs. 2(104) = 208.

Time = 5.44 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.40

method	result
default	$\frac{2\left(12\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-6E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-6\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\cdots\right.}{3\left(2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\cdots\right.}}$
parts	$-\frac{2a\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e}\,\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{-2\left(\sin^4\left(\cdots\right.}\right.}{e\sqrt{-e\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{e\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\cdots\right.}}$

```
[In] int((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*
e+e)^(1/2)/e*(12*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-6*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*sin(1/2*d*x+1/2*c)^2-6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+3*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)-I*sin(1/2*d*x+1/2*c))*a/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.70

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx = -\frac{2\left(2\sqrt{\frac{1}{2}}\left(3i\,ae^{(4i\,dx+4i\,c)}+i\,ae^{(2i\,dx+2i\,c)}\right)\sqrt{ee^{(2i\,dx+2i\,c)}+e}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}+3\left(i\sqrt{2}ae^{(4i\,dx+4i\,c)}+2i\sqrt{2}ae^{(2i\,dx+\cdots}\right.\right.}{3\left(de^{2}e^{(4i\,dx+4i\,c)}+2\,de^{2}e^{(2i\,dx+2i\,c)}\cdots\right.}$$

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(2*sqrt(1/2)*(3*I*a*e^(4*I*d*x + 4*I*c) + I*a*e^(2*I*d*x + 2*I*c))*sqr
t(e*e^(2*I*d*x + 2*I*c) + e)*e^(-1/2*I*d*x - 1/2*I*c) + 3*(I*sqrt(2)*a*e^(4
*I*d*x + 4*I*c) + 2*I*sqrt(2)*a*e^(2*I*d*x + 2*I*c) + I*sqrt(2)*a)*sqrt(e)*
weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x + I*c))))/(d*e^2
*e^(4*I*d*x + 4*I*c) + 2*d*e^2*e^(2*I*d*x + 2*I*c) + d*e^2)
```

Sympy [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx = ia\left(\int\left(-\frac{i}{(e\cos(c+dx))^{\frac{3}{2}}}\right)dx+\int\frac{\tan(c+dx)}{(e\cos(c+dx))^{\frac{3}{2}}}\,dx\right)$$

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))**(3/2),x)
```

```
[Out] I*a*(Integral(-I/(e*cos(c + d*x))**(3/2), x) + Integral(tan(c + d*x)/(e*cos
(c + d*x))**(3/2), x))
```

Maxima [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx = \int \frac{i\,a\tan(dx+c)+a}{(e\cos(dx+c))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)/(e*cos(d*x + c))^(3/2), x)
```

Giac [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx = \int \frac{i\,a\tan\left(dx+c\right)+a}{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)/(e*cos(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{3/2}}\,dx = \int \frac{a+a\tan(c+d\,x)\;1\mathrm{i}}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)/(e*cos(c + d*x))^(3/2),x)
[Out] int((a + a*tan(c + d*x)*1i)/(e*cos(c + d*x))^(3/2), x)
```

3.661 $\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 96

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx = \frac{2ia}{5d(e\cos(c+dx))^{5/2}} + \frac{2a\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d(e\cos(c+dx))^{5/2}} + \frac{2a\cos(c+dx)\sin(c+dx)}{3d(e\cos(c+dx))^{5/2}}$$

```
[Out] 2/5*I*a/d/(e*cos(d*x+c))^(5/2)+2/3*a*cos(d*x+c)^(5/2)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d/(e*cos(d
*x+c))^(5/2)+2/3*a*cos(d*x+c)*sin(d*x+c)/d/(e*cos(d*x+c))^(5/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3596, 3567, 3853, 3856, 2720}

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx = \frac{2ia}{5d(e\cos(c+dx))^{5/2}} + \frac{2a\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d(e\cos(c+dx))^{5/2}} + \frac{2a\sin(c+dx)\cos(c+dx)}{3d(e\cos(c+dx))^{5/2}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (((2*I)/5)*a)/(d*(e*Cos[c + d*x])^(5/2)) + (2*a*Cos[c + d*x]^(5/2)*Elliptic
F[(c + d*x)/2, 2])/(3*d*(e*Cos[c + d*x])^(5/2)) + (2*a*Cos[c + d*x]*Sin[c +
 d*x])/(3*d*(e*Cos[c + d*x])^(5/2))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int (e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))\,dx}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}\\
&= \frac{2ia}{5d(e\cos(c+dx))^{5/2}}+\frac{a\int (e\sec(c+dx))^{5/2}\,dx}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}\\
&= \frac{2ia}{5d(e\cos(c+dx))^{5/2}}+\frac{2a\cos(c+dx)\sin(c+dx)}{3d(e\cos(c+dx))^{5/2}}+\frac{(ae^2)\int \sqrt{e\sec(c+dx)}\,dx}{3(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}\\
&= \frac{2ia}{5d(e\cos(c+dx))^{5/2}}+\frac{2a\cos(c+dx)\sin(c+dx)}{3d(e\cos(c+dx))^{5/2}}+\frac{\left(a\cos^{\frac{5}{2}}(c+dx)\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3(e\cos(c+dx))^{5/2}}\\
&= \frac{2ia}{5d(e\cos(c+dx))^{5/2}}+\frac{2a\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3d(e\cos(c+dx))^{5/2}}+\frac{2a\cos(c+dx)\sin(c+dx)}{3d(e\cos(c+dx))^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.59

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx=\frac{a\left(6i+10\cos^{\frac{5}{2}}(c+dx)\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)+5\sin(2(c+dx))\right)}{15d(e\cos(c+dx))^{5/2}}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (a*(6*I + 10*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 5*Sin[2*(c + d*
x)]))/(15*d*(e*Cos[c + d*x])^(5/2))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(107) = 214.

Time = 6.70 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.71

method	result
parts	$-\frac{2a(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,F(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{2})\sqrt{2(\sin^2(\frac{dx}{2}+\frac{c}{2}))-1}\,(\sin^2(\frac{dx}{2}+\frac{c}{2}))-2\cos(\frac{dx}{2}+\frac{c}{2})(\sin^2(\frac{dx}{2}+\frac{c}{2}))+\sqrt{2(\sin^2(\frac{dx}{2}}}{3e^2\sqrt{-e(2(\sin^4(\frac{dx}{2}+\frac{c}{2}))-(\sin^2(\frac{dx}{2}+\frac{c}{2})))}\,(2(\cos^2(\frac{dx}{2}+\frac{c}{2}))-1)\sin(}$
default	$-\frac{2(20\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,F(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{2})\sqrt{2(\sin^2(\frac{dx}{2}+\frac{c}{2}))-1}\,(\sin^4(\frac{dx}{2}+\frac{c}{2}))+20(\sin^4(\frac{dx}{2}+\frac{c}{2}))\cos(\frac{dx}{2}+\frac{c}{2})-20\sqrt{\frac{1}{2}-\frac{\cos(d}{2}}}{15(4(\sin^4(\frac{dx}{2}+\frac{c}{2}}$

```
[In] int((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*a*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-2*cos(1/2*d*x+1/2*
c)*sin(1/2*d*x+1/2*c)^2+(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2))/e^2*(e*(2*cos(1/2*d*x+1/2
*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-e*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*
x+1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(e*(2*cos(
1/2*d*x+1/2*c)^2-1))^(1/2)/d+2/5*I*a/d/(e*cos(d*x+c))^(5/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.98

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx =$$
$$-\frac{2\left(2\sqrt{\frac{1}{2}}\left(5i\,ae^{(5i\,dx+5i\,c)}-12i\,ae^{(3i\,dx+3i\,c)}-5i\,ae^{(i\,dx+i\,c)}\right)\sqrt{ee^{(2i\,dx+2i\,c)}+e}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}+5\left(i\sqrt{2}ae^{(6i\,dx+6i\,c)}\right.}{15\left(de^{3}e^{(6i\,dx+6i\,c)}+3\,de^{3}e^{(4i\,dx+4i\,c)}+3\right.}$$

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/15*(2*sqrt(1/2)*(5*I*a*e^(5*I*d*x + 5*I*c) - 12*I*a*e^(3*I*d*x + 3*I*c)
- 5*I*a*e^(I*d*x + I*c))*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*e^(-1/2*I*d*x - 1/
2*I*c) + 5*(I*sqrt(2)*a*e^(6*I*d*x + 6*I*c) + 3*I*sqrt(2)*a*e^(4*I*d*x + 4*
I*c) + 3*I*sqrt(2)*a*e^(2*I*d*x + 2*I*c) + I*sqrt(2)*a)*sqrt(e)*weierstrass
PInverse(-4, 0, e^(I*d*x + I*c)))/(d*e^3*e^(6*I*d*x + 6*I*c) + 3*d*e^3*e^(4
*I*d*x + 4*I*c) + 3*d*e^3*e^(2*I*d*x + 2*I*c) + d*e^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx = \int \frac{i\,a\tan\left(dx+c\right)+a}{\left(e\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)/(e*cos(d*x + c))^(5/2), x)
```

Giac [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx = \int \frac{i\,a\tan\left(dx+c\right)+a}{\left(e\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)/(e*cos(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{5/2}}\,dx = \int \frac{a+a\tan(c+d\,x)\ 1\mathrm{i}}{{\left(e\ \cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)/(e*cos(c + d*x))^(5/2),x)
[Out] int((a + a*tan(c + d*x)*1i)/(e*cos(c + d*x))^(5/2), x)
```

3.662 $\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 130

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{7/2}}\,dx = \frac{2ia}{7d(e\cos(c+dx))^{7/2}} - \frac{6a\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d(e\cos(c+dx))^{7/2}} + \frac{2a\cos(c+dx)\sin(c+dx)}{5d(e\cos(c+dx))^{7/2}} + \frac{6a\cos^3(c+dx)\sin(c+dx)}{5d(e\cos(c+dx))^{7/2}}$$

```
[Out] 2/7*I*a/d/(e*cos(d*x+c))^(7/2)-6/5*a*cos(d*x+c)^(7/2)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/(e*cos(d
*x+c))^(7/2)+2/5*a*cos(d*x+c)*sin(d*x+c)/d/(e*cos(d*x+c))^(7/2)+6/5*a*cos(d
*x+c)^3*sin(d*x+c)/d/(e*cos(d*x+c))^(7/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3596, 3567, 3853, 3856, 2719}

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{7/2}}\,dx = \frac{2ia}{7d(e\cos(c+dx))^{7/2}} - \frac{6a\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d(e\cos(c+dx))^{7/2}} + \frac{6a\sin(c+dx)\cos^3(c+dx)}{5d(e\cos(c+dx))^{7/2}} + \frac{2a\sin(c+dx)\cos(c+dx)}{5d(e\cos(c+dx))^{7/2}}$$

```
[In] Int[(a + I*a*Tan[c + d*x])/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (((2*I)/7)*a)/(d*(e*Cos[c + d*x])^(7/2)) - (6*a*Cos[c + d*x]^(7/2)*Elliptic
E[(c + d*x)/2, 2])/(5*d*(e*Cos[c + d*x])^(7/2)) + (2*a*Cos[c + d*x]*Sin[c +
```

```
d*x])/(5*d*(e*Cos[c + d*x])^(7/2)) + (6*a*Cos[c + d*x]^3*Sin[c + d*x])/(5*
d*(e*Cos[c + d*x])^(7/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\text{integral} = \frac{\int (e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))\,dx}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$= \frac{2ia}{7d(e\cos(c+dx))^{7/2}} + \frac{a\int (e\sec(c+dx))^{7/2}\,dx}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$= \frac{2ia}{7d(e\cos(c+dx))^{7/2}} + \frac{2a\cos(c+dx)\sin(c+dx)}{5d(e\cos(c+dx))^{7/2}} + \frac{(3ae^2)\int (e\sec(c+dx))^{3/2}\,dx}{5(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$= \frac{2ia}{7d(e\cos(c+dx))^{7/2}} + \frac{2a\cos(c+dx)\sin(c+dx)}{5d(e\cos(c+dx))^{7/2}} + \frac{6a\cos^3(c+dx)\sin(c+dx)}{5d(e\cos(c+dx))^{7/2}} - \frac{(3ae^4)\int \frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{5(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$= \frac{2ia}{7d(e\cos(c+dx))^{7/2}} + \frac{2a\cos(c+dx)\sin(c+dx)}{5d(e\cos(c+dx))^{7/2}} + \frac{6a\cos^3(c+dx)\sin(c+dx)}{5d(e\cos(c+dx))^{7/2}} - \frac{\left(3a\cos^{\frac{7}{2}}(c+dx)\right)\int \sqrt{\cos(c+dx)}\,dx}{5(e\cos(c+dx))^{7/2}}$$

$$= \frac{2ia}{7d(e\cos(c+dx))^{7/2}} - \frac{6a\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d(e\cos(c+dx))^{7/2}} + \frac{2a\cos(c+dx)\sin(c+dx)}{5d(e\cos(c+dx))^{7/2}} + \frac{6a\cos^3(c+dx)\sin(c+dx)}{5d(e\cos(c+dx))^{7/2}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.52 (sec) , antiderivative size = 666, normalized size of antiderivative = 5.12

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{7/2}}\,dx = \frac{\cos^5(c+dx)\left(\csc(c)\sec(c)\left(\frac{6\cos(c)}{5} - \frac{6}{5}i\sin(c)\right) + \sec^4(c+dx)\left(\frac{2}{7}i\cos(c) + \frac{2\sin(c)}{7}\right.\right.}{}$$

$$- \frac{3i\cos^{\frac{9}{2}}(c+dx)\left(\frac{{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\arctan(\tan(c)))\right)\sin(dx+\arctan(\tan(c)))\tan(c)}{\sqrt{1-\cos(dx+\arctan(\tan(c)))}\sqrt{1+\cos(dx+\arctan(\tan(c)))}\sqrt{\cos(c)\cos(dx+\arctan(\tan(c)))}\sqrt{1+\tan^2(c)}\sqrt{1+\tan^2(c)}} - \right.}{5d(e\cos(c+dx))^{7/2}(\cos(dx)+i\sin(}$$

$$+ \frac{3\cos^{\frac{9}{2}}(c+dx)\cot(c)\left(\frac{{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\arctan(\tan(c)))\right)\sin(dx+\arctan(\tan(c)))\tan(c)}{\sqrt{1-\cos(dx+\arctan(\tan(c)))}\sqrt{1+\cos(dx+\arctan(\tan(c)))}\sqrt{\cos(c)\cos(dx+\arctan(\tan(c)))}\sqrt{1+\tan^2(c)}\sqrt{1+\tan^2(}}\right.}{5d(e\cos(c+dx))^{7/2}(\cos(dx)+i\sin}$$

```
[In] Integrate[(a + I*a*Tan[c + d*x])/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (Cos[c + d*x]^5*(Csc[c]*Sec[c]*((6*Cos[c])/5 - ((6*I)/5)*Sin[c]) + Sec[c +
d*x]^4*(((2*I)/7)*Cos[c] + (2*Sin[c])/7) + Sec[c]*Sec[c + d*x]^3*((2*Cos[c]
)/5 - ((2*I)/5)*Sin[c])*Sin[d*x] + Sec[c]*Sec[c + d*x]*((6*Cos[c])/5 - ((6*
I)/5)*Sin[c])*Sin[d*x] + Sec[c + d*x]^2*((2*Cos[c])/5 - ((2*I)/5)*Sin[c])*T
an[c])*(a + I*a*Tan[c + d*x]))/(d*(e*Cos[c + d*x])^(7/2)*(Cos[d*x] + I*Sin[
d*x])) - (((3*I)/5)*Cos[c + d*x]^(9/2)*((HypergeometricPFQ[{-1/2, -1/4}, {3
/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1
 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[
c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Si
```

```
n[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x +
ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[
d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]])*(a + I*a*Tan[c + d*x]))/(d*(e*Co
s[c + d*x])^(7/2)*(Cos[d*x] + I*Sin[d*x])) + (3*Cos[c + d*x]^(9/2)*Cot[c]*(
(HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*
x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + C
os[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Ta
n[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 +
 Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos
[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]
])*(a + I*a*Tan[c + d*x]))/(5*d*(e*Cos[c + d*x])^(7/2)*(Cos[d*x] + I*Sin[d*
x]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(137) = 274$.

Time = 11.27 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.96

method	result
parts	$-\frac{2a\sqrt{e\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-12E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\cdots}\right.}{\cdots}$
default	$\frac{2\left(336\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-168E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-504\left(\sin^6\left(\frac{dx}{2}\cdots\right.\right.\right.}{\cdots}$

```
[In] int((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*a*(e*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/e^4/sin(1/
2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*
x+1/2*c)^2-1)*(24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*cos(1/2*d*x+1/2*c)*sin(1
/2*d*x+1/2*c)^2-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2))*(-2*sin(1/2*d*x+1/2*c)^4*e+sin(
1/2*d*x+1/2*c)^2*e)^(1/2)/(e*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d+2/7*I*a/d/
(e*cos(d*x+c))^(7/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.81

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{7/2}}\,dx = -\frac{2\left(2\sqrt{\frac{1}{2}}\left(21i\,ae^{(8i\,dx+8i\,c)}+77i\,ae^{(6i\,dx+6i\,c)}+23i\,ae^{(4i\,dx+4i\,c)}+7i\,ae^{(2i\,dx+2i\,c)}\right)\sqrt{ee^{(2i\,dx+2i\,c)}+e}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}\cdots}{35\left(de^{4}e^{(8i\,dx+8\cdots}\right.}$$

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -2/35*(2*sqrt(1/2)*(21*I*a*e^(8*I*d*x + 8*I*c) + 77*I*a*e^(6*I*d*x + 6*I*c)
 + 23*I*a*e^(4*I*d*x + 4*I*c) + 7*I*a*e^(2*I*d*x + 2*I*c))*sqrt(e*e^(2*I*d*
x + 2*I*c) + e)*e^(-1/2*I*d*x - 1/2*I*c) + 21*(I*sqrt(2)*a*e^(8*I*d*x + 8*I
*c) + 4*I*sqrt(2)*a*e^(6*I*d*x + 6*I*c) + 6*I*sqrt(2)*a*e^(4*I*d*x + 4*I*c)
 + 4*I*sqrt(2)*a*e^(2*I*d*x + 2*I*c) + I*sqrt(2)*a)*sqrt(e)*weierstrassZeta
(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x + I*c))))/(d*e^4*e^(8*I*d*x + 8
*I*c) + 4*d*e^4*e^(6*I*d*x + 6*I*c) + 6*d*e^4*e^(4*I*d*x + 4*I*c) + 4*d*e^4
*e^(2*I*d*x + 2*I*c) + d*e^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{7/2}}\,dx = \int \frac{i\,a\tan\left(dx+c\right)+a}{\left(e\cos\left(dx+c\right)\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*tan(d*x + c) + a)/(e*cos(d*x + c))^(7/2), x)
```

Giac [F]

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{7/2}}\,dx = \int \frac{i\,a\tan\left(dx+c\right)+a}{\left(e\cos\left(dx+c\right)\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))/(e*cos(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)/(e*cos(d*x + c))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+ia\tan(c+dx)}{(e\cos(c+dx))^{7/2}}\,dx = \int \frac{a+a\tan(c+d\,x)\ 1\mathrm{i}}{\left(e\ \cos\left(c+d\,x\right)\right)^{7/2}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)/(e*cos(c + d*x))^(7/2),x)
[Out] int((a + a*tan(c + d*x)*1i)/(e*cos(c + d*x))^(7/2), x)
```

3.663 $\int \frac{(e\cos(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 190

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{2(e\cos(c+dx))^{7/2}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{7a^2d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\cos(c+dx)(e\cos(c+dx))^{7/2}\sin(c+dx)}{15a^2d} + \frac{6(e\cos(c+dx))^{7/2}\tan(c+dx)}{35a^2d} + \frac{2(e\cos(c+dx))^{7/2}\sec^2(c+dx)\tan(c+dx)}{7a^2d} + \frac{4i\cos^2(c+dx)(e\cos(c+dx))^{7/2}}{15d\left(a^2+ia^2\tan(c+dx)\right)}$$

```
[Out] 2/7*(e*cos(d*x+c))^(7/2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/cos(d*x+c)^(7/2)+2/15*cos(d*x+c)*
(e*cos(d*x+c))^(7/2)*sin(d*x+c)/a^2/d+6/35*(e*cos(d*x+c))^(7/2)*tan(d*x+c)/
a^2/d+2/7*(e*cos(d*x+c))^(7/2)*sec(d*x+c)^2*tan(d*x+c)/a^2/d+4/15*I*cos(d*x
+c)^2*(e*cos(d*x+c))^(7/2)/d/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3596, 3581, 3854, 3856, 2720}

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{2\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(e\cos(c+dx))^{7/2}}{7a^2d\cos^{\frac{7}{2}}(c+dx)} + \frac{4i\cos^2(c+dx)(e\cos(c+dx))^{7/2}}{15d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{2\sin(c+dx)\cos(c+dx)(e\cos(c+dx))^{7/2}}{15a^2d} + \frac{6\tan(c+dx)(e\cos(c+dx))^{7/2}}{35a^2d} + \frac{2\tan(c+dx)\sec^2(c+dx)(e\cos(c+dx))^{7/2}}{7a^2d}$$

```
[In] Int[(e*Cos[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (2*(e*Cos[c + d*x])^(7/2)*EllipticF[(c + d*x)/2, 2])/(7*a^2*d*Cos[c + d*x]^
(7/2)) + (2*Cos[c + d*x]*(e*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(15*a^2*d) +
(6*(e*Cos[c + d*x])^(7/2)*Tan[c + d*x])/(35*a^2*d) + (2*(e*Cos[c + d*x])^(7
/2)*Sec[c + d*x]^2*Tan[c + d*x])/(7*a^2*d) + (((4*I)/15)*Cos[c + d*x]^2*(e*
Cos[c + d*x])^(7/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\text{integral} = \left((e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\right)\int \frac{1}{(e\sec(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx$$

$$= \frac{4i\cos^2(c+dx)(e\cos(c+dx))^{7/2}}{15d(a^2+ia^2\tan(c+dx))} + \frac{\left(11e^2(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\right)\int\frac{1}{(e\sec(c+dx))^{11/2}}dx}{15a^2}$$

$$= \frac{2\cos(c+dx)(e\cos(c+dx))^{7/2}\sin(c+dx)}{15a^2d} + \frac{4i\cos^2(c+dx)(e\cos(c+dx))^{7/2}}{15d(a^2+ia^2\tan(c+dx))} + \frac{\left(3(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\right)\int\frac{1}{(e\sec(c+dx))^{7/2}}dx}{5a^2}$$

$$= \frac{2\cos(c+dx)(e\cos(c+dx))^{7/2}\sin(c+dx)}{15a^2d} + \frac{6(e\cos(c+dx))^{7/2}\tan(c+dx)}{35a^2d} + \frac{4i\cos^2(c+dx)(e\cos(c+dx))^{7/2}}{15d(a^2+ia^2\tan(c+dx))} + \frac{\left(3(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\right)\int\frac{1}{(e\sec(c+dx))^{3/2}}dx}{7a^2e^2}$$

$$= \frac{2\cos(c+dx)(e\cos(c+dx))^{7/2}\sin(c+dx)}{15a^2d} + \frac{6(e\cos(c+dx))^{7/2}\tan(c+dx)}{35a^2d} + \frac{2(e\cos(c+dx))^{7/2}\sec^2(c+dx)\tan(c+dx)}{7a^2d} + \frac{4i\cos^2(c+dx)(e\cos(c+dx))^{7/2}}{15d(a^2+ia^2\tan(c+dx))} + \frac{\left((e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\right)\int\sqrt{e\sec(c+dx)}\,dx}{7a^2e^4}$$

$$= \frac{2\cos(c+dx)(e\cos(c+dx))^{7/2}\sin(c+dx)}{15a^2d} + \frac{6(e\cos(c+dx))^{7/2}\tan(c+dx)}{35a^2d} + \frac{2(e\cos(c+dx))^{7/2}\sec^2(c+dx)\tan(c+dx)}{7a^2d} + \frac{4i\cos^2(c+dx)(e\cos(c+dx))^{7/2}}{15d(a^2+ia^2\tan(c+dx))} + \frac{(e\cos(c+dx))^{7/2}\int\frac{1}{\sqrt{\cos(c+dx)}}dx}{7a^2\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2(e\cos(c+dx))^{7/2}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{7a^2d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\cos(c+dx)(e\cos(c+dx))^{7/2}\sin(c+dx)}{15a^2d} + \frac{6(e\cos(c+dx))^{7/2}\tan(c+dx)}{35a^2d} + \frac{2(e\cos(c+dx))^{7/2}\sec^2(c+dx)\tan(c+dx)}{7a^2d} + \frac{4i\cos^2(c+dx)(e\cos(c+dx))^{7/2}}{15d(a^2+ia^2\tan(c+dx))}$$

Mathematica [A] (verified)

Time = 2.63 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.82

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{e^3\sqrt{e\cos(c+dx)}\Big(-240\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(\cos(2(c+dx))+i\sin(2(c+$$

[In] Integrate[(e*Cos[c + d*x])^(7/2)/(a + I*a*Tan[c + d*x])^2,x]

[Out] (e^3*Sqrt[e*Cos[c + d*x]]*(-240*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]) + Sqrt[Cos[c + d*x]]*((-296*I)*Cos[c + d*x] + (68*I)*Cos[3*(c + d*x)] + (4*I)*Cos[5*(c + d*x)] + 134*Sin[c + d*x] - 117*Sin[3*(c + d*x)] - 11*Sin[5*(c + d*x)])))/(840*a^2*d*Cos[c + d*x]^(5/2)*(-I + Tan[c + d*x])^2)

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 386 vs. 2(192) = 384.

Time = 9.44 (sec) , antiderivative size = 387, normalized size of antiderivative = 2.04

method	result
default	$-\frac{2e^4\left(25088i\left(\sin^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3584\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{16}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-3584i\left(\sin^{17}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12544\left(\sin^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}{}$

[In] int((e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] -2/105/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^4*(25088*I*sin(1/2*d*x+1/2*c)^11+3584*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^16-3584*I*sin(1/2*d*x+1/2*c)^17-12544*sin(1/2*d*x+1/2*c)^14*cos(1/2*d*x+1/2*c)+6272*I*sin(1/2*d*x+1/2*c)^7+19264*sin(1/2*d*x+1/2*c)^12*cos(1/2*d*x+1/2*c)-14*I*sin(1/2*d*x+1/2*c)-16800*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-1568*I*sin(1/2*d*x+1/2*c)^5+9104*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+224*I*sin(1/2*d*x+1/2*c)^3-3128*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+14336*I*sin(1/2*d*x+1/2*c)^15+700*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-15680*I*sin(1/2*d*x+1/2*c)^9-90*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+15*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-25088*I*sin(1/2*d*x+1/2*c)^13)/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.79

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{\left(-480i\sqrt{2}e^{\frac{7}{2}}e^{(7i\,dx+7i\,c)}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{\frac{1}{2}}\left(-15i\,e^{3}e^{(10}\right.\right.}{}$$

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/1680*(-480*I*sqrt(2)*e^(7/2)*e^(7*I*d*x + 7*I*c)*weierstrassPInverse(-4,
0, e^(I*d*x + I*c)) + sqrt(1/2)*(-15*I*e^3*e^(10*I*d*x + 10*I*c) - 185*I*e^
3*e^(8*I*d*x + 8*I*c) + 430*I*e^3*e^(6*I*d*x + 6*I*c) + 162*I*e^3*e^(4*I*d*
x + 4*I*c) + 49*I*e^3*e^(2*I*d*x + 2*I*c) + 7*I*e^3)*sqrt(e*e^(2*I*d*x + 2*
I*c) + e)*e^(-1/2*I*d*x - 1/2*I*c))*e^(-7*I*d*x - 7*I*c)/(a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*cos(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\cos(dx+c))^{\frac{7}{2}}}{(i\,a\tan(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*cos(d*x + c))^(7/2)/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\cos(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\,\cos(c+d\,x))^{7/2}}{(a+a\tan(c+d\,x)\,1\mathrm{i})^2}\,dx$$

```
[In] int((e*cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^2,x)
[Out] int((e*cos(c + d*x))^(7/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

3.664 $\int \frac{(e\cos(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 154

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{42(e\cos(c+dx))^{5/2}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{65a^2d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\cos(c+dx)(e\cos(c+dx))^{5/2}\sin(c+dx)}{13a^2d} + \frac{14(e\cos(c+dx))^{5/2}\tan(c+dx)}{65a^2d} + \frac{4i\cos^2(c+dx)(e\cos(c+dx))^{5/2}}{13d\,(a^2+ia^2\tan(c+dx))}$$

```
[Out] 42/65*(e*cos(d*x+c))^(5/2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/cos(d*x+c)^(5/2)+2/13*cos(d*x+c
)*(e*cos(d*x+c))^(5/2)*sin(d*x+c)/a^2/d+14/65*(e*cos(d*x+c))^(5/2)*tan(d*x+
c)/a^2/d+4/13*I*cos(d*x+c)^2*(e*cos(d*x+c))^(5/2)/d/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3596, 3581, 3854, 3856, 2719}

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{42E\left(\frac{1}{2}(c+dx)\middle|2\right)(e\cos(c+dx))^{5/2}}{65a^2d\cos^{\frac{5}{2}}(c+dx)} + \frac{4i\cos^2(c+dx)(e\cos(c+dx))^{5/2}}{13d\,(a^2+ia^2\tan(c+dx))} + \frac{2\sin(c+dx)\cos(c+dx)(e\cos(c+dx))^{5/2}}{13a^2d} + \frac{14\tan(c+dx)(e\cos(c+dx))^{5/2}}{65a^2d}$$

```
[In] Int[(e*Cos[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (42*(e*Cos[c + d*x])^(5/2)*EllipticE[(c + d*x)/2, 2])/(65*a^2*d*Cos[c + d*x
]^(5/2)) + (2*Cos[c + d*x]*(e*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(13*a^2*d)
+ (14*(e*Cos[c + d*x])^(5/2)*Tan[c + d*x])/(65*a^2*d) + (((4*I)/13)*Cos[c +
 d*x]^2*(e*Cos[c + d*x])^(5/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\text{integral} = \left((e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\right)\int\frac{1}{(e\sec(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx$$

$$=\frac{4i\cos^2(c+dx)(e\cos(c+dx))^{5/2}}{13d\left(a^2+ia^2\tan(c+dx)\right)}+\frac{\left(9e^2(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\right)\int\frac{1}{(e\sec(c+dx))^{9/2}}\,dx}{13a^2}$$

$$= \frac{2\cos(c+dx)(e\cos(c+dx))^{5/2}\sin(c+dx)}{13a^2d} + \frac{4i\cos^2(c+dx)(e\cos(c+dx))^{5/2}}{13d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\left(7(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\right)\int\frac{1}{(e\sec(c+dx))^{5/2}}dx}{13a^2}$$

$$= \frac{2\cos(c+dx)(e\cos(c+dx))^{5/2}\sin(c+dx)}{13a^2d} + \frac{14(e\cos(c+dx))^{5/2}\tan(c+dx)}{65a^2d} + \frac{4i\cos^2(c+dx)(e\cos(c+dx))^{5/2}}{13d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\left(21(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\right)\int\frac{1}{\sqrt{e\sec(c+dx)}}dx}{65a^2e^2}$$

$$= \frac{2\cos(c+dx)(e\cos(c+dx))^{5/2}\sin(c+dx)}{13a^2d} + \frac{14(e\cos(c+dx))^{5/2}\tan(c+dx)}{65a^2d} + \frac{4i\cos^2(c+dx)(e\cos(c+dx))^{5/2}}{13d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\left(21(e\cos(c+dx))^{5/2}\right)\int\sqrt{\cos(c+dx)}dx}{65a^2\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{42(e\cos(c+dx))^{5/2}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{65a^2d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\cos(c+dx)(e\cos(c+dx))^{5/2}\sin(c+dx)}{13a^2d} + \frac{14(e\cos(c+dx))^{5/2}\tan(c+dx)}{65a^2d} + \frac{4i\cos^2(c+dx)(e\cos(c+dx))^{5/2}}{13d\left(a^2+ia^2\tan(c+dx)\right)}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.00 (sec) , antiderivative size = 464, normalized size of antiderivative = 3.01

$$\int\frac{(e\cos(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}dx = \frac{(e\cos(c+dx))^{5/2}\sec^5(c+dx)(\cos(dx)+i\sin(dx))^2\left(-21\cos(c)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4}\right.\right.}{}$$

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((e*Cos[c + d*x])^(5/2)*Sec[c + d*x]^5*(Cos[d*x] + I*Sin[d*x])^2*(-21*Cos[c
]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d
*x + ArcTan[Tan[c]]] - (42*I)*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*
x + ArcTan[Tan[c]]]^2]*Sin[c]*Sin[d*x + ArcTan[Tan[c]]] + (21*I)*(3*Cos[c -
 d*x - ArcTan[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Sqrt[Sin[d*x + ArcT
an[Tan[c]]]^2] + (21*(3*Cos[c - d*x - ArcTan[Tan[c]]] + Cos[c + d*x + ArcTa
n[Tan[c]]])*Cot[c]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])/2 - (Cos[c + d*x]*Csc
[c]*Sqrt[Sec[c]^2]*(Cos[2*d*x] - I*Sin[2*d*x])*(178*Cos[c + 2*d*x] + 158*Co
s[3*c + 2*d*x] - 9*Cos[3*c + 4*d*x] + 9*Cos[5*c + 4*d*x] - (88*I)*Sin[c] +
(208*I)*Sin[c + 2*d*x] + (128*I)*Sin[3*c + 2*d*x] - (4*I)*Sin[3*c + 4*d*x]
+ (4*I)*Sin[5*c + 4*d*x])*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])/8 + 21*Hyperge
```

```
ometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[c]*Sin[d*x
 + ArcTan[Tan[c]]]*Tan[c] - (21*(3*Cos[c - d*x - ArcTan[Tan[c]]] + Cos[c +
d*x + ArcTan[Tan[c]]])*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]*Tan[c])/2))/(65*d*
Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]*(a + I*a*Tan[c + d*x])^2)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(160) = 320$.

Time = 8.42 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.28

method	result
default	$\frac{2e^3\left(840i\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1280\left(\sin^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-140i\left(\sin^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-3840\left(\sin^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-6720i\left(\sin\right.\right.}{}$

```
[In] int((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/65/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^3*(840*I*
sin(1/2*d*x+1/2*c)^5+1280*sin(1/2*d*x+1/2*c)^14*cos(1/2*d*x+1/2*c)-140*I*si
n(1/2*d*x+1/2*c)^3-3840*sin(1/2*d*x+1/2*c)^12*cos(1/2*d*x+1/2*c)-6720*I*sin
(1/2*d*x+1/2*c)^11+4960*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-1280*I*sin
(1/2*d*x+1/2*c)^15-3520*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-2800*I*sin(
1/2*d*x+1/2*c)^7+1496*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+10*I*sin(1/2*
d*x+1/2*c)-376*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+5600*I*sin(1/2*d*x+1
/2*c)^9+44*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+21*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)+4480*I*sin(1/2*d*x+1/2*c)^13)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.91

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{\left(336i\sqrt{2}e^{\frac{5}{2}}e^{(6i\,dx+6i\,c)}\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx-}\right.\right.\right.}{}$$

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/520*(336*I*sqrt(2)*e^(5/2)*e^(6*I*d*x + 6*I*c)*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, e^(I*d*x + I*c))) + sqrt(1/2)*(-13*I*e^2*e^(8*I*d*x
 + 8*I*c) + 386*I*e^2*e^(6*I*d*x + 6*I*c) + 88*I*e^2*e^(4*I*d*x + 4*I*c) +
30*I*e^2*e^(2*I*d*x + 2*I*c) + 5*I*e^2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*e^(
-1/2*I*d*x - 1/2*I*c))*e^(-6*I*d*x - 6*I*c)/(a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*cos(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\cos(dx+c))^{\frac{5}{2}}}{(ia\tan(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\cos(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\,\cos(c+d\,x))^{5/2}}{(a+a\tan(c+d\,x)\,1\mathrm{i})^2}\,dx$$

```
[In] int((e*cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^2,x)
```

```
[Out] int((e*cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

3.665 $\int \frac{(e\cos(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 154

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{10(e\cos(c+dx))^{3/2}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{33a^2d\cos^{\frac{3}{2}}(c+dx)} + \frac{2\cos(c+dx)(e\cos(c+dx))^{3/2}\sin(c+dx)}{11a^2d} + \frac{10(e\cos(c+dx))^{3/2}\tan(c+dx)}{33a^2d} + \frac{4i\cos^2(c+dx)(e\cos(c+dx))^{3/2}}{11d\,(a^2+ia^2\tan(c+dx))}$$

```
[Out] 10/33*(e*cos(d*x+c))^(3/2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/cos(d*x+c)^(3/2)+2/11*cos(d*x+c
)*(e*cos(d*x+c))^(3/2)*sin(d*x+c)/a^2/d+10/33*(e*cos(d*x+c))^(3/2)*tan(d*x+
c)/a^2/d+4/11*I*cos(d*x+c)^2*(e*cos(d*x+c))^(3/2)/d/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3596, 3581, 3854, 3856, 2720}

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{10\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(e\cos(c+dx))^{3/2}}{33a^2d\cos^{\frac{3}{2}}(c+dx)} + \frac{4i\cos^2(c+dx)(e\cos(c+dx))^{3/2}}{11d\,(a^2+ia^2\tan(c+dx))} + \frac{2\sin(c+dx)\cos(c+dx)(e\cos(c+dx))^{3/2}}{11a^2d} + \frac{10\tan(c+dx)(e\cos(c+dx))^{3/2}}{33a^2d}$$

```
[In] Int[(e*Cos[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (10*(e*Cos[c + d*x])^(3/2)*EllipticF[(c + d*x)/2, 2])/(33*a^2*d*Cos[c + d*x
]^(3/2)) + (2*Cos[c + d*x]*(e*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(11*a^2*d)
+ (10*(e*Cos[c + d*x])^(3/2)*Tan[c + d*x])/(33*a^2*d) + (((4*I)/11)*Cos[c +
 d*x]^2*(e*Cos[c + d*x])^(3/2))/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\text{integral} = \left((e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\right)\int \frac{1}{(e\sec(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx$$

$$= \frac{4i\cos^2(c+dx)(e\cos(c+dx))^{3/2}}{11d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\left(7e^2(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\right)\int \frac{1}{(e\sec(c+dx))^{7/2}}\,dx}{11a^2}$$

$$= \frac{2\cos(c+dx)(e\cos(c+dx))^{3/2}\sin(c+dx)}{11a^2d} + \frac{4i\cos^2(c+dx)(e\cos(c+dx))^{3/2}}{11d\,(a^2+ia^2\tan(c+dx))} + \frac{\left(5(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\right)\int\frac{1}{(e\sec(c+dx))^{3/2}}\,dx}{11a^2}$$

$$= \frac{2\cos(c+dx)(e\cos(c+dx))^{3/2}\sin(c+dx)}{11a^2d} + \frac{10(e\cos(c+dx))^{3/2}\tan(c+dx)}{33a^2d} + \frac{4i\cos^2(c+dx)(e\cos(c+dx))^{3/2}}{11d\,(a^2+ia^2\tan(c+dx))} + \frac{\left(5(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\right)\int\sqrt{e\sec(c+dx)}\,dx}{33a^2e^2}$$

$$= \frac{2\cos(c+dx)(e\cos(c+dx))^{3/2}\sin(c+dx)}{11a^2d} + \frac{10(e\cos(c+dx))^{3/2}\tan(c+dx)}{33a^2d} + \frac{4i\cos^2(c+dx)(e\cos(c+dx))^{3/2}}{11d\,(a^2+ia^2\tan(c+dx))} + \frac{\left(5(e\cos(c+dx))^{3/2}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{33a^2\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{10(e\cos(c+dx))^{3/2}\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{33a^2d\cos^{\frac{3}{2}}(c+dx)} + \frac{2\cos(c+dx)(e\cos(c+dx))^{3/2}\sin(c+dx)}{11a^2d} + \frac{10(e\cos(c+dx))^{3/2}\tan(c+dx)}{33a^2d} + \frac{4i\cos^2(c+dx)(e\cos(c+dx))^{3/2}}{11d\,(a^2+ia^2\tan(c+dx))}$$

Mathematica [A] (verified)

Time = 1.95 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.85

$$\int\frac{(e\cos(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{(e\cos(c+dx))^{3/2}\left(-20\operatorname{EllipticF}\left(\frac{1}{2}(c+dx),2\right)(\cos(2(c+dx))+i\sin(2(c+\right.}{66a^2d\cos}$$

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((e*Cos[c + d*x])^(3/2)*(-20*EllipticF[(c + d*x)/2, 2]*(Cos[2*(c + d*x)] +
I*Sin[2*(c + d*x)]) + Sqrt[Cos[c + d*x]]*((-28*I)*Cos[c + d*x] + (4*I)*Cos[
3*(c + d*x)] + 13*Sin[c + d*x] - 7*Sin[3*(c + d*x)])))/(66*a^2*d*Cos[c + d*
x]^(7/2)*(-I + Tan[c + d*x])^2)
```

Maple [A] (verified)

Time = 7.26 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.05

method	result
default	$-\frac{2e^2\left(-384i\left(\sin^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+384\left(\sin^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+1152i\left(\sin^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-960\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-1440i\left(\right.}{}$

```
[In] int((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/33/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e^2*(-384*
I*sin(1/2*d*x+1/2*c)^13+384*sin(1/2*d*x+1/2*c)^12*cos(1/2*d*x+1/2*c)+1152*I
*sin(1/2*d*x+1/2*c)^11-960*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)-1440*I*
sin(1/2*d*x+1/2*c)^9+1008*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+960*I*sin
(1/2*d*x+1/2*c)^7-552*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-360*I*sin(1/2
*d*x+1/2*c)^5+176*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+72*I*sin(1/2*d*x+
1/2*c)^3-28*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+5*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1
/2)-6*I*sin(1/2*d*x+1/2*c))/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.75

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = \frac{\left(-40i\sqrt{2}e^{\frac{3}{2}}e^{(5i\,dx+5i\,c)}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)+\sqrt{\frac{1}{2}}\left(-11i\,ee^{(6i\,dx}\right.\right.}{}$$

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/132*(-40*I*sqrt(2)*e^(3/2)*e^(5*I*d*x + 5*I*c)*weierstrassPInverse(-4, 0,
 e^(I*d*x + I*c)) + sqrt(1/2)*(-11*I*e*e^(6*I*d*x + 6*I*c) + 41*I*e*e^(4*I*
d*x + 4*I*c) + 15*I*e*e^(2*I*d*x + 2*I*c) + 3*I*e)*sqrt(e*e^(2*I*d*x + 2*I*
c) + e)*e^(-1/2*I*d*x - 1/2*I*c))*e^(-5*I*d*x - 5*I*c)/(a^2*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate((e*cos(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**2,x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\cos(dx+c))^{\frac{3}{2}}}{(ia\tan(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*cos(d*x + c))^(3/2)/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\cos(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\cos(c+d\,x))^{3/2}}{(a+a\tan(c+d\,x)\,1\mathrm{i})^2}\,dx$$

```
[In] int((e*cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^2,x)
[Out] int((e*cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

3.666 $\int \frac{\sqrt{e\cos(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 120

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx = \frac{2\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\sqrt{\cos(c+dx)}} + \frac{2i\sqrt{e\cos(c+dx)}}{9d(a+ia\tan(c+dx))^2} + \frac{2i\sqrt{e\cos(c+dx)}}{9d\left(a^2+ia^2\tan(c+dx)\right)}$$

```
[Out] 2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))*(e*cos(d*x+c))^(1/2)/a^2/d/cos(d*x+c)^(1/2)+2/9*I*(e*cos(d*x
+c))^(1/2)/d/(a+I*a*tan(d*x+c))^2+2/9*I*(e*cos(d*x+c))^(1/2)/d/(a^2+I*a^2*t
an(d*x+c))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3596, 3581, 3854, 3856, 2719}

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx = \frac{4i\cos^2(c+dx)\sqrt{e\cos(c+dx)}}{9d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{2\sin(c+dx)\cos(c+dx)\sqrt{e\cos(c+dx)}}{9a^2d} + \frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{e\cos(c+dx)}}{3a^2d\sqrt{\cos(c+dx)}}$$

```
[In] Int[Sqrt[e*Cos[c + d*x]]/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (2*Sqrt[e*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(3*a^2*d*Sqrt[Cos[c + d*
x]]) + (2*Cos[c + d*x]*Sqrt[e*Cos[c + d*x]]*Sin[c + d*x])/(9*a^2*d) + (((4*
I)/9)*Cos[c + d*x]^2*Sqrt[e*Cos[c + d*x]])/(d*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\text{integral} = \left(\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int \frac{1}{\sqrt{e\sec(c+dx)}(a+ia\tan(c+dx))^2}\,dx$$

$$= \frac{4i\cos^2(c+dx)\sqrt{e\cos(c+dx)}}{9d\,(a^2+ia^2\tan(c+dx))} + \frac{\left(5e^2\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int \frac{1}{(e\sec(c+dx))^{5/2}}\,dx}{9a^2}$$

$$= \frac{2\cos(c+dx)\sqrt{e\cos(c+dx)}\sin(c+dx)}{9a^2d} + \frac{4i\cos^2(c+dx)\sqrt{e\cos(c+dx)}}{9d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\left(\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int\frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{3a^2}$$

$$= \frac{2\cos(c+dx)\sqrt{e\cos(c+dx)}\sin(c+dx)}{9a^2d} + \frac{4i\cos^2(c+dx)\sqrt{e\cos(c+dx)}}{9d\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\sqrt{e\cos(c+dx)}\int\sqrt{\cos(c+dx)}\,dx}{3a^2\sqrt{\cos(c+dx)}}$$

$$= \frac{2\sqrt{e\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d\sqrt{\cos(c+dx)}} + \frac{2\cos(c+dx)\sqrt{e\cos(c+dx)}\sin(c+dx)}{9a^2d} + \frac{4i\cos^2(c+dx)\sqrt{e\cos(c+dx)}}{9d\left(a^2+ia^2\tan(c+dx)\right)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.25 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.23

$$\int\frac{\sqrt{e\cos(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx$$

$$= \frac{\sqrt{e\cos(c+dx)}\sec^3(c+dx)(\cos(dx)+i\sin(dx))^2\left(6\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\arctan(\tan(c)))\right)\sec(c)(\cos\right.}{}$$

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] (Sqrt[e*Cos[c + d*x]]*Sec[c + d*x]^3*(Cos[d*x] + I*Sin[d*x])^2*(6*Hypergeom
etricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sec[c]*(Cos[2*c]
 + I*Sin[2*c])*Sin[d*x + ArcTan[Tan[c]]] + Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2
]*(Cos[c + d*x]*Csc[c]*Sqrt[Sec[c]^2]*(Cos[2*d*x] - I*Sin[2*d*x])*(7*Cos[c
+ 2*d*x] + 5*Cos[3*c + 2*d*x] - (4*I)*(Sin[c] - 2*Sin[c + 2*d*x] - Sin[3*c
+ 2*d*x])) - 3*Cos[c + d*x + ArcTan[Tan[c]]]*(I + Cot[c])^2*Tan[c] + 9*Cos[
c - d*x - ArcTan[Tan[c]]]*(-2*I - Cot[c] + Tan[c]))))/(18*a^2*d*Sqrt[Sec[c]
^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]*(-I + Tan[c + d*x])^2)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 276 vs. 2(128) = 256.

Time = 6.17 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.31

method	result
default	$\frac{2e\left(-64i\left(\sin^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+64\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+160i\left(\sin^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-128\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-160i\left(\sin^{7}\left(\frac{dx}{2}\right.\right.\right.}{}$

```
[In] int((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/9/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*e*(-64*I*sin
(1/2*d*x+1/2*c)^11+64*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+160*I*sin(1/
2*d*x+1/2*c)^9-128*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-160*I*sin(1/2*d*
x+1/2*c)^7+104*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+80*I*sin(1/2*d*x+1/2
*c)^5-40*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-20*I*sin(1/2*d*x+1/2*c)^3+
6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+3*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)+2*I*si
n(1/2*d*x+1/2*c))/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx = \frac{\left(\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\left(15i\,e^{(4i\,dx+4i\,c)}+4i\,e^{(2i\,dx+2i\,c)}+i\right)e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}+12i\,\sqrt{2}\sqrt{e}e^{(4i\,dx+4i\,c)}\mathrm{weierstrassZe}\right.}{18\,a^2d}$$

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/18*(sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*(15*I*e^(4*I*d*x + 4*I*c) +
 4*I*e^(2*I*d*x + 2*I*c) + I)*e^(-1/2*I*d*x - 1/2*I*c) + 12*I*sqrt(2)*sqrt(
e)*e^(4*I*d*x + 4*I*c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^
(I*d*x + I*c))))*e^(-4*I*d*x - 4*I*c)/(a^2*d)
```

Sympy [F]

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{\sqrt{e\cos(c+dx)}}{\tan^2(c+dx)-2i\tan(c+dx)-1}\,dx}{a^2}$$

```
[In] integrate((e*cos(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**2,x)
[Out] -Integral(sqrt(e*cos(c + d*x))/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)
/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{\sqrt{e\cos(dx+c)}}{(i\,a\tan(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
[Out] integrate(sqrt(e*cos(d*x + c))/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e\cos(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{\sqrt{e\cos(c+d\,x)}}{(a+a\tan(c+d\,x)\,1\mathrm{i})^2}\,dx$$

```
[In] int((e*cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^2,x)
[Out] int((e*cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^2, x)
```

3.667 $\int \frac{1}{\sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 120

$$\int \frac{1}{\sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))^2}\,dx = \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{7a^2d\sqrt{e\cos(c+dx)}} + \frac{2i}{7d\sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))^2} + \frac{2i}{7d\sqrt{e\cos(c+dx)}\,(a^2+ia^2\tan(c+dx))}$$

```
[Out] 2/7*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)/a^2/d/(e*cos(d*x+c))^(1/2)+2/7*I/d/(e*cos(d
*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2+2/7*I/d/(e*cos(d*x+c))^(1/2)/(a^2+I*a^2*t
an(d*x+c))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3596, 3581, 3854, 3856, 2720}

$$\int \frac{1}{\sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))^2}\,dx = \frac{4i\cos^2(c+dx)}{7d\,(a^2+ia^2\tan(c+dx))\,\sqrt{e\cos(c+dx)}} + \frac{2\sin(c+dx)\cos(c+dx)}{7a^2d\sqrt{e\cos(c+dx)}} + \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{7a^2d\sqrt{e\cos(c+dx)}}$$

```
[In] Int[1/(Sqrt[e*Cos[c + d*x]]*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(7*a^2*d*Sqrt[e*Cos[c + d*
x]]) + (2*Cos[c + d*x]*Sin[c + d*x])/(7*a^2*d*Sqrt[e*Cos[c + d*x]]) + (((4*
I)/7)*Cos[c + d*x]^2)/(d*Sqrt[e*Cos[c + d*x]]*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\sqrt{e\sec(c+dx)}}{(a+ia\tan(c+dx))^2}\,dx}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$= \frac{4i\cos^2(c+dx)}{7d\sqrt{e\cos(c+dx)}\,(a^2+ia^2\tan(c+dx))} + \frac{(3e^2)\int \frac{1}{(e\sec(c+dx))^{3/2}}\,dx}{7a^2\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$= \frac{2\cos(c+dx)\sin(c+dx)}{7a^2d\sqrt{e\cos(c+dx)}} + \frac{4i\cos^2(c+dx)}{7d\sqrt{e\cos(c+dx)}\,(a^2+ia^2\tan(c+dx))} + \frac{\int\sqrt{e\sec(c+dx)}\,dx}{7a^2\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}}$$

$$= \frac{2\cos(c+dx)\sin(c+dx)}{7a^2d\sqrt{e\cos(c+dx)}} + \frac{4i\cos^2(c+dx)}{7d\sqrt{e\cos(c+dx)}\,(a^2+ia^2\tan(c+dx))} + \frac{\sqrt{\cos(c+dx)}\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{7a^2\sqrt{e\cos(c+dx)}}$$

$$= \frac{2\sqrt{\cos(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{7a^2d\sqrt{e\cos(c+dx)}} + \frac{2\cos(c+dx)\sin(c+dx)}{7a^2d\sqrt{e\cos(c+dx)}} + \frac{4i\cos^2(c+dx)}{7d\sqrt{e\cos(c+dx)}\,(a^2+ia^2\tan(c+dx))}$$

Mathematica [A] (verified)

Time = 1.90 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.32

$$\int \frac{1}{\sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))^2}\,dx = \frac{\left(-i\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sqrt{\cos(c+dx)}\left(3\cos\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{3}{2}(c+dx)\right)+4i\sin^3\left(\frac{1}{2}(c\right.\right.\right.}{7a^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{e\cos(c+dx)}(-i+\text{t}}$$

```
[In] Integrate[1/(Sqrt[e*Cos[c + d*x]]*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (((-I)*Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(Sqrt[Cos[c + d*x]]*(3*Cos[(c +
 d*x)/2] + Cos[(3*(c + d*x))/2] + (4*I)*Sin[(c + d*x)/2]^3) + 2*EllipticF[(
c + d*x)/2, 2]*((-I)*Cos[(3*(c + d*x))/2] + Sin[(3*(c + d*x))/2])))/(7*a^2*
d*Cos[c + d*x]^(3/2)*Sqrt[e*Cos[c + d*x]]*(-I + Tan[c + d*x])^2)
```

Maple [A] (verified)

Time = 5.09 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.99

method	result
default	$-\frac{2\left(-32i\left(\sin^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+32\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+64i\left(\sin^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-48\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-48i\left(\sin^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{7a^{2}}$

[In] int(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] -2/7/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(-32*I*sin(1/2*d*x+1/2*c)^9+32*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+64*I*sin(1/2*d*x+1/2*c)^7-48*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-48*I*sin(1/2*d*x+1/2*c)^5+28*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+16*I*sin(1/2*d*x+1/2*c)^3-6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)-2*I*sin(1/2*d*x+1/2*c))/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))^2}\,dx$$
$$= \frac{\left(\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\left(3i\,e^{(2i\,dx+2i\,c)}+i\right)e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}-2i\,\sqrt{2}\sqrt{e}e^{(3i\,dx+3i\,c)}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx-}\right.\right.}{7\,a^{2}de}$$

[In] integrate(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")

[Out] 1/7*(sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*(3*I*e^(2*I*d*x + 2*I*c) + I)*e^(-1/2*I*d*x - 1/2*I*c) - 2*I*sqrt(2)*sqrt(e)*e^(3*I*d*x + 3*I*c)*weierstrassPInverse(-4, 0, e^(I*d*x + I*c)))*e^(-3*I*d*x - 3*I*c)/(a^2*d*e)

Sympy [F]

$$\int \frac{1}{\sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{1}{\sqrt{e\cos(c+dx)}\tan^2(c+dx)-2i\sqrt{e\cos(c+dx)}\tan(c+dx)-\sqrt{e\cos(c+dx)}}\,dx}{a^2}$$

```
[In] integrate(1/(e*cos(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**2,x)
[Out] -Integral(1/(sqrt(e*cos(c + d*x))*tan(c + d*x)**2 - 2*I*sqrt(e*cos(c + d*x)
)*tan(c + d*x) - sqrt(e*cos(c + d*x))), x)/a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima"
)
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{1}{\sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{\sqrt{e\cos(dx+c)}(ia\tan(dx+c)+a)^2}\,dx$$

```
[In] integrate(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
[Out] integrate(1/(sqrt(e*cos(d*x + c))*(I*a*tan(d*x + c) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{e\cos(c+dx)}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{\sqrt{e\,\cos\left(c+d\,x\right)}\,{\left(a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^2}\,dx$$

```
[In] int(1/((e*cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^2),x)
[Out] int(1/((e*cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

3.668 $\int \frac{1}{(e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 92

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = \frac{2\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d(e\cos(c+dx))^{3/2}} + \frac{4i\cos^2(c+dx)}{5d(e\cos(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)}$$

```
[Out] 2/5*cos(d*x+c)^(3/2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/(e*cos(d*x+c))^(3/2)+4/5*I*cos(d*x+c)
^2/d/(e*cos(d*x+c))^(3/2)/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3596, 3581, 3856, 2719}

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = \frac{2\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d(e\cos(c+dx))^{3/2}} + \frac{4i\cos^2(c+dx)}{5d\left(a^2+ia^2\tan(c+dx)\right)(e\cos(c+dx))^{3/2}}$$

```
[In] Int[1/((e*Cos[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (2*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2])/(5*a^2*d*(e*Cos[c + d*x])^
(3/2)) + (((4*I)/5)*Cos[c + d*x]^2)/(d*(e*Cos[c + d*x])^(3/2)*(a^2 + I*a^2*
Tan[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{(e\sec(c+dx))^{3/2}}{(a+ia\tan(c+dx))^2}\,dx}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$= \frac{4i\cos^2(c+dx)}{5d(e\cos(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)} + \frac{e^2\int \frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{5a^2(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}}$$

$$= \frac{4i\cos^2(c+dx)}{5d(e\cos(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\cos^{\frac{3}{2}}(c+dx)\int \sqrt{\cos(c+dx)}\,dx}{5a^2(e\cos(c+dx))^{3/2}}$$

$$= \frac{2\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d(e\cos(c+dx))^{3/2}} + \frac{4i\cos^2(c+dx)}{5d(e\cos(c+dx))^{3/2}\left(a^2+ia^2\tan(c+dx)\right)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.55 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.24

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = \frac{2\cos^2(c+dx)\left(1+2^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{3}{4},\frac{1}{2}(1-\right.\right.}{5a^2d(e\cos(c+}$$

```
[In] Integrate[1/((e*Cos[c + d*x])^(3/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (2*Cos[c + d*x]^2*(1 + 2^(3/4)*Hypergeometric2F1[-1/4, 1/4, 3/4, (1 + I*Tan
[c + d*x])/2]*(1 - I*Tan[c + d*x])^(1/4)*(1 + I*Tan[c + d*x]) - I*Tan[c + d
*x]))/(5*a^2*d*(e*Cos[c + d*x])^(3/2)*(-I + Tan[c + d*x]))
```

Maple [A] (verified)

Time = 4.18 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.24

method	result
default	$\frac{-\frac{32i\left(\sin^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{32\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{5}+\frac{48i\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{32\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{5}-\frac{24i\left(\sin^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{8\cos\left(\frac{dx}{2}+\right.}{}}{e\,a^2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e\,}}$

```
[In] int(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/5/e/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(-16*I*sin
(1/2*d*x+1/2*c)^7+16*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+24*I*sin(1/2*d
*x+1/2*c)^5-16*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-12*I*sin(1/2*d*x+1/2
*c)^3+4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)+2*
I*sin(1/2*d*x+1/2*c))/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.02

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx =$$

$$-\frac{2\left(\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\left(-2i\,e^{(2i\,dx+2i\,c)}-i\right)e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}-i\,\sqrt{2}\sqrt{e}e^{(2i\,dx+2i\,c)}\text{weierstrassZeta}\left(-4,0,\text{weiers}\right.\right.}{5\,a^2de^2}$$

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas"
)
```

```
[Out] -2/5*(sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*(-2*I*e^(2*I*d*x + 2*I*c) -
 I)*e^(-1/2*I*d*x - 1/2*I*c) - I*sqrt(2)*sqrt(e)*e^(2*I*d*x + 2*I*c)*weiers
trassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x + I*c))))*e^(-2*I*d*x
- 2*I*c)/(a^2*d*e^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*cos(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}\left(i\,a\tan\left(dx+c\right)+a\right)^{2}}\,dx$$

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(3/2)*(I*a*tan(d*x + c) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{3/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{\left(e\,\cos\left(c+d\,x\right)\right)^{3/2}\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^2}\,dx$$

```
[In] int(1/((e*cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^2),x)
[Out] int(1/((e*cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

3.669 $\int \frac{1}{(e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 92

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = -\frac{2\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d(e\cos(c+dx))^{5/2}} + \frac{4i\cos^2(c+dx)}{3d(e\cos(c+dx))^{5/2}\left(a^2+ia^2\tan(c+dx)\right)}$$

```
[Out] -2/3*cos(d*x+c)^(5/2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/(e*cos(d*x+c))^(5/2)+4/3*I*cos(d*x+c
)^2/d/(e*cos(d*x+c))^(5/2)/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3596, 3581, 3856, 2720}

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = -\frac{2\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d(e\cos(c+dx))^{5/2}} + \frac{4i\cos^2(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)(e\cos(c+dx))^{5/2}}$$

```
[In] Int[1/((e*Cos[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (-2*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2])/(3*a^2*d*(e*Cos[c + d*x])
^(5/2)) + (((4*I)/3)*Cos[c + d*x]^2)/(d*(e*Cos[c + d*x])^(5/2)*(a^2 + I*a^2
*Tan[c + d*x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{(e\sec(c+dx))^{5/2}}{(a+ia\tan(c+dx))^2}\,dx}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$= \frac{4i\cos^2(c+dx)}{3d(e\cos(c+dx))^{5/2}\left(a^2+ia^2\tan(c+dx)\right)} - \frac{e^2\int \sqrt{e\sec(c+dx)}\,dx}{3a^2(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$= \frac{4i\cos^2(c+dx)}{3d(e\cos(c+dx))^{5/2}\left(a^2+ia^2\tan(c+dx)\right)} - \frac{\cos^{\frac{5}{2}}(c+dx)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3a^2(e\cos(c+dx))^{5/2}}$$

$$= -\frac{2\cos^{\frac{5}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d(e\cos(c+dx))^{5/2}} + \frac{4i\cos^2(c+dx)}{3d(e\cos(c+dx))^{5/2}\left(a^2+ia^2\tan(c+dx)\right)}$$

Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.26

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = \frac{2\sqrt{\cos(c+dx)}(\cos(dx)+i\sin(dx))^2\left(\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right.\right.}{3a^2d(e\cos(c+a}$$

```
[In] Integrate[1/((e*Cos[c + d*x])^(5/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(Cos[d*x] + I*Sin[d*x])^2*(EllipticF[(c + d*x)/2, 2]*
(Cos[2*c] + I*Sin[2*c]) + 2*Sqrt[Cos[c + d*x]]*((-I)*Cos[c - d*x] + Sin[c -
 d*x])))/(3*a^2*d*(e*Cos[c + d*x])^(5/2)*(-I + Tan[c + d*x])^2)
```

Maple [A] (verified)

Time = 3.52 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.86

method	result
default	$-\frac{2\left(-8i\left(\sin^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+8i\left(\sin^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}}{3e^2a^2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\,d}$

```
[In] int(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/e^2/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(-8*I*s
in(1/2*d*x+1/2*c)^5+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+8*I*sin(1/2*d
*x+1/2*c)^3-4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1
/2)-2*I*sin(1/2*d*x+1/2*c))/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.86

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx =$$
$$-\frac{2\left(-i\sqrt{2}\sqrt{e}e^{(i\,dx+i\,c)}\text{weierstrassPInverse}\left(-4,0,e^{(i\,dx+i\,c)}\right)-2i\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}\right)e^{(-i\,dx-i\,c)}}{3\,a^2de^3}$$

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas"
)
```

```
[Out] -2/3*(-I*sqrt(2)*sqrt(e)*e^(I*d*x + I*c)*weierstrassPInverse(-4, 0, e^(I*d*
x + I*c)) - 2*I*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*e^(-1/2*I*d*x - 1
/2*I*c))*e^(-I*d*x - I*c)/(a^2*d*e^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*cos(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{(e\cos(dx+c))^{\frac{5}{2}}\,(i\,a\tan(dx+c)+a)^2}\,dx$$

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(5/2)*(I*a*tan(d*x + c) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{5/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{\left(e\,\cos\left(c+d\,x\right)\right)^{5/2}\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^2}\,dx$$

```
[In] int(1/((e*cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^2),x)
[Out] int(1/((e*cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

3.670 $\int \frac{1}{(e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 122

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx = \frac{6\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(e\cos(c+dx))^{7/2}} - \frac{6\cos^3(c+dx)\sin(c+dx)}{a^2d(e\cos(c+dx))^{7/2}} + \frac{4i\cos^2(c+dx)}{d(e\cos(c+dx))^{7/2}\left(a^2+ia^2\tan(c+dx)\right)}$$

```
[Out] 6*cos(d*x+c)^(7/2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/(e*cos(d*x+c))^(7/2)-6*cos(d*x+c)^3*sin
(d*x+c)/a^2/d/(e*cos(d*x+c))^(7/2)+4*I*cos(d*x+c)^2/d/(e*cos(d*x+c))^(7/2)/
(a^2+I*a^2*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3596, 3581, 3853, 3856, 2719}

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx = \frac{6\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(e\cos(c+dx))^{7/2}} - \frac{6\sin(c+dx)\cos^3(c+dx)}{a^2d(e\cos(c+dx))^{7/2}} + \frac{4i\cos^2(c+dx)}{d\left(a^2+ia^2\tan(c+dx)\right)(e\cos(c+dx))^{7/2}}$$

```
[In] Int[1/((e*Cos[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (6*Cos[c + d*x]^(7/2)*EllipticE[(c + d*x)/2, 2])/(a^2*d*(e*Cos[c + d*x])^(7
/2)) - (6*Cos[c + d*x]^3*Sin[c + d*x])/(a^2*d*(e*Cos[c + d*x])^(7/2)) + ((4
*I)*Cos[c + d*x]^2)/(d*(e*Cos[c + d*x])^(7/2)*(a^2 + I*a^2*Tan[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{(e\sec(c+dx))^{7/2}}{(a+ia\tan(c+dx))^2}\,dx}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}\\
&= \frac{4i\cos^2(c+dx)}{d(e\cos(c+dx))^{7/2}\,(a^2+ia^2\tan(c+dx))} - \frac{(3e^2)\int(e\sec(c+dx))^{3/2}\,dx}{a^2(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}\\
&= -\frac{6\cos^3(c+dx)\sin(c+dx)}{a^2d(e\cos(c+dx))^{7/2}} + \frac{4i\cos^2(c+dx)}{d(e\cos(c+dx))^{7/2}\,(a^2+ia^2\tan(c+dx))}\\
&\quad + \frac{(3e^4)\int\frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{a^2(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}
\end{aligned}$$

$$= -\frac{6\cos^3(c+dx)\sin(c+dx)}{a^2d(e\cos(c+dx))^{7/2}} + \frac{4i\cos^2(c+dx)}{d(e\cos(c+dx))^{7/2}\left(a^2+ia^2\tan(c+dx)\right)} + \frac{\left(3\cos^{\frac{7}{2}}(c+dx)\right)\int\sqrt{\cos(c+dx)}\,dx}{a^2(e\cos(c+dx))^{7/2}}$$

$$= \frac{6\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(e\cos(c+dx))^{7/2}} - \frac{6\cos^3(c+dx)\sin(c+dx)}{a^2d(e\cos(c+dx))^{7/2}} + \frac{4i\cos^2(c+dx)}{d(e\cos(c+dx))^{7/2}\left(a^2+ia^2\tan(c+dx)\right)}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.39 (sec) , antiderivative size = 1106, normalized size of antiderivative = 9.07

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx = \frac{\sqrt{\cos(c+dx)}(\cos(dx)+i\sin(dx))^2\left(-2i\cos(c-dx)\sqrt{\cos(\right.}{d(e\cos(c+dx))^{7/2}(a+ia\tan(}$$

$$+\frac{3\cos(c)\cos^{\frac{3}{2}}(c+dx)(\cos(dx)+i\sin(dx))^2\left(-\frac{\cos(dx-\arctan(\cot(c)))\cot(c)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\sin^2(dx-\mathrm{arc}\right.}{\sqrt{1+\cot^2(c)}\sqrt{1-\sin(dx-\arctan(\cot(c)))}\sqrt{-\sqrt{1+\cot^2(c)}\sin(c)\sin(dx-\arctan(}}\right.}{d(e\cos(c+dx))^{7/2}(a+ia\tan(c}$$

$$+\frac{3i\cos^{\frac{3}{2}}(c+dx)\sin(c)(\cos(dx)+i\sin(dx))^2\left(-\frac{\cos(dx-\arctan(\cot(c)))\cot(c)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\sin^2(dx-\mathrm{ar}\right.}{\sqrt{1+\cot^2(c)}\sqrt{1-\sin(dx-\arctan(\cot(c)))}\sqrt{-\sqrt{1+\cot^2(c)}\sin(c)\sin(dx-\arctan(}}\right.}{d(e\cos(c+dx))^{7/2}(a+ia\tan(c}$$

$$-\frac{3i\cos(c)\cos^{\frac{3}{2}}(c+dx)(\cos(dx)+i\sin(dx))^2\left(\frac{{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\arctan(\tan(c)))\right)\sin(dx+\arctan(\mathrm{ta}}{\sqrt{1-\cos(dx+\arctan(\tan(c)))}\sqrt{1+\cos(dx+\arctan(\tan(c)))}\sqrt{\cos(c)\cos(dx+\mathrm{arcta}}}\right.}{d(e\cos(c+dx))^{7/2}(a+ia\tan(c}$$

$$+\frac{3\cos^{\frac{3}{2}}(c+dx)\sin(c)(\cos(dx)+i\sin(dx))^2\left(\frac{{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\arctan(\tan(c)))\right)\sin(dx+\arctan(\mathrm{tan}}{\sqrt{1-\cos(dx+\arctan(\tan(c)))}\sqrt{1+\cos(dx+\arctan(\tan(c)))}\sqrt{\cos(c)\cos(dx+\arctan(\mathrm{t}}}\right.}{d(e\cos(c+dx))^{7/2}(a+ia\tan(c}$$

```
[In] Integrate[1/((e*Cos[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(Cos[d*x] + I*Sin[d*x])^2*((-2*I)*Cos[c - d*x]*Sqrt[Cos
[c + d*x]] + 2*Sqrt[Cos[c + d*x]]*Sin[c - d*x]))/(d*(e*Cos[c + d*x])^(7/2)*
(a + I*a*Tan[c + d*x])^2) + (3*Cos[c]*Cos[c + d*x]^(3/2)*(Cos[d*x] + I*Sin[
d*x])^2*(-((Cos[d*x - ArcTan[Cot[c]]]*Cot[c]*HypergeometricPFQ[{-1/2, -1/4}
```

, {3/4}, Sin[d*x - ArcTan[Cot[c]]]^2])/(Sqrt[1 + Cot[c]^2]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])) + ((Cos[d*x - ArcTan[Cot[c]]]*Cot[c])/Sqrt[1 + Cot[c]^2] + (2*Sqrt[1 + Cot[c]^2]*Sin[c]^2*Sin[d*x - ArcTan[Cot[c]]])/(Cos[c]^2 + Sin[c]^2))/Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]))/(d*(e*Cos[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^2) + ((3*I)*Cos[c + d*x]^(3/2)*Sin[c]*(Cos[d*x] + I*Sin[d*x])^2*(-((Cos[d*x - ArcTan[Cot[c]]]*Cot[c]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Sin[d*x - ArcTan[Cot[c]]]^2])/(Sqrt[1 + Cot[c]^2]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])) + ((Cos[d*x - ArcTan[Cot[c]]]*Cot[c])/Sqrt[1 + Cot[c]^2] + (2*Sqrt[1 + Cot[c]^2]*Sin[c]^2*Sin[d*x - ArcTan[Cot[c]]])/(Cos[c]^2 + Sin[c]^2))/Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]))/(d*(e*Cos[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^2) - ((3*I)*Cos[c]*Cos[c + d*x]^(3/2)*(Cos[d*x] + I*Sin[d*x])^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(d*(e*Cos[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^2) + (3*Cos[c + d*x]^(3/2)*Sin[c]*(Cos[d*x] + I*Sin[d*x])^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(d*(e*Cos[c + d*x])^(7/2)*(a + I*a*Tan[c + d*x])^2)

Maple [A] (verified)

Time = 3.72 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.11

method	result
default	$-\frac{2(4i(\sin^3(\frac{dx}{2}+\frac{c}{2}))+2\cos(\frac{dx}{2}+\frac{c}{2})(\sin^2(\frac{dx}{2}+\frac{c}{2}))-3E(\cos(\frac{dx}{2}+\frac{c}{2}),\sqrt{2})\sqrt{2(\sin^2(\frac{dx}{2}+\frac{c}{2}))-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}-2i\sin(\frac{dx}{2}+\frac{c}{2})}{e^3a^2\sin(\frac{dx}{2}+\frac{c}{2})\sqrt{-2(\sin^2(\frac{dx}{2}+\frac{c}{2}))e+e}\,d}$

[In] int(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] -2/e^3/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)*(4*I*sin(1/2*d*x+1/2*c)^3+2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)-2*I*sin(1/2*d*x+1/2*c))/d

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.93

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx = -\frac{2\left(2\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\left(-3i\,e^{(2i\,dx+2i\,c)}-2i\right)e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}+3\left(-i\sqrt{2}e^{(2i\,dx+2i\,c)}-i\sqrt{2}\right)\sqrt{e}\mathrm{weierstrassZe}\right.}{a^2de^4e^{(2i\,dx+2i\,c)}+a^2de^4}$$

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas"
)
```

```
[Out] -2*(2*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*(-3*I*e^(2*I*d*x + 2*I*c) -
 2*I)*e^(-1/2*I*d*x - 1/2*I*c) + 3*(-I*sqrt(2)*e^(2*I*d*x + 2*I*c) - I*sqrt
(2))*sqrt(e)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, e^(I*d*x + I
*c))))/(a^2*d*e^4*e^(2*I*d*x + 2*I*c) + a^2*d*e^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*cos(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{7}{2}}\left(i\,a\tan\left(dx+c\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
[Out] integrate(1/((e*cos(d*x + c))^(7/2)*(I*a*tan(d*x + c) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{7/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{\left(e\,\cos\left(c+d\,x\right)\right)^{7/2}\,\left(a+a\tan\left(c+d\,x\right)\,1\mathrm{i}\right)^2}\,dx$$

```
[In] int(1/((e*cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)^2),x)
[Out] int(1/((e*cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

3.671 $\int \frac{1}{(e\cos(c+dx))^{9/2}(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 126

$$\int \frac{1}{(e\cos(c+dx))^{9/2}(a+ia\tan(c+dx))^2}\,dx = \frac{10\cos^{\frac{9}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d(e\cos(c+dx))^{9/2}} + \frac{10\cos^3(c+dx)\sin(c+dx)}{3a^2d(e\cos(c+dx))^{9/2}} - \frac{4i\cos^2(c+dx)}{d(e\cos(c+dx))^{9/2}\,(a^2+ia^2\tan(c+dx))}$$

```
[Out] 10/3*cos(d*x+c)^(9/2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/(e*cos(d*x+c))^(9/2)+10/3*cos(d*x+c)
^3*sin(d*x+c)/a^2/d/(e*cos(d*x+c))^(9/2)-4*I*cos(d*x+c)^2/d/(e*cos(d*x+c))^
(9/2)/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3596, 3581, 3853, 3856, 2720}

$$\int \frac{1}{(e\cos(c+dx))^{9/2}(a+ia\tan(c+dx))^2}\,dx = \frac{10\cos^{\frac{9}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d(e\cos(c+dx))^{9/2}} + \frac{10\sin(c+dx)\cos^3(c+dx)}{3a^2d(e\cos(c+dx))^{9/2}} - \frac{4i\cos^2(c+dx)}{d\,(a^2+ia^2\tan(c+dx))\,(e\cos(c+dx))^{9/2}}$$

```
[In] Int[1/((e*Cos[c + d*x])^(9/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (10*Cos[c + d*x]^(9/2)*EllipticF[(c + d*x)/2, 2])/(3*a^2*d*(e*Cos[c + d*x])
^(9/2)) + (10*Cos[c + d*x]^3*Sin[c + d*x])/(3*a^2*d*(e*Cos[c + d*x])^(9/2))
 - ((4*I)*Cos[c + d*x]^2)/(d*(e*Cos[c + d*x])^(9/2)*(a^2 + I*a^2*Tan[c + d*
x]))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{(e\sec(c+dx))^{9/2}}{(a+ia\tan(c+dx))^2}\,dx}{(e\cos(c+dx))^{9/2}(e\sec(c+dx))^{9/2}}\\
&= -\frac{4i\cos^2(c+dx)}{d(e\cos(c+dx))^{9/2}\left(a^2+ia^2\tan(c+dx)\right)}+\frac{(5e^2)\int(e\sec(c+dx))^{5/2}\,dx}{a^2(e\cos(c+dx))^{9/2}(e\sec(c+dx))^{9/2}}\\
&= \frac{10\cos^3(c+dx)\sin(c+dx)}{3a^2d(e\cos(c+dx))^{9/2}}-\frac{4i\cos^2(c+dx)}{d(e\cos(c+dx))^{9/2}\left(a^2+ia^2\tan(c+dx)\right)}\\
&\quad+\frac{(5e^4)\int\sqrt{e\sec(c+dx)}\,dx}{3a^2(e\cos(c+dx))^{9/2}(e\sec(c+dx))^{9/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{10\cos^3(c+dx)\sin(c+dx)}{3a^2d(e\cos(c+dx))^{9/2}} - \frac{4i\cos^2(c+dx)}{d(e\cos(c+dx))^{9/2}\,(a^2+ia^2\tan(c+dx))} \\
&\quad + \frac{\left(5\cos^{\frac{9}{2}}(c+dx)\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3a^2(e\cos(c+dx))^{9/2}} \\
&= \frac{10\cos^{\frac{9}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)}{3a^2d(e\cos(c+dx))^{9/2}} + \frac{10\cos^3(c+dx)\sin(c+dx)}{3a^2d(e\cos(c+dx))^{9/2}} \\
&\quad - \frac{4i\cos^2(c+dx)}{d(e\cos(c+dx))^{9/2}\,(a^2+ia^2\tan(c+dx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.57 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.53

$$
\int \frac{1}{(e\cos(c+dx))^{9/2}(a+ia\tan(c+dx))^2}\,dx = \frac{2\left(-6i\cos(c+dx)+5\cos^{\frac{3}{2}}(c+dx)\,\text{EllipticF}\left(\frac{1}{2}(c+dx),2\right)\right.}{3a^2de^3(e\cos(c+dx))^{3/2}}
$$

```
[In] Integrate[1/((e*Cos[c + d*x])^(9/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (2*((-6*I)*Cos[c + d*x] + 5*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] -
Sin[c + d*x]))/(3*a^2*d*e^3*(e*Cos[c + d*x])^(3/2))
```

Maple [A] (verified)

Time = 4.56 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.65

method	result
default	$\frac{-\frac{20\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,F\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-8i\left(\sin^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{4\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{10\sqrt{2(\mathrm{si}}}{}}{\left(2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)a^2\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)e+e}\,e^4d}$

```
[In] int(1/(e*cos(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/(2*sin(1/2*d*x+1/2*c)^2-1)/a^2/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c
)^2*e+e)^(1/2)/e^4*(-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-12*I*
sin(1/2*d*x+1/2*c)^3+2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+5*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1
/2*c)^2)^(1/2)+6*I*sin(1/2*d*x+1/2*c))/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.21

$$\int \frac{1}{(e\cos(c+dx))^{9/2}(a+ia\tan(c+dx))^2}\,dx = -\frac{2\left(2\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\left(5i\,e^{(3i\,dx+3i\,c)}+7i\,e^{(i\,dx+i\,c)}\right)e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}+5\left(i\sqrt{2}e^{(4i\,dx+4i\,c)}+2i\sqrt{2}e^{(2i\,dx+2i\,c)}+\right.}{3\left(a^2de^5e^{(4i\,dx+4i\,c)}+2\,a^2de^5e^{(2i\,dx+2i\,c)}+a^2de^5\right)}$$

```
[In] integrate(1/(e*cos(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas"
)
```

```
[Out] -2/3*(2*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*(5*I*e^(3*I*d*x + 3*I*c)
+ 7*I*e^(I*d*x + I*c))*e^(-1/2*I*d*x - 1/2*I*c) + 5*(I*sqrt(2)*e^(4*I*d*x +
 4*I*c) + 2*I*sqrt(2)*e^(2*I*d*x + 2*I*c) + I*sqrt(2))*sqrt(e)*weierstrassP
Inverse(-4, 0, e^(I*d*x + I*c)))/(a^2*d*e^5*e^(4*I*d*x + 4*I*c) + 2*a^2*d*e
^5*e^(2*I*d*x + 2*I*c) + a^2*d*e^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{9/2}(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*cos(d*x+c))**(9/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\cos(c+dx))^{9/2}(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/(e*cos(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{1}{(e\cos(c+dx))^{9/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{(e\cos(dx+c))^{\frac{9}{2}}\,(ia\tan(dx+c)+a)^2}\,dx$$

```
[In] integrate(1/(e*cos(d*x+c))^(9/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
[Out] integrate(1/((e*cos(d*x + c))^(9/2)*(I*a*tan(d*x + c) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{9/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{(e\,\cos(c+d\,x))^{9/2}\,(a+a\tan(c+d\,x)\;1\mathrm{i})^2}\,dx$$

```
[In] int(1/((e*cos(c + d*x))^(9/2)*(a + a*tan(c + d*x)*1i)^2),x)
[Out] int(1/((e*cos(c + d*x))^(9/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

3.672 $\int \frac{1}{(e\cos(c+dx))^{11/2}(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 164

$$\int \frac{1}{(e\cos(c+dx))^{11/2}(a+ia\tan(c+dx))^2}\,dx = -\frac{14\cos^{\frac{11}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d(e\cos(c+dx))^{11/2}}+\frac{14\cos^3(c+dx)\sin(c+dx)}{15a^2d(e\cos(c+dx))^{11/2}}+\frac{14\cos^5(c+dx)\sin(c+dx)}{5a^2d(e\cos(c+dx))^{11/2}}-\frac{4i\cos^2(c+dx)}{3d(e\cos(c+dx))^{11/2}\left(a^2+ia^2\tan(c+dx)\right)}$$

```
[Out] -14/5*cos(d*x+c)^(11/2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d/(e*cos(d*x+c))^(11/2)+14/15*cos(d*x+c)^3*sin(d*x+c)/a^2/d/(e*cos(d*x+c))^(11/2)+14/5*cos(d*x+c)^5*sin(d*x+c)/a^2/d/(e*cos(d*x+c))^(11/2)-4/3*I*cos(d*x+c)^2/d/(e*cos(d*x+c))^(11/2)/(a^2+I*a^2*tan(d*x+c))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3596, 3581, 3853, 3856, 2719}

$$\int \frac{1}{(e\cos(c+dx))^{11/2}(a+ia\tan(c+dx))^2}\,dx = -\frac{14\cos^{\frac{11}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d(e\cos(c+dx))^{11/2}}+\frac{14\sin(c+dx)\cos^5(c+dx)}{5a^2d(e\cos(c+dx))^{11/2}}+\frac{14\sin(c+dx)\cos^3(c+dx)}{15a^2d(e\cos(c+dx))^{11/2}}-\frac{4i\cos^2(c+dx)}{3d\left(a^2+ia^2\tan(c+dx)\right)(e\cos(c+dx))^{11/2}}$$

```
[In] Int[1/((e*Cos[c + d*x])^(11/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (-14*Cos[c + d*x]^(11/2)*EllipticE[(c + d*x)/2, 2])/(5*a^2*d*(e*Cos[c + d*x
])^(11/2)) + (14*Cos[c + d*x]^3*Sin[c + d*x])/(15*a^2*d*(e*Cos[c + d*x])^(1
1/2)) + (14*Cos[c + d*x]^5*Sin[c + d*x])/(5*a^2*d*(e*Cos[c + d*x])^(11/2))
- (((4*I)/3)*Cos[c + d*x]^2)/(d*(e*Cos[c + d*x])^(11/2)*(a^2 + I*a^2*Tan[c
+ d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3581

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[2*d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
 f*x])^(n + 1)/(b*f*(m + 2*n))), x] - Dist[d^2*((m - 2)/(b^2*(m + 2*n))), I
nt[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{
a, b, d, e, f, m}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, -1] && ((ILtQ[n/2, 0]
&& IGtQ[m - 1/2, 0]) || EqQ[n, -2] || IGtQ[m + n, 0] || (IntegersQ[n, m + 1
/2] && GtQ[2*m + n + 1, 0])) && IntegerQ[2*m]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{(e\sec(c+dx))^{11/2}}{(a+ia\tan(c+dx))^2}\,dx}{(e\cos(c+dx))^{11/2}(e\sec(c+dx))^{11/2}}$$

$$= -\frac{4i\cos^2(c+dx)}{3d(e\cos(c+dx))^{11/2}\left(a^2+ia^2\tan(c+dx)\right)} + \frac{(7e^2)\int (e\sec(c+dx))^{7/2}\,dx}{3a^2(e\cos(c+dx))^{11/2}(e\sec(c+dx))^{11/2}}$$

$$= \frac{14\cos^3(c+dx)\sin(c+dx)}{15a^2d(e\cos(c+dx))^{11/2}} - \frac{4i\cos^2(c+dx)}{3d(e\cos(c+dx))^{11/2}\left(a^2+ia^2\tan(c+dx)\right)} + \frac{(7e^4)\int (e\sec(c+dx))^{3/2}\,dx}{5a^2(e\cos(c+dx))^{11/2}(e\sec(c+dx))^{11/2}}$$

$$= \frac{14\cos^3(c+dx)\sin(c+dx)}{15a^2d(e\cos(c+dx))^{11/2}} + \frac{14\cos^5(c+dx)\sin(c+dx)}{5a^2d(e\cos(c+dx))^{11/2}} - \frac{4i\cos^2(c+dx)}{3d(e\cos(c+dx))^{11/2}\left(a^2+ia^2\tan(c+dx)\right)} - \frac{(7e^6)\int \frac{1}{\sqrt{e\sec(c+dx)}}\,dx}{5a^2(e\cos(c+dx))^{11/2}(e\sec(c+dx))^{11/2}}$$

$$= \frac{14\cos^3(c+dx)\sin(c+dx)}{15a^2d(e\cos(c+dx))^{11/2}} + \frac{14\cos^5(c+dx)\sin(c+dx)}{5a^2d(e\cos(c+dx))^{11/2}} - \frac{4i\cos^2(c+dx)}{3d(e\cos(c+dx))^{11/2}\left(a^2+ia^2\tan(c+dx)\right)} - \frac{\left(7\cos^{\frac{11}{2}}(c+dx)\right)\int \sqrt{\cos(c+dx)}\,dx}{5a^2(e\cos(c+dx))^{11/2}}$$

$$= -\frac{14\cos^{\frac{11}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d(e\cos(c+dx))^{11/2}} + \frac{14\cos^3(c+dx)\sin(c+dx)}{15a^2d(e\cos(c+dx))^{11/2}} + \frac{14\cos^5(c+dx)\sin(c+dx)}{5a^2d(e\cos(c+dx))^{11/2}} - \frac{4i\cos^2(c+dx)}{3d(e\cos(c+dx))^{11/2}\left(a^2+ia^2\tan(c+dx)\right)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.21 (sec) , antiderivative size = 414, normalized size of antiderivative = 2.52

$$\int \frac{1}{(e\cos(c+dx))^{11/2}(a+ia\tan(c+dx))^2}\,dx = \frac{\cos^3(c+dx)(\cos(dx)+i\sin(dx))^2\left(7\cos(c)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\mathrm{c}\right.\right.}{}$$

```
[In] Integrate[1/((e*Cos[c + d*x])^(11/2)*(a + I*a*Tan[c + d*x])^2),x]
```

```
[Out] (Cos[c + d*x]^3*(Cos[d*x] + I*Sin[d*x])^2*(7*Cos[c]*HypergeometricPFQ[{-1/2
, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]] - (7
*(3*Cos[c - d*x - ArcTan[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Cot[c]*S
qrt[Sin[d*x + ArcTan[Tan[c]]]^2])/2 + (Csc[c]*Sqrt[Sec[c]^2]*Sec[c + d*x]^2
*(Cos[2*c] + I*Sin[2*c])*(36*Cos[d*x] + 27*Cos[2*c + d*x] + 21*Cos[2*c + 3*
d*x] + (20*I)*Sin[d*x] - (20*I)*Sin[2*c + d*x])*Sqrt[Sin[d*x + ArcTan[Tan[c
```

```
]]]^2])/6 + (7*I)*(2*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTa
n[Tan[c]]]^2]*Sin[c]*Sin[d*x + ArcTan[Tan[c]]] - (3*Cos[c - d*x - ArcTan[Ta
n[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])
+ (7*(-2*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2
]*Sin[c]*Sin[d*x + ArcTan[Tan[c]]] + (3*Cos[c - d*x - ArcTan[Tan[c]]] + Cos
[c + d*x + ArcTan[Tan[c]]])*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])*Tan[c])/2))/
(5*d*(e*Cos[c + d*x])^(11/2)*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^
2]*(a + I*a*Tan[c + d*x])^2)
```

Maple [A] (verified)

Time = 7.55 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.96

method	result
default	$\frac{\frac{112\left(\sin^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)}{5}-\frac{56E\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{2\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{112\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right.}{5}}{\left(4\left(\sin^{4}\left(\frac{c}{\right.\right.\right.}}$

```
[In] int(1/(e*cos(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/a^2/sin(1/2*d*x+1/2*
c)/(-2*sin(1/2*d*x+1/2*c)^2*e+e)^(1/2)/e^5*(168*sin(1/2*d*x+1/2*c)^6*cos(1/
2*d*x+1/2*c)-84*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-168*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+84*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*
c)^2+20*I*sin(1/2*d*x+1/2*c)^3+36*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2
1*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)-10*I*sin(1/2*d*x+1/2*c))/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.21

$$\int \frac{1}{(e\cos(c+dx))^{11/2}(a+ia\tan(c+dx))^2}\,dx =$$

$$-\frac{2\left(2\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\left(21i\,e^{(6i\,dx+6i\,c)}+56i\,e^{(4i\,dx+4i\,c)}+47i\,e^{(2i\,dx+2i\,c)}\right)e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}+21\left(i\sqrt{2}e^{(6i\,dx+6i\right.}\right.}{15\left(a^2de^6e^{(6i\,dx+6i\,c)}+3\,a^2de^6e^{(\right.}}$$

```
[In] integrate(1/(e*cos(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -2/15*(2*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*(21*I*e^(6*I*d*x + 6*I*c
) + 56*I*e^(4*I*d*x + 4*I*c) + 47*I*e^(2*I*d*x + 2*I*c))*e^(-1/2*I*d*x - 1/
2*I*c) + 21*(I*sqrt(2)*e^(6*I*d*x + 6*I*c) + 3*I*sqrt(2)*e^(4*I*d*x + 4*I*c
) + 3*I*sqrt(2)*e^(2*I*d*x + 2*I*c) + I*sqrt(2))*sqrt(e)*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, e^(I*d*x + I*c))))/(a^2*d*e^6*e^(6*I*d*x +
6*I*c) + 3*a^2*d*e^6*e^(4*I*d*x + 4*I*c) + 3*a^2*d*e^6*e^(2*I*d*x + 2*I*c)
+ a^2*d*e^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{11/2}(a+ia\tan(c+dx))^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*cos(d*x+c))**(11/2)/(a+I*a*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(e\cos(c+dx))^{11/2}(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/(e*cos(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{1}{(e\cos(c+dx))^{11/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{11}{2}}\left(i\,a\tan\left(dx+c\right)+a\right)^{2}}\,dx$$

```
[In] integrate(1/(e*cos(d*x+c))^(11/2)/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(11/2)*(I*a*tan(d*x + c) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{11/2}(a+ia\tan(c+dx))^2}\,dx = \int \frac{1}{\left(e\;\cos\left(c+d\,x\right)\right)^{11/2}\,{\left(a+a\tan\left(c+d\,x\right)\;1\mathrm{i}\right)}^2}\,dx$$

```
[In] int(1/((e*cos(c + d*x))^(11/2)*(a + a*tan(c + d*x)*1i)^2),x)
[Out] int(1/((e*cos(c + d*x))^(11/2)*(a + a*tan(c + d*x)*1i)^2), x)
```

3.673 $\int (e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 179

$$\int (e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{12ia(e\cos(c+dx))^{7/2}\sec^2(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}} + \frac{32ia(e\cos(c+dx))^{7/2}\sec^4(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}} - \frac{2i(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}{7d} - \frac{16i(e\cos(c+dx))^{7/2}\sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{35d}$$

```
[Out] 12/35*I*a*(e*cos(d*x+c))^(7/2)*sec(d*x+c)^2/d/(a+I*a*tan(d*x+c))^(1/2)+32/3
5*I*a*(e*cos(d*x+c))^(7/2)*sec(d*x+c)^4/d/(a+I*a*tan(d*x+c))^(1/2)-2/7*I*(e
*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^(1/2)/d-16/35*I*(e*cos(d*x+c))^(7/2)*
sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3596, 3578, 3583, 3569}

$$\int (e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{2i\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{7/2}}{7d} + \frac{32ia\sec^4(c+dx)(e\cos(c+dx))^{7/2}}{35d\sqrt{a+ia\tan(c+dx)}} - \frac{16i\sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{7/2}}{35d} + \frac{12ia\sec^2(c+dx)(e\cos(c+dx))^{7/2}}{35d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[(e*Cos[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((12*I)/35)*a*(e*Cos[c + d*x])^(7/2)*Sec[c + d*x]^2)/(d*Sqrt[a + I*a*Tan[c
 + d*x]]) + (((32*I)/35)*a*(e*Cos[c + d*x])^(7/2)*Sec[c + d*x]^4)/(d*Sqrt[a
 + I*a*Tan[c + d*x]]) - (((2*I)/7)*(e*Cos[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[
c + d*x]])/d - (((16*I)/35)*(e*Cos[c + d*x])^(7/2)*Sec[c + d*x]^2*Sqrt[a +
I*a*Tan[c + d*x]])/d
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left((e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\right)\int\frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{7/2}}\,dx\\ &= -\frac{2i(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}{7d}\\ &\quad+\frac{\left(6a(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\right)\int\frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx}{7e^2}\end{aligned}$$

$$= \frac{12ia(e\cos(c+dx))^{7/2}\sec^2(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}} - \frac{2i(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}{7d} + \frac{\left(24(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\right)\int\frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}dx}{35e^2}$$

$$= \frac{12ia(e\cos(c+dx))^{7/2}\sec^2(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}} - \frac{2i(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}{7d} - \frac{16i(e\cos(c+dx))^{7/2}\sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{35d} + \frac{\left(16a(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\right)\int\frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}dx}{35e^4}$$

$$= \frac{12ia(e\cos(c+dx))^{7/2}\sec^2(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}} + \frac{32ia(e\cos(c+dx))^{7/2}\sec^4(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}} - \frac{2i(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}{7d} - \frac{16i(e\cos(c+dx))^{7/2}\sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{35d}$$

Mathematica [A] (verified)

Time = 2.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.45

$$\int(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{e^3\sqrt{e\cos(c+dx)}(35i\cos(c+dx)+i\cos(3(c+dx))+70\sin(c+dx)+6\text{s}}{70d}$$

```
[In] Integrate[(e*Cos[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] (e^3*Sqrt[e*Cos[c + d*x]]*((35*I)*Cos[c + d*x] + I*Cos[3*(c + d*x)] + 70*Si
n[c + d*x] + 6*Sin[3*(c + d*x)])*Sqrt[a + I*a*Tan[c + d*x]])/(70*d)
```

Maple [A] (verified)

Time = 8.88 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.44

method	result	size
default	$\frac{2i\sqrt{e\cos(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}\,e^3\left(-6i(\cos^2(dx+c))\sin(dx+c)+\cos^3(dx+c)-16i\sin(dx+c)+8\cos(dx+c)\right)}{35d}$	78

```
[In] int((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/35*I/d*(e*cos(d*x+c))^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)*e^3*(-6*I*cos(d*x+
c)^2*sin(d*x+c)+cos(d*x+c)^3-16*I*sin(d*x+c)+8*cos(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.56

$$\int (e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{\sqrt{2}\sqrt{\frac{1}{2}}\left(-5i\,e^3e^{(6i\,dx+6i\,c)}-35i\,e^3e^{(4i\,dx+4i\,c)}+105i\,e^3e^{(2i\,dx+2i\,c)}+7i\,e^3\right)\sqrt{}}{140\,d}$$

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/140*sqrt(2)*sqrt(1/2)*(-5*I*e^3*e^(6*I*d*x + 6*I*c) - 35*I*e^3*e^(4*I*d*x
 + 4*I*c) + 105*I*e^3*e^(2*I*d*x + 2*I*c) + 7*I*e^3)*sqrt(e*e^(2*I*d*x + 2*
I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-5/2*I*d*x - 5/2*I*c)/d
```

Sympy [F(-1)]

Timed out.

$$\int (e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((e*cos(d*x+c))**(7/2)*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.43 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.13

$$\int (e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{\left(7i\,e^3\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)-5i\,e^3\cos\left(\frac{7}{5}\,\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\left(\frac{5}{2}\,dx+\right.\right.\right.\right.}{}$$

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] 1/140*(7*I*e^3*cos(5/2*d*x + 5/2*c) - 5*I*e^3*cos(7/5*arctan2(sin(5/2*d*x +
 5/2*c), cos(5/2*d*x + 5/2*c))) - 35*I*e^3*cos(3/5*arctan2(sin(5/2*d*x + 5/
2*c), cos(5/2*d*x + 5/2*c))) + 105*I*e^3*cos(1/5*arctan2(sin(5/2*d*x + 5/2*
c), cos(5/2*d*x + 5/2*c))) + 7*e^3*sin(5/2*d*x + 5/2*c) + 5*e^3*sin(7/5*arc
tan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 35*e^3*sin(3/5*arctan2(
sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 105*e^3*sin(1/5*arctan2(sin(
5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*sqrt(a)*sqrt(e)/d
```

Giac [F]

$$\int (e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}\,dx=\int (e\cos(dx+c))^{\frac{7}{2}}\sqrt{i\,a\tan(dx+c)+a}\,dx$$

```
[In] integrate((e*cos(d*x+c))^(7/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((e*cos(d*x + c))^(7/2)*sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [B] (verification not implemented)

Time = 6.15 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.54

$$\int (e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}\,dx=\frac{e^3\sqrt{e\cos(c+d\,x)}\sqrt{\frac{a\,(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1i)}{\cos(2\,c+2\,d\,x)+1}}\left(\sin(c+d\,x)+\frac{3\sin(3\,c+\dots}{35}\right.}{d}$$

```
[In] int((e*cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] (e^3*(e*cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1
))/(cos(2*c + 2*d*x) + 1))^(1/2)*((cos(c + d*x)*1i)/2 + sin(c + d*x) + (cos
(3*c + 3*d*x)*1i)/70 + (3*sin(3*c + 3*d*x))/35))/d
```

3.674 $\int (e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 132

$$\int (e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{8ia(e\cos(c+dx))^{5/2}\sec^2(c+dx)}{15d\sqrt{a+ia\tan(c+dx)}} - \frac{2i(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}{5d} - \frac{16i(e\cos(c+dx))^{5/2}\sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{15d}$$

```
[Out] 8/15*I*a*(e*cos(d*x+c))^(5/2)*sec(d*x+c)^2/d/(a+I*a*tan(d*x+c))^(1/2)-2/5*I
*(e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(1/2)/d-16/15*I*(e*cos(d*x+c))^(5/
2)*sec(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3596, 3578, 3583, 3569}

$$\int (e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{2i\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{5/2}}{5d} - \frac{16i\sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{5/2}}{15d} + \frac{8ia\sec^2(c+dx)(e\cos(c+dx))^{5/2}}{15d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[(e*Cos[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((8*I)/15)*a*(e*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2)/(d*Sqrt[a + I*a*Tan[c
+ d*x]]) - (((2*I)/5)*(e*Cos[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]])/d
- (((16*I)/15)*(e*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2*Sqrt[a + I*a*Tan[c + d
*x]])/d
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left((e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\right)\int\frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx\\
&= -\frac{2i(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}{5d}\\
&\quad+\frac{\left(4a(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\right)\int\frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx}{5e^2}
\end{aligned}$$

$$= \frac{8ia(e\cos(c+dx))^{5/2}\sec^2(c+dx)}{15d\sqrt{a+ia\tan(c+dx)}} - \frac{2i(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}{5d} + \frac{\left(8(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\right)\int\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}dx}{15e^2}$$

$$= \frac{8ia(e\cos(c+dx))^{5/2}\sec^2(c+dx)}{15d\sqrt{a+ia\tan(c+dx)}} - \frac{2i(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}{5d} - \frac{16i(e\cos(c+dx))^{5/2}\sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{15d}$$

Mathematica [A] (verified)

Time = 1.71 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.48

$$\int (e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{ie^2\sqrt{e\cos(c+dx)}(-15+\cos(2(c+dx))-4i\sin(2(c+dx)))\sqrt{a+ia\tan(c+dx)}}{15d}$$

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] ((I/15)*e^2*Sqrt[e*Cos[c + d*x]]*(-15 + Cos[2*(c + d*x)] - (4*I)*Sin[2*(c + d*x)])*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Maple [A] (verified)

Time = 8.99 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.47

method	result	size
default	$\frac{2\left(i\left(\cos^2(dx+c)\right)+4\sin(dx+c)\cos(dx+c)-8i\right)\sqrt{e\cos(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}\,e^2}{15d}$	62
risch	$-\frac{ie^2\sqrt{2}\,\sqrt{e\cos(dx+c)}\,\sqrt{\frac{a\,e^{2i(dx+c)}}{e^{2i(dx+c)}+1}}\,(30-2\cos(2dx+2c)+8i\sin(2dx+2c))}{30d}$	74

```
[In] int((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/15/d*(I*cos(d*x+c)^2+4*sin(d*x+c)*cos(d*x+c)-8*I)*(e*cos(d*x+c))^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)*e^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.65

$$\int (e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{\sqrt{2}\sqrt{\frac{1}{2}}\left(-3i\,e^2e^{(4i\,dx+4i\,c)}-30i\,e^2e^{(2i\,dx+2i\,c)}+5i\,e^2\right)\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\,e^{\left(-\frac{3}{2}i\,dx-\frac{3}{2}i\,c\right)}}{30\,d}$$

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/30*sqrt(2)*sqrt(1/2)*(-3*I*e^2*e^(4*I*d*x + 4*I*c) - 30*I*e^2*e^(2*I*d*x
+ 2*I*c) + 5*I*e^2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*
I*c) + 1))*e^(-3/2*I*d*x - 3/2*I*c)/d
```

Sympy [F(-1)]

Timed out.

$$\int (e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((e*cos(d*x+c))**(5/2)*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.12

$$\int (e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{\left(5i\,e^2\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)-3i\,e^2\cos\left(\frac{5}{3}\,\arctan\left(\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right),\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\right)\right) \cdots}{\cdots}$$

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] 1/30*(5*I*e^2*cos(3/2*d*x + 3/2*c) - 3*I*e^2*cos(5/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) - 30*I*e^2*cos(1/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) + 5*e^2*sin(3/2*d*x + 3/2*c) + 3*e^2*sin(5/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 30*e^2*sin(1/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sqrt(a)*sqrt(e)/d
```

Giac [F]

$$\int (e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}\,dx = \int (e\cos(dx+c))^{\frac{5}{2}}\sqrt{ia\tan(dx+c)+a}\,dx$$

```
[In] integrate((e*cos(d*x+c))^(5/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)*sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.64

$$\int (e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{e^2\sqrt{e\cos(c+dx)}\sqrt{\frac{a(\cos(2c+2dx)+1+\sin(2c+2dx)\,1i)}{\cos(2c+2dx)+1}}\left(\cos(2c+2dx)\,1i+\right.}{15d}$$

```
[In] int((e*cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] (e^2*(e*cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1
))/(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(2*c + 2*d*x)*1i + 4*sin(2*c + 2*d*x)
- 15i))/(15*d)
```

3.675 $\int (e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 85

$$\int (e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{4iae\sqrt{e\cos(c+dx)}\sec(c+dx)}{3d\sqrt{a+ia\tan(c+dx)}} - \frac{2i(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{3d}$$

```
[Out] 4/3*I*a*e*sec(d*x+c)*(e*cos(d*x+c))^(1/2)/d/(a+I*a*tan(d*x+c))^(1/2)-2/3*I*
(e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3596, 3578, 3569}

$$\int (e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{4ia\sec^2(c+dx)(e\cos(c+dx))^{3/2}}{3d\sqrt{a+ia\tan(c+dx)}} - \frac{2i\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{3/2}}{3d}$$

```
[In] Int[(e*Cos[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((4*I)/3)*a*(e*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2)/(d*Sqrt[a + I*a*Tan[c +
 d*x]]) - (((2*I)/3)*(e*Cos[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
```

```
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left((e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\right)\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx \\
&= -\frac{2i(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{3d} \\
&\quad+\frac{\left(2a(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\right)\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{3e^2} \\
&= \frac{4ia(e\cos(c+dx))^{3/2}\sec^2(c+dx)}{3d\sqrt{a+ia\tan(c+dx)}}-\frac{2i(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.66

$$\int (e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx=\frac{2e\sqrt{e\cos(c+dx)}(i\cos(c+dx)+2\sin(c+dx))\sqrt{a+ia\tan(c+dx)}}{3d}$$

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] (2*e*Sqrt[e*Cos[c + d*x]]*(I*Cos[c + d*x] + 2*Sin[c + d*x])*Sqrt[a + I*a*Ta
n[c + d*x]])/(3*d)
```

Maple [A] (verified)

Time = 8.58 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.59

method	result	size
default	$\frac{2(i\cos(dx+c)+2\sin(dx+c))\sqrt{e\cos(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}\,e}{3d}$	50
risch	$-\frac{ie\sqrt{2}\,\sqrt{e\cos(dx+c)}\,\sqrt{\frac{a\,e^{2i(dx+c)}}{e^{2i(dx+c)}+1}}\,(-2\cos(dx+c)+4i\sin(dx+c))}{3d}$	65

```
[In] int((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/d*(I*cos(d*x+c)+2*sin(d*x+c))*(e*cos(d*x+c))^(1/2)*(a*(1+I*tan(d*x+c)))
^(1/2)*e
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.80

$$\int (e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\left(-i\,ee^{(2i\,dx+2i\,c)}+3i\,e\right)\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{3\,d}$$

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/3*sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*(-I*e*e^(2*I*d*x + 2*
I*c) + 3*I*e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/d
```

Sympy [F(-1)]

Timed out.

$$\int (e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx = \text{Timed out}$$

```
[In] integrate((e*cos(d*x+c))**(3/2)*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.73 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.69

$$\int (e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{\left(-i\,e\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3i\,e\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+e\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3\,e\sin\ldots}{3\,d}$$

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] 1/3*(-I*e*cos(3/2*d*x + 3/2*c) + 3*I*e*cos(1/2*d*x + 1/2*c) + e*sin(3/2*d*x
 + 3/2*c) + 3*e*sin(1/2*d*x + 1/2*c))*sqrt(a)*sqrt(e)/d
```

Giac [F]

$$\int (e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx = \int (e\cos(dx+c))^{\frac{3}{2}}\sqrt{i\,a\tan(dx+c)+a}\,dx$$

```
[In] integrate((e*cos(d*x+c))^(3/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((e*cos(d*x + c))^(3/2)*sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.04

$$\int (e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx = \frac{2\,e\,\sqrt{e\,\left(2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,2\mathrm{i}+2\,\sin\left(c+d\,x\right)-\mathrm{i}\right)\,\sqrt{\ldots}}{3\,d}$$

```
[In] int((e*cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] (2*e*(e*(2*cos(c/2 + (d*x)/2)^2 - 1))^(1/2)*(2*sin(c + d*x) + cos(c/2 + (d*
x)/2)^2*2i - 1i)*((a*(sin(2*c + 2*d*x)*1i + 2*cos(c + d*x)^2))/(2*cos(c + d
*x)^2))^(1/2))/(3*d)
```

3.676 $\int \sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 36

$$\int \sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx=-\frac{2i\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}$$

```
[Out] -2*I*(e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3596, 3569}

$$\int \sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx=-\frac{2i\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{d}$$

```
[In] Int[Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-2*I)*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
```

```
+ b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx\\ &= -\frac{2i\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{2i\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}{d}$$

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] ((-2*I)*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/d
```

Maple [A] (verified)

Time = 7.31 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{2i\sqrt{e\cos(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}}{d}$	32
risch	$-\frac{2i\sqrt{2}\,\sqrt{e\cos(dx+c)}\,\sqrt{\frac{a\,\mathrm{e}^{2i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}}{d}$	46

```
[In] int((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*I/d*(e*cos(d*x+c))^(1/2)*(a*(1+I*tan(d*x+c)))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.44

$$\int \sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{2i\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{d}$$

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] -2*I*sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c)/d
```

Sympy [F]

$$\int \sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{e\cos\left(c+dx\right)}\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}\,dx$$

```
[In] integrate((e*cos(d*x+c))**(1/2)*(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(e*cos(c + d*x))*sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(28) = 56.

Time = 0.50 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.11

$$\int \sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx = -\frac{2i\sqrt{a}\sqrt{e}\sqrt{-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}}{d\sqrt{-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}}$$

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] -2*I*sqrt(a)*sqrt(e)*sqrt(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 - 1)/(d*sqrt(-sin(d*x + c)^2/(cos(d*x + c) + 1)^2
 - 1))
```

Giac [F]

$$\int \sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{e\cos(dx+c)}\sqrt{i\,a\tan(dx+c)+a}\,dx$$

```
[In] integrate((e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(e*cos(d*x + c))*sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{e\,\cos(c+d\,x)}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}\,dx$$

```
[In] int((e*cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] int((e*cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.677 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 335

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx = \frac{i\sqrt{2}\sqrt{a}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{d\sqrt{e}} - \frac{i\sqrt{2}\sqrt{a}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{d\sqrt{e}} - \frac{i\sqrt{a}\log\left(a\sqrt{e}-\sqrt{2}\sqrt{a}\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}+\sqrt{e}\cos(c+dx)(a+ia\tan(c+dx))\right)}{\sqrt{2}d\sqrt{e}} + \frac{i\sqrt{a}\log\left(a\sqrt{e}+\sqrt{2}\sqrt{a}\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}+\sqrt{e}\cos(c+dx)(a+ia\tan(c+dx))\right)}{\sqrt{2}d\sqrt{e}}$$

```
[Out] -1/2*I*ln(a*e^(1/2)-2^(1/2)*a^(1/2)*(e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))
^(1/2)+cos(d*x+c)*e^(1/2)*(a+I*a*tan(d*x+c)))*a^(1/2)/d*2^(1/2)/e^(1/2)+1/2
*I*ln(a*e^(1/2)+2^(1/2)*a^(1/2)*(e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/
2)+cos(d*x+c)*e^(1/2)*(a+I*a*tan(d*x+c)))*a^(1/2)/d*2^(1/2)/e^(1/2)+I*arcta
n(1-2^(1/2)*(e*cos(d*x+c))^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/e^(1/2))*
2^(1/2)*a^(1/2)/d/e^(1/2)-I*arctan(1+2^(1/2)*(e*cos(d*x+c))^(1/2)*(a+I*a*ta
n(d*x+c))^(1/2)/a^(1/2)/e^(1/2))*2^(1/2)*a^(1/2)/d/e^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {3594, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx = \frac{i\sqrt{2}\sqrt{a}\arctan\left(1-\frac{\sqrt{2}\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{d\sqrt{e}} - \frac{i\sqrt{2}\sqrt{a}\arctan\left(1+\frac{\sqrt{2}\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{d\sqrt{e}} - \frac{i\sqrt{a}\log\left(-\sqrt{2}\sqrt{a}\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}+\sqrt{e}\cos(c+dx)(a+ia\tan(c+dx))+a\sqrt{e}\right)}{\sqrt{2}d\sqrt{e}} + \frac{i\sqrt{a}\log\left(\sqrt{2}\sqrt{a}\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}+\sqrt{e}\cos(c+dx)(a+ia\tan(c+dx))+a\sqrt{e}\right)}{\sqrt{2}d\sqrt{e}}$$

```
[In] Int[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (I*Sqrt[2]*Sqrt[a]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Ta
n[c + d*x]])/(Sqrt[a]*Sqrt[e])])/(d*Sqrt[e]) - (I*Sqrt[2]*Sqrt[a]*ArcTan[1
+ (Sqrt[2]*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e
])])/(d*Sqrt[e]) - (I*Sqrt[a]*Log[a*Sqrt[e] - Sqrt[2]*Sqrt[a]*Sqrt[e*Cos[c
+ d*x]]*Sqrt[a + I*a*Tan[c + d*x]] + Sqrt[e]*Cos[c + d*x]*(a + I*a*Tan[c +
d*x])])/(Sqrt[2]*d*Sqrt[e]) + (I*Sqrt[a]*Log[a*Sqrt[e] + Sqrt[2]*Sqrt[a]*Sq
rt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]] + Sqrt[e]*Cos[c + d*x]*(a + I
*a*Tan[c + d*x])])/(Sqrt[2]*d*Sqrt[e])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3594

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[-4*(b/f), Subst[Int[x^2/(a^2*d^2 + x^4), x], x,
Sqrt[d*Cos[e + f*x]]*Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}
, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(4ia)\text{Subst}\left(\int \frac{x^2}{a^2e^2+x^4}\,dx, x, \sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\right)}{d}\\ &= \frac{(2ia)\text{Subst}\left(\int \frac{ae-x^2}{a^2e^2+x^4}\,dx, x, \sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\right)}{d}\\ &\quad-\frac{(2ia)\text{Subst}\left(\int \frac{ae+x^2}{a^2e^2+x^4}\,dx, x, \sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\right)}{d}\end{aligned}$$

$$= -\frac{(ia)\text{Subst}\left(\int \frac{1}{ae-\sqrt{2}\sqrt{a}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\right)}{d}$$
$$-\frac{(ia)\text{Subst}\left(\int \frac{1}{ae+\sqrt{2}\sqrt{a}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\right)}{d}$$
$$-\frac{\left(i\sqrt{a}\right)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{a}\sqrt{e}+2x}{-ae-\sqrt{2}\sqrt{a}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\right)}{\sqrt{2}d\sqrt{e}}$$
$$-\frac{\left(i\sqrt{a}\right)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{a}\sqrt{e}-2x}{-ae+\sqrt{2}\sqrt{a}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}\right)}{\sqrt{2}d\sqrt{e}}$$

$$=$$
$$-\frac{i\sqrt{a}\log\left(a\sqrt{e}-\sqrt{2}\sqrt{a}\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}+\sqrt{e}\cos(c+dx)(a+ia\tan(c+dx\right.}{\sqrt{2}d\sqrt{e}}$$
$$+\frac{i\sqrt{a}\log\left(a\sqrt{e}+\sqrt{2}\sqrt{a}\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}+\sqrt{e}\cos(c+dx)(a+ia\tan(c+dx\right.}{\sqrt{2}d\sqrt{e}}$$
$$-\frac{\left(i\sqrt{2}\sqrt{a}\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{d\sqrt{e}}$$
$$+\frac{\left(i\sqrt{2}\sqrt{a}\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{d\sqrt{e}}$$

$$=\frac{i\sqrt{2}\sqrt{a}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{d\sqrt{e}}$$
$$-\frac{i\sqrt{2}\sqrt{a}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{d\sqrt{e}}$$
$$-\frac{i\sqrt{a}\log\left(a\sqrt{e}-\sqrt{2}\sqrt{a}\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}+\sqrt{e}\cos(c+dx)(a+ia\tan(c+dx\right.}{\sqrt{2}d\sqrt{e}}$$
$$+\frac{i\sqrt{a}\log\left(a\sqrt{e}+\sqrt{2}\sqrt{a}\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}+\sqrt{e}\cos(c+dx)(a+ia\tan(c+dx\right.}{\sqrt{2}d\sqrt{e}}$$

Mathematica [A] (verified)

Time = 1.79 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx = \frac{ie^{-\frac{3}{2}idx}\left(-e^{-2ic}\right)^{3/4}\left(1+e^{2i(c+dx)}\right)\left(\arctan\left(\frac{e^{\frac{idx}{2}}}{\sqrt[4]{-e^{-2ic}}}\right)-\operatorname{arctanh}\left(\frac{e^{\frac{idx}{2}}}{\sqrt[4]{-e^{-2ic}}}\right)\right)\sqrt{a+ia\tan(c+dx)}}{d\sqrt{e\cos(c+dx)}}$$

```
[In] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/Sqrt[e*Cos[c + d*x]],x]
```

```
[Out] (I*(-E^((-2*I)*c))^(3/4)*(1 + E^((2*I)*(c + d*x)))*(ArcTan[E^((I/2)*d*x)/(-
E^((-2*I)*c))^(1/4)] - ArcTanh[E^((I/2)*d*x)/(-E^((-2*I)*c))^(1/4)])*Sqrt[a
 + I*a*Tan[c + d*x]])/(d*E^(((3*I)/2)*d*x)*Sqrt[e*Cos[c + d*x]])
```

Maple [A] (verified)

Time = 9.28 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.46

method	result	si
default	$\frac{(-1-i)\sqrt{a(1+i\tan(dx+c))}\left(i\operatorname{arctanh}\left(\frac{\cos(dx+c)+\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)+\operatorname{arctanh}\left(\frac{-\cos(dx+c)+\sin(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)\right)\cos(dx+c)}{d(-i\cos(dx+c)+\sin(dx+c)-i)\sqrt{e\cos(dx+c)}\sqrt{\frac{1}{\cos(dx+c)+1}}}$	15

```
[In] int((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1-I)/d*(a*(1+I*tan(d*x+c)))^(1/2)*(I*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1
)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))+arctanh(1/2*(-cos(d*x+c)+sin(d*x
+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2)))*cos(d*x+c)/(-I*cos(d*x+c)+
sin(d*x+c)-I)/(e*cos(d*x+c))^(1/2)/(1/(cos(d*x+c)+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx$$
$$= -\frac{1}{2}\sqrt{\frac{4i\,a}{d^2e}}\log\left(\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}+\frac{1}{2}i\,de\sqrt{\frac{4i\,a}{d^2e}}\right)$$
$$+\frac{1}{2}\sqrt{\frac{4i\,a}{d^2e}}\log\left(\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}-\frac{1}{2}i\,de\sqrt{\frac{4i\,a}{d^2e}}\right)$$
$$-\frac{1}{2}\sqrt{-\frac{4i\,a}{d^2e}}\log\left(\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}+\frac{1}{2}i\,de\sqrt{-\frac{4i\,a}{d^2e}}\right)$$
$$+\frac{1}{2}\sqrt{-\frac{4i\,a}{d^2e}}\log\left(\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}-\frac{1}{2}i\,de\sqrt{-\frac{4i\,a}{d^2e}}\right)$$

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] -1/2*sqrt(4*I*a/(d^2*e))*log(sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) +
 e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c) + 1/2*I*d*e*
sqrt(4*I*a/(d^2*e))) + 1/2*sqrt(4*I*a/(d^2*e))*log(sqrt(2)*sqrt(1/2)*sqrt(e
*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x -
 1/2*I*c) - 1/2*I*d*e*sqrt(4*I*a/(d^2*e))) - 1/2*sqrt(-4*I*a/(d^2*e))*log(s
qrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c
) + 1))*e^(-1/2*I*d*x - 1/2*I*c) + 1/2*I*d*e*sqrt(-4*I*a/(d^2*e))) + 1/2*sq
rt(-4*I*a/(d^2*e))*log(sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c) - 1/2*I*d*e*sqrt(-
4*I*a/(d^2*e)))
```

Sympy [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx = \int \frac{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}{\sqrt{e\cos\left(c+dx\right)}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2)/(e*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))/sqrt(e*cos(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1400 vs. $2(247) = 494$.

Time = 0.50 (sec) , antiderivative size = 1400, normalized size of antiderivative = 4.18

$$\int \frac{\sqrt{a + ia\tan(c + dx)}}{\sqrt{e\cos(c + dx)}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] 1/4*(-2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + 1, sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + 1) - 2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1, -sqrt(2)*sin(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 2*I*sqrt(2)*arctan2(sqrt(2)*
cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 1, sqrt(2)*s
in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 2*I*sqrt
(2)*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) - 1, -sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) + 1) - 2*sqrt(2)*arctan2(sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c))) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))), sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)
 + 2*sqrt(2)*arctan2(-sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))), -sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
+ cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + I*sqr
t(2)*log(2*sqrt(2)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c)))*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*(sqrt
(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(2
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(2/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*cos(1/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c))) + 1) - I*sqrt(2)*log(-2*sqrt(2)*sin(2/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) - 2*(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) - 1)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^
2 + 2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(
2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arct
```

```
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + sqrt(2)*log(2*cos(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - sqrt(2)*log(2*cos(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + sqrt(2)*log(2*cos(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - sqrt(2)*log(2*cos(1/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2))*sqrt(a)/(d*sqrt(e))
```

Giac [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx=\int \frac{\sqrt{i\,a\tan\left(dx+c\right)+a}}{\sqrt{e\cos\left(dx+c\right)}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)/sqrt(e*cos(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx=\int \frac{\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{\sqrt{e\,\cos\left(c+d\,x\right)}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2)/(e*cos(c + d*x))^(1/2),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)^(1/2)/(e*cos(c + d*x))^(1/2), x)
```

3.678 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 524

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx = \frac{ia}{d(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} - \frac{ia^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e}}\right)\sec(c+dx)}{\sqrt{2}de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{ia^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e}}\right)\sec(c+dx)}{\sqrt{2}de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{ia^{3/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{2\sqrt{2}de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{ia^{3/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{2\sqrt{2}de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] I*a/d/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2)-1/2*I*a^(3/2)*arctan(1-
2^(1/2)*(e*cos(d*x+c))^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/e^(1/2))*sec(
d*x+c)/d/e^(3/2)*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+
1/2*I*a^(3/2)*arctan(1+2^(1/2)*(e*cos(d*x+c))^(1/2)*(a-I*a*tan(d*x+c))^(1/2
)/a^(1/2)/e^(1/2))*sec(d*x+c)/d/e^(3/2)*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a
+I*a*tan(d*x+c))^(1/2)+1/4*I*a^(3/2)*ln(a-2^(1/2)*a^(1/2)*(e*cos(d*x+c))^(1
/2)*(a-I*a*tan(d*x+c))^(1/2)/e^(1/2)+cos(d*x+c)*(a-I*a*tan(d*x+c)))*sec(d*x
+c)/d/e^(3/2)*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-1/4
*I*a^(3/2)*ln(a+2^(1/2)*a^(1/2)*(e*cos(d*x+c))^(1/2)*(a-I*a*tan(d*x+c))^(1/
2)/e^(1/2)+cos(d*x+c)*(a-I*a*tan(d*x+c)))*sec(d*x+c)/d/e^(3/2)*2^(1/2)/(a-I
*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 620, normalized size of antiderivative = 1.18, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3596, 3579, 3580, 3576, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx = -\frac{ia^{3/2}e^{3/2}\sec(c+dx)\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}} + \frac{ia^{3/2}e^{3/2}\sec(c+dx)\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}} + \frac{ia^{3/2}e^{3/2}\sec(c+dx)\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{2\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}} - \frac{ia^{3/2}e^{3/2}\sec(c+dx)\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{2\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}} + \frac{ia}{d\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{3/2}}$$

```
[In] Int[Sqrt[a + I*a*Tan[c + d*x]]/(e*Cos[c + d*x])^(3/2),x]
```

```
[Out] (I*a)/(d*(e*Cos[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]) - (I*a^(3/2)*e^
(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt
[e*Sec[c + d*x]])]*Sec[c + d*x])/(Sqrt[2]*d*(e*Cos[c + d*x])^(3/2)*(e*Sec[c
 + d*x])^(3/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + (I*
a^(3/2)*e^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sq
rt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(Sqrt[2]*d*(e*Cos[c + d*x])^(3/2
)*(e*Sec[c + d*x])^(3/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*
x]]) + ((I/2)*a^(3/2)*e^(3/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a
*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]
*Sec[c + d*x])/(Sqrt[2]*d*(e*Cos[c + d*x])^(3/2)*(e*Sec[c + d*x])^(3/2)*Sqr
t[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - ((I/2)*a^(3/2)*e^(3/2
)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c
 + d*x]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/(Sqrt[2]*d*(e
*Cos[c + d*x])^(3/2)*(e*Sec[c + d*x])^(3/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt
[a + I*a*Tan[c + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
```

```
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3580

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Dist[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]])), Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int (e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx}{(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}} \\
&= \frac{ia}{d(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} + \frac{a\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{2(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}} \\
&= \frac{ia}{d(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} \\
&\quad + \frac{(ae\sec(c+dx))\int \sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}\,dx}{2(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&= \frac{ia}{d(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} \\
&\quad + \frac{(2ia^2e^3\sec(c+dx))\,\text{Subst}\left(\int \frac{x^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&= \frac{ia}{d(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} \\
&\quad - \frac{(ia^2e^2\sec(c+dx))\,\text{Subst}\left(\int \frac{a-ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&\quad + \frac{(ia^2e^2\sec(c+dx))\,\text{Subst}\left(\int \frac{a+ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}
\end{aligned}
$$

$$= \frac{ia}{d(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} + \frac{(ia^2 e\sec(c+dx))\,\text{Subst}\left(\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{2d(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{(ia^2 e\sec(c+dx))\,\text{Subst}\left(\int \frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{2d(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{\left(ia^{3/2}e^{3/2}\sec(c+dx)\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}+2x}{-\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{2\sqrt{2}d(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{\left(ia^{3/2}e^{3/2}\sec(c+dx)\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}-2x}{-\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{2\sqrt{2}d(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{ia}{d(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} + \frac{ia^{3/2}e^{3/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{2\sqrt{2}d(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{ia^{3/2}e^{3/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{2\sqrt{2}d(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{\left(ia^{3/2}e^{3/2}\sec(c+dx)\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{\left(ia^{3/2}e^{3/2}\sec(c+dx)\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}d(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{ia}{d(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}} - \frac{ia^{3/2}e^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{\sqrt{2}d(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{ia^{3/2}e^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{\sqrt{2}d(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{ia^{3/2}e^{3/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{2\sqrt{2}d(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{ia^{3/2}e^{3/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{2\sqrt{2}d(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A] (verified)

Time = 5.15 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx = \frac{ie^{-\frac{1}{2}i(c+dx)}\cos^2(c+dx)\left(2\sqrt{2}\cos\left(\frac{1}{2}(c+dx)\right)+2\arctan\left(1-\sqrt{2}e^{\frac{1}{2}i(c+dx)}\right)\cos\right.}{}$$

[In] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/(e*Cos[c + d*x])^(3/2),x]

[Out] (I*Cos[c + d*x]^2*(2*Sqrt[2]*Cos[(c + d*x)/2] + 2*ArcTan[1 - Sqrt[2]*E^((I/2)*(c + d*x))]*Cos[c + d*x] - 2*ArcTan[1 + Sqrt[2]*E^((I/2)*(c + d*x))]*Cos[c + d*x] + Cos[c + d*x]*Log[1 - Sqrt[2]*E^((I/2)*(c + d*x)) + E^(I*(c + d*x))] - Cos[c + d*x]*Log[1 + Sqrt[2]*E^((I/2)*(c + d*x)) + E^(I*(c + d*x))] - (2*I)*Sqrt[2]*Sin[(c + d*x)/2])*(Cos[c + d*x] + I*Sin[c + d*x])*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[2]*d*E^((I/2)*(c + d*x))*(1 + E^((2*I)*(c + d*x)))*(e*Cos[c + d*x])^(3/2))

Maple [A] (verified)

Time = 10.44 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.61

method	result
default	$-\frac{i\sqrt{a(1+i\tan(dx+c))}\left(i\cos(dx+c)\operatorname{arctanh}\left(\frac{\cos(dx+c)-\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)+i\cos(dx+c)\operatorname{arctanh}\left(\frac{\cos(dx+c)+\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}}\right.\right.}{2}$

[In] int((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/2*I/d*(a*(1+I*tan(d*x+c)))^(1/2)*(I*cos(d*x+c)*arctanh(1/2*(cos(d*x+c)-sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))+I*cos(d*x+c)*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))-2*I*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-2*I*(1/(cos(d*x+c)+1))^(1/2)+arctanh(1/2*(cos(d*x+c)-sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)-2*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2))/(I*cos(d*x+c)+I-sin(d*x+c))/(1/(cos(d*x+c)+1))^(1/2)/e/(e*cos(d*x+c))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 470, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx = \frac{4i\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}-\left(de^{2}e^{(2i\,dx+2i\,c)}+de^{2}\right)\sqrt{\frac{i}{d}\cdots}}{\cdots}$$

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(3/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(4*I*sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d
*x + 2*I*c) + 1))*e^(1/2*I*d*x + 1/2*I*c) - (d*e^2*e^(2*I*d*x + 2*I*c) + d*
e^2)*sqrt(I*a/(d^2*e^3))*log(d*e^2*sqrt(I*a/(d^2*e^3)) + sqrt(2)*sqrt(1/2)*
sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I
*d*x - 1/2*I*c)) + (d*e^2*e^(2*I*d*x + 2*I*c) + d*e^2)*sqrt(I*a/(d^2*e^3))*
log(-d*e^2*sqrt(I*a/(d^2*e^3)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*
c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)) + (d*e^
2*e^(2*I*d*x + 2*I*c) + d*e^2)*sqrt(-I*a/(d^2*e^3))*log(d*e^2*sqrt(-I*a/(d^
2*e^3)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)) - (d*e^2*e^(2*I*d*x + 2*I*c) +
 d*e^2)*sqrt(-I*a/(d^2*e^3))*log(-d*e^2*sqrt(-I*a/(d^2*e^3)) + sqrt(2)*sqrt
(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(
-1/2*I*d*x - 1/2*I*c)))/(d*e^2*e^(2*I*d*x + 2*I*c) + d*e^2)
```

Sympy [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx = \int \frac{\sqrt{ia\left(\tan\left(c+dx\right)-i\right)}}{\left(e\cos\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2)/(e*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral(sqrt(I*a*(tan(c + d*x) - I))/(e*cos(c + d*x))**(3/2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1836 vs. 2(396) = 792.

Time = 0.47 (sec) , antiderivative size = 1836, normalized size of antiderivative = 3.50

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(3/2),x, algorithm="maxim
a")
```

```
[Out] -8*(2*(sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arc
tan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, sqrt
(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 2*(sqrt(2)*
cos(2*d*x + 2*c) + I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*co
s(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 2*(sqrt(2)*cos(2*d*x + 2*
c) + I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*cos(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 1) + 2*(sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*
sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 1) - 2*(-I*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)*sin(2*d*x + 2
*c) - I*sqrt(2))*arctan2(sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), sqrt(2)*
cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 2*(I*sqrt(2)*cos(2*d*x + 2*c) - sqr
t(2)*sin(2*d*x + 2*c) + I*sqrt(2))*arctan2(-sqrt(2)*sin(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))), -sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (sqrt(2)*cos(2*
d*x + 2*c) + I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*log(2*sqrt(2)*sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - (
sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*log(-2*sqr
t(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 1) - (-I*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)*sin(2*d*x + 2*c) - I*s
qrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*si
n(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - (I*sqrt(2)*cos(2*d*x + 2*c) - sqrt(
2)*sin(2*d*x + 2*c) + I*sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), co
```

```
s(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqr
t(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - (-I*sqrt(2
)*cos(2*d*x + 2*c) + sqrt(2)*sin(2*d*x + 2*c) - I*sqrt(2))*log(2*cos(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 2) - (I*sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(2*d*x + 2*c) + I*
sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*s
in(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 16*cos(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) - 16*I*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))))*sqrt(a)*sqrt(e)/((-64*I*e^2*cos(2*d*x + 2*c) + 64*e^2*sin(2*d*x + 2
*c) - 64*I*e^2)*d)
```

Giac [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx=\int \frac{\sqrt{i\,a\tan\left(dx+c\right)+a}}{\left(e\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)/(e*cos(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{3/2}}\,dx=\int \frac{\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2)/(e*cos(c + d*x))^(3/2),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)^(1/2)/(e*cos(c + d*x))^(3/2), x)
```

3.679 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 512

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{5/2}}\,dx = \frac{3i\sqrt{a}e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} - \frac{3i\sqrt{a}e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} - \frac{3i\sqrt{a}e^{5/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{8\sqrt{2}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} + \frac{3i\sqrt{a}e^{5/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{8\sqrt{2}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} + \frac{ia}{2d(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}} - \frac{3i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d(e\cos(c+dx))^{5/2}}$$

```
[Out] 3/8*I*e^(5/2)*arctan(1-2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*
sec(d*x+c))^(1/2))*a^(1/2)/d/(e*cos(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2)*2^(1
/2)-3/8*I*e^(5/2)*arctan(1+2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)
/(e*sec(d*x+c))^(1/2))*a^(1/2)/d/(e*cos(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2)*
2^(1/2)-3/16*I*e^(5/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2
)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))*a^(1/2)/d/(e*cos(d*x+
c))^(5/2)/(e*sec(d*x+c))^(5/2)*2^(1/2)+3/16*I*e^(5/2)*ln(a+2^(1/2)*a^(1/2)*
e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a+I*a*tan
(d*x+c)))*a^(1/2)/d/(e*cos(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2)*2^(1/2)+1/2*I
*a/d/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2)-3/4*I*cos(d*x+c)^2*(a+I*
a*tan(d*x+c))^(1/2)/d/(e*cos(d*x+c))^(5/2)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3596, 3579, 3582, 3576, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{5/2}}\,dx = \frac{3i\sqrt{a}e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} - \frac{3i\sqrt{a}e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} - \frac{3i\sqrt{a}e^{5/2}\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{8\sqrt{2}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} + \frac{3i\sqrt{a}e^{5/2}\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{8\sqrt{2}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} - \frac{3i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d(e\cos(c+dx))^{5/2}} + \frac{ia}{2d\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{5/2}}$$

```
[In] Int[Sqrt[a + I*a*Tan[c + d*x]]/(e*Cos[c + d*x])^(5/2),x]
```

```
[Out] (((3*I)/4)*Sqrt[a]*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c +
 d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])])/(Sqrt[2]*d*(e*Cos[c + d*x])^(5/2)*
(e*Sec[c + d*x])^(5/2)) - (((3*I)/4)*Sqrt[a]*e^(5/2)*ArcTan[1 + (Sqrt[2]*Sq
rt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])])/(Sqrt[2]
*d*(e*Cos[c + d*x])^(5/2)*(e*Sec[c + d*x])^(5/2)) - (((3*I)/8)*Sqrt[a]*e^(5
/2)*Log[a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec
[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c + d*x])])/(Sqrt[2]*d*(e*Cos[c + d*
x])^(5/2)*(e*Sec[c + d*x])^(5/2)) + (((3*I)/8)*Sqrt[a]*e^(5/2)*Log[a + (Sqr
t[2]*Sqrt[a]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos
[c + d*x]*(a + I*a*Tan[c + d*x])])/(Sqrt[2]*d*(e*Cos[c + d*x])^(5/2)*(e*Sec
[c + d*x])^(5/2)) + ((I/2)*a)/(d*(e*Cos[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c
+ d*x]]) - (((3*I)/4)*Cos[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Cos[
c + d*x])^(5/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\int (e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}\,dx}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$= \frac{ia}{2d(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}} + \frac{(3a)\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{4(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$= \frac{ia}{2d(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}} - \frac{3i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d(e\cos(c+dx))^{5/2}} + \frac{(3e^2)\int \sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx}{8(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$= \frac{ia}{2d(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}} - \frac{3i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d(e\cos(c+dx))^{5/2}} - \frac{(3iae^4)\,\text{Subst}\left(\int \frac{x^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{2d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$= \frac{ia}{2d(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}} - \frac{3i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d(e\cos(c+dx))^{5/2}} + \frac{(3iae^3)\,\text{Subst}\left(\int \frac{a-ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{4d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} - \frac{(3iae^3)\,\text{Subst}\left(\int \frac{a+ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{4d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$= \frac{ia}{2d(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}} - \frac{3i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d(e\cos(c+dx))^{5/2}}$$
$$- \frac{(3iae^2)\operatorname{Subst}\left(\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{8d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$
$$- \frac{(3iae^2)\operatorname{Subst}\left(\int \frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{8d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$
$$- \frac{\left(3i\sqrt{a}e^{5/2}\right)\operatorname{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}+2x}{-\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$
$$- \frac{\left(3i\sqrt{a}e^{5/2}\right)\operatorname{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}-2x}{-\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$= -\frac{3i\sqrt{a}e^{5/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{8\sqrt{2}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$
$$+ \frac{3i\sqrt{a}e^{5/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{8\sqrt{2}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$
$$+ \frac{ia}{2d(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}} - \frac{3i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d(e\cos(c+dx))^{5/2}}$$
$$- \frac{\left(3i\sqrt{a}e^{5/2}\right)\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$
$$+ \frac{\left(3i\sqrt{a}e^{5/2}\right)\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$= \frac{3i\sqrt{a}e^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} - \frac{3i\sqrt{a}e^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$
$$- \frac{3i\sqrt{a}e^{5/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{8\sqrt{2}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$
$$+ \frac{3i\sqrt{a}e^{5/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{8\sqrt{2}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$
$$+ \frac{ia}{2d(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}} - \frac{3i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{4d(e\cos(c+dx))^{5/2}}$$

Mathematica [A] (verified)

Time = 3.71 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\left(\frac{3ie^{-\frac{1}{2}i(2c+5dx)}\left(-e^{-2ic}\right)^{3/4}\left(1+e^{2i(c+dx)}\right)^2\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(\arctan\left(\frac{e}{\sqrt[4]{-}}\right.\right.}{4\sqrt{2}}\right.}{}$$

[In] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/(e*Cos[c + d*x])^(5/2),x]

[Out] (Sqrt[Cos[c + d*x]]*((((3*I)/4)*(-E^((-2*I)*c))^(3/4)*(1 + E^((2*I)*(c + d*x)))^2*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*(ArcTan[E^((I/2)*d*x)/(-E^((-2*I)*c))^(1/4)] - ArcTanh[E^((I/2)*d*x)/(-E^((-2*I)*c))^(1/4)]))/(Sqrt[2]*E^((I/2)*(2*c + 5*d*x))) - (3*I)*Cos[c + d*x]^(3/2) + 2*Sqrt[Cos[c + d*x]]*(I*Cos[c + d*x] + Sin[c + d*x]))*Sqrt[a + I*a*Tan[c + d*x]])/(4*d*(e*Cos[c + d*x])^(5/2))

Maple [A] (verified)

Time = 10.12 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.70

method	result
default	$\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\sqrt{a(1+i\tan(dx+c))}\left(3i\cos(dx+c)\operatorname{arctanh}\left(\frac{\cos(dx+c)+\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)-3i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\cos(dx+c)+3i\sqrt{\frac{1}{\cos(dx+c)+1}}\,\mathrm{s}\right.}{}$

[In] int((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)

[Out] (1/8+1/8*I)/d*(a*(1+I*tan(d*x+c)))^(1/2)/(I*cos(d*x+c)+I-sin(d*x+c))/(1/(cos(d*x+c)+1))^(1/2)/e^2/(e*cos(d*x+c))^(1/2)*(3*I*cos(d*x+c)*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))-3*I*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+3*I*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)-I*(1/(cos(d*x+c)+1))^(1/2)+2*I*tan(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+3*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))*cos(d*x+c)+3*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)+3*sin(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+2*I*sec(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+(1/(cos(d*x+c)+1))^(1/2)+2*tan(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)-2*sec(d*x+c)*(1/(cos(d*x+c)+1))^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 569, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-3i\,e^{(4i\,dx+4i\,c)}+i\,e^{(2i\,dx+2i\,c)}\right)e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}\right.}}{}$$

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(5/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1))*(-3*I*e^(4*I*d*x + 4*I*c) + I*e^(2*I*d*x + 2*I*c))*e^(-1/2*I*
d*x - 1/2*I*c) - (d*e^3*e^(4*I*d*x + 4*I*c) + 2*d*e^3*e^(2*I*d*x + 2*I*c) +
 d*e^3)*sqrt(9/16*I*a/(d^2*e^5))*log(4/3*I*d*e^3*sqrt(9/16*I*a/(d^2*e^5)) +
 sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I
*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)) + (d*e^3*e^(4*I*d*x + 4*I*c) + 2*d*e^3*
e^(2*I*d*x + 2*I*c) + d*e^3)*sqrt(9/16*I*a/(d^2*e^5))*log(-4/3*I*d*e^3*sqrt
(9/16*I*a/(d^2*e^5)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sq
rt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)) - (d*e^3*e^(4*I*d
*x + 4*I*c) + 2*d*e^3*e^(2*I*d*x + 2*I*c) + d*e^3)*sqrt(-9/16*I*a/(d^2*e^5)
)*log(4/3*I*d*e^3*sqrt(-9/16*I*a/(d^2*e^5)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2
*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*
I*c)) + (d*e^3*e^(4*I*d*x + 4*I*c) + 2*d*e^3*e^(2*I*d*x + 2*I*c) + d*e^3)*s
qrt(-9/16*I*a/(d^2*e^5))*log(-4/3*I*d*e^3*sqrt(-9/16*I*a/(d^2*e^5)) + sqrt(
2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
1))*e^(-1/2*I*d*x - 1/2*I*c)))/(d*e^3*e^(4*I*d*x + 4*I*c) + 2*d*e^3*e^(2*I*
d*x + 2*I*c) + d*e^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2)/(e*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2249 vs. 2(384) = 768.

Time = 0.57 (sec) , antiderivative size = 2249, normalized size of antiderivative = 4.39

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(5/2),x, algorithm="maxim
a")
```

```
[Out] -32*(6*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*s
in(4*d*x + 4*c) + 2*I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*c
os(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, sqrt(2)*sin(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 6*(sqrt(2)*cos(4*d*x + 4*
c) + 2*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(4*d*x + 4*c) + 2*I*sqrt(2)*
sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 1) + 6*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*
c) + I*sqrt(2)*sin(4*d*x + 4*c) + 2*I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*a
rctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, sq
rt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 6*(sqrt(2
)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(4*d*x + 4*c
) + 2*I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 1) + 6*(-I*sqrt(2)*cos(4*d*x + 4*c) - 2*I*s
qrt(2)*cos(2*d*x + 2*c) + sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x +
2*c) - I*sqrt(2))*arctan2(sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), sqrt(2)
*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 6*(I*sqrt(2)*cos(4*d*x + 4*c) + 2*
I*sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(4*d*x + 4*c) - 2*sqrt(2)*sin(2*d*x
 + 2*c) + I*sqrt(2))*arctan2(-sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), -sq
rt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 3*(sqrt(2)*cos(4*d*x + 4*c) +
 2*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(4*d*x + 4*c) + 2*I*sqrt(2)*sin(
2*d*x + 2*c) + sqrt(2))*log(2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*(s
qrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arcta
```

n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 3*(sqrt(2)*cos(4*d*x + 4*c)
+ 2*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(4*d*x + 4*c) + 2*I*sqrt(2)*sin
(2*d*x + 2*c) + sqrt(2))*log(-2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*
(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1)*cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 3*(I*sqrt(2)*cos(4*d*x + 4
*c) + 2*I*sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(4*d*x + 4*c) - 2*sqrt(2)*s
in(2*d*x + 2*c) + I*sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)
*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 3*(-I*sqrt(2)*
cos(4*d*x + 4*c) - 2*I*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)*sin(4*d*x + 4*c)
+ 2*sqrt(2)*sin(2*d*x + 2*c) - I*sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 3
*(I*sqrt(2)*cos(4*d*x + 4*c) + 2*I*sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(4
*d*x + 4*c) - 2*sqrt(2)*sin(2*d*x + 2*c) + I*sqrt(2))*log(2*cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 2) + 3*(-I*sqrt(2)*cos(4*d*x + 4*c) - 2*I*sqrt(2)*cos(2*d*x + 2*c) +
sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c) - I*sqrt(2))*log(2*co
s(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 2) + 48*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) - 16*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 48*I*sin(7
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 16*I*sin(3/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))))*sqrt(a)*sqrt(e)/((-1024*I*e^3*cos(4*d*x +
 4*c) - 2048*I*e^3*cos(2*d*x + 2*c) + 1024*e^3*sin(4*d*x + 4*c) + 2048*e^3*
sin(2*d*x + 2*c) - 1024*I*e^3)*d)

Giac [F]

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{i\,a\tan\left(dx+c\right)+a}}{\left(e\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)/(e*cos(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{5/2}}\,dx = \int \frac{\sqrt{a+a\tan\left(c+d\,x\right)\,1{}\mathrm{i}}}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2)/(e*cos(c + d*x))^(5/2),x)
```

```
[Out] int((a + a*tan(c + d*x)*1i)^(1/2)/(e*cos(c + d*x))^(5/2), x)
```

3.680 $\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 719

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{7/2}}\,dx = \frac{ia}{3d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} + \frac{5ia\cos^2(c+dx)}{8d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} - \frac{5ia^{3/2}e^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{8\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{5ia^{3/2}e^{7/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{8\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{5ia^{3/2}e^{7/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{16\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{5ia^{3/2}e^{7/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{16\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{5i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{12d(e\cos(c+dx))^{7/2}}$$

```
[Out] 1/3*I*a/d/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2)+5/8*I*a*cos(d*x+c)^
2/d/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2)-5/16*I*a^(3/2)*e^(7/2)*ar
ctan(1-2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2
))*sec(d*x+c)/d/(e*cos(d*x+c))^(7/2)/(e*sec(d*x+c))^(7/2)*2^(1/2)/(a-I*a*ta
n(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+5/16*I*a^(3/2)*e^(7/2)*arctan(1+2^
(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))*sec(d*
x+c)/d/(e*cos(d*x+c))^(7/2)/(e*sec(d*x+c))^(7/2)*2^(1/2)/(a-I*a*tan(d*x+c))
```

```
^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+5/32*I*a^(3/2)*e^(7/2)*ln(a-2^(1/2)*a^(1/2)
*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a-I*a*ta
n(d*x+c)))*sec(d*x+c)/d/(e*cos(d*x+c))^(7/2)/(e*sec(d*x+c))^(7/2)*2^(1/2)/(
a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-5/32*I*a^(3/2)*e^(7/2)*ln(
a+2^(1/2)*a^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos
(d*x+c)*(a-I*a*tan(d*x+c)))*sec(d*x+c)/d/(e*cos(d*x+c))^(7/2)/(e*sec(d*x+c)
)^(7/2)*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-5/12*I*co
s(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)/d/(e*cos(d*x+c))^(7/2)
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 719, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {3596, 3579, 3582, 3580, 3576, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{7/2}}\,dx =$$
$$-\frac{5ia^{3/2}e^{7/2}\sec(c+dx)\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$
$$+\frac{5ia^{3/2}e^{7/2}\sec(c+dx)\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$
$$+\frac{5ia^{3/2}e^{7/2}\sec(c+dx)\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{16\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$
$$-\frac{5ia^{3/2}e^{7/2}\sec(c+dx)\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{16\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$
$$-\frac{5i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{12d(e\cos(c+dx))^{7/2}}+\frac{5ia\cos^2(c+dx)}{8d\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{7/2}}$$
$$+\frac{ia}{3d\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{7/2}}$$

```
[In] Int[Sqrt[a + I*a*Tan[c + d*x]]/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] ((I/3)*a)/(d*(e*Cos[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]) + (((5*I)/8
)*a*Cos[c + d*x]^2)/(d*(e*Cos[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]) -
 (((5*I)/8)*a^(3/2)*e^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a*Tan[c
+ d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(Sqrt[2]*d*(e*Cos[c
+ d*x])^(7/2)*(e*Sec[c + d*x])^(7/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*
a*Tan[c + d*x]]) + (((5*I)/8)*a^(3/2)*e^(7/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e]*S
qrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(S
qrt[2]*d*(e*Cos[c + d*x])^(7/2)*(e*Sec[c + d*x])^(7/2)*Sqrt[a - I*a*Tan[c +
```

```
d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + (((5*I)/16)*a^(3/2)*e^(7/2)*Log[a - (S
qrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + C
os[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/(Sqrt[2]*d*(e*Cos[c + d*x
])^(7/2)*(e*Sec[c + d*x])^(7/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan
[c + d*x]]) - (((5*I)/16)*a^(3/2)*e^(7/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*
Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I*a*Ta
n[c + d*x])]*Sec[c + d*x])/(Sqrt[2]*d*(e*Cos[c + d*x])^(7/2)*(e*Sec[c + d*x
])^(7/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - (((5*I)/1
2)*Cos[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]])/(d*(e*Cos[c + d*x])^(7/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3580

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Dist[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]])), Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\int (e\sec(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}\,dx}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$=\frac{ia}{3d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}+\frac{(5a)\int\frac{(e\sec(c+dx))^{7/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{6(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$=\frac{ia}{3d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}-\frac{5i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{12d(e\cos(c+dx))^{7/2}}+\frac{(5e^2)\int(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx}{8(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$=\frac{ia}{3d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}+\frac{5ia\cos^2(c+dx)}{8d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}-\frac{5i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{12d(e\cos(c+dx))^{7/2}}+\frac{(5ae^2)\int\frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{16(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$=\frac{ia}{3d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}+\frac{5ia\cos^2(c+dx)}{8d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}-\frac{5i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{12d(e\cos(c+dx))^{7/2}}+\frac{(5ae^3\sec(c+dx))\int\sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}\,dx}{16(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$=\frac{ia}{3d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}+\frac{5ia\cos^2(c+dx)}{8d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}-\frac{5i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{12d(e\cos(c+dx))^{7/2}}+\frac{(5ia^2e^5\sec(c+dx))\operatorname{Subst}\left(\int\frac{x^2}{a^2+e^2x^4}\,dx,x,\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{4d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$=\frac{ia}{3d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}+\frac{5ia\cos^2(c+dx)}{8d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}-\frac{5i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{12d(e\cos(c+dx))^{7/2}}-\frac{(5ia^2e^4\sec(c+dx))\operatorname{Subst}\left(\int\frac{a-ex^2}{a^2+e^2x^4}\,dx,x,\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{8d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}+\frac{(5ia^2e^4\sec(c+dx))\operatorname{Subst}\left(\int\frac{a+ex^2}{a^2+e^2x^4}\,dx,x,\frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{8d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{ia}{3d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} + \frac{5ia\cos^2(c+dx)}{8d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} - \frac{5i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{12d(e\cos(c+dx))^{7/2}} + \frac{(5ia^2e^3\sec(c+dx))\,\text{Subst}\left(\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{16d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{(5ia^2e^3\sec(c+dx))\,\text{Subst}\left(\int \frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{16d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{(5ia^{3/2}e^{7/2}\sec(c+dx))\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}+2x}{-\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{16\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{(5ia^{3/2}e^{7/2}\sec(c+dx))\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}-2x}{-\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{16\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{ia}{3d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} + \frac{5ia\cos^2(c+dx)}{8d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} + \frac{5ia^{3/2}e^{7/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{16\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{5ia^{3/2}e^{7/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{16\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{5i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{12d(e\cos(c+dx))^{7/2}} + \frac{(5ia^{3/2}e^{7/2}\sec(c+dx))\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{(5ia^{3/2}e^{7/2}\sec(c+dx))\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{ia}{3d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} + \frac{5ia\cos^2(c+dx)}{8d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} - \frac{5ia^{3/2}e^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{8\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{5ia^{3/2}e^{7/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{8\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{5ia^{3/2}e^{7/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{16\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{5ia^{3/2}e^{7/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{16\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{5i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{12d(e\cos(c+dx))^{7/2}}$$

Mathematica [A] (verified)

Time = 4.51 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.42

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{7/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\left(-\frac{40}{3}i\cos^{\frac{3}{2}}(c+dx)+\frac{5}{8}ie^{-\frac{7}{2}i(c+dx)}\left(1+e^{2i(c+dx)}\right)^3\sqrt{e^{-i(c+dx)}\left(1\right.}\right.}{}$$

```
[In] Integrate[Sqrt[a + I*a*Tan[c + d*x]]/(e*Cos[c + d*x])^(7/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(((-40*I)/3)*Cos[c + d*x]^(3/2) + (((5*I)/8)*(1 + E^((2
*I)*(c + d*x)))^3*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*(2*ArcTan
[1 - Sqrt[2]*E^((I/2)*(c + d*x))] - 2*ArcTan[1 + Sqrt[2]*E^((I/2)*(c + d*x)
)] + Log[1 - Sqrt[2]*E^((I/2)*(c + d*x)) + E^(I*(c + d*x))] - Log[1 + Sqrt[
2]*E^((I/2)*(c + d*x)) + E^(I*(c + d*x))]))/E^(((7*I)/2)*(c + d*x)) + (32*S
qrt[Cos[c + d*x]]*(I*Cos[c + d*x] + Sin[c + d*x]))/3 + 20*Cos[c + d*x]^(5/2
)*(I*Cos[c + d*x] + Sin[c + d*x]))*Sqrt[a + I*a*Tan[c + d*x]])/(32*d*(e*Cos
[c + d*x])^(7/2))
```

Maple [A] (verified)

Time = 8.77 (sec) , antiderivative size = 459, normalized size of antiderivative = 0.64

method	result
default	$\frac{\left(\frac{1}{48}-\frac{i}{48}\right)\sqrt{a(1+i\tan(dx+c))}\left(10i\tan(dx+c)\sqrt{\frac{1}{\cos(dx+c)+1}}+15i\cos(dx+c)\operatorname{arctanh}\left(\frac{\cos(dx+c)+\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)-8i\left(\sec^2(dx+c\right.\right.}{}$

```
[In] int((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/48-1/48*I)/d*(a*(1+I*tan(d*x+c)))^(1/2)/(I*cos(d*x+c)+I-sin(d*x+c))/(1/(
cos(d*x+c)+1))^(1/2)/e^3/(e*cos(d*x+c))^(1/2)*(10*I*tan(d*x+c)*(1/(cos(d*x+
c)+1))^(1/2)+15*I*cos(d*x+c)*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x
+c)+1)/(1/(cos(d*x+c)+1))^(1/2))-8*I*sec(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2)-
5*I*(1/(cos(d*x+c)+1))^(1/2)+8*I*tan(d*x+c)*sec(d*x+c)*(1/(cos(d*x+c)+1))^(
1/2)-15*arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)/(1/(cos(d*x+c
)+1))^(1/2))*cos(d*x+c)-15*(1/(cos(d*x+c)+1))^(1/2)*cos(d*x+c)-15*sin(d*x+c
)*(1/(cos(d*x+c)+1))^(1/2)+15*I*(1/(cos(d*x+c)+1))^(1/2)*sin(d*x+c)+2*I*sec
(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)-5*(1/(cos(d*x+c)+1))^(1/2)-10*tan(d*x+c)*(
1/(cos(d*x+c)+1))^(1/2)-15*I*cos(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+2*sec(d*x+
c)*(1/(cos(d*x+c)+1))^(1/2)-8*tan(d*x+c)*sec(d*x+c)*(1/(cos(d*x+c)+1))^(1/2
)-8*sec(d*x+c)^2*(1/(cos(d*x+c)+1))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 657, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{7/2}}\,dx = \frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-5i\,e^{(5i\,dx+5i\,c)}+42i\,e^{(3i\,dx+3i\,c)}+15i\,e^{(i\,d}\right.}{}$$

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(7/2),x, algorithm="frica
s")
```

```
[Out] 1/12*(sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x
+ 2*I*c) + 1))*(-5*I*e^(5*I*d*x + 5*I*c) + 42*I*e^(3*I*d*x + 3*I*c) + 15*I*
e^(I*d*x + I*c))*e^(-1/2*I*d*x - 1/2*I*c) - 6*(d*e^4*e^(6*I*d*x + 6*I*c) +
3*d*e^4*e^(4*I*d*x + 4*I*c) + 3*d*e^4*e^(2*I*d*x + 2*I*c) + d*e^4)*sqrt(25/
64*I*a/(d^2*e^7))*log(8/5*d*e^4*sqrt(25/64*I*a/(d^2*e^7)) + sqrt(2)*sqrt(1/
2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/
2*I*d*x - 1/2*I*c)) + 6*(d*e^4*e^(6*I*d*x + 6*I*c) + 3*d*e^4*e^(4*I*d*x + 4
*I*c) + 3*d*e^4*e^(2*I*d*x + 2*I*c) + d*e^4)*sqrt(25/64*I*a/(d^2*e^7))*log(
-8/5*d*e^4*sqrt(25/64*I*a/(d^2*e^7)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x
```

```
+ 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)) +
 6*(d*e^4*e^(6*I*d*x + 6*I*c) + 3*d*e^4*e^(4*I*d*x + 4*I*c) + 3*d*e^4*e^(2*
I*d*x + 2*I*c) + d*e^4)*sqrt(-25/64*I*a/(d^2*e^7))*log(8/5*d*e^4*sqrt(-25/6
4*I*a/(d^2*e^7)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a
/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)) - 6*(d*e^4*e^(6*I*d*x
 + 6*I*c) + 3*d*e^4*e^(4*I*d*x + 4*I*c) + 3*d*e^4*e^(2*I*d*x + 2*I*c) + d*e
^4)*sqrt(-25/64*I*a/(d^2*e^7))*log(-8/5*d*e^4*sqrt(-25/64*I*a/(d^2*e^7)) +
sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*
c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)))/(d*e^4*e^(6*I*d*x + 6*I*c) + 3*d*e^4*e^
(4*I*d*x + 4*I*c) + 3*d*e^4*e^(2*I*d*x + 2*I*c) + d*e^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + ia\tan(c + dx)}}{(e\cos(c + dx))^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((a+I*a*tan(d*x+c))**(1/2)/(e*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2661 vs. $2(549) = 1098$.

Time = 0.59 (sec) , antiderivative size = 2661, normalized size of antiderivative = 3.70

$$\int \frac{\sqrt{a + ia\tan(c + dx)}}{(e\cos(c + dx))^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(7/2),x, algorithm="maxim
a")
```

```
[Out] -192*(30*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)
*cos(2*d*x + 2*c) + I*sqrt(2)*sin(6*d*x + 6*c) + 3*I*sqrt(2)*sin(4*d*x + 4*
c) + 3*I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*cos(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, sqrt(2)*sin(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 1) + 30*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(
2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(6*d*x + 6*
c) + 3*I*sqrt(2)*sin(4*d*x + 4*c) + 3*I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))
*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1,
-sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 30*(sq
rt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x +
 2*c) + I*sqrt(2)*sin(6*d*x + 6*c) + 3*I*sqrt(2)*sin(4*d*x + 4*c) + 3*I*sqr
```

t(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 30*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(6*d*x + 6*c) + 3*I*sqrt(2)*sin(4*d*x + 4*c) + 3*I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 30*(-I*sqrt(2)*cos(6*d*x + 6*c) - 3*I*sqrt(2)*cos(4*d*x + 4*c) - 3*I*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c) - I*sqrt(2))*arctan2(sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 30*(I*sqrt(2)*cos(6*d*x + 6*c) + 3*I*sqrt(2)*cos(4*d*x + 4*c) + 3*I*sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(6*d*x + 6*c) - 3*sqrt(2)*sin(4*d*x + 4*c) - 3*sqrt(2)*sin(2*d*x + 2*c) + I*sqrt(2))*arctan2(-sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), -sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 15*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(6*d*x + 6*c) + 3*I*sqrt(2)*sin(4*d*x + 4*c) + 3*I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*log(2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 15*(sqrt(2)*cos(6*d*x + 6*c) + 3*sqrt(2)*cos(4*d*x + 4*c) + 3*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(6*d*x + 6*c) + 3*I*sqrt(2)*sin(4*d*x + 4*c) + 3*I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*log(-2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 15*(-I*sqrt(2)*cos(6*d*x + 6*c) - 3*I*sqrt(2)*cos(4*d*x + 4*c) - 3*I*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c) - I*sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 15*(I*sqrt(2)*cos(6*d*x + 6*c) + 3*I*sqrt(2)*cos(4*d*x + 4*c) +

```
3*I*sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(6*d*x + 6*c) - 3*sqrt(2)*sin(4*
d*x + 4*c) - 3*sqrt(2)*sin(2*d*x + 2*c) + I*sqrt(2))*log(2*cos(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 2) - 15*(-I*sqrt(2)*cos(6*d*x + 6*c) - 3*I*sqrt(2)*cos(4*d*x + 4*c) -
3*I*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d
*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c) - I*sqrt(2))*log(2*cos(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 2) - 15*(I*sqrt(2)*cos(6*d*x + 6*c) + 3*I*sqrt(2)*cos(4*d*x + 4*c) + 3*
I*sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(6*d*x + 6*c) - 3*sqrt(2)*sin(4*d*x
 + 4*c) - 3*sqrt(2)*sin(2*d*x + 2*c) + I*sqrt(2))*log(2*cos(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 2) + 80*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 672*cos(5/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 240*cos(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 80*I*sin(9/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) - 672*I*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) - 240*I*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sqrt(a)*sqr
t(e)/((-36864*I*e^4*cos(6*d*x + 6*c) - 110592*I*e^4*cos(4*d*x + 4*c) - 1105
92*I*e^4*cos(2*d*x + 2*c) + 36864*e^4*sin(6*d*x + 6*c) + 110592*e^4*sin(4*d
*x + 4*c) + 110592*e^4*sin(2*d*x + 2*c) - 36864*I*e^4)*d)
```

Giac [F]

$$\int \frac{\sqrt{a + ia\tan(c + dx)}}{(e\cos(c + dx))^{7/2}}\,dx = \int \frac{\sqrt{i\,a\tan\left(dx + c\right) + a}}{\left(e\cos\left(dx + c\right)\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((a+I*a*tan(d*x+c))^(1/2)/(e*cos(d*x+c))^(7/2),x, algorithm="giac"
)

[Out] integrate(sqrt(I*a*tan(d*x + c) + a)/(e*cos(d*x + c))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\cos(c+dx))^{7/2}}\,dx = \int \frac{\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}{\left(e\,\cos\left(c+d\,x\right)\right)^{7/2}}\,dx$$

```
[In] int((a + a*tan(c + d*x)*1i)^(1/2)/(e*cos(c + d*x))^(7/2),x)
[Out] int((a + a*tan(c + d*x)*1i)^(1/2)/(e*cos(c + d*x))^(7/2), x)
```

3.681 $\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 175

$$\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i(e\cos(c+dx))^{5/2}}{7d\sqrt{a+ia\tan(c+dx)}} + \frac{16i(e\cos(c+dx))^{5/2}\sec^2(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}} - \frac{12i(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}{35ad} - \frac{32i(e\cos(c+dx))^{5/2}\sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{35ad}$$

```
[Out] 2/7*I*(e*cos(d*x+c))^(5/2)/d/(a+I*a*tan(d*x+c))^(1/2)+16/35*I*(e*cos(d*x+c)
)^(5/2)*sec(d*x+c)^2/d/(a+I*a*tan(d*x+c))^(1/2)-12/35*I*(e*cos(d*x+c))^(5/2
)*(a+I*a*tan(d*x+c))^(1/2)/a/d-32/35*I*(e*cos(d*x+c))^(5/2)*sec(d*x+c)^2*(a
+I*a*tan(d*x+c))^(1/2)/a/d
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3596, 3583, 3578, 3569}

$$\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{12i\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{5/2}}{35ad} + \frac{2i(e\cos(c+dx))^{5/2}}{7d\sqrt{a+ia\tan(c+dx)}} - \frac{32i\sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{5/2}}{35ad} + \frac{16i\sec^2(c+dx)(e\cos(c+dx))^{5/2}}{35d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[(e*Cos[c + d*x])^(5/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((2*I)/7)*(e*Cos[c + d*x])^(5/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((16*I
)/35)*(e*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2)/(d*Sqrt[a + I*a*Tan[c + d*x]])
 - (((12*I)/35)*(e*Cos[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]])/(a*d) -
(((32*I)/35)*(e*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x
]])/(a*d)
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \left((e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\right)\int \frac{1}{(e\sec(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx$$

$$= \frac{2i(e\cos(c+dx))^{5/2}}{7d\sqrt{a+ia\tan(c+dx)}} + \frac{\left(6(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\right)\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{5/2}}\,dx}{7a}$$

$$= \frac{2i(e\cos(c+dx))^{5/2}}{7d\sqrt{a+ia\tan(c+dx)}} - \frac{12i(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}{35ad} + \frac{\left(24(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\right)\int \frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx}{35e^2}$$

$$= \frac{2i(e\cos(c+dx))^{5/2}}{7d\sqrt{a+ia\tan(c+dx)}} + \frac{16i(e\cos(c+dx))^{5/2}\sec^2(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}} - \frac{12i(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}{35ad} + \frac{\left(16(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}\right)\int \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx}{35ae^2}$$

$$= \frac{2i(e\cos(c+dx))^{5/2}}{7d\sqrt{a+ia\tan(c+dx)}} + \frac{16i(e\cos(c+dx))^{5/2}\sec^2(c+dx)}{35d\sqrt{a+ia\tan(c+dx)}} - \frac{12i(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}{35ad} - \frac{32i(e\cos(c+dx))^{5/2}\sec^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{35ad}$$

Mathematica [A] (verified)

Time = 2.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.46

$$\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{ie^3(35\cos(c+dx)+\cos(3(c+dx))+70i\sin(c+dx)+6i\sin(3(c+dx)))}{70d\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[(e*Cos[c + d*x])^(5/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-1/70*I)*e^3*(35*Cos[c + d*x] + Cos[3*(c + d*x)] + (70*I)*Sin[c + d*x] +
(6*I)*Sin[3*(c + d*x)]))/(d*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]
)
```

Maple [A] (verified)

Time = 7.94 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.40

method	result	size
default	$-\frac{2e^{2} \sqrt{e \cos (d x+c)}\, \left(i \left(\cos^{2}(d x+c)\right)-6 \sin (d x+c) \cos (d x+c)+8 i-16 \tan (d x+c)\right)}{35 d \sqrt{a (1+i \tan (d x+c))}}$	70

[In] int((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -2/35/d*e^2*(e*cos(d*x+c))^(1/2)/(a*(1+I*tan(d*x+c)))^(1/2)*(I*cos(d*x+c)^2 -6*sin(d*x+c)*cos(d*x+c)+8*I-16*tan(d*x+c))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.59

$$\int \frac{(e \cos(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{2}\sqrt{\frac{1}{2}}\left(-7i\,e^{2}e^{(6i\,dx+6i\,c)} - 105i\,e^{2}e^{(4i\,dx+4i\,c)} + 35i\,e^{2}e^{(2i\,dx+2i\,c)} + 5i\,e^{2}\right)\sqrt{ee^{(2i\,dx}}}{140\,ad}$$

[In] integrate((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")

[Out] 1/140*sqrt(2)*sqrt(1/2)*(-7*I*e^2*e^(6*I*d*x + 6*I*c) - 105*I*e^2*e^(4*I*d*x + 4*I*c) + 35*I*e^2*e^(2*I*d*x + 2*I*c) + 5*I*e^2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-7/2*I*d*x - 7/2*I*c)/(a*d)

Sympy [F(-1)]

Timed out.

$$\int \frac{(e \cos(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Timed out}$$

[In] integrate((e*cos(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**(1/2),x)

[Out] Timed out

Maxima [A] (verification not implemented)

none

Time = 0.80 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.15

$$\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\left(5i\,e^2\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)-7i\,e^2\cos\left(\frac{5}{7}\,\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right)\right)\right.}{}$$

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] 1/140*(5*I*e^2*cos(7/2*d*x + 7/2*c) - 7*I*e^2*cos(5/7*arctan2(sin(7/2*d*x +
 7/2*c), cos(7/2*d*x + 7/2*c))) + 35*I*e^2*cos(3/7*arctan2(sin(7/2*d*x + 7/
2*c), cos(7/2*d*x + 7/2*c))) - 105*I*e^2*cos(1/7*arctan2(sin(7/2*d*x + 7/2*
c), cos(7/2*d*x + 7/2*c))) + 5*e^2*sin(7/2*d*x + 7/2*c) + 7*e^2*sin(5/7*arc
tan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 35*e^2*sin(3/7*arctan2(
sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 105*e^2*sin(1/7*arctan2(sin(
7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*sqrt(e)/(sqrt(a)*d)
```

Giac [F]

$$\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\cos(dx+c))^{\frac{5}{2}}}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

```
[In] integrate((e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((e*cos(d*x + c))^(5/2)/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [B] (verification not implemented)

Time = 6.14 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.63

$$\int \frac{(e\cos(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{e^2\,\sqrt{e\,\cos\left(c+d\,x\right)}\,\sqrt{\frac{a\,\left(\cos\left(2\,c+2\,d\,x\right)+1+\sin\left(2\,c+2\,d\,x\right)\,1\mathrm{i}\right)}{\cos\left(2\,c+2\,d\,x\right)+1}}\,\left(\cos\left(2\,c+2\,d\,x\right)\,28\mathrm{i}+\cos\left(\right.\right.}{140\,a\,d}$$

```
[In] int((e*cos(c + d*x))^(5/2)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] (e^2*(e*cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1
))/(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(2*c + 2*d*x)*28i + cos(4*c + 4*d*x)*5
i + 42*sin(2*c + 2*d*x) + 5*sin(4*c + 4*d*x) - 105i))/(140*a*d)
```

3.682 $\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 126

$$\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i(e\cos(c+dx))^{3/2}}{5d\sqrt{a+ia\tan(c+dx)}} + \frac{16i(e\cos(c+dx))^{3/2}\sec^2(c+dx)}{15d\sqrt{a+ia\tan(c+dx)}} - \frac{8i(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{15ad}$$

```
[Out] 2/5*I*(e*cos(d*x+c))^(3/2)/d/(a+I*a*tan(d*x+c))^(1/2)+16/15*I*(e*cos(d*x+c)
)^(3/2)*sec(d*x+c)^2/d/(a+I*a*tan(d*x+c))^(1/2)-8/15*I*(e*cos(d*x+c))^(3/2)
*(a+I*a*tan(d*x+c))^(1/2)/a/d
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3596, 3583, 3578, 3569}

$$\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{8i\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{3/2}}{15ad} + \frac{2i(e\cos(c+dx))^{3/2}}{5d\sqrt{a+ia\tan(c+dx)}} + \frac{16i\sec^2(c+dx)(e\cos(c+dx))^{3/2}}{15d\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[(e*Cos[c + d*x])^(3/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((2*I)/5)*(e*Cos[c + d*x])^(3/2))/(d*Sqrt[a + I*a*Tan[c + d*x]]) + (((16*I
)/15)*(e*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2)/(d*Sqrt[a + I*a*Tan[c + d*x]])
 - (((8*I)/15)*(e*Cos[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]])/(a*d)
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3578

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/(
a*f*m)), x] + Dist[a*((m + n)/(m*d^2)), Int[(d*Sec[e + f*x])^(m + 2)*(a + b
*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^
2, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \left((e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\right)\int \frac{1}{(e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx$$

$$= \frac{2i(e\cos(c+dx))^{3/2}}{5d\sqrt{a+ia\tan(c+dx)}} + \frac{\left(4(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\right)\int \frac{\sqrt{a+ia\tan(c+dx)}}{(e\sec(c+dx))^{3/2}}\,dx}{5a}$$

$$= \frac{2i(e\cos(c+dx))^{3/2}}{5d\sqrt{a+ia\tan(c+dx)}} - \frac{8i(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{15ad} + \frac{\left(8(e\cos(c+dx))^{3/2}(e\sec(c+dx))^{3/2}\right)\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{15e^2}$$

$$= \frac{2i(e\cos(c+dx))^{3/2}}{5d\sqrt{a+ia\tan(c+dx)}} + \frac{16i(e\cos(c+dx))^{3/2}\sec^2(c+dx)}{15d\sqrt{a+ia\tan(c+dx)}} - \frac{8i(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}{15ad}$$

Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.50

$$\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{ie^2(-15+\cos(2(c+dx))+4i\sin(2(c+dx)))}{15d\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[(e*Cos[c + d*x])^(3/2)/Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] ((-1/15*I)*e^2*(-15 + Cos[2*(c + d*x)] + (4*I)*Sin[2*(c + d*x)]))/(d*Sqrt[e
*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] (verified)

Time = 8.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.47

method	result	size
default	$-\frac{2e\sqrt{e\cos(dx+c)}\,(i\cos(dx+c)-4\sin(dx+c)-8i\sec(dx+c))}{15d\sqrt{a(1+i\tan(dx+c))}}$	59

```
[In] int((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/15/d*e*(e*cos(d*x+c))^(1/2)/(a*(1+I*tan(d*x+c)))^(1/2)*(I*cos(d*x+c)-4*s
in(d*x+c)-8*I*sec(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.66

$$\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{2}\sqrt{\frac{1}{2}}\left(-5i\,ee^{(4i\,dx+4i\,c)}+30i\,ee^{(2i\,dx+2i\,c)}+3i\,e\right)\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}}{30\,ad}$$

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
[Out] 1/30*sqrt(2)*sqrt(1/2)*(-5*I*e*e^(4*I*d*x + 4*I*c) + 30*I*e*e^(2*I*d*x + 2*
I*c) + 3*I*e)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) +
 1))*e^(-5/2*I*d*x - 5/2*I*c)/(a*d)
```

Sympy [F]

$$\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\cos(c+dx))^{\frac{3}{2}}}{\sqrt{ia\left(\tan(c+dx)-i\right)}}\,dx$$

```
[In] integrate((e*cos(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Integral((e*cos(c + d*x))**(3/2)/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.69 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.08

$$\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\left(3i\,e\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)-5i\,e\cos\left(\frac{3}{5}\,\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\right)\right)\right.}{}$$

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
[Out] 1/30*(3*I*e*cos(5/2*d*x + 5/2*c) - 5*I*e*cos(3/5*arctan2(sin(5/2*d*x + 5/2*
c), cos(5/2*d*x + 5/2*c))) + 30*I*e*cos(1/5*arctan2(sin(5/2*d*x + 5/2*c), c
os(5/2*d*x + 5/2*c))) + 3*e*sin(5/2*d*x + 5/2*c) + 5*e*sin(3/5*arctan2(sin(
5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 30*e*sin(1/5*arctan2(sin(5/2*d*x
 + 5/2*c), cos(5/2*d*x + 5/2*c))))*sqrt(e)/(sqrt(a)*d)
```

Giac [F]

$$\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\cos(dx+c))^{\frac{3}{2}}}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

```
[In] integrate((e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
[Out] integrate((e*cos(d*x + c))^(3/2)/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [B] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.79

$$\int \frac{(e\cos(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{e\,\sqrt{e\,\cos\left(c+d\,x\right)}\,\sqrt{\frac{a\,\left(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i}\right)}{\cos(2\,c+2\,d\,x)+1}}\,\left(35\,\sin\left(c+d\,x\right)+3\,\sin\left(3\,c+\right.\right.}{30\,a\,d}$$

```
[In] int((e*cos(c + d*x))^(3/2)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] (e*(e*cos(c + d*x))^(1/2)*((a*(cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))
/(cos(2*c + 2*d*x) + 1))^(1/2)*(cos(c + d*x)*25i + 35*sin(c + d*x) + cos(3*
c + 3*d*x)*3i + 3*sin(3*c + 3*d*x)))/(30*a*d)
```

3.683 $\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 80

$$\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i\sqrt{e\cos(c+dx)}}{3d\sqrt{a+ia\tan(c+dx)}} - \frac{4i\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}{3ad}$$

```
[Out] 2/3*I*(e*cos(d*x+c))^(1/2)/d/(a+I*a*tan(d*x+c))^(1/2)-4/3*I*(e*cos(d*x+c))^
(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a/d
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3596, 3583, 3569}

$$\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i\sqrt{e\cos(c+dx)}}{3d\sqrt{a+ia\tan(c+dx)}} - \frac{4i\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{3ad}$$

```
[In] Int[Sqrt[e*Cos[c + d*x]]/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (((2*I)/3)*Sqrt[e*Cos[c + d*x]])/(d*Sqrt[a + I*a*Tan[c + d*x]]) - (((4*I)/3
)*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])/(a*d)
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int\frac{1}{\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx\\
&= \frac{2i\sqrt{e\cos(c+dx)}}{3d\sqrt{a+ia\tan(c+dx)}}+\frac{\left(2\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}\right)\int\frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\,dx}{3a}\\
&= \frac{2i\sqrt{e\cos(c+dx)}}{3d\sqrt{a+ia\tan(c+dx)}}-\frac{4i\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}{3ad}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.60

$$
\int\frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx=\frac{2\sqrt{e\cos(c+dx)}(-i+2\tan(c+dx))}{3d\sqrt{a+ia\tan(c+dx)}}
$$

```
[In] Integrate[Sqrt[e*Cos[c + d*x]]/Sqrt[a + I*a*Tan[c + d*x]],x]
[Out] (2*Sqrt[e*Cos[c + d*x]]*(-I + 2*Tan[c + d*x]))/(3*d*Sqrt[a + I*a*Tan[c + d*
x]])
```

Maple [A] (verified)

Time = 7.82 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.52

method	result	size
default	$-\frac{2\sqrt{e\cos(dx+c)}\,(i-2\tan(dx+c))}{3d\sqrt{a(1+i\tan(dx+c))}}$	42
risch	$-\frac{i\sqrt{2}\,\sqrt{e\cos(dx+c)}\,(3\,\mathrm{e}^{2i(dx+c)}-1)}{3(\mathrm{e}^{2i(dx+c)}+1)\sqrt{\frac{a\,\mathrm{e}^{2i(dx+c)}}{\mathrm{e}^{2i(dx+c)}+1}}\,d}$	72

```
[In] int((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3/d*(e*cos(d*x+c))^(1/2)/(a*(1+I*tan(d*x+c)))^(1/2)*(I-2*tan(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-3i\,e^{(2i\,dx+2i\,c)}+i\right)e^{\left(-\frac{3}{2}i\,dx-\frac{3}{2}i\,c\right)}}{3\,ad}$$

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="frica
s")
[Out] 1/3*sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x +
2*I*c) + 1))*(-3*I*e^(2*I*d*x + 2*I*c) + I)*e^(-3/2*I*d*x - 3/2*I*c)/(a*d)
```

Sympy [F]

$$\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{ia\left(\tan(c+dx)-i\right)}}\,dx$$

```
[In] integrate((e*cos(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Integral(sqrt(e*cos(c + d*x))/sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= \frac{\sqrt{e}\left(i\,\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)-3i\,\cos\left(\frac{1}{3}\,\arctan\left(\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right),\cos\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)\right)\right)+\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3\,\sin\left(\frac{1}{3}\right.\right.}{3\,\sqrt{a}d}$$

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] 1/3*sqrt(e)*(I*cos(3/2*d*x + 3/2*c) - 3*I*cos(1/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) + sin(3/2*d*x + 3/2*c) + 3*sin(1/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))/(sqrt(a)*d)
```

Giac [F]

$$\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{\sqrt{e\cos\left(dx+c\right)}}{\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

```
[In] integrate((e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(e*cos(d*x + c))/sqrt(I*a*tan(d*x + c) + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{e\cos(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx$$
$$= \frac{\sqrt{e\,\cos\left(c+d\,x\right)}\,\left(\cos\left(2\,c+2\,d\,x\right)\,1\mathrm{i}+\sin\left(2\,c+2\,d\,x\right)-3\mathrm{i}\right)\,\sqrt{\frac{a\,\left(\cos(2\,c+2\,d\,x)+1+\sin(2\,c+2\,d\,x)\,1\mathrm{i}\right)}{\cos(2\,c+2\,d\,x)+1}}}{3\,a\,d}$$

```
[In] int((e*cos(c + d*x))^(1/2)/(a + a*tan(c + d*x)*1i)^(1/2),x)
```

```
[Out] ((e*cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*1i + sin(2*c + 2*d*x) - 3i)*((a*(
cos(2*c + 2*d*x) + sin(2*c + 2*d*x)*1i + 1))/(cos(2*c + 2*d*x) + 1))^(1/2))
/(3*a*d)
```

3.684 $\int \frac{1}{\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 36

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i}{d\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 2*I/d/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {3596, 3569}

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i}{d\sqrt{a+ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}$$

```
[In] Int[1/(Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]),x]
[Out] (2*I)/(d*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
```

```
+ b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
n}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\int \frac{\sqrt{e\sec(c+dx)}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{\sqrt{e\cos(c+dx)}\sqrt{e\sec(c+dx)}} \\ &= \frac{2i}{d\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i}{d\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[1/(Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]),x]
[Out] (2*I)/(d*Sqrt[e*Cos[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] (verified)

Time = 10.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{2i}{d\sqrt{e\cos(dx+c)}\,\sqrt{a(1+i\tan(dx+c))}}$	32
risch	$\frac{i\sqrt{2}}{\sqrt{e\cos(dx+c)}\,\sqrt{\frac{a\,e^{2i(dx+c)}}{e^{2i(dx+c)}+1}}\,d}$	46

```
[In] int(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE
)
[Out] 2*I/d/(e*cos(d*x+c))^(1/2)/(a*(1+I*tan(d*x+c)))^(1/2)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(28) = 56.

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.61

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{ade}$$

```
[In] integrate(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fri
cas")
```

```
[Out] 2*I*sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x +
2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)/(a*d*e)
```

Sympy [F]

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{\sqrt{e\cos(c+dx)}\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

```
[In] integrate(1/(e*cos(d*x+c))**(1/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(e*cos(c + d*x))*sqrt(I*a*(tan(c + d*x) - I))), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(28) = 56.

Time = 0.33 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{2i\sqrt{-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}}{\sqrt{a}d\sqrt{e}\sqrt{-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}}$$

```
[In] integrate(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="max
ima")
```

```
[Out] 2*I*sqrt(-sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)/(sqrt(a)*d*sqrt(e)*sqrt(
-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1)^2
- 1))
```

Giac [F]

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{\sqrt{e\cos\left(dx+c\right)}\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

```
[In] integrate(1/(e*cos(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/(sqrt(e*cos(d*x + c))*sqrt(I*a*tan(d*x + c) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{e\cos(c+dx)}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{\sqrt{e\,\cos\left(c+d\,x\right)}\,\sqrt{a+a\tan\left(c+d\,x\right)\,1\mathrm{i}}}\,dx$$

```
[In] int(1/((e*cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(1/2)*(a + a*tan(c + d*x)*1i)^(1/2)), x)
```

3.685 $\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 495

$$\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{i\sqrt{2}\sqrt{a}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e}}\right)\sec(c+dx)}{de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$+\frac{i\sqrt{2}\sqrt{a}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e}}\right)\sec(c+dx)}{de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$+\frac{i\sqrt{a}\log\left(a\sqrt{e}-\sqrt{2}\sqrt{a}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}+\sqrt{e}\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+}{\sqrt{2}de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$-\frac{i\sqrt{a}\log\left(a\sqrt{e}+\sqrt{2}\sqrt{a}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}+\sqrt{e}\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c-}{\sqrt{2}de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] 1/2*I*ln(a*e^(1/2)-2^(1/2)*a^(1/2)*(e*cos(d*x+c))^(1/2)*(a-I*a*tan(d*x+c))^
(1/2)+cos(d*x+c)*e^(1/2)*(a-I*a*tan(d*x+c)))*sec(d*x+c)*a^(1/2)/d/e^(3/2)*2
^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-1/2*I*ln(a*e^(1/2)
+2^(1/2)*a^(1/2)*(e*cos(d*x+c))^(1/2)*(a-I*a*tan(d*x+c))^(1/2)+cos(d*x+c)*e
^(1/2)*(a-I*a*tan(d*x+c)))*sec(d*x+c)*a^(1/2)/d/e^(3/2)*2^(1/2)/(a-I*a*tan(
d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-I*arctan(1-2^(1/2)*(e*cos(d*x+c))^(1
/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/e^(1/2))*sec(d*x+c)*2^(1/2)*a^(1/2)/d/
e^(3/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+I*arctan(1+2^(1/2
)*(e*cos(d*x+c))^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/e^(1/2))*sec(d*x+c)
*2^(1/2)*a^(1/2)/d/e^(3/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2
)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3595, 3594, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{i\sqrt{2}\sqrt{a}\sec(c+dx)\arctan\left(1-\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{i\sqrt{2}\sqrt{a}\sec(c+dx)\arctan\left(1+\frac{\sqrt{2}\sqrt{a-ia\tan(c+dx)}\sqrt{e\cos(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{i\sqrt{a}\sec(c+dx)\log\left(-\sqrt{2}\sqrt{a}\sqrt{a-ia\tan(c+dx)}\sqrt{e\cos(c+dx)}+\sqrt{e}\cos(c+dx)(a-ia\tan(c+dx))+a\right.}{\sqrt{2}de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{i\sqrt{a}\sec(c+dx)\log\left(\sqrt{2}\sqrt{a}\sqrt{a-ia\tan(c+dx)}\sqrt{e\cos(c+dx)}+\sqrt{e}\cos(c+dx)(a-ia\tan(c+dx))+a\right.}{\sqrt{2}de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[1/((e*Cos[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] ((-I)*Sqrt[2]*Sqrt[a]*ArcTan[1 - (Sqrt[2]*Sqrt[e*Cos[c + d*x]]*Sqrt[a - I*a
*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e])]*Sec[c + d*x])/(d*e^(3/2)*Sqrt[a - I*a*Ta
n[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + (I*Sqrt[2]*Sqrt[a]*ArcTan[1 + (Sq
rt[2]*Sqrt[e*Cos[c + d*x]]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e])]*S
ec[c + d*x])/(d*e^(3/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x
]]) + (I*Sqrt[a]*Log[a*Sqrt[e] - Sqrt[2]*Sqrt[a]*Sqrt[e*Cos[c + d*x]]*Sqrt[
a - I*a*Tan[c + d*x]] + Sqrt[e]*Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c
+ d*x])/(Sqrt[2]*d*e^(3/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c +
d*x]]) - (I*Sqrt[a]*Log[a*Sqrt[e] + Sqrt[2]*Sqrt[a]*Sqrt[e*Cos[c + d*x]]*Sq
rt[a - I*a*Tan[c + d*x]] + Sqrt[e]*Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec
[c + d*x])/(Sqrt[2]*d*e^(3/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c
 + d*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3594

```
Int[Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]]/Sqrt[cos[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[-4*(b/f), Subst[Int[x^2/(a^2*d^2 + x^4), x], x,
Sqrt[d*Cos[e + f*x]]*Sqrt[a + b*Tan[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}
, x] && EqQ[a^2 + b^2, 0]
```

Rule 3595

```
Int[1/((cos[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)*Sqrt[(a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]]), x_Symbol] :> Dist[1/(d*Cos[e + f*x]*Sqrt[a - b*Tan[e + f*x]]
*Sqrt[a + b*Tan[e + f*x]]), Int[Sqrt[a - b*Tan[e + f*x]]/Sqrt[d*Cos[e + f*x
]], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\sec(c+dx)\int \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\cos(c+dx)}}\,dx}{e\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{(4ia\sec(c+dx))\text{Subst}\left(\int \frac{x^2}{a^2e^2+x^4}\,dx, x, \sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}\right)}{de\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= -\frac{(2ia\sec(c+dx))\text{Subst}\left(\int \frac{ae-x^2}{a^2e^2+x^4}\,dx, x, \sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}\right)}{de\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{(2ia\sec(c+dx))\text{Subst}\left(\int \frac{ae+x^2}{a^2e^2+x^4}\,dx, x, \sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}\right)}{de\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{\left(i\sqrt{a}\sec(c+dx)\right)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{a}\sqrt{e}+2x}{-ae-\sqrt{2}\sqrt{a}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}\right)}{\sqrt{2}de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{\left(i\sqrt{a}\sec(c+dx)\right)\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{a}\sqrt{e}-2x}{-ae+\sqrt{2}\sqrt{a}\sqrt{e}x-x^2}\,dx, x, \sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}\right)}{\sqrt{2}de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{(ia\sec(c+dx))\text{Subst}\left(\int \frac{1}{ae-\sqrt{2}\sqrt{a}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}\right)}{de\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{(ia\sec(c+dx))\text{Subst}\left(\int \frac{1}{ae+\sqrt{2}\sqrt{a}\sqrt{e}x+x^2}\,dx, x, \sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}\right)}{de\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{i\sqrt{a}\log\left(a\sqrt{e}-\sqrt{2}\sqrt{a}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}+\sqrt{e}\cos(c+dx)(a-ia\tan(c+dx))\right)}{\sqrt{2}de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{i\sqrt{a}\log\left(a\sqrt{e}+\sqrt{2}\sqrt{a}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}+\sqrt{e}\cos(c+dx)(a-ia\tan(c+dx)\right.}{\sqrt{2}de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{\left(i\sqrt{2}\sqrt{a}\sec(c+dx)\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{\left(i\sqrt{2}\sqrt{a}\sec(c+dx)\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e}}\right)}{de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= -\frac{i\sqrt{2}\sqrt{a}\arctan\left(1-\frac{\sqrt{2}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e}}\right)\sec(c+dx)}{de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$+\frac{i\sqrt{2}\sqrt{a}\arctan\left(1+\frac{\sqrt{2}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e}}\right)\sec(c+dx)}{de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$+\frac{i\sqrt{a}\log\left(a\sqrt{e}-\sqrt{2}\sqrt{a}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}+\sqrt{e}\cos(c+dx)(a-ia\tan(c+dx\right.}{\sqrt{2}de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$
$$-\frac{i\sqrt{a}\log\left(a\sqrt{e}+\sqrt{2}\sqrt{a}\sqrt{e\cos(c+dx)}\sqrt{a-ia\tan(c+dx)}+\sqrt{e}\cos(c+dx)(a-ia\tan(c+dx\right.}{\sqrt{2}de^{3/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A] (verified)

Time = 2.59 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.42

$$\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{ie^{\frac{1}{2}i(c+dx)}\left(2\arctan\left(1-\sqrt{2}e^{\frac{1}{2}i(c+dx)}\right)-2\arctan\left(1+\sqrt{2}e^{\frac{1}{2}}\right.\right.}{\sqrt{2}de\sqrt{\frac{ae^{2i(}}{1+e^{2i(}}}}$$

```
[In] Integrate[1/((e*Cos[c + d*x])^(3/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] (I*E^((I/2)*(c + d*x))*(2*ArcTan[1 - Sqrt[2]*E^((I/2)*(c + d*x))] - 2*ArcTa
n[1 + Sqrt[2]*E^((I/2)*(c + d*x))] + Log[1 - Sqrt[2]*E^((I/2)*(c + d*x)) +
E^(I*(c + d*x))] - Log[1 + Sqrt[2]*E^((I/2)*(c + d*x)) + E^(I*(c + d*x))]))
/(Sqrt[2]*d*e*Sqrt[(a*E^((2*I)*(c + d*x)))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[
(e*(1 + E^((2*I)*(c + d*x))))/E^(I*(c + d*x))])
```

Maple [A] (verified)

Time = 12.92 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.33

method	result	s
default	$\frac{\left(-\frac{1}{2}-\frac{i}{2}\right)\left(i\,\mathrm{arctanh}\left(\frac{\cos(dx+c)+\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)-\mathrm{arctanh}\left(\frac{-\cos(dx+c)+\sin(dx+c)-1}{2(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)\right)(\cos(dx+c)+1+i\sin(dx+c))}{d(\cos(dx+c)+1)\sqrt{a(1+i\tan(dx+c))}\sqrt{\frac{1}{\cos(dx+c)+1}}\,e\sqrt{e\cos(dx+c)}}$	1

```
[In] int(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] (-1/2-1/2*I)/d*(I*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(
cos(d*x+c)+1))^(1/2))-arctanh(1/2*(-cos(d*x+c)+sin(d*x+c)-1)/(cos(d*x+c)+1)
/(1/(cos(d*x+c)+1))^(1/2)))*(cos(d*x+c)+1+I*sin(d*x+c))/(cos(d*x+c)+1)/(a*(
1+I*tan(d*x+c)))^(1/2)/(1/(cos(d*x+c)+1))^(1/2)/e/(e*cos(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.69

$$\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{1}{2}\sqrt{\frac{4i}{ad^2e^3}}\log\left(\frac{1}{2}\,ade^2\sqrt{\frac{4i}{ad^2e^3}} + \sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}\right) + \frac{1}{2}\sqrt{\frac{4i}{ad^2e^3}}\log\left(-\frac{1}{2}\,ade^2\sqrt{\frac{4i}{ad^2e^3}} + \sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}\right) + \frac{1}{2}\sqrt{-\frac{4i}{ad^2e^3}}\log\left(\frac{1}{2}\,ade^2\sqrt{-\frac{4i}{ad^2e^3}} + \sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}\right) - \frac{1}{2}\sqrt{-\frac{4i}{ad^2e^3}}\log\left(-\frac{1}{2}\,ade^2\sqrt{-\frac{4i}{ad^2e^3}} + \sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}\right)$$

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fri
cas")
```

```
[Out] -1/2*sqrt(4*I/(a*d^2*e^3))*log(1/2*a*d*e^2*sqrt(4*I/(a*d^2*e^3)) + sqrt(2)*
sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))
*e^(-1/2*I*d*x - 1/2*I*c)) + 1/2*sqrt(4*I/(a*d^2*e^3))*log(-1/2*a*d*e^2*sqr
t(4*I/(a*d^2*e^3)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)) + 1/2*sqrt(-4*I/(a*
d^2*e^3))*log(1/2*a*d*e^2*sqrt(-4*I/(a*d^2*e^3)) + sqrt(2)*sqrt(1/2)*sqrt(e
*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x -
 1/2*I*c)) - 1/2*sqrt(-4*I/(a*d^2*e^3))*log(-1/2*a*d*e^2*sqrt(-4*I/(a*d^2*e
^3)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x
 + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c))
```

Sympy [F]

$$\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{(e\cos{(c+dx)})^{\frac{3}{2}}\sqrt{ia(\tan{(c+dx)}-i)}}\,dx$$

```
[In] integrate(1/(e*cos(d*x+c))**(3/2)/(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Integral(1/((e*cos(c + d*x))**(3/2)*sqrt(I*a*(tan(c + d*x) - I))), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.49 (sec) , antiderivative size = 714, normalized size of antiderivative = 1.44

$$\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="max
ima")
[Out] -1/4*(2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1, sqrt(2)*sin(1/2
*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/2*d*x + 1/2*c) + 1,
-sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*arctan2(sqrt(2)*cos(1/2*d*
x + 1/2*c) - 1, sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) + 2*I*sqrt(2)*arctan2(sqr
t(2)*cos(1/2*d*x + 1/2*c) - 1, -sqrt(2)*sin(1/2*d*x + 1/2*c) + 1) - 2*sqrt(
2)*arctan2(sqrt(2)*sin(1/2*d*x + 1/2*c) + sin(d*x + c), sqrt(2)*cos(1/2*d*x
 + 1/2*c) + cos(d*x + c) + 1) + 2*sqrt(2)*arctan2(-sqrt(2)*sin(1/2*d*x + 1/
2*c) + sin(d*x + c), -sqrt(2)*cos(1/2*d*x + 1/2*c) + cos(d*x + c) + 1) + I*
sqrt(2)*log(2*sqrt(2)*sin(d*x + c)*sin(1/2*d*x + 1/2*c) + 2*(sqrt(2)*cos(1/
2*d*x + 1/2*c) + 1)*cos(d*x + c) + cos(d*x + c)^2 + 2*cos(1/2*d*x + 1/2*c)^
2 + sin(d*x + c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) + 1) - I*sqrt(2)*log(-2*sqrt(2)*sin(d*x + c)*sin(1/2*d*x + 1/2*c) - 2*(
sqrt(2)*cos(1/2*d*x + 1/2*c) - 1)*cos(d*x + c) + cos(d*x + c)^2 + 2*cos(1/2
*d*x + 1/2*c)^2 + sin(d*x + c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/
2*c) + 2) + sqrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - s
qrt(2)*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*
cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))/(sqrt(a)*d*e^(3/2))
```

Giac [F]

$$\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{(e\cos(dx+c))^{\frac{3}{2}}\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

```
[In] integrate(1/(e*cos(d*x+c))^(3/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(3/2)*sqrt(I*a*tan(d*x + c) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{(e\cos(c+d\,x))^{3/2}\sqrt{a+a\tan(c+d\,x)\ 1\mathrm{i}}}\,dx$$

```
[In] int(1/((e*cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(3/2)*(a + a*tan(c + d*x)*1i)^(1/2)), x)
```

3.686 $\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 470

$$\begin{aligned}
\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx &= \frac{ie^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} \\
&- \frac{ie^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} \\
&- \frac{ie^{5/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{2\sqrt{2}\sqrt{a}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} \\
&+ \frac{ie^{5/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{2\sqrt{2}\sqrt{a}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} \\
&- \frac{i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{ad(e\cos(c+dx))^{5/2}}
\end{aligned}$$

```
[Out] 1/2*I*e^(5/2)*arctan(1-2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*
sec(d*x+c))^(1/2))/d/(e*cos(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2)*2^(1/2)/a^(1
/2)-1/2*I*e^(5/2)*arctan(1+2^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)/a^(1/2)
/(e*sec(d*x+c))^(1/2))/d/(e*cos(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2)*2^(1/2)/
a^(1/2)-1/4*I*e^(5/2)*ln(a-2^(1/2)*a^(1/2)*e^(1/2)*(a+I*a*tan(d*x+c))^(1/2)
/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a+I*a*tan(d*x+c)))/d/(e*cos(d*x+c))^(5/2)
/(e*sec(d*x+c))^(5/2)*2^(1/2)/a^(1/2)+1/4*I*e^(5/2)*ln(a+2^(1/2)*a^(1/2)*e^
(1/2)*(a+I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a+I*a*tan(d
*x+c)))/d/(e*cos(d*x+c))^(5/2)/(e*sec(d*x+c))^(5/2)*2^(1/2)/a^(1/2)-I*cos(d
*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)/a/d/(e*cos(d*x+c))^(5/2)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3596, 3582, 3576, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{ie^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} - \frac{ie^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} - \frac{ie^{5/2}\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{2\sqrt{2}\sqrt{a}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} + \frac{ie^{5/2}\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))+a\right)}{2\sqrt{2}\sqrt{a}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} - \frac{i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{ad(e\cos(c+dx))^{5/2}}$$

```
[In] Int[1/((e*Cos[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] (I*e^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/(Sqrt[a]
*Sqrt[e*Sec[c + d*x]])])/(Sqrt[2]*Sqrt[a]*d*(e*Cos[c + d*x])^(5/2)*(e*Sec[c
 + d*x])^(5/2)) - (I*e^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[e]*Sqrt[a + I*a*Tan[c
 + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])])/(Sqrt[2]*Sqrt[a]*d*(e*Cos[c + d*
x])^(5/2)*(e*Sec[c + d*x])^(5/2)) - ((I/2)*e^(5/2)*Log[a - (Sqrt[2]*Sqrt[a]
*Sqrt[e]*Sqrt[a + I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a
 + I*a*Tan[c + d*x])])/(Sqrt[2]*Sqrt[a]*d*(e*Cos[c + d*x])^(5/2)*(e*Sec[c +
 d*x])^(5/2)) + ((I/2)*e^(5/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a + I*
a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a + I*a*Tan[c + d*x])
])/(Sqrt[2]*Sqrt[a]*d*(e*Cos[c + d*x])^(5/2)*(e*Sec[c + d*x])^(5/2)) - (I*C
os[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]])/(a*d*(e*Cos[c + d*x])^(5/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
 x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] && !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{(e\sec(c+dx))^{5/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$= -\frac{i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{ad(e\cos(c+dx))^{5/2}} + \frac{e^2\int\sqrt{e\sec(c+dx)}\sqrt{a+ia\tan(c+dx)}\,dx}{2a(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$= -\frac{i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{ad(e\cos(c+dx))^{5/2}} - \frac{(2ie^4)\,\text{Subst}\left(\int\frac{x^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$= -\frac{i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{ad(e\cos(c+dx))^{5/2}} + \frac{(ie^3)\,\text{Subst}\left(\int\frac{a-ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} - \frac{(ie^3)\,\text{Subst}\left(\int\frac{a+ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$= -\frac{i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{ad(e\cos(c+dx))^{5/2}} - \frac{(ie^2)\,\text{Subst}\left(\int\frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{2d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} - \frac{(ie^2)\,\text{Subst}\left(\int\frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{2d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} - \frac{(ie^{5/2})\,\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}+2x}{-\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{2\sqrt{2}\sqrt{a}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} - \frac{(ie^{5/2})\,\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}-2x}{-\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx, x, \frac{\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{2\sqrt{2}\sqrt{a}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}}$$

$$
\begin{aligned}
&= -\frac{ie^{5/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{2\sqrt{2}\sqrt{a}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} \\
&\quad+\frac{ie^{5/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{2\sqrt{2}\sqrt{a}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} \\
&\quad-\frac{i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{ad(e\cos(c+dx))^{5/2}} \\
&\quad-\frac{\left(ie^{5/2}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} \\
&\quad+\frac{\left(ie^{5/2}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} \\
&= \frac{ie^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} \\
&\quad-\frac{ie^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{\sqrt{2}\sqrt{a}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} \\
&\quad-\frac{ie^{5/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{2\sqrt{2}\sqrt{a}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} \\
&\quad+\frac{ie^{5/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a+ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a+ia\tan(c+dx))\right)}{2\sqrt{2}\sqrt{a}d(e\cos(c+dx))^{5/2}(e\sec(c+dx))^{5/2}} \\
&\quad-\frac{i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{ad(e\cos(c+dx))^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 3.37 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.53

$$
\int\frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx=\frac{ie^{ic-\frac{idx}{2}}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(-2e^{\frac{3idx}{2}}+\left(-e^{-2ic}\right)^{3/4}\left(1+e^{2i(c+dx)}\right)\text{arc}\right.}{d\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\sqrt{\cos(c+dx)}(e}
$$

```
[In] Integrate[1/((e*Cos[c + d*x])^(5/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] (I*E^(I*c - (I/2)*d*x)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(-2*
E^(((3*I)/2)*d*x) + (-E^((-2*I)*c))^(3/4)*(1 + E^((2*I)*(c + d*x)))*ArcTan[
E^((I/2)*d*x)/(-E^((-2*I)*c))^(1/4)] - (-E^((-2*I)*c))^(3/4)*(1 + E^((2*I)*
(c + d*x)))*ArcTanh[E^((I/2)*d*x)/(-E^((-2*I)*c))^(1/4)]))/(d*Sqrt[(1 + E^(
(2*I)*(c + d*x)))/E^(I*(c + d*x))]*Sqrt[Cos[c + d*x]]*(e*Cos[c + d*x])^(5/2
)*Sec[c + d*x]^(5/2)*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 764 vs. $2(370) = 740$.

Time = 12.39 (sec) , antiderivative size = 765, normalized size of antiderivative = 1.63

method	result	size
default	Expression too large to display	765

```
[In] int(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/4/d*(I*csc(d*x+c)^2*arctanh(1/2*(-cot(d*x+c)+csc(d*x+c)+1)*2^(1/2)/(csc(
d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2))*2^(1/2)*(1-cos(d*x+c))^2+I*csc(d*x+c)^2
*arctanh(1/2*(-cot(d*x+c)+csc(d*x+c)-1)*2^(1/2)/(csc(d*x+c)^2*(1-cos(d*x+c)
)^2+1)^(1/2))*2^(1/2)*(1-cos(d*x+c))^2-csc(d*x+c)^2*arctanh(1/2*(-cot(d*x+c
)+csc(d*x+c)+1)*2^(1/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2))*2^(1/2)*(1
-cos(d*x+c))^2+csc(d*x+c)^2*arctanh(1/2*(-cot(d*x+c)+csc(d*x+c)-1)*2^(1/2)/
(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2))*2^(1/2)*(1-cos(d*x+c))^2-I*arctanh
(1/2*(-cot(d*x+c)+csc(d*x+c)+1)*2^(1/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(
1/2))*2^(1/2)-I*arctanh(1/2*(-cot(d*x+c)+csc(d*x+c)-1)*2^(1/2)/(csc(d*x+c)^
2*(1-cos(d*x+c))^2+1)^(1/2))*2^(1/2)-4*I*(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^
(1/2)*(csc(d*x+c)-cot(d*x+c))+2^(1/2)*arctanh(1/2*(-cot(d*x+c)+csc(d*x+c)+1
)*2^(1/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2))-2^(1/2)*arctanh(1/2*(-co
t(d*x+c)+csc(d*x+c)-1)*2^(1/2)/(csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2))-4*(
csc(d*x+c)^2*(1-cos(d*x+c))^2+1)^(1/2))*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)*(
-csc(d*x+c)+cot(d*x+c)+I)/(-a*(2*I*(csc(d*x+c)-cot(d*x+c))-csc(d*x+c)^2*(1-
cos(d*x+c))^2+1)/(csc(d*x+c)^2*(1-cos(d*x+c))^2-1))^(1/2)/(csc(d*x+c)^2*(1-
cos(d*x+c))^2+1)^(5/2)/(-e*(csc(d*x+c)^2*(1-cos(d*x+c))^2-1)/(csc(d*x+c)^2*
(1-cos(d*x+c))^2+1))^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.07

$$\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{-4i\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}e^{\left(\frac{3}{2}i\,dx+\frac{3}{2}i\,c\right)}-\left(ade^{3}e\right.}{}$$

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fri
cas")
```

```
[Out] 1/2*(-4*I*sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*
d*x + 2*I*c) + 1))*e^(3/2*I*d*x + 3/2*I*c) - (a*d*e^3*e^(2*I*d*x + 2*I*c) +
```

```
a*d*e^3)*sqrt(I/(a*d^2*e^5))*log(I*a*d*e^3*sqrt(I/(a*d^2*e^5)) + sqrt(2)*s
qrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*
e^(-1/2*I*d*x - 1/2*I*c)) + (a*d*e^3*e^(2*I*d*x + 2*I*c) + a*d*e^3)*sqrt(I/
(a*d^2*e^5))*log(-I*a*d*e^3*sqrt(I/(a*d^2*e^5)) + sqrt(2)*sqrt(1/2)*sqrt(e*
e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x -
1/2*I*c)) - (a*d*e^3*e^(2*I*d*x + 2*I*c) + a*d*e^3)*sqrt(-I/(a*d^2*e^5))*lo
g(I*a*d*e^3*sqrt(-I/(a*d^2*e^5)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*
I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)) + (a*
d*e^3*e^(2*I*d*x + 2*I*c) + a*d*e^3)*sqrt(-I/(a*d^2*e^5))*log(-I*a*d*e^3*sq
rt(-I/(a*d^2*e^5)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt
(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)))/(a*d*e^3*e^(2*I*d*
x + 2*I*c) + a*d*e^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*cos(d*x+c))**(5/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2147 vs. 2(358) = 716.

Time = 0.49 (sec) , antiderivative size = 2147, normalized size of antiderivative = 4.57

$$\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="max
ima")
```

```
[Out] -8*(2*(sqrt(2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
 + I*sqrt(2)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
 sqrt(2))*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + 1, sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + 1) + 2*(sqrt(2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + I*sqrt(2)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) + sqrt(2))*arctan2(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 1, -sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 1) + 2*(sqrt(2)*cos(4/3*arctan2(sin(3/2*d*x + 3
```

/2*c), cos(3/2*d*x + 3/2*c))) + I*sqrt(2)*sin(4/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) + sqrt(2))*arctan2(sqrt(2)*cos(1/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 1, sqrt(2)*sin(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 2*(sqrt(2)*cos(4/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + I*sqrt(2)*sin(4/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2))*arctan2(sqrt(2)*cos(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 1, -sqrt(2)*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 2*(-I*sqrt(2)
*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2)*sin
(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - I*sqrt(2))*arct
an2(sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))), sqrt(2)*cos(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(2/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 2*(I*sqrt(2)*cos(4/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - sqrt(2)*sin(4/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + I*sqrt(2))*arctan2(-sqrt(2)
*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sin(2/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))), -sqrt(2)*cos(1/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(2/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - (sqrt(2)*cos(4/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + I*sqrt(2)*sin(4/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2))*log(2*sqrt(2)*sin(2/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(1/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + 2*(sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))) + cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c)))^2 + 2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
^2 + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + (sqrt(2)*c
os(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + I*sqrt(2)*sin
(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sqrt(2))*log(-2
*sqrt(2)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*(sqrt(2)*cos(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 1)*cos(2/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(2/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + 1) + (I*sqrt(2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) - sqrt(2)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + I*sqrt(2))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*

```
c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) + 2) + (-I*sqrt(2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + sqrt(2)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) - I*sqrt(2))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
 - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
 2) + (I*sqrt(2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) - sqrt(2)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
 I*sqrt(2))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 -
2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*
sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) +
 (-I*sqrt(2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
 sqrt(2)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - I*s
qrt(2))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^
2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sq
rt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt
(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 16*
cos(3/2*d*x + 3/2*c) + 16*I*sin(3/2*d*x + 3/2*c))*sqrt(a)*sqrt(e)/((-64*I*a
*e^3*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 64*a*e^
3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 64*I*a*e^3
)*d)
```

Giac [F]

$$\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{(e\cos(dx+c))^{\frac{5}{2}}\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

```
[In] integrate(1/(e*cos(d*x+c))^(5/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gia
c")
[Out] integrate(1/((e*cos(d*x + c))^(5/2)*sqrt(I*a*tan(d*x + c) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{5/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{1}{(e\,\cos(c+d\,x))^{5/2}\,\sqrt{a+a\tan(c+d\,x)\,1\mathrm{i}}}\,dx$$

```
[In] int(1/((e*cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
[Out] int(1/((e*cos(c + d*x))^(5/2)*(a + a*tan(c + d*x)*1i)^(1/2)), x)
```

3.687 $\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 682

$$\begin{aligned}
\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx &= \frac{3i\cos^2(c+dx)}{4d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{3i\sqrt{a}e^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{4\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&+ \frac{3i\sqrt{a}e^{7/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{4\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&+ \frac{3i\sqrt{a}e^{7/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{8\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{3i\sqrt{a}e^{7/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{8\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} \\
&- \frac{i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2ad(e\cos(c+dx))^{7/2}}
\end{aligned}$$

```
[Out] 3/4*I*cos(d*x+c)^2/d/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2)-3/8*I*e^
(7/2)*arctan(1-2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+
c))^(1/2))*sec(d*x+c)*a^(1/2)/d/(e*cos(d*x+c))^(7/2)/(e*sec(d*x+c))^(7/2)*2
^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+3/8*I*e^(7/2)*arct
an(1+2^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/a^(1/2)/(e*sec(d*x+c))^(1/2))
*sec(d*x+c)*a^(1/2)/d/(e*cos(d*x+c))^(7/2)/(e*sec(d*x+c))^(7/2)*2^(1/2)/(a-
I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)+3/16*I*e^(7/2)*ln(a-2^(1/2)*
a^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1/2)+cos(d*x+c)*(a
-I*a*tan(d*x+c)))*sec(d*x+c)*a^(1/2)/d/(e*cos(d*x+c))^(7/2)/(e*sec(d*x+c))^
(7/2)*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^(1/2)-3/16*I*e^(7
```

```
/2)*ln(a+2^(1/2)*a^(1/2)*e^(1/2)*(a-I*a*tan(d*x+c))^(1/2)/(e*sec(d*x+c))^(1
/2)+cos(d*x+c)*(a-I*a*tan(d*x+c)))*sec(d*x+c)*a^(1/2)/d/(e*cos(d*x+c))^(7/2
)/(e*sec(d*x+c))^(7/2)*2^(1/2)/(a-I*a*tan(d*x+c))^(1/2)/(a+I*a*tan(d*x+c))^
(1/2)-1/2*I*cos(d*x+c)^2*(a+I*a*tan(d*x+c))^(1/2)/a/d/(e*cos(d*x+c))^(7/2)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 682, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {3596, 3582, 3579, 3580, 3576, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx =$$
$$-\frac{3i\sqrt{a}e^{7/2}\sec(c+dx)\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$
$$+\frac{3i\sqrt{a}e^{7/2}\sec(c+dx)\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$
$$+\frac{3i\sqrt{a}e^{7/2}\sec(c+dx)\log\left(-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$
$$-\frac{3i\sqrt{a}e^{7/2}\sec(c+dx)\log\left(\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))+a\right)}{8\sqrt{2}d\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$
$$-\frac{i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2ad(e\cos(c+dx))^{7/2}}+\frac{3i\cos^2(c+dx)}{4d\sqrt{a+ia\tan(c+dx)}(e\cos(c+dx))^{7/2}}$$

```
[In] Int[1/((e*Cos[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
```

```
[Out] (((3*I)/4)*Cos[c + d*x]^2)/(d*(e*Cos[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d
*x]]) - (((3*I)/4)*Sqrt[a]*e^(7/2)*ArcTan[1 - (Sqrt[2]*Sqrt[e]*Sqrt[a - I*a
*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d*x])/(Sqrt[2]*d*(e
*Cos[c + d*x])^(7/2)*(e*Sec[c + d*x])^(7/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt
[a + I*a*Tan[c + d*x]]) + (((3*I)/4)*Sqrt[a]*e^(7/2)*ArcTan[1 + (Sqrt[2]*Sq
rt[e]*Sqrt[a - I*a*Tan[c + d*x]])/(Sqrt[a]*Sqrt[e*Sec[c + d*x]])]*Sec[c + d
*x])/(Sqrt[2]*d*(e*Cos[c + d*x])^(7/2)*(e*Sec[c + d*x])^(7/2)*Sqrt[a - I*a*
Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) + (((3*I)/8)*Sqrt[a]*e^(7/2)*Log[
a - (Sqrt[2]*Sqrt[a]*Sqrt[e]*Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x
]] + Cos[c + d*x]*(a - I*a*Tan[c + d*x])]*Sec[c + d*x])/(Sqrt[2]*d*(e*Cos[c
 + d*x])^(7/2)*(e*Sec[c + d*x])^(7/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I
*a*Tan[c + d*x]]) - (((3*I)/8)*Sqrt[a]*e^(7/2)*Log[a + (Sqrt[2]*Sqrt[a]*Sqr
t[e]*Sqrt[a - I*a*Tan[c + d*x]])/Sqrt[e*Sec[c + d*x]] + Cos[c + d*x]*(a - I
*a*Tan[c + d*x])]*Sec[c + d*x])/(Sqrt[2]*d*(e*Cos[c + d*x])^(7/2)*(e*Sec[c
```

```
+ d*x])^(7/2)*Sqrt[a - I*a*Tan[c + d*x]]*Sqrt[a + I*a*Tan[c + d*x]]) - ((I/
2)*Cos[c + d*x]^2*Sqrt[a + I*a*Tan[c + d*x]])/(a*d*(e*Cos[c + d*x])^(7/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 3576

```
Int[Sqrt[(d_.)*sec[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*tan[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-4*b*(d^2/f), Subst[Int[x^2/(a^2 + d^2*x^4), x],
```

```
x, Sqrt[a + b*Tan[e + f*x]]/Sqrt[d*Sec[e + f*x]]], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3579

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^(n
 - 1)/(f*(m + n - 1))), x] + Dist[a*((m + 2*n - 2)/(m + n - 1)), Int[(d*Sec
[e + f*x])^m*(a + b*Tan[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, d, e, f,
m}, x] && EqQ[a^2 + b^2, 0] && GtQ[n, 0] && NeQ[m + n - 1, 0] && IntegersQ[
2*m, 2*n]
```

Rule 3580

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_) + (b_.)*tan[(e_.) + (f_
.)*(x_)]], x_Symbol] :> Dist[d*(Sec[e + f*x]/(Sqrt[a - b*Tan[e + f*x]]*Sqrt
[a + b*Tan[e + f*x]])), Int[Sqrt[d*Sec[e + f*x]]*Sqrt[a - b*Tan[e + f*x]],
x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{(e\sec(c+dx))^{7/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$= -\frac{i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2ad(e\cos(c+dx))^{7/2}} + \frac{(3e^2)\int (e\sec(c+dx))^{3/2}\sqrt{a+ia\tan(c+dx)}\,dx}{4a(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$= \frac{3i\cos^2(c+dx)}{4d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} - \frac{i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2ad(e\cos(c+dx))^{7/2}} + \frac{(3e^2)\int \frac{(e\sec(c+dx))^{3/2}}{\sqrt{a+ia\tan(c+dx)}}\,dx}{8(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}}$$

$$= \frac{3i\cos^2(c+dx)}{4d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} - \frac{i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2ad(e\cos(c+dx))^{7/2}} + \frac{(3e^3\sec(c+dx))\int \sqrt{e\sec(c+dx)}\sqrt{a-ia\tan(c+dx)}\,dx}{8(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{3i\cos^2(c+dx)}{4d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} - \frac{i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2ad(e\cos(c+dx))^{7/2}} + \frac{(3iae^5\sec(c+dx))\,\text{Subst}\left(\int \frac{x^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{2d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{3i\cos^2(c+dx)}{4d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} - \frac{i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2ad(e\cos(c+dx))^{7/2}} - \frac{(3iae^4\sec(c+dx))\,\text{Subst}\left(\int \frac{a-ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{4d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{(3iae^4\sec(c+dx))\,\text{Subst}\left(\int \frac{a+ex^2}{a^2+e^2x^4}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{4d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{3i\cos^2(c+dx)}{4d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} - \frac{i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2ad(e\cos(c+dx))^{7/2}} + \frac{(3iae^3\sec(c+dx))\,\text{Subst}\left(\int \frac{1}{\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{8d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{(3iae^3\sec(c+dx))\,\text{Subst}\left(\int \frac{1}{\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}+x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{8d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{(3i\sqrt{a}e^{7/2}\sec(c+dx))\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}+2x}{-\frac{a}{e}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{(3i\sqrt{a}e^{7/2}\sec(c+dx))\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{a}}{\sqrt{e}}-2x}{-\frac{a}{e}+\frac{\sqrt{2}\sqrt{a}x}{\sqrt{e}}-x^2}\,dx, x, \frac{\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}\right)}{8\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{3i\cos^2(c+dx)}{4d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} + \frac{3i\sqrt{a}e^{7/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{8\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{3i\sqrt{a}e^{7/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{8\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2ad(e\cos(c+dx))^{7/2}} + \frac{\left(3i\sqrt{a}e^{7/2}\sec(c+dx)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{\left(3i\sqrt{a}e^{7/2}\sec(c+dx)\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)}{4\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}}$$

$$= \frac{3i\cos^2(c+dx)}{4d(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}} - \frac{3i\sqrt{a}e^{7/2}\arctan\left(1-\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{4\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{3i\sqrt{a}e^{7/2}\arctan\left(1+\frac{\sqrt{2}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{a}\sqrt{e\sec(c+dx)}}\right)\sec(c+dx)}{4\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} + \frac{3i\sqrt{a}e^{7/2}\log\left(a-\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{8\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{3i\sqrt{a}e^{7/2}\log\left(a+\frac{\sqrt{2}\sqrt{a}\sqrt{e}\sqrt{a-ia\tan(c+dx)}}{\sqrt{e\sec(c+dx)}}+\cos(c+dx)(a-ia\tan(c+dx))\right)\sec(c+dx)}{8\sqrt{2}d(e\cos(c+dx))^{7/2}(e\sec(c+dx))^{7/2}\sqrt{a-ia\tan(c+dx)}\sqrt{a+ia\tan(c+dx)}} - \frac{i\cos^2(c+dx)\sqrt{a+ia\tan(c+dx)}}{2ad(e\cos(c+dx))^{7/2}}$$

Mathematica [A] (verified)

Time = 3.42 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.36

$$\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\left(\frac{3}{4}ie^{\frac{1}{2}i(c+dx)}\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^{5/2}\left(2\,\text{arct}\right.\right.}{}$$

```
[In] Integrate[1/((e*Cos[c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]]),x]
[Out] (Sqrt[Cos[c + d*x]]*(((3*I)/4)*E^((I/2)*(c + d*x))*((1 + E^((2*I)*(c + d*x)
))/E^(I*(c + d*x)))^(5/2)*(2*ArcTan[1 - Sqrt[2]*E^((I/2)*(c + d*x))] - 2*Ar
```

```
cTan[1 + Sqrt[2]*E^((I/2)*(c + d*x))] + Log[1 - Sqrt[2]*E^((I/2)*(c + d*x))
 + E^(I*(c + d*x))] - Log[1 + Sqrt[2]*E^((I/2)*(c + d*x)) + E^(I*(c + d*x))
]) + 4*Sqrt[Cos[c + d*x]]*(I*Cos[c + d*x] + 2*Sin[c + d*x])))/(16*d*(e*Cos[
c + d*x])^(7/2)*Sqrt[a + I*a*Tan[c + d*x]])
```

Maple [A] (verified)

Time = 12.96 (sec) , antiderivative size = 502, normalized size of antiderivative = 0.74

method	result
default	$\frac{\left(-\frac{1}{16}-\frac{i}{16}\right)\left(3i\sin(dx+c)\operatorname{arctanh}\left(\frac{\cos(dx+c)-\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)+3i\cos(dx+c)\operatorname{arctanh}\left(\frac{\cos(dx+c)+\sin(dx+c)+1}{2(\cos(dx+c)+1)\sqrt{\frac{1}{\cos(dx+c)+1}}}\right)+3i\operatorname{arctan}\right.}{}$

```
[In] int(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x,method=_RETURNVERBOSE
)

[Out] (-1/16-1/16*I)/d/(cos(d*x+c)+1)/(a*(1+I*tan(d*x+c)))^(1/2)/(1/(cos(d*x+c)+1
))^(1/2)/e^3/(e*cos(d*x+c))^(1/2)*(3*I*sin(d*x+c)*arctanh(1/2*(cos(d*x+c)-s
in(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))+3*I*cos(d*x+c)*arctan
h(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))+3*
I*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(
1/2))-2*I*(1/(cos(d*x+c)+1))^(1/2)+4*I*tan(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+
3*arctanh(1/2*(cos(d*x+c)-sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(
1/2))*cos(d*x+c)-3*sin(d*x+c)*arctanh(1/2*(cos(d*x+c)+sin(d*x+c)+1)/(cos(d*
x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))-2*I*sec(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+4
*I*tan(d*x+c)*sec(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)+3*arctanh(1/2*(cos(d*x+c)
-sin(d*x+c)+1)/(cos(d*x+c)+1)/(1/(cos(d*x+c)+1))^(1/2))-2*(1/(cos(d*x+c)+1)
)^(1/2)-4*tan(d*x+c)*(1/(cos(d*x+c)+1))^(1/2)-2*sec(d*x+c)*(1/(cos(d*x+c)+1
))^(1/2)-4*tan(d*x+c)*sec(d*x+c)*(1/(cos(d*x+c)+1))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 604, normalized size of antiderivative = 0.89

$$\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{ee^{(2i\,dx+2i\,c)}+e}\sqrt{\frac{a}{e^{(2i\,dx+2i\,c)}+1}}\left(-i\,e^{(3i\,dx+3i\,c)}+3i\,e^{(i\,dx\cdots}\right.}{}$$

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fri
cas")

[Out] 1/2*(sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x +
 2*I*c) + 1))*(-I*e^(3*I*d*x + 3*I*c) + 3*I*e^(I*d*x + I*c))*e^(-1/2*I*d*x
```

```
- 1/2*I*c) - (a*d*e^4*e^(4*I*d*x + 4*I*c) + 2*a*d*e^4*e^(2*I*d*x + 2*I*c) +
 a*d*e^4)*sqrt(9/16*I/(a*d^2*e^7))*log(4/3*a*d*e^4*sqrt(9/16*I/(a*d^2*e^7))
 + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2
*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)) + (a*d*e^4*e^(4*I*d*x + 4*I*c) + 2*a*
d*e^4*e^(2*I*d*x + 2*I*c) + a*d*e^4)*sqrt(9/16*I/(a*d^2*e^7))*log(-4/3*a*d*
e^4*sqrt(9/16*I/(a*d^2*e^7)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c)
 + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)) + (a*d*e^
4*e^(4*I*d*x + 4*I*c) + 2*a*d*e^4*e^(2*I*d*x + 2*I*c) + a*d*e^4)*sqrt(-9/16
*I/(a*d^2*e^7))*log(4/3*a*d*e^4*sqrt(-9/16*I/(a*d^2*e^7)) + sqrt(2)*sqrt(1/
2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*e^(-1/
2*I*d*x - 1/2*I*c)) - (a*d*e^4*e^(4*I*d*x + 4*I*c) + 2*a*d*e^4*e^(2*I*d*x +
 2*I*c) + a*d*e^4)*sqrt(-9/16*I/(a*d^2*e^7))*log(-4/3*a*d*e^4*sqrt(-9/16*I/
(a*d^2*e^7)) + sqrt(2)*sqrt(1/2)*sqrt(e*e^(2*I*d*x + 2*I*c) + e)*sqrt(a/(e^
(2*I*d*x + 2*I*c) + 1))*e^(-1/2*I*d*x - 1/2*I*c)))/(a*d*e^4*e^(4*I*d*x + 4*
I*c) + 2*a*d*e^4*e^(2*I*d*x + 2*I*c) + a*d*e^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(e*cos(d*x+c))**(7/2)/(a+I*a*tan(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2254 vs. $2(522) = 1044$.

Time = 0.87 (sec) , antiderivative size = 2254, normalized size of antiderivative = 3.30

$$\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="max
ima")
```

```
[Out] -32*(6*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*s
in(4*d*x + 4*c) + 2*I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*c
os(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1, sqrt(2)*sin(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 6*(sqrt(2)*cos(4*d*x + 4*
c) + 2*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(4*d*x + 4*c) + 2*I*sqrt(2)*
sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c
```

```
), cos(2*d*x + 2*c))) + 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 1) + 6*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*
c) + I*sqrt(2)*sin(4*d*x + 4*c) + 2*I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*a
rctan2(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, sq
rt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 6*(sqrt(2
)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(4*d*x + 4*c
) + 2*I*sqrt(2)*sin(2*d*x + 2*c) + sqrt(2))*arctan2(sqrt(2)*cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1, -sqrt(2)*sin(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 1) - 6*(-I*sqrt(2)*cos(4*d*x + 4*c) - 2*I*s
qrt(2)*cos(2*d*x + 2*c) + sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x +
2*c) - I*sqrt(2))*arctan2(sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), sqrt(2)
*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 6*(I*sqrt(2)*cos(4*d*x + 4*c) + 2*
I*sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(4*d*x + 4*c) - 2*sqrt(2)*sin(2*d*x
 + 2*c) + I*sqrt(2))*arctan2(-sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))), -sq
rt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 3*(sqrt(2)*cos(4*d*x + 4*c) +
 2*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(4*d*x + 4*c) + 2*I*sqrt(2)*sin(
2*d*x + 2*c) + sqrt(2))*log(2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*(s
qrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 3*(sqrt(2)*cos(4*d*x + 4*c)
+ 2*sqrt(2)*cos(2*d*x + 2*c) + I*sqrt(2)*sin(4*d*x + 4*c) + 2*I*sqrt(2)*sin
(2*d*x + 2*c) + sqrt(2))*log(-2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*
(sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1)*cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 + 2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 3*(-I*sqrt(2)*cos(4*d*x +
4*c) - 2*I*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*
sin(2*d*x + 2*c) - I*sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
+ 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2
)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 3*(I*sqrt(2)*
cos(4*d*x + 4*c) + 2*I*sqrt(2)*cos(2*d*x + 2*c) - sqrt(2)*sin(4*d*x + 4*c)
- 2*sqrt(2)*sin(2*d*x + 2*c) + I*sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
```

```
 + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 3
*(-I*sqrt(2)*cos(4*d*x + 4*c) - 2*I*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)*sin(
4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c) - I*sqrt(2))*log(2*cos(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 2) - 3*(I*sqrt(2)*cos(4*d*x + 4*c) + 2*I*sqrt(2)*cos(2*d*x + 2*c) -
sqrt(2)*sin(4*d*x + 4*c) - 2*sqrt(2)*sin(2*d*x + 2*c) + I*sqrt(2))*log(2*co
s(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 2) + 16*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) - 48*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*I*sin(5
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 48*I*sin(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))))*sqrt(a)*sqrt(e)/((-1024*I*a*e^4*cos(4*d*x
 + 4*c) - 2048*I*a*e^4*cos(2*d*x + 2*c) + 1024*a*e^4*sin(4*d*x + 4*c) + 204
8*a*e^4*sin(2*d*x + 2*c) - 1024*I*a*e^4)*d)
```

Giac [F]

$$\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{1}{\left(e\cos\left(dx+c\right)\right)^{\frac{7}{2}}\sqrt{i\,a\tan\left(dx+c\right)+a}}\,dx$$

```
[In] integrate(1/(e*cos(d*x+c))^(7/2)/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/((e*cos(d*x + c))^(7/2)*sqrt(I*a*tan(d*x + c) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(e\cos(c+dx))^{7/2}\sqrt{a+ia\tan(c+dx)}}\,dx=\int \frac{1}{{\left(e\,\cos\left(c+d\,x\right)\right)}^{7/2}\,\sqrt{a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}}}\,dx$$

```
[In] int(1/((e*cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)^(1/2)),x)
```

```
[Out] int(1/((e*cos(c + d*x))^(7/2)*(a + a*tan(c + d*x)*1i)^(1/2)), x)
```

3.688 $\int (e\cos(c+dx))^m (a+ia\tan(c+dx))^n\,dx$

Optimal result

Integrand size = 26, antiderivative size = 105

$$\int (e\cos(c+dx))^m (a+ia\tan(c+dx))^n\,dx =$$
$$-\frac{i2^{-\frac{m}{2}+n}(e\cos(c+dx))^m \operatorname{Hypergeometric2F1}\left(-\frac{m}{2},\frac{1}{2}(2+m-2n),1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\,\mathrm{ta}}{dm}$$

```
[Out] -I*2^(-1/2*m+n)*(e*cos(d*x+c))^m*hypergeom([-1/2*m, 1+1/2*m-n],[1-1/2*m],1/
2-1/2*I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/2*m-n)*(a+I*a*tan(d*x+c))^n/d/m
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3596, 3586, 3604, 72, 71}

$$\int (e\cos(c+dx))^m (a+ia\tan(c+dx))^n\,dx =$$
$$-\frac{i2^{n-\frac{m}{2}}(a+ia\tan(c+dx))^n(e\cos(c+dx))^m(1+i\tan(c+dx))^{\frac{1}{2}(m-2n)} \operatorname{Hypergeometric2F1}\left(-\frac{m}{2},\frac{1}{2}(m-\right.}{dm}$$

```
[In] Int[(e*Cos[c + d*x])^m*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] ((-I)*2^(-1/2*m + n)*(e*Cos[c + d*x])^m*Hypergeometric2F1[-1/2*m, (2 + m -
2*n)/2, 1 - m/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^((m - 2*n)/2)
*(a + I*a*Tan[c + d*x])^n)/(d*m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = ((e\cos(c+dx))^m(e\sec(c+dx))^m)\int (e\sec(c+dx))^{-m}(a+ia\tan(c+dx))^n\,dx$$

$$= \left((e\cos(c+dx))^m(a -ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}\right)\int (a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-\frac{m}{2}+n}\,dx$$

$$= \frac{\left(a^2(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}\right)\mathrm{Subst}\left(\int (a-iax)^{-1-\frac{m}{2}}(a+i\right.}{d}$$

$$= \frac{\left(2^{-1-\frac{m}{2}+n}a(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^n\left(\frac{a+ia\tan(c+dx)}{a}\right)^{\frac{m}{2}-n}\right)\text{Sub}}{d}$$

$$= -\frac{i2^{-\frac{m}{2}+n}(e\cos(c+dx))^m\text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{1}{2}(2+m-2n),1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{dm}$$

Mathematica [A] (verified)

Time = 14.50 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.92

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^n\,dx$$
$$= \frac{i2^{-m+n}\left(e^{idx}\right)^n\left(\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}\right)^n\left(1+e^{2i(c+dx)}\right)^{-m+n}\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^m\cos^{-m}(c+dx)(e\cos(c+dx))^m}{}$$

```
[In] Integrate[(e*Cos[c + d*x])^m*(a + I*a*Tan[c + d*x])^n,x]
```

```
[Out] (I*2^(-m + n)*(E^(I*d*x))^n*(E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))))^n*(
1 + E^((2*I)*(c + d*x)))^(-m + n)*((1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x)
))^m*(e*Cos[c + d*x])^m*Hypergeometric2F1[-m + n, -1/2*m + n, 1 - m/2 + n,
-E^((2*I)*(c + d*x))]*(a + I*a*Tan[c + d*x])^n)/(d*(m - 2*n)*Cos[c + d*x]^m
*Sec[c + d*x]^n*(Cos[d*x] + I*Sin[d*x])^n)
```

Maple [F]

$$\int (e\cos(dx+c))^m(a+ia\tan(dx+c))^n\,dx$$

```
[In] int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x)
```

```
[Out] int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x)
```

Fricas [F]

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^n\,dx = \int (e\cos(dx+c))^m(i\,a\tan(dx+c)+a)^n\,dx$$

```
[In] integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1))^n*e^(I*d*m*x +
 I*c*m + m*log(a*e) - m*log(2*a*e^(2*I*d*x + 2*I*c)/(e^(2*I*d*x + 2*I*c) +
1))), x)
```

Sympy [F]

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^n\,dx = \int (e\cos{(c+dx)})^m\,(ia(\tan{(c+dx)}-i))^n\,dx$$

```
[In] integrate((e*cos(d*x+c))**m*(a+I*a*tan(d*x+c))**n,x)
[Out] Integral((e*cos(c + d*x))**m*(I*a*(tan(c + d*x) - I))**n, x)
```

Maxima [F]

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^n\,dx = \int (e\cos{(dx+c)})^m\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

```
[In] integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((e*cos(d*x + c))^m*(I*a*tan(d*x + c) + a)^n, x)
```

Giac [F]

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^n\,dx = \int (e\cos{(dx+c)})^m\,(i\,a\tan{(dx+c)}+a)^n\,dx$$

```
[In] integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^n,x, algorithm="giac")
[Out] integrate((e*cos(d*x + c))^m*(I*a*tan(d*x + c) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^n\,dx = \int (e\,\cos{(c+d\,x)})^m\,(a+a\tan(c+d\,x)\,1\mathrm{i})^n\,dx$$

```
[In] int((e*cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^n,x)
[Out] int((e*cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^n, x)
```

3.689 $\int (e\cos(c+dx))^m (a+ia\tan(c+dx))^2\,dx$

Optimal result

Integrand size = 26, antiderivative size = 86

$$\int (e\cos(c+dx))^m (a+ia\tan(c+dx))^2\,dx = -\frac{i2^{2-\frac{m}{2}}a^2(e\cos(c+dx))^m \operatorname{Hypergeometric2F1}\left(\frac{1}{2}(-2+m),-\frac{m}{2},1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c}{dm}$$

```
[Out] -I*2^(2-1/2*m)*a^2*(e*cos(d*x+c))^m*hypergeom([-1/2*m, -1+1/2*m],[1-1/2*m],
1/2-1/2*I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/2*m)/d/m
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3596, 3586, 3604, 72, 71}

$$\int (e\cos(c+dx))^m (a+ia\tan(c+dx))^2\,dx = -\frac{ia^2 2^{2-\frac{m}{2}}(1+i\tan(c+dx))^{m/2}(e\cos(c+dx))^m \operatorname{Hypergeometric2F1}\left(\frac{m-2}{2},-\frac{m}{2},1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+d}{dm}$$

```
[In] Int[(e*Cos[c + d*x])^m*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((-I)*2^(2 - m/2)*a^2*(e*Cos[c + d*x])^m*Hypergeometric2F1[(-2 + m)/2, -1/2
*m, 1 - m/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(m/2))/(d*m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
```

```
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = ((e\cos(c+dx))^m(e\sec(c+dx))^m)\int(e\sec(c+dx))^{-m}(a+ia\tan(c+dx))^2\,dx$$

$$= ((e\cos(c+dx))^m(a - ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2})\int(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{2-\frac{m}{2}}\,dx$$

$$= \frac{(a^2(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2})\,\text{Subst}\left(\int(a-iax)^{-1-\frac{m}{2}}(a+i\right.}{d}$$

$$= \frac{\left(2^{1-\frac{m}{2}}a^3(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{m/2}\right)\text{Subst}\left(\int\left(\frac{1}{2}+\frac{ix}{2}\right)^{1-\frac{m}{2}}(a-\right.}{d}$$

$$= -\frac{i2^{2-\frac{m}{2}}a^2(e\cos(c+dx))^m \text{Hypergeometric2F1}\left(\frac{1}{2}(-2+m),-\frac{m}{2},1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(1+}{dm}$$

Mathematica [A] (verified)

Time = 9.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^2\,dx = -\frac{i2^{2-\frac{m}{2}}a^2(e\cos(c+dx))^m \text{Hypergeometric2F1}\left(\frac{1}{2}(-2+m),-\frac{m}{2},1-\frac{m}{2},-\frac{1}{2}i(i+\tan(c+dx))\right)(1+i\tan(}{dm}$$

```
[In] Integrate[(e*Cos[c + d*x])^m*(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((-I)*2^(2 - m/2)*a^2*(e*Cos[c + d*x])^m*Hypergeometric2F1[(-2 + m)/2, -1/2
*m, 1 - m/2, (-1/2*I)*(I + Tan[c + d*x])]*(1 + I*Tan[c + d*x])^(m/2))/(d*m)
```

Maple [F]

$$\int (e\cos(dx+c))^m (a+ia\tan(dx+c))^2\,dx$$

```
[In] int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x)
```

Fricas [F]

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^2\,dx = \int (i\,a\tan(dx+c)+a)^2(e\cos(dx+c))^m\,dx$$

```
[In] integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(4*(1/2*(e*e^(2*I*d*x + 2*I*c) + e)*e^(-I*d*x - I*c))^m*a^2*e^(4*I*
d*x + 4*I*c)/(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^2\,dx = -a^2\left(\int\left(-(e\cos(c+dx))^m\right)dx + \int (e\cos(c+dx))^m\tan^2(c+dx)\,dx + \int\left(-2i(e\cos(c+dx))^m\tan(c+dx)\right)dx\right)$$

```
[In] integrate((e*cos(d*x+c))**m*(a+I*a*tan(d*x+c))**2,x)
[Out] -a**2*(Integral(-(e*cos(c + d*x))**m, x) + Integral((e*cos(c + d*x))**m*tan
(c + d*x)**2, x) + Integral(-2*I*(e*cos(c + d*x))**m*tan(c + d*x), x))
```

Maxima [F]

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^2\,dx = \int (i\,a\tan(dx+c)+a)^2(e\cos(dx+c))^m\,dx$$

```
[In] integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)^2*(e*cos(d*x + c))^m, x)
```

Giac [F]

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^2\,dx = \int (i\,a\tan(dx+c)+a)^2(e\cos(dx+c))^m\,dx$$

```
[In] integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)^2*(e*cos(d*x + c))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))^2\,dx = \int (e\,\cos(c+d\,x))^m\,(a+a\tan(c+d\,x)\,1\mathrm{i})^2\,dx$$

```
[In] int((e*cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^2,x)
[Out] int((e*cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^2, x)
```

3.690 $\int (e\cos(c+dx))^m(a+ia\tan(c+dx))\,dx$

Optimal result

Integrand size = 24, antiderivative size = 82

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))\,dx = -\frac{i2^{1-\frac{m}{2}}a(e\cos(c+dx))^m \text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{m}{2},1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))^{m/}}{dm}$$

```
[Out] -I*2^(1-1/2*m)*a*(e*cos(d*x+c))^m*hypergeom([-1/2*m, 1/2*m],[1-1/2*m],1/2-1
/2*I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/2*m)/d/m
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3596, 3586, 3604, 72, 71}

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))\,dx = -\frac{ia2^{1-\frac{m}{2}}(1+i\tan(c+dx))^{m/2}(e\cos(c+dx))^m \text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{m}{2},1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)}{dm}$$

```
[In] Int[(e*Cos[c + d*x])^m*(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((-I)*2^(1 - m/2)*a*(e*Cos[c + d*x])^m*Hypergeometric2F1[-1/2*m, m/2, 1 - m
/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(m/2))/(d*m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
```

```
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] && !IntegerQ[m]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = ((e\cos(c+dx))^m(e\sec(c+dx))^m)\int (e\sec(c+dx))^{-m}(a+ia\tan(c+dx))\,dx$$

$$= \left((e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}\right)\int (a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{1-\frac{m}{2}}\,dx$$

$$= \frac{\left(a^2(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}\right)\text{Subst}\left(\int (a-iax)^{-1-\frac{m}{2}}(a+i\ldots\right.}{d}$$

$$= \frac{\left(2^{-m/2}a^2(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{m/2}\right)\text{Subst}\left(\int\left(\frac{1}{2}+\frac{ix}{2}\right)^{-m/2}(a-\ldots\right.}{d}$$

$$= -\frac{i2^{1-\frac{m}{2}}a(e\cos(c+dx))^m \text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{m}{2},1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx)}{dm}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))\,dx = -\frac{a(e\cos(c+dx))^m\left(i(1+m)+m\cot(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}\right)}{dm(1+m)}$$

```
[In] Integrate[(e*Cos[c + d*x])^m*(a + I*a*Tan[c + d*x]),x]
[Out] -((a*(e*Cos[c + d*x])^m*(I*(1 + m) + m*Cot[c + d*x]*Hypergeometric2F1[1/2,
(1 + m)/2, (3 + m)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2]))/(d*m*(1 + m)))
```

Maple [F]

$$\int (e\cos(dx+c))^m (a+ia\tan(dx+c))\,dx$$

```
[In] int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c)),x)
[Out] int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c)),x)
```

Fricas [F]

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))\,dx = \int (i\,a\tan(dx+c)+a)(e\cos(dx+c))^m\,dx$$

```
[In] integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c)),x, algorithm="fricas")
[Out] integral(2*(1/2*(e*e^(2*I*d*x + 2*I*c) + e)*e^(-I*d*x - I*c))^m*a*e^(2*I*d*
x + 2*I*c)/(e^(2*I*d*x + 2*I*c) + 1), x)
```

Sympy [F]

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))\,dx = ia\left(\int \left(-i(e\cos{(c+dx)})^m\right)\,dx + \int (e\cos{(c+dx)})^m\tan{(c+dx)}\,dx\right)$$

```
[In] integrate((e*cos(d*x+c))**m*(a+I*a*tan(d*x+c)),x)
[Out] I*a*(Integral(-I*(e*cos(c + d*x))**m, x) + Integral((e*cos(c + d*x))**m*tan
(c + d*x), x))
```

Maxima [F]

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))\,dx = \int (i\,a\tan{(dx+c)}+a)(e\cos{(dx+c)})^m\,dx$$

```
[In] integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c)),x, algorithm="maxima")
[Out] integrate((I*a*tan(d*x + c) + a)*(e*cos(d*x + c))^m, x)
```

Giac [F]

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))\,dx = \int (i\,a\tan{(dx+c)}+a)(e\cos{(dx+c)})^m\,dx$$

```
[In] integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c)),x, algorithm="giac")
[Out] integrate((I*a*tan(d*x + c) + a)*(e*cos(d*x + c))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\cos(c+dx))^m(a+ia\tan(c+dx))\,dx = \int {\left(e\,\cos\left(c+d\,x\right)\right)}^m\,\left(a+a\,\mathrm{tan}\left(c+d\,x\right)\,1{}\mathrm{i}\right)\,dx$$

```
[In] int((e*cos(c + d*x))^m*(a + a*tan(c + d*x)*1i),x)
[Out] int((e*cos(c + d*x))^m*(a + a*tan(c + d*x)*1i), x)
```

3.691 $\int \frac{(e\cos(c+dx))^m}{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 86

$$\int \frac{(e\cos(c+dx))^m}{a+ia\tan(c+dx)}\,dx = -\frac{i2^{-1-\frac{m}{2}}(e\cos(c+dx))^m \text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{4+m}{2},1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx))$$
$$adm$$

```
[Out] -I*2^(-1-1/2*m)*(e*cos(d*x+c))^m*hypergeom([-1/2*m, 2+1/2*m],[1-1/2*m],1/2-
1/2*I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/2*m)/a/d/m
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3596, 3586, 3604, 72, 71}

$$\int \frac{(e\cos(c+dx))^m}{a+ia\tan(c+dx)}\,dx = -\frac{i2^{-\frac{m}{2}-1}(1+i\tan(c+dx))^{m/2}(e\cos(c+dx))^m \text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{m+4}{2},1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+dx\right.$$
$$adm$$

```
[In] Int[(e*Cos[c + d*x])^m/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] ((-I)*2^(-1 - m/2)*(e*Cos[c + d*x])^m*Hypergeometric2F1[-1/2*m, (4 + m)/2,
1 - m/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(m/2))/(a*d*m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] && !IntegerQ[m]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \left((e\cos(c+dx))^m(e\sec(c+dx))^m\right)\int\frac{(e\sec(c+dx))^{-m}}{a+ia\tan(c+dx)}\,dx$$

$$= \left((e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}\right)\int(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-1-\frac{m}{2}}\,dx$$

$$= \frac{\left(a^2(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}\right)\text{Subst}\left(\int(a-iax)^{-1-\frac{m}{2}}(a+i\right.}{d}$$

$$= \frac{\left(2^{-2-\frac{m}{2}}(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{m/2}\right)\text{Subst}\left(\int\left(\frac{1}{2}+\frac{ix}{2}\right)^{-2-\frac{m}{2}}(a-i\right.}{d}$$

$$= -\frac{i2^{-1-\frac{m}{2}}(e\cos(c+dx))^m\,\text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{4+m}{2},1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c}{adm}$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 433 vs. 2(86) = 172.

Time = 7.52 (sec) , antiderivative size = 433, normalized size of antiderivative = 5.03

$$\int \frac{(e\cos(c+dx))^m}{a+ia\tan(c+dx)}\,dx =$$
$$-\frac{2^{-m/2}\cos(c+dx)(e\cos(c+dx))^m\left(1-2\cos^2(c+dx)+i\sin(2(c+dx))\right)^{m/2}\left(2^{m/2}(2+m)\,\text{Hypergeomet}\right.}{ad(1+m)(2+m)\left(-i\sin(c+dx)\left(\left(1-2\cos^2(c+dx)+i\sin(2(c+dx))\right)^{m/2}((\cos(c)+i\sin(c\right.\right.}$$

```
[In] Integrate[(e*Cos[c + d*x])^m/(a + I*a*Tan[c + d*x]),x]
```

```
[Out] -((Cos[c + d*x]*(e*Cos[c + d*x])^m*(1 - 2*Cos[c + d*x]^2 + I*Sin[2*(c + d*x
)])^(m/2)*(2^(m/2)*(2 + m)*Hypergeometric2F1[1 + m, (2 + m)/2, 2 + m, 2*Cos
[c + d*x]*(Cos[c + d*x] - I*Sin[c + d*x])]*((Cos[c] - I*Sin[c])*Sin[c]*(I +
 Tan[d*x]))^(m/2) - 2*(1 + m)*Hypergeometric2F1[-1 - m/2, m/2, -1/2*m, (1 +
 I*Tan[c + d*x])/2]*((Cos[c] + I*Sin[c])*Sin[c]*(-I + Tan[d*x]))^(m/2)*(1 -
 I*Tan[c + d*x])^(m/2)))/(2^(m/2)*a*d*(1 + m)*(2 + m)*((-I)*Sin[c + d*x]*((
1 - 2*Cos[c + d*x]^2 + I*Sin[2*(c + d*x)])^(m/2)*((Cos[c] + I*Sin[c])*Sin[c
]*(-I + Tan[d*x]))^(m/2) - ((Cos[c] - I*Sin[c])*Sin[c]*(I + Tan[d*x]))^(m/2
)) + Cos[c + d*x]*((1 - 2*Cos[c + d*x]^2 + I*Sin[2*(c + d*x)])^(m/2)*((Cos[
c] + I*Sin[c])*Sin[c]*(-I + Tan[d*x]))^(m/2) + ((Cos[c] - I*Sin[c])*Sin[c]*
(I + Tan[d*x]))^(m/2)))*(-I + Tan[c + d*x])))
```

Maple [F]

$$\int \frac{(e\cos(dx+c))^m}{a+ia\tan(dx+c)}dx$$

```
[In] int((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c)),x)
```

```
[Out] int((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c)),x)
```

Fricas [F]

$$\int \frac{(e\cos(c+dx))^m}{a+ia\tan(c+dx)}\,dx = \int \frac{(e\cos(dx+c))^m}{i\,a\tan(dx+c)+a}\,dx$$

[In] integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c)),x, algorithm="fricas")

[Out] integral(1/2*(1/2*(e*e^(2*I*d*x + 2*I*c) + e)*e^(-I*d*x - I*c))^m*(e^(2*I*d
*x + 2*I*c) + 1)*e^(-2*I*d*x - 2*I*c)/a, x)

Sympy [F]

$$\int \frac{(e\cos(c+dx))^m}{a+ia\tan(c+dx)}\,dx = -\frac{i\int \frac{(e\cos(c+dx))^m}{\tan(c+dx)-i}\,dx}{a}$$

[In] integrate((e*cos(d*x+c))**m/(a+I*a*tan(d*x+c)),x)

[Out] -I*Integral((e*cos(c + d*x))**m/(tan(c + d*x) - I), x)/a

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cos(c+dx))^m}{a+ia\tan(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

[In] integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c)),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.

Giac [F]

$$\int \frac{(e\cos(c+dx))^m}{a+ia\tan(c+dx)}\,dx = \int \frac{(e\cos(dx+c))^m}{i\,a\tan(dx+c)+a}\,dx$$

[In] integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c)),x, algorithm="giac")

[Out] integrate((e*cos(d*x + c))^m/(I*a*tan(d*x + c) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\cos(c+dx))^m}{a+ia\tan(c+dx)}\,dx = \int \frac{(e\ \cos\left(c+d\,x\right))^m}{a+a\tan\left(c+d\,x\right)\ 1\mathrm{i}}\,dx$$

```
[In] int((e*cos(c + d*x))^m/(a + a*tan(c + d*x)*1i),x)
[Out] int((e*cos(c + d*x))^m/(a + a*tan(c + d*x)*1i), x)
```

3.692 $\int \frac{(e\cos(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 86

$$\int \frac{(e\cos(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = -\frac{i2^{-2-\frac{m}{2}}(e\cos(c+dx))^m \text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{6+m}{2},1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx)}{a^2dm}$$

```
[Out] -I*2^(-2-1/2*m)*(e*cos(d*x+c))^m*hypergeom([-1/2*m, 3+1/2*m],[1-1/2*m],1/2-
1/2*I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/2*m)/a^2/d/m
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3596, 3586, 3604, 72, 71}

$$\int \frac{(e\cos(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = -\frac{i2^{-\frac{m}{2}-2}(1+i\tan(c+dx))^{m/2}(e\cos(c+dx))^m \text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{m+6}{2},1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+d}{a^2dm}$$

```
[In] Int[(e*Cos[c + d*x])^m/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((-I)*2^(-2 - m/2)*(e*Cos[c + d*x])^m*Hypergeometric2F1[-1/2*m, (6 + m)/2,
1 - m/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^(m/2))/(a^2*d*m)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = \left((e\cos(c+dx))^m(e\sec(c+dx))^m\right)\int \frac{(e\sec(c+dx))^{-m}}{(a+ia\tan(c+dx))^2}\,dx$$

$$= \left((e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}\right)\int (a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-2-\frac{m}{2}}\,dx$$

$$= \frac{\left(a^2(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}\right)\text{Subst}\left(\int (a-iax)^{-1-\frac{m}{2}}(a+ia\right.}{d}$$

$$= \frac{\left(2^{-3-\frac{m}{2}}(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{m/2}\right)\text{Subst}\left(\int\left(\frac{1}{2}+\frac{ix}{2}\right)^{-3-\frac{m}{2}}(a-\right.}{ad}$$

$$= -\frac{i2^{-2-\frac{m}{2}}(e\cos(c+dx))^m\,\text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{6+m}{2},1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan}{a^2dm}$$

Mathematica [A] (verified)

Time = 3.56 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.16

$$\int\frac{(e\cos(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = -\frac{i2^{-m/2}(e\cos(c+dx))^m\,\text{Hypergeometric2F1}\left(-2-\frac{m}{2},\frac{2+m}{2},-1-\frac{m}{2},\frac{1}{2}(1+i\tan(c+dx))\right)(1-i\tan(c}{a^2d(4+m)(-i+\tan(c+dx))^2}$$

```
[In] Integrate[(e*Cos[c + d*x])^m/(a + I*a*Tan[c + d*x])^2,x]
```

```
[Out] ((-I)*(e*Cos[c + d*x])^m*Hypergeometric2F1[-2 - m/2, (2 + m)/2, -1 - m/2, (
1 + I*Tan[c + d*x])/2]*(1 - I*Tan[c + d*x])^(m/2))/(2^(m/2)*a^2*d*(4 + m)*(
-I + Tan[c + d*x])^2)
```

Maple [F]

$$\int\frac{(e\cos(dx+c))^m}{(a+ia\tan(dx+c))^2}dx$$

```
[In] int((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x)
```

```
[Out] int((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x)
```

Fricas [F]

$$\int\frac{(e\cos(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = \int\frac{(e\cos(dx+c))^m}{(i\,a\tan(dx+c)+a)^2}\,dx$$

```
[In] integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(1/4*(1/2*(e*e^(2*I*d*x + 2*I*c) + e)*e^(-I*d*x - I*c))^m*(e^(4*I*d
*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1)*e^(-4*I*d*x - 4*I*c)/a^2, x)
```

Sympy [F]

$$\int \frac{(e\cos(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = -\frac{\int \frac{(e\cos{(c+dx)})^m}{\tan^2{(c+dx)}-2i\tan{(c+dx)}-1}\,dx}{a^2}$$

```
[In] integrate((e*cos(d*x+c))**m/(a+I*a*tan(d*x+c))**2,x)
[Out] -Integral((e*cos(c + d*x))**m/(tan(c + d*x)**2 - 2*I*tan(c + d*x) - 1), x)/
a**2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(e\cos(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{(e\cos(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\cos{(dx+c)})^m}{(i\,a\tan{(dx+c)}+a)^2}\,dx$$

```
[In] integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*cos(d*x + c))^m/(I*a*tan(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\cos(c+dx))^m}{(a+ia\tan(c+dx))^2}\,dx = \int \frac{(e\,\cos{(c+d\,x)})^m}{(a+a\tan{(c+d\,x)}\;1\mathrm{i})^2}\,dx$$

```
[In] int((e*cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^2,x)
[Out] int((e*cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^2, x)
```

3.693 $\int (e\cos(c+dx))^m \sqrt{a+ia\tan(c+dx)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 105

$$\int (e\cos(c+dx))^m \sqrt{a+ia\tan(c+dx)}\,dx = -\frac{i2^{\frac{1}{2}-\frac{m}{2}}a(e\cos(c+dx))^m \text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{1+m}{2},1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx}{dm\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] -I*2^(1/2-1/2*m)*a*(e*cos(d*x+c))^m*hypergeom([-1/2*m, 1/2+1/2*m],[1-1/2*m]
,1/2-1/2*I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/2+1/2*m)/d/m/(a+I*a*tan(d*x+c))^
(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3596, 3586, 3604, 72, 71}

$$\int (e\cos(c+dx))^m \sqrt{a+ia\tan(c+dx)}\,dx = -\frac{ia2^{\frac{1}{2}-\frac{m}{2}}(1+i\tan(c+dx))^{\frac{m+1}{2}}(e\cos(c+dx))^m \text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{m+1}{2},1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+}{dm\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[(e*Cos[c + d*x])^m*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-I)*2^(1/2 - m/2)*a*(e*Cos[c + d*x])^m*Hypergeometric2F1[-1/2*m, (1 + m)/
2, 1 - m/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^((1 + m)/2))/(d*m*
Sqrt[a + I*a*Tan[c + d*x]])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = ((e\cos(c+dx))^m(e\sec(c+dx))^m)\int(e\sec(c+dx))^{-m}\sqrt{a+ia\tan(c+dx)}\,dx$$

$$= \left((e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}\right)\int(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{\frac{1}{2}-\frac{m}{2}}\,dx$$

$$= \frac{\left(a^2(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2}\right)\text{Subst}\left(\int(a-iax)^{-1-\frac{m}{2}}(a+ia\right.}{d}$$

$$= \frac{\left(2^{-\frac{1}{2}-\frac{m}{2}}a^2(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{\frac{1}{2}+\frac{m}{2}}\right)\text{Subst}\left(\int\left(\frac{1}{2}+\frac{ix}{2}\right)^{-\frac{1}{2}-\frac{m}{2}}\right.}{d\sqrt{a+ia\tan(c+dx)}}$$

$$= -\frac{i2^{\frac{1}{2}-\frac{m}{2}}a(e\cos(c+dx))^m\,\text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{1+m}{2},1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan}{dm\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A] (verified)

Time = 2.50 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.16

$$\int(e\cos(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx$$
$$= \frac{i2^{-m}\left(1+e^{2i(c+dx)}\right)^{\frac{1}{2}-m}\left(ee^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^m\text{Hypergeometric2F1}\left(\frac{1}{2}-m,\frac{1-m}{2},\frac{3-m}{2},-e^{2i(c+dx)}\right)\sqrt{}}{d(-1+m)}$$

```
[In] Integrate[(e*Cos[c + d*x])^m*Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (I*(1 + E^((2*I)*(c + d*x)))^(1/2 - m)*((e*(1 + E^((2*I)*(c + d*x))))/E^(I*
(c + d*x)))^m*Hypergeometric2F1[1/2 - m, (1 - m)/2, (3 - m)/2, -E^((2*I)*(c
 + d*x))]*Sqrt[a + I*a*Tan[c + d*x]])/(2^m*d*(-1 + m))
```

Maple [F]

$$\int(e\cos(dx+c))^m\sqrt{a+ia\tan(dx+c)}dx$$

```
[In] int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] int((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int(e\cos(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx = \int\sqrt{i\,a\tan(dx+c)+a}(e\cos(dx+c))^m\,dx$$

```
[In] integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2)*(1/2*(e*e^(2*I*d*x + 2*I*c) + e)*e^(-I*d*x - I*c))^m*sqrt(
a/(e^(2*I*d*x + 2*I*c) + 1))*e^(I*d*x + I*c), x)
```

Sympy [F]

$$\int (e\cos(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx = \int (e\cos{(c+dx)})^m\,\sqrt{ia\,(\tan{(c+dx)}-i)}\,dx$$

```
[In] integrate((e*cos(d*x+c))**m*(a+I*a*tan(d*x+c))**(1/2),x)
[Out] Integral((e*cos(c + d*x))**m*sqrt(I*a*(tan(c + d*x) - I)), x)
```

Maxima [F]

$$\int (e\cos(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{i\,a\tan{(dx+c)}+a}(e\cos{(dx+c)})^m\,dx$$

```
[In] integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*(e*cos(d*x + c))^m, x)
```

Giac [F]

$$\int (e\cos(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx = \int \sqrt{i\,a\tan{(dx+c)}+a}(e\cos{(dx+c)})^m\,dx$$

```
[In] integrate((e*cos(d*x+c))^m*(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(I*a*tan(d*x + c) + a)*(e*cos(d*x + c))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (e\cos(c+dx))^m\sqrt{a+ia\tan(c+dx)}\,dx = \int (e\,\cos{(c+d\,x)})^m\,\sqrt{a+a\tan{(c+d\,x)}\,1\mathrm{i}}\,dx$$

```
[In] int((e*cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] int((e*cos(c + d*x))^m*(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.694 $\int \frac{(e\cos(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 104

$$\int \frac{(e\cos(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{i2^{-\frac{1}{2}-\frac{m}{2}}(e\cos(c+dx))^m \text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{3+m}{2},1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan(c+dx}{dm\sqrt{a+ia\tan(c+dx)}}$$

```
[Out] -I*2^(-1/2-1/2*m)*(e*cos(d*x+c))^m*hypergeom([-1/2*m, 3/2+1/2*m],[1-1/2*m],
1/2-1/2*I*tan(d*x+c))*(1+I*tan(d*x+c))^(1/2+1/2*m)/d/m/(a+I*a*tan(d*x+c))^(
1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3596, 3586, 3604, 72, 71}

$$\int \frac{(e\cos(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx = -\frac{i2^{-\frac{m}{2}-\frac{1}{2}}(1+i\tan(c+dx))^{\frac{m+1}{2}}(e\cos(c+dx))^m \text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{m+3}{2},1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+\right.}{dm\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Int[(e*Cos[c + d*x])^m/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] ((-I)*2^(-1/2 - m/2)*(e*Cos[c + d*x])^m*Hypergeometric2F1[-1/2*m, (3 + m)/2
, 1 - m/2, (1 - I*Tan[c + d*x])/2]*(1 + I*Tan[c + d*x])^((1 + m)/2))/(d*m*S
qrt[a + I*a*Tan[c + d*x]])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 3586

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_.), x_Symbol] :> Dist[(d*Sec[e + f*x])^m/((a + b*Tan[e + f*x])^(m/
2)*(a - b*Tan[e + f*x])^(m/2)), Int[(a + b*Tan[e + f*x])^(m/2 + n)*(a - b*T
an[e + f*x])^(m/2), x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 +
b^2, 0]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3604

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*tan[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a*(c/f), Subst[Int[(a + b*x)^(m - 1)*(c
 + d*x)^(n - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = ((e\cos(c+dx))^m(e\sec(c+dx))^m)\int\frac{(e\sec(c+dx))^{-m}}{\sqrt{a+ia\tan(c+dx)}}\,dx$$

$$= ((e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2})\int(a-ia\tan(c+dx))^{-m/2}(a+ia\tan(c+dx))^{-\frac{1}{2}-\frac{m}{2}}\,dx$$

$$= \frac{(a^2(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}(a+ia\tan(c+dx))^{m/2})\,\text{Subst}\Big(\int(a-iax)^{-1-\frac{m}{2}}(a+}{d}$$

$$= \frac{\left(2^{-\frac{3}{2}-\frac{m}{2}}a(e\cos(c+dx))^m(a-ia\tan(c+dx))^{m/2}\left(\frac{a+ia\tan(c+dx)}{a}\right)^{\frac{1}{2}+\frac{m}{2}}\right)\text{Subst}\Big(\int\left(\frac{1}{2}+\frac{ix}{2}\right)^{-\frac{3}{2}-\frac{m}{2}}(}{d\sqrt{a+ia\tan(c+dx)}}$$

$$= -\frac{i2^{-\frac{1}{2}-\frac{m}{2}}(e\cos(c+dx))^m\,\text{Hypergeometric2F1}\left(-\frac{m}{2},\frac{3+m}{2},1-\frac{m}{2},\frac{1}{2}(1-i\tan(c+dx))\right)(1+i\tan}{dm\sqrt{a+ia\tan(c+dx)}}$$

Mathematica [A] (verified)

Time = 2.72 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.38

$$\int\frac{(e\cos(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx = \frac{i4^{-m}\left(1+e^{2i(c+dx)}\right)\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^m\left(ee^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^m\cos^{-m}(c+dx)\,\text{Hypergeometric}}{d(1+m)\sqrt{a+ia\tan(c+dx)}}$$

```
[In] Integrate[(e*Cos[c + d*x])^m/Sqrt[a + I*a*Tan[c + d*x]],x]
```

```
[Out] (I*(1 + E^((2*I)*(c + d*x)))*((1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x)))^m*
((e*(1 + E^((2*I)*(c + d*x))))/E^(I*(c + d*x)))^m*Hypergeometric2F1[1, (2 +
 m)/2, (1 - m)/2, -E^((2*I)*(c + d*x))])/(4^m*d*(1 + m)*Cos[c + d*x]^m*Sqrt
[a + I*a*Tan[c + d*x]])
```

Maple [F]

$$\int\frac{(e\cos(dx+c))^m}{\sqrt{a+ia\tan(dx+c)}}dx$$

```
[In] int((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x)
```

```
[Out] int((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int \frac{(e\cos(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\cos(dx+c))^m}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

[In] integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(1/2*sqrt(2)*(1/2*(e*e^(2*I*d*x + 2*I*c) + e)*e^(-I*d*x - I*c))^m*sqrt(a/(e^(2*I*d*x + 2*I*c) + 1))*(e^(2*I*d*x + 2*I*c) + 1)*e^(-I*d*x - I*c)/a, x)

Sympy [F]

$$\int \frac{(e\cos(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\cos(c+dx))^m}{\sqrt{ia(\tan(c+dx)-i)}}\,dx$$

[In] integrate((e*cos(d*x+c))**m/(a+I*a*tan(d*x+c))**(1/2),x)

[Out] Integral((e*cos(c + d*x))**m/sqrt(I*a*(tan(c + d*x) - I)), x)

Maxima [F]

$$\int \frac{(e\cos(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\cos(dx+c))^m}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

[In] integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((e*cos(d*x + c))^m/sqrt(I*a*tan(d*x + c) + a), x)

Giac [F]

$$\int \frac{(e\cos(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\cos(dx+c))^m}{\sqrt{i\,a\tan(dx+c)+a}}\,dx$$

[In] integrate((e*cos(d*x+c))^m/(a+I*a*tan(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((e*cos(d*x + c))^m/sqrt(I*a*tan(d*x + c) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(e\cos(c+dx))^m}{\sqrt{a+ia\tan(c+dx)}}\,dx = \int \frac{(e\,\cos(c+d\,x))^m}{\sqrt{a+a\tan(c+d\,x)\,1i}}\,dx$$

```
[In] int((e*cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^(1/2),x)
[Out] int((e*cos(c + d*x))^m/(a + a*tan(c + d*x)*1i)^(1/2), x)
```

3.695 $\int (d\cos(e+fx))^m (a+b\tan(e+fx))^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 183

$$\int (d\cos(e+fx))^m (a+b\tan(e+fx))^3\,dx = -\frac{a(3b^2-a^2(1-m))\,(d\cos(e+fx))^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{3}{2},-\tan^2(e+fx)\right)\sec^2(e+fx)^{m/2}\tan(e+fx)}{f(1-m)} + \frac{b(d\cos(e+fx))^m(a+b\tan(e+fx))^2}{f(2-m)} + \frac{b(d\cos(e+fx))^m\,(2(b^2-a^2(3-m))\,(1-m)+ab(4-m)m\tan(e+fx))}{fm\,(2-3m+m^2)}$$

```
[Out] -a*(3*b^2-a^2*(1-m))*(d*cos(f*x+e))^m*hypergeom([1/2, 1+1/2*m],[3/2],-tan(f
*x+e)^2)*(sec(f*x+e)^2)^(1/2*m)*tan(f*x+e)/f/(1-m)+b*(d*cos(f*x+e))^m*(a+b*
tan(f*x+e))^2/f/(2-m)+b*(d*cos(f*x+e))^m*(2*(b^2-a^2*(3-m))*(1-m)+a*b*(4-m)
*m*tan(f*x+e))/f/(1-m)/(2-m)/m
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used

$= \{3596, 3593, 757, 794, 251\}$

$$\int (d\cos(e+fx))^m (a+b\tan(e+fx))^3\,dx$$

$$= \frac{a\left(a^2-\frac{3b^2}{1-m}\right)\tan(e+fx)\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{3}{2},-\tan^2(e+fx\right.}{f} + \frac{b(d\cos(e+fx))^m\left(2(1-m)\left(b^2-a^2(3-m)\right)+ab(4-m)m\tan(e+fx)\right)}{fm\left(m^2-3m+2\right)} + \frac{b(a+b\tan(e+fx))^2(d\cos(e+fx))^m}{f(2-m)}$$

```
[In] Int[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (a*(a^2 - (3*b^2)/(1 - m))*(d*Cos[e + f*x])^m*Hypergeometric2F1[1/2, (2 + m
)/2, 3/2, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x])/f + (b*(d*C
os[e + f*x])^m*(a + b*Tan[e + f*x])^2)/(f*(2 - m)) + (b*(d*Cos[e + f*x])^m*
(2*(b^2 - a^2*(3 - m))*(1 - m) + a*b*(4 - m)*m*Tan[e + f*x]))/(f*m*(2 - 3*m
 + m^2))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
```

```
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = ((d\cos(e+fx))^m(d\sec(e+fx))^m)\int(d\sec(e+fx))^{-m}(a+b\tan(e+fx))^3\,dx$$

$$= \frac{((d\cos(e+fx))^m\sec^2(e+fx)^{m/2})\,\text{Subst}\left(\int(a+x)^3\left(1+\frac{x^2}{b^2}\right)^{-1-\frac{m}{2}}\,dx,x,b\tan(e+fx)\right)}{bf}$$

$$= \frac{b(d\cos(e+fx))^m(a+b\tan(e+fx))^2}{f(2-m)} + \frac{(b(d\cos(e+fx))^m\sec^2(e+fx)^{m/2})\,\text{Subst}\left(\int(a+x)\left(-2+\frac{a^2(2-m)}{b^2}+\frac{a(4-m)x}{b^2}\right)\left(1+\frac{x^2}{b^2}\right)^{-1-\frac{m}{2}}\,d}{f(2-m)}$$

$$= \frac{b(d\cos(e+fx))^m(a+b\tan(e+fx))^2}{f(2-m)} + \frac{b(d\cos(e+fx))^m\left(2(b^2-a^2(3-m))(1-m)+ab(4-m)m\tan(e+fx)\right)}{fm(2-3m+m^2)} - \frac{(a(3b^2-a^2(1-m))(d\cos(e+fx))^m\sec^2(e+fx)^{m/2})\,\text{Subst}\left(\int\left(1+\frac{x^2}{b^2}\right)^{-1-\frac{m}{2}}\,dx,x,b\tan(e+\right.}{bf(1-m)}$$

$$= -\frac{a(3b^2-a^2(1-m))(d\cos(e+fx))^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{3}{2},-\tan^2(e+fx)\right)\sec^2(e+f}{f(1-m)} + \frac{b(d\cos(e+fx))^m(a+b\tan(e+fx))^2}{f(2-m)} + \frac{b(d\cos(e+fx))^m\left(2(b^2-a^2(3-m))(1-m)+ab(4-m)m\tan(e+fx)\right)}{fm(2-3m+m^2)}$$

Mathematica [A] (verified)

Time = 6.57 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.16

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^3\,dx = \frac{\cos(e+fx)(d\cos(e+fx))^m\left(-\frac{b^3}{-2+m}+\frac{b(-3a^2+b^2)\cos^2(e+fx)}{m}-\frac{a(a^2-3b^2)\cos^3(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cdots\right)}{(1+m)\sqrt{\sin^2(e+fx)}}\cdots\right.}{f(a\cos(e+fx)+b\sin\cdots}$$

```
[In] Integrate[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x])^3,x]
```

```
[Out] (Cos[e + f*x]*(d*Cos[e + f*x])^m*(-(b^3/(-2 + m)) + (b*(-3*a^2 + b^2)*Cos[e
 + f*x]^2)/m - (a*(a^2 - 3*b^2)*Cos[e + f*x]^3*Hypergeometric2F1[1/2, (1 +
m)/2, (3 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/((1 + m)*Sqrt[Sin[e + f*x]^2
]) - (3*a*b^2*Hypergeometric2F1[1/2, (-1 + m)/2, (1 + m)/2, Cos[e + f*x]^2]
*Sin[2*(e + f*x)])/(2*(-1 + m)*Sqrt[Sin[e + f*x]^2]))*(a + b*Tan[e + f*x])^
3)/(f*(a*Cos[e + f*x] + b*Sin[e + f*x])^3)
```

Maple [F]

$$\int \left(d\cos\left(fx+e\right)\right)^m\left(a+b\tan\left(fx+e\right)\right)^3\,dx$$

```
[In] int((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^3,x)
```

```
[Out] int((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^3,x)
```

Fricas [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^3\,dx = \int \left(b\tan\left(fx+e\right)+a\right)^3\left(d\cos\left(fx+e\right)\right)^m\,dx$$

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*tan(f*x + e)^3 + 3*a*b^2*tan(f*x + e)^2 + 3*a^2*b*tan(f*x + e
) + a^3)*(d*cos(f*x + e))^m, x)
```

Sympy [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^3\,dx = \int (d\cos{(e+fx)})^m\,(a+b\tan{(e+fx)})^3\,dx$$

```
[In] integrate((d*cos(f*x+e))**m*(a+b*tan(f*x+e))**3,x)
[Out] Integral((d*cos(e + f*x))**m*(a + b*tan(e + f*x))**3, x)
```

Maxima [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^3\,dx = \int (b\tan{(fx+e)}+a)^3(d\cos{(fx+e)})^m\,dx$$

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^3,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e) + a)^3*(d*cos(f*x + e))^m, x)
```

Giac [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^3\,dx = \int (b\tan{(fx+e)}+a)^3(d\cos{(fx+e)})^m\,dx$$

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^3,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)^3*(d*cos(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^3\,dx = \int (d\,\cos{(e+f\,x)})^m\,(a+b\tan(e+f\,x))^3\,dx$$

```
[In] int((d*cos(e + f*x))^m*(a + b*tan(e + f*x))^3,x)
[Out] int((d*cos(e + f*x))^m*(a + b*tan(e + f*x))^3, x)
```

3.696 $\int (d\cos(e+fx))^m(a+b\tan(e+fx))^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 155

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^2\,dx = -\frac{ab(2-m)(d\cos(e+fx))^m}{f(1-m)m} + \frac{(b^2-a^2(1-m))\cos(e+fx)(d\cos(e+fx))^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(e+fx)\right)\sin(e+}{f(1-m)(1+m)\sqrt{\sin^2(e+fx)}} + \frac{b(d\cos(e+fx))^m(a+b\tan(e+fx))}{f(1-m)}$$

```
[Out] -a*b*(2-m)*(d*cos(f*x+e))^m/f/(1-m)/m+(b^2-a^2*(1-m))*cos(f*x+e)*(d*cos(f*x
+e))^m*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],cos(f*x+e)^2)*sin(f*x+e)/f/(-
m^2+1)/(sin(f*x+e)^2)^(1/2)+b*(d*cos(f*x+e))^m*(a+b*tan(f*x+e))/f/(1-m)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3596, 3589, 3567, 3857, 2722}

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^2\,dx = \frac{(b^2-a^2(1-m))\sin(e+fx)\cos(e+fx)(d\cos(e+fx))^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\cos^2(e+fx\right.}{f(1-m)(m+1)\sqrt{\sin^2(e+fx)}} - \frac{ab(2-m)(d\cos(e+fx))^m}{f(1-m)m} + \frac{b(a+b\tan(e+fx))(d\cos(e+fx))^m}{f(1-m)}$$

```
[In] Int[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] -((a*b*(2 - m)*(d*Cos[e + f*x])^m)/(f*(1 - m)*m)) + ((b^2 - a^2*(1 - m))*Co
s[e + f*x]*(d*Cos[e + f*x])^m*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2,
Cos[e + f*x]^2]*Sin[e + f*x])/(f*(1 - m)*(1 + m)*Sqrt[Sin[e + f*x]^2]) + (b
*(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x]))/(f*(1 - m))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3589

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^2, x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])/(f*(m
 + 1))), x] + Dist[1/(m + 1), Int[(d*Sec[e + f*x])^m*(a^2*(m + 1) - b^2 + a
*b*(m + 2)*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, m}, x] && NeQ[a^2
 + b^2, 0] && NeQ[m, -1]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left((d\cos(e+fx))^m(d\sec(e+fx))^m\right)\int(d\sec(e+fx))^{-m}(a+b\tan(e+fx))^2\,dx\\ &= \frac{b(d\cos(e+fx))^m(a+b\tan(e+fx))}{f(1-m)}\\ &\quad+\frac{\left((d\cos(e+fx))^m(d\sec(e+fx))^m\right)\int(d\sec(e+fx))^{-m}\left(-b^2+a^2(1-m)+ab(2-m)\tan(e+\right.}{1-m}\end{aligned}$$

$$= -\frac{ab(2-m)(d\cos(e+fx))^m}{f(1-m)m} + \frac{b(d\cos(e+fx))^m(a+b\tan(e+fx))}{f(1-m)} + \frac{((-b^2+a^2(1-m)))\,(d\cos(e+fx))^m(d\sec(e+fx))^m)\int(d\sec(e+fx))^{-m}\,dx}{1-m}$$

$$= -\frac{ab(2-m)(d\cos(e+fx))^m}{f(1-m)m} + \frac{b(d\cos(e+fx))^m(a+b\tan(e+fx))}{f(1-m)} + \frac{\left((-b^2+a^2(1-m))\left(\frac{\cos(e+fx)}{d}\right)^{-m}(d\cos(e+fx))^m\right)\int\left(\frac{\cos(e+fx)}{d}\right)^m\,dx}{1-m}$$

$$= -\frac{ab(2-m)(d\cos(e+fx))^m}{f(1-m)m} + \frac{(b^2-a^2(1-m))\cos(e+fx)(d\cos(e+fx))^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(e+fx)\right)}{f(1-m)(1+m)\sqrt{\sin^2(e+fx)}} + \frac{b(d\cos(e+fx))^m(a+b\tan(e+fx))}{f(1-m)}$$

Mathematica [A] (verified)

Time = 5.48 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.90

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^2\,dx = \frac{(d\cos(e+fx))^m\left(2ab(-1+\sec^2(e+fx)^{m/2}) + b^2m\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m}{2},\frac{3}{2},-\tan^2(e+fx)\right)\sec^2(e\right.}{}$$

```
[In] Integrate[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x])^2,x]
```

```
[Out] ((d*Cos[e + f*x])^m*(2*a*b*(-1 + (Sec[e + f*x]^2)^(m/2)) + b^2*m*Hypergeome
tric2F1[1/2, m/2, 3/2, -Tan[e + f*x]^2]*(Sec[e + f*x]^2)^(m/2)*Tan[e + f*x]
 + (a^2 - b^2)*m*Hypergeometric2F1[1/2, (2 + m)/2, 3/2, -Tan[e + f*x]^2]*(S
ec[e + f*x]^2)^(m/2)*Tan[e + f*x]))/(f*m)
```

Maple [F]

$$\int (d\cos(fx+e))^m\,(a+b\tan(fx+e))^2\,dx$$

```
[In] int((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^2,x)
```

```
[Out] int((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^2,x)
```

Fricas [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^2\,dx = \int (b\tan(fx+e)+a)^2(d\cos(fx+e))^m\,dx$$

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^2,x, algorithm="fricas")
[Out] integral((b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)*(d*cos(f*x + e))^m
, x)
```

Sympy [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^2\,dx = \int (d\cos(e+fx))^m\,(a+b\tan(e+fx))^2\,dx$$

```
[In] integrate((d*cos(f*x+e))**m*(a+b*tan(f*x+e))**2,x)
[Out] Integral((d*cos(e + f*x))**m*(a + b*tan(e + f*x))**2, x)
```

Maxima [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^2\,dx = \int (b\tan(fx+e)+a)^2(d\cos(fx+e))^m\,dx$$

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e) + a)^2*(d*cos(f*x + e))^m, x)
```

Giac [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^2\,dx = \int (b\tan(fx+e)+a)^2(d\cos(fx+e))^m\,dx$$

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^2,x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)^2*(d*cos(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^2\,dx = \int (d\,\cos(e+f\,x))^m\,(a+b\tan(e+f\,x))^2\,dx$$

```
[In] int((d*cos(e + f*x))^m*(a + b*tan(e + f*x))^2,x)
[Out] int((d*cos(e + f*x))^m*(a + b*tan(e + f*x))^2, x)
```

3.697 $\int (d\cos(e+fx))^m(a+b\tan(e+fx))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 90

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))\,dx = -\frac{b(d\cos(e+fx))^m}{fm} - \frac{a(d\cos(e+fx))^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(e+fx)\right)\sin(e+fx)}{df(1+m)\sqrt{\sin^2(e+fx)}}$$

```
[Out] -b*(d*cos(f*x+e))^m/f/m-a*(d*cos(f*x+e))^(1+m)*hypergeom([1/2, 1/2+1/2*m],[
3/2+1/2*m],cos(f*x+e)^2)*sin(f*x+e)/d/f/(1+m)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3596, 3567, 3857, 2722}

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))\,dx = -\frac{a\sin(e+fx)\cos(e+fx)(d\cos(e+fx))^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\cos^2(e+fx)\right)}{f(m+1)\sqrt{\sin^2(e+fx)}} - \frac{b(d\cos(e+fx))^m}{fm}$$

```
[In] Int[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x]),x]
```

```
[Out] -((b*(d*Cos[e + f*x])^m)/(f*m)) - (a*Cos[e + f*x]*(d*Cos[e + f*x])^m*Hyperg
eometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(f*(1
+ m)*Sqrt[Sin[e + f*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] && !IntegerQ[m]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left((d\cos(e+fx))^m(d\sec(e+fx))^m\right)\int (d\sec(e+fx))^{-m}(a+b\tan(e+fx))\,dx\\
&= -\frac{b(d\cos(e+fx))^m}{fm}+\left(a(d\cos(e+fx))^m(d\sec(e+fx))^m\right)\int (d\sec(e+fx))^{-m}\,dx\\
&= -\frac{b(d\cos(e+fx))^m}{fm}+\left(a\left(\frac{\cos(e+fx)}{d}\right)^{-m}(d\cos(e+fx))^m\right)\int \left(\frac{\cos(e+fx)}{d}\right)^m dx\\
&= -\frac{b(d\cos(e+fx))^m}{fm}\\
&\quad -\frac{a\cos(e+fx)(d\cos(e+fx))^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(e+fx)\right)\sin(e+fx)}{f(1+m)\sqrt{\sin^2(e+fx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.83

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))\,dx = -\frac{(d\cos(e+fx))^m\left(b+bm+am\cot(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},\cos^2(e+fx)\right)\sqrt{\sin^2(e+}\right.}{fm(1+m)}$$

```
[In] Integrate[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x]),x]
[Out] -(((d*Cos[e + f*x])^m*(b + b*m + a*m*Cot[e + f*x]*Hypergeometric2F1[1/2, (1
 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*Sqrt[Sin[e + f*x]^2]))/(f*m*(1 + m)))
```

Maple [F]

$$\int (d\cos(fx+e))^m (a+b\tan(fx+e))\,dx$$

```
[In] int((d*cos(f*x+e))^m*(a+b*tan(f*x+e)),x)
[Out] int((d*cos(f*x+e))^m*(a+b*tan(f*x+e)),x)
```

Fricas [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))\,dx = \int (b\tan(fx+e)+a)(d\cos(fx+e))^m\,dx$$

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e)),x, algorithm="fricas")
[Out] integral((b*tan(f*x + e) + a)*(d*cos(f*x + e))^m, x)
```

Sympy [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))\,dx = \int (d\cos(e+fx))^m\,(a+b\tan(e+fx))\,dx$$

```
[In] integrate((d*cos(f*x+e))**m*(a+b*tan(f*x+e)),x)
[Out] Integral((d*cos(e + f*x))**m*(a + b*tan(e + f*x)), x)
```

Maxima [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))\,dx = \int (b\tan(fx+e)+a)(d\cos(fx+e))^m\,dx$$

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e)),x, algorithm="maxima")
[Out] integrate((b*tan(f*x + e) + a)*(d*cos(f*x + e))^m, x)
```

Giac [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))\,dx = \int (b\tan(fx+e)+a)(d\cos(fx+e))^m\,dx$$

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e)),x, algorithm="giac")
[Out] integrate((b*tan(f*x + e) + a)*(d*cos(f*x + e))^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))\,dx = \int (d\,\cos(e+f\,x))^m\,(a+b\tan(e+f\,x))\,dx$$

```
[In] int((d*cos(e + f*x))^m*(a + b*tan(e + f*x)),x)
[Out] int((d*cos(e + f*x))^m*(a + b*tan(e + f*x)), x)
```

3.698 $\int \frac{(d\cos(e+fx))^m}{a+b\tan(e+fx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 140

$$\int \frac{(d\cos(e+fx))^m}{a+b\tan(e+fx)}\,dx=\frac{b(d\cos(e+fx))^m\,\text{Hypergeometric2F1}\left(1,-\frac{m}{2},1-\frac{m}{2},\frac{b^2\sec^2(e+fx)}{a^2+b^2}\right)}{(a^2+b^2)\,fm}+\frac{\text{AppellF1}\left(\frac{1}{2},1,\frac{2+m}{2},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\cos(e+fx))^m\sec^2(e+fx)^{m/2}\tan(e+fx)}{af}$$

```
[Out] b*(d*cos(f*x+e))^m*hypergeom([1, -1/2*m],[1-1/2*m],b^2*sec(f*x+e)^2/(a^2+b^
2))/(a^2+b^2)/f/m+AppellF1(1/2,1,1+1/2*m,3/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+
e)^2)*(d*cos(f*x+e))^m*(sec(f*x+e)^2)^(1/2*m)*tan(f*x+e)/a/f
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3596, 3593, 771, 440, 455, 70}

$$\int \frac{(d\cos(e+fx))^m}{a+b\tan(e+fx)}\,dx$$
$$=\frac{\tan(e+fx)\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m\,\text{AppellF1}\left(\frac{1}{2},1,\frac{m+2}{2},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{af}+\frac{b(d\cos(e+fx))^m\,\text{Hypergeometric2F1}\left(1,-\frac{m}{2},1-\frac{m}{2},\frac{b^2\sec^2(e+fx)}{a^2+b^2}\right)}{fm\,(a^2+b^2)}$$

```
[In] Int[(d*Cos[e + f*x])^m/(a + b*Tan[e + f*x]),x]
```

```
[Out] (b*(d*Cos[e + f*x])^m*Hypergeometric2F1[1, -1/2*m, 1 - m/2, (b^2*Sec[e + f*
x]^2)/(a^2 + b^2)])/((a^2 + b^2)*f*m) + (AppellF1[1/2, 1, (2 + m)/2, 3/2, (
b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Cos[e + f*x])^m*(Sec[e + f*x]^
2)^(m/2)*Tan[e + f*x])/(a*f)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = ((d\cos(e+fx))^m(d\sec(e+fx))^m)\int \frac{(d\sec(e+fx))^{-m}}{a+b\tan(e+fx)}\,dx$$

$$= \frac{((d\cos(e+fx))^m\sec^2(e+fx)^{m/2})\,\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^{-1-\frac{m}{2}}}{a+x}\,dx, x, b\tan(e+fx)\right)}{bf}$$

$$= \frac{((d\cos(e+fx))^m\sec^2(e+fx)^{m/2})\,\text{Subst}\left(\int \left(\frac{a\left(1+\frac{x^2}{b^2}\right)^{-1-\frac{m}{2}}}{a^2-x^2}+\frac{x\left(1+\frac{x^2}{b^2}\right)^{-1-\frac{m}{2}}}{-a^2+x^2}\right)\,dx, x, b\tan(e+fx)\right)}{bf}$$

$$= \frac{((d\cos(e+fx))^m\sec^2(e+fx)^{m/2})\,\text{Subst}\left(\int \frac{x\left(1+\frac{x^2}{b^2}\right)^{-1-\frac{m}{2}}}{-a^2+x^2}\,dx, x, b\tan(e+fx)\right)}{bf} + \frac{(a(d\cos(e+fx))^m\sec^2(e+fx)^{m/2})\,\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^{-1-\frac{m}{2}}}{a^2-x^2}\,dx, x, b\tan(e+fx)\right)}{bf}$$

$$= \frac{\text{AppellF1}\left(\frac{1}{2},1,\frac{2+m}{2},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\cos(e+fx))^m\sec^2(e+fx)^{m/2}\tan(e+fx)}{af} + \frac{((d\cos(e+fx))^m\sec^2(e+fx)^{m/2})\,\text{Subst}\left(\int \frac{\left(1+\frac{x}{b^2}\right)^{-1-\frac{m}{2}}}{-a^2+x}\,dx, x, b^2\tan^2(e+fx)\right)}{2bf}$$

$$= \frac{b(d\cos(e+fx))^m\,\text{Hypergeometric2F1}\left(1,-\frac{m}{2},1-\frac{m}{2},\frac{b^2\sec^2(e+fx)}{a^2+b^2}\right)}{(a^2+b^2)\,fm} + \frac{\text{AppellF1}\left(\frac{1}{2},1,\frac{2+m}{2},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\cos(e+fx))^m\sec^2(e+fx)^{m/2}\tan(e+f}{af}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 14.21 (sec) , antiderivative size = 1132, normalized size of antiderivative = 8.09

$$\int \frac{(d\cos(e+fx))^m}{a+b\tan(e+fx)}\,dx$$

$$= \frac{}{f(a+b\tan(e+fx))\left(am\,\text{Hypergeometric2F1}\left(\frac{1}{2},1+\frac{m}{2},\frac{3}{2},-\tan^2(e+fx)\right)\sec^2(e+fx)-bm\sec^2(e+f\right.}$$

```
[In] Integrate[(d*Cos[e + f*x])^m/(a + b*Tan[e + f*x]),x]
```

```
[Out] ((d*Cos[e + f*x])^m*(b*(-1 + (Sec[e + f*x]^2)^(-1/2*m)) + a*m*Hypergeometri
c2F1[1/2, 1 + m/2, 3/2, -Tan[e + f*x]^2]*Tan[e + f*x] - (b*AppellF1[m, m/2,
 m/2, 1 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])
]*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I + Tan[e + f*x
]))/(a + b*Tan[e + f*x]))^(m/2))/(Sec[e + f*x]^2)^(m/2)))/(f*(a + b*Tan[e +
 f*x])*(a*m*Hypergeometric2F1[1/2, 1 + m/2, 3/2, -Tan[e + f*x]^2]*Sec[e + f
*x]^2 - (b*m*Tan[e + f*x])/(Sec[e + f*x]^2)^(m/2) + (b*m*AppellF1[m, m/2, m
/2, 1 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*
Tan[e + f*x]*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I +
Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2))/(Sec[e + f*x]^2)^(m/2) - (b*((b
*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I + Tan[e + f*x]))/(
a + b*Tan[e + f*x]))^(m/2)*(-1/2*((a - I*b)*b*m^2*AppellF1[1 + m, 1 + m/2,
m/2, 2 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]
*Sec[e + f*x]^2)/((1 + m)*(a + b*Tan[e + f*x])^2) - ((a + I*b)*b*m^2*Appell
F1[1 + m, m/2, 1 + m/2, 2 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a
 + b*Tan[e + f*x])]*Sec[e + f*x]^2)/(2*(1 + m)*(a + b*Tan[e + f*x])^2)))/(S
ec[e + f*x]^2)^(m/2) - (b*m*AppellF1[m, m/2, m/2, 1 + m, (a - I*b)/(a + b*T
an[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*((b*(-I + Tan[e + f*x]))/(a +
 b*Tan[e + f*x]))^(-1 + m/2)*((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^
(m/2)*(-((b^2*Sec[e + f*x]^2*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x])^2) +
 (b*Sec[e + f*x]^2)/(a + b*Tan[e + f*x])))/(2*(Sec[e + f*x]^2)^(m/2)) - (b*
m*AppellF1[m, m/2, m/2, 1 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a
 + b*Tan[e + f*x])]*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((
b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(-1 + m/2)*(-((b^2*Sec[e + f*x]
^2*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x])^2) + (b*Sec[e + f*x]^2)/(a + b*
Tan[e + f*x])))/(2*(Sec[e + f*x]^2)^(m/2)) + a*m*Sec[e + f*x]^2*(-Hypergeom
etric2F1[1/2, 1 + m/2, 3/2, -Tan[e + f*x]^2] + (1 + Tan[e + f*x]^2)^(-1 - m
/2))))
```

Maple [F]

$$\int \frac{(d\cos(fx+e))^m}{a+b\tan(fx+e)}dx$$

```
[In] int((d*cos(f*x+e))^m/(a+b*tan(f*x+e)),x)
```

```
[Out] int((d*cos(f*x+e))^m/(a+b*tan(f*x+e)),x)
```

Fricas [F]

$$\int \frac{(d\cos(e+fx))^m}{a+b\tan(e+fx)}\,dx = \int \frac{(d\cos{(fx+e)})^m}{b\tan{(fx+e)}+a}\,dx$$

```
[In] integrate((d*cos(f*x+e))^m/(a+b*tan(f*x+e)),x, algorithm="fricas")
[Out] integral((d*cos(f*x + e))^m/(b*tan(f*x + e) + a), x)
```

Sympy [F]

$$\int \frac{(d\cos(e+fx))^m}{a+b\tan(e+fx)}\,dx = \int \frac{(d\cos{(e+fx)})^m}{a+b\tan{(e+fx)}}\,dx$$

```
[In] integrate((d*cos(f*x+e))**m/(a+b*tan(f*x+e)),x)
[Out] Integral((d*cos(e + f*x))**m/(a + b*tan(e + f*x)), x)
```

Maxima [F]

$$\int \frac{(d\cos(e+fx))^m}{a+b\tan(e+fx)}\,dx = \int \frac{(d\cos{(fx+e)})^m}{b\tan{(fx+e)}+a}\,dx$$

```
[In] integrate((d*cos(f*x+e))^m/(a+b*tan(f*x+e)),x, algorithm="maxima")
[Out] integrate((d*cos(f*x + e))^m/(b*tan(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{(d\cos(e+fx))^m}{a+b\tan(e+fx)}\,dx = \int \frac{(d\cos{(fx+e)})^m}{b\tan{(fx+e)}+a}\,dx$$

```
[In] integrate((d*cos(f*x+e))^m/(a+b*tan(f*x+e)),x, algorithm="giac")
[Out] integrate((d*cos(f*x + e))^m/(b*tan(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\cos(e+fx))^m}{a+b\tan(e+fx)}\,dx = \int \frac{(d\,\cos\left(e+f\,x\right))^m}{a+b\tan\left(e+f\,x\right)}\,dx$$

```
[In] int((d*cos(e + f*x))^m/(a + b*tan(e + f*x)),x)
[Out] int((d*cos(e + f*x))^m/(a + b*tan(e + f*x)), x)
```

3.699 $\int \frac{(d\cos(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 227

$$\int \frac{(d\cos(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx = \frac{2ab(d\cos(e+fx))^m \operatorname{Hypergeometric2F1}\left(2,-\frac{m}{2},1-\frac{m}{2},\frac{b^2\sec^2(e+fx)}{a^2+b^2}\right)}{(a^2+b^2)^2 fm} + \frac{\operatorname{AppellF1}\left(\frac{1}{2},2,\frac{2+m}{2},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\cos(e+fx))^m \sec^2(e+fx)^{m/2}\tan(e+fx)}{a^2 f} + \frac{b^2 \operatorname{AppellF1}\left(\frac{3}{2},2,\frac{2+m}{2},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)(d\cos(e+fx))^m \sec^2(e+fx)^{m/2}\tan^3(e+fx)}{3a^4 f}$$

```
[Out] 2*a*b*(d*cos(f*x+e))^m*hypergeom([2, -1/2*m],[1-1/2*m],b^2*sec(f*x+e)^2/(a^
2+b^2))/(a^2+b^2)^2/f/m+AppellF1(1/2,2,1+1/2*m,3/2,b^2*tan(f*x+e)^2/a^2,-ta
n(f*x+e)^2)*(d*cos(f*x+e))^m*(sec(f*x+e)^2)^(1/2*m)*tan(f*x+e)/a^2/f+1/3*b^
2*AppellF1(3/2,2,1+1/2*m,5/2,b^2*tan(f*x+e)^2/a^2,-tan(f*x+e)^2)*(d*cos(f*x
+e))^m*(sec(f*x+e)^2)^(1/2*m)*tan(f*x+e)^3/a^4/f
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used

$= \{3596, 3593, 771, 440, 455, 70, 524\}$

$$\int \frac{(d\cos(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx$$
$$= \frac{\tan(e+fx)\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m \operatorname{AppellF1}\left(\frac{1}{2},2,\frac{m+2}{2},\frac{3}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{a^2f}$$
$$+ \frac{2ab(d\cos(e+fx))^m \operatorname{Hypergeometric2F1}\left(2,-\frac{m}{2},1-\frac{m}{2},\frac{b^2\sec^2(e+fx)}{a^2+b^2}\right)}{fm\,(a^2+b^2)^2}$$
$$+ \frac{b^2\tan^3(e+fx)\sec^2(e+fx)^{m/2}(d\cos(e+fx))^m \operatorname{AppellF1}\left(\frac{3}{2},2,\frac{m+2}{2},\frac{5}{2},\frac{b^2\tan^2(e+fx)}{a^2},-\tan^2(e+fx)\right)}{3a^4f}$$

```
[In] Int[(d*Cos[e + f*x])^m/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] (2*a*b*(d*Cos[e + f*x])^m*Hypergeometric2F1[2, -1/2*m, 1 - m/2, (b^2*Sec[e
+ f*x]^2)/(a^2 + b^2)])/((a^2 + b^2)^2*f*m) + (AppellF1[1/2, 2, (2 + m)/2,
3/2, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Cos[e + f*x])^m*(Sec[e +
 f*x]^2)^(m/2)*Tan[e + f*x])/(a^2*f) + (b^2*AppellF1[3/2, 2, (2 + m)/2, 5/2
, (b^2*Tan[e + f*x]^2)/a^2, -Tan[e + f*x]^2]*(d*Cos[e + f*x])^m*(Sec[e + f*
x]^2)^(m/2)*Tan[e + f*x]^3)/(3*a^4*f)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
```

```
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 771

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - e*(x/(d^2 - e^2*x^2)))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = ((d\cos(e+fx))^m(d\sec(e+fx))^m)\int\frac{(d\sec(e+fx))^{-m}}{(a+b\tan(e+fx))^2}\,dx$$

$$=\frac{((d\cos(e+fx))^m\sec^2(e+fx)^{m/2})\,\text{Subst}\left(\int\frac{\left(1+\frac{x^2}{b^2}\right)^{-1-\frac{m}{2}}}{(a+x)^2}\,dx,x,b\tan(e+fx)\right)}{bf}$$

$$=\frac{((d\cos(e+fx))^m\sec^2(e+fx)^{m/2})\,\text{Subst}\left(\int\left(\frac{a^2\left(1+\frac{x^2}{b^2}\right)^{-1-\frac{m}{2}}}{(a^2-x^2)^2}-\frac{2ax\left(1+\frac{x^2}{b^2}\right)^{-1-\frac{m}{2}}}{(a^2-x^2)^2}+\frac{x^2\left(1+\frac{x^2}{b^2}\right)^{-1-\frac{m}{2}}}{(-a^2+x^2)^2}\right)\,d}{bf}$$

$$= \frac{((d\cos(e+fx))^m \sec^2(e+fx)^{m/2})\,\text{Subst}\left(\int \frac{x^2\left(1+\frac{x^2}{b^2}\right)^{-1-\frac{m}{2}}}{(-a^2+x^2)^2}\,dx, x, b\tan(e+fx)\right)}{bf} - \frac{(2a(d\cos(e+fx))^m \sec^2(e+fx)^{m/2})\,\text{Subst}\left(\int \frac{x\left(1+\frac{x^2}{b^2}\right)^{-1-\frac{m}{2}}}{(a^2-x^2)^2}\,dx, x, b\tan(e+fx)\right)}{bf} + \frac{(a^2(d\cos(e+fx))^m \sec^2(e+fx)^{m/2})\,\text{Subst}\left(\int \frac{\left(1+\frac{x^2}{b^2}\right)^{-1-\frac{m}{2}}}{(a^2-x^2)^2}\,dx, x, b\tan(e+fx)\right)}{bf}$$

$$= \frac{\text{AppellF1}\left(\frac{1}{2}, 2, \frac{2+m}{2}, \frac{3}{2}, \frac{b^2\tan^2(e+fx)}{a^2}, -\tan^2(e+fx)\right)(d\cos(e+fx))^m \sec^2(e+fx)^{m/2}\tan(e+fx}{a^2 f} + \frac{b^2\,\text{AppellF1}\left(\frac{3}{2}, 2, \frac{2+m}{2}, \frac{5}{2}, \frac{b^2\tan^2(e+fx)}{a^2}, -\tan^2(e+fx)\right)(d\cos(e+fx))^m \sec^2(e+fx)^{m/2}\tan^3(e}{3a^4 f} - \frac{(a(d\cos(e+fx))^m \sec^2(e+fx)^{m/2})\,\text{Subst}\left(\int \frac{\left(1+\frac{x}{b^2}\right)^{-1-\frac{m}{2}}}{(a^2-x)^2}\,dx, x, b^2\tan^2(e+fx)\right)}{bf}$$

$$= \frac{2ab(d\cos(e+fx))^m\,\text{Hypergeometric2F1}\left(2, -\frac{m}{2}, 1-\frac{m}{2}, \frac{b^2\sec^2(e+fx)}{a^2+b^2}\right)}{(a^2+b^2)^2 fm} + \frac{\text{AppellF1}\left(\frac{1}{2}, 2, \frac{2+m}{2}, \frac{3}{2}, \frac{b^2\tan^2(e+fx)}{a^2}, -\tan^2(e+fx)\right)(d\cos(e+fx))^m \sec^2(e+fx)^{m/2}\tan(e+}{a^2 f} + \frac{b^2\,\text{AppellF1}\left(\frac{3}{2}, 2, \frac{2+m}{2}, \frac{5}{2}, \frac{b^2\tan^2(e+fx)}{a^2}, -\tan^2(e+fx)\right)(d\cos(e+fx))^m \sec^2(e+fx)^{m/2}\tan^3(e}{3a^4 f}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 17.71 (sec) , antiderivative size = 2502, normalized size of antiderivative = 11.02

$$\int \frac{(d\cos(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(d*Cos[e + f*x])^m/(a + b*Tan[e + f*x])^2,x]
```

```
[Out] ((d*Cos[e + f*x])^m*((2*a*b*(-1 + (Sec[e + f*x]^2)^(-1/2*m)))/m + a^2*Hyper
geometric2F1[1/2, 1 + m/2, 3/2, -Tan[e + f*x]^2]*Tan[e + f*x] - b^2*Hyperge
ometric2F1[1/2, 1 + m/2, 3/2, -Tan[e + f*x]^2]*Tan[e + f*x] - (2*a*b*Appell
F1[m, m/2, m/2, 1 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan
```

[e + f*x])]*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I + T
an[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2))/(m*(Sec[e + f*x]^2)^(m/2)) - (b*
(a^2 + b^2)*AppellF1[1 + m, m/2, m/2, 2 + m, (a - I*b)/(a + b*Tan[e + f*x])
, (a + I*b)/(a + b*Tan[e + f*x])]*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f
x]))^(m/2)((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2))/((1 + m)*(
Sec[e + f*x]^2)^(m/2)*(a + b*Tan[e + f*x]))))/(f*(a + b*Tan[e + f*x])^2*(a^
2*Hypergeometric2F1[1/2, 1 + m/2, 3/2, -Tan[e + f*x]^2]*Sec[e + f*x]^2 - b^
2*Hypergeometric2F1[1/2, 1 + m/2, 3/2, -Tan[e + f*x]^2]*Sec[e + f*x]^2 - (2
*a*b*Tan[e + f*x])/(Sec[e + f*x]^2)^(m/2) + (2*a*b*AppellF1[m, m/2, m/2, 1
+ m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*Tan[e
+ f*x]*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I + Tan[e
+ f*x]))/(a + b*Tan[e + f*x]))^(m/2))/(Sec[e + f*x]^2)^(m/2) + (b^2*(a^2 +
b^2)*AppellF1[1 + m, m/2, m/2, 2 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a +
I*b)/(a + b*Tan[e + f*x])]*(Sec[e + f*x]^2)^(1 - m/2)*((b*(-I + Tan[e + f*x
]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]
))^(m/2))/((1 + m)*(a + b*Tan[e + f*x])^2) + (b*(a^2 + b^2)*m*AppellF1[1 +
m, m/2, m/2, 2 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e
+ f*x])]*Tan[e + f*x]*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*
((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2))/((1 + m)*(Sec[e + f*x]
^2)^(m/2)*(a + b*Tan[e + f*x])) - (2*a*b*((b*(-I + Tan[e + f*x]))/(a + b*Ta
n[e + f*x]))^(m/2)*((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*(-1/
2*((a - I*b)*b*m^2*AppellF1[1 + m, 1 + m/2, m/2, 2 + m, (a - I*b)/(a + b*Ta
n[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*Sec[e + f*x]^2)/((1 + m)*(a +
b*Tan[e + f*x])^2) - ((a + I*b)*b*m^2*AppellF1[1 + m, m/2, 1 + m/2, 2 + m,
(a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*Sec[e + f*x
]^2)/(2*(1 + m)*(a + b*Tan[e + f*x])^2)))/(m*(Sec[e + f*x]^2)^(m/2)) - (b*(
a^2 + b^2)*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I + Ta
n[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*(-1/2*((a - I*b)*b*m*(1 + m)*Appel
lF1[2 + m, 1 + m/2, m/2, 3 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(
a + b*Tan[e + f*x])]*Sec[e + f*x]^2)/((2 + m)*(a + b*Tan[e + f*x])^2) - ((a
 + I*b)*b*m*(1 + m)*AppellF1[2 + m, m/2, 1 + m/2, 3 + m, (a - I*b)/(a + b*T
an[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])]*Sec[e + f*x]^2)/(2*(2 + m)*(a
 + b*Tan[e + f*x])^2)))/((1 + m)*(Sec[e + f*x]^2)^(m/2)*(a + b*Tan[e + f*x]
)) - (a*b*AppellF1[m, m/2, m/2, 1 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a +
 I*b)/(a + b*Tan[e + f*x])]*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^
(-1 + m/2)*((b*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*(-((b^2*Sec[
e + f*x]^2*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x])^2) + (b*Sec[e + f*x]^2
)/(a + b*Tan[e + f*x])))/(Sec[e + f*x]^2)^(m/2) - (b*(a^2 + b^2)*m*AppellF1
[1 + m, m/2, m/2, 2 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*T
an[e + f*x])]*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(-1 + m/2)*((b
*(I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*(-((b^2*Sec[e + f*x]^2*(-I
 + Tan[e + f*x]))/(a + b*Tan[e + f*x])^2) + (b*Sec[e + f*x]^2)/(a + b*Tan[e
 + f*x])))/(2*(1 + m)*(Sec[e + f*x]^2)^(m/2)*(a + b*Tan[e + f*x])) - (a*b*A
ppellF1[m, m/2, m/2, 1 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a +
b*Tan[e + f*x])]*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(

```
I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(-1 + m/2)*(-((b^2*Sec[e + f*x]^2*
(I + Tan[e + f*x]))/(a + b*Tan[e + f*x])^2) + (b*Sec[e + f*x]^2)/(a + b*Tan
[e + f*x])))/(Sec[e + f*x]^2)^(m/2) - (b*(a^2 + b^2)*m*AppellF1[1 + m, m/2,
 m/2, 2 + m, (a - I*b)/(a + b*Tan[e + f*x]), (a + I*b)/(a + b*Tan[e + f*x])
]*((b*(-I + Tan[e + f*x]))/(a + b*Tan[e + f*x]))^(m/2)*((b*(I + Tan[e + f*x
]))/(a + b*Tan[e + f*x]))^(-1 + m/2)*(-((b^2*Sec[e + f*x]^2*(I + Tan[e + f*
x]))/(a + b*Tan[e + f*x])^2) + (b*Sec[e + f*x]^2)/(a + b*Tan[e + f*x])))/(2
*(1 + m)*(Sec[e + f*x]^2)^(m/2)*(a + b*Tan[e + f*x])) + a^2*Sec[e + f*x]^2*
(-Hypergeometric2F1[1/2, 1 + m/2, 3/2, -Tan[e + f*x]^2] + (1 + Tan[e + f*x]
^2)^(-1 - m/2)) - b^2*Sec[e + f*x]^2*(-Hypergeometric2F1[1/2, 1 + m/2, 3/2,
 -Tan[e + f*x]^2] + (1 + Tan[e + f*x]^2)^(-1 - m/2))))
```

Maple [F]

$$\int \frac{(d\cos(fx+e))^m}{(a+b\tan(fx+e))^2}dx$$

```
[In] int((d*cos(f*x+e))^m/(a+b*tan(f*x+e))^2,x)
[Out] int((d*cos(f*x+e))^m/(a+b*tan(f*x+e))^2,x)
```

Fricas [F]

$$\int \frac{(d\cos(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\cos(fx+e))^m}{(b\tan(fx+e)+a)^2}\,dx$$

```
[In] integrate((d*cos(f*x+e))^m/(a+b*tan(f*x+e))^2,x, algorithm="fricas")
[Out] integral((d*cos(f*x + e))^m/(b^2*tan(f*x + e)^2 + 2*a*b*tan(f*x + e) + a^2)
, x)
```

Sympy [F]

$$\int \frac{(d\cos(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\cos(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx$$

```
[In] integrate((d*cos(f*x+e))**m/(a+b*tan(f*x+e))**2,x)
[Out] Integral((d*cos(e + f*x))**m/(a + b*tan(e + f*x))**2, x)
```

Maxima [F]

$$\int \frac{(d\cos(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\cos(fx+e))^m}{(b\tan(fx+e)+a)^2}\,dx$$

```
[In] integrate((d*cos(f*x+e))^m/(a+b*tan(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((d*cos(f*x + e))^m/(b*tan(f*x + e) + a)^2, x)
```

Giac [F]

$$\int \frac{(d\cos(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\cos(fx+e))^m}{(b\tan(fx+e)+a)^2}\,dx$$

```
[In] integrate((d*cos(f*x+e))^m/(a+b*tan(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*cos(f*x + e))^m/(b*tan(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d\cos(e+fx))^m}{(a+b\tan(e+fx))^2}\,dx = \int \frac{(d\,\cos(e+f\,x))^m}{(a+b\tan(e+f\,x))^2}\,dx$$

```
[In] int((d*cos(e + f*x))^m/(a + b*tan(e + f*x))^2,x)
[Out] int((d*cos(e + f*x))^m/(a + b*tan(e + f*x))^2, x)
```

3.700 $\int (d\cos(e+fx))^m(a+b\tan(e+fx))^n\,dx$

Optimal result

Integrand size = 23, antiderivative size = 187

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^n\,dx = \frac{\text{AppellF1}\left(1+n,\frac{2+m}{2},\frac{2+m}{2},2+n,\frac{a+b\tan(e+fx)}{a-\sqrt{-b^2}},\frac{a+b\tan(e+fx)}{a+\sqrt{-b^2}}\right)\cos^2(e+fx)(d\cos(e+fx))^m(a+b\tan(e+fx)}{bf(1+n)}$$

```
[Out] AppellF1(1+n,1+1/2*m,1+1/2*m,2+n,(a+b*tan(f*x+e))/(a-(-b^2)^(1/2)),(a+b*tan
(f*x+e))/(a+(-b^2)^(1/2)))*cos(f*x+e)^2*(d*cos(f*x+e))^m*(a+b*tan(f*x+e))^(
1+n)*(1+(-a-b*tan(f*x+e))/(a-(-b^2)^(1/2)))^(1+1/2*m)*(1+(-a-b*tan(f*x+e))/
(a+(-b^2)^(1/2)))^(1+1/2*m)/b/f/(1+n)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3596, 3593, 774, 138}

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^n\,dx = \frac{\cos^2(e+fx)(d\cos(e+fx))^m\left(1-\frac{a+b\tan(e+fx)}{a-\sqrt{-b^2}}\right)^{\frac{m+2}{2}}\left(1-\frac{a+b\tan(e+fx)}{a+\sqrt{-b^2}}\right)^{\frac{m+2}{2}}(a+b\tan(e+fx))^{n+1}\text{Appel}}{bf(n+1)}$$

```
[In] Int[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x])^n,x]
```

```
[Out] (AppellF1[1 + n, (2 + m)/2, (2 + m)/2, 2 + n, (a + b*Tan[e + f*x])/(a - Sqr
t[-b^2]), (a + b*Tan[e + f*x])/(a + Sqrt[-b^2])]*Cos[e + f*x]^2*(d*Cos[e +
f*x])^m*(a + b*Tan[e + f*x])^(1 + n)*(1 - (a + b*Tan[e + f*x])/(a - Sqrt[-b
```

```
^2]))^((2 + m)/2)*(1 - (a + b*Tan[e + f*x])/(a + Sqrt[-b^2]))^((2 + m)/2))/
(b*f*(1 + n))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 774

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[(a + c*x^2)^p/(e*(1 - (d + e*x)/(d + e*(q/c)))^p*
(1 - (d + e*x)/(d - e*(q/c)))^p), Subst[Int[x^m*Simp[1 - x/(d + e*(q/c)), x
]^p*Simp[1 - x/(d - e*(q/c)), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, c, d,
 e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 3593

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Dist[d^(2*IntPart[m/2])*((d*Sec[e + f*x])^(2*FracP
art[m/2])/(b*f*(Sec[e + f*x]^2)^FracPart[m/2])), Subst[Int[(a + x)^n*(1 + x
^2/b^2)^(m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 + b^2, 0] &&  !IntegerQ[m/2]
```

Rule 3596

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)])^(n_.), x_Symbol] :> Dist[(d*Cos[e + f*x])^m*(d*Sec[e + f*x])^m, Int[(a
 + b*Tan[e + f*x])^n/(d*Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m,
 n}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \left((d\cos(e+fx))^m(d\sec(e+fx))^m\right)\int(d\sec(e+fx))^{-m}(a+b\tan(e+fx))^n\,dx$$

$$= \frac{\left((d\cos(e+fx))^m\sec^2(e+fx)^{m/2}\right)\text{Subst}\left(\int(a+x)^n\left(1+\frac{x^2}{b^2}\right)^{-1-\frac{m}{2}}\,dx,x,b\tan(e+fx)\right)}{bf}$$

$$= \frac{\left(\cos^2(e+fx)(d\cos(e+fx))^m\left(1-\frac{a+b\tan(e+fx)}{a-\frac{b^2}{\sqrt{-b^2}}}\right)^{1+\frac{m}{2}}\left(1-\frac{a+b\tan(e+fx)}{a+\frac{b^2}{\sqrt{-b^2}}}\right)^{1+\frac{m}{2}}\right)\text{Subst}\left(\int x^n\left(1-\right.\right.}{bf}$$

$$= \frac{\text{AppellF1}\left(1+n,\frac{2+m}{2},\frac{2+m}{2},2+n,\frac{a+b\tan(e+fx)}{a-\sqrt{-b^2}},\frac{a+b\tan(e+fx)}{a+\sqrt{-b^2}}\right)\cos^2(e+fx)(d\cos(e+fx))^m(a+b}{bf(1+n)}$$

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 23.41 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.95

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^n\,dx$$

$$= \frac{2(a-ib)(a+ib)(2+n)\,\text{AppellF1}\left(1+n,1+\frac{m}{2},1\right.}{bf(1+n)\left(2\left(a^2+b^2\right)(2+n)\,\text{AppellF1}\left(1+n,1+\frac{m}{2},1+\frac{m}{2},2+n,\frac{a+b\tan(e+fx)}{a-ib},\frac{a+b\tan(e+fx)}{a+ib}\right)+(2+m\right.}$$

```
[In] Integrate[(d*Cos[e + f*x])^m*(a + b*Tan[e + f*x])^n,x]
```

```
[Out] (2*(a - I*b)*(a + I*b)*(2 + n)*AppellF1[1 + n, 1 + m/2, 1 + m/2, 2 + n, (a
 + b*Tan[e + f*x])/(a - I*b), (a + b*Tan[e + f*x])/(a + I*b)]*Cos[e + f*x]*(
d*Cos[e + f*x])^m*(a*Cos[e + f*x] + b*Sin[e + f*x])*(a + b*Tan[e + f*x])^n)
/(b*f*(1 + n)*(2*(a^2 + b^2)*(2 + n)*AppellF1[1 + n, 1 + m/2, 1 + m/2, 2 +
n, (a + b*Tan[e + f*x])/(a - I*b), (a + b*Tan[e + f*x])/(a + I*b)] + (2 + m
)*((a - I*b)*AppellF1[2 + n, 1 + m/2, 2 + m/2, 3 + n, (a + b*Tan[e + f*x])/
(a - I*b), (a + b*Tan[e + f*x])/(a + I*b)] + (a + I*b)*AppellF1[2 + n, 2 +
m/2, 1 + m/2, 3 + n, (a + b*Tan[e + f*x])/(a - I*b), (a + b*Tan[e + f*x])/(
a + I*b)])*(a + b*Tan[e + f*x])))
```

Maple [F]

$$\int (d\cos(fx+e))^m\,(a+b\tan(fx+e))^n\,dx$$

```
[In] int((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^n,x)
```

```
[Out] int((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^n,x)
```

Fricas [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^n\,dx = \int (d\cos(fx+e))^m (b\tan(fx+e)+a)^n\,dx$$

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^n,x, algorithm="fricas")
[Out] integral((d*cos(f*x + e))^m*(b*tan(f*x + e) + a)^n, x)
```

Sympy [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^n\,dx = \int (d\cos(e+fx))^m (a+b\tan(e+fx))^n\,dx$$

```
[In] integrate((d*cos(f*x+e))**m*(a+b*tan(f*x+e))**n,x)
[Out] Integral((d*cos(e + f*x))**m*(a + b*tan(e + f*x))**n, x)
```

Maxima [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^n\,dx = \int (d\cos(fx+e))^m (b\tan(fx+e)+a)^n\,dx$$

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((d*cos(f*x + e))^m*(b*tan(f*x + e) + a)^n, x)
```

Giac [F]

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^n\,dx = \int (d\cos(fx+e))^m (b\tan(fx+e)+a)^n\,dx$$

```
[In] integrate((d*cos(f*x+e))^m*(a+b*tan(f*x+e))^n,x, algorithm="giac")
[Out] integrate((d*cos(f*x + e))^m*(b*tan(f*x + e) + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d\cos(e+fx))^m(a+b\tan(e+fx))^n\,dx = \int (d\,\cos\left(e+f\,x\right))^m\,\left(a+b\tan(e+f\,x)\right)^n\,dx$$

```
[In] int((d*cos(e + f*x))^m*(a + b*tan(e + f*x))^n,x)
[Out] int((d*cos(e + f*x))^m*(a + b*tan(e + f*x))^n, x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_result


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```